Doctoral Thesis

Single electron transport through AFM defined quantum dots

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Single electron transport through AFM defined quantum dots

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH

for the degree of
Doctor of Natural Sciences

presented by

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2001
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Abstract

This thesis discusses the fabrication, the characterization and the transport properties of quantum dots formed in the two-dimensional electron gas of a GaAs/AlGaAs heterostructure by local oxidation of the heterostructure's surface with an atomic force microscope. It was experimentally demonstrated that this technique leads to small lateral depletion lengths. In addition, planar gates can easily be combined with top gates and multiply connected geometries are straightforward to pattern.

In this thesis, it is investigated how this novel technology can be exploited to fabricate quantum dots with desirable properties, like highly accurate transfer of the lithographic pattern in the two-dimensional electron gas, steep confining potential walls, or reduced electron-electron interactions.

Measurements on such quantum dots in high magnetic field are used to establish that the potential walls can be significantly steeper than those in quantum dots defined with conventional lithographic techniques, such as electron beam or ion beam lithography. Furthermore, we find that the shape of our quantum dots is not strongly modified when a gate voltage is tuned. A high-density two-dimensional electron gas as a starting material thus leads to high quality quantum dots with large electron densities and therefore reduced electron-electron interactions compared to earlier experiments on semiconductor quantum dots.

As a consequence the parametric evolution of Coulomb blockade peaks shows a pronounced pairwise correlation in both the position and the amplitude in small magnetic fields. This is interpreted as spin pairing. We develop a model in order to explain the distribution of the nearest-neighbor spacings and conclude that interactions beyond the constant interaction model are significant.

As an example for a twofold connected geometry, we have fabricated and measured a quantum ring in the Coulomb blockade regime. Here, modulations periodic in magnetic field of the amplitude and the position of Coulomb blockade resonances are found.
Zusammenfassung

In dieser Arbeit werden die Herstellung, die Charakterisierung und die Transporteigenchaften von Quantenpunkten, die durch laterales Einschnüren in einem zweidimensionalen Elektronengas definiert sind, besprochen. Das laterale Einschnüren wird durch lokale Oxidation der Oberfläche einer Heterostruktur mit einem Rasterkraftmikroskop realisiert. Experimentell wurde gezeigt, dass diese Technik zu kleinen elektronischen Verarmungsängen führt. Zusätzlich können planare Elektroden mit Elektroden, die sich auf der Heterostruktur (also über dem Quantenpunkt) befinden, kombiniert werden. Weiter erlaubt es, diese Technik auf einfachste Weise mehrfach zusammenhängende Strukturen zu definieren.

Es wird untersucht, wie diese neue Technologie genutzt werden kann, um Quantenpunkte mit hervorragenden Eigenschaften herzustellen. Diese Eigenschaften sind zum Beispiel, die exakte Übertragung der lithografischen Struktur in das zweidimensionale Elektronengas, steile Potentialwände oder reduzierte Elektron-Elektron-Wechselwirkung.

Transportmessungen dieser Quantenpunkte in hohen Magnetfeldern zeigen, dass die Potentialwände in diesen Nanostrukturen um eine Größenordnung steiler sind, als jene bei herkömmlichen Lithografiemethoden wie zum Beispiel bei der Ionen- oder Elektronenstrahl lithografie. Weiter finden wir, dass die Form des Quantenpunktes nur wenig verändert wird, wenn man die Zahl der Elektronen auf dem Punkt variiert. Da die hier besprochenen Quantenpunkte in einem zweidimensionalem Elektronengas mit hoher Elektronendichte definiert werden, ist die Elektron-Elektron-Wechselwirkung, verglichen zu jener in früheren Experimenten an ähnlichen Quantenpunkten, reduziert.


# Lists of abbreviations

<table>
<thead>
<tr>
<th>Physical Constants</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_B$</td>
<td>Bohr radius</td>
</tr>
<tr>
<td>$-e$</td>
<td>electron charge</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>dielectric permittivity</td>
</tr>
<tr>
<td>$\epsilon_0$</td>
<td>vacuum dielectric constant</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>$h/e$ flux quantum</td>
</tr>
<tr>
<td>$h = 2\pi\hbar$</td>
<td>Planck’s constant</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
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</table>

