Creep and Fatigue Behaviour of Eutectic Sn62Pb36Ag2 Solder

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH
for the degree of
Doctor of Technical Sciences

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1997
"[Stagnum] may be produced also by other means, mixing one part tin with one part lead; some call this mixture 'argentarium'. The same say 'tertiarium' to a mixture of two parts lead and one part tin. [...] This is used for soldering of plumbings."

Plinius Gaius Secundus, around 75 AD

Acknowledgements

I am deeply grateful to Prof. Dr. Alessandro Birolini for having given me the opportunity to do this work and for supporting it. I would also like to thank Prof. Dr. Peter J. Uggowitzer for accepting to co-examine this thesis.

The support of Dr. K. Heiduschke and his encouraging as well as scientifically sound comments are gratefully appreciated. The cheerful company of my peers G. Grossmann and M. Held was a source of relaxing moments in hard times.

My thanks go also to Prof. Dr. H. Hieber and Dr. Ing. T. Ahrens of Centrum für Mikroelektronik in Neumünster, Germany for helpful suggestions and valuable technical discussions.

I wish to thank also all my colleagues at Reliability Lab. and, especially, my wife Nathalie for having endured my wavering mood and responding to it with steady friendship and love.

This work was partly funded by the KTI of the Swiss government.
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Abstract

Broken solder joints represent a frequently observed mode of field failure of surface mounted devices. Finite Element Modelling (FEM) of service conditions has been established as tool for lifetime assessment of solder joints. For physically meaningful simulation, an appropriate constitutive description of the solder is crucial. In this work, a comprehensive constitutive description of eutectic lead-tin-silver solder is presented including microstructural features such as coarsened shear band forming and recrystallisation.

The constitutive behaviour in terms of stress-strain rate relation has been determined using single lap joint specimens. After soldering, strain rates depend on stresses by a power-law with exponent 7. The solder softened during further deformation and the exponent of the stress decreased gradually to a value of 3.3 within 100% engineering strain. Both features were related to the development of coarsened shear bands in the solder joint. These shear bands formed owing to local recrystallisation. The thickness of the shear band depended on the rate of cooling during solidification and on the applied strain rate being the thicker the slower the cooling rate and the higher the strain rate.

The kinetics of the development of coarsened shear bands were determined by monitoring the softening in tensile tests. The extent of recrystallisation within the shear band was found to depend on the amount of local deformation. The imposed deformation was increasingly localised in the shear band as recrystallisation proceeded. Hence, strains and strain rates were determined with respect to the shear band thickness.

Damage in form of homogeneously distributed reduction in load bearing area accumulated as deformation increased. In uniaxial deformation, the rate of damage per unit strain was proportional to the applied stress and the remaining load-bearing area.

Creep strain-controlled cyclic loading was performed in order to compare evolution of the microstructure in unidirectional and cyclic deformation.
Both, shear band formation as well as transition from high to low stress exponent was observed. However, quantitative modelling by the equations determined in uniaxial testing fell short. Damage evolved according to a Coffin-Manson approach and depended thus on the deformation path.
Zusammenfassung


Schädigung in Form von homogen verteilter Abnahme des tragenden Querschnitts entwickelt sich mit zunehmender Deformation. Die Zunahme in der Schädigung pro Deformationseinheit ist proportional zur angelegten Spannung und dem verbleibenden tragenden Querschnitt.

Schädigung entwickelt sich in Wechsellastransuche gemäß eines Coffin-Manson Ansatzes und ist somit wegbabhängig.
1

Introduction

1.1 Presentation of the problem

The miniaturisation of electronic devices and the increasingly compact design of printed circuits in the electronic industry has substantially been supported by the introduction of the surface mount technology (SMT), as replacement of the 'through-hole'-technique. This change adds a new element to the requirement profile of the connection between device and printed circuit board (PCB): beside the electrical contact it has to ensure the mechanical adhesion as well. These interconnections are traditionally made of (low-melting) eutectic or near-eutectic lead-tin(-silver)-solders. As the performance of the integrated circuits further increases, a further increase in number of pins per device is required. Since the production technology is restricted to a certain minimum distance between the pins, the increase of pins is gained by enlarging the casing of the integrated circuit.

By switching the device on and off, and thus heating it up and letting it cool down again, the solder joint is cyclically loaded due to the difference in the thermal expansion coefficients of both device and PCB, which results in fatigue of the solder material. Since the introduction of the SMT, thermo-mechanical fatigue failure of solder joints has been found to be one of the main life-limiting factors for surface mounted devices (SMD). Consequently, a lot of work has been focused on this topic in recent years. There are mainly two ways to deal with the problem: some research has
been directed to the development of new connecting materials, e.g. filled glue or other low-melting solders, in order to achieve higher fatigue resistance. However, the success in doing so has been rather limited so far. On the other hand, efforts have been made to develop models for life-time prediction of stressed interconnections by quantifying the impact of the main influencing parameters—e.g. thermal expansion mismatch, temperature amplitude, connection geometry, dimension of the device—on the fatigue behaviour. Herein, two different approaches have been followed: On the one hand, several assemblies are tested under varying test conditions until failure. Using these data enables to estimate the number of temperature cycles to failure under given field conditions by interpolation [Eng92]; on the other hand (triggered by the fast increasing availability of computing performance), the more general approach of the Finite Element Analysis (FEA) has gained importance in recent years. In a FEA, a geometry-independent material model allows to calculate the number of cycles to failure for given geometry and test condition. Obviously, the quality of the material model is of major importance for any physically meaningful simulation.

Preliminary investigation of different alternative materials have revealed that the resistance against fatigue is at its best the same as in eutectic lead-tin but is usually worse than in the currently used alloy. Hence, the focus is set on the characterisation of actual solder material. While the former of the above described approaches offers merely information about an assembly after very time-consuming testing, the latter may also be helpful in designing and provide reliable data in shorter times with only some few tests.

The difficulty of this approach lies in finding an appropriate description of the material. Many models are given in the literature, ranging from pure elastic-plastic behaviour to various creep-behaviour descriptions. A possible reason for this is that there is not a unique mechanical behaviour of eutectic lead-tin solder. Depending on the condition of solidification and subsequent mechanical treatment, the mechanical behaviour varies from superplastic to seemingly normal-plastic.

To further complicate the matter, eutectic lead-tin in soldered joints exhibits a highly unstable microstructure, which results in coarsening and recrystalli-
sation. The latter gives rise to inhomogeneous deformation patterns. However, neither inhomogeneous deformation nor microstructural instabilities leading to transitional behaviour has so far been accounted for in constitutive modelling.

1.2 Aim of this thesis

The aim of this thesis is to investigate the mechanical behaviour of solder material over a wide range of temperatures and stresses. In contrast to previous work, the focus is set on the use of specimen with solder layers of about 60 µm, instead of bulk or thick-layered specimen, in order to be as close as possible to the conditions encountered in SMT solder joints. Furthermore, the work is aiming at the integration of both inhomogeneous deformation and microstructural instabilities in the framework of the material model. Comparing studies of unidirectional straining behaviour and cyclic straining shall offer some basic insight about the connection between these two types of loading conditions.

1.3 Structure of the thesis

Current knowledge of mechanical behaviour of lead-tin eutectic is reviewed in Chapter 2. The various currently used constitutive models and lifetime-estimation methods are critically assessed on their physical basis and their aptitude for FEA. Testing strategies and set-ups as well as specimen designs used in the literature are reviewed.

Chapter 3 deals with the specimen preparation and the presentation of the testing procedures. Moreover, a sample preparation technique without etching is presented that reveals grain boundaries in light microscopy.

In Chapter 4 the evolution of the microstructure in soldered joints during straining and the accompanying changes in the mechanical behaviour are conveyed. The differences in fracture behaviour of quenched and slowly cooled specimen is phenomenologically described. The elastic behaviour of lead-tin-silver solder is determined.
Using the results of Chapter 4, the constitutive behaviour incorporating both inhomogeneous deformation and microstructural instabilities is developed in Chapter 5. Evolution of damage is described as well. Subsequently, some predictions and direct consequences of the model are discussed. The last part of the chapter deals with the generalisation of the model necessary for implementation in FEA.

Analogies and differences between unidirectional straining and cyclic deformation are outlined and discussed in Chapter 6 in connection with difficulties to be expected in FEA.

General conclusions are given in the last Chapter.
In spite of all those Material Handbooks that offer whole sets of mechanical data about eutectic lead-tin solder, the matter is not as settled as it might seem. In fact, we now know that 'there is no such thing as the mechanical behaviour of a particular solder; it depends on the microstructure, which depends ... on how the solder is processed' [Mor94]. It is, therefore, necessary to give first an idea what kind of microstructure is present in solidified solder, how it depends on the cooling rate and how it may be influenced by mechanical treatment. We then will see what mechanical behaviour results from the different microstructures. This shall be treated in section 2.1.

The constitutive behaviour of materials—i.e., their general response to stress in function of temperature and of structural parameters such as grain size—is required for mechanical modelling by means of Finite Elements. The mathematical form of the constitutive equation and the incorporated parameters are not simply a matter of curve-fitting but reflect the assumed phenomenology of the behaviour as well. Therefore, various frequently used types of constitutive models for tin-lead solder and the corresponding phenomenologies are discussed in section 2.2. Since cracking due to cyclic deformation is the main limiting factor in solder joint's life, fatigue lifetime evaluation techniques are discussed in section 2.3. Finally, different currently used testing strategies, specimen geometries and specimen
preparation techniques for mechanical testing are presented and discussed in section 2.4.

2.1 Microstructure and properties

2.1.1 Microstructures after solidification and after working

Alloys of lead and tin have been in use for over 2000 years. Even the Romans were using at least two different types of lead-tin alloys though none was of eutectic composition. The alloys were employed as mirror coatings and as solder materials in plumbings [Pli89]. Other applications were in medicine. The first investigation reported on the binary phase diagram of tin-lead dates from the end of the last century. Roberts-Austen [Rob97] determined the eutectic temperature to be 190 °C and the composition to be 31 mass-% lead and 69 mass-% tin. He suggested that the

![Binary phase diagram of lead-tin](image)

Fig. 2.1 : The binary phase diagram of lead-tin as generally accepted in the literature, drawn from the 'Handbook of Binary Phase Diagrams' [Mas92].
solid solution of tin in lead and vice versa was negligible. Ten years later, Rosenhain and Tucker [Ros09] published their data correcting the eutectic composition to 37 mass-% lead and the remainder tin. The eutectic melting point was given at 183°C. As concerns the solubility of lead in tin, they agreed with the previous investigation while for tin in lead they determined the solid solution limit to be 18 mass-% at the eutectic temperature. Nowadays, the eutectic composition is accepted to be at 61.9% tin and 38.1% lead. The binary phase diagram is given in Fig. 2.1 [Mas92]. For simplicity's sake, the tin-rich phase is called tin-phase while the lead-rich phase is referred to as lead-phase. The ternary alloy containing 2 mass-% silver is slightly off the ternary eutectic composition being at 1.4 mass-% silver and 62 mass-% tin the remainder being lead. The melting point of the ternary alloy is at 178°C [Vil95].

Since the density of the tin-phase and the lead-phase differs considerably\(^1\), the volume fraction of the phases cannot be derived from the composition by a simple lever law. Density-corrected calculation leads to 66 vol.-% tin-phase and 34 vol.-% lead-phase. This results in a structure where the tin-phase builds the matrix in which the lead-phase is embedded. Rosenhain and Tucker [Ros09], Portevin [Por23] and Brady [Bra22] investigated the morphology of the phases and reported a lamellar structure after casting. Kurnakoff and Achnazaroff [Kur22] investigated the variation in hardness and microstructure as a function of the cooling rate. As for the microstructure, they found a transition from distinct lamellar structures to irregular, globular lead-phases for increasing cooling rates. These two microstructures are conveyed in Fig. 2.2. Rosenhain and Tucker [Ros09] reported also on a gradual change during ageing from the lamellar to the globular form of the embedded lead-phases.

One point that caused much dissent was what should be considered the microstructural unit. In fact, the lamellar structure was often found to extend coherently over up to 1 mm\(^2\), while the individual lamellae were but a few microns in width. Some considered the phases to be the microstructural units while others supported the idea of macroscopic grains, so called 'eutectic colonies'. To further complicate the matter, Hargreaves [Har27a,b] showed by means of etching and oblique illumination that tin as matrix

\(^1\) 10.4 g/cm\(^3\) for lead and 7.3 g/cm\(^3\) for tin
Fig. 2.2: Different microstructures in eutectic lead-tin after solidification. Lamellar structure in large-scale colonies after slow solidification (cooling rate 0.1°C/s) (left) and fine globular structure after quenching (100°C/s) (right). Owing to small solder masses and quick processing in electronics resulting in fast cooling, the microstructure in solder joints resembles the quenched material on the right.

material in a colony had one single crystallographic orientation. Finally, Hargreaves [Har28] provided further support to the latter view by showing that crystallographic twins within the tin-phase induced by deformation extended throughout the whole colony causing the embedded lead-phases to deform accordingly.

This picture of the microstructures after solidification has often been confirmed since. Due to the very small solder masses and the quick processing present in SMT soldering, the microstructure in electronic solder joints resembles that of quick solidification.

Already during the studies in the 20's of our century it was recognised that working the material would significantly alter the microstructure in lead-tin eutectic solder. In 1923, Tamman and Dahl [Tam23] reported on recrystallisation phenomena in eutectic structures after cold-working. Hargreaves [Har27c,Har28], performing microstructural investigations after hammering experiments noted that the former coherent orientation of the tin within one colony got lost after cold working. He concluded that cold working would lead to recrystallisation, which was perfectly in line with the findings of Tamman and Dahl. Furthermore, he pointed out that the tendency to
recrystallise was dependent of the spacing between two lead-phases being higher in coarse structures than in very fine ones. He gave a lower limit of 30 percent cold-working to cause recrystallisation of the relatively coarse colony boundaries while rather fine structured intracolonial areas did not recrystallise before 80 percent of deformation. While the lamellar structure was obviously removed totally by an equiaxed structure, the globular structure seemed to show only general coarsening after recrystallisation. However, it has been shown recently in [Gro96] that after rapid cooling the globular structure consists like the lamellar microstructure of large scale colonies with one single orientation within the tin matrix. As in the work of Hargreaves on lamellar structures, not only the globular lead phases coarsens after recrystallisation but also the tin colony breaks down to a very fine grained structure, cf. Fig. 2.3.

In line with these findings of a general change in microstructure after recrystallisation, it has been reported by various authors (e.g. [Har27a, Ewi00]) that the deformation in as-cast lead-tin eutectic is localised within the deformed body and these areas of localised strain have been referred to as 'slip lines'. The slip lines are obviously of recrystallised microstructure and result from partial recrystallisation probably triggered by the size of the initial lamellae spacing.
The said formation of slip lines has recently become a commonly reported feature in solder joint investigations. They appear as bands of about 10 μm width of coarsened and equiaxed microstructure and are therefore often referred to as 'coarsened shear bands'. Deformation and ultimate failure is usually localised in these bands as will be discussed in the next subsection.

2.1.2 Mechanical properties of the different microstructures

The mechanical behaviour of any microstructure is dominantly affected by the testing temperature. In case of lead-tin eutectic testing at ambient temperature is a high temperature treatment with regard to the low melting point of the alloy. Therefore, typical high-temperature phenomena, e.g. creep, recovery, and recrystallisation, are present. Hence, mechanical response to stress is given in terms of steady state creep rate rather than strain rate independent yield stress or tensile strength models.

Early qualitative investigations on changes in mechanical behaviour of different microstructures in lead-tin date back to the twenties of our century. Hargreaves [Har27c,Har28] pointed out that cold working did significantly reduce the Brinell-hardness compared to the as-cast state, but that this effect was partly removed after annealing. In 1928, Jenkins [Jen28] performed comparing studies of lead-tin and cadmium-zinc eutectic. The tensile strength of both alloys was highest in the as-cast state and was considerably lowered if cold working was applied. Both alloys exhibited elongations to fracture of several hundred percent though in the cold-worked condition and at relatively low strain rates only. In the as-cast state, creep deformations to rupture were limited to some 50% accompanied by significant necking in the final stage.

The next step, probably the most famous one, was the publication of Pearson's data in 1934 [Pea34]. He performed creep tests with constant stress on cold-worked lead-tin and bismuth-tin reaching up to 2000% elongation without appreciable necking at low stresses while specimen tested at high stresses and strain rates behaved normally plastic, i.e., they exhibited fracture elongations of about 40% and necking. The picture of his tensile test specimen is given in Fig. 2.4 and has regularly been quoted since. Pearson found that this viscous behaviour extends even to relatively
Fig. 2.4: Photograph of the tensile test specimen elongated to 20 times its initial length. No significant necking can be detected. While the actual photograph was taken of a tin-bismuth alloy similar behaviour was shown for eutectic lead-tin (after [Pea34]).

High strain rates up to $10^{-3}\text{s}^{-1}$, especially if the specimen is tested immediately after cold working. Ageing lowers the upper limit of strain rates at which excessive straining is observed. Straight forwardly, he ascribed this effect to the increase of grain size during the ageing treatment. Microstructural investigation led him to the conclusion that neither dynamic recrystallisation nor intragranular flow was present in the structure but that the deformation was rather due to intergranular flow.

This anomalous flow behaviour was in general only observed if the structure consisted of two small phases to about equal parts. It was suggested, since the presence of many grain or interphase boundaries was observed to be crucial, that this kind of phase distribution inhibited the grains to coarsen as it would happen in a single phase structure. Later, the term 'superplasticity' was proposed by [Boc45] to refer to such extraordinary ductility.
It was not before the 60's that the obviously strongly differing creep behaviour of as-cast and rolled lead-tin was quantitatively described in a general way using a so-called 'power-law' description originating from Norton [Nor29], cf. (2.1):

$$\dot{\gamma} = A \cdot \left(\frac{\tau}{G}\right)^n \cdot \exp\left[-\frac{Q}{RT}\right]$$

(2.1)

where the stress $\tau$ is divided by the shear modulus $G$ in order to omit units other than $s^{-1}$ in the pre-factor $A$. The temperature dependence is accounted for by an Arrhenius factor and the temperature dependence of the shear modulus. A major advantage of this kind of description is that the parameters $Q$ and $n$ have been recognised to be indicative for the micro-mechanisms governing the creep deformation.

Since the reported superplastic behaviour seemed to offer new ways to accomplish complex structures by deep drawing, much effort was dedicated to understanding the micromechanisms responsible for the superplastic behaviour of the structure of rolled lead-tin. Thus, a large number of publications was concerned with this microstructural state. The findings of these investigations can be summarised as follows:

**Mechanical behaviour of the as-cast structure**

The steady-state creep rates of the as-cast structure obey a Norton-law with one single set of parameters (cf. e.g. [Cli67]) for temperatures between 0 and 100°C and strain rates ranging from $10^{-7}$ to $10^{-2}$ s$^{-1}$. This comes up with a straight line for each temperature in a double-logarithmic plot of stress vs. strain rate. The slope of the line is identified with $1/n$, the inverse of the stress exponent. This value is usually referred to as the 'strain rate sensitivity' (SRS). Typical values for $n$ are 7...8 and, thus, the SRS is about 0.12...0.15. The activation energy $Q$ is in the order of 70-90 kJ/mol. These values indicate that dislocation climb controlled by intragranular diffusion is the rate controlling step in creep deformation.

In creep tests, the following features are encountered: considerable primary creep, necking, and elongated grains after testing. The elongation to fracture is in the range of 20% to 50%.
Mechanical behaviour of lead-tin after rolling

While at strain rates above $10^{-3}\text{s}^{-1}$, the behaviour of rolled eutectic lead-tin is comparable to that of the as-cast material, significant changes are encountered at intermediate strain rates between $10^{-4}\text{s}^{-1}$ and $10^{-6}\text{s}^{-1}$. In this range the slope in the double logarithmic strain rate vs. stress plot increased from $1/8$ to $1/2$ leading to considerably higher creep-rates at the same stress levels in the deformed than in the as-cast condition. At strain rates below $10^{-5}\text{s}^{-1}$, the SRS slightly dropped to $1/3$. This behaviour is schematically drawn in Fig. 2.5.

Before we turn to the features of the different deformation regimes we will shortly discuss what the relation of the different underlying mechanisms to each other must be in order to turn up in this order at increasing stress or strain rate. For convenience's sake we call them stages I, II and III according to Fig. 2.5.

The transition from stage II to III is simply a matter of which of them is the faster. It means in terms of a rheological analogue that two different non-linear damping elements, i.e., damping elements having a strain rate

![Fig. 2.5: Schematic log τ vs. log $\dot{\gamma}$ behaviour for worked lead-tin eutectic (full line) and in the as-cast condition (dotted line).](image-url)
sensitivity other than unity, are working in parallel. At the intersection of lines II and III, both mechanisms give way at equal speed. Increasing the stress gives rise to a higher acceleration in mechanism guiding stage III than in that of stage II. On the other hand, lowering the stress decelerates III stronger than II. Applying the same arguments to the transition from stage I to stage II leads to the conclusion that there always the slower mechanism is the one that determines the strain rate. Therefore, in our rheological model these two damping elements are connected in series. The combination of these findings is drawn in Fig. 2.6.

The features of the distinct stages I to III in the log τ vs. log \( \dot{\gamma} \) diagram are the following: In stage III, the usual features of creep are present: primary creep, steady state and tertiary creep including internal necking by evolution of voids. The strain rate sensitivity is about 0.12 - 0.15. The thermal activation energy is in the range of 70 to 90 kJ/mol, about the value for bulk diffusion in tin. Neither the morphology nor the size of the phases seem to influence the strain rate significantly. The initially equiaxed grains are elongated in the direction of stress. Typical elongations to fracture are about 50\%, and, therefore, comparable to those of as-cast material.

