Doctoral Thesis

Unsteady flow fields in a high specific speed centrifugal pump

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Unsteady Flow Fields
in a
High Specific Speed Centrifugal Pump

ABHANDLUNG
zur Erlangung des Titels
Doktor der Technischen Wissenschaften
der EIDGENÖSSISCHEN TECHNISCHEN HOCHSCHULE ZÜRICH

vorgelegt von
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aus London, Kanada

Angenommen auf Antrag von
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Dr. T. Staubli, Korreferent
Dr. U. Bolleter, Korreferent

Zürich 1997
Acknowledgments

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Prof. Dr. G. Gyarmathy I thank for permitting me to perform this work within his laboratory and having the confidence to allow someone with a Canadian engineering education to taste a piece of Switzerland.

Dr. U. Bolleter I thank for donating his guru like expertise in the field of pumps. His energetic acceptance and willingness to assist with many helpful suggestions from an industry stand point have made this dissertation an extra notch better.

Dr. P. Holbein I thank for his concentrated work in computational fluid dynamics which was further used by the author as a foundation for the numerical results and interpretation in this dissertation.

Mr. H. Suter I acknowledge for making use of his impressive mechanical skills and wealth of experience in construction.

From the industrial side my appreciation goes to Dr. J. Gülich for the many tips during our discussions and for drawing on his wealth of experience in pump hydraulics. Also Dr. M. Casey and Dr. A. Schachenmann I thank for their interest in this work.
Abstract

The unsteady flow fields within a non-cavitating high specific speed centrifugal pump ($\omega_s=1.7$) were the object of an experimental, numerical, and theoretical investigation in which new signal analysis techniques for unsteady phenomena analysis were implemented. Three particular aspects were subject to detailed concentration, all known to be problematic in the pump industry,

- part load behavior and impeller flow recirculation,
- unsteady impeller blade loading,
- acoustic generation.

Part Load Behavior and Impeller Flow Recirculation

The measured characteristics for the high specific speed centrifugal pump revealed distinct discontinuities in part load operation and were shown to coincide with abrupt unsteady flow field changes in the pump. These discontinuities occurred at different threshold volume fluxes when increasing or decreasing the pump discharge and make up a hysteresis loop. Discrepancies in the magnitude of the pressure discontinuities and shaft power discontinuities indicated, shown theoretically to be plausible, the commencement/cessation of dissipater mechanisms within the flow as the recirculation commences/ceases.

Transient behavior of the hysteresis in the pump pressure discharge characteristic was evaluated as a function of changing volume flux rate experimentally and accordingly theoretically. Quasi steady behavior was found for $|d\phi/dt|<0.005\text{s}^{-1}$. A 2nd order nonlinear dependence on the changing volume flux rate was determined for the change in useful hydraulic power during the commencement/cessation of the impeller recirculation.

The pump impeller pressure discharge characteristic was evaluated experimentally and numerically by taking the difference between the integrated impeller outlet and impeller inlet total pressure. The experimental and numerical characteristics agree well including the volume flux location and magnitude of the pressure discontinuities in the hysteresis loop. Experimental and numerical comparisons are made at the impeller inlet/outlet with emphasis on the changing flow field in the hysteresis loop flow regime and its coupling to the onset of recirculation flow zones. This combined application of numerical and experimental tools provided insight for the hysteresis flow field of a pump impeller and for abrupt changes in the associated unsteady flow field.
Unsteady Impeller Blade Loading

For investigating unsteady impeller blade loading twenty five piezoresistive pressure transducers were mounted within a single blade passage and sampled in the rotating impeller frame with a telemetry system. The influence of varying volume flux on the pressure transducers was evaluated in terms of pressure fluctuation magnitudes and phase differences. The magnitude information reveals pressure fluctuations from the impeller-volute interaction grow as the volume flux became further removed from the best efficiency point and as the trailing edge of the impeller blade was approached. These fluctuations reached 35% of the pump head in deep part load. The upstream influence of the volute steady pressure field dominates the unsteady pressure field within the impeller at all off design load points. Acquired signal phase information permitted identification of the pressure field unsteadiness within the impeller passage as fundamentally synchronized simultaneously with tongue passing frequency. Special emphasis was placed on the volume flux regime where the pump and impeller characteristics undergo hysteresis as impeller recirculation commence and cease.

A synthesis of the rotating blade transducers was performed to obtain unsteady blade loading parameters. The value of the unsteady blade lift coefficient varies to near 200% the steady lift coefficient at 55% best efficiency point volume flux, an abrupt fluctuation occurring as the fore running blade suction side passes a volute tongue. The unsteady moment coefficient and center of pressure are also shown to vary significantly during the impeller-volute tongue interaction. The unsteady moment coefficient was shown to be directly related to the unsteady torsional vibrations on the pump shaft providing a link between hydrodynamics and rotordynamics.

Acoustic Generation

Focused on was the quantification of acoustic generation mechanisms within the pump on the fundamental level. These consisted of unsteady flow blockage at the impeller outlet by the volute tongue, unsteady flow forces on the volute tongue, and the classic rotor (Gutin) noise. Of special interest were the unsteady flow forces on the volute tongue which generated an acoustic field. The analytic manipulations performed here incorporate Powell’s theory of vortex sound being equivalent to that of Curle and Lighthill for acoustic generation but providing more physical insight into the concept of vortex formation and interaction found within hydraulic turbomachinery. Measurements taken from nineteen pressure transducers on the volute tongue were used to determine the unsteady force from the impeller outlet flow.
impingement. These were applied as boundary conditions for the vortex induced sound.

The acoustic generation from unsteady mass flow due to volute tongue blockage is known to be the main source of acoustic generation. However, unsteady forces on the volute tongue were not insignificant comprising near 40% of the total acoustic generation. The quantification of the unsteady mass flow acoustic generation was approximated here to permit comparison to the unsteady tongue forces acoustic generation. Further the rotor (Gutin) noise was also included in the comparison and found to have a minimal contribution. The results were interpreted using measured acoustic pressure fluctuations at the pump outlet.
Zusammenfassung

Die Strömungsfluktuationen einer schnellläufigen radialen Pumpe (\(\omega_s=1.7\)) in kavitationsfreiem Betrieb sind experimentell, numerisch, und theoretisch untersucht worden. Dabei sind neue Methoden zur Signalanalyse von transienten Vorgängen angewendet worden. Schwerpunkte dieser Untersuchung liegen bei drei bekannten Problemen der Pumpenindustrie:
- Teillastverhalten und Laufradrezirkulation
- Zeitlich veränderliche Schaufelbelastung im Laufrad
- Akustische Druckschwankungen und Lärmquelle.

Teillastverhalten und Laufradrezirkulation


Das transiente Verhalten der Druckhysterese ist in Abhängigkeit vom zeitlich veränderlichen Volumenstrom experimentell und theoretisch untersucht worden. Für eine Drosselgeschwindigkeit von \(|d\varphi/dt|<0.005\text{s}^{-1}\) ist das quasi stationäre Verhalten der Druckzifferhysterese dokumentiert worden. Eine nicht lineare Abhängigkeit zweiter Ordnung vom veränderlichen Volumenstrom ist in der abgegebenen hydraulischen Leistung während des Beginns und Endes der Laufradrezirkulation festgestellt worden.

Zeitlich veränderliche Laufradschaufelbelastung


Akustische Druckschwankungen und Lärmquelle

montierten Druckgebern dienten als Basis für die Auswertung der zeitlich veränderlichen Kräfte auf den Sporn die durch die Laufradaustrittsströmung hervorgerufen werden. Diese Kräfte dienten als Randbedingung für die wirbelinduzierten Akustikberechnungen.

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### Nomenclature

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<th>Definition</th>
<th>Unit</th>
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<tr>
<td>$a$</td>
<td>wavelet transform dilation parameter</td>
<td>[-]</td>
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<tr>
<td>$n_o$</td>
<td>local isentropic acoustic speed</td>
<td>1326±2% m/s</td>
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<td>$A$</td>
<td>minimum $\phi$ point without recirculation</td>
<td>0.1213</td>
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<tr>
<td>$A_T$</td>
<td>local tongue area segment</td>
<td>[m$^2$]</td>
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<tr>
<td>$A_T$</td>
<td>volute tongue gap cross sectional area</td>
<td>[m$^2$]</td>
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<tr>
<td>$b$</td>
<td>wavelet transform time shift parameter</td>
<td>[s]</td>
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<td>maximum $\phi$ operating point with recirculation</td>
<td>0.1220</td>
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<td>impeller outlet height</td>
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<td>volute height at impeller outlet</td>
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<td>velocity in absolute system</td>
<td>[m/s]</td>
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<td>blade chord at midheight</td>
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<td>meridional velocity in absolute system</td>
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<td>[m/s]</td>
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<td>tangential velocity abs. at impeller inlet</td>
<td>[m/s]</td>
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<td>$c_{t2}$</td>
<td>tangential velocity abs. at impeller outlet tip</td>
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<td>$c_z$</td>
<td>axial velocity in absolute system</td>
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<td>coefficient of pressure, (3.5), (3.6), (3.7)</td>
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<td>$C_{p_{a}}$</td>
<td>coefficient of pressure phase averaged, (3.8)</td>
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<tr>
<td>$C_{p_{t}}$</td>
<td>coefficient of pressure tongue area averaged</td>
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<tr>
<td>$d$</td>
<td>traversal length along the probe</td>
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<td>$D$</td>
<td>dipole strength</td>
<td>[m$^4$/s]</td>
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<td>$D_0$</td>
<td>pump outlet flange diameter</td>
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<td>$E_k$</td>
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<td>$f$</td>
<td>frequency</td>
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<td>$f$</td>
<td>body force</td>
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<td>blade passing frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$f_{o}$</td>
<td>pump shaft frequency</td>
<td>[Hz]</td>
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<tr>
<td>$f_{tp}$</td>
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<tr>
<td>$F$</td>
<td>concentrated blade force</td>
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<td>$F_i$</td>
<td>the $i^{\text{th}}$ body force per unit volume</td>
<td>[N/m$^3$]</td>
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<tr>
<td>$F_r$</td>
<td>dimensionless radial pressure induced force</td>
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<td>$F_{ro}$</td>
<td>radial pressure induced force on impeller</td>
<td>[N]</td>
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<tr>
<td>$g$</td>
<td>gravitational acceleration in Zurich</td>
<td>9.80665 m/s$^2$</td>
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<td>$h_o$</td>
<td>total enthalpy</td>
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<td>Unit</td>
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<td>---------</td>
<td>------</td>
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<tr>
<td>i,j</td>
<td>indices</td>
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</tr>
<tr>
<td>i</td>
<td></td>
<td>[-]</td>
</tr>
<tr>
<td>J_n</td>
<td>Bessel function</td>
<td>[-]</td>
</tr>
<tr>
<td>H</td>
<td>pump head</td>
<td>[m]</td>
</tr>
<tr>
<td>k</td>
<td>acoustic wave number 1/λ</td>
<td>[m⁻¹]</td>
</tr>
<tr>
<td>K</td>
<td>free vortex constant found from measurements</td>
<td>[-]</td>
</tr>
<tr>
<td>K_1</td>
<td>constant within hysteresis regime, (dϕ/dr*)</td>
<td>[-]</td>
</tr>
<tr>
<td>K_2</td>
<td>constant within hysteresis regime, ϕ(6.5)</td>
<td>[-]</td>
</tr>
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<td>K_3</td>
<td>constant within hysteresis regime, (6.6)</td>
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<td>K_stat</td>
<td>probe static pressure calibration coefficient</td>
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</tr>
<tr>
<td>K_tot</td>
<td>probe total pressure calibration coefficient</td>
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</tr>
<tr>
<td>K_Φ</td>
<td>probe yaw angle calibration coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>K_γ</td>
<td>probe pitch angle calibration coefficient</td>
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<tr>
<td>L</td>
<td>power, (6.2)</td>
<td>[W]</td>
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<tr>
<td>L_o</td>
<td>log RMS blade passing pressure fluctuation</td>
<td>[-]</td>
</tr>
<tr>
<td>L_T</td>
<td>total power dissipation from recirculation zones</td>
<td>[W]</td>
</tr>
<tr>
<td>L_1</td>
<td>impeller leading edge length</td>
<td>[m]</td>
</tr>
<tr>
<td>L_3</td>
<td>total distance along tongue midline</td>
<td>[m]</td>
</tr>
<tr>
<td>M</td>
<td>M_e z, pump shaft moment (torque)</td>
<td>[N m]</td>
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<td>Mach number</td>
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<td>mass flux</td>
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<tr>
<td>N</td>
<td>rotational speed</td>
<td>[rpm]</td>
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<tr>
<td>n</td>
<td>shaft harmonic n ∈ {1,2,3,...}</td>
<td>[-]</td>
</tr>
<tr>
<td>n_i</td>
<td>normal vector to a surface</td>
<td>[-]</td>
</tr>
<tr>
<td>P</td>
<td>pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>P(r,θ,z,t)</td>
<td>spatial and temporal pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>P_a(x,t)</td>
<td>acoustic pressure fluctuation position x, time t</td>
<td>[Pa]</td>
</tr>
<tr>
<td>P_o</td>
<td>ambient far field pressure</td>
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</tr>
<tr>
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</tr>
<tr>
<td>P_stat</td>
<td>pressure static</td>
<td>[Pa]</td>
</tr>
<tr>
<td>P_tot</td>
<td>pressure total</td>
<td>[Pa]</td>
</tr>
<tr>
<td>ΔP_2-1</td>
<td>static pressure rise across the pump</td>
<td>[Pa]</td>
</tr>
<tr>
<td>Q</td>
<td>volume flux</td>
<td>[m³/s]</td>
</tr>
<tr>
<td>r</td>
<td>radial distance</td>
<td>[m]</td>
</tr>
<tr>
<td>r</td>
<td>radial coordinate in a cylindrical system</td>
<td>[m]</td>
</tr>
<tr>
<td>r_cp</td>
<td>radial distance to blade center of pressure</td>
<td>[m]</td>
</tr>
<tr>
<td>R_v</td>
<td>recirculation vortex minor radius</td>
<td>[m]</td>
</tr>
<tr>
<td>R_1</td>
<td>impeller inlet tip radius</td>
<td>0.134 m</td>
</tr>
<tr>
<td>R_2</td>
<td>impeller outlet tip radius</td>
<td>0.162 m</td>
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<td>Variable</td>
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<tr>
<td>------------</td>
<td>--------------------------------------------------</td>
<td>--------------</td>
</tr>
<tr>
<td>$R_3$</td>
<td>volute tongue inlet radius</td>
<td>0.202 m</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number</td>
<td>[-]</td>
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<tr>
<td>$s$</td>
<td>distance along line (i.e., wall streamline)</td>
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</tr>
<tr>
<td>$S$</td>
<td>surface area</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$S_2$</td>
<td>impeller outlet area, $\pi D_2 B_2$</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>[s]</td>
</tr>
<tr>
<td>$t_{rlao}$</td>
<td>the retardation time</td>
<td>[s]</td>
</tr>
<tr>
<td>$T^*$</td>
<td>time parameter for duration of hysteresis regime</td>
<td>[s]</td>
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<tr>
<td>$u_2$</td>
<td>impeller outlet tip velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$u_{lt}$</td>
<td>impeller inlet tip velocity</td>
<td>[m/s]</td>
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<tr>
<td>$V$</td>
<td>volume</td>
<td>[m$^3$]</td>
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<tr>
<td>$V_o$</td>
<td>vortex sheet stretching velocity</td>
<td>[m/s]</td>
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<tr>
<td>$w$</td>
<td>velocity in relative system</td>
<td>[m/s]</td>
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<td>$w_i$</td>
<td>velocity of a moving surface</td>
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<td>$W$</td>
<td>fluid work</td>
<td>[J]</td>
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<tr>
<td>$x$</td>
<td>distance from surface center to the observer</td>
<td>[m]</td>
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<td>$x$</td>
<td>directional vector</td>
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<tr>
<td>$y$</td>
<td>distance surface center to point in the volume</td>
<td>[m]</td>
</tr>
<tr>
<td>$z$</td>
<td>axial coordinate in a cylindrical system</td>
<td>[m]</td>
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**Greek symbols**

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>$\alpha$</td>
<td>Gabor transform window width control parameter</td>
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<tr>
<td>$\alpha_2$</td>
<td>impeller absolute outlet flow angle</td>
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</tr>
<tr>
<td>$\beta$</td>
<td>impeller relative flow angle</td>
<td>[°]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>on axis alignment for rotor acoustic</td>
<td>[-, °]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>pressure probe pitch angle</td>
<td>[-, °]</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>circulation</td>
<td>[m$^2$/s]</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta function</td>
<td>[-]</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>meridian angle transducer makes with surface</td>
<td>[-, °]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>constant percentage fluctuation in $C_L$ and $C_M$</td>
<td>[-]</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>vorticity $(\nabla \times \mathbf{v})$</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>shaft efficiency, (3.4)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta_h$</td>
<td>impeller hydraulic efficiency</td>
<td>[-]</td>
</tr>
<tr>
<td>$\vartheta_i$</td>
<td>inlet angle transducer makes with blade surface</td>
<td>[-, °]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>tangential coordinate in a cylindrical system</td>
<td>[-, °]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>power coefficient, (3.3)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
<td>[m]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>viscosity dynamic</td>
<td>[N/(m s)]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>viscosity kinematic</td>
<td>[m$^3$/s$^3$]</td>
</tr>
</tbody>
</table>
\(\rho\) local fluid density \([\text{kg/m}^3]\)

\(\xi\) constant within hysteresis regime, \((d\phi/dt*)/\phi\) [-]

\(\sigma\) cavitation coefficient [-]

\(\sigma_{ij}\) stress tensor, (6.1) [-]

\(\tau_{ij}\) the viscous portion of Stokes stress tensor (6.1) \([\text{N/m}^2]\)

\(\phi\) discharge coefficient, (3.1) [-]

\(\varphi\) phase in pressure transducer signals [-]

\(\Phi\) pressure probe yaw angle \([-\degree]\)

\(\psi\) pump pressure coefficient, (3.2) [-]

\(\psi_o\) pump pressure coefficient, (3.2) stationary jump 0.0383

\(\Psi\) wavelet transform mother wavelet [-]

\(\omega\) pump shaft angular frequency \([\text{s}^{-1}]\)

\(\omega_s\) dimensionless specific speed at bep \(\omega Q^{0.5}(gH)^{-0.75}\) [-]

\(\Omega\) rotating tonal source frequency \([\text{s}^{-1}]\)

**acronyms**

bep best efficiency point [-]

ISO international standards organization

PSS pressure side shroud

PSH pressure side hub

rpm rotations per minute \([\text{rpm}]\)

SSS suction side shroud

SSH suction side hub

STD standard deviation normally distributed variable

**overscripts**

\(\sim\) unsteady quantity with time averaged removed

\(-\) steady quantities

\(*\) complex conjugate

\(*\) phenomena characterizing parameter

**operators**

\(\text{D}/\text{D}t\) material derivative, Lagrange system

\(d/dt\) derivative with respect to time, Euler system

\(\partial/\partial x_i\) gradient in tensor form (\(\nabla\) vector form)

\(x_i\) vector in tensor form

\(l\) magnitude

\(\times\) curl
1.0 Introduction

Application of centrifugal pump technology is required for fluid transportation in a wide range of diverse technological fields such as the power generation, petroleum, irrigation, paper, and aerospace industries. Of pertinence herein are tendencies industry is pursuing for centrifugal pump development which demand from science, three dimensional unsteady flow field insight. This dissertation provides a portion of this insight through the application of new technologies.

1.1 Industrial Motivation: Economical Considerations

A trend which emerged in the centrifugal pump industry during the 1960’s was increasing power concentration per stage. Fig. 1.1 [Florjancic & Simon 1989] details advances made in centrifugal pump stage power output concentration over sixty years in thermal power plants. This indicates an increase in power concentration during the 60's and 70's, demanded by innovations in boiler and steam turbine design. Since the 1980’s their power concentrations have remained relatively constant as cavitation limits were reached and booster pumps for the boiler feed pumps grew to uneconomical proportions. The increased power concentrations in the 60’s and 70’s today still prove to be problematic in terms of the produced unsteady flow field, in particular pump unsteady system behavior [Telfer 1993, Hartlen et al. 1993]. These problems manifest themselves in reduced pump system reliability and difficulties in the design of hydraulically predictable pumps with regards to “unstable” operating characteristics, unacceptable to industrial clients.

The centrifugal turbopumps of the rocket industry are extreme examples of the desire for continued increasing pump power concentrations [Biggs 1989]. Weight constrictions are the prime driving factor. Although rocket turbopumps typically have lifetimes less than a few thousand seconds they must behave reliably and predictably over a wide operational regime with respect to volume flux.

In reversible pump turbine development a single stage compact unit was recently unveiled providing 750 m head, 412 MW, for the planned Kazunogawa installation. Fig. 1.2 [Tamatsukuri 1992] details a history of reversible pump turbine development showing ever increasing stage head. These increasingly higher heads imply larger unsteady flow field magnitudes which are responsible for decreasing reliability and complicating pump predictability. This coupled with the recent emphasis on short term transient capabilities of pump turbines [Rustad & Sundsvold 1992] to enable the
almost instantaneous reaction to power grids for supplying peak power, absorbing excess power, maintaining a stable grid frequency, and providing quick phase angle compensation make any knowledge of unsteady phenomena within pumps particularly valuable.

![Diagram of impeller and disk area](image)

**Fig. 1.1** The power concentration development of boiler feed pumps [Florjancic & Simon 1989].

![Diagram of pump turbine stage head](image)

**Fig. 1.2** The maximum head for single-stage pump turbines [Tamatsukuri et al. 1992].

In the paper industry strict guidelines limit the magnitude of acoustic pressure fluctuations permitted in the pump system (i.e., a maximum of ±0.5% the dynamic pump pressure based on the impeller outlet tip velocity [Bolleter et al. 1981]). The quality of paper produced from the system may be poor if these guideline limits are exceeded. The necessity for predictable, efficient paper pumps maintaining an unsteady flow field within prescribed guidelines is recognized (the pump examined within this dissertation is taken from the paper industry).

Economically construction trends in centrifugal pumps must lie in greater reliability [Smith 1993, Jantunen et al. 1993], better predictability [Favre 1995], and improved efficiency [Drake & Sims 1996] over the entire operational regime. This coupled with the mentioned increases in pump power concentration, impeller tip speeds, head, and the desire for greater transient capabilities inherently applies larger unsteady flow field magnitudes within pumps. These larger unsteady flow field magnitudes are responsible for larger dynamic flow forces [Tamatsukuri et al. 1992], greater vibrational difficulties [Guo 1986], and larger acoustic pressure generation [Telfer 1993], all implying sacrifices in pump reliability and more difficult design predictability. Industrial demands should no longer, and often can no longer, be attained by "trial and error" or "build and bust" philosophies but rather with scientific understanding and development which provides long term financial rewards.
1.2 Scientific Motivation: The Unsteady Pump Flow Field

The significance of unsteady flow fields in pump design have long been recognized [Dean 1959] as dictating the work transferred from the pump shaft to a fluid. Considering a fluid particle in the absolute system as it passes through the impeller, the total enthalpy change in Lagrangian coordinates is expressed as,

\[ \frac{Dh_o}{Dt} = \frac{1}{\rho} \frac{\partial P}{\partial t} \]  

(1.1)

for reversible, adiabatic flow. The specific work performed on the fluid is known to be,

\[ W = \int Dh_o. \]

These expressions state that if work is to be performed to change a fluid particles total enthalpy then it must be performed by an unsteady pressure field. The underlying principle of pump operation, work being performed on the fluid, is achieved through the unsteady flow field. Fig. 1.3 reveals this on a simplified level. Left is shown a particle entering (1) the impeller blade passage and experiencing an increasing pressure while traveling through the blade passage to the exit (2). The rotating pressure field is depicted to the right of Fig. 1.3 where the coriolis force \( F_{cor} \) causes the particle to deflect towards the blade passage pressure side. The fluid particle experiences a \( \frac{\partial P}{\partial t} \) in the blade passage which when integrated results in work being done on the particle. To simplify this event, commonly numerical simulations are performed in the rotating pump impeller frame. Little numerical success has however been reported in accounting for the unsteady flow field which interact between the stationary and rotating frame.

