A measurement of $K^0, \bar{K}^0 \rightarrow 3\pi^0$ and an improved test of CPT

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Abstract

The CPLEAR experiment at CERN is aimed to study CP-, T- and CPT-symmetries in the neutral-kaon system. The neutral kaons are produced in $p\bar{p}$ annihilations at rest:

$$p\bar{p} \rightarrow K^0K^+\pi^-, K^0K^-\pi^+.\ $$

The observation of the sign of the accompanying charged kaon allows one to determine for each event, whether the neutral kaon is a $K^0$ or a $K^0$. This gives to CPLEAR the capacity of extracting relevant physical parameters through time-dependent asymmetries between the rates of initially pure $K^0$ and $\bar{K}^0$ decaying into various final states.

In this work an analysis of $K^0,\bar{K}^0 \rightarrow \pi^0\pi^0\pi^0$ decays is presented. This analysis has lead to the determination of the real and imaginary part of the CP-violation parameter $\eta_{\pi\pi\pi}$ which are measured to be:

$$Re(\eta_{\pi\pi\pi}) = 0.18 \pm 0.14(stat) \pm 0.06(syst)$$

$$Im(\eta_{\pi\pi\pi}) = 0.15 \pm 0.20(stat) \pm 0.03(syst).$$

These values, although still compatible with CP-conservation, represent the best sensitivity to CP-violation in this decay mode; they lead to an upper limit $|\eta_{\pi\pi\pi}| < 0.58$ at 90% confidence level. Using the result on $Re(\eta_{\pi\pi\pi})$ and $Im(\eta_{\pi\pi\pi})$ together with unitarity relations in the neutral-kaon system, the imaginary part of the CPT-violation parameter $\delta$ is found to be:

$$Im(\delta) = (1.3 \pm 3.6) \cdot 10^{-5}.\ $$

This value is in agreement with CPT-invariance; it leads to a mass difference between the neutral kaon and its antiparticle:

$$M_{K^0} - M_{\bar{K}^0} = (1.8 \pm 5.1) \cdot 10^{-19} GeV/c^2.\ $$

The limit on this mass difference is of the order of the inverse of the Planck scale and experimentally confirms the CPT-invariance in the neutral-kaon system up to these scales.
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Chapter 1

INTRODUCTION

1.1 Discrete symmetries and kaons

Symmetry is one of the most important concepts in physics [1]. Symmetries of physical laws often lead to observable symmetries in nature. They can also generate conserved quantities. Violation of symmetry often indicates some dynamical mechanism acting behind the current understanding of physics. In this thesis, we consider three discrete transformations: Parity transformation (P), charge conjugation (C) and time reversal operation (T). Under P, all the space coordinates are inverted; C interchanges a particle to its anti-particle; T reverses the direction of time.

In nature, both electromagnetic and strong interactions are found to be invariant under C, P and T and any combination of them. However, the weak interaction was discovered to be not invariant under P, so called Parity violation. This lead to the conclusion that the structure of the charged weak current was of the type V-A (vector - axial). The weak interaction is also non-invariant under C.

An unexpected violation of CP was first discovered in the neutral-kaon decays [2]. Although the Standard Model [3, 4] can accommodate this through a complex mass mixing matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix [5, 6], no decisive experimental test has been made to exclude mechanisms of CP violation as suggested by theories which go beyond the Standard Model. This is why the field of CP-violation is still one of the most active areas in particle physics both experimentally and theoretically.

While no particular reason exists why nature has to remain invariant under P, C, T, CP, TP and CT, it has been shown [7] that a few very fundamental assumptions such as the Lorentz invariance lead to a conclusion that the laws of the physics should stay invariant under the CPT transformation. Any sign of violation indicates a breakdown of one or more of those fundamental assumptions. Therefore, the search for CPT-violation is an important subject in physics. The quantum gravity [8] and the string theory [9] no more fulfill all these fundamental assumptions, and have thus revived the interest for accurate experimental tests
of CPT.

Since CP symmetry is already violated in the neutral-kaon system, this system appears as one of the most promising places to look for CPT-violation. CPT-violation would result in the mass and decay width differences between particle and anti-particle. In the neutral-kaon system, the $\overline{K}^0-K^0$ oscillations generate an interference effect that could amplify such differences. This makes this system especially attractive.

For such studies, many parameters describing the neutral-kaon system have to be well measured. One of the most poorly known parameters is $\eta_{000}$, the CP-violation parameter in $\overline{K}^0, K^0 \rightarrow 3\pi^0$ decays; the determination of this parameter is all the more interesting since no CP-violation in $K_S^0$ decays has been observed. The main motivation of the thesis is to improve this measurement and investigate its influence on the knowledge about the validity of CPT invariance.

### 1.2 Thesis outline

One aim of this work is to test the CPT-symmetry in the neutral-kaon system. In the coming chapter, we will present this system and focus on a method using the Bell-Steinberger relations to do a test of CPT invariance. We will see that the parameter $\eta_{000}$ describing CP-violation in $K_S^0 \rightarrow \pi^0\pi^0\pi^0$ decays is one of the important parameters for this test.

In the third chapter we will present the CPLEAR detector, with particular emphasis on the Calorimeter which is essential to reconstruct neutral-kaon decays into $\pi^0$'s.

Chapter four is devoted to the analysis of $K^0, \overline{K}^0 \rightarrow \pi^0\pi^0\pi^0 \rightarrow 6\gamma$ decays. Special attention has been paid to the reconstruction of the neutral-kaon decay time and to the discrimination of the background; the photon-pairing problem and the use of the angles of photon showers has been studied in detail. This analysis will lead to an improved value of the CP-violation parameter $\eta_{000}$.

In chapter five finally, we will perform an indirect test of CPT-symmetry where the newly measured value of $\eta_{000}$ leads to a better precision on the CPT-violation parameter than previously obtained.
Chapter 2

PHENOMENOLOGY OF THE $K^0$-MESON SYSTEM

In this chapter we will first explain the dynamics of the neutral-kaon system and express it in term of observable states. In addition of providing an understanding of this system, the unitarity relations will give an interesting way to compute some of its relevant parameters. The study of neutral-kaon decays will permit us to define the input parameters to a CPT-test through the unitarity relations.

2.1 The neutral-kaon system

2.1.1 Time evolution of the $\bar{K}^0-K^0$ system

Let $|K^0\rangle$ and $|\bar{K}^0\rangle$ be the stationary neutral-kaon eigenstates of the strong and electromagnetic hamiltonian,

$$(H_S + H_E)|K^0\rangle = M_0 |K^0\rangle , \quad (H_S + H_E)|\bar{K}^0\rangle = \bar{M}_0 |\bar{K}^0\rangle$$

where $M_0$ and $\bar{M}_0$ are the rest masses of $K^0$ and $\bar{K}^0$. Since $K^0$ and $\bar{K}^0$ are CP-conjugated states, we have:

$$\text{CP}|K^0\rangle = e^{i\phi_{CP}} |\bar{K}^0\rangle , \quad \text{CP}|\bar{K}^0\rangle = e^{-i\phi_{CP}} |K^0\rangle (2.1)$$

where $\phi_{CP}$ is a phase. Since $K^0$ and $\bar{K}^0$ are defined as stationary states, the $T$ operator introduces only phases when applied to them:

$$T|K^0\rangle = e^{i\phi_T} |K^0\rangle , \quad T|\bar{K}^0\rangle = e^{i\phi_T} |\bar{K}^0\rangle . (2.2)$$

By demanding that $\text{CPT}|K^0\rangle = T\text{CP}|K^0\rangle$, we obtain the following relation between the phases:

$$2\phi_{CP} = \phi_T - \phi_T . (2.3)$$
Since the weak interaction does not conserve strangeness, the neutral-kaon states can decay into a final state without strange particles as shown in figure 2.1, or oscillate to each other as shown in figure 2.2.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.1}
\caption{\(K^0 \rightarrow \pi^0 \pi^0 \pi^0\) decay through first order diagram in the Standard Model.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.2}
\caption{An example of \(K^0 - \bar{K}^0\) oscillations through second order diagram in the Standard Model.}
\end{figure}

The time-dependent general state is then expressed as

\[ |\Psi(t)\rangle = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle + \sum_f c_f(t)|f\rangle \]  \hspace{1cm} (2.4)

where \(a(t), b(t)\) and \(c_f(t)\) are time dependent functions. This state is a solution of the Schrödinger equation:

\[ i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = (H_S + H_E + V)|\Psi(t)\rangle \]  \hspace{1cm} (2.5)

where \(V\) is the weak interaction hamiltonian.
2.1. **THE NEUTRAL-KAON SYSTEM**

Since the weak interaction is much smaller than the other interactions, we can solve equation 2.5 by applying perturbation theory. Furthermore, weak interactions between the final states are neglected (Wigner-Weisskopf approach [10]). It can be shown that \( a(t) \) and \( b(t) \) follow

\[
i\frac{\partial}{\partial t}|\psi(t)\rangle = \Lambda|\psi(t)\rangle
\]

(2.6)

where

\[
|\psi(t)\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}
\]

and where \( \Lambda \) is given by a non-hermitian \( 2 \times 2 \) matrix; \( \Lambda \) may be decomposed as:

\[
\Lambda = M - \frac{i}{2} \Gamma
\]

(2.7)

where \( M \) and \( \Gamma \) are hermitian. Their elements are

\[
M_{ij} = M_{0}\delta_{ij} + \langle i|V|j\rangle + \sum_{f} \mathcal{P} \left( \frac{\langle i|V|f\rangle\langle f|V|j\rangle}{M_{0} - E_{f}} \right)
\]

\[
\Gamma_{ij} = 2\pi \sum_{f} \langle i|V|f\rangle\langle f|V|j\rangle\delta(M_{0} - E_{f})
\]

(2.8)

with \( i, j = 1, 2 \) where \( 1 \equiv K^{0}, 2 \equiv \bar{K}^{0} \) and \( \mathcal{P} \) means principal part.

From the first of the above equation one can see how the weak oscillation of the neutral kaon also contributes to the mass. The second term of \( M_{12} \) representing a direct \( K^{0}-\bar{K}^{0} \) transition, demands *Flavour Changing Neutral Currents* in the first order and thus vanishes if we consider only the standard weak interaction [11]. The hypothetical superweak interaction [12] occurring at much higher energy scale can generate such currents. The third term of \( M_{12} \) represents second-order interactions, here oscillations through all possible virtual states (fig. 2.2). The \( \Gamma \)-matrix is constructed from the decays of neutral kaons (see section 2.1.2) to all real final states as seen from the second equation.

If the weak interactions \( (V) \) are T-, CPT- or CP-invariant, then the \( \Lambda \) matrix must satisfy the following conditions [13]:

\[
T : |\Lambda_{12}| = |\Lambda_{21}|
\]

(2.9)

\[\text{CPT} : \Lambda_{11} = \Lambda_{22}\]

\[\text{CP} : |\Lambda_{12}| = |\Lambda_{21}|, \Lambda_{11} = \Lambda_{22}.
\]

We observe that T-violation \( (|\Lambda_{12}| \neq |\Lambda_{21}|) \) or CPT-violation \( (\Lambda_{11} \neq \Lambda_{22}) \) are always accompanied by CP-violation.

By assuming that CP-violation is small [2], the solutions of equation 2.6 for initially pure \( K^{0} \) and \( \bar{K}^{0} \) are given by

\[
|K^{0}(t)\rangle = \frac{1}{\sqrt{2}}[e^{-i\Delta s^{t}}|K_{S}\rangle + e^{-i\Delta L^{t}}|K_{L}\rangle]
\]

(2.10)

\[
|\bar{K}^{0}(t)\rangle = e^{i\phi_{L}}\frac{1 + 2\delta t}{\sqrt{2}}[(1 - 2\delta)e^{-i\Delta s^{t}}|K_{S}\rangle - (1 + 2\delta)e^{-i\Delta L^{t}}|K_{L}\rangle]
\]
where $\epsilon_T$ and $\delta$ are T- and CPT-violation parameters and

\begin{align}
\Lambda_{L,S} &= M_{L,S} - \frac{i}{2} \Gamma_{L,S} \\
M_{L(S)} &= \frac{M_{11} + M_{22}}{2} + (-)|M_{12}| \\
\Gamma_{L(S)} &= \frac{\Gamma_{11} + \Gamma_{22}}{2} - (+)|\Gamma_{12}|.
\end{align}

The $e^{-iM_{s,t}\tau}$ term in equation 2.10 exhibits the oscillatory character of the $K^0\bar{K^0}$ system, while the $e^{-\Gamma_{s,t}\tau/2}$ shows their decay to final states. The states $|K_S\rangle$ and $|K_L\rangle$ are the eigenvectors of the $\Lambda$ matrix with eigenvalues $\Lambda_{S,L}$ and are thus the physical neutral-kaon states with definite masses $M_{S,L}$ and decay widths $\Gamma_{S,L}$. They are found to be

\begin{align}
|K_S\rangle &= \frac{1}{\sqrt{2}}[(1 + \epsilon_S)|K^0\rangle + (1 - \epsilon_S)e^{-i\phi_T}|\bar{K^0}\rangle] \\
|K_L\rangle &= \frac{1}{\sqrt{2}}[(1 + \epsilon_L)|K^0\rangle - (1 - \epsilon_L)e^{-i\phi_T}|\bar{K^0}\rangle]
\end{align}

with $\epsilon_S = \epsilon_T + \delta$ and $\epsilon_L = \epsilon_T - \delta$. In the limit of CP-conservation ($\epsilon_S = \epsilon_L = 0$), $|K_S\rangle$ and $|K_L\rangle$ are CP-eigenstates with eigenvalues +1 and −1, respectively.

The small CP-violation parameter

\begin{equation}
\epsilon_T = \sin(\phi_{SW}) \frac{|\Lambda_{12}|^2 - |\Lambda_{21}|^2}{\Delta m \Delta \Gamma} e^{i\phi_{SW}} = \frac{i(2\Delta m - i\Delta \Gamma)}{4\Delta m^2 + \Delta \Gamma^2} \sin(\phi_T - \phi_M)
\end{equation}

is also a T-violation parameter as seen from equation 2.9. The phase $\phi_{SW}$ is defined by $\phi_{SW} \equiv \tan^{-1}(2\Delta m/\Delta \Gamma)$ and $\phi_M$ and $\phi_T$ are respectively the phases of $M_{12}$ and $\Gamma_{12}$. From the above equation one can see that this CP-violation is in fact generated by the interference between oscillations through box diagrams (fig. 2.2) which gives $M_{12}$ and through common final states (see for example fig. 2.3) which gives $\Gamma_{12}$ [14].

The small CP-violation parameter

\begin{equation}
\delta = \frac{\Lambda_{22} - \Lambda_{11}}{2\Delta m + i\Delta \Gamma} = \frac{i2\Delta m + \Delta \Gamma}{4\Delta m^2 + \Delta \Gamma^2} \left[\frac{(\Gamma_{11} - \Gamma_{22})}{2} + i(M_{11} - M_{22})\right]
\end{equation}

also violates CPT as seen from equation 2.9. Since the mass-matrix $M$ represents transition through virtual states (see third term of $M_{ij}$ in equation 2.8), a new and possibly CPT-violating effect can occur in a $K^0\bar{K^0}$ oscillation; however, an oscillation through real final states, as represented by the decay-matrix $\Gamma$, occurs through weak interaction which is a field interaction and thus CPT-conserving (see section 1.1); moreover, the superweak interaction which can be CPT-violating is only present in the $M$-matrix (see second term of $M_{ij}$ in equation 2.8). Therefore, in our investigation of CPT-invariance, we will consider the possibility to neglect the CPT-violation in the decays of neutral kaons ($\Gamma_{11} = \Gamma_{22}$).
2.1. THE NEUTRAL-KAON SYSTEM

Figure 2.3: An example of $K^0\bar{K}^0$ oscillation through common final state, here $\pi^+\pi^-$, in the Standard Model.

From the eigenstates of the strong and electromagnetic interactions $K^0$ and $\bar{K}^0$, we have defined the physical states of the neutral kaon $K_S$ and $K_L$, which are eigenstates of the weak interaction. We have defined the T- and CPT-violation parameters $\epsilon_T$ and $\delta$ which can both generate CP-violation. Previous experiments have often evaluated $\epsilon_T$ under the assumption of CPT-symmetry. In order to identify whether CP-violation is accompanied by T- or CPT-violation, it is interesting to measure $\epsilon_T$ and $\delta$ without any assumption.

2.1.2 The Unitarity relations

$K_S, K_L$ basis Exressing the time development of only $K^0$ and $\bar{K}^0$ as function of eigenstates $K_S$ and $K_L$, we have

$$\psi(t) = Ae^{-i\Lambda_L t|K_L} + Be^{-i\Lambda_S t|K_S}$$ (2.15)

where $A$ and $B$ are constants. The norm of the general $K$-meson state is then

$$N(t) = |A|^2e^{-\Gamma_L t} + |B|^2e^{-\Gamma_S t}$$

$$+A^*Be^{i(\Lambda_L^* - \Lambda_S)t}\langle K_L|K_S\rangle + AB^*e^{i(\Lambda_S^* - \Lambda_L)t}\langle K_S|K_L\rangle$$ (2.16)

and at $t = 0$ we obtain

$$-\frac{dN(0)}{dt} = \Gamma_L|A|^2 + \Gamma_S|B|^2$$

$$-i(\Lambda_L^* - \Lambda_S)A^*B\langle K_L|K_S\rangle - i(\Lambda_S^* - \Lambda_L)AB^*\langle K_S|K_L\rangle.$$ (2.17)

We consider that this decrease of the norm of the general state should be equal to its transition rate in all possible final states $f$ (unitarity condition):

$$-\frac{dN(0)}{dt} = \sum_f |(f|V|\psi(0))|^2.$$ (2.18)
Since this relation is required to be true for any \( A \) and \( B \), we obtain using 2.15

\[
\Gamma_L = \sum_f |\langle f|V|K_L\rangle|^2 \\
\Gamma_S = \sum_f |\langle f|V|K_S\rangle|^2 \\
-i(\Lambda_L^* - \Lambda_S)(K_L|K_S\rangle = \sum_f \langle f|V|K_L\rangle^*\langle f|V|K_S\rangle.
\]

The two first equations give the definition of the decay width of the two physical states. Using equations 2.11 and 2.12, the last equation gives

\[
\text{Re}(\epsilon_T) + i\text{Im}(\delta) = \frac{\Gamma + i\Delta m}{2(\Gamma^2 + \Delta m^2)} \sum_f A_{L,f}^* A_{S,f}
\]

where \( \Gamma = \frac{\Gamma_L + \Gamma_S}{2} \) and \( A_{L,f} (A_{S,f}) \) denote the \( K_L (K_S) \) decay amplitudes:

\[
A_{L(S),f} = \langle f|V|K_{L(S)}\rangle
\]

**\( \bar{K}^0, K^0 \) basis** In a similar way, we obtain

\[
\Gamma_11 = \sum_f |\langle f|V|K^0\rangle|^2 \\
\Gamma_22 = \sum_f |\langle f|V|\bar{K}^0\rangle|^2 \\
-i(\Lambda_{21} - \Lambda_{12}^*) = \sum_f \langle f|V|K^0\rangle^*\langle f|V|\bar{K}^0\rangle
\]

in the \( \bar{K}^0, K^0 \) basis. The two first equations are familiar since they give the definition of decay widths. The third equation gives the antihermitian part of the \( \Lambda \) matrix, namely the decay width \( \Gamma_{12} \), as function of the decay amplitudes of \( K^0 \) and \( \bar{K}^0 \) in the final states \( f \):

\[
\Gamma_{12} = \sum_f \langle f|V|K^0\rangle^*\langle f|V|\bar{K}^0\rangle = \sum_f A_f^* A_f.
\]

The relations of equations 2.19 and 2.21, which are derived from unitarity are referred as *Bell-Steinberger* [15] relations. They give a relation between the CPT-violation parameter \( \text{Im}(\delta) \) and the decay amplitudes, thus permitting to *test the CPT symmetry*. They are the main theoretical basis of this work.

### 2.2 Neutral-kaon decays

The aim of this section is to define all relevant parameters of neutral-kaon decays to test CPT. In the definition of these parameters, there will be no assumption about the CPT-symmetry and CP-violation is assumed to be small. Finally, we will emphasise the determination of the CP-violation parameter \( \eta_{000} \).
2.2. NEUTRAL-KAON DECAYS

2.2.1 Two-pion decays

- $K^0, K^0 \rightarrow \pi^+ \pi^-$ decays

The $\pi^+ \pi^-$ final state has always a CP eigenvalue of +1. In the limit of CP-conservation ($\epsilon_S = \epsilon_L = 0$), the $K_S$ and $K_L$ states become CP eigenstates with respective eigenvalues +1 and −1 (see eq. 2.12). Therefore the parameter defined as

$$\eta_{+-} = \frac{\langle \pi^+ \pi^- | \mathcal{V} | K_L \rangle}{\langle \pi^+ \pi^- | \mathcal{V} | K_S \rangle}$$

is a measure of CP-violation in $\pi^+ \pi^-$ decays of neutral kaons.

Using equation 2.12, we have

$$\eta_{+-} = \epsilon_T - \delta + \frac{A_{+-} - e^{-i\delta_T} A_{+-}^*}{A_{+-} + e^{-i\delta_T} A_{+-}^*}$$

where $A_{+-}$ and $A_{+-}^*$ represent the decay-amplitude of $K^0$ and $\bar{K}^0$ into $\pi^+ \pi^-$. The isospin decomposition of the $\pi^+ \pi^-$ state leads to

$$A_{+-} = \sqrt{\frac{1}{3}} A_2 + \sqrt{\frac{2}{3}} A_0$$

where $A_{0,2}$ and $A_{0,2}^*$ represent the decay-amplitude of $K^0$ and $\bar{K}^0$ into the isospin $I = 0, 2$ two pion final state. These amplitudes are defined as follows

$$A_{0,2} = (a_{0,2} + b_{0,2}) e^{i\delta_{0,2}}$$
$$A_{0,2}^* = (a_{0,2}^* - b_{0,2}^*) e^{i\delta_{0,2}} e^{i(\phi_{CP} - \phi_T)}$$

where $a_{0,2}$ and $b_{0,2}$ are respectively CPT-conserving and -violating weak decay-amplitudes. This parametrisation of $A_I$ and $A_I^*$ amplitudes [16] permits to express the CPT-invariance and non-invariance in term of CP- and T- invariant and non-invariant amplitudes as it can be seen from table 2.1. The results of this table can be checked by considering equations 2.12 and 2.26. The parameters $\delta_0$ and $\delta_2$ are phase shift values due to strong interactions of the pions in the final state [17]. Since $|A_0^2| \simeq 0.045$ [18], we obtain

$$\frac{A_{+-} - A_{+-}^*}{A_{+-} + A_{+-}^*} = e^{i\frac{1}{2} \frac{|A_0| - |A_0^*|}{|A_0|}} + i \frac{1}{2} \Phi$$

Table 2.1: CPT-, CP- and T- invariance and non-invariance of neutral kaon decay amplitudes.
where

\[ \Phi = \phi_1 - \arg A_0^* A_0 \]  

(2.28)

can be derived from unitarity relations (see section 2.3.1); these relations will show that \( \Phi \ll 1 \), used to obtain the above equation. The parameter

\[ e^+ = \frac{1}{2\sqrt{2}} \left[ \frac{A_2}{A_0} - \frac{\overline{A}_2}{\overline{A}_0} \right] \]  

(2.29)

represents CP-violation in two-pion decay amplitudes. In the limit of CPT-conservation, we obtain the well known relation

\[ e^+ = e^{i(\delta_2 - \delta_1)} \frac{1}{\sqrt{2}} \left[ i \text{Im} \left( \frac{a_2}{a_0} \right) + \text{Re} \left( \frac{b_2}{a_0} \right) - \text{Re} \left( \frac{a_2 b_0^*}{|a_0|^2} \right) \right] \]  

(2.29)

where the interference between the weak decays from \( I = 2 \) and \( I = 0 \) isospin states generates CP-violation in the decay amplitudes.

Finally, using equations 2.24 and 2.27, we obtain the expression of the CP-violation parameter \( \eta_{+-} \) :

\[ \eta_{+-} = e_T - \delta + e^+ + \text{Re} \left( \frac{b_0}{a_0} \right) + i \frac{\Phi}{2} \]  

(2.30)

The best measurement of the norm of \( \eta_{+-} \) is given by [19] :

\[ |\eta_{+-}| = (2.30 \pm 0.035) \cdot 10^{-3} \]  

(2.30)

showing CP violation in the \( K^0 \rightarrow \pi^+\pi^- \) decay mode.