In stage II, the elongation to fracture is significantly enhanced compared to

![Diagram](image)

Fig. 2.6: The rheological analogue for the three regimes of different strain rate sensitivity. The always present parallel elastic element has been omitted in the discussion.
stage III. Proper testing results in a fracture elongation of as much as 50 times the initial length [Moh75]. The grains do not change their shape seemingly but slide along each other. This phenomenon is called grain boundary sliding. Interestingly enough, void formation as would be expected to occur at triple-points is virtually absent in tensile testing while cyclic bending led to some extent to formation of cavities [Ram87]. For the activation energy values varying from 40 kJ/mol to 90 kJ/mol are reported. The former is in the order of the activation energy for grain boundary diffusion while the latter is close to that for bulk diffusion. The strain rate sensitivity is about 0.5. There is, as would be expected, a strong dependence of the strain rate on the grain size $d$ in this region cf. table 2 in [Kas81]. Equation (2.1) is then extended to

$$\dot{\gamma} = A \cdot \left( \frac{\tau}{G} \right)^n \cdot \left( \frac{d}{b} \right)^p \cdot \exp[-Q/RT]$$

(2.2)

where $b$, the Burger's vector, is introduced in order to retain the dimension $s^{-1}$ for the factor $A$. The grain size exponent $p$ is $-2...-3$ [Zeh68], which is between the value for Herring-Nabarro creep [Her50], where the diffusion path is through the bulk, and Coble creep [Cob63], where grain boundaries are carrying the diffusion. In stage II, no or only little primary creep is encountered. Therefore, dislocation movement within the grain is considered to be not rate controlling for the mechanism present.

The features of stage I are somehow a mixture of the characteristics of stage II and III: at one hand the activation energy is similar to that of bulk diffusion and the strain rate sensitivity is about 1/3. Moreover, little primary creep and hardening is present. On the other hand, there is a grain size dependence similar to stage II. Some researchers [Ash73], [Hor74] deny the existence of this stage invoking some internal backstress to account for the decreasing strain rate sensitivity at very low stresses. Though the postulated values for the backstress have been found to be too high. However, considerable disagreement about numerical parameters and variations in experimental data in this stage makes it difficult to rule out one or the other point of view. Weertman in 1967 [Wee67] pointed out that improper testing, i.e., measuring strain rates over too small deformation ranges, might be the source of these equivocal results as other non-steady-state mechanisms may account for it.
Although most of the publications on worked lead-tin agree about the general features in the deformation behaviour, there are significant differences in numerical values of the various parameters of equation (2.2). In Fig. 2.7, measurements of various authors are plotted together. Since some of the data stem from testing in unidirectional tension they had to be converted using the von-Mises factor $\sqrt{1/3}$. A variation within one order of magnitude was the best fit that could be accomplished. Arrowood [Arr91] pointed out that there might be some inconsistency due to different techniques in measuring the grain size. However, Murty et al. [Mur75] reported that pre-straining would enhance steady-state creep rates by an order of magnitude. Therefore, different testing procedures are the source of major differences. We will tackle this point in Section 2.4.

Since grain boundary sliding seems to play a crucial role in superplasticity, it was argued that this deformation mechanism did not or considerably less

![Fig. 2.7: Superposition of creep data of worked lead-tin from different authors conveying agreement about the sigmoidal form in the log stress vs. log strain rate diagram. Numerical differences may be due to different testing procedure.](image)
than other mechanisms damage the structure. Therefore, a superplastic material should exhibit outstanding fatigue resistance as well.

On the other hand, the analysis of the stability of a tensile test specimen against necking revealed that the SRS-value becomes the key-factor if net work-hardening is zero. This is the case in steady state creep deformation. Hence, there has been some arguing whether the outstanding ductility of superplastic material is due to the non-damaging nature of grain boundary sliding or merely a consequence of the low SRS suppressing necking. The former view has been experimentally supported when first in shear tests deformations to fracture of several hundred percent were reported [Mei91a]. Moreover, Aldrich and Avery [Ald70] performing low cycle fatigue experiments on worked lead-tin reported that superplastic material exhibited a higher fatigue resistance than other materials.

*Mechanical behaviour of eutectic lead-tin in soldered joints*

Recent investigations on eutectic lead-tin solder joints call in question whether all three of the above described regimes are present. Most of the data [So186], [Dar92], [Wil90], [Lee93] determined in the temperature range from 0°C to 100°C indicate that merely strain rate sensitivities of 1/3 (low stresses) and 1/7 to 1/10 (high stresses) are present, i.e., the intermediate or 'superplastic' stage is dropped or probably shifted to higher temperatures. Moreover, the transition from stage I to stage III varies up to a factor of 10^3 in strain rate indicating that other not yet qualified influences are relevant.

The presence of so called 'coarsened shear bands' in the commonly used shear test specimen may partly account for this uncertainty. Depending on the relative width of the shear band compared to the solder joint thickness the strain rate present in the shear band can significantly exceed the 'overall' strain rate.

These shear bands form along the boundaries of the eutectic colonies which evolve during solidification. They are parallel to the direction of stress, as depicted in Fig. 2.8.
Fig. 2.8: Shear bands along the colony boundaries parallel to the direction of stress. Extensive changes in the microstructure indicate that recrystallisation is involved.

The coarsened slip bands consist of recrystallised material and, thus, exhibit superplastic behaviour at appropriate strain rates. Following Aldrich and Avery, they should offer enhanced resistance to fatigue. However, fatigue failure usually takes place within the coarsened shear bands. The remainder of the specimen is not damaged. This is due to the fact that the recrystallised structure deforms at much higher rates than the as-solidified material at a given intermediate stress level. Therefore, almost the entire deformation is localised in the slip bands.

[Fen94] wrongly proposed that the higher deformation rate in the shear band was due to its coarsened structure invoking some sort of a Hall-Petch relationship. In fact, the matter is exactly inverse: the shear band, even though the lead phase might look coarsened, consists of very fine grained tin phases in contrast to the large scale eutectic colonies present in the still as-soldered parts. This structure enables grain boundary sliding to take place and contribute significantly to the deformation.

Hence, however high the fatigue resistance of the recrystallised structure might be, it is of no help as long as it does not persist over the whole joint and is restricted to but a tiny part of the cross-section.
Facing these difficulties, Morris and Mei [Mor94] have reported on various approaches to get homogeneously distributed recrystallised structures within a solder joint:

- Inhibiting the slip band to extend over the whole area of the joint by introducing large stable phases as present in off-eutectic solders [Sum90] or by adding second phase dispersants [Kra92]. The off-eutectic alloy was reported to exhibit a not quantified increase in fatigue resistance. The addition of gold-tin dispersants lowered the fatigue resistance due to the brittle dispersants.

- Refining the structure by rapid cooling, i.e., quenching in water [Mei92]. As was shown in Fig. 2.2, the microstructure of quenched specimen is equiaxed and resembles very much the structure found in worked material, though the structure is finer after quenching. Quenching resulted in an increase of fatigue resistance by up to a factor of 3.

- Suppressing the evolution of eutectic colonies or broadening the colony boundaries, i.e., the future shear bands, using minor alloy additions (about 2%), e.g. In, Sb, Bi, and Cd [Tri90]. While Bi and Sb have no beneficial effect, the In and Cd additions resulted in quite homogenous deformation within the first 50% strain in unidirectional shear testing.

In Chapter 5, we will give a model that incorporates local recrystallisation and thus inhomogeneous deformation in solder joints in a constitutive equation framework.

2.1.3 Summary

The deformation behaviour of eutectic lead-tin is strongly influenced by its microstructure. In one special state, after deformation and subsequent recrystallisation, the material is able to deform in a stable manner over several hundred percent. In this state, the fatigue resistance is increased as well. The benefit of this structure is diminished as it is restricted to but a fraction of the joint, the shear bands. The evolution of shear bands is recognised to be due to the unstable microstructure after soldering.
2.2 Constitutive equations for eutectic tin-lead solder

The constitutive equation of a material is a quantitative description of its deformation response to a given load condition. Skimming through the literature about constitutive equations of lead-tin eutectics reveals that a lot of different mathematical forms have successfully been used to describe different types of mechanical behaviour, e.g. relaxation tests, creep data and hysteresis loops in cyclic deformation.

However, as was pointed out by Stone and Rashid [Sto94], the constitutive equation of a material should have 'a well defined phenomenology, a basis in empirical data, and a basis in mechanism'. While most of the proposed constitutive equations come more or less up to these requirements individually, they vary significantly from each other. So, 'What is truth?" (John 18:38)

200 years ago, the German philosopher Immanuel Kant postulated that the perception of an object's quality depends not only on the object itself but also on the means by which the quality is perceived. In our century, quantum mechanics have taught us that Kant's statement was not restricted to philosophical matters only. Accordingly, it is not surprising that different ways of assessing a material's behaviour—according to the assumed phenomenology—would produce 'reasonable' sets of parameters. It is, however, commonplace that these models often fail in predicting data they were not made for. The main reason is the inconsistency between postulated mechanism and phenomenology and mechanism(s) effective in the material.

Therefore, various types of constitutive equations are currently used for solder description. They are reviewed in the following with respect to their appropriateness for modelling different features occurring in lead-tin solder at temperatures and microstructures present in solder joints in electronic applications.

2.2.1 Norton's power-law and some extensions

Already in 1929 Norton [Nor29] proposed an empirical description of steady-state creep rate \( \dot{\gamma} \) dependence on stress \( \tau \) of the form
\[ \gamma_{cr} \propto \tau^n \] (2.3)

where \( \propto \) denotes proportionality. Equation (2.3) reduces for \( n=1 \) to Newtonian viscous flow. The description corresponds to a linear relationship in double-logarithmic stress vs. strain rate plot, where \( 1/n \) equals the slope.

The influence of the temperature is accounted for by an Arrhenius term. The stress is normalised by the shear modulus \( G \) in order to preserve the units of the pre-factor \( A \) as s\(^{-1} \) leading to

\[ \dot{\gamma}_{cr} = A \cdot \left( \frac{\tau}{G} \right)^n \cdot \exp \left( -\frac{Q}{RT} \right) \] (2.4)

Originally proposed as an empirical description, later work revealed the physical meaning of the power \( n \) and the activation energy \( Q \): \( n \)- and \( Q \)-values are indicative for which deformation mechanism is rate controlling. Some of the established combinations of \( n \) and \( Q \) are compiled in Tab. 2.1.

In 1964, Dorn's equation, a special form of Norton's power-law (2.4) was proposed [Bir64]

\[ \dot{\gamma}_{cr} = A \cdot \left( b \cdot G \cdot \frac{D_{\text{eff}}}{kT} \right) \cdot \left( \frac{\tau}{G} \right)^n \] (2.5)

where \( b \) denotes the Burger's vector, \( G \) is the shear modulus, \( k \) is Boltzmann's constant, \( T \) denotes the temperature, and \( A, n \) are material constants. The effective rate of diffusion \( D_{\text{eff}} \) is derived from a combination of bulk diffusion rate originating from Nabarro [Nab48] and Herring [Her50] and the rate of diffusion along phase or grain boundaries going back to Coble [Cob63]:

\[ D_{\text{eff}} = \left( \frac{D_v}{d} + \pi \cdot \frac{\delta \cdot D_b}{d^3} \right) \] (2.6)

with \( d \) being the grain size, \( D_v \) denoting the bulk diffusion rate, \( D_b \) being the diffusion rate along grain or phase boundaries, and \( \delta \) standing for the width of the grain or phase boundary. A third diffusion path, along dislocation
<table>
<thead>
<tr>
<th>n-values</th>
<th>Activation energy $Q$</th>
<th>deformation mechanism</th>
<th>reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Q_m$</td>
<td>Nabarro-Herring-creep</td>
<td>[Her50]</td>
</tr>
<tr>
<td>1</td>
<td>$Q_{gb}$</td>
<td>Coble-creep</td>
<td>[Cob63]</td>
</tr>
<tr>
<td>2</td>
<td>$Q_{gb}$</td>
<td>Grain boundary sliding</td>
<td>[Bal69]</td>
</tr>
<tr>
<td>3</td>
<td>$Q_m$</td>
<td>viscous glide</td>
<td>[Wee60]</td>
</tr>
<tr>
<td>5</td>
<td>$Q_m$</td>
<td>climb, recovery</td>
<td>[Lan83]</td>
</tr>
<tr>
<td>5</td>
<td>$Q_m$</td>
<td>viscous glide low temp</td>
<td>[Lan83]</td>
</tr>
<tr>
<td>3-10</td>
<td>$Q_m$</td>
<td>dislocation climb</td>
<td>[Muk69]</td>
</tr>
</tbody>
</table>

Tab. 2.1: Characteristic $n$- and $Q$-values for various creep mechanisms. $Q_m$ and $Q_{gb}$ stand for activation energy of matrix diffusion and grain boundary diffusion, respectively.

cores, as proposed by Harper and Dorn [Har57], is only considered, if creep at comparatively low temperatures and in structures exhibiting large grains is modelled.

In Dorn's equation, the temperature is accounted for in the hidden values of $D_v$ and $D_b$ which have distinct activation energies and, thus, cannot be handled together. Ignoring this fact, equation (2.5) has later turned up in the form

$$\dot{\gamma}_{cr} = A \cdot \frac{b \cdot G \cdot D_0}{kT} \cdot \left(\frac{\tau}{G}\right)^n \cdot \exp \left[-\frac{Q}{RT}\right]$$

which caused some irritation, as activation energies determined for (2.4) and (2.7) were not the same and seemed to suggest different rate controlling steps. Another point that causes confusion about the activation energy is whether the temperature dependence of the shear modulus $G$ is accounted for or not. Moreover, as pointed out by Padmanabhan and Davies [Pad74], the activation energy is sensitive to whether it is determined at constant stress or at constant strain rate.

---

2For the dislocation climb mechanism as initially described by Weertmann [Wee57,68] a $n$-value of 3-6 was predicted. Experimental evidence rather points to the value $n=7$ for solder as given by Grivas et al. [Gri78,79], though a theoretical explanation is still lacking.
As pointed out in section 2.1, the mechanical behaviour of lead-tin or lead-tin-silver eutectics cannot be described by a single power-law. The rheological analogue (Fig 2.6) offers some help in constructing a combined constitutive equation. However, considering all three stages, there is no analytical solution for general parameters to lead to a constitutive equation. An expedient is provided by the fact that stage I is important at very low strain rates only and may therefore be ignored. Thus, the usual form, including only stages II and III only, is given by

\[ \dot{\gamma}_{cr} = A \cdot \left( \frac{\tau}{G} \right)^n \cdot \exp\left[ -\frac{Q_{dc}}{RT} \right] + B \cdot \left( \frac{\tau}{G} \right)^m \cdot \exp\left[ -\frac{Q_{gb}}{RT} \right] \] (2.8)

However, even this simplification is not easy to treat mathematically in terms of inverting and solving for stress \( \tau \) instead of strain rate, as analytical solutions exist for some special combinations of \( n \) and \( m \) only.

The advantage of (2.8) lies in the separation of mechanisms as each term stands for a distinct mechanism. A constitutive equation of the form of (2.8) has been used by [Ros91], [Kas81], [Gri79], and [Shi88] for modelling the mechanical behaviour of eutectic lead-tin solder while (2.4) has been adopted by Püttnner et al. [Püt94] and Schmidt and Seaman [Schmi90].

A slightly modified model has been presented by Knecht and Fox in 1990 [Kne90]. It featured a separation of the overall strain rate in an elastic, a plastic, and a creep component

\[ \dot{\gamma} = \dot{\gamma}_{el} + \dot{\gamma}_{pl} + \dot{\gamma}_{cr} \] (2.9)

The elastic \( \dot{\gamma}_{el} \) term was related to the stress change rate by Hook's Law while the plastic part \( \dot{\gamma}_{pl} \) was accounted for by a Ramberg-Osgood-type of equation which is usually introduced to model plastic strain assuming strain hardening effects

\[ \dot{\gamma}_{pl} = \frac{\partial}{\partial t} \left( \left( \frac{\tau}{\tau_p} \right)^{\eta_1} \right) \] (2.10)
The time-dependent or creep deformation $\dot{\gamma}_{cr}$ was described by a sum of two power-laws using exponents as previously published by Grivas, Murty, and Morris [Gri79]

$$\dot{\gamma}_{cr} = C_0 \left( \left( \frac{\tau}{\tau_0} \right)^2 + \left( \frac{\tau}{\tau_0} \right)^{7.1} \right)$$  \hspace{1cm} (2.11)$$

with $C_0$ and $\tau_0$ being materials parameter. The temperature dependence is accounted for by varying both $C_0$ and $\tau_0$ with temperature. The usually reported difference in activation energy of the two assumed mechanisms is ignored in that work.

### 2.2.2 The approach of Garofalo

The power law description for steady state creep applies only up to intermediate stress levels while at higher stresses the strain rate sensitivity steadily decreases. This phenomenon is usually referred to as 'power-law breakdown'. In 1963, Garofalo [Gar63] offered a way to describe both regimes with one single equation:

$$\dot{\gamma}_{cr} = C_1 \cdot [\sinh(C_2 \cdot \tau)]^n$$  \hspace{1cm} (2.12)$$

where $\dot{\gamma}_{cr}$ denotes creep strain rate, $\tau$ is the shear stress and $C_1$, $C_2$ and $n$ are material dependent parameters. He took advantage of the fact, that

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \approx x$$  \hspace{1cm} (2.13)$$

for small arguments $x$. The above equation reduces to Norton's law for small stresses $\tau$. Hence, the parameter $A$ from Norton's description can be identified as

$$A = C_1 \cdot C_2^n \quad \text{and} \quad n = n$$  \hspace{1cm} (2.14)$$

Although the model was originally proposed for the description of the power-law breakdown, it has also been successfully applied for modelling
bilinear solder-data in a double-logarithmic plot, cf. [Dar92], [Lau93b], [Pan91]. A similar—though a bit more sophisticated—approach for a creep steady state behaviour is implemented in the commercial FE-programme Ansys.

The main advantage of the above mathematical description is that the function can be inverted, i.e., there are two equivalent descriptions of the creep behaviour, being a strain rate as function of stress and vice versa. Garofalo's description allows an analytical solution while in equation (2.8), the stress in order to achieve a certain strain rate can but be approximated numerically.

On the other hand, the description of Garofalo is not based on mechanisms and phenomenologies. Therefore, significant changes in the constitutive behaviour, as they occur due to recrystallisation, cannot be handled by Garofalo's description.

2.2.3 Hart's Model

While Garofalo's mathematical description provided a single equation for a changing strain rate-sensitivity, i.e., a steadily changing slope in a double-logarithmic stress vs. strain rate plot, it could not solve the problem of incorporating stage I as well. In 1967, Hart [Har67] proposed a mathematical description that incorporated phenomenological aspects as well. The general idea is that grain boundary sliding (gbs) is governed by a linear-viscous law, as later modelled by Raj and Ashby [Raj71], while matrix creep is guided by a power-law. At high stresses, matrix creep dominates the grain boundary sliding, leading to uniform deformation. At very low stresses—owing to the high strain rate sensitivity—grain boundary sliding would be much faster than matrix creep. As gbs alone without any accommodation by matrix creep cannot lead to continuous deformation, the rate of sliding is restricted by the accommodation process. Thereby, the grain or phase boundaries act like flaws reducing thus the load bearing area. Stage II is viewed as a mere transition between stages I and III both governed by matrix creep.
In order to determine the according constitutive equation he investigated two possible rheological analoga which are depicted in Fig. 2.9. The model on the top is just a special case of that shown in Fig. 2.6 (two similar nonlinear damping elements, one linear damping element) and, thus, allows to some extent an analytical treatment.

For the rheological analogue on the bottom of Fig. 2.9 he found the following constitutive equation (for the version on the top the equation is similar but a bit more complicated)

\[
\sigma = \sigma_o \cdot \left[ y + (1 - y) \cdot Z \right] \cdot \dot{\varepsilon}^\mu
\]  

(2.15)

where \( \sigma \) and \( \dot{\varepsilon} \) are stress and strain rate, respectively, and \( \sigma_o \) and \( \mu \) are material constants while \( y \) is a model parameter that includes the grain boundary density or the grain size. The variable \( Z \) denotes the ratio of the stresses transmitted through the matrix \( \sigma_m \) and over the grain boundary \( \sigma_b \)

\[
Z = \frac{\sigma_b}{\sigma_m}
\]  

(2.16)
The ratio that defines $Z$ is obviously not trivial. However, Hart provides a numerical method to obtain values for $Z$. He was thus able to model a set of data published by Cline and Alden [Cli67].

2.2.4 The $\lambda$-law and other theories of unified plasticity

Another idea that owes its origins to a publication of Hart [Har70] is the so called '\(\lambda\)-law'. The original idea is to describe unified plasticity, i.e., plastic and creep strain, by state variables, using the 'hardness-parameter' $\sigma^*$ as state variable. The plastic strain rate $\dot{\varepsilon}$ is correlated to the stress $\sigma$ by

$$\ln\left(\frac{\sigma^*}{\sigma}\right) = \left(\frac{\dot{\varepsilon}^*}{\dot{\varepsilon}}\right)^\lambda$$

(2.17)

While $\lambda$ is a numerical parameter, the variable $\dot{\varepsilon}^*$ is connected to the hardness-parameter $\sigma^*$ by a power-law as given in equation (2.4). The hardness-parameter $\sigma^*$ evolves by a rate equation in order to account for hardening and recovery. In case of stationary structure, i.e., $\sigma^*=\text{const.}$, (2.17) describes a steady decrease of strain rate-sensitivity with increasing stress. This framework has been adopted for modelling of eutectic solder by Stone et al. [Sto85]. He used different sets of parameters for matrix creep and grain boundary flow by applying Hart's model. He was able to account for Hall's data of stress temperature evolution of a simple SMD quite well.