Fig. 1.3 Left, a particle path in the absolute system entering (1) and exiting (2) the impeller. Right the unsteady pressure field experienced by the fluid particle as it travels through the blade passage.
A scientific knowledge of the unsteady flow fields in pumps is lacking. The prediction processes for unsteady flow field behavior including model tests and numerical simulation cannot currently substitute full scale pump testing due to an uncertainty in parameters affecting the complex generation, interaction and dissipation of the unsteady flow field [Glattfelder 1982]. A greater knowledge of the unsteady flow field will permit targeted unsteady numerical and model simulation development.

1.3 Validation of Numerically Predicted Pump Flow Fields

Numerical prediction of the pump steady and unsteady flow field reduces industry’s design costs. Much effort has been spent predicting steady flow quantities, obtaining limited success near pump best efficiency point operation. These quantities are useful for pump performance predictions, particularly for evaluating geometric changes, even though agreement in part load is generally poor [e.g. Staubli et al. 1995]. Little success has been reported in the numerical prediction of the pump unsteady flow field due greatly to a nonexistence of extensive experimental data to perform any validation [Brennen 1994]. With the previously mentioned considerations of increased pump stage head and greater dynamic capabilities, experimental validation of the numerically predicted unsteady flow field becomes increasingly necessary.

1.4 Centrifugal Pump Performance

Centrifugal pump performance measurements and numerical predictions have been ample in the past [IMechE 1982] and will thus only receive attention as a manifestation of the pump unsteady flow field. Specifically the operational regime where the pump characteristics undergo discontinuities and hysteresis is found. The changing flow behavior a pump undergoes in the discontinuity and questions concerning unstable pump operation which may accompany the discontinuity are currently not satisfactorily answered.

1.5 Terminology Distinctions

Due to the broad application of various terms to describe steady and unsteady signals in the engineering community [Staubli 1993] the flow chart in Fig. 1.4 defines the usage in this dissertation. To note is that a stationary system and stationary signal are unrelated phrases. Also of importance is the realization that a steady signal in a stationary frame of reference is unsteady when viewed in the rotating frame of reference. These classified unsteady signals are a manifestation of the unsteady flow fields active within the pump.
1.6 Problem Definition and Methodology

This dissertation addresses the problems encountered with increased pump stage head, increased power concentrations and greater pump transient capabilities by providing an analysis and synthesis of the unsteady flow field within a centrifugal pump, special emphasis being placed on the pump characteristic hysteresis. Pump performance is investigated with greater resolution in this operating regime because the flow field undergoes abrupt changes. This dissertation also provides a benchmark case for validation of unsteady numerical predictions with detailed experimental quantitative and qualitative data for the complex flow within a centrifugal pump of high specific speed ($\omega_s=1.7$).

**Fig. 1.4 Classification of signals and terminology definitions**

*Steady* in the stationary frame is *Unsteady* in the rotating frame.

*Steady* in the rotating frame is *Unsteady* in the stationary frame.
2.0 Pump and Test Facility

Described are the investigated pump and modifications fabricated to permit measurement access for unsteady flow field assessment in the most nonobtrusive manner. Measurement locations in the pump and the designed test facility are presented.

2.1 Pump Specifications

Scrutinized was a single stage centrifugal pump of high specific speed ($\omega_s=1.7$, $n_q=90$) constructed for the paper industry to transport slurry inhomogeneous substances. It belongs to a caliber of pumps known to exhibit highly three dimensional complex flow fields because of the relatively short blade chord and wide blade passages, the impeller outlet tip solidity is 1.4. The impeller shown in Fig. 2.1 has an outlet diameter $D_2=324$ mm and ran within a double spiral volute to minimize radial forces. The impeller meridian and inlet view are also shown. In Fig. 2.2 the section view of the pump double volute spiral is seen. The pump mechanical section view is provided in Fig. 2.3. In Fig. 2.4 the blade plan is provided.

![Fig. 2.1 The pump impeller with its schematic meridian and inlet views.](image)

Geometric specifications were,
- 7 blades, shrouded impeller,
- $D_1/D_2 = 0.83$ impeller inlet tip diameter,
- $B_2/D_2 = 0.27$ blade outlet height,
- $D_3/D_2 = 1.22$ volute tongue inlet diameter,
- $33^\circ$ blade outlet back lean angle, $20^\circ$ blade outlet rake.

Material specifications were,
- impeller, cast iron 25,
- volute, cast iron 40.
Fig. 2.2 The pump double spiral volute.

Fig. 2.3 The centrifugal pump section view.
Fig. 2.4 The blade plan for the investigated impeller. Exact detailed dimensions are not included due to copyright limitations.
2.2 Pump Modifications

To permit the measurement of flow quantities within the pump structural modifications were performed. The volute of the pump was fitted with pressure probe fixation apparatus for traversals at the pump impeller inlet and outlet, Fig. 2.5. The volute also contained five traversal locations axially aligned, Fig. 2.6 and Fig. 2.7, for investigation of the flow within a spiral of the pump double spiral. The pressure probe traversal locations were all located at the shown circumferential position between the two volute tongues. To evaluate the circumferential variation in the impeller outlet flow and the pressure induced radial thrust 32 wall flush pressure taps, 16 on the hub side and 16 on the shroud side, were installed in the volute at a radius ratio of r/R_2=1.05 shown also in Fig. 2.7.

The pump shaft was hollowed for electrical access required by the wires of 25 pressure transducers mounted in a pump impeller blade passage. The wires were fed into a telemetry device which was mounted on the pump shaft, Fig. 2.8, for measurements in the rotating frame of reference. To the right an external battery pack can be seen which permitted uninterrupted testing for 16 hours at 18V. To the left a metallic box was used to hold the plugs for those pressure transducers not currently being sampled.

![Fig. 2.5 Pressure probe traversal locations at the pump impeller inlet and outlet.](image)

![Fig. 2.6 Pressure probe traversal locations in the volute.](image)
2.3 The Test Facility

The working fluid for the open circuit pump system was water which was drawn from an 80 m³ reservoir into the pump inlet through a flow straightener to provide uniform inlet flow, Fig. 2.9. A booster pump located 53D₁ upstream increased the NPSH of the pump investigated thereby avoiding cavitation. Throttling up and down of the volume flux was performed with a slice throttle located 16D₁ downstream of the volute outlet to reservoir.

\[
Q = Q_{\text{out}} + Q_{\text{pack}} \quad Q_{\text{pack}} \ll Q_{\text{out}}
\]

2.4 Chapter Summary

Detailed modifications on an industrial high specific speed centrifugal pump were made for the progression to an experimental test facility. The locations of interest for scrutinizing the unsteady flow field within the pump were presented.
3.0 Instrumentation and Data Analysis Tools

In the investigation of the centrifugal pump both global and local flow data required assessment in order to gain an appreciation for the unsteady flow field within the pump and its system. To this means global pump data for pump performance evaluation were obtained with measurement instrumentation in accordance with ISO [ISO 5189, 1987] precision grade acceptance norms. Local pump data were obtained at local positions within the flow to provide detailed flow field knowledge and an evaluation of the dynamics the flow field contains. All measurements were taken during constant rotational speed of the impeller.

In the treatment of unsteady pump flow fields which include transient events associated with variable volume flux, it is necessary to determine the flow fields dynamic behavior qualitatively and quantitatively with the frequencies, magnitudes, and phases at which unsteady measured or computed signals exist, these being a manifestation for the possible flow induced excitation of various system components. While Fourier methods are a powerful tool for unsteady signal analysis they provide information localized in frequency, not sufficient for analyzing a signal in which non-periodic or non-deterministic frequencies are evident. Quasi joint time frequency analysis techniques provide non-periodic or non-deterministic signal information concurrently in time and frequency establishing signatures of a specific measured phenomena occurring within the pump for future diagnostic classification.

3.1 Global Pump Data

For evaluation of pump performance four global parameters were measured, seen in Fig. 2.8 on the test facility,
- the volume flux, measured with a magnetic inductive flow meter \( Q \),
- the static pressure rise, measured differentially \( \Delta P_{2-1} \),
- the shaft torque, measured inductively with strain gages \( M \),
- the shaft angular velocity, measured optically with 3600 TTL pulses per rotation \( \omega \).

Errors associated with these measurements lie under ±0.5%. From these and the pump’s geometrical parameters the flow coefficient \( \phi \), pressure discharge coefficient \( \psi \), power coefficient \( \lambda \), and shaft efficiency \( \eta \) characteristics of the pump were evaluated using,
3.2 Local Pump Data

Local pump data were obtained as steady and unsteady pressure and velocity vectors at various spatial locations. The applied local measurement instrumentation consisted of devices in two categories, maneuverable 5 hole and fast response pressure probes, surface mounted pressure taps and transducers.

3.2.1 Coefficients of Pressure

The local pressure measured by both the pressure probes and pressure transducers will for comparison convenience be presented in a nondimensional form using the dynamic pressure based on the impeller outlet tip velocity to form the steady coefficient of pressure,

\[ C_p = \frac{\bar{p}(r, \theta, z)}{0.5 \rho u_2^2} \]

and the superposed unsteady coefficient of pressure,

\[ \tilde{C}_p = \frac{\tilde{p}(r, \theta, z, t)}{0.5 \rho u_2^2} \]

written for a cylindrical coordinate system. Equation (3.5) is similar to a local \( \psi \) value defined globally for the pump by equation 3.2. Summation of the steady and unsteady coefficients yields the coefficient of pressure,

\[ C_p = \bar{C}_p + \tilde{C}_p \] (3.7)

The phase averaged coefficient of pressure is of the form,

\[ C_{pa}(j) = \frac{1}{N} \sum_{i=1}^{N} \frac{C_p(i, j)}{N} \] (3.8)

where \( j \)th data point of the sampling period for the \( i \)th sampling period is averaged over \( N \) sampling periods (i.e., revolutions).
3.2.2 The 5 Hole Pressure Probe

A 5 hole pressure probe was constructed, seen in Fig. 3.1, for steady flow field evaluation within the pump. A mechanical drawing of the 5 hole probe is found in Appendix A. From 5 measured steady pressures and the appropriate calibration surfaces the three dimensional velocity vector at a local point was calculated. The calibration surfaces for the probe were obtained from a static calibration performed in the ETH Zurich’s free air jet calibration facility [Kupferschmied & Gossweiler 1992] at a Reynolds number of $3 \times 10^4$ appropriate for the measurements carried out in water. The automatic calibration process was performed over $-20^\circ$ to $+36^\circ$ in pitch ($\gamma$) angle and $\pm 26^\circ$ in yaw angle ($\Phi$) with steps of $2^\circ$. Four coefficients were defined and evaluated for all pitch, yaw angle combinations. These coefficients were interpolated with a polynomial fit of 5, 6 or 7th order in 2 dimensions, the fit order being chosen to minimize the existing residuals. Fig. 3.3 details such calibration surfaces with the definitions for the calibration coefficients,

\[
K_\Phi (\Phi , \gamma ) = \frac{P_2 - P_3}{P_1 - \frac{P_2 + P_3}{2}} \\
K_\gamma (\Phi , \gamma ) = \frac{P_4 - P_5}{P_1 - \frac{P_2 + P_3}{2}} \\
K_{tot} (\Phi , \gamma ) = \frac{P_{tot} - P_1}{P_1 - \frac{P_2 + P_3}{2}} \\
K_{stat} (\Phi , \gamma ) = \frac{P_{tot} - P_{stat}}{P_1 - \frac{P_2 + P_3}{2}}
\]
The calibration coefficients and surfaces for the 5 hole probe.

Application of these calibration surfaces permits the calculation of the three dimensional velocity vector at a particular point in the flow. The measurement of the 5 probe pressures allows calculation of \( K_\phi \) and \( K_\gamma \) which were used in conjunction with the plots in Fig. 3.3 to calculate both \( \Phi \) and \( \gamma \), the flow velocity angles. Thereafter the \( \Phi \) and \( \gamma \) angles may be used accordingly to calculate \( K_{stat} \) and \( K_{tot} \), thus allowing \( \bar{P}_{tot} \) and \( \bar{P}_{stat} \) for the flow to be calculated. The magnitude of the flow velocity is then calculated using \( \bar{P}_{tot} = \bar{P}_{stat} + 0.5 \rho c^2 \) which is based on the assumption of incompressible flow (a Mach number of 0.25 was used for the static calibration implying a maximum error of +1.6% [Chue 1975] for a perfect gas undergoing isentropic expansion). For the pressure probe measurements care was taken to rotate the probe yaw angle so as to zero the probe, meaning \( \bar{P}_2 = \bar{P}_3 \) and application of the calibration surface central region. No geometrical pitch angle variation was possible for the given geometry thus the flow angle in the pitch direction had to lie within the calibrated range \(-20^\circ \) to \(+36^\circ \). The pressure measured at each local point by the probe was recorded over 1 minute in time as a steady coefficient of pressure, equation 3.5.

A correction to the data was made for the blockage provided by the probe in the flow field. A standard technique [Wyler 1975] was used to
correct both the flow velocities and pressures calculated. No attempt was made to correct for wall effects as the probe head neared a solid boundary or the secondary flow which existed along the probe shaft. Errors associated with the 5 hole probe were ±2% in the flow angles for less disturbed flows, in complex flow zones the error rose substantially estimated near ±5% due to larger flow field fluctuations and associated secondary flow gradients [Humm & Verdegaal 1992].

3.2.3 Wall Pressure Taps

Pressure taps at various positions within the pump were installed for an accurate measurement of a steady surface pressure. Each tap was sharp edged, carefully constructed to avoid mechanical burrs, rounded edges, and other mechanical imperfections. The residual error arising from measurements with pressure taps was dictated by the local change in the boundary layer conditions due to the disturbance caused by the tap. The parameters identified as those responsible for erroneous pressure tap measurements were pressure tap diameter, length, fluid density, and fluid shear stress at the wall. Using this information [Franklin, R. & Wallace, J. 1969] all pressure taps were designed to provide measurements with a less than ±1% error. Results of these pressure measurements were recorded as steady coefficients of pressure (\( \overline{C_P} \)) given in equation 3.5.

3.2.4 The Fast Response Pressure Probe

A single hole fast response probe was constructed for unsteady flow field evaluation within the pump, seen in Fig. 3.1. It can be found as a mechanical drawing in Appendix A. A microelectronic pressure transducer used in medical research (Keller Cardiology pressure transducer type MV) was packaged into the probe tip and the sensing hole filled with oil to avoid an air filled resonant cavity. The calibration performed for this probe was static similar to the 5 hole probe, a dynamic calibration was not necessary because of the high (100kHz) transducer eigenfrequency in air and the precautionary oil filled cavity. The eigenfrequency in water was calculated [Blevins 1987] to remain high (near 25 kHz), not near the maximum frequency of interest here, 1 kHz. The nonlinearity of the pressure transducer was found to be less than 0.2%. The unsteady pressure measured at each local point by the probe was recorded as an unsteady coefficient of pressure (\( \overline{C_P} \)), equation 3.6.

The errors associated with the unsteady coefficient of pressure were dependent on inertial and Magnus flow effects that influence the pressure distribution on the cylindrical probe body [Blevins 1977, Kovasznay et al.
More precisely the aerodynamic effects related to the influence of Reynolds number, velocity gradients, vortex interaction, blockage, turbulence, and unsteady flow angles were investigated [Humm et al. 1994] for the cylindrical geometry of the probe used in this investigation. Since the flow field fluctuation frequencies are relatively small in the pump (i.e., blade passing frequency was 88 Hz at 750 rpm) the influence of the fluctuations on the measured $\Phi$ angle were found to be $\pm 4^\circ$ [Humm 1996] for a reduced frequency of 0.04 and a turbulence intensity of 8%. For the total pressure a maximum of -4% error will be obtained for velocity fluctuations of $\pm 15^\circ$ in the $\gamma$ angle away from the zeroed perpendicular to the flow. The data presented here were not corrected for inertial effects because no true measurement of solely the fluid inertia exists, only the errored coupled with the other mentioned effects. In more three dimensional flows the probe was used sparingly as the application of a 1 hole probe becomes questionable and errors may grow to unacceptable proportions.

The technique of using a single hole fast response probe as a simulated three hole fast response probe was implemented for determination of two dimensional velocity vectors. The results of phase averaged measurements for three pressures located at yaw angles of $-42^\circ$, $0^\circ$, and $+42^\circ$ relative to the probe shaft mechanical zero were then used to calculate the phase averaged $\Phi$ flow angle and the two dimensional velocity magnitude [Goto 1988]. The $0^\circ$ location was set from the zeroing criteria previously found using the 5 hole probe at the same position in the flow. The periodicity of the flow was a crucial assumption made that permits phase averaging of the three pressures not measured simultaneously but at the same pump shaft angle position. Each point in the measured pressure signals at $-42^\circ$, $0^\circ$, and $+42^\circ$ was triggered by a shaft position signal, 3600 TTL shaft trigger signals were provided per rotation and an accuracy of $\pm 0.02^\circ$ exists in the shaft angle. The signal information lost in this phase averaging approach is discussed in section 3.3.

3.2.5 Pressure Transducers

The selection and application of modern pressure transducer technology is goal oriented depending on the specific measurement criteria. A compromise between temporal, spatial, and sensitivity resolution was considered. Whenever feasible pressure was measured differentially to increase the measurement accuracy.

For the measurement of the pressure on the impeller blades and volute tongue piezoresistive pressure transducers were implemented in a full bridge alignment, Fig. 3.4. The bridge existed in the chip which was bonded using epoxy and silicon rubber into the stainless steel pressure transducer hull
(Keller AG Series 2Mi) permitting long duration exposure to water. This transducer was selected for its small size (diameter 4.5 mm), high eigenfrequency (>100 kHz), good sensitivity (unamplified maximum 30 mV/bar), and its ability to provide both the steady and unsteady pressure. The eigenfrequency of the transducers was sufficiently high in air that the frequencies of interest in water, a maximum of 1.0 kHz, were not detrimentally influenced with any undesired amplification or phase shift. A dynamic calibration was not necessary since the eigenfrequency of the transducers was sufficiently large and the transducer surface was always flush mounted on a surface. A static calibration of all transducers was performed to verify manufacturing specifications, linearity was within ±0.2% over the full pressure range of 0 to 5 bar. Errors associated with pressure measurements from these transducers include drift, hysteresis, nonlinearity, eigenfrequency interference, and creep [Gossweiler 1994]. For the measurements performed creep was found to be the largest culprit in error introduction which had a detrimental effect on the transducer steady pressure value, up to ±5% revealed in error analysis. The unsteady component of the pressure however was found to be significantly better, not effected by the pressure transducer creep, providing errors of less than ±1.0%.

The mounting of such chip pressure transducers is critical to a reliable pressure measurement. No mechanical surface stresses in the material from vibrational and centrifugal force should be transferred to the piezoresistive bridge. Thus mounting schemes were investigated to quantify the error introduced by surface stresses for various epoxies. This revealed that a flush mounted surface transducer with the sides soft epoxied (silicon rubber) and the underside hard epoxied (Araldit AV138) provided little transfer of mechanical stresses to the pressure transducer hull, under a large deformation of 0.4 mm the error in the pressure reading was 0.04% of the 5 bar pressure transducer range. Linearity of the errored pressure readings induced by mechanical stress for each mount was found as a function of strain to have a correlation coefficient greater than 0.96. For the case of an all silicon rubber mount zero errored pressure reading was found but the strength of the bond was feared too weak to maintain the transducer position for the testing duration, thus the hard-soft epoxy combination in Fig. 3.4 was opted for. Thermal effects were not present due to the small flow field temperature gradients near the transducers (< 0.01 °C/cm across the pump) and small power (< 2mW) used by the transducers.
3.2.6 Telemetry System

On the impeller 25 piezoresistive pressure transducers were strategically mounted within a single blade passage, Fig. 3.5. Their location, Fig. 3.6, was selected to follow the path of 2 “wall streamlines” on the blade passage pressure side, Pressure Side Hub (PSH) and Pressure Side Shroud (PSS), and 2 “wall streamlines” on the blade passage suction side, Suction Side Hub (SSH) and Suction Side Shroud (SSS). Transducers were also mounted near the blade passage suction side leading edge with the anticipation that this region undergoes interesting unsteady flow field changes with consequences to the hysteresis loop in the pump characteristic [Kaupert et al., 1996]. In the blade surface wire passages were grooved for planting transducer wires and epoxyed smooth afterwards. To reduce the transfer of any mechanical
surface stresses from blade vibration mechanical stress the transducers were mounted with the detailed technique described in section 3.2.5. The wires for the pressure transducers were fed through a hollow pump shaft (acting as a Faraday cage protecting the unamplified signal) to amplifiers and a telemetry data sending device.

![Diagram of pressure transducer locations](image)

**Fig. 3.6 Location of the pressure transducers mounted within the impeller blade passage.**

The telemetry device was mounted on the pump shaft to send the sampled pressure transducer signals to the stationary system as a high band frequency modulated signal, shown in Fig. 3.7. The telemetry device features 16 channels with amplifiers, anti-aliasing filters at 2.4 kHz, and sample and hold acquisition circuitry. In the stationary frame the decoder provided a 12 bit parallel data stream at slightly more than 6 kHz per channel implying 482 data points per impeller rotation at 750 rpm for test series A. An optical shaft encoder providing 3600 TTL pulses per rotation was also incorporated into the measuring system for an exact location of the pump shaft position once per rotation. Since 25 pressure transducers existed each operating point was sampled twice, once with 16 transducers connected and once with 9 connected; reproducibility of results was confirmed with the remaining 7 channels. Digital I/O was appropriately programmed for computer acquisition. These measurements provided steady and unsteady pressures in the impeller frame and were recorded in the nondimensional form of equation 3.5 and 3.6.
All telemetry signal analysis performed was performed for a sampling set of $2^{17}$ points per channel over 271.9 rotations for the case of impeller rotation at 750 rpm. Since 482 points per rotation is not an integer power of 2 care was taken to apply a Hann window over $2^{13}$ points within 16 windows over the sampling set. The influence of window functions and signal leakage was indepthly investigated apriori on both signal magnitude and phase.

A single shaft position provided by the optical shaft position encoder was used as a start point for all analysis to allow signal phase comparison between transducers. While changes in the geodetic height of the rotating pressure transducers were a cause of weak static pressure fluctuations these effects are well quantified and thus subtracted from all measured results. Centrifugal field effects were also removed from the rotating pressure transducer measurements by first measuring their magnitude while rotating the impeller in air. The centrifugal field’s presence only influenced the steady state value of the transducer.

**Fig. 3.7** Telemetry system used for sending pressure transducer signals from the rotating to the stationary frame.

### 3.2.7 Volute Tongue Measurements

On the top tongue of the double spiral volute 19 pressure transducers were mounted for investigating the flow fields interaction. Their orientation is seen in Fig. 3.8 on the side of the tongue which faces the impeller outlet, the tongue side which faces the volute throat is not shown. The measured signals were acquired by an A/D card at a sampling frequency of 3.2 kHz over a wide range of volume fluxes and pump rotational speeds corresponding to operating points measured previously in the rotating system described in section 3.2.6. The number of points sampled per rotation was $2^8$ over 512 rotations implying $2^{17}$ points in total. Anti-aliasing filters were used at 1 kHz.
3.3 Phase Averaging

Performing a phase averaging of unsteady measured signals is common practice in turbomachinery applications to eliminate the part of the signal often linked to stochastic (i.e., turbulence) effects and to reduce the data size for more convenient signal analysis. However, the remaining signal information after phase averaging not only has the stochastic part of the signal removed but rather everything which is not periodic with the pump rotational speed such as stochastic or transient behavior. This implies a possible loss of valuable signal information in flow instability zones (i.e., recirculation) or any subharmonics (i.e., subsynchronous whirl) the signal may be providing. This is illustrated mathematically by considering the following time domain signal $h(t)$ written as the sum of two Fourier series,

$$h(t) = \sum_{f_k} A_k e^{i2\pi f_k t} + \sum_{f_j} B_j e^{i2\pi f_j t}$$

The lost signal information could be of great value to study unsteady pump flow fields and thus signal analysis was performed on raw signal data. Two paths were followed in the signal analysis,

- phase averaging in time the periodic portion,
- no phase averaging for the non-periodic or non-deterministic portion.
Measured pressure signals typically converge to their phase averaged value as a function of the number of rotations applied in the averaging. A representative convergence for most unsteady phase averaged signals found in the pump gave an rms deviation of 0.4% for near 64 rotations implying a rapid convergence. The individual points sampled at the same shaft angle had a 2 standard deviations value generally within 10% of the phase averaged point assuming a normally distributed random variable, Fig. 3.9.

