Doctoral Thesis

Fall velocity and shape of snowflakes

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Fall Velocity and Shape of Snowflakes

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presented by
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Abstract

The fall velocity of precipitation particles was measured with the 2D–Video–Distro-meter built by Joanneum Research (Schönhuber et al., 1994). The instrument records images produced by the shading of photodiode arrays by particles. This method had proven to be successful for raindrops. Some adjustments had to be made to obtain snow data of sufficient quality.

With the help of these data, the shape parameters of snow aggregates were identified which influence the fall velocity. This was done by theoretical considerations about the drag force and by statistical analysis of the data. The results of theory and experiment agree very well: the two shape parameters with the largest influence on the velocity are the vertical extension of the snow aggregate (H) and the ratio of the cross-sectional area to the circumscribed area (α). The circumscribed area which is opposed to the fall (A · B) also has a significant influence. The parameters of the equation describing the velocity \( v = a_1 \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \) are investigated. The variation in the data not explained by this equation is expected to be mainly caused by the varying density of the aggregates.

In a next step, equations of this form were fitted to the data of precipitation events with aggregates of different crystal types and different degrees of riming. Crystal type turned out to be of minor importance. The degree of riming, however, changes the fitted equation considerably by changing the relative importance of the two parameters H and α. An improved determination of the fitted coefficients can be obtained by transforming H and (A · B) into two parameters which are less correlated. Equations are given which relate these parameters to the velocity of hydrometeors. The missing parameter in the description of a snow aggregate, its density, can be estimated from the measured data. The variability of the density for large aggregates turns out to be lower than for small aggregates.

There is a wide field of applications for the methods developed in this work. Relevant hydrometeor parameters can be determined for an individual snowflake: size, shape, density and velocity. Depending on the objectives of the research, these individual snowflake data can be used or fitted values can be applied to classes of particles. Yielding high quality input data to models simulating the precipitation evolution will have the highest priority at the moment. These models may then help to improve the interpretation of radar measurements.
Zusammenfassung

Die Fallgeschwindigkeit von Niederschlagspartikeln wurde mit dem 2D-Video-Distrometer der Firma Joanneum Research bestimmt (Schönhubcr et al., 1994). Dieses Messgerät zeichnet Bilder auf, die durch die Abschattung eines Photodiodenarrays durch die Niederschlagspartikel entstehen. Die Methode hatte sich für Regentropfen als erfolgreich erwiesen. Um auch im Schneefall Daten von genügender Qualität zu erhalten, mussten einige Anpassungen vorgenommen werden.

Mit Hilfe dieser Daten wurden Formparameter von Schneeaggregaten bestimmt, die ihre Fallgeschwindigkeit beeinflussen. Zum einen wurden theoretische Überlegungen zur Reibungskraft angestellt, zum anderen wurden die Daten mit statistischen Methoden analysiert. Die Ergebnisse aus Theorie und Experiment stimmen gut überein: die zwei Formparameter mit dem größten Einfluss auf die Geschwindigkeit sind die vertikale Ausdehnung des Schneeaggregates (H) und das Verhältnis von Schattenfläche zu umschriebener Fläche (α). Weiterhin hat die umschriebene Fläche in Fallrichtung (A · B) einen signifikanten Einfluss. Die Parameter der Gleichung zur Beschreibung der Geschwindigkeit \(v = a_1 \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d\) wurden untersucht. Die Variabilität der Daten, die durch diese Gleichung nicht erklärt werden kann, ist wohl hauptsächlich durch die Dichteunterschiede zwischen den Aggregaten verursacht.


Chapter 1

Introduction

1.1 Motivation

Precipitation is a weather phenomenon of high interest to the public. Though exceptional events like storms accompanied by hail damages or flooding caused by long lasting precipitation attract the most attention, the less spectacular events are also of great interest. Hydrologists would like to get accurate information on rain amounts and agriculture depends on precipitation as well as the plans people make for their spare time. Switzerland lies within a climatic zone where precipitation even in summer starts its development as ice crystals which melt on their way to the ground. Commonly, precipitation is monitored by weather radars. These instruments often yield data for a height where the particles are not yet melted. To obtain information on the precipitation reaching the ground the microphysical development of the particles on their way down has to be known. An important parameter is the fall velocity of the hydrometeors. It determines the number of particles present in the measuring volume. Relationships between the velocity and the size of hydrometeors are necessary for understanding parameters like rain rates or Doppler spectra (Atlas et al., 1973). Velocity–size relations are also used for the determination of particle sizes (Heymsfield, 1972) and the deduction of vertical air velocities (Heymsfield, 1978) from Doppler radar measurements. Some methods to deduce air velocity are sensitive to errors in hydrometeor velocity (Matejka and Bartels, 1998).

Not only the interpretation of measurements is facilitated by accurate velocity data, they are also important for the modeling of microphysical processes. These processes interact with the atmospheric dynamics and influence large-scale processes. It is, therefore, important for numerical simulations in all scales to adequately describe the processes of hydrometeor growth.

Important processes in ice crystal growth are riming and aggregation. Rimming is the growth by freezing of supercooled water droplets onto the surface of the crystal. Velocity is one of the input parameters in riming modeling (Heymsfield and Kajikawa, 1987) because the rime growth rate is critically dependent on fall speed (Lew et al., 1986). Aggregation occurs when two crystals collide and stick together. As for the riming, the collision rate is dependent on the relative velocity of the two particles. The development of precipitation in cold clouds can, therefore, be better understood with
the help of precise velocity data. The same is valid for raindrops developing in warm clouds because drops also grow by collisions.

The methods of simulating clouds and precipitation have undergone a rapid development in recent years. Some examples for this development are listed below:

- Rutledge and Hobbs (1983) described a parameterized numerical model of the growth processes which can lead to the "seed–feeder" process. This model can be used to predict radar reflectivities and rainfall rates. One of the input parameters is the mass–weighted fall speed which the authors calculate from a velocity–size relation given by Locatelli and Hobbs (1974, see Section 1.2.1).

- Yokoyama and Tanaka (1984) discussed the possibility for two–wavelength radar to detect microphysical processes in the melting layer. Their work is based on model calculations using the fall velocity relations given by Langlæben (1954, see Section 1.2.1).

- Mitchell (1988) developed a snow growth model based on the stochastic collection equation and calculated the aggregation efficiency between two levels in a cloud. He also used the velocity–size relations of Locatelli and Hobbs (1974). Later, he extended this work by admitting the deviation of hydrometeor size spectra from the exponential form (Mitchell, 1991).

- Hardaker et al. (1995) used a dynamic microphysical model to obtain vertical reflectivity profiles. These model profiles were applied to correct for the radar bright band. The velocity of a snowflake was derived from the velocity of the raindrop to which it melts by assuming the increase of the velocity to be proportional to the decrease of the diameter.

- Szyrmer (1996) introduced a model of the melting layer in which dynamic, thermodynamic and microphysical processes are fully coupled and used it to simulate air velocities around the melting layer. In this work, a fixed velocity–size function is used whose derivation is not given.

Hydrometeor velocities are important input parameters in these models. The used velocity–size functions, however, do not distinguish between different particle types. Partly, better information on hydrometeor velocities is available. The present knowledge on velocities is given in the next section. There are two ways to approach the question: by measurement (Section 1.2.1, 1.2.2) and by calculation (Section 1.2.3).

The variability of the phenomenon, however, turned out to be high and a more accurate description will be needed which has to take into account other snowflake parameters than the size only. The work done in this thesis aims at closing this gap.
1.2 Current knowledge

1.2.1 Experimental data

The amount and reliability of available velocity data vary for different types of hydrometeors. In most cases, experimental results are described by relations of the form \( v = a \cdot D^b \), with \( v \) the terminal fall velocity, \( D \) the diameter of the hydrometeor and \( a, b \) fit parameters. The power law relationship is often chosen because it facilitates analytical solutions in models, e.g. for Doppler spectra calculations (Atlas et al., 1973).

Raindrop terminal velocities were determined already decades ago by measuring natural raindrops (Gunn and Kinzer, 1949) and by the means of wind funnel experiments (Beard and Pruppacher, 1969). These data are widely accepted as accurate representations of drop fall velocities.

The interest in hail has been large because of its potential destructivity. Consequently, a lot of research has been done on graupel and hail. Velocity was for example investigated by Roos and Carte (1973), Heymsfield (1978), Matson and Huggins (1980) and Knight and Heymsfield (1983).

Much work has also been done on ice crystal velocity measurements (e.g. Jayaweera and Cottis, 1969; Zikmunda and Vali, 1972; Kajikawa, 1976, 1992; Kajikawa and Okuhara, 1997). The different ice crystal shapes exhibit a different fall behaviour. Therefore, separate relationships had to be found for the different types of crystals and degrees of riming.

Aggregates of crystals are the type of hydrometeors whose fall velocities are the least investigated. The available velocity-size relations were derived from less than 50 measured particles (Zikmunda, 1972; Locatelli and Hobbs, 1974) or are only valid for small aggregates, consisting of 2–6 crystals (Kajikawa, 1989). In radar studies, however, the larger aggregates play an important role as the energy backscattered by a particle depends on the sixth power of its diameter. Small sample sizes lead to the problem that correlations between velocity and size might be accidental (Locatelli and Hobbs, 1974). This difficulty is caused by technical problems arising when size and velocity of snow aggregates have to be measured simultaneously (Section 1.2.2).

The available information on fall speeds is even more scarce in the melting layer. Work on this subject was done by Mitra et al. (1990). These authors used a wind tunnel to analyse the fall velocity during melting for flakes of 10 mm and 5 mm diameter.

1.2.2 Measurement techniques

Most questions addressed in precipitation physics require the knowledge of more than one snow characteristic: size, velocity and shape of the particles as well as the size distribution within a given volume. Different techniques have been developed to measure these characteristics.

The first size distributions for snow were determined as melted particle sizes by means of filter papers (Ohtake, 1969). Currently, most size measuring systems are based on the shadowing of a light beam by the precipitating particles. These instruments register the occultation of a light beam by means of one photodiode (Hauser et al.,
1984) or are based on a linear array of photodiodes acting as a size measuring grid (Knollenberg, 1973).

The earliest measurements of snow velocity were made by means of a stopwatch while visually observing the falling particles (e.g. Nakaya and Terada, 1935), a method used until recently for snow studies (Mellor and Mellor, 1988). The ice crystal and snow velocity data used today were mainly obtained by photographic techniques with stroboscopic illumination (Langleben, 1954; Zikmunda and Vali, 1972; Kajikawa, 1992).

Another optical method was used by Locatelli and Hobbs (1974). They placed two light beams at a defined vertical distance and recorded the time difference between changes in the intensity of the two beams. Size was measured by these authors after catching the snowflakes on a plastic sheet. Sasvo and Matsuo (1980) measured the fall speeds of snowflakes in a similar manner, but determined size by photographing the falling particles from below. With the help of these two methods, velocity–size relations could be derived but not the size distribution in a given volume.

The easiest way to measure size distributions and velocity simultaneously is to use one optical instrument for both purposes. This solution was realized by using TV cameras (Muramoto et al., 1995) and light beam techniques (Hauser et al., 1984). The latter technique was also chosen by Donat Högl of the Institute for Atmospheric Science of the Swiss Federal Institute of Technology (ETH) who built a first version of a snow spectrometer in 1987 (Steiner, 1988). This instrument records shadow images of hydrometeors by means of a photodiode array illuminated by an incandescent lamp. It is a drawback of this technique that velocity is determined assuming equal vertical and horizontal extension of the particles. Consequently, errors occur in the velocity measurements of asymmetrical particles and the shape of the particles remains unknown. This problem can be solved by mounting a second optical system at a given vertical distance. This solution was chosen for two instruments:

- The existing ETH spectrometer was improved by mounting a second photodiode array parallel to the first one (Chapter 3).

- The measuring system of the 2D–Video–Distrometer, built by Joanneum Research (Schönhuber et al., 1994), consists of two cameras at an angle of 90 degrees (Chapter 2.1).

With these instruments, the fall velocity of the snowflakes can be determined and, hence, scaled images of the snowflakes can be derived. For the spectrometer, the two images of one snowflake show the same view whereas the 2D–Video–Distrometer yields views from two orthogonal directions (Schönhuber et al., 1995). These instruments are, therefore, able to give information on size distributions, velocity and shape of a large number of particles.

1.2.3 Calculation of velocity values

As an alternative to costly experiments, methods for the calculation of velocity values were developed. One common approach was originally derived by Beard (1976) for drops at very small Reynolds numbers. It is based on empirically obtained relationships between the Reynolds number and the Davies number (Davies, 1945).
1.2. CURRENT KNOWLEDGE

which can be used to calculate velocities. The Davies number has the advantage to
depend only on particle characteristics and on atmospheric parameters, but not on
speed or drag.

For ice crystals, the relationships between Reynolds number and Davies number
were found empirically in tank experiments (Jayaweera and Cottis, 1969) where the
fall of ice crystal models is observed in fluids of varying viscosity. List and Schemenauer
(1971) extended this type of laboratory experiment to graupel and small hailstones.

The applicability of the approach to other hydrometeors than raindrops
was then tested by comparing the calculated velocities with measured velocities. Jayaweera (1972) and Jayaweera and Ryan (1972) found satisfying agreement in the case
of plates and columns, but not for the more complicated stellar crystals. The same
test for graupel particles lead to a ratio of measured to calculated velocity of 0.6 to
1.8 (Heymsfield and Kajikawa, 1987). The Davies number – Reynolds number appro¬
ach has, therefore, limited value for particles with complex shapes or high Reynolds
numbers.

Another approach was introduced by Abraham (1970). He used some simplifica¬
tions to derive a functional dependence of the drag coefficient on the Reynolds
number. In his model, a rigid body together with its boundary layer forms one body
passing through an inviscid fluid. Originally, this model was developed for spheres.
Based on Abraham’s results, Böhm (1989) derived a functional dependence of velocity
on hydrometeor characteristics and atmospheric parameters valid for all types of solid
hydrometeors. In the case of snowflakes, however, he had to make assumptions about
their shape and the density of the individual crystals building up the flake. Therefore,
he assumed all aggregates to have the shape of oblate spheroids. To obtain better
results, this approach needs data on snowflake shape. Shape-independent formulas
were proposed by Mitchell (1996), based also on the boundary layer theory of Abra¬
ham (1970). The possibilities to compare his considerations to measured values were
scarce for snow aggregates. Furthermore, he was forced to make assumptions on the
orientation of the particles.

The calculation of the fall velocity of snow could be improved by using better data
on the shape, orientation and density of natural particles. The description of the
velocity of snow aggregates needs, therefore, field measurements of these parameters.

On the other hand, we may try to invert the problem. Theoretically derived equa¬
tions for the fall velocity can be solved for other parameters (e.g. the mass or density of
the particles). In Section 7.5, for example, the density is estimated for particles whose
velocity, size and shape were measured.

1.2.4 The role of hydrometeor size and shape

The influence of hydrometeor size on the velocity is widely accepted, not, however,
the definition of this size. The equivolumetric diameter is commonly used for drops
(Pruppacher and Klett, 1997) and, therefore, similar definitions were proposed for
snowflakes (Locatelli and Hobbs, 1974). Some authors use the melted diameter (e.g.
Langleben, 1954) which corresponds to the mass of the particle. But other definitions
were brought up also by several authors (Zikmunda, 1972; Heymsfield and Kajikawa,
1987; Kajikawa, 1992), without giving reasons for their choice. A unified definition, however, would facilitate comparisons of data (Castellano and Nasello, 1997). This study deals with the question which parameter is appropriate to be used as the size of a solid hydrometeor in velocity studies (Section 6.1).

Some studies on snow showed the influence of other hydrometeor characteristics than size and mass on the terminal velocity. For plates, Heymsfield (1987) stated that the ratio of crystal area to the area of the hexagonal disc with same diameter has importance. List and Schemenauer (1971) found (during their experiments on graupel) an empirical relationship between the drag coefficient and the ratio of the area opposed to the flow to the circumscribed area. Consequently, the velocity formula of Böhm (1989) includes this area ratio. The area ratio had not been measured for the particles Böhm used to validate his theory. Thus, he had to approximate it by other parameters. The measurement of the area ratio and the inclusion of the measured value in the velocity-size relation might considerably improve the description of the velocity distributions in models.

1.3 The contribution of this study

An effort is made to interpret experimentally determined velocities of snow aggregates. Relevant shape parameters for the description of the velocity are identified. The first part (Chapter 2 to 4) deals with the measurement techniques which had to be further developed to obtain the data and with the arrangement of the measurements. Part II (Chapter 5–7) describes the experimental results and puts them in a theoretical context. An expression for the terminal fall velocity of snow aggregates is presented and results for different types of snow are given.

Part I (Chapter 2–4)
The report starts presenting the instrumentation used for the measurements. Chapter 2 describes the 2D–Video–Dilometer built by Joanneum Research (Schönhuber et al., 1994) for rain measurements. In order to use it for snow, tests and software development were necessary. In Chapter 3, the ETH optical spectrometer is described. The old version of this instrument allowed an approximate determination of the fall velocity of hydrometeors. During this work, an improved version was developed which measures the velocity and, therefore, yields data of higher quality than the old instrument.

The measurements were part of a field campaign of the Institute for Atmospheric Science of the Swiss Federal Institute of Technology. The project was called Swiss Alpine Melting layer Measurements (SAMM). During two winters, precipitation and other meteorological measurements were carried out along mountain slopes in the Swiss Alps to investigate the vertical evolution of precipitation. The precipitation events yielding the data for the present study are described in Chapter 4 with the meteorological context necessary to classify the data.

Part II (Chapter 5–7)
As explained in Section 1.2, the fall velocity of a precipitation particle is influenced by its size and shape. In Chapter 5, the theory around relevant snowflake parameters is discussed. Some conclusions of Chapter 5 are validated in Chapter 6 by analysing the
experimental results. Those two chapters give the **theoretical and experimental justification for a velocity formula** based on the discussed snowflake parameters. Chapter 6 closes with an analysis of other influences (relevant snowflake parameters which were not measured, wind) on the velocity which might be responsible for the variations in the data not explained by the fitted functions.

Chapter 7 shows the **derived relationships** for aggregates of different types of crystals and degrees of riming. They are of direct use for the interpretation of radar measurements and for the simulation of microphysical processes in precipitation. An attempt is made to derive the density of individual snowflakes from the measured data. Thus, all snowflake parameters are made available which are necessary as input for precipitation models.
Chapter 2

The 2D–Video–Distrometer

2.1 Operating principle

A 2D–Video–Distrometer was lent to our group by Joanneum Research, Graz, Austria. A schematic drawing of the instrument can be seen in Figure 2.1 (Schönhuber et al., 1994). The sensor unit and the outdoor electronics unit are exposed to the weather at the measuring site. They are connected to the indoor user terminal with two cables: the power supply and the data transfer cable. At the indoor user terminal, the measurement can be observed in real time.

The operating principle of the sensor unit is shown in Figure 2.2 (Schönhuber et al., 1994). Two line-scan cameras are positioned at an angle of 90° to each other.
They are directed towards the openings of two illumination devices. The two light beams overlap below the opening of the sensor unit. Each particle passing through this virtual measuring area will be recorded as a shadow in both cameras. Since the vertical distance of the two light beams is known, it is possible to determine the fall velocity of the particles. The relative error is less than 5%.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal resolution</td>
<td>better than 0.22 mm</td>
</tr>
<tr>
<td>vertical resolution</td>
<td>better than 0.3 mm</td>
</tr>
<tr>
<td>(vertical velocity &lt; 10 m/s)</td>
<td></td>
</tr>
<tr>
<td>vertical velocity accuracy</td>
<td>better than 5 %</td>
</tr>
<tr>
<td>sampling area</td>
<td>approx. 100 x 100 mm</td>
</tr>
<tr>
<td>integration time</td>
<td>15 sec. to 24 hours</td>
</tr>
<tr>
<td>data rate</td>
<td>1 - 2 MB/mm rain (typ.)</td>
</tr>
<tr>
<td>mains voltage</td>
<td>110/220/240 V at 50/60 Hz</td>
</tr>
<tr>
<td>power consumption</td>
<td>500 W</td>
</tr>
<tr>
<td>length (SU with OEU mounted)</td>
<td>1500 mm</td>
</tr>
<tr>
<td>width</td>
<td>600 mm</td>
</tr>
<tr>
<td>height</td>
<td>1100 mm</td>
</tr>
<tr>
<td>weight SU + OEU</td>
<td>approx. 130 kg</td>
</tr>
<tr>
<td>weight IUT</td>
<td>approx. 60 kg</td>
</tr>
</tbody>
</table>

Table 2.1: Specifications of the 2D–Video–Distrometer
Figure 2.3: The 2D-Video-Distrometer at the measuring site on Mount Rigi. The large case is the sensor unit, the smaller one houses the Outdoor Electronics Unit. Behind the Outdoor Electronics Unit, an ultrasonic anemometer was mounted. It measured temperature and three-dimensional wind every minute. At right, a humidity sensor and a tipping bucket raingauge can be seen.
2.2 Data

2.2.1 Example pictures

![Figure 2.4: Snowflakes as recorded by the 2D-Video-Distrometer on December 14, 1996.](image)

The software of the 2D-Video-Distrometer had been developed for rain. In snow problems arose to match the images of the two cameras (different point of view of the two cameras), e.g. particles with branches were not accepted or divided into several snowflakes. Since snowflakes often develop branches and may have holes, the cutting of parts of the particles leads to intolerable errors in the further evaluation of the data. A new software was developed to recognize particles and accept them as a whole. This software uses the raw data as a basis for further processing. The raw data consist of a list of the shadowed pixels for every line. The line scan frequency of the two cameras is 34.1 kHz. If the velocity of a particle is known, its vertical extension can be
2.2. DATA

calculated. Figure 2.4 shows four examples of scaled snowflakes recorded by the 2D-Video-Distrometer and processed by the new software. The snowflakes with branches and holes are now accepted as a whole.

The pictures in the two cameras may differ substantially. For the fourth flake in Figure 2.4, for example, the shadowed areas are quite different. Therefore, the matching of two images is not easy. Section 2.3.2 describes how this difficulty is overcome.

It should also be noted in Figure 2.4 that the first and second flake fall with their largest axis oriented vertically. The expected fall orientation would correspond to a maximum projected area perpendicular to the direction of fall. Most single ice crystals of planar shape and most columnar ice crystals fulfill this expectation (Zikmunda and Vali, 1972). Large aggregates show a more complex fall behaviour (Figure 2.10). Zikmunda and Vali (1972) describe the influence of the particle orientation on the fall velocity of ice crystals. For snow aggregates, this influence is reflected by the dependence of the velocity on the horizontal extension of the particle (Sections 5.4 and 6.1).

2.2.2 Shape parameters

![Shape parameters A, B and H of snowflakes determined from the 2D-Video-Distrometer pictures.](image)

Figure 2.5: Shape parameters A, B and H of snowflakes determined from the 2D-Video-Distrometer pictures.

Figure 2.5 illustrates the shape parameters determined from the silhouettes of the precipitation particles. Each camera yields a width. The width is defined as the distance between the first and the rearmost pixel shadowed by the particle at any time during its passage. The two widths are named A and B. The height H is defined as the distance between the first and the last line in which at least one pixel is shadowed by the particle. Height H is equal in both cameras. Three more parameters are defined: The width W is the maximum of A and B. (The true maximum width cannot be
determined, as the third view of the particle – in the vertical direction – cannot be recorded.) The areas of the shadows in camera A and B are calculated ($M_A$ and $M_B$). The geometric mean of $M_A$ and $M_B$ is called $M$.

The ratio of the shaded areas $M_A$ and $M_B$ to the areas of the ellipses defined by the width and height in the corresponding camera is called $\alpha$. Here, the geometric mean of the two cameras is taken:

$$\alpha = \sqrt{\left(\frac{4 \cdot M_A}{H \cdot A \cdot \pi} \cdot \frac{4 \cdot M_B}{H \cdot B \cdot \pi}\right)}$$

### 2.3 The matching problem

#### 2.3.1 The cause of the problem

![Figure 2.6: Measuring area of the 2D-Video-Distrometer viewed from above.](image)

The scaled pictures shown in Sections 2.2.1 and 2.2.2 can only be produced for particles with known velocity. To determine the velocity of a particle, its corresponding images in the two cameras have to be found. The instrument records the two images from different points of view. Furthermore, there are particles seen by one camera only. This is illustrated in Figure 2.6. The cameras record particles passing within an area of 400 cm$^2$. The area which is seen by both cameras, however, is only 100 cm$^2$. Hence, only one out of four recorded hydrometeors is seen by both cameras. This is valid in still air only, with wind the percentage seen by both cameras becomes slightly smaller. Ambiguities arise when several particles of similar size and velocity are passing the
2.3. THE MATCHING PROBLEM

cameras at the same time. At first approximation, size distributions of hydrometeors obey a negative exponential law. As a consequence, a large number of small particles is recorded (Figure 2.7) and the risk of a wrong matching increases with decreasing particle size.

Figure 2.7: Size distribution of ice and snow particles measured by one camera of the 2D-Video-Distrometer, December 14, 1996, 1:00 – 1:15 pm, \(N(D)\) is the number per millimetre size interval and per cubic metre volume.

2.3.2 Matching criteria

To match raindrop images three selection criteria were used: the size of the particle, the fall velocity and the oblateness. The fall velocity should be close to the values given by Gunn and Kinzer (1949). The probability of the estimated oblateness is judged using the values of Pruppacher and Beard (1970).