State variable approaches are usually used in unified plasticity theories, i.e., theories that combine plasticity and creep features. To this class of models count among others the model of Korhonen, Hannula and Li [Kor87], which is a generalisation and extension of Hart's model and the approach of Bodner and Partom (Bod87). Both models are well elaborated and hence provide a complete framework of constitutive equations. There are even models to account for specific straining effects such as Bauschinger effect, microplasticity, directional hardening, and anelastic effects provided. The description of these effects, however, requires a good deal of experimental evaluation while their quantitative contribution in modelling of solder joints may be negligible.

The form of the stress vs. strain rate relation is similar to that produced by Garofalo's sine hyperbolic-law. It may be shifted according to the evolution
of state variables but does not change its general form unless it is incorporated in Hart's model and the state variables for matrix and grain boundary creep are changed independently as performed by Stone et al. [Sto85] for the $\lambda$-law. However, the approach of Bodner and Partom has recently been applied to constitutive modelling of eutectic lead-tin solder [Ski96].

2.2.5 Summary

The common constitutive descriptions for creep phenomena have been presented. As most of them have shown their aptitude to model eutectic lead-tin solder behaviour, the choice of which to take must be based on the effects to be modelled.

As it was reported by Guo et al. [Guo92a], the strain rate sensitivity of lead-tin eutectic increased with increasing accumulated strain, while the stresses in order to achieve high strain rates changed but insignificantly. Preliminary experiments have confirmed such behaviour. Furthermore, microstructural changes due to recrystallisation have been found to accompany this transition having a significant impact on the creep behaviour of the solder. As none of the above presented model is able to account for this in a direct manner, a new model has to be established basing on the extended version of Norton's law (equation (2.8)).

The phenomenological description and the equation's framework for this new model including transient phenomena is presented in Chapter 5.

2.3 Fatigue lifetime estimations

In microelectronic applications the solder joint is usually exposed to cyclic temperature changes and thus, because of the CTE mismatch, to mechanical cyclic loading as well. As a large number of field failures of electronic assemblies are due to broken solder joints, lifetime estimation models for solder in a thermomechanical fatigue environment are crucial. The customary models are shortly presented in the following with emphasis on their aptitude for FE-modelling.
2.3.1 The approach of Coffin and Manson and related models

Fatigue lifetime assessment owes its origins to the German engineer August Wöhler, who published some basic results on the topic, e.g. a relationship of stress amplitude to cycles to failure, in 1856. His results were restricted to the elastic deformation range and hence talking about cycles to failure >10⁴. At the beginning of our century, Basquin [Bas10] established the equation

\[ N_f^{\beta} \cdot \Delta \sigma = C_B \]  

which related the stress amplitude \( \Delta \sigma \) to the number of cycles to failure \( N_f \), where \( \beta \) and \( C_B \) are materials parameter, and \( \beta \) is in the range of 0.1...0.15. Half a century later, Coffin [Cof54] and Manson [Man53] proposed independently a method to assess fatigue lifetime for structural materials based on the plastic strain amplitude. As under those conditions, the number of cycles to failure is usually <10⁴, their results provided an extension of Wöhler's and Basquin's description. Accordingly, there is a distinction between low cycle fatigue (LCF) for failure caused by plastic strain amplitudes and high cycle fatigue (HCF) for failure caused by cyclic elastic strains only. As cyclic deformation in solder joints is of predominantly plastic character, models describing LCF are generally applied.

The relation of number of cycles to failure \( N_f \) and the plastic strain amplitude \( \Delta \varepsilon_{pl} \) proposed by Coffin and Manson is

\[ N_f^{\alpha'} \cdot \Delta \varepsilon_{pl} = C_{c-m} \]  

where \( C_{c-m} \) and \( \alpha' \) are materials constants\(^3\). Mark the similar structure of (2.18) and (2.19). As \( \alpha' \) has usually values of \( = 0.5 \), doubling of the plastic strain amplitude \( \Delta \varepsilon_{pl} \) reduces the number of cycles to failure by a factor of about 4.

In the following, some modifications of the Coffin-Manson relation are considered.

\(^3\)\( C_{c-m} \) is sometimes related to the elongation to fracture \( \varepsilon_f \) by \( C_{c-m} = (1/4)^{\alpha'} \varepsilon_f \)
1. The frequency-modified Coffin-Manson relationship

While (2.19) is able to model most of the data set of various metals and alloys, significant deviation occur if testing conditions namely cycle shape and cycle frequency are varied. In case of viscoplastic materials (e.g. solders at and above ambient temperature) the impact of frequency is especially important. There is great need for a modification of the Coffin-Manson relation that includes the test frequency \( v \). A first approach to come up to this requirement has been proposed by Coffin [Cof71], [Cof73] as

\[
\left( N_f \cdot v^{k-1} \right)^\alpha \cdot \Delta \varepsilon_{pl} = C_{c-m} 
\]

(2.20)

where \( v \) denotes the test frequency and \( k \) is a numerical parameter. Obviously, for \( k=1 \), (2.20) transforms to (2.19). This is the case for high frequencies and low temperatures, whereas at high temperatures and low frequencies (\( v \) in the order of \( 10^{-3} \) s\(^{-1} \)), \( k \) comes close to zero [Sol88] and, hence, the time to failure becomes constant at a fixed plastic strain amplitude. The frequency dependent results are often due to difficulties in separating the elastic and plastic strains. Especially when the total strain—which is usually controlled—is small, the relaxation at longer times or lower frequencies may change the ratio of elastic to plastic strain. This problem is accentuated if thin film shear specimen are used where the elastically stored deformation of the substrate exceeds by far the plastic deformation in the solder joint.

2. The strain range partitioning approach

An alternative way to account for the shape and the frequency of the load cycle is the strain rate partitioning (SRP) approach first proposed by Manson et al. [Man71] and later extended by Halford and Manson [Hal76]. The general idea is to distinguish whether a cycle shape produces instantaneous plastic strain or creep strain and whether the one or the other is encountered in tension or compression. A further assumption is that for every 'pure' form of the duty cycle (i.e., creep in tension and compression or instantaneous plastic flow in both or creep in tension and plastic flow in compression or vice versa) an equation of the form of (2.19) holds. Be \( F_{ij} \)
the part of the cycle with condition $i$ in tension and $j$ in compression (with $ij...c(\text{creep}), p(\text{lastic flow}))$ then the number of cycles to failure $N_f$ is:

$$\frac{1}{N_f} = \frac{F_{cc}}{N_{cc}} + \frac{F_{pp}}{N_{pp}} + \frac{F_{cp}}{N_{cp}} + \frac{F_{pc}}{N_{pc}}$$  \hspace{1cm} (2.21)$$

with $N_{ij}$ being the number of cycles to failure for the pure cycle shape $ij$. This is obviously a special case of the Palmgren-Miner-law that predicts failure if

$$\sum_i \frac{N_i}{N_f} = \theta$$  \hspace{1cm} (2.22)$$

with $\theta = 1$, if no interaction of the various steps or duty parts $N_i$ occurs.

There are two applications of the SRP-approach which deserve special mentioning: The first is that proposed by Solomon (SoI88) in order to account for the case where $k$ (from (2.20)) becomes zero and thus the time to failure becomes constant (for a given plastic strain amplitude). Then, for symmetric cycles (2.21) reduces to

$$\frac{1}{t_f} = \frac{F_{cc}}{t_{cc}} + \frac{F_{pp}}{N_{pp}} \cdot \nu$$  \hspace{1cm} (2.23)$$

where $t_f$ is the time to failure, $t_{cc}$ denotes the time to creep failure, and $\nu$, $F_{cc}$, $F_{pp}$, and $N_{pp}$ have their above defined meaning.

Another interesting situation follows from the approach presented by Shine and Fox [Shi88]. They assume that according to the constitutive behaviour two distinct mechanisms have part in deformation which they call matrix creep and grain boundary sliding. While matrix creep (mc) takes place at higher stresses, the grain boundary sliding part (gbs) is prominent at low strain rates. They formulated analogously to the SRP approach:

$$\frac{1}{N_f} = \frac{F_{mc}}{N_{mc}} + \frac{F_{gbs}}{N_{gbs}}$$  \hspace{1cm} (2.24)$$

where $F_i$ and $N_i$ have similar meanings as in (2.21) with $i...gbs$ or mc. As in superplastic materials the large deformations are only achieved if extensive
grain boundary sliding is present, it was further assumed that $N_{gb} \gg N_{mc}$. Then (2.24) reduces to

$$N_f \cdot \gamma_{mc} = C_{S-F} \tag{2.25}$$

as $\gamma_{mc} \propto F_{mc}$. $C_{S-F}$ is determined to 890%. This result is especially interesting for FEA implementation as it allows for assessment of damage by simple accumulation of the overall strain independent from the strain amplitude per cycle. For Coffin-Manson parameters with values $\neq 1$, the damage parameter $\omega$, with $\omega = 0$ for no damage and $\omega = 1$ for complete damage, may be estimated using the Palmgren-Miner-law and sum over the different strain amplitudes $i$ already encountered:

$$\sum_i \frac{N_i}{N_f} = \omega \tag{2.26}$$

It is thus possible to apply a 'localised Coffin-Manson approach' for local damage evolution to a general joint geometry where the strain amplitude varies throughout the joint.

The main difficulty with the whole SRP approach and its variations is the determination of the portion of the various 'pure' cycle shapes. Moreover, for the general description as in (2.21) at least 8 parameters have to be determined independently.

3. The deformation energy approach

Another variation of the Coffin-Manson equation (2.19) is given by

$$N_f^{\alpha} \cdot E_{cyc} = C_{de} \tag{2.27}$$

where the plastic strain amplitude is replaced by the deformation energy, given by the area of the hysteresis loop expressed by $E_{cyc}$ and the remaining symbols keep their usual meaning. This description is ascribed to Morrow [Mor65] although he provides even earlier sources. It seems to be a matter of taste whether the original approach of Coffin and Manson or the Morrow's model is used.
There is also a comprehensive approach to solder joint lifetime prediction presented by Engelmaier, cf. e.g. [Eng83], [Eng85], [Eng92]. Starting from a Coffin-Manson kind of equation, his model derives the total strain amplitudes from geometrical data of the assembly. For leadless components, he gives

\[ N_f = \frac{1}{2} \left[ \frac{F \cdot L_D \cdot \Delta \alpha \cdot \Delta T_e}{C_{c-m} \cdot h} \right]^{1/\alpha'} \]  

(2.28)

and for leaded components results

\[ N_f = \frac{1}{2} \left[ \frac{F \cdot K_D \cdot (L_D \cdot \Delta \alpha \cdot \Delta T_e)^2}{C_{c-m} \cdot A \cdot h \cdot 1.38 \text{MPa}} \right]^{1/\alpha'} \]  

(2.29)

where \( L_D \) denotes half the distance between two solder joints, \( \Delta \alpha \) is difference of the thermal expansion coefficient of PCB and device, \( \Delta T_e \) stands for the effective temperature range, \( h \) is the height of the solder joint, \( A \) is the load bearing area, \( K_D \) stands for the diagonal flexural stiffness of the lead, and \( F \) represents a free numerical parameter, which does not vary considerably from 1. The Coffin-Manson parameter \( \alpha' \) is given as

\[ \alpha' = -0.42 - 0.0006 \cdot T_a + 0.0174 \cdot (\ln(1 + 360/t_D)) \]  

(2.30)

where \( T_a \) denotes the average temperature in °C and \( t_D \) stands for half the cycle dwell time in minutes.

While the accuracy of the result depends to some extent on the free numerical parameter \( F \), Engelmaier's model is a good rule of thumb for a wide range of applications, though extrapolation to not yet assessed geometries is to be called in question.

### 2.3.2 Crack propagation and fracture mechanics

In the Coffin-Manson approach the way by which the solder joint fails is not specified. However, evolution of damage may well be considered as the
growth of a single crack. This is the case in joint failure estimations based on fracture mechanics considerations.

The basic approach has been proposed by Paris and Erdogan \[Par63\].

\[
\frac{da}{dN} = C_P \cdot \Delta K_{II}^{n_P} \tag{2.31}
\]

where the stress intensity range $\Delta K_{II}$ is related to the crack propagation per cycle $da/dN$. $C_P$, $n_P$ are material and environment dependent parameters. As the stress intensity depends on the actual crack length $a$ by $\Delta K_{II} \propto \tau \cdot \sqrt{a}$, the number of cycles to failure $N_f$ can be determined by integration of (2.31) leading to

\[
N_f = \int_{a_o}^{a_f} \frac{da}{C_P \cdot \left(\tau \cdot \sqrt{a}\right)^{n_P}} \tag{2.32}
\]

where $a_o$ is the initial crack length and $a_f$ is the crack length at failure, i.e., the length of the solder joint. The crucial assumption is the correct choice of the initial crack length, as most of the life will be spend while the crack is still small. However, as was shown by Solomon \[Sol72\] this approach may lead to erroneous results as it is based simply on stress state considerations neglecting both plastic deformation range and time. Hence, fatigue life prediction based on stress intensity ranges only should be treated with caution.

These problems might be overcome by adopting the J-integral approach cf. \[Ric68\], \[Dow76,77,79\]. Instead of stress intensity, the J-integral uses the energy dissipated by plastic deformation in the region ahead of the crack. Thus, the crack growth rate is given as

\[
\frac{da}{dN} = C_J \cdot \Delta J^{n_J} \tag{2.33}
\]

where again $C_J$ and $n_J$ are material and environmental parameters. Although the J-integral is not defined for unloading, equation (2.33) has successfully been applied to modelling eutectic lead-tin solder \[Guo92b,92c\]. The
correlation of the cyclic strain energy, i.e., the area of the hysteresis loop, to the crack growth rate has been comparatively successful [Guo92c].

Recently, the viscous analogue to the J-integral approach, the C*-integral, has been adopted to fatigue crack growth modelling cf. [Pao90,92]. The data sets investigated by this approach are still rare, although preliminary results exhibit good agreement with data from the literature.

A major advantage of the modelling of the crack growth is that it is well-suited for FEA. However, the implemented constitutive equation has a tremendous impact on the crack growth rate, as stresses and strains are of major importance. Furthermore, the assumption of one single crack growing through the solder joint might be called in question; there is rather some microcrack growth and coalescence reported [Sol94].

2.3.3 Homogeneous damage

The direct opposite to the single crack growth modelling is the assumption of damage which is homogeneously distributed, i.e., it has no specific form. The damage parameter $\omega$ can be given as reduced load-bearing area:

$$\omega = \frac{A_o - A}{A_o} \quad (2.34)$$

where $A_o$ is the initial and $A$ the actual load-bearing area. This concept has first been proposed by Kachanov [Kac58] (see also [Kac86] and [Lem92]).

The value of the damage is not necessarily constant throughout the whole structure under consideration, i.e., the solder joint. The homogeneous distribution is assumed for a volume element with given damage parameter, but it may well differ for any neighbouring volume element. The effective stress acting on the material is then

$$\sigma_{\text{eff}} = \frac{\sigma}{1 - \omega} \quad (2.35)$$
where \( \sigma \) denotes the smeared stress. As the constitutive behaviour is considered to be independent of the damage state, the damage may be introduced in any constitutive equation by replacing the applied stress by the effective stress. This kind of description is well-suited for FE-applications.

The evolution of the damage parameter \( \omega \) is usually given in form of a rate equation. The main challenge is therefore to determine the variables and their influence on the damage evolution. Another difficulty is to assess the state of damage which is performed by monitoring depending variables, e.g. electric resistivity, load drop, Young's modulus, density, hardness etc., as discussed by Pluvinage and Raguet [Plu83]. Direct access might be provided by ultrasonic microscopy as shown by Solomon [Sol86].

Solomon [Sol91] assessed the damage evolution in eutectic (60-40) tin-lead solder in shear by monitoring the shear load drop in function of the number of cycles. Provided the constitutive behaviour does not change its characteristic throughout the test, the load drop is related to the damage parameter \( \omega \) by

\[
\omega = \frac{\Delta P_m - \Delta P}{\Delta P_m} \tag{2.36}
\]

where \( \Delta P \) denotes the actual loading range and \( \Delta P_m \) the initial (or after early strain hardening: the maximal) loading range. He finds that the damage evolves according to

\[
\omega = A_\omega \cdot N^{B_\omega} \tag{2.37}
\]

where \( N \) is the number of cycles and \( A_\omega, B_\omega \) are numerical parameter. \( B_\omega \) varies between 0.4 and 1.4 being smaller for smaller strain ranges.

In general, the state of damage \( \omega \) can be expressed by a modified Palmgren-Miner-law:

\[
\omega = \sum_i N_i \cdot \left( \frac{\partial \omega}{\partial N_i} \right) \tag{2.38}
\]
where \( i \) denotes the different strain ranges in terms of size and \( N^i \) the number of cycles performed of each type \( i \). Equation (2.38) requires that the damage rate per cycle is independent of the actual damage state parameter \( \omega \).

The homogeneous damage approach differs considerably from the ideas presented in the previous subsections. On the one hand, the form of damage evolution is distinct from that assumed in single crack growth: While in crack propagation theories a volume element is broken (or not), the homogeneous damage approach allows for a continuous transition from \( \omega = 0 \) to \( \omega = 1 \). On the other hand, there is no scaling applied in the homogeneous damage approach in order to transform results determined on large specimen onto small scale specimen, e.g. solder joints. This implies that the form of the damage can be scaled arbitrarily, while microstructural investigations usually report on cracks along grain-boundaries, which are the same in large scale specimen as in small ones. On the other hand, single crack propagation as required for the analysis in subsection 2.3.2 is usually the case neither. Microcrack coalescence as 'real' damage behaviour lies somewhere in between the two approaches discussed.

### 2.4 Mechanical testing strategies

One of the basic assumptions in mechanical testing is the independence of specimen geometry and mechanical behaviour. While this assumption holds at a macroscopic level, it is no longer valid if the dimensions of the deforming volume are of the order of the microstructural unit of the material, i.e., grain or phase size. As was shown in Fig. 2.4, a solder joint in the SMT is made up of but a few eutectic colonies. Therefore, the mechanical behaviour of the solder in SMT solder joints may differ from that encountered in bulk specimen.

In recent years, several publications have dealt with determining the mechanical behaviour of solder material in thin solder joints. The various specimen geometries and testing procedures in use are shortly reviewed and discussed in the following.
2.4.1 Specimen design

As solder joints in SMD's are supposed to deform mainly in simple shear, most of the proposed specimen geometries are designed for shear testing. A synopsis of these various geometries is given in Fig. 2.10.

The solder joint are in general compared to the whole testing structure consisting of the loading substrate and the joint. The assemblies either have an axis (or a point) of symmetry in the specimen, Fig. 2.10a,b or there is a plane of symmetry, Fig. 2.10c,d. The former are referred to as 'single lap-joint specimen' while the latter are called 'double lap-joint specimen'. The single lap joint has often been criticised since there is a bending moment in the substrate due to the non-collinearity of the load. This causes additional forces perpendicular to the solder joint and rotation of the solder joint plane resulting in complex states of stress. Undoubtedly, design b in Fig. 2.10 is an attempt to reduce this problem. However, the problem is not solved since

![Fig. 2.10: Different designs of test specimen for thin solder joint shear testing. a) single lap-joint specimen [Mei90] b) single lap-joint [Guo92a] c) double lap-joint [Tri88] and d) double lap-joint specimen [Mei92].](image-url)
close to the solder joint the forces do not act on the same line.

Double lap-joint specimen, on the other hand, are suggested to solve the problem owing to their higher symmetries. They don't really. They simply inhibit rotation of the solder joint while the problem of bending still remains. As mechanical analysis of the single lap-joint specimen in appendix A shows, the rotation of the solder joint plane leads to normal forces in the solder joint of about one thousandth part of the shear force while bending resulted in normal stresses of a twentieth of the shear stress.

Since there is in fact no significant difference between single lap-joints and double lap-joint specimen in terms of the mechanical situation, a distinction may result from other features, e.g. production process and/or testing requirements. One important point is the constancy of the solder joint's thickness. Although at solder joint thickness of 200 to 300 μm the problem of co-planarity is not that severe, the designs a,b, and d of Fig. 2.10 lack the possibility to control the achieved solder joint thickness before testing. Moreover, non-destructive microstructural investigations before and during testing are not feasible. For even smaller joint thickness of about 50 μm as encountered in SMT, it is very difficult to produce double lap-joint specimen with two solder joints of identical thickness.

The state of stress is far from simple shear close to the edge of the solder joint as shear stresses must be zero at free surfaces and the stress tensor is symmetric. As a rule of thumb the simple shear state is achieved at a distance thrice the joint thickness from the solder joint's edge. Thus, a large coherent soldered area minimises these boundary effects.

However, special care is required to avoid internal voids and bubbles in soldering large areas. This becomes a severe problem for solder joints thinner than 80 μm. As the retaining forces acting onto the bubbles by surface friction are about constant, the driving force owing to the buoyancy decreases with decreasing joint's thickness.

As mentioned above, the joint thickness has usually been kept in the range between 150 and 300 μm in recent investigations [Mei92, Fre88, Guo92a]. Thus, its dimension is about three orders of magnitude smaller than that of the load frame. The elastic strain of the latter (typically about 0.1%) may
well lead to displacements in the order of the plastic strains within the solder joint, as discussed in the following.

2.4.2 Unidirectional loading

Isothermal testing at various temperatures is considered the most appropriate way to determine constitutive behaviour. If ever thermomechanical fatigue is to be modelled by FEA, the material's behaviour will be described in terms of isothermal data.

There are three testing procedures for the determination of time-dependent plastic behaviour: 1) the creep test, during which the load (or the stress) is kept constant and the strain is monitored versus time; 2) the relaxation test, where the total strain (or more likely the displacement) is kept constant and the stress level at different times is recorded; and 3) the tensile test, where the strain rate (or in fact the cross-head speed) is kept constant and the stress is measured in function of the strain.

The existence of a constitutive equation suggests that determining the mechanical behaviour by one single testing procedure should enable to predict the behaviour in any other testing procedure. However, as was reviewed by Stone and Rashid [Sto94] there are significant differences in constitutive parameters by determining them in different testing procedures.

