Doctoral Thesis

Composite ceramic-metal rods and plates systematic investigations on the correlations between local flaws and the global mechanical characteristics

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Composite Ceramic-Metal Rods and Plates: Systematic Investigations on the Correlations between local Flaws and the global Mechanical Characteristics

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH
for the degree of Doctor of Technical Sciences

presented by
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accepted on the recommendation of
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Prof. Dr. P. Jacquot, co-examiner
Prof. Dr. P. Ermanni, co-examiner

Zürich 2001
Für meine Familie
structural wave

heterodyne interferometer; highest resolution: 0.25 nm.

holography and vibrometry

pulse holography
2 Nd:YAG cavities
14 ns pulse duration
1 μs...80 ms pulse separation, phase step techniques

positioning stage
4 μm accuracy and repeatability 1 μm resolution

composite ceramic metal plates

asymptotic approximation
Acknowledgments

I would like to express my sincere gratitude to my advisor, Prof. Dr. M. B. Sayir, for his valuable guidance throughout this work. Many stimulating discussion resulted in deep insights of the physics. Thank to his engagement, this project was supported by the council of the Swiss Federal Institute of Technology.

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Many thanks Dr. Cuche for his valuable advises concerning the optics, W. Guggi and K. Vögeli for their friendly help concerning the linguistic aspects of this thesis.

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Abstract

Non-destructive testing and characterization of metallic components are mostly standardized, however, for ceramic components there are only a few standardized methods available. The standardized non destructive testing methods, e.g. X-rays, thermography, ultrasound and coloring agents which penetrate hair cracks, are usually selected for quality control of ceramics. This approach permits measurements of characteristics of ceramic specimen and allows to localize flaws. However, it is not possible to draw quantitative conclusions on the comprehensive mechanical properties (rigidity of the entire structure, large areas of flaws), because so far, no systematic investigation has been performed. Furthermore, little is known about the characterization of ceramic specimen or ceramic-metal composites by means of structural wave propagation or structural vibration. A quantitative correlation between the standardized non-destructive testing methods and the effective allover mechanical characteristics would be a valuable tool for defining technical standards. Therefore, a systematic investigation based on simple structures such as rods and plates is the object of the present study:

First, the influence of adhesive joints on the dynamic parameters of rods is investigated. Evidently, the quality of the joint influences somehow the resonance frequency and the overall internal damping. According to the mode shapes, the magnitude of these influences is variable. However, for structural resonance modes the wavelength is usually much larger than the thickness of the adhesive film. In order to detect the presence and quality of this film, new experimental schemes to attain the required high precision of the measurements are developed. In fact, the resonance frequencies are measured with an accuracy higher than 0.05 %.
All the knowledge from the rod specimen experiments are adapted to the experiments with composite ceramic-metal plates. As far as structural vibration is concerned, the mode shapes are made visible by means of quasi-heterodyne strobo-holographic interferometry. The main differences between two different optical set-ups, i.e with reference beams close together vs. well separated reference beams, will be discussed in detail. The resonance frequencies are measured with an accuracy of 2%. As a consequence, mode shapes and resonance frequencies coincide quite accurately with calculations based on Kirchhoff’s theory. Moreover, the influence of voids and flaws is partly measurable.

As a première in Switzerland, the evaluation of transient flexural waves has been realized by using the application of quasi-heterodyne holographic interferometry employing pulsed lasers. It was demonstrated that the influence of voids and flaws is a function of frequency, in other words, the wavelength of the structural flexural wave. These influences produce more distinctive results than obtained with vibration experiments. The experimental interferograms (phase maps) coincide quite accurately with the analytic results derived from Mindlin’s theory. The influence of shear and rotary inertia had to be taken into account, since the flexural wave field was measured with extremely high accuracy. Furthermore, Mindlin’s theory will be compared with an asymptotic approximation of the three-dimensional equations of elasticity considering shear stresses and rotary inertia. The detailed architecture of the models leads to a full understanding of the physical aspects behind. These mathematical approaches are a valuable tool for the quantitative interpretation of results obtained from standardized non-destructive testing methods, because an implementation in finite differentiation or integration routines can be carried out easily.
Zusammenfassung

Im Gegensatz zu den Metallen, wo die Prüfung und Charakterisierung von Werkstoffen und Bauteilen weitgehend durch Normen festgelegt sind, existieren derartige Regelwerke für keramische Werkstoffe nur spärlich. Meist werden heute noch die klassisch anwendbaren Prüfmöglichkeiten für metallische Werkstoffe (Durchstrahlungs-, Ultraschall- und Feinbindprüfung), allenfalls angepasst auf die zu findende Fehlergröße, eingesetzt. Mit diesen Verfahren lässt sich zwar ein Teil der wünschenswerten Informationen (Einzelfehler) gewinnen, die Auskunft über den Gesamtzustand (z.B. Struktursteifigkeit, grössere Schwachstellenbereiche) ist ziemlich schwierig zu erarbeiten. Diesbezüglich fehlen weitgehend systematische Untersuchungen. Ferner ist noch wenig bekannt über den Einsatz der Strukturwellenausbreitung und der Strukturresonanz zur globalen Beurteilung ganzer Bauteilabschnitte und die Korrelation der entsprechenden Resultate mit der Ultraschallprüfung. Der Schwerpunkt dieser Arbeit liegt in der systematischen Untersuchung von Stäben und Rohren. Die Auswirkungen diverser Fehlstellen im Verbund werden aufgezeigt und diskutiert:

Zuerst wird anhand von Stäben der Einfluss einer Verbindungsstelle auf das dynamische mechanische Verhalten aufgezeigt, d.h. Resonanzfrequenzen und Dämpfung werden je nach vorherrschender Eigenschwingungsform beeinträchtigt. Allerdings ist der Einfluss der Naht sehr klein, so dass die Resonanzfrequenzen mit hoher Genauigkeit erfasst werden muss. Beispielsweise konnte eine Schwingung bei 10 kHz auf 0.5 Hz genau gemessen werden. Der Einfluss der Naht ist klein, weil sie im Verhältnis zur Wellenlänge extrem schmal ist. Damit dieser Einfluss nicht von anderen Einflussgrössen, wie Lagerung, Temperatur und Geometrie,
überschattet wird, mussten die Messmethoden entsprechend angepasst werden.


List of Symbols

\(a, \, a'\) length, corrected length
\(a_r, \, a_t\) twist amplitude of an incident, reflected and transmitted wave at a flaw in a rod.
\(a_1, \, a_2\) constants
\(A\) Amplitude of a vibration
\(A_{\text{max}}, \, A_{\text{min}}\) maximal and minimal Amplitude of vibration
\(\alpha\) angle
\(\alpha\) phase
\(\alpha\) in-plane index 1,2
\(\alpha T\) center time of a strobo pulse
\(\Delta \alpha\) phase difference
\(b, \, b'\) width, corrected width
\(B\) magnetic field
\(\beta\) in-plane index 1,2
\(c\) index of ceramic
\(C\) constant
\(C, C^*\) spring constant, complex spring constant
\(C_1, \, C_2\) electric capacity
\(\gamma\) mechanical strain
\(\gamma\) in-plane index 1,2
\(d\) piezoelectric charge constant
\(D\) bending stiffness in a plate
\(D^e\) electric displacement
\(\delta\) error in distance
\(\delta\) damping parameter
\(\delta_{\alpha \beta}\) Kronecker symbol
\(\delta_{\text{lim}}\) detection limit of the vibrometer
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta )</td>
<td>Laplace operator</td>
</tr>
<tr>
<td>( e_{ijk} )</td>
<td>permutation tensor</td>
</tr>
<tr>
<td>( E, E' )</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>( E^e )</td>
<td>electric field</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>scale</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>permittivity constant</td>
</tr>
<tr>
<td>( \hbar \nu )</td>
<td>energy of the photon</td>
</tr>
<tr>
<td>( \eta )</td>
<td>quantum efficiency</td>
</tr>
<tr>
<td>( \eta )</td>
<td>value indicating the importance of shear and rotary inertia of a flexural wave in a Mindlin’s type plate</td>
</tr>
<tr>
<td>( \zeta_n )</td>
<td>PLL damping factor</td>
</tr>
<tr>
<td>( \vartheta_1, \vartheta_2 )</td>
<td>phase of the signals in the electronic Phase Locked Loop (PLL) circuit</td>
</tr>
<tr>
<td>( \Theta, \Theta_1, \Theta_2 )</td>
<td>twist amplitude</td>
</tr>
<tr>
<td>( f )</td>
<td>frequency</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>resonance frequency</td>
</tr>
<tr>
<td>( F(s) )</td>
<td>filter transfer function</td>
</tr>
<tr>
<td>( F )</td>
<td>force</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>force magnitude</td>
</tr>
<tr>
<td>( g )</td>
<td>piezoelectric voltage constant</td>
</tr>
<tr>
<td>( G )</td>
<td>shear modulus</td>
</tr>
<tr>
<td>( G^* )</td>
<td>complex shear modulus</td>
</tr>
<tr>
<td>( G )</td>
<td>transfer function</td>
</tr>
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<td>( G(s) )</td>
<td>transfer function of the closed loop</td>
</tr>
<tr>
<td>( h )</td>
<td>thickness</td>
</tr>
<tr>
<td>( H(s) )</td>
<td>transfer function of the open loop</td>
</tr>
<tr>
<td>( H_0^{(1)}, H_0^{(2)} )</td>
<td>Hankel functions</td>
</tr>
<tr>
<td>( i )</td>
<td>( \sqrt{-1} )</td>
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<td>( i )</td>
<td>general index: 1,2,3</td>
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<tr>
<td>( j )</td>
<td>index</td>
</tr>
<tr>
<td>( I_{I, II} )</td>
<td>index of the components in a compound plate</td>
</tr>
<tr>
<td>( I_C )</td>
<td>polar moment of inertia of a mass</td>
</tr>
<tr>
<td>( I_p )</td>
<td>polar moment of inertia of a rod</td>
</tr>
<tr>
<td>( j )</td>
<td>general index: 1,2,3</td>
</tr>
<tr>
<td>( J_0 )</td>
<td>Bessel function</td>
</tr>
<tr>
<td>( k )</td>
<td>duty cycle factor</td>
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<td>general index: 1,2,3</td>
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<tr>
<td>Symbol</td>
<td>Description</td>
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<tr>
<td>k</td>
<td>index</td>
</tr>
<tr>
<td>k</td>
<td>index of kovar (nickel alloy)</td>
</tr>
<tr>
<td>$k_2$</td>
<td>factor</td>
</tr>
<tr>
<td>$K_D$</td>
<td>gain factor</td>
</tr>
<tr>
<td>$K_0$</td>
<td>PLL transfer coefficient</td>
</tr>
<tr>
<td>$\kappa, \kappa_n, \kappa_m$</td>
<td>wave number</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>correction factor in Mindlin’s equations</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>damping coefficient</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>general index 1...6</td>
</tr>
<tr>
<td>$L, L_1, L_2, L_N$</td>
<td>length of a rod</td>
</tr>
<tr>
<td>$m$</td>
<td>mass</td>
</tr>
<tr>
<td>$m$</td>
<td>mode number in compound rods</td>
</tr>
<tr>
<td>$m/n$</td>
<td>mode number in plates</td>
</tr>
<tr>
<td>$M$</td>
<td>index of the backing mass</td>
</tr>
<tr>
<td>$M_x, M_y$</td>
<td>bending moment per unit length</td>
</tr>
<tr>
<td>$M_{xy}$</td>
<td>twisting moment per unit length</td>
</tr>
<tr>
<td>$\mu$</td>
<td>general index 1...6</td>
</tr>
<tr>
<td>$n$</td>
<td>mode number in rods</td>
</tr>
<tr>
<td>$n$</td>
<td>index of nickel alloy</td>
</tr>
<tr>
<td>$N$</td>
<td>index of the n-th data point</td>
</tr>
<tr>
<td>$N$</td>
<td>index of the n-th element in a stack</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$Q$</td>
<td>quality factor</td>
</tr>
<tr>
<td>$Q_x, Q_y$</td>
<td>shear force per unit length</td>
</tr>
<tr>
<td>$r$</td>
<td>radius</td>
</tr>
<tr>
<td>$r_1, r_2$</td>
<td>radius of a spherical surface</td>
</tr>
<tr>
<td>$R$</td>
<td>radius</td>
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<tr>
<td>$R$</td>
<td>radial distance</td>
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<tr>
<td>$R_i, R_o$</td>
<td>inner and outer radius of a cylinder</td>
</tr>
<tr>
<td>$R_1, R_2, R_3, R_4$</td>
<td>electric resistance</td>
</tr>
<tr>
<td>$\rho$</td>
<td>specific mass</td>
</tr>
<tr>
<td>$s$</td>
<td>Laplace transform of the time</td>
</tr>
<tr>
<td>$s_E$</td>
<td>mechanical compliance</td>
</tr>
<tr>
<td>$s_D, s_{D33}$</td>
<td>mechanical compliance</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>mechanical stress</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>stress for scaling purpose</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
</tbody>
</table>
Δt: discrete time increment
T: period of a harmonic oscillation
T: time for scaling purpose
T₀, T₀, T₀: torque amplitude
τ₁, τ₂: time constants of the PLL filter
Y: scale
u, u: displacement
u: in-plane displacement
u₃: displacement
v: in-plane displacement
v₀: velocity magnitude
V_max, V_min: maximal and minimal voltage
V₀: electro static potential
φ: circumferential coordinate
χ: value considering the correction of rotary inertia due to shear correction in a Mindlin’s type plate
ψ: shear strain correction
w, w: transversal displacement
Wₘₙ: mode shape
ω: circular frequency
ω_max, ω_min: maximal and minimal circular frequency
ωₙ: PLL natural circular frequency
ωₘₙ: circular resonance frequency in a plate
ω₀: circular resonance frequency
ω₁: circular frequency below resonance
ω₂: circular frequency above resonance
Δω: circular frequency difference
x: displacement
x: in-plane coordinate
x: vector defining a point in a space
x₀: distance
y: in-plane coordinate
y₀: distance
Y₀: Bessel function
z: out-of-plane coordinate
z₀: coordinate of the “neutral plane” in a plate
Zp: relation between transversal force and displacement
1. Introduction

This project is supported by the council of the Swiss Federal Institute of Technology (ETH-Rat). Ceramic-metal composites are designed for various applications, e.g. they are used in turbines or in lasers. But if the bonding between the metal and ceramic component is defect, the use with respect to mechanics of the composite plate is lost. Therefore, the ceramic-metal composites have to be tested. By the combination of standardized non destructive testing methods, such as ultrasound, X-rays and thermography, flaws in such composite structures are detected with high resolution. However, from the visualized flaw alone, it is not possible to decide if the composite fulfills its technical specification. Therefore, a correlation to the mechanics has to be taken into account.

Such a correlation can be achieved by systematic investigations. The influence of voids and flaws in composites structures on the global mechanical behavior is the focus of attention in this work rather than their localization. Correspondingly, the specimen is set under mechanical load. In order to investigate the influence mentioned above as a function of frequency, dynamic load is applied.

One well known method is modal analysis with vibration experiments. Another method is structural wave propagation. The velocity of the wave and the wavelength is a function of the frequency. The common main feature of these methods is that the characteristic wavelength of the structural wave or of the oscillation is much larger than the thickness of the plate or the diameter of the rod. Thus, global mechanic characteristics of the structure are investigated rather than local discontinuities. The main difference between these two methods is that the boundaries of the specimen do not
influence the structural wave as far as the wave front has not reached
them. In contrast, structural vibration are influenced by all boundaries.

The propagation of structural waves in thin structures such as beams,
tubes, plates and shells has been the subject of several studies at the Insti-
tute of Mechanics since 1980 (see [63]). The characteristic wavelengths
are large in comparison with the thickness of the structure. In contrast to
through thickness information usually obtained from ultrasonic measure-
ments, structural waves have proved to be quite useful for evaluating glo-
bal characteristics of the structure, e.g stiffness and relaxation effects. In
spite of the relatively large magnitude of the wavelengths, such structural
waves have been shown to allow detection of hidden defects too [15].

In the present work, the influences of such flaws on the dynamic be-
havior of ceramic-metal rods and plates are studied by means of structural
vibrations and structural wave propagation. As a consequence, the results
can be theoretically interpreted with the help of quantitative analytical
models. In combination with standardized non-destructive testing meth-
ods (Ultrasound, X-rays and thermography) a valuable quantitative char-
acterization of the global mechanical behavior of ceramic-metal
composite is possible. This is the main condition to define technical spec-
ifications of ceramic-metal composites.

1.1 The Rod Specimen

The experiments are designed to being theoretically interpreted with the
help of a quantitative analytical model. This requires structures with sim-
ple geometries, e.g. beams, tubes and plates. Furthermore, the same crite-

The physics of rods is easily interpretable and the phenomena can be
quantitatively characterized using simple analytic models. Thus, the dy-
namic characteristics of brazed joints in composite ceramic-metal and ce-
ramic-ceramic rods are investigated first. Then, all the knowledge is
adapted to composite ceramic-metal plates.
1.2 Composites Ceramic-Metal Plates

1.2.1 Brazing

The composite plates were manufactured at the EMPA, Dübendorf. Dr. A. Satir [58] managed to braze plates with top surfaces of 101 x 76 mm². The test samples iron-nickel alloy plate named kovar (Ni 30%, Co 17%, Fe 53%) were combined with Al 23 ceramic plates (typical 99.7% Al₂O₃). As brazing material AgCuTi filler metal foils (Ti 3%) of 60 µm thickness were used. In order to ensure good wetting, the surfaces were polished. After brazing in a vacuum furnace at 900°C and 5 x 10⁻⁶ mbar the compounds are bent such that the mid-point is 2.5 mm above the corners, because the thermal coefficients of ceramic and kovar are different.

The specifications of the composite plate may be affected either by non-bonding areas in the brazed joint, because the wetting did not take place, or by hair cracks in the ceramic layer, because the ceramic is under permanent tensile force. The flaws are detected with the combination of following standardized non destructive testing methods:

1.2.2 Ultrasonic Imaging

The measurements are made on a high-resolution immersion scanner using the pulse-echo technique at 20 MHz or 50 MHz. With the help of the C-scan method the magnitude of the echo from the bond region is evaluated and displayed as shown in Fig. 1.1. Flaws, such as missing solder in the brazed joint or hair cracks in the ceramic component, are easily detected. Apart from designed non-bonding areas with circular shapes looking like ink blots in Fig. 1.1, some insufficient wetting led to missing solder, in particular along the boarders where the evenness of each plate-component is slightly different.

However, with an ultrasonic image alone, it is not possible to decide if a hair crack has been detected or a non-bonding area or a combination of both. For unambiguous interpretation, the images have to be compared with results obtained by another test method.

Fig. 1.1: Ultrasonic images of metal-ceramic compound plates with different main characteristics. The samples on the left hand side are used for structural resonance studies (see “Structural Resonance in Plates” on page 43) and the samples on the right hand side are investigated with the structural wave propagation method (see “Structural Wave Propagation in Plates” on page 81).
1.2.3 Microfocus Radiography

![Micrographs of composite ceramic-metal plates with cracks in the ceramic layer (left), which cannot be visualized by this method, and a non-bonding area in the brazed joint (right).]

A 200 kV 1.0 mA microfocus X-ray tube is applied in conjunction with an image intensifier allowing an on-line viewing on the X-rays image. The detection limit under optimum conditions is about 0.01 mm. The detection of flaws is only possible if the brazeing material has a comparable contrast to the base material. Missing solders, i.e. non-bonding areas, may be detected, however, no cracks in the ceramic, as far as they are oriented in the direction of the X-rays, will not.

1.2.4 Thermography

![Infrared thermogram. In order to increase the emissivity, the specimen is blackened in its center area (light gray scale area).]
The ceramic layer is heated up by a flash lamp. A infrared camera (8-12 μm wavelength) captures up to 30 images per second with a resolution of 0.1°C (see Fig. 1.3). Non-bonding areas may be detected because heat is built up in such regions. However, the lateral elongation has to be at least 3-4 times bigger than the depth of the flaw.

1.2.5 Conclusions

With standardized non destructive measuring methods flaws in ceramic-metal composites can precisely be localized with a high resolution. Moreover, it can be determined which type of flaw (hair crack, inclusion, defect bonding) is detected, by combination of different standardized non-destructive methods.

Nevertheless, no quantitative information of the influence of such flaws on the global mechanical behavior of the structure is given. Therefore, a correlation between these standard measurements and the mechanical characteristics of the structure, has to be taken into account. The global mechanical behavior will be determined with the help of structural waves or structural vibrations experiments. This allows to determine the influence of voids and flaws in ceramic-metal composites.

The table on the next page gives an overview of the work done in this thesis. The conclusions concerning the techniques are given in each cell. This table should help to keep the orientation about which techniques are used and combined.
### Structural Resonance in Torsion in Rods
- Brazed joint as a visco-elastic torsional spring

### Structural Resonance in Plates
- Kirchhoff

### Structural Wave Propagation in Plates
- Kirchhoff, Mindlin
- Asymptotic Approximation incl. shear and rotary inertia

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<th>Applied Techniques</th>
<th>Topic</th>
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<td>- brazed joint as a visco-elastic</td>
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<td>- torsional spring</td>
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<td>- influence of the mass used for frequencies over 10 kHz</td>
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<td>- high impedance above 10 kHz</td>
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<td>- simplest setup with standard</td>
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<tr>
<td>- deviation 0.05%</td>
<td>- 10000 data points within 100 μs</td>
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<tr>
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<td>- limited range in magnitude 0.3 μm</td>
<td>- 0.3 μm ... 3 μm</td>
</tr>
<tr>
<td>- double pulse</td>
<td>- laser pulse used also as actuator</td>
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2. **Structural Resonance in Torsion in Rods**

2.1 **Introduction**

This method is based on studying torsional vibrations of composite rods in the vicinity of their first few resonance modes. Evidently, the quality of the joint influences somehow the resonance frequency and the overall internal damping. But this small influence may be overshadowed by other spurious effects which have to be eliminated in order to reach the required high precision allowing its detection. The influence of the adhesive film on the dynamic parameters is small mainly because the wave lengths of the first few resonance modes are much larger than the thickness of the film. Hence, the resonance frequencies and the damping must be measured with very high precision and the experiments must be designed very carefully.

2.2 **Methods to Measure the Frequency and Damping of an Mechanical Oscillator**

Experimental modal analysis refers to the process of determining the modal parameters (frequencies, damping factors, and mode shapes) of a linear, time invariant system by way of an experimental approach. Naturally, the modal parameters may be determined by analytical means and one of the common reasons for experimental modal analysis is the verification of the results of the analytical approach.
In order to understand modal analysis, the complete comprehension of a single-degree-of-freedom system with mass $m$, spring constant $C$ and damping coefficient $\lambda$ is necessary. The well-known differential equation of the displacement $x$ due to a harmonically oscillating force $F(t) = F_0 e^{i\omega t}$ and $f(t) = F(t)/m$ is

$$\ddot{x} + 2\delta \dot{x} + \omega_0^2 x = f e^{i\omega t} \quad \text{with} \quad \omega_0 = \sqrt{\frac{C}{m}} \quad \text{and} \quad \delta = \frac{\lambda}{2m} \quad (2.1)$$

An equivalent equation of motion may be determined for the Fourier or frequency ($\omega$) domain. This representation has the advantage of converting a differential equation to an algebraic equation. This is accomplished by taking the Fourier transformation of equation (2.1) leading to a complex transfer function

$$G(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{(\omega_0^2 - \omega^2) + i \cdot 2\delta \omega} \quad (2.2)$$
Amplitude and phase spectra of equation (2.2) are sketched in Fig. 2.1. In case that the angular frequency of the force equals $\omega_0$, following important statements are valid [61],[14]:

- Displacement and force are $90^\circ$ out of phase
- The magnitude of the transfer function is equal to $1/(2\omega_0\delta)$.
- $\omega_0$ is the resonance frequency of the system with $(\delta \to 0)$
- The slope of the phase vs. frequency curve is equal to $1/\delta$.