The matching of snow particles is complicated because of their complex shapes. Criteria had to be defined to find the matching images for these particles. The extension of the hydrometeor in the vertical direction is the only measured parameter which is equal in both cameras. Therefore, it can be used as a criterion (see Section 2.2.2). A rather large interval of velocities has to be accepted, the velocity being the parameter to be determined.

A software was developed to test our matching criteria. A snow shower of 30 minutes duration was chosen for this test (14.12.96, 12:45 – 13:15).

a) Selection of the particles for the test

Images which touch the edge of the measuring area are excluded from this analysis because their exact shape and size cannot be determined. This leads to a decrease of the measuring area with increasing size of the particles. But this procedure improves the accuracy of the shape parameters for all analysed particles.

For the first test, only particles with a height of less than 100 lines were taken into account (corresponding to a time of 2.93 milliseconds). If the velocity of a snowflake is 1 m/s (1.5 m/s), its dimension in the vertical direction is 2.9 mm (4.4 mm). Particles
with diameters in this size range do not occur in large numbers within the time needed by both cameras to see a single particle falling (Section 2.3.1). Therefore, ambiguities are rare.

During the chosen time period, camera A recorded 1753, camera B 1806 particles. This difference could be explained by the statistical distribution of the particles (areas of the two cameras do not coincide, see Section 2.3.1). It might also be caused by the horizontal wind, blowing in the direction of view of camera A. At the edge of the measuring area of camera A, particles may be blocked by the top cover of the sensor unit.

If the geometry of the measuring areas is considered (see Figure 2.6), 10/25 of the particles recorded by one camera should fall through the common measuring area. 701 particles would then be seen by camera A, 722 by camera B. As these particles are seen by both cameras, different numbers of particles can only be explained by the influence of the wind. Particles falling through the common region in one camera can be carried by the wind and may miss the region seen by the second camera. The number of particles with two matched images should, therefore, be equal to or less than 701.

Matching images were searched applying the following criteria:
- The difference between the number of lines in the two cameras should not exceed a limit, here arbitrarily set to 11 per cent of the larger number.
- The velocity calculated from the two images should be more than 0.5 m/s and less than 10 m/s.

When these criteria were applied, matching was successful for 603 particles. Eight of them were ambiguously matched, i.e. for an image in the one camera, two or more images were found in the other camera. In order to eliminate ambiguities, the following selection criteria were applied:
- The difference in the height H between the two images is minimal.
- The calculated velocity is comparable to the velocity of similar sized particles which were successfully matched.

Applying these criteria, only one ambiguity remained. There was no possibility to find out which of the two images in the one camera was produced by the particle seen by the other camera. This particle was excluded from the analysis.

b) Results of the test
The images of the two cameras can differ substantially (see Section 2.2.1). Nevertheless, the size of the images has to be used to find corresponding images. Therefore, the 602 particles which could be matched were analysed (for the definition of the parameters see Section 2.2.2).

Ratio of the two widths A and B
First, the difference between the two measured widths was determined (Figure 2.8a). The knowledge of this difference is important to judge the results of one camera or if both pictures are taken from one side, e.g. by the optical spectrometer described in Chapter 3. For about one fourth of the analysed particles, the two widths differ by no more than 10 %. 10 % of the particles have a difference of more than 50 % between the two widths. For only 1 % of the particles, the width in camera B is more than
two times the width in camera A. The horizontal orientation of the particles should be random. It will be rare that the cameras A and B measure the major and the minor axis of the particles. However, the number of analysed particles is high enough to be representative and to conclude that there will only be a negligible number of particles with differences between the minor and major axis larger than 120%. To avoid the influence of corrupted lines a minimum value for the ratio of the smaller to the larger width is set. A ratio of 0.1 was chosen because this is the smallest ratio determined from the Formvar plates (Section 4.1). It was found for single needles.

**Ratio of the measured heights $H_A$ and $H_B$.**

Figure 2.8b shows the difference between the heights measured by the two cameras. The height is the only dimension of the particle seen by both cameras and, therefore, suitable to be used as a matching criterion. The two cameras should measure almost equal heights. Slight differences can occur caused by the threshold value which has to be achieved to mark a pixel as shaded. The figure shows a maximum measured difference of 11% of the larger height. This percentage was chosen for the limit of particles to be accepted. Probably even larger differences can be found, but only 0.7% of the particles
show a difference in height of more than 10 % of the height whereas the difference is less than 5 % for more than 80 % of the particles. The threshold of 11 %, therefore, seems to be reasonable for particles of average size. This threshold may be too low for small particles and too high for large particles, Therefore, a graduated threshold for the difference between the two measured heights was implemented (Figure 2.9). It depends on the number of lines shaded by the particle. Up to 20 lines, the maximum difference is 3 lines. From 20 to 45 lines, the maximum difference changes linearly from 15 % to 11 % of the number of lines. For 45 to 182 lines, the maximum difference is 11 %. For even larger particles, the threshold is fixed at 20 lines.

![Figure 2.9: The maximum accepted difference between the two measured particle heights in dependence on the number of lines shaded by the particle. For illustration, the corresponding particle height in mm for a velocity of 1.5 m/s is given.](image)

Ratio of width to height

The matching program accepts an upper and lower threshold for the oblateness $\epsilon$ (the ratio of width to height, see page 111) of a particle. These thresholds should not exclude real particles. The histogram in Figure 2.10 shows the frequency distribution of the ratio of maximum width $W$ to height $H$ for the 602 particles used in the test. Values are between 0.3 and 2.6.

As for the ratio of the two measured widths, the thresholds were set to 0.1 and 10 because this ratio was the maximum found by analysing the Formvar plates.

The histogram (Figure 2.10) also reflects the orientation of the particles. If the orientations would be randomly distributed, the mean ratio of width to height should be 1.0. Obviously, the mean value is higher. Both, mean and median value are 1.25, thus indicating that the snowflakes tend to fall with their major axis oriented horizontally. But there is also a considerable number of snowflakes not obeying this rule (Figure 2.4). Furthermore, the mass of some snowflakes may not be evenly distributed. Neither mass nor mass distribution can be determined by the measurements made during this study.

As described before, only particles were accepted which exceeded a minimum vertical extension. A sample which contains all particles would, therefore, have a higher mean value of this ratio. The tendency of the snowflakes to fall with their major axis
2.3. THE MATCHING PROBLEM

![Graph showing frequency distribution of the ratio of maximum width W to height H. The mean value is 1.25, indicating the tendency of the particles to fall with their major axis in horizontal orientation.](image)

Figure 2.10: Frequency distribution of the ratio of maximum width W to height H. The mean value is 1.25, indicating the tendency of the particles to fall with their major axis in horizontal orientation.

oriented horizontally is even larger than concluded from Figure 2.10.

**Velocity range**
The last threshold which had to be set was the velocity range. If a wide range of velocities is accepted, the time window increases which is searched for matching particles in the second camera. This increases the risk of wrong matching, especially if the lower velocity threshold is set too low. Additionally, a smaller range of accepted velocities accelerates computation. For snowflakes, the range is 0.2 m/s to 4.0 m/s. For graupel, the upper limit is set to 8.0 m/s.

**Number of particles lost due to the matching criteria**
The maximum ratios of A to B and W to H (Section 2.2.2) which are applied as selection criteria were chosen according to the maximum axes ratios found on the Formvar plates (Section 4.1). These plates, however, are too small to yield a representative sample of the particles present during a precipitation event. It is likely that particles with smaller ratios occur in precipitation events where single needles are recorded. The number of particles which will be excluded from the analysis can be influenced by setting the two thresholds for width to height and for width A to width B.

The particle width W (Section 2.2.2) resolved by one pixel is about 0.2 mm. If a selection criterion $W/H < 0.1$ is applied, the maximum allowed vertical axis will be 2 mm. If the particle is slightly larger, it will be excluded as long as its deviation from the vertical is small enough not to shade a second pixel. The shading of a second pixel would occur at an angle of $\gamma = \sin^{-1}\left(\frac{0.2}{2}\right) = 13^\circ$ (compare Figure 2.11). If the particle is falling in a horizontal orientation, the same consideration is valid because particles which are thinner than 0.2 mm will not produce enough shading to be recorded.

All particles deviating more than $\gamma$ from the vertical or horizontal orientation are recorded in spite of their extreme axes ratios (grey region in Figure 2.11). Particles at other orientations are rejected by the selection criteria (regions with stripes in Figure 2.11). If orientation is assumed to be randomly distributed, about $\frac{30}{100} \times 100\% = 30\%$ of the particles with extreme axes ratios will be lost.
This estimate is rather crude because the orientation of the particles is not randomly distributed. Columnar crystals are known to fall preferably with nearly horizontal orientations of their major axes. Zikmunda and Vali (1972) found that only particles with axes ratios up to 3 fall steadily in a horizontal orientation. Columnar crystals with larger axes ratios exhibit rotatory oscillations. According to the data of Zikmunda and Vali (1972), the angles are smaller than 15° for more than 80 % of the observed particles. 80 % of the particles with extreme axes ratios would then be excluded by the chosen matching criteria. On the other hand, Kajikawa found in 1976 that a single oscillating needle was deviated from the horizontal orientation by an angle of more than 13° for 68 % of the observing time. This would imply an exclusion of only 32 % of the particles with extreme axes ratios. These widely differing literature values show that it is not possible to give the correct number of particles lost due to the axes ratio thresholds.

These problems do only affect single crystals which are not in the center of interest in this work. Snowflakes consisting of several particles do not show extreme axes ratios (Figure 2.10). But if the interpretation of the results is extended to smaller sized particles these insecurities should always be kept in mind.

The following list gives an overview of all criteria which were applied in order to record every snowflake in the measuring area and to achieve correct matching:

- Images that touch the edge of the measuring area are excluded.
- The velocity of the particle has to be between 0.2 m/s and 4.0 m/s.
- The ratio of maximum width to height has to be between 0.1 and 10.
- The ratio of the smaller to the larger width has to be larger than 0.1.
2.3. THE MATCHING PROBLEM

- The accepted difference between the two measured heights is varied with the height of the particle (Figure 2.9):

<table>
<thead>
<tr>
<th>maximum accepted difference</th>
<th>height of particle in lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 lines</td>
<td>up to 20 lines</td>
</tr>
<tr>
<td>15 % to 11 % (linearly)</td>
<td>21 to 44 lines</td>
</tr>
<tr>
<td>11 %</td>
<td>45 to 181 lines</td>
</tr>
<tr>
<td>20 lines</td>
<td>182 or more lines</td>
</tr>
</tbody>
</table>
Chapter 3

The optical spectrometer

During the winter period 1997/98, measurements were made on a cable car. This setup provided the possibility to vary the measuring height above sea level. The use of a cable car required an instrument which was easy to transport and insensitive to movement. The Institute of Atmospheric Science built an instrument fulfilling these requirements in 1987 (Steiner, 1988; Barthazy et al., 1998). In its first version, the instrument was not able to measure velocity directly.

In this chapter, the first version of the instrument will be described and the error in velocity measurements is analysed with the help of 2D–Video–Distrometer data (Section 3.1). In Section 3.2 follows a description of the improved instrument used for the measurements in winter 1997/98, which yields a direct estimate of the velocity.

3.1 First version of the instrument

Description
An incandescent lamp illuminates a line of 256 photosensitive elements. The number of photo elements shaded by a precipitation particle is sampled at a frequency of 7300 Hertz. The measuring area is 37.23 mm x 100 mm. The horizontal resolution is 0.146 mm.

Cause of the error in the velocity measurement
One photodiode array can only see one dimension of the particle: its horizontal extension. Velocity cannot be determined directly. An estimate of the velocity is calculated from the line scan frequency and the maximum number of lines shaded by a given particle. The calculation is based on the assumption that the horizontal and vertical extension of the particle are equal.

Determination of the error
The 2D–Video–Distrometer yields the opportunity to determine the error produced by the assumption of spherical particles discussed before. For each particle, the true velocity determined from the displacement of the two cameras can be compared to the velocity which would have been determined by one camera only.

This analysis was made for the snowflakes whose images had been matched beyond doubt in Section 2.3.2. As mentioned there, the procedure used to search for the matching particles leads to an underestimation of the ratio of width to height for the
Figure 3.1: Size distribution of the matched particles (seen by both cameras; diamonds). \( N(D) \) is the number of particles per millimetre size interval and per cubic metre volume. Asterisks show the size distribution from single camera data. From 5 mm size on, the matched particles show a similar negative exponential behaviour, i.e. all particles with sizes of 5 mm or more are matched.

Figure 3.2: Analysis of the error in velocity of the old spectrometer. Crosses mark the velocity difference for single particles if their exact velocities are subtracted from the velocities estimated by one camera assuming spherical particles. The dotted line indicates the mean value for all analysed particles. The solid line is fitted to the data: \( \delta v = 0.034 \frac{m}{mm^3} \cdot \text{width} \). Only particles larger than 5 mm were used because below this size not all particles were matched (Figure 3.1).
particles with small widths. If these particles are taken into account the estimate of the error will be biased. This problem can be avoided by restricting the analysis to particle sizes for which all particles could be matched.

Size distributions of snow particles show approximately a negative exponential behaviour (Figure 2.7). The size distribution of the test particles (Figure 3.1) starts to show this behaviour at a width of about 5 mm. It is, therefore, assumed that all particles with a width of 5 mm or more are matched. For these particles, the distribution of axes ratios should not be biased and, consequently, they can be used to assess the systematic error in the velocity measurement by the old spectrometer.

**Results**

If the velocity determined by two cameras is subtracted from the velocity determined by one camera, the differences range from -1.0 m/s to +1.5 m/s (Figure 3.2). The mean value for all particles is 0.17 m/s (dotted line in Figure 3.2). Hence, the old method tends to overestimate velocities. Clearly, the value of 0.17 m/s is too low for larger particles. A fit was made with the function \( \delta v = f \cdot \text{width} \). Figure 2.10 shows the tendency of the particles to fall with their major axis oriented horizontally. Therefore, the vertical extension of the particles is biased towards larger values by the instrument. Consequently, the calculated velocities are greater than the actual ones and \( \delta v \) should not fall below zero. Therefore, the function was chosen to include the origin of the coordinate system. The fitted line (solid line in Figure 3.2, \( f = 0.034 \frac{\text{m}}{\text{mm} \cdot \text{s}} \)) indicates an overestimation of the vertical velocity of 0.35 m/s for particles of 1 cm size. These particles had a mean velocity of 1.55 m/s, the relative error is \( \left( \frac{0.35}{1.55} \right) \cdot 100\% = 23\% \).

**Influence of the velocity error on the size distributions**

Figure 3.3 shows the size distributions measured by the 2D–Video–Dustrometer during three time intervals of 5 minutes. The distributions obtained with the new matching program are given by triangles. The size distributions of single camera data are plotted in asterisks. For the computation of the number of particles per cubic metre of air and per millimetre size interval (N(D)) it is necessary to know the fall speed of the hydrometeors. For single camera data, the assumption of equal height and width was made.

For small particle sizes, the N(D) calculated by single camera data is larger than the N(D) calculated from two cameras. This difference can be explained by matching problems leading to data loss. From about 4 mm size on, the deviation is no longer systematic to one direction. It is concluded that the calculation of size distributions from the measurements by a single camera yields similar results than the calculation by using a set of correctly matched particles.

Two features of Figure 3.3 even point to a higher data quality of single camera data: They are less scattered and reach to higher particle sizes. This higher data quality is achieved by a larger measuring area (250 cm²) for one camera than for two cameras (100 cm², Figure 2.6). In the case of the improved spectrometer (Section 3.2), this higher data quality of single camera data will not occur because this instrument has the same measuring area in its old and its improved version.
Figure 3.3: A comparison of size distributions of single camera data (asterisks) with size distributions obtained by matching the particles measured by two cameras (triangles). \( N(D) \) is the number of particles per 1 mm size interval per cubic metre volume. Data recorded by the 2D–Video–Distrometer on December 14, 1996. The size distributions obtained by single camera data do not differ significantly from the distributions obtained by two cameras.

Conclusion
Size distributions measured by the old spectrometer can be used without correction whereas velocity values should be corrected according to Figure 3.2.

3.2 Improved optical spectrometer

Description
The improved version of the spectrometer produces two parallel light beams at a defined vertical distance. Therefore, velocity can be measured with higher accuracy. The beams are shielded by a tube except for a gap near the light source. The sensor geometry can be seen in Figure 3.4. Two CCD line scan sensors containing 512 photosensitive elements record the shadows cast by the particles falling through the gap. An interface unit converts the video signals to 16 bit words which are stored on a standard PC after compression. The storage capacity makes it possible to record seven hours of heavy
3.2. IMPROVED OPTICAL SPECTROMETER

Figure 3.4: Sensor geometry of the improved optical spectrometer built by the Institute for Atmospheric Science.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of measuring planes</td>
<td>(76.75 ± 4.25) mm x 108.5 mm</td>
</tr>
<tr>
<td>Resolution</td>
<td>0.15 mm</td>
</tr>
<tr>
<td>Line scan rate</td>
<td>9470 Hz</td>
</tr>
<tr>
<td>Number of planes</td>
<td>2</td>
</tr>
<tr>
<td>Distance of planes</td>
<td>9.45 mm</td>
</tr>
<tr>
<td>Number of picture elements per line</td>
<td>512</td>
</tr>
<tr>
<td>Power consumption:</td>
<td></td>
</tr>
<tr>
<td>Sensor and interface unit</td>
<td>20 Watt</td>
</tr>
<tr>
<td>PC</td>
<td>100 Watt</td>
</tr>
<tr>
<td>Weight:</td>
<td></td>
</tr>
<tr>
<td>Sensor</td>
<td>12 kg</td>
</tr>
<tr>
<td>Interface unit</td>
<td>2.5 kg</td>
</tr>
<tr>
<td>PC</td>
<td>19 kg</td>
</tr>
<tr>
<td>Dimensions:</td>
<td></td>
</tr>
<tr>
<td>Sensor</td>
<td>1.40 m x 0.44 m x 0.37 m</td>
</tr>
<tr>
<td>Interface unit</td>
<td>0.22 m x 0.21 m x 0.12 m</td>
</tr>
</tbody>
</table>

Table 3.1: Specifications of the improved optical spectrometer.
Advantages of the improved spectrometer

1. Mobility
The instrument can be transported to a measuring location and mounted there by one person. Calibration after transportation and mounting is generally not necessary because the instrument is very stable. Therefore, it was the ideal tool to be mounted on the cable car during the measurements in the second winter period (Section 4.4).

2. Data quality
The velocity measurement by the improved spectrometer is as accurate as by the 2D–Video–Distrometer. The matching of the images of the two sensors, however, is much easier because both pictures are taken from the same side and can be matched using computerized pattern recognition. Compared to the 2D–Video–Distrometer, some shape information gets lost but, on the other hand, the matching will be better for small particles and, consequently, the determined size distributions are more reliable.
Chapter 4

The SAMM measurement campaign

The SAMM (Swiss Alpine Melting Layer Measurements) campaign was initiated to study microphysical processes within and above the melting layer. Data from experiments should be evaluated to gain an improved knowledge of these processes. The experimental data should, furthermore, be compared to a model of the melting layer of precipitation (Göke, 1999). This study presents a part of the evaluation of the experimental data gained during this campaign. Measurements were made during two winter periods. In this chapter, the methods of crystal classification are explained and the setup of the experiments is presented. Section 4.3 describes the precipitation events yielding the data for this study.

4.1 Crystal classification

The physical properties of snow aggregates depend on the types of crystals building up the aggregates and on their degree of riming. Therefore, it was necessary to describe and classify the crystals occurring during the precipitation events. Two methods were used for this purpose: photography and Formvar technique.

Particles for photographs were collected on a piece of black mohair wool to avoid damage on impact. Macrophotographs were taken immediately after collecting the particles at ambient temperatures.

The Formvar technique is described by Schaefer (1941). Polyvinyl formal resin (Formvar 15/95) is dissolved in ethylene chloride (1,2 Dichloroethan). The solution is kept below 0 ° Celsius and brought onto glass slides shortly before exposing the slides to the precipitation. The area where particles are captured has a size of approximately 50 mm x 25 mm. After the exposure time (usually some seconds), the slides are placed in a box containing silica gel. The solvent evaporates and the snowflake is preserved within a shell of resin. The glass slides can later be examined under a microscope.

A coarse classification of crystals was used as shown in Table 4.1. The scheme proposed by Magono and Lee (1966) seemed too diversified for this purpose.

The degree of riming was determined according to Mosimann et al., 1994. Rime is a deposit of white rough ice crystals which forms when a frozen particle comes into contact with supercooled water droplets. By examining the crystals by eye, the rime coverage of the crystal's surface area is expressed on a scale ranging from 0 to 5.
Table 4.1: *Classification of crystal shapes.*

Table 4.2 lists the evaluation criteria applied to every single crystal. The bulk degree of riming was then calculated as the mean value for all crystals on one glass slide.

<table>
<thead>
<tr>
<th>R</th>
<th>degree of riming</th>
<th>planar crystals</th>
<th>needlelike crystals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>unrimed</td>
<td>no rime on surface</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>lightly rimed</td>
<td>a maximum of 25 % of the crystal’s surface is covered with rime</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>moderately rimed</td>
<td>edges and branches covered with rime, inner part of crystal only lightly rimed or unrimed</td>
<td>rime coverage of surface is around 50 %</td>
</tr>
<tr>
<td>3</td>
<td>densely rimed</td>
<td>whole crystal surface covered with one layer of rime; crystal shape can still be identified at first glance</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>heavily rimed</td>
<td>multiple layers of rime</td>
<td>original crystal shape barely recognizable</td>
</tr>
<tr>
<td>5</td>
<td>graupel-like</td>
<td>all kind of graupel</td>
<td></td>
</tr>
</tbody>
</table>
Waldvogel disdrometer (Joss and Waldvogel, 1967) and meteorological instruments measuring wind, temperature, pressure and humidity. The X-Band radar is mobile and mounted on a van. It is vertically pointing and has a wavelength of 3.2 cm. Every 30 seconds, a full Doppler spectrum is measured. The vertical range resolution was 50 m, the velocity resolution was 0.125 m/s for this measurement campaign. A description of the radar is given by Mosimann et al. (1993).

The X-Band radar was used to classify the precipitation as stratiform or convective. The criterion for stratiform precipitation was the appearance of a zone of high reflectivity marking the melting layer. This zone cannot be seen in convective precipitation. A good example of a descending melting layer is shown in Figure 4.2. There, temperature dropped and consequently, the melting layer descended. The descent of the melting layer can also be seen in the plot of the Doppler velocity (Figure 4.3). The fall velocity of the hydrometeors increases sharply when they are melting to raindrops. At 10:00 am, raindrops can be identified in the lower part of the profile. Later, the melting layer is too low to be seen by the radar.

If a precipitation event was classified as stratiform, the radar was also used to identify the position in the melting layer for the instruments at the other stations. This is especially important for the description of changes of physical properties during the melting process.

At the top station, the 2D–Video–Distrometer (Section 2.1) recorded the precipitation particles and, at the same time, photographs and Formvar replica were taken to observe particle type and the degree of riming defined in Section 4.1. The top station...
Figure 4.2: X-Band Doppler radar, February 15, 1997. The zone of high reflectivity marking the melting layer (dark band in the picture) descends between 10:00 and 11:30 local time.

Figure 4.3: X-Band Doppler radar, February 15, 1997.
was equipped with an ultrasonic anemometer measuring temperature and wind, and with a heated tipping bucket raingauge (Figure 2.3).

The development of the precipitation system was observed by the C-Band weather radar of the ETH in Zurich (Li et al., 1995), about 40 km from the measuring site. This radar has a wavelength of 5.3 cm. Data are transformed to a Cartesian coordinate system of 1 km x 1 km grid size. In this study, the plan position indicator (PPI) is used. It shows the horizontal distribution of the radar reflectivity in a circle around the radar at a given time. PPI pictures were available every ten minutes. An example can be seen in Figure 4.5.

Figure 4.4: Weather chart. December 14th, 1996, 1:00 pm (redrawn from Berliner Wetterkarte). The cross marks the location of Mount Rigi.

4.3 Precipitation events in winter 96/97

The general weather situations leading to an interesting precipitation event were situations with westerly or northwesterly wind which were prevailingly cyclonic in middle Europe. Fronts approaching the Alps in such a situation are often stationary, because
they are impeded to move further to the South. Thus, a large amount of precipitation develops at the northern slope of the Alps.

4.3.1 December 14, 1996

On December 14, 1996, a cold front passed Mount Rigi around noon. The meteorological situation can be seen in the weather chart of that day at 1:00 pm (Fig. 4.4). During the day, several snow showers passed by, moving from West to East.

This precipitation event was the first event measured throughout the SAMM measurement campaign. The methods used to characterize the individual precipitation events are illustrated with the shower passing Mount Rigi at 1:00 pm. The same methods were used for the other events but only the results will be mentioned briefly.

The shower at 1:00 pm was chosen for further analysis because the PPI pictures measured by the C-Band radar in Zurich indicated that the shape and intensity of the shower did not change much with time. As an example, the PPI picture at 12:50 am is shown in Figure 4.5.

The sequence of PPI pictures can be used to observe the direction and velocity of movement of the shower. In this case, the shower was moving from West to East. As the instrument setup also follows a line from West to East, the measurements of the different instruments can be compared by calculating a time shift. For this purpose, the velocity of the shower was determined by comparing the position of highest

![Diagram](image-url)
reflectivity at several times. Furthermore, the time of the highest reflectivity in the X–Band measurements at the height of the 2D–Video–Distrometer was compared to the time of the highest reflectivity near the ground and also to the time where the 2D–Video–Distrometer recorded the highest number of large particles (> 5 mm). All three methods led to the same result: the time of the radar measurement had to be shifted five minutes to compare radar data and 2D–Video–Distrometer data.