The main reason for these differences are due to the fact that general assumptions usually connected with different testing methods are no longer valid if testing is performed at small solder joint specimen instead of comparably large bulk specimen.

As for the creep test the general assumption is that the strain rate decreases in a first stage until it reaches a minimum value, the so-called steady-state or stage II, followed by the stage of tertiary creep where the strain rate steadily increases until rupture takes place. This picture is encountered in a large number of typical structural materials and is schematically drawn in Fig. 2.11. Without going into further detail, it results from building of a stable dislocation network in primary creep, deforming in a state where strain hardening (i.e., further production of dislocations) is consumed by
recovery (i.e., annihilation of dislocations) in secondary or steady state creep, and ending in tertiary creep where internal and external necking increase the effective stress causing increased strain rates. Since at constant stress most of the lifetime is experienced in the secondary creep stage, the steady state creep rate is usually used to describe the creep behaviour of a material.

Eutectic tin-lead solder experiences a significant change in the microstructure due to local recrystallisation after some percent creep strain, resulting in a considerable increase in the creep rate as we will show in Chapter 4. Unlike other materials, eutectic tin-lead solder encounters a large part of its strain to fracture in the seemingly tertiary creep state. The key to this is that the increase of the strain rate is but a transition to another steady state and not a continuous acceleration to rupture.

As for the testing procedure, the creep test causes the following difficulties: since the load is kept constant, the elastic deformation of the load frame and

![Diagram](image)

**Fig. 2.11**: Schematic creep behaviour of a typical structure material: primary creep (I), secondary creep (II) with constant deformation rate, and deformation until rupture in tertiary creep (III).
the substrate are constant as well except for some anelastic effects at the beginning of the test. If stepped creep testing is applied, i.e., tests in which the stress is changed after a certain amount of strain or time, there might arise some problems especially at low strain rates (<5·10^{-6}s^{-1}) owing to anelastic effects, i.e., time dependent elasticity, in the substrate.

If stepped-load creep testing is applied, the problem arises whether there is a change in microstructure during the test that would result in a change in constitutive behaviour. This can be reduced if strain rates are measured over small strain ranges only. Then, however, anelastic effects might influence the results considerably.

Measuring strain rates over small strains is the characteristic feature of the relaxation test. In fact, for the relaxation test the eminent assumption is that the total strain is constant during the test, i.e.

\[ \varepsilon_{pl} + \varepsilon_{el} = \text{const}. \]  

(2.39)

where \( \varepsilon_{el} \) denotes the elastic and \( \varepsilon_{pl} \) the plastic strain during the test. Differentiation of the above equation with respect to time leads to

\[ \dot{\varepsilon}_{pl} + \dot{\varepsilon}_{el} = 0 \]  

(2.40)

Since the elastic deformation is proportional to the stress (Hook's law), equation (2.40) equals

\[ \dot{\varepsilon}_{pl} = -\sigma/E \]  

(2.41)

For bulk specimen, where the elastic strain is in the order of 10^{-3}, relaxation testing results in measuring strain rates over some orders of magnitude within a plastic strain range of 10^{-3} provided the elastic elongation of the specimen is larger or of the same order as that in the load frame. As was outlined above, the last assumption is no longer valid for thin solder joint specimen. In this case, the elastic strain in (2.40) is to be replaced by the elastic elongation of either the load frame if testing is controlled by cross-head displacement or the part of the substrate between the clips of the elongation measuring unit. Moreover, the Young's modulus is to be replaced by the ratio of assembly's stiffness \( k \) to the soldered area \( O \).
If relaxation testing is performed at constant cross-head displacement the plastic strain encountered by the solder joint may well be in the order of 100%. Thus, testing should not be expected to have taken place at constant microstructure. Controlling the test by keeping the elongation of the clip gauge constant may reduce the total plastic strain encountered by the solder joint down to some few percents and comes quite close to testing at constant microstructure. However, such tests require highly sophisticated equipment.

On the other hand, relaxation testing within some few percent plastic strain reduces the influence of anelastic effects while it still provides creep data over a wide range of strain rates.

Tensile testing, i.e., testing at a given strain rate, is usually not supposed to raise any difficulties. However, if the stress changes during the test, it becomes crucial whether the test is controlled by a constant cross-head speed or by constant rate of elongation within the strain gauge. The plastic strain rate $\dot{\gamma}_{\text{eff}}$ encountered by the soldered joint can be written as

$$\dot{\gamma}_{\text{eff}} = \dot{\gamma}_o - \frac{\dot{t} \cdot O}{k \cdot h}$$

where $O$ is the soldered area and $h$ stands for the solder joint thickness. $\dot{\gamma}_o$ denotes the rate of displacement imposed by the controlled movement of the cross-head or the clips of the strain gauge. Again, $k$ is the stiffness of the load frame and $\dot{t}$ denotes the rate of load change. As the joint thickness becomes small, the strain rate due to the decreasing load may be well larger than the strain rate due to the regular displacement of the cross-head or clips, respectively. Hence, in order to investigate any strain-hardening or softening within lead-tin eutectic at constant strain rate, it is crucial to control the strain rate by means of a clip gauge fixed as close as possible to the solder joint. Alternatively, the additional strain rate due to load changes may be accounted for in the testing software.

### 2.4.3 Cyclic loading

While in unidirectional testing the aim is mainly to determine the parameters required for the constitutive equation, cyclic testing is applied in order to simulate field conditions and their impact on fatigue life and relia-
bility of an electronic assembly. Owing to the shift of load cells, the thermal expansion and the shift of the clip gauge and the thermal expansion of the load frame that accompanies temperature changes, cyclic testing is usually performed either isothermally by means of a cyclic changes of stress in a tensile test machine or a fatigue apparatus or by cyclic variation of the temperature in a heat chamber. In the latter case, the specimen is fixed in a load frame whose thermal expansion differs from that of the specimen. Thus, cyclic deformation is applied in phase with the temperature cycles.

**Isothermal testing**

Isothermal testing is the most common way to assess fatigue behaviour of solder material. If standard tensile test equipment and bulk specimen are used, the parameters of testing, i.e., stress levels, strain amplitudes, and strain rates, can easily be measured and controlled. For simplicity's sake total strain controlled cyclic tests are performed frequently, which is considered to provide results comparable to plastic strain controlled testing [Sol94]. Testing in the total strain control mode, the testing frequency becomes important, especially at low total strain levels [Vay89]. This is due to increasing fractions of plastic strain the more time is allowed to convert elastic in plastic strain by relaxation.

Using shear test specimens, problems arise at decreasing joint thickness. This is owing to the increasing stiffness of the solder joint compared to the load frame and substrate [Sub89], [Wil90b]. In this case, a displacement, measured by either clip gauge or cross-head movement, is not necessarily a plastic strain in the joint if the load changes as well. In displacement controlled testing, the displacement which is elastically stored in the substrate may exceed the plastic amplitude of the solder joint. Here, the cycle frequency is crucial in order to control the plastic strain owing to relaxation.

Hence, tests on thin solder joints should be performed with care. The simplest way to omit the mixing up of plastic strains in the joint and release or accumulation of elastic strains in the substrate or load frame is to test in constant load controlled mode.
Thermomechanical testing

While isothermal testing is accessible easily, thermomechanical testing cannot be performed with standard test equipment. In general, there are two ways of accomplishing it:

On the one hand, intensive testing on common device/PCB combinations is performed by means of thermal shock tests or in heat chambers with controlled temperature gradients and dwell times. While this produces quite reliable data for the combination under testing any extrapolation to other assemblies is difficult owing to the complex mechanical situation in leaded components. However, the reliability prediction model of Engelmaier is based on such data.

On the other hand, some testing is performed on very simple geometries in order to separate the impact of complex geometry and material behaviour. The general idea is to join two stripes of materials with different thermal expansion coefficients and expose them to a given temperature-time history. The assemblies used by Frear [Fre88] and Stone [Sto85] are shown in Fig. 2.12. The assembly proposed by Stone (a) exhibits a nearly homogeneous mechanical situation in the solder joint (provided the solder joint is small compared to the distance to the symmetry plane). On the other hand, Frear's specimen (b) bears some problems: the strain is not homogeneously distributed throughout the solder joint: it is highest at the edges and nearly

Fig. 2.12: Some simple specimen designed for thermomechanical fatigue testing. The thermal expansion mismatch is provided by an Al-Cu couple.
vanishing at the centre.

In the 80's, Hall [Hal83,84,87] published a method to measure the strains and stresses occurring in the solder joints of flat and stiff devices, e.g. a ceramic resistor or a ceramic capacitor or a LCC(C). This method is based on measuring the elastic deformation of the component, including bending, and its changes due to temperature changes, from which the corresponding stresses in the solder joint are derived. The bending is measured by differential strain method and holographic interferometry. His data have often been referenced since in order to validate the predictions of any constitutive equation determined in isothermal testing, see Chapter 5.3 for further comments.
3

Experimental procedure

3.1 Geometry of the test specimen

As discussed in the previous chapter, many different designs of shear test specimens have been proposed in the literature. Some of them are single-lap-joint specimen as shown in Fig. 3.1, while others for reasons of higher symmetry have been designed as double lap joint specimen (Fig. 2.10 d). However, mechanical analysis reveals that the state of stress in a single lap joint is quite close to simple shear, cf. Appendix A.1

In order to choose the appropriate specimen design, one has first to formulate the goals of the testing and, thus, the boundary conditions. The latter are in this case as follows:

Fig. 3.1: Picture of a single-lap-joint specimen as used in this work.
• Since real solder joints in electronics consist of but two or three large eutectic colonies and as it has been found in microstructural investigation that coarsened shear bands built at colony boundaries, the thickness of the solder joint in testing is to be kept in the same order of magnitude as in SMT applications, i.e. about 50 µm. Accordingly, a processing route is to be established that restricts solder joint thickness variations to be smaller than 5 µm.

• As microstructural investigation are planned to take place before, during, and after testing, a specimen's geometry has to be chosen that offers direct access to the joint by means of light microscopy and allows direct checking of the joint thickness as well.

• Measuring the creep rates in small shear test specimen always means measuring the rate of displacement at the substrate and divide it by the joint thickness assuming that there is no creep in the substrate. It is well known that usual creep rates of possible substrate materials, i.e. copper, PCB material (reinforced epoxy), are much slower than those of lead-tin solder. One though has to take into consideration the different dimensions of the creeping objects. The substrate usually creeps over a gauge length which exceeds the joint thickness by a factor of $10^3$. Thus, the creep rates of the substrate have to be slower than $10^{-5}$ times the solder creep rate at a given force. This problem is accentuated the more the solder joint area exceeds the load bearing area of the substrate.

• Since the data are evaluated in terms of shear stress and shear creep deformation, care is to be taken by inducing such a state of stress. However, a field of simple shear stress is usually not attained near an edge due to the symmetry of the stress tensor. Therefore, a solder joint should exhibit a high (>50) width-to-thickness ratio.

Although various specimen geometries and processing routes have been established in recent work (Section 2.4), none of them seems appropriate for our case. Thus, a single lap joint specimen is designed that fulfils the above mentioned requirements (Fig. 3.1). The requirement of a controlled solder joint thickness is accounted for by placing an aluminium foil mask of required thickness between the substrate plates defining the solder joint's geometry (Fig. 3.2). The whole assembly is clamped together during the
soldering process. Grinding and polishing on both sides allows to check the joint thickness as well as to monitor the evolution of the microstructure during testing. Since the creep resistance of pure copper is considered to be too small, a CuSn8 bronze has been chosen as substrate material.

3.2 Processing of the specimen

Two CuSn8 bronze platelets of 15-20 mm width, 45 mm length and 3 mm thickness were ground and polished on a copper/diamond disc since grinding on emery paper would impair the planarity of the platelets by significant cambering. The platelets were cleaned with acetone and covered with a Multicore FSW 33 flux. Then the aluminium mask was placed between the substrate plates and a screwable clamp was tightly set around the whole assembly. Subsequently, the whole assembly was dipped for 20 to 30 seconds into a solder bath at 230\(^\circ\)C that was vibrated at about 400 Hz in order to drive out any air bubbles that might still be in the joint.

To take care of the differences in fatigue behaviour depending on the cooling rate of soldered joints [Mei92], the specimen were either quenched in water or slowly cooled by placing them into a furnace at 230\(^\circ\)C and switching it off. Thus, cooling rates at 180\(^\circ\)C in the order of 100\(^\circ\)C/s and about 0.04\(^\circ\)C/s were achieved, cf. Fig 3.3.

After solidification, the solder was removed on both sides of the specimen making microstructural investigation and joint thickness inspection possible.
Fig. 3.3: Different cooling rates for quenched and furnace cooled specimen measured by placing a thermocouple in the solder joint.

On the areas where the specimen was clamped in the test machine any excess solder was removed as well.

The microstructure of eutectic lead-tin directly after solidification is thermodynamically unstable due to the large inner surface provided by the fine grained eutectic structure. Moreover, significant reduction in the solid solution limit of lead in tin and vice versa causes precipitation of secondary lead-rich and tin-rich phases. Therefore, some specimen were tested after ageing for 10 hours or 100 hours at 100°C while the bulk of the specimen were tested after ageing for 1 to 20 days at ambient temperature.

Since localised straining has often been reported in previous investigations, the impact of the joint thickness on overall straining behaviour should be appreciable as soon as localisation takes place. Therefore, some tests have been performed on specimen with 25μm, 60μm, and 240μm solder joint thickness in both the furnace cooled and the quenched condition as well as aged for 100 h at 100°C and as-soldered.
3.3 Testing strategies

3.3.1 Unidirectional testing

There are some constraints in testing since some crucial assumptions for tensile testing and relaxation testing do not hold in case of thin solder joints. The problem is generally due to the fact that the test conditions, i.e. constant rate of displacement and constant displacement, respectively, are controlled by cross-head speed and cross-head position. This problem can be reduced by controlling the test using an extensiometer.

The tests were run on a Zwick020 universal testing machine with a temperature chamber covering a temperature range from -20°C up to 270°C. The displacement measurement device is a two-sided Zwick inductive extensiometer with a resolution limit of 4 nm. The extensiometer is limited to a maximum displacement of ±2 mm and a maximum temperature of 80°C. The oven is able to keep the temperature constant within ±1°C.

For determination of general constitutive behaviour, load-step creep tests have been used. Starting at high strain rates the stress was decreased by a certain amount after the specimen had encountered a given plastic deformation, usually 1.5% plastic strain or, alternatively, had been tested for a given time, usually 1000 seconds. Within the whole deformation at a given stress the strain rate was constant, not giving rise to prolonged strain intervals as might be required in presence of extensive transition phenomena.

Thus, steady state creep data were determined between 0°C and 80°C for strain rates varying from $10^{-7}$ to $10^{-2}s^{-1}$.

In order to monitor the possible localisation of the strain, scratches were drawn across the solder joint using a razor-blade.

It has been recognised that fracture did not take place immediately after the onset of 'ternary creep' which was in line with findings of Morris and Mei [Mor91]. Therefore, creep tests have been performed far into the ternary
creep range (80% overall strain) in order to find out about the general behaviour in this stage.

As the creep rates increased during these tests easily by two orders of magnitude, the data recording became difficult due to a limited transient memory. Therefore, tensile tests controlled by the inductive extensiometer were used to monitor the creep behaviour over large deformations, i.e. 200% engineering strain. Since the elastic deformation in the substrate between the gauges was in the order of 10% overall strain of the joint, the displacement-rate of the extensiometer should be corrected in function of the stress release rate in the test programme as was outlined in the previous chapter. It turned out, however, that the combination of an internal and an external controlling loop was too difficult to adjust. Thus, the test was controlled by the extensiometer's displacement only, leading to variations in the strain rate of 10% of the given value.

Load-step creep tests after the tensile tests revealed that the constitutive behaviour had significantly changed not only in terms of general acceleration but also in the strain rate sensitivity. Therefore, a test was designed including both extended straining at constant rate and repeated monitoring of the constitutive behaviour by means of load-step creep tests. These tests were performed at a broad variety of test conditions, e.g. temperature, ageing condition, and processing path of the specimen.

Since on one hand, a strain rate sensitivity for the recrystallised, coarsened shear bands of 0.33 is reported in the literature and on the other hand, it was argued that the shear band may deform superplastic, there was a need to check whether grain boundary sliding (gbs), which is considered to be indicative for superplastic deformation, is present in the solder joint. Hence, the specimen were first polished down to 1/4 μm diamond paste and subsequently shortly set on a rotating polishing disc containing 3 μm diamond paste. Thus, a very fine pattern of scratches over the solder joint was produced. If grain boundary sliding was present, offsets in scratches over a grain or phase boundary would appear.
3.3.2 Cyclic Testing

In order to find a correlation between unidirectional behaviour and fatigue, some isothermal cyclic testing was performed at 40°C. Here again, the strain controlled testing was done either at constant stress or constant strain rate. The plastic strain ranges were varied between 1% and 15%.

Load-controlled cyclic straining was performed by applying a given load and holding it until a certain amount of creep strain was reached. Subsequently, the load was reversed and held until the same creep strain was achieved. Thus one cycle was performed. As the testing was performed on shear specimen, there is no meaningful difference between tension and compression as would exist while using tensile test specimen.

The strain rate-controlled testing was performed by applying a given load at which creep rates were still slow and thus determining the stiffness of the specimen. After having set the strain to zero, the machine was controlled by the rate of displacement of the clip gauge allowing for changes in load. The strain was calculated using the actual position of the clip gauge and the change in load combined with the machine's stiffness. As soon as the intended strain was reached, the actual load was reversed and, again, the measurement was continued by controlling the rate of displacement of the clip gauge.

3.4 Sample preparation for light microscopy

Sample preparation for light microscopy of tin-lead eutectics has been successfully performed from the beginning of our century. The steps were typically: grinding, polishing and subsequent etching. Difficulties arise in microelectronics as large metallic parts are in electric contact with the solder. Having a lower electro-chemical potential than Alloy42 or copper, lead-tin dilutes very quickly in any etching process. Thus, preparation of individual grains by grain boundary etching becomes difficult. In case of our bronze-substrate solder joint specimen the problem is even accentuated.

Hence, a preparation technique that managed without etching is required. Such a technique has been developed by Wulff at CEM in Neumünster
(Germany). It takes advantage of the fact that the tin crystals are tetragonal and have, thus, an optical axis. After careful polishing, individual tin grains are visible under polarised light if polariser and analyser are almost crossed. This technique has been described by Grossmann and Nicoletti at length [Gro96]. The major difficulty of this technique is the rather low contrast and the very low light intensity achievable with standard light microscope equipment.

Another technique using the same physical effect would be scanning electron microscopy using the back scattered electron intensity for contrast. However, powerful analyser for back scattered electron imaging are rare.
Ten years ago, Frear et al. [Fre88] reported on the forming of so-called 'coarsened bands' that occurred after some straining in shear test specimen. This bands feature a coarsened looking microstructure compared to their neighbouring zones. It is widely accepted that recrystallisation is involved in the process of coarsening. Before long, it has been found that strain is localised to some extent in these zones and that final fracture occurs always in these regions [Fre91]. Later, investigations on SMD have revealed that forming of coarsened shear bands is not unique in shear test specimen but is frequently encountered in soldered joints in electronics. However, not much effort has been undertaken since in order to characterise the recrystallisation behaviour and the localisation.

This Chapter is dedicated to quantifying the change in mechanical behaviour accompanying the shear band forming. In a first section, we will characterise the microstructure and the mechanical behaviour after soldering. Then, we turn to the changes in these quantities after straining, which are the contents of section 4.2. In section 4.3 we will report on the fracture behaviour present in soldered shear test specimen as function of the solidification condition. The last section is concerned with the elastic properties of eutectic solder alloy.
4.1 Characterisation of mechanical behaviour and microstructure after soldering

The mechanical behaviour of the eutectic lead-tin solder is significantly influenced by the microstructure present and the process it has been achieved by. As any kind of controlled working—which would change the microstructure—is not viable in electronics, the only way to influence the microstructure after soldering is by varying the rate of cooling during solidification. Variation of the solder's composition, which would also change the microstructure as proposed by Morris and Mei [Mor91] is not considered in this work.

The thesis in hand deals with specimen soldered by two different solidification processes, the one being quenching in water and the other being slow cooling in a switched-off furnace. We will first have a look on the microstructures present after both solidification processes and later the resulting mechanical properties is presented.

4.1.1 Microstructures after solidification

After quenching, the microstructure consists of very fine dispersed lead-phases, i.e., phase size of 1...2 μm, embedded in a tin matrix. The morphology of the lead phases is slightly elliptical, cf. the left hand picture in Fig. 4.1. In the furnace cooled specimen the lead phases are generally larger and more rod-like. The lead phases are oriented in the same direction over large areas. At some spots even the typical lamellar eutectic structure has evolved, as shown in Fig. 4.1 on the right. In the furnace cooled specimen, there are some primary lead phases and some large intermetallics presumably of the type Cu₆Sn₂ present. The reason for the formation of the former is the consumption of tin owing to the formation of the latter. In quenched specimen where only little time is allowed to form intermetallics neither large intermetallics nor primary lead-phases are present.

The large areas of correlated orientation of the lead phases found in the slowly cooled specimen are identified as eutectic colonies as frequently described in the literature. In fact, using the preparation technique given in section 3.4, the tin phase in these colonies is found to exhibit one single
crystallographic orientation being one single grain with the lead phases as embedded dispersants. This is shown in Fig. 4.2.

While the quenched specimen did not exhibit distinguishable areas of constant orientation of the lead phases, there were nevertheless large areas of the same crystallographic orientation in the tin matrix phase. They were even much larger as those encountered in the furnace cooled specimen. An example is given in Fig. 4.2. A quenched shear test specimen with a joint
thickness of 60 μm is even made up of one single layer of colonies.

