

Probabilistic models for the resistance of concrete anchors

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Probabilistic Models for the Resistance of Concrete Anchors

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Preface

Concrete anchors of various types and brands are widely used in the building industry. Anchors may act singly or in groups, may be positioned away from or close to an edge or a corner of the concrete element, and may be stressed by centric or eccentric tensile forces, or by shear and combined actions. Depending on its type, embedment depth, steel and concrete strength an anchor may fail by steel yielding or rupture, by concrete cone breakout, by pull-out, or by concrete splitting in various ways.

A key problem in dimensioning such anchors is the prediction of their failure load. In the development of the different types of anchors that are currently on the market, the aim is that the concrete cone failure precludes all other failure modes. Also, failures due to eccentric tensile forces, shear and combined actions may be traced back to concrete cone failure. This failure mode, therefore, has received much attention in experimental research during the last two decades in many laboratories around the world and yielded a large database including results from more than three thousand experiments.

To predict the concrete cone resistance a number of deterministic semi-empirical models have been developed. As concrete properties and other input variables are subject to physical and statistical uncertainty, stochastic modelling, however, seems appropriate. Using the existing mechanical models and the large database, stochastic models for the concrete cone resistance of single anchors and anchors in a group positioned in uncracked concrete were developed.

By probabilistic analyses a number of questions and uncertainties can be clarified which, in traditional anchor dimensioning, are hidden behind sometimes unspecific safety factors. For specific questions and problems, e.g., in the case of existing anchors where safety may be in question, the models and the methods proposed in this report may be of particular interest.

I am glad to thank the authors, Dipl. Ing. Pawlo Flores Hokkanen and Dr. sc. techn. Alex Scheiwiller for their endeavour.

May 1999

Prof. Dr. h.c. Jörg Schneider

Summary

The main failure mode of single anchors or anchors in a group under centric tensile load is the concrete cone breakout. A number of deterministic semi-empirical models has been developed to predict the respective concrete cone resistance. As concrete properties and other input data are subject to uncertainty, stochastic modelling of the problem seems appropriate. The aim of this study was to develop such stochastic models.

The experimental basis is a large database including results from over three thousand concrete cone breakout tests performed during the past two decades in many laboratories around the world. Included are tests on single anchors, anchors in a group and close to an edge in uncracked concrete.

The deterministic concrete cone breakout models were checked against the data. The Concrete Capacity Design Model (CCD-Model) seemed to predict best the mean concrete cone resistance and, hence, was chosen as the basis for developing the stochastic models for the concrete cone resistance. These included models for single anchors, as well as for groups of 2 and 4 anchors. Centric tensile loads and uncracked concrete were assumed.

By way of examples, probabilistic analyses were performed. Furthermore, the sensitivity of the results of the probabilistic analysis to changes of the model uncertainty variable is shown, as well as the sensitivity to changes of the spacing between the anchors in a group. Finally, it is demonstrated that the safety index of an anchor is largely influenced by the stochastic definition of the applied loads.

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1 Notation

1.1 Latin letters

a	empirical coefficient of the generalised model
a'	empirical coefficient of the generalised model
A_N	failure surface of a group
A_{N_0}	failure surface of a single anchor
b	empirical coefficient of the generalised model
c	empirical coefficient of the generalised model
c'	anchor geometry-dependent empirical coefficient of the fracture toughness model and the Sawade model
d_{\max}	maximum aggregate size of concrete
DL	dead load
E	elastic modulus of concrete
f_{cc}	cube compressive strength of concrete
$f_{cc,cube}$	standardised cube compressive strength of concrete ($200 \cdot 200 \cdot 200 \text{ mm}^3$)
$f_{cc,cyl}$	cylinder compressive strength of concrete
$f_{cc,cyl,k}$	characteristic value of the cylinder compressive strength of concrete
f_{ct}	tensile strength of concrete
G	limit state function
G_f	specific crack energy
h_0	empirical coefficient of the tensile strength model
h_{ef}	embedment depth
k	empirical coefficient of the CCD model
k'	empirical coefficient of the fracture toughness model
K_{IC}	fracture toughness
L	load variable
L_d	design value of the load variable
LL	live load
M	model uncertainty variable
n	sample size
N_d	design value of the concrete cone capacity
n_g	number of anchors in a group
N_u	concrete cone failure load
$N_{u,cal}$	calculated concrete cone failure load
$N_{u,obs}$	observed concrete cone failure load
p_f	probability of failure
Pw	probability of bad workmanship in placing an anchor

Notation

R_{cone}	concrete cone resistance
s	spacing between anchors
S_i	site imperfection variable
Sw	a variable introducing the sensitivity of the anchor resistance relating to bad workmanship
v	coefficient of variation
Y	a variable representing variations due to special placing, curing, and hardening conditions on in situ concrete

1.2 Greek Letters

α	weighting factor
α_T	constant which takes into account the concrete age at the loading time and the duration of loading
β	Hasofer-Lind safety index
γ	partial safety factor
δ	empirical coefficient of the tensile strength model
Λ	variable which takes into account the concrete age at the loading time and the duration of loading
μ	mean value
σ	standard deviation
ψ	group reduction factor of the concrete cone failure load to be applied to the single anchor
Ω	failure angle of the concrete cone with the horizontal axis

2 Introduction

Concrete anchors, like headed studs, expansion anchors and undercut anchors, are nowadays widely used in the building industry to carry loads in tension and/or shear. Apart from installation problems, the bearing capacity of anchors is of interest. Depending on its type, embedment depth, steel and concrete strength an anchor may fail by steel yielding or rupture, by concrete cone breakout, by pull-out, or by concrete splitting in various ways. All these failure modes are readily explained in CEB, 1991 and Fuchs et al., 1995.

In the development of the different types of anchors that are currently on the market, the aim is that the concrete cone failure precludes other failure modes. Also, failures due to eccentric tensile, shear and combined actions may be traced back to concrete cone failure. Thus, this failure mode has received much attention in experimental and theoretical research.

To predict the concrete cone resistance a number of deterministic semi-empirical models have been developed. As concrete properties and other input variables are subject to physical and statistical uncertainty, the development of stochastic models is appropriate. Such models can be of help in developing design methods and rules or in assessing failure probabilities of installed anchor systems.

In this report the existing mechanical models are screened against the data and the model selected that seemed to fit best to the data. The database used includes results from over three thousand concrete cone breakout tests in uncracked concrete performed in the past two decades in many laboratories around the world. Most of the tests were performed on single anchors in uncracked concrete under centric tensile loads. A relatively small part of the data relates to anchors in a group and close to an edge.

The following sections include

- a brief description of the anchor types found in the database,
- an analysis of the available experimental data for choosing the most appropriate mechanical model,
- an evaluation and definition of the respective model uncertainties,
- the definition of stochastic models of all relevant input data and
- the definition of concrete cone resistance models for single anchors and anchors in a group.

Finally, some numerical examples are shown and sensitivity studies performed.

3 The Database

3.1 Classification of Anchors

Anchors in general are classified in two broad categories: cast-in-place and post-installed. Cast-in-place anchors are positioned prior to the casting of concrete and transfer loads by means of mechanical interlock. The most common type is the headed stud consisting of a smooth shaft with a head at the tip. Post-installed anchors are set in preformed or drilled holes and can transfer loads by means of friction or mechanical interlock. They are subdivided into expansion anchors and undercut anchors. Figure 1 gives a general classification of the anchor types found in the database.

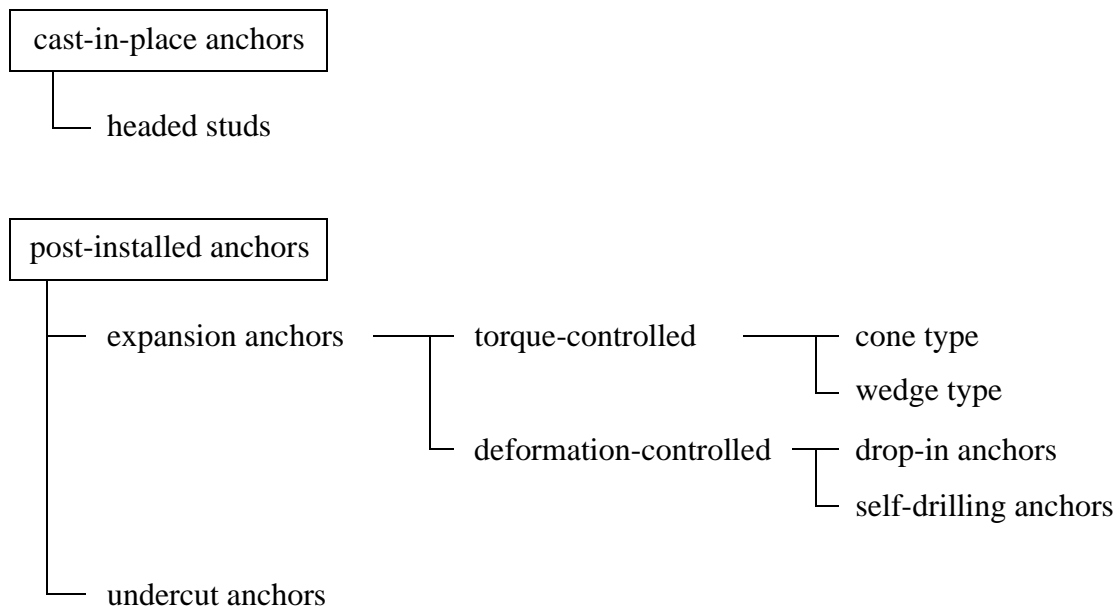


Figure 1: Classification of Anchors

Expansion anchors are again subdivided according to their expansion mechanism into torque- and deformation-controlled expansion anchors. In the database two types of torque-controlled expansion anchors are found: the cone type and the wedge type. The cone type are expanded by applying a specified torque to the bolt head that draws a cone between the spreading elements. The cone type are equipped either with a single cone or with a double cone. In the wedge type, the expansion element consists of a spring steel collar at the end. In deformation-controlled expansion anchors the cone is driven between the spreading elements by means of impact energy. There are two types of deformation-controlled expansion anchors in the database: the drop-in anchor and the self-drilling anchor. The drop-in anchor consists of an expansion sleeve and a cone. It is set by driving the cone into the expansion sleeve with a hammer or setting tool. The self-drilling anchor is a special type where the expansion sleeve is used to drill the hole in the concrete. The cone is then inserted into the tip

of the sleeve and placed in the hole. The sleeve is finally driven over the cone using the hammering action of the rotary percussion drill.

An undercut anchor is a post-installed system in which the hole is locally widened to accept the expansion element and create a mechanical interlock. This special hole geometry may be created before or during setting of the anchor.

A complete description of the different anchor types may be taken from the CEB Bulletin (1991).

3.2 Available Experimental Data

The database for this work was previously compiled by the ACI 349 task group and researchers at the Institut für Werkstoffe im Bauwesen at the University of Stuttgart. It contains results of failure load measurements of concrete cone breakout tests for different types of anchors in uncracked concrete under tensile force (Klingner, 1996). The database is subdivided according to the tests performed in the following categories:

- Single anchors tested far away from an edge (850 data sets)
- Single anchors tested close to an edge (171 tests)
- 2- and 4-anchor groups tested far away from an edge (190 tests)

The database lists the anchor type, its dimensions, its steel yield strength, the compressive strength of the concrete, the maximum aggregate size and the measured concrete cone failure load. Regarding the anchor types, headed studs are distinguished by the size of the head. For the compressive strength values of cube and cylinder compressive strength are given. Unfortunately, it is not possible to identify which of the two strength types was measured or which one was converted from the other type.

The database for single anchor tests on expansion and undercut anchors contains concrete cone failure loads which actually represent mean values of test series (oral communication by Fuchs and Eligehausen, Stuttgart, 1997). Usually, five to ten experiments built a typical test series. In order to create a homogeneous basis for the analysis, each the mean value was replaced by 5 single values chosen such that a coefficient of variation of 8% results, a value known from experience, while the known mean was maintained. In doing so about 2500 results on separate experiments constituted the basis for the analysis of single anchor tests.

Table 1 gives an overview of the database modified as mentioned above for single anchor tests, while table 2 covers the results of tests on anchors close to an edge.

The 188 tests on anchors in a group consist of 94 tests of 2-anchor groups and 94 of 4-anchor groups. For 37 tests the same value for cube and cylinder compressive strengths is given, which naturally cannot be true. These tests were excluded from the analysis, so that 89 tests of 2-anchor groups and 62 of 4-anchor groups remain (see tables 3 and 4).

Expansion anchors													Total
Cone type			Wedge type			Drop-in			Self-drilling				
Origin of data	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	
England	95	37-136	23-32	80	22-100	23-34	100	25-83	26-30	-	-	-	275
France	370	42-156	14-55	-	-	-	25	60-77	20-50	-	-	-	395
Germany	780	18-148	9-75	-	-	-	355	25-80	11-57	230	28-83	11-69	1365
Sweden	-	-	-	90	32-67	18-43	-	-	-	-	-	-	90
USA	155	25-170	27-52	70	36-87	15-35	5	41	30	140	28-94	25-50	37
Total	1400			240			485			370			2495

Undercut anchors			Studs							Total
			Headed studs			With undersized heads				
Origin of data	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	
Bucknell (USA)	-	-	-	2	203-305	23-24	-	-	-	2
CSFR (Prague)	-	-	-	11	50	28-34	15	150-450	28-34	26
Europe	-	-	-	13	50-150	21-47	-	-	-	13
Germany	455	36-220	13-67	202	43-525	11-72	-	-	-	657
Sweden	50	43-125	16-44	-	-	-	-	-	-	50
USA	-	-	-	28	76-251	24-43	-	-	-	28
Total	505			256			15			776

Table 1: Overview of single anchor tests grouped according to origin and anchor type

Expansion anchors													Total
Cone type			Wedge type			Drop-in			Self-drilling				
Origin of data	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	
England	1	37	26	2	65-100	23	-	-	-	-	-	-	3
France	2	64	18	-	-	-	-	-	-	-	-	-	2
Germany	13	38-148	25	-	-	-	3	30-37	25	5	53-83	25	21
Sweden	8	142-344	31-57	-	-	-	-	-	-	-	-	-	8
USA	2	83	24	6	65-100	17-34	-	-	-	6	52-83	24-30	14
Total	26			8			3			11			48

Undercut anchors			Studs							Total
			Headed studs			With undersized heads				
Origin of data	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	n	depth [mm]	f_{cc} [N/mm ²]	
Germany	11	37-220	24-59	62	43-525	14-48	-	-	-	73
USA	35	89-279	26-52	14	89-168	35-45	-	-	-	49
Total	46			76			-			122

Table 2: Overview of single anchor tests of anchors close to an edge grouped according to origin and anchor type

4 Review of Concrete Cone Resistance Models

4.1 Generalities

Assuming sufficient distances from edges and corners, a concrete cone failure in an anchor under tensile load is characterised by the formation of a fairly conical fracture surface initiating and radiating from the tip of the anchor head. The crack initiation begins already under very small loads. The crack propagation is small until about 90% of the bearing capacity is reached, then it increases considerably (Sawade, 1994). At the failure load, the crack length is about 50% of the conical fracture surface. Finally, the fracture cone breaks out of the concrete. The main influence factors of the bearing capacity are the mechanical properties of the concrete and the failure surface, as well as the embedment depth of the anchor. The fracture mechanics theory is suitable for describing such effects (CEB, 1991). In applying this theory, also size effects are considered (Bazant, 1984). Consequently the different models for the failure load include expressions that consider material properties, the shape and the extent of the failure surface and the size effects.