The height–time–indicator (HTI) measured by the X–Band radar at the bottom station is shown in Figure 4.6. The height of the 2D–Video–Distrometer is marked by a horizontal line. The 2D–Video–Distrometer was near the upper edge of the bright band, the temperature at its location being about 0 °C. The strong slant of the shower features is caused by the movement of the precipitation cell. Time in this picture is shifted five minutes to correspond to the time of the 2D–Video–Distrometer measurements.

The size distributions of hydrometeors for the time from 1:00 to 1:15 pm are shown in Figure 3.3. The precipitation consisted of needles, aggregates of needles and some dendrites. The particles were lightly rimed ($R = 0.7$).

Two other showers on this day were chosen for further analyses: From 5:36 to 5:43 am and from 8:00 to 8:15 am. Unfortunately, the X–band radar did not operate before 8:15 am. The two events were chosen with the help of the PPI–pictures of the C–band radar, the Formvar plates and by personal observations at the measuring site. Formvar and photographs showed dendrites and aggregates of dendrites with a degree of riming of 1.0 around 5:40 am and needles and aggregates of needles with a degree of riming of 0.4 between 8:00 and 8:15 am. Temperature was -1 and -0.5 °C, respectively.
4.3.2 February 5, 1997

From February 2 until February 10, the weather in central Europe was dominated by westerly winds. The event is an exceptional case as the situation was prevailing anticyclonic and most fronts were blocked by a high pressure area reaching from the Azores via the Alps to the Ukraine. The front on February 5, 1997, was the only one which could reach the measuring site. It drove the high towards Southeast and brought humid air from the Northwest.

The precipitation on that day was rather variable. The plan position indicators from the ETH C-Band radar do not show a defined band of precipitation but rather a large precipitation region. This region did not move into a definite direction but the intensity within the region changed with location and time. Therefore, the calculation of a time shift for a comparison of the radar and 2D-Video-Distrometer data as described in Section 4.3.1 was not possible. Consequently, the analysed time interval was not chosen with the help of the radar data but with the help of the Formvar plates, photographs and the log by searching for an interval with unchanging crystal type and degree of riming.

Between 5:10 and 5:18 pm rather large loose flakes were observed. The largest photographed particle was 10.5 mm big. The aggregates were built up by dendrites and needles, their degree of riming being 1.2.

Temperature during this measurement interval was steadily below $-6^\circ C$ (Fig.4.8).
4.3.3 March 28, 1997

Beginning March 26, a West wind situation dominated the weather processes in middle Europe. The weather chart of March 28 shows two high pressure regions: one from the Azores to Great Britain, the other over Scandinavia. An intermediate low was wandering West to East through Northern Europe. At the rear of this low, the cold air could advance South. Temperature dropped and the cold front approaching the Alps caused precipitation and even thunderstorms.

Figure 4.9 shows the reflectivity factor and the Doppler velocity measured from 3:30 to 4:30 pm. Unfortunately, a lightning hit the data transmission cable of the 2D–Video–Distrometer at 3:50 pm. C–band measurements with the ETH–radar were not made during this day. The C–band radar pictures of the Swiss Meteorological Institute were used instead. They show the arrival of a thunderstorm from Northwest. The storm stopped moving at 3:40 pm, when it reached Mt. Rigi, then it slowly died. The photographs show graupel with a size of up to 1 cm. Most of them are round, but some have a conical shape. The smaller particles are heavily rimed and crystal shape is not discernible for most of them with the exception of some densely rimed needles. Temperature at the 2D–Video–Distrometer was −2 °C.

4.4 The setup in winter 97/98

The fixed measuring sites in the first winter made it almost impossible to observe the changes in precipitation during melting. Only one case occurred where the melting layer was above the 2D–Video–Distrometer when precipitation started and then dropped below the top station rapidly. This event was accompanied by strong vertical winds which made an analysis of the fall velocities impossible. To have a higher chance of measurements within the melting layer, it was decided to place instruments on a cable railway in Linthal during the second winter (Figure 4.10). The mobile Doppler radar was placed at 800 m above sea level where the railway starts which reaches a height of
Figure 4.9: X-Band Doppler radar, March 28, 1997.
1700 m. Hence, it was possible to choose the height of the particle measurements with respect to the melting layer. It was also possible to pass through the melting layer within a small horizontal distance and to observe the changes in size distributions, velocities and shapes of the hydrometeors.

The particle measuring system on the railway was the improved version of the optical spectrometer described in Section 3.2. The data of the second winter are not included in this work.

Figure 4.10: Setup for the winter season 97/98.
Chapter 5

Theory

A snowflake falling in still air can be regarded as a particle moving through a fluid. Therefore, this chapter starts with a short physical description of moving fluids and the different types of flow. These types of flow are linked to the frictional force acting on the falling snowflakes. Different formulations for the frictional force are considered and inserted into the equation of motion of the snowflakes. This equation is then used to derive the shape parameters of the falling particles in order to find rules for determining the fall velocity. The theoretical conclusions made in this chapter will be verified by analysing experimental data in Chapter 6.

5.1 Description of fluid motion

![Diagram of fluid motion](image)

Figure 5.1: Motion of a volume element within a fluid.

\( \eta \): viscosity of the fluid, \( \rho \): density of the fluid, \( p \): pressure, \( u \): velocity of the volume element.

The motion of a volume element within continuous matter (Fig. 5.1) is described by the Navier–Stokes–equation which is derived from the Newtonian equation of motion. The acceleration \( \mathbf{a}_v \) of the volume element depends on the sum of the forces \( F \) acting on the volume element, divided by the mass \( m_v \) of the volume element: \( \mathbf{a}_v = (\sum F)/m_v. \)
The involved forces are:

- volume forces, i.e. external forces which are proportional to the volume (mass),
- forces caused by pressure differences and
- frictional forces.

If these forces are not balanced, the volume element is accelerated. The acceleration can be caused by a time dependence of the velocity at the location of the volume element or by moving to another location with different velocity. In stationary flow, only the second possibility applies. In the following discussion, stationary flow will be assumed. The Navier–Stokes–equation can then be written in terms of forces per unit volume:

\[ \rho_a \cdot \mathbf{a}_a = -\nabla p + \eta \Delta \mathbf{u} \quad (5.1) \]

with \( \rho_a \) and \( \eta \) the density and viscosity of the fluid respectively, \( p \) pressure, and \( \mathbf{u} \) the velocity of the flow.

In some cases, one of the forces can be neglected. If there is no frictional force, the fluid is regarded as ideal. If the flow is built by non–mixing layers of fluid, the acceleration caused by a change of location can be neglected and we call it laminar flow. In contrast, turbulent flow is given if the inertia term \( \rho_a \cdot \mathbf{a}_a \) influences the motion more than the frictional force. The following section describes how the type of flow can be specified.

### 5.2 Types of flow

As described above, laminar and turbulent flow differ by the relative importance of the inertia term \( \rho_a \cdot \mathbf{a}_a \) and the frictional term \( \eta \Delta \mathbf{u} \) in the Navier–Stokes–equation (5.1). The order of magnitude of these terms is estimated by regarding the distances over which the velocity changes significantly caused by the forces. Significantly in this case means that the change in \( u \) is in the order of \( u \). The change in \( u \) caused by the inertia term will take place over the distance \( d_1 \). The volume element needs the time \( t = d_1 / u \) to overcome this distance. Its acceleration is then \( a_a = u/t \approx u^2/d_1 \). Similarly, the distance \( d_2 \) for a significant change of velocity caused by friction, is estimated.

Two cases are then distinguished:

a) \( \eta \cdot u/d_2^2 \ll \rho_a \cdot u^2/d_1 \)

This case describes the ideal fluid without friction as well as turbulent flow if friction is negligible.

b) \( \rho_a \cdot u^2/d_1 \ll \eta \cdot u/d_2^2 \)

Inertia can be neglected. In this case, we have laminar flow and \( \nabla p = \eta \Delta \mathbf{u} \).

The transition between the states a) and b) takes place when the ratio \( \rho_a u d_1^2 / \eta d_2 \) is approximately 1. If the distances \( d_1 \) and \( d_2 \) are both set equal to \( d_0 \) we get the Reynolds number \( Re \):

\[ Re = \frac{\rho_a \cdot u \cdot d_0}{\eta} \quad (5.2) \]
5.2. TYPES OF FLOW

5.2.1 Reynolds numbers of snowflakes

The Reynolds numbers of the particles analysed in this study can be estimated if we insert dynamic viscosity and density of the air into the equation 5.2. The dynamic viscosity depends only on the temperature. It is described by the function $\eta = (\beta \cdot T^{3/2})/(T + Z)$, with $T$ the temperature in Kelvin, $Z$ a constant (110.4 °K) and $\beta = 1.458 \cdot 10^{-6} \text{kg/(m \cdot s)}$ (US Standard Atmosphere Supplements, 1966).

All measurements were made at temperatures between -6°C and +3°C. This temperature range corresponds to a range in viscosity from $1.687 \cdot 10^{-5}$ kg/(m s) to $1.730 \cdot 10^{-5}$ kg/(m s). At a given air density of $\rho_a = 1.011 \text{ kg/m}^3$ (1600 m above sea level), this leads to Reynolds numbers of 877 to 899 for a particle with D = 1 cm and a velocity of 1.5 m/s. For a smaller particle (0.1 cm) with a velocity of 1 m/s the resulting Reynolds numbers are 58 and 60. It can be concluded that Re is not significantly affected by the change of the dynamic viscosity of the air.

Air density variation has not to be accounted for because all data were gathered at the same height (1600 m above sea level). Hence, the main reason for differences in the Reynolds numbers is the range of particle sizes and velocities. The transition from laminar to turbulent flow might occur within this size range. This transition is treated in Section 5.3.4.
5.3 Frictional force on a body moving through a fluid

The frictional force on a body moving through a fluid can theoretically be calculated by solving the Navier–Stokes equation (Equation 5.1) for a given problem. This is only possible for some simple cases. Therefore, several concepts have been proposed to describe the frictional force. Each of these concepts is exactly valid only for a special case depending on the relative importance of the acting forces, i.e. on the type of flow (Section 5.2). In reality, the frictional force will lie somewhere in between these special cases. The equations for the frictional force derived for the special cases will be described in this section and their dependence on the shape parameters of the regarded particles will be investigated.

5.3.1 Stokes friction

If a sphere with radius $r$ is moving through a still fluid, the fluid layers very close to the sphere will stick to the surface of the sphere and move with its velocity. At a distance $d \sim r$ from the surface of the body, the velocity of the fluid is zero. The velocity gradient is then $dv/dz \sim v/r$. Between the layers of the fluid a viscous tension $\sigma = dv/dz$ is acting. The frictional force onto the surface $S = 4\pi r^2$ is then

$$F_{RS} \approx -\eta \frac{dv}{dz} \approx -4\pi \eta vr.$$ 

For this flow problem, the Navier–Stokes equation can be solved with neglect of the inertia term. Turbulence in this case does not exist. The solution (Lüst, 1978) is then

$$F_{RS} = -6\pi \eta vr. \tag{5.3}$$

The Stokes friction does only apply in laminar flow (case a in section 5.2), i.e. for small bodies falling at low velocities.
5.3. FRICTIONAL FORCE ON A BODY MOVING THROUGH A FLUID

Figure 5.4: A body with cross-section $A_{eff}$ moving with velocity $v$ through a fluid for the time interval $dt$ replaces the fluid in the column $A_{eff} v \, dt$.

5.3.2 Newton friction

Newton made the following consideration: If a body moves through a fluid of density $\rho_a$, it has to replace the fluid particles. These particles must be accelerated to a velocity approximately equal to the velocity $v$ of the body. During the time interval $dt$, the fluid has to be replaced within a column of length $v \, dt$ and cross-sectional area $A_{eff}$ of the body (Fig. 5.4). The kinetic energy which is necessary to accelerate the mass of this column ($m = \rho_a A_{eff} v dt$) is $E_{kin} = 1/2 \rho_a A_{eff} v^3 dt$. A power of $1/2 \rho_a A_{eff} v^3$ has to be supplied. The power corresponds to the frictional force multiplied by the velocity. Thus, the frictional force is

$$F_{RN} = 1/2 C_D \rho_a A_{eff} v^2.$$  \hspace{1cm} (5.4)

The drag coefficient $C_D$ depends on the shape of the body, the roughness of its surface, the Reynolds number and the degree of turbulence in the flow. It is approximately 1 for a sphere, smaller for streamlined bodies and larger for bodies with an aerodynamically unfavourable shape.

Combining the Newton and the Stokes formulation for a sphere with radius $r$ yields an approximation for the drag coefficient $C_D$ of a sphere in laminar flow.

$$1/2 C_D \rho_a r^2 \pi v^2 = 6 \pi \eta v \tau \Rightarrow C_D = \frac{12 \eta}{\rho_a v \tau} = \frac{24}{Re}.$$  \hspace{1cm} (5.5)

The Newton formulation can yield incorrect values if bodies with an aerodynamically favourable shape do not require the fluid particles to be accelerated to the velocity of the body. Smaller velocities may then be sufficient and consequently the frictional force will be lower.

5.3.3 Laminar boundary layer

Prandtl introduced the concept of the laminar boundary layer which sticks to every body moving through a fluid. Within this layer velocity changes gradually from the body's velocity to the velocity of the fluid. The velocity gradient can be assumed to
Figure 5.5: Velocity within the laminar boundary layer of a body moving through a fluid. $u$: velocity of the fluid, $v$: velocity of the body, $\delta$: thickness of the boundary layer, $z$: distance from the centre $r_0$ of the body.

be a linear function of $z$ if the thickness $\delta$ of the boundary layer is small compared to the body's size. The frictional force $F_{RP}$ is proportional to the velocity gradient $v/\delta$, the surface $S$ of the body, and the viscosity $\eta$ of the fluid.

$$F_{RP} = \frac{\eta Sv}{\delta} \quad (5.6)$$

The work $W_m$ to move the body by his own length $l$ is $W_m = F_{RP} \cdot l = \eta Slv/\delta$. With the energy set free, a new boundary layer is built with the surface $S$ and the kinetic energy $W_{kin} = 1/2 \int_0^S S\rho u dz(v^2/\delta)^2 = 1/6 \cdot S\rho u^2 v^2\delta$. By setting $W_m$ and $W_{kin}$ equal we may calculate the thickness of the boundary layer $\delta = \sqrt{(6\eta l)/(\rho uv)}$. Equation 5.6 becomes

$$F_{RP} = S\sqrt{\frac{\eta \rho u v^3}{6l}} \quad (5.7)$$

The main difficulty for the application of this formula to snowflakes is the complicated shape of these particles. The surface $S$ of snowflakes has indentations and the whole body is porous. Consequently, the boundary layer is also complicated.

### 5.3.4 Connection between the different concepts

The Stokes equation (5.3) can be regarded as a special case of equation 5.6 if the thickness of the boundary layer around a sphere is of the order of the radius of the sphere. The Newton formulation is the other extreme case with negligible boundary layer. Generally, the Stokes formulation yields too small values by excluding any turbulence and the Newton formulation yields too high values because it does not take into account the streamlined shape of a body. Between Newton and Stokes friction, there is a continuous transition. Equation 5.7 is one special case within this transition zone. This connection between the three concepts can be seen if all measures of length in the equations 5.3, 5.4 and 5.7 are set equal to a typical dimension $D$ of the particle. ($D^2 = r^2 = A = l^2 = S$). Stokes friction is then proportional to $vD\eta$ and Newton friction to $v^3D^2\rho_u$. $F_{RP}$ in equation 5.7 is proportional to $\sqrt{\eta v^3 D^3 \rho_u}$ which is the geometric mean of the other two expressions.
5.3. \textbf{FRICIONAL FORCE ON A BODY MOVING THROUGH A FLUID}

\begin{center}
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=\textwidth/2.5,
axis lines=center,
xlabel={Reynolds number},
ylabel={Drag coefficient},
grid=both,
]
\addplot[black,solid] coordinates {
(0.1,1000)
(1,100)
(10,10)
(100,1)
(1000,0.1)
(10000,0.01)
(100000,0.001)
};
\addplot[black,dashed] coordinates {
(0.1,1000)
(1,100)
(10,10)
(100,1)
(1000,0.1)
(10000,0.01)
(100000,0.001)
};
\addplot[black,dashdotted] coordinates {
(0.1,1000)
(1,100)
(10,10)
(100,1)
(1000,0.1)
(10000,0.01)
(100000,0.001)
};
\node at (axis cs:8,5) {$C_D = 24/\text{Re}$};
\node at (axis cs:7,0.5) {$C_D = 0.6$};
\end{axis}
\end{tikzpicture}
\end{center}

Figure 5.6: Dependence of the drag coefficient $C_D$ on $Re$, derived by Böhm (1989) for solid precipitation particles. At small Reynolds numbers, the function roughly coincides with equation 5.5 for laminar flow. At high Reynolds numbers, it approaches a value of $C_D = 0.6$.

Figure 5.6 illustrates the continuous transition between Stokes and Newton friction. It shows the relationship between $C_D$ and $Re$ which was empirically derived by Böhm (1989): $C_D = 0.6\left(1 + \frac{5.83}{Re^{3/2}}\right)^2$. At low Reynolds numbers, the function roughly coincides with the equation 5.5 for laminar flow. At high Reynolds numbers, the function is an asymptote to a constant value of $C_D = 0.6$ which was found for hailstones (Macklin and Ludlam, 1961). The frictional forces acting on snowflakes are in the transition between Stokes and Newton friction. Hence, all three formulations will be considered when the shape parameters are derived which influence the velocity.

5.3.5 \textbf{Shape parameters determining the frictional force}

In the following considerations, the dependence of the frictional force on the measured shape parameters $A$, $B$, $H$, $M$ and $\alpha$ and on the velocity $v$ is shown for each of the three concepts:

- The formulation found for Stokes friction (equation 5.3) is only valid for a sphere. Friction depends then on the radius of the sphere. The equivalent measured parameter is $(ABH)^{1/3}$.

$$F_{RS} \sim (ABH)^{1/3}v$$ \hfill (5.8)

- For Newton friction (equation 5.4), the force depends on the cross-sectional area opposed to the flow. It is approximated by multiplying the area of the ellipse set up by $A$ and $B$ with the area ratio $\alpha$.

$$F_{RN} \sim AB\alpha v^2$$ \hfill (5.9)

- In the example chosen to represent the transition zone between Stokes and Newton (Section 5.3.3), friction depends on the surface $S$ of the body in the numerator.
(equation 5.7). This surface is assumed equal to the surface of the boundary layer and can, in a first approximation, be represented by the surface of the ellipsoid set up by A, B and H multiplied by α. It is then proportional to \( (ABH)^{2/3} \cdot α \). \( H^{1/2} \) is used as denominator because the length l a particle has to fall to replace the whole boundary layer is its dimension in the direction of movement.

\[
F_{RP} \sim A^{2/3} B^{2/3} H^{1/6} α \nu^{3/2}
\]  

(5.10)

5.4 Shape parameters and fall velocity

The formulations of the frictional force can be inserted into the equation of motion to estimate the influence of the shape parameters on the velocity. The equation of motion of a snowflake is set up by regarding the forces acting on a body falling in the gravitational field of the earth: gravity, buoyancy and friction.

\[
m \cdot \alpha = -ρ_s V g + ρ_a V g + F_R
\]  

(5.11)

\( m \) is the mass of the snowflake, \( \alpha \) its acceleration, \( V \) its volume and \( ρ_s \) its density. \( g \) is the acceleration by gravity, and \( ρ_a \) the density of the surrounding air. \( F_R \) is the frictional force.

When the snowflake has reached terminal fall velocity, we have

\[
g(ρ_s - ρ_a)V = F_R.
\]  

(5.12)

The left side of the equation depends on the volume and density of the snowflake. The volume is proportional to \( ABH \), but generally not of ellipsoidal shape. \( α \) describes the difference of the actual cross-sectional view of the particle from the ellipse set up by A (B) and H. It is valid for two-dimensional sections of the particle. The deviation of the three-dimensional volume from the ellipsoid set up by A, B and H can, therefore, be described by multiplication with \( α^{3/2} \) \((V \sim ABHα^{3/2})\).

Snowflake density \( ρ_s \) is also a parameter which influences velocity. \( α^{3/2} \) describes density partly by giving a hint to the amount of space filled by the snowflake. But the actual density is not adequately described by \( α^{3/2} \) as the internal structure of the snowflake is not known. Density may vary over a large range while the snowflake produces the same shadow image. As density cannot be measured with the available instruments no additional parameter to represent it is introduced. A first approximation is to regard snowflake density as a constant during the short time intervals analysed. As a result, the question is open to what extent the individual events have to be regarded separately or a unified description of fall velocity for all events can be found.

Caused by the limitations in describing the internal structure of the snowflakes, the considerations are restricted to the parameters representing the shape of the particles seen by the instrument. The important shape parameters determining the fall velocity can be found by regarding equation 5.12 and the relations 5.8, 5.9 and 5.10.
5.5. CONCLUSIONS

The following relations are derived:

<table>
<thead>
<tr>
<th></th>
<th>( ABH_\alpha^{3/2} )</th>
<th>( ABH_{\alpha^{3/2}} )</th>
<th>( AB\alpha^{3/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stokes</td>
<td>( \sim (ABH)^{1/3}v )</td>
<td>( \Rightarrow v \sim (A \cdot B)^{2/3}H^{2/3}\alpha^{3/2} )</td>
<td></td>
</tr>
<tr>
<td>Newton</td>
<td>( \sim A\beta^{3/2} )</td>
<td>( \Rightarrow v \sim H^{1/2}\alpha^{1/4} )</td>
<td></td>
</tr>
<tr>
<td>Transition</td>
<td>( ABH_\alpha^{3/2} )</td>
<td>( \sim A^{2/3}B^{2/3}H^{1/6}\alpha^{3/2} )</td>
<td>( \Rightarrow v \sim (A \cdot B)^{2/9}H^{5/9}\alpha^{1/3} )</td>
</tr>
</tbody>
</table>

If the Stokes formulation for the frictional force is valid, all three dimensions of the particle will be equivalent because only spheres are taken into consideration. If the Newton formulation is valid, the shape parameters determining velocity should be \( H \) and \( \alpha \). In the transition between these two extremes, the area opposed to the flow, represented by \( A \) and \( B \), also influences the velocity but the influence of \( H \) will be larger than the influence of the area exposed to the flow. This can be concluded because \( A \) and \( B \) get unimportant for higher Reynolds numbers. Hence, the extensions of a falling snowflake in the vertical and horizontal direction have a different influence on the motion of a snowflake.

5.5 Conclusions

Three conclusions can be drawn with respect to the type of flow determining the fall behaviour of a particle (Section 5.2):

1. If the inertial force is dominating, the velocity will depend on the vertical extension of the particle (\( H \)) and on the ratio of the effective area exposed to the flow to the circumscribed area (\( \alpha \)).

2. When the influence of the viscous force cannot be neglected, the velocity depends also on the area exposed to the flow (\( A \cdot B \)).

3. The dependence of the velocity on both \( A \cdot B \) and \( \alpha \) grows with growing influence of the viscous force. The dominance of \( H \) as measure of the particle size gets lost in the case of laminar flow.

The velocity function resulting from these considerations is \( v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \). The exponents \( b \), \( c \) and \( d \) will change in dependence on the frictional force. The values expected from the theoretical considerations above are:

<table>
<thead>
<tr>
<th>Shape parameter</th>
<th>( H )</th>
<th>( \alpha )</th>
<th>( (A \cdot B) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponent</td>
<td>( b )</td>
<td>( c )</td>
<td>( d )</td>
</tr>
<tr>
<td>Stokes</td>
<td>( 2/3 )</td>
<td>( 3/2 )</td>
<td>( 2/3 )</td>
</tr>
<tr>
<td>Newton</td>
<td>( 1/2 )</td>
<td>( 1/4 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Example for transition zone</td>
<td>( 5/9 )</td>
<td>( 1/3 )</td>
<td>( 2/9 )</td>
</tr>
</tbody>
</table>

As snowflakes are in the transition zone, the values of \( b \) and \( c \) should be between the values of Stokes and Newton friction. One example is given in the tabular above. If the velocity function is fitted to experimental data, it should theoretically even be possible to estimate the relative importance of the frictional and inertia force by regarding the fitted exponents.
In Chapter 6, functions of the form \( v = a \cdot X_1^b \cdot X_2 \cdot X_3^{\frac{1}{2}} \) are fitted to the measured data with varying snowflake parameters \( X_1, X_2, X_3 \) to investigate the correlation of the parameters with the velocity and their significance. In Chapter 7, the velocity functions fitted to the data of snow aggregates with different crystal types and degrees of riming will be compared. The change of the exponents of the shape parameters reflects the change in the relative importance of the forces.
Chapter 6
Experimental results

In Section 5.4, the parameters which determine the velocity of a snowflake were derived from the equation of motion. According to these considerations, the vertical extension of a particle H (Section 2.2.2) should be the size measure with the highest influence on the fall velocity. This conclusion will be tested in Section 6.1 by comparing the correlations of velocity with different size measures as described in the literature. From theory we expect two other parameters to influence the velocity: the ratio of shaded area to circumscribed area α and the circumscribed area, represented by the product of the two measured widths ($A \cdot B$) (Section 5.5). The commonly used velocity-size relation is, therefore, complemented by these additional parameters and their significance is tested in Section 6.2. These three parameters explain part of the variability in the data, but not the total variability. Other parameters influencing the velocity will, therefore, be discussed in Section 6.3. The statistical methods applied in this chapter are described in Appendix A.1.