3.4 Signal Analysis Tools

Upon obtaining a measured signal, investigation of the unsteady flow field phenomena within the pump may only be delved into with the appropriate signal analysis tools. A wealth of literature [Kwakernaak & Sivan 1994, Newland 1993] explaining the classical Fourier techniques exists thus they will only be surveyed in a context directly related to unsteady pump flow fields. Quasi joint time frequency techniques are however, a novel technique to decompose turbomachinery signals [Kaupert 1996] into adaptive frequency bands and will be explained with a direct coupling to unsteady non-periodic or non-deterministic signals in pump flow fields.

3.4.1 Classic Fourier Techniques

Fourier techniques are the foundation for much of what is classified as modern signal analysis. These powerful engineering tools are common practice and thus only concepts necessary to the unsteady pump flow fields will be considered. The building blocks of signal analysis lies in the acquisition of the signal itself meeting the requirements of Shannon's sampling theorem which states that time domain signals $h(t)$ bandlimited to $\{ f | |f| < J \}$ are uniquely determined by the samples $h(J^{-1}k/2)$ where $J \in \text{Real}$, $k \in 1,...,2J$. In applying the classic FFT for evaluation of signal magnitude and phase in the frequency domain the signal start point is crucial for investigating the flow fields relation to the geometric orientation of the pump based on circumferential shaft position. For this purpose a shaft signal encoder was implemented with all unsteady measurements to better permit identification of acoustic, and convective flow effects (section 8.3) in the unsteady flow field. The signal coherence in relation to the cross spectral power was also of assistance when evaluating these effects as it permits linearity determination of a transfer function between various positions.
within the pump. A phase shift in a Fourier harmonic of two measured signals is only significant if the coherence between them is high which implies a linear relationship (i.e. a nondispersive wave), a condition stronger than requiring the cross correlation to be high because the coherence is frequency dependent. The coherence is essentially a correlation coefficient for each defined frequency [Stegen & Van Atta 1969]. In cases where the coherence is high the true phase speed (section 7.2) of the Fourier harmonic can be identified.

Together this repertoire of classic signal analysis tools provides a first approach for understanding the unsteady flow field but lacks in that the unsteady flow field within pumps is known to contain transient signal frequencies.

3.4.2 Quasi Joint Time Frequency Techniques

A basic understanding of the mathematics involved in quasi joint time frequency analysis is necessary in the engineering application of this diagnostic tool to better grasp the presented results in this dissertation. The mathematical techniques employed are described amply in the literature available [Chui 1992, Wickerhauser 1994].

The classic Fourier transform is a useful signal analysis tool localized only in frequency, Fig. 3.9a, making it not sufficient for analyzing transient or non-deterministic phenomenon within a signal. Consider only the case of a Dirac delta spike $\delta(t-t_0)$ in the time domain at time $t_0$, its Fourier transform is $e^{-i 2 \pi f t_0}$ whose magnitude covers the entire frequency domain. The time at which the delta spike frequencies exist is lost implying a loss of temporal resolution which is the case as the Fourier transform is localized in frequency. Signal information evolving with time is not reflected.
Inclusion of temporal resolution has only recently been focused upon due to increased computational complexity. The conceptually simplest technique is to divide the time domain signal into a number of short windows and perform time localized Fourier transforms, Fig. 3.9b. This technique, termed a short time Fourier transform (STFT), preserves the linearity of the operator but lacks in that the user must define the window size, associated leakage effects are evident, and no window adaptability is available. The Gabor transformation [Gabor 1946], which can be computationally less expensive than a STFT, has come to the forefront of modified STFT methods available [Chen 1994]. This transformation essentially chooses an minimal window for signal analysis based on Heisenberg’s uncertainty principle ($\Delta t \Delta f \geq 1/4\pi$). A fixed window function $g_\alpha(t-b)$, fixed in length $2\Delta t$, and width $2\Delta f$, is produced which is convolved with the time domain signal $h(t)$. The Gabor transform of a real valued signal $h(t)$ with respect to the analyzing function $g_\alpha(t-b)$ may be defined as,

$$ (G_b^\alpha h)(2\pi f) = \int_{-\infty}^{\infty} [h(t)e^{-2i\pi ft}]g_\alpha(t-b)dt \quad (3.9) $$

$$ g_\alpha(t-b) = \frac{1}{2\sqrt{\pi \alpha}}e^{-\frac{(t-b)^2}{4\alpha}} \quad (3.10) $$
on an open time and frequency scale where the function $g_\alpha(t - b)$ is recognized to be Gaussian. The $\alpha$ parameter is a constant which controls the window width. This technique provides a new alternative allowing a transformation to be localized in time and frequency but still the window dimensions have no adaptability, fixed in frequency band $2\Delta f$ and time band $2\Delta t$ (note, for discrete signals $\Delta t$ is the sampling interval).

A step beyond STFTs regarding adaptability is the wavelet transform. The wavelet transform of a real valued signal $h(t)$ with respect to an analyzing function $\Psi(t)$ is defined as,

$$
(W\Psi h)(b, a) = a^{-1/2} \int_{-\infty}^{\infty} h(t) \Psi^* \left( \frac{t - b}{a} \right) dt
$$

(3.11)
on an open time and frequency scale where the factors $a$ and $b$ are dilation and shift parameters of the $\Psi(t)$ function, which may be complex. The $\Psi(t)$ function is known as the "mother wavelet". The wavelet transform and associated mother wavelet function allow not only translation of the window function in the time domain with the $b$ factor but also dilation in the frequency domain with the $a$ factor providing adaptability. The wavelet function window is of a fixed area but not of fixed dimensions. In a time signal's low frequency portions the wavelet window's time resolution is large, frequency band small while in the higher frequency portions the dilation parameter causes the time resolution to become smaller and frequency band larger, an adaptability feature shown Fig. 3.9c. This is exactly what is desired in signal analysis techniques providing improved quasi joint time frequency resolution where required. The wavelet transform is however less accurate with regard to the uncertainty principle than the Gabor transform, accuracy depending on the chosen mother wavelet function.

The choices available for $\Psi(t)$ provide great flexibility to the user of equation 3.11 but also a great deal of possible confusion; a set of mother wavelet $\Psi(t)$ selection criteria is required. Essentially the compactness and smoothness of $\Psi(t)$ are the main consideration. Compactness refers to the class of functions $\Psi(t)$ which are not continuous. They are in general best suited for high resolution time localization within an $h(t)$ which contains discontinuities. However, the compact mother wavelet provides low resolution frequency localization. The smoothness of the mother wavelet refers to $\Psi(t)$ functions which are continuous. They are in general best suited for high resolution in the calculation of frequencies present in $h(t)$ but sacrifice time localization for signals containing discontinuities. The compactness and smoothness of the analyzing mother wavelet are a time
frequency trade off in signal localization which serves as a selection criterion prior to signal analysis depending upon the signal form to be analyzed.

Computational time and capacity serve as the other mother wavelet $\Psi(t)$ selection criteria for the signal analysis. The condition of mother wavelet $\Psi(t)$ orthogonality allows implementation of the discrete wavelet transform which operates on time signals having a length of an integral power of two [Wickerhauser 1994]. Computational time and capacity savings for the wavelet transforms calculation over non–orthogonal mother wavelet functions are large, often greater than the order of coefficients present. Orthogonal wavelets also allow quicker reconstruction algorithms to be implemented for time signal reconstruction from the wavelet transform (the inverse of an orthogonal matrix is simply its transpose). Only orthogonal wavelets will be applied here.

The graphical convention for presentation of computed quasi joint time frequency spectra is known as the time frequency plane, as in Fig. 3.9. It permits a qualitative and quantitative visual presentation of the calculated data over a large range of scale and dilation parameters that enables quick user inspection. Each time and frequency window computed for the wavelet transform corresponds to a shift value of $b$ and a dilation value of $a$ in equation 3. A discrete frequency axis can be evaluated according to the dilation parameter,

$$a_j = 2^{j/n} \quad j \in \text{Integer},$$

where $n$ represents the number of voices per octave and $j$ is termed the voice within the octave. Typically 8 to 12 voices per octave ($n = 8$ to 12) suffice for a clear view of the time frequency plane [Grossman 1989] thereby reducing the computational time and capacity for the time frequency plane presentation.

3.5 Chapter Summary

For the gathering of global pump data instrumentation was implemented which was in accordance with standardized high precision norms [ISO 1987]. For the acquisition of local flow field data pressure transducers are implemented in a variety of geometries (i.e., within fast response pressure probes, flush mounted on the impeller blades, outside the pump for time averaged flow field evaluation). The process of reducing the acquired signals by means of phase averaging was not always carried out in the signal analysis for fear of loosing the non-periodic and stochastic flow field components.

Quasi joint time frequency signal analysis provides a method for quick qualitative and quantitative analysis of unsteady signals containing non-
periodic or non-deterministic information. The necessity of an adaptive window for signal analysis as well as the compactness and smoothness of the mother wavelet function within the analyzing window serve as the most important selection criteria determined by the signal form. The signals analyzed here are not limited to the single pump on which the signals were measured but rather could serve as a diagnostic tool’s signature for a specific unsteady flow field phenomena within turbomachines.
4.0 Pump Performance

The pump performance measurements within this chapter form the foundation for the unsteady flow field investigation. Provided are the pump characteristics, an overview of the steady flow field within the pump, and hysteresis discontinuities in the pressure coefficient characteristic. Pump performance is dependent on the volute, impeller, and system design, the impeller geometry being dominant in the designer's desire to shape the characteristics [Chiappe 1982]. Both volute and impeller are coupled together by the angular momentum exchange which near best efficiency point is a match. However, clients demand pumps to perform for extended durations in off design resulting in impeller-volute mismatch, poor relative flow angles, and flow instabilities. These off design phenomena introduce greater flow gradients which leads to the formation and/or amplification of the unsteady flow field [Hureau et al. 1993].

4.1 Pump and Impeller Characteristics

![Graph showing pump operational characteristics.](image)

*Fig. 4.1 The pump operational characteristics.*
The pressure ($\psi$), power ($\lambda$), shaft efficiency ($\eta$) and cavitation ($\sigma$) coefficient characteristics as a function of the discharge coefficient ($\phi$) are shown in Fig. 4.1 for various rotational speeds. The tests at 1450 rpm were carried out on a Sulzer Pumps Ltd. test facility. The similitude for scaling between the various rotational speeds shows measurable deviations in the $\eta$ characteristic (-1.2% at 400rpm bep) below 500 rpm due to decreasing Reynolds number within a pump impeller blade passage. The fluid inertia becomes smaller relative to the viscous forces which cause the turbulent boundary layers to thicken implying greater fluid friction losses. Attempts to predict this effect have been numerous [Pfleiderer 1961] but no method has been able to provide the required accuracy based solely on theoretical development [Spurk & Grein 1993]. The depression in the $\psi$ characteristic for $\phi < 0.06$ (34.5%) was caused by prerotation in the suction pipe elevating the pressure in the suction pipe pressure taps, documented in section 4.2.

![Fig. 4.2a](image1) The $\psi$ characteristic and calculated volute matching.

![Fig. 4.2b](image2) The $\psi$ characteristic in the hysteresis for pump and impeller in a zoom.

The characteristic chosen for unsteady flow field investigation under variable volume flux and fixed rotational speed was 750 rpm ($u_2 = 12.7$ m/s), seen in Fig. 4.1 to be free of the Reynolds number effects. In Fig. 4.2a the impeller and pump $\psi$ characteristics are shown at 750 rpm; the bep volume flux was 196 l/s ($\phi = 0.174$) with a head of 0.59 bar ($\psi = 0.72$). The impeller $\psi$ characteristic was evaluated by integrating the local energy flux ($P/\rho + c^2/2$) through flow cross sections [Staubli et al., 1995] measured with the 5 hole pressure probe at the impeller inlet and outlet, Fig. 2.4. This procedure required many high resolution pressure probe traversals, thus only select points along the impeller characteristic have been evaluated. The measured impeller characteristic is larger than the pump characteristic due to total pressure losses encountered in the double spiral volute of the pump. Accompanying the pump and impeller characteristics in Fig. 4.2a is a calculated line representing the angular momentum in the volute [Lorette & Gopalakrishnan, 1986]. This calculation was based on the volute throat area, the continuity equation, and the assumption of free vortex flow in the volute.
(section 4.4) to determine $c_{u2}$. The impeller and the double spiral volute characteristics are according to classic theory a matching of the angular momentum exchange which determines a bep $\phi$. Agreement between the measured bep, marked bep, and the intersection of the volute-impeller lines is excellent.

A zoom of the hysteresis loop found in the $\psi$ characteristic is shown in Fig. 4.2b. Upon decreasing the volume flux from $\phi > 0.123$ (70.9% bep) the top branch of the pump hysteresis loop was followed reaching a $\psi$ discontinuity at the stability limit of $\phi = 0.121$ (69.4%) where an abrupt $\psi$ decrease occurs. Increasing the volume flux from $\phi < 0.121$ (69.4%) the bottom branch of the pump hysteresis loop was followed reaching a $\psi$ discontinuity at $\phi = 0.123$ (70.9%) where an abrupt $\psi$ increase occurs. These exact values were not measured in the impeller $\psi$ characteristic due to a lack of resolution in the selected pressure probe measurement points but agreement seems to be present. The path followed by $\psi$ in the hysteresis flow regime was dependent on $\phi$ increasing or decreasing (i.e., history of $\phi$).

4.2 Pump Inlet Flow

At the pump impeller inlet five hole pressure probe traversals were performed to quantify the steady inlet flow field. Figure 4.3 depicts the meridional and tangential velocity profiles for the 90° probe traversal orientation. The meridional velocity established continuity of mass at $\phi = 0.209$ (120%) and $\phi = 0.174$ (100%) with deviations of -3% and -1% respectively. In part load a continuity comparison is incompetent due to the formation of recirculation zones, negative $c_{m}$, which cause greater three dimensional flow structure (only one circumferential position for the probe traversal was used). Rectangular like meridional velocity profiles, similar to that of Fig. 4.3 at $\phi = 0.209$ (120%) and $\phi = 0.174$ (100%) with no preswirl and thus not shown, were measured at all volume fluxes slightly behind the test facility flow straightener to ensure that the profiles measured directly in front of the pump were formed by the pump. Uniform inlet velocity profiles also provided simple boundary conditions for numerical simulations.

![Fig. 4.3 Meridional and tangential flow velocity at the pump inlet.](image-url)
Examining the tangential velocity no preswirl existed at $\phi = 0.209$ (120%) and $\phi = 0.174$ (100%), reasonable if the impeller blade inlet angle was near the relative flow incidence angle. No preswirl existed in the outer radius regions which could have resulted from the impeller leakage flow. The preswirl became evident in the $\phi = 0.096$ (55%) and $\phi = 0.017$ (10%) case, penetrating upstream to the inlet pressure taps to elevate the pressure and create the low $\phi = 0.06$ (34.5%) dip in the pump $\psi$ characteristic, Fig. 4.1.

![Graph showing meridional and tangential velocity at the impeller inlet.](image)

Fig. 4.4 Meridional and tangential velocity at the impeller inlet.

The results for the pressure probe traversal in the $45^\circ$ traversal orientation along the impeller leading edge are shown in Fig. 4.4. The meridional velocity component, perpendicular to the probe axis, again shows recirculation zones at $\phi = 0.096$ (55%) and $\phi = 0.017$ (10%). The steady measured recirculation zones commence at $\phi = 0.121$ (69.4%) after decreasing $\phi$ and cease at $\phi = 0.123$ (70.9%) after increasing $\phi$ (not depicted). These values were determined (section 4.1) as the $\phi$ where the pump $\psi$ characteristic discontinuities in Fig. 4.2b occur. The steady measured preswirl was also found to commence/cease at the operating points where the steady measured recirculation flow commences/ceases as reported previously [del Valle et al. 1992].

The inlet preswirl was caused by the tangential motion of the impeller driven recirculation zones which through viscosity [Schiavello & Sen 1980] transmitted the motion upstream. The size of the recirculation zones increases with decreasing $\phi$ and penetrates further upstream. In the case of $\phi = 0.017$ (10%) the preswirl in Fig. 4.3 and Fig. 4.4 approaches rigid body rotation. The preswirl velocity near the impeller inlet tip reaches 10.20 m/s ($c_{u1}/u_{2} = 0.80$) for an inlet tip velocity of 10.52 m/s ($u_{1}/u_{2} = 0.83$). The commencement/cessation of the inlet recirculation and preswirl are an indicator of complex flow field changes occurring in the pump’s hysteresis flow regime.
4.3 Impeller Outlet Flow

Fig. 4.5 shows the impeller outlet, radial velocity and absolute flow angle distributions, clearance sections inclusive. Continuity of mass at $\phi = 0.209$ (120%) and $\phi = 0.174$ (100%) was reasonably established, a discrepancy of -6% and -5% respectively. A steady measured recirculation zone formed near the shroud side as $\phi$ was decreased (at $z/B_3 = 0.75$), the $\phi = 0.122$ (70.0%) case was measured after decreasing $\phi$. The $\phi$ of commencement and cessation of this steady measured outlet recirculation corresponded to $\phi = 0.121$ (69.4%) after decreasing $\phi$ and $\phi = 0.123$ (70.9%) after increasing $\phi$, not depicted. These values correspond to those for the commencement/cessation of the impeller inlet steady measured recirculation and the locations of the $\psi$ characteristic discontinuities. The absolute flow angle reveals similar behavior to the radial velocity implying the tangential velocity distribution was nearly uniform. Using simple velocity triangles and the continuity equation, the angle $\alpha_{2metal} = 15^\circ$ was calculated at bep.

4.4 Flow within the Volute

The cross section within the volute seen in Fig. 2.6 at the probe position in Fig. 2.5 was used for an evaluation of the volute flow. Fig 4.6 reveals the tangential velocity contour plots at four volume fluxes each implementing 105 equidistant measurement positions with a linear interpolation to fill the missing data.

Of interest is the decrease the tangential velocity undergoes as the radius is increased, excluding the $\phi = 0.017$ (10%) which has little tangential velocity. From these data free vortex flow, $c_u r = K$ where $K$ is a constant, is verified at $\phi = 0.174$ (100%) with $z/B_3 = 0.5$ shown plotted in Fig. 4.6. The plot depicts a line calculated with the free vortex flow equation using a $K$ value determined by $K = \sum (c_u r)_j / N$ where $j$ represents the measurement point and $N$ the total number of points. This agreement confirms that the pressure rise in the radial direction within the volute at $\phi = 0.174$ (100%), Fig. 4.7, stems from the centrifugal field’s pressure rise given by,

$$\frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{c_u^2}{r}$$  \hspace{1cm} (4.1)
which when used in conjunction with the steady momentum equation and an 
assumption of zero radial velocity permits the free vortex flow equation to be 
developed [Vavra 1960]. The coefficient of pressure in the contour plots of 
Fig. 4.7 reveal the dependence of the pressure on the volume flux deviation 
from bep operation, a manifestation of the impeller \( \psi \) characteristic shown in 
Fig. 4.2a. For decreasing \( \phi \) the radial pressure gradient is seen to decrease 
generally. Since \( \partial P/\partial r \) decreases as \( \phi \) decreases equation 4.1 implies that \( c_u \) 
also decreases which is the general tendency in Fig. 4.6, except directly at the 
impeller outlet near \( r/R_2 \approx 1 \). The tendency is in fact reversed here, the lower 
the volume flux the larger \( c_u \). This result is problematic considering that the 
volute characteristic desires increasing \( c_u \) for increasing \( \phi \) but the impeller 
provides, near \( r/R_2 \approx 1 \), decreasing \( c_u \) for increasing \( \phi \) due to its backswept 
blade angle (see also section 5.2, Fig. 5.6). The notion of a coupling of two 
components, impeller and volute, is thus introduced which together form a 
best point operating volume flux (see again Fig. 4.2a). This matching or 
mismatching of the impeller and volute characteristics are found (chapter 7) 
to be the prime cause of unsteady flow field generation within the impeller.

![Fig. 4.6 Tangential velocity in the volute cross section for four \( \phi \) ’s.](image-url)
**Fig. 4.7** Coefficient of pressure in the volute cross section for four $\phi$'s.

An important realization for the high specific speed pump is that the flow in most operating points was three dimensional at the impeller outlet and within the volute. This is quantified by contour plots in Fig. 4.8 capturing the axial secondary flow from pump hub to shroud. Interestingly this secondary flow was smallest at $\phi = 0.096$ (55%) below the hysteresis in the pump characteristic.

**Fig 4.8** Axial velocity in the volute cross section for four $\phi$'s.
4.5 Volute Outlet Flow

Pressure probe traversals were performed at the volute outlet to verify that both spirals of the double spiral volute were operating symmetrically for various volume fluxes. This is revealed in Fig. 4.9 showing moderate variations across the traversal length but no drastic differences such as passage blockage or separation which could cause the pump characteristic hysteresis.

![Fig. 4.9 Meridional velocity at the volute outlet.](image)

4.6 Impeller Leakage Flow

The leakage flow in the impeller shroud clearance is a non negligible quantity since it may be several percent of the total pump flow and is injected locally in a very sensitive flow region upstream of the impeller inlet. The leakage volume flux was measured by blocking the pump inlet and applying a pressure gradient over the pump to simulate the head at a particular rotational speed. The results are depicted in Fig. 4.10 as the percentage in $\phi$ that the leakage flow represents vs. $\phi$. Agreement between empirical predictions for a smooth gap [Yamada et al. 1969] and measurements for 100% leakage volume flux reveal excellent accordance, less than $+1\%\Delta\phi$ difference. With this information the important outlet to inlet clearance flow may be simulated in computational calculations with the measured results as boundary conditions.

![Fig. 4.10 Measured leakage flow in the impeller leakage flow regions.](image)

4.7 Chapter Summary

The measured pump characteristics reveal Reynolds number effects at rotational speeds below 500 rpm (e.g. -1.2% efficiency at 400rpm bep). The $\psi$ characteristics contain two distinct discontinuities which make up a hysteresis loop. The $\psi$ discontinuities $\phi$ locations agree well with the commencement and cessation of steady measured recirculation zones at the impeller inlet and outlet. These volume fluxes also correspond to the commencement and cessation of inlet preswirl induced by the inlet
recirculation. The volute flow may be considered as free vortex flow near bep volume flux, deviations from bep result in appreciable changes in the volute pressure as the impeller $\psi$ characteristic would suggest. The flow at the impeller outlet was three dimensional since the centrifugal impeller is of high specific speed. Both passages of the double spiral volute function similarly over the entire operational volume flux.
5.0 Impeller Inlet and Outlet Recirculation

The discontinuities (often referred to as the instability) in the pump characteristics are caused by an abrupt change in flow properties within the impeller introducing significant formation and amplification of the unsteady flow field. Exact reasons for this abrupt unsteady flow field change lie in the complex interaction between centrifugal, coriolis, viscous, and blade loading forces acting on the relative flow. The \( \alpha_S = 1.7 \) pump has relatively large blade passages with short blades for a radial pump implying lesser flow guidance, greater secondary flow development, and greater susceptibility to flow instabilities (i.e., recirculation zones) typically accompanied by characteristic discontinuities [Hergt & Starke 1985]. In striving to obtain increased pump operational regimes with respect to the volume flux these recirculation zones are encountered more frequently [Fraser 1982]. Thus pump designers and users must evaluate the risks (i.e., dynamic loading, cavitation, flow induced vibrations) involved in the changing unsteady flow field caused by recirculation zones [Makay 1980] encountered with the wider pump operational regimes. Further, pump manufacturers must design pumps with predictable recirculation zones and a resistance to the produced unsteady flow field because recirculation zones are unavoidable over the entire pump operational regime.