The only concrete properties reported in the database are the concrete compressive strength and, but not in all cases, the maximum aggregate size. Additional concrete properties such as the tensile strength or the specific crack energy, therefore, can only be determined from the compressive strength using empirical formulas derived by regression analysis and reported in the literature.

4.2 Single Anchors

Several models exist to describe the concrete cone resistance of anchors (Bergmeister, 1990; Sawade, 1994). The four most important will be described here in detail. These are

- the Concrete Capacity Design (CCD) Model
- the Fracture Toughness Model
- the Sawade Model
- the Tensile Strength Model

The first two models are based on the theory of linear fracture mechanics, the other two on the theory of non-linear fracture mechanics. In the following, for each of the models, a short description of the model itself, a comparison with measured failure loads and the empirical coefficients derived from the data is shown.

The comparison was made with the results of tests performed in Germany on headed studs (202 tests). To visualize this comparison, the ratio between measured failure load ($N_{u,obs}$) and the corresponding calculated failure load ($N_{u,cal}$) was plotted against the embedment depth. To determine the calculated failure load the empirical coefficients were taken from the literature.

4.2.1 Concrete Capacity Design (CCD) Model

The Concrete Capacity Design model was developed by (Fuchs et al., 1995) in an attempt to provide a model that is easy to use and that lends itself to analysing group and edge effects. The fracture cone is idealized as a pyramid, assuming a quadratic base length of three times the embedment depth h_{ef} (see figure 2). The failure surface, therefore, corresponds to $(3 \cdot h_{ef})^2$.

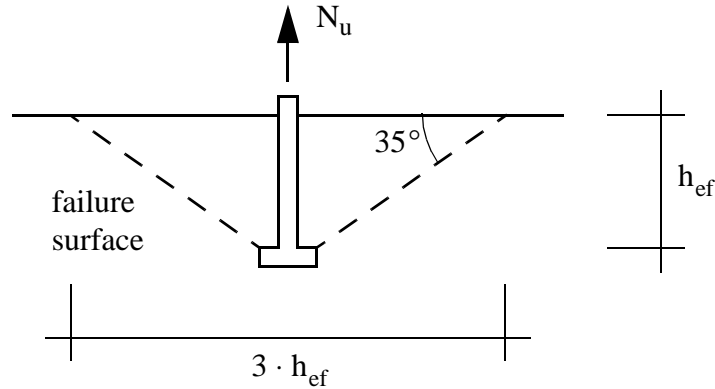


Figure 2: Concrete cone failure for a single anchor according to the CCD model

The concrete properties under tension are modelled by means of the tensile strength only, which is calculated from the concrete compressive strength using empirical formulas. The failure load is given by

$$N_u = k_1 \cdot f_{cc}^{0.5} \cdot k_2 \cdot h_{ef}^2 \cdot k_3 \cdot h_{ef}^{-0.5} \quad [\text{N}] \quad (1)$$

where

- f_{cc} : cube compressive strength [N/mm^2]
- h_{ef} : effective embedment depth [mm]
- k_1, k_2, k_3 : constant calibration coefficients
- $k_1 \cdot f_{cc}^{0.5}$: concrete tensile strength
- $k_2 \cdot h_{ef}^2$: area of the failure cone ($k_2 = 9$)
- $k_3 \cdot h_{ef}^{-0.5}$: size effect

With $k = k_1 \cdot k_2 \cdot k_3$ the failure load is

$$N_u = k \cdot f_{cc}^{0.5} \cdot h_{ef}^{1.5} \quad [\text{N}] \quad (2)$$

The value of k is obtained by fitting experimental data to this model. Proposed values are 13.5 for expansion and undercut anchors and 15.5 for headed studs (Fuchs et al., 1995).

The comparison of the model with the data from Germany for headed studs is shown in figure 3. The model, obviously, fits well to the data over all relevant embedment depths. The overall mean value is close to 1. The overall coefficient of variation is reasonably small and slightly larger for small embedment depths.

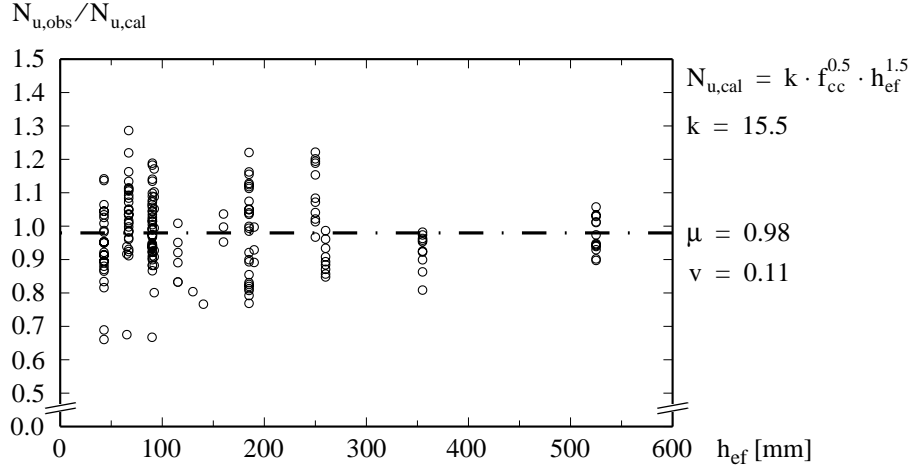


Figure 3: CCD model versus headed studs data from Germany

4.2.2 Fracture Toughness Model

According to the theory of linear fracture mechanics, the capacity of concrete in tension is described by the fracture toughness K_{IC} :

$$K_{IC} = (E \cdot G_f)^{0.5} \quad [N/mm^{1.5}] \quad (3)$$

where E is the elastic modulus in $[N/mm^2]$ and G_f the specific crack energy in $[N/mm]$.

Research carried out in this field by Eligehausen and Sawade (1989) resulted in the following model for the failure load of an anchor in concrete:

$$N_u = c' \cdot K_{IC} \cdot h_{ef}^{1.5} = c' \cdot (E \cdot G_f)^{0.5} \cdot h_{ef}^{1.5} \quad [N] \quad (4)$$

where c' is an anchor geometry-dependent empirical coefficient which has to be determined by data fitting, e.g., a value of 2.1 is valid for studs. This equation is, contrary to the CCD model, unit conform. Due to the difficulties in measuring the material parameters E and G_f empirical formulas linking these to the compressive strength f_{cc} are used to replace them. An approach for the elastic modulus E can be made with the well-known expression (Leonhardt, 1973)

$$E = 5600 \cdot f_{cc}^{0.5} \quad [N/mm^2] \quad (5)$$

whereas the crack energy G_f in $[N/mm]$ can be estimated with (CEB, 1993)

$$\begin{aligned} G_f &= 0.003 \cdot f_{cc}^{0.7} & \text{for } d_{max} \leq 8\text{mm} \\ G_f &= 0.006 \cdot f_{cc}^{0.7} & \text{for } d_{max} \leq 16\text{mm} \\ G_f &= 0.0012 \cdot f_{cc}^{0.7} & \text{for } d_{max} \leq 32\text{mm} \end{aligned} \quad (6)$$

where d_{max} is the maximum aggregate size of the concrete.

Replacing E and G_f in equation (4) results in

$$N_u = k' \cdot f_{cc}^{0.6} \cdot h_{ef}^{1.5} \quad [N] \quad (7)$$

In this form, the model differs from the CCD model in the exponent for the cube compressive strength and in the value of k' which depends on the maximum aggregate size, e.g. for headed studs and an aggregate size of 16 mm k' is equal to 12.17.

The comparison of the model with the data from Germany for headed studs shows that the model fits well to the data over all relevant embedment depths (see figure 4). The overall mean value is 0.9. Therefore, the model slightly overestimates the concrete cone resistance. The overall coefficient of variation is reasonably small and slightly larger for small embedment depths.

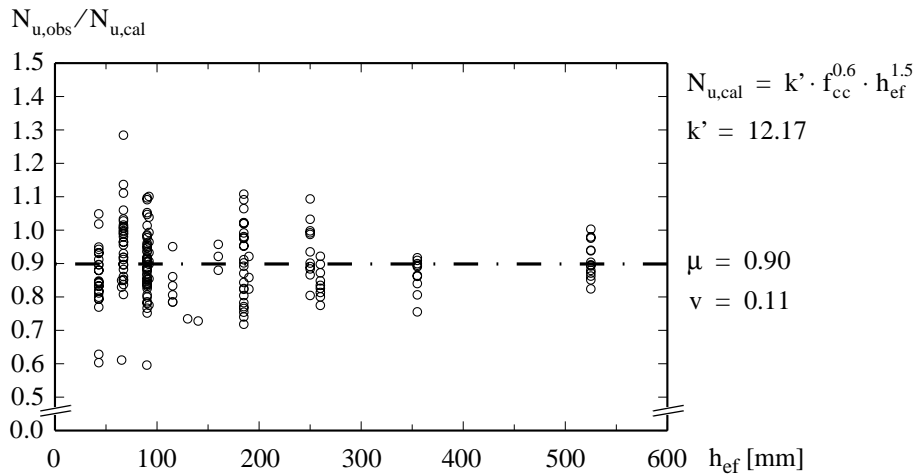


Figure 4: Fracture Toughness model versus headed studs data from Germany

4.2.3 The Sawade Model

The Sawade model (Sawade, 1994) is based on the theory of non-linear fracture mechanics and is given by

$$N_u = \frac{c' \cdot (E \cdot G_f)^{0.5} \cdot h_{ef}^2}{\left(\frac{c'^2 \cdot E \cdot G_f \cdot (\tan \Omega)^4}{(\pi \cdot f_{ct})^2} + h_{ef} \right)^{0.5}} \quad [N] \quad (8)$$

where

- c' : an anchor geometry-dependent empirical coefficient. For headed studs $c' \approx 2.1$
- f_{ct} : concrete tensile strength [N/mm^2]
- Ω : failure angle of the concrete cone with the horizontal axis, empirically found to be about 33 degrees

The denominator contains the size effect. For small anchor sizes, the value of the embedment depth in the denominator becomes negligible. Hence the size effect disappears and the failure load approaches the expression

$$N_u = \frac{\pi \cdot f_{ct} \cdot h_{ef}^2}{(\tan \Omega)^4} \quad [N] \quad (9)$$

This is called the plastic solution. For large anchor sizes, the concrete cone failure load approaches the fracture toughness model. The Sawade model, despite its strong theoretical background, is limited for practical purposes because of the difficulties in obtaining the material properties needed (E , G_f , f_{ct}). All these, however, can be expressed as functions of the compressive strength. Replacing the properties E and G_f by equations (5) and (6) respectively and the tensile strength by (Heilmann, 1976):

$$f_{ct} = 0.24 \cdot f_{cc}^{2/3} \quad [\text{N}/\text{mm}^2] \quad (10)$$

for a concrete with $d_{\max} = 16$ mm results in

$$N_u = \frac{5.8 \cdot c' \cdot f_{cc}^{0.6} \cdot h_{ef}^2}{\left(59.1 \cdot c'^2 \cdot f_{cc}^{-0.13} \cdot (\tan \Omega)^4 + h_{ef}\right)^{0.5}} \quad [\text{N}] \quad (11)$$

In this form the concrete cone failure load depends only on the concrete compressive strength and on the empirical coefficients c' and Ω .

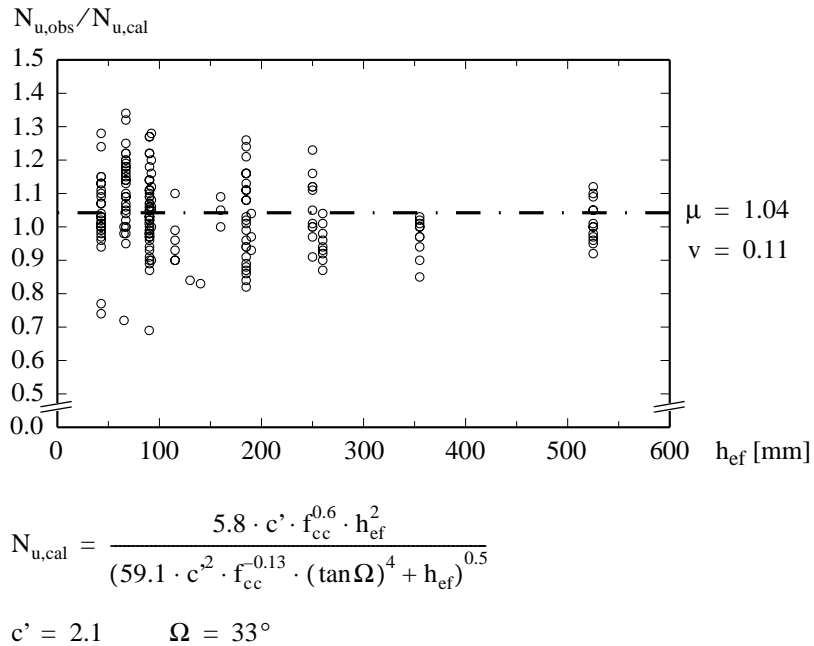


Figure 5: Sawade model versus the headed studs data from Germany

The comparison of the model with the data from Germany for headed studs is shown in figure 5. The model, obviously, is conservative for smaller embedment depths and may be on the unsafe side for larger ones. The overall mean value is close to 1. The overall coefficient of variation is reasonable but larger for small embedment depths.

4.2.4 The Tensile Strength Model

The Tensile Strength Model proposed by Eligehausen et al. (1992) is given by

$$N_u = \frac{\delta \cdot f_{cc}^{0.5} \cdot h_{ef}^2}{\left(1 + \frac{h_{ef}}{h_0}\right)^{0.5}} \quad [\text{N}] \quad (12)$$

where δ and h_0 are empirical coefficients which have to be determined from experimental data. For headed studs $\delta = 2.2$ and $h_0 = 100$ mm are proposed. Recent research by Eligehausen reports values of 2.5 and 50, respectively (personal communication). The material properties are modelled by the tensile strength, like in the CCD model, whereas the size effect considered is based on the theory of non-linear fracture mechanics.

The comparison of the model with the data from Germany for headed studs is shown in figure 6. Obviously, the model is conservative for smaller embedment depths and on the unsafe side for larger ones. The overall mean value is close to 1. The overall coefficient of variation is rather large and even larger for small embedment depths.