For later reference the mechanical Q-factor (quality factor) is defined:

$$Q = \frac{\omega_0}{(\omega_2 - \omega_1)} = \frac{\omega_0}{(2\delta)}$$

where $\omega_1$ and $\omega_2$ are the frequencies, where the phase of the transfer function is equal to $\pi/4$ and $3\pi/4$ respectively, which also correspond to the frequencies, where the dissipated power dropped to one half of its maximum value and thus the amplification shrinks to $1/\sqrt{2}$ of its maximum value. The damping of the oscillator can be determined from its behavior in the vicinity of resonance by measuring

- the power dissipated at resonance
- the magnitude of the transfer function at resonance
- two frequencies for given values of phase of the transfer function (Fig. 2.1). From the difference of these two frequencies, the damping can be computed. This method is especially suitable, because the damping measurement is reduced to an intrinsically digital measurement of two frequencies.

Following relationship may be useful for strongly damped systems:

$$\frac{\omega_2 - \omega_1}{\Delta \omega} = \frac{\tan(\pi/4)}{\tan \Delta \alpha}$$

where $\Delta \alpha$ is the phase shifted up and down around $\pi/2$ (Fig. 2.1) and $\Delta \omega$ is the measured difference of the frequencies.
2.3 Experimental Setup

As already mentioned in the “Introduction” on page 21, the resonant frequencies and damping must be measured with very high precision and the experiment must be designed very carefully.

Thus, the position of the supports must be chosen adequately, i.e. as close as possible to the nodal cross-sections of the oscillating mode. As a consequence, energy-loss is reduced to a minimum. Placing the supports at the nodal cross-section requires careful positioning for each resonance frequency. The supports are realized either by two tight threads or by a foam, which takes the advantage of having tolerance to some misplacement. However, the foam must have following characteristic, to be more suitable than threads: The contact surface between the foam and the rod measures between 5 mm and 10 mm and the rod, with 8 mm diameter and 300 mm length, sinks about 3 mm into the foam.

Furthermore, temperature changes have to be measured and controlled because they influence the resonance frequency by changing shear modulus of the material. On condition that the resonance frequency is at 10 kHz, the frequency decreases approximately by 2 Hz in steel and by 0.5 Hz in ceramic for each 1 °C increase. The temperature variations are not only recorded (directly) with a Firag PT100 elements, with a resolution of 0.1 °C, but also by measuring the resonance frequency of reference rods of ceramic and stainless steel. As far as the rods composite St 37-brass rods are concerned, the temperature was recorded only with a conventional quicksilver thermometer. All resonance frequencies are corrected such that their values corresponds to frequencies at 23 °C for ceramic and stainless steel rod and at 27 °C for St 37 and brass rod. The extreme temperatures in the laboratory were 19 °C and 27 °C respectively.

2.3.1 Excitation and Sensing

Torsional modes have been chosen to minimize acoustic radiation, which would increase the overall damping. Either a piezoelectric or an electromagnetical transducer was used to produce the necessary torque.
Experimental Setup

**Fig. 2.2:** Piezoelectric (left) or electromagnetic transducer (right) used for producing torsional vibrations.

**Coupling effects** between the electrical circuit and the mechanical oscillator may be avoided by using piezoelectric excitation. For frequencies far below the resonance frequency of the piezoelectric element, this effect is negligible [14]. The details are discussed in chapter 4.2.1 on page 81. The particular transducer is a modified ring of piezoelectric ceramic material (Ferroperm PZ 27) with inner and outer diameter of 5 mm and 10 mm respectively and a thickness of 2 mm. This element is polarized in axial direction and if an electric field is applied in a direction orthogonal to the polarization, shear is induced. Thus, circumferential shear can be produced by circumferential electric field. To apply the voltage, the ring has to be broken apart along a diameter. The fractured surfaces are silver-painted and serve as electrodes. In order to obtain a resultant torsional moment, one of the transducer halves has to be turned upside down. As a consequence, the resultant shear will be in the same direction for both halves, because opposite electric fields are applied to opposite polarization. The transducer is glued with a two component fast cure epoxy adhesive (Perma Bond double bubble) on an electrically isolating ceramic disk. Of course, the overall damping is enlarged by this adhesive.

The other transducer sketched in figure Fig. 2.2 is based on electromagnetic principle. The torque results from induced Lorentz forces. The magnetic field is realized by two permanent magnets with a thickness of 5 mm and a diameter of 8 mm (Magnetfabrik Bonn, Alnico 500). Unwanted damping may also be caused by the reaction force of the excitation. Therefore, they are held in a solid holder so that mechanical power radiated into neighboring structural part is minimized. A thin wire (0.08 mm) is fixed either by an epoxy (Ciba Geigy AV118, hardened in a furnace for one hour at 190 °C) or a two component adhesive based on ceramic powder (Firag 1500). Both adhesive are excellent as far as their low damping coefficient is concerned (see Fig. 2.12 on page 40). In order to get distinctive signals, the wire has to be wind three times around a ceramic rod or
at least eight times around a steel or a steel-ceramic rod (with a diameter of 8 mm).

The mass of the electromagnetical transducer amounts to only 0.01 g (fixed on the ceramic rod and therefore including the adhesive) whereas the weight of the piezoelectric transducer is 1.35 g (not fixed on the rod and therefore excluding the weight of the adhesive ~0.05 g). Thus, the mass of the electromagnetical transducer is negligible, even for the ceramic rods (~61 g). On the other hand the influence of the mass of the piezoelectric transducer is detectable and has to be taken into consideration.

The response of the vibrating structure is recorded either by the same transducer used for excitation or with a heterodyne laser interferometer. In the first case a patented procedure [28] is applied. A gate generator defines consecutive alternating sequences of excitation and measuring, with 8 cycles each, and allows the use of the same coil for both, actuator and sensor, to prevent cross-talk. Of course, using a laser interferometer to record the response is another way of avoiding cross-talk between excitation and sensing.

### 2.3.2 Electronic Control

As already mentioned in chapter “Methods to Measure the Frequency and Damping of an Mechanical Oscillator” on page 21 the frequency is stabilized by fixing the phase between force (excitation) and displacement (sensing). This can be done with the help of a phase locked loop [4] & [25].

The phase locked loop contains three basic components: Phase detector (PD), loop filter, with operation amplifier, and a voltage controlled oscillator, whose frequency is controlled by an external voltage (Fig. 2.3). The phase detector compares the phase of a rectangularized periodic signal (2) with the phase of the TTL signal of the VCO (1). In case of the PD being a simple EXOR-gate, a rectangular signal with duplicated frequency (3) is the input for the low-pass filter. If the signal, which is a rectangularized response from the excited oscillator, and the reference, which excites after amplification the actuator, are 90° out of phase, the VCO’s input (4) is exactly 2.5 V. Only at this level the frequency is stabilized, otherwise the VCO, which is integrated in the chip MM74HC4046, readjusts the frequency until this voltage input is reached. The corresponding frequency is known as the center frequency.
Fig. 2.3: Basic phase locked loop. The voltage controlled oscillator and the phase detector, which is a simple EXOR-gate, are integrated in the chip MM74HC4046.

In appendix “A TTL Phase Locked Loop” on page 117 the phased locked loop is described in details, including quantitative information about center frequencies, lock range and maximal sweep rate. The lock range is the bandwidth where the phase locked loop is able to lock in. The maximal sweep rate is the maximal change of frequency which allows the loop to lock in as soon as the frequency is in the vicinity of the center frequency. For further literature [4] is recommended.

Fig. 2.4: Experimental set-up with a phase-locked loop for measuring the frequency of an oscillating system.
It is much more comfortable working with commercial available gadgets. The principle is the same as discussed above in detail and will be summarized using Fig. 2.4: The Function Generator (Krohn-Hite 5920) produces a periodic signal whose frequency can be readjusted by an analog voltage controlled oscillator (VCO). The Lock-In Amplifier (EG&G 5210) records the difference between the phase of excitation and response and produces an analogue voltage signal: There is a built-in mixer, which multiplies the rectangularized reference with the sinusoidal response [18]. After passing a low-pass filter, the remaining DC voltage proportional to the desired phase difference $\Delta \alpha$ is used for readjustment of the frequency. Instead of a simply passive low-pass filter, an analogue PI-controller controls the loop. The frequency is displayed at the counter (Keithley 776) and fed into a computer. The main advantage of this set-up is that the measurements are controlled by a computer and hence some necessary calibration processes may be automated.

2.3.3 Calibration and Compensation Processes

In chapter 2.2 on page 23 we noticed that the phase difference between force and displacement is $90^\circ$. However, there are many hidden phase shifters, e.g. in the transducers for excitation and response or in some internal amplifiers and filters, which improve the signal noise ratio. Therefore the signal of the response is shifted by a unknown amount which has to be compensated in order to measure the resonance frequency and damping. Since the amount of this phase of compensation is unknown at the moment, it has to be searched by means of a calibration process.

The real oscillating system will be compared with an analytical model. The simplest model for all system has already been presented in chapter 2.2. In case of a completely manual measurement the phase is compensated until the amplitude of the response is maximal. Afterwards, the phase is switched by $45^\circ$ up and down. The amplitude shrinks to $0.707$ of its maximum value. This amount is hardly controllable on a analogue oscilloscope. Fortunately, the amplitude of the response with the switched down phase is equal to the amplitude with the switched up phase. Therefore, the phase is compensated until the amplitudes, associated with the switched phases are equal.
As already mentioned in chapter “Experimental Setup” on page 24, the temperature influences the frequency. Since the Young’s modulus decreases with increasing temperature, the measured frequency also decreases. In contrast to resonance frequency measurement, the temperature compensation for measurements of the damping or Q-factor is realized without any further temperature measurement:

The phase register first shifts $\Delta \alpha$ up and measures the higher frequency. Then it shifts $\Delta \alpha$ down and fixes the lower frequency. Finally it shifts $\Delta \alpha$ up and records again a higher frequency, which has a slightly different value due to some temperature changes. The definitive high frequency results from the mean value. The resonance frequency is near to the average of this high frequency and the measured low frequency. Fortunately, the difference between this definition of the resonance frequency and the ex-
act resonance frequency is smaller than the dispersion of the measurements (compare Fig. 2.8 on page 33 with Fig. 2.10 on page 37). As a result, the definitive high frequency and the resonance frequency are registered at the same time as the measured lower frequency. Therefore, the measured damping is temperature-compensated, as far as the assumption linearity of the frequency change is valid. This validity is evident because this measurement is done within four seconds and thus the change in temperature is maximal a tenth of one degree.

In a similar way the length is compensated. The frequency is inversely proportional to the length of the rod, as it is shown in equation (2.7). However, the standard deviation of all registered frequencies of repeated experiences is smaller than 0.05 %c (i.e. 0.5 Hz at 10 kHz). This usually requests an extremely accurate measurement of the length (e.g. 300 mm ±15 μm). Nevertheless, the measurement of the length may be left out if we focus our interest not on the resonance frequencies themselves but on the relations of resonance frequencies of several vibration modes.

2.4 Physical Model

Neglect first the damping and consider two rods of length $L_1$ and $L_2$ joined by an elastic torsional spring constant $C$, which gives the torque per unit twist angle (see Fig. 2.6). The spring represents mechanically the adhesive film whose mass can be neglected since this thickness is much smaller than the wavelength of the stationary vibration modes.

![Fig. 2.6: The adhesive film modelled as a torsional spring](image)

If the joint allows perfect adhesion, the spring constant $C$ should tend to infinity, so that for a given transmitted torque, the difference between the twist amplitude $\Theta_1$ and $\Theta_2$ on the left and right hand sides of the spring tends to zero. Thus, the quality of the adhesive joint is characterized by the magnitude of the spring constant $C$ and hence, for a given torque, by
the difference $\Theta_2 - \Theta_1$. In the simplest case of a homogeneous, linear elastic continuous rod of total length $2L = L_1 + L_2$ subjected at its left end to a sinusoidal torque of circular frequency $\omega$ and the amplitude $T_0$, the amplitude and twist angle measured at the right end can be calculated by elementary analysis and found to be

$$\Theta^{(2L)} = \frac{T_0}{Gl_p\kappa \sin(2\kappa L)} \quad (2.5)$$

where $G$ is the shear modulus and $l_p$ the polar moment of inertia of the cross section and

$$\kappa = \omega \sqrt{\rho / G} \quad (2.6)$$

is the wave number corresponding to the forced circular frequency $\omega$ ($\rho$ is the specific mass). The resonance frequency is obtained by setting the denominator in (2.5) equal to zero. Thus

$$\kappa_n = n \cdot \frac{\pi}{2L} \quad n = 1, 2, 3, \ldots$$

are the wave numbers corresponding to resonances. The modes with the odd numbers $n = 1, 3, 5, \ldots$ will be called the "odd modes" those with even numbers $n = 2, 4, 6, \ldots$ the "even modes". Odd modes have a node in the middle of the rod where the twist angle vanishes, whereas the torque tends to infinity. For even modes the middle of the rod is an antinode and the amplitude of the torque is just $T_0/2$.

For two rods with the same length $L_1 = L_2 = L$ joined by a spring, the amplitude of the measured twist angle at the right end follows also from elementary dynamic analysis, which in this case takes into account that the twist angles at the left and right ends of the spring $\Theta_1$ and $\Theta_2$ are different and that the torque in the spring is proportional to this difference. One obtains after some easy calculations

$$\Theta^{(2L)} = \frac{T_0}{Gl_p\kappa \sin(2\kappa L)} \cdot \frac{1}{1 - \frac{\kappa}{2C} \tan(\kappa L)} \quad (2.8)$$

Obviously, the presence of the spring introduces with respect to the result of (2.5) the factor
\[ \mu = \left(1 - \frac{GL_p \kappa}{2C} \tan(\kappa L)\right)^{-1} \]  

(2.9)

This leads to an additional set of resonance frequencies corresponding to the roots of the denominator of \( \mu \). The wave numbers associated with these additional frequencies follow from

\[ \tan(\kappa_m L) = \frac{2C}{GI_p \kappa_m} \quad m = 1, 2, 3, \ldots \]  

(2.10)

Of course these “additional modes” tend to the “odd” ones if the spring constant \( C \) tends to infinity (in this case \( \kappa_m \) becomes \( \pi/2L, 3\pi/2L, \ldots \)). Thus, in case of defective or weaker adhesion for which the spring constant of the joint is finite, for each odd mode, in addition to the odd resonance frequency, a second slightly lower one should appear in its vicinity.

Damping can be introduced in this analysis by assuming linear viscoelastic behavior both for the rods and the spring and hence by replacing in (2.5) and (2.8) the shear modulus \( G \) and the spring constant \( C \) with their complex counterparts \( G^* \) and \( C^* \). The transfer function between measured twist and the torque can be expressed as

\[ \Theta_{(2L)} = \frac{T_0}{Re\{D(\omega)\} + Im\{D(\omega)\}} \]  

(2.11)

Setting the real part of the denominator \( Re\{D(\omega)\} = 0 \) one obtains frequencies similar to the ones of the purely elastic case discussed above. Thus if we define the frequency dependent phase angle \( \alpha(\omega) \) between excitation and response as

\[ \tan\alpha(\omega) = \frac{Im\{D(\omega)\}}{Re\{D(\omega)\}} \]  

(2.12)

we can obtain the resonance frequencies corresponding to the idealized elastic model by setting \( \alpha(\omega) = \pi/2 \). This is the most comfortable way of determining the frequencies since our experimental setup is based on a phase locked loop (see chapter “Electronic Control” on page 26). The amplitude and phase spectra based on equation (2.11) and (2.12) are computed and shown in Fig. 2.7.
Fig. 2.7: Amplitude and phase spectra near a resonance frequency. The frequency $f_{\text{res}}$ corresponds to the resonance frequency of an idealized elastic rod. **Left:** “odd mode”; the resonance frequency of the rod with adhesive joint (black) is smaller than the resonance frequency of a rod of the same length but without any joint (gray). The dashed line is the phase spectra of an idealized elastic model of the rod with joint. **Right:** ”even mode”; no influence of the adhesive joint is visible.

Fig. 2.8 gives an impression of the coincidence of a real phase spectra of a steel rod, which is based on (2.12), with the best fit based on a single degree-of-freedom system with the transfer function (2.2). The fit parameters of the model are resonance frequency and the Q-factor. **Fig. 2.8:** Phase spectra near the resonance frequency of a steel rod (circles) and best least square fit of a single degree-of-freedom system with mass, spring and damping (line). Levenberg-Marquardt is the method used in fit. The fit parameters are the Q-factor and the resonance frequency. The Q-factor is 4762.
An additional mass such as a piezoelectric transducer makes the resonance frequencies smaller and thus the distance between the nodal cross-sections expands. To be precise, the rod is subjected at its left and right end to a torque $T_L$ and $T_R$

$$T_L = T_0 - \Theta(x = 0) \cdot I_C \quad T_R = \Theta(x = 2L) \cdot I_C$$  \hspace{1cm} (2.13)

where $I_C$ is the polar moment inertia of the mass: $I_C = mR^2/2$, where $R$ is the mean radius of the mass $m$. The amplitude of the twist angle captured by means of a piezoelectric element on the right end can be calculated by elementary analysis and found to be

$$\Theta_{(2L)} = \frac{T_0(I_C \omega^2 G \sin(2\kappa L) - G I_p \kappa \cos(2\kappa L))}{(-I_C \omega^4 + G^2 I_p^2 \kappa^2) \sin(2\kappa L) + 2I_C \omega^2 G I_p \kappa \cos(2\kappa L)}$$  \hspace{1cm} (2.14)

The resonance frequency is found by setting the denominator in (2.14) equal to zero:

$$\tan(2\kappa L) = \frac{2I_C \omega^2 G I_p \kappa}{G^2 I_p^2 \kappa^2 - I_C \omega^4}$$  \hspace{1cm} (2.15)

For an infinitesimal small mass $m$, the wave numbers $\kappa$ of equation (2.15) tend to the same values as given in solution (2.6).

### 2.5 Measurements

#### 2.5.1 The Rod Specimens

Each component of the composite rod specimens listed in table 1 has been measured for reference purposes before adhesion. Besides, two continuous rods of stainless steel and two continuous rods of Al$_2$O$_3$-ceramic with the same size and total length as the corresponding composite rods of line 5 to 8 of table 1 have also been measured for reference purposes.
Table 1: composite rod specimens and adhesives.

<table>
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<th>Diameter ((\phi))</th>
<th>Left Part (Length (L_1))</th>
<th>Right Part (Length (L_2))</th>
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<td>6 mm</td>
<td>St37 (299.5 mm)</td>
<td>St37 (299.5 mm)</td>
<td>Araldit</td>
</tr>
<tr>
<td>6 mm</td>
<td>St37 (299.6 mm)</td>
<td>St37 (299.6 mm)</td>
<td>Castolin 1802</td>
</tr>
<tr>
<td>6 mm</td>
<td>brass (299.2 mm)</td>
<td>brass (299.2 mm)</td>
<td>Castolin 1802</td>
</tr>
<tr>
<td>6 mm</td>
<td>brass (299.5 mm)</td>
<td>brass (299.5 mm)</td>
<td>Araldit</td>
</tr>
<tr>
<td>8 mm</td>
<td>(\text{Al}_2\text{O}_3) (150.0 mm)</td>
<td>(\text{Al}_2\text{O}_3) (150.0 mm)</td>
<td>CS1</td>
</tr>
<tr>
<td>8 mm</td>
<td>(\text{Al}_2\text{O}_3) (150.0 mm)</td>
<td>(\text{Al}_2\text{O}_3) (150.0 mm)</td>
<td>CS1 damaged</td>
</tr>
<tr>
<td>8 mm</td>
<td>(\text{Al}_2\text{O}_3) (148.7 mm)</td>
<td>stainless steel (73.7 mm)</td>
<td>CS1</td>
</tr>
<tr>
<td>8 mm</td>
<td>(\text{Al}_2\text{O}_3) (149.7 mm)</td>
<td>stainless steel (74.7 mm)</td>
<td>CS1 damaged</td>
</tr>
</tbody>
</table>

2.5.2 Accuracy of the Measurements

\[\Delta f = \pm 1.987 \times 10^{-4} \text{[Hz °C]}\]

\[\Delta f = \pm 4.874 \times 10^{-5} \text{[Hz °C]}\]

Fig. 2.9: Typical frequency decrease due to increase of temperature of a rod oscillating in its fourth mode. **Left:** stainless steel \(-2.0 \times 10^{-4} \text{[Hz °C]}\)  
**Right:** \(\text{Al}_2\text{O}_3\)-ceramic \(-4.9 \times 10^{-5} \text{[Hz °C]}\)

The modulus of elasticity is a quantity depending on the temperature. Thus, the frequency also changes with the temperature. A typical relation of these two parameters is shown in Fig. 2.9. On condition that a resonance frequency is found at 10 kHz the frequency of a steel rod decreases...
approximately by 2 Hz whereas in a ceramic rod the decrease of the frequency is only 0.5 Hz.

Another aspect which has to be considered, is the dissipation of energy in the transducers (compare Fig. 2.2 on page 25). Especially the loss of energy in the electromagnetic transducer, which is in fact a coil fixed on the rod, leads to a general increase of the temperature in the rod. Corresponding to heat conduction the end nearer to the excitation is warmer than the other end. Typical increase of the temperature on both ends of the rods are shown in Fig. 2.10. After a few minutes, the frequency is dropped to a considerable low value. Lets first discuss the behavior of the ceramic rods: The resonance frequencies of three different rods are measured twice, which leads to six different declining curves. Accordingly, two curves are close together and their exist three pairs of curves corresponding to the three individual rod. These pairs are most conspicuous in the third graphic of Fig. 2.10. Of course each rod has its own falling characteristic, because on the one hand the slightly different material characteristics of the individual rod and on the other hand because it is not possible to build exactly the same transducer twice.

The same is valid for the steel rod, with the exception that the scattering between the individual measurements are ten times bigger. In contrast to pure ceramic rods no pairs of curves can be made out from the top graphic of figure Fig. 2.10.

Consequently, the measurement of the reference rods are done few seconds just before the main measurement of the composite rods. It was paid attention that the time between the start of the excitation and the record of the resonance frequency was the same for all, the composite rods and both reference rods of ceramic and steel respectively.

On overall, taking into account all the facts mentioned above, an excellent accuracy is the result with a standard deviation of all registered resonance frequencies\(^1\) (based on more than 340 single measurements)

---

1. Measured are four different resonance modes of two different metal-ceramic composite rods and two different ceramic-ceramic composite rods. Each measurement was repeated at least four times. Furthermore, the frequencies of reference rods (ceramic and steel) are taken into account for each measure of the composite rods as well. Besides, the individual ceramic and steel rods are investigated too before cutting and brazing. Each resonance frequency was measured at least on four different days.
smaller than 0.05 % (i.e. < 0.5 Hz at 10 kHz) including the influence of the supports and the tolerance in manufacturing the transducers.