- \( K^0, K^0 \rightarrow \pi^0\pi^0 \)

The CP-violation parameter \( \eta_{00} \) is defined as

\[ \eta_{00} = \frac{\langle \pi^0\pi^0 | V | K_L \rangle}{\langle \pi^0\pi^0 | V | K_S \rangle} \]  

(2.33)

Using isospin decomposition of the \( \pi^0\pi^0 \) state

\[ |\pi^0\pi^0\rangle = \sqrt{\frac{2}{3}} |I = 2\rangle - \sqrt{\frac{1}{3}} |I = 0\rangle \]  

(2.34)

by a similar reasoning as for \( \pi^+\pi^- \) decays, we obtain

\[ \eta_{00} = e_T - \delta - 2e^+ + \text{Re} \left( \frac{b_0}{a_0} \right) + i \frac{\Phi}{2} \]  

(2.35)

The best measurement of the norm of \( \eta_{00} \) is given by [20] :

\[ |\eta_{00}| = (2.71 \pm 0.37) \cdot 10^{-3} \]  

(2.36)

showing CP violation in the \( K^0 \rightarrow \pi^0\pi^0 \) decay mode.
2.2. Neutral-Kaon Decays

2.2.2 Three-pion decays

Before defining the CP-violation parameters for the \( K^0, K^0 \to \pi^+\pi^-\pi^0, \pi^0\pi^0\pi^0 \) decays, we should consider the question of the CP-eigenvalue of these final states. Let us consider two pions \( \pi_1 \) and \( \pi_2 \) (charged or neutral pions); taking into account the eigenvalues of the charge (C) and parity (P) operators with, we can define the CP-eigenvalue of the three-pion final state as follows:

\[
CP(\pi_1\pi_2\pi^0) = CP(\pi^0) \cdot CP(\pi_1\pi_2) \cdot CP(\pi_1\pi_2 \leftrightarrow \pi^0) \\
= (-1) \cdot (-1)^{2l} \cdot C(\pi_1\pi_2 \leftrightarrow \pi^0)P(\pi_1\pi_2 \leftrightarrow \pi^0) \\
= (-1)^{l_{\pi_1\pi_2\pi^0}} \\
= (-1)^{l_{\pi_1\pi_2\pi^0}}
\]

where \( l_{\pi_1\pi_2\pi^0} \) is the angular momentum between the neutral pion and the system of two other pions. Considering the angular momentum of the whole final state:

\[
l_{\pi_1\pi_2\pi^0} \leq L_{3\pi} \leq l_{\pi_1\pi_2} + l_{\pi_1\pi_2\pi^0}
\]

and the conservation of angular momentum \( (L_{3\pi} = L_{K^0} = 0) \), we obtain \( l_{\pi_1\pi_2\pi^0} = l_{\pi_1\pi_2} \). Therefore, the CP eigenvalue of three-pion final state is defined by:

\[
CP(\pi^+\pi^-\pi^0) = CP(\pi^0\pi^0\pi^0) = (-1)^{l_{\pi_1\pi_2}+1}.
\]

Since the isospin part of the wave function must be symmetric under the exchange of \( \pi_1^0 \) and \( \pi_2^0 \), which corresponds to \( l_{\pi_1\pi_2} = 0 \) or 2, the \( 3\pi^0 \) final state can have a total isospin \( I_{3\pi} = 1 \) or 3. By demanding the total wavefunction to be symmetric under the same exchange, the angular momentum part of the wavefunction should be symmetric leading to an even \( L_{3\pi} \). Therefore, a \( 3\pi^0 \) final state has always a CP-eigenvalue \(-1\) and any \( K_S \to 3\pi^0 \) decay is a violation of the CP symmetry. This leads us to the definition of the \( \eta_{000} \) CP-violation parameter as follows:

\[
\eta_{000} = \frac{\langle \pi^0\pi^0\pi^0 | V | K_S \rangle}{\langle \pi^0\pi^0\pi^0 | V | K_L \rangle}.
\]

The \( \pi^+\pi^-\pi^0 \) final state can have an even \((I_{3\pi} = 0, 2)\) or an odd \((I_{3\pi} = 1, 3)\) total isospin, respectively leading to an antisymmetric and a symmetric wavefunction for the angular momentum under the exchange of two pions; therefore the \( \pi^+\pi^-\pi^0 \) final state can have a CP-eigenvalue +1 or \(-1\). The antisymmetric part \((CP=-1)\) vanishing by an integration over the whole phase-space, we define the \( \eta_{+-0} \) CP-violation parameter as follows:

\[
\eta_{+-0} = \frac{\int d\Omega \langle \pi^+\pi^-\pi^0 | V | K_S \rangle \langle \pi^+\pi^-\pi^0 | V | K_L \rangle^*}{\int d\Omega |\langle \pi^+\pi^-\pi^0 | V | K_L \rangle|^2}.
\]

More generally, only three-pion final states with isospin \( I_{3\pi} = 1, 3 \) can be CP-violating thus limiting the sample of \( K^0, K^0 \to \pi^+\pi^-\pi^0 \) decays for the search of CP-violation. According
to kaon studies [18], no evidence of $I_{3\pi} = 3$ in final state of kaon decays has been found.
Therefore, for the decay amplitudes of neutral kaons into $3\pi$, we will only consider $I_{3\pi} = 1$
($\Delta I = 1/2$ rule):

$A_{3\pi} = A_1 = (a_1 + b_1)e^{i\delta_1}$
$\overline{A_{3\pi}} = \overline{A_1} = -(a_1^* - b_1^*)e^{i\delta_1}e^{i(\phi_{CP} - \phi_T)}$ (2.40)

where $a_1$ and $b_1$ are respectively CPT-conserving and -violating weak decay amplitudes; the
minus sign in the expression of $A_{3\pi}$ is due to CP-eigenvalue $-1$ of the $3\pi$ state. Using equation
2.12 and 2.40, CP-violation parameters $\eta_{3\pi}$ become:

$\eta_{3\pi} = \epsilon_T + \delta + \frac{A_1 + e^{-i\phi_T}A_1}{A_1 - e^{-i\phi_T}A_1}$
$\eta_{3\pi} = \frac{(a_1 + b_1) - (a_1^* - b_1^*)e^{i(\phi_{CP} - \phi_T)}}{(a_1 + b_1) + (a_1^* - b_1^*)e^{i(\phi_{CP} - \phi_T)}}$ (2.41)

Considering small CPT-violation in two-pion decays ($b_{0,2} \sim 0$) and using equation 2.26 we
can write

$argA_0\overline{A_0} \simeq \phi_{CP} - \phi_T - 2\phi_0$ (2.42)

where $\phi_0 = arg(a_0)$. Therefore, we have

$\phi_{CP} - \phi_T - \phi \simeq 2\phi_0 - \Phi.$ (2.43)

Adopting the Wu-Yang [21] phase-convention ($\phi_0 = 0$), we obtain

$\phi_{CP} - \phi_T - \phi \simeq -\Phi$ (2.44)

where the quantity $\Phi$ is very small as we will show in section 2.3.1. Using this result and
equation 2.41, we can express the CP-violation parameters $\eta_{3\pi}$ as follows

$\eta_{000} = \eta_{+-0} = \epsilon_T + \delta + \frac{Re(a_1b_1^*)}{(Re(a_1))^2} + i\left\{\frac{Im(a_1)}{Re(a_1)} + \frac{\Phi}{2} \left[1 - \left(\frac{Im(a_1)}{Re(a_1)}\right)^2\right]\right\}$ (2.45)

Since there is only one isospin state, no direct CP-violation is present. Therefore, any deviation
of $\eta_{000}$ from $\eta_{+-0}$ would be a sign of $I_{3\pi} = 3$ in the final state of kaon decays. Equation 2.45
shows another interest in studying CP-violation in three-pion decays: any deviation of $Re(\eta_{3\pi})$
from $Re(\epsilon_T)$ would be a sign of CPT-violation.

The best limit on $\eta_{+-0}$ is provided by the CPLEAR experiment [22]:

$Re(\eta_{+-0}) = (-2 \pm 8) \cdot 10^{-3} \quad Im(\eta_{+-0}) = (-2 \pm 9) \cdot 10^{-3}$ (2.46)

showing no evidence of CP violation in $\pi^+\pi^-\pi^0$ decays of neutral kaons at the present exper¬
imental precision.
2.2. NEUTRAL-KAON DECAYS

The determination of \( \eta_{000} \). The \( \pi^0\pi^0\pi^0 \) decay has from a branching-ratio \( \sim 2 \) times larger than \( \pi^+\pi^-\pi^0 \) [23]. Furthermore, \( K_S^0 \rightarrow \pi^0\pi^0\pi^0 \) is a clear sign of CP-violation. Therefore, this decay mode is potentially a good channel to study CP violation. In the CPLEAR experiment we can measure the difference between the CP conjugate rates \( R(K^0 \rightarrow \pi^0\pi^0\pi^0)(t) \) and \( R(K^0 \rightarrow \pi^0\pi^0\pi^0)(\bar{t}) \) through the eigentime-dependent rate asymmetry

\[
A_{000}(t) = \frac{R(K^0 \rightarrow \pi^0\pi^0\pi^0)(t) - R(K^0 \rightarrow \pi^0\pi^0\pi^0)(\bar{t})}{R(K^0 \rightarrow \pi^0\pi^0\pi^0)(t) + R(K^0 \rightarrow \pi^0\pi^0\pi^0)(\bar{t})}
\]

where \( t \) is the neutral-kaon decay time.

Considering the time evolution of the initially pure \( K^0 \) and \( K^0 \) states from equations 2.10, we derive the rates

\[
R_{000}^{th}(t) = R(K^0 \rightarrow \pi^0\pi^0\pi^0)(t) = \left| \langle \pi^0\pi^0\pi^0 | V | K^0(t) \rangle \right|^2 (2.48)
\]

\[
R_{000}^{th}(t) = R(K^0 \rightarrow \pi^0\pi^0\pi^0)(t) = \left| \langle \pi^0\pi^0\pi^0 | V | K^0(t) \rangle \right|^2 (2.49)
\]

where \( \tau_S \) and \( \tau_L \) are the mean lives of \( K_S \) and \( K_L \) and \( \Delta m \) is the \( K_S-K_L \) mass difference.

The CP-violation parameter \( \eta_{000} \) is then derived from the time dependence of the asymmetry

\[
A_{000}(t) = 2Re(\epsilon_S) - 2[Re(\eta_{000}) \cos(\Delta mt) - Im(\eta_{000}) \sin(\Delta mt)] e^{-(1/\tau_S+1/\tau_L)t/2} (2.50)
\]

where we have assumed that \( \eta_{000} \) is small.

Let us illustrate the asymmetry given by equation 2.50. By assuming CPT conservation, it follows that \( \eta_{000} \approx \epsilon_T \) (see eq. 2.45). For \( Re(\epsilon_T) \) we can use the measurement of the semileptonic charge asymmetry [23]:

\[
\delta_l = \frac{\Gamma(K_L \rightarrow \pi^-l^+\nu) - \Gamma(K_L \rightarrow \pi^+l^-\bar{\nu})}{\Gamma(K_L \rightarrow \pi^-l^+\nu) + \Gamma(K_L \rightarrow \pi^+l^-\bar{\nu})} \approx 2Re(\epsilon_T)
\]

The phase of \( \epsilon_T \) is given by \( tan^{-1}(2\Delta m/\Delta \Gamma) \). They give asymmetries shown in figure 2.4; it has to be noticed that this time-dependent asymmetry is only observable at short lifetimes.

2.2.3 Semileptonic decays

Considering semileptonic decays of neutral kaons, we define four semileptonic rates depending on the initial strangeness of the neutral kaon and on the charge of the decay lepton:

\[
\begin{align*}
R^+_\pi(t) & = R(K^0_{t=0} \rightarrow (l^+\pi^-\nu)_{t=t}) \\
R^-_\pi(t) & = R(K^0_{t=0} \rightarrow (l^-\pi^+\bar{\nu})_{t=t}) \\
R^-_+(t) & = R(K^0_{t=0} \rightarrow (l^-\pi^+\bar{\nu})_{t=t}) \\
R^+_-\pi(t) & = R(K^0_{t=0} \rightarrow (l^+\pi^-\nu)_{t=t})
\end{align*}
\]
We denote $l$ to be all possible leptons, electrons and muons. The semileptonic decay amplitudes can be written as

\begin{align}
\langle l^+\pi^-\nu|V|K^0\rangle &= a + b \\
\langle l^-\pi^+\bar{\nu}|V|K^0\rangle &= a^* - b^* \\
\langle l^-\pi^+\bar{\nu}|V|K^0\rangle &= c + d \\
\langle l^+\pi^-\nu|V|K^0\rangle &= c^* - d^*
\end{align}

where $b$ and $d$ are CPT-violation terms and $c$ and $d$ violate the $\Delta S = \Delta Q$ rule; this rule means that between the final and initial states, the difference of strangeness should be equal to the difference of charge for the hadrons (fig. 2.5). Therefore, the quantities

\begin{align}
x &= e^{-i\phi_\tau}\frac{\langle l^+\pi^-\nu|V|K^0\rangle^* \cdot \langle l^+\pi^-\nu|V|K^0\rangle}{|\langle l^+\pi^-\nu|V|K^0\rangle|^2} = e^{-i\phi_\tau}\frac{c^* - d^*}{a + b} \\
\bar{x} &= e^{-i\phi_\tau}\frac{\langle l^-\pi^+\bar{\nu}|V|K^0\rangle^* \cdot \langle l^-\pi^+\bar{\nu}|V|K^0\rangle}{|\langle l^-\pi^+\bar{\nu}|V|K^0\rangle|^2} = e^{-i\phi_\tau}\frac{c^* + d^*}{a - b}
\end{align}

\(2.53\)

describe the violation of the $\Delta S = \Delta Q$ rule in decays into positive and negative leptons, respectively. In the framework of the Standard Model, this rule holds in the first order.
2.2. NEUTRAL-KAON DECAYS

diagrams (fig. 2.5) and can only be violated by the second or higher order diagrams. Therefore, it is expected to be very small. In the limit of the $\Delta S = \Delta Q$ rule, a positive and negative

![Diagram of the weak decay $K^0 \rightarrow l^+\pi^-\nu$](image)

Figure 2.5: First order diagram for the weak decay $K^0 \rightarrow l^+\pi^-\nu$ illustrating $\Delta S = \Delta Q = -1$.

lepton are respectively the signature of the $K^0$ and $\bar{K}^0$ decays.

Using this property, we consider the difference between the number of $\bar{K}^0$ oscillating into $K^0$ and the number of $K^0$ oscillating into $\bar{K}^0$, given that we know the strangeness of the neutral kaon at its production (sign of the charged kaon at the production point) and at the decay time $\tau$ ($\Delta S = \Delta Q$ rule). Therefore, the asymmetry

$$A_T(\tau) = \frac{R_+(\tau) - R_-(\tau)}{R_+(\tau) + R_-(\tau)}$$

(2.54)

is a measure of two time-reversed processes, a measure of the time-reversal (T) invariance. Taking into account different efficiencies to measure a decay rate of initially pure $K^0$ or $\bar{K}^0$ (see section 4.1), and different efficiencies to measure a decay rate with a $e^-$ or a $e^+$ in the final state, we define an experimental asymmetry [24] which is given by :

$$A_T^{exp}(\tau) \approx 4Re(\epsilon_T - y - x_-)$$

$$+ 2Re(x_+)(e^{-\Delta \Gamma\tau/2} + \cos(\Delta m\tau)) + Im(x_+)\sin(\Delta m\tau)$$

$$\cosh(\Delta \Gamma\tau/2) - \cos(\Delta m\tau)$$

(2.55)

where $y = -\frac{b}{a}$ describes CPT-violation in semileptonic decays when the $\Delta S = \Delta Q$ rule holds. The parameter $x_+ = \frac{a+ib}{2} \propto ae^\phi$ describes the violation of the $\Delta S = \Delta Q$ rule while $x_- = \frac{a-ib}{2} \propto (bc^* + d^*a)$ describes the violation of the same rule together with CPT-violation. For the results of this analysis, CPT-invariance in the semileptonic decays ($y = x_- = 0$) is assumed. The asymmetry is shown in figure 2.6 and has an average of [24] :

$$\langle A_T^{exp}(1-20)\rangle_{\tau\psi} = (6.6 \pm 1.6) \times 10^{-3}$$

(2.56)

showing a clear evidence for T violation.
Figure 2.6: The asymmetry $A_T^{\text{CP}}$ versus the neutral kaon decay time (in units of $\tau_S$). The solid line represents the fitted average $\langle A_T^{\text{CP}} \rangle$ [24].

Finally, the fit of the experimental asymmetry by the equation 2.55 gives:

$$Re(\epsilon_T) = (1.55 \pm 0.35_{\text{stat.}}) \cdot 10^{-3} , \quad Im(x_+) = (1.2 \pm 1.9_{\text{stat.}}) \cdot 10^{-3}$$  \hspace{1cm} (2.57)

confirming that the $\Delta S = \Delta Q$-violating parameter $x$ is small if not consistent with zero.

### 2.3 CPT Test

In this section, we will explain various methods to determine the real and imaginary part of the CPT-violation parameter $\delta$. Having defined the relevant parameters specific to each channel of the neutral-kaon decays, we can now compute the decay amplitudes necessary to test CPT (we will neglect all final states with a branching ratio smaller than 1%). At the end of this section, we will also explain the method to do a direct determination of the CPT-violation parameter $\delta$. 
2.3. CPT TEST

2.3.1 Indirect test of CPT

Using unitarity in $K_L, K_S$ basis, we have an information on CPT-violation parameter $\text{Im}(\delta)$ expanding equation 2.20 [15]:

\[
\frac{1}{2} \left[ \Gamma + i \Delta m \right] \times \left[ A_{L, \pi \nu} A_{S, \pi \nu} - A_{L, \pi^+ \pi^-} A_{S, \pi^+ \pi^-} + A_{L, \pi^0 \pi^0} A_{S, \pi^0 \pi^0} - A_{L, \pi^+ \pi^-} A_{S, \pi^0 \pi^0} - A_{L, \pi^0 \pi^0} A_{S, \pi^+ \pi^-} \right].
\]

Computing the decay amplitudes in all final states (see appendix A), we obtain

\[
\text{Re}(\epsilon_T) = \frac{1}{2} \left[ \Gamma^2 + \Delta m^2 - \Gamma L Br(K_L \rightarrow \pi l \nu) \right] \times \left( \Gamma S Br(K_S \rightarrow \pi^+ \pi^-) \cdot (\text{Re}(\eta_{+0}) - \text{Im}(\eta_{+0})) \right)
\]

\[
+ \Gamma S Br(K_S \rightarrow \pi^0 \pi^0) \cdot (\text{Re}(\eta_{00}) + \text{Im}(\eta_{00}))
\]

\[
+ \Gamma L Br(K_L \rightarrow \pi^+ \pi^- \pi^0) \cdot (\text{Re}(\eta_{+0}) - \text{Im}(\eta_{+0}))
\]

\[
+ \Gamma L Br(K_L \rightarrow \pi^0 \pi^0 \pi^0) \cdot (\text{Re}(\eta_{00}) + \text{Im}(\eta_{00}))
\]

\[
- 2 \Gamma L Br(K_L \rightarrow \pi l \nu) \cdot (\text{Re}(y) + \text{Im}(\delta + x_+))
\]

and:

\[
\text{Im}(\delta) = \frac{1}{2} \left[ \Gamma^2 + \Delta m^2 - \Gamma L Br(K_L \rightarrow \pi l \nu) \right] \times \left( \Gamma S Br(K_S \rightarrow \pi^+ \pi^-) \cdot (\text{Re}(\eta_{+0}) - \text{Im}(\eta_{+0})) \right)
\]

\[
+ \Gamma S Br(K_S \rightarrow \pi^0 \pi^0) \cdot (\text{Re}(\eta_{00}) - \text{Im}(\eta_{00}))
\]

\[
+ \Gamma L Br(K_L \rightarrow \pi^+ \pi^- \pi^0) \cdot (\text{Re}(\eta_{+0}) + \text{Im}(\eta_{+0}))
\]

\[
+ \Gamma L Br(K_L \rightarrow \pi^0 \pi^0 \pi^0) \cdot (\text{Re}(\eta_{00}) + \text{Im}(\eta_{00}))
\]

\[
+ 2 \Gamma L Br(K_L \rightarrow \pi l \nu) \cdot (\text{Re}(\epsilon_T - y) + \text{Im}(x_+)) \right].
\]

As discussed in section 5.1, the current precision on $\eta_{000}$ dominates the error on $\text{Im}(\delta)$.

Using unitarity in $K^0, \bar{K}^0$ basis, (eq. 2.22), we can compute the matrix element

\[
\Gamma_{12} = \frac{\Gamma_S}{2} e^{i \alpha \lambda_0 \lambda_0} + 2 \Gamma L Br(K_L \rightarrow l \pi l \nu) x_+ + \frac{\Gamma L Br(K_L \rightarrow \pi^+ \pi^- \pi^0)}{2} [2i \text{Im}(\eta_{+0}) - \delta - \epsilon_T - 1] + \frac{\Gamma L Br(K_L \rightarrow \pi^0 \pi^0 \pi^0)}{2} [2i \text{Im}(\eta_{000}) - \delta - \epsilon_T - 1]
\]

where calculations similar to the computations of appendix A have been used for the decay amplitudes; are also used the relations giving the $|K^0\rangle$ and $|\bar{K}^0\rangle$ states as function of the physical states $|\bar{K}_S\rangle$ and $|K_L\rangle$ (inverting eq. 2.12). The imaginary part of $\epsilon_T$ is given by

\[
\text{Im}(\epsilon_T) = \frac{2 \Delta m}{\Delta T} \text{Re}(\epsilon_T).
\]
Considering the imaginary part of this equation and taking into account the small size of all parameters, we obtain the angle $\Phi$ defined in equation 2.28:

$$\Phi = 2 \frac{\Gamma_L}{\Gamma_S} \times [2 \text{Br}(K_L \to \pi l \nu) \text{Im}(x_+) - \text{Br}(K_L \to \pi^+ \pi^- \pi^0) \text{Im}(\epsilon_T - \eta_{+0}) - \text{Br}(K_L \to \pi^0 \pi^0 \pi^0) \text{Im}(\epsilon_T - \eta_{000})]$$

(2.62)

which contributes to the CPT-violation parameter $\delta$ (see eq. 2.31 and 2.35) as follows:

$$2 \eta_{+-} + \eta_{00} = 3 \left[ \epsilon_T - \delta + \text{Re} \left( \frac{b_0}{a_0} \right) + \frac{i}{2} \Phi \right] + \Xi. \quad (2.63)$$

It has to be noted in equation 2.62 that the angle $\Phi$ depends only on small parameters. The parameter $\delta$ depends on experimentally determined parameters $\eta_{+-}$, $\eta_{00}$, $\text{Re}(\epsilon_T)$ and the indirectly determined parameter $\Phi$; the remaining unknown CPT-violation parameter $\text{Re}(\frac{b_0}{a_0})$ is difficult to access. In order to remove the dependence of $\delta$ on this parameter, let us consider the CPT-violation parameter $\delta_\perp$ defined as [16]$$\delta_\perp = -\delta - \frac{\Gamma_{22} - \Gamma_{11}}{2\Delta \Gamma} = \frac{2 \Delta m - i \Delta \Gamma}{\Delta \Gamma} \text{Im}(\delta). \quad (2.64)$$

It has to be noted that $\delta_\perp$ has its phase perpendicular to $\epsilon_T$ (see eq. 2.13). Thanks to the unitarity relations 2.21, we can compute the matrix elements $\Gamma_{11}$ and $\Gamma_{22}$ and therefore calculate their difference:

$$\Gamma_{22} - \Gamma_{11} = |A_0|^2 - |A_\phi|^2$$

$$+ |A_{3\pi}|^2 - |A_{3\pi}e^{i\phi}|^2 - |A_{3\pi}|^2$$

$$+ |A_{-3\pi}|^2 - |A_{3\pi}|^2$$

(2.65)

where we have neglected $(\Delta S = \Delta Q)$-violating terms in second order. Having calculated the decay amplitudes, we re-express $\text{Re}(\frac{b_0}{a_0})$:

$$-\delta + \text{Re}(\frac{b_0}{a_0}) = \delta_\perp + \Xi \quad (2.66)$$

with:

$$\Xi = \frac{\Gamma_L}{\Delta \Gamma} [\text{Br}(K_L \to \pi^+ \pi^- \pi^0) \text{Re}(\epsilon_T - \eta_{+0})$$

$$+ \text{Br}(K_L \to \pi^0 \pi^0 \pi^0) \text{Re}(\epsilon_T - \eta_{000}) + \text{Br}(K_L \to l \pi \nu) \text{Re}(y)]$$

(2.67)

which, like $\Phi$, can be computed from experimentally determined parameters. It should be noticed that frequently the $\Xi$ term in the expression of $\text{Im}(\delta)$ is ignored (see for example ref. [27]).