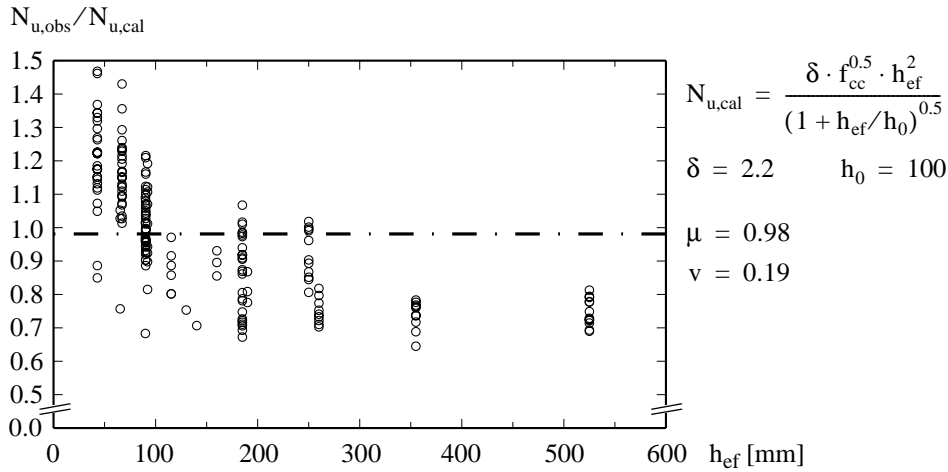


Figure 6: Tensile Strength model versus headed studs data from Germany.

4.2.5 Generalised Model

The four models presented above have the following structure in common:

$$N_u = \frac{a \cdot f_{cc}^b \cdot h_{ef}^2}{(c + h_{ef})^2} \approx a' \cdot f_{cc}^b \cdot h_{ef}^{1.5} \quad (13)$$

All the material properties are modelled by empirical formulas by means of the compressive strength raised to the power of $b = 0.5 - 0.6$. The size effect can be considered to be based on the theory of linear fracture mechanics (with $c = 0$) or on the theory of non-linear fracture mechanics. The latter should, theoretically, yield better results, especially for smaller embedment depths. The coefficients a and c may be obtained by fitting to experimental data.

4.2.6 Data Fitting

All models contain empirical coefficients, which can be determined by means of regression analysis. For this purpose, the measured failure loads were reduced, that is divided by the concrete cube compressive strength and theoretical coefficients. The resulting reduced failure loads are given in equations (14) to (17).

$$N_{u,red,CCD} = \frac{N_{u,obs}}{f_{cc}^{0.5}} = k \cdot h_{ef}^{1.5} \quad (14)$$

$$N_{u,\text{red,fracture toughness}} = \frac{N_{u,\text{obs}}}{(5600 \cdot f_{cc}^{0.5} \cdot G_f)^{0.5}} = c' \cdot h_{ef}^{1.5} \quad (15)$$

$$N_{u,\text{red,Sawade}} = \frac{N_{u,\text{obs}}}{5.8 \cdot f_{cc}^{0.6} \cdot h_{ef}^2} = \frac{c'}{(59.1 \cdot c' \cdot f_{cc}^{-0.13} \cdot (\tan \Omega)^4 + h_{ef})^{0.5}} \quad (16)$$

$$N_{u,\text{red,tensile strength}} = \frac{N_{u,\text{obs}}}{f_{cc}^{0.5} \cdot h_{ef}^2} = \frac{\delta}{\left(1 + \frac{h_{ef}}{h_0}\right)^{0.5}} \quad (17)$$

In equation (15) G_f had to be replaced for the regression analysis with one of the expressions given in equation (6). The reduced failure load of the Sawade model (equation (15)) contains the coefficient c' in the denominator and the numerator. For the regression analysis, c' in the denominator was looked at as a different parameter than the one in the numerator. With this assumption equation (16) can, finally, be written as

$$N_{u,\text{red}} = \frac{A}{(B + h_{ef})^{0.5}} \quad (18)$$

The term B depends on the cube compressive strength. To eliminate this dependency, the data was bundled in concrete compressive strength intervals of 5 N/mm^2 . For each interval the values of A and B , and c' and Ω , respectively, were determined by means of a regression analysis. Then a weighted average was calculated over all intervals. The weighting factor corresponded to the ratio of the number of tests in each interval divided by the number of all tests.

The results of the regression analysis are given in tables 5 to 8. The comparison of the calculated coefficients with those of the literature shows a good agreement in the case of the CCD model. The coefficients of the fracture toughness model and the Sawade model are somewhat lower than those given in the literature. And in the case of the tensile strength model the variability of the result is remarkable high, and the results for headed studs differ considerably from those given in the literature. The regression analysis of the wedge type expansion anchors for the Sawade model and the tensile strength model do not give consistent results. Therefore, this anchor type is not included in the results listing.

CCD model			
Anchor type	n	k_{data}	$k_{literature}$
Headed studs	256	15.0	15.5
Expansion anchors (all types) and undercut anchors	3000	13.8	13.5
Expansion anchors:			
• Wedge type	240	12.5	13.5
• Cone type	1400	13.9	13.5
• Self-drilling anchors	370	15.1	13.5
• Drop-in anchors	485	14.9	13.5
Undercut anchors	505	13.6	13.5

Table 5: CCD model: results of the determination of the coefficient k compared with the one given in the literature (Fuchs et al., 1995)

Fracture toughness model		
Anchor type	n	c'
Headed studs	213	1.9
Expansion anchors:		
• Cone type	945	1.6
• Wedge type	150	1.1
• Self-drilling anchors	230	1.8
• Drop-in anchors	460	1.8
Undercut anchors	455	1.7
Literature (Eligehausen and Sawade, 1989)	-	2.1

Table 6: Fracture toughness model: results of the determination of the coefficient c' .

Sawade model			
Anchor type	n	c'	Ω
Headed studs	213	1.9	27°
Expansion anchors:			
• Cone type	945	1.8	38°
• Self-drilling anchors	230	1.9	29°
• Drop-in anchors	460	1.7	28°
Undercut anchors	455	1.6	18°
Literature (Sawade, 1994)	-	2.1	33°

Table 7: Sawade model: results of the determination of the coefficients c' and Ω .

Tensile strength model			
Anchor type	n	δ	h_0
Headed studs	256	5.6	7.0
Expansion anchors (all types) and undercut anchors	3000	3.4	17.3
Expansion anchors:			
• Cone type	1400	1.8	104.9
• Self-drilling anchors	370	5.0	9.8
• Drop-in anchors	485	5.1	8.6
Undercut anchors	505	6.4	4.3
Literature (Eligehausen et al., 1992)	-	2.2	100

Table 8: Tensile strength model: results of the determination of δ and h_0

4.2.7 Model Choice

A stochastic model should be based on a mechanical model complying with the following requirements:

- good prediction capability of the mean concrete cone failure load. The ratio between predicted and observed loads should remain close to unity.
- transferable to anchors in a group and close to edges

From the models discussed in Chapter 3, the CCD model predicts best the mean concrete cone failure load. It has also the advantage of simplicity and suitability for design purposes. Further it lends itself to handling group effects. The Fracture Toughness model is similar to the CCD model. It has the inconvenience that the empirical coefficient depends on the maximum aggregate size which is only partly available in the database. The Sawade model has a strong theoretical background and coefficient stability. But it is complicated and less suitable for practical purposes. It also requires information about the maximum aggregate size. The Tensile Strength model greatly overestimates the failure load for smaller embedment depths while underestimating it at larger ones. In addition, this model does not provide stable coefficients.

In summary, the CCD model best fulfils the requirements. Therefore, it was chosen as the basis for stochastic modelling.

4.3 Anchors in a Group

4.3.1 General Model

As mentioned under 4.2.1, the CCD model lends itself to modelling the failure load of a group of anchors. According to this model, the failure surface in concrete for a single anchor is idealized as the base of a quadratic pyramid with length equal to three times the embedment depth. To calculate the failure surface of a group a similar assumption is made. Each anchor in the group develops the failure surface of a single anchor which may overlap with

the failure surface from its neighbour, depending on the spacing between them. The resulting failure surface is equal to the enclosed area of the failure surfaces of all anchors.

If the spacing between the anchors is smaller than $3 \cdot h_{ef}$, then the failure surface of the group is smaller than the sum of the failure surfaces of each single anchor and group failure may develop. This may be considered by reducing the single anchor failure load by a group reduction factor. This factor is equal to the ratio of failure surface of the group divided by the failure surface of a single anchor and by the number of anchors in a group. The group failure load is therefore given by:

$$N_{u,m} = n_g \cdot \left(\frac{A_N}{n_g \cdot A_{N_0}} \right) \cdot N_{u,single} = n_g \cdot \psi \cdot N_{u,single} \quad (19)$$

where

- A_N : failure surface of the group
- A_{N_0} : failure surface of a single anchor
- n_g : number of anchors in the group
- ψ : group reduction factor to be applied to the single anchor
- $N_{u,single}$: failure load of a single anchor

This way of treating the group effect is slightly different from the one normally used with the CCD model. In the CCD model, normally the failure load of the single anchor is increased by some factor in order to calculate the bearing capacity of the group. It seems, however, more logical to reduce the bearing capacity of each single anchor by some group reduction factor when an anchor acts in a group.

4.3.2 2-Anchor Group

In figure 7 the projected failure surfaces for the 2-anchor group are shown. Under an axial tensile force each of the anchors carries one half of the applied load. The group may fail because one of the two anchors fails individually or, if anchors are close enough, because the group as such fails.

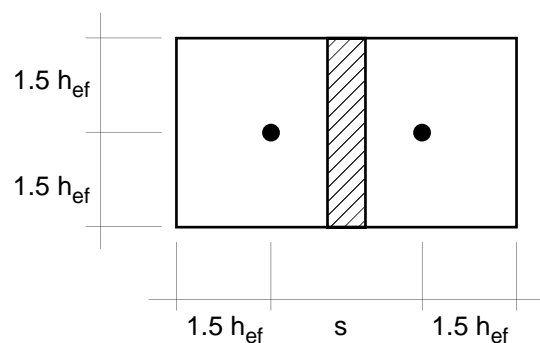


Figure 7: Group effect for a 2-anchor group according to the CCD model

The failure surface of a single anchor is

$$A_{N_0} = 9 \cdot h_{ef}^2 \quad (20)$$

while the failure surface of the group for $s/h_{ef} < 3$ is

$$A_N = 3 \cdot h_{ef} \cdot (3h_{ef} + s) \quad (21)$$

For the 2-anchor group, therefore, the group reduction factor to be applied to each of the single anchors is

$$\Psi_2 = \frac{A_N}{n_g \cdot A_{N_0}} = \frac{3h_{ef} \cdot (3h_{ef} + s)}{2 \cdot 9 \cdot h_{ef}^2} = \frac{1}{2} \cdot \left(1 + \frac{s}{3 \cdot h_{ef}}\right) = 0.5 + \frac{s}{6 \cdot h_{ef}} \quad (22)$$

For $s/h_{ef} \geq 3$, the group reduction factor is equal to one.

This group reduction factor was checked against all available test data and plotted against the relative spacing between anchors s/h_{ef} . The result of this analysis is shown in figure 8. The model shows rather good agreement with the test data. It can also be seen that for larger anchor spacing the model seems to be on the safe side.

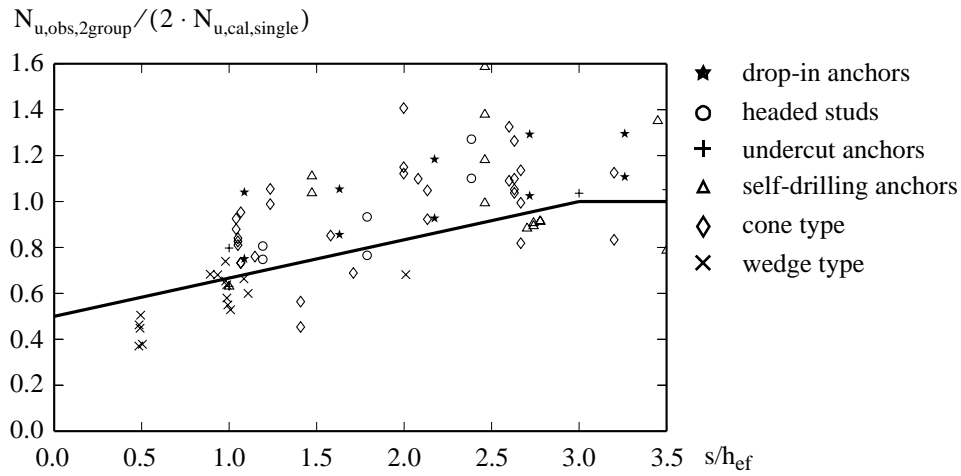


Figure 8: Comparison of the data with the model for the 2-anchor group

4.3.3 4-Anchor Group

For the 4-anchor group only the symmetric case (see figure 9) is considered, because the database contains only test results for this configuration. Other configurations may be treated in a similar manner.

The failure surface of a 4-anchor group for $s/h_{ef} < 3$ is

$$A_N = (3 \cdot h_{ef} + s)^2 \quad (23)$$

Under an axial tensile force each of the anchors carries one quarter of the load. The group may start to fail because at the tip of one of the four anchors radiating cracks occur. In this case the anchor diagonally opposite, to satisfy equilibrium, is bound to carry the same load as the failing one. Thus, a load redistribution takes place allocating more than half of the load to the other pair of anchors. One anchor of this pair will start to fail next. Only if neighbouring anchors in the group are so close that fracturing and radiating from the tip of one of the anchors triggers the neighbouring anchors to fail as well, does the group fracture cone develop and the group fails.

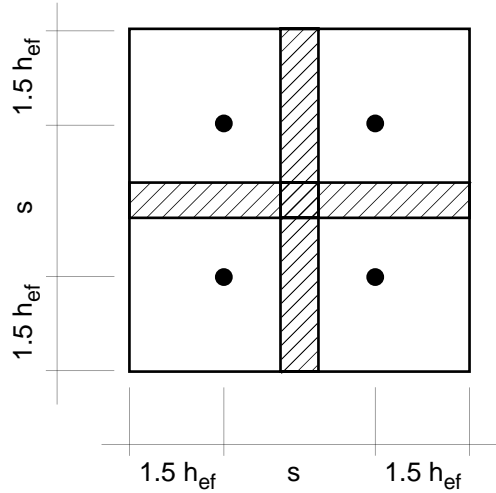


Figure 9: Group effect for a 4-anchor group according to the CCD model

For the 4-anchor group, therefore, the group reduction factor to be applied to each of the single anchors is

$$\begin{aligned} \Psi_4 &= \frac{A_N}{n_g \cdot A_{N_0}} = \frac{(3h_{ef} + s)^2}{4 \cdot 9 \cdot h_{ef}^2} = \frac{1}{4} \cdot \left(1 + \frac{s}{3 \cdot h_{ef}}\right)^2 \\ &= 0.25 + \frac{s}{6 \cdot h_{ef}} \cdot \left(1 + \frac{s}{6 \cdot h_{ef}}\right) = 0.25 + \frac{s}{6 \cdot h_{ef}} + \frac{s^2}{36 \cdot h_{ef}^2} \end{aligned} \quad (24)$$

Again, the group reduction factor is equal to one for $s/h_{ef} \geq 3$.