Fig. 2.10: Decrease of frequency compared to the increase of temperature. The temperature rises due to some thermal loss in the coil used for excitation. The temperature was captured on both ends of the rods. Two different steel rods and three different ceramic rods were investigated twice.
2.5.3 Dynamic Characteristics of Adhesive Joints in Rods of Brass and St37

The geometric features of these specimens are listed in table 1 on page 35 under number 1...4. In structural resonance experiments piezoelectric transducers (see table 2.2 on page 25) are used as actuators and sensors. The resonance frequencies of the whole rod has been measured before splitting and welding or gluing the two halves of equal length. The difference between the frequencies of the whole and the composite specimens are shown in Fig. 2.11. The influence of the transducers themselves is compensated automatically, since they remain fixed on the butt during the welding or gluing process.

Both upper graphics of figure Fig. 2.11 show a decrease in the frequency due to weaker adhesive joints (Araldit) for all odd modes. The frequencies of the even modes are practically the same for the unsplitted reference rods and the corresponding composite specimens of the same length. The welded joint didn't influence the resonance frequencies as much as Araldit. On the contrary, the frequencies for all odd modes in brass (see Fig. 2.11 left column lower raw) increased due to the presence of the welded joint. Apparently a bulge in the joint whose diameter was larger than the diameter of the rod reinforced the structure. Of course, in this case, the joint can not be modelled as a spring of finite stiffness, but rather as a rigid disc whose thickness depends on the degree of reinforcement introduced by the stronger joint. A similar behavior could be seen in the specimens of stainless steel joint with the same welding material Castolin 1802. But there, the influence of the temperature changes on the measured frequencies together with the unavoidable errors in determining the exact length of the specimens have been too strong to allow better quantitative evaluations.

2.5.4 Ceramic-Ceramic and Ceramic-Steel-Composites

The geometric features of these rods are listed in table 1 on page 35 at line 5...8. Five consecutive modes 2...6 with frequencies between 20 kHz and 60 kHz are studied. The frequencies of the modes 2,3,5, and 6 were compared with the frequency of the fourth mode. These relations were then confronted with the corresponding values obtained of reference rods, as mentioned in chapter 2.5.2. The standard deviation of all registered resonance frequencies was smaller than 0.05 % (i.e. < 0.5 Hz at 10 kHz) in-
including the influence of the supports and the tolerance in manufacturing the transducers.

- 27°C (all 9 modes were measured at two different days)
- upper limit of the frequency-difference as a result of uncertain temperature
- lower limit of the frequency-difference as a result of uncertain temperature
- upper limit of the frequency-difference as a result of uncertain temperature and length
- lower limit of the frequency-difference as a result of uncertain temperature and length

**Fig. 2.11:** The influence of Araldit (upper raw) and weld joints (lower raw) on the resonance frequency of brass (left column) and stainless steel (right column). The resonance frequencies of mode 2 to 14 were in the range of 3...21 kHz for brass and 5...35 kHz for steel.
Fig. 2.12: The Influence of the quality of a welded joint on the resonance frequency (bars) and damping (lines) between two ceramic rods (top) and between steel and ceramic (bottom)
As the previous subsection for odd modes figure Fig. 2.12 shows frequency decreases due to the joint, except for the ceramic-ceramic rod with high quality adhesive. However, after the bulge of the joint had been cut away, this rod showed a decrease in the frequency also (for the third mode 0.25 %). Of course the resonance frequencies for odd modes were much lower for the specimen with the “bad” joint and decreased even more when occasional bulges in the joint were removed.

The graphic of the ceramic-steel composite shows a similar behavior. Moreover, the Q-factor falls down for all odd modes (right-hand scale of the graphic on the right side of the figure). Such low values have not been registered for the Q-factors in the ceramic-ceramic composites (right-hand scale of the graphic on the left side of the figure). High-Q systems require careful handling since the amplitude change in the vicinity of resonance is very abrupt. Here the maximum Q-values of several measurements obtained both by varying the position of the supports (to reduce energy losses) and by replacing the transducers have been shown. The lower Q-factors at higher frequencies in ceramic are at least partly assumed to be due its porosity. In fact the Q-factor of a system is influenced by following factors:

- Material related losses due to internal damping
- Heat flow caused by expansion and compression
- Damping due to supports and boundary conditions
- Viscous damping caused by air
- Acoustic damping caused by air

### 2.6 Conclusions

Resonance frequencies of torsional vibrations were measured with an accuracy of 0.05 %. As a consequence, the influence of the quality of the brazed or adhesive joint in different composite rods (steel-steel, brass-brass, ceramic-ceramic and ceramic-metal) were measured without any ambiguity. Furthermore, the measured phenomena, i.e. the frequency loss due to low-quality of the joint in the composite rods, is qualitatively explained with a simple physical model. The brazed joint was modelled as a spring. The spring constant is a measure of the quality of the joint. All the experiences and insights will be useful for investigations of composite ceramic-metal plates.
Structural Resonance in Torsion in Rods
3. Structural Resonance in Plates

3.1 Introduction

The mode shapes of a structure are determined not only by its geometry but also by other boundary conditions such as excitation and supports. Furthermore, flaws in composite plates such as areas including weak or defective bonding or cracks influence the mode shape also.

In this chapter, the measured mode shapes of steel and ceramic plates are compared to FEM-models. The experiments were arranged very carefully with respect to the supporting pins, excitation and detection of the deflection, so that no unknown or only hardly to determinable parameters or boundary conditions are necessary for the corresponding FEM-model. Consequently, only mode shapes with four free boundaries were investigated. Specific difficulties in detecting the influence of flaws in composite plates are discussed at the end of this chapter.

3.2 Experimental Setup and Procedure

3.2.1 Supports

To ensure an undisturbed resonance mode of the plate with four free borders, the supporting arrangement must be positioned very carefully. Three pins must be placed within the assumed nodal line so that the overall damping of the oscillating system is minimized. Moreover, the pins have to be repositioned until the slope of the phase curve of the transfer function between excitation and response at the resonance frequency (see Fig. 2.5 on page 29) is maximized. Using foam pad in place of pins is also pos-
ssible. The main advantage of foams is a more comfortable adjustment procedure, since the foam do not require precise adjustments within the nodal line as needed with a pin type arrangement. The foam should not be very soft. In practice the plate should sink about 5 mm into the foam and the radius of the contact surface should be approximately 5 mm. However, if the mode shape is captured with stroboscopic holographic interferometry, the pins are the better supporting method. This prevents additional low frequency oscillations leading to undesired displacements, which could disturb the fringe pattern. However, these low frequency oscillations may be filtered out either by double pulsed holographic interferometry, which require a high power laser, or else a scanning vibrometer for detecting the mode shape (see “Scanning Vibrometer” on page 52). The procedure of measuring resonance frequency and damping is the same as described in “Calibration and Compensation Processes” on page 28.

3.2.2 Excitation

On a plate with four free boarders, many mode shapes have a maximal displacement at the corners. As a consequence, the excitation takes place at one corner alone, this way the power transferred for excitation is providing a maximum deflection. Since the amplitude is controlled by the power, the magnitude of the deflection may be adjusted for the measurements with holographic interferometry. For frequencies below 13 kHz an electromagnetic excitation power supplied by a KEPCO BOP 36-6M 200 W amplifier has been preferred. Either an EMAT or a simple pin steel-core solenoid was used to apply a sinusoidal force to the plate to avoid any direct contact with the plate. The transducers are sketched below:

Fig. 3.1: Transducers for contactless excitation: Solenoid with a steel core on the left hand side and EMAT (ElectroMAgnetic Transducer) on the right hand side.

The operating principle of the solenoid is based on a sinusoidal magnetic field (B) which attracts all plates containing ferromagnetic components. This arrangement was applied to all composite plates or nickel alloy plates. Of course, it does not
work with plain ceramic plates. Hence, an EMAT-transducer was used, which is based on a different physical principle: If a wire is placed in close proximity to a surface of an electric conducting object and supplied with an electric current (J) at the desired frequency, eddy current are induced within the near surface region. Lorentz forces, which excite the plate, emerge from a permanent magnetic field (B). Details of the physical principles are given in [72]. In order to use this principle for ceramics, a silver layer (polyscience H31) is burnt into the ceramic plate (at 120°C for one hour).

Solenoid details: 300 turns of Ø 0.5 mm copper wire are wound in 3 layers around a iron core with 55 mm length and Ø 5 mm. In order to excite the ceramic plates not only one but a two EMATs were used, located in opposite positions, facing either top or bottom of the ceramic plate. The copper wire of each EMAT has a diameter of 0.16 mm and was wound with 300 turns around the ferrite magnet. As a consequence, the impedance of such an element rises with increasing frequency, which leads to special difficulties concerning the needed power.

3.2.3 Electronic Control

As far as measurements of the damping and resonance frequency were concerned, the same method has been applied to the rods (see Fig. 2.4 on page 27). Details are given in "Electronic Control" on page 26. The procedures will be shortly repeated (see Fig. 3.2): The Phase-Locked-Loop consists on a voltage controlled function generator a Lock-In amplifier (phase sensitive detector) and a PI-Controller as a loop filter.

As soon as the phase loop is locked-in, the transfer function between excitation and response is measured with a PC which is connected via GPIB interface to the counter and to the Lock-in amplifier. Hence, the measured transfer function, i.e. magnitude and phase vs. frequency, in the vicinity of the resonance frequency is measured and stored. Consequently, some internal phase shifts (in the gadgets, transducers, filters and heterodyne interferometer) can be compensated by means of a physical model and curve fitting, so that the phase corresponding to a difference 90° between excitation (force) and response (displacement) is determined. Details are given in "Calibration and Compensation Processes" on page 28. The displacement is measured by means of a heterodyne interferometer. Details of this interferometer are given in [14].
Electronic set-up with synchronization of the strobo light to the extreme position of the oscillating plate (upper part) and the Phase Locked Loop for measuring the frequency and damping of the oscillating plate.
3.2.4 Stroboscopic Holographic Interferometry

The modes-shapes could actually be registered with time-average holographic interferometry, which was done first in the pioneering work of Powell and Stetson [51]. This has become an established tool because the experimental arrangement exists in its most simple form. However, there are two technical drawbacks: One is that only the amplitudes are registered, but not the sign of the displacement with respect to the reference state. The other is the loss of fringe intensity or visibility with increasing vibration amplitude. Stroboscopic holographic interferometry, which is experimentally more complex, solves this problem [68].

The common procedure is to store the hologram using stroboscopic techniques with the object’s surface in its extreme position of the vibration cycle. Therefore, the strobe pulse has to be synchronized with the vibration of the plate. All the electronics are an integral part of the Vib-Driver (shown on the top of Fig. 3.2). It produces a TTL signal, whose starting point and length can be chosen in comparison with the period of the oscillating plate. Whenever the signal is HIGH, potentials are applied to the acousto-optic modulator (IntraAction Corp. Model AOM - 403, 40 MHz), which are manufactured from high quality flint glass as the interaction medium. Epoxy bonded Lead Zirconium Titanium piezoelectric transducers generate the acoustic waves. Light incident to the acoustic wavefronts is diffracted and its frequency is either up- or down-shifted by the frequency of the acoustic wave. The setup of the optical elements are chosen, so that only this diffracted light passes the spacial filter, which consists of a microscopic objective and a pinhole.

The highest quality of the visibility of the interferogram is reached if the strobe pulses are infinitesimally short synchronized with the extreme deflection of the oscillating plate. In the book of Ostrovsky et al. [47] the fringe intensity function is studied in a systematic manner for the case of finite exposure time and of oscillatory processes. Fig. 3.3 shows the essential results. The duration of the strobo pulse, which is characterized with the duty cycle factor $k$ as $T/k$, where $T$ is the period of the harmonic vibration, is centered at an arbitrary moment of $\alpha T$ (or $\alpha T + T/2$) rather than at $T/4$ (or $3/4T$). Both parameters are synchronized with the vibration by means of the VIB-Driver. This gadget permits values of $k$ in the range of 2...20 and values of $\alpha$ in the range 0.1...0.95. The amplitude $A$ of the vibration, which is chosen typically three times larger than the wavelength of the Nd:YAG laser light (532 nm), is controlled by a power amplifier.
(see on the top left in Fig. 3.2. Type: Rohrer München, ±40 V, DC...100 kHz). To obtain interferograms with an almost perfectly fringe contrast, all parameters, i.e. duty cycle $k$, shift of the strobo pulse $\alpha T$ and the amplitude $A$ measured in one corner of the plate, are studied well by means of a digital storage oscilloscope (LeCroy 3504 shown on the top right in Fig. 3.2).

As seen in Fig. 3.3, the irradiance at the fringe maxima in the stroboscopic technique methods falls off much slower than with Powell-Stetson time-averaging technique, so that even at duty factor of $k \sim 10...20$ one can study vibrations with amplitudes of a few tens of wavelengths. By contrast, a shift of the strobing moment by only a few percent of the vibration period results in a drastic reduction of fringe intensity. An example is shown on the top of Fig. 3.10 on page 58.

![Diagram](image.png)

**Fig. 3.3:** Fringe maximum intensity vs. vibration amplitude $A$. **Left:** Time average method (P-S) and stroboscopic holograph for different stroboscopic pulse duty cycles $k$ (for time-average techniques, $k = 2$). **Right:** Fringe maximum intensity vs. vibration amplitude with the stroboscopic pulse, shifted by $\alpha T$ from the moment of transit through the equilibrium for different values of $\alpha$ ($\alpha = 0.25$ corresponds to the maximum displacement of the vibrating object). The duty cycle $k = 100$. (Figures: Ostrovsky et al. [47])

Both holographic setups used for the experiment are standard [11]. The optical arrangement are sketched in Fig. 3.4. Except for the two references, these setups are the same as for classical holography. The deformation analysis is well known and described in detail in [67]. First and second ob-
ject field, i.e. the light which is scattered from the diffuse surface of the plate, are recorded on the same hologram (H) in the same setup, but using two different reference waves (Ref. 1) and (Ref. 2), respectively. The two object fields are stored and accessible independently by corresponding reference. Interferometry takes place during reconstruction with both references simultaneously. Relative phase and fringe position can be controlled during reconstruction, which can now be used for accurate fringe interpolation by electronic interference phase measurement. Beside improved contrast of the interferogram (see upper row in comparison with the lower row in Fig. 3.6 on page 50) the main advantage of the phase map consists in its direct proportionality to the amplitude of the oscillating plate. Details of this phase step technique is minutely studied in [9] and [10]. The data are transferred into a computer with the help of a CCD-Camera (Sony, 8 Bit, 512×512 Pixels) and a real-time frame grabber (Imaging Technology Inc., MFG, 4 MB, 8 Bit by 1024 × 1024Pixels).
Fig. 3.5: Primary reconstructed images with two reference beams: **Left:** References close together yield overlapping cross-reconstructions. **Right:** Well separated reference sources yield separated cross-reconstructions. (Figure: Dändliker [11]).

Fig. 3.6: Influence of the interferograms due to the separation of the reference beams. **Left:** references close together. **Right:** Well separated references. The images on the **upper row** correspond to Dändliker’s results in Fig. 3.5. Both pictures on the **bottom** show the corresponding phase maps of the oscillating plate: St37, $141 \times 100 \times 0.5$ mm at 2603 Hz (left), nickel-alloy $100 \times 76 \times 1.0$ mm at 6656 Hz (right). The distance between two fringes corresponds to $\sim 0.3$ μm.
Expérimental Setup and Procedure

Illuminating the hologram with two references yields not only two but four primary reconstructions, namely two desired self-reconstructions and two undesired cross-reconstructions. To avoid disturbing overlapping of different cross-reconstructions, a setup with well separated reference beam is indispensable. Fig. 3.5, which is copied from Dändliker's work [11], illustrates this effect. The practical use of this fact concerning oscillating plates at resonance is illustrated in Fig. 3.6. With two well separated reference beams, the contrast of the interferogram is improved. As a consequence, the noise to signal ratio in the phase map is improved too. Therefore, a setup with well separated references is preferred.

However, the fringe pattern is not only determined by the deformation of the object itself, but also some modification of the setup during the reconstruction of the interferogram:

• **Difference in the wavelength of the laser light for exposure and the laser light used for reconstruction:** In the following experiments, the reconstruction was made with the same laser and with the acoustic modulation. As a consequence, this influence is neglected. Dändliker showed in [11], that a change in the wavelength introduces a linear phase deviation across the hologram which is proportional to difference of the direction between the wave vectors of the reference beams. Thus the reference sources close together reduce also the sensitivity to the wavelength changes.

• **Reposition error of the hologram upon reconstruction:** This error is usually thought to be created by rigid body motion of the hologram, and can be minimized with an appropriate holder after Abramson [2] (see Fig. 3.7).

Fig. 3.7: Gravity plate holder (After Abramson [2]. 1, 2, and 3 are ball-bearings: 4, 5 and 6 are pins. Thus the plate is located in all six freedoms.

• **Modification during the developing process of the hologram, such as shrinkage and bending:** The latter is negligible, since the photo emulsion is put on a rigid plate of glass. The shrinkage undergone by a transmission hologram with AGFA HOLOTEST 8 E 56 emulsion and GP 61 developer (see “Processing Formulas” on page 115) was not
investigated further, since it is on the one hand assumed to be negligible and on the other hand the shapes of resonance modes in plates are investigated rather than the absolute value of the amplitude. If the shrinkage is the same on the whole plate, the distortion of the interference pattern is on the whole interferogram the same so that the mode shape itself is not affected. In other words, the influence of the shrinkage on the appearance of the fringes is the same as if a shift in the wavelength of the laser happened.

- **The position change of the reference light sources**: Since the optical setup has not changed and the reconstructions were performed immediately, i.e. few minutes after the recordings, with the same acoustic modulated laser light, this influence is negligible.

- **Nonlinear recording effects**: The amplitude of the diffracted light during the reconstruction is a function of the amplitude transmittance of the photographic plate. The amplitude transmittance is a function of the received energy during the recordings. As a consequence, the brilliance of the primary-image wave field may be influenced by nonlinear recording effects.

As already mentioned, these influences will not be discussed and quantitatively evaluated. On the one hand, the experiments are designed carefully in order to minimize these effects so that on the other hand their evaluation is not necessary for the interpretation of the results in the chapters “Analytic Evaluation of Mode Shapes” on page 53 and “Influence of Flaws on Vibration Modes” on page 75.

However, all the facts are investigated in the literature. An overview is given by Dändliker in [11]. Very detailed discussion can be read in Dändliker’s works [12], or from another view of point with some additional aspects in Schumann’s et al. book [67].

### 3.2.5 Scanning Vibrometer

The mode shape of a vibrating plate was also detected by means of a heterodyne interferometer, which measures the displacement or velocity with high resolution in time at one point on the surface of the plate. Dual described in his thesis the physical principle [14]. With FFT-algorithm, the amplitude and phase spectra is evaluated and digitally stored in a computer. One can get the mode shape for a single frequency by scanning all over the plate.
The Polytec PSV-100 Scanning Vibrometer is a stand-alone turnkey system for non-contact, full-field (area) measurement of vibrating structures. The operating range is $1 \mu\text{m/s} \ldots 10 \text{ m/s}$. A fast dual channel A/D board with DSP type hardware FFT processor and -90 dB anti aliasing filter digitizes and processes the velocity signal from the vibrometer and a reference signal from the function generator. Phase is measured relative to the reference. This allows to distinguish between peaks and valleys in a mode shape. However, at this thesis only the magnitudes are evaluated. Maximum sampling rate is 400 kHz with up to 2048 samples. The exact shape of the area (plate) can be drawn with reference to the video image. Up to $128 \times 128$ points can be measured per scan. The built in software allows multiple FFT averaging for each scanning point. Furthermore, up to eight frequency bands, bandwidth and center frequency is determined by the user, (e.g. after a reference measurement at a single point) for each area scan may be stored.

This is in fact a filter method. As a consequence, bearings made of foams are used to support the plate (see “Supports” on page 43). Furthermore, higher harmonic oscillation can also be detected and compared with the desired oscillation. Beside the Q-factor (see “Methods to Measure the Frequency and Damping of an Mechanical Oscillator” on page 21) the presence of higher order oscillations is another hint of the quality of one single experiment.

There are two important disadvantages in comparison to the holographic method. The spacial resolution is limited and the scanning process is time consuming. In order to reduce the noise to signal ratio the average of 20 FFT spectras for each single point is stored. The measurement of the mode shape (including the evaluation of two higher order oscillations) of a rectangular plate with the dimensions $100 \times 76 \text{ mm}^2$ with a spatial resolution of $1 \text{ mm}^2$ takes four hours. As a consequence, the oscillation had to be stabilized for this period.

### 3.3 Thin Plates

#### 3.3.1 Analytic Evaluation of Mode Shapes

The experiments were designed, so that an analytic solution does not exceed the resources, such as time or the capacitance of the computer. Of
course, it is not always possible to design the experiment so that it fits to the easiest known analytical solution.

M. Sayir\(^1\) developed a simple method to find the nodal line for a vibrating thin plate with four free boarders. The solution is based on the well known formula for the eigenmodes of a vibrating plate with simply supported boarders. The method is shortly described in Fig. 3.8. Details are given in appendix “Nodal Line in a Vibrating Thin Plate” on page 124. P. Kosza showed the influence of a piezoelectric transducer with mass [37].

\[\text{Fig. 3.8: Development of a simplified model of a vibrating plate with four free boarders. Firstly, the solution of a substitute plate with width } b' \text{ and length } a \text{ is developed. The plate is free along } b' \text{ and simply supported along } a. \text{ Secondly, the solution of a substitute plate with length } a' \text{ and width } b \text{ is formulated, whereas now the plate is simply supported along } b. \text{ The final solution, which is sketched on the top left, is found by combination of the two partial solutions. Due to some simplifications, the nodal lines of the final solution are nearer to the boarders than in the reality (distance } \delta \text{ in the cross section on the top right).}\]

In particular, a ST 37 plate (141.2 x 100.0 x 0.5 mm\(^3\)) was investigated. The numerical results are listed in the table below, compared with the results from finite elements simulations (see “Finite Element Model” on page 55).

**Table 2: resonance frequencies and substitute dimensions a’ and b’**

<table>
<thead>
<tr>
<th>m/n</th>
<th>(a' = a - 2x_0)</th>
<th>(b' = b - 2y_0)</th>
<th>frequency</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1</td>
<td>79.0 mm</td>
<td>59.4 mm</td>
<td>554 Hz</td>
<td>573 Hz</td>
</tr>
<tr>
<td>2/1</td>
<td>104.0 mm</td>
<td>57.0 mm</td>
<td>846 Hz</td>
<td>871 Hz</td>
</tr>
<tr>
<td>2/2</td>
<td>101.2 mm</td>
<td>74.4 mm</td>
<td>1390 Hz</td>
<td>1423 Hz</td>
</tr>
<tr>
<td>4/2</td>
<td>117.4 mm</td>
<td>72.2 mm</td>
<td>2406 Hz</td>
<td>2352 Hz</td>
</tr>
<tr>
<td>3/3</td>
<td>111.8 mm</td>
<td>81.6 mm</td>
<td>2585 Hz</td>
<td>2601 Hz</td>
</tr>
</tbody>
</table>

### 3.3.2 Finite Element Model

The FEM computation was performed with the finite element program Marc Mentat. An eight node thick shell element with global displacements and rotations as degrees of freedom is used. The membrane strains are obtained from the displacement field; the curvatures from the rotation field. The transverse shear strains are calculated at ten special points and interpolated to the integration points. The stiffness of this element is formed using four point Gaussian integration.