Finally, from equations 2.63 and 2.66 we obtain the CPT-violation parameter $\delta_\perp$:

$$2 \eta_{+-} + \eta_{00} \simeq 3 \left( \epsilon_T + \delta_\perp + \frac{i}{2} \Phi + \Xi \right). \quad (2.68)$$
Decomposing the above equation in terms perpendicular to $\epsilon_T$, we obtain

$$\frac{\sqrt{4\Delta m^2 + \Delta \Gamma^2}}{\Delta \Gamma} Im(\delta) = |\delta_{11}| \approx \frac{2}{3} |\eta_{+-}|(\phi_{+-} - \phi_{SW}) + \frac{1}{3} |\eta_{00}|(\phi_{00} - \phi_{SW})$$

(2.69)

$$-\frac{1}{2} \cos(\phi_{SW}) + \Xi \sin(\phi_{SW})/ \Phi.$$}

Note that we do not use the direct CP-violation parameter $\epsilon$ in equation 2.63, but use the experimentally measured $\eta_{+-}$ and $\eta_{00}$.

The sensitivity of this test is currently dominated by the uncertainty on $\Phi$ and $\Xi$ which are determined by unitarity relations in the $K^0$-$\bar{K}^0$ basis. Therefore, with an improved determination of the CP-violation parameters $\eta_{+-0}$ and $\eta_{000}$ and of $(\Delta S = \Delta Q)$-violating parameter $Im(x_{+-})$, this test of the CPT-symmetry is based on the phase differences $(\phi_{+-} - \phi_{SW})$ and $(\phi_{00} - \phi_{SW})$.

If we assume CPT-conservation in the neutral-kaon decays ($\Gamma_{11} = \Gamma_{22}$), it follows that the $\Xi$ term should be equal to zero (see eq. 2.64 and 2.66) and $Im(\delta)$ depends on only eight parameters. In chapter five, we will determine $Im(\delta)$ using the formalism of the $K^0$-$\bar{K}^0$ basis.

### 2.3.2 Direct test of CPT

The direct test of the CPT symmetry is performed with the study of the semileptonic decays of neutral kaons. In order to test directly CPT, we consider an asymmetry measuring the difference between the number of $K^0$ remaining as $K^0$ and the number of $\bar{K}^0$ remaining as $\bar{K}^0$, the semileptonic decay serving (in the limit of the $\Delta S = \Delta Q$ rule) to tag the final strangeness of the kaon (see eq. 2.51):

$$A_{\text{CPT}}(\tau) = \frac{R_-(\tau) - R_+(\tau)}{R_- + R_+}(2.70)$$

When calculating the expression of $A_{\text{CPT}}$, asymmetry, which takes into account the difference of efficiencies to measure a decay rate of initially pure $K^0$ or $\bar{K}^0$, and the difference of efficiencies to measure a decay rate with an $e^-$ or an $e^+$ in the final state, the experimental asymmetry shows a dependence on $Re(\delta)$, $Im(\delta)$, $Re(x_{+-})$, $Im(x_{+-})$ and $Re(y)$. It should be pointed out that no measurement of $Re(y)$ has been performed. Therefore, to reduce as much as possible the number of unknowns in the fit we consider the following asymmetry:

$$A_{\delta}(\tau) = \frac{\bar{R}_+ - R_-(1 + 4 Re(e_{L}))}{\bar{R}_+ + R_-(1 + 4 Re(e_{L}))} + \frac{\bar{R}_- - R_+(1 + 4 Re(e_{L}))}{\bar{R}_- + R_+(1 + 4 Re(e_{L}))}$$

(2.71)

$$= 4 Re(\delta) + 2 \frac{Im(x_{+-}) e^{-T \tau} \sin(\Delta m \tau) + Re(x_{+-}) E_-(-\tau)}{E_{\pm}(\tau) - e^{-T \tau} \cos(\Delta m \tau)}$$

$$+ \frac{-4 Re(\delta) E_-(-\tau) - 2 Re(x_{+-}) E_-(-\tau) + [2 Im(x_{+-}) + 4 Im(\delta)] e^{-T \tau} \sin(\Delta m \tau)}{E_{\pm}(\tau) + e^{-T \tau} \cos(\Delta m \tau)}$$

with $E_{\pm}(\tau) = e^{-T \tau} + e^{-2 T \tau}$ and where second-order terms in small parameters have been neglected. This asymmetry has the advantage of being independent from $Re(y)$ while being
sensitive to CPT-violation parameter $\delta$. Moreover, the experimental asymmetry $A_{S}^{\mathrm{exp}}(\tau)$ is independent from any parameter determined by an other experiment than CPLEAR [26]. The asymmetry $A_{S}^{\mathrm{exp}}(\tau)$ is shown in figure 2.7. The fit of this asymmetry by equation 2.71 gives:

$$Re(\delta) = (3.0 \pm 3.4) \cdot 10^{-4}, \quad Im(\delta) = (-1.5 \pm 2.3) \cdot 10^{-2}$$ (2.72)

where statistical and systematic errors are taken into account; the precision on $Re(\delta)$ is better than $Im(\delta)$ since the asymmetry is sensitive to the first parameter over the whole range of decay time ($\tau = 1 - 20\tau_S$) while being sensitive to the second only in the region $\tau \sim 2/\Gamma_S + \Gamma_L$.

Using time-dependent semileptonic decay-rates, the CPLEAR experiment has, for the first time, directly determined the CPT-violation parameter $\delta$ [26]. Both real and imaginary part of $\delta$ are compatible with zero within one standard deviation, showing no sign of CPT-violation.

Figure 2.7: The asymmetry $A_{S}^{\mathrm{exp}}$ versus the neutral-kaon decay time (in units of $\tau_S$). The solid line represents the result of the fit.
Chapter 3

EXPERIMENTAL APPARATUS

3.1 The method of the CPLEAR experiment

The CPLEAR experiment was designed to study the CP-, T- and CPT-symmetries in the neutral-kaon system. The neutral kaons are produced by proton-antiproton annihilations at rest:

\[ p\bar{p} \rightarrow \overline{K^0}K^+\pi^- \]
\[ \rightarrow K^0K^-\pi^+ \]

with a total branching fraction of \( \sim 0.42\% \) [28]: these annihilations will be referred to as golden events. Due to the conservation of the strangeness in the strong interaction, the charge sign of the charged kaon tags the strangeness of the neutral kaon, i.e., \( K^0 \) and \( \overline{K^0} \). The antiprotons are provided by the Low Energy Antiproton Ring (LEAR) at CERN at a rate of 1MHz.

The experimental method to study symmetries is to measure asymmetries which are constructed using the time dependent decay rates of initially pure \( \overline{K^0} \) and \( K^0 \):

\[ A(\tau) = \frac{R_{\overline{K^0} \rightarrow f}(\tau) - R_{K^0 \rightarrow f}(\tau)}{R_{\overline{K^0} \rightarrow f}(\tau) + R_{K^0 \rightarrow f}(\tau)}. \]

Then:

- If \( f = f \), the asymmetry is a measure of the difference between two CP-conjugated rates into a common final state and is sensitive to \( CP \) violation.

- If \( f = K^0, f = \overline{K^0} \), the asymmetry is a measure of two time conjugated rates and is thus sensitive to \( T \) violation.

- If \( f = \overline{K^0}, f = K^0 \), the asymmetry is sensitive to \( CPT \) violation.

The advantage of this method is that the geometrical acceptance cancels to first order in the asymmetry, and only second order effects have to be controlled through simulation studies.
The antiprotons (see reaction 3.1) provided by LEAR have a momentum of 200 MeV/c. Their momentum is reduced with a beryllium degrader so that they stop inside the hydrogen target. Before reaching the target, the beam traverses a scintillator (Beam Counter) which gives the start signal to the trigger electronics. Two different targets were used in the experiment: a spherical target of radius 7 cm filled with 16 bar gaseous hydrogen was used until...
3.2. THE TRACKING DETECTORS

spring 1994 and then a 1.2cm cylindrical target filled with 26 bar gaseous hydrogen.

The charge signs, positions and momenta of the charged tracks \((K^\pm, \pi^\pm)\) are provided by a series of cylindrical tracking detectors. The charged tracks are curved in a uniform magnetic field of 0.44\(T\) along the z-axis of the detector, provided by a solenoidal magnet. The size of the tracking detector is designed so that the decays of the neutral kaons below 20\(\tau_S\) can be measured.

The particle identification is carried out by the Particle Identification Detector \textbf{PID} : it consists of a threshold Cherenkov counter sandwiched by two scintillators (S1 and S2) which measure the energy-loss \(dE/dX\) and the time of flight \(T_{of}\) specific to each particle.

In order to select the desired final state, one has to distinguish between the dominant neutral-kaon decay channels:

\[
\begin{align*}
\bar{K}^0, K^0 &\rightarrow \pi l \nu_l \\
&\rightarrow \pi^+\pi^-, \pi^0\pi^0 \\
&\rightarrow \pi^+\pi^-\pi^0, \pi^0\pi^0\pi^0
\end{align*}
\]  

where \(l\) and \(\nu_l\) mean the lepton and the associated neutrino. The \(l/\pi\) separation is performed by the energy-loss in the PID. The neutral pions are reconstructed through the electromagnetic decay \(\pi^0 \rightarrow \gamma\gamma\) where the photons are detected and measured by an Electromagnetic Calorimeter \textbf{ECAL}.

Finally, due to the small branching ratio of the golden events (reaction 3.1) and a high beam intensity, a multilevel trigger system is needed for a fast and effective event selection.

A complete description of each of the subdetectors of the CPLEAR experiment can be found in ref. [28]. In this chapter, we will describe the salient subdetectors for our analysis and present their performances.

3.2 The tracking detectors

The tracking device provides the position of charged tracks and is used to determine their momentum. It comprises, from the inside to the outside of the detector, three multiwire proportional chambers, six drift chambers and two layers of streamer tubes.

The Proportional Chambers (PC) [29] are cylindrical chambers which contain axially placed anodes and helical cathode strips. The proportional chamber PC0 is placed at 1.5\(cm\) radius from the centre of the detector. Since the \(K_S\) have a mean momentum of 480\(MeV/c\), they have a mean decay-length of 2.6\(cm\). Therefore no significant loss is made by triggering only those neutral kaons which decayed outside of PC0. This is done for the final states with charged particles by restricting the number of PC0 hits to be two, reducing the trigger rate very much. PC1 and PC2 are placed at 9.5\(cm\) and 12.7\(cm\) radius. If a track is reconstructed
by the drift chambers without associated hit in the PC’s, it will be considered by the trigger as originating from the secondary vertex, i.e., the neutral-kaon vertex. The PC’s can determine the position of charged tracks in the transversal plane with a resolution of 340\(\mu\text{m}\).

**The Drift Chambers (DC)** [30] are also cylindrical chambers with axially placed wires and helical cathode strips. They are placed between 25.5cm and 51cm. The right/left ambiguity in the transversal plane is removed by using a double-wire system where the distance between the wires is small (0.5mm) compared to the size of the drift-cell (5mm). The size of the drift-cell is dictated by the trigger requirement where the hit information has to be rapidly delivered to the trigger system. Supposing that a track originates from the centre of the detector, the information of DC1 and DC6 can rapidly compute the transversal momentum \(p_T\) of the charged particles [31]. This information coupled with the information of the PID is used by the trigger for the identification of the kaon candidate. The position of the tracks are determined with a resolution of 300\(\mu\text{m}\) in the transversal plane. The \(z\)-coordinate of each track is obtained with a precision better than 3\(\text{mm}\) by using the information of cathode strips.

**The Streamer Tubes (ST)** [32] provide a fast information on the \(z\)-position of the tracks. There are 192 ST’s being placed in two layers between radii 56.8 and 62.0cm. Each ST consists of a central anode wire surrounded by a gas mixture operating at a high-voltage. This high voltage induces a streamer when a charged particle ionises the gas, leading to a pulse on the wire. By measuring the difference of the propagation time of the pulse to the both ends of the tube, one can determine the \(z\)-position of each track with a resolution of 1.4cm. Providing an information on the \(z\)-position of the tracks, the ST’s give the possibility of computing the longitudinal component of the momentum \(p_z\) which is used in the online selection of events by the trigger.

In the offline analysis, the hit informations of PC1 and PC2 and all the six DC’s are taken into account to reconstruct a track: a minimum of three hits out of eight is demanded to have a pattern-track; then, the quality of this track is improved by a fit leading to a fit-track used in the analysis. The efficiency for finding a track by such a method is 99\%. The momentum resolution \(\Delta p/p\) for a track is between 5 and 10\%.

### 3.3 The Particle Identification Detector (PID)

The task of the PID [33] is to identify the charged pions and kaons and particularly, to provide the trigger a fast information about the charged kaon-candidate. It consists of 32 sectors located between radii 62.5 and 75cm.

**The Scintillators (S)** All charged particles passing through these plastic scintillators give a signal which is used by the trigger to count the number of tracks. This signal is also used
to compute the difference of time of flight $\delta T_{of}$ between the two charged particles; with a resolution of $\sim 0.26\text{ns}$, this allows to separate between events where $K\pi$- or $\pi\pi$-pairs from the primary vertex. Moreover, in the offline analysis, we consider the following quantity:

$$\Delta T_{of} = (\delta T_{of})_{exp} - (\delta T_{of})_{th}.$$  \hfill (3.4)

where $(\delta T_{of})_{exp}$ and $(\delta T_{of})_{th}$ are respectively the measured and expected time-of-flight differences. We demand that $|\Delta T_{of}| < 0.7\text{ns}$.

The scintillator $S_1$ is also used to measure the energy-loss of charged particles (per unit of length) $dE/dX$ \cite{33, 34}. As can be seen in figure 3.3, the $dE/dX$ permits a good separation between $K^\pm$ and $\pi^\mp$, particularly at low momentum. The $\delta T_{of}$- and the $dE/dX$-information are used both in the online and offline analysis of the events.

The Cherenkov ($C$) detector consists of a liquid radiator ($C_6F_{14}$). This medium has been chosen in such a way that charged kaons and pions with momenta higher than 650 and 200$\text{MeV/c}$ respectively produces light. Therefore, for charged particles with a momentum above 200$\text{MeV/c}$ a $SCS$ signal (meaning "light in the scintillators but no light in the cherenkov") can be used as a kaon-candidate signature by the trigger.

In addition, $\Delta T_{of}$ and the $dE/dX$ must be compatible with a kaon hypothesis. The amount of low-momentum pions which also give a $SCS$ signal is significantly reduced by a cut on their transversal momentum $p_T$ achieved in the trigger. By requiring $p_T > 270\text{MeV/c}$,
where \( p_T \) is measured by DC1 and DC6, the probability for a pion to be misidentified as a kaon by the trigger is lower than 0.6% [31] for pions with a momentum higher than 360 MeV/c.

### 3.4 The Electromagnetic Calorimeter (ECAL)

In order to reconstruct the photons in neutral-kaon decays with \( \pi^0 \)'s, an electromagnetic calorimeter has been included in the CPLEAR experiment. The design criteria for the calorimeter are dictated by the ability to reconstruct as accurately as possible the \( K^0 \rightarrow 2(3)\pi^0 \rightarrow 4(6)\gamma \) decay vertex. The design studies have shown that the spatial resolution when measuring the coordinates where \( \gamma \) interacts with the detector is the most important parameter for the \( K^0 \) decay vertex determination [35]. The calorimeter had to lie in between the end of the PID and the magnet coil at 1m radius. Performance requirement and space limitation lead to a lead/gas sampling calorimeter, rather than a crystal calorimeter such as CsI. The principle of the sampling calorimeter is the following: a layer of lead sandwiched between two layers of aluminium converts the \( \gamma \)-ray in an electromagnetic shower which is then measured by a layer of streamer tubes (fig. 3.4.a); the wire information (W) of these tubes provide position information in the transversal plane; each W-plane is sandwiched by an U- and V-plane which consist of strips crossing the W-plane at an angle of \( \pm 30^\circ \) (fig. 3.4.b) providing position information in the longitudinal direction. A total of 18 such layers, representing six radiation-length (6\( X_0 \)) permits a full reconstruction of the shower in the space.

A full description of the ECAL can be found in ref. [36]. In this section, we will focus on the performances (position, angle and energy resolution) of the calorimeter which are of great importance for our analysis.

#### 3.4.1 Shower reconstruction

A shower is defined by a pattern-recognition algorithm as a set of hits in the W-, U- and V-planes. A minimum of 3 hits in at least two W-layers and a minimum of 2 hits in both U- and V-layers are demanded to be considered as a shower. For a shower, the longitudinal position of a hit given by the W-projection has to be at less than 3cm from the position calculated from the U-V projection (matching condition). An ambiguity flag is set for showers having the same projection in one of the planes.

The shower-foot coordinates (fig. 3.5) are obtained from the first layer of the shower fulfilling the matching condition. The spatial resolution of the ECAL is studied by using simulated \( p\bar{p} \rightarrow \phi\pi^0 \rightarrow K^+K^-2\gamma \) decays. The error on \( r\phi \)- and \( z \)-coordinates are obtained by comparing the reconstructed with generated \( r\phi \)- and \( z \)-informations. They are found to be \( \sigma_r = 2.2 \text{ mm} \) and \( \sigma_z = 3 \text{ mm} \). In \( K^0 \rightarrow 3\pi^0 \) simulated data, for each photon, we have the coordinates of the generated conversion-point and can thus compare it to the reconstructed value in unit of \( \sigma_r \) (computed from \( r \) and \( \sigma_r \phi \)) and \( \sigma_z \): in figures 3.6.a and .b we can observe
that for almost 70% of the events the reconstructed coordinate of the shower-foot is within one standard deviation from the generated value. However, tails can be observed; these tails are mainly due to limited wire and strip granularity and to effects related to the shower-pattern recognition.

The shower angles ($\Phi, \Theta$) are determined by taking into account the hit distributions in the various planes [37] (fig. 3.5). The errors on these angles were determined with data where the directions of the photons are well known: in the $p\bar{p} \rightarrow \phi \pi^0 \rightarrow K^+K^-2\gamma$ decays, the true direction of a photon is given by the line between the primary vertex and the shower foot. The error of the shower-angle in the transversal and longitudinal plane $\Delta \Phi$ and $\Delta \Theta$ are respectively in the interval $[0.2, 0.5]\text{rad}$ and $[0.25, 0.63]\text{rad}$ (FWHM); it has to be noted that these errors are decreasing with increasing photons energies. Some consequences of the quality of the shower-angle information will be presented in Appendix D.

The shower energy. Simulation studies have shown that the best energy resolution for photons of a few MeV energy to be detected in a sampling calorimeter is obtained by counting the number of hits rather than measuring the energy deposited in the gas. Thus, measurement
Figure 3.5: Transversal and longitudinal view of the calorimeter. Geometrical definition of shower-foot given by $X$ and shower-angles $(\Phi, \Theta)$.

Figure 3.6: Simulated $K^0 \rightarrow 3\pi^0$ data. Pull distribution of the shower-foot angle (a) and $z$-coordinate (b) fitted by a gaussian function in the interval $[-2, 2]$. c) : Pull distribution of the shower energy fitted by a gaussian function in the interval $[-4, 1]$.

of the entire shower development allows the number of $W$ hits (tubes fired) to be related to the shower energy [36]. To calibrate the number of hits as a function of the photon energy, a sample of $p\bar{p} \rightarrow \phi\pi^0 \rightarrow K^+K^-2\gamma$ decays is used. The photon energy is determined with a good precision ($\sim 4\%$) with a constrained fit demanding energy and momentum conservation and assuming a missing $\pi^0$ mass at the annihilation vertex.
3.4. THE ELECTROMAGNETIC CALORIMETER (ECAL)

The hit distribution for photons with different energies is shown in figure 3.7. Low energy photons give less hits in the calorimeter than high energy photons. This can be easily understood since the number of hits is in fact the number of electrons and positrons produced by the interaction of a photon with the lead. The number of $e^-e^+$-pairs itself is increasing with the energy of the interacting photon [23]. At low energies the hit distribution is asymmetric because of the low number of hits. Therefore, it is possible to calibrate the energy as a function of the mean value of the number of hits $\langle w(E) \rangle$. The variation of $\langle w(E) \rangle$ as a function of the energy of a photon converted in different layers of the ECAL is shown in figure 3.8. The non-linear variation of $\langle w(E) \rangle$ reflects the moderate thickness of the ECAL: a photon which has converted in a more advanced layer is not totally contained in the ECAL and thus gives less hits than a photon of the same energy which has converted in previous layers.

The variance $\sigma_E$ on the photon energy determined by this method is given by

$$\sigma_E^2 = \sigma_0^2(E) + 2\Omega^2(E) \quad (3.5)$$

where $\sigma_0^2(E) = \sqrt{w(E)}$, and $\Omega$ takes into account the asymmetry of the hit distribution (fig. 3.7) at low energies; $\Omega$ is of the order of 1.5 and 1 for photons with an energy of 100MeV and 200MeV, respectively. It is clear that the relative error on energy is greater for low energy photons. This method permits to have a moderate energy resolution $\sigma_E/E = 13%/\sqrt{E(GeV)}$
3.4.2 Photon detection efficiency

The reconstruction of a photon being based on the detection of hits in the calorimeter, a low-energy photon gives less hits and has thus a lower probability to be detected. This is what we observe in figure 3.9 where photons of simulated $\phi\pi^0 \rightarrow K^+K^-2\gamma$ events. Photons with energies above $200\,\text{MeV}$ reach to a constant detection efficiency of 97% for a single photon. The residual inefficiency is due to the limited radiation-length of the calorimeter and to the shower reconstruction requirements (see section 3.4.1).

Since the energy of each shower is derived from its W-hit distribution, a W-ambiguity can lead to a bad energy assignment. Therefore in our analysis, we will demand all showers to have no W-ambiguities. When demanding no W-ambiguity, the detection efficiency drops to 90% for this decay mode.
3.4. THE ELECTROMAGNETIC CALORIMETER (ECAL)

3.4.3 Detection efficiency for six photons from $3\pi^0$ decays

In the perspective of our analysis, we determine the efficiency of the ECAL to reconstruct exactly six showers originating from neutral-kaon decays.

Given the size of the detector and the neutral-kaon momentum, around 96% of the $K^0_L$'s decay outside the apparatus [28]. To study the geometrical acceptance (fig. 3.10.a), we consider a sample of simulated $K^0_L \rightarrow 3\pi^0$ events with photons generated in $4\pi st.$ and with flat lifetime in the interval $[0, 50] \tau_3$, the upper limit corresponding to the lifetime of low-momentum neutral kaons with a maximal decay-length inside the detector; we will call this sample of data toy simulated data. For this acceptance, we take into account two effects:

- The absence of end-caps in the ECAL: The CPLEAR calorimeter is constructed in the form of a barrel without end-caps, with a solid angle coverage of nearly 75%. The loss of one or more photons in these end-caps will contribute to the geometrical inefficiency of the ECAL. In figure 3.10.b, we have represented the generated lifetime of events with no photon pointing to the end-caps, normalised to the flat lifetime of all $K^0_L \rightarrow 3\pi^0$ toy simulated events: we can observe that the absence of end-caps gives an efficiency of 20% when requiring six detected photons from $K^0_L \rightarrow 3\pi^0 \rightarrow 6\gamma$ decays. The absence of variation in lifetime can be explained by the isotropic decay of $\pi^0$'s in the neutral kaon frame and the neutral-kaon momentum distribution.