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>Aharonov-Bohm</td>
</tr>
<tr>
<td>AFM</td>
<td>atomic force microscope</td>
</tr>
<tr>
<td>CB</td>
<td>Coulomb blockade</td>
</tr>
<tr>
<td>ENNS</td>
<td>even nearest-neighbor spacings</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width at half maximum</td>
</tr>
<tr>
<td>Ga[Al]As</td>
<td>$Al_xGa_{(1-x)}As$ heterostructure</td>
</tr>
<tr>
<td>GOE</td>
<td>Gaussian orthogonal ensemble</td>
</tr>
<tr>
<td>GUE</td>
<td>Gaussian unitary ensemble</td>
</tr>
<tr>
<td>HF</td>
<td>Hartree Fock</td>
</tr>
<tr>
<td>IPG, ipg</td>
<td>in-plane gate</td>
</tr>
<tr>
<td>LL(i)</td>
<td>Landau Level i</td>
</tr>
<tr>
<td>LO</td>
<td>local oxidation</td>
</tr>
<tr>
<td>NNS</td>
<td>nearest-neighbor spacings</td>
</tr>
<tr>
<td>ONNS</td>
<td>odd nearest-neighbor spacings</td>
</tr>
<tr>
<td>PC, pc</td>
<td>point contact</td>
</tr>
<tr>
<td>RMT</td>
<td>random matrix theory</td>
</tr>
<tr>
<td>RPA</td>
<td>random phase approximation</td>
</tr>
<tr>
<td>SdH</td>
<td>Shubnikov de Haas</td>
</tr>
<tr>
<td>TG, tg</td>
<td>top gate</td>
</tr>
</tbody>
</table>
Symbol | Explanation
-------|----------------------------------
A       | area of the quantum dot          
B       | magnetic field                   
C       | self-capacitance of the quantum dot 
C_g     | capacitance between quantum dot and gate g 
$\Delta$ | spin degenerate single particle level spacing 
$E_F$  | Fermi energy                     
f      | frequency in $T^{-1}$            
G      | conductance                      
$h\Gamma$ | intrinsic level broadening     
k      | wavenumber                       
k_F    | Fermi wavenumber                 
L      | system size                      
$\lambda_F$ | Fermi wavelength              
l_d    | elastic mean free path          
m^*    | effective electron mass in GaAs 
$n_s$  | electron sheet density           
$\nu$  | filling factor                   
$\omega_c$ | cyclotron frequency            
r_c    | cyclotron radius                 
r_o    | radius of the quantum ring       
r_s    | gas parameter                    
T      | temperature                      
$\xi$  | interaction energy               

**List of samples**

<table>
<thead>
<tr>
<th>sample name</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCL</td>
<td>rectangular closed large quantum dot</td>
</tr>
<tr>
<td>RO</td>
<td>rectangular open large quantum dot</td>
</tr>
<tr>
<td>RS</td>
<td>rectangular small quantum dot</td>
</tr>
<tr>
<td>stadium</td>
<td>quantum dot with stadium shape</td>
</tr>
<tr>
<td>qring</td>
<td>quantum dot with a ring shape</td>
</tr>
</tbody>
</table>

1A more detailed description of these samples is given in table 4.2
Chapter 1

Introduction

Conductors are called mesoscopic, if they are much larger than microscopic objects like atoms, but not large enough to be governed by macroscopic properties such as diffusive transport. A conductor usually shows classically diffusive behavior if its dimensions \( L \) are much larger than each of the three characteristic lengths scales: (i) the de Broglie wavelength, which is related to the kinetic energy of the electrons, (ii) the elastic mean free path \( l_d \), which is the distance that an electron travels before its momentum direction is randomized and (iii) the phase relaxation length \( l_\phi \), which is the distance that an electron travels before its phase relation is destroyed. The field of mesoscopic physics originated in the study of phase coherent \( (l_\phi \geq L) \) disordered systems in which the electron motion is diffusive, i.e., the elastic mean free path \( l_d \) is small relative to the system size \( L \). In the late 1980s it became possible to produce high-mobility semiconductor microstructures that were sufficiently small and free of impurities to ensure that the elastic mean free path \( l_d \) exceeds the systems size \( L \). Such devices are termed ballistic. Transport in a ballistic quantum dot is dominated by electronic scattering not from impurities, but from the structure’s boundaries.