The plate was not absolutely plane. However, the surface was modelled spherically with the radii 12 m and 8 m (see Fig. 3.9).

![Fig. 3.9: All finite elements models approximate the real surface of the plate as an elliptical sphere.](image)

The radius resulted from a least square fit of measured surface data. In the specific problem, a commercial computerized shadow moire optical method was used (gom: ATOS). An example of results from a classical moire optical method, where a grid is projected to the specimen and com-
pared to a reference grid, is shown in Fig. 3.21 on page 71. Other more precise methods, such as ultrasonic or mechanical scanning and holographic interferometry, have not been further investigated, because the curvature is so small, that even a finite element model of an absolutely plane surface is exact enough.

In order to optimize the number of elements, the calculations are performed only with few elements first. The number of elements are then quadrupled so that both the length and width of a single element is halved. This procedure is finished, after the change of the calculated resonance frequencies is smaller or in the order of 1 %. This accuracy is absolutely sufficient, because the repeatability of the experiment is of that order since the supports are placed manually. Of course, the higher vibrational modes need more elements in order to reach the same precision than the lower modes. This tendency can be derived from table 3. The final results of the evaluated resonance frequencies are listed in the last column of table 2.

Table 3: relative change in resonance frequency in dependence of the number of elements

<table>
<thead>
<tr>
<th>m/n</th>
<th>8 x 8</th>
<th>16 x 16</th>
<th>32 x 32</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1</td>
<td></td>
<td>2.25 %</td>
<td>0.00 %</td>
</tr>
<tr>
<td>2/1</td>
<td></td>
<td>0.56 %</td>
<td>0.19 %</td>
</tr>
<tr>
<td>2/2</td>
<td></td>
<td>6.23 %</td>
<td>0.45 %</td>
</tr>
<tr>
<td>4/2</td>
<td></td>
<td>14.28 %</td>
<td>1.38 %</td>
</tr>
<tr>
<td>3/3</td>
<td></td>
<td>14.70 %</td>
<td>0.62 %</td>
</tr>
</tbody>
</table>

3.3.3 Results and Conclusions

In this chapter both the measurement by holographic stroboscopy and scanning vibrometry are compared to numerical results derived either from an analytic model based Kirchhoff’s thin plate theory or a finite element model based on a thick shell element. In order to do this, some image processing techniques was needed. The plate under consideration was the same as described in table 2 on page 55.

Firstly, the holographic amplitude interferogram is stored with the help of a CCD-camera and a real-time frame grabber as described in detail in “Stroboscopic Holographic Interferometry” on page 47. The resulting pictures are shown in Fig. 3.10 on page 58. The amplitude of the interfer-
ogram on the right hand side is twice the amplitude of the interferogram on the left hand side. The distance between two fringes corresponds to a transversal displacement of 264 nm. The nodal lines are lightened up due to the fact gone through in context of Fig. 3.3 on page 48. The strobe pulse was not exactly synchronized with the maximal deflection of the oscillating plate. As a consequence, the fringe contrast is diminished.

Secondly, with the help of an algorithm a phase map of the interferogram is evaluated. Beside the improved contrast (second row in Fig. 3.10) the main advantage of a phase map consists on its direct proportionality to the displacement modulo $2\pi$. This means, that the displacement is proportional to the optical path difference between the object beam scattered on the plate surface in a reference state and the object beam scattered on the oscillating plate. A phase of $2\pi$ corresponds in the specific case to a path difference of 264 nm. Moreover, the program (optocat for windows 1.5) performs the phase step algorithm with a depth of 8 Bit per pixel, so that a resolution in the amplitude of almost 1 nm is theoretically possible. However, noise during the reconstruction diminishes the high resolution. In order to get a concise phase map, the amplitude of the oscillation has to be adapted: For too high amplitudes, the fringes cannot be resolved by the CCD-camera (see right column in Fig. 3.10). In case the amplitudes are below 100 nm the mode shapes can hardly be recognized especially if the interferogram are deformed due to hologram misalignment.

Thirdly, the $2\pi$ jumps are eliminated by an algorithm. The resolution of the amplitude may be diminished since the demodulated phase map, which is called topography, has still a depth of 8 Bit. In return the displacement is represented in RGB-colors (third row in Fig. 3.10). Clear topography require phase maps with well separated fringes.

Finally, the topography is rescaled (bottom in Fig. 3.10), i.e. a new look-up table is used, in order to compare directly the measurements with analytical (as in Fig. 3.13) or FEM results (as in Fig. 3.15 and Fig. 3.14).

Each experiment is characterized by its damping. After having maximized Q-factor, i.e. searching the nodal points for the supports as described in “Supports” on page 43, an interferogram was recorded. However, the Q-factor shows no predictable behavior (see also page 41). For some modes it increases, while it decreases for other ones. Fortunately, the repeated experiments with other supports made of foam instead of pins led to almost the same results (Fig. 3.11).
Fig. 3.10: Steps in the image processing: 1. interferogram, 2. phase map, 3. topography, 4. topography with other colors. The distance between two fringes in the phase map corresponds to 264 nm. **Right column:** If the amplitude of the oscillation is too big, the CCD-Camera cannot resolve the fringes.
Fig. 3.11: Comparison of the mode shapes measured with holographic stroboscopy (left) and scanning vibrometry (right). On the right hand side only the magnitude is shown without distinguishing between peaks and valleys.
The influence of a little mass (weight: 2.65 g; Ø 10 mm, thickness: 4 mm) onto the different mode shapes is shown in Fig. 3.12. Again, the mode shape and frequency of both experiments stroboscopic holography and vibrometry coincide quite well. Furthermore, comparing the results on the top and middle row of Fig. 3.12, it can be seen, that the influence of a flaw is not for all modes the same. Therefore, by studying several mode shapes,
it may be possible to predict the location and to characterize the type of the flaw, e.g. cracks, holes, missing solder in a composite ceramic-metal plate. This method corresponds to the measurements described in “Dynamic Characteristics of Adhesive Joints in Rods of Brass and St37” and “Ceramic-Ceramic and Ceramic-Steel-Composites” on page 38.

The experiments should be designed so that they can be theoretically interpreted with the help of a quantitative analytic model. The influence of a flaw on a modal shape could be studied by comparing the numerical result with the measurement. Therefore, to use this procedure for non-destructive testing, the accuracy has to be proved on a simple model, namely a thin plate. Fig. 3.13 on page 62 shows such a comparison between the analysis developed in appendix “Nodal Line in a Vibrating Thin Plate” on page 124. The mode shapes, which are visualized with 32 colors (compare bottom right in Fig. 3.10 on page 58), so that a difference between them resulted in a green field, i.e. the relative error is almost below 10%. Of course, on the boarders, where the topography from the holographic measurement is disturbed, the difference is bigger. Finally, only an occasional flaw should disturb the mode shape, so that its influence is not overshadowed by other spurious effects as mentioned in detail in connection with rods (see “Experimental Setup” on page 24).

Despite the fact that the experiment is always influenced somehow by some unknown factors, the physical model may be inaccurate too. The model derived in “Analytic Evaluation of Mode Shapes” on page 53 suggests, that only straight but no curved nodal line exists. In contrast to the measurement for higher modes 3/3 and 4/2 (compare the four pictures on the top of Fig. 3.11 on page 59), where the nodal lines are convex and concave respectively. As a consequence, the measured mode shapes are compared to FEM results. Again the numerical results match quite well the measurement (Fig. 3.14 and Fig. 3.15). Furthermore, the frequencies evaluated by the finite element program match the measured ones as well. Moreover, a slight correction of the Young’s modulus, which is chosen to be 210 GPa in accordance to the values of table 2 on page 55 would yield a smaller difference.
Fig. 3.13: Mode 2/2. Top: Holographic stroboscopy. Middle: Analytical theory. Bottom: Difference between Measurement and Analysis.
Fig. 3.14: Mode 4/2. Top: Holographic stroboscopy, Middle: FEM. Bottom: Difference between Measurement and FEM-analysis.
Fig. 3.15: Mode 3/3. Top: Holographic stroboscopy. Middle: FEM. Bottom: Difference between Measurement and FEM-analysis.
Fig. 3.16 to Fig. 3.19 on the next four pages give an overview of the coincidence of both the simplified analytical model based on the equation for a thin plate and the finite element model based on thick shell elements with the phase map of a holographic interferogram. All figures are in scale of nearly 1:1. First of all, the finite element model maps the experiment quite well referring the mode shape, i.e. the nodal lines (with exception of Fig. 3.17), and the resonance frequency. Apart from the already mentioned fact, that the nodal lines of the analytical model are always straight, they coincide well with the ones of the finite element model and the measured ones. They are without exception always a little bit nearer to the borders of the plates, and therefore, the estimated resonance frequencies are always lower. The reason is that the boundary layer was neglected. A detailed explanation in accordance with equations is given in the appendix "Nodal Line in a Vibrating Thin Plate" on page 124.

Fig. 3.17 on page 67 shows, that some mode shape have almost the same frequency. Whereas in the measurement the mode with almost straight lines was forced by correspondent placing the supports, the finite element routine returned a mode shape with curved nodal lines, whose shape may be forced by the meshing and the characteristic of the elements themselves.

Summary: the experiments can be repeated with high accuracy so that a direct comparison with numerical results based on analytic or finite element model is possible. However, the accuracy in measuring the frequency is far below of that one gained in the experiments with rods. It is assumed the main reason is due to high sensitivity to the exact position of the supports. Therefore, an occasional flaw in a composite ceramic metal plate, should disturb the mode shape so that it is not overshadowed by some other hardly controllable effects.
Fig. 3.16: Phase map of the holographic stroboscopic measurement (881.8 Hz) compared with the nodal line of both a simple analytic model based on thin plate (red, 846 Hz) and a finite element model based on a thick shell element (blue, 870.1 Hz. FEM-mesh in yellow).

St 37: 141.2 mm x 100.0 mm x 0.5 mm.
Fig. 3.17: Phase map of the holographic stroboscopic measurement (1411.2 Hz) compared with the nodal line of both a simple analytic model based on thin plate (red, 1390 Hz) and a finite element model based on a thick shell element (blue, 1422.8 Hz, FEM mesh-yellow).
St 37: 141.2 mm x 100.0 mm x 0.5 mm.
Fig. 3.18: Phase map of the holographic stroboscopic measurement (2356.1 Hz) compared with the nodal line of both a simple analytic model based on thin plate (red, 2406 Hz) and a finite element model based on a thick shell element (blue, 2352.2 Hz. FEM-mesh in yellow)
St 37: 141.2 mm × 100.0 mm × 0.5 mm.
Fig. 3.19: Phase map of the holographic stroboscopic measurement (2602.7 Hz) compared with the nodal line of both a simple analytic model based on thin plate (red, 2585 Hz) and a finite element model based on a thick shell element (blue, 2601.1 Hz, FEM-mesh in yellow).
St 37: 141.2 mm x 100.0 mm x 0.5 mm.
3.4 Composite Ceramic-Metal Plate

With respect to the previous paragraph, the holographic interferograms are compared to results obtained from scanning vibrometry and finite element models. As far as holographic interferometry is concerned, the measured displacement field of the mode shape (see Fig. 3.10 on page 58) is coded by a RGB color scale with 8 shaded colors as shown on the top of Fig. 3.20. Black corresponds to a maximal negative amplitude $A_{\text{min}}$, white to the corresponding maximal positive Amplitude $A_{\text{max}}$. However, due to some noise in the measurements, the valid color scale may only be in the range between turquoise and yellow. Accordingly, the others colors, e.g. black and white, do not correspond to a measured amplitude and have to be ignored.

The results from the scanning vibrometry are mostly displayed as root mean square values (RMS), evaluated from the continuous sinus signal measured point to point. However, sometimes the amplitude of the sinus signal is shown. Accordingly, the phase was measured with respect to a reference source, e.g. from the function generator. The corresponding RGB color bar (six shaded colors) is shown in the middle row of Fig. 3.20. According to some noise in the measurements and the limited measuring range, the valid color scale is limited to violet or turquoise. Accordingly, the others colors, e.g. yellow, do not correspond to a measured amplitude and have to be ignored.

The last color bar (ten colors) in Fig. 3.20 belongs to the finite element calculations, where only the absolute amplitude without sign is displayed.

![Color coding for the mode shapes](https://example.com/fig320)

*Fig. 3.20:* Color coding for the mode shapes: **Top:** Holographic interferometry (compare Fig. 3.10); **Middle:** Scanning vibrometry, where the values are mostly given as root mean square values, so that the sign of the amplitude gets lost. **Bottom:** FEM, only the absolute amplitude, i.e. without sign, is displayed.
3.4.1 Determining of the Material Parameters

From the experimentally evaluated response of the system, e.g. resonance frequencies and damping, the main characteristic, e.g. the complex elastic constant, has to be determined. The experiment is carried out under defined conditions (inputs) and some relevant responses of the system is measured. The experimental conditions are simulated by a numerical model which needs an initial trial set of parameters. The numerical and experimental output are compared and the parameter set is optimized by an algorithm, which usually minimizes a cost function (also called error function). This new parameter set is than used in the simulation and this iterative process is repeated until some convergence criterion is met.

![Fig. 3.21: Topography of moiré-interferometry measurements of a composite ceramic-metal plate (left), a nickel-alloy plate (middle) and a ceramic plate (right). The values are determinable by the color bar. Measurements by Dr. E. Hack, EMPA Dübendorf, Switzerland.](image)

However, the structural damping cannot be measured with high reliability for reason mentioned on page 41. As a consequence, it makes no sense to develop a viscoelastic model. Furthermore, the experiments are performed only at low frequencies, i.e. the wavelengths are at least 70 times longer than the thickness of the plate. Accordingly a finite element model based on the Kirchhoff's thin shell elements produce excellent results. More details are discussed in "Asymptotic Analysis of Three Dimensional Basic Equation vs. Mindlin’s Theory" on page 91, Fig. 4.8. The proof is given in [43]. Consequently, 2158 linear triangle thin shell elements (lateral length: 2.8 mm) are used to evaluate the first few mode shapes of the plates with dimension 101 × 76 × 1 mm³. The mode shapes are evaluated by the finite element program ID-DEAS™ using a "Normal Mode Dynamics Lanczos" solver. The surface of the shell was modelled spherically with a radius of 10 m for nickel and 20 m for ceramic. The data are eval-
uated with the help of moire-interferometry, whose topographies are shown in Fig. 3.21. The radius of the composite shell, for which no FEM-results are evaluated (see chapter 4.3.1 on page 91) is 0.9 m.

The main reason of investigation at low frequencies is that the experiment should be designed so that it can be interpreted with a quantitative analytic model. Therefore, a contactless excitation is preferred. However, the impedance of electromagnetic transducers as sketched in Fig. 3.1 on page 44 grows with increasing frequencies so that the Amplifier (KEPCO BOP 36—6M: < 13kHz, 200 W or ROHRER MÜNCHEN: <100 kHz, 100 W) cannot deliver enough power for excitation over 10 kHz.

The resonance frequencies are measured several times, such that a statistical evaluation of the measurements is possible. The statistics give a measure for the reliability and repeatability of the measurements. The standard deviation of all 120 frequency measurements is 1.8 %. Therefore the compensation of the temperature was left, since it would be in the order of 0.2 %. Considering, that a mode shape was only registered by minimized overall damping (by accurate positioning of the supports) of the oscillating system, the results published in this thesis are more accurate than its characterization by the standard deviation.

Table 4: Resonance frequencies of a nickel alloy (Ni 30 %, Co 17 %, Fe 53 %) plate: 106.6 x 76.4 x0.92 mm³; E = 131 GPa; ρ = 8090 kg/m³;

<table>
<thead>
<tr>
<th>experiment (mean values) [Hz]</th>
<th>717.30</th>
<th>884.25</th>
<th>1095.4</th>
<th>1468.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM [Hz]</td>
<td>729.08</td>
<td>887.75</td>
<td>1072.7</td>
<td>1484.3</td>
</tr>
</tbody>
</table>

Table 5: Resonance frequencies of a ceramic (typically 99.7 % Al₂O₃) plate: 101.3 x 76.3 x1.01 mm³; E = 315 GPa; ρ = 3696 kg/m³; ν = 0.3

<table>
<thead>
<tr>
<th>experiment (mean values) [Hz]</th>
<th>1859.8</th>
<th>2270.4</th>
<th>2668.0</th>
<th>3781.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM [Hz]</td>
<td>1867.8</td>
<td>2272.2</td>
<td>2746.8</td>
<td>3787.3</td>
</tr>
</tbody>
</table>

The Young’s moduli and Poisson’s ratio found by the mixed numerical experimental techniques are given in the head line of tables 4 and 5. Of course, the other parameters, i.e. geometry and specific mass are measured with classical methods using a balance (resolution: 0.01 g). The contents of these tables give the resonance frequencies of the first four modes which are shown in Fig. 3.22 and Fig. 3.23 respectively. The measured mode shapes perfectly match the calculated ones, with exception of
the third one. This mode shape was extremely sensitive to misposition of
the supports. Accordingly, the Q-factor of the system is lower. The ex-
treme amplitudes of the oscillation are given in Fig. 3.22 and Fig. 3.23.
All other values can be deduced from the color bar in Fig. 3.20 on page 70.

![Fig. 3.22: Measured mode shape of two different nickel alloy-plates (middle and right) compared to FEM-results (left). The electromagnetic transducer (solenoid) was placed at the top left corner for each mode shape.](image)

**Parameters for the FEM-thin shell element:**
length: 101.6 mm; width: 76.4 mm; thickness: 0.92 mm; Young’s modulus: 131 GPa; density: 8090 kg/m³; Poisson’s ratio: 0.3.
Fig. 3.23: Topography of the mode shape of two different ceramic-plates (middle and right). The plate in the middle was measured again with scanning vibrometry (left). The EMAT masks the excited part of the plate. The results are compared to FEM-results, whose mode shapes are not shown but whose frequencies and amplitudes are given in brackets.

Parameters for the FEM-thin shell element:
- length: 101.2 mm
- width: 76.3 mm
- thickness: 1.01 mm
- Young's modulus: 315 GPa
- density: 3696 kg/m
- Poisson's ratio: 0.3
In Fig. 3.23 some areas (gray and black) near the boarders are not evaluated for all mode shapes. These areas are hidden by the EMAT. The left mode shape in the third row has very small amplitudes, such that the decoder of the vibrometer worked on its lower limit. As a consequence, only a small range of the colors is used. The amplitudes are small because the plate was excited over the nodal line of the corresponding mode shape. For most of all other mode shapes, the plates are excited in an appropriate place far from nodal lines, such that the amplitudes are controllable with the help of the function generator and amplifier.

### 3.4.2 Influence of Flaws on Vibration Modes

All the previous results give the impression that this method is suitable to detect flaws in composites ceramic-metal plates. Above all, the standard deviation of all 120 measured frequencies is 1.8%. Furthermore, the flaws seem to be rather big, as it can be seen from the ultrasonic C-scan images shown in the last row in Fig. 3.24 and described below:

Three different composite plates with distinguishing marks have been investigated: A perfect one. A second one with a built-in flaw, which resulted in a circular area with missing solder in the middle part of the composites plate. However, the corresponding plate had a defect bonding (missing solder) near the lower left corner marked with black areas of the ultrasonic C-scan image in the middle of the last row in Fig. 3.24. The third plate has an oval peripheral hair crack and an additional straight one in the ceramic layer of the composite plate. This resulted from the tension in the ceramic, because the plates are brazed in a high vacuum furnace of at most $5 \times 10^{-6}$ mbar. They were heated to 900°C at 10°C/min., held at the temperature and then cooled down slowly at max 10°C/min. Because of the difference in the thermal coefficient of the ceramic and the nickel alloy, the ceramic is under static tension and the composite plate is bent such the middle part is 2.4 mm above the corners (compare first moire interferogram in Fig. 3.21 on page 71 which was taken from the perfect composite). The corresponding ultrasonic C-scan image is shown on the right hand side at the bottom of Fig. 3.24.

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1. The plates were brazed by the EMPA, Dübendorf. Dr. A. Satir was responsible for the brazing process and Dr. J. Neuenschwander for classical nondestructive testing with ultrasonic, thermography and X-rays.
The mode shapes of all three composite plates are similar, especially the topographies in the first and second row in Fig. 3.24. The mode shape in the third row is a little different. The amplitude seems to be bigger at the weakened part of the plate, which is marked black in the ultrasonic C-scan images (last row). However, this mode is also very sensitive to the exact position of the supports, so that the difference between the plates is not unambiguously due to the flaws. The resonance frequencies of the perfect composite plate hardly differ from the resonance frequencies of the composite with missing solder. The difference is resulting more by chance than by the influence of the flaw. Only the crack diminishes the resonance frequencies, namely about 1.95 % for the mode in the first row in Fig. 3.24, 0.46 % for the mode in the second row and 2.56 % for the mode in the third row. Only the mode in the third row is not influenced by the crack. In fact, the cracks are in the vicinity of the nodal lines. In the corresponding part of the plate the local load is small, because of the vanishing bending moments, which are proportional to the local bow (see appendix “Asymptotic Development” on page 133). The internal damping of the composite plate with crack seems to be higher (lower Q-factors), than the corresponding damping of the other two plates. But as already mentioned on page 41, it is not possible to make conclusions to the flaw by measuring the damping.

In summary, the results in Fig. 3.24 do not characterize the influence of the corresponding flaw. The wavelength of the corresponding modes are to small in order to measure unambiguous influence of the flaws. Moreover, the stresses on the brazing layer are very small (see “Theory of Transversal Wave in Plate” on page 90). Consequently, measurements at higher frequencies have to be performed. An electromagnetic excitation without any physical contact to the plate was not possible at these higher frequencies, because the impedance of the transducer would require a high power amplifier (> 5 kW), which was not available at the institute.

The excitation at higher frequencies was realized by means of a piezoelectric transducer glued to one corner. However, the mass of the transducer will somehow influence the mode shape and resonance frequencies of the oscillating plate also. An impression was already given in Fig. 3.12 on page 60.
Fig. 3.24: Topography of three different composite ceramic-metal plates: A perfect one (left), one with missing solder in the brazed joint (middle) and one with hair cracks in the ceramic layer (right). The different flaws are visualized by ultrasonic C-Scan images (bottom). The influence of the flaws is overshadowed by other effects above all by the supports.
The mode shapes of all three composite plates are similar, especially the
topographies in the first and second row in Fig. 3.24. The mode shape in
the third row is a little different. The amplitude seems to be bigger at the
weakened part of the plate, which is marked black in the ultrasonic C-scan
images (last row). However, this mode is also very sensitive to the exact
position of the supports, so that the difference between the plates is not un-
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appendix “Asymptotic Development” on page 133). The internal damp-
ing of the composite plate with crack seems to be higher (lower Q-fac-
tors), than the corresponding damping of the other two plates. But as
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without any physical contact to the plate was not possible at these higher
frequencies, because the impedance of the transducer would require a
high power amplifier (> 5 kW), which was not available at the institute.