- The finite volume of the detector: given the internal radius of the ECAL, if the vertex of the $K^0_L \rightarrow 3\pi^0$ decay is greater than a value $R_{\text{max}}$, the event cannot be detected, mainly because of the incapacity of disentangling the showers. In figure 3.10.c, we have represented the generated lifetime (toy simulated data) of events for which the decay vertex is below $R_{\text{max}}$ normalised to the flat lifetime of all $K^0_L \rightarrow 3\pi^0$ toy simulated...
events: we can observe that the finite volume of the detector diminishes the detection efficiency for high lifetimes. The global detection efficiency due to this effect is 53.3%. The value of $R_{\text{max}}$ has been determined in order that the combined effect of finite volume of the calorimeter and absence of end-caps reproduce the acceptance of simulated $K_L^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ events (fig. 3.10.a); this value is found to be $(43 \pm 2)cm$ showing that given the dimensions of the ECAL (inner radius of 77.5cm) and the neutral-kaon momentum range, the effective volume to detect $K_L^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ decays is a sphere of 43cm radius.

![Figure 3.10](image)

Figure 3.10: a) : The acceptance for detecting $K^0, K^0 \rightarrow 3\pi^0$ events, after analysis cuts (see section 4.4.4), as a function of the generated decay-time obtained from simulated data (line) and from toy simulated data (dots) taking into account (b) and (c). The efficiency for detecting $K_L^0 \rightarrow 3\pi^0$ events in the calorimeter, taking into account the absence of ECAL in the end-caps (b) and the finite volume of the latter (c) obtained from toy simulated data.

Taking into account the photon detection efficiency as a function of the energy (for showers without W-ambiguity) from section 3.4.2, and the energy distribution of each of the six photons from simulated $K_L^0 \rightarrow 3\pi^0$ data, we derive the probability for each photon to be detected. Then, we derive the probability for a six-shower event to be detected by the calorimeter; this acceptance for reconstructing photons has a mean value of 24%.

Finally, the detection efficiency due to the analysis cuts (section 4.4.4), which improve the signal to background ratio, is 40% (normalised to the total number of events in the
3.5. THE TRIGGER

$K_L^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ simulated data). These cuts are lifetime independent: the acceptance shown in figure 3.10.a (solid line) represents events which have survived the analysis cuts.

The different detection efficiencies of the ECAL are reported in table 3.1. The geometrical acceptance of $\sim 11\%$ is the most limiting factor for the detection of desired events. The overall

<table>
<thead>
<tr>
<th>Source</th>
<th>Detection efficiency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometrical acceptance:</td>
<td></td>
</tr>
<tr>
<td>Absence of end-caps in the ECAL</td>
<td>20.0 ± 0.4</td>
</tr>
<tr>
<td>Finite volume of the ECAL</td>
<td>53.3 ± 1.0</td>
</tr>
<tr>
<td>Acceptance for reconstructing photons</td>
<td>23. ± 1.</td>
</tr>
<tr>
<td>Analysis cuts</td>
<td>40. ± 1.</td>
</tr>
</tbody>
</table>

Table 3.1: The detection efficiency of the electromagnetic calorimeter as function of different factors. For the geometrical acceptance, the normalisation is the total number of events in the toy simulated data and for the analysis cuts the normalisation is the total number of events in the $K_L^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ simulated data.

efficiency for detecting $K_L^0 \rightarrow 3\pi^0$ events is 1%.

3.5 The Trigger

The online event selection is an important step in the CPLEAR experiment. Since the branching ratio of the golden events (3.1) is very small ($\sim 0.42\%$), this event selection has to be very effective. Moreover, the need of high statistics requires a high $p\bar{p}$ annihilation rate (1MHz) and the selection has to be very fast. To achieve this, a multilevel hardwired processor system (HWP) is used to select the events as function of all informations provided by the CPLEAR subdetectors. The main levels of this trigger are:

1. The Early Decision Logic (EDL), which demands at least two hits in the S1 scintillator, one of which should be a kaon-candidate. The amount of low-momentum pions among kaon candidates (also giving an $S\bar{C}iS$ signal) is reduced by a cut on the transversal momentum $p_T$; this cut affects mainly the pions since low-momentum kaons are stopped in the Cherenkov without firing the $S_2$ scintillator.

2. The Intermediate Decision Logic (IDL) which uses the hit information of each chamber to do a first track computation and classify them as primary- or secondary-track depending on whether they have a hit in the PC's or not. Several levels in the IDL can select events with different number of primary and secondary tracks.

3. The HWP1 which performs a better track reconstruction (Track Parametrisation) and checks the number of primary tracks; it demands the total number of tracks not to
exceed four. Finally, the HWP1 performs a rapid check of compatibility of the event with a golden event; this check takes into account the number of tracks, kaon candidates and the sign of tracks.

4. The HWP2, taking into account the \(dE/dX\)- and the \(ToF\)-information, improves the \(K^\pm\pi^\mp\)-pair identification.

5. For events with only two primary tracks, i.e., neutral final states of reaction (3.3), the HWP2.5 level of the trigger can demand different minimum number of clusters \(n_0\) in the calorimeter.

The full logical-path of the information flow and particularly the time needed by each level of the trigger are shown in figure 3.11. With a total rejection factor of \(\sim 1100\), an average beam rate of \(800 kHz\) and taking into account the dead-time (about 25%) of the data acquisition system, we have a read-out of \(\sim 450\) events/second.

One of the advantages of the CPLEAR trigger is that the decision of different levels of the trigger can be changed. This has permitted for example to calibrate the different subdetectors of CPLEAR through the study of various channels like \(p\bar{p} \rightarrow \phi\pi^0\). The main trigger requirements used to obtain the data of this analysis corresponds to the following decisions:

![Figure 3.11: Logical path of all subdetector informations in the CPLEAR trigger. For each step, the needed time for online analysis and the reduction rate are reported.](image-url)
3.5. THE TRIGGER

- EDL requirements: A minimum of two scintillators S1 fired and at least one kaon candidate; transversal momentum of primary tracks greater than 270 MeV/c.

- IDL requirements: At least one kaon candidate; if the number of primary tracks is 2 or 3 or 4, the total number of tracks should at least be 2, 4 and 4 respectively.

- HWP1 requirement: The total momentum of the primary kaon and pion greater than 700 MeV/c.

- HWP2.5 requirement: A minimum of $n_0 = 6$ clusters seen in the ECAL.

It is important to evaluate the effect of different triggers on the data. To do that, let us consider an important variable of the analysis for different data taking periods corresponding to different triggers; we do this comparison for events having survived the preselection cuts (see 4.2.1). In figure 3.12.a, we can compare the neutral-kaon momentum for the periods P27 and P29. While other trigger conditions are same, IDL decision was not applied for the P29 data; we can observe a similarity between these distributions. In figure 3.12.b we can also compare the distribution of the invariant-mass of the six showers (see Appendix C) for the periods P24 and P27 where the minimal number of clusters $n_0$ in the ECAL is respectively six and five, the other trigger conditions being the same; we again observe a similarity between these distributions. Therefore, we conclude that different selections criteria of the trigger have not a major effect on our analysis and the applied cuts on the data do not have to be specific to each period.
Figure 3.12: $K^0, K^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ data. a) Neutral-kaon momentum for periods P27 (full triangle) and P29 (open triangle) where the IDL requirements are respectively active and passive. b) Invariant mass of six showers for periods P24 (full dot) and P27 (open dot) where the minimum number of showers $n_0$ in the ECAL is respectively six and five.
Chapter 4

SEARCH FOR CP VIOLATION IN $\bar{K}^0, K^0 \rightarrow \pi^0 \pi^0 \pi^0$ DECAYS

In this chapter, we will present the analysis leading to our determination of the CP violation parameter $\eta_{000}$ using the rate asymmetry of initially pure $\bar{K}^0$ and $K^0$ decaying to $\pi^0\pi^0\pi^0$ [38]. The tools of ref. [39, 40] have been used to perform this analysis.

We will discuss the experimental conditions and how they affect the determination of $\eta_{000}$. Then, we will describe our event selection procedure and its performances. We will present the signal and background determination method. Finally, we will describe the fit of the $A_{000}$ asymmetry leading to our final result.

4.1 Detection efficiencies for the $\bar{K}^0$ and $K^0$ signal

Due to strong interactions a $K^+$ and a $K^-$ interact differently with the detector material, leading to different detection probabilities for $K^+\pi^-$ and for $K^-\pi^+$: $\epsilon(K^+\pi^-) \neq \epsilon(K^-\pi^+)$. Since the strangeness of the neutral kaon is given by the sign of the charged kaon, this leads to a difference of detection probabilities for a $\bar{K}^0$ and a $K^0$. Therefore, the ratio defined as

$$\xi = \frac{\epsilon(K^+\pi^-)}{\epsilon(K^-\pi^+)} = \frac{\epsilon(\bar{K}^0)}{\epsilon(K^0)} \neq 1 \quad (4.1)$$

deviates from unity. In the observed asymmetry of initially pure $\bar{K}^0$ and $K^0$ (eq. 3.2) we have to take into account this difference of detection probabilities:

$$A^{obs}(t) = \frac{R^{obs}_{\bar{K}^0}(t) - R^{obs}_{K^0}(t)}{R^{obs}_{\bar{K}^0}(t) + R^{obs}_{K^0}(t)} = \frac{\epsilon(\bar{K}^0)R_{\bar{K}^0}(t) - \epsilon(K^0)R_{K^0}(t)}{\epsilon(\bar{K}^0)R_{\bar{K}^0}(t) + \epsilon(K^0)R_{K^0}(t)} \quad (4.2)$$

where $R^{obs}_{\bar{K}^0,K^0}(t)$ are the observed decay rates and $R_{\bar{K}^0,K^0}(t)$ are the theoretical decay rates.
Recalling the expression of $R_{\text{th}}^{000}(t)$ and $\overline{R}_{\text{th}}^{000}(t)$ which are the theoretical decay rates of $K^0$ and $\overline{K^0}$ into $3\pi^0$ (eq. 2.48 and 2.49), we define $R_{000}(t)$ and $\overline{R}_{000}(t)$ as follows:

$$R_{000}(t) = R_{\text{th}}^{000}(t)$$

$$\overline{R}_{000}(t) = (1 + 4Re(\epsilon_S))\overline{R}_{\text{th}}^{000}(t).$$

The observed asymmetry between decay rates of initially pure $\overline{K^0}$ and $K^0$ decaying into $3\pi^0$ becomes

$$A_{\text{obs}}(t) = \frac{\alpha_{3\pi^0}R_{\text{th}}^{000}(t) - R_{000}(t)}{\alpha_{3\pi^0}R_{\text{th}}^{000}(t) + R_{000}(t)}$$

where $\alpha_{3\pi^0}$ is the ratio of the number of $\overline{K^0}$ over the number of $K^0$, decaying into $3\pi^0$. Using equation 2.12 it follows that

$$\alpha_{3\pi^0} = \frac{N_{K^0\rightarrow 3\pi^0}}{N_{K^0\rightarrow \overline{3\pi^0}}} = \xi[1 + 4Re(\epsilon_S)]$$

The parameter $\alpha_{3\pi^0}$ is experimentally determined from the fit of the observed asymmetry (eq. 4.4).

As we will see later, the parameter $\alpha_{3\pi^0}$ deviates from unity by $\sim 10\%$ and $Re(\epsilon_S) \sim 10^{-3}$. Therefore, the parameter $\alpha$ essentially measures the difference of detection efficiencies between a $K^+$ and a $K^-$ and is function of variables measured at the primary vertex; thus, this normalisation factor should be the same for all channels with a neutral-kaon decay, independent from the final state of this decay. However, if a correlation would be introduced between the charged particles from the primary vertex and particles produced at the secondary vertex, $\alpha$ could depend on the final state. Therefore, a more specific study of the $\alpha$ parameter for various neutral-kaon decay channels will be done (section 4.5.1).

### 4.2 Feasibility study

The parameter $\eta_{000}$ will be obtained from the fit of the properly normalised experimental asymmetry to equation 2.50. Since the detection of gamma rays of a few $MeV$ and the measurement of their four-momenta is difficult, the observed asymmetry differs from the theoretical one of 2.50. The two effects which deteriorate the most this asymmetry are: (1) the resolution in reconstructing the neutral-kaon lifetime ($\tau$) which can differ from the generated lifetime ($\tau$), and (2) the background contamination. Therefore, the rates given by equations 2.48 and 2.49 are not the directly observed rates and we must take into account the background as function of lifetime $bckg(\tau)$ and the resolution function $\Gamma(\tau - t, t)$; the latter function varies with the lifetime as we will see in section 4.5.1, and is determined through a simulation study. The observed rates are then represented by:

$$R_{\text{obs}}^{000}(\tau) = N \int_0^{\tau_{\text{max}}} \epsilon(t)R_{000}(t)\Gamma(\tau - t, t)dt + bckg(\tau)$$

$$\overline{R}_{\text{obs}}^{000}(\tau) = \alpha_{3\pi^0}N \int_0^{\tau_{\text{max}}} \epsilon(t)\overline{R}_{000}(t)\Gamma(\tau - t, t)dt + bckg(\tau)$$
where $\varepsilon(t)$, the time-dependent acceptance function for $K^0 \rightarrow 3\pi^0$ events represents their
detection probability (fig. 3.10.a), $\alpha_{3\pi^0}$ is the normalisation factor of equation 4.5 and $\tau_{\text{max}}$
is the maximal reconstructed lifetime; $\text{bckg}(\tau)$, $\varepsilon(t)$ are determined by simulation studies.

Fitting the asymmetry $A_{000}(\tau)$ with these observed rates will affect the measure of $\eta_{000}$.
In order to study the effect of the lifetime resolution and the background, we generate an
asymmetry with different number of events $N$ for different resolution functions and amounts
of background; we then fit this asymmetry to see the sensitivity to the two fitted parameters.
For simplicity we take $\alpha_{3\pi^0} = 1$ and $\eta_{000} \sim \varepsilon_T$ as suggested by equation 2.45.

Effect of Lifetime Resolution $\Gamma(\tau - t, t)$: To study the effect of the lifetime resolution,
we consider the resolution function to be a gaussian with various widths $\sigma$. In figure 4.1, we

![Figure 4.1: $A_{000}$ asymmetry for a gaussian resolution function with $\sigma = 0, 1, 2$ and $3\tau_S$. Error
on $\text{Re}(\eta_{000})$ (a) and $\text{Im}(\eta_{000})$ (b) for the a gaussian resolution function with $\sigma = 0$ (full circle),
$1\tau_S$ (open circle), $2\tau_S$ (full triangle) and $3\tau_S$ (open triangle), versus number of events.]

represent simulated asymmetry functions with $\text{bckg}(\tau) = 0$ and $\sigma = 0, 1, 2$ and $3\tau_S$. One can
clearly see that an increase of $\sigma$ of the resolution function results in a deterioration of the
asymmetry; the limited lifetime resolution smears the asymmetry; this is the reason why, for
a limited lifetime resolution, the asymmetry $A_{000}$ populates the negative lifetime region. In figures 4.1.a and 4.1.b, we can observe the variation of the error on $Re(\eta_{000})$ and $Im(\eta_{000})$ as a function of the number of events, for fixed resolution functions $\sigma = 0, 1, 2, 3 \tau_S$. As expected, the error on $Re(\eta_{000})$ and $Im(\eta_{000})$ follows the following relation:

$$\sigma_{Re,Im} = \frac{C_{Re,Im}}{\sqrt{N}}$$

where $C_{Re,Im}$ are the intrinsic sensitivity on the fitted parameters for gaussian resolution functions with different $\sigma$. The different values of these parameters are reported in the figure 4.3. With a gaussian resolution function of $\sigma \approx 2.6 \tau_S$, we need ten times more events than if we had perfect lifetime resolution ($\sigma \approx 0 \tau_S$) to have the same error on $Re(\eta_{000})$.

Effect of background $bckg(\tau)$: In figure 4.2, we represent simulated asymmetry functions with perfect lifetime resolution ($\sigma = 0 \tau_S$) and with amounts of total background of 0%, 25%, 50% and 75%. The background shape as function of lifetime and the relative amount

![Figure 4.2: $A_{000}$ asymmetry for an amount background of 0%, 25%, 50% and 75%. Error on $Re(\eta_{000})$ (a) and $Im(\eta_{000})$ (b) for an amount background of 0% (full circle), 25% (open circle), 50% (full triangle) and 75% (open triangle), versus number of events.](image-url)
of the background channels are those explained in section 4.4, the only factor which has been varied is the proportion of the total background. One can see that an increase of the amount of background results in a deterioration of the asymmetry and that $\sigma_{Re\eta}$ and $\sigma_{Im\eta}$ obeys to the same kind of relation than 4.7. With an amount of background of $\sim 50\%$, we need three times more events than if we had no background to have the same error on $Re(\eta_{000})$.

Given the lifetime resolution ($\sigma \simeq 2.6\tau_3$) and the total amount of background ($\sim 50\%$) that we expect to have, we can observe that the lifetime resolution is the greater source of statistical error than the background. With the total number of $3\pi^0$ events that we expect to collect at CPLEAR experiment ($\sim 20000$ events), we can expect:

$$\sigma_{Re\eta} = 0.13, \quad \sigma_{Im\eta} = 0.19$$

Given the suggested size of $\eta_{000}$ by equation 2.45, we conclude that we will not see an effect of CP-violation in $K^0, K^0 \rightarrow 3\pi^0$ decays, and we will only be able to put an upper limit on $\eta_{000}$. 

Figure 4.3: (a) : Intrinsic sensitivity of $Re(\eta_{000})$ (full circles) and $Im(\eta_{000})$ (open circles) for gaussian resolution functions of various $\sigma$. (b) : Intrinsic sensitivity of $Re(\eta_{000})$ (full triangles) and $Im(\eta_{000})$ (open triangles) for various amount of background.
4.3 Event selection and reconstruction

For the reconstruction of signal events:

\[ pp \rightarrow K^0(K^0)K^\pm\pi^\mp \rightarrow 3\pi^0 \rightarrow 6\gamma \]

we first require the detection of two primary tracks identified as a kaon and a pion candidate and six electromagnetic showers (fig. 4.4). Then, in order to determine the neutral-kaon decay time, the position of the annihilation vertex and the four-momentum of the neutral kaon are calculated from the track parameters of the charged kaon and pion, whereas the decay vertex of the neutral kaon along its direction of flight is determined from the six photon showers measured in the calorimeter; this is achieved through a constrained-fit.

Now, we will describe the criteria to select the events. Then, we will define the constrained-fit and describe its properties; finally, we will focus on its performance in terms of \( \gamma-\gamma \) pairing.

4.3.1 Preselection

The charged tracks and the neutral showers of the events to be selected must fulfill some quality criteria before being sent to the time consuming constrained-fit process.

Selection of the charged tracks

- There should be two tracks (for the definition of a track, see in section 3.2).
- The total charge of the two tracks must be zero.
- At least one PC must be touched by both tracks.
- The primary vertex must be inside the target.
- There should be at least one kaon candidate (for the definition of a kaon candidate, see section 3.3).
- For both \( K^\pm \) and \( \pi^\mp \), the number of hits in the PC's and the DC's must be more than four.
- For both \( K^\pm \) and \( \pi^\mp \), the number of hits in the Z-plane of the DC's must be more than two.
- The charged kaons with momenta less than 300 MeV/c have a detection efficiency less than 50% because interacting in the Cherenkov and having difficulties to reach the \( S_2 \) scintillator; thus, to have a better quality for the kaon candidates, we require that their momenta \( P_K \) must be in the range \([350, 800]\) MeV/c.
Figure 4.4: Data events. $\bar{K}^0$ (a) and $K^0$ (b) decays into $3\pi^0$, detected in the CPLEAR apparatus. The charged kaons and pions respectively give an $S\bar{O}S$ and an $SCS$ signal in the PID; the photons give six electromagnetic showers, detected in the ECAL.

- The momenta of the charged pions $P_\pi$ must be in the range $[100, 750]\, MeV/c$.
- the energy-loss of the kaon candidate should be at less than four standard deviations from the theoretical energy-loss of a kaon with the same momenta.
Selection of the neutral showers

- By strong interaction, the charged particles can produce some additional photons around their interaction point in the calorimeter. Therefore, we define neutral showers [41] as showers at a distance greater than 25 cm from a charged pion and 50 cm from a charged kaon. For each event we demand six neutral showers.

- Due to a gas flow problem in some running periods, some sections of the calorimeter produce large noise [42], i.e., they produce an anomalously high number of shower candidates. We demand no shower to be in those regions.

4.3.2 The constrained fit

The neutral-kaon decay vertex is determined by a full geometrical and kinematical reconstruction of the annihilation $\bar{p}p \rightarrow K^0(K^0)K^{\pm}\pi^\mp$ and the neutral-particle cascade $K^0, K^0 \rightarrow \pi^0\pi^0\pi^0 \rightarrow 6\gamma$ through a constrained fit (for a more detailed description of constrained fits, see ref. [43]). The fit assumes that the six photons originate from a common point along the flight direction of the neutral kaon, namely the unknown neutral-kaon decay vertex.

The measured quantities used in the constrained fit are:

- the coordinates of the primary vertex,
- the momentum $p_K$ of the charged kaon and the momentum $p_\pi$ of the charged pion (see figures 4.5.a and .b),
- the shower-foot coordinates of the photons reconstructed in the ECAL,
- the energies $E_i$ of the six photons (see figures 4.5.c and .d) measured in the ECAL.

The constraints are energy- and momentum-conservation at the primary vertex ($c_{1-4}$), the equality of the $K^{\pm}\pi^{\mp}$ missing-mass with the neutral-kaon mass at the primary vertex ($c_5$) and the equality of the $\gamma\gamma$ invariant-mass with the $\pi^0$ mass for the three couples of photons ($c_6-8$):

\begin{align*}
  c_1 : & \quad 2m_p - (E_K + E_\pi + \sum_{i=1}^{6} E_i) = 0 \\
  c_{2-4} : & \quad p_K^2 + p_\pi^2 + \sum_{i=1}^{6} \vec{p_i} = \vec{0} \\
  c_5 : & \quad m^2_{K^0} - (2m_p - E_K - E_\pi)^2 + (p_K + p_\pi)^2 = 0 \\
  c_6 : & \quad \frac{1}{2}m^2_{\pi^0} - (E_1E_2 - \vec{p_1} \cdot \vec{p_2}) = 0 \\
  c_7 : & \quad \frac{1}{2}m^2_{\pi^0} - (E_3E_4 - \vec{p_3} \cdot \vec{p_4}) = 0 \\
  c_8 : & \quad \frac{1}{2}m^2_{\pi^0} - (E_5E_6 - \vec{p_5} \cdot \vec{p_6}) = 0
\end{align*}
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Figure 4.5: 3π° simulated data. Momentum distribution of the primary pion (a) and the primary kaon (b). Distribution of photons with minimal (c) and maximal (d) energy.

where \( m_p \), \( m_{K^0} \) and \( m_{\pi^0} \) are the mass of the proton, the \( K^0 \) and the \( \pi^0 \) respectively. This fit with eight constraints and one unknown is called the 7C-fit. The momenta \( p_i \) of the six photons are deduced from their energies, their shower-foot coordinates and the decay vertex of the \( K^0 \) which is the unknown of the constrained-fit.

One should note that there are fifteen possibilities to form three pairs with six photons. Therefore, the 7C-fit described in the previous paragraph is repeated over the fifteen possible pairings and sorted according to the probability of the fit result. The constraints \( c_{6-8} \) are directly related to the pairing; therefore the energy \( E_i \) of the six photons affects the pairing. We will discuss this question of pairing in more detail in the next section 4.3.3.

The chi-square of the 7C-fit, \( \chi^2_{7C} \), can be written as follows:

\[
\chi^2_{7C} = \chi^2_{\text{prim}} + \chi^2_{\text{sec}} + \chi^2_c
\]  

where \( \chi^2_{\text{prim}} \) is a function of measured quantities of primary tracks, namely their momentum; the unknown at the primary vertex is the neutral-kaon momentum and the physical constraints are energy and momentum conservation. \( \chi^2_{\text{sec}} \) is a function of measured quantities of photons, namely their energies and shower-foot coordinates; however, the relative error on the shower-foot coordinates being smaller to the error on the energy by a factor 100 (see section 3.4.1),
\( \chi_{sec}^2 \) is more sensitive to the energy and can be written as:

\[
\chi_{sec}^2 \approx \sum_{i=1}^{6} \left( \frac{E_i^{7C} - E_i}{\sigma_i} \right)^2
\]

(4.10)

where \( E_i \) and \( \sigma_i \) are the photon energies and their errors as determined by the calorimeter and \( E_i^{7C} \) are fitted photon energies. \( \chi_{sec}^2 \) measures in fact the performance of the 7C-fit in the subset of the photon-physics. In figures 4.6.a and 4.6.b we can see, for the best pairing (the pairing giving the highest probability), the probability distribution of the 7C-fit \( \text{Prob}(\chi_{7C}^2) \) and the secondary-probability distribution \( \text{Prob}(\chi_{sec}^2) \). Finally, \( \chi_c^2 \) is a function of both kind of variables, revealing the possible correlation between charged and neutral hemispheres. Apart from very particular mechanisms like albedo photons where a photon is sent back from the charged to the neutral hemisphere (see section 4.4.2), the measurement of charged quantities \((p_K, p_\pi, \text{ and } p_{\pi^0} = -(p_K + p_\pi))\) does not affect the measurement of neutral quantities; this is because different detectors are used to reconstruct charged and neutral particles. Furthermore, the precision of measurement of charged quantities is much higher than the precision of measurement of neutral quantities (see sections 3.2 and 3.4). Therefore, the 7C-fit determines the charged parameters independently from neutral parameters and we expect the \( \chi_c^2 \) to vanish. This is proven by figure 4.6.c where the secondary-probability is displayed versus the
primary-probability: one can observe that there is no correlation between this two variables, proving that the output parameters related to the neutral hemisphere (see relation 4.10) are not affected by charged parameters.