4.1.2 Mechanical behaviour after solidification

We will now come to the mechanical behaviour in terms of stress-strain rate characteristic. Following the work of Mei et al. [Mei91b] the creep behaviour of lead-tin eutectic with an equiaxed fine microstructure should be superplastic, i.e., the power of the stress should be \( n \approx 2 \). However, as shown in Fig. 4.3, the creep behaviour of a furnace cooled specimen is characterised by a much higher stress power, i.e., about 7, which is indicative for dislocation climb-controlled deformation. Thus, the stress-strain rate relation can be written as

\[
\dot{\gamma} = A \left( \frac{\tau}{G} \right)^{\gamma} \exp \left[ -\frac{Q_{dc}}{RT} \right]
\]  \hspace{1cm} (4.1)

Fig. 4.3: Stress-strain rate relation for a slowly cooled shear test specimen. The power of the stress is about 7 as usually reported for dislocation climb in eutectic lead-tin solder.
Fig. 4.4: Determination of the activation energy for the creep deformation in the as-soldered state.

While the exponent of the stress is constant over the temperature range under consideration, there are sometimes deviations from it at small creep rates. They are owing to the fact that coming to the small creep rates the specimen has already encountered some 10 to 15% strain in the stepped load test that might have caused already some changes in the microstructure. They are, however, not due to any systematic change in deformation mechanism since they occur rather randomly at any temperature. The value of the activation energy $Q_{dc}$ is determined to be 52 kJ/mol and for $A$ a value of $3.5 \cdot 10^{25} \text{s}^{-1}$ is found, cf. Fig 4.4.

For quenched specimen the situation is a bit different. Although there is at various temperatures at the beginning of testing—i.e., at high strain rates—a high stress power, its value drops already at intermediate strain rates to about 6...4. As this drop is quite non-uniform for the data at various temperatures, it is difficult to describe the data by a single equation. The data are conveyed in Fig. 4.5.
Fig. 4.5: Comprehensive drawing of the stress-strain rate behaviour of quenched shear specimen at various temperatures determined by stepped load tests.

However, as the creep rates at high stresses are quite the same as in furnace cooled specimen at constant stress, the deviations at lower strain rates are assumed to be due to beginning changes in microstructure. The reason for this might be provided by the higher instability of the structure as fine dispersants provide large interfaces. Moreover, as we will see below, the strain is much more localised in quenched specimen giving raise to major changes within the layer of localisation even at small overall strains.

Considering the arguments given above the behaviour of the undeformed quenched microstructure is assumed to be described accurately enough by the equation given for the slowly cooled specimen.

It is a basic assumption of mechanical testing that the quantity to be measured is independent of the specimen's geometry. In order to check this point, the stress vs. strain rate characteristic has been determined for solder joints of 25, 60 and 240 μm thickness. As can be seen in Fig. 4.6, the initial joint thickness has no influence on the creep characteristic of slowly cooled
specimen. This may proof indeed that the deformation at this point is still homogeneously distributed within the joint. As quenched specimen exhibit already in a 60 \( \mu \)m joint a non-uniform characteristic, a comparison with specimen of other joint thickness was not expected to offer further insight.

It has been suggested that ageing after the soldering process would stabilise the microstructure [Dar92]. The proposed ageing behaviour was 100h at 100°C or several weeks at ambient temperature. The influence of varying ageing time on the creep characteristic in furnace cooled specimen is given in Fig 4.7. While the slope in the log \( \tau \) vs. log \( \dot{\gamma} \) diagram is unchanged, the lines are shifted towards lower stresses as ageing time increases. This may be due to the coarsening of the lead-phase reducing the obstacles for dislocation motion.

As for the stabilising effect, however, the findings for the quenched

![Graph showing influence of joint thickness on creep characteristic in slowly cooled specimen.](image)
Fig. 4.7: Influence of the ageing time on the creep characteristic in slowly cooled specimen.

microstructure indicate that there is no significant influence of the ageing as the data at lower strain rates are still far from keeping to a straight line.

4.2 Changes due to straining

Performing creep tests to rupture, it has been recognised that the strain rate at constant load increased after about 5 to 10% deformation suggesting that the material had entered the stage of tertiary creep, which is usually considered to be the beginning of catastrophic failure. However, rupture did not take place within the next 80% of deformation. Subsequent testing revealed that the stress strain rate characteristic has significantly changed by the preceded deformation. While a power-law was still able to describe the data, the stress power had changed indicating that the basic rate controlling
deformation mechanism had changed during straining. An example for this change is given in Fig. 4.8. It conveys the stress-strain rate characteristic in the as-soldered state and after 70% overall deformation. Additional 100% deformation yielded no further change in the stress power being at a about constant value of 3.3 but gave rise to a general acceleration.

From these experimental results it is concluded that there are two things happening in tin-lead-solder due to straining:

- Firstly, there is a change in the rate controlling step in the deformation within the first 80 - 100% overall strain giving rise to a change in stress-strain rate characteristic.

- Secondly, straining causes a steady acceleration of the creep rate at constant load by accumulating damage and, thus, reducing the load bearing area.
Further research has revealed that the change in creep characteristic is accompanied by a change in microstructure of the soldered joint. Fig. 4.9 shows a cross-section through a shear test specimen before and after straining. The initially flat surface in the left-hand picture in Fig. 4.9 has evolved to a broken and bulged surface due to intensive intergranular deformation. The bulging is accentuated in the middle of the solder joint for slowly cooled specimen while for quenched specimen bulging is found near the solder/substrate interfaces only.

Subsequent polishing indicates that the bulged bands are made up of a microstructure which is coarsened compared to the remainder of the joint which still consists of virtually unchanged as-soldered microstructure. However, microstructural investigation under polarised light reveals that the eutectic colonies in the bulged region are replaced by a recrystallised microstructure while the remainder still consists of large eutectic colonies. This is shown in Fig. 4.10 for a quenched specimen tested at 40°C and a strain rate between $10^{-6}$ to $10^{-3}$ s$^{-1}$.

A pattern of fine scratches drawn across the solder joint before straining clearly indicates (by the form of the individual scratches) that deformation is localised in those bulged areas, i.e., in the recrystallised structure. These bands in which bulging and recrystallisation is present are identified as

![Fig. 4.9: Appearance of solder joint in a specimen before (left) and after (right) straining. The formerly flat surface is locally disturbed.](image-url)
Fig. 4.10: Optical micrograph of a quenched solder joint after 80% overall creep deformation. Thin layers of recrystallised bands are visible near the solder/substrate interface. Unchanged large colonies in the middle of the joint.

'coarsened shear bands' as described in the literature.

Fig. 4.11 shows a scratch across a soldered joint before and after straining. Mark the distinct orientations of microscratches on individual grains in the unpolished shear band after straining shown in Fig. 4.12.

Fig. 4.11: Localised deformation in a solder joint indicated by a formerly straight scratch (left) and its inhomogeneous deformation after straining.
Fig. 4.12 offers a new interpretation of the change in strain rate sensitivity after deformation, cf. Fig 4.8: while previous work [Dar92] ascribes the strain rate sensitivity of 0.3 to 'class-A behaviour', i.e., viscous glide of dislocations, the findings presented above indicate that deformation along interfaces, i.e., grain or phase boundary sliding, is characteristic for deformation in this stage. The latter view is further supported by the fact that the presence of a fine-grained structure in the tin-matrix is necessary as it does not appear in the as-soldered state. In contrast, 'viscous glide' is usually grain-size independent.

One might suspect that the apparent viscosity of the 'viscous glide' of dislocations is affected by the process of solidification and the according content of solute alloying elements. This argument, however, is weakened by the fact that ageing is not able to cause a similar change in creep characteristic while any supersaturation is removed by such a heat treatment.

A certain inconsistency with the theory of the above presented mechanism is that grain boundary sliding is usually found to show a strain rate dependence of 0.5 which differs significantly from the value found in this
work (0.3) and other investigations on solder joint shear specimen. However, the obvious presence of deformation along interfaces as shown in Fig. 4.12 is considered to offer ample evidence.

After having shown that a) the creep characteristic in a soldered joint specimen changes considerably after straining and b) that this change is accompanied by a change in microstructure, we will now turn to the evolution of this change. As was mentioned in connection with Fig. 4.8, the change in strain rate at constant stress is large in the first 50 to 100% increasing at a slower rate at further strain. This behaviour can be seen in both creep test data shown in Fig 4.13 as well as in tensile test data conveyed in Fig 4.14. As the localisation is more pronounced in quenched specimen, the considerable change in the strain rate takes place within less overall strain in creep testing of quenched specimen, cf. Fig 4.13, as in tensile testing of slowly cooled specimen, cf. Fig 4.14.

Fig. 4.13: Creep rate vs. Creep strain behaviour of a quenched specimen tested at constant stress at 40 °C. There is a significant acceleration visible in the first 50% of overall strain.
There is again ample evidence that the apparent softening of the solder can be divided in two different parts, the one featuring fast changes in strain rate at the beginning while the other extending to higher strains exhibits a slow but steady increase in strain rate at a given stress. The first part is assumed to account for the changes in microstructure while the second part reflects the ongoing damage in the solder joint introduced by straining. The numerical description of these two parts is given in Chapter 5.

Ageing has been proposed in the literature to stabilise the microstructure. However, testing at various ageing conditions revealed that the significant strain softening could not be suppressed by ageing. These results are conveyed in Fig. 4.15.
There is, as was expected from Fig. 4.7, a decrease of general stress level with increasing ageing time. The form and the kinetics of the transition, however, are not affected by ageing.

One of the direct implications of the above described localised deformation is that the initial joint thickness is important, as measurement is usually performed in terms of relative displacement of the two substrate platelets. The strain rate in the solder is then calculated by dividing the rate of displacement by the joint's thickness. If localised strain is present in a shear band, the remainder of the joint contributes virtually nothing to the displacement compared to the strain in the shear band. Therefore, it is the thickness of the shear band that influences the measurable rate of displacement rather than the joint thickness. The calculation of the strain rate, however, is significantly affected.

![Stress-strain behaviour of slowly cooled specimen at various ageing conditions tested at constant overall strain rate at 40°C. The decrease in applied stress in the specimen is of similar nature for all specimen.](image)
Experimental evidence for this is given in Fig. 4.16: while the testing procedure was the same for all three specimen—having 25µm, 60µm, and 240µm joint thickness, respectively—the change in strain rate vs. overall strain was the faster the larger the joint thickness. Eliminating the joint thickness, i.e., monitoring the stress vs. displacement produced, three parallel lines indicating that the width of the shear band is about the same in all three specimen, cf. Fig. 4.17.

The test procedure was a tensile test at constant overall strain rate. The variation of the stress levels for the different joint thickness may be explained by the higher local strain rate present in the shear band the thicker the solder joint. However, this argument is not able to account for the data of the specimen of 25 µm joint thickness; they would be expected at lower
stress levels than those of the 60 μm specimen. This inconsistency might be due to the fact that the shear band in the 25 μm specimen is a bit thinner than those in the other specimens. This is supported by the somewhat steeper decrease in applied stress in the 25 μm specimen compared to the others.

The features outlined above clearly show that creep data in presence of localised deformation and evolving damage are difficult to be compared to each other. As we will see in the next chapter, the apparent creep rate depends on the thickness of the shear band, the state of recrystallisation, and the state of damage.

Therefore, it is difficult to determine the temperature dependence as well as the pre-factor of the mechanism present after recrystallisation. However,
Fig. 4.18: Modelling of stepped load test data on slowly cooled specimen at 0°C, 40°C, 80°C leading to an activation energy of 32 kJ/mol and a pre-factor of $6.7 \cdot 10^{12}$.

direct determination of the pre-factor $B$ from stepped load experiments after 90% overall strain at 60 µm solder joints yielded $6.7 \cdot 10^{12} \text{s}^{-1}$ and the activation energy was 32 kJ/mol, which fitted data at 0°C, 40°C, and 80°C quite well, cf. Fig. 4.18.

The overall behaviour after recrystallisation can then be described by a power-law as in (4.1) as follows:

$$\dot{\gamma}_{\text{rec}} \left[ \text{s}^{-1} \right] = 6.7 \cdot 10^{12} \cdot \left( \frac{\tau}{G} \right)^{3.3} \cdot \exp \left[ -32000 \right] \left/ RT \right.$$  \hspace{3cm} (4.2)

where $\tau, G, R$, and $T$ have their usual meaning as previously introduced.
4.3 Modes of fracture

As mentioned above, the localisation of the deformation is usually much more pronounced in quenched than in slowly cooled specimen. Moreover, the strain localisation in quenched specimen is usually near the solder/substrate interfaces while in slowly cooled specimen the shear band is found in the middle of the solder joint.

Inspection of broken joints indicated that the fracture surface in quenched specimen is the result of one single crack propagation through the solder joint as the fracture path is flat and deflection only occurred near inner surfaces to encapsulated air bubbles. This is shown in Fig. 4.19.

For slowly cooled specimen the situation is different. There, the fracture surface is furrowed and the crack path is repeatedly deflected. This suggests that fracture is controlled by coalescence of microcracks, which are not necessarily in the same plane. Such a fracture behaviour is quite difficult to account for by means of fracture mechanics. An example of such a fracture surface is given in Fig. 4.20.

![Fracture surface of a quenched specimen](image)

Fig. 4.19: Fracture surface of a quenched specimen after about 200% deformation and forced final fracture. At the mesoscale, the topography of the fracture surface is featureless and flat except from some bubbles due to the soldering process.
Taking a closer look at the fracture surfaces reveals also different deformation patterns for the solidification routes under investigation: While the quenched specimen exhibits intragranular deformation and shear lips that reflect the direction of deformation, cf. Fig. 4.21, the slowly cooled specimen shows substantial presence of intergranular fracture. Moreover,
Fig. 4.22: Fracture morphology of a slowly cooled specimen: intergranular fracture and traces of grain boundary sliding visible in adjacent tin-tin boundaries.

Traces of grain boundary sliding can be seen along tin-tin grain boundaries, cf. Fig. 4.22.

4.4 Elastic behaviour of eutectic Sn62Pb36Ag2 solder

In order to describe the mechanical behaviour comprehensively, the elastic constants are to be determined as well. Measuring the elastic properties by means of tensile and shear testing is not appropriate for tin-lead solder owing to its visco-plastic behaviour even at ambient temperature. Therefore, the elastic constants have been determined in connection to this work by measuring the propagation velocity of both longitudinal and transversal elastic waves as suggested by Rashid and Stone [Sto94].

Measurements in the temperature range between -20°C to 100°C yielded the following results:

- the value of the Young's modulus can be given as

\[ E = 42.23 - 0.059 \cdot T[^\circ C] \text{ [GPa]} \]
Tab. 4.1: Comparison of data for elastic properties of eutectic lead-tin solder as given in the literature and as determined in this work.

- For the shear modulus $G$ holds

$$15.29 - 0.023 \cdot T[^{\circ}\text{C}] \text{ [GPa]}$$

- The Poisson's ratio was thus in the range of 0.38 to 0.4

The presented results are as expected at the upper limit of the range of values given in the literature as can be seen from Tab. 4.1. The considerably lower values in [War93] are supposed to be owing to inappropriate testing, i.e., determination using the elastic range in a tensile test.
Chapter 4 has dealt with the evolution of microstructure and mechanical behaviour in conjunction with straining. In this chapter, we develop how this behaviour can be integrated in a constitutive equation's framework. We intentionally wrote 'can' since there might be different ways to incorporate the features presented in the last section, although it has not been performed so far. However, we choose a form in which the parameters in use have a physical meaning.

As postulated by Stone and Rashid [Sto94], a constitutive description has to be based on 'a well defined phenomenology, a basis in empirical data, and a basis in mechanism'. Having provided the basis in empirical data and having discussed to some extent the underlying mechanisms we will now turn to the phenomenology.

We start at the mechanical behaviour present in the unstrained state after soldering. We postulate that in this state the stress-strain rate behaviour can be described by a typical power-law expression, i.e.

\[ \dot{\gamma}_{\text{as}} = 3.5 \cdot 10^{25} \cdot \left( \frac{\tau}{G} \right)^7 \cdot \exp \left[ -52'000 / RT \right] \]  

(5.1)
as confirmed by Fig. 4.3. We further postulate, that straining—whether localised or not—causes changes in the microstructure which mainly result from recrystallisation. The recrystallised part, then, exhibits a characteristic mechanical behaviour distinct from that in the as-soldered parts. This characteristic behaviour of the recrystallised part is also described by a power-law (i.e. appears as a linear relationship in a double logarithmic plot) as demonstrated in Fig. 4.18. We further postulate that deforming the recrystallised structure causes damage. In the following section we will develop the mathematics necessary to model the transition from the as-soldered to the recrystallised state. The introduction of damage will be the contents of section 5.2. In section 5.3 the capability of the model in describing independent data sets is tested. Section 5.4 deals with the generalisation of the results presented in the preceding sections.

5.1 Modelling of the transition from the as-soldered to the recrystallised state

Consider a 2.5-dimensional model of a solder joint as shown in Fig. 5.1. divided in \( k \times l \) rod-like volume elements which are identified by the subscript indices \( i \) and \( j \) of their position with \( 1 \leq i \leq k \) and \( 1 \leq j \leq l \). For simplification's sake we assume that a) the stress transmitted from the \( y \)'th layer to the \( y+1 \)'th is constant and b) for reasons of continuity the shear deformation of all elements \( i \) in a layer \( j \) must be equal.

Consider the solder joint in unidirectional shear at constant temperature and moderate strain rates, i.e., \(< 10^{-4}\text{s}^{-1}\). Assumed a random volume element \( a_{gh} \)

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![Fig. 5.1: Schematic drawing of the mechanical model. The model extends in the direction perpendicular to the drawing plane by a thickness of unity.](image)
would recrystallise first. By testing in the range of creep rates where grain boundary sliding is faster than dislocation climb the recrystallised volume element should need less stress to deform at an equal rate as its neighbouring elements in the same layer. At constant overall load and invoking assumption a), there should be some excess load which is to be transmitted by the other volume elements $a_{ijh}$ of the same layer $h$ causing them to creep faster. Thus, the driving force to recrystallise in layer $h$—i.e., the accumulated strain—will increase faster than in other layers and will lead to even more recrystallised volume elements.

We now develop a set of equations guiding the deformation of such a structure. Be $x$ the recrystallised fraction of the layer $h$. For simplification's sake, we introduce the stress partitioning ratio $P$ denoting the ratio between the stresses necessary to give rise to the same strain rate in the recrystallised and in the as soldered state represented by $\tau_{rec}$ and $\tau_{as}$, respectively, which is given as follows:

$$\tau_{as} = P \cdot \tau_{rec} \quad (5.2)$$

The applied stress $\tau$ can then be expressed as

$$\tau = x \cdot \tau_{rec} + (1-x) \cdot \tau_{as} = x \cdot \tau_{rec} + (1-x) \cdot P \cdot \tau_{rec} \quad (5.3)$$

For the creep rate $\dot{\gamma}_h$ in layer $h$ holds (see assumption b)

$$\dot{\gamma}_h = A_T \cdot \left( \frac{\tau_{as}}{G} \right)^n = B_T \cdot \left( \frac{\tau_{rec}}{G} \right)^m \quad (5.4)$$

with $A_T = A \cdot \exp\left[-Q_{dc}/RT\right]$ and $B_T = B \cdot \exp\left[-Q_{gvs}/RT\right]$.

Combining (5.3) and (5.4) and using the stress partitioning ratio $P$ as introduced in (5.2) leads to

$$\dot{\gamma}_h = A_T \cdot \left( \frac{\tau_{as}}{G \cdot (1-x) \cdot P + x} \right)^n = B_T \cdot \left( \frac{\tau_{rec}}{G \cdot (1-x) + x / P} \right)^m \quad (5.5)$$

or with respect to the creep rate $\dot{\gamma}_o$ of the other layers holds
\[
\dot{\gamma}_h = \dot{\gamma}_o \cdot \left( \frac{1}{(1-x) + x/P} \right)^\alpha
\]  
(5.6)

For \(P > 1\), the whole expression in brackets is larger than 1 and the creep rate in layer \(h\) is accelerated compared to the neighbouring layers as a function of \(P\) and the recrystallised part \(x\) of layer \(h\). For \(x=1\), the creep rate in layer \(h\) is that of pure grain boundary sliding, cf. (5.5).

In order to predict the total displacement behaviour, which is usually measured in creep testing, the thickness of the layer \(h\) in which the strain is localised becomes important. Be \(\lambda\) by way of definition the—with respect to the joint thickness \(l\)—normalised thickness of the layer \(h\). Then, for the total strain rate \(\dot{\gamma}_{\text{tot}}\) holds

\[
\dot{\gamma}_{\text{tot}} = \dot{\gamma}_o \left[ \lambda \left( \frac{1}{(1-x) + x/P} \right)^\alpha - 1 \right] + 1
\]  
(5.7)

Obviously, there are three parameters which determine the (measurable) increase in overall strain rate: the slip band thickness \(\lambda\), the stress partitioning ratio \(P\) and the recrystallised fraction \(x\) of layer \(h\).

We will now have a look at those different quantities. The slip band thickness for one is accessible using light microscopy as is shown in the previous chapter. However, one has to assume that its value does not change throughout the test. Another difficulty is that the thickness of the evolving shear band seems to depend on the condition, i.e., strain rate and temperature, at which the deformation is run. We will come to that point in the discussion chapter.

As for the stress partitioning factor \(P\) the situation is a bit complicated: it depends on the (local) strain rate and on the temperature, as can be seen in Fig. 5.2 showing also that the distance between the two lines obtained from (4.1) and (4.2) is equal to \(\log\,P\). While in a tensile test the overall strain rate is kept constant, the local strain rate in the recrystallising shear band may well change causing a change in \(P\), too. In a creep test, the situation is much more complex as the strain rate in the shear band changes significantly as
Fig. 5.2: Schematic drawing of the stress-strain rate behaviour for as-soldered (---) and recrystallised (——) microstructure. The distance between the two lines equals logP.

the recrystallised part \( x \) increases, cf. Fig. 5.2. Thus, in modelling the stress strain behaviour the calculation of \( P \) must base on the local strain rate. Its value can then be calculated as described below.