The excitation at higher frequencies was realized by means of a piezo-
electric transducer glued to one corner. However, the mass of the trans-
ducer will somehow influence the mode shape and resonance frequencies
of the oscillating plate also. An impression was already given in Fig. 3.12
on page 60.
Fig. 3.25: Topography of three different composite ceramic-metal plates: A perfect one (left), one with missing solder in the brazed joint (middle) and one with hair cracks in the ceramic layer (right). The mode shapes are disturbed by the mass of the piezo-electric transducer (Ø 5mm).
Composite Ceramic-Metal Plate

length: 3mm, weight: 0.3g) glued on the lower left corner. The axial symmetric symmetry is disturbed additionally by the plate with the cracks, which becomes clearly visible at the second and third topography on the right hand side.

Nonetheless, measurements with high accuracy and repeatability are performed at 19 kHz and 36 kHz. The maximal difference of five measurements with two different plates at 19 kHz is 46 Hz and of three measurements with two different plates at 36 kHz is only 2 Hz. The corresponding mode shapes are shown in the middle and left hand side of the first two rows in Fig. 3.25. Indeed, the influence of the missing solder is not visible. The mode shapes of both composites plates, the good one at the left column and the one with missing solder in the middle column are disturbed by the influence of the mass of the piezoelectric transducer glued at the lower left corner. Due to some problems by the development of the hologram, i.e. the glass plate with the emulsion, the images are very noisy at the lower left corner. But both mode shapes at 19 kHz and 36 kHz have the same characteristics: A region with very high amplitudes on the lower left corner, where the piezoelectric transducer is glued to the composite plates.

The mode shape of the plate with the hair crack on the ceramic layer (right column in Fig. 3.25) has exactly the same characteristics at 19 kHz as the ones of the other two plates. The frequency is diminished about 2.19 %, like in the measurements at low frequencies in Fig. 3.24. Furthermore, the amplitude is very small compared to the results of the other plates. By contrast, the mode shape of the plate with the hair cracks is completely different at 36 kHz. Again, the amplitude is ten times smaller, although the excitation of all three plates was performed the same way. In the following mode shape at 41 kHz, the extremes are in the region of the hair crack whereas for the other plate (in the middle column) the axial-symmetric characteristic remains at 42 kHz and even at 58 kHz.

In conclusion, the influence on the mode shape and resonance frequencies of some missing solder was not measured, whereas the influence of the hair cracks in the ceramic layer was visualized somehow. But the interpretation of the measurements is difficult, since the mode shape is a result of all boundary conditions, namely the mass of the piezoelectric transducer, the position of the supports, and the dimensions of the composite plate rather than the exact geometry of the hair crack. Therefore, an
FEM calculation was not executed, because on the one hand it was already shown in chapter 3.3.3 on page 56 that the experiments are handled very carefully so that they are directly comparable to numerically results and on the other hand the final results of finite element models would not lead to deeper insights of the mechanics.

3.4.3 Conclusions

In this chapter it was shown, that mode shapes and resonance frequencies can be measured with high accuracy and repeatability. The numerically evaluated mode shapes of corresponding analytical and finite element models map quite well with the measurements. The standard deviation of the resonance frequencies of 120 individual experiments is lower than 1.8 %. In spite of this good achievement it was not possible to show the influence of missing solder in form of a circle with 12 mm diameter in the middle of a composite ceramic-metal plate with the dimensions $101 \times 76 \times 2$ mm$^3$. Indeed, the stresses on the brazing layer are very small (see “Theory of Transversal Wave in Plate” on page 90).

By contrast, the influence of an elliptical shaped hair crack (diameters: $-48$ mm and $-61$ mm) in the ceramic layer with thickness 1 mm becomes visible at frequencies over 36 kHz. For that frequency, the wavelength is 40 mm along the length of the plate and 10 mm along the width of the plate. But even there, the interpretation of the measurement is cumbersome, since the mode shapes are influenced by all boundary conditions, i.e. not only by the flaw alone, but also by the dimension of the plate, the positions of the supports and the mass of the glued piezoelectric transducer needed for high frequencies. No influence of the hair crack is visible for frequencies below 20 kHz.

In order not to get unambiguous measures of the influence of flaws to the global dynamic behavior of metal ceramic plates a method based on wave propagation in elastic structures is preferred, because the boundary conditions such as dimension of the plate and supports do not affect the measurement as far as the front of the wave has not reached or passed the boundaries.
4. Structural Wave Propagation in Plates

4.1 Introduction

Similar composite ceramic-metal plates are investigated as in the precedent chapter. The influence of flaws in ceramic-metal plates is expected to be detectable with the structural wave propagation method.

The experiments should be designed so that they can be understood with analytical model and the physical interpretation remains always evident. Such mathematics were firstly done by Sneddon [69].

However, the dimension of the plate is rather small, so that the experiment has to be designed carefully, such that the mathematics of a straight crested wave in a very large plate remains valid. Otherwise, the analysis would become complex and the clarity of the model would be lost. Consequently, a computational simulation based on finite differences or finite integration techniques would be the most economic technique to compare the numerical results with the experiments. However, only a detailed discussion of an analytical solution ends in a deep understanding of the physics.

4.2 Experimental Setup and Procedure

4.2.1 Excitation

Two different methods have been used. An excitation with a short time pulse and on the contrary an excitation with narrow-band frequency spectra.
If a short laser pulse, typically few tens of nanoseconds, is focused on the surface of a material, a small amount of this material will be evaporated. The laser pulse causes a nearly instantaneous surface evaporation, so that the rapid expansion of the vaporized material produces an effect similar to a blast or explosion. As a consequence, a shock wave propagates and will be absorbed within the material [24]. The effects are depending on the pulse energy, the peak power and the power density, the reflectivity and absorption coefficient of the material at a given wavelength, the density and melting/boiling points of the material at the focused spot. Details are given in [24], [33] and [32]. A typical response of such an impact in a 0.2 mm thin brass plate is shown in Fig. 4.1.

In order to ensure the same conditions between the experiments of different composite ceramic-metal plates and the individual ceramic and nickel alloy plate, a little brass rod (Ø 3 mm and 5 mm thickness) was glued (Perma Bond double bubble) to the plate. This mass works as an optoacoustic generator, which transforms the laser pulse to a shock wave. The bandwidth of the impact it is shown in Fig. 4.2.

![Fig. 4.1: A circular propagating transversal bending wave in a brass plate (0.2 mm). The interferogram was taken 30 µs after a Nd:YAG laser pulse impact (140 mJ), which is almost a “Dirac-pulse” in space (Ø 0.4 mm) and time (14 ns).](image-url)
A Nd:Yag laser is used to generate a laser pulse of 14 ns and 140 mJ energy. Thus the peak power is in the order of 10 MW. Of course, the laser pulse does not have a sinusoidal time function in a rectangular window, but a sinusoidal function in a gaussian window (see Fig. 4.6 on page 89). The laser pulse is focused to a spot of 0.4 mm. Most material will melt under such an intense irradiation. The vapor will leave the surface at a very high velocity according to the high temperature rise created, similar to an impact of very short duration. A very sharp "noise" like a micro-explosion can be heard when focusing the laser pulse is shot to the target.

However, the displacement of a bending wave created by an impact with the characteristics shown in Fig. 4.2 is rather small, above all for frequencies below 100 kHz. In order to get concise interferograms the magnitude of the maximum displacement due to the flexural wave should be in the range of micrometers. This should be taken into account by tailoring the first resonance frequency of the transducer. Therefore, a transversal wave is generated with the help of multi layers piezoelectric transducers (Ferroperm CMA-PZ 27-Rings) as shown in Fig. 4.3. These transducers are designed such that its efficiency is optimized for frequencies of about 50 kHz.
Fig. 4.3: Stack of ten multi layers actuators (Ferroperm CMA PZ 27 Ring). Each piezo-ceramic consists of 18 layers of 104 μm thickness. The individual elements are bonded with a two component adhesive based on ceramic powder (HBM X-60).

The constitutive differential equation of harmonic oscillation with circular frequency \( \omega \) for one single element (e.g. the element \( N \) in the stack) of the piezo ceramic is

\[
\ddot{u}_{3,33}^N + s_{D33} \cdot \rho \cdot \omega^2 \cdot u_3^N = 0
\]

whereby the model is simplified: An uniaxial state will be assumed for both, electrical and mechanical quantities. All coupling effects with other components of stress, strains, electric displacements, and electric fields will be neglected. \( u_3 \) is the displacement along the axes, \( \rho \) the density and \( s_{D33} \) is the mechanical compliance for constant electric displacement. Such an element has a resonance frequency which is inversely proportional to the length of the element. The amplification at the resonance will produce higher amplitudes. Higher amplitudes can also be produced by increasing the mechanical compliance \( s_{D33} \), which in fact can be done by stacking many piezo elements, so that the voltages are applied to each single element. The corresponding constitutive equation of an optional backing mass is:

\[
\ddot{u}_{3,33}^M + \frac{\rho M}{E M} \cdot \omega^2 \cdot u_3^M = 0
\]

where \( E^M \) is the elasticity modulus of the brass. With these two equations and the boundary conditions listed in detail in the appendix “Piezoelectric Transducers used to Excite Mechanical Vibrations on thin Plates” on page 123 the transfer functions can easily be calculated. The results are shown in Fig. 4.4.
It was assumed that the piezoelectric transducer was fixed on a composite ceramic-metal plate with a thickness of 1.92 mm. For simplicity, shear stresses and rotary inertia of the plate are neglected. Furthermore, the glue between the stack is modelled to be rigid and infinitesimally thin. The same assumption was made for all 17 anodes in each of the ten multi layer actuators. Therefore the results in Fig. 4.4 are only a qualitative hint for the final design of the actuator.

For holographic measurements, it is an advantage to use the transducer in its resonance, since the additional amplification yields bigger amplitudes in the transversal displacement field of the bending wave in the composite plate. Thus a better signal to noise ratio is obtained.
If such an additional amplification is not needed, the transducer is advantageously excited far below its own resonance frequency, because the transfer function between the applied voltages and the displacement remains constant. For low frequencies shown on the lower left in Fig. 4.4 the transfer function corresponds to that of a high-pass filter. The limiting frequency where the amplitude spectra of the high pass reaches its 3 dB point is lower if a backing mass is used. A simple model was published in Staudenmann’s thesis [70] on page 67.

Consequently, there exists a frequency bandwidth with constant transfer function with all the advantages for excitation with narrow-band amplitude spectra discussed in the beginning of this section.

4.2.2 Electronic Control

The time between the impact and the process of freezing the displacement field onto the holographic plate is an important parameter in wave propagation experiments. Hence, the excitation was synchronized with the illumination. A photodiode captures some laser light and triggers the arbitrary function generator (Krohn Hite 5920). The time pulses to drive the transducer are sinusoidal functions in a rectangular window. Nevertheless, the frequency spectra obtained for various central frequencies given by the sinusoidal part of the pulse are quite narrow, due to the filtering effect of the resonance behavior of the transducer. The amplified voltages (Rohrer München, ±40 V, DC...100 kHz) are applied with 1.95 μs delay to the piezoelectric transducer. A heterodyne interferometer records the displacement in the center of the contact area between transducer and composite ceramic-metal plate. An internal delay of 3.55 μs has to be taken into account. With the help of a digital storage oscilloscope (LeCroy 3504, 300MHz, 100 Ms/s, 50 kPts) the measured data of the displacement and the second exposure are stored and used as boundary conditions for the numerical analysis.

In case of the impact with laser light, a synchronization is automatically realized since the light for holography and impact are generated by the same source (see Fig. 4.6 on page 89). The heterodyne interferometer scans the displacement over the whole plate. The position of the measuring point is fixed by a positioning system. Both the horizontal and vertical position (Aerotech ATS0200 and Aerotech ATS0600) is controllable by the computer with an accuracy and repeatability of 4 μm. The grid was
chosen to be 1 mm and at each point the signal was stored with 10000 points in the time axes, so that the total time window is 100 μs.

As far as the heterodyne interferometer is concerned, the resolution of such a system is mainly dependent on the light intensity, which is reflected from the plate and captured by a built-in photo detector, and the bandwidth of an additional low-pass filter. The RMS detection limit $\delta_{\text{lim}}$ corresponding to a signal to noise ratio of 1 is [13]:

$$\delta_{\text{lim}} = \frac{\lambda}{4\pi} \cdot \sqrt{\frac{h\nu}{\eta}}$$  \hspace{1cm} (4.3)

where $\eta$ is the quantum efficiency of the built-in detector, $h\nu$ is the photon energy and $\lambda$ its wavelength. This detection limit for one single measurement is typically $5 \times 10^{-7}$ nm√(W/Hz). By averaging N times the noise

**Fig. 4.5:** Electronic set-up with the synchronization of the excitation (PZT) and the laser light.
level is reduced according to \(1/\sqrt{N}\). Therefore, the measurement is syn-
chronized to the laser pulse, so that after the averaging 100 times the
stored signal was fed automatically to a computer. Afterwards the next
point on the plate is fixed by the positioning system. By a repetition rate
of the laser pulses of 12.5 Hz, ten seconds are needed for a single point,
so that the totally 21 hours are needed to scan over the whole area of
100 \(\times\) 75 mm\(^2\). Beside the long measuring time, these measurements need
a lot of memory: The resolution of the amplitude is 8 bit. Consequently,
the whole measurement needs 75 MB.

### 4.2.3 Double Pulsed Holographic Interferometry

The holographic arrangement is a standard [11], and the plate is held ver-
tically using a clamp at one corner. The reference beams are close toget-
er. A setup with well separated reference beams yields interferograms
with better contrast [11] & [12], as we have confirmed in preliminary ex-
periments with stationary plate vibrations (see Fig. 4.6). However, such a
setup does not seem appropriate for transient wave propagation measure-
ments with our system. The reason is as follows: On the one hand, the rec-
cordings are made with a double cavity Nd:YAG laser system (Quantel
YG980, 140 mJ at 532 nm and 14 ns pulse length) as sketched in Fig. 4.6.
On the other hand, the interferograms are reconstructed by means of a sin-
gle line continuous Nd:YAG laser (Adlas DPY305II, Nd:YAG, 532 nm).
The consequence of a large separation of the reference sources is high sen-
sitivity to hologram misalignment [11]. A quantitative analysis is given in
[67] chapter 3. Besides, the fringe contrast is affected too, which is shown
in [67] on page 141. Therefore, reference sources close together are pre-
ferred since the inevitable misalignment between the pulsed and the con-
tinuous laser beam does not distort the interference pattern as much.

Relative phase and fringe position can be controlled during recostruc-
tion. In order to determine the phase in the reconstructed interferogram a
phase-stepping technique is used [9] & [10]. All the advantages of this
technique cannot be used if the hologram would be reconstructed with the
help of the pulsed laser system, whose repetition rate of 12.5 Hz is adapted
to the standard video rate. But the fringe pattern of the interferogram nev-
er stops moving, because of the water cooling circuit of the flash lamp
pumped cavities.
Experimental Setup and Procedure

Fig. 4.6: Arrangement for two-reference-beam holographic interferometry with a double cavity Nd:YAG laser. The holographic light has a wavelength of 532 nm. Besides, there is an output at 1064 nm, which was sometimes used for excitation of the bending wave on the plate instead of the piezoelectric transducer (PZT). The interferogram are reconstructed with the help of a single line continuous Nd:YAG laser, which has to be aligned to the references beam (Ref.1 & 2).

The second laser pulse (2) is deliberately delayed with respect to the first pulse (1) by a time interval between 1 μs and 80 ms. There is not only one but two oscillators. Thus, two different Nd:YAG rods (SF 611-06, 6 mm diameter by 115 mm length with ends cut at 2°/2°) generate the
pulses. Consequently, each oscillator has its own characteristics and hence the beam profiles of the two laser pulses in the near field (i.e. on the hologram) are different. This fact has to be taken into account for the interpretation of the phase map (see Fig. 4.7 left), since the two different characteristics of the Nd:YAG rods give rise to an additional phase map. Accordingly, this additional phase map (middle) is measured by leaving the plate in its static reference state for both exposures. Finally, one gets the desired phase map of the flexural wave propagating in the plate by subtracting the precedent phase maps (right).

![Fig. 4.7: Compensation of the characteristic of the two YAG rods: left: phase map as a first result. The fringes are distorted. middle: phase map owing to the different characteristics of the two YAG rods. right: phase map of the flexural wave in the plate. The distance between two fringes corresponds to ~0.3 μm.](image)

### 4.3 Theory of Transversal Wave in Plate

The frequencies of the excitation are chosen under the consideration of the results in chapter “Measurements with Holography in Comparison with the Analytic Model” on page 103, especially in Fig. 3.25 on page 78. The wavelength of the structural flexural wave is about 20 mm at 60 kHz. The thickness of the three composite plate specimens considered here is either 1.9 or 2.1 mm. Hence, to obtain a precise theoretical evaluation of the phase velocity (better than 1 %) the influence of shear and rotary inertia has to be taken into account. (see Fig. 4.9 on page 94). Mindlin’s type simplified theoretical models [43] are applied and compared to the results obtained from an asymptotic approach based on the three dimensional
4.3.1 Asymptotic Analysis of Three Dimensional Basic Equation vs. Mindlin’s Theory

Mindlin’s theory including shear deformation and rotary inertia effects is well adapted to model propagation of structural waves in plates and delivers excellent results for practically the whole range of realizable frequencies. The dispersion curve resulting from Mindlin’s approximate theory maps quite accurately the curve of the exact solution given by Lamb (see [43] on page 36). In order to adapt Mindlin’s theory to composite ceramic-metal plates, one has to consider the following points:

- The bending and twisting moments and the transverse shearing forces, all per unit length and with respect to the “neutral plane” with vanishing in-plane displacements, are defined by the integral of the corresponding stresses over the plate’s thickness.

- The stresses are found with the help of Hooke’s law. Due to the different material characteristics of each component of the composite plate, the moments and shear forces are the sum of two integrals.

- Since bending waves are considered, the membrane forces are zero. Hence, the “neutral plane” with vanishing in-plane displacements is generally not in the mid-plane of the composite plate.

- This “neutral plane” is in the vicinity to the contact surface between the single components. Accordingly the stresses are small.

- The shear force is connected with an averaged shear strain. Hooke’s law and a factor $\kappa$ is used. This factor is chosen so that the phase velocity tends to the velocity of Rayleigh’s wave for the limiting case of high frequencies.

- The transverse displacement is assumed to be constant over the thickness and the in-plane displacements are proportional to the distance from the “neutral plane”. The cross-sections remain plain and steady.
The stress equation of motion of the three-dimensional elasticity is integrated over the thickness too. The three equations for the unknown two in-plane displacement components and the transversal out-of-plane component \( w(x,y,t) \) leads to the final equation of motion

\[
\frac{\partial^2 w}{\partial t^2} + D \Delta w = \eta \frac{\partial^2 w}{\partial t^2} + \chi \frac{\partial^4 w}{\partial t^4}
\]  

(4.4)

with the bending stiffness \( D \). Shear and rotatory inertia are included in the term with the parameter \( \eta \). The last term with \( \chi \) takes the inertia due to the shear correction (with respect to the Kirchhoff's plate) into account.

There are two specific characteristics, which are not easy to understand: First of all, the last term in (4.4) is in fact a "correction of a correction". In case such a correction is needed, there should probably another physical aspect to be taken under consideration. Otherwise the last term could be left out. Secondly, only bending and twisting moments and shear forces per unit length are used to find the main equation of motion. It was shown, that the boundary conditions with respect to the displacement field on the contact surface are fulfilled, but the stresses on the contact surface with components perpendicular to the surface has not to be explicitly fulfilled.

In order to validate this adaption of Mindlin's theory to composite ceramic-metal plates, another physical model based on an asymptotic approximation of the three dimensional equations of elasticity was developed and compared to the Mindlin's type of theory. The main characteristics of an asymptotic approximation are:

- A scale \( \varepsilon = 2\pi h/\lambda \) is introduced, where \( h \) is the thickness of the plate and \( \lambda \) the wavelength.

- The three dimensional equations of elasticity, i.e. the kinematic relations, Hooke's law and the equation of motions, are transformed in a dimensionless form. The stresses are scaled by a reference stress, the displacements by a reference displacement, the time by a reference time, the in-plane coordinate by the wave number and the out-of-plane coordinate by the thickness of the plate. Only the relations between reference displacement, time and stress are determined. Their explicit quantities are not of further interest.
• The reference time is determined by the scaling factor $\epsilon$, because the equation of motion in transverse direction need a coupling between stress and inertia.

• The transversal displacement is expanded in the form

$$w(x, y, z, t) = w^{(0)}(x, y, z, t) \cdot \epsilon^0 + w^{(2)}(x, y, z, t) \cdot \epsilon^2 + w^{(4)}(x, y, z, t) \cdot \epsilon^4 + O(\epsilon^6)$$

• $w^{(0)}$ is the first approximation of the transversal displacement, corresponding to the results of Kirchhoff’s plate. $w^{(2)}$, $w^{(4)}$ … are correction needed for shorter wavelengths.

• All other physical parameters such as in-plane displacement and stresses are determinable with the transversal displacement and its derivatives, because flexural waves are considered and the equations of elasticity are valid.

• For each approximation exists a set equations of elasticity, they are solved under consideration of all boundary conditions (i.e. considering displacement and stresses) on the top surfaces and on the contact surface between the two components.

The main advantage of the asymptotic development is, that the physics are easily interpretable. For example, the first approximation $w^{(0)}$ leads to a differential equation corresponding to Kirchhoff plate: $\ddot{w} + D \Delta w = 0$. The second approximation takes shear deformation and rotary inertia into account leading to a differential equation for $w = w^{(0)} + w^{(2)} \epsilon^2$

$$\frac{\partial^2 w}{\partial t^2} + D \Delta w = \eta \frac{\partial^2 w}{\partial t^2}$$

(4.5)

With the exception of the last term in (4.4) equation (4.5) is the same. A third approximation would introduce beside other terms a term with the fourth derivative in time. Fig. 4.8 and Fig. 4.9 show the phase velocity vs. wavelength and frequency respectively. The correspondence between Mindlin’s type of plate and the second order of approximation is quite good. Since the Rayleigh’s velocities of ceramic and nickel alloy are different, the phase velocity of Mindlin’s type plate will tend to a value between them with $\lambda \rightarrow 0$. 
Fig. 4.8: Dispersion relation in a composite ceramic-metal plate (thickness of $Al_2O_3$: 0.98 mm; thickness of nickel alloy 1.01 mm). The influence of shear and rotary inertia has to be taken into account for wavelengths shorter than 18 mm.

Fig. 4.9: Phase velocity vs. frequency in a composite ceramic-metal plate (thickness of $Al_2O_3$: 0.98 mm; Thickness of nickel alloy 1.01 mm).

Fig. 4.10 illustrates the consequence of shearing force on the deformation of the cross-section: The cross-sections of the linear Kirchhoff's theory remain plane and perpendicular to the "neutral" plane with vanishing in-plane components. Accordingly, the cross-sections are perpendicular to
the contact surface between the single components. The cross-sections of the model based on Mindlin’s theory remain plane, but they are not perpendicular to the “neutral” plane. The slope is the same for both components. Furthermore, the in-plane displacement field is not zero on the contact surface. The same conclusions follow from the asymptotic approximation.