The relation 4.9 then becomes:

\[ \chi_{7C}^2 = \chi_{\text{prim}}^2 + \chi_{\text{sec}}^2. \tag{4.11} \]

The property of the 7C-fit described by the above relation can also be seen in the figure 4.7 where we apply a cut of 10% on \( \text{Prob}(\chi_{\text{sec}}^2) \) and on \( \text{Prob}(\chi_{7C}^2) \) and observe the \( K^\pm\pi^\mp \) missing-mass squared spectrum at the primary vertex; as can be seen from this figure, cutting on \( \text{Prob}(\chi_{\text{sec}}^2) \) does not affect the missing-mass squared distribution, while cutting on \( \text{Prob}(\chi_{7C}^2) \) selects the region corresponding to the \( K^0 \) mass. Therefore, in our analysis we will not cut on \( \text{Prob}(\chi_{7C}^2) \) in order to be able to monitor the background through the missing-mass squared distribution (see section 4.4.5), while cutting on \( \text{Prob}(\chi_{\text{sec}}^2) \) to reduce background originating from the secondary vertex.

![Figure 4.7: 3π⁰ data. K^±π^± missing mass square distribution after a cut on energy probability (solid line) and after a cut on 7C-fit probability (dashed line). Events have been selected in the lifetime interval [−1, 20]τ_S. The peak in the higher missing-mass region corresponds to channels where, in addition to the neutral kaon, a π⁰ has been produced at the primary vertex (see section 4.4.3).](image-url)
In order to improve the quality of the selected events, we apply the following cuts:

- The fit for the primary vertex has to converge. We do not cut severely on the probability of this fit in order to not affect the missing-mass spectrum when selecting events.

- The momentum of the primary particles $K^\pm$, $\pi^\mp$ and $K^0$ as computed by the fit for the primary vertex must respectively be smaller than 750\,MeV/c, 670\,MeV/c and 750\,MeV/c.

- The secondary-probability $\text{Prob}(\chi^2_{\text{sec}})$ should be greater than 10%.

- Due to the limited lifetime resolution, some signal events are also reconstructed at negative lifetimes (see section 4.3.4); the background channels which fulfil the 7C-fit requirements are mainly reconstructed at negative lifetimes (see section 4.4.5). Therefore, to study both the signal and background in the largest lifetime region, we demand the lifetime $\tau_{7C}$ reconstructed by the 7C-fit for the best pairing to be in the $[-40, 60] \tau_5$ range.

4.3.3 $\gamma\gamma$ Pairing

In our analysis we want to reconstruct the $K^0, K^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ decays. For this, we require two neutral showers with the preselection cuts of section 4.3.1 for each $\pi^0$. However, we do not know beforehand which pair of two photons originates from which $\pi^0$.

With a simulation study, we can evaluate how the probability of the 7C-fit for different pairings is correlated with a correct pairing, and then see the consequence of wrong pairings on the lifetime reconstruction.

**Matching of generated and reconstructed photons** We consider a sample of $3\pi^0$ Monte-Carlo Data. For this sample, we have all the informations concerning the generated MC photons and particularly their *conversion point* in the calorimeter. Computing the distance between the conversion points of generated photons and the shower-foot of the reconstructed showers, we can determine whether the pairing found by the 7C-fit is correct (for the matching between generated and reconstructed photons, see Appendix B).

In the determination of the correct pairing rate, we should not take into account events for which a shower has been reconstructed far from a generated photon. In figure 4.8 we can see the distance between the conversion point of a generated photon and the shower-foot of a reconstructed photon in the transversal plane: around 80% of the showers are reconstructed at less than 2.5\,cm from the generated photons; however, non-negligible tails can be observed. For this study, we require that all the six reconstructed shower-foots should be at less than 1\,cm from a generated photon conversion point.

**Pairing** Repeating the procedure described above for the pairings given by the $n$ highest probabilities of the 7C-fit, we can determine in how many cases they match with the correct
4.3. EVENT SELECTION AND RECONSTRUCTION

Figure 4.8: $3\pi^0$ simulated data. Shower-foot difference in transversal plane between generated and reconstructed photons.

pairing of the generated events. In table 4.1, we can see the percentage of correct pairing as function of decreasing probability for the different pairings of the $7C$-fit. One can observe that

<table>
<thead>
<tr>
<th>Pairing of highest probability</th>
<th>Correct pairing rate [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pairing of $2^{nd}$ highest probability</td>
<td>40. ± 1.</td>
</tr>
<tr>
<td>Pairing of $3^{rd}$ highest probability</td>
<td>24. ± 2.</td>
</tr>
<tr>
<td>Pairing of $4^{th}$ highest probability</td>
<td>15. ± 2.</td>
</tr>
<tr>
<td>Pairing of $5^{th}$ highest probability</td>
<td>9. ± 1.</td>
</tr>
</tbody>
</table>

Table 4.1: Proportion of correct pairing for various pairings of the $7C$-fit.

the $7C$-fit gives the right pairing by a decisive manner: the correct pairing rate given by the highest $7C$-fit probability is 60% higher than the rate given by the $2^{nd}$ probability. Applying the cuts of our analysis (see section 4.4.4) improves the percentage of correct pairing for the highest probability to 55%.

The question now is if there is a criteria other than the highest probability which would allow us to improve on the pairing.

Let us consider the $7C$-reconstructed lifetime $\tau_{7C}(i)$ for the $i^{th}$ highest probability and $\tau_A$, where $\tau_A$ is the reconstructed lifetime by the intersection of shower directions (as measured in the ECAL) and the neutral-kaon flight direction (see Appendix C); $\tau_A$ is pairing independent.
We can then build the pairing-dependent variable $\Delta \tau(i)$:

$$\Delta \tau(i) = \tau_{7C}(i) - \tau_A. \quad (4.12)$$

The smaller this difference of lifetime for a given pairing, the higher the probability that this pairing is the correct one. This is what we can observe in figure 4.9.a where the correct pairing rate is higher for small values of $\Delta \tau(i)$. However, for a given value of lifetime difference, the correct-pairing rate of the $(i + 1)^{th}$ highest probability never exceeds the correct-pairing rate of the $i^{th}$ highest probability; it is thus not possible to improve the correct pairing.

Figure 4.9: a) : Correct-pairing rate as function of the time difference $\tau_{7C} - \tau_A$ (of eq. 4.12) for the first (full circle), second (open circle), third (full square), forth (open square) and fifth (full triangle) highest probability. b) : Correct-pairing rate as function of the probability difference $\Delta \text{Prob}$ (of eq. 4.13) between the first and the second (full circle) and between the second and the third (open circle) highest probability.

When sorting the probabilities of the 7C-fit, there could be cases where the difference between two consecutive probabilities is not big, revealing the weakness of the 7C-fit to choose between the corresponding pairings; we thus consider the difference of probability:

$$\Delta \text{Prob}(i) = \text{Prob}_{7C}(i) - \text{Prob}_{7C}(i + 1) \quad (4.13)$$

where $\text{Prob}_{7C}(i)$ is higher than $\text{Prob}_{7C}(i + 1)$. We can estimate that if $\text{Prob}_{7C}(i)$ is much higher than $\text{Prob}_{7C}(i + 1)$, the pairing $i$ has a big probability to be the correct one. This is
what we can observe in figure 4.9.b where the correct pairing rate is increasing with $\Delta Prob(i)$; however, it is again not possible to improve the total correct pairing.

Having not found a criteria which allows to improve the correct-pairing rate, we will use the pairing associated with the highest probability of the 7C-fit.

### 4.3.4 Lifetime resolution

The effect of a bad pairing on the lifetime resolution can be seen by comparing the figures 4.10.a and 4.10.b: fitting the lifetime resolution distribution with a gaussian between $-5\tau_S$ and $+5\tau_S$, for the correct pairing we have a resolution function of $\sigma \simeq 1.5\tau_S$, while for the bad pairing we have a resolution function of $\sigma \simeq 3\tau_S$ with more events in the tails (greater RMS of the resolution distribution). Moreover, in figure 4.10.c, we can see that the lifetime resolution improves when increasing the number of events with correct pairing. This correlation between the lifetime resolution and the pairing can be understood by equation 4.8: a bad association between photons through the constraints $c_6 \sim 8$ will result in a badly reconstructed decay vertex.

When comparing the lifetime resolution of events for correct and bad pairing (figures 4.10.a and .b), we are comparing different events; the deterioration of the lifetime resolution could be due to the topology of the event (geometrical or kinematical configuration of photons and $K^{\pm}\pi^{\mp}$) which could favour a bad association between photons. In order to be independent of such effects, we select those events for which the 2nd probability gives the correct photon-pairing and we compare the lifetime resolution ($\tau_{(2)} - t$) with the lifetime resolution ($\tau_{(1)} - t$) given by the highest probability, for the same events. In figures 4.11.a and .b we observe a better lifetime resolution for the correct-pairing probability than for the highest probability, confirming that the deterioration of the lifetime resolution is mainly due to a bad pairing.

The best function to fit the lifetime resolution is given by:

$$g(x) = N \exp\left[\frac{(x - \mu)^2}{2\alpha_x^2(1 + \beta|x|^\gamma)}\right]$$

with $x = \tau - t$, where $\alpha_x$, $\beta$, $\gamma$, $\mu$, and $N$ are free parameters. This function is a kind of gaussian function where the $\sigma$ is symmetrically increased with $x$. Fixing $\beta$ at 0.7 and $\gamma$ at 1.5, we obtain $\alpha_x$ by fitting $g(x)$ to the distribution of $\tau - t$. There are five such distributions corresponding to the five highest probabilities given by the 7C-fit; each distribution is filled when the corresponding probability gives the good pairing. In the limit of $\beta = 0$, the $\alpha_x$ parameter is the $\sigma$ of a gaussian distribution; its value for the various probabilities is reported in table 4.2. It is interesting to observe that even if all five lifetime resolutions correspond to a correct pairing, there is a slow deterioration of the lifetime resolution towards lower probabilities, which indicates that the event topology also affects the lifetime resolution.

Therefore, we can conclude that the lifetime resolution is mainly a function of the pairing but also decreases with decreasing 7C-fit probabilities. The final lifetime resolution (see figure
Figure 4.10: $3\pi^0$ simulated data. 1st pairing of 7C-fit: a) : Lifetime resolution for correct pairing events. b) : Lifetime resolution for bad pairing events. c) : Lifetime resolution as a function of the percentage of correct pairing for the highest 7C-fit probability.

4.12.b), includes both correct (4.10.a) and bad pairings (4.10.b), resulting in a total lifetime resolution of $\sigma \approx 2.6\tau_S$. The tails of the lifetime resolution distribution are mainly due to a bad pairing. We have slightly improved this resolution by demanding that the momentum of the neutral-kaon must be greater than $180\,\text{MeV}/c$: since neutral-kaons of low momentum give low-energy photons with very few hits in the calorimeter, their energy is badly reconstructed (see section 3.4.1) resulting in a bad pairing and though in a bad lifetime reconstruction.

In figure 4.12.a, we have the reconstructed neutral-kaon decay-time from $K^0 \rightarrow 3\pi^0$ simulated data after all the analysis cuts (see next section). This lifetime is the $K_L$ lifetime which includes the effect of the acceptance of our detector (see section 3.4.3) and the limited lifetime resolution which gives non-negligible number of signal events reconstructed in the negative lifetime region.

In order to improve the lifetime resolution and thus the statistical error of the fitted
4.3. EVENT SELECTION AND RECONSTRUCTION

![Graphs showing lifetime resolution](image)

Figure 4.11: $3\pi^0$ simulated data. Lifetime resolution of the $2^{nd}$ (a) and $1^{st}$ (b) probability when the $2^{nd}$ gives the correct pairing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime resolution of $1^{st}$ probability</td>
<td>$0.68 \pm 0.01$</td>
</tr>
<tr>
<td>Lifetime resolution of $2^{nd}$ probability</td>
<td>$0.68 \pm 0.02$</td>
</tr>
<tr>
<td>Lifetime resolution of $3^{rd}$ probability</td>
<td>$0.72 \pm 0.02$</td>
</tr>
<tr>
<td>Lifetime resolution of $4^{th}$ probability</td>
<td>$0.76 \pm 0.03$</td>
</tr>
<tr>
<td>Lifetime resolution of $5^{th}$ probability</td>
<td>$0.76 \pm 0.03$</td>
</tr>
</tbody>
</table>

Table 4.2: The $\alpha_\sigma$ parameter of the correct-pairing lifetime resolution (eq. 4.14) for various 7C-fit probabilities.

parameters $Re(\gamma_{000})$ and $Im(\gamma_{000})$, we have studied the possibility of reconstructing the events with the measured angles of showers in the ECAL (see Appendix C). Such a constrained-fit improves slightly the lifetime resolution, while losing signal events, as discussed in Appendix C. This 13C-fit will therefore not be included in our analysis.
4.4 Signal and background determination

By requiring two charged tracks identified as kaon- and pion-candidate and six electromagnetic showers for the selection of $3\pi^0$ golden events, we allow some other background channels fulfilling these conditions, to be in the final data-sample from which we will determine the CP-violation parameter $\eta_{000}$. Sources of background are the pionic annihilation channel $p\bar{p} \rightarrow \pi^+\pi^- + n\pi^0 (n \geq 0)$ with a charged pion mistaken for a charged kaon, the neutral-kaon decay $K^0 \rightarrow 4\gamma$ with two additional photons that originate from secondary interactions and the kaonic annihilation channel $p\bar{p} \rightarrow \overline{K^0}(K^0)K^\pm \pi^\mp + \pi^0$.

We need to control the level of background in order to take into account its effect in the analysis. The number of $\overline{K^0}, K^0 \rightarrow \pi^0\pi^0\pi^0$ events and the contribution of background to the data are determined from the reconstructed decay-time distribution and from the $K^\pm\pi^\mp$ missing-mass spectrum, by fitting reference distributions for signal and background to the measured ones. The reference distributions are obtained from simulations of the signal and of the background with neutral-kaon decays. For the pionic-annihilation background these distributions are obtained from data by studying the energy-loss distribution of the charged particles in the inner scintillator of the particle-identification detector. The proportion of signal to background events in the data is determined from a simultaneous fit of the reference
distributions to the measured missing-mass and decay-time distributions, leaving the number of events for each contribution as a free parameter in the fit.

In this section, we will explain the reasons why these channels can contaminate the \(3\pi^0\) data and we will discuss the cuts allowing their reduction. Then, we will discuss the simultaneous fit permitting the determination of the amount of signal and the level of backgrounds in the final data sample.

### 4.4.1 The pionic annihilation channel

In the process:

\[
\bar{p}p \rightarrow \pi^+\pi^-n\pi^0
\]

one of the charged pions can be misidentified as a kaon and the \(n\pi^0\) can give the total photon multiplicity 6. In figure 4.13, we have the energy-loss of the kaon- and pion-candidate in the inner scintillator \(S1\) for our data: we can observe that at high momenta, the energy-loss of the charged pion is very close to the energy-loss of the kaon-candidate, increasing thus the pionic contamination of our data. Since the branching ratio of this channel is around hundred times higher than the signal events, even a small probability of bad identification results in a significant pionic background.

![Figure 4.13: \(3\pi^0\) data. Energy-loss \(dE/dX\) versus momenta for kaon- and pion-candidates. For each momentum interval, the \(dE/dX\) of kaon-candidates is higher than the \(dE/dX\) of pion-candidates.](image)

To study the pionic background and in order not to rely on simulation for the \(dE/dX\) measurement, we use a reference data sample where we know that a primary charged kaon-candidate is a kaon and the same for the pion. Such a data-sample is the \(\bar{p}p \rightarrow K^0(K^0)K^\pm\pi^\mp\),
$K^0(K^0) \rightarrow \pi^+\pi^-$ channel, where the above requirements are fulfilled through a good separation between primary and secondary vertices [44].

To reduce the pionic background, we demand that the energy-loss of the kaon-candidate, which includes both kaons and pions, should be at more than two standard deviations from the theoretical energy-loss of a kaon for the same momenta:

$$\frac{(dE/dX)_K - (dE/dX)_{\text{expected}}}{\sigma_K} > -2$$

where $\sigma_K$ is the error on the expected energy-loss of the kaon. In figures 4.14.a and .b, we can see the reference energy-loss spectra of a kaon and a pion respectively, before and after the $dE/dX$ cut for four regions of momenta between 400MeV/c and 800MeV/c. As expected, the cut (4.15) affects particularly the pions.

![Figure 4.14: $\bar{p}p \rightarrow K^0(K^0)K^\pm\pi^\mp$, $K^0(K^0) \rightarrow \pi^+\pi^-$ data. Energy-loss of kaons (a) and pions (b) in the four regions of momentum [400, 500]MeV/c, [500, 600]MeV/c, [600, 700]MeV/c and [700, 800]MeV/c, before (solid line) and after the $dE/dX$ cut (stars).](image)

In order to determine how much of this background remains in our data, we analyse the energy-loss of the kaon-candidate in the S1 scintillator. We perform a fit of the $dE/dX$ spectrum of the kaon-candidate in the $3\pi^0$ data by a reference spectra of a charged kaon and pion, which are obtained from the $K^0(K^0) \rightarrow \pi^+\pi^-$ data-sample. The result of the fit of the data by these reference spectra is shown in figure 4.15. Repeating this procedure for each bin of lifetime and missing-mass, we can obtain the reference shapes of the pionic background in these two variables.

**Variation of the pionic background versus data-taking periods** During the different data-taking periods of the CPLEAR experiment (P17 until P29), there have been an increasing freon leakage in the radiator of a few Cherenkov sectors, decreasing their capacity to produce
Figure 4.15: $3\pi^0$ data. Fit (solid line) of the measured $dE/dX$ spectrum of kaon-candidates in the four regions of momentum with the reference distribution, revealing pionic contribution (dashed area).

Therefore, a charged pion traversing such a Cherenkov detector has a greater probability to trigger a $S\bar{C}\bar{S}$ signal, appearing thus as a kaon-candidate and increasing the amount of the pionic background. In figure 4.16, we can observe the evolution of different PID-sectors through the number of their kaon-candidates in the $3\pi^0$ data: one can clearly see that some sectors have an abnormally high number of kaon-candidates compared to others; the sector 22, which had lost nearly half of its freon, is not taken into account by the trigger.

Figure 4.16: $3\pi^0$ data. Distribution of kaon-candidates for each PID-sector, for different periods.

In order to reduce the amount of pionic background produced by this mechanism, for each period, we demand that the kaon-candidate should not have given a $S\bar{C}\bar{S}$ signal in a "noisy" PID-sector, i.e., a sector giving more than 4% of the total number of kaon-candidates in the $3\pi^0$ data of a given period. However by doing that, we do not take into account that there
can be some smaller but increasing inefficiency in all other sectors, resulting in a flat kaon-candidate distribution of PID-sectors. This is the reason why the pionic contamination is globally increasing during the time as we can see in figure 4.17, some recalibrations (radiator refilling) being done on the Cherenkovs before P24 and P28.

![Figure 4.17: Percentage of pionic background as a function of period](image)

**4.4.2 $\bar{K}^0, K^0 \rightarrow 2\pi^0$ decay with 2 secondary photons**

In the $p\bar{p}$ annihilations, the neutral kaon can decay into $2\pi^0$ and two additional photons can be produced by secondary interactions, giving thus the total photon multiplicity 6 :

$$p\bar{p} \rightarrow \bar{K}^0(K^0)K^{\pm}\pi^\mp + 2\gamma_{sec.} \quad \leftrightarrow 2\pi^0 \rightarrow 4\gamma$$

Considering $2\pi^0$ simulated data, by matching generated and reconstructed photons (see Appendix B), we can tag among the six reconstructed showers the two originating from secondary photons and study their characteristics.

Before describing the mechanisms giving secondary photons, let us discuss their common characteristics. The point which characterises all of them is that they produce low energy photons. In figure 4.18, we can see that the mean energy of secondary photons is $\sim 131MeV$ while the mean energy of photons from the $K^0 \rightarrow 2\pi^0$ decay is $\sim 167MeV$, resulting in a mean energy of $\sim 156MeV$ for the six photons of $2\pi^0$ events while this mean is around $127MeV$ for $3\pi^0$ events, each photon having less available energy. We can make use of this property with computing the invariant mass of the six reconstructed photons (see Appendix D); this invariant mass corresponds to the mass of the neutral-kaon for $3\pi^0$ events and to a higher invariant mass for $2\pi^0 + 2\gamma_{sec.}$ events where the four photons from the $K^0$ decay already give the neutral-kaon mass (see figure 4.19). In order to reduce the amount of $2\pi^0 + 2\gamma_{sec.}$ events in the final data, we require that the invariant mass of the six showers should be smaller than
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Figure 4.18: Simulated data. Mean energy for: the two secondary photons of the $2\pi^0(a)$, the four generated photons of the $2\pi^0(b)$, the six photons of the $2\pi^0$ (dashed line) and the six generated photons of the $3\pi^0$ (solid line) (c).

800 MeV/c$^2$. According to the simulation, this cut suppresses $\sim 27\%$ of the $2\pi^0 + 2\gamma_{sec.}$ background while suppressing only $\sim 6\%$ of the signal.

We describe now the three different mechanisms producing secondary photons and the variables permitting their further reduction.

"Charged" showers are produced by charged primary particles around their interaction point in the ECAL. In order not to count charged showers in the total number of photons, for each event we demand six showers with a distance greater than 25cm from a $\pi^\pm$ and 50cm from a $K^\pm$.

Showers from $\gamma$ conversions Before entering the calorimeter, some photons can convert into a $e^+e^-$ pair in some massive part of the detector (Cherenkov, Scintillators); the minimal probability of such a conversion for a photon with an energy above 100 MeV is 40% [23]. This gives two electromagnetic showers into the calorimeter which are indistinguishable from photon showers and can thus give the total photon multiplicity 6 in the following cases:

- Among the four $\gamma$ of the $2\pi^0$, two are detected by the calorimeter and the two other convert in a $e^+e^-$ pair.
Among the four $\gamma$ of the $2\pi^0$, three are detected by the calorimeter, the fourth photon has converted in a $e^+e^-$ pair seen as photons in the ECAL, while there is an additional albedo photon (for a definition of albedo photons, see next paragraph).

It is also possible that among the four $\gamma$ of the $2\pi^0$, one is not detected by the calorimeter because being a low energy photon or because pointing outside the geometrical acceptance (see section 3.4.3), while three other photons convert in $e^+e^-$ pairs.

When computing, for simulated events, the minimal $\gamma\gamma$ invariant mass $m_{\gamma\gamma}^{\text{min}}$ out of all possible $\gamma\gamma$ pairs (see Appendix D), such background shows a clear peak below $10\text{MeV}/c^2$ while the signal has a broad distribution with a mean value of $\sim 31\text{MeV}/c^2$ (see figure 4.20.a). One can observe in figure 4.20.b that the distribution of this variable for the data reflects the characteristics of both signal and $2\pi^0+2\gamma_{\text{sec}}$ background. In order to reduce this background, we require $m_{\gamma\gamma}^{\text{min}} \geq 12\text{MeV}/c^2$. According to the simulation, this cut suppresses $\sim 48\%$ of the $2\pi^0+2\gamma_{\text{sec}}$ background while suppressing only $\sim 12\%$ of the signal.