Various studies [Gülich 1995, Hergt & Jaberg 1989, Tanaka 1980] which deal with pump characteristic discontinuities and their flow fields indicate a uniqueness for each case examined, the case studied here is not exemplary of all cases. However, in all cases without exception recirculation zones form at the impeller inlet and outlet as the volume flux is decreased from bep. These zones have been postulated [e.g. Gülich 1995] to be a self healing of the flow field as the relative flow deceleration becomes too great. Recirculation zones form and act as blockage contracting the flow area, the remaining through flow accelerates.

Within this chapter the impeller inlet and outlet flow recirculation are examined experimentally and using computational fluid dynamic (CFD) simulations as a tool to aid the interpretation of experimental results, providing greater insight into the three dimensional flow physics.

5.1 Computational Fluid Dynamics

The computational fluid dynamics were performed for the same operating conditions as the experiments with the commercially available TASCflow code version 2.3 which performs Reynolds averaging of the Navier Stokes equations in strong conservative form to obtain steady state
solutions. A standard k-ε model was used for turbulence modeling [ASC, 1994]. A coarse grid with 41,000 nodes was implemented for the impeller, outlet pressure pipe, and inlet suction pipe, shown in Fig. 5.1*. No attempt was made to model the double spiral volute in which the experimental pump impeller operated. A fine grid was also used for computations containing 200,000 nodes but due to excessive computation times was put into limited use to evaluate grid refinements. The concept of energy flux analysis over flow cross sections, was applied to check the physical consistency of the computed flow fields [Staubli & Holbein 1995]. The impeller clearance leakage flow was also included in the numerical calculations taken however, as a boundary condition from experimental measurements. Greater detail pertaining to the applied numerics has previously been expanded upon [Staubli et al. 1995, Kaupert et al. 1996].

**Fig. 5.1 The impeller and suction pipe numerical coarse grids Impeller shroud is removed for viewing purposes [Kaupert et al 1996].**

### 5.2 Stationary Frame Inlet Recirculation

A quantitative comparison of the steady meridional velocity at the pump inlet is shown in Fig. 5.2 for the φ=0.174 (100% bep) φ =0.096 (55%) operating points [Staubli et al., 1995] at the 90° traversal position. The inlet reverse flow at φ =0.096 (55%) has accurately been predicted. A clear explanation however to the three dimensional flow changes occurring at the inlet to produce recirculation is not provided.

**Fig. 5.2 Impeller inlet meridional velocity at two flow coefficients.**

* Both Fig. 5.1 and 5.3 were diligently computed by Dr. P. Holbein.
Greater insight into the recirculation flow mechanisms are obtained by viewing the numerical impeller flow streamlines at points corresponding to increasing and decreasing $\phi$, Fig. 5.3. While decreasing $\phi$ the flow streamlines at the impeller inlet tip undergo an abrupt change. At $\phi = 0.1191$ (68.4%) the impeller tip streamlines pass through the impeller, at $\phi = 0.1184$ (68.0%) they begin to deviate radially along the inlet blade suction side, and at $\phi = 0.1167$ (67.0%) they no longer have enough energy to traverse an adverse pressure gradient and do not pass through the impeller. Upon exacter observation streamlines can be seen at the leading edge slightly entering the impeller and then be directed radially outward to reverse in flow direction.
and exit the impeller at a larger radius. This is the numerical flow recirculation zone at the impeller inlet tip responsible for reverse flow. While increasing $\phi$ this process repeats itself in reverse order but at a larger volume flux implying a hysteresis process (section 6.2). Note the computational values of $\phi$ corresponding to the commencement/cessation of inlet recirculation don’t exactly match those measured (section 4.2 and 4.3).

![Impeller meridian view](image)

**Fig. 5.4** The change in the blade suction surface pressure along the impeller leading edge during decreasing $\phi$ [Kaupert et al. 1996].

The numerical blade suction surface pressure along the leading edge (1 mm behind the leading edge) for decreasing $\phi$, Fig. 5.4, reveals an abrupt change in the pressure gradient perpendicular to the healthy flow direction as numerical inlet recirculation commences. Excessive rates of flow deceleration at the inlet lead to the formation of this pressure gradient along the leading edge, acting towards larger $s$ near the impeller tip ($0.79 < s / L_1 < 0.98$). The pressure gradient disappears abruptly while increasing $\phi$, not shown but occurring in a similar manner, as the numerical inlet recirculation ceases. This effect is responsible for the radial outward flow portion of the inlet recirculation occurring in the numerical impeller. A further manifestation hereto is the increase in pressure at the shroud ($s / L_1 = 0.99$), a redirection of the streamlines occurs here.

<table>
<thead>
<tr>
<th>operating point</th>
<th>observations in impeller inlet spectra</th>
</tr>
</thead>
</table>
| A, $\phi = 0.1213$ (69.7%) | • blade passing frequency dominant  
• stochastic components near 50 Hz  
• small pump shaft frequency $f_o$ component  
• at $f_{bp}$ FFT reveals $\bar{C}_p = 0.04$ |
| B, $\phi = 0.1220$ (70.0%) | • blade passing frequency dominant  
• greater stochastic variation (noisier than in A)  
• small pump shaft frequency $f_o$ component indicates blade passages have inlet recirculation concurrently  
• at $f_{bp}$ FFT reveals $\bar{C}_p = 0.06$ |
| $\phi < A$ (not shown) | • stochastic levels grew as $\phi$ decreased, inlet recirculation intensified |

**Table 5.1 Observations for stationary frame unsteady inlet recirculation.**
The formation of the impeller inlet recirculation causes an abrupt change in the measured unsteady inlet pressure. Revealed in Fig. 5.5 are the Gabor spectra for a pressure signal on the inlet flange wall upstream of the impeller seen schematically to the bottom right. At the top right of Fig. 5.5 a zoom of the pump $\psi$ characteristic is shown. Table 5.1 summarizes the observations made.

![Gabor spectra magnitudes at the impeller inlet](image)

<table>
<thead>
<tr>
<th>operating point</th>
<th>inlet recirculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>No</td>
</tr>
<tr>
<td>B</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Fig 5.5 The Gabor spectra magnitudes at the impeller inlet without recirculation A (top) and with recirculation B (bottom).

Results similar to these have been previously reported [Stachnik 1991] using Fourier spectra and with flow visualization [Weiβ 1995] where the inlet recirculation was commented as a strong stationary phenomena always present in time.

5.3 Stationary Frame Outlet Recirculation

A qualitative experimental and numerical comparison of the impeller outlet steady velocity triangles for $\phi = 0.174, 0.122$, and $0.110$ (100%, 70% and 63%) operating points is provided in Fig. 5.6 at $r/R_2=1.05$, the axial velocity not being shown. Agreement is seen to be reasonable at $\phi = 0.174$ (100%), in part load however there are significant discrepancies in the radial velocity. Experimentally determined outlet recirculation is seen to form near the
shroud side just below the $\phi=0.122$ (70%) operating point for the decreasing $\phi$ case. The corresponding numerical frame in Fig. 5.6 does not show this impeller outlet recirculation. There numerically exists impeller inlet recirculation only, not impeller outlet recirculation.

Fig. 5.6 Comparison of experimental and numerical impeller outlet velocity triangles. Circular magnifying glass indicates region where impeller outlet recirculation commences, $\phi < 0.122$ [Kaupert et al. 1996].

To further this point Fig. 5.7 reveals the steady static pressure at the impeller outlet $r/R_2=1.05$, for 4 volume fluxes over the blade height. The
effect of the abrupt pressure drop manifested in the impeller characteristic $\psi$ discontinuity (see Fig. 4.2b) occurs between $\phi = 0.122$ (70%) and $\phi = 0.110$ (63%). This pressure drop also occurs at the numerical impeller outlet, Fig. 5.7, but without any outlet recirculation. Thus the outlet flow recirculation and any volute tongue flow interaction are not a necessary condition for the $\psi$ discontinuity in the numerical model for the impeller characteristic.

![Graph](image)

**Fig. 5.7** Left probe measured impeller outlet pressure coefficient for a decreasing volume flux. Right computational results.

The formation of impeller outlet recirculation causes an abrupt change in the measured unsteady flow field. In Fig. 5.8 the Gabor spectra are shown for pressure signals obtained with the fast response probe at the impeller outlet ($r/R_2 = 1.05$, $z/B_3 = 0.72$) where the outlet recirculation first forms. Caution must be exercised in interpreting these spectra as they represent the unsteady flow field the immersed probe was experiencing, not necessarily the unsteady flow field the impeller produces. The frequency band near 340 Hz corresponds well to the frequency of a vortex street behind a cylinder with a Strouhal number of 0.2 [Blevins 1977]. Table 5.2 summarizes the observations.

<table>
<thead>
<tr>
<th>operating point</th>
<th>observations in impeller outlet spectra</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, $\phi = 0.1213$ (69.7%)</td>
<td>• blade passing frequency dominant • low ($-f_p/2$) and high ($\sim 340$ Hz) frequency bands • small pump shaft frequency $f_p$ component • at $f_{bp}$ FFT reveals $C_p = 0.06$</td>
</tr>
<tr>
<td>• minimum $\phi$ without inlet recirculation</td>
<td></td>
</tr>
<tr>
<td>B, $\phi = 0.1220$ (70.0%)</td>
<td>• blade passing frequency dominant • intenser low ($-f_p/2$) and high ($\sim 340$ Hz) bands • greater stochastic variation (noisier than A) • small pump shaft frequency $f_p$ component indicates blade passages have outlet recirculation concurrently • at $f_{bp}$ FFT reveals $C_p = 0.09$</td>
</tr>
<tr>
<td>• maximum $\phi$ with inlet recirculation</td>
<td></td>
</tr>
<tr>
<td>$\phi &lt; A$ (not shown)</td>
<td>• stochastic variation grew as $\phi$ decreased, outlet recirculation intensified.</td>
</tr>
</tbody>
</table>

**Table 5.2** Observations for stationary frame unsteady outlet recirculation.

These results are contrary to some reports in the literature [Stachnik 1991] where no abrupt change in the outlet Fourier spectra was reported as outlet recirculation commenced while in accordance with others [Fraser
1982] where the technique of increased stochastic portions in spectra was shown to accurately detect outlet recirculation. Apparently the strength of the outlet recirculation and its detectability with frequency spectra varies between cases examined.

<table>
<thead>
<tr>
<th>operating point</th>
<th>inlet recirculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>No</td>
</tr>
<tr>
<td>B</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Fig 5.8** The Gabor spectra magnitudes at the impeller outlet without recirculation A (top) and with recirculation B (bottom).

### 5.4 Rotating Frame Recirculation

From the pressure transducers mounted within the rotating blade passage the inlet and outlet recirculation with their coupling may be determined. Concentrating on the inlet recirculation first the transducers along the blade suction side leading edge are presented in the center of Fig. 5.9. To the top left a 3D plot is presented of the pressure signal fluctuation magnitude (statistically evaluated as 2 standard deviations of the measured signal magnitude, not the peak to peak value) as a function of \( \phi \) and dimensionless transducer position \( s/L_1 \). At large \( \phi \) (0.209 (120%) > \( \phi \) > 0.1254 (72%)) the fluctuations are approximately equal across the blade height. As \( \phi \) was decreased an abrupt change in the signal magnitudes occurs just below point A (\( \phi = 0.1213 \) (69.7%)), the inlet recirculation commenced. The outer two transducers (large \( s/L_1 \)) were affected by this inlet recirculation with magnitude changes from near 0.05 to 0.11, shown
quantitatively in Fig. 5.9 top right. As $\phi$ was decreased further to $\phi = 0.0957$ (55%) and the inlet recirculation intensified the two inner transducers (small $s/L_1$) can be seen to increase in magnitude as they become influenced by the inlet recirculation.

The change in the inlet flow field is examined for the tip transducer at the bottom of Fig. 5.9 with a Daubechies 8 coefficient [Appendix B] wavelet spectrum. To the left the spectrum with no inlet recirculation (A) showing a strong pump shaft and second harmonic frequency. To the right the inlet recirculation with $\phi$ on the bottom branch of the hysteresis (B). Both the pump shaft and the second harmonic were present but a frequency band higher (40 to 70 Hz) is seen to exist. This appears to be the frequency in the vortex formed, rotating at the impeller inlet and influencing the unsteady flow field within the pump.

**Fig 5.9** Leading edge transducers magnitude (top) for varying $\phi$. Wavelet spectrum (bottom) for the tip transducer at points A and B.
The remaining question of how impeller inlet and outlet recirculation influence each other is examined with the aid of Fourier spectra. At the top of Fig 5.10 a zoom of the pump ψ characteristic is again show, points A and B retaining their positions. All the Fourier spectra are chopped at a value of 0 to 0.015 to qualitatively compare the stochastic portions of the spectra over a frequency band from 0 to 100 Hz, the $f_0$ being the pump shaft frequency. Table 5.3 summarizes the observations. The signal content at $f_0/2$ could be the result of rotating stall but has not been fully investigated.

<table>
<thead>
<tr>
<th>operating point</th>
<th>observations in rotating impeller spectra</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, $\phi = 0.1213$ (69.7%) • minimum $\phi$ without inlet recirculation</td>
<td>suction side • shaft passing frequency $f_0$ and multiples dominant • near leading edge (2A, 3A, 4A, 5A) lower shaft harmonics prominent and weak</td>
</tr>
<tr>
<td></td>
<td>pressure side • shaft passing frequency and multiples dominant</td>
</tr>
<tr>
<td>B, $\phi = 0.1220$ (70.0%) • maximum $\phi$ with inlet recirculation</td>
<td>suction side • increased stochastic level in 3B, 4B, 5B, 8B, 10B, 12B generated by the recirculation zone • 2B shows little change, revealing shroud to hub influence of recirculation • 10B and 12B show decrease in stochastic level compared with 5B, 8B. Deemed the end of the stationary recirculation zone • 11B on hub side shows little change</td>
</tr>
<tr>
<td></td>
<td>pressure side • 15B, 16B, 17B, 18B reveal no significant change</td>
</tr>
</tbody>
</table>

Table 5.3 Observations for rotating frame impeller unsteady recirculation.

In terms of the frequency spectra and stochastic levels it can thus be stated that the inlet recirculation was a stationary phenomena more intense than the outlet recirculation. It is put forth here that the outlet recirculation is coupled to the inlet recirculation through the impeller but is highly non-stationary when compared to the strong stationary inlet recirculation. This is supported by the fluctuating forward and reverse flow found at the impeller outlet (section 8.2). The hypothesis has been previously reported with flow visualization tests [Weiß 1995] on a pump of medium specific speed. It implies that the inlet recirculation penetrates downstream to a minimum depth of transducer 8 and fluctuates deeper to transducer 10 and 12. This penetration downstream was enough to register with the steady outlet recirculation detected in Fig. 5.6. Hereto the pressure distribution in the volute of the pump has an upstream influence on the impeller flow which can act on the recirculation zones depth of impeller penetration. In contrast for the inlet portion of recirculation a case was reported [Caignaert & Descamps
1997] of trimming the volute tongue radial gap incrementally 4 times from $R_3/R_2 = 1.024$ to 1.07 obtaining no change in the recirculation commencement volume flux. In diffuser pumps the selection of a diffuser is known to have a significant influence on the impeller outlet recirculation but little to none on the inlet recirculation [Stachnik 1991, Breugelmans & Sen 1982].

5.5 Chapter Summary

Agreement in the measured and numerical inlet recirculation is excellent in spite of the missing numerical impeller outlet recirculation, which has been measured to commence simultaneously with the onset of inlet recirculation. The computed pressure distribution at the impeller outlet however agrees well with the measured distributions which will manifest itself in any $\psi$ characteristic calculation for the impeller. Computationally the impeller inlet tip streamlines reveal the formation/cessation of a recirculation zone with hysteresis occurring over the volume flux. A pressure gradient parallel to the blade leading edge at the impeller tip was found to abruptly form/cease as the inlet recirculation formed/ceased. Further investigation of the vorticity in this region is necessary.

The commencement/cessation of the recirculation flow is detectable with Gabor spectra at both the impeller inlet and outlet. It manifests itself by an increase in the stochastic level of the spectrum which stems from the recirculating vortex. At both the inlet and outlet the recirculation was found to commence/cease abruptly in all blade passages upon decreasing/increasing $\phi$ at the pump $\psi$ characteristic discontinuities.

The recirculation zone at the inlet was a strong phenomena which can be considered stationary. For the highest $\phi$ of its existence, lowest intensity, it extends through approximately half the impeller passage suction side shroud side retaining a strong circulating behavior. In the later half of the impeller the recirculation takes on a non stationary form considerably weaker in intensity than at the inlet. The inlet and outlet recirculation zone are one recirculation zone coupled stochastically and transiently through the impeller. The outlet recirculation was influenced by the volute configuration and thus was not predicted in the numerics, performed without any volute.
Fig 5.10 The changing flow in the rotating impeller without (A) and with (B) recirculation.
6.0 Pump and Impeller Characteristic Hysteresis

The hysteresis in the pump characteristics provides an operating regime containing complex flow field changes within the pump. Interest in this regime exists because this operational regime often has,

- positive slope characteristics implying system instability [Greitzer 1981],
- a significant decrease in efficiency (3% for the pump investigated here),
- increased pressure fluctuations [Stoffel 1991],
- an increased danger of cavitation [Pfleiderer & Petermann 1991].

During the design process manufactures must pay special attention to the hysteresis operational regime as clients don’t normally accept pumps having large characteristic discontinuities making an understanding and prediction of this phenomena economically necessary.

The prediction of the pump characteristics with numerical computations is a valuable tool to pump manufactures expediting new impeller designs and modification processes. These new impellers must later be proven on site over the operational regime and meet commercial customer guarantees. The discontinuities of a pump’s characteristics provides a problematic regime, complex to predict empirically and numerically. Techniques for influencing the discontinuities and hysteresis have been well established [Güllich 1995] however their influence must be evaluated during the design stages.

In aerodynamics, hysteresis effects in both the steady and unsteady lift coefficient with regard to static and dynamic stall are recognized [McCroskey 1982] on two dimensional profiles. This is caused by flow separation and reattachment occurring at different angles of incidence. Flow separation may also occur within the investigated impeller here, hysteresis in the blade lift coefficient would directly manifests itself in the pump characteristics. The three dimensionality, rotation, geometry, and unsteadiness of the pump flow field coupled with transient pump operational load adds to the complexity of the flow making local separation an enlightening interpretation but much oversimplified.

6.1 Hysteresis in the Characteristics

Revealed in Fig. 6.1 are the pressure, power, and efficiency coefficient in the zoomed hysteresis regime of the characteristics. Discontinuities and hysteresis are seen to be found in all three characteristics occurring at the same volume fluxes. Movement from the top to bottom branch of the hysteresis is accompanied by flow field changes consisting of inlet prerotation, impeller recirculation, and inlet flow contraction as the impeller recirculation blocks the larger radius portions of the impeller (section 4.2).
Two important observations can be made from Fig. 6.1.,
- discontinuities in the characteristics exist
- discontinuities in $\psi$ and $\lambda$ are different in magnitude, conservation of energy seems defied.

Fig 6.1 Pump characteristics discontinuities and hysteresis zoomed for the pressure power and efficiency coefficients at 750 rpm.

6.1.1 The Characteristics Discontinuities
The discontinuities in the characteristics of Fig. 6.1 correspond exactly to those operating points where impeller recirculation commences/ceases (section 6.5). For the case of recirculation commencing the impeller experiences a partially blocked flow area at larger radius portions and the resulting through flow experiences a sudden path change resulting in a different energy transfer path. This sudden flow path change [Güllich & Egger 1992] results in the discontinuities in $\psi$ and $\lambda$. Simple argumentation with the Euler turbomachinery equation can be made by considering the specific energy transferred to the flow by the impeller, written as $c_{u2}u_2-$
Before recirculation $c_{u1} = 0$ meaning for a constant rotational speed the specific energy transferred depends only on $c_{u2}$. After recirculation starts $c_{u1} \neq 0$ at the impeller inlet but only at the blocked through flow locations. In the through flow $c_{u1} = 0$ still [Hureau et al. 1993, section 4.2 at $\phi = 0.096$ (55%)]. Again the specific energy transferred depends only on $c_{u2}$. However because of the abrupt flow change occurring within the impeller $c_{u2}$ before recirculation does not necessarily equal $c_{u2}$ after recirculation.

6.1.2 The Discrepancy in Discontinuities Magnitude

To provide a physical understanding for the discrepancy in the magnitude of the $\psi$ and $\lambda$ discontinuities the concept of changing power will be applied. Along the top branch of the hysteresis for decreasing $\phi$ at the $\psi$ discontinuity near $\phi = 0.12$ the power (the product $Q \Delta P_{2-1}$, interpreted as the useful power transferred to the flow) drops 340W, the $\lambda$ coefficient (the power on the pump shaft $M \omega$) drops 250W meaning that after dropping down the discontinuity the flow receives 340W less useful power for 250W less power on the pump shaft. Missing are 90W of shaft power not transferred to the useful flow power. Generalizing this case for a $-\lambda_d$ discontinuity in shaft power a $-4/3 \lambda_d$ discontinuity in the useful flow power occurs. For increasing $\phi$ a $\lambda_d$ discontinuity in shaft power results in an $4/3 \lambda_d$ increase of transferred useful flow power.

To heuristically explain this paradox consider the definition of the shaft efficiency,

$$\eta = \frac{Q \Delta P}{M \omega} = \frac{\eta_h \dot{m}(u_2 c_{u2} - u_1 c_{u1})}{M \omega}$$

where the hydraulic efficiency with the Euler turbomachinery equation has replaced the useful flow power. Implementation of the Euler equation allows a simplified physical interpretation though it is certainly not sufficient for the case at hand (i.e., unsteady, three dimensional, viscous flow). At a characteristic discontinuity $\eta$ and $M \omega$ are measured to change and the term $u_2 c_{u2} - u_1 c_{u1}$ changes appropriately with the change in shaft power and flow structure (i.e., recirculation). The hydraulic efficiency $\eta_h$ however also changes. The commencement/cessation of the impeller driven recirculation and prerotation produces a change in the flow field which influences the power transferred to the fluid from the impeller meaning a change in $\eta_h$, the recirculation and prerotation can be considered as power dissipation mechanisms having viscous stresses and mixing to overcome while rotating (section 4.2). Thus the discontinuity in $Q \Delta P_{2-1}$ can be larger than the $\lambda$ discontinuity through a discontinuity in $\eta_h$.