In figure 10 a comparison of the 4-anchor group model with the available test data is shown. The model shows rather good agreement with the test data.

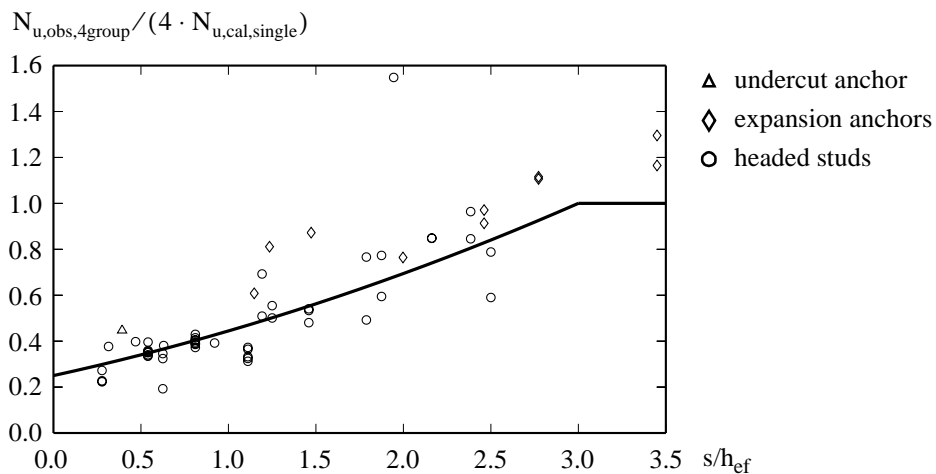


Figure 10: Comparison of the data with the model for the 4-anchor group

5 Stochastic Modelling

5.1 Single Anchors

5.1.1 The Model

According to the CCD model the concrete cone resistance of an anchor in uncracked concrete can be written as:

$$R_{\text{cone, single}} = S_i \cdot M \cdot k \cdot F_{\text{cc}}^{0.5} \cdot H_{\text{ef}}^{1.5} \quad (25)$$

where

F_{cc}	:	cube compressive strength of concrete [N/mm ²]
H_{ef}	:	effective embedment depth [mm]
k	:	empirical coefficient [(N/mm) ^{0.5}]
M	:	model uncertainty variable [-]
$R_{\text{cone, single}}$:	concrete cone resistance of a single anchor [N]
S_i	:	site imperfection variable [-]

As usual in reliability analysis, stochastic variables are indicated by capital letters while deterministic values appear in small letters. In the following, only the model uncertainty variable M will be treated. The other stochastic variables will be explained later in chapter 5.3.

5.1.2 The Model Uncertainty Variable

The model uncertainty variable M takes into account the deviation between the test results and the model used to predict them. The model uncertainty variable M , therefore, is determined by the ratios of the results of concrete cone failure load tests, $n_{\text{u, obs, i}}$ and the calculated failure load $n_{\text{u, cal, i}}$ derived from the respective model. These ratios are given as follows:

$$m_i = \frac{n_{\text{u, obs, i}}}{n_{\text{u, cal, i}}} \quad (26)$$

From a large number of tests a histogram for M can be obtained, as well as the mean value and the standard deviation of M . Finally, a suitable statistical distribution type can be assigned.

a) Mean Value

First of all the mean value of the model uncertainty variable was fixed. In order to check for dependence on the embedment depth, the data for a given origin of the data and anchor type (see table 1) was grouped according to embedment depth intervals of 10 mm. The mean val-

ues of each interval were calculated under the condition that it contained more than three single test results. Figures 11 to 16 show the result of this examination.

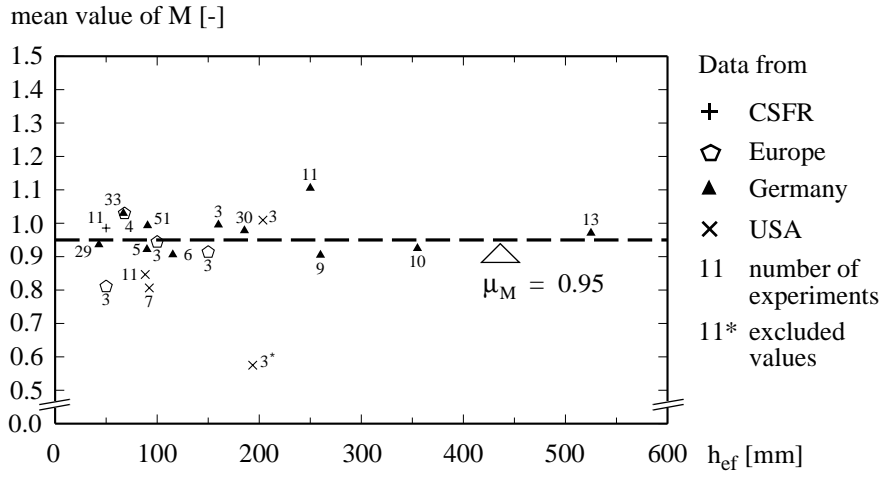


Figure 11: Mean values of the model uncertainty variable M for headed studs

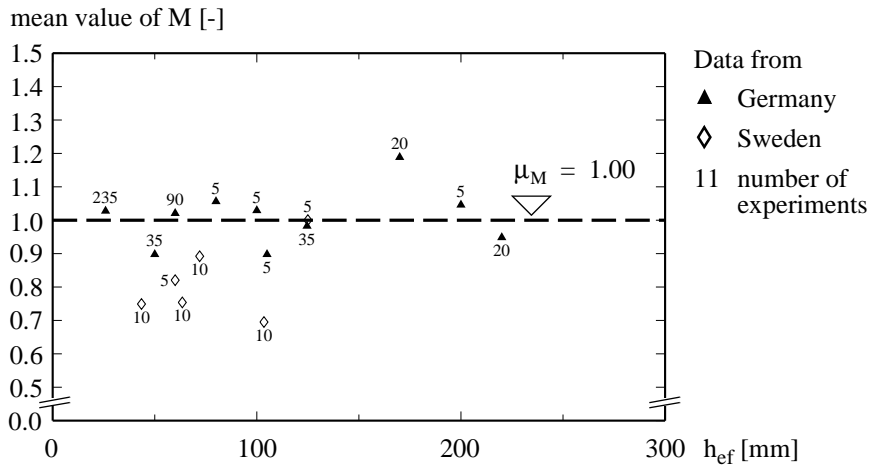


Figure 12: Mean values of the model uncertainty variable M for undercut anchors

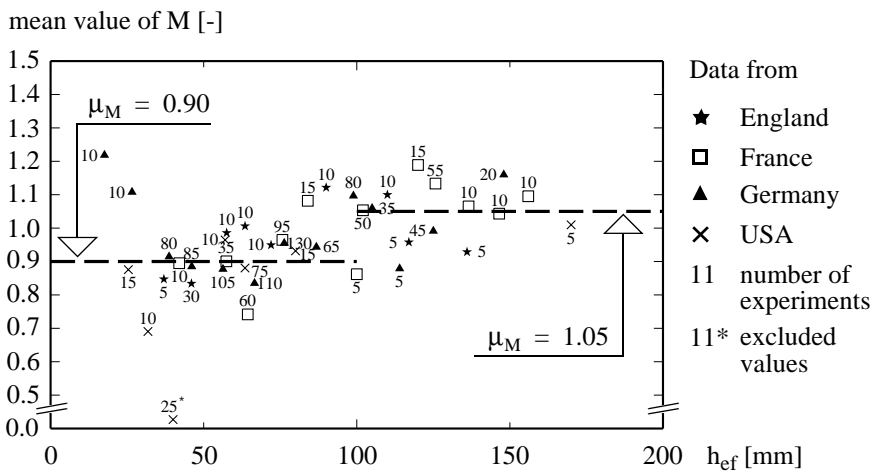


Figure 13: Mean values of the model uncertainty variable M for cone type torque-controlled expansion anchors

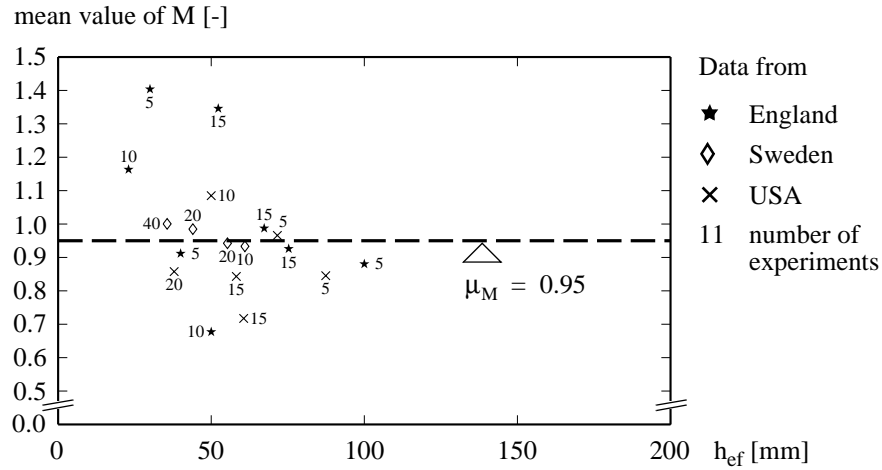


Figure 14: Mean values of the model uncertainty variable M for wedge type torque-controlled expansion anchors

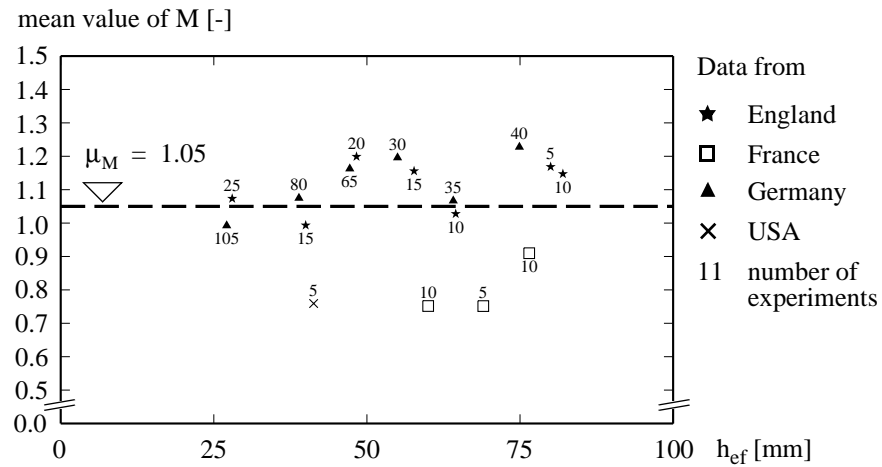


Figure 15: Mean values of the model uncertainty variable M for drop-in anchors

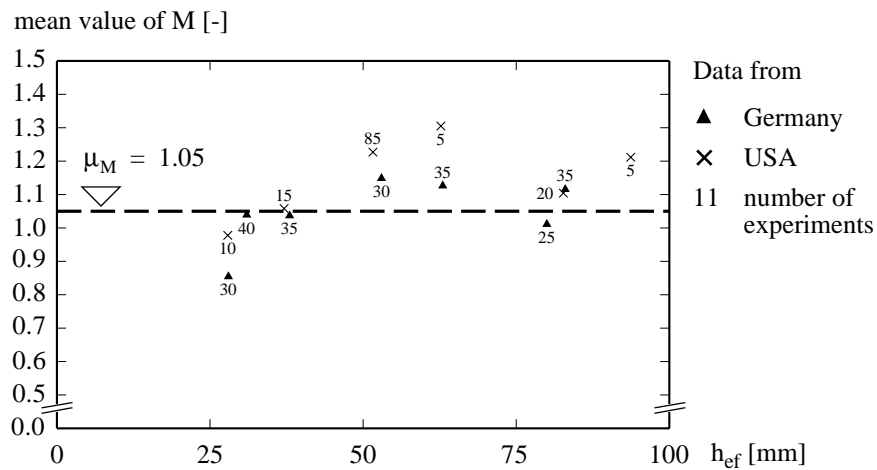


Figure 16: Mean values of the model uncertainty variable M for self-drilling anchors

Only for cone type expansion anchors do the mean values of the model uncertainty variable show a trend towards smaller mean values for smaller embedment depths (see figure 13). Instead of fitting a curve to the different mean values, two constant mean values for the range $h_{ef} < 100\text{mm}$ and the range $h_{ef} \geq 100\text{mm}$ are proposed.

The mean values for the other anchor types do not show a significant dependence on the embedment depth (see figures 11 to 16). Consequently, a constant mean value over the whole range of the embedment depth was chosen. The respective values for all anchor types and the standard deviation of the mean values are given in table 9.

Anchor type	μ_M	σ_μ
Headed studs	0.95	0.065
Undercut anchors	1.00	0.088
Expansion anchors:		
• Wedge type	0.95	0.160
• Cone type ($h_{ef} < 100\text{mm}$)	0.90	0.107
• Cone type ($h_{ef} \geq 100\text{mm}$)	1.05	0.069
• Self-drilling anchors	1.05	0.117
• Drop-in anchors	1.05	0.111

Table 9: Mean values and corresponding standard deviation of the model uncertainty variable M

b) Coefficient of Variation

The coefficient of variation was also analysed for dependence on the embedment depth. For this purpose, the coefficients of variation of each interval were calculated with the corresponding mean values. For all anchor types, the coefficient of variation decreases with increasing embedment depth (see figures 17 to 21).