![neutral plane with vanishing in-plane displacement field](image)

**Fig. 4.10: Consequence of shearing force on the deformation of the cross-section**

The in-plane displacement field \((u,v)\) do not remain plane over the thickness but it follows a cubical function, which can be written for \(u\) as:

\[
\begin{align*}
    u_{\text{ceramic}} &= u_0 - a \cdot z + b_{\text{ceramic}} \cdot z^2 \cdot (c + z) \\
    u_{\text{nickel}} &= u_0 - a \cdot z + b_{\text{nickel}} \cdot z^2 \cdot (c + z)
\end{align*}
\]

\(a, b, c\) are functions of the derivatives of the third order of the transversal displacement \(w(x,y,z,t)\) and the material constant and thicknesses of the components. The value integration constant \(u_0\) is defined by the condition of no resulting membrane force. In contrast to the model based on Mindlin’s theory, no “neutral” plane has to be introduced. The slope \(a\) is the same for both components, whereas the square and cubical term are different. The exact form of equation (4.6) can be found by the integration of the equation (A.37) on page 133 in appendix “Asymptotic Development”.

### 4.3.2 Transversal Displacement Field

With the help of the differential equation (4.5) the displacement field of a radially propagating flexural wave due to a harmonically oscillating point source is easily determinable. Details are given in the appendix “Wave
due to a Harmonically Oscillating Point” on page 134. In fact the transfer function is a combination of Bessel functions:

\[
w(r, t) = \frac{i \cdot v_0 \cdot e^{i \cdot 2 \cdot \pi \cdot f \cdot t}}{2 \cdot \pi \cdot f} \cdot \left[ J_0 \left( \frac{2 \pi}{\lambda} \cdot r \right) - i \cdot Y_0 \left( \frac{2 \pi}{\lambda} \cdot r \right) \right] (4.7)
\]

where \( v_0 \) is the magnitude of the velocity of the point source and \( \lambda \) the wavelength, which is dependent on the frequency \( f \). With the combination of (4.7) with the transfer function of a piezoelectric element (see Fig. 4.4 on page 85), the relation between the applied voltage and the displacement field is given. The transfer function between the voltages applied to the piezoelectric transducer and the resulting load on the plate would require detailed information about the transducer, i.e. an experimentally confirmed theoretical model of each piezoelectric element (including the anodes), the coupling between the elements and the coupling of the whole transducer and the plate.

All these problems are avoided by feeding the measured velocity (of the upper right of Fig. 4.11) into the calculations based on (4.7). However, some assumptions of the piezoelectric transducer are needed, because the velocity was measured only of a single point instead over the entire ring shaped area. Fortunately, very good results were obtained although a simple model of the transducer was used:

- An uniaxial state is assumed for both, electrical and mechanical quantities. All coupling effects with other components of stress, strain electric displacement and electric field are neglected.

- Consequently, the ring shaped transducer can be modulated by many thin rods with infinitesimally small front areas. This assumptions leads to better results than modelling the whole tube-shaped transducer as a point source. The difference is shown in Fig. 4.11.

Furthermore, the electric field is assumed to remain constant over the whole ring-area, because of the conducting characteristics of the anode. Thus, the mechanical stress remains constant too, which can easily been proved with equation (A.8) on page 121. Under this condition, the whole velocity field vs. time is determinable with a measured velocity in the
middle of the tube-shaped transducer (as shown on the upper right in Fig. 4.11). By superposing the displacements from each point source with the help of (4.7) and Fourier synthesis, one gets the displacement field. Details are given in the appendix “Numerical Solution” on page 136.

Fig. 4.11: Influence of shear and rotatory inertia (gray) on the displacement field vs. the radius 25 μs after the excitation (upper right). The black curve corresponds to the response of a Kirchhoff-plate. The dashed line is the corresponding displacement field if the ring-shaped piezoelectric transducer is modelled as an infinitesimal thin rod (point source).

Fig. 4.11 also shows the influence of shear and rotatory inertia on the displacement field. The small difference is measurable with holographic interferometry. The interferogram is digitally stored with a resolution of $512 \times 512$ pixels. The sizes of a captured picture are related as 4:3, which exactly corresponds to the relation of length:width of the composite ceramic-metal plates. Thus, the resolution of one pixel corresponds to an area of $0.20 \times 0.15 \text{ mm}^2$. 
4.4 Composite Ceramic-Metal Plates

4.4.1 Specimen Description

Three different composite ceramic-metal plates with distinguishing marks have been investigated. The plates were manufactured\textsuperscript{1} and tested with ultrasound\textsuperscript{2} at EMPA. Iron-nickel alloy (Ni 30\%, Co 17\%, Fe 53\%) were brazed together with a ceramic aluminium-oxide ceramic plates (typically 99.7\% Al\textsubscript{2}O\textsubscript{3}), whose dimensions are 101 \times 76 \times 1.01 \text{ mm}\textsuperscript{3}. The thickness of the metal plate was 1.2 mm for the first two specimen and 1.0 mm for the third one. The material properties are listed below:

Table 6: Material properties of the components

<table>
<thead>
<tr>
<th></th>
<th>iron-nickel-alloy (kovar)</th>
<th>Al\textsubscript{2}O\textsubscript{3}-ceramic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>131 GPa</td>
<td>315 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>density</td>
<td>8090 kg/m\textsuperscript{3}</td>
<td>3696 kg/m\textsuperscript{3}</td>
</tr>
</tbody>
</table>

Young’s moduli were determined by a combination of structural resonance measurements of single component plates under free boundary conditions (simple pin supports were placed at the nodes of the free plate modes) with FEM-simulations (see “Determining of the Material Parameters” on page 71). AgCuTi filler metal foils of 60 \textmu m thickness were used as brazing material. A perfect bonding between the two components is assumed in the analytic model.

The composite plate specimens did not have perfectly plane surfaces due to differences in the thermal expansion coefficients of the ceramic and the metal. After brazing at 900 °C and cooling down to the room temperature, the bending of the composite plates is such that the plate mid-point is about 2.5 mm above the corners. Thus, both plates are under stress even without external loads. Careless manipulation of the composite plate may also lead to defects caused by the combination of small impacts and the internal stresses mentioned above. This happened to the third specimen.

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The defect pattern in form of an elliptical peripheral crack and an additional straight one in the middle of the ceramic part of the composite specimen is shown in Fig. 4.12. This crack was visible by the naked eye!

![Fig. 4.12: Plate No 1: Cracks in the ceramic layer, visible with the naked eye.](image)

A C-scan ultrasonic image is shown in Fig. 4.13. The main characteristic of this composite plate is a black hole with 18 mm diameter on the left side from the center. It is a field with missing solder on the brazing layer. Thus, there is no bonding between the ceramic and metal component in this field. There are further details, such as missing solder near the hole and near the boarders. Lastly, there are very thin hair cracks in the ceramic layer.
Fig. 4.13: Plate No 2: Missing solder in the brazing layer at left Ø 18 mm

Fig. 4.14: Plate No 3: Missing solder in the brazing layer at bottom left Ø 23 mm
Defect bonding was enforced on plate No 3 (see Fig. 4.14) too. The corresponding hole with 23 mm is placed on the lower left of the plate’s center. Beside this main characteristics there are other small region with weakened bonding.

3.4.2 Measurements with Heterodyne Interferometer and Excitation with Laser Pulse

The main advantage of the heterodyne interferometer is its very high sensitivity. As a consequence, the amplitude of the flexural wave need not to be very large, i.e. an excitation with the laser pulse (see figure 4.2 on page 83) is powerful enough even for low frequencies (< 100 kHz). Although differences in the measurements between the plates shown in Fig. hit on the eyes an interpretation is not possible. Only the superposed oscillation at high frequencies on the blue curve in Fig. bottom can be explained with the resonance frequencies of the brass rod (Ø 3 mm; 5 mm length) which was fixed on the back side of the plate as an optoacoustic generator.

*Fig. 4.15: Transversal displacement as a response of an impact shown in figure 4.2 on page 83. Measurements at the vicinity of the impact and at distances of 6, 12, 18, 24, and 30 mm from the impact center. Top: nickel alloy 1.2 mm thick. Middle: Composite plate shown in Fig. 4.12. Bottom: Composite plate shown in Fig. 4.13*
However, there is no such a superposed oscillation at high frequencies (blue curve in the graphic) of the composite plate with hair cracks (middle). This was filtered out by the crack passing through the plates center. As far as nickel alloy is concerned, this superposed high frequency oscillation is present too, but with lower amplitude. In contrast to the composite ceramic-metal plates, there have no measurement been done exactly below the optoacoustic transducer.

Fig. 4.16: Displacement field 10 μs and 15 μs after impact. The wave front is disturbed (middle row) by the flaws (bottom) in the composite plates.
Easy interpretation is possible with respect to displacement fields shown in Fig. 4.16. Due to the enormous amount of data stored in the computer (The heterodyne interferometer was scanned over the whole plate, with a very fine grid of 1 mm$^2$ and a sampling rate of 10000 points within 100 μs), it was possible to simulate holographic interferograms. A shutter was simulated to be opened for 200 ns.

The phase velocity at a flow, such as elliptical peripheral crack (left column in Fig. 4.16) or defect bonding between the layers (right column) is slower, because the global stiffness of the composite ceramic-metal plate is weakened. Thus, the wave front is disturbed (middle row of Fig. 4.16) corresponding to the characteristic of the main flaw. However, there are not any scattered waves from the flaw explicitly visible. Of course, the whole displacement field is radially symmetrical (top row) as far as the wave front has not reached any flaws.

These measurements are helpful for the preparation of the final holographic measurements. Therefore, a quantitative interpretation of the displacement field has not been done yet.

### 4.4.3 Measurements with Holography in Comparison with the Analytic Model

In order to compare the measurements with the analytically evaluated displacement field, the velocity in the center of the composite plate was measured. The measurements in figure 4.17 on page 104 are different although the same voltages were applied to the transducers, namely a sinusoid at 45 kHz and 61 kHz respectively with three cycles within a rectangular window. It is on the one hand not possible to manufacture the same transducer (shown in figure 4.3 on page 84) twice, on the other hand the characteristic of the plate just below the transducer will influence the velocity. First of all, the nickel-alloy component of the first plate is only 1.0 mm thick whereas the nickel-alloy component of the second plate was 1.2 mm. Moreover, the first plate has a crack in the ceramic layer passing just below the transducer. The velocities on the top raw in Fig. 4.17 are

---

1. This topic is described in the thesis of M. Staudenmann [70], chapter 6. He established a connection between the repeatability of the measurements and the visibility of the scattered wave from a hole in a thin plate. The area of his plate was big enough for the scattered wave only being superposed by the incident wave, but not by reflections from the boundaries.
correspondingly influenced. Furthermore, the phase velocity is between 800 and 1000 m/s (compare figure 4.9 on page 94). Thus the distance of the wave front from the source is >100 mm after 100 $\mu$s. Thus, after less than 70 $\mu$s there are reflections from the boundary superposed to the measurements. Scattered waves from flaws had influenced the measured velocity even earlier. This has to be taken into account by interpreting the holographic interferograms in this section.

Besides, the delay of the heterodyne interferometer (3.55 $\mu$s) has to be compensated before feeding the data of Fig. 4.17 into the analysis.

In the following pages many interferograms are shown. However, not the full area of the composite plates have been illuminated by the expanded laser light, but only a circular area with 76 mm diameter. Furthermore, in order to get high spacial resolution of the stored interferograms, the CCD camera was placed near to the specimen, so that not the whole plate was captured but only a rectangular field of $83 \times 62$ mm$^2$. 
Each holographic measurement is compared to the analytic model. The result is represented by a phase map of the interferogram in such a way that a direct comparison is possible. In order to make the comparison easy, a sector of the measured interferogram was cut out so that the numerical result behind appears in this sector in the lower right quarter of the circular area. This area with least of flaws for all specimen catches the eye because there is no noise in the numerics whereas the measurements are quite noisy.

Fig. 4.18: Plate No 2 (see Fig. 4.13); 61 kHz, 30 µs. Defect bonding (area within the black circle) yields higher amplitudes. Besides, the wave front is deformed. The sector without noise on the right lower quarter shows the analytic results. The measurement itself is quite noisy.

Areas with missing solder, which are marked by circles in the interferograms of Fig. 4.18 and Fig. 4.19, are in fact two independent plates lying on the top of each other. Thus, this part of the structure is weakened which leads primarily to a higher amplitude and a locally shorter wavelength. As in the previous section, the presence of the wave scattered from this weakened part could not be measured with this method either (see Footnote on page 103). It is not possible to measure the whole displacement field and the scattered wave field at the same time because interferograms are too
noisy. If the power of the excitation increases ten times, a scattered wave field could be resolved. But the fringes near the center are too thin and the noise is such that they cannot be spatially resolved.

As far as areas of intact bonding are concerned, the measured transversal displacement field (i.e. the noisy phase maps from the holographic interferometry) and the calculated phase map (shown in the sector without noise on the lower right) coincide perfectly.

Cracks in the ceramic layer have a similar effect. Moreover, there are areas with slight discontinuities along the peripheral hair crack in Fig. 4.20. The crack across the mid-part leads to two additional local maximums above and below the center. An impression of how these two local maximums emerge from a almond-shaped maximum, which is a consequence to the anisotropy in the mid-part of the structure at the impact source, is shown in Fig. 4.21. In fact, the discontinuities along the oval crack are a consequence of the local anisotropy too.
Since there is no quarter without any flaw in this plate, a very good coincidence, such as shown in Fig. 4.19, between analytic results and measurement cannot be expected. Nonetheless, the analytic evaluation maps the measurement quite well.

Fig. 4.20: Plate No 1 (see Fig. 4.12): 61 kHz, 30 μs: The oval peripheral crack (marked with black lines) leads to a locally higher amplitude and discontinuities. Straight crack causes additional local maxima above and below the center. The sector without noise on the right lower quarter shows the analytic results. The measurement itself is quite noisy.

The influence of the flaw is only detectable if the wave front is propagated far enough so that the stresses in the vicinity of the flaw are big enough to make out any difference (with respect to the displacement on the plate’s surface) to the circumferential areas. Therefore, the influence of the missing solder is detectable only 25 μs after the impact, as shown in Fig. 4.22 on page 109. The correspondence between the holographic measurement and the analytic results coincide once more perfectly for all interferograms in Fig. 4.22.
Fig. 4.21: Influence of the crack to the interferograms in Plate No.1. These series give an impression of how the two local maximums emerge from an almond-shaped maximum.
Fig. 4.22: The influence of the non-bonding area of plate No 3 (see Fig. 4.14) depends on the center frequency of excitation (45 kHz on the left and 61 kHz on the right) and on the location of the wavefront. The sector without noise on the right lower quarter shows the analytic results.
The main difference between the two series in Fig. 4.22 (at 45 kHz and at 61 kHz respectively) is due to the wavelength. The influences of the flaw, i.e. the locally higher amplitude and the locally shorter wavelength, are more distinctive for wave fronts with short wavelengths. As already Fällström mentioned in [21], it is concluded that the magnitude of the wavelength should still remain in the range of defect size. Indeed, with the structural resonance measurement, no influences of such flaws were detected for low frequencies (compare Fig. 3.24 with figure 3.25 on page 78).

4.4.4 Conclusions

We have shown that it is possible to partly localize flaws in composite ceramic-metal plates by studying the propagation of transient flexural waves by means of dynamic holographic interferometry. Both the cracks in the ceramic layer and the defective bonding, i.e. hidden flaws between the two components were detected. However, these flaws are only detectable if the corresponding area is under load. Therefore, not only the frequency, i.e. the length of the structural wave which should remain in the range of the defect size, but also the energy of the impact are considered, so that a good resolution of the whole transversal displacement field (> 200 nm) is available.

A physical model based on Mindlin's equation of motion was developed and compared with an asymptotic approximation of the three dimensional equation of elasticity. Both models take shear and rotary inertia into account. The coincidence of the phase velocity, which depends on the frequency, is very good. Therefore, the physics are well done. Moreover, the numerical solution based on measurements of the velocity (vs. time with high resolution) in the middle of the plate was compared to interferograms. Finally, both models proved to be accurate because the numerical solution coincides perfectly with the holographic interferogram as far as intact areas of the plates are concerned.

In principle, the influence of flows in composite ceramic-metal plates onto the overall mechanical behavior is shown. However, interesting details, such as scattered waves, have not been measured yet for reasons presented in the following chapter “Conclusions and Outlook” on page 111.
5. Conclusions and Outlook

The results from chapter “Structural Wave Propagation in Plates” on page 81 are quite encouraging. First of all, holographic interferograms of flexural waves in composite ceramic-metal plates were recorded. The maximum magnitude in the center was about 3 μm and the idealized resolution is about 3 nm. However, the real images are noisy but the resolution was still high enough to detect even hidden flaws, such as non-bonding areas in the brazing layer, i.e. flaws near the mid-plane of the compound plate where stresses may be weak in the vicinity of the flaw.

Furthermore, Mindlin’s theory is well adapted to composite ceramic-metal plates. The results are compared with an asymptotic approximation of the three dimensional equations of elasticity. The physics in the asymptotic approximation are easy interpretable. The comparison of these two models leads to a complete understanding of composite plates with many layers. However, waves scattered from the flaws are not further investigated. This could be done in a further project, with two main challenges:

- **Improvement of the sensitivity of the measurement:** The magnitude of the scattered wave is generally small (see footnote on page 103) so that the measurement should have been adapted, either by increasing the excitation and taking the risk of damaging the composite ceramic-metal plate, or by improving the sensitivity of the measuring, e.g. by heterodyne holographic interferometry [12] (the resolution is 1/1000 of a fringe) instead of quasi heterodyne holographic interferometry with phase step techniques [9] (the resolution is 1/100). The phase is evaluated with the help of optoacoustic detectors and phase meters and is built up by scanning over the hologram. This alternative
method of measurement is similar to the principle of a heterodyne Michelson interferometer. Scanning with a heterodyne Michelson interferometer increases the sensitivity too, as far as the excitation can exactly be reproduced several thousands times. Otherwise, the transfer function between excitation and response has to be determined separately for each measured point. The performance of the heterodyne interferometer could be improved by using a photo refractive element [13]. The interference takes place not on the Michelson principle but on the holographic principle. The speckled image of the object in a reference state is stored on the photo refractive element and compared to the speckled image of the moving object. Since the speckle pattern is the same, the result corresponds to interferometry with smooth (i.e. not speckled) wave front of the light. Consequently, the signal to noise ratio is improved.

- The theory of the scattered wave at a non-bonding area has not been developed yet although a solution could be found. Indeed, this is a very interesting problem from the viewpoint of mechanics. It helps to understand the physics. Paul Fromme investigates at present time the influence of a damaged hole, with a hair crack, on the scattered wave. A similar method could be adapted to composite ceramic-metal plates in combination with the equations presented in appendix “Equations of Ceramic-Metal Plate” on page 127. Furthermore, circular non bonding areas could be handled by an analytic model. However, in the reality, the shapes of non-bonding areas are seldom circular. Waves scattered from bonding areas with any shape could be handled with a finite difference method or with finite integration techniques. Both methods are presently used for structural wave propagation in tubes and plates by Tobias Leutenegger and Daniel Gsell.

This research is valuable because it allows to define a tolerable amount of flaws in composite plates, so that the structure fulfills the mechanical specifications. *This means that images obtained e.g. with ultrasonic C-scan could be interpreted from a mechanical point of view.* Indeed, the main interest of ultrasonic images is not how many flaws are in the structure. The engineer has to check if the global mechanical characteristic of the structure is spoilt too much.

The main advantage of using holographic interferometry is that the displacement field is recorded at once. The results are easy interpretable
as far as discontinuities in the displacement field are clearly visible and influences from boundaries can be excluded. However, it is not the best method to find flaws in structures. Other methods such as ultrasonic imaging thermography, and X-rays are standardized. The holographic interferometry combined with structural wave propagation in plates should be automatized and improved:

- The handling with a chemical photo emulsion is cumbersome. Speckle interferometry is not a good alternative because its resolution is about ten times lower than that with holography. At present a lot of research is done on holographic interferometry stored directly on a CCD-chip, i.e. the photographic plate is replaced by the CCD chip takes the phase of. This type of recording is known as electro-optic holography [53], TV-holography [49] and digital holography [74]. The main advantage of this fully digital measurement is the reconstruction being done with the help of Fourier synthesis directly in the computer. However the grain size of a CCD chip is nowadays about 8 \( \mu \text{m} \), whereas a photographic emulsion resolves 5000 lines per millimeters. This strongly limits the flexibility of the holographic arrangement and thus the range of applications [49]. But as soon as the grain size of the CCD-chip is about 1 \( \mu \text{m} \), holographic interferometry may be used as easy as ultrasound and becomes a standardized application in non-destructive testing. Another alternative optical storage medium are photo refractive elements, e.g. \( \text{Bi}_{12}\text{SiO}_{20} \), \( \text{Bi}_{12}\text{GeO}_{20} \) and \( \text{Bi}_{12}\text{TiO}_{20} \). The optical set-ups are usually the same as for classical holographic interferometry except that the photo emulsion is replaced by these elements. As a consequence, the whole measurement may be automatized. For further literature are [26], [39], [52] and [56] recommended.

- The quality of the laser light can be improved by using phase conjugate mirrors in the cavity. In high power solid-state lasers the transverse mode TEM00 is degraded due to thermally-induced optical phase aberrations. The loss of the mode quality has several consequences: The brightness of the interferograms is locally reduced (see Fig. 4.7 on page 90). Hot spots can develop and therefore lead to damage of optical components. Focusablity is also diminished. An interesting work about phase conjugation in a continuous-wave diode-pumped Nd:YVO\(_4\) laser was published by A. Brignon et al.
1999 [6]. They demonstrate phase conjugation and dynamic holography by degenerate four-wave mixing in the laser medium itself. Implementing such techniques in holographic interferometry would lead to results corresponding to interferometry with smooth light fronts.

- The excitation should have been done with either contactless electromagnetic excitation (see Fig. 3.1 on page 44), or with a pendulum [30] or a laser light pulse. The first method was not successful for reasons mentioned in chapter “Excitation” on page 44. The second method is presented in the thesis of M. Isay [30], chapter 2. Unfortunately, our pulsed laser can not be triggered on a single event. Consequently, the pendulum has to be connected with the laser trigger output, so that the jitter of the start of the impact is within 20 μs. Applying the third method, a synchronization of the excitation with the holographic laser light (see “Excitation” on page 81) was realized. However, an adequate optoacoustic transducer has to be found, which at first produces most energy of the shock wave at the desired frequencies and which secondly should exist as a “color” to be painted on the surface.