Be \( \tau^a \) the normalised, effective stress at which the deformation rate of both mechanisms is equal, i.e.

\[
A_r \cdot \tau^{\alpha_n} = B_r \cdot \tau^{\alpha_m}
\]  
(5.8)

Rearranging leads to

\[
\tau^* = \left( \frac{B_r}{A_r} \right)^{\frac{1}{\alpha_m - \alpha_n}}
\]  
(5.9)

and invoking (5.1) follows

\[
\dot{\gamma}^* = A_r \cdot \left( \frac{B_r}{A_r} \right)^{\frac{1}{\alpha_m - \alpha_n}}
\]  
(5.10)
Fig. 5.3: Temperature dependence of the point of equal strain rate as used for the determination of the stress partitioning $P$.

where $\dot{\gamma}^*$ denotes the strain rate at which both mechanisms contribute by the same amount to the strain rate. Its value is temperature dependent as conveyed in Fig. 5.3 since $A_T$ and $B_T$ both depend on the temperature as well.

With regard to Fig. 5.2, the stress partitioning $P$ is then given as

$$\log P = \left(\log \dot{\gamma}^* - \log \dot{\gamma}\right) \cdot \left(\frac{1}{m} - \frac{1}{n}\right)$$

(5.11)

with $\log \dot{\gamma}$ being the strain rate in the shear band.

Alternatively, the stress partitioning $P$ is related to the applied stress $\tau$ by

$$\tau = G \cdot \frac{B_T}{A_T} \left(\frac{1}{n-m}\right) \cdot \frac{\left(1-x\right) \cdot P + x}{P^{\left(\frac{1}{m-n}\right)}}$$

(5.12)
In this equation, however, the recrystallised part of the shear band is to be known in order to calculate the stress partitioning $P$.

We, therefore, turn now to the kinetics of the evolution of the recrystallised part. Consider the tensile test data in Fig. 5.4: There is after some percent initial strain a continuous, steep drop in load that changes into a steady decrease after about 80 to 100% overall strain. While the decrease in stress at large strains is considered to be due to evolving damage, the drop at the beginning is ascribed to the change in microstructure demonstrated in section 4.2.

In a first order approach, we assume that the strain rate in the shear band is constant throughout the straining at $\dot{\gamma}_o/\lambda$ with $\dot{\gamma}_o$ being the applied strain rate. This assumption does not hold in the very beginning of the straining but is a reasonable approach at strains $>20\%$ especially if a) $\lambda$ is not too

![Fig. 5.4: Stress-strain behaviour at constant total strain rate tested at 40 °C. The load drop within the first 100% is markedly triggered by the strain rate reflecting the evolution of the stress partitioning $P$ with respect to strain rate.](image)
small and b) the stress partitioning is large, i.e. \( P > 2 \). We further assume that the linear decrease in stress at high strains can be extrapolated to low strains denoting the stress present in the recrystallised part of the shear band.

The stress needed to produce the intended strain rate is then related to the recrystallised part \( x \) by

\[
\frac{\tau}{\tau_{\text{rec}}} = (P + x \cdot (1 - P)) \tag{5.13}
\]

using the above introduced notation. Taking the logarithmic form of the above equation gives

\[
\log \tau - \log \tau_{\text{rec}} = \log(P + x \cdot (1 - P)) \tag{5.14}
\]

Using the previously mentioned extrapolation, the left hand side of the above equation can be determined from Fig. 5.4. Assuming, that at zero strain the structure is completely in the as soldered state \( (x = 0) \) enables to determine \( \log P \) and further on \( x \) as function of the overall strain.

Talking about 'kinetics' usually means to give an evolution of a state in function of time. The most common form of phase transformation or recrystallisation kinetics is that widely known as the Johnson-Mehl-Avrami equation given by

\[
x = 1 - \exp\left[ -\frac{4\pi}{3} \cdot \dot{r}^3 \cdot t^4 \right] \tag{5.15}
\]

where \( \dot{r} \) denotes the growth rate of the transformed phase and \( t \) stands for the time. The critical parameter in this equation is the phase size change rate \( \dot{r} \). It is usually proportional to the Debye-frequency \( \nu \) and the energy gained by the phase transformation, i.e.

\[
\dot{r} \propto \nu \cdot \Delta g \quad \text{if} \quad \Delta g \ll RT
\]

In case of recrystallisation, the energy gain \( \Delta g \) is, generally speaking, proportional to the dislocation density, the density of inner surfaces, i.e., interphase boundaries, and the oversaturation due to fast cooling. However,
the last two terms seem to be negligible since simple ageing does not lead to any appreciable recrystallisation. It, hence, remains the dislocation density $\rho$, which is under conditions of fast recovery proportional to the square of the applied stress \cite{Arg70}. As function of strain rate, $\Delta g$ may be given as

$$\Delta g \propto \rho \propto \tau^2 \propto \dot{\gamma}^{2/n} \quad (5.16)$$

On the other hand, the time and total strain are connected as follows

$$t = \frac{\gamma_{\text{tot}}}{\dot{\gamma}}$$

Thus, the Johnson-Mehl-Avrami equation becomes

$$x = 1 - \exp \left[ -\Gamma \cdot \dot{\gamma}^{6/n} \cdot \left( \frac{\gamma_{\text{tot}}}{\dot{\gamma}} \right)^{4} \right] \quad (5.17)$$

One, therefore, should expect a strong dependency of the total strain to recrystallise on the deformation rate. This, however, is not observed as indicates Fig. 5.4. There is rather a direct dependence of the recrystallisation on the total strain observed.

Let us assume that the dependence of the recrystallisation on the total strain is of the form

$$x = 1 - \exp \left( -\beta \cdot \gamma_{\text{tot}}^\alpha \right) \quad (5.18)$$

The evaluation of the parameters is performed using a Weibull-plot as shown in Fig. 5.5. On the x-axis is then $\log(\gamma_{\text{tot}})$ while on the y-axis there is $\log(-\log(1-x))$. Strikingly enough, all of the three data sets fall on about the same line exhibiting a slope of $\approx 2$, i.e., the value of parameter $\alpha$. For $\beta$ one finds values of $2\ldots 5$.

---

\footnote{For very low stresses, however, the dislocation density $\rho$ is independent from the stress. On the other hand, Ishida and McLean \cite{Ish67} reported on a dependence to the third power at high stresses for steel.}
Fig. 5.5: Evaluation of the recrystallisation-guiding parameters by means of a Weibull-plot. Variation of the strain rate within two orders of magnitude had virtually no impact on the evaluation of the parameter $a$. For the parameter $\beta$, however, a variation according to the thickness of the shear band is found.

In other words, the recrystallisation rate per unit strain is proportional to the not yet recrystallised remainder of the shear band as well as the total strain encountered, i.e.

$$\frac{\partial x}{\partial \gamma} = \frac{\beta}{2} \cdot (1 - x) \cdot \gamma_{tot}$$

(5.19).

We now come back once again to the normalised shear band thickness $x$. We might well assume that the quantity relevant for the recrystallisation is the local strain rather than the overall strain as used above. This, however, implies that the parameter $\beta$ in (5.19) is very likely to change as the shear band thickness varies. This variation is accounted for by the '$\lambda$-independent' form given as

$$x = 1 - \exp(-R \cdot \gamma_{loc}^2)$$

(5.20)
For $\lambda$ one finds a value of 0.9.

A most obvious case of the influence of $\lambda$ is the differing recrystallisation behaviour of quenched and furnace cooled specimen as given in Fig. 5.6. As was described in the previous chapter, the localisation is more pronounced in quenched than in furnace cooled specimen. At equal overall strain, the localised band has encountered more strain in the quenched specimen leading to faster recrystallisation. When the whole band is recrystallised, there is a higher local strain rate required to produce a similar overall strain rate the thinner the localised band. This, accordingly, requires higher stresses. Third, if localisation is pronounced, catastrophic failure which is characterised by ongoing deviation from the linear decrease in $\log \tau$ takes place at lower overall strain levels. All these features can be recognised in Fig. 5.6.

![Graph](image)

Fig. 5.6: Comparison of the recrystallisation behaviour of slowly cooled (---) and quenched (-----) specimen.
5.2 Evolution and modelling of damage

As was pointed out in section 2.3, the modelling of damage and fatigue damage might be performed by different approaches. The thesis in hand deals with the problem as outlined in section 2.3.3 based on work by Kachanov [Kac86] and Lemaitre [Lem92]. The damage \( \omega \) is described as a reduction of the load-bearing cross-section—cf. (2.34) and (2.35)—in function of strain, temperature and strain rate. It is thus

a) per definitionem homogeneously distributed throughout a shear band since strain and strain rate is at any time constant within a plane parallel to the applied stress.

b) an internal variable. It is given by a rate equation to account for the influence of the strain rate (or the stress, rather).

As was pointed out in sections 4.2 and 4.3, the fracture behaviour of quenched and slowly cooled specimen differs to some extent. We, therefore will treat these two states of solidification separately in the following.

5.2.1 Damage evolution in slowly cooled specimen

The evolution of damage is easily monitored assuming that, after having reached a completely recrystallised structure within a shear band, further decrease in load at constant strain rate is due to damage. For slowly cooled specimen, the load drop appears as a linear decrease of the logarithm of the applied stress vs. strain over a wide range of strain rates and temperatures, i.e., as an exponential decrease. This behaviour is demonstrated in Fig. 5.7.

Such an exponential decrease can be described by an equation of the form

\[
\partial \ln \tau = \partial \ln (1 - \omega) = -\Omega \cdot \partial \gamma_{\text{tot}}
\]  

(5.21)

where the slope \( -\Omega \) denotes the slope in the log stress vs. strain diagram. However, the value for \( -\Omega \) of the distinct data sets varies considerably. On the one hand, this might be ascribed to the differing strain rates while on the
Fig. 5.7: Linear decrease of log $\tau$ in a log $\tau$ vs. $\gamma$ plot at constant strain rate suggests that the evolution of damage follows an exponential law as given in equation (5.21).

As above, we extend (5.21) to

$$\partial \ln \tau = \partial \ln(1 - \omega) = -s \cdot \partial \gamma_{loc}$$

(5.22)

As $-\Omega$—the effective slope in the log $\tau$ vs. $\gamma_{tot}$ plot—can be directly determined and $\lambda$ may be measured using light microscopy, the dependence of $s$ on strain rate and temperature can be evaluated. The results of the analysis can be summarised as follows:

- The damage parameter $s$ increases with increasing strain rate. An attempt to ascribe the enhanced damaging influence to a higher fraction of strain due to dislocation climb present at higher strain rates as proposed by Shine and Fox [Shi88] failed as the latter changed by several orders of magnitude while the damage parameter varied but within a decade.
- The damage parameter was lowered if the same strain rate was applied at higher temperatures. The effect, however, was difficult to quantify as the thickness of the shear band and, thus, localisation of the strain changed simultaneously.

- The relation between damage parameter \( s \) and local strain rate was found to follow a power law as confirmed in Fig. 5.8 and given in (5.23).

\[
\dot{\gamma}_{\text{loc}} = 0.87 \cdot s^{3.3}
\]  

(5.23)

Since the power of the damage parameter \( s \) is the same as that of the stress in the recrystallised structure (cf. (4.2)), it can be derived from (5.23) that

\[
s = s_o \cdot \tau_{\text{eff}} = s_o \cdot \frac{\tau}{(1 - \omega)}
\]  

(5.24)

with \( s_o = 0.014 \text{ MPa}^{-1} \) and \( \tau_{\text{eff}} \) denoting the effective stress on a volume element.

Fig. 5.8: Influence of the local strain rate on the rate of damage determined by a best-fit evaluation of tensile data. The influence of the temperature is not accounted for.
The rate of damage per strain is therefore proportional to the effective shear stress $\tau_{\text{eff}}$. This relationship can be seen from Fig. 5.9. The values for $s$ have been determined by a best-fit evaluation of tensile test data.

Equation (5.24) in combination with (5.22) allows for two alternative ways to describe the evolution of damage, cf. equations (5.25) and (5.26):

\[
\ln(1 - \omega) = -s_o \cdot \int_0^\gamma_{\text{loc}} \tau_{\text{eff}}(\gamma_{\text{loc}}) \cdot \partial \gamma_{\text{loc}} \quad (5.25)
\]

\[
\omega = s_o \cdot \int_0^\gamma_{\text{loc}} \tau(\gamma_{\text{loc}}) \cdot \partial \gamma_{\text{loc}} \quad \text{as} \quad \partial \ln(1 - \omega) = \frac{\partial(1 - \omega)}{(1 - \omega)} \quad (5.26)
\]

While both descriptions represent the same material characteristic, they reveal differences in distinct testing procedures. In (5.25), the integral may be removed if the effective stress $\tau_{\text{eff}}$ is constant throughout the test, i.e., in a tensile test. There, an exponential evolution of the damage vs. strain is expected. On the other hand, (5.26) applies if the load is held constant as in

![Graph showing linear dependence of damage parameter $s$ on effective stress $\tau_{\text{eff}}$.](image)

Fig. 5.9: Linear dependence of the damage parameter $s$ on the effective stress.
Fig. 5.10: Comparison of damage evolution in tensile testing (cf. (5.25)) and creep (cf. (5.26)). Significant deviations occur at damage values \( \omega > 0.4 \). Initial stress = 17 MPa.

a creep test. The damage accumulation is then linear in the accumulated local strain. This is shown for a shear test specimen in Fig. 5.10 modelled at an initial stress of 17 MPa.

5.2.2 Damage evolution in quenched specimen

While for slowly cooled specimen an exponential evolution of the damage in the localised strain is found, the load drop is rather linear in the localised strain for quenched specimen at constant strain rate. In analogy to (5.21) for quenched specimen it holds

\[
\partial \tau = \partial (1 - \omega) = - \partial \omega = - \xi \cdot \partial \gamma_{\text{loc}}
\]

(5.27)

where \( \xi \) denotes a dimensionless factor of proportionality. Again, \( \xi \) depends on the local strain rate and thus on the effective stress. This is
conveyed in Fig. 5.11. The values for $\xi$ have been determined by a best-fit modelling of tensile tests at various strain rates and temperatures.

For $\xi$ the stress dependence may be expressed by

$$\xi = \xi_0 \cdot \tau_{eff}^2$$

with $\xi_0 \sim 0.00043 \text{ MPa}^{-2}$. The evolution of damage for quenched specimen is then given by

$$\omega = \xi_0 \cdot \int_0^{\gamma_{loc}} \tau_{eff}^2 \cdot \partial \gamma_{loc}$$

(5.28)

Fig. 5.11: Relation between $\xi$ and $\tau_{eff}$ for quenched specimen as determined by a best-fit modelling of tensile data at various temperatures and strain rates.
Epilogue

With regard to the stress dependence of the damage evolution it is obvious that identical testing procedures in terms of strain rate and overall strain would not yield the same damage state if testing is performed at different temperatures, i.e., stress levels. Hence, the assumption of identical internal state for the three specimen used for determination of the activation energy as well as the pre-factor \( B \) for the grain boundary sliding regime as used in Fig. 4.18 is not valid. In fact, the lower the testing temperature the more damage is accumulated after the same amount of strain. Thus, the data sets are closer to each other feigning a lower activation energy. Re-evaluation of the data with regard to the damage state yielded \( 3 \cdot 10^{15} \text{s}^{-1} \) for the pre-factor \( B \) and 48.0 kJ/mol for the activation energy \( Q_{gbs} \). All 'best-fit' evaluations as described above have been performed using these parameters instead of those given in section 4.2.

5.3 Verification of the model

In section 5.2, we have developed a model for describing various phenomena occurring during deformation in eutectic lead-tin solder based on physical parameters, e.g. state of recrystallisation, state of damage, and localisation of strain. The comprehensive description of the creep behaviour in a single lap joint is given as

\[
\dot{\gamma}_{tot} = A \cdot \left( \frac{\tau}{(1 - \omega) \cdot G} \right)^n \cdot \exp \left[ -\frac{Q_{gbs}}{RT} \right] \cdot \left[ \lambda \cdot \left( \left(1 - x + \frac{x}{P} \right)^{-n} - 1 \right) + 1 \right]
\]  (5.29)

where \( P, x, \) and \( \omega \) are given by the equations (5.11), (5.18), and (5.25) or (5.28), respectively.

Equation (5.29) is able to fit the data sets of tensile tests at 40°C of both quenched and slowly cooled specimen as shown in Fig. 5.12 and Fig. 5.13, respectively. Furthermore, tensile test data at various temperatures and strain rates can be modelled as well, cf. Fig. 5.14.

As described in Chapter 3, creep tests have been performed at various temperatures and stresses. Those independent data (i.e. data which were not
Fig. 5.12: Modelling of tensile test data of quenched specimen at 40°C. 60 μm joints tested at various overall strain rates.

Fig. 5.13: Modelling of tensile data at 40°C for slowly cooled specimen at various overall strain rates.
Fig. 5.14: Modelling of creep test data at various initial loads. Measurements performed at 40 °C.

Fig. 5.15: Modelling of creep data at various initial loads. Measurements at 40°C and 0°C.
used for determination of model parameters) can be reproduced by the model fairly well, though not as accurate as the tensile data, see Fig. 5.15.

While the model presented in the previous sections is able to account for a fair amount of experimental data collected in various testing procedures, it fails to reproduce the data set published by Hall [Hal84] determined on a 'real' device/substrate-combination as described in chapter 2.4. The data were modelled using the following equations

\[
\tau = \left( \Delta \alpha \cdot (T - T_{\text{ref}}) - \frac{h}{L} \cdot \gamma_{\text{tot}} \right) \cdot \frac{k \cdot L}{O}
\]

and

\[
\gamma_{\text{tot}} = \int_{T_{\text{ref}}}^{T} \dot{\gamma} \, dt
\]

Fig. 5.16: Comparison of various attempts of modelling experimental data published by Hall [Hal84]. Numerical simulations provided by Stone et al. [Sto85], Subrahmanyan et al. [Sub89], and based on the constitutive equations used in this thesis.
where $O$...soldered area, $T_{\text{ref}}$...starting temperature of the cycle, $T$...actual temperature, $t$...time, $h$...solder joint height, $C$...stiffness of the assembly, and $L$...half the distance between the solder joints. The stiffness of the solder joint as such is assumed to be much higher than the stiffness if the device and substrate, respectively. Hence, the change in elastic constants of the solder is not taken into account as was proposed by [Sto94].

The geometry has been adopted from Hall's original paper while the apparent device/substrate-stiffness is determined by graphical evaluation of Hall's results. The comparison of both modelled and measured data is given in Fig. 5.16. Other attempts to model Hall's data, published by Stone et al. [Sto85] and Subrahmanyan et al. [Sub89] are also included in Fig. 5.16. Instead of a power-law description of the stress-strain rate description they both used a $\lambda$-law-model as presented in section 2.2.3. Their numerical modelling yielded better results as the description evolved in this work. To what extent this inferiority in quantitative modelling is owing to the choice of the constitutive model will be subject of the discussion.

### 5.4 Generalisation to any given geometry

As the aim of the thesis in hand is to provide a model that is suitable for FEA application, one has to formulate the model independent from the geometry on which its parameters have been determined. There will be for instance neither such thing as a normalised shear band thickness $\lambda$ nor something as a recrystallised fraction of a shear band in a general solder joint geometry. However, one can assign to every volume element a value $x$ denoting the state of recrystallisation and a value $\omega$ representing the state of damage. Both of these values can be calculated using the 'X,-independent' forms for both evolution of $x$ and $\omega$ given in equations (5.20) and (5.25) or (5.28), respectively. Equation (5.29) is then reduced to

$$\dot{\gamma} = A \cdot \left( \frac{\tau}{(1 - \omega) \cdot G} \right)^n \cdot \exp \left[ -\frac{Q_d}{RT} \cdot ((1 - x) + \frac{x}{\Gamma})^n \right]$$  \hspace{1cm} (5.30)

Localisation, as it is frequently observed in field solder joints, will evolve due to deformation gradients and, thus, recrystallisation gradients owing to the solder joint's geometry and the shape of the loading cycle.
Below, an overview on the equations necessary for comprehensive description of unidirectional straining of eutectic lead-tin-silver is given. The list of parameters used in equations (5.31) to (5.34) is provided in Tab. 5.1.

\[
\dot{\gamma} = A \left( \frac{\tau}{(1 - \omega) \cdot G} \right)^n \cdot \exp \left[ -\frac{Q_{dc}}{RT} \right] \cdot \left( (1 - x) + \frac{x}{p} \right)^n
\]  
(5.31)

\[
\log P = (\log \dot{\gamma}^* - \log \dot{\gamma}) \cdot \left( \frac{1}{m} - \frac{1}{n} \right)
\]  
(5.32)

with \[
\dot{\gamma}^* = A_T \left( \frac{B_T}{A_T} \right)^{\left( \frac{1}{n} - \frac{1}{m} \right)}
\]

and \[
A_T = A \cdot \exp\left[ -\frac{Q_{gbs}}{RT} \right] \quad \text{and} \quad B_T = B \cdot \exp\left[ -\frac{Q_{gbs}}{RT} \right]
\]

\[
\ln(1 - \omega) = -s_o \cdot \int_{\gamma_{loc}}^{\gamma_{loc}} \tau_{eff} (\gamma_{loc}) \cdot \partial \gamma_{loc}
\]  
(5.33)

\[
x = 1 - \exp\left( -\mathcal{R} \cdot \gamma_{loc}^2 \right)
\]  
(5.34)

<table>
<thead>
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<th>Parameter</th>
<th>value</th>
<th>unit</th>
<th>Parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
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<td>s⁻¹</td>
<td>(Q_{gbs})</td>
<td>48.0</td>
<td>kJ/mol</td>
</tr>
<tr>
<td>B</td>
<td>3 E 15</td>
<td>s⁻¹</td>
<td>(\mathcal{R})</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>n</td>
<td>7</td>
<td>-</td>
<td>(s_o)</td>
<td>0.014</td>
<td>MPa⁻¹</td>
</tr>
<tr>
<td>m</td>
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<td>-</td>
<td>(G)</td>
<td>15'290</td>
<td>MPa</td>
</tr>
<tr>
<td>(Q_{dc})</td>
<td>52.0</td>
<td>kJ/mol</td>
<td>(\mathcal{R}\cdot[°C])</td>
<td>-23</td>
<td>MPa</td>
</tr>
</tbody>
</table>

Tab. 5.1: Values of the invariant parameters used in equations (5.31) to (5.34) as previously determined in chapters 4&5.
Leer - Vide - Empty
Cyclic straining and fatigue experiments

In the preceding chapters we have concentrated on unidirectional shear deformation only. As in microelectronic applications there is usually cyclic straining encountered, we will now assess the evolution of damage and microstructural changes in cyclic deformation, which are the contents of section 6.1. In section 6.2, differences and analogies between unidirectional and cyclic straining in terms of the evolution of the state variables \( x \) and \( \omega \) are investigated. Finally, the incorporation of fatigue damage evolution in a FEA environment is discussed.