In this thesis we focus on the physics of such mesoscopic devices, namely ballistic semiconductor quantum dots. Quantum dots are artificially defined "droplets" of charge that can contain anything from a single electron to a collection of several thousand. The typical dimensions of these quantum dots range from nanometers to a few microns, and their size, shape and interactions can be precisely controlled by the use of advanced nanofabrication technology. Most experimental research on quantum dots is focused on systems defined by forming a two-dimensional electron gas in the interface region of a semiconductor heterostructure and applying an electrostatic potential to metal gates to further confine the electrons laterally to a small region in the interface plane. Because the electronic motion is restricted in all three dimensions, a quantum dot is sometimes referred to as a zero-dimensional system. The transport properties of a quantum dot can be measured by coupling it to leads. The electron phase is preserved over distances that are large compared with the size of the system. The coupling between a
quantum dot and its leads can be experimentally controlled by the point contacts. A point contact is a narrow constriction which separates the dot and the leads. By applying voltages to the gates defining these narrow constrictions the coupling of the quantum dot to its environment can be tuned. In an open dot, the coupling is strong and the movement of electrons across the dot-lead junctions is classically allowed. When the point contacts are pinched off, effective barriers are formed and conductance occurs only by tunneling. In these almost-isolated or closed quantum dots, the electron number on the dot is quantized. As in atoms, the energy levels in a quantum dot become quantized due to the confinement of electrons and the widths of the levels is smaller than their spacing. The physics of these systems shows many similarities to the behavior of naturally occurring quantum systems in atomic and nuclear physics [1].

We like to mention here that besides these quantum dots there are other mesoscopic systems which display strikingly similar phenomena. Experiments have been performed recently on even smaller structures such as very clean metallic nanoparticles [2], $C_{60}$ molecules [3], and carbon nanotubes [4, 5, 6].

However, the advantage of these artificially defined quantum dots is that their transport properties are readily measured, with the strength of the dot-lead coupling, the number of electrons in the dot, and the dots size and shape all under experimental control. This control over the electron number allows experiments on quantum dots to address questions about the role of electron number and electron-electron interactions in quantum systems. Furthermore, the total spin of the electrons on the quantum dot can be tuned.

This is of importance for the implementation of quantum dots in the new quantum information technologies, since, the spin of the electron is a most natural candidate for the qubit - the fundamental unit of quantum information. These spin qubits, when located in quantum-confined structures such as semiconductor quantum dots, atoms or molecules, satisfy all requirements needed for a scalable quantum computer. Therefore controlling the spin in a quantum system offers unique possibilities for finding novel mechanisms for information processing and communication. Spin-based devices in conventional as well as in quantum computer hardware have been proposed.

In transport experiments two different types of quantum dots can be investigated, corresponding to lateral and vertical geometries. Here vertical and lateral defines the current flow with respect to the two-dimensional electron gas plane. For example, vertical dots were used for spectroscopic studies of systems with few electrons ($N \leq 20$). Such dots can be prepared in regular shapes, such as a disk, with a confining potential close to being harmonic. Here, the single-particle levels are arranged in shells. This shell structure is observed by measuring the Coulomb blockade peak separation as a function of the number of electrons in the dot [7]. The energy level spacing determined in these small artificial atoms exhibits clear maxima at $N=2,6$ and $12$. This corresponds to completely filled shells of a two-dimensional spin-degenerate harmonic oscillator. Furthermore ad-
ditional maxima in the level separation can be seen at \( N = 4, 9 \) and \( 16 \), describing half-filled shells with parallel spins, in agreement with Hund’s rules from atomic physics. We conclude that in these systems the spin of the electrons is observable, and furthermore the total spin of the electrons on the quantum dot can be controlled.

While the physics of the few electron quantum dots can well be modeled and understood, there is still a debate especially about the spin physics of many electron quantum dots.

These many-electron quantum dots are normally lateral quantum dots defined by the depletion of the two-dimensional electron gas through top gate electrodes. Usually they have no particular symmetry. Scattering of an electron from the irregular boundaries of such dots leads to single-particle dynamics that are mostly chaotic. Measured quantities such as the dot’s conductance and addition spectrum display random fluctuations when various parameters (e.g. the dot shape or the magnetic field) are varied. We enter the statistical regime, in which new kinds of questions are of interest. For example, rather than trying to calculate the precise, observed sequence of conductance peaks in a specific dot, we can study the statistical properties of the dot’s conductance or its single particle level spacing sampled from different shapes and applied magnetic fields. One method used to describe the statistical fluctuations of a chaotic quantum dot’s conductance peak and level spacing distribution is random matrix theory (chapter 2). The predictions of this theory have been experimentally tested, with different results. Namely, the predictions of random matrix theory for the peak height fluctuations were experimentally confirmed, while on the other hand the nearest neighbor spacing distributions were not. One important result of this distributions was the absence of even/odd structure, expected from the presence of spin. Therefore, spin effects in quantum dots with many electrons have not been experimentally clearly identified.