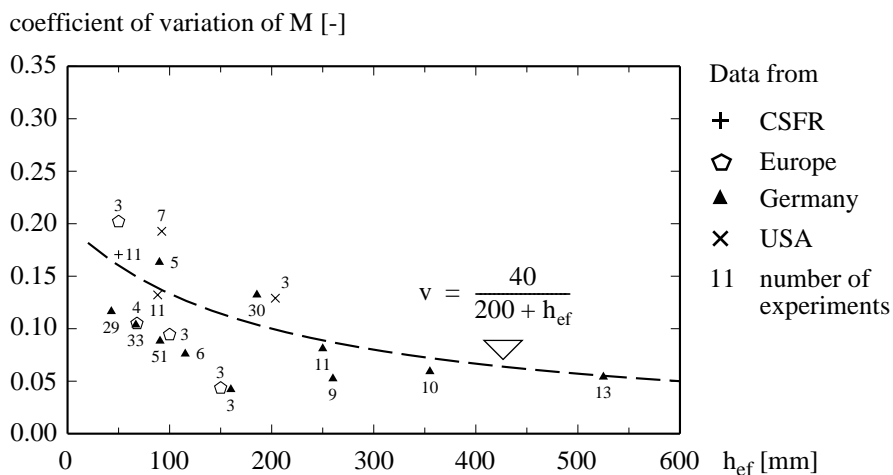


Figure 17: Coefficients of variation of the model uncertainty variable M for headed studs

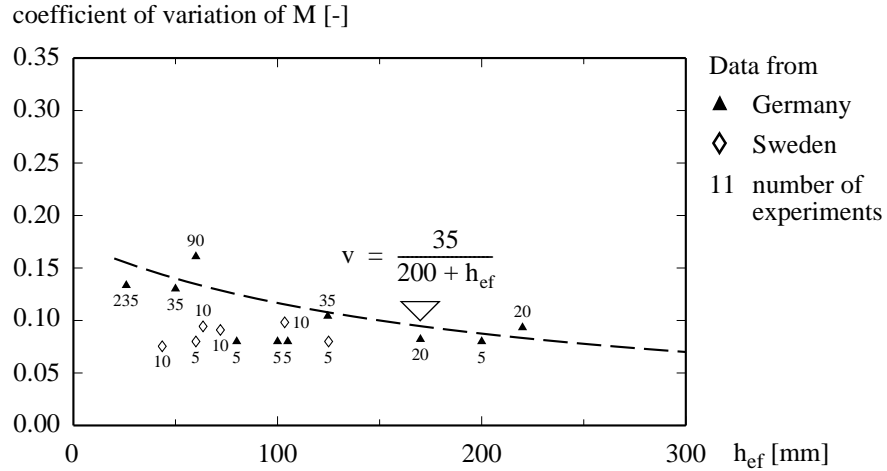


Figure 18: Coefficients of variation of the model uncertainty variable M for undercut anchors

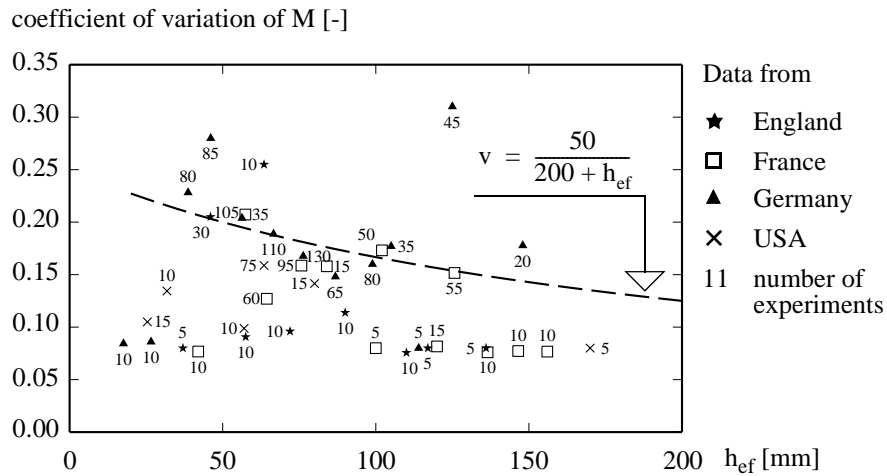


Figure 19: Coefficients of variation of the model uncertainty variable M for cone type torque-controlled expansion anchors

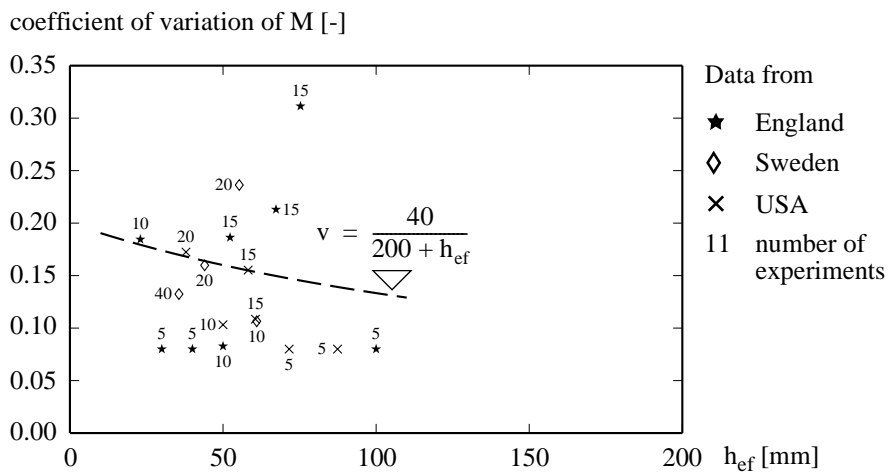


Figure 20: Coefficients of variation of the model uncertainty variable M for wedge type torque-controlled expansion anchors

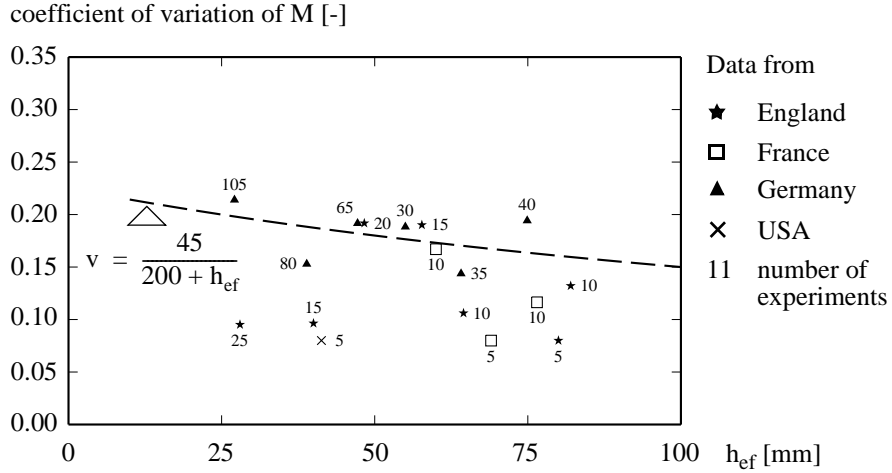


Figure 21: Coefficients of variation of the model uncertainty variable M for drop-in anchors

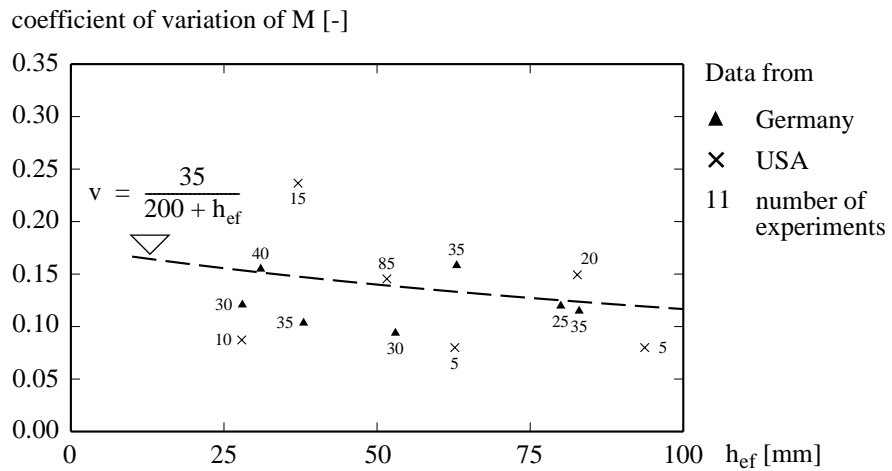


Figure 22: Coefficients of variation of the model uncertainty variable M for self-drilling anchors

The dependence of the coefficient of variation on the embedment depth can be described by the following equation:

$$v = \frac{q}{200 + h_{ef}} \quad (27)$$

where q is an empirical coefficient which depends on the anchor type and h_{ef} is given in [mm]. In table 10 the empirical coefficients for each anchor type together with the corresponding standard deviations of the coefficient of variation are given. The standard deviations were calculated without considering a possible dependence on the embedment depth. For such an analysis the number of tests, especially on anchors with larger embedment depths, is too small.

Anchor type	v_M/e	σ_v
Headed studs	40	0.040
Undercut anchors	35	0.039
Expansion anchors:		
• Wedge type	40	0.076
• Cone type	50	0.053
• Self-drilling anchors	35	0.073
• Drop-in anchors	45	0.051
$e = 200 + h_{ef}$ (h_{ef} in mm)		

Table 10: Coefficient of variation and the corresponding standard deviation of the model uncertainty variable M

5.1.3 Summary

For all anchor types, the mean value of the model uncertainty variable does not seem to be dependent on the embedment depth. The mean values were calculated using the weighted least square method where the weights were equal to the ratio number of tests in an interval divided by the number of all tests. Finally, the resulting values were rounded to the next lower five hundredth.

The coefficients of variation of the model uncertainty variable, however, do show a dependence on the embedment depth, decreasing with increasing embedment depths. This behaviour is described by an empirical function given in equation (27). The chosen function and the corresponding empirical coefficients are a result of a regression analysis combined with expert judgement.

The moments of the model uncertainty variable M for all anchor types are given in table 11. A lognormal distribution for M is proposed based on the fact that negative values are impossible.

Anchor type	μ_M	v_M/e
Headed studs	0.95	40
Undercut anchors	1.00	35
Expansion anchors:		
• Wedge type	0.95	40
• Cone type ($h_{ef} < 100\text{mm}$)	0.90	50
• Cone type ($h_{ef} \geq 100\text{mm}$)	1.05	50
• Self-drilling anchors	1.05	35
• Drop-in anchors	1.05	45
$e = 200 + h_{ef}$ (h_{ef} in mm)		

Table 11: Moments of the model uncertainty variable M for the different anchor types

Naturally, the proposed moments depend on the data. For single anchors the database is sufficiently large so that new data should not change the model uncertainty variable. But it should be mentioned that a great part of the database consists of generated values. This generation was made under the assumption that the test series show a coefficient of variation of 8%. An increase of this coefficient of variation will result in a greater coefficient of variation of the model uncertainty variable, thus influencing the results of a probabilistic analysis, that is to a smaller safety index β .

5.2 Anchors in a Group

5.2.1 The Model

The number of tests on anchors in a group is insufficient for deriving consistent stochastic models. Nevertheless, in order to indicate the paths to follow stochastic models for the 2-anchor and the 4-anchor groups on the basis of the CCD group model are proposed. New test series performed in the future may change the resulting moments of the model uncertainty variable and consequently the results of a probabilistic analysis.

For the concrete cone resistance of a 2-anchor group the following stochastic model is proposed:

$$R_{\text{cone},2\text{group}} = 2 \cdot \psi_2 \cdot Si \cdot M_2 \cdot k \cdot F_{\text{cc}}^{0.5} \cdot H_{\text{ef}}^{1.5} \quad (28)$$

Similarly, the stochastic model of a 4-anchor group can be written as:

$$R_{\text{cone},4\text{group}} = 4 \cdot \psi_4 \cdot Si \cdot M_4 \cdot k \cdot F_{\text{cc}}^{0.5} \cdot H_{\text{ef}}^{1.5} \quad (29)$$

where

- F_{cc} : cube compressive strength of concrete [N/mm²]
- H_{ef} : effective embedment depth [mm]
- k : empirical coefficient of the single anchor [(N/mm)^{0.5}]
- M_2 : model uncertainty variable of the 2-anchor group [-]
- M_4 : model uncertainty variable of the 4-anchor group [-]
- $R_{\text{cone},2\text{group}}$: concrete cone resistance of a 2-anchor group [N]
- $R_{\text{cone},4\text{group}}$: concrete cone resistance of a 4-anchor group [N]
- S : space between anchors [mm]
- Si : site imperfection variable [-]
- ψ_2 : group reduction factor of a 2-anchor group (see equation (22))
- ψ_4 : group reduction factor of a 4-anchor group (see equation (24))

Assuming that the group fails as a whole, the concrete cone resistance of one anchor in a 2-anchor group is therefore:

$$R_{\text{cone},1\text{of}2\text{group}} = \psi_2 \cdot Si \cdot M_2 \cdot k \cdot F_{\text{cc}}^{0.5} \cdot H_{\text{ef}}^{1.5} \quad (30)$$

and in a 4-anchor group it is:

$$R_{\text{cone},1\text{of}4\text{group}} = \psi_4 \cdot Si \cdot M_4 \cdot k \cdot F_{\text{cc}}^{0.5} \cdot H_{\text{ef}}^{1.5} \quad (31)$$

As stochastic model for the model uncertainty variables M_2 and M_4 , a lognormal distribution is proposed. In the following, only the model uncertainty variables will be treated. The other stochastic variables will be explained in chapter 5.3. The proposed models are only valid for ratios $s/h_{ef} < 3$. If $s/h_{ef} \geq 3$, the single anchor model applies.

5.2.2 Model Uncertainty Variable of a 2-Anchor Group

The procedure to obtain the moments of the model uncertainty variable for the 2-anchor group was similar to that for the single anchor. Measured failure loads were divided by calculated values and grouped in categories, for which mean values and coefficients of variation were calculated. For single anchors the grouping was made according to anchor type, origin of the data and embedment depth. Due to the small amount of data available for the 2-anchor group, a grouping in origin and in anchor type was not feasible. Further, the data was grouped according to the ratio of spacing between anchors and embedment depth s/h_{ef} instead of embedment depth alone. The moments of the model uncertainty variable for the 2-anchor group was calculated from a weighted average of the results in each category. The weighting factor was the number of tests in each category divided by the total number of the tests. Table 12 gives the results for each category. Although a slight trend of the mean values is visible, it is not taken into account because of the small and unequal number of tests in each category. It is remarkable that the mean value of the model uncertainty variable is larger than one. In other words, the model underestimates the failure load and is therefore conservative.

To be exact, the model uncertainty variable M_2 is the model uncertainty variable of the whole system. This remark is based on the fact, that only the whole system can be tested and not each element separately. However, for $s/h_{ef} = 1$ the probability that only a single anchor fails is very small. Therefore, the model uncertainty variable for this case is treated as the model uncertainty variable of the anchor group.

s/h_{ef}	n	μ_{M_2}	v_{M_2}
0.25 - 0.75	5	0.74	0.13
0.75 - 1.25	28	1.12	0.18
1.25 - 1.75	8	1.09	0.29
1.75 - 2.25	11	1.21	0.20
2.25 - 2.75	20	1.20	0.18

Table 12: Moments of the model uncertainty variable for the 2-anchor group

5.2.3 Model Uncertainty Variable of a 4-Anchor Group

The data for the 4-anchor group tests contains almost exclusively German tests on headed studs. The data was again grouped according to the ratio of spacing between anchors and embedment depth s/h_{ef} . The moments of the model uncertainty variable for the 4-anchor group were calculated in the same way as for the 2-anchor group. The results are given in table 13. Compared with the model uncertainty variable of the 2-anchor group, the mean value of the 4-anchor group agrees rather well with the CCD model. This may be due to the more homogeneous database for 4-anchor groups.

s/h_{ef}	n	μ_{M_4}	v_{M_4}
0.25 - 0.75	19	0.96	0.20
0.75 - 1.25	21	0.97	0.24
1.25 - 1.75	4	1.07	0.31
1.75 - 2.25	8	1.18	0.37
2.25 - 2.75	6	1.01	0.19

Table 13: Moments of the model uncertainty variable for the 4-anchor group

As mentioned before, M_4 is effectively the model uncertainty variable of the system. Again, it was assumed that for the ratio $s/h_{ef} = 1$ the probability that a single anchor fails alone is very small and that the model uncertainty variable for this case can be treated as the model variable of the anchor group.