6.1 Definition of the snowflake size

When comparing the published fall velocity-size distributions for snow we find different definitions for the size of a snowflake. This variety of definitions is not astonishing since aggregates of ice crystals have irregular shapes. The definition depends on the aim of a study. Here, the size parameter having the largest influence on the fall velocity is investigated. Contrary to popular believe, the theoretical considerations in Section 5.4 showed the extension of the particle in the direction of fall to be the dimension of the snowflake with the largest influence on velocity.

To test the theory, fit functions between measured fall velocities and different particle dimensions were compared. A fifteen-minute period of snowfall on the 14th December 1996 (1:00–1:15 pm), yielding 5616 particles, was selected for the analysis. The different dimensions $D$ were chosen considering the definitions commonly used in the literature (for the definition of the used shape parameters see Figure 2.2.2):

a) The width in camera A.

b) The width in camera B.
c) The maximum width $W$.

d) The height $H$.


f) The geometric mean of $W$ and $H$.

g) The diameter of the smallest circle totally containing the snowflake picture (Kajikawa, 1989). Here, the maximum value of the three parameters $A$, $B$, and $H$ was taken.

h) The diameter of the circle having the same area as the shadow, as proposed by Locatelli and Hobbs (1974). The area in this case was the geometric mean of the shadowed areas in both cameras.

The function $v(D) = aD^b$ was assumed to describe the relation between the velocity and the size. For the correlation analysis, the function was linearized to $\log(v) = \log(a) + b \cdot \log(D)$. The method of analysis is described in Appendix A.1.1. The coefficient of determination $R^2$ was used as an indicator to find the dimension having the best correlation to the velocity. It measures the proportion of total variation about the mean explained by the regression (Draper and Smith, 1966). Larger $R^2$ indicates a better fit (explains more of the variation in the data). The fitted equations for the different definitions of $D$, the residual standard deviations of the regression $s_r$ (equation A.2) and the coefficients of determination $R^2$ (equation A.3) are listed in Table 6.1. The tables in Appendix A.2 give a more comprehensive overview of the fit results and a characterization of the used data populations.

<table>
<thead>
<tr>
<th>Definition for $D$</th>
<th>fitted equation</th>
<th>$s_r$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) width $A$</td>
<td>$\log(v) = 0.046 + 0.099 \cdot \log(D)$</td>
<td>0.106</td>
<td>0.19</td>
</tr>
<tr>
<td>b) width $B$</td>
<td>$\log(v) = 0.049 + 0.086 \cdot \log(D)$</td>
<td>0.106</td>
<td>0.19</td>
</tr>
<tr>
<td>c) maximum width $W$</td>
<td>$\log(v) = 0.038 + 0.101 \cdot \log(D)$</td>
<td>0.107</td>
<td>0.18</td>
</tr>
<tr>
<td>d) height $H$</td>
<td>$\log(v) = 0.053 + 0.134 \cdot \log(D)$</td>
<td>0.094</td>
<td>0.36</td>
</tr>
<tr>
<td>e) $\frac{W+H}{2}$</td>
<td>$\log(v) = 0.042 + 0.125 \cdot \log(D)$</td>
<td>0.101</td>
<td>0.27</td>
</tr>
<tr>
<td>f) $\sqrt{W \cdot H}$</td>
<td>$\log(v) = 0.044 + 0.125 \cdot \log(D)$</td>
<td>0.100</td>
<td>0.28</td>
</tr>
<tr>
<td>g) maximum $(W, H)$</td>
<td>$\log(v) = 0.032 + 0.122 \cdot \log(D)$</td>
<td>0.103</td>
<td>0.24</td>
</tr>
<tr>
<td>h) $\sqrt[\pi]{\frac{A}{M}}$</td>
<td>$\log(v) = 0.053 + 0.131 \cdot \log(D)$</td>
<td>0.099</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 6.1: Comparison of the different definitions for the size of a snowflake. Data from December 14, 1996, 1:00–1:15. The size definitions $D$ are given in mm, the velocity in m/s. $s_r$ is the standard deviation of $\log(v)$ from the fitted function $\log(v) = a + b \cdot \log(D)$, $R^2$ the coefficient of determination (Appendix A.1.1). The number of analysed particles is 5616.
6.1. DEFINITION OF THE SNOWFLAKE SIZE

The higher \( R^2 \)-value and the smaller standard deviation \( s_p \) for the fit with \( H \) compared to the other definitions support the conclusion of Chapter 5: The vertical extension of a particle has a larger influence on its velocity than the horizontal extensions. The \( p \)-value of the \( t \)-test (see Appendix A.1.3) is nearly zero for all the fits, thus proving the statistical significance of the influence of size on velocity for each of the size definitions.

![Figure 6.1: Fitted velocity-size relationship for different definitions of the size of a snowflake (Table 6.1).](image)

Figure 6.1 visualizes the regression equations (of the form \( v = a \cdot D^b \)) for the velocity \( v \), for the different size definitions \( D \). The curve for the fit with \( H \) is best correlated. There is no difference between using the width of the shadow recorded by one camera (\( A \) or \( B \)) or the maximum width of the shadows recorded by both cameras (\( W \)) in the fit. But all of them yield regression parameters which are quite different from the ones fitted to \( H \). Vertical and horizontal dimension are not equivalent. If these dimensions can be determined, the vertical extension \( H \) can be chosen to represent the size. It can be concluded that information on the orientation of the particle during fall improves the velocity-size relations derived from measured data. The original definitions of Zikmunda (1972), Kajikawa (1989) and Locatelli and Hobbs (1974) referred to photographs of snowflakes which had been caught on a soft medium. The orientation of the falling particle could, therefore, not be measured. However, the function obtained in this work for the size as defined by Locatelli and Hobbs (1974) is similar to the one obtained with \( H \). The definition of Locatelli and Hobbs (1974) should, therefore, be chosen in cases where information on the orientation of the particle cannot be obtained.
<table>
<thead>
<tr>
<th>Fit parameters ($x_i$)</th>
<th>Fitted equation</th>
<th>$s_r$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(A \cdot B)$</td>
<td>$\log(v) = 0.048 + 0.049 \cdot \log(A \cdot B)$</td>
<td>0.105</td>
<td>0.20</td>
</tr>
<tr>
<td>$\log(M)$</td>
<td>$\log(v) = 0.0602 + 0.0656 \cdot \log(M)$</td>
<td>0.099</td>
<td>0.30</td>
</tr>
<tr>
<td>$\log(\alpha)$</td>
<td>$\log(v) = 0.029 - 0.275 \cdot \log(\alpha)$</td>
<td>0.117</td>
<td>0.02</td>
</tr>
<tr>
<td>$\log(H)$ and $\log(A \cdot B)$</td>
<td>$\log(v) = 0.058 + 0.266 \cdot \log(H) - 0.071 \log(A \cdot B)$</td>
<td>0.089</td>
<td>0.43</td>
</tr>
<tr>
<td>$\log(H)$ and $\log(M)$</td>
<td>$\log(v) = 0.040 + 0.342 \log(H) - 0.114 \cdot \log(M)$</td>
<td>0.091</td>
<td>0.40</td>
</tr>
<tr>
<td>$\log(H)$ and $\log(\alpha)$</td>
<td>$\log(v) = 0.100 + 0.190 \log(H) + 0.741 \cdot \log(\alpha)$</td>
<td>0.087</td>
<td>0.45</td>
</tr>
<tr>
<td>$\log(H)$ and $\log(A \cdot B)$ and $\log(M)$</td>
<td>$\log(v) = 0.101 - 0.022 \cdot \log(H) - 0.204 \cdot \log(A \cdot B) + 0.295 \cdot \log(M)$</td>
<td>0.087</td>
<td>0.45</td>
</tr>
<tr>
<td>$\log(H)$ and $\log(A \cdot B)$ and $\log(\alpha)$</td>
<td>$\log(v) = 0.094 + 0.276 \cdot \log(H) - 0.051 \cdot \log(A \cdot B) + 0.615 \cdot \log(\alpha)$</td>
<td>0.084</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Table 6.2: Fit with $A \cdot B$, $M$ and $\alpha$ as parameters, data from December 14, 1996, 1:00-1:15. $H$ is given in mm, $(A \cdot B)$ in mm$^2$, $\alpha$ is the dimensionless area ratio and $v$ is given in m/s. $s_r$ is the residual standard deviation of the regression (equation A.2, A.5), $R^2$ the coefficient of determination. The number of analysed particles is 5616.
6.2 Additional parameters in the velocity function

Using the height yields the best result for the fit compared to other size measures but, nevertheless, a high percentage of the variability remains unexplained. According to the results of Section 5.5, the use of additional parameters will improve the fit. The shape parameters affecting the velocity are \((A \cdot B)\) (the circumscribed area) and \(\alpha\) (the ratio of \(M\) and \((A \cdot B)\), where \(M\) is the shaded area). The influence of additional parameters is analysed by adding them to the equation: 
\[
\log(v) = \log(a) + b \cdot \log(H) + c \cdot \log(X_2) + d \cdot \log(X_3)
\]
Table 6.2 shows the values for the residual standard deviation on the regression \(s_r\) and the coefficient of determination \(R^2\) when the velocity is fitted with varying shape parameters. A list of the standard errors of the coefficients is given in the Appendix (Table A.3).

The results on the whole confirm the expectations drawn from theoretical considerations: The product of \(A\) and \(B\) does not yield a much better result than \(A\) or \(B\) alone (compare Table 6.1). \(M\) is equivalent to the size definition of Locatelli and Hobbs (1974), and leads to a similar \(R^2\). Combining \((A \cdot B)\) or \(M\) with the height \(H\) explains more of the variation in the data than the height \(H\) alone. The negative exponent for \((A \cdot B)\) may be reasonable because a larger area opposed to the flow slows the movement down. But it should be kept in mind that a larger extension in the vertical direction is in the average strongly correlated with a larger area opposed to the flow (compare Fig.6.2).

The parameter \(\alpha\) (division of two size parameters) estimates the roughness of the particle. It is slightly correlated with size (Figure 6.3). Fitting the data with \(H\) and \(\alpha\) as parameters improves the coefficient of determination by 10% as compared to \(H\) alone, although \(\alpha\) alone explains nearly nothing of the variation in the data (Table 6.2).
The statistical significance of the influence of $\alpha$ on the velocity is proven by a p-value near zero (Appendix A.1.3).

$\alpha$ is calculated by dividing $M$ through $(A \cdot B)$. We may, therefore, expect that a fit with $H$, $(A \cdot B)$ and $M$ should yield a similar result as a fit with $H$ and $\alpha$. If only the coefficient of determination is considered, this appears to be true (Table 6.2). But the fit results in an equation where $v$ decreases with $H$ and, therefore, makes no physical sense. This example demonstrates that using two parameters related to the size of the snowflakes may bear problems, although the fit with $H$, $(A \cdot B)$ and $\alpha$ has the highest value of $R^2$ and yields a reasonable equation.

Figure 6.4 illustrates the dependence of the velocity on $H$ and $\alpha$. The $v$–$H$ relations for fixed $\alpha$–values exhibit the usual curve shape. With growing value of $\alpha$ velocities are shifted towards higher values. Higher $\alpha$–values indicate particles with smoother surfaces. Rimming and melting both cause a smoother appearance of the images and both processes lead to higher fall velocities.

The results of the fitting might be influenced by the peculiarities of the special precipitation event. Therefore, the fitting was repeated for another case. Tables 6.3 and 6.4 summarize the results for February 5, 1997 which lead to the same conclusions as the analysis for December 14, 1996. The height is the best size definition for estimating the fall velocity. The addition of the parameter $\alpha$ raises the percentage of explained variation significantly. But still, the unexplained variation is large. Figure 6.5 shows the surface built up by the function $v = 1.42 \cdot H^{0.29} \cdot \alpha^{1.35}$. In Figure 6.6, the region for $0.7 < \alpha < 0.8$ is cut out of Figure 6.5. The measured values within this region are added to illustrate the deviation from the fitted surface. Adding $(A \cdot B)$ raises the percentage of explained variation by only 1 percent in this case.

The results of the statistical analyses in this section confirm the choice of a velocity function of the form $v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d$ which had been derived theoretically.
6.2. ADDITIONAL PARAMETERS IN THE VELOCITY FUNCTION

Figure 6.4: Fitted relationship of velocity, $H$, and $\alpha$. Data from December 14, 1996, 1:00-1:15. Meshes: $H$ 0.5 mm, $\alpha$ 0.05. Function: $v = 1.26 \cdot H^{0.19} \cdot \alpha^{0.74}$.

<table>
<thead>
<tr>
<th>Definition for D</th>
<th>fitted equation</th>
<th>$s_r$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) width $A$</td>
<td>$\log(v) = 0.092 + 0.105 \cdot \log(D)$</td>
<td>0.158</td>
<td>0.11</td>
</tr>
<tr>
<td>b) width $B$</td>
<td>$\log(v) = 0.089 + 0.099 \cdot \log(D)$</td>
<td>0.159</td>
<td>0.11</td>
</tr>
<tr>
<td>c) maximum width $W$</td>
<td>$\log(v) = 0.082 + 0.103 \cdot \log(D)$</td>
<td>0.159</td>
<td>0.10</td>
</tr>
<tr>
<td>d) height $H$</td>
<td>$\log(v) = 0.090 + 0.164 \cdot \log(D)$</td>
<td>0.150</td>
<td>0.21</td>
</tr>
<tr>
<td>e) $\frac{W + H}{2}$</td>
<td>$\log(v) = 0.085 + 0.143 \cdot \log(D)$</td>
<td>0.155</td>
<td>0.16</td>
</tr>
<tr>
<td>f) $\sqrt{W \cdot H}$</td>
<td>$\log(v) = 0.086 + 0.138 \cdot \log(D)$</td>
<td>0.155</td>
<td>0.15</td>
</tr>
<tr>
<td>g) maximum($W, H$)</td>
<td>$\log(v) = 0.076 + 0.145 \cdot \log(D)$</td>
<td>0.155</td>
<td>0.15</td>
</tr>
<tr>
<td>h) $\sqrt{\frac{4M}{\pi}}$</td>
<td>$\log(v) = 0.096 + 0.150 \cdot \log(D)$</td>
<td>0.153</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 6.3: Same as Table 6.1 for the data from February 5, 1997. The number of analysed particles is 1420.
### Table 6.4

Same as Table 6.2 for the data from February 5, 1997. The number of analysed particles is 1420.

<table>
<thead>
<tr>
<th>Fit parameters ((x_i))</th>
<th>Fitted equation</th>
<th>(s_r)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\log(A \cdot B))</td>
<td>(\log(v) = 0.092 + 0.054 \cdot \log(A \cdot B))</td>
<td>0.158</td>
<td>0.12</td>
</tr>
<tr>
<td>(\log(M))</td>
<td>(\log(v) = 0.104 + 0.075 \cdot \log(M))</td>
<td>0.153</td>
<td>0.17</td>
</tr>
<tr>
<td>(\log(\alpha))</td>
<td>(\log(v) = 0.060 - 0.207 \log(\alpha))</td>
<td>0.168</td>
<td>0.01</td>
</tr>
<tr>
<td>(\log(H)) and (\log(A \cdot B))</td>
<td>(\log(v) = 0.080 + 0.291 \cdot \log(H) - 0.061 \cdot \log(A \cdot B))</td>
<td>0.148</td>
<td>0.23</td>
</tr>
<tr>
<td>(\log(H)) and (\log(M))</td>
<td>(\log(v) = 0.067 + 0.365 \cdot \log(H) - 0.103 \cdot \log(M))</td>
<td>0.149</td>
<td>0.22</td>
</tr>
<tr>
<td>(\log(H)) and (\log(\alpha))</td>
<td>(\log(v) = 0.152 + 0.287 \cdot \log(H) + 1.35 \cdot \log(\alpha))</td>
<td>0.137</td>
<td>0.34</td>
</tr>
<tr>
<td>(\log(H)) and (\log(A \cdot B)) and (\log(M))</td>
<td>(\log(v) = 0.172 - 0.319 \cdot \log(H) - 0.356 \cdot \log(A \cdot B) + 0.634 \cdot \log(M))</td>
<td>0.142</td>
<td>0.28</td>
</tr>
<tr>
<td>(\log(H)) and (\log(A \cdot B)) and (\log(\alpha))</td>
<td>(\log(v) = 0.145 + 0.337 \cdot \log(H) - 0.029 \cdot \log(A \cdot B) + 1.251 \cdot (\alpha))</td>
<td>0.135</td>
<td>0.35</td>
</tr>
</tbody>
</table>
6.2. ADDITIONAL PARAMETERS IN THE VELOCITY FUNCTION

Figure 6.5: Fitted relationship of velocity, $H$, and $\alpha$. Data from February 05, 1997, 5:10–5:18 pm. Meshes: $H = 0.5$ mm, $\alpha = 0.05$. Function: $v = 1.42 \cdot H^{0.29} \cdot \alpha^{1.35}$. The two arrows mark the lines where the grey band of Figure 6.6 was cut out.

Figure 6.6: February 5, 1997. Particles with $\alpha$-values between 0.7 and 0.8 are marked by asterisks. The grey band indicates the region of fitted values for those particles, cut out of Figure 6.5.
in Chapter 5. The parameter \( \alpha \) significantly raises the percentage of the explained variation whereas the influence of \((A \cdot B)\) varies for the two analysed cases. This variation of the influence of \((A \cdot B)\) on the velocity is not in contradiction with the conclusions from theory (Section 5.5). Theory predicts an influence of \((A \cdot B)\) equal to the influence of \(H\) if the flow type is near Stokes friction. The influence of \((A \cdot B)\) disappears at high Reynolds numbers. The influence of \(\alpha\), however, is effective for all flow types.

### 6.3 Other influences on the velocity

The previous section confirmed the choice of \(H\), \(\alpha\) and \((A \cdot B)\) as the measured shape parameters determining the velocity. But it also showed that the use of these three parameters leaves a high percentage (51% and 65% for the two precipitation events analysed in this chapter) of the variation in the data unexplained. This disagreement between measured and fitted values can have several reasons:

- the density of the particles is unknown,
- change of crystal type and degree of riming,
- mixture of crystal types,
- melting of particles,
- influence of wind,
- incorrectly matched particles.

The instruments used in this work do not provide the possibility to measure density. A possible approximation to real densities would be to weigh the snow fallen on a given area during a given (short) time interval and to derive a mean density by either using the data of the optical instruments (Muramoto et al., 1995) or by measuring snow cover depth (Fujiyoshi et al., 1990). These methods, however, yield a mean density for all particles. To explain the velocity variations, we would like to know the density of the individual particles. These values could be found by melting the particles and measuring the volume of the melted drop. This method would not be applicable automatically to a large number of particles. In Section 7.5, the question is inverted and the densities of the analysed particles are estimated from the measured velocities.

As far as possible, changes of crystal type and degree of riming during analysed precipitation intervals are avoided. For this purpose, direct observations were recorded in a log and the Formvar plates and photographs were analysed (Section 4.1). When degree of riming and crystal types remained constant for some minutes, the time interval was chosen for further analysis. Degree of riming and crystal type for each individual precipitation event are indicated in Chapter 4.

Most of the time, the aggregates consist of mixtures of crystals. As long as the fractions of the crystal types within the aggregates were constant over a certain time interval, this time interval was accepted. But the density and hence the velocity of
the crystals depends on the proportion of the types within one aggregate. This leads to an unavoidable variability during chosen time intervals which can be attributed to density variations.

The influence of melting can be recognized in the data of the shower on December 14, 1996, 1:00–1:15 pm. A thorough analysis cannot be done in the frame of this work. But the main reason for higher velocities during melting is again the higher density of melting particles. The density of the particles is a parameter in the equation of motion (Section 5.4) which was not taken into account when the parameters for the velocity equation were chosen. \( \alpha \) reflects the density only partly because the internal structure of the snowflake is not known. If the density could be added as a fit parameter in the equation, it might be possible to increase the percentage of explained variation considerably. An attempt could then be made to derive an equation for all events instead of dividing the events according to the degree of riming. As measured density values are not available, this work has to restrict itself on treating the individual events separately.

The influence of wind on the data is negligible for the precipitation events analysed here (Section 6.3.1).

The problem of incorrectly matched particles is addressed in Section 6.3.2.

### 6.3.1 Vertical wind

The 2D–Video–Distrometer was placed at the measuring site without additional wind shielding (Figure 2.3). As the measuring area of the instrument is located only about 5 cm below the opening of the instrument, larger particles will not fall in a still environment long enough to neglect wind effects on the velocity. Strong upward currents were not expected as the 2D–Video–Distrometer was placed at ground level. Furthermore, particles are shielded from upward currents as soon as they enter the instrument. But strong downward currents have to be collected for. They can influence the particles even inside the instrument and can thus lead to inaccurate velocity values.

Figures 6.7 and 6.8 show the vertical winds measured at the location of the Distrometer during the events on March 28 and February 5. Positive values indicate an upward current, negative values a downward current. The mean value over the indicated time is in both cases pointing upward. During the short time when measurements were made on March 28 (3:48–3:49 pm), no negative wind values were measured. Owing to the shielding by the instrument, the particles can be assumed to have reached their terminal velocity at the time of measurement.

The situation is more complicated on February 5. The maximum absolute values of vertical wind velocities fall below zero two times. Theoretically, a correction should be made for these downwind periods. But it is impossible to find the exact correction value for each individual particle. Even if the vertical velocity would be measured every second, we cannot know which value applies to the particle falling through the 2D–Video–Distrometer because the velocity measurement takes place one metre above the instrument (Figure 2.3). Therefore, no correction was made for this case. But the occurrence of downward wind could be an explanation for the high variability of the velocity values measured during this event (Tables 6.3 and 6.4 and Figure 7.4).
Figure 6.7: Ultrasonic anemometer, March 28, 1997. Solid line: mean values of vertical wind over one minute. Dashed line: mean over the whole period. Dotted line: Maximum absolute value of the vertical wind for every minute.

Figure 6.8: Same as Figure 6.7 for February 5, 1997.
On December 14, 1996, the anemometer was not yet mounted at the measuring site. But strong winds as on February 5 were not observed. Horizontal wind is assumed not to influence the velocity measurements. It can considerably influence particle size distributions caused by the shielding of the measuring area. These errors are not further analysed here as they do not affect the velocity analysis.

### 6.3.2 Optimizing the fit function for aggregates

The main interest in this work lies on snow aggregates. Figure 6.6 shows large discrepancies between fit and measured values especially for larger aggregates. The values for the aggregates lie below the fitted surface. This is mainly caused by the high velocities measured for the particles between 2 mm and 4 mm height. But partly it is also caused by the large number of small particles (<1 mm). These particles are single crystals and there are reasons to eliminate them when the falling behaviour of aggregates is considered:

- Single crystals are compact bodies which might react in a different manner on the flow resistance than the porous aggregates of crystals. (Single crystals have low Reynolds numbers and are quite near to the Stokes regime; Section 5.3.4).

- The matching of these small particles is not reliable (Section 2.3.1). This leads for example to impossibly high $\alpha$-values (Figure 6.3) and low $(A \cdot B)$-values (Figure 6.2) of particles shading only a few pixels.

If these particles are eliminated from the data of February 5 (Figures 6.5, 6.6), the fit gets closer to the measured values for large particles but still overestimates their velocity (Figure 6.9). This case was extremely inhomogeneous (Sections 4.3.2 and 6.3.1) but one is faced with these problems in nearly every natural precipitation event.
In Chapter 7, the dependence of the velocity of snow aggregates on crystal type will be investigated. Particles with one of their extensions smaller than 1 mm are excluded from the analyses. This is done to exclude most of the single crystal data and to avoid incorrectly matched particles to be taken into account during the fit procedure.

6.4 Summary

The theoretical considerations in Chapter 5 lead to the choice of the shape parameters $H$, $\alpha$ and $(A \cdot B)$ for the velocity function. In this chapter, correlation analysis was used to test if these parameters really influence the fall velocity.

First, the correlation of the vertical extension $H$ to the velocity was tested in comparison to other size definitions, partly drawn from the literature (Section 6.1). The coefficient of determination $R^2$ was highest for $H$ and the residual standard deviation on the regression $s_r$ was lowest. It was concluded that $H$ is the best size definition if the velocity of snowflakes is regarded because it takes the orientation of the particle into consideration.

The inclusion of the area ratio $\alpha$ in the velocity function increased the percentage of explained variation in the data considerably. The same effect was observed for the inclusion of $(A \cdot B)$, but to a smaller extent. The highest coefficients of determination were found for the combination of all three parameters $H$, $\alpha$ and $(A \cdot B)$. Hence, the analysis of the measured data confirmed the choice of these three snowflake parameters for the velocity equation $v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d$ which had been theoretically derived in Section 5.5.

Two main influences on the velocity not taken into account by this velocity equation were identified in Section 6.3 to be the vertical wind and the density of the measured particles. The influence of the vertical wind can be diminished by analysing only time intervals during which the vertical wind is negligible (Section 6.3.1). The lack of information on snowflake density, however, cannot be compensated for. It leads to high percentages of unexplained variability in the data and is the main problem in the search for equations which explain the dependence of fall velocity on snowflake parameters.
Chapter 7

Applications

The shape parameters of a snowflake influencing the velocity were derived in Chapter 5 to be the vertical extension $H$, the circumscribed area exposed to the flow $(A \cdot B)$ and the ratio of the shaded area to the circumscribed area $\alpha$ (Section 2.2.2). In Chapter 6, the choice of these parameters was confirmed by correlation analysis of measured data. In this chapter, the influence of these parameters on the fall velocity of snow aggregates is discussed for the data gathered during five particular precipitation events. To exclude single crystal data, only particles with $A$, $B$ and $H$ larger than 1 mm are analysed. First, velocity–size relations of the form $v = a \cdot H^b$ are regarded because these functions can be compared to literature values (Section 7.1). Section 7.2 shows how the parameter $\alpha$ is distributed for the different events. In Section 7.3, the influence of $\alpha$ and $(A \cdot B)$ on the velocity is discussed and the fit results are compared to the theoretically expected values (Section 5.5). To avoid the correlation between $(A \cdot B)$ and $H$ which can cause higher standard errors of the coefficients in the regression analysis, an alternative formulation of the velocity function is proposed in Section 7.4. The chapter is closed by an estimation of the particle densities (Section 7.5).