There were several explanations for this absence of spin signatures in the nearest-neighbor spacings statistics. Among these are electron-electron interactions, which become important in closed dots [8, 9, 10, 11, 12, 13, 14, 15]. Furthermore - related to this - scrambling of the energy spectrum when electrons are added [16, 17]. On the other hand it was proposed that gate-voltage induced shape deformations of the dot can modify the nearest-neighbor spacings distribution as well [18, 19]. In order to test the validity of random matrix theory it is necesserary to understand these effects in more detail:

Quantum dots defined by the depletion of the two-dimensional electron gas through top gate electrodes, are always strongly deformed as the occupation number is tuned. Furthermore the confining potential in these nanostructures is roughly parabolic and the electron densities in these structures are about \( \approx 30\% \) reduced compared to the sheet density of the unconfined two-dimensional electron gas.

In this thesis we show, that it is possible to overcome these problems by defin-
ing quantum dots with AFM Lithography (chapter 4). This is a new technique recently developed in our group. The principle of AFM Lithography is local oxidation of the heterostructure’s surface with an AFM (chapter 3). In this work, we show that the resulting confining potential is up to one order of magnitude steeper than potential walls defined by depletion of the two-dimensional electron gas through top gate electrodes (chapter 5). Furthermore, the electron-electron interactions are reduced in these quantum dots because of the sample geometry (metallic top gate electrode leading to improved screening) and the intrinsic high electron densities of the heterostructures we pattern (chapter 2, 3). We show that reducing shape deformation and electron-electron interactions indeed enables the observation of spin effects. This will be discussed in chapter 6.
Chapter 2

Theory

This chapter is divided into several sections where we discuss the theoretical background of this thesis. Therefore, in section 2.1 we will start by explaining what Coulomb blockade is. First, we will consider systems, where the charging energy is much larger than the confinement energy, followed by a discussion of quantum dots, i.e. systems where the confinement energy is measurably large. Once we have established that Coulomb blockade is an appropriate tool to measure the ground state energy spectrum of a quantum dot, we will discuss what one expects this spectrum to look like. This is the subject of section 2.2 and section 2.3. In Section 2.2 we focus on the description of chaotic quantum dots in a non-interacting picture while in section 2.3 we discuss what the effect of interactions on the quantum dot spectrum are, and we will mention some of the theoretical treatments of interactions. In the last section of this chapter the effects of strong magnetic fields on quantum dots are discussed.

2.1 Single electron tunneling

The concept of Coulomb blockade refers to the phenomenon that tunneling through a metallic grain with small capacitance is inhibited at low enough temperatures and sufficiently small applied voltages. The reason is that the addition of a single electron to such a system requires an electrostatic charging energy $e^2/2C \gg k_B T$, where C is the system's capacitance and T is the temperature.

Basically, this is the explanation first given by Gorter [20] to explain an observed [21, 22] anomalous increase of the resistance of thin granular metallic films with decreasing temperature. The systematic investigation of Coulomb blockade started in 1987 when single charge tunneling was observed for the first time in microfabricated samples by Fulton and Dolan [23]. Meanwhile, a large amount of literature focusing on closed mesoscopic systems and Coulomb blockade exists, and an excellent review about Coulomb blockade has been written by Kouwenhoven et al. [24].
2.1. Single electron tunneling

Figure 2.1: Schematic diagram of a single electron transistor. A metallic island is connected to source and drain via tunnel barriers, and capacitively coupled to a gate electrode. Here $C_g, C_s$, or $C_d$ are the capacitances between the island and the gate, the source or the drain, respectively. The electrochemical potential of the island can be tuned by changing the gate voltage $V_g$. In addition, a bias voltage is applied across the island.