To further this hypothesis an order of magnitude approximation of the power dissipated in the impeller driven recirculation and prerotation was
performed (better evaluation would require the full constitutive equations and looses physical insight, section 6.2). Of interest are the viscous terms in the momentum of a fluid which are contained in the momentum equations (C.2),

$$\rho \frac{Dc_i}{Dt} = \frac{\partial \sigma_{ij}}{\partial x_i} + \rho f_i$$  \hspace{1cm} (6.1)

where \( f_i \) represents the net external body forces on the fluid and \( \sigma_{ij} \) is the stress tensor. For the Navier Stokes equations this \( \sigma_{ij} \) is of the form,

$$\sigma_{ij} = -P \delta_{ij} + \tau_{ij} = \begin{bmatrix} -P & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & -P & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & -P \end{bmatrix}$$

Here \( P \) is the static pressure, \( \tau_{ij} \) is the shear stress, and \( \delta_{ij} \) the Kronecker delta. The interest in this analysis lies in answering the question “how much power is dissipated by the impeller driven recirculation and prerotation?”. The answer lies in the stress tensor and an evaluation of the shear stresses the recirculation and prerotation must overcome. This was approximated by considering the shear stress where the fluid moves adjacent to a solid boundary, providing a sizable portion of the power dissipated which is not required without the recirculation and prerotation zones. The power further dissipated through fluid fluid interaction and mixing are neglected from this analyses, the other sizable portion of the power dissipation. The power required to move a lump of fluid adjacent to a solid boundary may be determined as the rate of change in the kinetic energy of the fluid when a solid boundary is introduced, written as,

$$L = \frac{dE_k}{dt}$$  \hspace{1cm} (6.2)

where \( E_k \) is the kinetic energy of the fluid being slowed by the boundary. If the fluid has a velocity \( c_i \) near the solid boundary then the kinetic energy equation [Panton 1984] may be written for a volumetric arbitrary region \( V \) enclosed by a surface \( S \) (the surface of the recirculation zone) by considering the combined continuity equation (C.1) and the Navier Stokes equations (C.3) written as,

$$\rho \frac{\partial c_i}{\partial t} + \rho c_j \frac{\partial c_i}{\partial x_j} + c_i \left[ \frac{\partial P}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho c_j \right) \right] = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_i} + \rho f_i,$$

where the last two terms on the left side are zero. Taking the dot product of \( c_i \) with this equation provides,
\[ \frac{\partial}{\partial t} (\rho c^2) + \frac{\partial}{\partial x_j} (\rho c_i c_j) = -c_i \frac{\partial P}{\partial x_i} + c_i \frac{\partial \tau_{ij}}{\partial x_i} + \rho c_i f_i . \]

Substituting the following differential chain rule results,
\[ c_i \frac{\partial P}{\partial x_i} = \frac{\partial}{\partial x_i} (c_i P) - P \frac{\partial c_i}{\partial x_i} \]
\[ c_i \frac{\partial \tau_{ij}}{\partial x_i} = \frac{\partial}{\partial x_i} (c_i \tau_{ij}) - \tau_{ij} \frac{\partial c_i}{\partial x_i} \]

and using Leibnitz's Theorem (C.5) provides after integration the kinetic energy equation,
\[ \frac{d}{dt} \int_V \frac{\rho c^2}{2} dV = \int_S \left( \rho n_i (c_i - w_i) c_i^2 + n_i (c_i P - \tau_{ij} c_j) \right) dS + \int_V \left( \rho f_i c_i + P \frac{\partial c_i}{\partial x_i} - \tau_{ij} \frac{\partial c_j}{\partial x_i} \right) dV \]

where \( n_i \) is the normal to the surface, and \( w_i \) is the velocity of the solid surface. For the case of incompressible flow in the boundary layer over a stationary surface with no body forces and no flow normal to the surface this reduces to,
\[ \frac{d}{dt} \frac{\rho c^2}{2} dV = -\int_V \tau_{ij} \frac{\partial c_j}{\partial x_i} dV \quad (6.3) \]

where \( dV \) is the volume of fluid effected by the shear stress in the boundary layer. The pressure term is zero from continuity of mass. The shear stress may be evaluated with,
\[ \tau_{ij} = \mu \left( \frac{\partial c_i}{\partial x_j} + \frac{\partial c_j}{\partial x_i} \right) . \quad (6.4) \]

The total power dissipated in a boundary layer may then be evaluated from equation (6.2) implementing (6.3) and (6.4).

For the power dissipation order of magnitude evaluation the recirculation and prerotation are considered to rotate over a portion of the blade surface \( (S_b) \), the impeller shroud surface \( (S_s) \), and the inlet suction pipe surface \( (S_p) \) as shown in Fig. 6.2.
The exact geometry of these surfaces may not be experimentally determined in the complex unsteady flow field but their range is known, thus the dissipation power will be evaluated in terms of a variable radius term \( R_v \) representing the minor radius of the assumed elliptical shaped impeller recirculation vortices. The eccentricity is taken as 0.95 for the oblong shape. The impeller recirculation is modeled only at the inlet because of its highly transient behavior at the impeller outlet. For the approximation the fluid in contact with surfaces \( S_b, S_s, \) and \( S_p \) is assumed to have a turbulent boundary layer of the flat plate nature with a laminar sublayer, written as,

\[
\delta(x) = 0.37 \left( \frac{\mu c_{\infty}}{\rho} \right)^{1/5} x^{4/5}, \quad c/c_{\infty} = \left( \frac{y}{\delta(x)} \right)^{1/7}
\]

\[
laminar sublayer: \quad \frac{c}{c_{\infty}} = \frac{3}{2} \left( \frac{y}{\delta} \right) + \left( \frac{y}{\delta} \right)^3 \frac{3}{2}
\]

where the transition conditions are standard [Schlichting 1974] between the turbulent and laminar sublayer.

The simplified problem is then reduced to finding the boundary layer thickness found from the 1/5 exponent \( \delta(x) \) relation. The shear stress equations in (6.4) may then be used with the boundary layer velocity distributions and equation (6.3) to determine the power dissipated.

Summing over the surfaces provides the total power dissipation \( (LT) \) in Fig. 6.4 as a function of the recirculation zones size. Individual surface contributions are also shown. Each of the surface contributions has nearly equal magnitude, increasing with \( R_v \) in a parabolic fashion. Impeller recirculation at the discontinuities in the characteristics experimentally has a size of 0.02m<\( R_v <0.025 \)m implying a power dissipation on the wall boundary layers of near 40W. This corresponds to near half of the 90W difference between the discontinuities magnitude in \( Q \Delta P_{2,1} \) and \( \Delta \), meaning the proposed hypothesis for power dissipation can be viewed as plausible. To obtain a better approximation the Navier Stokes equations can be
numerically solved for a stationary flow without any geometric simplifications but still requiring a turbulent boundary layer model.

Fig. 6.4 Results of power dissipation model within the boundary layers.

6.2 The Numerical Characteristics

In Fig. 6.5 the experimental pump and impeller \( \psi \) characteristics are shown. A zoom of the flow regime where the hysteresis loop was found in the \( \psi \) characteristics is shown to the right in Fig. 6.5. The numerically calculated impeller \( \psi \) characteristic is also shown in Fig. 6.5 for the fine and coarse grid case without any impeller leakage flow. The fine grid has been computed for fewer operating points and shows a definite upward shift from the coarse grid obtaining better agreement with the measured impeller characteristic.

Fig. 6.5 Experimental and numerical characteristics. Left pump and impeller \( \psi \) characteristics, the latter evaluated with the shown formula. Right a zoom of the \( \psi \) discontinuity hysteresis loop [Kaupert et al. 1996].

Outside the two \( \psi \) discontinuities on both the upper and lower branch of the hysteresis good numerical convergence was always obtained. At the discontinuity between the two \( \psi \) branches numerical convergence was not obtained. A typical plot of the residual maximum pressure and three velocity components at the discontinuity from the upper branch to the lower branch, Fig. 6.6, reveals the nonconvergence. This problem can be attributed to the code trying to numerically integrate the nonlinear constitutive equations having no mathematically stable solution, in agreement with the flow physics
also having no stable solution at the $\psi$ discontinuity in the unsteady flow.

With the missing double spiral volute in the numerics only part of the pump flow is numerically solved. However, important statements may be made concerning the agreement of the measured pump and impeller $\psi$ characteristics and the calculated coarse grid impeller characteristic. Firstly the numerical volume flux at which the impeller characteristic hysteresis loop has its stability limits, the $\psi$ discontinuities, are in excellent agreement with those measured for the pump. Secondly the magnitude of the $\psi$ increase/decrease at the impeller characteristics stability limits are in excellent agreement with those measured for the pump and impeller.

Another consideration is the mentioned magnitude agreement of the $\psi$ discontinuity, useful flow power transferred from the impeller. The numerics predicted the inlet recirculation and prerotation accurately (chapter 5) for the viscous three dimensional flow meaning the concept of these flow structures as dissipaters (section 6.1) of power is well predicted (i.e., the 340W).

![Fig. 6.6 Non convergence of four variables at lower $\psi$ discontinuity.](image)

![Fig. 6.7 The numerical impeller $\psi$ characteristic with leakage flow [Kaupert et al. 1996].](image)

### 6.3 The Characteristic's Hysteresis Loop

Both experimental and numerical hysteresis loops in Fig. 6.5 provide a non-unique $\psi$ vs. $\phi$ relationship in which nonlinear transition between two branches occurs [Mayergouz 1991]. This hysteresis $\psi$ vs. $\phi$ relationship may not be held indefinitely at all path states. Two stability limits exist where $\psi$ discontinuities occur. The location of the two stability limits and the dynamics of the $\psi$ discontinuities transition are system dependent (i.e., impeller geometry, inlet flow skewness, throttling rate). A further characterization of $\psi$ vs. $\phi$ hysteresis lies in directional dependence, the location of branch transition from an increasing or decreasing $\phi$ is influenced by the previous $\phi$ value meaning the hysteresis has local memory. In the experiments data on the branches were taken from steady measurements. Additional tests confirmed quasi-steady behavior of the hysteresis loop for slowly increasing and decreasing $\phi$ (section 6.5).
6.4 Impeller Shroud Leakage Flow

The measured leakage flow (section 4.6) was used as a boundary condition for the numerics to determine its influence on the impeller \( \psi \) characteristic hysteresis. The numerical re-entering leakage flow at an impeller inlet notch was given a circumferential velocity equal to half the impeller tip velocity.

The numerical impeller \( \psi \) characteristic with leakage flow calculated on a coarse grid is not significantly changed, Fig. 6.7. The numerical hysteresis loop shifts to a marginally greater \( \phi \) retaining however the magnitude of its \( \psi \) discontinuity. No change in the impeller outlet or inlet flow was found, the numerical data previously presented remains virtually unchanged. However, the numerical complexity and convergence times were greatly increased compared to the no leakage flow case. It should be noted that this result is not to be generalized as the literature [Eisele 1994] includes cases where the leakage flow has a significant impact on the numerical \( \psi \) characteristic.

6.5 Transient behavior of the Hysteresis Regime

Typically a pump will not remain at a constant operational point but rather be used in transient operation with respect to changing volume flux. The hysteresis flow regime, which usually lies above the 65% bep [Fraser 1982] thus undergoes transient behavior influencing the pump system.

![Fig 6.8 Left probe insertion point at impeller inlet. Right \( \psi \) characteristic of pump with point A (without recirculation) and B (with recirculation).](image)

To qualitatively evaluate the transient behavior of the hysteresis the fast response probe was inserted into the flow, Fig. 6.8, near the impeller leading edge tip. A link between the occurrence of impeller recirculation with prerotation and the \( \psi \) discontinuity in the hysteresis loop of the pump characteristic was sought. Fig. 6.9 shows two pressure signals acquired by the fast response probe while slowly decreasing (top left) and increasing (top right) \( \phi \) over 40 impeller rotations at 750 rpm. Below each signal are the computed Haar wavelet [Appendix B] transform magnitudes in the time frequency plane representation. The frequency in the time frequency plane is a log scale on the top half, linear scale on the bottom half. The corresponding
points in a static measurement of the pump characteristic are also shown in a zoom in Fig. 6.8. The observations for the spectra are summarized in Table 6.1,

<table>
<thead>
<tr>
<th>decreasing $\phi$</th>
<th>increasing $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>• stability limit at point A</td>
<td>• stability limit at point B</td>
</tr>
<tr>
<td>• above A, $f_{bp}$ dominant</td>
<td>• above B, $f_{bp}$ dominant</td>
</tr>
<tr>
<td>• below A, $f_{bp}$ is smeared, more stochastic signal content</td>
<td>• below B, $f_{bp}$ is smeared, more stochastic signal content</td>
</tr>
<tr>
<td>• below A, transient impeller recirculation and prerotation commence</td>
<td>• above B, transient impeller recirculation and prerotation cease</td>
</tr>
</tbody>
</table>

**Table 6.1 Observations of impeller recirculation transient behavior.**

The excellent $\phi$ localization provided by a Haar function mother wavelet establishes that $\phi_A \neq \phi_B$. The impeller recirculation with prerotation commences at exactly the location of $\psi$ discontinuity A and ceases at exactly the location of $\psi$ discontinuity B.

![Image of wavelet spectrum](image1)

**Fig 6.9 The transient probe signal in a wavelet spectrum for commencement and cessation of the inlet recirculation.**

The quantitative transient behavior of the pump $\psi$ characteristic for
increasing $\phi$ is revealed in Fig. 6.10, retaining the shape of the steady measured $\psi$ characteristic but with points measured along the $\psi$ discontinuity, not possible in the steady case because of the unstable pump system behavior. This is the path $\psi$ takes as flow reattachment occurs within the impeller and the recirculation with prerotation cease. The individual time history of $\psi$ and $\phi$ is also shown, Fig. 6.10, within the hysteresis flow regime. The constant $T^*$ is introduced as the hysteresis regime transition time, a constant for each test. The sudden transition (jump) in $\psi$ at the stability limit has been quantitatively evaluated with a best fit linear interpolation providing a slope of $d\psi/dt = 0.034 \text{ s}^{-1}$ when $d\phi/dt = 0.0025 \text{ s}^{-1}$. This evaluation determines the transient behavior of the useful power transferred to the fluid, proportional to $\phi \psi$, during the flow reattachment within the impeller. The behavior is similar, not shown, for the case of decreasing $\phi$ as the flow separates in the impeller.

\[ d(Q\Delta P) \frac{dt}{dt} = dL \frac{dt}{dt}. \]

Taking the derivative with the dimensionless coefficients and introducing $t^* = t/T^*$ provides,

\[ \frac{S_2 \rho}{2} \frac{u_2^3}{t^*} \left( \frac{d\phi}{dt^*} \psi + \frac{d\psi}{dt^*} \phi \right) = \frac{dL}{dt^*} \tag{6.5} \]

at constant rotational speed. This is a quantifying linear differential equation to describe a single aspect of the hysteresis in the $\psi$ transition regime. It appears contradictory since hysteresis is a nonlinear phenomena in general but for the single quantity hydraulic power it was found to be suitable.

For the case at hand throttling was performed at a constant rate meaning $d\phi/dt^* = K_1$ within the transition regime, a 0.97 linear correlation coefficient. The $\phi$ at which the $\psi$ transition begins is the stability limit $\phi = K_2$, also a
constant so that for a series of tests performed at various $K_1$ for increasing and decreasing $\phi$ within the $\psi$ transition regime equation 6.5 becomes,

$$\frac{d\psi}{dt^*} + \xi \psi = \frac{2}{S_2 \rho u_1^3 K_2} \frac{dL}{dt^*}$$

where $\xi = K_1/K_2 = (d\phi/dt*)/\phi$ and $K_2$ is taken to be constant over the transition regime (it varies less than 3%). The $t^*=0$ is defined as the start point of the $\psi$ transition regime. For the case $dL/dt$ constant (assumed here for simplicity to be 340 W/1.5 s from Fig. 6.1 and Fig. 6.10) this equation becomes linear and first order,

$$\frac{d\psi}{dt^*} + \xi \psi = K_3$$

and may be integrated to obtain,

$$\psi = (\psi_o - K_3) \exp(-\xi t^*) - \xi^{-1} [T^* \text{sgn}(\xi) \psi_o \exp(-\xi t^*) - K_3]$$

$$\frac{d\psi}{dt^*} = [T^* \text{sgn}(\xi) \psi_o - \xi \psi_o + K_3 \xi] \exp(-\xi t^*)$$

where $\psi_o$ is the steady value $\psi$ discontinuity magnitude for an infinitely slow change in $\phi$ (i.e., $d\phi/dt^*\approx 0$). For all tests performed $d\phi/dt^*<<\phi$ and the $\psi$ transition regime time satisfies the condition $T^*<2s$ meaning the exponential in equation (6.7) satisfies $t,t^*<<1$ and further for a given test $d\psi/dt^*$ depends primarily on $\xi$ and weakly on $t^*$. This predicts well the nearly constant slope for $\psi$ in the transition regime of Fig. 6.10 right.

![Fig 6.11 Results of the model from equation 6.7 and measurements on the pump within the $\psi$ transition regime at a $t=1.5$ s.](image)

The results from equation (6.7) revealing the behavior of the pump system during the $\psi$ transition regime are plotted in Fig. 6.11 for the case $t^*=1.5$ s (as mentioned the model is only weakly dependent on time). The model reveals that as $|d\phi/dt^*|$ increases the $|d\psi/dt^*|$ decreases. The results of
various tests are also shown in Fig. 6.11 revealing for small $|d\phi/dt^*| < 0.005$ the model and experiments are in good agreement. The experiments show a greater nonlinearity in the relation than found in equation (6.7) which becomes significant for larger $|d\phi/dt^*|$. Reason for the discrepancy lies in the assumed constant $dL/dt^*$, which implies that as the flow separates/reattaches within the impeller the pump system useful power is changing at a constant rate.

A parameter variation on the inhomogeneity of equation (6.6) for the case $dL/dt^* = dL/dt^* [ (d\phi/dt^*)^2, d\phi/dt^*]$ was performed to quantitatively obtain better agreement and permit physical interpretation, results are plotted in Fig. 6.11. The rate of change of useful flow power in the hysteresis transient regime during the commencement/cessation of recirculation and prerotation has a nonlinear dependence, exponent 2, on the throttling rate. This characterizes the lost useful hydraulic power due to recirculation and prerotation with a path dependency on $d\phi/dt^*$ (i.e., the throttling rate). The simplistic model of equation (6.6) thus provides insight into the $\psi$ transition regimes nonlinear $dL/dt^*$ with the useful result that quasi steady behavior of the $|d\psi/dt^*|$ can be expected for small $|d\phi/dt^*| < 0.005$.

6.6 Chapter Summary

The discontinuities in the pump $\psi$, $\lambda$, $\eta$ characteristics occur at the same $\phi$, resulting in a 3% change in $\eta$, to form a hysteresis loop. The concept of the impeller driven recirculation and prerotation as pump power dissipaters was hypothesized to explain the magnitude difference between the characteristic discontinuity in $\lambda$ and $\psi$. The hypothesis is deemed plausible after making an order of magnitude approximation of the power dissipated in the boundary layers of the recirculation and prerotation. The numerical predictions showing the same $\psi$ discontinuity magnitude reinforce this hypothesis as a more exact determination of the power dissipated is performed.

Agreement between numerical impeller and both experimental pump and impeller $\psi$ characteristics is excellent with respect to the two stability limits in $\phi$, the local memory for $\psi$ in the hysteresis loop, and the magnitude of the two $\psi$ discontinuities. The mathematics of the numerical flow model becomes unstable as the unsteady experimental flow becomes unstable at the $\psi$ discontinuities. Impeller outlet reverse flow is not a necessary condition for a pump impeller $\psi$ discontinuity in the numerical flow as this was shown (section 5.2) to be numerically not predicted. The inclusion of impeller leakage flow marginally effects the numerical $\psi$ characteristic but adds greatly to the numerical complexity.
The inlet recirculation was shown to commence/cease transiently at the exact $\phi$ values where the $\psi$ discontinuity exists, useful for the interpretation of the transient behavior of $\psi$.

The behavior of the pump $\psi$ discontinuity in transient operation was experimentally and theoretically evaluated. In particular $\psi$ was found not to undergo a discontinuity but rather had a defined $d\psi/dt$. This was related to the $d\phi/d\tau$ during increasing and decreasing $\phi$ tests. The theoretically development bases itself on the change in useful power the flow receives in the hysteresis flow regime, as the impeller flow is separating or reattaching. The linear model shows good agreement with experimental results for $|d\phi/d\tau*| < 0.005$, and predicts the decreasing $|d\psi/d\tau*|$ as $|d\phi/d\tau*|$ increases with quasi steady $|d\psi/d\tau*|$ behavior for small $|d\phi/d\tau*|$. An exponential relation was empirically determined for the rate of changing useful flow power as $dL/d\tau* = dL/d\tau* [(d\phi/d\tau*)^2, d\phi/d\tau*]$ over the entire $d\phi/d\tau*$ regime governing the physical mechanism of transient transferred hydraulic power.
7.0 Unsteady Blade Loading

The unsteady flow field in pumps is rarely taken into consideration during the design and development process [Arndt et al. 1989]. Selection of geometrical parameters (i.e., blade thickness, volute inlet diameter) are based on steady flow, often empirical relations (i.e., Pfleiderer 1991, Stepanoff 1959), which have historically proven effective. With the previously mentioned industrial construction trends (section 1.1) of larger power concentrations and greater transient load operation a necessity has arisen for a better understanding of the unsteady flow within the pump impeller and the unsteady blade loading [Tamatsukuri et al. 1992] it provokes causing, • mechanical vibrations which can be traced in the rotordynamics, • unsteady impeller operation influencing the pump efficiency, • possible cavitation damage [Arndt et al. 1990].

Several authors provide case studies [MaKay 1988, Stanmore 1988] wherein component defect or failure were directly related to the impeller unsteady blade loading.

Within this chapter the unsteady flow field in the rotating impeller is presented as it relates to unsteady blade loading. The importance of the well known circumferential pressure distribution in the volute at off design was quantified as the dominant factor of unsteady blade loading. A synthesis of the rotating blade pressure signals was performed to obtain unsteady blade loading parameters, the lift and moment coefficient. This provides an appropriate boundary condition for any numerical blade loading computations being performed.

7.1 Impeller Outlet Circumferential Pressure Distribution

The impeller and the double spiral volute according to classic theory form a matching of the angular momentum exchange to determine a best efficiency point (bep)$\phi$ (section 4.1). Any mismatch in this angular momentum exchange causes the flow near the impeller outlet in the volute to either be accelerated for $\phi > \phi_{bep}$ or decelerated for $\phi < \phi_{bep}$. The measured circumferential pressure distribution of the volute wall pressure taps at $r/R_2=1.05$ for four volume fluxes are presented in Fig. 7.1 with observations found in Table 7.1.
Fig. 7.1 The circumferential pressure distribution at four volume fluxes.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$\bar{C}_p$</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi &gt; \phi_{bep}$</td>
<td>$\bar{C}_p$ sunk between tongues, volute flow accelerated</td>
<td></td>
</tr>
<tr>
<td>$\phi = \phi_{bep}$</td>
<td>$\bar{C}_p$ variation small, volute flow relatively constant</td>
<td></td>
</tr>
<tr>
<td>$\phi &lt; \phi_{bep}$</td>
<td>$\bar{C}_p$ rose between tongues, volute flow decelerated</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1 Observations in the impeller outlet pressure distribution.

These results are in accordance with previously reported experimental results [Adkins & Brennen 1988, Wesche 1987] for single spiral volutes. A comparison of the values for $\bar{C}_p$ averaged over the circumference, Fig. 7.1 the dotted lines, reveal a rise as the volume flux is sunk. This is a manifestation of the pump impeller characteristic, the definition of $\bar{C}_p$ being in fact a local $\psi$ value at the measurement point. A summation scheme for the circumferential $\bar{C}_p$ distribution was used to evaluate the dimensionless pressure induced radial thrust on the impeller as,
F_r = \frac{F_o}{0.5 \rho u_2^2 \pi R_2 B_3}

|F_r| = \frac{1}{2\pi R_2} \left\{ \sum_{i=1}^{16} (C_{pis} + \overline{C}_{piH}) \Delta \theta_i \cos \theta_i \right\}^{2} + \left\{ \sum_{i=1}^{16} (C_{pis} + \overline{C}_{piH}) \Delta \theta_i \sin \theta_i \right\}^{2} \left( \sum_{i=1}^{16} (C_{pis} + \overline{C}_{piH}) \Delta \theta_i \cos \theta_i \right)^{-1/2},

\text{Arg}\{F_r\} = \tan \left( \sum_{i=1}^{16} \frac{(C_{pis} + \overline{C}_{piH}) \Delta \theta_i \sin \theta_i}{\sum_{i=1}^{16} (C_{pis} + \overline{C}_{piH}) \Delta \theta_i \cos \theta_i} \right),

where a surface was obtained by revolving a line at constant radius of length equal to the volute height at \( r_{tap} \) through the angle \( \Delta \theta_i = (\theta_{i+1} - \theta_{i-1})/2 \). The pressure acting on the surface is a linear interpolation of the hub \( (C_{pis}) \) and shroud \( (C_{piS}) \) side measured values, the subscript \( i \) refers to the \( i \)th pressure tap. The results of such a summation are the pressure induced radial thrust on the impeller. This pressure induced radial thrust is known to be the dominant contribution to the total induced radial thrust [Adkins & Brennen 1988, Brennen 1994] in single spiral volutes.