"Albedo" showers The albedo photons are those produced by charged tracks but sent back and detected in the ECAL as neutral showers (see figure 4.21); the probability for a photon to be detected as neutral shower is roughly $9\%$ [41].
Once the cut of 50 cm around $K^\pm$ and 25 cm around $\pi^\pm$ is done (mainly eliminating charged showers) and the cut of 12 MeV on the $\gamma \gamma$ minimal mass $m_{\gamma\gamma}^{\text{min}}$ is done (mainly eliminating photon conversions into $e^+e^-$), the remaining events have photons with an energy distribution very similar to the one of figure 4.18.a with a mean energy very comparable with the one of photons of $2\pi^0 + 2\gamma_{\text{sec}}$ events; with variables computed with the energy of the photons it is not possible to distinguish between the remaining photons and those originating from the $K^0 \rightarrow 3\pi^0 \rightarrow 6\gamma$ decay. This is what motivates us to find a geometrical variable to characterise these photons.

Let us compare the direction of flight of the neutral-kaon reconstructed using the charged tracks with the direction of flight reconstructed using the six showers. Let $\alpha$ be the direction of flight of the neutral-kaon in the transversal plane given by the charged tracks ($p_{K^0} = -(p_{K^+} + p_{K^-})$). Using the pairing of the six photons obtained with the highest 7C-fit probability, and using the shower-angles ($\Phi, \Theta$) and shower-foot coordinates in the calorimeter, we intersect the directions of each pair of photons to obtain three independent "secondary vertices" corresponding to the decay vertices of the three $\pi^0$s; we then define $\beta(i)_{i=1,3}$ as three possible directions of flight of the neutral-kaon in the transversal plane, directions obtained using the primary vertex and the three independent "secondary vertices". Having calculated the associated errors $\sigma_{\beta(i)}$, we can compute the weighted direction of flight of the $K^0$ in the

Figure 4.20: a) : Simulated data : $2\gamma$ minimal invariant mass for $3\pi^0$ (solid line) and for $2\pi^0 + 2\gamma_{\text{sec}}$ (dashed line). b) : $3\pi^0$ data : $2\gamma$ minimal invariant mass for data events
transversal plane given by the photons:

\[ \beta = \frac{\sum_{i=1}^{3} \beta(i)/\sigma_\beta^2(i)}{\sum_{i=1}^{3} 1/\sigma_\beta^2(i)}. \]  

(4.16)

It is now interesting to observe the correlation between \( \alpha \) and \( \beta \), which both give the flight direction of the \( K^0 \) in the transversal plane \( (x, y) \) of the detector: in figure 4.22.a we can see that for signal events, there is a clear correlation between the angles \( \alpha \) and \( \beta \) while for background events (4.22.b) the two angles are uncorrelated. The angle \( \beta \) being a weighted mean of neutral-kaon flight-directions given by photons, the peak at \( \pi \) shows the pure random character of the shower distribution in the \( 2\pi^0 + 2\gamma_{\text{sec}} \) background; one should notice a smaller but non-negligible similar effect for signal events.

We can define the angle \( \delta = |\alpha - \beta| \) which measures the difference between the two angles giving the neutral-kaon flight-direction. For signal events figure 4.22.c, we can observe a distribution with a clear peak at zero and decreasing but non-negligible tails, while the distribution for \( 2\pi^0 + 2\gamma_{\text{sec}} \) background is flat. It is interesting to note that for signal events with \( \delta < 1 \text{rad} \), we have for the \( \delta \) distribution a \( FWHM \approx 0.25 \text{ rad} \) which is very comparable to the error on the shower-angle in the transversal plane \( (\Theta) \) of photons with an energy of \( \sim 150 \text{ MeV} \) (see section 3.4.1). It has to be noted that no correlation between \( \delta \) and \( m_{\gamma\gamma}^\text{min} \) has been observed so if we eliminate a large part of photon-conversions by cutting on \( m_{\gamma\gamma}^\text{min} \), the
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Figure 4.22: Simulated data. Correlation between $\alpha$, the $K^0$-line angle, determined in the transversal plane by the charged tracks, and by the neutral showers $\beta$, for $3\pi^0$ (a) and for $2\pi^0 + 2\gamma_{\text{sec.}}$ (b). c) $\delta = |\alpha - \beta|$ angle for $3\pi^0$ (solid line) and for $2\pi^0 + 2\gamma_{\text{sec.}}$ (dashed line).

remaining events, i.e., those with albedo photons show the same characteristics of figure 4.22. This variable shows the characteristic of albedo photons for the $2\pi^0$ channel: their random distribution in the calorimeter. However if we can learn about the geometrical topology of signal and background, unfortunately this variable does not allow us to discriminate between signal and $2\pi^0$ background with albedo photons.

Possibility of an Anti-6C-fit In order to study the possibility to reject the $2\pi^0 + 2\gamma_{\text{sec.}}$ background, we can apply a constrained-fit like in equation 4.8, but here, we demand the equality of the $\gamma\gamma$ invariant-mass with the $\pi^0$ mass only for two couples of photons (we omit the constraint $c_8$ to reconstruct the $K^0, K^0 \rightarrow 2\pi^0 \rightarrow 4\gamma$ channel; this is a 6C-fit. Applying this constrained-fit both on the $2\pi^0 + 2\gamma_{\text{sec.}}$ background and on the signal, we expect that the background and the signal will respectively be reconstructed with a high and a low probability, permitting a rejection of the background.

Applying such a constrained-fit to data where the photon multiplicity is six, we have 15 different ways of choosing four photons among six; for each way, we have 3 possibilities of coupling two photons among four. Thus, for each event we have 45 different probabilities. According to the simulation study, for each of the 15 possibilities, the 6C-fit gives the highest
probability by a decisive way (i-e, the highest probability is much higher than the two others); this is the reason why, for each of the 15 possibilities, we take the highest probability. In figure 4.23, we can see for simulated data, the distribution of this highest probability both for background and signal: One can observe that the $2\pi^0 + 2\gamma$ channel populates the high-

**Figure 4.23:** Simulated data. Highest 6C-fit probability for $2\pi^0 + 2\gamma$ (solid line) and signal (dashed line).

probability region while the signal populates both the high- and low-probability regions. This means that for the signal, there are some cases where four photons are sufficient to reconstruct the event and give it a $"2\pi^0"$-profile. Therefore, we can not use the probability of this 6C-fit to reject efficiently the $2\pi^0 + 2\gamma$ background.

### 4.4.3 Kaonic annihilation channels with an additional $\pi^0$

In addition of a neutral kaon, a neutral pion can be produced at the primary vertex:

$$p\bar{p} \rightarrow K^0(K^0)K^\pm\pi^\mp\pi^0 \rightarrow 2\gamma$$

$$\rightarrow 2\pi^0 \rightarrow 4\gamma$$

$$\rightarrow 3\pi^0 \rightarrow 6\gamma$$

When the neutral-kaon decays into $2\pi^0$ and all photons are detected, we have the total photon multiplicity 6; it is the same when the neutral kaon decays into $3\pi^0$ and two photons are undetected because of the finite detector acceptances (see section 3.4.3).

When we consider the $K^\pm\pi^\mp$ missing-mass square spectrum, the golden $K^0$ decay channels (like the signal or $K^0 \rightarrow 2\pi^0$) have a distribution which corresponds to $m_{K^0}^2$ while the kaonic annihilation channels with an additional $\pi^0$ have a distribution corresponding to $(m_{\pi^0} + m_{K^0})^2$.
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(see figure 4.24); it is interesting to compare these MC distributions with the data missing-mass square distribution of figure 4.7 where the contribution of both kind of channels is observed.

![Figure 4.24: Simulated data. $K^+\pi^-$ missing mass square distribution for signal (solid line) and for kaonic annihilation (dashed line) events.](image)

This property will be used to discriminate between signal events and events originating from kaonic annihilation channels with an additional $\pi^0$ (see section 4.4.5).

### 4.4.4 Summary of analysis cuts

Before explaining the method of determining the level of background, we recall the important cuts applied on the data in order to maximise the signal to background ratio and have the best quality of event reconstruction:

- $P_{K^0} > 180\text{MeV}/c$: to improve the lifetime resolution.
- $\text{Prob}(\chi^2_{2\text{sec}}) > 10\%$: to reduce all background channels.
- $(dE/dX)_{K^-}(dE/dX)_{\text{expected}} < -2$: to reduce the amount of pionic background.
- $\text{Invariant-mass}(6\gamma) < 800\text{MeV}/c^2$: to reduce the $2\pi^0 + 2\gamma_{\text{sec.}}$ background, and the kaonic annihilation channel background with an additional $\pi^0$ and a $K^0 \rightarrow 2\pi^0$ decay.
- $m^{\gamma\gamma}_{\text{min}} \geq 12\text{MeV}/c^2$: to reduce $2\pi^0 + 2\gamma_{\text{sec.}}$ channel where photons have converted in $e^+e^-$ pairs.

We have not observed any major correlation between the variables mentioned above, permitting an efficient rejection of the different background channels.
4.4.5 The signal to background ratio

The proportion of signal to background in the data is determined from a simultaneous fit of reference distributions for signal and for all background contributions, to the measured $K^\pm\pi^\mp$ missing mass (at primary vertex) and decay time distributions (taken from the 7C-fit); the number of events for each contribution is left free in the fit. The reference distributions are obtained from simulation for channels with neutral-kaon decays; for the pionic-annihilation channel these distributions are obtained from data by studying the energy-loss of charged particles in the scintillator $S1$ (see section 4.4.1). In order to control the level of background over a large range of lifetime, the fit is performed in the lifetime region $[-40, 60] \tau_S$, this for events in the missing-mass region $[0.15, 0.35] GeV^2/c^4$; in order to disentangle contributions overlapping in the lifetime distribution, the fit is simultaneously performed in the missing-mass region $[-0.1, 0.7] GeV^2/c^4$, this for events in the three lifetimes $[-20, -10] \tau_S$, $[-10, 0] \tau_S$ and $[0, 20] \tau_S$. The fit is performed using a $\chi^2$-minimisation method and we obtain a good description of the data by the reference distributions (see figure 4.25) with $\chi^2/Ndf = 2.4$.

In the decay-time distribution (fig. 4.25.a), the signal events populate the positive lifetime region while events reconstructed at negative lifetime are mainly background. This is because background channels, with the presence of an additional $\pi^0$ or secondary photons for example, fulfil the requirements of the 7C-fit only for very small or negative lifetime regions. In the missing-mass square spectrum (fig. 4.25.b, c and d), the signal and $2\pi^0 + 2\gamma_{sec.}$ channel events correspond to the neutral-kaon mass squared $m^2_{K^0}$ while events from kaon-decay channels with an additional $\pi^0$ populate the high-mass region and those from the pionic annihilation channel populate the low-mass region. It is interesting to note again the dominance of background and signal respectively at negative (fig. 4.25.b and c) and positive (fig. 4.25.d) lifetime regions. Fitting both the missing-mass spectra and the decay-time distribution simultaneously allows to separate the signal from background events: the signal and the $2\pi^0 + 2\gamma_{sec.}$ events have a very similar distribution for the missing-mass (fig. 4.25.d) but have different lifetime distributions (fig. 4.25.a); therefore, the correlation coefficients between signal and background do not exceed 0.32 (see table 4.3). The correlation coefficient between the various background channels vary from 0.02 to 0.68, the latter corresponding to the correlation between the pionic annihilation channel and the $2\pi^0 + 2\gamma_{sec.}$ channel.

For the final data sample, to retain most of the signal events we select the lifetime interval $[-1, 20] \tau_S$, and to reject a significant amount of background only events in the missing-mass square interval $[0.15, 0.35] GeV^2/c^4$ are accepted. Table (4.4) summarises the contribution of signal and background to the final data sample in the missing-mass interval $[0.15, 0.35] GeV^2/c^4$ and the decay-time interval $[-1, 20] \tau_S$. We can observe that the greatest contamination is the pionic background at a level of $\sim 25\%$. The signal, with a total of 17'300 events, represents $\sim 50\%$ of the final data sample.
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Figure 4.25: Fits to the measured neutral-kaon decay time distribution (a) in the missing-mass squared interval $[0.15, 0.35] \text{GeV}^2/c^4$. Fits to the measured $K^\pm \pi^\mp$ missing-mass squared in the lifetime intervals $[-20, -10] \tau_S$ (b), $[-10, 0] \tau_S$ (c) and $[0, 20] \tau_S$ (d). The measured spectra (points) are compared with the fit result (crosses); the contribution of the signal and background channels are shown.
Table 4.3: Correlation between signal and background channels.

<table>
<thead>
<tr>
<th>Channels</th>
<th>Pionic</th>
<th>Signal</th>
<th>2π⁰</th>
<th>π⁰3π⁰</th>
<th>π⁰2π⁰</th>
</tr>
</thead>
<tbody>
<tr>
<td>nπ⁰</td>
<td>1.</td>
<td>-0.04</td>
<td>-0.68</td>
<td>0.02</td>
<td>-0.09</td>
</tr>
<tr>
<td>Signal</td>
<td>1.</td>
<td>-0.32</td>
<td>-0.21</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>2π⁰</td>
<td>1.</td>
<td>0.03</td>
<td>-0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π⁰3π⁰</td>
<td></td>
<td>1.</td>
<td>-0.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>π⁰2π⁰</td>
<td></td>
<td></td>
<td>1.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4: Contribution of signal and backgrounds to the final data sample in the decay-time interval [-1, 20]τ₅ and within a missing-mass square interval of [0.15, 0.35]GeV²/c⁴. The errors are statistical.
4.5 Determination of $\eta_{000}$

The real and imaginary part of the CP-violation parameter $\eta_{000}$ in $\bar{K}^0, K^0 \rightarrow \pi^0\pi^0\pi^0$ decays are extracted from the fit of the theoretical asymmetry (2.50) to the observed lifetime dependent asymmetry:

$$A_{000}^{\text{obs}}(\tau) = \frac{R_{000}^{\text{obs}}(\tau) - R_{000}^{\text{obs}}(\tau)}{R_{000}^{\text{obs}}(\tau) + R_{000}^{\text{obs}}(\tau)}$$

(4.17)

where $R_{000}^{\text{obs}}(\tau)$ and $R_{000}^{\text{obs}}(\tau)$ are the observed decay rates of initially pure $\bar{K}^0$ and $K^0$ states given by equation 4.6 and shown in figure 4.26. Therefore, the observed asymmetry depends on

![Graphs](image)

Figure 4.26: $3\pi^0$ data: Lifetime-distribution of $\bar{K}^0$ (a) and $K^0$ (b) decays including 50% background contamination (see table 4.4).

the resolution function $\Gamma(\tau - t, t)$, the amount of background, the background shape $bckg(\tau)$, the normalisation factor $\bar{K}^0/K^0$ and, in second order, on the acceptance function $\epsilon(t)$. While leaving the $\bar{K}^0/K^0$ factor as a free parameter in the fit of the asymmetry, we should know these functions and parameters in order to extract the information about the CP-violation parameter $\eta_{000}$ contained in the theoretical rates $\bar{R}_{000}^{\text{th}}(t)$ and $R_{000}^{\text{th}}(t)$ (equations 2.48 and 2.49).

We will first study and determine these "components", necessary to the fit of the asymmetry. Then, we will present the fit procedure; we will finally discuss the systematic errors of our analysis and present our final result.
4.5.1 Components of the asymmetry

The resolution function \( \Gamma(\tau - t, t) \) in reconstructing the neutral-kaon decay-time

We derive the resolution function from a simulation study. As can be seen from figure 4.27, the resolution function \( \Gamma(\tau - t, t) \) varies as a function of the generated lifetime \( t \): the RMS of the resolution distribution varies from 4.2\( \tau_S \) for small decay-times to 4.8\( \tau_S \) for large decay-times.

In addition, the resolution function is asymmetric and this asymmetry also varies with \( t \). We parametrise the decay-time resolution by the following function:

\[
p(x) = N \exp\left[ \frac{(x - \mu)^2}{2\sigma^2} \right] \left( 1 + \beta |x| - \gamma x \right) \]

(4.18)

where \( x = \tau - t \). This function takes into account the asymmetric tails of the lifetime resolution distribution (compare with resolution function given in equation 4.14). Moreover, for each bin of generated lifetime (\( \Delta t = 1\tau_S \)), we will fold the theoretical rates (eq. 4.6) by a specific \( p(x) \) parametrisation in order to take into account the variation of the resolution distribution with \( t \).

Figure 4.27: \( 3\pi^0 \) simulated data. The decay-time resolution for four different decay-time regions. The solid line shows the parametrisation (eq. 4.18) of the decay-time resolution.

Uncertainties in the lifetime resolution, resulting in systematic errors will be discussed in section 4.5.3.
The amount of background contributing to the sum of $\bar{K}^0$ and $K^0$. In the expression of the observed rates (eq. 4.6), we take into account the amount of signal and background given by table 4.4. The precision with which we know the level of all contributions will be taken into account as a source of systematic errors in section 4.5.3.

Shape of background as function of lifetime ($bckg(\tau)$). We obtain the lifetime distribution for the background channels from simulated data for channels with a neutral-kaon decay, and from the $dE/dX$-study for the pionic background. A fit to these distributions is shown in figure 4.28, together with the fit intervals and the parametrisations which fit best these lifetime shapes for the lifetime region $[-1,20] \tau_S$ (see table 4.5).

Having determined $bckg(\tau)$ from simulation, we obtain $\bar{bckg}(\tau)$ through the relation:

$$\bar{bckg}(\tau) = \alpha_{bckg} \cdot bckg(\tau)$$  \hspace{1cm} (4.19)

where $\alpha_{bckg}$ is the normalisation of the relative contribution of background to the $K^0$ and $\bar{K}^0$ signal; it will be determined by the method explained in the next paragraph.

In order to study the effect of the parametrisation as a source of systematic errors, we will fit the decay-time distributions of background channels with different functions, as discussed in section 4.5.3.
CHAPTER 4. SEARCH FOR CP VIOLATION IN $K^0$,$K^0 \rightarrow \pi^0\pi^0\pi^0$ DECAYS

<table>
<thead>
<tr>
<th>Channel</th>
<th>Parametrisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pionic</td>
<td>exponential + constant</td>
</tr>
<tr>
<td>$2\pi^0 + 2\gamma_{sec.}$</td>
<td>exponential + exponential</td>
</tr>
<tr>
<td>$\pi^0\pi^0$</td>
<td>gaussian + gaussian</td>
</tr>
<tr>
<td>$\pi^02\pi^0$</td>
<td>exponential</td>
</tr>
</tbody>
</table>

Table 4.5: Parametrisation of the decay-time of background channels.

Difference of signal and background contributions to the $K^0$ and $K^0$ signal The observed asymmetry depends on the normalisation $\overline{K^0}/K^0$ for signal and background as it can be seen from equations 4.6 and 4.19. In figure 4.29 we can observe that, within statistics, the relative contribution of background to the $K^0$ and $\overline{K^0}$ signal is independent of the reconstructed decay time for all contributions, this in a large interval of decay-time $[-20, 20] \tau_S$; this justifies that we consider this normalisation as a time-independent parameter in the fit interval $[-1, 20] \tau_S$. To derive the absolute value of the normalisation of the relative contribution of background to the $K^0$ and $\overline{K^0}$ signal for all contributions, we repeat the simultaneous fit described in section 4.4.5 separately for $\overline{K^0}$- and $K^0$-data. With this method, which fits reference distributions to simulation and data, we obtain the number of $K^0$- and $K^0$-events for each channel and can thus determine $\alpha$ for each contribution (see table 4.6).

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\overline{K^0}/K^0$ normalisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal</td>
<td>$1.13 \pm 0.02$</td>
</tr>
<tr>
<td>$2\pi^0 + 2\gamma_{sec.}$</td>
<td>$1.14 \pm 0.05$</td>
</tr>
<tr>
<td>$\pi^0\pi^0$</td>
<td>$1.16 \pm 0.04$</td>
</tr>
<tr>
<td>$\pi^02\pi^0$</td>
<td>$1.18 \pm 0.06$</td>
</tr>
<tr>
<td>Pionic</td>
<td>$1.18 \pm 0.02$</td>
</tr>
</tbody>
</table>

Table 4.6: Normalisation of the relative contribution of background to the $K^0$ and $\overline{K^0}$ signal for the signal and for the background channels, obtained from the simultaneous lifetime and missing-mass fit.

Recalling that for signal $\alpha_{3\pi^0} = \xi[1 + 4\text{Re}(\epsilon_S)]$ and that $\epsilon_S \sim 10^{-3}$, we can observe that the deviation of $\alpha_{3\pi^0}$ from unity is dominated by the difference of the detection probabilities for $\overline{K^0}$ and $K^0$. More generally, for any channel with a neutral-kaon decay, the normalisation $\alpha$ is dominated by $\xi$, the difference of detection probabilities for $\overline{K^0}$ and $K^0$, but also includes a possible CP-violation effect of the order of $10^{-3}$. Therefore, the $\overline{K^0}/K^0$ normalisation is expected to be equal for all channels involving a neutral-kaon that has been tagged by...
4.5. DETERMINATION OF $\eta_{000}$

The normalisation $K^0/K^0$ for contributions with a kaon-decay (from simulated data) and for the pionic contribution (from the $dE/dX$-study).

In Table 4.6, we can observe that the $\alpha$'s of all kaonic backgrounds are compatible with the $\alpha$ of the signal within their statistical error. Therefore, we will consider the $\alpha_{bckg}$ normalisation of all kaonic backgrounds to be equal to the normalisation of the signal ($\alpha_{3\pi^0}$) and leave it as a free parameter $\alpha$ in the fit of the $A_{000}$ asymmetry. However, for the pionic background, we have a charged pion instead of a charged
kaon at the primary vertex: the strong interaction acting differently for a π± than for a K±, can a priori lead to a different normalisation of the relative contribution to the K° and $\bar{K}^0$. Therefore, in the fit of the $A_{000}$ asymmetry, we fix $\alpha_{\text{bg}}$ to 1.18 for the pionic background (see table 4.6).

The uncertainty in the determination of $\alpha$ for signal and background channels will be taken into account in the evaluation of systematic errors (section 4.5.3).

**The acceptance $\epsilon(t)$** We derive the acceptance function for $K^0 \to 3\pi^0$ events versus the generated lifetime $t$ from a simulation study. This probability of detecting a $\bar{K}^0, K^0 \to \pi^0\pi^0\pi^0$ event has already been described in section 3.4.3. As already stated, the geometrical acceptance cancels at first order in the observed asymmetry (eq. 4.17).

**The fit interval** Due to a finite lifetime resolution, we have around 9% of signal events reconstructed at negative lifetimes (see figure 4.12.a). Therefore, extending the fit interval to negative lifetimes increases the number of signal events and diminishes the errors on Re($\eta_{000}$) and Im($\eta_{000}$). However, as shown in figure 4.25, the background is mainly present in the negative-lifetime region; extending the fit interval to this region also increases the errors on the fitted parameters as seen in section 4.2. Therefore, the fit interval has been chosen to be $[-1, 20]\tau_S$ by minimising the errors on Re($\eta_{000}$) and Im($\eta_{000}$).