2.1.1 Principle of Coulomb blockade

A suitable model system to investigate the Coulomb blockade in more detail consists of a small metallic island weakly coupled by tunnel barriers to two leads, as depicted in Fig. 2.1. The total charge on the dot consists of two contributions, namely $Q = -Ne$ from the number $N$ of electrons on the dot, and a displacement charge $Q_g$ induced by external potentials, e.g. a gate with potential $V_g$, therefore $Q_g = C_g V_g$. This displacement charge can be varied continuously, while the charge of the $N$ electrons $Q = -Ne$ can only change in discrete units of $e$. The total energy of the island is then given by

$$U(N) = \frac{(C_g V_g - Ne)^2}{2C}$$  \hspace{1cm} (2.1)

Here $C = C_s + C_d + C_g$ is the total capacitance and $C_g, C_s$, or $C_d$ are the capacitances between the island and the gate, the source or the drain, respectively (c.f. Fig. 2.1). The minimum energy needed to add an electron to the island is given by the electrochemical potential $\mu(N) = U(N+1) - U(N)$:

$$\mu(N) = \frac{e^2}{C} \left( N + \frac{1}{2} \right) - eV_g \frac{C_g}{C}$$  \hspace{1cm} (2.2)
At fixed gate voltage, the number of electrons on the dot $N$ is the largest integer for which $\mu(N) < \mu_s \approx \mu_d$, where $\mu_s$ and $\mu_d$ are the electrochemical potentials of source and drain, respectively. When at fixed gate voltage the number of electrons is changed by one, the resulting change in electrochemical potential is

$$\mu(N + 1) - \mu(N) = \frac{e^2}{C} = E_{\text{gap}}$$

(2.3)

The addition energy $\mu(N + 1) - \mu(N)$ is large for small capacitance $C$ and it leads to blockade of tunneling if it is larger than the thermal energy of the electrons in the system. By tuning $\mu(N)$ with the gate voltage $V_g$, $\mu(N)$ can be brought into resonance with the electrochemical potential of source $\mu_s$ and drain $\mu_d$. In this situation, the electrons can tunnel through the dot, one by one. The Coulomb blockade is lifted, and the (zero bias) conductance exhibits a peak. Therefore, the conductance oscillates as a function of the gate voltage. This effect is referred to as “Coulomb-blockade oscillations” (c.f. Fig. 2.2). They have been well studied experimentally and understood theoretically in the past years since their first observation in 1987.

Figure 2.2: Coulomb blockade oscillations as a function of gate voltage as observed with an aluminium single electron transistor. The peak spacing as well as the peak amplitude is independent of $V_g$. (Data taken from Furlan et al. [25]).

Note that $E_{\text{gap}} = \frac{e^2}{C} = e \Delta V_g \alpha$, where $\alpha$ is the “leverarm” which converts the applied gate voltage into the actual change in island energy. The period of these Coulomb blockade oscillations is:

$$\Delta V_g = \frac{E_{\text{gap}}}{e\alpha} = \frac{e}{C\alpha}$$

(2.4)

With $\Delta V_g C_g = e$, we find that

$$\alpha = \frac{C_g}{C}$$

(2.5)

Observation of this charging energy effect requires that the island is sufficiently isolated from the rest of the environment. This means in the experiment, that the
tunnel resistances $R_T$ to the source and drain are both larger than the quantum resistance $R_Q > h/(e^2) = 25.8 k\Omega$.\footnote{To see this, consider the typical time to charge or discharge the island $\Delta t = R_T C$. The Heisenberg uncertainty relation: $\Delta E \Delta t = (e^2/C)R_T C \geq h$ implies that $R_T$ should be much larger than the resistance quantum $R_Q = h/e^2 = 25.813 k\Omega$ in order for the energy uncertainty to be much smaller than the charging energy.}

The source-drain bias voltage can also lift the Coulomb blockade if it is sufficiently high. From electrostatic considerations one finds the following stability condition for $N$ electrons on the island [26].

$$e(N - \frac{1}{2}) \leq C_g V_g - (C_s + \frac{1}{2} C_g)V_{sd} \leq e(N + \frac{1}{2})$$ (2.6)

$$e(N - \frac{1}{2}) \leq C_g V_g + (C_d + \frac{1}{2} C_g)V_{sd} \leq e(N + \frac{1}{2})$$ (2.7)

Consequently, one observes diamond-shaped regions of zero current when the conductance is measured as a function of both the gate voltage and the bias voltage. From the slopes of the boundaries, the individual capacitances $C_s$ and $C_d$ can be calculated.

### 2.1.2 Coulomb blockade oscillations in quantum dots

A quantum dot is a conducting island, with a typical size that is comparable to the Fermi wavelength $\lambda_F$. Usually these quantum dots are artificially defined in a semiconductor material. There are different kinds of quantum dots such as self-assembled quantum dots [27], vertical (e.g. [1, 7]) and lateral quantum dots (e.g. [28, 16, 29]). In vertical (lateral) quantum dots the transport is vertical (lateral) to the two-dimensional electron gas (c.f. section 3.1). The quantum dots studied in this thesis are all lateral quantum dots.