The results of the summation scheme are presented in Fig. 7.2, similar in form to other double spirals reported [Shiels 1995]. The point of minimum pressure induced radial thrust was found to be below the pump bep, at \( \phi = 0.146 \) (84%). A region of hysteresis exists in the \( |F_r| \) not previously reported to the authors knowledge. It corresponds to those \( \phi \) values at which the pump and impeller characteristic undergo hysteresis (section 4.1). This hysteresis is not within the experimental error in the evaluated points (±6%) and can be considered a manifestation of the impeller \( \psi \) hysteresis. No prominent change in the \( \overline{C}_p \) vs. \( \theta \) distribution was observed within the hysteresis flow regime.

7.2 Influence of the Volute on the Impeller Pressure Field

Interpretation of the unsteady pressure field within the impeller must be made keeping the results of Fig. 7.1 in mind. They represent the average pressure at a blade passage outlet as it passed a particular point in the volute.
The stationary frame steady pressure distribution was experienced as an unsteady pressure distribution in the impeller frame. Missing is the unsteady pressure field within the volute caused by the impeller rotation, this however would be mostly steady in the impeller frame. Fig. 7.3 reveals the phase averaged unsteady pressure coefficient for 7 transducers at 5 operating points over two rotations, observations are summarized in Table 7.2 where $f_{tp}$ is the tongue passing frequency. The positions bottom tongue and top tongue in Fig. 7.3 indicate the position where the tongue was radially aligned with the suction side hub corner of the blade passage.

| $\phi > \phi_{bep}$ | mild $f_{tp}$  
| | $C_p$ sinking between tongues, as $C_p$ in volute |
| $\phi = \phi_{bep}$ [0.174] | marginally present $f_{tp}$  
| | $C_p$ little change between tongues, as $C_p$ in volute |
| $\phi < \phi_{bep}$ | relatively strong $f_{tp}$, for small $\phi$ reaches 30% pump head  
| | $C_p$ rose between tongues, as $C_p$ in volute  
| | $f_{bp}$, weakly present |

Table 7.2 Observations for rotating frame unsteady pressure signals.

Other authors [Arndt 1988] have obtained similar results with even greater fluctuation magnitudes being reported.

Previous studies [Caignaert et al. 1991, Tourret et al. 1988] reported the unsteady pressure within a pump impeller grew in magnitude as the volume flux was further removed from the bep and as the trailing edge of the blade was approached. This has also been found to be the case in this study where Fig. 7.4 reveals the $C_p$ as 2 standard deviations of a pressure signal magnitude as a function of $\phi$ and position along the four impeller “wall streamlines”, observations are summarized in Table 7.3.

| $\phi > \phi_{bep}$ | $C_p$ magnitudes small |
| $\phi = \phi_{bep}$ [0.174] | $C_p$ magnitudes small |
| $\phi < 0.122 (70\%)$ hysteresis regime | $C_p$ magnitudes rose significantly as trailing edge approached, generally greater on the PS than SS |

Table 7.3 Observations in rotating frame unsteady pressure magnitudes.
Fig. 7.3 Phase averaged unsteady pressure for 7 transducers at 5 volume fluxes.
In Fig. 7.4 the hysteresis flow regime is recognized by a concentration of measuring points and connecting lines near $\phi \approx 0.12$, this regime was measured with particularly good resolution in $\phi$.

![Fig. 7.4 Unsteady pressure magnitude along 4 wall streamlines.](image)

The circumferential pressure distribution at the impeller outlet influenced the impeller blade passage in a periodic fashion dominant at the tongue passing frequency. Four pressure transducer locations were selected for supporting this statement, seen in Fig. 7.5. Shown are the harmonic magnitudes from a Fourier analysis where $n$ represents the pump shaft harmonics over 8 volume fluxes, $n=2$ is the tongue passing harmonic. It can be seen that $n=2$ was the dominant harmonic at these four pressure transducer positions, often a factor of near five greater than the shaft harmonic ($n=1$) and the blade passing harmonic ($n=7$) at most $\phi$. This is typical of all pressure transducer positions in the blade channel excluding a few positions near the impeller inlet for $\phi > 0.15$ where the shaft harmonic ($n=1$) can become the same size as the tongue passing harmonic ($n=2$) but both are then, as seen in Fig. 7.4, relatively small. This fact suggests interpretation of signal phase information at the dominant tongue passing harmonic is of prime interest.
Figure 7.6 reveals the phase information from an FFT for the tongue passing harmonic along the four wall streamlines within the impeller blade passage for 13 volume fluxes. The zero phase position occurred when the blade passage suction side trailing edge hub was radially aligned with the top tongue. Observations for the phase measurements are presented in Table 7.4.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \geq \phi_{bep}$</td>
<td>no clear trend but $\tilde{C}_p$ magnitudes small</td>
</tr>
<tr>
<td>$\phi &lt; \phi_{bep}$</td>
<td>PSH, PSS - first 3 transducers have no phase change - last 2 have phase lag from blade curvature SSH, SSS - first 2 transducers have phase lead, but $\tilde{C}_p$ magnitudes small - last 3 have no phase change</td>
</tr>
</tbody>
</table>

Table 7.4 Observations for the phase measurements in rotating system.

Interpretation of the phase is made by considering a mathematical solution to the planar wave equation [French 1971] for the $n^{th}$ harmonic within a homogeneous region containing no sources written as,

$$\tilde{C}_{pn}(s, t) = |\tilde{C}_{pn}| \sin(2\pi s/\lambda_n + \varphi) \cos(\omega_n t)$$

where

$$\tilde{C}_p(s, t) = \sum_n \tilde{C}_{pn}(s, t) ,$$
The phase information for the tongue passing (n=2) harmonic along the 4 wall streamlines.

The phase shift in distance is chosen because the pressure transducers are at different spatial locations concurrently in time, the \( \phi \) varies with position \( s \). Separating this equation to reveal the classic right (\( R \)) and left (\( L \)) moving waves provides,

\[
\tilde{C}_{pn}(s, t) = |\tilde{C}_{pnL}| \sin \left[ \frac{2\pi}{\lambda_n} (s - vt) + \phi \right] + |\tilde{C}_{pnR}| \sin \left[ \frac{2\pi}{\lambda_n} (s + vt) + \phi \right] 
\]

(7.1)

where the wave velocity \( v = \omega_n \lambda_n / 2\pi \). This is further reduced to the form,

\[
\tilde{C}_{pn}(s, t) = |\tilde{C}_{pnR,L}| \sin \left[ \pm 2\pi f_n t + 2\pi s / \lambda_n + \phi \right] .
\]

(7.2)

The terms are interpreted as,

1) negative, right moving pressure wave (increasing \( s \)) downstream in impeller fluid flow,
2) positive, left moving pressure wave (decreasing \( s \)) upstream in impeller fluid flow.

In this manner changes in the phase with position in both the upstream and downstream moving waves in equation (7.2) are interpreted as,

1) \( d\phi/ds > 0 \), upstream (decreasing \( s \)) moving wave,
2) \( d\phi/ds < 0 \), downstream (increasing \( s \)) moving wave.
External noise and wave dispersion has been neglected in the above phase considerations. They are evaluated in general [Stegen & Van Atta 1969] for two transducer signals $x$ and $y$ with the coherence and phase lead defined respectively as,

$$\eta^2_{xy}(f) = \frac{S_{xy}(f)S_{xy}^{*}(f)}{S_x(f)S_y(f)}$$

$$\varphi_{xy}(f) = -\tan^{-1}\left[\frac{Q_{xy}(f)}{C_{xy}(f)}\right]$$  \hspace{1cm} (7.3)

where $S$ represents the cross spectral density with $C_{xy}$ and $Q_{xy}$ as the co- and quadrature spectrum. The coherence is interpreted as a frequency dependent correlation coefficient. The time delay and wave velocity, as in equation (7.1), between two transducers may also be obtained using,

$$\Delta t = \frac{\varphi_{xy}(f)}{2\pi f}$$

$$v(f) = 2\pi f s / \varphi_{xy}(f).$$  \hspace{1cm} (7.4)

The relative phase in equation (7.3) provides the same results as the FFT results in Fig. 7.6 but with no fixed zero reference phase (i.e., relative phases are determined) and with the concept of coherence introduced. The coherence between all transducers in Fig. 7.6 was high (>0.93) at all volume fluxes $\phi < \phi_{bep}$ indicating little disturbance in the form of noise and nonlinearities, the wave was nondispersive.

The transducers undergoing small phase changes ($\Delta \varphi < 0.05$) are classified as experiencing an acoustic wave in the blade passage since the acoustic wave propagation speed in water was measured (section 8.1) to be 1326.0 ± 2% m/s meaning pressure transducers and blade loading are influenced simultaneously. Both the pump shaft harmonic ($n=1$) and twice the tongue harmonic ($n=4$) have also been analyzed to reveal similar, not shown here, phase relations confirming the nondispersive nature of wave. The wave group and phase velocity are both equal to the acoustic velocity. Near the trailing edge ($s/PSH=1$, $s/PSS=1$) however, $d\varphi/ds < 0$ exists because these transducers arrive at the tongue later due to impeller blade curvature; they experience the change in the pressure field across the tongue at a later time meaning a phase lag.

The implications of this acoustic pressure wave in the impeller can be realized from simple two dimensional classical aeroacoustics [Goldstein 1977]. Consider the starting transient vortex formation on an impeller blade impulsively accelerated from rest to some definite velocity, Fig 7.7a. The main flow shall be taken as inviscid, the boundary layer viscid. Kelvin’s theorem states for an inviscid incompressible flow the circulation about the blade and surrounding fluid must at all times remain constant, $d\Gamma/dt = 0$. In the transient acceleration the action of viscosity causes the formation of a thin boundary layer on the blade surface. Experimental observations [JSME
1988, Freymuth 1985] reveal high velocities at the blade trailing edge creating a low pressure region while at the rear stagnation point high pressure forms. This pressure gradient across the trailing edge causes separation and formation of a trailing edge vortex, Fig. 7.7b. This vortex induces (i.e., according to Kelvin’s theory) a circulatory flow about the blade which shifts the rear stagnation point to the trailing edge eliminating the trailing edge pressure gradient. The vortex formed then leaves the blade surface and is swept downstream, Fig 7.7c to infinity. This behavior is a description of the classic Kutta-Joukowski condition. The circulation about the blade $\Gamma$ is proportional to the blade lift also in the unsteady case [Theodorsen 1939].

In the case of the pump impeller blade passage the acoustic pressure wave travelling upstream from the impeller-volute interaction changes the pressure distribution instantaneously over the impeller blade suction side or pressure side which changes the blade lift. The instantaneous change in the blade lift requires a reaction from the circulation about the blade and thus the simplified case in Fig 7.7 occurs with a periodicity. A continuous trail of vorticity creating vorticity in the wake must be formed, strength satisfying the unsteady Kutta-Joukowski condition. Since the volute pressure dominates the unsteady blade pressures at the $n=2$ harmonic it is conceivable (it has never been documented) that a synchronization between vortex wake frequency and the $n=2$ harmonic would exist. The fact that the blade outlet pressure is unsteady further complicates matters beyond Fig. 7.7 regarding the formation of the vortex wake. The acoustic propagation upstream in the blade passage thus induces a changing circulatory flow about the impeller blade which in turn changes the circulation in the downstream flow. This is termed here a circulatory flow effect.

To further clarify the above explanation Fig. 7.8 depicts time domain phase averaged pressure signals in part load with a two dimensional sketch of the blade passage and tongue. The arrows at the bottom tongue and the top tongue represent times when the blade passage suction side hub was aligned with a tongue. Examining the interaction with the bottom tongue at,

- $t = 30$ ms, the suction side of the blade passage approached the bottom tongue, influenced by the pressure in the region High.
- $t = 34$ ms, the blade suction side hub passed the bottom tongue, came under
the influence of the region Low, the entire suction side was immediately influenced by the abrupt change in pressure at the blade passage outlet and reacts with change in circulation about the blade instantaneously. The blade passage pressure side however was now under the influence of pressure in region High and Low. The first three pressure transducers (18,19,22) near the pressure side inlet react to Low while last two transducers (15,23) near the trailing edge were still, due to blade curvature, in High’s influence.

\( t = 45 \text{ ms} \), as the blade passage continued to rotate the last two transducers (15,23) moved past the tongue coming into Low’s influence. This blade curvature effect is removed from the phase information of the last 2 pressure side transducers shown in Fig. 7.6 labelled “curvature correction” to demonstrate the geometrical phase lags.

![Graph](image)

**Fig. 7.8 Two wall streamlines time domain pressure signals and sketch of blade passage as it moves past a volute tongue in part load operation.**

### 7.3 Synthesis of the Impeller Pressure Field

The circumferentially varying pressure field within the volute causes an unsteady pressure field within the impeller blade passage and was the dominant cause of unsteady blade loading. To quantify this unsteady blade loading coefficients were evaluated where the lifting line of action was taken as the circumferential direction of rotation, Fig. 7.8. This evaluation involves a summation over the pressure and suction side using Haar function interpolation for each pressure transducer acting over a given surface. Errors were introduced into the evaluation from,
the surface resolution (i.e., only 25 pressure transducers exist within the blade passage), did not capture any local pressure gradient effects,
• relied on extrapolation to the wall regions,
• used the phase averaged pressures,
• a single blade passage, not a single blade was investigated. A shift of the blade passage suction side data by one blade passage backward was performed to remedy this for single blade loading.

Fig. 7.8 The unsteady blade loading coefficients defined geometrically.

The lift coefficient ($C_L$), moment coefficient ($C_{Mo}$), and the resulting center of pressure ($r_{cp}$) about the impeller axis, were evaluated according to,

\[
C_L(\phi, \theta) = \sum_{i=1}^{25} C_{pi}(\phi, \theta) A_i \sin \vartheta_i \cos \delta_i \left( \sum_{i=1}^{25} A_i \sin \vartheta_i \sin \delta_i \right)
\]

\[
C_{Mo}(\phi, \theta) = \sum_{i=1}^{25} C_{pi}(\phi, \theta) A_i r_i \sin \vartheta_i \cos \delta_i \right) \left( \frac{25}{R_2} \sum_{i=1}^{25} A_i \sin \vartheta_i \sin \delta_i \right)
\]

where $A_i$, $\vartheta_i$, and $\delta_i$, are geometrical parameters for the $i^{th}$ transducer. The impeller radius $R_2$ has been used as a characteristic length. The $r_{cp}$ is with respect to the impeller axis of rotation.

The unsteady value $\tilde{C}_L(\phi, \theta)$ is shown in nondimensional form using the steady $C_L(\phi)$ at a particular $\phi$ over one rotation of the impeller seen in Fig. 7.9 top. Values in the plot are shown for an impeller angle $\theta$ corresponding to the alignment of the blade suction side hub with the volute tongue. Errors were estimated to be $\pm 15\%$, observations are summarized in Table 7.5.
Fig. 7.9 The unsteady blade loading parameters for a single impeller blade representing the unsteady loading.
| \( \phi > \phi_{bep} \) | \( \overline{C}_L(\phi, \theta) \) sunk between tongues, as \( \overline{C}_p \) in volute |
| \( \phi = \phi_{bep} \) | \( \overline{C}_L(\phi, \theta) \) little change between tongues, as \( \overline{C}_p \) in volute |
| \( \phi < \phi_{bep} \) | \( \overline{C}_L(\phi, \theta) \) rose between tongues, as \( \overline{C}_p \) in volute, reaching maximum as suction side of previous blade aligned with volute tongue |

| Significant variation in deep part load, from 2 to -2 in 20° |

**Table 7.5 Observations of the unsteady blade lift coefficient.**

The unsteady value of \( \overline{C}_m(\phi, \theta) \) is also shown in a nondimensionalised form over one impeller rotation. A similar interpretation to that for the \( \overline{C}_L(\phi, \theta)/\overline{C}_L(\phi) \) can be applied with the \( \overline{C}_m(\phi, \theta)/\overline{C}_m(\phi) \) for a single blade undergoing a change from 0.6 to -0.4 during deep part load operation.

Of interest is also the variation of the blade center of pressure \( r_{cp}(\phi, \theta)/R_2 \). Between tongues the plot values remain relatively constant near a value of 1.7 except during the impeller blade-volute tongue interaction where it becomes comparatively large. In this region the value of \( C_L(\phi, \theta) \) takes on smaller values because of the large \( \overline{C}_L(\phi, \theta) \) and thus a large \( r_{cp}(\phi, \theta) \), particularly in part load. In the computation of unsteady blade loading often the results of a stationary code are taken as a percent magnitude fluctuation, (i.e., the entire blade surface fluctuates in phase). This implies a value of \( r_{cp}(\phi, \theta) \) independent of \( \theta \) shown by considering equations (7.4) with a percentage fluctuation \( \varepsilon(\theta) \) in the steady value (i.e., all pressures fluctuate in phase) at a particular volume flux.

\[
C_L(\phi, \theta) = \sum_{i=1}^{25} \left( \varepsilon(\theta) C_{pi}(\phi) + C_{pi}(\phi) \right) G_i / \left( \sum_{i=1}^{25} G_i \right)
\]

\[
C_{Mo}(\phi, \theta) = \sum_{i=1}^{25} \left( \varepsilon(\theta) C_{pi}(\phi) + C_{pi}(\phi) \right) G_i r_i / R_2 \left( \sum_{i=1}^{25} G_i \right)
\]

\[
r_{cp} = \left( \sum_{i=1}^{25} \left( \varepsilon(\theta) C_{pi}(\phi) + C_{pi}(\phi) \right) r_i G_i \right) / \left( \sum_{i=1}^{25} \left( \varepsilon(\theta) C_{pi}(\phi) + C_{pi}(\phi) \right) G_i \right),
\]

where \( G_i = A_i \sin \delta_i \cos \delta_i \). After some rearrangement this provides,
Unsteady Blade Loading

\[ \begin{align*}
    r_{cp} &= \frac{\varepsilon(\theta)\left(\sum_{i=1}^{25} \overline{C_{pi}}(\phi) r_i G_i\right)}{(1 + \varepsilon(\theta))\left(\sum_{i=1}^{25} \overline{C_{pi}}(\phi) G_i\right)} + \frac{\left(\sum_{i=1}^{25} \overline{C_{pi}}(\phi) r_i G_i\right)}{(1 + \varepsilon(\theta))\left(\sum_{i=1}^{25} \overline{C_{pi}}(\phi) G_i\right)} \\
    \text{or} \quad r_{cp} &= \left(\sum_{i=1}^{25} \overline{C_{pi}}(\phi) r_i G_i\right) / \left(\sum_{i=1}^{25} \overline{C_{pi}}(\phi) G_i\right),
\end{align*} \]

which is independent of the rotation angle \( \theta \).

The results presented in Fig. 7.9 at first glance seem to support such an assumption since the large fluctuations in \( r_{cp}(\phi, \theta) \) are found where the \( C_L(\phi, \theta) \) was small and could be taken as negligible. However, in regions near the jump in \( r_{cp}(\phi, \theta) \) where the \( C_L(\phi, \theta) \) is no longer small the value of \( r_{cp}(\phi, \theta)/R_2 \) can still be appreciable. The fluctuations in \( r_{cp}(\phi, \theta) \) stem from two sources,

1) the blade pressure side was influenced earlier by the change in pressure across the volute tongue than the blade suction side,
2) the previously shown pressure wave phase lag near the blade pressure side trailing edge, a direct result of the impeller blade passage curvature.

7.4 Rotordynamic Reaction

The unsteady pressure field in the rotating system causes a rotordynamic reaction in the form of mechanical vibrations on the pump shaft. In particular the unsteady moment coefficient on the impeller blade causes torsional fluctuations. To compare a shaft torsion transducer’s fluctuations with the impeller blade unsteady moment coefficient an impeller of 7 equal blades was assumed each blade having the same \( \overline{C}_M(\phi, \theta) \) with a 1/7 rotation shift in shaft angle \( \theta \). A superposition of 7 unsteady moment coefficients was performed to obtain a modeled impeller moment coefficient seen in Fig. 7.10 for the case \( \phi = 0.125 \) (72%) over one impeller rotation. Also shown are a comparison of the fluctuation magnitudes (2 STD and FFT magnitude at blade passing frequency) and the phase of the unsteady moment as a function of volume flux.

Agreement in the fluctuation magnitude and phase seems reasonable for \( \phi > 0.07 \) (40%), the phase for both fluctuations was triggered at the same pump shaft position from the optical angle encoder. A compensation for any shaft rotational damping was not made, shaft twist from the stiffness module (i.e., \( GI = O(10^5) \) Nm) was calculated at less than 0.1° over the shaft length. The
harmonic component at \( f_{bp} \) is evident. Notice that both shaft and modeled phases change by near \( \pi \) at the \( \phi_{bep} \) due to the changing form in the volute circumferential pressure distribution, Fig. 7.1, above and below \( \phi_{bep} \).

\[ \phi = 0.126 \text{ (72%)} \]

\[ \text{impeller modeled} - \text{shaft measured} \]

![Fig. 7.10 Comparison of model and shaft measured moment coefficient.](image)

Not only does this impeller modeled to shaft measured comparison verify the quality of the unsteady lift and moment coefficients determined on the impeller blade but exposes the link from pump hydrodynamics (impeller outlet pressure distribution Fig. 7.1) to rotodynamic torsional vibrations.

Using the same modeling means the steady torque evaluated from the steady blade pressures is compared with the steady torque measured from the shaft mounted torque transducer. The results shown in Fig. 7.11 reveal a tendencial agreement between the two with near \( \pm 15\% \) discrepancy as mentioned previously.

**Fig. 7.11 Steady torque comparison.**

7.5 Rotational Speed Dependence

The variation in \( \bar{C}_p \) distributions at \( r/R_2 = 1.05 \) is shown in Fig. 7.12 at 700 rpm and 600 rpm, 750 rpm is found in Fig. 7.1, revealing little rotational speed dependence at bep operation. The variation in \( \bar{C}_p(\phi, \theta) \) within the impeller blade passage at \( \phi = 0.174 \text{ (100%)} \) is seen in Fig. 7.13 to not depend
on rotational speed along the four "wall streamlines". This behavior was also verified at $\phi = 0.209$ (120%) and $\phi = 0.096$ (55%) to hold true, not shown. The pressure fluctuations are thus reasonably scalable with the square of the rotational speed, $u_2^2$.

**Fig. 7.12** Circumferential pressure distribution for 2 rotational speeds at bep.

**Fig. 7.13** At bep fluctuation magnitudes dependence on rotational speed.
7.6 Chapter Summary

The circumferential pressure variation within the volute of centrifugal pumps resulting from a mismatch of angular momentum exchange is well documented in the literature. This pressure variation was experienced in the rotating system as unsteady, occurring predominately at the tongue passing frequency.

The volute tongue acts as a boundary separating two distinctly different flow regimes providing a sharp circumferential pressure gradient responsible for an abrupt flow field fluctuation in the impeller. The magnitude of this flow field unsteadiness grew as the volume flux was further removed from the bep volume flux, particularly below the flow coefficient at which pump characteristic hysteresis occurred. In deep part load the pressure fluctuations near the impeller outlet grew in magnitude to 30% of the pump head. The pressure fluctuations propagated upstream through the blade passage as acoustic pressure waves changing the blade loading and blade circulation instantaneously excluding those locations where blade curvature provided a phase lag.