7.1 Velocity–size relations

In this section, the velocity–size relations ($v = aH^b$) found during the SAMM measurements are presented for five events (Table 7.1). A description of the meteorological situation during the events can be found in Section 4.3. The fitted functions and correlation coefficients are listed in Table 7.2. The comprehensive list of the standard errors (Table A.8) and a characterization of the fitted populations (Table A.7) is found in Appendix A.3. Apart from the graupel measured during the thunderstorm on March 28, 1997, all other events yielded particles with low degrees of riming. Temperature also does not vary more than $2^\circ$C. Nevertheless, the velocity–size relations of the regarded events differ considerably.

The particles with the smallest degree of riming ($R = 0.4$) are the aggregates of needles whose velocity–size relation is shown in Figure 7.1 (14 Dec 96, 8:00–8:15). The velocity of these particles does not change with size but is about 1 m/s for particles between 1 mm and 9 mm size. The lack of correlation between velocity and size is reflected by a coefficient of determination of 0.001. The p-value of the t-test for the
Table 7.1: Analysed events. Crystal type, degree of riming $R$ (Section 4.1) and temperature at the measuring site.

<table>
<thead>
<tr>
<th>date</th>
<th>time</th>
<th>crystal type</th>
<th>degree of riming $R$</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 Dec 96</td>
<td>8:00–8:15 am</td>
<td>needles</td>
<td>0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>14 Dec 96</td>
<td>1:00–1:15 pm</td>
<td>needles</td>
<td>0.7</td>
<td>0</td>
</tr>
<tr>
<td>14 Dec 96</td>
<td>5:36–5:43 am</td>
<td>dendrites</td>
<td>1.0</td>
<td>-1</td>
</tr>
<tr>
<td>05 Feb 97</td>
<td>5:10–5:18 pm</td>
<td>dendrites &amp; needles</td>
<td>1.2</td>
<td>-1.5</td>
</tr>
<tr>
<td>28 Mar 97</td>
<td>3:48–3:49 pm</td>
<td>graupel</td>
<td>5</td>
<td>-2</td>
</tr>
</tbody>
</table>

Table 7.2: Number of analysed particles $N$, fitted functions and coefficients of determination ($R^2$) for the precipitation events described in Table 7.1.

<table>
<thead>
<tr>
<th>date</th>
<th>time</th>
<th>$N$</th>
<th>Fitted function</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 Dec 96</td>
<td>8:00–8:15 am</td>
<td>567</td>
<td>$\log(v) = 0.02 \cdot \log(H)$</td>
<td>0.001</td>
</tr>
<tr>
<td>14 Dec 96</td>
<td>1:00–1:15 pm</td>
<td>2747</td>
<td>$\log(v) = 0.047 + 0.16 \cdot \log(H)$</td>
<td>0.23</td>
</tr>
<tr>
<td>14 Dec 96</td>
<td>5:36–5:43 am</td>
<td>637</td>
<td>$\log(v) = 0.002 + 0.14 \cdot \log(H)$</td>
<td>0.16</td>
</tr>
<tr>
<td>05 Feb 97</td>
<td>5:10–5:18 pm</td>
<td>479</td>
<td>$\log(v) = 0.15 + 0.015 \cdot \log(H)$</td>
<td>0.00</td>
</tr>
<tr>
<td>28 Mar 97</td>
<td>3:48–3:49 pm</td>
<td>70</td>
<td>$\log(v) = 0.14 + 0.60 \cdot \log(H)$</td>
<td>0.49</td>
</tr>
</tbody>
</table>

The coefficient of $\log(H)$ is larger than 0.05 (Appendix A.1.3). The regression coefficient $b$ might, therefore, be accidental. For these particles, the velocity can be regarded as independent from the size. Included in Figure 7.1 are the function given by Locatelli and Hobbs (1974) for aggregates of unrimed radiating assemblages of dendrites or dendrites and the fit made by Zikmunda (1972) for mixed aggregate types. The original function given by Zikmunda was $v = 102 + 37 \cdot \log(H)$ with $v$ in cm/s and $H$ in cm. It was corrected for the height as described in Appendix A.4 because the measurements had been carried out at a pressure of 680 mbar. With the help of the photographs shown in Zikmunda’s work the degree of riming of the particles was determined to be 0 (Section 4.1). The curve fitted here is close to the two lines taken from the literature which relate to unrimed crystals of different types. This accordance leads to the conclusion that the influence of the crystal type on the fall velocity of aggregates is small.

The lack of correlation between velocity and size of unrimed aggregates which was found for this case was also stated by Locatelli and Hobbs (1974). The gravitational and the frictional force seem to grow at the same extent if the degree of riming of the particles is near zero.

The aggregates of needles recorded during the shower on **December 14, 1:00–1:15**, yielded the velocity–size relation shown by the solid line in Figure 7.2. Although the degree of riming is not much higher than during the event on December 14, 8:00–8:15 ($R= 0.7$ instead of $R= 0.4$) and although needles were recorded during both events, the influence of size on the velocity is significant in this case. Apart from the
7.1. VELOCITY–SIZE RELATIONS

Figure 7.1: Velocity–height relation for the particles recorded on December 14, 8:00–8:15. Asterisks mark the measured particles. The fitted curve is given by the solid line. Standard deviation $s_r$ (equation A.1) of the measured velocity values from the function $v = H^{0.02}: 0.28 \text{ m/s}$. Dashed line: fit of Locatelli and Hobbs (1974) for aggregates of unrimed radiating assemblages of dendrites or dendrites: $v = 0.8 \cdot D^{0.16}$. Dash-dotted line: fit of Zikmunda (1972) for 44 aggregates.

Figure 7.2: Velocity–height relation for particles recorded on December 14, 1:00–1:15. Asterisks mark the measured particles. The fitted curve is given by the solid line. Standard deviation of the measured velocities from the function $v = 1.11 \cdot H^{0.16}: 0.22 \text{ m/s}$. 
degree of riming, there are two more differences between these two events: between 8:00 and 8:15, some dendrites were found on the Formvar plates and the temperature was lower. If the additional presence of dendrites caused the higher dependence of the fall speed on the size, the particles recorded on February 5, 5:10–5:18 should show the same effect, because a similar amount of dendrites and needles was present in this case. But the velocity of these particles is again not significantly correlated to size in spite of an even higher degree of riming. (Figure 7.1). The higher velocities on December 44, 1:00-1:15 are probably caused by partial melting of the particles, seen on the Formvar plates.

Figure 7.3: Velocity–height relation for the particles recorded on December 14, 5:36–5:43. Asterisks mark the measured particles. The fitted curve is given by the solid line. Standard deviation of the measured velocities from the function $v = 1.00 \cdot H^{0.14}$: 0.21 m/s. Dotted and dashed lines are functions given by Locatelli and Hobbs (1974) for aggregates of densely rimed radiating assemblages of dendrites or dendrites (dotted, $v = 0.79 \cdot H^{0.27}$) and for aggregates of unrimed radiating assemblages of dendrites or dendrites (dashed, $v = 0.8 \cdot H^{0.16}$).

Aggregates built up by dendrites only were recorded on December 14, 5:36–5:43. The degree of riming is similar to the precipitation event on February 5, 5:10–5:18 and the velocity is significantly correlated to size. The fitted function and two curves given by Locatelli and Hobbs (1974) are shown in Figure 7.3. The shape of the curve for the unrimed aggregates is similar to the curve measured here but the values are considerably lower. The curve for rimed aggregates has similar values for particle sizes between 4 and 10 mm but the change of velocity with size is larger. Despite a degree of riming of 1.0, the function determined in this work is nearer to the function for densely rimed particles than to the function for unrimed particles given by Locatelli and Hobbs (1974). This apparent discrepancy is probably caused by differing descriptions of the riming.

Figure 7.4 shows the velocity–size relation for the aggregates consisting of a mixture
of dendrites and needles (February 5, 5:10–5:18). For a comparison, again the function of Locatelli and Hobbs (1974) for densely rimed radiating assemblages of dendrites or dendrites is plotted. The two curves approach each other for large particles. The curve measured in this work remains steady because there are a lot of small particles with highly varying velocities. This might be caused by varying percentages of the two crystal types inside the individual aggregates or by the variability of the vertical wind. As shown in Section 6.3.1, the vertical wind was rather variable and there were even short downwind periods during the measuring interval. The high variability in the velocity leads to standard deviations for this precipitation event which are about twice the standard deviations for the three events on December 14, 1996.

The velocity–size relation of the graupel particles recorded on March 28, 3:48–3:49, has a coefficient of determination $R^2$ of 0.49 (Table 7.2) which is much higher than the $R^2$–values of the aggregates. The fitted relation is shown in Figure 7.5. The dotted line is the function of Locatelli and Hobbs (1974) for lump graupel with a density of 0.1 to 0.2 g/cm$^3$. These authors measured particles within a size range from 0.5 to 3.0 mm. The line fitted here might be seen as a continuation of their function to larger sizes.

The comparison of the measured data with literature data shows good agreement of the results for unrimed aggregates and for graupel. Intermediate degrees of riming cannot be compared because the classifications used in the literature differ from the classification used here. It is not possible to make any conclusions about the influence of crystal type from these measurements alone but the comparison with data from other researchers (Figure 7.1) shows no obvious differences between the crystal types. The comparison of unrimed particles indicates only a small influence of the crystal type on the fall velocity. The influence of the degree of riming is dominating.
Figure 7.5: Velocity–height relation for the particles recorded on March 28, 3:48–3:49. Asterisks mark the measured particles. The fitted curve is given by the solid line. Standard deviation of the measured velocities from the function \( v = 1.38 \cdot H^{0.60} \), 0.45 m/s. Dotted line: Fit of Locatelli and Hobbs (1974) for lump graupel with a density of 0.1 to 0.2 g/cm³.

Figure 7.6: Comparison of the velocity–size relations of the five analysed events as shown in Figures 7.1–7.5.
7.2. AREA RATIO $\alpha$

In Figure 7.6, the data of the five analysed events are shown together. The high
dependence of velocity on the degree of rime can clearly be seen. The dependence of
velocity on size is large for heavily rime particles whereas lightly rime particles show
only a small change of their velocity during growth. Melting increases the fall velocity
considerably. With further experiments, the gap between the degrees of rime of 1.2
and 5 should be investigated. Then it might be possible to find an empirical relationship
between the degree of rime and the velocity–size–relation, e.g. by analysing the
change in the fit parameters $a$ and $b$.

7.2 Area ratio $\alpha$

![Figure 7.7: Frequency distribution of the $\alpha$-values for the data from December 14, 8:00–8:15.](image)

In Section 6.2 the dependence of the velocity on the area ratio $\alpha$ could be proven. $\alpha$
represents the deviation of the cross-sectional area of the particle from a smooth ellipse
(for the definition of $\alpha$ see Section 2.2.2). For the five events listed in Tables 7.1 and 7.2
frequency distributions of the $\alpha$-values are shown in Figures 7.7–7.11. The broadest
distribution is found for the nearly unrimed particles (Figure 7.7). This distribution
seems to have more than one maximum. The other distributions are narrower and have
clearly discernible maxima. The $\alpha$-range with the largest number of particles usually
includes the mean and median value.

The graupel particles (Figure 7.11) have a mean $\alpha$-value of 0.916. This high value
is not surprising as the formation of graupel leads to more compact shapes. The range
of $\alpha$-values is smaller than the one for aggregates because the variation in shape is less
for the graupel particles. The relatively broad distribution of $\alpha$ for the aggregates is
caused by the high variability of shapes and leads to a high variability of the velocity
(Section 7.1). Part of this variability can be explained by introducing $\alpha$ to the velocity
equation (Section 7.3).
As mentioned in Section 7.1, gravitational and frictional force are growing to the same extent with size for unrimed flakes whereas in the case of rimed particles, the gravitational force grows significantly faster. Unrimed flakes have a rather loose structure, thus experiencing more friction than rimed particles (more compact shape). This effect is more pronounced for larger sizes. \(\alpha\) is, therefore, expected to be smaller for unrimed particles and to decrease more with size for these particles than for rimed particles. This is confirmed by the measurements. In a log–log plot of \(\alpha\) and \(H\) (Figure 6.3), a straight line fits the data reasonably well. Figure 7.12 shows these lines for the five events analysed. The nearly unrimed particles recorded on December 14, 8:00–8:15, show generally a lower area ratio \(\alpha\) and the change with size is more pronounced than for the events with larger degrees of riming. For the graupel particles, the change of the average \(\alpha\)-value with size is small. \(\alpha\) never drops below 0.9. This leads to the
7.3 Velocity–size–shape relations

In this section, the parameters \( \alpha \) and \((A \cdot B)\) are added to the velocity function used in Section 7.1. Regression analyses are made with different combinations of the parameters to assess their relative importance. Table 7.3 shows the \( R^2 \)-values of these regressions.

As yet discussed in Section 7.1, the height \( H \) has little influence on the velocity in two cases (Dec 14, 8:00–8:15 and Feb 5, 5:10–5:18). In these cases, \( \alpha \) is the variable

Figure 7.10: Frequency distribution of the \( \alpha \)-values for the data from February 05, 5:10–5:18.

Figure 7.11: Frequency distribution of the \( \alpha \)-values for the data from March 28, 3:48–3:49.
7.3. VELOCITY–SIZE–SHAPE RELATIONS

was found (Figure 7.1). Taken the shape into account by adding \( \alpha \) to the equation, the increase of velocity with increasing height \( H \) can be seen. If particles are not stratified with area ratio \( \alpha \), this dependence is masked by the high variability of the shape.

\[
v = 1.19 \cdot H^{0.31} \cdot \alpha^{1.04}
\]

Figure 7.13: Velocity values measured on December 14, 8:00–8:15, for aggregates with area ratio \( \alpha \) between 0.6 and 0.7 (asterisks). The grey band marks the expected velocities for this \( \alpha \)-range: \( v = 1.19 \cdot H^{0.31} \cdot \alpha^{1.04} \). The inclusion of \( \alpha \) in the fit reveals the dependence of the velocity on the vertical extension \( H \) which could not be seen in the \( v-H \) function (Figure 7.1).

In the case of graupel (March 28, 3:48–3:49) a high dependence of velocity on size is found and the inclusion of the area ratio \( \alpha \) causes less change in the coefficient of determination. \( \alpha \) is the more important the less compact the flakes are. This fact can easily be explained: the gravitational force acting on the less compact flakes is lower because of their lower mass. At the same time, the forces acting against the fall get stronger because friction is higher for the loose aggregates. Nevertheless, the use of the area ratio \( \alpha \) to describe graupel improves the results. This finding agrees with the results of List and Schmrcnauer (1971) who found an empirical relationship between the drag coefficient and the area ratio \( \alpha \) for graupel.

\( (A \cdot B) \) alone explains little of the variation. An exception is the case on March 28, where graupel particles were recorded. Graupel particles have similar vertical and horizontal dimension. Hence, \( (A \cdot B) \) is nearly equivalent to the vertical extension. The combination of \( (A \cdot B) \) with \( H \) is generally not an improvement compared to \( H \) alone, with the exception of the event on December 14, 1:00–1:15. In two cases (December 14, 5:36–5:43 and March 28, 3:48–3:49), the t-test for the coefficient of \( A \cdot B \) (Appendix A.1.3) indicates that the influence of \( (A \cdot B) \) could be accidental (\( p > 0.05 \)). This might be caused by the rather large correlation between \( (A \cdot B) \) and \( H \). Only on December 14, 1:00–1:15, the number of recorded particles was large enough to calculate a well defined coefficient for \( (A \cdot B) \) in spite of this correlation. As a measure of the correlation between \( (A \cdot B) \) and the other parameters, the mutual coefficients of determination \( R^2 \)
**CHAPTER 7. APPLICATIONS**

<table>
<thead>
<tr>
<th>Precipitation event</th>
<th>Fit parameters $H$</th>
<th>$\alpha$</th>
<th>$H$ and $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 8:00–8:15</td>
<td>0.57</td>
<td>0.28</td>
<td>0.60</td>
</tr>
<tr>
<td>Dec 14, 1:00–1:15</td>
<td>0.70</td>
<td>0.26</td>
<td>0.71</td>
</tr>
<tr>
<td>Dec 14, 5:36–5:43</td>
<td>0.59</td>
<td>0.23</td>
<td>0.61</td>
</tr>
<tr>
<td>Feb 05, 5:10–5:18</td>
<td>0.63</td>
<td>0.23</td>
<td>0.64</td>
</tr>
<tr>
<td>Mar 28, 3:48–3:49</td>
<td>0.90</td>
<td>0.03</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 7.4: Coefficients of determination $R^2$ for the regression of $(A \cdot B)$ on the parameters $H$ and $\alpha$. The correlation between $(A \cdot B)$ and $H$ is high for all cases.

<table>
<thead>
<tr>
<th>precipitation event</th>
<th>degree of riming</th>
<th>exponent for $H$</th>
<th>$\alpha$</th>
<th>$(A \cdot B)$</th>
<th>coefficient of determination $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 8:00</td>
<td>0.4</td>
<td>1.19</td>
<td>0.25</td>
<td>1.01</td>
<td>-0.03</td>
</tr>
<tr>
<td>Dec 14, 1:00</td>
<td>0.7</td>
<td>1.21</td>
<td>0.30</td>
<td>0.56</td>
<td>-0.06</td>
</tr>
<tr>
<td>Dec 14, 5:36</td>
<td>1.0</td>
<td>1.10</td>
<td>0.19</td>
<td>0.66</td>
<td>0.01</td>
</tr>
<tr>
<td>Feb 05, 5:10</td>
<td>1.2</td>
<td>1.68</td>
<td>0.25</td>
<td>1.70</td>
<td>-0.04</td>
</tr>
<tr>
<td>Mar 28, 3:48</td>
<td>5.0</td>
<td>1.38</td>
<td>0.40</td>
<td>1.19</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Theoretical values (Section 5.4)
- Stokes: $0.67$, $1.50$, $0.67$
- Newton: $0.50$, $0.25$, $0.00$

Table 7.5: Fitted coefficients of the functions $v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d$ in comparison to the theoretical values of the exponents derived in Section 5.4.

are calculated (Appendix A.1.2). They are listed in Table 7.4.

When all three parameters are used, the t-test indicates that the coefficient of log$(A \cdot B)$ is not significant for all cases except for December 14, 1:00–1:15, where the number of particles is higher than in the other cases (Table 7.2). In this case, the additional parameter $(A \cdot B)$ improves the regression (Table 7.3). In the other cases, $R^2$ is not or only slightly (+1%) increased by adding $(A \cdot B)$. It is possible that an increase could be observed if more particles would have been recorded in these cases.

The exponents found by fitting the complete equation $v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d$ to the data are compared to the exponents derived theoretically (Section 5.4) in Table 7.5. The coefficients $a$, $b$, $c$ and $d$ and their standard deviations are listed in Table A.10 in the Appendix. For a fully deterministic situation the values of the exponents should lie between the values for the Stokes–formulation and for the Newton–formulation (Table 7.5). $b$, the exponent for $H$, is not within this range when derived from the fit. It is smaller because there is still unexplained variation in the data.

The values of $c$, the exponent for $\alpha$, are within the expected range. Only on
February 05, 5:10–5:18, c is too high. In the derivation of the theoretical exponents, only the shape of the particles was regarded. But α is also influenced by the density of the particle. Therefore, the value of c might be larger than the derived one.

The values for d, the exponent for \((A \cdot B)\), are small or even negative. The relatively small values compared to the theoretically derived values may have the same reason than for b: a high percentage of unexplained variability. Negative values for d might be justified because the area exposed to the flow causes the forces acting against the fall.

The high variability which is still not explained is mainly caused by the impossibility to measure the density of the particles (Section 6.3). α is an insufficient representative for the density because the images show shadows, not sectional views of the particles. This limits the ability to explain the fall velocity of an individual snowflake by measuring the relevant snowflake parameters.

Summary
The regression analyses made in this section confirm the influence of the three parameters H, α and \((A \cdot B)\) on the velocity. Whereas for samples containing single crystals (Chapter 6) H is the most important parameter, α can be more important for samples of aggregates only (Chapter 7). For a large data base, the additional use of \((A \cdot B)\) significantly improves the regression. For smaller data bases, the high correlation between H and \((A \cdot B)\) makes a conclusion difficult because the coefficient d is not well determined (i.e. its standard error is high, Table A.10). There are two possible solutions for this problem. Either the variable \((A \cdot B)\) is left out or other variables are used which are less correlated. Leaving out \((A \cdot B)\) is not satisfying as the percentage of explained variation will then be smaller. Therefore, an alternative function with other variables is proposed in the next section.

7.4 An alternative velocity function

Caused by the correlation between the two shape parameters H and \((A \cdot B)\) the statistical tests made for the regressions in Section 7.3 indicated for most of the cases that \((A \cdot B)\) should be eliminated from the velocity function. On the other hand, \((A \cdot B)\) is known to be a relevant parameter from the theoretical considerations (Chapter 5) and from fits with larger data bases (Section 6.2 and Section 7.3, event on December 14, 1:00–1:15).

As an alternative it is proposed to use linear combinations of the parameters \(\log(H)\) and \(\log(A \cdot B)\): \(\log(H) + \log(A \cdot B)\) and \(\log(H) - \log(A \cdot B)\). The first one corresponds to the volume of the circumscribed ellipsoid. If \(\sqrt{A \cdot B}\) is used in the second one, it corresponds to the oblateness of the ellipsoid. This leads to two new shape parameters for the fit:

\[
\theta = H \cdot A \cdot B \quad \text{and} \quad \epsilon = \frac{H}{\sqrt{(A \cdot B)}}.
\]

\(\alpha\), which describes the deviation from the ellipsoidal shape, is used as before (Section 2.2.2).
### Table 7.6: Coefficients of determination $R^2$ for the regression between the velocity $v$ and the alternative fit parameters $\theta$, $\epsilon$, and $\alpha$.

<table>
<thead>
<tr>
<th>Fit parameters</th>
<th>Dec 14 8:00–8:15</th>
<th>Dec 14 1:00–1:15</th>
<th>Dec 14 5:36–5:43</th>
<th>Feb 05 5:10–5:18</th>
<th>Mar 28 3:48–3:49</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.01</td>
<td>0.13</td>
<td>0.12</td>
<td>0.00</td>
<td>0.48</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.05</td>
<td>0.13</td>
<td>0.06</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>$\theta$ and $\epsilon$</td>
<td>0.06</td>
<td>0.27</td>
<td>0.17</td>
<td>0.02</td>
<td>0.50</td>
</tr>
<tr>
<td>$\theta$ and $\alpha$</td>
<td>0.36</td>
<td>0.18</td>
<td>0.22</td>
<td>0.17</td>
<td>0.56</td>
</tr>
<tr>
<td>$\epsilon$ and $\alpha$</td>
<td>0.34</td>
<td>0.13</td>
<td>0.07</td>
<td>0.17</td>
<td>0.08</td>
</tr>
<tr>
<td>$\theta$ and $\epsilon$ and $\alpha$</td>
<td>0.41</td>
<td>0.34</td>
<td>0.27</td>
<td>0.20</td>
<td>0.57</td>
</tr>
</tbody>
</table>

### Table 7.7: Coefficients of determination $R^2$ for the correlation between the alternative shape parameters $\theta$, $\epsilon$, and $\alpha$. For the case of March 28, the correlations might be accidental ($p$-value > 0.05, see Appendix A.1.3).

<table>
<thead>
<tr>
<th>Dec 14, 8:00–8:15</th>
<th>Dec 14, 1:00–1:15</th>
<th>Dec 14, 5:36–5:43</th>
<th>Feb 05, 5:10–5:18</th>
<th>Mar 28, 3:48–3:49</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>$\alpha$</td>
<td>$\epsilon$</td>
<td>$\alpha$</td>
<td>$\epsilon$</td>
</tr>
<tr>
<td>$\theta$ 0.00</td>
<td>0.29</td>
<td>$\theta$ 0.00</td>
<td>0.29</td>
<td>$\theta$ 0.01</td>
</tr>
<tr>
<td>$\epsilon$ 0.00</td>
<td>$\epsilon$</td>
<td>0.01</td>
<td>$\epsilon$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The data can be fitted with the new parameter combination of $\theta$, $\epsilon$, and $\alpha$. As no additional information is gained, the coefficient of determination (Table 7.6, last line) stays the same as in the fit with $H$, $(A \cdot B)$, and $\alpha$ (Table 7.3, last line) but all parameters are statistically relevant because the correlation between the fit parameters is smaller. Table 7.7 shows the mutual coefficients of determination for $\theta$, $\epsilon$, and $\alpha$. For the case of March 28, 3:48–3:49, the t-tests show that the correlations between the parameters might be accidental. For the other cases, $\theta$ and $\epsilon$ are nearly independent from each other. There is a correlation between $\theta$, the volume of the circumscribed ellipsoid, and $\alpha$, the deviation from the ellipsoidal shape. This was expected as $\alpha$ depends on the size of the particle (Figure 7.12).