5.3 Stochastic variables

5.3.1 Embedment Depth

Investigations have shown that the coefficient of variation of the embedment depth H_{ef} is very small as long as checking procedures on the site are adequate (Bergmeister, 1990). Thus, the embedment depth can normally be introduced as a deterministic value.

5.3.2 Spacing between Anchors

The same consideration as for the embedment depth can be made for the spacing between anchors. Therefore, the spacing may be introduced as a deterministic value.

5.3.3 Concrete Strength

The data available for defining the models and the model uncertainties come from laboratory tests and refer to the concrete properties observed in the tests. For design purposes, a link between the cube compressive strength from the tests and the standardised concrete resistance classes and the respective characteristic values $f_{cc,cube,k}$ must be established. A model drawn from JCSS (1996) is as follows:

$$F_{cc} = \alpha_T \cdot F_{cc,cube}^\Lambda \cdot Y \quad (32)$$

where

α_T : a constant which takes into account the concrete age at the loading time and the duration of loading

$F_{cc,cube}$: standardised cube compressive strength of concrete ($200 \cdot 200 \cdot 200 \text{ mm}^3$), normally at 28 days [N/mm^2]

- Λ : a variable representing the variation of in situ compressive strength compared to strength derived from standard tests
- Y : a variable representing variations due to special placing, curing, and hardening conditions on in situ concrete

According to Bieger (1990) the cylinder compressive strength has a mean value of $(f_{cc,cyl,k} + 8) \text{ N/mm}^2$ where $f_{cc,cyl,k}$ represents the characteristic value. If the characteristic value is interpreted as a 5% fractile, a normal distribution with a standard deviation of 5 N/mm^2 can be assumed. For cube compressive strength the same transformation can be used. In other publications a lognormal distribution is chosen to describe the compressive strength (JCSS, 1996). The variable Λ is described by a lognormal distribution with a mean value of 0.96 and a coefficient of variation of 0.005 (Bergmeister, 1990; JCSS, 1996). Generally it is sufficient to take Λ as a constant. According to JCSS (1996) Y is described by a lognormal distribution with a mean value of 1.0 and a standard deviation of 0.06.

In addition, the age of the concrete at loading may be of interest, as well as the influence of the load duration. Here, this influence was taken into account with the constant α_T . This constant is derived from a deterministic function (see JCSS, 1996) applicable for normal conditions with respect to the type of cement and climatic environment. A more sophisticated integration of these effects and a description of the respective variables can be taken from JCSS (1996), Wood (1991), and CEB (1993).

When assessing anchors in existing structures, the concrete strength may be tested using suitable methods such as core drilling, rebound hammer and the like. Of course, uncertainties inherent in such methods and uncertainties in translating the test results into concrete cube compressive strength must be taken into consideration.

5.3.4 Influence of Site Imperfections

The quality of workmanship on site is different from laboratory conditions. Bad workmanship includes, for example, oversize hole diameter, improper cleaning of drilled hole or application of insufficient torque moment. It is therefore suggested to introduce a site imperfection variable S_i applied to the concrete strength model to cover such effects. However, gross errors are not considered. Obviously, this variable also depends on the anchor type. Further, there is not much data available so that expert judgement is largely applied in order to define the type of distribution and its parameters.

If necessary, the site imperfection variable S_i may be described in more detail as follows:

$$S_i = 1 - P_w \cdot (1 - S_w) \quad (33)$$

where

- P_w : probability of bad workmanship in placing the anchor
- S_w : a variable representing the sensitivity of the anchor resistance relating to bad workmanship

Fixing P_w is a matter of expert judgement. The variable may be best modelled by a Beta distribution $\text{Beta}(r;s;a;b)$ with bounds $[a;b]$ within the interval $[0;1]$. The variable S_w is established by appropriate tests (EOTA, 1997) and may be modelled by a Beta distribution as

well. In CEB (1997) under the notion of installation safety, three classes are introduced to describe the sensitivity of a system relating to bad workmanship. In accordance with this classification, the following parameters for Sw may be used:

- A : systems with low sensitivity: Beta(10;4;0.90;1.00)
- B : systems with normal sensitivity: Beta(3;3;0.75;1.00)
- C : systems with high but still acceptable sensitivity: Beta(5;11;0.65;1.00)

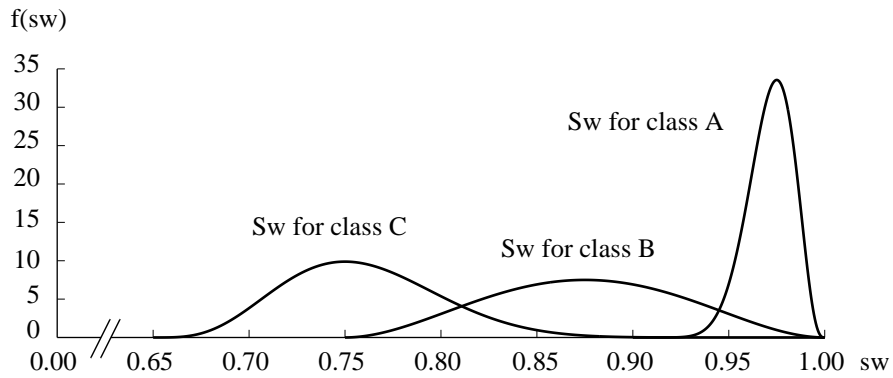


Figure 23: Density function of Sw for the different classes A, B and C

The 5% fractiles of these Beta distributions correspond to the bounds given in EOTA (1997). Class A would comprise cast-in-place headed anchors and most undercut anchors, class B most torque-controlled expansion anchors and class C would mainly consist of deformation-controlled expansion anchors (CEB, 1997).

6 The Probabilistic Model

6.1 The Single Anchor as a Series System

Anchors, depending on its type, embedment depth, steel and concrete strength, may fail by steel yielding or rupture, by concrete cone breakout, by pullout, or by concrete splitting in various ways. All these failure modes build a series system which fails if either one of the modes occurs.

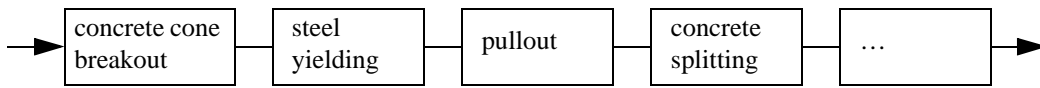


Figure 24: Failure modes of a single anchor

The probability of failure of the system $p_{f,\text{system}}$ cannot be easily calculated since the probabilities of failure of the elements of the system depend on the same load and the same concrete quality. Therefore the elements are correlated to some extent. However, a lower and an upper bound of the probability of failure are given as follows:

$$\max[p_{f,\text{cone}}, p_{f,\text{steel}}, \dots] \leq p_{f,\text{system}} \leq p_{f,\text{cone}} + p_{f,\text{steel}} + \dots \quad (34)$$

where $p_{f,i}$ are the probabilities of failure associated with one of the above modes. The left part of the above inequation applies if all modes are perfectly correlated. The right part is only valid for completely independent modes and only if the probabilities of failure are small.

In the development of the different types of anchors that are currently on the market, the aim is that concrete cone breakout failure precludes other failure modes. Steel failure is excluded by providing an adequate shaft diameter or proper steel quality in relation to the predicted concrete bearing capacity. Concrete bursting is avoided by limiting edge distances and splitting by maintaining a proper member thickness. Other failure types like pullout due to local concrete crushing (in the case of headed studs and undercut anchors) can also be avoided by keeping a proper ratio of the head (or the expansion element) and the shaft diameter. In the case of expansion anchors, preventing pullout relies on a proper design of the load transfer mechanism. The assessment is made by standardized suitability tests.

If the above-mentioned design criteria are properly applied, the concrete cone failure is the most likely failure to occur. The associated probability of failure $p_{f,\text{cone}}$ is the largest of all modes and the probabilities associated with the other modes, generally, are much smaller. This leads to the statement that the probability of failure of the system $p_{f,\text{system}}$ is only marginally larger than the one associated with the concrete cone failure mode, and therefore $p_{f,\text{system}}$ can be approximated by $p_{f,\text{cone}}$:

$$p_{f,\text{system}} \approx p_{f,\text{cone}} \quad (35)$$

In the following, only the concrete cone breakout failure is considered.

6.2 Anchors in a Group as a Series System

The behaviour of anchors in a group is largely determined by equilibrium conditions. First, under increasing load, each single anchor in the group shares the applied load according to equilibrium. This share remains stable even if at the tip of one of the anchors in a group the fracture cone develops. The ultimate load of the group, therefore, is reached if one of the anchors in the group fails under its share of the load. Only if neighbouring anchors in the group are so close that fracturing and radiating from the tip of one of the anchors triggers the neighbouring anchors to fail as well the respective group fracture cone develops and the group fails.

To describe this complex behaviour in a probabilistic model, it is assumed that the failure of an anchor is independent of the other ones, in other words, an anchor can fail without influencing the behaviour of the other ones. Under this assumption a 2-anchor group builds a series system which fails if either one of the anchors fails or both anchors fail at the same time (see figure 25).

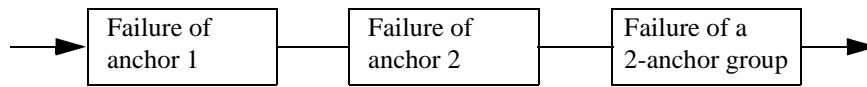


Figure 25: 2-anchor group as a series system

The lower and upper bounds of the probability of failure of this series system are as follows:

$$\max[p_{f,\text{single}1}, p_{f,\text{single}2}, p_{f,2\text{group}}] \leq p_{f,\text{system}} \leq p_{f,\text{single}1} + p_{f,\text{single}2} + p_{f,2\text{group}} \quad (36)$$

where $p_{f,\text{single}1}$ and $p_{f,\text{single}2}$ are the probabilities of failure of one anchor and $p_{f,2\text{group}}$ is the probability of failure of the anchor group. Given the characteristics of the concrete in which the anchors are placed are the same, the anchors being the same, and, essential as well, the load applied remains the same, the elements of the series system may be considered as perfectly correlated. Therefore the probability of failure of the system is close to the lower bound, e.g., close to the largest of the failure probabilities of either the single anchor, or the group.

$$\max[p_{f,\text{single}}, p_{f,2\text{group}}] \approx p_{f,\text{system}} \quad (37)$$

For the 4-anchor group the same consideration holds as for the 2-anchor group. The 4-anchor group builds a series system which fails if one anchor fails or a group of two, three or all four anchors fails. For the single anchor, the 2-anchor group (anchors side by side) and for the 4-anchor group the probability of failure can be calculated. A 2-anchor group with anchors placed in the diagonal has the smaller probability of failure than a group with anchors side by side. This fact is due to the greater space between the anchors. Due to the equilibrium condition the failure of three anchors at the same time is rather unlikely. Hence, the series system of a 4-anchor group has three elements considering only one of the multiple failure modes (see figure 26).

The lower bound of the failure probability of this system can be written as follows:

$$\max[p_{f,\text{single}}, p_{f,2\text{group}}, p_{f,4\text{group}}] \approx p_{f,\text{system}} \quad (38)$$

where $p_{f,\text{single}}$ is the probability of failure of one anchor, $p_{f,2\text{group}}$ that of a 2-anchor group and $p_{f,4\text{group}}$ the one of a 4-anchor group.

Anchor groups with any number of anchors can be treated in a similar way.

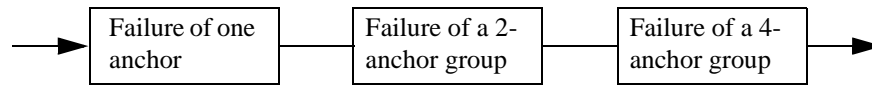


Figure 26: 4-anchor group as a series system

6.3 Probabilistic Analysis

The essential step to proceed with a probabilistic analysis is setting up the so-called limit state function G . This function is normally of the form

$$G(a_i, X_i) = R - S \quad (39)$$

Here R is the stochastic resistance model and S the stochastic stress model. Both models may contain several stochastic variables X_i and/or deterministic coefficients a_i . Failure is defined by the failure condition

$$G(a_i, X_i) < 0 \quad (40)$$

and the probability of failure can be written as:

$$p_f = P[G(a_i, X_i) < 0] \quad (41)$$

With the mean value and the standard deviation of G the so-called Hasofer-Lind safety index β can be determined:

$$\beta = \frac{\mu_G}{\sigma_G} \quad (42)$$

The safety index β shows how often the standard deviation of G may be placed between zero and the mean value of G (see figure 27). The probability of failure is, as mentioned above, the probability that G is smaller than zero. Assuming that G is normally distributed, the failure probability can be estimated as follows:

$$p_f \approx \Phi(-\beta) \quad (43)$$

where $\Phi(x)$ is the standard normal distribution function.

In order to calculate the failure probability or the safety index analytical methods are readily available. More about these methods and the theory behind them can be found in Spaethe (1992), Ditlevsen and Madsen (1996), and Schneider (1997).

In this study the crude Monte Carlo method and the first-order reliability method (FORM) as for example implemented in VaP (1997) are used. The results of the Monte Carlo method are the moments of G and the probability of failure. In addition a histogram can be plotted from the simulation which gives an idea of the form of the limit state function G .

The results of the first-order reliability method are the safety index β , the probability of failure calculated from β , and, for all variables, the sensitivity factors α_i and the design values $x_{i,*}$. The sensitivity factors indicate the relative importance of the variables included in the analysis. The larger the sensitivity factor α_i , the greater its influence on the safety index β .

The design values of the variables correspond to the values for which failure most likely occurs.