The unsteady blade loading coefficients provide an indicator of the strong unsteady flow field an impeller blade experiences and reveals the necessity to include these effects in any blade loading computations. The results provide an appropriate boundary condition for such numerical calculations. A comparison between impeller modeled and shaft measured unsteady torsional moment exposes a direct link between pump hydrodynamics and torsional rotordynamics while verifying the unsteady blade loading coefficients.

Rotational speed variation influences the unsteady blade loading pressures in a scalable fashion with $u_2^2$ like the pump head from classic similarity rules [Stepanoff 1959]. The steady pressure in the stationary frame is predominantly the unsteady pressure in the rotating frame.
8.0 Pump Acoustic Generation

Due to a fluid's low capability to absorb acoustic energy, distant fluid containing structures in pump systems which have low resonant frequencies (i.e., pipes, collection chambers) can be acoustically excited to the point of material failure [Schwartz & Nelson 1984, Telfer 1993, Bolleter 1993, Jaeger 1963]. Inside pumps acoustic resonance is less problematic [Brennen 1994] since low frequency acoustic waves are generally produced in fluids, and pump internal cavities are small (excluding pump crossovers). This is in direct contrast to the role played by resonant frequencies in gas turbines and compressors where concerns are internal [Cumptsy 1977] duct modes. Complicating matters in pump system components are higher resonant frequencies which may be excited (i.e., Jaeger demonstrated a 19th pipeline harmonic excitation) making prediction processes cumbersome.

The volute tongue of a radial pump is a known main source of acoustic pressure generation [Güllich & Bolleter 1992, Den Hartog 1929] greatly disturbing the unsteady impeller outlet flow field (cavitation is ignored as no cavitation was found in this pump). The radial positioning of the tongue and gap size has been the subject of experimental acoustic investigations, Fig. 8.1 [Leidel 1969], in centrifugal blowers and recently in centrifugal pumps [Caignaert & Descamps 1997]. The fluid-body interaction process between an impeller and volute tongue can be viewed as an unsteady flow blockage with vorticity interaction seen in Fig. 8.2. (the dotted lines represent negative vorticity and solid positive) [Chu et al. 1995] which induces further flow fluctuations, the acoustic field being part thereof.

![Fig. 8.1 Centrifugal blower blade passing acoustic power dependence on tongue gap [Leidel 1969].](image)

![Fig. 8.2 Vorticity contours near the tongue of a volute pump [Chu et al. 1995].](image)
This chapter focuses on quantification of unsteady forces on the volute tongue which generate an acoustic field. The acoustic generation from unsteady mass flow due to volute tongue blockage is known [Chantel et al 1995, Leist 1979] to be the main source of acoustic generation however the former is not insignificant. The quantification of the unsteady mass flow acoustic generation is performed here on a rough basis to permit comparison to the unsteady tongue forces acoustic generation. Further the so called rotor noise is also included in the comparison. The results are interpreted using measured acoustic pressure fluctuations at the pump outlet.

The presented theoretical and experimental analysis for the acoustic generation from unsteady tongue forces incorporates Powell's theory of vortex sound stating "vorticity induces the whole flow field, of which distant acoustic is an integral part" [Powell 1994, Powell 1964]. The theory of Curle and Lighthill [Curle 1955, Lighthill 1952] are more statistical in nature while that of Powell more physical, providing insight into the concept of vortex formation and interaction found within hydraulic turbomachinery. Measurements taken on the volute tongue to determine the unsteady force from the impeller outlet flow are applied as boundary conditions for the vortex induced sound. This methodology eliminates the need to assume an impacting wake profile (i.e., the common Silverstein [Silverstein 1939] wake), and a fluctuating lift force [Kemp & Sears 1955] on a volute tongue. Further the potential and viscous contributions found in the impeller outlet flow [Yuasa & Hinata 1979] need not be separately evaluated as in Simpson's [Simpson et al. 1967] benchmark work. The influence of acoustic loading on the acoustic generation mechanisms was not investigated although this is an important facet of the problem.

8.1 Acoustic Field

The acoustic field is defined as flow field fluctuations carried by an acoustic medium as a local compression. It manifests itself in local density fluctuations due to induced particle motion which may be linked to the pressure fluctuations in an isentropic fluid as,

\[ a_o^2 = \left( \frac{\partial P}{\partial \rho} \right)_s. \]  

(8.1)

Experimentally this value was determined in the pump outlet pipe, using the phase velocity measurement technique [Bollette 1981, Margolis & Brown 1976], to obtain \( a_o = 1326.0 \pm 2\% \) m/s. While the acoustic field stems from changes in fluid density the convective field can be considered hydrodynamic [Lamb 1945], meaning it is incompressible. Important is the concept that acoustic energy need not be conserved in a flow process,
generation, absorption and reflection being complicated interactive phenomena. This that implies information about the acoustic field outside a source region does not reveal the nature of a source distribution. Classic acoustical theories allow for unsteady mass flow (monopole), unsteady fluid forces (dipole), and unsteady fluid shear stresses (quadrupole) as sources for a generated far field acoustic (section 8.4).

8.2 The Physical Situation

Fig. 8.3 details the pump situation to be mathematically modeled. The identifiable acoustic pressure generators resulting from fluid-solid body interaction are:

1) Monopole - impeller blade passage unsteady blockage caused by the volute tongue, approximated on smallest 2D cross section area between tongue and impeller based on experience [Leist 1979].

2) Dipole - unsteady force distribution on the tongue caused by the impeller outlet unsteady flow impingement, locations and phases schematically based on volute tongue measurements of the unsteady pressure (section 8.6.2) representing unsteady fluid forces.

3) Dipole - moving impeller blades create rotor (Gutin) noise [Gutin 1936], locations and phases schematically based on rotating frame steady pressure distributions, representing unsteady forces in the stationary frame.

4) Quadrupole - impeller blade trailing edge mixing ≈ 0,

5) Quadrupole - boundary layer flow over any nearby surfaces ≈ 0.

Both 4 and 5 are negligible in comparison to the acoustic generators of 1, 2, and 3 at the pump shaft harmonics. This is based on the low flow Mach number implying quadrupoles are comparatively small to monopole and dipole effects. They can become pertinent when dealing with broad band acoustics or the production of aeolean tones not handled here.
Acoustic generators: Monopole(+) , Dipole (+ -), Quadrupole
Monopole represents unsteady displacement thickness blockage (1)
Dipoles represent the unsteady fluid forces from rigid bodies (2,3)
Quadrupole =0, $M<<1$

Fig. 8.3 Unsteady acoustic generators in the pump, a function of impeller rotational position in the stationary frame.

8.3 Impeller Outlet Unsteady Flow Quantification
8.3.1 Impeller Outlet Unsteady Flow Formation

The wake formation behind an impeller blade provides an unsteady flow zone which moves downstream to impart unsteady forces on the volute tongue. Near the blade trailing edge ($r/R_2=1.05$) in the stationary system the outlet flow was quantified using phase averaged fast response probe results, the radial velocity and absolute flow angle shown in Fig. 8.4 and Fig. 8.5 over 1.5 blade passages at $\phi = 0.209(120\%)$ and $\phi = 0.174(100\%)$. The radial velocity dents localize the trailing edge wake. The absolute flow angle reveals the strong unsteady flow nature near the blade trailing edge. Similar unsteady flow field fluctuations at the outlet of impellers are found in more detailed analysis [Gyarmathy et al. 1991]. These newly formed unsteady flow structures travel downstream and evolve in geometric shape.
8.3.2 Impeller Outlet Unsteady Flow Evolution

As the impeller outlet flow travels downstream its structure decays in strength and changes in form. Presented in Fig. 8.6 are the phase averaged unsteady total pressures behind the impeller at mid passage height for $\phi = 0.174$ (100%). These measurements assume an outlet flow circumferential symmetry in the wake formation and evolution permitting the phase averaging. There are 10 radial positions shown over one rotation of the impeller. The distinct phenomena of a viscous region and a circulation induced peak [Yuasa & Hinata 1979] are identified. The circulation peak propagates a pressure disturbance at acoustic velocity decaying with increased radial distance. This peak is due to the rotation of the blades past a stationary observer. The blade loading dictates a blade circulation (section 7.2) which when moved induces an unsteady flow field at acoustic velocity according to equation C.8,
\[ \mathbf{c}(\mathbf{x}) = -\frac{1}{4\pi} \oint_{\gamma} \frac{\mathbf{x} \times d\mathbf{l}}{r^3} \]

where \(d\mathbf{l}\) is an incremental length along any given vortex line. This equation is the hydrodynamic equivalent of the classical Biot-Savart law. It reveals how an unsteady flow field may be induced, as measured in Fig. 8.6, from a moving circulation (i.e., \(r\) varies). Important to recall is the purely kinematic nature of this relation meaning it is valid in inviscid and viscous flows. In the far field the circulation peak is registered as rotor acoustic (Gutin noise).

The more dominant effect in the near field is viscous from the trailing edge wake. The viscous region having an outlet flow angle between 30° to 80°, \(\beta_2\) blade outlet is 30°, becomes wider and decreases with amplitude as the radius increases, dissipating more slowly initially than the circulation peak.

\(\beta = 80° \quad \beta = 30° \quad r/R_2 = 1.290 \quad 1.259 \quad 1.228 \quad 1.197 \quad 1.167 \quad 1.135 \quad 1.105 \quad 1.074 \quad 1.043 \quad 1.012 \)

\(\phi = 0.174 \text{ (100%)} \quad z/B_3 = 0.55 \quad \beta_2 = 30° \)

**Fig. 8.6** Flow evolution downstream near impeller outlet midheight. Viscous and circulation induced unsteady total pressure are quantified.

### 8.3.3 Unsteady Flow Impingement on Volute Tongue

The impingement of the impeller outlet unsteady flow on the volute tongue surface creates an unsteady pressure distribution. This is presented in two dimensional fashion along the middle row of pressure transducers at four distinct impeller angular positions covering one blade passage, Fig. 8.7, and continuously over the whole blade passage, Fig. 8.8, at \(\phi = 0.174 \text{ (100%)}.\) In Fig. 8.8 a nearest neighbour interpolation was implemented between transducers. Observations are summarized in Table 8.1. The unsteady flow from the impeller at this larger radial distance may be interpreted as divisible into the classic jet-wake structure [Brennen 1994].
Table 8.1 Impeller outlet flow impingement on volute tongue observations.

<table>
<thead>
<tr>
<th>$\theta &lt; 24^\circ$</th>
<th>jet flow region $s/L_3 &gt; 0.6$, high pressure region indicating flow impingement.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta &gt; 24^\circ$</td>
<td>abrupt change in pressure distribution in $\theta$ direction, transition from classical jet to wake flow impingement.</td>
</tr>
<tr>
<td>$25^\circ &lt; \theta &lt; 40^\circ$</td>
<td>wake flow region $s/L_3 \geq 0.5$, wake strikes tongue (c large in wake).</td>
</tr>
</tbody>
</table>

Fig 8.7 Unsteady pressure for discrete impeller positions on the center line of the volute tongue (1 blade passage is near $52^\circ$).

Fig 8.8 Unsteady pressure distribution along the volute tongue midline as a function of shaft position over one blade passage.

8.4 Wave Equation

The wave equation is developed for acoustic field generation in a manner similar to Lighthill’s [Lighthill 1952] work but in a form providing physical appreciation for acoustic source modeling terms. Starting point is the continuity equation C.1 and the Navier Stokes equations C.3 written as,
\[
\frac{D\rho}{Dt} + \rho \frac{\partial c_i}{\partial x_j} = 0 \quad (8.2)
\]
\[
\rho \frac{Dc_i}{Dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + F_i \quad (8.3)
\]
which contains the mass flow, specific body force, and shear stress. Multiplying equation (8.2) by \(c_i\) and adding to equation (8.3) provides after differentiation with respect to \(x_i\),
\[
\frac{\partial}{\partial x_i} \left( \frac{\partial}{\partial t} \rho c_i \right) = -\frac{\partial^2}{\partial x_i \partial x_j} (\rho c_i c_j) - \frac{\partial^2 P}{\partial x_i^2} + \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j} + \frac{\partial (F_i)}{\partial x_i} . \quad (8.4)
\]
Now differentiating equation (8.2) with respect to \(t\) and subtracting away equation (8.4) provides,
\[
\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2 P}{\partial x_i^2} = \frac{\partial}{\partial t} \left( \frac{\partial (\rho c_i)}{\partial x_i} \right) + \frac{\partial^2}{\partial x_i \partial x_j} (\rho c_i c_j) - \frac{\partial}{\partial x_i} (F_i) - \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j} \quad (8.5)
\]
Taking equation (8.1) in a linearized form (i.e., \(P_a = a_o^2 \rho \) for small perturbations) and treating the fluid as inviscid (i.e., \(\tau_{ij} = 0\)) provides the final form wave equation,
\[
\frac{1}{a_o^2} \frac{\partial^2 P_a}{\partial t^2} - \nabla^2 P_a = \frac{\partial \dot{m}}{\partial t} + \frac{\partial F_i}{\partial x_i} + \frac{\partial^2}{\partial x_i \partial x_j} (\rho c_i c_j) \quad (8.6)
\]
where \(\dot{m}\) is mass flux per volume and \(F_i\) the body forces. The pressure \(P = P_o + P_a\) where \(P_o\) is the ambient far field pressure which when differentiated becomes zero, allowing the appearance of only \(P_a\) in equation (8.6). The source terms of the right hand side of equation (8.6) are interpreted as:
1) Monopole - Unsteady mass flux in flow (unsteady flow blockage).
2) Dipole - Unsteady force acting externally on flow (structure interaction).
3) Quadrupole - Unsteady shear force in the flow (free jet turbulence).
These three unsteady sources generate the acoustic pressure for any disturbance within the flow or on a flow surface boundary.

### 8.5 General Acoustic Assumptions
The linearized acoustic equation (8.1) \([\text{Goldstein 1977}]\) was implemented to arrive at equation (8.6) meaning an adiabatic process occurs. This further implies convective velocities are assumed much smaller than acoustic velocities (i.e., \(c << a_o\)).

Care is taken to use the general far field assumption adapted to Powell’s theory of vortex sound. The far field assumption (i.e., pressure and velocity
are in phase for a time harmonic field) is satisfied if a region of space is far enough away from the sources and interacting objects (relative to both wavelength and source size) to ensure pressure fluctuations are of the form

\[ P_a(x, t) = \frac{1}{4\pi r} q \left( t - \frac{r}{a_o}, \theta, z \right) \]

where \( q \) is a volume source distribution. For Powell’s theory the far field assumption is satisfied [Powell 1964] if the observation point is far off with respect to a vortex ring size.

It is further assumed the volute tongues and impeller blades under consideration in the flow are rigid, meaning no velocity exists perpendicular to the surface. While it is known that flow induced forces on bodies generate vibration which can induce/damp the acoustic field [Chu et al. 1995], this effect is neglected.

8.6 Impeller-Volute Tongue Interaction
8.6.1 Tongue Blockage - Monopole Approximation

The volute tongue provides unsteady blockage of the impeller outlet flow as a blade passage rotates. This creates an unsteady mass flux near the volute tongue causing other blade passages to experience unsteady mass flux [Loret & Gopalakrishnan 1986] (the non uniform pressure distribution at the impeller outlet in Fig. 7.4 also induces unsteady mass flux but in the rotating frame).

A situation with an unsteady mass flux near the tongue corresponds to a monopole acoustic generator found in equation (8.6). This unsteady mass flow is approximated using the area \( A_T \) [Yeow 1966], Fig 8.3, and the probe measurements at the impeller outlet, Fig. 8.4 & 8.5. The tangential velocity was mass averaged, using the mass flux, over \( A_T \). This method has previously been used [Leist 1979, Yeow 1966] for approximating the monopole nature of the tongue in centrifugal blowers. It is used here for an approximation of the monopole acoustic generator to permit comparison to dipole sources. Detailed evaluations of this acoustic generator have recently been performed numerically [Chatel et al. 1995]. The averaged tangential velocity fluctuation was used in conjunction with a simple spherical monopole source written as,

\[ P_a(x, t) = -\frac{i}{4\pi r} \left( \frac{d}{dt} \dot{m}(t) \right) \exp[ik_o(r - R_2) - i\omega t] \] (8.7)

where \( \dot{m}(t) = \rho c_u(t) A_T \).

The \( c_u(t) \) was replaced with the area averaged harmonic series,
\[ c_u = \overline{c_u} + \sum_{n} |c_{v,n}| \sin(2\pi f_n t + \phi_n) \]

for calculation of the discrete shaft passing frequency harmonics to arrive at the approximate results in Fig 8.9 labeled "monopole approx.", shown once as a \( \overline{C_p} \) value and once on a logarithmic scale. The reference pressure is 1 Pa. For the dependence on the distance \( r \) a pressure transducer was mounted 0.1m from the volute tongue in the volute. A transfer function was measured between this near volute tongue transducer and the pump outlet, 1m apart. In equation (8.7) \( r = 0.1m \) was used coupled with a transfer function to the pump outlet. The transfer function was found to well relate to acoustic wave propagation. For \( \phi = 0.174 \) (100\%) at blade passing frequency the transfer function revealed a phase change of -0.4 which at acoustic velocity is the 1m distance. The magnitude was 0.8 with a coherence of 0.96. This behavior was similar for other operating points and permits the interpretation that most of the acoustic energy downstream of the impeller was collected and transmitted down the discharge pipe.

**Fig. 8.9** Predicted and measured acoustic magnitude shown linearly and logarithmically as a function of volume flux at blade passing frequency. Measured and calculated acoustic are located 1 m from the pump outlet.

**Fig. 8.10** Predicted and measured acoustic magnitudes shown linearly and logarithmically as a function of shaft harmonic.
The acoustic fluctuations were measured using standard techniques [Bolletier et al. 1973] with three pressure transducers. These results for the blade passing frequency at the pump outlet are shown labeled “measured” and also an evaluation of the global hydraulic acoustic formula,

\[ L_p = -7.1 + 20\log[(\rho HQ)/(\omega_s \omega B_2^2 B_2)] \]

where \( L_p \) represents the RMS acoustic pressure level at blade passing frequency for the pump outlet [Jaremczak & Caignaert 1993, Pempie & Metail 1985]. Fig 8.10 details the results as a function of the pump shaft harmonics.

8.6.2 Flow Impact on Volute Tongue - Dipole

According to Powell’s [Powell 1964] theory of vortex sound the acoustic pressure field described by equation (8.6) for a body in a flow may be evaluated as,

\[
P_a(x, t) = \frac{1}{4\pi} \left[ \int \rho(\xi \times e) x_i \frac{dV(y, t-r/a_o)}{r^2} + \frac{1}{a_o^2} \left( \mathcal{N} \mathcal{S}^2 \mathcal{N} \mathcal{S} \mathcal{N} \mathcal{S} \right) \right] \]

where \( S \) is the body surface and \( V \) the fluid volume in which the surface rests written in acoustic retardation variables. Only first order terms have been included in Powell’s work. This equation is analogous to that derived by Curle [Curle 1955] but with exposure of vorticity changes generating the acoustic pressure field.

\[
P_a(x, t) = \frac{1}{4\pi} \left[ \int \rho(\xi \times e) x_i \frac{dV(y, t-r/a_o)}{r^2} + \frac{1}{a_o^2} \left( \mathcal{N} \mathcal{S}^2 \mathcal{N} \mathcal{S} \mathcal{N} \mathcal{S} \right) \right] \]

\[
+ \frac{1}{a_o^2} \int_S \left( \rho \mathcal{S} \mathcal{S} \mathcal{S} \mathcal{S} \right) \frac{dS(y, t-r/a_o)}{r^2} x_i - \frac{1}{a_o^2} \int_S \rho c \frac{dS(y, t-r/a_o)}{r^2} \]  (8.8)

where \( S \) is the body surface and \( V \) the fluid volume in which the surface rests written in acoustic retardation variables. Only first order terms have been included in Powell’s work. This equation is analogous to that derived by Curle [Curle 1955] but with exposure of vorticity changes generating the acoustic pressure field.

\[
P_a(x, t) = \frac{1}{4\pi} \left[ \int \rho(\xi \times e) x_i \frac{dV(y, t-r/a_o)}{r^2} + \frac{1}{a_o^2} \left( \mathcal{N} \mathcal{S}^2 \mathcal{N} \mathcal{S} \mathcal{N} \mathcal{S} \right) \right] \]

\[
+ \frac{1}{a_o^2} \int_S \left( \rho \mathcal{S} \mathcal{S} \mathcal{S} \mathcal{S} \right) \frac{dS(y, t-r/a_o)}{r^2} x_i - \frac{1}{a_o^2} \int_S \rho c \frac{dS(y, t-r/a_o)}{r^2} \]  (8.8)

where \( S \) is the body surface and \( V \) the fluid volume in which the surface rests written in acoustic retardation variables. Only first order terms have been included in Powell’s work. This equation is analogous to that derived by Curle [Curle 1955] but with exposure of vorticity changes generating the acoustic pressure field.

\[
P_a(x, t) = \frac{1}{4\pi} \left[ \int \rho(\xi \times e) x_i \frac{dV(y, t-r/a_o)}{r^2} + \frac{1}{a_o^2} \left( \mathcal{N} \mathcal{S}^2 \mathcal{N} \mathcal{S} \mathcal{N} \mathcal{S} \right) \right] \]

\[
+ \frac{1}{a_o^2} \int_S \left( \rho \mathcal{S} \mathcal{S} \mathcal{S} \mathcal{S} \right) \frac{dS(y, t-r/a_o)}{r^2} x_i - \frac{1}{a_o^2} \int_S \rho c \frac{dS(y, t-r/a_o)}{r^2} \]  (8.8)

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\[
P_a(x, t) = \frac{1}{4\pi} \left[ \int \rho(\xi \times e) x_i \frac{dV(y, t-r/a_o)}{r^2} + \frac{1}{a_o^2} \left( \mathcal{N} \mathcal{S}^2 \mathcal{N} \mathcal{S} \mathcal{N} \mathcal{S} \right) \right] \]

\[
+ \frac{1}{a_o^2} \int_S \left( \rho \mathcal{S} \mathcal{S} \mathcal{S} \mathcal{S} \right) \frac{dS(y, t-r/a_o)}{r^2} x_i - \frac{1}{a_o^2} \int_S \rho c \frac{dS(y, t-r/a_o)}{r^2} \]  (8.8)

where \( S \) is the body surface and \( V \) the fluid volume in which the surface rests written in acoustic retardation variables. Only first order terms have been included in Powell’s work. This equation is analogous to that derived by Curle [Curle 1955] but with exposure of vorticity changes generating the acoustic pressure field.

8.6.2 Flow Impact on Volute Tongue - Dipole

According to Powell’s [Powell 1964] theory of vortex sound the acoustic pressure field described by equation (8.6) for a body in a flow may be evaluated as,

\[
P_a(x, t) = \frac{1}{4\pi} \left[ \int \rho(\xi \times e) x_i \frac{dV(y, t-r/a_o)}{r^2} + \frac{1}{a_o^2} \left( \mathcal{N} \mathcal{S}^2 \mathcal{N} \mathcal{S} \mathcal{N} \mathcal{S} \right) \right] \]

\[
+ \frac{1}{a_o^2} \int_S \left( \rho \mathcal{S} \mathcal{S} \mathcal{S} \mathcal{S} \right) \frac{dS(y, t-r/a_o)}{r^2} x_i - \frac{1}{a_o^2} \int_S \rho c \frac{dS(y, t-r/a_o)}{r^2} \]  (8.8)

where \( S \) is the body surface and \( V \) the fluid volume in which the surface rests written in acoustic retardation variables. Only first order terms have been included in Powell’s work. This equation is analogous to that derived by Curle [Curle 1955] but with exposure of vorticity changes generating the acoustic pressure field.

The four terms in equation (8.8) are interpreted as,

1) A volume distribution of dipoles within the flow near the tongue whose strength is proportional to the vorticity motion, often coupled with the concept of a dipole sheet in the flow, Fig. 8.11. This is essentially the key to the vortex theory of sound, the dipole strength is the same as the vortex strength integrated over the area [Powell 1994]. This can then be coupled
to the momentum transferred to the fluid by $\mathbf{M} = \rho \mathbf{D} = \rho \mathbf{TS}$.