Furthermore, the optical setup was adapted to out-of-plane measurements. The positioning of the photo plate, with respect to the reference wave source, has been replaced extremely exactly. Moreover, the immense efforts have been made with respect of the alignment of the laser beam for reconstruction with the laser beam needed for recordings. There was even never a change in the wavelength of the laser light between reconstruction and recording. Lastly, since reference beam and illumination beam have come from the same direction, the grid on the photo emulsion is perpendicular to the top surface and therefore, a modification due to shrinkage can also be left out. If one of these conditions could not be fulfilled anymore, all the phenomena in [67], chapter 3 have to be taken into account. Moreover, one has to know exactly which physical aspects have to be considered in a similar way as it has been done in a similar way with the asymptotic approximation of the three dimensional equation of elasticity.
A: Appendix

A.1 Processing Formulas

The first process is optimized for AGFA HOLOTEST 8E56 HD emulsion [44]. This emulsion resolves 5000 lines per mm. AGFA recommends to illuminate the hologram ~50 μJ/cm² which leads to an optical density ~2 (before bleaching). In case the bleaching process is left out, an illumination at ~5 μJ/cm² is necessary. The signal to noise ratio of an amplitude hologram is better but the efficiency of the diffracted light amplitude is reduced, compared to a phase hologram. Indeed, the brightness of the hologram is quite good which allows to optimize the storage of the interferograms with the help of a CCD-camera. However, if the developer is not fresh anymore, the hologram has to be exposed longer. The second process is proposed by P. Hariharan [29]. The hologram needs ~5 μJ/cm² energy of illumination to get an optical density of ~2 (before bleaching). Consequently less light is needed. However, the interferograms are noisier than the holograms developed by AGFA’s formula. Further processes are listed in Graham Saxby’s book [65], appendix 6.

A.1.1 Phase Holograms with GP61 Developer

- Expose in hologram set-up (50 J/m²)
- Develop for 2 min. in GP 61 (plate’s optical density is ~2)
- Rinse in distilled water for 3 min.
- Fix with GP 328 (1 part in 4 part water) for 2 min. The fixative is commercial available.
- Rinse in distilled water for 3 min.
- Bleach in GP 62 until plates’s optical density is zero.
- Rinse in distilled water with AGEPON wetting agent (200:1).
Table A.1: AGFA-Process, developer for GP 61

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>distilled water</td>
<td>1 l</td>
</tr>
<tr>
<td>metol</td>
<td>6 g</td>
</tr>
<tr>
<td>hydroquinone</td>
<td>7 g</td>
</tr>
<tr>
<td>phenidone</td>
<td>0.8 g</td>
</tr>
<tr>
<td>Na₂SO₃</td>
<td>30 g</td>
</tr>
<tr>
<td>Na₂CO₃</td>
<td>60 g</td>
</tr>
<tr>
<td>KBr</td>
<td>2 g</td>
</tr>
<tr>
<td>Na₄EDTA</td>
<td>1 g</td>
</tr>
</tbody>
</table>

Table A.2: AGFA-process; Bleach GP 62

<table>
<thead>
<tr>
<th>stock A</th>
<th>stock B</th>
</tr>
</thead>
<tbody>
<tr>
<td>dest. water</td>
<td>dest. water</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>metol</td>
<td>Na₂CO₃</td>
</tr>
<tr>
<td>15 g</td>
<td>60 g</td>
</tr>
<tr>
<td>pyrogalic acid</td>
<td>7 g</td>
</tr>
<tr>
<td>Na₂SO₃</td>
<td>20 g</td>
</tr>
<tr>
<td>KBr</td>
<td>4 g</td>
</tr>
<tr>
<td>Na₄EDTA</td>
<td></td>
</tr>
</tbody>
</table>

The stock solutions A and B are durable. The final bleach is a mixture: 1 part of A + 2 parts of distilled water + 1 part of B.

A.1.2 Phase Holograms and Reversal bleach
- Expose in hologram set-up (5 J/m²).
- Develop for 5 min. in GP 61 (plate’s optical density is ~2).
- Rinse in distilled water for 3 min.
- Bleach for 5 min.
- Rinse in distilled water with AGEPON wetting agent (200:1).

Table A.3: Hariharan-Process, developer for GP 61

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>distilled water</td>
<td>1 l</td>
</tr>
<tr>
<td>metol</td>
<td>2 g</td>
</tr>
<tr>
<td>sodium sulfite</td>
<td>95 g</td>
</tr>
<tr>
<td>hydroquinone</td>
<td>8 g</td>
</tr>
<tr>
<td>sodium carbonate</td>
<td>45 g</td>
</tr>
<tr>
<td>potassium bromide</td>
<td>5 g</td>
</tr>
<tr>
<td>sodium thiosulfate</td>
<td>0.5 g</td>
</tr>
</tbody>
</table>
Table A.4: Hariharan-process; Bleach GP 62

<table>
<thead>
<tr>
<th>stock A</th>
<th>stock B</th>
</tr>
</thead>
<tbody>
<tr>
<td>water</td>
<td>water</td>
</tr>
<tr>
<td>potassium dichromate</td>
<td>potassium iodide</td>
</tr>
<tr>
<td>conc. sulfuric acid</td>
<td>10 ml</td>
</tr>
</tbody>
</table>

Mix 1 part of A, 1 part of B and 8 parts of destilled water just before use.

A.2 A TTL Phase Locked Loop

Fig. A.1: Basic phase locked loop. The voltage-controlled oscillator and the phase detector, which is a simple EXOR-gate, are integrated in the chip MM74HC4046. A ceramic rod with the length of 300 mm and Ø 8 mm oscillating at 30 kHz yields signals shown in the lower part.
The center frequency and the frequency range are determined by the resistors \(R_1\) and \(R_2\) and by the capacitor \(C_1\). The resistance of \(R_1\) is variable between 10 k\(\Omega\) and 100 k\(\Omega\), the resistance of \(R_2\) is variable between 10 k\(\Omega\) and 200 k\(\Omega\) and the capacitance is fixed at 32 nF. An important control parameter is the transfer coefficient

\[
K_0 = \frac{\omega_{\text{max}} - \omega_{\text{min}}}{V_{\text{max}} - V_{\text{min}}}
\]

This coefficient gives the change in the circular frequency if the VCO input signal (4 in Fig. A.1) varies ± 1 V. \(V_{\text{max}}\) and \(V_{\text{min}}\) are the Minimum HIGH Level Input Voltage (3.5 V) and the Maximum LOW Level Voltage (1.5 V) respectively.

![Typical Center Frequency](image1)

![Typical \(f_{\text{max}}/f_{\text{min}}\) vs \(R_2/R_1\)](image2)

![Typical Offset Frequency](image3)

![Fig. A.2: Typical performance characteristics of the IC MM74HC4046. The particular values of the resistors and capacitors used for vibration measurements in rod are marked with little rings. (Figures: Fairchild Semiconductor Corporation, Feb. 1984, rev. Feb. 1999)](image4)

The resistance \(R_1\) simultaneously influences both the center circular frequency \(\omega_0\) and the maximal circular frequency \(\omega_{\text{max}}\). The ratio of \(\omega_{\text{max}}/\omega_{\text{min}}\) is determined by the ratio of \(R_2/R_1\). Hence, varying the resistance \(R_2\) changes the minimal circular frequency \(\omega_{\text{min}}\), and thus the transfer coefficient \(K_0\). Furthermore, the center frequency \(\omega_0\), which is the mean value of the maximal and minimal circular frequency, is adjusted to a resonance frequency of the structure by varying \(R_2\). Table A.5 shows the frequency range for four different resonance frequencies used for the ceramic and composite ceramic-metal rods.
Table A.5: Upper and lower limit of the transfer coefficient

<table>
<thead>
<tr>
<th>$f_0$ [kHz]</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$ [kHz/V] if R2 = 10 kΩ</td>
<td>~2.5</td>
<td>~6</td>
<td>~9.5</td>
<td>~16</td>
</tr>
<tr>
<td>$K_0$ [kHz/V] if R2 = 200 kΩ</td>
<td>~10.5</td>
<td>~18.5</td>
<td>~25.5</td>
<td>~33.5</td>
</tr>
</tbody>
</table>

Only if the phase loop is locked frequency measurements are possible. However, the stability can only be reached if the center frequency of the voltage controlled oscillator is in the vicinity of the resonance frequency. The lock range $\Delta \omega_L$ defines the maximum circular frequency difference between circular resonance frequency and circular center frequency so that the loop gets locked. This dynamic characteristic is mainly determined by the time constants of the active lag filter ($\tau_1$, $\tau_2$) and the phase detector gain factor $K_D$.

$$K_D = \frac{(V_{max} - V_{min})}{\pi} \quad \tau_1 = R3 \cdot C2 \quad \tau_2 = R4 \cdot C2$$

With these parameters the loop characteristics are determined as

$$\omega_n = \sqrt{\frac{K_0 K_D}{\tau_1}} \quad \zeta_n = \frac{\omega_n}{2} \cdot \tau_2$$

The resonance frequency is found by sweeping the resistance $R_I$ and thus the VCO frequency $\omega_0$. If the sweep is applied correctly, the loop will lock up as soon as the VCO frequency sweeps into coincidence with the signal. However, the sweep rate $d\omega_0/dt$ has to be smaller than $\omega_n^2/2$ [4].

Table A.6: Lock range vs. transfer coefficient

<table>
<thead>
<tr>
<th>transfer coeff. $K_0$</th>
<th>Lock range</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 kHz/V</td>
<td>8.8 Hz</td>
</tr>
<tr>
<td>33.5 kHz/V</td>
<td>125 Hz</td>
</tr>
</tbody>
</table>

Table A.6 shows the lock ranges vs. the minimal and maximal transfer coefficient from table A.5. Since the range is extremely small it was quite
cumbersome to find the resonance frequencies in the experiment. However, following values are chosen (compare figure Fig. A.1): R = 100 Ω, R3 = 18 kΩ, C2 = 100 nF. These values may be improved by adjusting the filter characteristics:

The open-loop transfer function G(s) and the closed-loop transfer function H(s) of any PLL are:

\[ G(s) = \frac{K_D \cdot K_D \cdot F(s)}{s} \quad H(s) = \frac{G(s)}{1 + G(s)} \]  \hspace{1cm} (A.5)

F(s) is the active filter. For a high-gain second loop, with \( \tau_1 >> \tau_2 \) and the filter transfer function \( F(s) = -\left(\frac{s\tau_2 + 1}{s\tau_1}\right) \) [25], the error response is

\[ \frac{\theta_2 - \theta_1}{\theta_2} = 1 - H(s) = \frac{s^2}{s^2 + 2\zeta_n \omega_n \cdot s + \omega_n^2} \]  \hspace{1cm} (A.6)

The error response is plotted in Fig. A.3 for a high-gain loop. A high pass characteristic is obtained, which means that the loop tracks low frequency changes but cannot track high frequencies.

The last two resistances R5 and R6 in Fig. A.1 may be used as a voltage divider to set the reference phase of the loop. This enables adjustment of the phase at lock. Equal resistance R5 = R6 get locked at 90° phase difference between reference (2) and signal (1). Additionally, a phase shift register ahead of the IC MM74HC4046 shifts the phase of signal (1) before the comparison in the phase detector. This phase shift register is used for measuring the Q-factor, whereby the phase is shifted up and down ±Δα from the value \( \pi/2 \) which corresponds to resonance mode.

Fig.A.3: Error response of high-gain loop with transfer coefficient \( K_0 = 2.5 \text{ kHz/V} \). The damping factor \( \xi_n = 0.011 \) is far below the recommended damping value (\( \xi_n = 0.707 \)). As a consequence, the lock range is rather small, (compare table A.6 and equation (A.4)).
A.3 On the behavior of Piezoelectric Materials

A.3.1 Basic Equations for Piezoelectric Materials

The piezoelectric constitutive relations relate electrical and mechanical variables. The four quantities (given together with their SI units)

- $E_e$ electric field: $E_e^i$ [N/C]
- $D_e$ electric displacement: $D_e^i$ [C/m²]
- $\gamma$ mechanical strain: $\gamma_{ij} = (u_{i,j} + u_{j,i})/2$ [-]
- $\sigma$ mechanical stress: $\sigma_{ij}$ [N/m²]

are related by the material properties

- $s_E$ mechanical compliance for constant electric field [m²/N]
- $d$ piezoelectric charge constant [C/N]
- $\epsilon$ permittivity for constant mechanical stress [C²/m²N]

The constitutive equations for a piezoelectric material may then be given in the form (compare [54], page 9, [57], page 184 and [71], page 68).

$$\gamma_\lambda = s_{E\lambda\mu}\sigma_\mu + d_{k\lambda}E^e_k$$

$$D^e_i = d_{i\mu}\sigma_\mu + \epsilon_{ik}E^e_k$$

where the components are referred to an orthogonal system of coordinates and Einstein’s summation convention for repeated indices is invoked. Latin and Greek indices takes the values from 1...3 and 1...6 respectively, and the correspondence between Greek matrix indices and Latin tensor indices is given by

<table>
<thead>
<tr>
<th>$ij$</th>
<th>11</th>
<th>22</th>
<th>33</th>
<th>23,32</th>
<th>13,31</th>
<th>12,21</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

For simplicity, an uniaxial state will be assumed for both, electrical and mechanical quantities. All coupling effects with other components of stress, strain electric displacement and electric field will be neglected. Equation (A.7) can be now rewritten as

$$u_{3,3} = s_{D33}\sigma_{33} + g_{33}D^e_3$$

$$E^e_3 = -g_{33}\sigma_{33} + \frac{1}{\epsilon_{33}}D^e_3$$
where

- \( g = \frac{d}{\epsilon} \) piezoelectric voltage constant \([m^2/C]\)
- \( s_D = s_E \cdot d^2/\epsilon \) mechanical compliance for constant electric displacement

Values for the material constants in extension for Ferroperm PZ 67 and PZ 27 piezo ceramics are given in table A.7.

<table>
<thead>
<tr>
<th>Table A.7: material properties of piezoceramic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>mech. compliance ( s_{D33} )</td>
</tr>
<tr>
<td>charge constant ( d_{33} )</td>
</tr>
<tr>
<td>Voltage Constant ( g_{33} )</td>
</tr>
<tr>
<td>density</td>
</tr>
</tbody>
</table>

In addition, the equation of mechanical equilibrium for harmonical vibration with circular frequency \( \omega \) is [61]

\[
\sigma_{ij,j} + \rho\cdot \omega^2 \cdot u_i = 0
\]  

(A.9)

Since there are not any free charges in piezoelectrics, it follows from Maxwell’s first equation that \( D_{ei;i} = 0 \). As a consequence, the dielectric displacement is assumed to be constant in the uniaxial case:

\[
D_{e3}^e = D_{0e}.
\]  

(A.10)

Maxwell’s second equation \( \epsilon_{ijk}E_{i,j}^e = B_{k,l} (\epsilon_{ijk}: \text{permutation tensor}, B: \text{magnetic field, } t: \text{time}) \) is simplified to the form \( \epsilon_{ijk}E_{i,j}^e = 0 \), because the magnetic flux density is associated mainly with electromagnetic waves, which, in the presence of piezoelectricity, are coupled to waves in elastic solids. Since the velocity of electromagnetic waves is approximately \( 10^4 \) times higher than longitudinal waves in elastic solids, this coupling is very weak, i.e., the contribution of the magnetic field associated with the longitudinal wave to the total magnetic field is very small. In this case, an electrostatic potential is defined by similarity to electrostatic as

\[
V_0 = -\int E_3^e dx_3.
\]  

(A.11)
A.3.2 Piezoelectric Transducers used to Excite Mechanical Vibrations on thin Plates

The constitutive differential equations of harmonical oscillation for the piezo system with backing mass are:

\[ u_{3,33}^M + \rho^M \cdot \omega^2 \cdot u_{3}^M = 0 \]

\[ u_{3,33}^N + s_{D33} \cdot \rho^N \cdot \omega^2 \cdot u_{3}^N = 0 \]

where the indices \( M \) and \( N \) indicate the mass and the n-th Piezo in the stack respectively. \( E^M \) is the Young’s modulus of the mass and \( \rho^M \) its density. The differential equation of the piezo is deduced from the combination of equation (A.8) with (A.9) under consideration of (A.10).

Fig.A.4: System of stacked piezoelectric transducer with backing mass fixed on a thin plate.

The displacement between the individual piezo-electric transducers, the mass, and the thin plate must be the same and the principle of the reaction with respect of the force has to be fulfilled. The boundary conditions are therefore between piezo stack and backing mass:

\[ u^M_{3(x_3 = -L_M), 3} = 0 \]

\[ E^M u^M_{3(x_3 = 0), 3} = (u^1_{3(x_3 = 0), 3} - g^1_{33} \cdot D^1_{00}) / s_{D33} \]

\[ u^M_{3(x_3 = 0)} = u^1_{3(x_3 = 0)} \]

The boundary conditions between the thin plate and the cylindric piezo electric transducer with radius \( r \) are

\[ Z_p \cdot u^p_{3, r} = \pi r^2 (u^N_{3(x_3 = L_N), 3} - g^N_{33} \cdot D^0_{00}) / s_{D33} \]

\[ u^p_{3} = u^N_{3(x_3 = L_N)} \]

(A.14)
where \( Z_p \) determines the relation between the force and velocity in the center of the excitation in a plate. For thin plates with density \( \rho \), thickness \( h \) and stiffness \( D \) follows \( Z_p = 8\sqrt{\rho h D} \), as it is deduced in appendix A.5.3. The boundary conditions between the individual piezo electric transducer are:

\[
\left( u_3^{N} (x_3 = L_{N-1}), 3 - g_{33}^{N} \cdot D_0^{eN} \right) / s_{D33}^{N} = \left( u_3^{N-1} (x_3 = L_{N-1}), 3 - g_{33}^{N-1} \cdot D_0^{e(N-1)} \right) / s_{D33}^{N-1}
\]

(A.15)

This system has been solved by using the program Mathematica\textsuperscript{®}. The displacements in the piezo electric transducer are dependent on the dielectric displacements \( D_0^{eN} \). The dielectric displacements can be replaced by the electrostatic potential \( V_0 \) [V], which is in fact applied to each piezo by using (A.11) i.e. the integration of the second equation of (A.8).

### A.4 Nodal Line in a Vibrating Thin Plate

In case of a thin plate with Young’s modulus \( E \), density \( \rho \), length \( a \), width \( b \) and thickness \( h \) with all side simply supported, i.e. the displacements and moments are zero along the boarders, the mode shapes \( W_{mn} \) and circular frequency \( \omega_{mn} \) follow the well known equations [40]:

\[
W_{mn} = A_{mn} \cdot \sin \frac{m \pi x}{a} \cdot \sin \frac{n \pi y}{b}
\]

\[
\omega_{mn} = \sqrt{\frac{Eh^3}{12(1-\nu^2) \cdot \rho \cdot h} \left( \frac{(m \pi a)^2}{a} + \frac{(n \pi b)^2}{b} \right)}
\]

(A.16)

where \( A_{mn} \) is the amplitude-coefficient determined from the initial conditions of the problem and \( m \) and \( n \) are integers. This solution is adapted to the problem of an oscillating plate with four free boarders. In fact, for some modes there are nodal lines with distances \( x_0 \) and \( y_0 \) from the boarders (see top left in Fig. A.5) where displacements and moments are zero. In other words, a substitute plate with length \( a' = a - 2x_0 \) and width \( b' = a - 2y_0 \) has to be found. Those lengths will be found in two steps as described in detail in [37] following to [62]:
Fig. A.5: Development of a simplified model of a vibrating plate with four free boarders. Firstly, the solution of a substitute plate with width $b'$ and length $a$ is developed. The plate is free along $b'$ and simply supported along $a$. Secondly, the solution of a substitute plate with length $a'$ and width $b$ is formulated, whereas now the plate is simply supported along $b$. The final solution, which is sketched on the top left, is found by combination of the two partial solution. The nodal lines of the final solution are due to some simplifications nearer to the boarders than in the reality (distance $\delta$ in the cross section on the top right).

First, the mode shape of a substitute plate with width $b'$ and length $a$ (see bottom left in Fig. A.5) which is simply supported along the length and with free boundary along the width may be formulated as

$$w(x, y) = g(x) \cdot \sin \frac{n \pi y}{b'}$$

$$g(x) = A \cdot \sin \frac{m \pi x}{a'} + B \cdot \cos \frac{m \pi x}{a'} + C e^{-(k_2 \cdot x)}, \quad (A.17)$$

where $k_2$ is unknown at present. The coefficients $A$ and $B$ are determined by the boundary condition on the free edges, i.e. the momentum per unit
length $M_x = 0$ (which arises from the distribution of normal stresses $\sigma_x$, as
described in appendix A.5.2) and the combination of the twisting moment
(which arises from the distribution of the shear forces $\tau_{xy}$) with the shear
force (which arises from shearing stresses $\tau_{xz}$) $M_{xy, y} + Q_x = 0$.

$$A \cdot \frac{m \pi}{a'} \left[ \left( \frac{m \pi}{a'} \right)^2 + (2 - \nu) \left( \frac{n \pi}{b'} \right)^2 \right] = C \cdot k_2 \left[ k_2^2 - (2 - \nu) \left( \frac{n \pi}{b'} \right)^2 \right]$$  \hspace{1cm} (A.18)

$$B \cdot \left[ \left( \frac{m \pi}{a'} \right)^2 + \nu \left( \frac{n \pi}{b'} \right)^2 \right] = C \cdot \left[ k_2^2 - \nu \left( \frac{n \pi}{b'} \right)^2 \right]$$  \hspace{1cm} (A.19)

With the help of the differential equation of thin plates $k_2$ is determined as

$$k_2^2 = \left( \frac{m \pi}{a'} \right)^2 + 2 \left( \frac{n \pi}{b'} \right)^2$$  \hspace{1cm} (A.20)

$x_0$ is found by neglecting the boundary layer term $Ce^{-\left( k_2 \cdot x \right)}$ in (A.17)
which ends in

$$\tan \left( \frac{m \pi}{a'} \cdot x_0 \right) = -\frac{B}{A}$$  \hspace{1cm} (A.21)

which includes still a unknown constant $y_0$.

As a consequence similar equations to a corresponding problem of an os¬
cillating plate which is simply supported along the width $b$ and with free
boundary along the boarder with length $a'$ have to be formulated in order
to find a solution for $y_0$ corresponding to (A.21):

$$\tan \left( \frac{n \pi}{b'} \cdot y_0 \right) = -\frac{\tilde{B}}{\tilde{A}}$$  \hspace{1cm} (A.22)

where $\tilde{A}$ and $\tilde{B}$ are dependent from a factor $\tilde{k_2}$ defined in correspondence
to (A.20):

$$\tilde{k_2}^2 = \left( \frac{n \pi}{b'} \right)^2 + 2 \left( \frac{m \pi}{a'} \right)^2$$  \hspace{1cm} (A.23)

The distances $x_0$ and $y_0$ of the first nodal lines from the boarders are found
by solving the transcendental system of equations (A.21) and (A.22).

It must be mentioned again, that the boundary layer terms $Ce^{-\left( k_2 \cdot x \right)}$
and $Ce^{-\left( k_2 \cdot y \right)}$ were neglected with respect of the equations (A.21) and
(A.22). Therefore, the calculated nodal lines are nearer to the boarders as
sketched on the top right in Fig. A.5 (δ stands for the corresponding error). As a consequence, the resonance frequencies, which are found by replacing \( a \) by \( a' \) and \( b \) by \( b' \) in (A.16), are the lower bounds of the on vibration frequencies.