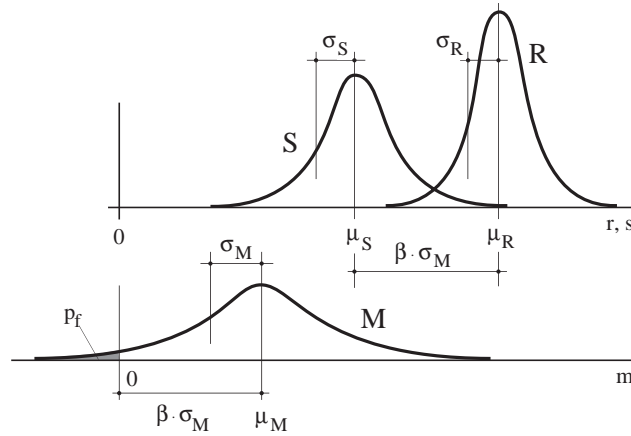


Figure 27: Definition of the safety index β

6.4 Limit State Functions

The limit state function of a single anchor may be written as

$$\begin{aligned} G_{\text{single}} &= R_{\text{cone, single}} - L = Si \cdot M \cdot k \cdot F_{\text{cc}}^{0.5} \cdot H_{\text{ef}}^{1.5} - L \\ &= [1 - Pw \cdot (1 - Sw)] \cdot M \cdot k \cdot [\alpha_T \cdot F_{\text{cc, cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{\text{ef}}^{1.5} - L \end{aligned} \quad (44)$$

- α_T : a constant which takes into account of the concrete age at the loading time and the duration of loading [-]
- F_{cc} : cube compression strength of concrete [N/mm^2]
- $F_{\text{cc, cube}}$: standardised cube compression strength of concrete ($200 \cdot 200 \cdot 200 \text{ mm}^3$), normally at 28 days [N/mm^2]
- H_{ef} : effective embedment depth [mm]
- k : empirical coefficient [$(\text{N}/\text{mm})^{0.5}$]
- L : applied load [N]
- Λ : a variable addressing the variation of in situ compressive strength compared to strength derived from standard tests [-]
- M : model uncertainty variable of a single anchor [-]
- Pw : probability of bad workmanship in placing the anchor [-]
- $R_{\text{cone, single}}$: concrete cone resistance of a single anchor [N]
- Si : site imperfection variable [-]
- Sw : a variable introducing the sensitivity of the anchor relating to bad workmanship [-]
- Y : a variable representing variations due to special placing, curing, and hardening conditions on in situ concrete [-]

The definitions of all the variables may be found in chapter 5. Similarly, the limit state functions of a 2-anchor group is

$$\begin{aligned}
G_{2\text{group}} &= R_{\text{cone},2\text{group}} - L \\
&= 2 \cdot \psi_2 \cdot Si \cdot M_2 \cdot k \cdot F_{cc}^{0.5} \cdot H_{ef}^{1.5} - L \\
&= 2 \cdot \psi_2 \cdot [1 - Pw \cdot (1 - Sw)] \cdot \\
&\quad M_2 \cdot k \cdot [\alpha_T \cdot F_{cc, \text{cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{ef}^{1.5} - L
\end{aligned} \tag{45}$$

where $R_{\text{cone},2\text{group}}$ is the concrete cone resistance, M_2 the model uncertainty variable and ψ_2 the group reduction factor of a 2-anchor group (see equation (22)). The other variables are the same as mentioned above.

The limit state function of each anchor in a 2-anchor group alone is identical to that of a single anchor. Naturally, the load has to be divided by 2.

$$\begin{aligned}
G_{1\text{of}2\text{group}} &= R_{\text{cone},\text{single}} - L/2 \\
&= [1 - Pw \cdot (1 - Sw)] \cdot M \cdot k \cdot [\alpha_T \cdot F_{cc, \text{cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{ef}^{1.5} - L/2
\end{aligned} \tag{46}$$

with M as the model variable of the single anchor.

Finally, the limit state function of the 4-anchor group is given by

$$\begin{aligned}
G_{4\text{group}} &= R_{\text{cone},4\text{group}} - L \\
&= 4 \cdot \psi_4 \cdot Si \cdot M_4 \cdot k \cdot F_{cc}^{0.5} \cdot H_{ef}^{1.5} - L \\
&= 4 \cdot \psi_4 \cdot [1 - Pw \cdot (1 - Sw)] \cdot \\
&\quad M_4 \cdot k \cdot [\alpha_T \cdot F_{cc, \text{cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{ef}^{1.5} - L
\end{aligned} \tag{47}$$

with $R_{\text{cone},4\text{group}}$ as the concrete cone resistance, M_4 as the model uncertainty variable and ψ_4 as the group reduction factor of a 4-anchor group (see equation (24)). For the failure of two anchors side by side of the 4-anchor group, the limit state function is given by

$$\begin{aligned}
G_{2\text{of}4\text{group}} &= R_{\text{cone},2\text{group}} - L/2 \\
&= 2 \cdot \psi_2 \cdot [1 - Pw \cdot (1 - Sw)] \cdot \\
&\quad M_2 \cdot k \cdot [\alpha_T \cdot F_{cc, \text{cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{ef}^{1.5} - L/2
\end{aligned} \tag{48}$$

and for the failure of a single anchor of the 4-anchor group by

$$\begin{aligned}
G_{1\text{of}4\text{group}} &= R_{\text{cone},\text{single}} - L/4 \\
&= [1 - Pw \cdot (1 - Sw)] \cdot M \cdot k \cdot [\alpha_T \cdot F_{cc, \text{cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{ef}^{1.5} - L/4
\end{aligned} \tag{49}$$

7 Examples and Sensitivity Analyses

The following examples and sensitivity studies are based on torque-controlled expansion anchors (cone type) available on the market. Its embedment depth is set to 80 mm. The concrete class is assumed to be C20/25.

7.1 Determination of the Variables

7.1.1 Loads

The stochastic definition of loads, be it live load LL, dead load DL, or a load L reflecting the combined action, depends on the specific application of an anchor. According to classical design rules, the design value L_d of the loads is given by:

$$L_d = \gamma_{DL} \cdot DL + \gamma_{LL} \cdot LL \quad (50)$$

According to Eurocode 1 (SIA, 1995) the partial load factors are $\gamma_{DL} = 1.35$ and $\gamma_{LL} = 1.5$, while the values of DL and LL are assumed to be 95% and 98% fractiles, respectively (JCSS, 1996 b). It is obvious that these fractiles depend on the means and standard deviations of the respective loads and of their distribution types. As a consequence, L_d , though not evident from the formula, depends on the moments of the load variables and so does the probability of failure p_f and the reliability index β (see chapter 7.3.2).

To avoid this dependence in view of examples and sensitivity analyses the load variable L is taken as a deterministic value and is set equal to the design value of the concrete cone capacity N_d . For a torque-controlled expansion anchor, applying the partial safety factors explained in EOTA (1997) N_d is:

$$N_{d,single} = k_1 \cdot f_{cc,cube,k}^{0.5} \cdot h_{ef}^{1.5} \cdot \psi_{ucr} / (\gamma_c \cdot \gamma_1 \cdot \gamma_2) \quad (51)$$

where k_1 is a factor equal to 7.2 [(N/mm)^{0.5}] and $f_{cc,cube,k}$ is the characteristic value of the cube compressive strength of concrete. The factors ψ and γ serve for covering the following effects:

- $\psi_{ucr} = 1.4$: factor to take into account that an anchor is placed in uncracked concrete.
For cracked concrete $\psi_{ucr} = 1.0$.
- $\gamma_c = 1.5$: partial safety factor for concrete loaded under compression
- $\gamma_1 = 1.2$: partial safety factor taking into account the scatter of the tensile strength of in situ concrete
- $\gamma_2 = 1.2$: partial safety factor taking into account the uncertainties due to anchor installation

For the concrete class C20/25 and the embedment depth of 80 mm the design value of the concrete cone capacity and therefore the load variable results in:

Examples and Sensitivity Analyses

$$L_{\text{single}} = N_{\text{d, single}} = 7.2 \cdot 25^{0.5} \cdot 80^{1.5} \cdot 1.4 / (1.5 \cdot 1.2 \cdot 1.2) = 16.7 \text{ kN} \quad (52)$$

The design value of the concrete cone capacity of a 2-anchor group is (EOTA, 1997):

$$N_{\text{d, 2group}} = k_1 \cdot f_{\text{cc, cube, k}}^{0.5} \cdot h_{\text{ef}}^{1.5} \cdot \left(1 + \frac{s}{3 \cdot h_{\text{ef}}}\right) \cdot \Psi_{\text{ucr}} / (\gamma_c \cdot \gamma_1 \cdot \gamma_2) \quad (53)$$

and for a 4-anchor group it is:

$$N_{\text{d, 4group}} = k_1 \cdot f_{\text{cc, cube, k}}^{0.5} \cdot h_{\text{ef}}^{1.5} \cdot \left(1 + \frac{s}{3 \cdot h_{\text{ef}}}\right)^2 \cdot \Psi_{\text{ucr}} / (\gamma_c \cdot \gamma_1 \cdot \gamma_2) \quad (54)$$

For the given example, the dependence of the load variable on the spacing between the anchors results for a 2-anchor group in:

$$\begin{aligned} L_{2\text{group}} = N_{\text{d, 2group}} &= 7.2 \cdot 25^{0.5} \cdot 80^{1.5} \cdot \left(1 + \frac{s}{3 \cdot 80}\right) \cdot 1.4 / (1.5 \cdot 1.2 \cdot 1.2) \\ &= 16.7 + 0.070 \cdot s \quad [\text{kN}] \text{ und } s \text{ in } [\text{mm}] \end{aligned} \quad (55)$$

and for the 4-anchor group in:

$$\begin{aligned} L_{4\text{group}} = N_{\text{d, 4group}} &= 7.2 \cdot 25^{0.5} \cdot 80^{1.5} \cdot \left(1 + \frac{s}{3 \cdot 80}\right)^2 \cdot 1.4 / (1.5 \cdot 1.2 \cdot 1.2) \\ &= 16.7 \cdot \left(1 + \frac{s}{240}\right)^2 \quad [\text{kN}] \text{ und } s \text{ in } [\text{mm}] \end{aligned} \quad (56)$$

The above formulae hold for $s < 3 \cdot 80\text{mm} = 240\text{mm}$. For larger spacings the design loads are bound to $L_{2\text{group}} = 33.4 \text{ kN}$ and $L_{4\text{group}} = 66.8 \text{ kN}$, respectively. For a spacing of 120 mm the design loads become $L_{2\text{group}} = 25 \text{ kN}$ and $L_{4\text{group}} = 37.5 \text{ kN}$.

7.1.2 Model Uncertainty Variables

The moments of the model uncertainty variable M of a single anchor can be determined using the recommendation of table 11. The mean value is set to $\mu_M = 0.90$ and the coefficient of variation to $v_M = 0.18$. Thus the standard deviation calculates to 0.16.

For the 2-anchor group the moments of the model uncertainty variable M_2 are given in table 12. The mean value is 1.12 and the standard deviation calculates to 0.20. Finally the moments of the model uncertainty variable M_4 of the 4-anchor group are given in table 13. The mean value is 0.97 and the standard deviation calculates to 0.23.

7.1.3 Overview of all Variables

In table 14 the definitions and the parameters of all variables are summarized in view of the subsequent examples and sensitivity analyses.

Variable	Distribution type	Mean	Standard deviation	Lower bound	Upper bound
α_T [-]	Deterministic	0.80	-	-	-
$F_{cc,cube}$ [N/mm ²]	Lognormal	33	5	0	-
H_{ef} [mm]	Deterministic	80	-	-	-
k [(N/mm ²) ^{0.5}]	Deterministic	13.5	-	-	-
L_{single} [N]	Deterministic	16700	-	-	-
L_{2group} [N]	Deterministic	25000	-	-	-
L_{4group} [N]	Deterministic	37500	-	-	-
Λ [-]	Deterministic	0.96	-	-	-
M [-]	Lognormal	0.90	0.16	0	-
M_2 [-]	Lognormal	1.12	0.20	0	-
M_4 [-]	Lognormal	0.97	0.23	0	-
Pw [-]	Beta	0.2	0.1	0	0.6
S [mm]	Deterministic	120	-	-	-
Sw [-]	Beta	0.88	0.05	0.75	1.0
Y [-]	Lognormal	1.0	0.06	0	-

Table 14: Definition of the variables

7.2 Examples

7.2.1 Single Anchor

In accordance with equation (44) the limit state function for a single anchor for concrete cone failure is given by

$$G_{single} = [1 - Pw \cdot (1 - Sw)] \cdot M \cdot k \cdot [\alpha_T \cdot F_{cc,cube}^\Lambda \cdot Y]^{0.5} \cdot H_{ef}^{1.5} - L_{single} \quad (57)$$

The parameters of all variables are given in table 14.

The probability density function of the concrete cone resistance $R_{cone, single}$ can be visualized in a histogram form by performing a crude Monte Carlo analysis. In figure 28 the result of a simulation with 100'000 runs is shown. The concrete cone resistance $R_{cone, single}$ has a mean value of 40.5 kN and a standard deviation of 7.9 kN. This results in a coefficient of variation of 0.20.

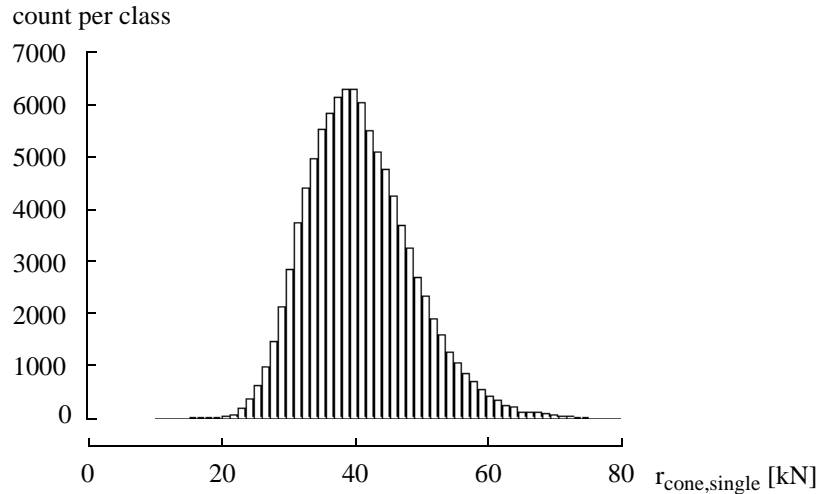


Figure 28: Histogram of the concrete cone resistance generated with a Monte Carlo simulation

Proceeding with a FORM analysis, the Hasofer-Lind safety index β , and, for all variables, the sensitivity factors α and the design values of the variables x^* can be obtained. In table 15 the results for the limit state function G_{single} are shown. Because the load L is considered as a deterministic quantity, the probability of failure which can be estimated from the safety index β corresponds to the probability that the anchor fails given L_{single} is applied to the anchor. To emphasize this fact, the safety index β is appended with the index L .

Variable	α	x^*
$F_{\text{cc,cube}}$	-0.372	25.4
M	-0.907	0.43
P_w	0.093	0.25
S_w	-0.077	0.86
Y	-0.154	0.96
$\beta_L = 4.5$		

Table 15: Results of FORM analysis for G_{single} given L_{single} is applied to the anchor

7.2.2 2-Anchor Group

The limit state functions of a 2-anchor group and of each anchor in the group are given in equations (45) and (46):

$$G_{2\text{group}} = 2 \cdot \left(0.5 + \frac{S}{6 \cdot H_{\text{ef}}} \right) \cdot [1 - P_w \cdot (1 - S_w)] \cdot \quad (58)$$

$$M_2 \cdot k \cdot [\alpha_T \cdot F_{\text{cc,cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{\text{ef}}^{1.5} - L_{2\text{group}}$$

$$G_{1\text{of}2\text{group}} = [1 - P_w \cdot (1 - S_w)] \cdot M \cdot k \cdot [\alpha_T \cdot F_{\text{cc,cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{\text{ef}}^{1.5} - \quad (59)$$

$$L_{2\text{group}}/2$$

The definitions of all variables are given in table 14.

In figure 29 a histogram of the concrete cone resistance of such a 2-anchor group is shown resulting from a crude Monte Carlo simulation with 100'000 runs. The mean value is 75.6 kN and the standard deviation 14.8 kN. The coefficient of variation is equal to 0.20. The results from the FORM analyses are given in table 16. The safety index of the 2-anchor group is smaller than that of one anchor in the group. Thus, with reference to equation (37) the safety index of the system is roughly equal to that of the 2-anchor group. In other words, if the system fails, it is most probably a group failure.