6.1 Cyclic deformation experiments

Mei and Morris [Mei92] have reported on significant differences in fatigue resistance of eutectic lead-tin solder depending on the solidification condition, i.e., the cooling rate. Hence, cyclic straining was performed on both quenched and slowly cooled specimen. The conditions of solidification are given in chapter 3, Fig. 3.3. As has shortly been mentioned in section 2.4, cyclic deformation may be introduced in a specimen either by holding the load constant and controlling the strain or by holding the overall strain rate constant and again controlling the strain. While the constant load testing procedure raises no difficulties as the elastic deformation of the substrate is constant throughout the test, the changing elastic deformation of the
substrate as the load level changes has to be separated from the creep strain in the joint in constant strain rate testing, as discussed in sections 2.4 and 3.3.

6.1.1 Cyclic straining of slowly cooled specimen

Cyclic straining was applied in the strain rate-controlled mode using strain amplitudes ranging from 1% to 15%. The overall strain rate was set to be $10^{-3}\text{s}^{-1}$ and testing was performed at 40°C. The peak load of every fifth cycle was taken to evaluate the evolution of load vs. number of cycles. After some few cycles at constant load, the specimen softened resulting in continuous decrease in load. The load-drop obeyed an exponential law which corresponds to a linear decrease in a log $\tau$ vs. number of cycles diagram as depicted in Fig. 6.1.

As the drop in the logarithm of the applied stress is linear, a virtual failure criterion in terms of a certain fraction of damage would reflect the relations

![Log shear stress $\tau$ (MPa) vs. number of cycles $N$ [-]](image)

Fig. 6.1: Exponential decrease in load with increasing number of cycles at constant strain rate for strain amplitudes of 1% to 15%.
Fig. 6.2: Coffin-Manson evaluation of 50%-load-drop data of slowly cooled specimen. Good linear relationship of log strain amplitude vs. number of cycles to failure; data from unidirectional testing are included as well.

of numbers of cycles to final fracture. Hence, the number of cycles at 50% initial load are taken as numbers of cycles to failure, \( N_f \). The data are evaluated in a Coffin-Manson plot. The result of this evaluation is shown in Fig. 6.2. In order to allow for a comparison to unidirectional data, the data point derived from the tensile test depicted in Fig. 5.4 has been included at \( N_f = 0.5 \).

As can be seen from Fig. 6.2, the \( N_f \)-values—including the one derived from unidirectional testing—fall on one line suggesting the accordance to a Coffin-Manson relationship. The Coffin-Manson parameter \( \alpha' \) is found to be \( \approx 0.6 \) and \( C_{CM} = 75\% \).

\[ \text{pl}[\%] \]

\[ 40^\circ C \]

\[ 60 \mu m \text{ joint} \]

\[ \text{slowly cooled} \]

\[ 0.1 \ 1 \ 10 \ 100 \ 1000 \ 10000 \]

\[ \text{number of cycles } N_f \text{ to 50% load drop [-]} \]

\[ \text{plastic strain amplitude } \gamma_{pl} \]

---

\[ ^5 \text{The strain to fracture from unidirectional testing is frequently incorporated in fatigue evaluation, though usually at } N_f = 0.25, \text{ as the loading is regarded as a quarter of a full cycle consisting of loading in tension, unloading to zero, loading in compression, and again unloading to zero. However, damage is introduced mainly in the loading parts of the cycle and, hence, } N_f = 0.5 \text{ in unidirectional straining.} \]
6.1.2 Cyclic straining of quenched specimen

Cyclic straining was performed by load-controlled testing as described in section 3.3. The testing was done at 40°C using strain amplitudes of 3% and 5%. Two distinct load levels were applied in order to investigate the impact of stress on fatigue damage accumulation.

As encountered in unidirectional testing the strain rate accelerated as testing proceeded. Again, the increase in strain rate was exponential in the number of cycles, which is represented by a straight line in a log strain rate vs. number of cycles diagram as given in Fig. 6.3.

It has been suggested by Shine and Fox [Shi88] that damage would only accumulate due to deformation in the dislocation climb (dc) regime while deformation controlled by grain boundary sliding (gbs) was assumed to be virtually non-damaging.

![Fig. 6.3: Exponential increase in the strain rate with increasing number of cycles at constant macroscopic stress.](image)
However, as can be seen from Fig. 5.3, the controllable deformation rates in both unidirectional and cyclic straining, i.e., $10^{-6}\text{s}^{-1}$ to $10^{-2}\text{s}^{-1}$ are always predominantly in the gbs regime provided the structure is already recrystallised. One might argue that the small but always present part of dislocation climb is responsible for the increasing damage. This, however, would imply that the increase in damage monitored as increase in strain rate would decrease if the overall strain rate and, thus, the fraction of dislocation climb controlled deformation was lowered. This is disproved by experimental evidence as shown in Fig. 6.4: two specimen tested at an equal strain amplitude but at different load levels exhibit an about equal relative increase in strain rate vs. numbers of cycles. While there might well be a difference in the thickness of the shear band in which the strain is localised, as found in unidirectional testing, the variation in damage parameter would not exceed a factor of 2. The dc controlled fraction yet varies by as much as a factor of 10. Therefore, the idea of Shine and Fox of gbs being an undamaging deformation mechanism should be rejected. This is, however, not to say that the damage rate in the gbs regime might not be

![Graph](image)

**Fig. 6.4**: Increase of strain rate vs. number of cycles at different load levels but constant strain amplitude. Quenched specimen tested at 40°C.
In the Coffin-Manson evaluation of the fatigue data, again the unidirectional damage rate has been included as well, cf. Fig. 6.5. The increase of the strain rate is exponential for all test conditions after evolution of the shear band. An increase of strain rate of one order of magnitude has been chosen to determine the number of cycles to failure. The Coffin-Manson parameters are determined to be $\alpha' = 0.6$ and $C_{C-M} = 48\%$.

6.1.3 Comparison of fatigue damage evolution in quenched and slowly cooled specimen

In order to be able to compare the damage evolution in quenched and in slowly cooled specimen, the arbitrary failure criterion has to be related to a
damage state. In case of strain rate controlled testing, as performed on slowly cooled specimen, the load drop is assumed to reflect the damage evolution. The formal approach is

$$\tau = \tau_o \cdot (1 - \omega)$$  \hspace{1cm} (6.1)

where \(\tau_o\) denotes the initial value of applied stress. Differentiation with respect to \(\omega\) yields

$$\frac{\partial \tau}{\partial \omega} = -\tau_o = -\frac{\tau}{1 - \omega}$$ \hspace{1cm} (6.2)

and rearranging leads to

$$\partial \ln \tau = \partial \ln (1 - \omega)$$ \hspace{1cm} (6.3)

However, this implies that recrystallisation has no part in the decrease of the load. We will discuss this point in the next section.

For quenched specimen, the evolution of damage as function of the strain rate may be determined as follows. Be the strain rate \(\dot{\gamma}\) given in the general form

$$\dot{\gamma} = C_T \cdot \left( \frac{\tau}{G \cdot (1 - \omega)} \right)^{n'}$$ \hspace{1cm} (6.4)

where the \(C_T\) is the isothermal value of the pre-factor. Differentiation with respect to \(\omega\) leads to

$$\frac{\partial \dot{\gamma}}{\partial \omega} = C_T \cdot \left( \frac{\tau}{G \cdot (1 - \omega)} \right)^{n'-1} \cdot n' \cdot \frac{\tau}{G \cdot (1 - \omega)^2} = \frac{\dot{\gamma} \cdot n'}{(1 - \omega)}$$ \hspace{1cm} (6.5)

equalling \(\partial \ln \dot{\gamma} = -n' \cdot \partial \ln (1 - \omega)\)

and the state of damage is the same if
\[ \Delta \ln \dot{\gamma} = -n' \cdot \Delta \ln \tau \] (6.6)

where \( \Delta \ln \tau \) denotes the load drop and \( \Delta \ln \dot{\gamma} \) stands for the increase in strain rate. To be able to compare these two kinds of damage evolution requires that the value of \( n' \) is known. For the specimen tested at 3\% strain amplitude the stress exponent was determined in stepped load testing to be 3.3 after 100 cycles.

As at 50\% load drop \( \Delta \ln \tau \approx -0.3 \) and an increase in strain rate of one order of magnitude results in \( \Delta \ln \dot{\gamma} = 1 \), (6.6) is fulfilled for the evaluations performed in subsection 6.1.1 and 6.1.2. Thus, the fatigue data determined for quenched and slowly cooled specimen can be compared to each other. This is done in Fig. 6.6. The slowly cooled specimen exhibit a slightly better fatigue resistance than the quenched specimen.

![Fig. 6.6](image)

Fig. 6.6: Comparison of fatigue data of both, quenched and slowly cooled specimen. Testing at 40°C revealing slightly higher fatigue resistance for slowly cooled specimen.
6.2 Differences and analogies between unidirectional and cyclic straining

While in both, unidirectional and cyclic straining, load drop indicative for accumulation of damage is found at continuous straining or cycling, respectively, there are some differences in the early stage of straining. In unidirectional testing, the load drop can be divided into parts, one at the beginning of straining related to local recrystallisation and another persisting over the whole test indicating accumulation of damage. In cyclic straining, however, there is no sign of such a partition. There is rather one steady decrease in load observed.

With regard to Fig. 5.4, one might argue that the load drop due to recrystallisation is usually not very large at strain rates of $10^{-3}\text{s}^{-1}$, especially if strain is highly localised as reported for cyclic straining by Morris and Mei [Mor91]. Hence, testing at a strain amplitude of 5% was performed at 80°C and a strain rate of $10^{-3}\text{s}^{-1}$ as well as at 40°C and a strain rate of $2\times10^{-4}\text{s}^{-1}$, where the load drop due to recrystallisation was expected to be more pronounced. As can be seen from Fig. 6.7, while there is a variation in initial stress level according to the constitutive behaviour, the load drop behaviour is virtually not affected by the altered testing parameters. This suggests that either the damage evolution is unaffected by stress in cyclic straining—which would contrast the established models of strain energy approach and stress intensity influenced crack growth—or the localisation is the more pronounced the lower the stress so that the lower stress and the higher local strain level out each other.

A third alternative would be that recrystallisation—for reasons still to be found—did not occur in cyclic straining. However, quenched specimen exhibited a strain rate sensitivity of 0.3 after 50 cycles at 3% strain amplitude. This indicates that a shear band has been built, i.e., recrystallisation has taken place within the first 50 cycles.

In cyclic straining, the monitoring of qualitative changes as carried out for unidirectional deformation is not easily accessible. Both, the high strain rate sensitivity as well as the evolution of a shear band during cyclic deformation indicate, that in general the same qualitative changes—i.e. recrystallisation and localisation—occur as in unidirectional testing, but that
Fig. 6.7: Evolution of peak load at 5% strain amplitude tested at various strain rates and load levels. The rate of load drop is virtually the same for all testing parameters suggesting stress-independent damage evolution.

numerical description and production of experimental evidence is much more difficult to accomplish than in unidirectional straining. Further research has to be carried out in this direction.

6.3 Fatigue damage evolution modelling in a FEA environment

As was already outlined in section 2.3, the description of damage evolution in cyclic straining is usually performed either in terms of Coffin-Manson evaluation or using crack propagation kinetics as given in e.g. the Paris-law. While the former describes accurately most of the field data, the latter is more appropriate for FEA. The difficulty in using the Coffin-Manson approach in FEA is that there is no single strain amplitude in a general solder joint geometry (see also Section 2.3). In contrast, using crack
propagation kinetics, the situation is defined at any point on the time axis. However, crack propagation models usually incorporate physical concepts which are appropriate neither for tin-lead eutectic nor for cyclic deformation. In the following a set of equations is presented that enables to implement a local damage evolution in FEA based on the Coffin-Manson description, cf. equation (2.19).

Similarly as the description in unidirectional deformation—and supported by the experimental evidence confirmed in Figs. 6.1 to 6.7—we postulate that the damage increase \( \Delta \omega \) during one thermocycle is proportional to the present damage \( (1-\omega) \) and to a function \( \varsigma \) of the strain amplitude \( \Delta \gamma_{pl} \) resulting in

\[
\Delta \omega = (1-\omega) \cdot \varsigma(\Delta \gamma_{pl}).
\] (6.7)

For \( \varsigma(\Delta \gamma_{pl}) \) one finds

\[
\varsigma(\Delta \gamma_{pl}) = \varsigma_0 \cdot (\Delta \gamma_{pl})^\alpha
\] (6.8)

The Coffin-Manson parameter \( \alpha' \) as well as \( \varsigma_0 \) have been determined to be 0.6 and 0.49, respectively, from Fig. 6.2. The damage rate parameter \( \varsigma_0 \) is calculated for \( \Delta \gamma_{pl} \) given in [\( \mu \text{m}/\text{\mu m} \)]. As the plastic deformation is in general not symmetric for a specific volume element and, thus, the meaning of \( \Delta \gamma_{pl} \) is not clear, the plastic strain amplitude may be replaced by the half of the equivalent strain encountered in one thermocycle.

Using the mathematical framework outlined above enables to calculate iteratively the damage state of any individual volume element. As the damage function approaches final failure asymptotically, an upper limit of damage has to be set in order to define final fracture of a volume element.
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Discussion and conclusions

7.1 Relation to previous work

The results presented in the previous chapters contain some points that contrast with what is said about eutectic lead-tin solder or metallic materials in general in the literature. We will hence have a closer look at these points in order to provide further evidence or bring the apparent inconsistency in line to previous research.

7.1.1 The constitutive equation

The bilinear appearance of stress vs. strain rate data for eutectic lead-tin solder is widely accepted in the literature, although there are various mathematical descriptions to account for it as shown in section 2.2. The main difference in this work is that the range of strain rates where the strain rate sensitivity is high is shifted to considerably higher strain rates (by a factor of up to $10^4$) compared to the values reported in the literature (cf. [Sto94], [Dar92], [Wil90], [Lee93]). On the other hand, it is in good qualitative agreement to the literature about the superplastic behaviour of lead-tin eutectics, cf. [Cli67], [Kay81]. The strain rate sensitivity of 0.3 as found in this study, however, is in good agreement with data determined on model solder joints ([Sol86], [Dar92], [Wil90], [Mor91]), while the 'superplastic' literature usually favours an SRS of 0.5.
The difference of the creep rates reported in the literature to those found in this thesis may be explained by the different grain or eutectic cell sizes used in the various studies. Following the findings of Lee and Stone [Lee93] the eutectic cell size in the as-solidified state is about 200 to 300 μm and the reported grain boundary sliding was in fact a cell boundary sliding, cf. Fig. 7.2. Wilcox [Wil90] reported on eutectic cell sizes of 30 to 120 μm after air cooling. In a study performed by Morris and Mei, the grain size was about 8 μm. After recrystallisation, however, the grain size is but 3 to 5 μm as shown in Fig. 7.2. Using a grain size exponent of \( \approx -2 \), differences of about a factor 10 to 10^4 are well in line with theoretically based expectations.

With regard to this fact, an acceptable explanation might be provided for the unsatisfactory modelling of Hall’s data performed in section 5.3. A structure consisting of very small grains as present after recrystallisation deforms faster than a structure consisting of larger grains if any such thing as
movement along grain boundaries is present. Assumed the grain size or more general the size of the microstructural unit along whose boundaries relative movement occurs was 10 times as much as in this study. Then, the pre-factor of the grain boundary sliding mechanism may be reduced by a factor of 100 to account for the more coarse structure.

As can be seen in Fig. 7.3, the model using a pre-factor according to the more coarse microstructure would come quite close to the experimental data determined by Hall, though it is not as good as the modelling provided by Subrahmanyan shown in section 5.3. However, there are some critical remarks to be made about such qualifying a constitutive model on data like Hall's. In fact, the form of a stress vs. temperature cycle is determined by three parameters only: firstly, the stiffness of the assembly, which is usually determined by evaluation of Hall's data, secondly by the 'overall' activation energy (including temperature dependence of the shear modulus) to account for the proportions of the graph and, third, by the pre-factor of the operating deformation mechanism to get the stress levels right. What is generally not important is the form of the stress-strain rate relationship. Therefore, the striking success of Subrahmanyan and co-workers might be due to the fact, that 'the effective activation energy was chosen to be 68 kJ/mol' while the values reported elsewhere by the same authors were 30 and 82 kJ/mol for gbs and matrix creep, respectively.
The evidence presented above further supports the point of view that a wide variety of mathematical descriptions of soft solder constitutive behaviour are able to account for experimental results at a given microstructure. It is the set of parameters that matters.

However, as the behaviour in the as-solidified state differs significantly from that after recrystallisation the question arises which set of parameters should be used for modelling Hall's data. As the test set-up used by Hall would induce some few percent of deformation in the joint throughout the test, the as-solidified data should be appropriate. The model describing the as-solidified behaviour, however, is far from being able to account for the data measured by Hall, as can be seen in Fig. 5.16. As the temperature change rate in Hall's experiments was slow resulting in creep strain rates of about 10⁻⁶s⁻¹ and the parameter of the model were determined in the range of 3·10⁻⁶s⁻¹ to 5·10⁻³s⁻¹ only, the question arises whether the low strain rate sensitivity determined at relatively high strain rates extends to low strain rates.
rates below $10^{-6}\text{s}^{-1}$ as well. Moreover, the constitutive models used by other authors as depicted in Fig. 7.1 as well as the experimental data shown in Fig. 4.3 exhibit a steady increase in strain rate sensitivity at low strain rates. Hence, low strain rate measurements were carried out by means of a load relaxation test in order to qualify the aptitude of the model for as-solidified material at low strain rates. The results of this check are conveyed together with stepped-load data and the models prediction in Fig. 7.4. As can be seen, the strain rate sensitivity, i.e., the slope of the data points, is constant down to strain rates as low as $10^{-7}\text{s}^{-1}$. The unsatisfactory modelling of Hall's data may hence not be ascribed to improper extrapolation of high strain rate behaviour to low strain rates. The obvious deviations are due to effects not yet identified.

Another point that needs some discussion is that of the strain rate sensitivity after straining. Although the microstructure is fine and globular in the shear band after recrystallisation, the strain rate sensitivity of 0.3 still differs

![Fig. 7.4: Comparison of low strain rate creep data measured in load relaxation testing and extrapolation from high strain rate data measured in stepped-load testing. Total strain range 5%, slowly cooled specimen tested at 40°C.](image)
considerably from that encountered in worked (bulk) material, i.e. 0.5. As the microstructure is virtually the same, cf. Fig. 7.5, and, moreover, grain boundary sliding is predominant in both materials, there is no direct evidence for a reason to account for such a difference. As was outlined in section 4.2, viscous glide of dislocations has been invoked in the literature as rate controlling deformation mechanism that would explain the high strain rate sensitivity at low strain rates. However, this mechanism is considered to be grain size independent [Wee60]. At last, the high strain rate sensitivity might be considered as a mere transition phenomenon from as postulated by Hart [Har67,70] and later numerically confirmed by Gharemani [Gar80]. Hereby, the transition is shifted to higher strain rates as the grain size decreases which would be in line with the changes in behaviour after recrystallisation. An explanation for the different strain rate sensitivity of worked bulk material and recrystallised shear specimen, however, is still lacking.

At last, in this thesis, the constitutive behaviour is assumed to be affected by strain only, not by time. While this may hold for the evolution of the recrystallised part as well as the damage, it takes not care of for instance grain coarsening as a consequence of exposure at high temperature and time. However, as was pointed out by Grivas [Gri78] the strain rate is the slower at a given effective stress the larger the grain size. As testing takes

Fig. 7.5: Globular microstructure in worked and annealed lead-tin eutectic (left, taken from [Gri78]). Microstructure of a recrystallised band in a solder joint revealing the same morphology as in the extruded material (right).
place at high homologous temperature, grain coarsening may be of some importance.

7.1.2 The change in microstructure after deformation

While the constitutive equation presented in this thesis seemingly lacks the accurateness to account for Hall's data, it incorporates for—to our knowledge—the first time the description of microstructural instabilities in eutectic lead-tin solder. The significant change in constitutive behaviour due to evolving instabilities reported in this study contrasts somewhat with results of Wilcox [Wil90] and Lee [Lee93], who both reported no or only little softening after repeated load relaxation. (Another widely used technique to determine constitutive behaviour, i.e., the evaluation of steady state creep data, is inappropriate to account for such changes as any softening is regarded as onset of the tertiary creep stage.) The key to the findings of Wilcox and Lee is that their repeated relaxation testing introduced but some few percent plastic strain, which in line with the model presented in this study does not lead to extensive changes.