In metallic islands (with a diameter $\geq 20nm$) the Coulomb blockade oscillations are essentially a classical phenomenon since the energy spectrum of the confined region can be considered as a continuum. This is not the case for quantum dots defined in semiconductor nanostructures because the dimension of the confined region may be comparable to the Fermi wavelength. The energy spectrum of such a quantum dot is determined by the interplay of the two energy scales: firstly by the capacitive charging energy $E_c = e^2/2C$, due to the discrete and countable number ($N \approx 1$-- up to a few hundred) of electrons in the system. The second contribution to the energy spectrum comes from the quantum mechanical confinement energy due to spatial localization of the electrons within the system boundaries.

A particle confined in any finite system can occupy only levels with discrete eigenenergies and in spatial eigenfunctions. These eigenenergies and eigenfunctions are solutions of the Schrödinger equation for the particular confining potential. Typical semiconductor quantum dots can effectively be described as a very
small two-dimensional system. The density of states of any two-dimensional system is constant as a function of energy, with an average energy spacing between spin-degenerate levels of

$$\Delta = \frac{2\pi \hbar^2}{m^* A}$$  \hspace{1cm} (2.8)

where $m^*$ is the electron effective mass and $A$ the two-dimensional area of the device. Since, in this approximation the quantum dot effectively treated as a two-dimensional system, this estimation becomes better the more steep the potential walls of the system are and the larger the quantum dot is. The typical dimensions of the quantum dots studied in this thesis vary between 150nm to 450nm. For example in a quantum dot with area $(190nm)^2$, according to eq. 2.8 the spin degenerate level spacing is estimated to $\Delta = 200\mu eV$. Spectroscopic resolution of the single particle spacings requires that the thermal smearing of the Fermi surface is less than this level spacing; $\Delta < kT$. Another condition to resolve the single
particle levels is that the intrinsic level broadening $h\Gamma$ should be much smaller than the level spacings, i.e. the dot should be measured in the weak coupling regime $h\Gamma \ll \Delta$. In the "zero bias" regime an electron can only tunnel into the lowest available quantum level when its Fermi energy is equal to the Coulomb gap $E_{\text{gap}}$ plus the additional quantum energy spacing $\Delta_{N+1}$ between the highest occupied and lowest unoccupied states, $E_F = E_{\text{gap}} + \Delta_{N+1}$, as depicted in Fig. 2.3 (a). In this charge degeneracy situation the highest occupied state in the dot has the same energy as the Fermi energy of source and drain, and therefore an electron can leave the dot (see Fig. 2.3 (b)). Thus, transport can take place via sequential tunneling processes and a current flows through the quantum dot as the system's energy oscillates back and forth between the $N$ and $N + 1$ ground states. Fig. 2.3 (c) shows the quantum dot after tuning the gate voltage out of resonance, where the device is back in the Coulomb blockaded mode.

In the weak coupling regime at low temperatures $\Delta < kT$ and if an infinitesimal source-drain bias is applied across the quantum dot all electron tunneling at a degeneracy point occurs into and out of a single quantum level in the dot.

![Graph](image)

Figure 2.4: Conductance $G$ through a quantum dot as a function of gate voltage. The spacing between two consecutive peaks is not constant. Furthermore the peak heights fluctuate, too.

In Fig. 2.4 a typical conductance through a quantum dot is shown as a function of gate voltage. The conductance is now only quasi-periodic in the gate voltage. In the constant interaction model it is assumed, that $C$ is independent of the number of electrons on the dot, and therefore the contribution of the peak spacing caused by the charging energy $e^2/C$ stays constant when electrons are added. This is a good approximation as long as the screening length is much smaller than the dot size. Therefore, variations in the separation between successive peaks reflect the distribution of the quantum level spacings. Including the single particle energies in the equation 2.4 for the period of the Coulomb oscillations, we get for the distance in gate voltage $\Delta V_g$: 

$$\Delta V_g = \frac{C}{eC_g} \left( \frac{e^2}{C} + \Delta \right)$$
\[ \Delta V_g = \frac{1}{e \cdot \alpha} (E_{gap} + \Delta_{N+1}) \] (2.9)

We note that within the constant interaction model, a spin degeneracy of each level would yield an even-odd structure in the Coulomb blockade peak spacings.