### A.5 Equations of Ceramic-Metal Plate

#### A.5.1 Model based on Mindlin’s Theory

The main feature of composite plates is the plate displacement field \((u,v,w)\) have to be formulated on a “neutral” plane which is not generally in the midplane of the composite plate because the material parameters of each component are different. It is assumed, that \( u \) and \( v \) are proportional to \( z \), with constant slopes \( \psi_x \) and \( \psi_y \), and the transversal displacement \( w \) remains independent of \( z \).

\[
    u = (z - zo)\psi_x(x, y, t) \quad v = (z - zo)\psi_y(x, y, t) \quad w = \bar{w}(x, y, t) \tag{A.24}
\]

The origin is on the contact area between the two components (see Fig. A.6), and \( zo \) the coordinate of the “neutral” plane with vanishing in-plane displacements \( u \) and \( v \).

Fig. A.6: Coordinate in the metal-ceramic plate. The origin \((z=0)\) is on the contact area between the two components.

Since flexural waves are investigated, there are not any resulting membrane forces.

\[
    \int_{0}^{0} (\sigma_{xx}, \sigma_{yy}, \tau_{xy}) dz + \int_{(z=-h_n)}^{0} (\sigma_{xx}, \sigma_{yy}, \tau_{xy}) dz = (0, 0, 0) \tag{A.25}
\]

where the stresses are derived from the general Hooke’s law. The bending twisting moments and the transverse shearing forces, all per unit length, are defined in the customary manner:
\[(M_x, M_y, M_{xy}) = \int_{0}^{(z = h_c)} (\sigma_x, \sigma_y, \tau_{xy})(z-z_0)dz + \int_{(z = h_c)}^{0} (\sigma_x, \sigma_y, \tau_{xy})(z-z_0)dz\]

\[(Q_x, Q_y) = \int_{0}^{(z = h_c)} (\tau_{xx}, \tau_{yy})dz + \int_{(z = h_c)}^{0} (\tau_{xx}, \tau_{yy})dz\]

(A.26)

As in [43] a constant \( \kappa \) is introduced for the shear forces in terms of the shear strain:

\[Q_x = \frac{E_c}{2(1 + \nu)} \cdot \kappa^2 \cdot \int_{0}^{(z = h_c)} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)dz + \frac{E_n}{2(1 + \nu)} \cdot \kappa^2 \cdot \int_{(z = h_c)}^{0} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)dz\]

\[Q_x = \frac{E_c}{2(1 + \nu)} \cdot \kappa^2 \cdot \int_{0}^{(z = h_c)} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)dz + \frac{E_n}{2(1 + \nu)} \cdot \kappa^2 \cdot \int_{(z = h_c)}^{0} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)dz\]

(A.27)

Mindlin obtains the value of \( \kappa \) from the limiting case of very large frequencies leading to Rayleigh waves and corresponding wave speeds. The dependence of \( \kappa^2 \) on Poisson’s ratio is illustrated by noting its ranging almost linearly from 0.76 for \( \nu = 0 \) to 0.91 for \( \nu = 1/2 \).

The stresses in the integral are dependent on the general Hooke’s law in ceramic and nickel alloy respectively. The stress equations of motion of three-dimensional elasticity theory

\[\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}\]

\[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2}\]

\[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}\]

(A.28)

are converted to plate-stress equations of motions in the following manner. The first two equations are multiplied by \((z-z_0)\) and integrated over the composite plate thickness. The third equation is integrated over the thickness.
The final differential equation of motion is obtained by elimination of all terms containing $\psi_x$ and $\psi_y$:

$$\frac{\partial^2 w}{\partial t^2} + D_{(E_i, \rho_i, h_i, E_s, \rho_s, h_s)} \Delta \Delta w = \eta_{(E_s, \rho_s, h_s, E_i, \rho_i, h_i)} \Delta \frac{\partial^2 w}{\partial t^2} + \chi \frac{\partial^4 w}{\partial t^4}$$  (A.30)

where $D$ is the final bending stiffness of the composite plate, $\eta$ a value taking the rotary inertia and shear stress into account and $\chi$ a very small value which considers the inertia of the shear strain correction $\psi_x$ and $\psi_y$. Since those shear corrections are introduced to improve the model for higher frequencies, the last term in (A.30) is a “correction of a correction” and can be omitted. Indeed, the asymptotic development in the next section does not include any terms with the fourth derivative in time.

**A.5.2 Asymptotic Development**

We introduce asymptotic expansions of the full three-dimensional dynamic equations of linear elastic isotropic behavior and obtain solutions of the resulting simplified relation for two steps of approximation. These solutions are given explicitly for the case of flexural waves due to an oscillatory point load transverse to the surface of a composite ceramic-metal plate whose coordinate and material properties are defined below:
In case of isotropy, the whole metal-ceramic plate is characterized by six material constants \( E_c, E_k, \rho_c, \rho_k, \nu_c \) and \( \nu_k \).

E is the elastic modulus of the linear elastic isotropic material, \( \rho \) the density and \( \nu \) Poisson's ratio. The indices \( c \) and \( k \) stand for ceramic and kovar alloy respectively. We shall use greek indices \( \alpha, \beta, \ldots \) for "in plane" coordinates \( \bar{x}_1, \bar{x}_2 \) throughout. The latin indices \( i, j, \ldots \) will designate as usual all three space coordinates and the corresponding component (the summation convention on repeated indices will hold for both greek and latin indices).

The following dimensionless quantities are defined in accordance with scaling considerations:

- **stresses**: \( \sigma_{ij} := \sigma_{ij} / \sigma_0 \)
- **displacement**: \( u_i := u_i E_c / \sigma_0 h_c \)
- **coordinates**: \( x_3 := x_3 / h_c \quad x_\alpha := x_\alpha 2\pi / \lambda \)
- **time**: \( t := \tau / T \)

\( \lambda \) represents the wavelength. The quantities \( \sigma_0 \) and \( T \) are introduced only for scaling considerations and their values are unknown at present. The overlined quantities do have dimensions.

The two layers are characterized with roman indices I,II, \ldots and their properties are scaled as follow:

- **elastic modulus**: \( e^I := E_c / E_c \quad e^{II} := E_k / E_c \)
- **density**: \( \rho^I := \rho_c / \rho_c \quad \rho^{II} := \rho_k / \rho_c \)
- **Poisson's ratio**: \( \nu^I := \nu_c \quad \nu^{II} := \nu_k \)
- **thickness**: \( h^I := h_c / h_c \quad h^{II} := h_k / h_c \)

The roman indices are usually left out, because the main formalism are the same for each layer with the corresponding values.
The relations of linear isotropic elasticity for linearized deformations and the equilibrium conditions can be written as follows in terms of dimensionless quantities which have just been introduced:

\[ u_{3,3} \cdot \varepsilon = \sigma_{33} - \nu \cdot \sigma_{\gamma \gamma} \]
\[ (\varepsilon \cdot u_{3,3} + u_{a,3}) \cdot \varepsilon = 2 \cdot (1 + \nu) \cdot \sigma_{\alpha \beta} \]
\[ \varepsilon \cdot (u_{a,\beta} + u_{\beta,a}) \cdot \varepsilon = 2 \cdot (1 + \nu) \cdot \sigma_{\alpha \beta} \]
\[ -2 \cdot \nu \cdot (\sigma_{\gamma \gamma} + \sigma_{33}) \cdot \delta_{\alpha \beta} \]

(A.31)

with \( \varepsilon = 2 \cdot \pi \cdot h_c / \lambda \), \( \Gamma = \rho_c h_c^2 / E_c T^2 \) and the Kronecker symbol \( \delta_{\alpha \beta} \).

The boundary conditions on the contact surface \( x_3 = 0 \) are:

\[ u_3(x_0, y, z = 0) = u_3(x_0, y, z = 0) \]
\[ u_3(x_0, y, z = 0) = u_3(x_0, y, z = 0) \]
\[ \sigma_{a3}(x_0, y, z = 0) = \sigma_{a3}(x_0, y, z = 0) \]
\[ \sigma_{33}(x_0, y, z = 0) = \sigma_{33}(x_0, y, z = 0) \]

(A.32)

The boundary conditions on the top surfaces are:

\[ \sigma_{a3}(x_0, y, z = -h') = 0 = \sigma_{a3}(x_0, y, z = h'') \]
\[ \sigma_{33}(x_0, y, z = -h') = 0 = \sigma_{33}(x_0, y, z = h'') \]

(A.33)

We define in the usual manner the dimensionless normal force of sectioning by integration over the whole thickness. As we are investigating a bending wave and not a longitudinal one, this force is equal to zero:

\[ \int_{x_3 = 0}^{x_3 = h''} \sigma_{\alpha \beta} d\lambda_3 + \int_{x_3 = 0}^{x_3 = h'} \sigma_{\alpha \beta} d\lambda_3 = 0 \]

(A.34)

To proceed with asymptotic expansions of these equations, the orders of occurring functions must be determined as \( \varepsilon \to 0 \). The main interest lies in the transversal displacement. Thus, \( u_3 \) is a quantity of the first order: \( u_3 = O(\varepsilon^0) \). From second equation in (A.31) one infers that \( u_\alpha = O(\varepsilon^1) \).
The following equations in (A.31) require that $\sigma_{\alpha\beta} = O(\varepsilon^2)$, $\sigma_{\alpha 3} = O(\varepsilon^3)$ and $\sigma_{33} = O(\varepsilon^4)$. The last equation of the momentum in (A.31) is of the order $\varepsilon^4$. One has to take into account that we consider a dynamic problem. Thus, $Y$ must be of the order $\varepsilon^4$, e.g. $Y = \varepsilon^4$. These orders of magnitude are the only ones which are compatible with the basic equations and the problem under consideration. Inserting expansions of the form

\begin{align*}
    u_3(y, t) &= u_3^{(0)} + u_3^{(2)} \cdot \varepsilon^2 + \ldots \\
    u_\alpha(y, t) &= u_\alpha^{(1)} + u_\alpha^{(3)} \cdot \varepsilon^3 + \ldots \\
    \sigma_{\alpha\beta}(y, t) &= \sigma_{\alpha\beta}^{(2)} \cdot \varepsilon^2 + \sigma_{\alpha\beta}^{(4)} \cdot \varepsilon^4 + \ldots \\
    \sigma_{\alpha 3}(y, t) &= \sigma_{\alpha 3}^{(3)} \cdot \varepsilon^3 + \sigma_{\alpha 3}^{(5)} \cdot \varepsilon^5 + \ldots \\
    \sigma_{33}(y, t) &= \sigma_{33}^{(4)} \cdot \varepsilon^4 + \sigma_{33}^{(6)} \cdot \varepsilon^6 + \ldots
\end{align*}

(A.35)

with $x = \{x_1, x_2, x_3\}^T$ in the equations of (A.31) ordering with respect to $\varepsilon$ and passing $\varepsilon \to 0$, one gets for the first step of approximation

\begin{align*}
    u_3^{(0)} \cdot \varepsilon &= 0 \\
    (u_3^{(0)} + u_\alpha^{(1)} \cdot \varepsilon^1) \cdot \varepsilon &= 0 \\
    (u_{\alpha\beta} + u_{\beta\alpha} \cdot \varepsilon) \cdot \varepsilon &= 2 \cdot (1 + \nu) \cdot \sigma_{\alpha\beta}^{(2)} \\
    -2 \cdot \nu \cdot \sigma_{\gamma\gamma}^{(2)} &\cdot \delta_{\alpha\beta}
\end{align*}

(A.36)

and for the second step of approximation (on the left side of the equations are the quantities corresponding to the second approximation, whereas on the right side are the functions which are known from (A.36), except for stresses in the third equation).
The integration of equations (A.36) using the boundary conditions (A.32) to (A.34) leads to an equation for the out-of-plane displacement of the whole composite in the first approximation:

\[
\begin{align*}
\int_{x_3 = 0}^{x_3 = h_3} \sigma_{\alpha\beta}^I \cdot x_3 dx_3 + \int_{x_3 = 0}^{x_3 = h_3} \sigma_{\alpha\beta}^{II} \cdot x_3 dx_3 &= M_{\alpha\beta} \\
\int_{x_3 = -h_3}^{x_3 = 0} \sigma_{\alpha3}^I dx_3 + \int_{x_3 = -h_3}^{x_3 = 0} \sigma_{\alpha3}^{II} dx_3 &= Q_{\alpha3} = M_{\alpha\beta}\beta
\end{align*}
\]

The equation (A.38) corresponds with the classical theory of thin plates. Following conclusions may be formulated:

• Bending displacements \( u_3 \) are independent of the thickness.
• In-plane displacements \( u_\alpha \) are linearly distributed over the thickness; the cross-sections are perpendicular to the surfaces.
• The Moment \( M_{\alpha\beta} \) of sectioning is proportional to the second in-plane derivative of the transversal displacement \( u_3 \), corresponding to the local bending.
• The distribution of shear stresses \( \sigma_{\alpha3} \) are parabolic. The resulting transversal force of sectioning is connected to the Moment.
• Equation (A.38) results by solving \( Q_{a3,\alpha} = (\rho_l^I \cdot h_l^I + \rho_l^{II} \cdot h_l^{II}) \cdot \ddot{u}_3 \)

The assumption of plane cross-sections is valid only for thin plates or long waves: \( \lambda > 15 (h_c + h_n) \).

The solution of second approximation (A.37) results in

\[
\begin{align*}
    u_3^{(2)}(x_\alpha, \tau_3 = -1, t) + D \cdot \Delta u_3^{(2)}(x_\alpha, \tau_3 = -1, t) = \eta \cdot \Delta u_3^{(0)}(x_\alpha, t) \\
    D = D(h_c, h_n, \rho_l, \rho_l^{II}, \rho_l', \rho_l^{II}', c', c''', v', v''') \\
    \eta = \eta(h_c, h_n, \rho_l, \rho_l^{II}, \rho_l', \rho_l^{II}', c', c''', v', v''')
\end{align*}
\]

(A.39)

The combination of (A.39) with (A.38), under consideration of (A.35) and the dimensions, leads to an equation of motion similar to the solution of Mindlin (equation (37) in [43]):

\[
\begin{align*}
    u_3^{(2)}(\bar{x}_\alpha, \bar{x}_3 = -\bar{h}_c, i) + D \cdot \frac{E_c}{\rho_c} \cdot \bar{h}_c^2 \cdot \Delta u_3^{(2)}(\bar{x}_\alpha, \bar{x}_3 = -\bar{h}_c, i) \\
    = \eta \cdot \bar{h}_c^2 \cdot \Delta u_3^{(0)}(\bar{x}_\alpha, \bar{x}_3 = -\bar{h}_c, i) + O(\epsilon^4)
\end{align*}
\]

(A.40)

On the left side of (A.40) are terms of a thin plate and on the right side there is a correction due to shear stresses which have to be taken into account for short wavelengths \( \lambda < 15 (h_c + h_n) \). There is one main difference to Mindlin’s equation: Mindlin takes the rotary inertia of the shear deformation into account, which is in fact of the order \( O(\epsilon) \).

**A.5.3 Wave due to a Harmonically Oscillating Point**

In this chapter we derive the solution of (A.40) for flexural waves radiating to infinity from the exciting point located at the origin. The transversal displacement is rotation-symmetrical: \( \bar{u}_3(r, i) \). Thus, the Laplace-operator is simplified to

\[
\Delta(,) = (,)_{rr} + \frac{(,)}{r}
\]

(A.41)

Time-harmonic functions are considered to obtain the solution:

\[
\bar{u}_3(r, i) = w_{(r)} \cdot e^{j2 \cdot \pi \cdot f \cdot i}
\]

(A.42)

We first neglect the influence of the shear stresses and derive a solution for the first approximation. With the abbreviation
\[ \kappa_0^4 = \frac{(2 \cdot \pi \cdot f)^2 \cdot \rho_e}{h_e^2 \cdot D \cdot E_e} \quad (A.43) \]

one gets

\[ \Delta \Delta w_{\dot{r}} - \kappa_0^4 w_{\dot{r}} = 0 \quad (A.44) \]

The solution is a linear combination of the Bessel functions \( J_0 \) and \( Y_0 \):

\[ w_{\dot{r}} = a_1 \cdot [J_0(\kappa_0 \cdot \dot{r}) - i \cdot Y_0(\kappa_0 \cdot \dot{r})] + a_3 \cdot [J_0(i \cdot \kappa_0 \cdot \dot{r}) + i \cdot Y_0(i \cdot \kappa_0 \cdot \dot{r})] \quad (A.45) \]

The first term on the right hand side characterizes a wave propagating in direction with longer distances from the source. The second term prevents an increase of the function with increased distance from the source. The constants \( a_1 \) and \( a_3 \) are determined by the boundary conditions:

\[ u_{\dot{3}, \dot{r}}(\dot{r} = 0, \dot{r}) = 0 \]

\[ v_{\dot{3}, \dot{r}}(\dot{r} = 0, \dot{r}) = -v_0 \cdot e^{i \cdot 2 \cdot \pi \cdot f \cdot \dot{r}} \quad (A.46) \]

The second condition corresponds with the velocity in the center of a forced symmetrical vibration on a plate, which can be measured easily with the help of the heterodyne interferometer. In case the plate is excited by a harmonically oscillating force with amplitude \( F_0 \). The equilibrium in the center of the plate has to be fulfilled:

\[ \lim_{\dot{r} \to 0} [2\pi \dot{r} Q_{r(i, \dot{r})}] = -F_0 \cdot e^{i \omega t} \quad (A.47) \]

where \( Q_r \) is the transversal force of sectioning. For a thin plate and harmonical time function, it can be shown that \( Q = -D(\Delta w) \dot{r} = D \kappa_0^2 w_{\dot{r}} \). Consequently, it follows

\[
\begin{align*}
-u_{\dot{3}, \dot{r}} &= \frac{-i \cdot v_0 \cdot e^{i \cdot 2 \cdot \pi \cdot f \cdot \dot{r}}}{2 \cdot \pi \cdot f} \cdot [J_0(\kappa_0 \cdot \dot{r}) - i \cdot Y_0(\kappa_0 \cdot \dot{r}) + J_0(i \cdot \kappa_0 \cdot \dot{r}) + i \cdot Y_0(i \cdot \kappa_0 \cdot \dot{r})] \\
&= \frac{-i \cdot F_0 \cdot e^{i \cdot 2 \cdot \pi \cdot f \cdot \dot{r}}}{8 \cdot 2 \cdot \pi \cdot f \cdot \sqrt{\rho_e h_e D}} \cdot [J_0(\kappa_0 \cdot \dot{r}) - i \cdot Y_0(\kappa_0 \cdot \dot{r}) + J_0(i \cdot \kappa_0 \cdot \dot{r}) + i \cdot Y_0(i \cdot \kappa_0 \cdot \dot{r})]
\end{align*}
\]
Thus an impedance $Z_p$ is defined as the relation between velocity $v_0$ and force in the center of a forced symmetrical vibration on a plate as

$$Z_p = 8\sqrt{\rho_c h_c D}.$$  (A.49)

In order to derive a solution for the second step of approximation, we modify the time-harmonic function (A.42)

$$\tilde{u}_{3(r,\tau)} = w_{(r)} \cdot [J_{0(k \cdot r)} - i \cdot Y_{0(k \cdot r)}] \cdot e^{i \cdot 2 \cdot \pi \cdot f \cdot \tau}$$  (A.50)

and insert this function into (A.40). This results in a dispersion relation for the wave number

$$\kappa^4 = \kappa_0^4 \cdot (1 + \eta \cdot h_c^2 \cdot \kappa_0^2).$$  (A.51)

The second term on the right hand side is a correction due to the shear stresses. In order to evaluate this quantity, we use the result from the first step of approximation. Accordingly, quantities of the order $\varepsilon^4$ are ignored. Using $\kappa^2 = \kappa_0^2$ in the right hand side yields

$$\kappa = \kappa_0 \cdot (1 + \eta \cdot h_c^2 \cdot \kappa_0^2)^{\frac{1}{4}}$$  (A.52)

By the substitution of $\kappa_0$ in (A.48) with $\kappa$, one gets the transversal displacement as a solution of (A.40). The displacement-field of any point sources is determinable with Fourier synthesis.

### A.5.4 Numerical Solution

In the experiments, $N$ data points (with the value of the velocity $v_k$, $k = 1 \ldots N$) are recorded and stored with the help of the heterodyne interferometer and a storage oscilloscope. The time difference between two data points is $\Delta t$. The value $v_k$ is determined by superposing all primary waves (based on equation (A.48)), generated due to an infinitesimal point force $dF = a_0^j R d\varphi dR$. This superposition is on the one hand a Fourier synthesis over all discrete frequencies $\tilde{f}$, $j = 1 \ldots N$, on the other hand an integration over the circular area of the piezoelectric transducer with the inner radius $R_i$ and the outer radius $R_o$, because the velocity is measured.
in the center of the circular contact area between the plate and the cylindric transducer. Thus, the mathematics of the data $v_k$ are

$$v_k = \frac{1}{2} \int \int \left\{ \sum_{j=1}^{N} e^{i \cdot 2 \cdot \pi \cdot j \cdot (k \cdot \Delta t)} \right\} \times \left[ H_0^{(2)}(\kappa \cdot R_0) + H_0^{(1)}(i \cdot \kappa \cdot R_0) \right] \frac{\sigma_j}{Z_p^j} \cdot R \cdot d\phi \cdot dR$$

$$v_0^j = \frac{\sigma_j \cdot R \cdot d\phi \cdot dR}{Z_p^j}$$

where $Z_p^j$ determines the relation between the infinitesimal force $dF^j$ and the transversal velocity $v_0^j$ of the plate at the point source. This relation depends on frequency because for higher frequencies, the influence of shear and rotatory inertia has to be considered in the boundary condition (A.47). The values $v_0^j$ (or $\sigma_0^j$) are calculated with the measured $v_k$ and the inversion of equation (A.53).

The field of the transversal displacement at a specific time $t_c = k \Delta t$, which corresponds to the delay time between the first and second exposure of the hologram, is evaluated by performing the same superpositions as above, namely

$$\tilde{u}_3(\tilde{r}, k \cdot \Delta t) = \frac{1}{2} \int \int \left\{ \sum_{j=1}^{N} \frac{-i \cdot e^{i \cdot 2 \cdot \pi \cdot j \cdot (k \cdot \Delta t)}}{2 \cdot \pi \cdot j^2} \right\} \times \left[ H_0^{(2)}(\kappa \cdot \tilde{R}) + H_0^{(1)}(i \cdot \kappa \cdot \tilde{R}) \right] \frac{\sigma_j}{Z_p^j} \cdot R \cdot d\phi \cdot dR$$

$$-\tilde{R}^2 = \tilde{R}^2 + \tilde{r}^2 - 2 \cdot R \cdot \tilde{r} \cdot \cos \phi$$

where $\tilde{R}$ is the specific distance from the individual point source and $\tilde{r}$ the distance from the center point of the contact area between transducer and composite plate. Since the final result depends on the measured values $v_k$, the value $Z_p^j$ is shortened by the combination of (A.53) with (A.54).
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Curriculum Vitae

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1975 — 1980 Primary school in Remetschwil/AG, Switzerland
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1981 — 1985 Bezirksschule in Baden/AG, Switzerland
1985 — 1989 Kantonsschule in Baden/AG, Switzerland Matura Type A (literature and philosophy)
1989 — 1994 Studies at the Department of Mechanical Engineering at the Swiss Federal Institute of Technology (ETH) in Zurich, Switzerland, Graduation with the degree Dipl. Masch. Ing. ETH
1994 —2000 Doctoral Student, teaching and research assistant at the Institute of Mechanics of the ETH Zurich