### 4.5.2 Fit of the $A_{000}$ asymmetry

To fit the $A_{000}$ asymmetry we use a maximum likelihood method. The likelihood that we will maximise is given by:

$$-\ln L = \sum_i N_i \cdot \ln R_i + \sum_i \bar{N}_i \cdot \ln \bar{R}_i.$$

where $N_i$ and $\bar{N}_i$ are respectively the number of events in the $i^{th}$ lifetime bin of initially pure $\bar{K}^0$ and $K^0$ measured rates; $N_i\bar{R}_i$ and $N_iR_i$ are the functions describing bin-per-bin the observed rates $R_{000}(\tau)$ and $\bar{R}_{000}(\tau)$ (see eq. 4.6). We can re-express this likelihood as a function of the measured binned asymmetry $A_i = (\bar{R}_i - R_i) / (\bar{R}_i + R_i)$:

$$-\ln L = \sum_i N_i \cdot \ln (1 - A_i) + \sum_i \bar{N}_i \cdot \ln (1 + A_i)$$

$$+ \sum_i (N_i + \bar{N}_i) \cdot \ln \left(\frac{R_i + \bar{R}_i}{2}\right).$$

The two first terms, through the asymmetry, measure a difference between two CP-conjugate processes and are thus very sensitive to the CP-violation parameter $\eta_{000}$; at the same time, they are not sensitive to the acceptance $\epsilon(t)$ since it is cancelled to first order in the asymmetry. The last term is not very sensitive to CP-violation parameters but depends strongly on the acceptance, which could lead to systematic errors in maximising the likelihood. This
is the reason why, to obtain the CP-violation parameters $Re(\eta_{000})$ and $Im(\eta_{000})$, we will only maximise the following likelihood:

$$-lnL' = \sum_i N_i \cdot ln(1 - A_i) + \sum_i N_i^* \cdot ln(1 + A_i)$$  \hspace{1cm} (4.22)

The measured asymmetry $A_i$ in the $i^{th}$ lifetime bin takes into account:

- the theoretical $\bar{K}^0, K^0 \rightarrow 3\pi^0$ decay rates (eq. 2.48 and 2.49) which contain the CP-violation parameter $\eta_{000}$.
- the folding of the theoretical decay rates (eq. 4.6) by the acceptance $\epsilon(t)$ (fig. 3.10) and the lifetime resolution function $\Gamma(\tau - t, t)$ (see section 4.5.1).
- the amount of signal and background given in tab. 4.4.
- the lifetime distributions of all background channels $bckg(\tau)$ (tab. 4.5) contributing to the observed rates (eq. 4.6).
- the normalisation of the relative contribution of background to the $\bar{K}^0$ and $K^0$ signal for signal and background decay rates (see eq. 4.6 and 4.19). For all channels with a neutral-kaon decay, namely the signal, the $K^0 \rightarrow 2\pi^0 + 2\gamma_{sec}, \pi^0 + K^0 \rightarrow 3\pi^0$ and $\pi^0 + K^0 \rightarrow 2\pi^0$ decays, this factor is left as a free parameter $\alpha$; for the pionic background, it is fixed at the value given by table 4.6.

Figure 4.30 shows the experimental asymmetry $A_{000}$. To fit the observed asymmetry by maximising the likelihood of equation 4.22, we fix the mass difference and the $K_S$ mean life at $\Delta m = (530.7 \pm 1.3) \times 10^7$ $\text{hs}^{-1}$ and $\tau_S = (89.22 \pm 0.10)$ $\text{ps}$ as determined in ref. [45]; the value of the $K_L$ mean life is fixed at $\tau_L = (51.7 \pm 0.4)$ $\text{ns}$ as given by ref. [23]. $Re(\eta_{000}), Im(\eta_{000})$ and the normalisation factor are left as free parameters in the fit. The fit yields:

$$Re(\eta_{000}) = 0.18 \pm 0.14_{\text{stat.}}$$  \hspace{1cm} (4.23)
$$Im(\eta_{000}) = 0.15 \pm 0.20_{\text{stat.}}$$

with $\alpha = 1.10 \pm 0.03$ where the error is statistical. The correlations between fitted parameters are given in table 4.7.

Given the obtained value for $\alpha$, one can observe that the measured asymmetry has a total offset of $(\alpha - 1)/(\alpha + 1)$ (see eq. 4.4).

We can observe that the obtained values for the CP-violation parameters are compatible with zero; we can only put an upper limit on the CP-violation parameter $\eta_{000}$ (see section 4.5.4). The limited lifetime resolution, coupling strongly the sinus and the cosine term in equation 2.50, leads to a correlation of $\rho = 79\%$ between $Re(\eta_{000})$ and $Im(\eta_{000})$. We can make use of this strong correlation by fixing $Re(\eta_{000})$ at $Re(\epsilon_S)$ as suggested by equation 2.45, and get a better precision for $Im(\eta_{000})$; however, there is no experimental value of
Figure 4.30: The measured time-dependent CP asymmetry between decay rates of initially pure $K^0$ and $K^0$ into $3\pi^0$, between $-1$ and $20\tau_s$. The dots are the experimental asymmetry and the solid line shows the result of the fit to the data.

Table 4.7: Correlation coefficients between parameters obtained from the fit of the $A_{000}$ asymmetry.

<table>
<thead>
<tr>
<th></th>
<th>$Re(\eta_{000})$</th>
<th>$Im(\eta_{000})$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re(\eta_{000})$</td>
<td>1</td>
<td>0.79</td>
<td>-0.09</td>
</tr>
<tr>
<td>$Im(\eta_{000})$</td>
<td></td>
<td>1</td>
<td>-0.56</td>
</tr>
<tr>
<td>$\alpha$</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

$Re(\epsilon_S)$. In order to be able to use the measurement of the semileptonic charge asymmetry [23]:

$$\delta_l = \frac{\Gamma(K_L \rightarrow \pi^- l^+ \nu) - \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- l^+ \nu) + \Gamma(K_L \rightarrow \pi^+ l^- \bar{\nu})} = 2Re(\epsilon_L - x_+ - y), \quad (4.24)$$

we have to assume CPT-conservation ($x_+ = y = \delta = 0$, see sections 2.1 and 2.2); we can then fix $Re(\eta_{000})$ to $\delta_l/2 = Re(\epsilon_L) = Re(\epsilon_S)$ in the fit of the asymmetry. This improves the statistical precision of the $Im(\eta_{000})$ by a factor $\sqrt{1 - \rho^2}$:

$$Im(\eta_{000}) = -0.05 \pm 0.12_{\text{stat}}. \quad (4.25)$$
4.5.3 Systematic errors

The "components" in the fit of the asymmetry (section 4.5.1) are known with some uncertainty. To take into account the effects of these uncertainties in the final result, we will vary each component of the asymmetry within its error interval and observe the resulting variation on the fitted CP-violation parameters.

- Neutral-kaon decay time resolution

In order to take into account a possible underestimation of the lifetime resolution by the simulation study, the resolution function has been increased by 10% in each bin of generated decay-time; the error on $\alpha_\sigma$ is below 10%, where $\alpha_\sigma$ is the parameter which represents the width of the resolution function given by equation 4.18. This has an effect of 0.01 on both $Re(\eta_{0000})$ and $Im(\eta_{0000})$.

Moreover, possible deviations in the decay-time dependence of the resolution function (fig. 4.27) from the simulation have been considered by fitting the asymmetry with a resolution function (fig. 4.12.b) independent of the decay time. The effect of such a variation has been found to be negligible.

- Lifetime shape of background channels

In order to study the effect of the parametrisation as a source of systematic errors, we will fit the decay-time distributions of background channels with different functions (tab. 4.8). The effect of the variation of the parametrisation of lifetime-distributions has been found to be negligible.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Parametrisation 1</th>
<th>Parametrisation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$</td>
<td>exponential</td>
<td>exponential + constant</td>
</tr>
<tr>
<td>$2\pi^0 + 2\gamma_{sec.}$</td>
<td>exponential</td>
<td></td>
</tr>
<tr>
<td>$\pi^03\pi^0$</td>
<td>gaussian + exponential</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.8: Alternative parametrisations of the decay-time for various background channels.

- Background contribution

To evaluate the systematic error induced by the method used to determine the level of signal and background, we replace the simultaneous fit by an independent missing-mass and lifetime fit to the reference distributions. The proportions of signal and background channels agree within better than 3% with the proportions given by the simultaneous fit (table 4.4). The level of the pionic background is obtained from a simultaneous fit where its amount is fixed, in each bin of lifetime and squared missing-mass, at the level determined by the $dE/dX$-study in each bin of momentum (see section 4.4.1); the level
of the pionic background is then 7% smaller than the result of the regular fit (table 4.4). Therefore, to determine the systematic errors on Re(η°°°) and Im(η°°°), we vary the level of contributions with a kaon decay by ±3%, and reduce the level of the pionic background by 7%, taking into account the correlations between all contributions (see table 4.9).

<table>
<thead>
<tr>
<th>Variation of the amount of the contribution</th>
<th>ΔRe(η°°°)</th>
<th>ΔIm(η°°°)</th>
<th>ΔIm(η°°°) Re(η°°°) = Re(eT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal : ±3%</td>
<td>±0.01</td>
<td>±0.01</td>
<td>±0.00</td>
</tr>
<tr>
<td>2π° + 2γ : ±3%</td>
<td>±0.00</td>
<td>±0.01</td>
<td>±0.01</td>
</tr>
<tr>
<td>π°π° : ±3%</td>
<td>±0.01</td>
<td>±0.01</td>
<td>±0.01</td>
</tr>
<tr>
<td>nπ° : ±3%</td>
<td>±0.00</td>
<td>±0.00</td>
<td>±0.01</td>
</tr>
<tr>
<td>nπ° : ±7%</td>
<td>±0.01</td>
<td>±0.01</td>
<td>±0.02</td>
</tr>
</tbody>
</table>

Table 4.9: Contribution of the variation of the level of signal and background in systematic errors on Re(η°°°), Im(η°°°); the last column represents the systematic error on Im(η°°°) when Re(η°°°) = Re(eT).

- The relative contribution of background to the K° and the K° signal
  The K°/K° normalisation of signal and background channels with a kaon decay are all compatible within 5%; the systematic errors on Re(η°°°) and Im(η°°°) are then evaluated by varying the K°/K° normalisation within ±5% for signal and background channels with a kaon decay. We recall that the mechanism giving different detection efficiencies for K° and K° is expected to be the same for all channels with a neutral-kaon decay; the K°/K° normalisation of the signal and background channels with a kaon decay is left as a free parameter α in the fit of the asymmetry; therefore, for these channels, we only take the biggest error on Re(η°°°) and Im(η°°°) (table 4.10).

The normalisation of the relative contribution of pionic background to the K° and K° is determined with a precision of 2% (table 4.6). Taking into account the systematic uncertainty in determining the level of the pionic background, we also vary the normalisation of this channel within ±5% to evaluate the systematic errors on Re(η°°°) and Im(η°°°).

We can observe that these systematic errors are dominated by uncertainties on α_{2π°} and α_{nπ°} (table 4.10). This is due to the fact that the pionic and the 2π° + 2γ channels are the dominating backgrounds (table 4.4).

- Uncertainties related to Δm, τL and τS
  The variation of Δm, τL and τS within their errors (ref. [23, 45]) have a negligible effect on the result of the fit.
4.5. DETERMINATION OF $\eta_{000}$

<table>
<thead>
<tr>
<th>Variation of $\alpha$ for each contribution</th>
<th>$\Delta Re(\eta_{000})$</th>
<th>$\Delta Im(\eta_{000})$</th>
<th>$\Delta Im(\eta_{000})<em>{Re(\eta</em>{000})=Re(\nu_T)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{2\pi^0} = \alpha_{3\pi^0} \pm 5%$</td>
<td>$\pm 0.035$</td>
<td>$\pm 0.01$</td>
<td>$\pm 0.035$</td>
</tr>
<tr>
<td>$\alpha_{3\pi^0} = \alpha_{3\pi^0} \pm 5%$</td>
<td>$\pm 0.00$</td>
<td>$\pm 0.01$</td>
<td>$\pm 0.00$</td>
</tr>
<tr>
<td>$\alpha_{2\varepsilon^0} = \alpha_{3\varepsilon^0} \pm 5%$</td>
<td>$\pm 0.02$</td>
<td>$\pm 0.01$</td>
<td>$\pm 0.01$</td>
</tr>
<tr>
<td>$\alpha_{\nu_T^0} = 1.18 \pm 5%$</td>
<td>$\pm 0.04$</td>
<td>$\pm 0.01$</td>
<td>$\pm 0.03$</td>
</tr>
</tbody>
</table>

Table 4.10: Contribution of uncertainty on $\alpha$ in systematic errors on $Re(\eta_{000})$ and $Im(\eta_{000})$.

- Uncertainties due to regeneration effects

The effect of neutral-kaon regeneration as measured by CPLEAR [46] is negligible at short decay times where the $A_{000}$ asymmetry is maximal (fig. 4.1).

A summary of systematic errors on $Re(\eta_{000})$ and $Im(\eta_{000})$ are reported in table 4.11. For the systematic error on $Re(\eta_{000})$, we can observe that it is dominated by the uncertainty on the $K^0/L$ normalisation.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta Re(\eta_{000})$</th>
<th>$\Delta Im(\eta_{000})$</th>
<th>$\Delta Im(\eta_{000})<em>{Re(\eta</em>{000})=Re(\nu_T)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decay-time resolution</td>
<td>0.01</td>
<td>0.01</td>
<td>$\ll 0.01$</td>
</tr>
<tr>
<td>Lifetime shape of background channels</td>
<td>$\ll 0.01$</td>
<td>$\ll 0.01$</td>
<td>$\ll 0.01$</td>
</tr>
<tr>
<td>Background contribution</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$K^0/L$ of backgrounds</td>
<td>0.05</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Delta m$, $\tau_L$, and $\tau_S$</td>
<td>$\ll 0.01$</td>
<td>$\ll 0.01$</td>
<td>$\ll 0.01$</td>
</tr>
<tr>
<td>Regeneration</td>
<td>$\ll 0.01$</td>
<td>$\ll 0.01$</td>
<td>$\ll 0.01$</td>
</tr>
<tr>
<td>Total</td>
<td>0.06</td>
<td>0.03</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 4.11: Contribution of systematic errors on $Re(\eta_{000})$ and $Im(\eta_{000})$, and on $Im(\eta_{000})$ when we fix $Re(\eta_{000}) = Re(\nu_T)$.

**Comparison with feasibility study and the ITEP experiment** The intrinsic sensitivity (eq. 4.7) to the fitted parameters due to the decay-time resolution and due to background are reported in table 4.12; are also reported the total intrinsic sensitivity of the CPLEAR experiment as derived from the total number of events and the statistical errors on $Re(\eta_{000})$ and $Im(\eta_{000})$ (eq. 4.23 and 4.25).

The numbers of table 4.12 agree within 3% for $C(Re(\eta_{000}))$ and $C(Im(\eta_{000}))$ with the relation:

$$\frac{C(\eta)_{\text{CPLEAR}}}{C(\eta)_{\text{Ideal}}} \approx \frac{C(\eta)_{\text{Background}}}{C(\eta)_{\text{Ideal}}} \times \frac{C(\eta)_{\text{Resolution}}}{C(\eta)_{\text{Ideal}}}.$$
Intrinsic sensitivity | Ideal | Background | Resolution | CPLEAR | ITEP |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(Re(\eta_{1000}))$</td>
<td>2.8</td>
<td>4.9</td>
<td>10.2</td>
<td>18.4</td>
<td>4.5</td>
</tr>
<tr>
<td>$C(Im(\eta_{1000}))$</td>
<td>5.6</td>
<td>9.3</td>
<td>16.2</td>
<td>26.3</td>
<td>6.8</td>
</tr>
<tr>
<td>$C(Im(\eta_{1000})) Re(\eta) Re(\epsilon)$</td>
<td>4.7</td>
<td>8.0</td>
<td>7.9</td>
<td>15.8</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 4.12: Intrinsic sensitivity on $Re(\eta_{1000})$ and $Im(\eta_{1000})$ for ideal case (no background, perfect resolution), for a background of 50% and for a decay-time resolution like in our analysis as derived from the feasibility study (section 4.2). The total sensitivities for the CPLEAR and ITEP experiment are also given.

This shows that our statistical errors are well understood in terms of loss of intrinsic significance due to a finite decay-time resolution and background contamination.

The only experiment up to date, apart from CPLEAR, to have searched for CP violation in decays of neutral-kaons in $3\pi^0$ is the ITEP experiment ref. [25]. It is a bubble-chamber experiment without magnetic-field where Xenon nuclei are targeted by a $K^+$.beam, producing neutral-kaons $K^0$. The signal event to reconstruct is the following:

$$K^+ + Xe \rightarrow K^0 p$$
$$\rightarrow 3\pi^0 \rightarrow 6\gamma \rightarrow 6(e^-e^+)$$

The secondary vertex being reconstructed through the intersection of six $e^-e^+$ pairs, the neutral-kaon decay-length is known with a high accuracy. Moreover, there is no background contamination. Therefore, this experiment has a better intrinsic sensitivity to $Re(\eta_{1000})$ and $Im(\eta_{1000})$ than CPLEAR (table 4.12). However, due to the very low probability to detect such events only 632 events have been collected (for a total of $6 \times 10^6 K^+_S$). This is the reason why, despite a finite decay-time resolution and a background contamination, but with 30 times more events, the CPLEAR experiment has a better sensitivity on $Re(\eta_{1000})$ and $Im(\eta_{1000})$.

4.5.4 Final results and conclusions

Determining the CP-violation parameters $Re(\eta_{1000})$ and $Im(\eta_{1000})$, the final result from 17'300 reconstructed $\overline{K}^0, K^0 \rightarrow \pi^0\pi^0\pi^0$ is:

$$Re(\eta_{1000}) = 0.18 \pm 0.14_{\text{stat.}} \pm 0.06_{\text{syst.}} \quad (4.26)$$
$$Im(\eta_{1000}) = 0.15 \pm 0.20_{\text{stat.}} \pm 0.03_{\text{syst.}}$$

where the total error is dominated by the statistical error; the correlation coefficient (see table 4.7) between $Re(\eta_{1000})$ and $Im(\eta_{1000})$ is 0.79.

This is the first determination of $\eta_{1000}$ using the rate asymmetry of initially pure $\overline{K}^0$ and $K^0$ decaying to $\pi^0\pi^0\pi^0$ [38] and is independent on any assumption on CPT invariance. We have
the best sensitivity for \( \eta_{000} \) since we have improved the previous measurement [25] by \( \sim 24\% \) as shown in figure 4.31. On this figure are also represented the equal likelihood contours in

![Contour plot for Re(\( \eta_{000} \)) and Im(\( \eta_{000} \))](image)

Figure 4.31: The contour plot for Re(\( \eta_{000} \)) and Im(\( \eta_{000} \)) obtained in our analysis; the error bars represent the statistical and systematic uncertainties added in quadrature. For comparison, the result of ref. [25] is also shown.

the \( \eta_{000} \) complex plane: one can observe that up to three-standard deviation, the contour remains an ellipse representing the correlation (\( \sim 80\% \)) of two gaussian parameters. It proves that our statistical approach in the fit of the asymmetry is not altered by any systematical effect.

**Upper limit for** \( B_{K_S \to 3\pi^0} \). Assuming CPT invariance, we fix Re(\( \eta_{000} \)) at Re(\( \epsilon_T \)) (eq. 2.45) in the fit of the asymmetry, obtaining:

\[
\text{Im}(\eta_{000}) = -0.05 \pm 0.12_{\text{stat.}} \pm 0.05_{\text{syst.}}.
\]

(4.27)

Given the order of magnitude of Re(\( \eta_{000} \)) fixed at Re(\( \epsilon_T \)) \( \sim 10^{-3} \), and Im(\( \eta_{000} \)) measured to be \( 5 \cdot 10^{-2} \), we can consider that:

\[
|\eta_{000}|^2 = \text{Re}(\eta_{000})^2 + \text{Im}(\eta_{000})^2 \approx \text{Im}(\eta_{000})^2
\]

(4.28)

and compute the branching ratio of \( K_S \) decaying in \( 3\pi^0 \) as follows:

\[
B_{K_S \to 3\pi^0} = \frac{\Gamma(K_S \to 3\pi^0)}{\Gamma(K_S \to 3\pi^0)} \times \frac{\Gamma(K_L \to 3\pi^0)}{\Gamma_L} \times \frac{\Gamma_L}{\Gamma_S}
\]

(4.29)
where the \( \Gamma \)'s are the partial and full decay widths of neutral-kaons and \( B_{K_L \rightarrow 3\pi^0} \) is the branching ratio of \( K_L \rightarrow \pi^0\pi^0\pi^0 \) decay. Using the approximation of equation 4.28 and \( B_{K_L \rightarrow 3\pi^0} = (21.12 \pm 0.27)\% \) from ref. [23], we deduce an upper limit for the branching ratio of the \( K_S \rightarrow \pi^0\pi^0\pi^0 \) decay:

\[
B_{K_S \rightarrow 3\pi^0} < 1.9 \times 10^{-5} \tag{4.30}
\]

at the 90\% confidence level, which is an improvement by a factor two compared to the measurement of [25] and the best upper limit for \( B_{K_S \rightarrow 3\pi^0} \).
Chapter 5

INDIRECT CPT TEST

5.1 Assuming CPT-conservation in decays

In this chapter we focus on the oscillations of the neutral kaons to search for CPT-violation in the mass matrix, assuming CPT-conservation in the decay amplitudes ($\Gamma_{11} = \Gamma_{22}$).

Assuming CPT-conservation in the neutral-kaon decays, it is more convenient to use the $\bar{K}^0-K^0$ basis to determine $Im(\delta)$, as explained in section 2.3.1. We recall the expression of $Im(\delta)$ (eq. 2.69):

$$Im(\delta) \approx \frac{\Delta \Gamma}{\sqrt{4\Delta m^2 + \Delta \Gamma^2}} \left[ \frac{2}{3} |\eta_{+}|(\phi_{+} - \phi_{SW}) + \frac{1}{3} |\eta_{00}|(\phi_{00} - \phi_{SW}) - \frac{\Phi}{2} \cos(\phi_{SW}) \right]$$

with:

$$\Phi = 2 \frac{\Gamma_{L}}{\Gamma_{S}} \times [2Br(K_L \rightarrow \pi l\nu)Im(x_{+})$$

$$-Br(K_L \rightarrow \pi^{+}\pi^{-}\pi^{0})Im(\epsilon_{T} - \eta_{++0}) - Br(K_L \rightarrow \pi^{0}\pi^{0}\pi^{0})Im(\epsilon_{T} - \eta_{000})].$$

Before computing CPT-violation parameters $Im(\delta)$, we explain our choice of all relevant parameters which are summarised in table 5.1.

The phase $\phi_{+-}$ of the CP-violation parameter $\eta_{+-}$ and the mass-difference $\Delta m$ are determined from the study of neutral kaon decays into $\pi^{+}\pi^{-}$ final states by various experiments [45]. It has to be noted that the value of $\phi_{+-}$ and $\Delta m$ were obtained assuming CPT in decay amplitudes and can thus be used as input for our indirect CPT-test.

The parameter $\eta_{+-0}$ is exclusively taken from CPLEAR [22] where our experiment has clearly the best sensitivity. $Re(\epsilon_{T})$ and $Im(x_{+})$ are extracted from the fit of $A^{exp}_{T}$ asymmetry [24] only by the CPLEAR experiment (section 2.2.3) with the assumption of CPT-conservation in the semileptonic decay amplitude.

For CP-violation parameter $\eta_{000}$, the CPLEAR experiment has the best sensitivity [38] by only $\sim 24\%$; therefore, we combine our result with that of another experiment [25].

For the modulus of the CP-violation parameter $\eta_{+-}$, the CPLEAR experiment has the second best sensitivity [44] which we combine with the result given in ref. [19].
Finally, $\eta_{00}$ is obtained from the ratio $\frac{\eta_{00}}{\eta_{+-}}$ and the phase difference $\phi_{00} - \phi_{+-}$, using the world average [23].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{+-}$</td>
<td>$(43.6 \pm 0.6)^\circ$</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>$(530.1 \pm 1.1) \cdot 10^{-7} h/s$</td>
</tr>
<tr>
<td>$\phi_{SW}$</td>
<td>$(43.46 \pm 0.08)^\circ$</td>
</tr>
<tr>
<td>$</td>
<td>\eta_{+-}</td>
</tr>
<tr>
<td>$\frac{\eta_{00}}{\eta_{+-}}$</td>
<td>$(995.6 \pm 2.3) \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\phi_{00} - \phi_{+-}$</td>
<td>$(-1 \pm 8)^\circ \cdot 10^{-1}$</td>
</tr>
<tr>
<td>$Re(\eta_{+-})$</td>
<td>$(-2 \pm 8) \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$Im(\eta_{+-})$</td>
<td>$(-2 \pm 9) \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$Re(\eta_{00})$</td>
<td>$(7 \pm 12) \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$Im(\eta_{00})$</td>
<td>$(8 \pm 16) \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$Re(\epsilon_T)$</td>
<td>$(1.55 \pm 0.35) \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$Im(x_+)$</td>
<td>$(1.2 \pm 1.9) \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$Br(K_L \to \pi \nu)$</td>
<td>$(65.95 \pm 0.37)$%</td>
</tr>
<tr>
<td>$Br(K_L \to \pi^+ \pi^- \pi^0)$</td>
<td>$(12.56 \pm 0.20)$%</td>
</tr>
<tr>
<td>$Br(K_L \to \pi^0 \pi^0 \pi^0)$</td>
<td>$(21.12 \pm 0.27)$%</td>
</tr>
<tr>
<td>$\tau_L$</td>
<td>$(5.17 \pm 0.04) \cdot 10^{-8}$ s</td>
</tr>
<tr>
<td>$\tau_S$</td>
<td>$(8.934 \pm 0.008) \cdot 10^{-11}$ s</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters necessary to perform an indirect test of CPT.