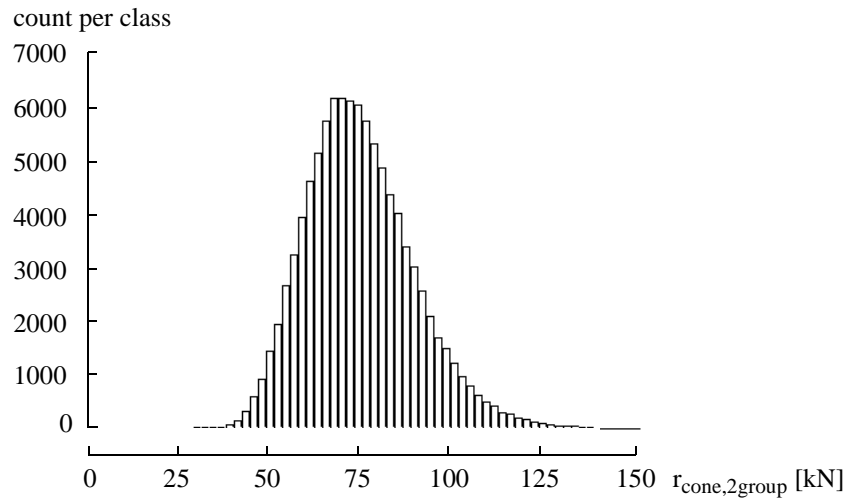


Figure 29: Histogram of the concrete cone resistance of a 2-anchor group generated with a Monte Carlo simulation

G_{2group}			$G_{1of2group}$		
Variable	α	x^*	Variable	α	x^*
$F_{cc,cube}$	-0.370	23.9	$F_{cc,cube}$	-0.372	23.4
M_2	-0.908	0.45	M	-0.907	0.34
P_w	0.097	0.26	P_w	0.099	0.27
S_w	-0.080	0.85	S_w	-0.081	0.86
Y	-0.153	0.95	Y	-0.153	0.95
$\beta_L = 5.6$			$\beta_L = 6.0$		

Table 16: Results of FORM analysis for G_{2group} and for $G_{1of2group}$ given L_{2group} is applied to the 2-anchor group

7.2.3 4-Anchor Group

The three limit state functions as given in equations (47) - (49) are

$$G_{4\text{group}} = 4 \cdot \left(0.25 + \frac{S}{6 \cdot H_{\text{ef}}} + \frac{S^2}{36 \cdot H_{\text{ef}}^2} \right) \cdot [1 - P_w \cdot (1 - S_w)] \cdot \quad (60)$$

$$M_4 \cdot k \cdot [\alpha_T \cdot F_{\text{cc, cube}}^\Lambda \cdot Y]^{0.5} \cdot H_{\text{ef}}^{1.5} - L_{4\text{group}}$$

$$G_{2\text{of}4\text{group}} = 2 \cdot \left(0.5 + \frac{S}{6 \cdot H_{\text{ef}}} \right) \cdot [1 - P_w \cdot (1 - S_w)] \cdot \quad (61)$$

$$M_2 \cdot k \cdot [\alpha_T \cdot F_{\text{cc}}^\Lambda \cdot Y]^{0.5} \cdot H_{\text{ef}}^{1.5} - L_{4\text{group}}/2$$

$$G_{1\text{of}4\text{group}} = [1 - P_w \cdot (1 - S_w)] \cdot M \cdot k \cdot [\alpha_T \cdot F_{\text{cc}}^\Lambda \cdot Y]^{0.5} \cdot H_{\text{ef}}^{1.5} - L_{4\text{group}}/4 \quad (62)$$

The definition of all variables are given in table 14.

A crude Monte Carlo simulation with 100'000 runs results in a mean value of the concrete cone resistance of 98.1 kN and a standard deviation of 24.6 kN. The coefficient of variation calculates to 0.25. Figure 30 shows the histogram of the simulation. The results of the FORM analyses are given in table 17. The 4-anchor group as a whole has the lowest safety index which is with reference to equation (38) roughly equal to the safety index of the system. Thus, the failure of the system is most probably the failure of all four anchors together.

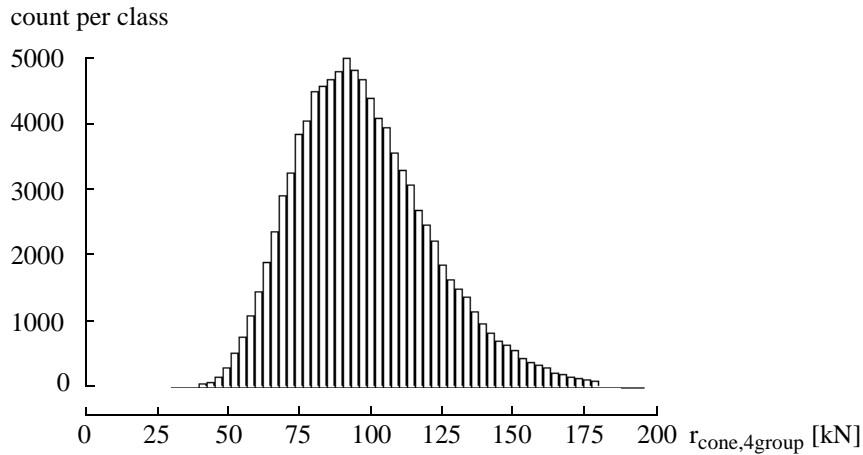


Figure 30: Histogram of the concrete cone resistance of a 4-anchor group generated with a Monte Carlo simulation

G_{4group}			$G_{2of4group}$			$G_{1of4group}$		
Variable	α	x^*	Variable	α	x^*	Variable	α	x^*
$F_{cc,cube}$	-0.293	27.6	$F_{cc,cube}$	-0.370	22.0	$F_{cc,cube}$	-0.371	21.5
M_4	-0.946	0.41	M_2	-0.907	0.36	M	-0.906	0.27
Pw	0.051	0.19	Pw	0.102	0.28	Pw	0.103	0.28
Sw	-0.043	0.88	Sw	-0.084	0.85	Sw	-0.086	0.84
Y	-0.121	0.97	Y	-0.153	0.94	Y	-0.154	0.93
$\beta_L = 3.8$			$\beta_L = 7.1$			$\beta_L = 7.4$		

Table 17: Results of FORM analysis for G_{4group} , $G_{2of4group}$ and $G_{1of4group}$ given L_{4group} is applied to the 4-anchor group

7.3 Sensitivity Analyses

7.3.1 Sensitivity of Results to Changes of the Model Uncertainty Variable

According to the results of the FORM analyses (see tables 15 to 17), the model uncertainty variables have the largest weighting factor α and therefore the greatest influence on the results. To show the order of magnitude of this influence a sensitivity analysis on the basis of the example for the single anchor is undertaken (see chapter 7.2.1).

First the mean value of the model uncertainty variable is varied keeping the coefficient of variation constant at 0.18, then the coefficient of variation is varied keeping the mean constant at 0.90. The results of this analysis are shown in figure 31. As expected, even small changes have large effects on the safety index β_L . The same consideration holds also for anchors in a group.

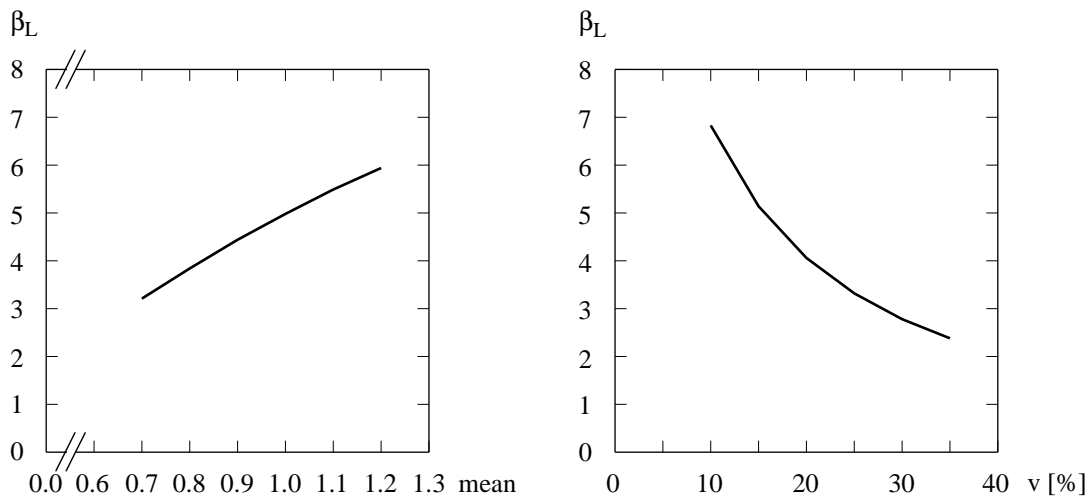


Figure 31: Results of the sensitivity analysis of the mean and the coefficient of variation

7.3.2 Sensitivity of Results relating to the Definition of Loads

The choice of the distribution type of the load variables and its parameters exerts also a great influence on the resulting safety level. To show this, the load which in the example for the single anchor is fixed at $L = N_d$ is now introduced as a stochastic variable.

Corresponding to the code specifications (JCSS, 1996 b) the design requirement is as follows

$$1.5 \cdot L_{95\%} \leq N_d \quad (63)$$

wherein

$L_{95\%}$: 95% fractile of load

N_d : design value of anchor resistance

Hence

$$L_{95\%} = N_d/1.5. \quad (64)$$

The left hand side of figure 32 shows, for a normal, a lognormal and a Gumbel distribution the mean values and the coefficients of variation resulting in the same 95% fractile values. The differences in the mean values do not appear to be large. However, the tails of the distributions are quite different. This results in the fact that the safety index β largely depends on the distribution type (see figure 32 right hand side). The same observation is made if the load variable is split into dead and live loads. Again, this consideration holds also for anchors in a group.

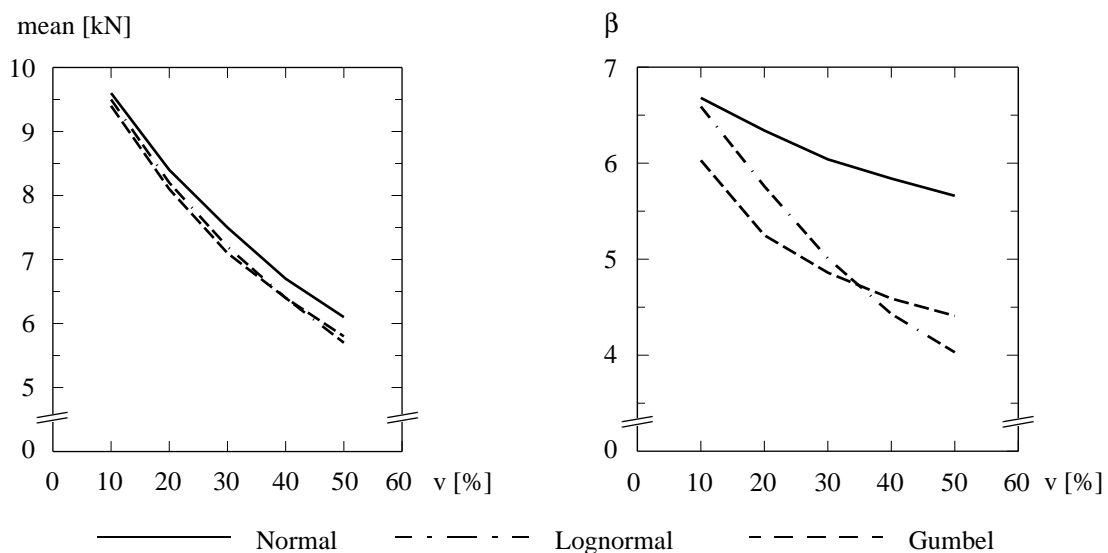


Figure 32: Load parameters and safety index β for different load distributions all resulting in the same 95% fractile

7.3.3 Sensitivity of Results to Changes of the Spacing between the Anchors in a Group

In the following sensitivity analysis, the embedment depth is kept constant at 80 mm and the spacing s is varied. As a consequence, the load which is defined as the design value of the concrete cone resistance also changes with the ratio s/h_{ef} . Therefore, the safety index β_L of the anchor group (limit state function G_{2group} and G_{4group}) remains constant. That corre-

sponding to the failures of one or two anchors of the group, respectively, decreases with increasing ratio s/h_{ef} (see figure 33).

According to equations (37) and (38), the probability of failure of the system corresponds roughly to the largest value of all failure modes. As a consequence, the lowest safety index of all elements is determinant. Thus for the 2-anchor group, the safety index of the system is approximately equal to the safety index of the group up to the ratio $s/h_{ef} \approx 1.8$ and, for larger spacings, equal to the one of the single anchor $\beta_L = 6.0$. For the 4-anchor group, the safety index of the system is approximately equal to that of the 4-anchor group up to the ratio $s/h_{ef} = 3$ and then equal to the failure of one anchor of the group.

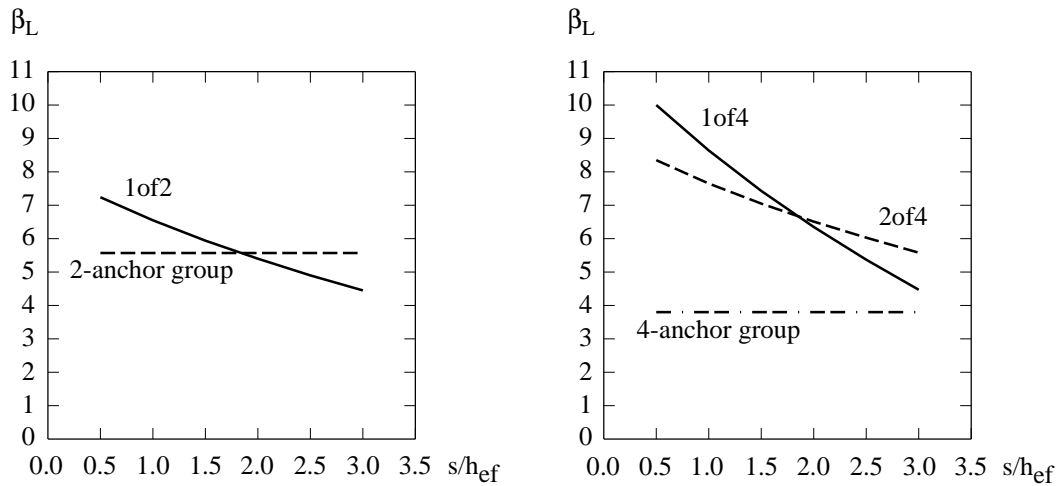


Figure 33: Safety index β_L in dependency with the ratio s/h_{ef}

Theoretically, the safety index for all elements of an anchor group should be the same for $s/h_{ef} = 3$. This is obviously not the case, because the model uncertainty variables of the elements involved have different moments.

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