If the coefficients of determination $R^2$ of the different fits in Table 7.6 are compared, similar conclusions can be derived as for the fits with $H$, $\alpha$, and $A \cdot B$ (Table 7.3). In the case from December 14, 8:00–8:15, and from February 05, 5:10–5:18, the deviation from a smooth shape ($\alpha$) is the dominant parameter. In two other cases (December 14, 5:36–5:43 and March 28, 3:48–3:49), the size (represented by $\theta$) is the most important influence on velocity. A special case is the event on December 14, 1:00–1:15 where the size ($\theta$) of the particles and their oblateness ($\epsilon$) have an equally
7.4. AN ALTERNATIVE VELOCITY FUNCTION

<table>
<thead>
<tr>
<th>precipitation event</th>
<th>fitted function</th>
<th>$s_e$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 14, 8:00–8:15</td>
<td>$v = 1.19 \cdot \theta^{0.08} \cdot \epsilon^{0.18} \cdot \alpha^{1.01}$</td>
<td>0.23</td>
</tr>
<tr>
<td>December 14, 1:00–1:15</td>
<td>$v = 1.21 \cdot \theta^{0.06} \cdot \epsilon^{0.24} \cdot \alpha^{0.56}$</td>
<td>0.21</td>
</tr>
<tr>
<td>December 14, 5:36–5:43</td>
<td>$v = 1.10 \cdot \theta^{0.07} \cdot \epsilon^{0.12} \cdot \alpha^{0.44}$</td>
<td>0.20</td>
</tr>
<tr>
<td>February 05, 5:10–5:18</td>
<td>$v = 1.68 \cdot \theta^{0.05} \cdot \epsilon^{0.19} \cdot \alpha^{1.70}$</td>
<td>0.47</td>
</tr>
<tr>
<td>March 28, 3:48–3:49</td>
<td>$v = 1.38 \cdot \theta^{0.22} \cdot \epsilon^{0.19} \cdot \alpha^{1.19}$</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 7.8: Functions fitted to the data with the alternative parameters $\theta$, $\epsilon$ and $\alpha$ and standard deviation $s_e$ of the measured velocities from these functions (equation A.1).

high correlation with velocity. Yet, the addition of $\alpha$ improves the fit considerably for all cases. For the case on March 28, 3:38–3:49, the t-test indicates that the parameter $\epsilon$ is superfluous. This might be caused by the similar horizontal and vertical dimension of the particles. But it should be kept in mind that only 70 particles were recorded during this event.

Table 7.8 lists the velocity functions fitted with the alternative shape parameters and the standard deviations $s_e$ of the measured velocities from these functions. The coefficients of the logarithmic fit and their standard errors are listed in Table A.11 in the Appendix. For the snow aggregates, all coefficients are well determined here in contrast to the coefficients for $(A \cdot B)$ in the previous Section 7.3 (Table A.10) which had standard errors in the order of magnitude of the coefficients (apart from the case of Dec 14, 1:00–1:15).

An exception is the event on March 28, 3:48–3:49. The t-test for the coefficient of log($\epsilon$) has a p-value larger than 0.05, thus indicating that the parameter $\epsilon$ might be left away. This is probably caused by the nearly round shape of the graupel particles. A parameter indicating the oblateness is not necessary. For graupel particles, the combination of $H$ and $\alpha$ or of $\theta$ and $\alpha$ is the best choice of parameters. $H$ or $\theta$ stands for the size of the particle and $\alpha$ for its deviation from a smooth ellipsoid. Both combinations explain 56% of the variation in the data (Tables 7.3 and 7.6) and the influence of these parameters on the velocity is significant.

In the case of snow aggregates, the combination of $\theta$, $\epsilon$ and $\alpha$ is the best choice of parameters to describe the fall velocity of the recorded particles. These parameters are calculated from the theoretically derived parameters $H$, $(A \cdot B)$ and $\alpha$ and have the advantage to be less correlated with each other.

The theoretical exponents of the parameters $\theta$, $\epsilon$ and $\alpha$ can be derived from the theoretical exponents for $H$, $\alpha$ and $A \cdot B$ (Section 5.5). They are compared to the fitted exponents in Table 7.9. As for the exponents of the parameters $H$ and $(A \cdot B)$ (Table 7.5), the fitted exponent of $\theta$ is below the range of expected values. The three parameters do only explain a part (20% - 57%) of the variation in the data (compare last line in Table 7.6).

The ‘improvement’ of the velocity estimation by using the three proposed parameters for snowflakes can be judged by looking at the reduction of scattering in the values. The standard deviations for the case on February 05 are rather high, probably
Table 7.9: Comparison of theoretical exponents for $\theta$, $\epsilon$ and $\alpha$ to the fitted exponents. The fitted exponents for $\theta$ are too small because only a part of the variation in the data can be explained by these three variables (compare last line of Table 7.6).

<table>
<thead>
<tr>
<th>precipitation event</th>
<th>degree of riming</th>
<th>exponent for $\theta$</th>
<th>$\epsilon$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 14, 8:00</td>
<td>0.4</td>
<td>0.06</td>
<td>0.18</td>
<td>1.01</td>
</tr>
<tr>
<td>December 14, 1:00</td>
<td>0.7</td>
<td>0.06</td>
<td>0.24</td>
<td>0.56</td>
</tr>
<tr>
<td>December 14, 5:36</td>
<td>1.0</td>
<td>0.07</td>
<td>0.12</td>
<td>0.44</td>
</tr>
<tr>
<td>February 05, 5:10</td>
<td>1.2</td>
<td>0.05</td>
<td>0.19</td>
<td>1.70</td>
</tr>
<tr>
<td>March 28, 3:48</td>
<td>5</td>
<td>0.22</td>
<td>0.19</td>
<td>1.19</td>
</tr>
</tbody>
</table>

Theoretical values
- Stokes: $0.66$, $0$, $1.50$
- Newton: $0.17$, $0.33$, $0.25$
- Example for transition zone: $0.33$, $0.22$, $0.33$

Table 7.10: Comparison of the standard deviation of $\log(v)$ from its mean value to the standard deviation $s_r$ of $\log(v)$ from the fitted functions with $H$ as single parameter (equation A.2) and with the alternative parameter combination $\theta$, $\epsilon$ and $\alpha$ (equation A.5).

<table>
<thead>
<tr>
<th>event</th>
<th>standard deviation of $\log(v)$ from mean $\log(v)$</th>
<th>fit with $H$</th>
<th>fit with $\theta$, $\epsilon$ and $\alpha$</th>
<th>total reduction of the standard deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 8:00–8:15</td>
<td>0.11</td>
<td>0.1065</td>
<td>0.0817</td>
<td>26</td>
</tr>
<tr>
<td>Dec 14, 1:00–1:15</td>
<td>0.075</td>
<td>0.0661</td>
<td>0.0613</td>
<td>18</td>
</tr>
<tr>
<td>Dec 14, 5:36–5:43</td>
<td>0.076</td>
<td>0.0693</td>
<td>0.0648</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 7.10: Comparison of the standard deviation of $\log(v)$ from its mean value to the standard deviation $s_r$ of $\log(v)$ from the fitted functions with $H$ as single parameter (equation A.2) and with the alternative parameter combination $\theta$, $\epsilon$ and $\alpha$ (equation A.5).
caused by the variability of the vertical wind (Section 6.3.1). This case can, therefore, not be used to give a velocity equation for the respective particle type. The three cases on December 14, however, were not affected by this problem. In Table 7.10, the standard deviation of the logarithms of the measured velocities from their mean value is shown together with their standard deviation \( s \), from the fitted logarithmic functions (Tables in Appendix A.3). The fitted functions reduce the deviation by 15–26 %. This is only a slight reduction but a higher reduction could not be expected because most of the variability in the data is still unexplained. Reasons why the improvement is modest:

- The density is a parameter in the equation of motion (Section 5.4) which was not taken into account when deriving the parameters for the velocity equation.
- The other parameters are affected by errors. The ellipsoid \( \theta \), for example, would better be described by the minor and major axis which are not known from the measurements. The same is valid for the oblateness \( \epsilon \).
- The problem of the fall velocity of snowflakes is complex and external influences cannot totally be avoided.

Conclusion

If velocity values for nearly unrimed (\( R = 0.4 \)) or lightly rimed (\( R = 1 \)) snowflakes are needed, the equations given in Table 7.8 for Dec 14, 8:00 and Dec 14, 5:36 can be used, respectively. (The case of December 14, 1:00, is not used because the snowflakes were partially melted.) As the data of other researchers did not differ much from the data measured here with respect to the fitted velocity–size relations (Section 7.1), it can be assumed that these equations will be applicable to other events as well. The standard deviation \( s_e \) (equation A.1) of the measured velocity values from the equations

\[ v = a \cdot R^k \cdot e^\epsilon \cdot \alpha^\alpha \]

is 0.23 m/s for \( R = 0.4 \) and 0.20 m/s for \( R = 1.0 \). To obtain equivalent equations for particles with higher degrees of riming, additional measurements are necessary which have to yield the circumscribed volume, the oblateness and the ratio of cross-sectional to circumscribed area for each particle.

7.5 An estimate for the density of snowflakes

As mentioned in Section 1.2.3, Böhm (1989) derived a general equation for the velocity of hydrometeors. His derivation was based on the theory of Abraham (1970) which regards hydrometeors together with their boundary layer as a rigid body moving through an inviscid fluid. The formula for the calculation of the velocity derived by Böhm (1989) is then:

\[ v = \frac{Re \cdot \eta}{2 \cdot \rho_a \cdot \left( \frac{\pi}{A_{\text{eff}}} \right)} \]  

(7.1)

with \( Re \) the Reynolds number, \( \eta \) and \( \rho_a \) the viscosity and density of the air and \( A_{\text{eff}} \) the area of the ellipse circumscribed to the area presented to the flow. \( Re \) is calculated from the Davies number \( X \) (introduced by Davies, 1945):

\[ Re = 8.5 \cdot \left[ \sqrt{\left(1 + 0.1519 \cdot \sqrt{X}\right)} - 1 \right]^2 \]  

(7.2)
X is given by:

\[ X = \frac{8 \cdot m \cdot g \cdot \rho_v}{\pi \cdot \eta^2} \cdot \left( \frac{A_{\text{eff}}}{A_{\text{eff}}} \right)^{1/4} \]  

(7.3)

with \( m \) the mass of the snowflake, \( g \) the gravitational acceleration, and \( A_{\text{eff}} \) the effective projected area presented to the flow.

The possibilities for Böhm to test his theory with data on snow aggregates were scarce. He used the data of Locatelli and Hobbs (1974) and made additional assumptions on the shape of the particles. Values for the ratio \( A_{\text{eff}}/A_{\text{eff}} \) were not available and had to be approximated. This ratio is the inverse of the parameter \( \alpha \) defined in this work (Section 2.2.2) if we assume \( \alpha \) to be isotropic, i.e., to be independent of the direction of view. Böhm (1989) compared velocity-size relations fitted with his calculated velocities to the relations fitted to the measured velocity values and stated a difference of less than 10% between the two curves.

If the formulae derived by Böhm are assumed to be generally valid, they can be solved for the density of the snow particles and the densities can be calculated. For this purpose, equations 7.1, 7.2 and 7.3 are solved for \( R_c \), \( X \), and \( m \), respectively.

\[ R_c = \frac{2 \cdot v \cdot \rho_v \cdot \sqrt{A_{\text{refl}}}}{\eta \cdot \sqrt{\pi}} \]  

(7.4)

\[ X = \left( \frac{\sqrt{R_c}}{8.5} + 1 \right)^2 \]  

(7.5)

\[ m = \frac{X \cdot \pi \cdot \eta^2}{8 \cdot g \cdot \rho_v \cdot \alpha^{1/4}} \]  

(7.6)

Figure 7.14: Density of the particles measured on Dec 14, 8:00–8:15, calculated using the formulae 7.4–7.7.

The density depends on the definition of the volume. Here, the volume of the circumscribed ellipsoid was assumed to represent the body together with its boundary
Table 7.11: Functions for the dependence of snowflake density on size. Size \( H \) in mm, density \( \rho_s \) in kg/m\(^3\). \( R^2 \) is the coefficient of determination (equation A.3).

<table>
<thead>
<tr>
<th>Data</th>
<th>Fitted function</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 8:00–8:15</td>
<td>( \rho_s = 107 \cdot H^{-0.99} )</td>
<td>0.36</td>
</tr>
<tr>
<td>Dec 14, 1:00–1:15</td>
<td>( \rho_s = 121 \cdot H^{-0.80} )</td>
<td>0.46</td>
</tr>
<tr>
<td>Dec 14, 5:36–5:43</td>
<td>( \rho_s = 100 \cdot H^{-0.70} )</td>
<td>0.39</td>
</tr>
<tr>
<td>Feb 05, 5:10–5:18</td>
<td>( \rho_s = 204 \cdot H^{-0.03} )</td>
<td>0.29</td>
</tr>
<tr>
<td>Muramoto et al. (1995)</td>
<td>( \rho_s = 48 \cdot H^{-0.496} )</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7.14 shows the densities calculated for the particles recorded on Dec 14, 8:00–8:15, and the function fitted to these values. The large scattering of the calculated density values was expected because of the large scattering of the velocity values. Two particles of the same size can have densities which differ by one order of magnitude. For the larger aggregates, however, scattering is less. They all have densities between 10 and 20 kg/m\(^3\).

![Diagram of density vs. size](image)

Figure 7.15: Calculated densities in comparison with the literature curve of Muramoto et al. (1995). The functions for the curves are listed in Table 7.11.

The large scattering of the values for small particles was also found for the other events. Figure 7.15 shows a comparison of the densities calculated for the three events on December 14 and the event on February 5 as well as the curve found by Muramoto et al. (1995). The fitted functions are listed in Table 7.11. Muramoto et al. (1995) determined snow density by dividing the total volume of optically recorded particles by
the total weight of these particles as measured by an electronic balance. The volume was determined as the sum of the volumes of spheres with the horizontal diameters of the measured particles. The relation of density and size was then determined as the relation between the mean diameter and the mean density during several time intervals. The definition of particle volume is comparable to the definition in this work. Although Muramoto et al. (1995) used mean values for the densities, the scattering of their values for particles smaller than 4 mm also spans one order of magnitude. Hence, the calculated density values are reasonable.

As expected, the densities increase with the degree of riming (Figure 7.15). For the graupel particles, the density values also scatter considerably (Figure 7.16). The values of most of the particles are between 100 and 200 kg/m³, in agreement with the density values of the lump graupel measured by Locatelli and Hobbs (1974) which had a similar velocity-size function than the particles recorded on March 28, 3:48–3:49 (Figure 7.5).

The measurement of the density of individual snow particles is not possible in this experiment. Hence, the shown results cannot be compared to independent data but they are in reasonable agreement with literature values.

### 7.6 Summary

In this chapter, the data recorded during five particular events were analysed. The analysis is restricted to particles larger than 1 mm to exclude most of the single crystal data.

In Section 7.1, the v–H relations fitted to the data are compared to v–H relations in the literature. There was generally good agreement with the literature data. It is concluded that the crystal type has not much influence on the fall velocity of aggregates.

The distributions of the area ratio \( \alpha \) are discussed in Section 7.2. For unrimed
particles $\alpha$ is found to be smaller and to decrease more rapidly with size than for rimed particles. The inclusion of $\alpha$ in the velocity equation is more important for the unrimed particles. This conclusion can also be drawn from the regression analyses in Section 7.3.

These regression analyses are used to assess the relative importance of the parameters in the velocity equation. In the cases where the height $H$ of the snowflake had a poor correlation with size, $\alpha$ has the highest coefficient of determination $R^2$. Unrimed particles thus showed to be more influenced by the viscous force than by inertia. The two parameters $H$ and $\alpha$ proved to be the best combination in most cases. The additional inclusion of $(A \cdot B)$ in the velocity equation improves the fit only if the number of particles $N$ is high ($N > 1000$).

The comparison of the fitted coefficients with the theoretically derived coefficients does not allow a conclusion about the “position” of the snowflakes within the transition zone between Stokes and Newton friction (Section 3.3.4). This may be caused by the lack of an important parameter in the velocity equation: the density of the particles. This lack of information leads to a high percentage of unexplained variability in the data and to low values of the coefficients.

The fact that $(A \cdot B)$ does not perform well as a fit parameter can be explained by the high correlation between $H$ and $(A \cdot B)$. Looking for parameters with less correlation, $\theta$ (representing the volume of the circumscribed ellipsoid) and $\epsilon$ (representing the oblateness of the particle) were defined in Section 7.4. These parameters are less correlated and, consequently, the coefficients of the fit with $\theta$, $\epsilon$ and $\alpha$ are better defined than those of the fit with $H$, $(A \cdot B)$ and $\alpha$. For particles with degrees of riming $R = 0.4$ or $R = 1$, equations are found which relate the shape parameters to the velocity with a standard deviation of the measured velocities from the velocities calculated with the fitted equation of 0.23 m/s and 0.20 m/s, respectively.

In Section 7.5, a method is described to estimate the density of particles recorded by instruments working on the basis of photodiode arrays. The results cannot be verified by measurements but the calculated densities are within a reasonable range compared to literature values.
Chapter 8

Discussion

If the velocity of hydrometeors is described in the literature, it is usually in terms of their size dependence. Snow aggregates show a high scattering in their velocity–size relations. The possibility to reduce this scattering by

a) using the size definition most correlated with the velocity and

b) adding other relevant hydrometeor parameters

was tested in this work.

Theoretical derivation of the relevant parameters

The relevant parameters were derived theoretically by considerations about the drag force to be (Section 5.4)

- the vertical extension of the particle (H),
- the circumscribed area opposed to the flow (A \cdot B),
- the ratio of the actual area to the circumscribed area (\alpha), and
- the density of the hydrometeor.

There was no possibility to measure the density of the individual particles. Hence, the analysis was restricted to parameters reflecting the shape and orientation of the particles. An equation describing the dependence of the velocity from these shape parameters has the form

\[ v = a_1 \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \]

with the coefficients \( a_1, b, c \) and \( d \) determined by regression.

The dependence of the exponents \( b, c \) and \( d \) on the drag force is estimated in Section 5.5. Snowflakes are neither in the range of Reynolds numbers where only Stokes friction is relevant nor is it possible to neglect the viscous drag in applying the formulation of the drag force given by Newton (Section 5.2.1 and Figure 5.6). The exponents should, therefore, have values between the values derived for Stokes and Newton friction.
H (the vertical extension) is a relevant parameter in both formulations whereas $A \cdot B$ (a measure of the horizontal extension) has the exponent zero in the Newton formulation. It was concluded that $H$ would be the size measure which is most correlated to the fall velocity (Section 5.4). $\alpha$ should also be an important parameter for all flow types whereas the importance of $(A \cdot B)$ gets stronger in laminar flow.

Test of the theory by correlation analyses

The parameters expected to be relevant from theoretical considerations (Chapter 5) were tested by analysing measured data. The 2D–Video–Distrometer (Chapter 2) records shadow images of precipitation particles from two orthogonal directions of view and determines their velocity. As the available shape information is recorded during the fall, the orientation of the particle is known—a disadvantage compared to methods of photographing the particles after they have reached the ground. The 2D–Video–Distrometer made it possible to measure the shape parameters $H$, $\alpha$ and $(A \cdot B)$ automatically for a large number of particles. Additional information on riming and crystal types was gained by evaluating photographs and Formvar replica sampled at the same time (Section 4.1).

First the assumption was tested whether the vertical extension $H$ is correlated to the velocity more than other size measures (Section 6.1). This was done by fitting equations of the form $v = a \cdot D^\beta$ where $v$ stands for the fall velocity. Different definitions of the snowflake size $D$ were investigated using two independent data sets. As expected, the fit with the height $H$ showed the highest coefficient of determination $R^2$ (equation A.3) and, consequently, the lowest standard deviation $s_e$ (equation A.1) for the estimated velocity. The higher correlation between the velocity and the vertical extension $H$ compared to other size measures shows the importance of measuring the particles during fall and, thus, determining their orientation.

In a next step, the dependence of the velocity on the parameters $\alpha$ and $(A \cdot B)$ was tested (Section 6.2). The relative importance of the parameters was judged by the change in $R^2$. Both parameters lead to a significant increase of the percentage of explained variation (measured by $R^2$) compared to the fit with the height $H$ only. In one case, the increase in $R^2$ by including $(A \cdot B)$ was nearly as high as the increase by including $\alpha$. In the other case, $(A \cdot B)$ had less importance. These results agree with the conclusion from theory that $\alpha$ is important for all flow types whereas the importance of $(A \cdot B)$ varies.

All three parameters ($H$, $\alpha$ and $A \cdot B$) together lead to the highest $R^2$ for both analysed precipitation events (lowest standard deviation $s_e$ of the data from the fitted logarithmic equations in Tables 6.2 and 6.4). The main result of these correlation analyses is, therefore, the dominant influence of the parameters $H$, $\alpha$ and $(A \cdot B)$ in the velocity equation. This result was expected from theory (Section 5.5).

Application of the results to snow aggregates

This result was then applied to measured events with the objective to derive velocity equations for different types of crystals and different degrees of riming. Difficulties in choosing the events for the analysis were found (Section 6.3):

- Little change of precipitation should occur during the event, i.e. the degree of
riming and the type of crystal building up the aggregates should be uniform.

- Time intervals with strong winds had to be avoided because wind may influence the velocity.

- Aggregates rather than ice crystals were analysed. To investigate single ice crystals would need a higher size resolution (one millimetre in horizontal distance is resolved in only 5 pixels in the used instrument).

These requirements reduced the number of analysed events to five cases (Table 7.1). Apart from one graupel measurement (28 Mar 97, 3:48), the degree of riming was low (< 1.5) for all other events. The graupel event yielded only 70 particles because a lightning struck the data transfer cable shortly after the onset of precipitation. Nevertheless, this case was analysed for the purpose of comparisons.

First, velocity–size relations were fitted to these cases using the height $H$ as size measure (Section 7.1). The fitted equations were compared to findings in the literature. There was a good agreement for unrimed particles and graupel. The results for unrimed aggregates showed that for aggregates of different crystal types the curves differ less from each other than the standard deviations of the measured particles from the curve fitted in this work (Figure 7.1). Crystal types seem to have a minor influence on the velocity of snow aggregates. Riming is far more important.

If only aggregates (particles > 1 mm) are analysed, the dependence of velocity on size is smaller than in the cases with single crystals and aggregates. Mainly for unrimed aggregates, the gravitational and the frictional force seem to grow to the same extent with growing size of the particle. Variations in fall speed are then partly explained by the shape parameter $\alpha$.

Restricting the analysis to particles larger than 1 mm decreased the number of analysed particles. Problems arose from correlations between $H$ and $(A \cdot B)$ for the regression analysis with the three shape parameters $H$, $\alpha$ and $(A \cdot B)$. The coefficients for $(A \cdot B)$ were not well determined with exception of the case on December 14, 1:00, with a large number of particles >1 mm. To reduce the influence of these correlations, a mathematical transformation was carried out. $H$ and $(A \cdot B)$ were replaced by the alternative parameters $\theta$ ($= H \cdot A \cdot B$) and $\varepsilon$ ($= \frac{H}{\sqrt{(A\cdot B)}}$).

For the parameter combination $\theta$, $\varepsilon$ and $\alpha$, the regression yielded well determined coefficients. Only in the case of graupel, the coefficient for $\varepsilon$ was not significant. This might be caused either by the low number of analysed particles or by the nearly equal horizontal and vertical dimension of graupel.

For the snow aggregates, the combination of shape parameters in the velocity equation yielding the best results is:

- $\theta$, the circumscribed volume,
- $\varepsilon$, the oblateness of the snowflake, and
- $\alpha$, the ratio of cross-sectional to circumscribed area.
Two of the events with snow aggregates cannot be interpreted. On Dec 14, 1:00–1:15, particles were partially melted but it is not known to which extent. On February 05, the vertical wind led to high standard deviations. But the equations fitted to the measured data with degrees of riming $R = 0.4$ (Dec 14, 8:00) and $R = 1$ (Dec 14, 5:36, Table 7.1) can be used to derive velocity values from the shape parameters listed above. It is expected that the velocity of snowflakes of other events with the same degree of riming will deviate less than the standard deviation, also because the velocity–size relations of other researchers agreed well with the data measured here.

Most of the variation which is not explained by the fitted equations is probably caused by the density variations of the particles. At the moment, no method to determine the density of individual particles is applicable to a number of particles high enough for a reliable statistical analysis. This is a severe restriction in the determination of the velocity.

Can the problem be inverted? The equations derived by Böhm for the fall velocity of solid hydrometeors may be solved for the density. This method was applied to the measured particles and yielded reasonable values. Yet, it has to be kept in mind that no independent verification of the density values was possible.
Chapter 9
Conclusions and Outlook

1. With the data sets gathered during this study it was possible to support the use of theoretically derived (Section 5.4) shape parameters in the velocity function (Section 6.2)

\[ v = a_1 \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \]

with \( H \) the vertical extension of the particle, \( \alpha \) the ratio of cross-sectional to circumscribed area and \( (A \cdot B) \) the circumscribed area opposed to the flow. The coefficients \( a_1, b, c \) and \( d \) are determined by regression analysis. The fitted functions have a smaller standard error and explain more of the variability in the data than the generally used velocity-size functions (Sections 7.1 and 7.3). This improvement is achieved because shape and orientation of the particles are taken into account by the use of these parameters.

2. The analysis of the data measured during this study and their comparison with literature values showed that the crystal type has only minor influence on the fall velocity of aggregates (Section 7.1).