### 2.1.3 Amplitude and lineshape of the Coulomb resonance peaks

In this zero bias regime the conductance of a Coulomb blockade peak is well described by first order resonant tunneling processes. Beenakker [30] calculated this conductance for low temperatures in the limit of negligible intrinsic level broadening \( \hbar \Gamma \to 0 \) for both the quantum Coulomb blockade (\( \hbar \Gamma \ll kT \ll \Delta < e^2/C \)) and the classical Coulomb blockade (\( \hbar \Gamma, \Delta \ll kT < e^2/C \)) regime. In the classical regime, when tunneling occurs via many quantum levels within the smeared Fermi energy window, the lineshape of the conductance has the following form:

\[ G = G_\infty \frac{1}{2} \cosh^{-2}(\frac{\delta}{2.5kT}) \] (2.10)

where \( \delta = eC_g \cdot |V_{g,\text{res}} - V_g| \) measures the distance to the center of the peak in units of energy and \( V_{g,\text{res}} \) is the position of the peak maximum in gate voltage. \( G_\infty \) is the conductance through the quantum dot at high temperatures \( kT \gg e^2/C \).

In the quantum Coulomb blockade regime the tunneling occurs through a single level \( i \). If the partial widths of this level to decay into source or drain are \( \hbar \Gamma^s_i \) and \( \hbar \Gamma^d_i \), then the total width of this level is \( \hbar \Gamma_i = \hbar \Gamma^s_i + \hbar \Gamma^d_i \) and the lineshape reads

\[ G^i = G_{\text{max}}^i \cosh^{-2}(\frac{\delta}{2kT}) \] (2.11)

where,

\[ G_{\text{max}}^i = \frac{e^2}{\hbar} \frac{\pi \Gamma^s_i}{4kT} g_i \] (2.12)

and

\[ g_i = \frac{2}{\bar{\Gamma}} \frac{\Gamma^s_i \Gamma^d_i}{\Gamma^s_i + \Gamma^d_i} \] (2.13)

The lineshape in the classical and quantum regimes are virtually the same, but their widths differ in that the full width at half maximum (FWHM) is 3.5kT in the quantum Coulomb blockade regime, but in the classical regime the FWHM is 4.35kT. On the other hand, a peak maximum \( G_{\text{max}}^i \propto 1/T \) decreases with increasing temperature in the quantum regime, while it is constant in the classical regime. In the low temperature limit the amplitude of a single Coulomb blockade peak is simply modeled as being proportional to the overlap of the wavefunction in
2.2 Energy spectrum of quantum dots within a single particle picture

The spectrum of quantum levels for a single particle in any given potential is an exactly (at least numerically) solvable problem. Schrödinger’s equation describes the behavior of the particle confined in a potential $V(\vec{x})$, and yields a set of allowed eigenenergies and eigenfunctions. For a particle with a de Broglie wavelength comparable to the size of the system, this set of energies forms a discrete ladder. The particle has one of the energies on this ladder and its probability is spatially distributed in the corresponding spatial wavefunction. The time-independent non-relativistic Schrödinger equation reads:

$$\left[ -\frac{\hbar^2}{2m^*} \nabla^2 + V(\vec{x}) \right] \Psi = E\Psi \quad (2.14)$$

where the first term yields the kinetic energy of the particle, the second term is the potential energy of the confinement potential, $\Psi(\vec{x})$ is the eigenfunction and $E$ is the corresponding eigenvalue.

The simplicity of the solution enables powerful calculations of the single-particle spectral properties. One property is the continuous evolution of both wavefunctions and eigenenergies as a function of an external parameter. The confinement potential shape and the magnetic field $B$ both perturb the quantum dot spectrum parametrically. A uniform magnetic field changes the eigenvalues and eigenfunctions symmetrically around $B = 0$ T. Individual levels evolve smoothly as a function of $B$, some ascending in energy and some descending in energy. For a symmetric confining potential (c.f section 2.4.1) this yields a large number of crossings between states with different velocities $dE/dB$. At each crossing the states are degenerate in energy, a direct consequence of the symmetric potential and the resulting symmetric wavefunctions. A shell structure similar to that of atoms exists.