Another point that might cause some objections is the evolution of the recrystallised fraction in the shear band given in section 5.1. As recrystallisation is usually considered as continuous process if ample internal energy is stored, one might argue that the simultaneous straining covers the dependence on time, feigning merely a relation to strain. In order to validate this argument a tensile test was run at 40°C and at a strain rate of 2·10⁻⁴s⁻¹. The test was stopped after 30% overall strain and the specimen unloaded. After 5000 seconds, the testing was restarted and again stopped after additional 20% strain. This procedure was repeated. After another 5000 seconds waiting the testing was restarted and run to another 70% overall strain. If recrystallisation was governed by time, the waiting between straining should produce an additional drop of load. If it was governed by strain, the load vs. time diagram should exhibit discrete horizontal steps caused by the waiting. As can be seen in Fig. 7.6 the load vs. time diagram reflects the features characteristic for strain controlled recrystallisation, i.e., horizontal steps at the waiting times, supporting the description as given in section 5.1. Similar behaviour was found in identical testing at 80°C.
Fig. 7.6: Tensile testing with repeated unloading and waiting in order to validate the recrystallisation kinetics as presented in section 5.1.

7.1.3 Strain localisation

In the model developed in section 5.1, we postulated that the thickness $\lambda$ of the recrystallised band was constant throughout the joint and not changing with time. In spite of the successful modelling, light microscopy indicates that this is an oversimplification. In fact, the localised band thickness $\lambda$ is rather an average value. Moreover, it is likely to be narrowed in close vicinity to a propagating microcrack. Nevertheless, the outlined simplifications represent an acceptable first order approach.

As the model can accurately account for the experimental data using the measured band thickness, it provides knowledge about the system \textit{a posteriori} only. It is, hence, of great interest to know which testing parameters influence the band thickness $\lambda$. 
Intuitively, we would expect that the recrystallised band thickness is the smaller the larger the ratio of local strain in the shear band to that in the as-solidified part of the joint. This ratio changes from 1 (x=0) to \( P^n \) (x=1), cf. equation (5.6) and, thus, depends mainly on the stress partitioning factor \( P \). In Fig. 7.7, the dependence of the shear band thickness on \( P \) is conveyed. While there is in fact a tendency towards thicker shear bands at low \( P \)-values, the data scatter considerably. As concerns the very narrow shear bands in quenched specimen, the first-principles-approach proposed above cannot take care for it.

As can be seen from Fig. 7.7, the solidification condition of the specimen plays a crucial role in determining the strain localisation. A reason for this influence may be provided by the difference in initial cell size after solidification. As was shown in Fig. 4.2, the microstructure after slow solidification exhibits eutectic cell boundaries in any direction, i.e., parallel

![Diagram](image)

Fig. 7.7: Dependence of the shear band thickness \( \lambda \) on the stress partitioning factor \( P \) determined at various temperatures and strain rates on 60\( \mu \)m joint specimen. For the slowly cooled specimen a inverse proportionality holds quite well. The quenched specimen exhibit narrower shear bands.
and perpendicular to the applied stress, while after quenching the cell boundaries are predominantly perpendicular to the direction of applied stress except at the solder/substrate interface. As shear bands form predominantly in eutectic cell boundary regions, they can evolve at various places throughout the solder joint in slowly cooled specimen, while in quenched specimen shear bands can form along solder/substrate-interfaces only. However, the shear band thickness is not affected by the overall joint thickness neither in quenched nor in slowly cooled specimen. This indicates, that the evolution of a shear band is affected but not completely controlled by the microstructure. The findings presented above contrast with work published by Mei et al. [Mei91a,b] where quenched material in solder joints have been reported to exhibit higher fatigue resistance than slowly cooled specimen. This has been ascribed to the decreased proneness to localise strain owing to the fine and equiaxed structure in quenched specimen. However, repeated investigation performed in this study supports that the inverse is the case.

Although the shear band thickness could be determined by light microscopy after testing there is some uncertainty about the actual value of $\lambda$ as it varied throughout the solder joint and was influenced by large primary phases and intermetallics. Thus, the value taken for $\lambda$ was rather 'the most likely average' then an exact value. Moreover, it is not clear, whether the shear band thickness changed during testing as only the situation after testing was accessible. This at last raises the question to what extent the determined values of the damage rate $s$ are reliable as merely the ratio of $s$ to $\lambda$ was represented by the relative decrease in load $\Omega$ per unit strain, which was determined by experiment. However, tensile testing allow a further cross-check as the stress is related to the local strain rate, which in turn depends on the shear band thickness $\lambda$ and the overall strain rate applied.

7.1.4 Damage evolution and fracture behaviour

As was shown in section 5.2, there are differences between slowly cooled and quenched specimen in terms of damage evolution. These differences are accounted for in the distinct damage evolution descriptions as given in (5.25), (5.26), and (5.28), respectively. While both damage evolution laws were developed as purely numerical descriptions they nevertheless may
allow for a physical interpretation. The damage evolution description for slowly cooled specimen, equations (5.25) and (5.26), comes up to a damage evolution law governed by the deformation energy as frequently found in state variable approaches, cf. [Bod87]. On the other hand, quenched specimen follow an equation that emphasises the influence of stress as typical for approaches backing on stress intensity calculations.

Comparing the damage evolution during tensile testing for quenched and slowly cooled specimen, cf. Figs. 5.12 and 5.13, one may argue that the deviation from the logarithmic damage evolution, as given in (5.25) for slowly cooled specimen, found in quenched specimen is significant at high local strain values only, i.e. in highly damaged structures. Hence, damage evolution during tensile testing in quenched specimen may be described by a logarithmic damage evolution law up to damage values of 0.5. In turn, as tensile testing in slowly cooled specimen usually resulted in damage values of about 0.5, the question arises whether at higher damage values the logarithmic damage evolution law still would hold or whether a deviation similar to that encountered in quenched specimen testing would occur.

Therefore, a tensile test has been performed on slowly cooled specimen at 350% shear strain resulting in a final damage value of 0.75 at which the onset of deviation from the logarithmic law should be visible. However as depicted in Fig. 7.8, the logarithmic damage evolution, represented by a linear decrease in log τ, persists throughout the whole test. Thus, the damage evolution of quenched and slowly cooled specimen is distinct indeed, which corresponds to the distinct fracture behaviour presented in chapter 4.

There are two more points that have to be mentioned with respect to the damage behaviour:

- Some tests have indicated that damage does not evolve from the beginning of straining. There is rather evidence for a threshold strain until which no damage is introduced being at about 30% local strain. This may be accounted for by changing the lower integration limit in equations (5.25), (5.26), and (5.28) from 0 to 30%.

- There have been general criticism about the load-drop being a damage indicator by Subrahmanyan and Wilcox and co-workers [Sub89],
Fig. 7.8: Evaluation of the damage evolution behaviour at high damage values in slowly cooled specimen. The models for both quenched and slowly cooled specimen are given as full and doted line revealing persisting logarithmic increase in damage.

[Wil90b]. Their arguments were directed towards the situation where the stiffness of the solder joint was smaller than or equal or to the stiffness of the substrate and testing was controlled by total strain. However, in this study the stiffness of the solder joint given by \( O \cdot G / l \) is much higher than that of the substrate being at about \( 10^5 \) N/mm. Moreover, the fatigue testing is controlled by plastic strain amplitudes as described in section 3.3. Under these conditions, the load drop is considered to be indicative for damage evolution.

### 7.1.5 Comparison of unidirectional and cyclic testing

In Fig. 7.9, fatigue data determined in this study are compared to those published by Solomon [Sol94]. The values from the literature exhibit slightly higher fatigue resistance, which might be due to the total strain
Fig. 7.9: Comparison of fatigue data determined in this work and those digitised from Solomon [Sol94] tested at 35°C showing quite good accordance. The somewhat higher number of cycles to failure found by Solomon may be due to total strain controlled testing.

controlled testing procedure. However, the value of the Coffin-Manson exponent \( \alpha' \) is about the same in both data sets, i.e. \( =0.6 \).

Comparing the damage evolution laws for both cyclic and unidirectional straining reveals that there are important differences, cf. (7.1) and (7.2):

\[
\Delta \ln \omega = \zeta_o \cdot \Delta \gamma_{pl}^{1/\alpha} \tag{7.1}
\]

\[
\Delta \ln \omega = s_o \cdot \tau_{eff} \cdot \Delta \gamma_{pl} \tag{7.2}
\]

While in the unidirectional case the stress level is found to affect the damage rate no such influence could be determined for cyclic straining.
However, there might be an effect of stress, cf. Fig. 6.4, but data are too rare to allow for a reliable determination.

On the other hand, the deformation amplitude is found to affect the damage per cycle other than linearly, which contrasts the findings in unidirectional testing. At this stage, it is thus not possible to provide a unified theory of damage evolution for both unidirectional and cyclic straining.

### 7.1.6 Implications for production and testing in the SMT

As has been outlined in the previous sections, the solidification process strongly affects the resulting mechanical properties of the solder. Due to the thinner shear bands, quenched solder joints exhibit a poorer resistance against fatigue damage than slowly cooled solder joints. It is hence advisable to use as slow solidification processes as possible in SMT production.

The obviously widespread forming of shear bands in eutectic lead-tin solder bears an implication for technical applications. As was suggested in Engelmaier's equation, cf. (2.28), the height of the solder joint would influence the number of cycles to failure. This is true if the strain amplitude is calculated by division of the relative thermal displacement by the solder height. However, if the thickness of the shear band, where virtually all of the displacement is absorbed by local straining, is not affected by the solder joint height, a further increase in joint thickness should not change the fatigue resistance of the joint. Hence, applying of underfiller and gap-provider in order to increase the fatigue resistance of the assembly may not necessarily be successful.

A similar problem arises by using ball grid arrays and column grid arrays as recently introduced: owing to differences in alloy composition and, thus, strength, the strain will localise in the low-melting part of the solder joint, cancelling the advantage of the higher solder joint. Generally speaking, the solder height is important only if the joint geometry allows for significant elastic bending of the solder post or column, i.e., if the column is very thin.
Testing considerations are guided by the same arguments as presented in section 2.4: Thermomechanical cycling leads imposes primarily an overall strain amplitude on the device/PCB/solder joint combination. The amount of plastic strain induced in the solder joint depends on the time during which the stress is applied and on the stress level as the rate of plastic deformation depends on the applied stress. The applied stress is influenced by the temperature swing, the thermo-mismatch, and the stiffness of the assembly. For compliant devices such as a QFP even large temperature amplitudes lead to small stresses only and hence require long dwell times in order to result in appreciable plastic strain. On the other hand, stiff devices such as ceramic capacitors transform most of the elastically stored strain in very short dwell times (< 300s) at the upper temperature limit of the cycle. If the lower temperature limit is below 0°C, the dwell time is of no use in compliant devices while again a short dwell time is required for stiff devices, especially if the temperature change rate is fast.

As for temperature change rates, the situation is just inverse: While compliant devices are more or less unaffected by fast temperature change rates, stiff components induce high stresses in the solder joint, provoking damage rates which are larger than those at intermediate stresses including the risk of promoting artificial failure mechanisms. Hence, in field tests a temperature change rate larger than 10°C should not be exceeded.

Obviously, the choice of the test parameters prejudices which kind of component in terms of stiffness is more likely to fail. It is thus, crucial to understand the results of field fatigue testing with respect to the testing parameters.

7.1.7 Direction of future work

As has been shown in this thesis, the proneness to microstructural changes and the accompanying localisation of strain and premature failure characterises the constitutive behaviour as well as the damage evolution in eutectic lead-tin solder. It will, hence, be of major interest to learn more about the relevant parameters that trigger the localised recrystallisation. Some efforts in this direction have already been made by Morris and co-
workers, cf. [Mor94]. However, basic understanding of these features is still on demand.

Another point that needs some further investigation is the recrystallisation behaviour in cyclic testing. As these preliminary studies have shown, there is no pronounced transition to a completely recrystallised state in cyclic testing. However, as unidirectional straining may be considered as limiting case of cyclic straining the evolution of a shear band would be expected to take place in cyclic deformation, too, resulting in a unified theory for both, cyclic and unidirectional straining.

7.2 Conclusions

The thesis in hand has dealt with the creep deformation and fatigue behaviour of a lead-tin-silver eutectic solder, as it is in use for soldering of electronic devices. Therein, the focus has been set on the deformation behaviour at large strains and on microstructural changes accompanying the creep deformation. The contribution to the state-of-the-art-knowledge on eutectic solder of this work can be summarised as follows:

1. The constitutive behaviour of eutectic lead-tin-silver solder and its dependence on the microstructure is investigated at length. It is shown, that straining causes significant changes in the creep characteristic in terms of increased strain rate sensitivity compared to the stress vs. strain rate behaviour after solidification.

2. The microstructural changes accompanying this transition, i.e. recrystallisation, are qualified and demonstrated experimentally. It is recognised that these changes are restricted to a fraction of the solder joint only and this fraction is identified as coarsened shear band as being regularly encountered in mechanically tested solder structures. In these shear bands grain and phase boundary movement contributes significantly to the deformation.

3. The evolution of damage is extracted from unidirectional as well as cyclic deformation tests. For slowly cooled specimen, it is found that the
rate of damage evolution is proportional to the strain energy in unidirectional testing. The damage evolution in cyclic straining is described with a localised and discretised Coffin-Manson approach.

4. The structural changes as well as the localisation are incorporated—to our knowledge—for the first time in an overall constitutive description. The damage evolution following the description proposed by Kachanov and Lemaitre is accounted for as well. This results in a state variable approach, where three variables—the recrystallisation state $x$, the damage state $\omega$, and the temperature $T$—completely define the (strain-) response of a volume element to a given load.

5. It is shown that the rate of recrystallisation is proportional to the not yet recrystallised volume fraction in the shear band and the accumulated local strain. This contrasts other kinetic models of phase transformation such as the Johnson-Mehl-Avrami equation, where transformation is governed by time.
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Mechanical Analysis of a Single-Lap-Joint Specimen

Consider the single-lap-joint specimen given in Fig. A.1. Since the length is an order of magnitude larger than the thickness, the problem is treated in terms of simple beam theory.

In a tensile test machine the specimen is clamped on both sides. Taking advantage of the symmetry, one can confine the problem on the situation presented in Fig. A.2: a force with components parallel and perpendicular to the intersecting plane in point A substitutes for the second half. The moment in A, which could occur as well, can be set to zero due to the symmetry (if the load frame is OK). In the clamp a normal force, a shear force and a bending moment is allowed. The shear component in the direction perpendicular to the drawing plane may not be zero—due to lateral contraction—but does not influence the result and, therefore, is neglected in the following.

![Fig. A.1: Single-Lap-Joint specimen clamped in a tensile test machine](image-url)
Since the two forces $T$ are not co-linear a bending moment is present in the beam. Consequently, a displacement of $A$ in direction perpendicular to the beam but within the drawing plane is expected. Due to the symmetry, this displacement must be cancelled by the normal force $S$ in $A$. The magnitude of $S$ is estimated in the following:

The momentum of bending $M$ is given as a function of $T$ and $S$ and the $x$-coordinate as follows:

$$M(x) = T \cdot \frac{d + \delta}{2} - S \cdot (l - x) \quad (A.1)$$

The differential equation of the bending line is

$$y'' = \frac{M}{E \cdot J} = \frac{A - S \cdot (l - x)}{E \cdot J} \quad (A.2)$$

with $E$...Young's modulus of the substrate, $J$... the moment of inertia about the $z$-axis of the beam and

$$A = T \cdot \frac{d + \delta}{2}$$

Integration leads to

$$y' = \frac{1}{E \cdot J} \left( \frac{1}{2} \cdot S \cdot x^2 + A \cdot x - l \cdot S \cdot x + C_1 \right) \quad (A.3)$$
and the parameter $C_1$ follows from the boundary condition $y'(0) = 0$ (rigid fixation!) to be zero. Further integration leads to the bending line:

$$y = \frac{1}{E \cdot J} \cdot \left( \frac{1}{6} S \cdot x^3 + \frac{1}{2} A \cdot x^2 - \frac{1}{2} l \cdot S \cdot x^2 + C_1 \right)$$  \hspace{1cm} (A.4)$$

As above, the parameter $C_2$ follows from the boundary condition $y(0) = 0$ to be zero. The value of $S$ follows from the boundary condition $y(l) = 0$:

$$S = \frac{3}{2} \cdot A \cdot \frac{1}{l} = \frac{3}{4} \cdot \frac{d + \delta}{l} \cdot T$$  \hspace{1cm} (A.5)$$

Interestingly enough, the ratio of $T$ and $S$ does not depend on the bending stiffness of the substrate. Therefore, the substrate is to keep as thin as possible (since $S$ is proportional to $d + \delta$).

A further source of normal forces in the solder joint might be the distortion of the solder plane with respect to the shear forces $T$. This distortion can be estimated using (A.3) and (A.5) for $x = l$:

$$y' = \frac{1}{8} \cdot \frac{l \cdot (d + \delta) \cdot T}{E \cdot J}$$

Using the explicit form of the moment of inertia, $J$, the tangent of the distortion angle and, thus, the additional part of the normal force can be calculated:

$$y' = \frac{3}{2} \cdot \frac{l \cdot (d + \delta) \cdot a}{d^3} \cdot \frac{\tau}{E}$$

$$\Delta S = y' \cdot T$$  \hspace{1cm} (A.6)$$

With respect to the used specimen geometry an additional normal force $\Delta S$ of about $10^{-3}$ times the shear force $T$ develops due to the distortion.

The analysis presented above assumed the forces $T$ and $S$ to act on one single point $A$. In fact, the normal force $S$ is introduced via a finite area—the solder joint—and, hence, the distribution of that force over the area has
to be discussed. While any boundary effects due to the symmetry of the stress tensor are treated by [Yan92], we will restrict our considerations to normal forces due to any moments of bending present in the joint. As it was mentioned above, in the plane of intersection the moment is zero, due to symmetry. At the boundary of solder and substrate, however, the bending moment $m$ is not zero but may be approximated by

$$m = M(l) \cdot \frac{\delta}{(d + \delta)}$$

(A.7)

Inserting $M(l)$ from (A.1), the moment $m$ is given as

$$m = \frac{T \cdot \delta}{2}$$

In order to account for this moment $m$ an additional pair of forces, $s^+$ and $s^-$ has to be introduced as given in Fig. A.3. The ratio of $S$ to $s^+$ equals the ratio of the level of homogeneously distributed normal stress and the additional stress amplitude accounting for the moment $m$. Hence,

$$m = \frac{T \cdot \delta}{2} = \frac{2 \cdot s^+ \cdot a}{3}$$

Fig. A.3: Distribution of normal stresses in the solder joint (left) and its substitution by a normal force and a pair of forces (right).
and the ratio of $S$ to $s^+$ equals

$$S = \frac{(d + \delta) \cdot a}{l \cdot \delta} \cdot s^+$$  \hspace{1cm} (A.8)

which inserting the actual geometrical values—i.e. $d = 3\text{mm}$, $\delta = 0.06\text{mm}$, $a = 4\text{mm}$, and $l = 30\text{mm}$—leads to $S \approx 7 \cdot s^+$. 
A2

List of Symbols and Abbreviations

A  power-law pre-factor for dislocation climb regime
A_T  temperature integrated pre-factor A
b  Burgers vector
B  power-law pre-factor for grain boundary sliding
B_T  temperature integrated pre-factor B
d  grain size
dc  dislocation climb
fc  furnace cooled
G  shear modulus
gbs  grain boundary sliding
k  stiffness of the assembly
l  solder joint thickness
m  stress exponent in the power-law description of grain boundary sliding controlled creep
n  stress exponent in the power-law description of dislocation climb controlled creep
P  stress partitioning factor
PCB  printed circuit board
Q(i) thermal activation energy
Q_{dc} thermal activation energy for dislocation climb
Q_{gbs} thermal activation energy for grain boundary sliding
qu quenched
R universal gas constant
\dot{r} rate of grain growth
\mathcal{K} pre-factor in the Weibull plot in recrystallisation kinetics
s rate of damage accumulation
SMT surface mount technology
SMD surface mount devices
SRS strain rate sensitivity
T temperature
x recrystallised part of the shear band
\alpha Weibull exponent in recrystallisation
\beta pre-factor in the Weibull plot in recrystallisation kinetics
\gamma_{tot} overall shear strain
\gamma_{loc} local shear strain
\dot{\gamma} total shear strain rate
\dot{\gamma}^* shear strain rate of equal contribution of gbs and dc
\dot{\gamma}' applied shear strain rate
\dot{\gamma}_{cr} creep shear strain rate
\dot{\gamma}_{eff} effective shear strain rate
\dot{\gamma}_{el} elastic shear strain rate
\dot{\gamma}_{loc} local shear strain rate
\dot{\gamma}_{pl} plastic shear strain rate
\dot{\gamma}_0 initial shear strain rate
\Gamma numerical constant
\Delta g Gibb's free energy of recrystallisation
\( \lambda \) shear band thickness
\( \nu \) Debye frequency, cycle frequency in fatigue testing
\( \rho \) dislocation density
\( \tau \) applied shear stress
\( \tau^* \) stress at which dc and gbs contribute to equal parts to the creep rate
\( \tau_{as} \) shear stress in the as soldered regions
\( \tau_{rec} \) shear stress in the recrystallised regions
\( \dot{\tau} \) rate of shear stress change
\( \omega \) damage variable
I low stress creep regime
II intermediate stress creep regime
III high stress creep regime
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Curriculum Vitae

I was born in Wettingen (Switzerland) on April 5, 1969. In 1971, my family moved to Stein am Rhein, where I attended primary and secondary school. Starting in 1983, I visited the Kantonsschule in Schaffhausen, graduating with the Matura in 1988. After military service, I studied from 1989 to 1994 at the Swiss Federal Institute of Technology (ETH), Zurich, where I received the diploma in materials engineering (Dipl. Werkst.-Ing. ETH). Since 1994, I worked as a research scientist at the Reliability Laboratory of the ETH on the field of constitutive equations and damage behaviour of solder material.