3. The parameter \( (A \cdot B) \) has more influence on the velocity for unrimed particles than for rimed particles because the flow around unrimed particles is nearer to the Stokes regime (Section 7.3). Furthermore, the parameter \( \alpha \) is more dependent on size for unrimed particles (Section 7.2).

4. To reduce the influence of the correlation between \( H \) and \( (A \cdot B) \), they are substituted by alternative parameters. \( \theta \) is defined as \( H \cdot A \cdot B \), \( \epsilon \) as \( \frac{H}{\sqrt{A \cdot B}} \). Functions fitted with the parameters \( \theta \), \( \epsilon \) and \( \alpha \) explain the same amount of variability in the data as those fitted with \( H, \alpha \) and \( (A \cdot B) \) but the coefficients of the regression are better determined (Section 7.4). Therefore, the velocity function which is proposed for snow aggregates is

\[ v = a_1 \cdot \theta^k \cdot \epsilon^c \cdot \alpha^e \]
From the measured data, the values of the coefficients can be given for degrees of riming \( R = 0.4 \) and \( R = 1 \). \( s_e \) is the standard deviation of the measured velocities from the function (Tables 7.1 and 7.8).

<table>
<thead>
<tr>
<th>( R )</th>
<th>function</th>
<th>( s_e )</th>
<th>precipitation event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>( v = 1.19 \cdot \theta^{0.06} \cdot \alpha^{0.18} \cdot e^{1.10} )</td>
<td>0.23</td>
<td>December 14, 8:00</td>
</tr>
<tr>
<td>1</td>
<td>( v = 1.10 \cdot \theta^{0.07} \cdot \epsilon^{0.12} \cdot \alpha^{0.44} )</td>
<td>0.20</td>
<td>December 14, 5:36</td>
</tr>
</tbody>
</table>

5. The percentage of variation in the data which can be explained by the functions given above is 41% and 27%, respectively (Section 7.4). Hence, the standard deviations of \( \log(v) \) from the fitted logarithmic functions (\( s_r \)) is only 26% (15%) less than their standard deviation from the mean \( \log(v) \). Although the dependence of velocity on the derived shape parameters could be proven, their use is only a modest step in the explanation of velocity variations. This is partly caused by limitations in the measurement techniques, partly by the complexity of snowflake structure and fall behaviour.

6. Lump graupel has similar vertical and horizontal extension. The parameter \( \epsilon \), which represents the oblateness, is therefore left away. The best combination of shape parameters for this sort of particles is the combination of the size measure \( H \) with the area ratio \( \alpha \).

7. An estimate of the density from the 2D–Video–Distrometer data by inverting the velocity formula of Böhm (1989) yields reasonable values. The wide range of densities for the same aggregate size found by other estimation methods is confirmed here (Section 7.5). Density varies less for larger aggregates, therefore, their velocity also varies less.

8. **Outlook**

To find velocity functions for aggregates with higher degrees of riming and to analyse the dependence of these functions from the degrees of riming and melting, further measurements will be necessary. The gathering of the data is the largest problem in field investigations of the fall velocity of snowflakes. Measurements are time consuming especially because one person has to be present to do the analysis of crystal type and riming degree.

An opportunity to obtain further data will be given in the frame of the Mesoscale Alpine Programme. During this project, the optical spectrometer of the ETH will be placed on a mountain in the Southern Alps. It will be important to build a windbreak around the spectrometer. Thus, wind influence will be reduced and a larger number of data sets can be obtained. It would also be desirable to automatically sample aggregates for the analysis of crystal type and riming degree. In cloud investigations, such instruments exist (Murakami and Matsuo, 1990; Miloshevich and Heymsfield, 1997). It would be necessary to build analogous instruments which are able to sample the larger aggregates. The problem of unknown density, however, would still not be solved. A solution will require the development of new techniques to measure the density of the individual particles.
Appendix A

A.1 Statistical methods

In Chapter 6 and Chapter 7, statistical methods are used to analyze the measured data. The used methods in these two chapters are similar, but the purpose is different. The question to be answered in Chapter 6 is which parameters are best suited to be used in an equation describing the velocity. The answer is given by comparing the correlations between the velocity and different snowflake parameters $X_i$. The dependence of the fall velocity $v$ of a particle from the parameters $X_i$ is described by a function of the form $v = a \cdot X_1^b \cdot (X_2^c \cdot X_3^d)$. This formula is linearized via the logarithm to the base 10:

$$\log(v) = \log(a) + b \cdot \log(X_1) + (c \cdot \log(X_2) + d \cdot \log(X_3)).$$

For the following description of the statistical methods, $\log(a)$ is named $a_1$, $\log(v)$ is named $y$ and $\log(X_i)$ is named $x_i$. The function used for the analysis is then $y = a_1 + b \cdot x$ for the simple correlation and regression analysis and $y = a_1 + b \cdot x_1 + c \cdot x_2 + (d \cdot x_3)$ for the multiple correlation and regression.

As a measure of the correlation of the $x$-values and the $y$-values, the coefficients of determination ($R^2$) are calculated. $R^2$ measures how much of the variation of the measured data is explained by the fitted function. It is given by the ratio of $(measured \ y - mean \ y)^2$ and $(calculated \ y - mean \ y)^2$. The calculation scheme is given below for the case of one regressor ($v = a \cdot X_1^b$, equation A.3) and for the case of multiple regressors (equation A.6).

In Appendix A.2, additional information is given on the populations which were used for the fit. They are characterized by giving the maximum, minimum and mean values as well as the standard deviation from the mean (equation A.4) and the coefficient of variation for the input parameters, e.g. $\log(v)$, $\log(H)$ and $\log(\alpha)$. The calculations are shown below in Section A.1.1. As a result of Chapter 6, the snowflake parameters to be used in the velocity equation are chosen. It is the height $H$, the area ratio $\alpha$ and the circumscribed area exposed to the flow $A \cdot B$ (Section 2.2.2). The purpose of Chapter 7 is the fit of a regression equation to the measured data for different types of snowflakes. This is first done for one regressor (the height $H$) to be able to compare the data measured during this project to the data of other authors. Then, the regression analysis is repeated by adding the other regressors. To indicate the quality of the equation, the standard deviations $s_r$ are given (equations A.2, A.5). $s_r$ is a measure for the deviation of the logarithm of the measured velocity values to the logarithm of the velocity values calculated by the fit equation.
In Chapter 7 the fitted equations are given in the form of power laws. For practical applications it is important to know the accuracy of these equations. For this purpose an 'effective' standard deviation $s_e$ of the calculated from the measured velocities is determined:

$$s_e = \sqrt{\frac{1}{N - n_{coeff}} \sum_{j=1}^{N} (v_j - \hat{v}_j)^2} \quad (A.1)$$

with $v_j$ the measured velocity values and $\hat{v}_j = a \cdot X_1^4 \cdot X_2^6 \cdot X_3^4$. $n_{coeff}$ is the number of coefficients.

The statistical methods described here are drawn from the book of Sachs (1978) for the simple linear regression (Section A.1.1) and from the book of Hartung (1992) for the multiple linear regression (Section A.1.2). Comprehensive derivations of the equations can be found there.

### A.1.1 Correlation and linear regression

Equation: $y = a_1 + b \cdot x$

$\hat{y}$ is the $y$-value calculated with this function for a given $x$-value: $\hat{y} = a_1 + b \cdot x$

$N$ is the number of data points.

**Preparatory calculations:**

- sum of all $x$-values: $\sum x$
- sum of all $y$-values: $\sum y$
- sum of the squares: $\sum x^2$, $\sum y^2$
- sum of the product $x \cdot y$: $\sum xy$

**mean values of $x$ and $y$:** $\bar{x} = \frac{1}{N} \sum x$, $\bar{y} = \frac{1}{N} \sum y$

**sum of the squared deviations from the mean:**

$Q_x = \sum (x - \bar{x})^2 = \sum x^2 - \frac{1}{N} (\sum x)^2$

$Q_y = \sum (y - \bar{y})^2 = \sum y^2 - \frac{1}{N} (\sum y)^2$

**sum of the products of the deviations:**

$Q_{xy} = \sum (x - \bar{x})(y - \bar{y}) = \sum xy - \frac{1}{N}(\sum x)(\sum y)$

**Calculation of the intercept and the regression coefficient**

regression coefficient $b = \frac{Q_{xy}}{Q_x}$

intercept $a_1 = \bar{y} - b \cdot \bar{x}$

**standard deviation**

$$s_r = \sqrt{\frac{\sum (y - \hat{y})^2}{(N - 2)}} \quad (A.2)$$
A.1.1 Statistical methods

Standard error of intercept and regression coefficient

$$s_b = \frac{s_r}{\sqrt{Q_x}}$$
$$s_{a_i} = s_r \sqrt{\frac{1}{N} + \frac{x^2}{Q_x}}$$

Coefficient of determination $R^2$

$$R^2 = \left( \frac{Q_{xy}}{Q_x Q_y} \right)^2 \quad (A.3)$$

Standard deviation of $x$ and $y$

$$s_x = \sqrt{\frac{Q_x}{n - 1}} \quad (A.4)$$
$$s_y = \sqrt{\frac{Q_y}{n - 1}}$$

A.1.2 Multiple linear regression

The multiple regression analysis can best be shown in matrix notation. $Y$ is the vector of the $N$ measured $y$-values. $X$ is the matrix of the measured $x_{ji}$-values (j from 1 to N, i number of variables from 0 to 3) and $\hat{b}$ is the vector of the coefficients (for example $a_i$, $b$, $c$ and $d$ in the equation $y_j = a_i + b \cdot x_{j1} + c \cdot x_{j2} + d \cdot x_{j3}$).

$$Y = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}, \quad X = \begin{pmatrix} 1 & X_{11} & X_{12} & X_{13} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & X_{N1} & X_{N2} & X_{N3} \end{pmatrix}, \quad \hat{b} = \begin{pmatrix} a_i \\ b \\ c \\ d \end{pmatrix}$$

The systems of equations which has to be solved is then:

$$X^T X \hat{b} = X^T Y,$$

with $X^T$ the transposed matrix of $X$.

The standard deviation is

$$s_r = \sqrt{\frac{1}{N - 4} \sum_{j=1}^{N} (y_j - \bar{y}_j)^2} \quad (A.5)$$

with $\bar{y}_j = a_i + b \cdot x_{j1} + c \cdot x_{j2} + d \cdot x_{j3}$.

The multiple coefficient of determination $R^2$ is

$$R^2 = 1 - \frac{\sum_{j=1}^{N} (y_j - \bar{y}_j)^2}{\sum_{j=1}^{N} (y_j - \bar{y})^2} \quad (A.6)$$

$\bar{y}$ is the mean value of the $y_j$. 
A.1.3 Statistical tests

t-test

To test the significance of a regressor \( \log(X_i) \) in the equation \( \log(v) = a_1 + b \cdot \log(X_1) + c \cdot \log(X_2) + d \cdot \log(X_3) \), a statistical test is made. If for example the significance of the influence of \( \log(X_1) \) on the regressand \( \log(v) \) is tested, a test quantity \( T \) is calculated:

\[
T = \frac{b - b_0}{s_b}
\]

\( b \) is the value calculated by regression, \( s_b \) its standard error, \( b_0 \) is set to the value against which we want to test (generally zero). The assumption that \( b_0 \) is valid instead of \( b \) is the so-called null hypothesis. \( T \) has a \( t \)-distribution. This distribution is symmetric and its shape depends on the degrees of freedom. For a large number of data points it approximates the normal distribution. The threshold value \( t \) marks the region wherein a random value will lie with a probability of 95%. If the calculated \( T \) is outside this region, the null hypothesis can be rejected. These \( t \)-values are tabulated. To simplify the test procedure, standardized \( p \)-values are used which are calculated from the \( t \)-values. The \( p \)-value is the lowest level at which the test would have rejected the null hypothesis. Here, it is required that the probability of the null hypothesis to be valid is below 5%. A \( p \)-value less than 0.05 is then equivalent to the statement that the influence of the variable \( \log(X_1) \) on the regressand \( y \) is statistically ensured.

The same test is made for the other variables. The calculated \( p \)-values may indicate the relative importance of the variables. If the \( p \)-value of one variable is below 0.05 it is concluded that this variable has no statistically ensured influence on the velocity. (This is not equivalent to the statement that the variable physically has no influence on the velocity.)

F-test

In the case of more than one regressor, the influence of all variables together on the velocity can be tested. The test statistic is then:

\[
T = \frac{\sum(y - \hat{y})^2 - \sum(y - \bar{y})^2}{\frac{3}{N-4}}
\]

In this case, \( T \) has a \( F \)-distribution. The \( F \)-distribution is not symmetric. Its shape depends on two degrees of freedom: The number of coefficients minus 1 (here: 3) and the number of data points minus the number of coefficients (here: \( N-4 \)). Analogous to the \( t \)-test above, the significance of the influence of all the \( x_i \) together is given if the \( p \)-value is less than 0.05.

A more thorough explanation of the test statistics and tabulated \( t \)- and \( F \)-values can, for example, be found in the books of Sachs (1978) and Hartung (1991).


A.2 Tables to Chapter 6

The data sets used in Chapter 6 include all particle sizes.

<table>
<thead>
<tr>
<th>Definition for D</th>
<th>a</th>
<th>$s_a$</th>
<th>b</th>
<th>$s_b$</th>
<th>$s_r$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) width $A$</td>
<td>0.0462</td>
<td>0.0014</td>
<td>0.0990</td>
<td>0.0027</td>
<td>0.1061</td>
<td>0.1890</td>
</tr>
<tr>
<td>b) width $B$</td>
<td>0.0487</td>
<td>0.0014</td>
<td>0.0858</td>
<td>0.0024</td>
<td>0.1063</td>
<td>0.1870</td>
</tr>
<tr>
<td>c) maximum width $W$</td>
<td>0.0377</td>
<td>0.0014</td>
<td>0.1005</td>
<td>0.0029</td>
<td>0.1068</td>
<td>0.1789</td>
</tr>
<tr>
<td>d) height $H$</td>
<td>0.0534</td>
<td>0.0013</td>
<td>0.1341</td>
<td>0.0024</td>
<td>0.09415</td>
<td>0.3617</td>
</tr>
<tr>
<td>e) $\frac{W+H}{2}$</td>
<td>0.0419</td>
<td>0.0013</td>
<td>0.1254</td>
<td>0.0028</td>
<td>0.1007</td>
<td>0.2702</td>
</tr>
<tr>
<td>f) $\sqrt{W \cdot H}$</td>
<td>0.0444</td>
<td>0.0013</td>
<td>0.1251</td>
<td>0.0027</td>
<td>0.09999</td>
<td>0.2801</td>
</tr>
<tr>
<td>g) maximum$(W, H)$</td>
<td>0.0321</td>
<td>0.0014</td>
<td>0.1223</td>
<td>0.0029</td>
<td>0.1026</td>
<td>0.2423</td>
</tr>
<tr>
<td>h) $\frac{4M}{\pi}$</td>
<td>0.0534</td>
<td>0.0013</td>
<td>0.1312</td>
<td>0.0027</td>
<td>0.0986</td>
<td>0.3000</td>
</tr>
</tbody>
</table>

Table A.1: Supplement to Table 6.1. Comparison of the different definitions for the size $D$ of a snowflake. Coefficients $(a, b)$ and their standard errors $(s_a, s_b)$ for the fit function $\log(v) = a + b \cdot \log(D)$. Standard deviation $s_r$ of $\log(v)$ from the logarithmic function (equation A.2) and coefficient of determination $R^2$. Data from December 14, 1996, 1:00 – 1:15 pm. Number of analysed particles: 5616.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(A)$</td>
<td>-1.42</td>
<td>-0.01342</td>
<td>1.138</td>
<td>0.5176</td>
</tr>
<tr>
<td>$\log(B)$</td>
<td>-1.523</td>
<td>-0.04466</td>
<td>1.099</td>
<td>0.5940</td>
</tr>
<tr>
<td>$\log(W)$</td>
<td>-1.42</td>
<td>0.07095</td>
<td>1.138</td>
<td>0.4960</td>
</tr>
<tr>
<td>$\log(H)$</td>
<td>-1.854</td>
<td>-0.06348</td>
<td>1.273</td>
<td>0.5286</td>
</tr>
<tr>
<td>$\log(\frac{W+H}{2})$</td>
<td>-1.561</td>
<td>0.02388</td>
<td>1.074</td>
<td>0.4885</td>
</tr>
<tr>
<td>$\log(\sqrt{W \cdot H})$</td>
<td>-1.602</td>
<td>0.003756</td>
<td>1.066</td>
<td>0.4986</td>
</tr>
<tr>
<td>$\log(\text{maximum}(W, H))$</td>
<td>-1.42</td>
<td>0.1047</td>
<td>1.273</td>
<td>0.4743</td>
</tr>
<tr>
<td>$\log(\frac{4M}{\pi})$</td>
<td>-1.448</td>
<td>-0.06474</td>
<td>0.985</td>
<td>0.4918</td>
</tr>
<tr>
<td>$\log(A \cdot B)$</td>
<td>-2.932</td>
<td>-0.05802</td>
<td>2.237</td>
<td>1.0725</td>
</tr>
<tr>
<td>$\log(M)$</td>
<td>-3.0</td>
<td>-0.2344</td>
<td>1.866</td>
<td>0.9837</td>
</tr>
<tr>
<td>$\log(o)$</td>
<td>-0.295</td>
<td>-0.05599</td>
<td>0.355</td>
<td>0.0632</td>
</tr>
<tr>
<td>$\log(v)$</td>
<td>-0.223</td>
<td>0.04486</td>
<td>0.602</td>
<td>0.1178</td>
</tr>
</tbody>
</table>

Table A.2: Data used for the fits in Tables 6.1 and 6.2. Characterization of the fit parameters. Minimum, median, mean and maximum values and standard deviation of the parameters from their mean values (equation A.4). Number of particles: 5616. The sizes $A, B, W, H$ are in mm, the area $M$ is in mm$^2$. 
Table A.3: Supplement to Table 6.2. Fit with $A \cdot B$, $M$ and $\alpha$ as parameters, data from December 14, 1996, 1:00-1:15. Coefficients and their standard errors, standard deviation $s_e$ of $\log(v)$ from the logarithmic function (equations A.2, A.5) and coefficient of determination $R^2$ for the functions of the form $\log(v) = a + b \cdot \log(x_1) + c \cdot \log(x_2) + d \cdot \log(x_3)$. The number of analysed particles is 5616.

<table>
<thead>
<tr>
<th>Fit parameters ($x_i$)</th>
<th>$a$</th>
<th>$s_a$</th>
<th>$b$</th>
<th>$s_b$</th>
<th>$c$</th>
<th>$s_c$</th>
<th>$d$</th>
<th>$s_d$</th>
<th>$s_e$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(A \cdot B)$</td>
<td>0.0477</td>
<td>0.0014</td>
<td>0.0494</td>
<td>0.0013</td>
<td></td>
<td></td>
<td></td>
<td>0.1053</td>
<td>0.2019</td>
<td></td>
</tr>
<tr>
<td>$\log(M)$</td>
<td>0.0602</td>
<td>0.0014</td>
<td>0.0656</td>
<td>0.0013</td>
<td></td>
<td></td>
<td></td>
<td>0.0986</td>
<td>0.3001</td>
<td></td>
</tr>
<tr>
<td>$\log(\alpha)$</td>
<td>0.0291</td>
<td>0.0021</td>
<td>-0.2746</td>
<td>0.0248</td>
<td></td>
<td></td>
<td></td>
<td>0.1166</td>
<td>0.0214</td>
<td></td>
</tr>
<tr>
<td>$\log(H)$ and $\log(A \cdot B)$</td>
<td>0.0579</td>
<td>0.0012</td>
<td>0.2660</td>
<td>0.0057</td>
<td>-0.0709</td>
<td>0.0028</td>
<td></td>
<td>0.08916</td>
<td>0.4278</td>
<td></td>
</tr>
<tr>
<td>$\log(H)$ and $\log(M)$</td>
<td>0.0389</td>
<td>0.0014</td>
<td>0.3420</td>
<td>0.0112</td>
<td>-0.1142</td>
<td>0.0060</td>
<td></td>
<td>0.09126</td>
<td>0.4005</td>
<td></td>
</tr>
<tr>
<td>$\log(H)$ and $\log(\alpha)$</td>
<td>0.0966</td>
<td>0.0019</td>
<td>0.1900</td>
<td>0.0028</td>
<td>0.7411</td>
<td>0.0239</td>
<td></td>
<td>0.08703</td>
<td>0.4548</td>
<td></td>
</tr>
<tr>
<td>$\log(H)$ and $\log(A \cdot B)$ and $\log(M)$</td>
<td>0.1007</td>
<td>0.0030</td>
<td>-0.0221</td>
<td>0.0191</td>
<td>-0.2043</td>
<td>0.0089</td>
<td>0.2946</td>
<td>0.0187</td>
<td>0.08726</td>
<td>0.4520</td>
</tr>
<tr>
<td>$\log(H)$ and $\log(A \cdot B)$ and $\log(\alpha)$</td>
<td>0.0939</td>
<td>0.0018</td>
<td>0.2759</td>
<td>0.0054</td>
<td>-0.0508</td>
<td>0.0028</td>
<td>0.6151</td>
<td>0.0243</td>
<td>0.08448</td>
<td>0.4863</td>
</tr>
</tbody>
</table>
Table A.4: Supplement to Table 6.3. Comparison of the different definitions for the size of a snowflake. Same as Table A.1 for the data from February 5, 1997, 5:10 – 5:18 pm. The number of analysed particles is 1420.

<table>
<thead>
<tr>
<th>Definition for D</th>
<th>a</th>
<th>$a_0$</th>
<th>b</th>
<th>$s_b$</th>
<th>$s_r$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) width A</td>
<td>0.0922</td>
<td>0.0046</td>
<td>0.1054</td>
<td>0.0075</td>
<td>0.1583</td>
<td>0.1143</td>
</tr>
<tr>
<td>b) width B</td>
<td>0.0881</td>
<td>0.0045</td>
<td>0.0987</td>
<td>0.0075</td>
<td>0.1588</td>
<td>0.1091</td>
</tr>
<tr>
<td>c) maximum width W</td>
<td>0.0821</td>
<td>0.0044</td>
<td>0.1033</td>
<td>0.0083</td>
<td>0.1588</td>
<td>0.09803</td>
</tr>
<tr>
<td>d) height H</td>
<td>0.0898</td>
<td>0.0042</td>
<td>0.1643</td>
<td>0.0086</td>
<td>0.1499</td>
<td>0.2056</td>
</tr>
<tr>
<td>e) $\frac{W+H}{2}$</td>
<td>0.0851</td>
<td>0.0043</td>
<td>0.1427</td>
<td>0.0088</td>
<td>0.1546</td>
<td>0.1558</td>
</tr>
<tr>
<td>f) $\sqrt{W \cdot H}$</td>
<td>0.0863</td>
<td>0.0043</td>
<td>0.1380</td>
<td>0.0087</td>
<td>0.1550</td>
<td>0.1507</td>
</tr>
<tr>
<td>g) maximum$(W,H)$</td>
<td>0.0763</td>
<td>0.0042</td>
<td>0.1452</td>
<td>0.0091</td>
<td>0.1550</td>
<td>0.1516</td>
</tr>
<tr>
<td>h) $\sqrt[4]{\frac{4M}{\pi}}$</td>
<td>0.0962</td>
<td>0.0044</td>
<td>0.1504</td>
<td>0.0088</td>
<td>0.1532</td>
<td>0.1709</td>
</tr>
</tbody>
</table>

Table A.5: Data used for the fits in Tables 6.3 and 6.4. Characterization of the fit parameters. Minimum, median, mean and maximum values and standard deviation of the parameters from their mean values (equation A.4). Number of particles: 1420. The sizes A, B, W, H are in mm, the area $M$ is in mm$^2$. 