Using the values given in table 5.1, the phase difference between $\phi_T$ and $arg A_T^0 \overline{A}_0$ (see eq. 2.28) is found to be

$$\Phi = (6.1 \pm 11.7) \cdot 10^{-5} rad$$ (5.1)

where correlations between

- $\phi_{+-}, \Delta m$ and $\phi_{SW}$ (determined from ref. [45])
- $Re(\epsilon_T)$ and $Im(x_+)$ (measured from the fit of $A_T^{exp}$)

are taken into account in the error calculation. The imaginary part of the CPT-violation parameter $\delta$ is then found to be

$$Im(\delta) = (1.3 \pm 3.6) \cdot 10^{-5}.$$ (5.2)

From the definition of $\delta$ (eq. 2.14) it follows that

$$M_{K^0} - M_{\overline{K}^0} = (1.8 \pm 5.1) \cdot 10^{-19} GeV/c^2.$$ (5.3)
We emphasise that the results of equations 5.1, 5.2 and 5.3 have been obtained taking the experimental values of CP-violation parameters \( \eta_{\pm 0} \) and \( \eta_{000} \), with assumptions of unitarity and CPT-conservation in the decay amplitude. The precision of this test is dominated by the precision of the phase differences \( \Phi \) as it can be seen from table 5.2, mainly due to the uncertainty in \( \eta_{000} \).

<table>
<thead>
<tr>
<th>( \eta_{000} ) from</th>
<th>( \Phi ) [rad.]</th>
<th>( Im(\delta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITEP</td>
<td>((-0.3 \pm 2.0) \cdot 10^{-4})</td>
<td>((-1.2 \pm 5.5) \cdot 10^{-5})</td>
</tr>
<tr>
<td>ITEP and CPLEAR</td>
<td>((6.1 \pm 11.7) \cdot 10^{-5})</td>
<td>((1.3 \pm 3.6) \cdot 10^{-5})</td>
</tr>
</tbody>
</table>

Table 5.2: Values of the phase difference \( \Phi \) and CPT-violation parameter \( Im(\delta) \) as function of different experimental values of \( \eta_{000} \).

From the result obtained in equation 5.3, an upper limit on the neutral-kaon mass difference can be derived to be:

\[
|M_{K^0} - M_{\bar{K}^0}| < 10.2 \cdot 10^{-19} \text{GeV}/c^2
\]

(5.4)

at the 90% confidence level, which represents the best limit on the \( K^0 - \bar{K}^0 \) mass difference. If we assume that there is no \( I = 3 \) decay amplitude in three-pion decays, it follows that \( \eta_{000} = \eta_{\pm 0} \), and we obtain

\[
\Phi = (1.4 \pm 7.7) \cdot 10^{-6} \text{ rad}
\]

\[
Im(\delta) = (-0.3 \pm 1.9) \cdot 10^{-5}
\]

leading to

\[
|M_{K^0} - M_{\bar{K}^0}| < 4.8 \cdot 10^{-19} \text{GeV}/c^2
\]

at the 90% confidence level.

### 5.2 Without assuming CPT-conservation in decays

If we do not assume CPT-invariance in the neutral-kaon decay amplitude, the parameter \( Re(y) \) must be included in the expression of \( Im(\delta) \) (eq. 2.60). Moreover, values of \( Re(e_T) \) and \( Im(x_+) \) obtained without assuming CPT are required. This can be done by fitting simultaneously asymmetries constructed with same rates than asymmetries \( A_T \) (eq. 2.54) and \( A_CPT \) (eq. 2.70) simultaneously, leaving \( \delta, Re(e_T), Re(y) \) and \( Im(x_+) \) as free parameters and using the unitarity equations 2.59 and 2.60 as constraints [47].
Results of this fit give

\[
\begin{align*}
Re(\delta) &= (2.4 \pm 2.8) \cdot 10^{-4} \\
Im(\delta) &= (2.4 \pm 5.0) \cdot 10^{-5} \\
Re(\epsilon_T) &= (164.9 \pm 2.5) \cdot 10^{-5} \\
Im(x+) &= (-2.0 \pm 2.7) \cdot 10^{-3} \\
Re(y) &= (0.3 \pm 3.1) \cdot 10^{-3} \\
Re(x-) &= (-0.5 \pm 3.0) \cdot 10^{-3}.
\end{align*}
\]

This is the first determination of CPT-violation parameter $\delta$ and T-violation parameter $Re(\epsilon_T)$ without an assumption on CPT. The errors on $Re(\delta)$ and $Im(\delta)$ are respectively reduced by a factor 1.2 and $5 \cdot 10^2$ compared to the results obtained by the fit of the $A_{\text{exp}}$ asymmetry (eq. 2.72), showing the capacity of unitarity relations in constraining the neutral-kaon system.

Using the above results on $Re(\delta)$, $Im(\delta)$ and equation 2.14, we determine the mass- and width-differences of the neutral kaon:

\[
\begin{align*}
M_{K^0} - M_{\bar{K}^0} &= (-1.5 \pm 2.0) \cdot 10^{-18}\text{GeV}/c^2 \\
\Gamma_{K^0} - \Gamma_{\bar{K}^0} &= (3.9 \pm 4.2) \cdot 10^{-18}\text{GeV}
\end{align*}
\]

This is the best limit on a particle-antiparticle mass and width difference [23], without any assumption on the CPT-symmetry.
Chapter 6

CONCLUSION

The CPLEAR experiment, by measuring asymmetries between the time-dependent rates of initially pure $\overline{K}^0$'s and $K^0$'s decaying into $\pi^0\pi^0\pi^0$, gives the best measurement of the CP-violation parameter $\eta_{000}$:

\[
\begin{align*}
Re(\eta_{000}) & = 0.18 \pm 0.15 \\
Im(\eta_{000}) & = 0.15 \pm 0.20
\end{align*}
\]

where the errors are dominated by the statistical errors; the correlation coefficient (see table 4.7) between $Re(\eta_{000})$ and $Im(\eta_{000})$ is 0.79. Assuming CPT symmetry, the measurement of $Im(\eta_{000})$ improves the current value [25] of the upper limit of the branching ratio of the $K_S^0 \rightarrow \pi^0\pi^0\pi^0$ decay by a factor two:

\[B_{K_S-3\pi^0} < 1.9 \times 10^{-5}\]

at 90% confidence level.

A limiting factor in the sensitivity of the measurement of $\eta_{000}$ is the lifetime resolution. The lifetime resolution is mainly determined by the excellent spatial resolution of the shower-position determination of the calorimeter. It is limited, however, by the wrong $\gamma\gamma$ combinations forming $\pi^0$'s, which is mainly caused by the moderate energy resolution of the electromagnetic calorimeter. The shower-angle information does not improve the photon pairing and thus improves only very slightly the lifetime resolution.

Looking for a possible CPT-violation, we use the unitarity relations and recent results on the parameters of the neutral-kaon system as given in table 5.1 to determine the CPT-violation parameter $Im(\delta)$ which is found to be:

\[Im(\delta) = (1.3 \pm 3.6) \cdot 10^{-5}.\]

Our determination of $\eta_{000}$ has improved the upper limit on the neutral-kaon mass difference by $\sim 30\%$ and yields:

\[|M_{K^0} - M_{\overline{K}^0}| < 10.2 \cdot 10^{-19} \text{GeV}/c^2\]

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at 90% confidence level. This shows the power of the neutral-kaon system which, strongly constrained by the unitarity relations, is the best microscopic laboratory for testing the CPT symmetry.

Assuming CPT-invariance in the decays, we have found an upper limit for the neutral-kaon mass-difference which is only a factor of twelve away from the inverse of the Planck mass $M_{pl} = \sqrt{\hbar c/G_N} \simeq 1.2 \cdot 10^{19} GeV/c^2$. This is a strong experimental verification of the CPT-theorem. CP-violation in weak interactions can be understood as a result of CPT-conservation and T-violation.
Appendix A

Neutral-kaon decay amplitudes

A.1 Two-pion decays

Considering neutral-kaon decays in final states with two pions, we have

\[ A_{L,2\pi}^* \cdot A_{S,2\pi} = \langle 2\pi|V|K_L\rangle^* \cdot \langle 2\pi|V|K_S\rangle \]

\[ = \left( \frac{\langle 2\pi|V|K_L\rangle}{\langle 2\pi|V|K_S\rangle} \right)^* \cdot \left| \frac{\langle 2\pi|V|K_S\rangle}{\langle 2\pi|V|K_S\rangle} \right|^2 \]

\[ = \eta_{2\pi}^* \cdot \Gamma_S \cdot Br(K_S \rightarrow 2\pi) \]

\[ = \Gamma_S \cdot Br(K_S \rightarrow 2\pi) \cdot [Re(\eta_{2\pi}) - iIm(\eta_{2\pi})] \]

using relations 2.23 or 2.33. Therefore, for the two-pion final states \( \pi^+\pi^- \) and \( \pi^0\pi^0 \), we obtain

\[ A_{L,\pi^+\pi^-}^* \cdot A_{S,\pi^+\pi^-} = \Gamma_S \cdot Br(K_S \rightarrow \pi^+\pi^-) \cdot [Re(\eta_{\pi+\pi}) - iIm(\eta_{\pi+\pi})] \] (A.2)

\[ A_{L,\pi^0\pi^0}^* \cdot A_{S,\pi^0\pi^0} = \Gamma_S \cdot Br(K_S \rightarrow \pi^0\pi^0) \cdot [Re(\eta_{\pi^0\pi^0}) - iIm(\eta_{\pi^0\pi^0})]. \]

A.2 Three-pion decays

Considering neutral-kaon decays in final states with three pions, we have

\[ A_{L,3\pi}^* \cdot A_{S,3\pi} = \langle 3\pi|V|K_L\rangle^* \cdot \langle 3\pi|V|K_S\rangle \]

\[ = \eta_{3\pi}^* \cdot \Gamma_L \cdot Br(K_L \rightarrow 3\pi) \]

using relations 2.39 or 2.38. Therefore, for the three-pion final states \( \pi^+\pi^-\pi^0 \) and \( \pi^0\pi^0\pi^0 \), we obtain

\[ A_{L,\pi^+\pi^-\pi^0}^* \cdot A_{S,\pi^+\pi^-\pi^0} = \Gamma_L \cdot Br(K_L \rightarrow \pi^+\pi^-\pi^0) \cdot [Re(\eta_{\pi^+\pi^-\pi^0}) + iIm(\eta_{\pi^+\pi^-\pi^0})] \] (A.4)

\[ A_{L,\pi^0\pi^0\pi^0}^* \cdot A_{S,\pi^0\pi^0\pi^0} = \Gamma_L \cdot Br(K_L \rightarrow \pi^0\pi^0\pi^0) \cdot [Re(\eta_{\pi^0\pi^0\pi^0}) + iIm(\eta_{\pi^0\pi^0\pi^0})]. \]
A.3 Semileptonic decays

Considering semileptonic decays of neutral kaons, we take into account contributions of positive and negative leptons:

\[ A_{L,f}^* \cdot A_{S,l} = (A_{L,f}^* \cdot A_{S,l+}) + (A_{L,f}^* \cdot A_{S,l-}) \quad (A.5) \]

where:

\[ A_{L,S,l^\pm} = \langle l^\pm \pi^0 | V | K_0 \rangle. \quad (A.6) \]

Using equations of relation 2.12, we have

\[ A_{S,l+} = \frac{1}{\sqrt{2}} [(1 + e_T + \delta) \langle l^+ \pi^- \nu | V | K_0^0 \rangle + (1 - e_T - \delta) e^{-i\phi_T} \langle l^+ \pi^- \nu | V | K_0^0 \rangle] \quad (A.7) \]

\[ A_{L,l+}^* = \frac{1}{\sqrt{2}} [(1 + e_T^* - \delta^*) \langle l^+ \pi^- \nu | V | K_0^0 \rangle^* - (1 - e_T^* + \delta^*) e^{-i\phi_T} \langle l^+ \pi^- \nu | V | K_0^0 \rangle^*]. \]

Using equation 2.52, 2.53, A.7 and neglecting second-order contributions of CPT-violating and \( \Delta S = \Delta Q \) violating terms, we obtain

\[ A_{L,l+}^* \cdot A_{S,l+} = \frac{|\langle l^+ \pi^- \nu | V | K_0^0 \rangle|^2}{2} \times \]

\[ [(1 + 2Re(\frac{b}{a})) \cdot (1 - (\delta^* - \delta) + (e_T^* + e_T)) + x - x^* \]

\[ \simeq \Gamma_L Br(K_L \to l^+ \pi^- \nu) \cdot [1 - 2Re(y) + i2Im(x) + i2Im(\delta) + 2Re(e_T)]. \]

By a similar way, we have

\[ A_{L,l-}^* \cdot A_{S,l-} \simeq \Gamma_L Br(K_L \to l^- \pi^+ \nu) \cdot [-1 - 2Re(y) + i2Im(\bar{x}) + i2Im(\delta) + 2Re(e_T)]. \quad (A.9) \]

Using equations A.5, A.8, A.9 and having \( Br(K_L \to l^+ \pi^- \nu) = Br(K_L \to l^- \pi^+ \nu) = \frac{1}{2} Br(K_L \to l\pi\nu) \), we finally obtain

\[ A_{L,f}^* \cdot A_{S,l} = 2\Gamma_L Br(K_L \to l\pi\nu) \cdot [Re(e_T) - Re(y) + iM(x_+) + iM(\delta)]. \quad (A.10) \]
Appendix B

Photon Matching

For matching the six generated photons with the reconstructed photons, we first consider the conversion point of the generated photon (see figure B.1).

\[
\delta r(r) = R_{\text{shower}} \delta \phi
\]  \hspace{1cm} (B.1)

where \( R_{\text{shower}} \) is the reconstructed shower-foot radius and \( \delta \phi \) is the angle between the reconstructed shower-foot and the generated conversion point in the transversal plane. The

Figure B.1: Conversion point of the generated photon and reconstructed shower.
distance in three dimensions is then given by:

\[ \delta_i = \sqrt{(\delta r \phi)_i^2 + (\delta z)_i^2} \]  

(B.2)

where \( \delta z \) is the longitudinal distance between the conversion point of the generated photon and the shower-foot of the reconstructed photon. Then, we can compute the sum of all distances between six generated and six reconstructed photons:

\[ \Delta = \sum_{i=1}^{6} \delta_i. \]  

(B.3)

For six generated photons, we have \( 6! = 720 \) possibilities to combine generated and reconstructed photons; among these 720 pairing possibilities, the one giving the smallest \( \Delta \) corresponds to the right combination.

In order to appreciate the quality of this matching-method, we can consider the distributions of the transversal and longitudinal distances between the 6 generated photons and their matched reconstructed photons:

![Graphs showing the distributions of transversal and longitudinal distances between 6 generated photons and their matched reconstructed photons.](image)

Figure B.2: 3\( \pi^0 \) simulated data. Transversal (full line) and longitudinal (dashed lines) distance between the 6 generated photons and their matched reconstructed photons.

One can observe that the widths are very comparable (\( \sigma_{r \phi} \approx 3 \text{mm} \) and \( \sigma_z \approx 4 \text{mm} \)) to those of the calorimeter simulation study given in section 3.4.
Appendix C

Possibility of a 13C-fit

In the event-reconstruction procedure, we can also use the information of the shower angles measured in the calorimeter. For each photon, we can consider a unitary vector of flight $\mathbf{U}_{fl}$ and a unitary vector of the shower-direction $\mathbf{U}_{sh}$ as shown in figure C.1. The flight-vector $\mathbf{U}_{fl}$ is a function of the shower-foot and the secondary vertex iterated in the 7C-fit, while the shower-direction vector $\mathbf{U}_{sh}$ is a function of the shower-angles $\Phi$ and $\Theta$ in the transversal and the longitudinal plane. We can reconstruct one event with six additional constraints as compared to equation 4.8: for all six photons, we demand the collinearity between their flight...
directions and their shower directions:

$$c_{s+i} = \frac{\overline{U}_{sh}^i \cdot \overline{U}_{sh}^i - 1}{\sigma_{\phi}^j \cdot \sigma_{\phi}^j}$$  \hspace{1cm} \text{(C.1)}$$

for $i = 1, 6$. This is a fit with 14 constraints and one unknown, namely a $13C$-fit.

In this section, we will first study the performance of the $13C$-fit in terms of signal acceptance and background rejection and then we will study its lifetime resolution.

### C.1 The acceptance of signal

When applying the $13C$-fit on simulated signal data, many of the events never converge while all converge in the $7C$-fit. This leads us to study the shower angle information.

Let us consider, for simulated signal data, the reconstructed shower direction vector $\overline{U}_{sh}^i$ and the true flight direction $\overline{U}_{true}^i$. The first vector is a function of the reconstructed shower-angles ($\phi, \theta$), their respective errors ($\sigma_{\phi}, \sigma_{\theta}$) and the shower-foot coordinates; the second is a function of the secondary vertex as given by the simulation, and the shower-foot coordinates as measured in the calorimeter. Therefore, a measure of the angle between $\overline{U}_{sh}^i$ and $\overline{U}_{true}^i$ is a very good approximation of the difference between the generated and reconstructed shower-angles $\phi$ and $\theta$. In figure C.2 are represented these angular differences divided by the corresponding errors $\sigma_{\phi}$ and $\sigma_{\theta}$; these pull-distributions are fitted by two gaussian functions. One can see that these distributions have tails which are fitted by gaussian functions of $\sigma = 1.9$ and $2.2$; these tails represent photons for which there is a big difference between the generated and the reconstructed shower angles in comparison to the corresponding error. In fact, they show that some showers are well developed in a very different direction compared to the incoming photon. Thus, it is natural to see that the collinearity constraint will not be fulfilled for these photons. In table C.1 are represented the amount of events for which the $13C$-fit has diverged, as a function of the upper limit of standard deviations of the reconstructed angle from the generated angle of the shower in the transversal plane, this for all six showers. One can clearly see the correlation between the proportion of events for which the $13C$-fit diverges and the number of well reconstructed photons.

Because of the loss of signal events (25%), we have to take into account a loss of significance for both real and imaginary part of $\eta_{\phi,0}$:

$$\sigma_{\eta}^{13C} = \sigma_{\eta}^{7C} \cdot 1.15. \hspace{1cm} \text{(C.2)}$$
C.2. THE SIGNAL/BACKGROUND RATIO

Because of the sensitivity of the 13C-fit to the shower-angle information, we can expect a good rejection of background channels where the flight-vector $\vec{U}_{fl}$ and shower-vector $\vec{U}_{sh}^{k}$ are highly non-collinear. In fact, this is what we see when comparing the signal/$2\pi^0$-background rejection of the 7C- and the 13C-fit (table C.2). The signal/background ratio is 34% better for
Table C.2: Simulated data. Amount of remaining signal and $2\pi^0 + 2\gamma_{sec.}$ background after different cuts within the 7C- and the 13C-fit framework.

<table>
<thead>
<tr>
<th>Prob(C-fit) &gt; 0</th>
<th>7C-fit</th>
<th>13C-fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>98%</td>
<td>92%</td>
<td>75%</td>
</tr>
<tr>
<td>Analysis cuts</td>
<td>58%</td>
<td>14%</td>
</tr>
</tbody>
</table>

The 13C-fit than for the 7C-fit, but the 13C-fit looses 25% of the signal. All the key variables of the analysis ($\text{Prob}(\chi^2_F)$, Invariant-mass($6\gamma$) and $m_{\gamma\gamma}^{\text{min}}$) having very similar distributions when coming from a 7C- or a 13C-fit, the signal/background ratio improves by the same factor when applying the analysis cuts (table C.2).

Due to the improved background rejection by the 13C-fit, the signal contributes at a level of 58% in the decay-time interval $[-1, 20]\tau_S$, to be compared with 50% for the case of the 7C-fit; according to the feasibility study (section 4.2), we can thus expect a better significance for the determination of $Re(\eta_{000})$ and $Im(\eta_{000})$:

$$\sigma^{13C}_{Re(\eta)} = \frac{\sigma^{7C}_{Re(\eta)}}{1.11}$$  \hspace{1cm} (C.3)$$

$$\sigma^{13C}_{Im(\eta)} = \frac{\sigma^{7C}_{Im(\eta)}}{1.10}.$$  

### C.3 Decay-time resolution of the 13C-fit

The resolution of the 13C- and the 7C-fit are shown in figure C.3 and their characteristics compared in table C.3. Despite the fact that the secondary vertex is determined with the additional constraints of shower angles, we do not note a great change in the core of the decay-time resolution ($[-5, 5]\tau_S$); however, we observe a sensible reduction of the tails (see values of RMS in table C.3).

In order to understand this limited amelioration, let us first consider a lifetime $\tau_A$ determined by the shower angles $\Phi_i$ and their error $\sigma_{\Phi_i}$: for each event, we consider the intersection of the neutral-kaon flight-line (determined by $K^\pm$ and $\pi^\pm$) and the direction of the shower.
Figure C.3: $3\pi^0$ simulated data. Lifetime resolution of the 13C-fit (solid line) and of the 7C-fit (dashed line).

$i$ in the transversal plane (defined by the angle $\Phi$ and the shower-foot coordinates) and we project this point on the neutral-kaon flight-line in full space; this results in a lifetime $\tau_i$ and its error $\sigma_i$. We then define the angle-lifetime $\tau_A$ as the weighted mean of all $\tau_i$'s. The lifetime resolution of the 13C-fit (see fig. C.4) can be reproduced when combining the lifetime given by the 7C-fit $\tau_{7C}$ and the lifetime determined using the shower angle informations $\tau_A$. This is due to the error of $\tau_{7C}$ which is much smaller than the error of $\tau_A$, making the weighted lifetime quasi-totally dominated by the 7C-fit information.

Even if the amelioration of the lifetime resolution is not very important with the 13C-fit, fitting the asymmetry $A_{000}$ with its lifetime resolution improves the significance on $Re(\eta_{000})$ and $Im(\eta_{000})$ by:

\[
\sigma_{Re(\eta)}^{13C} = \sigma_{Re(\eta)}^{7C} / 1.11 \\
\sigma_{Im(\eta)}^{13C} = \sigma_{Im(\eta)}^{7C} / 1.10.
\]

C.4 The overall effect of the 13C-fit

Taking into account the effect of the 13C-fit on the signal reduction (eq. C.2), on the signal/background ratio (eq. C.3) and on the decay-time resolution (eq. C.4), one expects the error of $Re(\eta_{000})$ and $Im(\eta_{000})$ are respectively reduced by only 7% and 5%.
Figure C.4: $3\pi^0$ simulated data. Lifetime resolution of the $13C$-fit (solid line) and of the lifetime obtained by the weighted mean of $\tau_{7C}$ and $\tau_A$ (dashed line).
Appendix D

Photon-energy related variables

We compute the invariant mass of the six photons $M(6\gamma)$ by the following way:

$$M(6\gamma) = \sqrt{\sum_{i\neq j} m_{ij}^2} = \sqrt{\sum_{i\neq j} 2E_i E_j (1 - \vec{U}_i \cdot \vec{U}_j)}$$

where $m_{ij}$ is the invariant mass of the photon-pair $ij$. $\vec{U}_i$ is the unitary vector giving the direction of the photon $i$; it is computed using the secondary vertex given by the highest probability of the 7C-fit, the shower-foot coordinates of the photon $i$ and the energy of this photon as given by the 7C-fit; thus, the vector $\vec{U}_i$ is calculated from variables given by the 7C-fit. The energy $E_i$ of the photon $i$ is the one measured by the calorimeter and not returned by the 7C-fit because even if the energies are different for $2\pi^0$ and $3\pi^0$ photons (see figure D.1.a), the 7C-fit "forces" the output energies to be like the energies of $3\pi^0$ photons (see figure D.1.b).

The $\gamma\gamma$-invariant mass for the $ij$ couple of photons is computed by the following way:

$$M_{ij} = \sqrt{E_{ij}^2 - P_{ij}^2}$$

where $E_{ij} = E_{i}^{7C} + E_{j}^{7C}$ is the total energy of the $ij$ pair and $P_{ij}^{x,y,z} = P_{i}^{x,y,z} + P_{j}^{x,y,z}$ are the corresponding momenta computed using the same variables than used for $\vec{U}_i$. 
Figure D.1: Simulated data. Mean energy before (a) and after (b) 7C-fit for $3\pi^0 \rightarrow 6\gamma$ photons (solid line) and for $2\pi^0 \rightarrow 4\gamma$ photons (dashed line)
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