2) A volume distribution of quadrupole sources, evaluated by approximations for $c$ from measurements and found to be negligibly small.

3) A surface distribution of dipoles over the volute tongue, strength being related to the total pressure. The pressure term here includes both the acoustic and hydrodynamic pressure.

4) A distribution of surface monopoles for modeling the volute tongues vibration within the flow.

Equation (8.7) may be further simplified with the assumption of small flow Mach numbers and the rigid volute tongue, terms 2 and 4 become negligible. The remaining equation is,

$$P_a(x, t) = \frac{1}{4\pi} \left[ \frac{1}{a_o \partial_r} \int \int \int_{V} \frac{\rho(\xi \times c) x_i}{r^2} dV(y, t - r/a_o) \right] + \frac{1}{a_o \partial_r} \int \int_{S} \left[ (p + \frac{\rho c^2}{2}) \frac{dS(y, t - r/a_o) x_i}{r^2} \right]$$

(8.9)

Solving equation (8.9) requires the unsteady total pressure and the unsteady vorticity for each impeller blade-volute tongue interaction. It should be mentioned here that the unsteady total pressure which creates unsteady forces on the volute tongue is the cause of acoustic pressure generation but not directly then, as indicated by Powell, the fluctuating force itself cannot possibly generate acoustic energy, rather the eddies and changing vorticity as a result of this action generate the acoustic pressure.

A simpler form of equation (8.9) is required for evaluation from measurements performed. Unfortunately the vorticity revealed in equation (8.9) becomes hidden but its importance is tantamount. The vorticity equation (C.4) may be substituted in equation 8.9. A form of the divergence theorem derived for retardation variables, equation (C.7), is then applied to the volumetric total pressure term resulting from the combined equation (8.9) and (C.4). After simplification of equation (8.9) with (C.4) and (C.7) the surface integrals cancel to leave,

$$P_a(x, t) = \frac{1}{4\pi} \left[ \frac{1}{a_o \partial_r} \int \int \int_{V} \left( \frac{\partial c_i}{\partial t} x_i \right) \frac{dV(y, t - r/a_o)}{r^2} \right] + \frac{1}{a_o \partial_r} \int \int_{S} \left[ (p + \frac{\rho c^2}{2}) \frac{dV(y, t - r/a_o)}{r^2} \right]$$

where the second term is seen to be of second order, a quadrupole effect, and will be neglected. This leaves then,

$$P_a(x, t) = -\frac{1}{4\pi} \left[ \frac{1}{a_o \partial_r} \int \int \int_{V} \frac{\partial c_i}{\partial t} x_i \frac{dV(y, t - r/a_o)}{r^2} \right][1 + O(M)]$$

This is interpreted as the impulse applied by the fluid on the surface and may be written in terms of the force as
where $F_i$ are the forces imparted on the fluid by the solid surface. This reveals the necessity for force measurements on the volute tongue. The equivalent form to equation (8.12) can be derived from equation (8.6) but the vorticity which sticks within equation (8.12) then remains hidden.

For the evaluation of equation (8.12) magnitude spectra of an area averaged pressure coefficient $\tilde{C}_{pt}$ from a summation over the tongue surface pressure transducers is provided in Fig. 8.12 at 5 volume fluxes. The value of $\tilde{C}_{pt}$ is interpreted as the dimensionless unsteady pressure per unit area.

\begin{align*}
P_a(x, t) &= -\frac{1}{4\pi a_o r^2} \frac{x_i \partial F_i(t - r/a_o)}{\partial t} [1 + O(M)] \quad (8.12)
\end{align*}

Fig. 8.12 Magnitude spectra of the surface pressure summation acting on the volute tongue.
The results from equation (8.12) at blade passing frequency are shown as a function volume flux in Fig. 8.9 labeled "dipole". The same transfer function for the acoustic emission was used as in the monopole evaluation. The results as a function of pump harmonic are seen in Fig. 8.10. This dipole contribution is seen to be smaller than the monopole but still significant.

8.7 Rotor Acoustic - Gutin Noise

In the rotating impeller reference frame the blade forces a rotor produces may be considered steady however, relative to the acoustic medium outside the impeller these forces are periodic and generate an acoustic field, commonly termed Gutin noise. Any machine implementing rotating blades produces this noise which stems from the circulation around the blades required for the blade lift. Quantification is made with the approximation of Blake [Blake 1986] found by the simple kinematics of the blade interaction with an acoustic medium. In Blake's approach when the forces are considered concentrated acting on the blades the following expression for the acoustic pressure is obtained,

$$ P_a(x, t) = \frac{\cos \gamma}{4\pi r} \sum_{n = -\infty}^{\infty} (-i)^{n+1} k_o F(\Omega) e^{-i(\Omega + n\omega)t} e^{i(k_o r + n\theta)} J_n(k_o R_2 \sin \gamma) $$

where the off shaft angle alignment \(\gamma\) to the observer is seen to be pertinent, the axis of rotation is \(0^\circ\). At the impeller outlet this angle is near \(90^\circ\) implying the rotor acoustic generation is small in agreement with recent literature [Chatel et al. 1993] where more detailed quantification of the Gutin noise has been made. The results from the above equation are plotted in Fig. 8.9. This quantification is verified by the magnitude of the circulation peak in Fig. 8.6 which at larger radius is of minor strength. The same transfer function used for both the monopole and dipole sources is applied to the rotor acoustic.

8.8 Chapter Summary

The impeller outlet flow revealed an unsteady nature through its radial velocity and absolute flow angle fluctuations. Within this outlet flow a viscous region and circulation induced peak were identified as physical mechanisms for flow field fluctuations. Impingement of the impeller outlet flow on the volute tongue provided unsteady surface forces on the tongue and unsteady blockage of the blade passage. These two factors are dominant for acoustic generation.

It is often suggested that the volute tongue is the main culprit responsible for acoustic generation. The mathematical coupling to the unsteady blockage of the impeller outlet flow and unsteady vorticity
interaction provided reveals this fundamentally. The physical mechanisms for dominant acoustic generators were evaluated as monopole for the unsteady fluid mass and dipole for the unsteady fluid forces.

The synthesis of the results reveals some agreement to the measured acoustic field at the pump outlet for the blade passing frequency. The monopole is, as documented [Chantel et al 1995], of greatest importance and was only approximated. The dipole generator from unsteady volute tongue forces are shown not to be negligible, approximately 40% the magnitude of the monopole generator. This is in contrast to centrifugal blowers [Leist 1979] where monopole generators are considered dominant. At the other pump shaft harmonics agreement of the model is rather poor due to the simplicity of the model. The influence of acoustic loading on these generation sources for example has not been taken into account. The results provided here could serve as the bases for a new study investigating the reaction of acoustic generation to acoustic loading.
9.0 Conclusions

This dissertation provides first hand qualification and quantification of the dominant mechanisms responsible for complex three dimensional unsteady flow fields in a centrifugal pump. The experimental, numerical, and theoretical results may improve the predictability, reliability, and efficiency of future pumps. These turbomachines must minimize the detrimental influence of generated unsteady flow fields to provide long term financial rewards for pump manufactures.

9.1 Characteristic Hysteresis and Pump Performance

- Evaluation of the pump impeller characteristic with a stationary numerical code is a useful tool for the design and modification of centrifugal impellers.
- A stationary numerical code may be used to effectively predict the impeller head near bep operation including the location and magnitude of characteristic discontinuities. It is however still a time intensive procedure.
- The unsteady flow field undergoes an abrupt amplification/attenuation as impeller recirculation commences/ceases at the pump characteristic discontinuities.
- The pump recirculation and prerotation act as energy dissipaters which change the flow path in the impeller and reduce the amount of useful hydraulic power transferred to the flow, resulting in a 3% discontinuity in efficiency.
- In transient operation the hysteresis in the pressure discharge characteristic does not undergo a discontinuity but rather had a definite slope. The change in useful flow power received in the hysteresis flow regime, as the impeller recirculation and prerotation was commencing/ceasing, has been successfully dynamically modeled with a second order nonlinear dependence on throttling rate.

9.2 Unsteady Blade Loading

- The circumferential pressure variation within the volute of centrifugal pumps resulting from a mismatch of fluid angular momentum exchange is well documented in the literature. This steady pressure in the stationary frame was dominatly the unsteady pressure in the rotating frame resulting in unsteady blade loading at tongue passing frequency.
- The volute tongue was a boundary separating two distinctly different flow
regimes providing a sharp circumferential pressure gradient responsible for an abrupt pressure field fluctuation in an impeller passage. These pressure fluctuations propagated upstream in the blade passage with acoustic velocity changing the blade loading and blade circulation.

- The unsteady blade loading coefficients provide an indicator of the strong unsteady flow field an impeller blade experiences at off design and reveals the necessity for inclusion in blade loading computations.

9.3 Acoustic Generation
- It has been suggested that the volute tongue is the main culprit responsible for acoustic generation. The mathematical coupling to the tongue’s unsteady flow blockage and its unsteady force interaction reveals this on a fundamental level. The former provides the larger acoustic generator but the latter is not negligible.

9.4 General Comments
9.4.1 Industrial Applications
Industry’s current focus in centrifugal pump design driven by economical considerations was documented in the introduction as lying in improved predictability, reliability, and efficiency. This is forced by customer demands requiring powerful compact machines, short term dynamic capabilities, and wide operating regimes. These factors combine to make detailed investigation of unsteady flow fields within pumps of financial interest to industry. Made clear from this dissertation is:
- Of primary necessity is the flowering of nonstationary codes for centrifugal pumps. This manifests itself in the large blade loading coefficients revealing why a stationary code failed to be useful deep in part load operation.
- Numerical prediction of the discontinuities in the impeller characteristics with the direct link to the consequences of experimentally measured increased flow field unsteadiness is directly applicable to current impellers. These considerations must be made in any impeller design or modification. Further geometrical changes may be made first in the numerics for evaluating their influence on the impeller characteristic.
- Assessment of the unsteady flow field was partly performed with new signal analysis techniques which are employable to turbomachinery in general [Kaupert 1996]. Their ease of implementation and interpretation make them a well suited alternative to Fourier methods for transient signal analysis available for immediate application.
- Unsteady blade loading data presented here can be implemented by
impeller designers currently attempting to perform numerical blade loading calculations.

- Acoustically the dipole generator from unsteady surface forces on the tongue was shown to be significant when compared (near 40%) to the monopole from unsteady mass flux. It is these two mechanisms on which the focus of emphasis should be concentrated to obtain a reduction in acoustic generation while maintaining the desired pump performance parameters.

9.4.2 Scientific Applications

In supporting industry science must strive to provide applicable solutions permitting further pump development and satisfaction of customer requirements. In so doing new topics of interest are unveiled and come to the forefront of interest. Advances of this form in the dissertation are:

- Hysteresis effects in both the steady and unsteady lift coefficients with regard to static and dynamic local stall on two dimensional airfoil profiles are well documented [McCroskey 1982]. The three dimensionality, rotation, geometry, and unsteadiness of the pump flow field coupled with transient pump operational load makes local flow separation an enlightening interpretation for the static and transient pump characteristics hysteresis but too oversimplified. In fact scientific work in this field is so lacking [Tobak & Peake 1982] that it is not clear what separation is in the general case. Fundamental fluid dynamic research is required in this area including vorticity evaluation.

- Knowledge gained regarding the unsteady flow field permits targeted unsteady numerical and model simulation development. A current available option would be to model the saw tooth form of the nonuniform circumferential pressure around the impeller (Fig. 7.1) caused by the volute as a boundary condition on the impeller outlet pressure (i.e., in the rotating frame the outlet pressure distribution would rotate). Through this the volute would not have to be initially modeled.

- The unsteady blade loading was found to cause an unsteady circulation about the impeller blades which must cause unsteady vortex generation at the blade trailing edge if the Kutta condition is to remain satisfied. A coupling between the unsteady pressure in the volute and the frequency of vortex shedding should be undertaken. This not only permits a deep understanding of the unsteady flow in the impeller but provides a necessary test case for any nonstationary code development.

- The study here is limited to volute pumps, the unsteady blade loading should also be investigated in a similar fashion within diffuser pumps
where the distance between impeller blades and diffuser blades is usually significantly smaller and the circumferential pressure gradient at the impeller outlet (Fig. 7.1) is much milder.

- The influence of acoustic loading (i.e., pump system behavior) on the acoustic generators quantified in this pump was neglected. This important boundary condition changes from system to system and would have a significant impact on the results previously presented. Acoustic loading from the system could form the bases for a new study.
- A full modeling of pump acoustic generation, transmission, absorption, and reflection is far beyond the reach of current computational techniques. Future unsteady numerical development will aid the situation only marginally. The many uncertainties involved with material properties and complex geometries make a full simulation within a pump not realistic in the near future.

9.5 Closing Remarks

Pumps with ever increasing reliability and efficiency can no longer be constructed on trial and error concepts with empirical relations currently in existence that do not incorporate unsteady flow field knowledge. They must be designed using fundamentally solid physics obtained from continued targeted experimental, numerical, and theoretical high technology research which will in turn improve pump predictability during design.
10.0 References


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Appendix A: Probes Constructed

5 Hole Probe

- Ø 3.1
- Ø 6.3
- Ø 4

1 Hole Probe

- Ø 3.1
- Ø 6.3

Detail

- Sensor Keller MV-040 MS28
- Pressure sensitive zone
- Oil filled hole
- Semi-conductor electronics
- Epoxy
- Rubber stopper
- Rubber fastener
- Sensor membrane
- Wires
- PVC seal
- 5 pin plug
- Soldered position
Appendix B: Applied Wavelets

The following is a brief summary of the wavelets used for signal analysis in this dissertation [Kaupert 1996]. The wavelet transform is described by equation (3.11).

\[ \Psi(t) \]

\[ \Psi(t) \]

\[ \psi\left(\frac{t - b}{a}\right) \]

\[ \psi\left(\frac{t - b}{a}\right) \]

\[ |\Psi(f)| \]

\[ |\Psi(f)| \]
Appendix C: Equations

The following is a list of equations useful for the theoretical portions of this dissertation. These equations are merely provided as a reference, for development of them the reader is encouraged to check the literature.

Continuity [Currie 1974, page 13]
\[
\frac{D\rho}{Dt} + \rho \frac{\partial c_i}{\partial x_j} = 0 \quad (C.1)
\]

Momentum [Currie 1974, page 17]
\[
\rho \frac{Dc_i}{Dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \quad (C.2)
\]

where \(f_i\) represents the net external body forces on the fluid and \(\tau_{ij}\) is the stress tensor.

Navier Stokes (incompressible, viscosity constant) [Currie 1974, page 31]
\[
\rho \frac{Dc_i}{Dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \quad (C.3)
\]

Vorticity (incompressible, inviscid) [Currie 1974, page 59]
\[
\rho(\zeta \times \mathbf{c}) = -\rho \frac{\partial c_i}{\partial t} - \frac{\partial}{\partial x_i} \left( P + \frac{\rho c^2}{2} \right) \quad (C.4)
\]

Leibnitz’s Theorem [Panton 1995, page 58]
\[
\frac{d}{dt} \int_V H_{ij}(x_i, t) \, dV = \int_V \frac{\partial}{\partial t} H_{ij}(x_i, t) \, dV + \int_S n_k w_k H_{ij}(x_i, t) \, dS \quad (C.5)
\]

where \(H_{ij}(x_i, t)\) is any scalar, vector, or tensor function in the volume \(V\). The vector \(w_k\) is the velocity of the surface component surrounding the volume.

Gauss’s Theorem [Panton 1995, page 56]
\[
\int_V \frac{\partial c_i}{\partial x_i} \, dV = \int_S n_k c_i \, dS \quad (C.6)
\]
Divergence for Retardation Variables [Blake 1986, page 73]

\[
\int \int \int \frac{1}{r^2} \frac{\partial}{\partial x_i} \left( P + \frac{\rho c^2}{2} \right) dV(y, t-r/a_o)
\]

or with Gauss’s theory

\[
\int \int \int \frac{P + \rho c^2}{r^2} dS(y, t-r/a_o) + \frac{\partial}{\partial x_i} \int \int \int \left( \frac{P + \rho c^2}{r^2} \right) dV(y, t-r/a_o) \quad (C.7)
\]

This is also obtainable using the well known chain rule for differentiation of a retardation variable \( F(y, t-r/a_o) \) [Goldstein 1977, Page 73]

\[
\left[ \frac{\partial}{\partial y_i} \right]_{t,R} = \frac{\partial}{\partial y_i} \bigg|_{x,t} + \frac{\partial}{\partial x_i} \bigg|_{y,t} \right] F(y, t-r/a_o)
\]

where \( R = |x-y| \).

Biot-Savart Law Hydroacoustic Form [Powell 1994, Lamb 1932]

\[
c(x) = -\frac{1}{4\pi} \oint \frac{x \times dl}{r^3} \quad (C.8)
\]
Appendix D: Impeller Outlet Unsteady Blockage

The following analytical development describes mathematically the behavior of flow in an impeller blade passage as it rotates past a volute tongue which provides an unsteady flow blockage. Of particular interest is a physical evaluation of the unsteady mass flux for equations 8.6 and 8.7 which quantify the acoustic generation termed the monopole contribution. The differential equation governing the unsteady mass flux in a blade passage is developed in a manner similar to existing methods [Gikadi 1981] and further applied concretely to acoustic generation.

Consider the impeller passage sketch shown in Fig. D.1. The following assumptions are made:
- two dimensional flow,
- irrotational flow,
- incompressible flow
- no flow slip along the blades,
- circumferential symmetry,
- no gravity forces present,
- Bernoulli constant zero.

The flow situation is analyzed in the rotating system using the unsteady Bernoulli equation.

As the impeller rotates the blade passage outlet area is described by the relation,

\[ S_2(t) = S_{20}[1 + \zeta(t)] \]

where \( S_{20} \) represents the open passage area at the outlet and \( \zeta(t) \) an area blockage parameter. Within the passage continuity of mass provides

\[ S(l)w(l, t) = S_2(t)w_2(t) \quad (D.1) \]

which uses the no slip condition interpreted as the flow is tangential to the blade surface at all positions and times meaning,

\[ w(l, t) = w(l, t)e_l \]

along the \( l \) direction. The unsteady Bernoulli equation along a streamline can then be written as

\[
\frac{\partial}{\partial t} w(l, t) + \nabla \left( \frac{dP}{\rho} + \frac{w(l, t) \cdot w(l, t)}{2} \right) = 0
\]

\[
\int_1^2 \frac{\partial}{\partial t} w(l, t) dl + \int_1^2 \frac{dP}{\rho} + \frac{w_2(t)^2(1 - \kappa^2)}{2} = 0 \quad (D.2)
\]

Fig. D.1 Blade passage geometry.
where body forces are taken as negligible, the Bernoulli constant is taken as zero and the substitution \( w(t) = k \omega(t) \) was made. A further convention is the concept of the unblocked passage outlet flow velocity given by,

\[
w_{20} = \left( 2 \int_{0}^{1} \frac{dP}{2 \rho (1 - \kappa^2)} \right)^{1/2} \tag{D.3}
\]

To evaluate the first term of equation D.2 the expression in equation D.1 may be used to write,

\[
\int \frac{d}{dt} w(l, t) dl = \int \frac{d}{dt} \left[ \frac{S_2(t) w_2(t)}{S(l)} \right] dl
\]

\[
= \frac{d}{dt} \left[ \frac{S_2(t) w_2(t)}{S_{20}} \right] \int \frac{S_{20}}{S(l)} dl
\]

which then puts equation D.2 in the form,

\[
\int \frac{dP}{2 \rho} = \frac{w_2(t)^2}{2} \left( 1 - \kappa^2 \right) + \frac{d}{dt} \left[ \frac{S_2(t) w_2(t)}{S_{20}} \right] \int \frac{S_{20}}{S(l)} dl \tag{D.4}
\]

Now the situation can be further simplified with the unblocked passage outlet velocity by introducing a dimensionless time \( \tau \), defined as,

\[
\tau = t w_{20} / \int \frac{S_{20}}{S(l)} dl
\]

and using the chain rule for differentiation provides,

\[
\frac{d}{dt} = \frac{d}{d\tau} w_{20} / \int \frac{S_{20}}{S(l)} dl . \tag{D.5}
\]

The concrete problem at hand lies in the determination of the unsteady mass flux. For the unsteady mass flux an expression can be written,

\[
\frac{\rho Q}{\rho Q_{20}} = \frac{S_2(t) w_2(t)}{S_{20} w_{20}} = 1 + \chi(t)
\]

Using equation D.3 and the area blockage factor \( \zeta(t) \) the unsteady outlet velocity may be then written as,

\[
w_2(t) = \left( 2 \int_{0}^{1} \frac{dP}{2 \rho (1 - \kappa^2)} \right)^{1/2} \frac{[1 + \chi(t)]}{[1 + \zeta(t)]} \tag{D.6}
\]

After substitution of equations D.5 and D.6 the unsteady Bernoulli equation D.4 becomes with simplification,
\[
\frac{d\chi(t)}{dt} + \frac{(1 - \kappa^2)}{2}\left\{ \frac{(1 + \chi)^2}{[1 + \zeta(t)]^2} - 1 \right\} = 0. \tag{D.7}
\]

This is the nonlinear differential equation describing the blade passage volume flux. Similar results have previously been obtained in analogous geometric configurations [Gikadi 1981]. For the unsteady mass flux the following relation is applied,

\[
\frac{dm}{dt} = \frac{d[\rho S_2(t)w_2(t)]}{dt},
\]

\[
\frac{dm}{dt} = \rho Q_{20}\left(\frac{d\chi(t)}{dt}\right). \tag{D.8}
\]

Equations (D.7) and (D.8) are numerically solved to quantify the monopole acoustic source found in equation 8.6 and 8.7.

Monopole Acoustic Generation

The problem of determining an unsteady area blockage factor $\zeta(t)$ is a difficult aspect in solving equation (D.7) and (D.8). Once this has been determined numerical integration is possible. To overcome this an area blockage function is assumed over one impeller rotation, shown in Fig. D.3a. This blockage function assumes that near 40% of the effective blade passage is blocked when the tongue is in the middle of the passage. The blade passage undergoes the blockage twice per rotation. Using this area blockage factor with the velocity factor $\kappa = 0.5$ the dimensionless mass flux and its derivative in time for a single blade passage are shown in Fig. D.3b,c over one rotation.

The mathematical model predicts a rapid decrease in the mass flux as the blade passage rotates into the region of outlet area blockage. Hereafter the mass flux approaches a quasi steady value, no longer changing in time. Upon further impeller rotation the region of outlet area blockage is departed resulting in a sudden increase of the mass flux through the passage. Then a settling time to a quasi steadiness sets in until the impeller rotates into the influence of the second volute tongue.
Fig. D.3 a) The assumed area blockage factor over one impeller rotation. b) The dimensionless unsteady mass flux in a single blade passage. c) The rate of change of mass flux in a single blade passage.

A summation of 7 equal blade passages out of phase with each other by 1/7 rotation then simulates the entire impeller blockage. Neglected is the increase in mass flux through the 5 impeller passages as 2 of the 7 passages undergo blocking. The FFT for the resulting superposition of the mass flux derivative (i.e., Fig D.3b translated and superposed 7 times) for the whole impeller is shown in Fig. D.4.
Fig. D.4 Magnitude spectrum of the rate of change of mass flux.

The results of Fig. D.4 coupled with equation 8.7 for the simple monopole acoustic generator then indicate 206.8 Pa of acoustic generation from the unsteady flow blockage. This single value, in the form of $\bar{C}_p = 0.00256$ and $L_p = 46$ dB, fits into Fig. 8.9 appropriately.

The model thus provides insight into the physical mechanism of the monopole acoustic generator caused by the volute tongues unsteady blockage. No attempt is made to fine tune this model by changing the impeller outlet area blockage factor $\zeta(t)$ for various volume fluxes. The simple model merely provides intuitive demonstration for a complex phenomena.