<table>
<thead>
<tr>
<th>Parameter</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(A)</td>
<td>-1.42</td>
<td>-0.2395</td>
<td>1.06</td>
<td>0.5383</td>
</tr>
<tr>
<td>log(B)</td>
<td>-1.523</td>
<td>-0.2221</td>
<td>0.98</td>
<td>0.5623</td>
</tr>
<tr>
<td>log(W)</td>
<td>-1.42</td>
<td>-0.1429</td>
<td>1.06</td>
<td>0.5067</td>
</tr>
<tr>
<td>log(H)</td>
<td>-1.796</td>
<td>-0.1401</td>
<td>0.964</td>
<td>0.4642</td>
</tr>
<tr>
<td>log($\frac{W+H}{2}$)</td>
<td>-1.545</td>
<td>-0.1283</td>
<td>1.011</td>
<td>0.4653</td>
</tr>
<tr>
<td>log($\sqrt{W \cdot H}$)</td>
<td>-1.592</td>
<td>-0.1415</td>
<td>1.007</td>
<td>0.4733</td>
</tr>
<tr>
<td>log(maximum(W,H))</td>
<td>-1.42</td>
<td>-0.06584</td>
<td>1.06</td>
<td>0.4511</td>
</tr>
<tr>
<td>log($\sqrt[4]{\frac{4M}{\pi}}$)</td>
<td>-1.448</td>
<td>-0.1953</td>
<td>0.928</td>
<td>0.4625</td>
</tr>
<tr>
<td>log(A · B)</td>
<td>-2.943</td>
<td>-0.4615</td>
<td>2.031</td>
<td>1.0685</td>
</tr>
<tr>
<td>log(M)</td>
<td>-3.00</td>
<td>-0.4956</td>
<td>1.751</td>
<td>0.9249</td>
</tr>
<tr>
<td>log($\alpha$)</td>
<td>-0.259</td>
<td>-0.03343</td>
<td>0.345</td>
<td>0.0625</td>
</tr>
<tr>
<td>log(ρ)</td>
<td>-0.228</td>
<td>0.06731</td>
<td>0.587</td>
<td>0.1671</td>
</tr>
<tr>
<td>Fit parameters ((x_i))</td>
<td>(a)</td>
<td>(s_a)</td>
<td>(b)</td>
<td>(s_b)</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>(\log(A \cdot B))</td>
<td>0.0918</td>
<td>0.0046</td>
<td>0.0541</td>
<td>0.0039</td>
</tr>
<tr>
<td>(\log(M))</td>
<td>0.1040</td>
<td>0.0046</td>
<td>0.0752</td>
<td>0.0044</td>
</tr>
<tr>
<td>(\log(\alpha))</td>
<td>0.0601</td>
<td>0.0048</td>
<td>-0.2074</td>
<td>0.0597</td>
</tr>
<tr>
<td>(\log(H)) and (A \cdot B)</td>
<td>0.0795</td>
<td>0.0044</td>
<td>0.2910</td>
<td>0.0201</td>
</tr>
<tr>
<td>(\log(H)) and (\log(M))</td>
<td>0.0667</td>
<td>0.0059</td>
<td>0.3651</td>
<td>0.0379</td>
</tr>
<tr>
<td>(\log(H)) and (\log(\alpha))</td>
<td>0.1521</td>
<td>0.0052</td>
<td>0.2870</td>
<td>0.0106</td>
</tr>
<tr>
<td>(\log(H)) and (\log(A \cdot B)) and (\log(M))</td>
<td>0.1722</td>
<td>0.0113</td>
<td>-0.3192</td>
<td>0.0731</td>
</tr>
<tr>
<td>(\log(H)) and (\log(A \cdot B)) and (\log(\alpha))</td>
<td>0.1449</td>
<td>0.0059</td>
<td>0.3367</td>
<td>0.0187</td>
</tr>
</tbody>
</table>

Table A.6: Supplement to Table 6.4. Same as Table A.3 for the data from February 5, 1997. The number of analysed particles is 1420.
### A.3 Tables to Chapter 7

The data sets include only particles larger than 1 mm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>min</th>
<th>mean</th>
<th>max</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>December 14, 8:00–8:15, N=567, riming 0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(H)</td>
<td>0.008</td>
<td>0.2991</td>
<td>0.961</td>
<td>0.20</td>
</tr>
<tr>
<td>log(α)</td>
<td>-0.361</td>
<td>-0.1268</td>
<td>0.032</td>
<td>0.075</td>
</tr>
<tr>
<td>log(A·B)</td>
<td>0.014</td>
<td>0.5089</td>
<td>1.753</td>
<td>0.365</td>
</tr>
<tr>
<td>log(θ)</td>
<td>0.024</td>
<td>0.898</td>
<td>2.635</td>
<td>0.528</td>
</tr>
<tr>
<td>log(ε)</td>
<td>-0.424</td>
<td>-0.0004</td>
<td>0.654</td>
<td>0.134</td>
</tr>
<tr>
<td>log(v)</td>
<td>-0.22</td>
<td>0.006053</td>
<td>0.523</td>
<td>0.11</td>
</tr>
<tr>
<td>December 14, 1:00–1:15, N=2747, riming 0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(H)</td>
<td>0.008</td>
<td>0.3505</td>
<td>1.273</td>
<td>0.22</td>
</tr>
<tr>
<td>log(α)</td>
<td>-0.298</td>
<td>-0.08337</td>
<td>0.021</td>
<td>0.042</td>
</tr>
<tr>
<td>log(A·B)</td>
<td>0.029</td>
<td>0.8149</td>
<td>2.237</td>
<td>0.43</td>
</tr>
<tr>
<td>log(θ)</td>
<td>0.05</td>
<td>1.195</td>
<td>3.232</td>
<td>0.628</td>
</tr>
<tr>
<td>log(ε)</td>
<td>-0.497</td>
<td>-0.0270</td>
<td>0.734</td>
<td>0.125</td>
</tr>
<tr>
<td>log(v)</td>
<td>-0.211</td>
<td>0.1078</td>
<td>0.594</td>
<td>0.075</td>
</tr>
<tr>
<td>December 14, 5:36–5:43, N=637, riming 1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(H)</td>
<td>0.005</td>
<td>0.377</td>
<td>1.093</td>
<td>0.21</td>
</tr>
<tr>
<td>log(α)</td>
<td>-0.264</td>
<td>-0.1006</td>
<td>0</td>
<td>0.043</td>
</tr>
<tr>
<td>log(A·B)</td>
<td>0.023</td>
<td>0.7302</td>
<td>1.74</td>
<td>0.37</td>
</tr>
<tr>
<td>log(θ)</td>
<td>0.036</td>
<td>1.107</td>
<td>2.664</td>
<td>0.551</td>
</tr>
<tr>
<td>log(ε)</td>
<td>-0.384</td>
<td>0.01194</td>
<td>0.583</td>
<td>0.138</td>
</tr>
<tr>
<td>log(v)</td>
<td>-0.208</td>
<td>0.05607</td>
<td>0.551</td>
<td>0.076</td>
</tr>
<tr>
<td>February 05, 5:10–5:18, N=479, riming 1.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(H)</td>
<td>0.008</td>
<td>0.3188</td>
<td>0.964</td>
<td>0.19</td>
</tr>
<tr>
<td>log(α)</td>
<td>-0.264</td>
<td>-0.06886</td>
<td>0.018</td>
<td>0.040</td>
</tr>
<tr>
<td>log(A·B)</td>
<td>0.026</td>
<td>0.526</td>
<td>2.031</td>
<td>0.33</td>
</tr>
<tr>
<td>log(θ)</td>
<td>0.069</td>
<td>0.9028</td>
<td>2.986</td>
<td>0.499</td>
</tr>
<tr>
<td>log(ε)</td>
<td>-0.312</td>
<td>0.0268</td>
<td>0.706</td>
<td>0.118</td>
</tr>
<tr>
<td>log(v)</td>
<td>-0.228</td>
<td>0.1609</td>
<td>0.579</td>
<td>0.14</td>
</tr>
<tr>
<td>March 28, 3:48–3:49, N=70, riming 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(H)</td>
<td>0.569</td>
<td>0.753</td>
<td>0.923</td>
<td>0.089</td>
</tr>
<tr>
<td>log(α)</td>
<td>-0.095</td>
<td>-0.03843</td>
<td>0.004</td>
<td>0.018</td>
</tr>
<tr>
<td>log(A·B)</td>
<td>1.121</td>
<td>1.473</td>
<td>1.843</td>
<td>0.16</td>
</tr>
<tr>
<td>log(θ)</td>
<td>1.705</td>
<td>2.226</td>
<td>2.766</td>
<td>0.248</td>
</tr>
<tr>
<td>log(ε)</td>
<td>-0.091</td>
<td>0.01633</td>
<td>0.076</td>
<td>0.028</td>
</tr>
<tr>
<td>log(v)</td>
<td>0.344</td>
<td>0.5923</td>
<td>0.761</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Table A.7: Characterization of the populations used in Chapter 7: Minimum, mean, and maximum value, and standard deviation from the mean (equation A.4). N is the number of analysed particles.
<table>
<thead>
<tr>
<th>Event</th>
<th>N</th>
<th>riming</th>
<th>log(a)</th>
<th>$s_{\log(a)}$</th>
<th>b</th>
<th>$s_b$</th>
<th>c</th>
<th>$s_c$</th>
<th>$s_r$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 8:00</td>
<td>567</td>
<td>0.4</td>
<td>0.0001</td>
<td>0.0082</td>
<td>0.0200</td>
<td>0.0229</td>
<td>0.1665</td>
<td>0.0013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dec 14, 1:00</td>
<td>2747</td>
<td>0.7</td>
<td>0.0470</td>
<td>0.0025</td>
<td>0.1595</td>
<td>0.0056</td>
<td>0.06607</td>
<td>0.2273</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dec 14, 5:36</td>
<td>637</td>
<td>1.0</td>
<td>0.0017</td>
<td>0.0056</td>
<td>0.1443</td>
<td>0.0129</td>
<td>0.06927</td>
<td>0.1646</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feb 05, 5:10</td>
<td>479</td>
<td>1.2</td>
<td>0.1561</td>
<td>0.0124</td>
<td>0.0149</td>
<td>0.0332</td>
<td>0.14</td>
<td>0.0004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mar 28, 3:48</td>
<td>70</td>
<td>5</td>
<td>0.1386</td>
<td>0.0561</td>
<td>0.6025</td>
<td>0.0740</td>
<td>0.05454</td>
<td>0.4933</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.8: Same as Table A.10 but for the function $\log(v) = \log(a) + b \cdot \log(H)$ (Section 7.1).

<table>
<thead>
<tr>
<th>Event</th>
<th>N</th>
<th>riming</th>
<th>log(a)</th>
<th>$s_{\log(a)}$</th>
<th>b</th>
<th>$s_b$</th>
<th>c</th>
<th>$s_c$</th>
<th>$s_r$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 8:00</td>
<td>567</td>
<td>0.4</td>
<td>0.0742</td>
<td>0.0073</td>
<td>0.2142</td>
<td>0.0202</td>
<td>1.0427</td>
<td>0.0527</td>
<td>0.0819</td>
<td>0.4106</td>
</tr>
<tr>
<td>Dec 14, 1:00</td>
<td>2747</td>
<td>0.7</td>
<td>0.0046</td>
<td>0.0028</td>
<td>0.2191</td>
<td>0.0063</td>
<td>0.6026</td>
<td>0.0339</td>
<td>0.06257</td>
<td>0.3073</td>
</tr>
<tr>
<td>Dec 14, 5:36</td>
<td>637</td>
<td>1.0</td>
<td>0.0429</td>
<td>0.0068</td>
<td>0.2067</td>
<td>0.0137</td>
<td>0.6440</td>
<td>0.0675</td>
<td>0.06482</td>
<td>0.2696</td>
</tr>
<tr>
<td>Feb 05, 5:10</td>
<td>479</td>
<td>1.2</td>
<td>0.2194</td>
<td>0.0126</td>
<td>0.1929</td>
<td>0.0342</td>
<td>1.7427</td>
<td>0.1631</td>
<td>0.1258</td>
<td>0.1938</td>
</tr>
<tr>
<td>Mar 28, 3:48</td>
<td>70</td>
<td>5</td>
<td>0.1598</td>
<td>0.0532</td>
<td>0.6318</td>
<td>0.0702</td>
<td>1.1237</td>
<td>0.3553</td>
<td>0.05125</td>
<td>0.5591</td>
</tr>
</tbody>
</table>

Table A.9: Same as Table A.10 but for the function $\log(v) = \log(a) + b \cdot \log(H) + c \cdot \log(\alpha)$. 
Table A.10: Coefficients and their standard errors for the fits of Section 7.3. Function: \( \log(v) = \log(a) + b \cdot \log(H) + c \cdot \log(\alpha) + d \cdot \log(A \cdot B) \). \( s_r \) is the standard deviation of the measured \( \log(v) \) from the fitted logarithmic function (equation A.5) and \( R^2 \) the coefficient of determination.

<table>
<thead>
<tr>
<th>Event</th>
<th>riming</th>
<th>( \log(a) )</th>
<th>( s_{\log(a)} )</th>
<th>( b )</th>
<th>( s_b )</th>
<th>( c )</th>
<th>( s_c )</th>
<th>( d )</th>
<th>( s_d )</th>
<th>( s_r )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 14, 8:00</td>
<td>0.4</td>
<td>0.0772</td>
<td>0.0075</td>
<td>0.2481</td>
<td>0.0270</td>
<td>1.0144</td>
<td>0.0547</td>
<td>-0.0280</td>
<td>0.0149</td>
<td>0.0817</td>
<td>0.4143</td>
</tr>
<tr>
<td>Dec 14, 1:00</td>
<td>0.7</td>
<td>0.0839</td>
<td>0.0029</td>
<td>0.3024</td>
<td>0.0098</td>
<td>0.5553</td>
<td>0.0334</td>
<td>-0.0552</td>
<td>0.0051</td>
<td>0.0613</td>
<td>0.3361</td>
</tr>
<tr>
<td>Dec 14, 5:36</td>
<td>1.0</td>
<td>0.0414</td>
<td>0.0070</td>
<td>0.1943</td>
<td>0.0192</td>
<td>0.6567</td>
<td>0.0689</td>
<td>0.0102</td>
<td>0.0111</td>
<td>0.0648</td>
<td>0.2706</td>
</tr>
<tr>
<td>Feb 05, 5:10</td>
<td>1.2</td>
<td>0.2242</td>
<td>0.0130</td>
<td>0.2484</td>
<td>0.0501</td>
<td>1.6998</td>
<td>0.1653</td>
<td>-0.0437</td>
<td>0.0288</td>
<td>0.1257</td>
<td>0.1976</td>
</tr>
<tr>
<td>Mar 28, 3:48</td>
<td>5</td>
<td>0.1396</td>
<td>0.0564</td>
<td>0.4061</td>
<td>0.2243</td>
<td>1.1889</td>
<td>0.3603</td>
<td>0.1307</td>
<td>0.1234</td>
<td>0.0512</td>
<td>0.5665</td>
</tr>
</tbody>
</table>

Table A.11: Same as Table A.10 for the fits in Section 7.4. Function: \( \log(v) = \log(a) + k \cdot \log(\theta) + n \cdot \log(\epsilon) + c \cdot \log(\alpha) \).
A.4 Data correction for height above sea level

All hydrometeor data used in this work were obtained at a height of 1600 metres above
sea level (at Rigi Staffel). They have to be corrected for the height above sea level
before they can be compared with data of other researchers. A correction formula for
raindrops was given by Foote and du Toit (1969): \( v = v_0 \left( \frac{\rho_0}{\rho} \right)^{0.4} \). \( v \) and \( \rho \) are the
velocity and air density at the given height, \( v_0 \) and \( \rho_0 \) the velocity and air density at
sea level.

Foote and du Toit (1969) had been fitting velocity data measured at low air densities.
They found that drag coefficients for raindrops are higher at low air densities caused by
enhanced flattening of the drops. It is not known how the drag coefficients of snowflakes
vary with changing air density. As a first approximation it is assumed here that they
do not change with higher elevations. The formula proposed by Zikmunda (1972) can
then be used: \( v = v_0 \left( \frac{\rho_0}{\rho} \right)^{0.5} \). According to the results of Beard (1980), this correction
factor does only apply for Reynolds numbers higher than 1000. He proposes a more
accurate formula depending on the Reynolds number of the particle. The Reynolds
number for porous aggregates is, however, not well defined.

Instead of air density which is generally not measured, air pressure can be used
in the correction formula. At a given temperature, density and pressure of the air
are connected by the function \( \rho = \frac{p}{RT} \) with \( R^* \) the universal gas constant and \( T \)
the absolute temperature in Kelvin. Assuming a standard atmosphere, the correction
factor for the height of 1600 m (Rigi–Staffel) is 1.12. This means that snowflakes fall
about 12% faster at Rigi–Staffel than at sea level.

In Section 7.1, the measured values are compared to the values of Locatelli and Hobbs
(1974). These authors made their measurements between 750 and 1500 metres above
sea level. A medium height of their measurements is assumed at 1125 metres. The
difference to values measured at the height of the 2D–Video–Distrometer (1600 m) is,
therefore, below 3% and is not corrected for.
Bibliography


List of symbols

\( \alpha \) Ratio of the area shaded by a particle to the area circumscribed to the shadow. As two areas are recorded, the geometric mean of both is taken. \( \alpha = \sqrt{\frac{A_{H-A} \cdot A_{H-B}}{H-A \cdot H-B}} \) (Section 2.2.2). Dimensionless.

\( \beta \) Constant in the formula for the calculation of air viscosity (Section 5.2.1). \( \eta = (\beta \cdot T^{3/2})/(T + Z) \). \( \beta = 1.458 \cdot 10^{-6} \frac{kg}{sm \cdot \sqrt{R}} \).

\( \gamma \) Deviation of the major axis of a particle from the horizontal or vertical orientation (Section 2.3.2). [°]

\( \delta \) Thickness of the boundary layer (Section 5.3.3). [m]

\( \delta \) Error of the fall velocity (Section 3.1). [m/s]

\( \epsilon \) \( \epsilon = \frac{H}{\sqrt{A \cdot B}} \). \( \epsilon \) represents the oblateness of a particle (Section 7.4). Dimensionless

\( \eta \) Viscosity of the air. [\( \frac{kg}{m \cdot s} \)]

\( \theta \) \( \theta = A \cdot B \cdot H \). \( \theta \) is proportional to the volume of the ellipsoid circumscribed to a particle (Section 7.4). [m/m³]

\( \rho_0 \) Density of the air at sea level (Section A.4). [\( \frac{kg}{m^3} \)]

\( \rho_0 \) Density of the air. [\( \frac{kg}{m^3} \)]

\( \rho_s \) Density of a precipitation particle. [\( \frac{kg}{m^3} \)]

\( \sigma \) Tension (Section 5.3). [m/s²]

\( \Delta \) Laplace operator (Section 5.1). \( \Delta = \nabla^2 \).

\( \nabla \) Nabla operator (Section 5.1). Gradient.

\( a \) Coefficient in the equation \( v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \) or \( v = a \cdot \theta^b \cdot e^c \cdot \alpha^d \).

\( a_a \) Acceleration of a volume element within a fluid (Sections 5.1 and 5.2). [m/s²]

\( a_i \) \( a_i = \log(a) \) (Appendix A.1).

\( a_s \) Acceleration of a precipitation particle (Section 5.4). [m/s²]

\( b \) Exponent of \( H \) in the equation \( v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \).

\( b_0 \) The value to which \( b \) is set in the null hypothesis of the \( t \)-test (Appendix A.1.3).

\( \hat{b} \) Vector of estimated coefficients in the multiple linear regression (Appendix A.1.2).

\( c \) Exponent of \( a \) in the equation \( v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \) or \( v = a \cdot \theta^b \cdot e^c \cdot \alpha^d \).

\( d \) Exponent of \( (A \cdot B) \) in the equation \( v = a \cdot H^b \cdot \alpha^c \cdot (A \cdot B)^d \).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_0$</td>
<td>Distance (Section 5.2). [m]</td>
</tr>
<tr>
<td>$d_1$</td>
<td>Distance for a significant change of $u$ due to inertia (Section 5.2). [m]</td>
</tr>
<tr>
<td>$d_2$</td>
<td>Distance for a significant change of $u$ due to frictional forces (Section 5.2). [m]</td>
</tr>
<tr>
<td>$f$</td>
<td>Gradient of the function describing the error in velocity of the old snow spectrometer of the ETH in dependence on particle size. $\delta v = f \cdot $ (width of the particle) (Section 3.1). [m/s²]</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration by gravity (Sections 5.4 and 7.5). [m/s²]</td>
</tr>
<tr>
<td>$i$</td>
<td>Index for the number of regressands in the multiple linear regression (Appendix A.1.2).</td>
</tr>
<tr>
<td>$j$</td>
<td>Index for the number of measured particles in the multiple linear regression (Appendix A.1.2).</td>
</tr>
<tr>
<td>$k$</td>
<td>Exponent of $\theta$ in the equation $v = a \cdot \theta^k \cdot e^n \cdot \alpha^c$ (Section 7.4).</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of a body in the direction of its movement (Section 5.3.3). [m]</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of a body. [kg]</td>
</tr>
<tr>
<td>$m_a$</td>
<td>Mass of a volume element within a fluid (Section 5.1). [kg]</td>
</tr>
<tr>
<td>$n$</td>
<td>Exponent of $e$ in the equation $v = a \cdot \theta^k \cdot e^n \cdot \alpha^c$ (Section 7.4).</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure. [N/m²]</td>
</tr>
<tr>
<td>p-value</td>
<td>Value calculated for statistical tests. A p-value less than 0.05 indicates that the null-hypothesis can be rejected (Appendix A.1.3).</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Location of the centre of gravity of a particle (Section 5.3).</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of a particle (Section 5.3). [m]</td>
</tr>
<tr>
<td>$s_r$</td>
<td>Residual standard deviation of the regression (Equations A.2, A.5).</td>
</tr>
<tr>
<td>$s_e$</td>
<td>Standard deviation of the measured velocity values from the equation $v = a \cdot X_1^b \cdot X_2^c \cdot X_3^d$. Equation A.1.</td>
</tr>
<tr>
<td>$s_b$</td>
<td>Standard error of $b$.</td>
</tr>
<tr>
<td>$s_c$</td>
<td>Standard error of $c$.</td>
</tr>
<tr>
<td>$s_{\log(a)}$</td>
<td>Standard error of $\log(a)$.</td>
</tr>
<tr>
<td>$t$</td>
<td>Time. [s]</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity of a volume element within the flow (Sections 5.1 and 5.2). [m/s]</td>
</tr>
<tr>
<td>$v$</td>
<td>Fall velocity of a precipitation particle. [m/s]</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Fall velocity of a particle at sea level (Section A.4). [m/s]</td>
</tr>
<tr>
<td>$x_i$</td>
<td>$x_i = \log(X_i)$. Variables used for the correlation and regression analysis (Appendix A.1).</td>
</tr>
<tr>
<td>$y$</td>
<td>Regressand in the function $y = a_1 + b \cdot x_1 + c \cdot x_2 + d \cdot x_3$ (Appendix A.1).</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>Mean value of measured $y$-values (Appendix A.1).</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>$y$-value calculated with the fitted function for given values of $x_i$ (Appendix A.1).</td>
</tr>
<tr>
<td>$z$</td>
<td>Distance from the centre of gravity of a particle (Section 5.3). [m]</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

A \quad \text{Width of a particle measured by the camera named A (Section 2.2.2). [mm]}

A_{\text{eff}} \quad \text{Effective area of a particle opposed to the flow (Section 5.3). [m}^2]\]

A_{\text{ell}} \quad \text{Area of the ellipse circumscribed to } A_{\text{eff}} \text{ (Section 7.5). [m}^2]\]

B \quad \text{Width of a particle measured by the camera named B (Section 2.2.2). [mm]}

C_D \quad \text{Drag coefficient: } F_{RN} = 1/2 \cdot C_D \cdot \rho_a \cdot A_{\text{eff}} \cdot v^2 \text{ (Section 5.3). Dimensionless.}

D \quad \text{Size of a precipitation particle, if method of determining this size is not defined.}

F \quad \text{Force. [N]}

F_R \quad \text{Frictional force (Section 5.3). [N]}

F_{RS} \quad \text{Stokes' frictional force on a sphere. } F_{RS} = 6 \cdot \pi \cdot \eta \cdot v \cdot r \text{ (Section 5.3.1). [N]}

F_{RN} \quad \text{Frictional force as derived by Newton. } F_{RN} = 1/2 \cdot C_D \cdot \rho_a \cdot A_{\text{eff}} \cdot v^2 \text{ (Section 5.3.2). [N]}

F_{RP} \quad \text{Frictional force in boundary layer theory. } F_{RP} = \eta \cdot S \cdot v/\delta \text{ (Section 5.3.3). [N]}

H \quad \text{Vertical extension of a particle (Section 2.2.2). [mm]}

M \quad \text{Geometric mean of the two areas shaded by a particle: } M = \sqrt{M_A \cdot M_B} \text{ (Section 2.2.2). [mm}^2]\]

M_A \quad \text{Area shaded by a particle in camera A (Section 2.2.2). [mm}^2]\]

M_B \quad \text{Area shaded by a particle in camera B (Section 2.2.2). [mm}^2]\]

N \quad \text{Number of particles measured during one precipitation event.}

N(D) \quad \text{Number of precipitation particles per millimetre size interval and per cubic metre volume. [mm}^{-5} \text{m}^3\text{.]}

R \quad \text{Degree of riming as defined in Section 4.1. Dimensionless.}

R^2 \quad \text{Coefficient of determination (Appendix A.1, Equations A.3 and A.6).}

R^* \quad \text{Universal gas constant: } 8.314 \cdot \text{kJ/kg} \cdot \text{K} \text{ (Section A.4).}

Re \quad \text{Reynolds number (Section 5.2). } Re = \frac{v \cdot d \cdot \rho}{\eta}. \text{ Dimensionless.}

S \quad \text{Total surface area of a particle (Section 5.3). [m}^2]\]

T \quad \text{Temperature. [°Kelvin]}

V \quad \text{Volume of a body (Section 5.4). [m}^3]\]

W \quad \text{Maximum of the two measured horizontal extensions of a particle (Section 2.2.2). [mm]}

W_{\text{kin}} \quad \text{Kinetic energy of a body. } W_{\text{kin}} = \frac{1}{2} \cdot m \cdot v^2 \text{ (Section 5.3.3). [J]}

W_m \quad \text{Work to move a body against the frictional force. } W_m = F_R \cdot d \text{ (Section 5.3.3). [Nm] = [J]}

X \quad \text{Davies number: } X = C_D \cdot Re^2 \text{ (Section 7.5). Dimensionless.}

X_{\text{matrix}} \quad \text{Matrix of measured values of } x_i \text{ (Appendix A.1.2).}

X_i \quad \text{Parameters in the velocity equation } v = a \cdot X_1^b \cdot X_2^c \cdot X_3^d \text{ (Section 6.2).}

Y \quad \text{Vector of measured } y-\text{values (Appendix A.1.2).}

Z \quad \text{Constant in the formula for the calculation of air viscosity } \eta = (\beta \cdot T^{3/2})/(T + Z) \text{ (Section 5.2.1). } Z = 110.4 °\text{Kelvin.}
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