In a disordered or irregularly shaped quantum dot the spectrum of energy levels appears random, due to the underlying quantum chaotic nature. No shell structure exists to impose special energy degeneracies. The wavefunctions themselves lack any spatial symmetry. Instead they extend over the entire confinement volume in a random pattern of nodes and antinodes. As a result, the response of the spectrum to a parametric magnetic field perturbation also appears random and the magnetic field alters the quantum interference pattern randomly. For the description of these chaotic quantum dots a different approach has been pursued: it is called random matrix theory (RMT) and it will be introduced in
the next section. In particular random matrix theory [33] has yielded many valuable statistical results including the distributions of tunneling amplitudes [34], level spacings [35] and the parametric correlation functions for the tunneling amplitudes [36]. In each case the generic quantum properties are different at zero and non-zero magnetic field, and random matrix theory provides results for both cases.

2.2.1 Quantum chaos

Classically, a system is chaotic if small differences in the initial conditions result in differences in the final outcome growing exponentially with time. A stadium billiard (c.f. Fig. 2.5 (b)) is such an example. Two billiard balls starting on nearby paths have exponentially diverging trajectories. For a given system one may look at a bundle of trajectories originating from a narrow cloud of points in phase space. The distance between any two such trajectories grows exponentially with time in the chaotic case. For regular motion in a non-chaotic system, on the other hand, the distance in question may increase like a power of the time, but never exponentially. Fig. 2.5 shows examples for systems whose classical dynamics is integrable (ring) respectively non-integrable (stadium billiard).

![Examples for a classically non-chaotic (a) ring and a classically chaotic (b) stadium cavity. The lines inside the billiards are trajectories of a particle.](image)

When quantum effects are important for a physical system the notion of phase-space looses its meaning. Therefore, when turning from classical chaos to quantum mechanics one looses the classical distinction between regular and chaotic motion. How can we distinguish by quantum mechanical criteria whether a system is regular or chaotic? Several intrinsic quantum mechanical distinction criteria have been developed [31, 32]. One of them is based on the energy spectrum of a quantum mechanical system, namely on the statistics of the energy level spacings. Generic classically integrable systems have levels that tend to cluster and are not prohibited from crossing when a parameter in the Hamiltonian is
varied. Classically nonintegrable systems with their phase spaces dominated by chaos have levels that are correlated such that crossings are strongly avoided. The energy level spacing distribution can be calculated theoretically by using the above mentioned random matrix theory [33].

2.2.2 Random Matrix Theory

In 1950 Wigner and Dyson [37] first proposed random matrix theory as a tool for the statistical analysis of resonance spectra of heavy nuclei. Since then random matrix theory has found applications in many other fields of physics [38]. In mesoscopic physics, the Wigner-Dyson random matrix theory is appropriate for a statistical description of non-interacting chaotic quantum dots. The term “chaotic” is used for ballistic quantum dots whose shape leads to classically chaotic motion.

Random Hamiltonian

A closed quantum dot can be characterized by its energy levels and wavefunctions. The precise value of the level energy and the amplitude of the wavefunction is very sensitive to the impurity configuration or the sample boundaries. Therefore, one usually considers a statistical ensemble of chaotic quantum dots, which have slightly different shapes, Fermi energies, or impurity configurations. The statistical properties of the energy levels and wavefunctions for such an ensemble turn out to be universal. They are independent of the size or shape of the quantum dot, or the impurity concentration, and depend entirely on the basic symmetries of the system: time-reversal symmetry, spin-rotational symmetry, and spatial symmetries. The statistical properties of a chaotic quantum dot are the same as those of a big random hermitian matrix $H$ that has the same symmetries as the microscopic Hamiltonian of the quantum dot. For ballistic dots, numerical evidence for this correspondence was first given by Bohigas et al. [39], while an analytical justification was given by Andreev et al. [40]. The energy levels of the quantum dot correspond to the eigenvalues of the random matrix $H$, and the wavefunction to its eigenvectors.

Wigner originally proposed an ensemble of Hermitian matrices $H$ where all matrix elements are independent Gaussian distributed random numbers. More generally, the matrices $H$ have the following probability distribution:

$$P(H) = c \cdot \exp\left[-\beta \text{tr} V(H)\right], \quad (2.15)$$

where $V$ is a function of $H$. The choice $V(H) \propto H^2$ corresponds to the Gaussian ensemble. $\beta$ is the universality index. Let $E_1, \ldots, E_M$ be the eigenvalues of the $M \times M$ hermitian matrix $H$ and let $U$ be the matrix of its eigenvectors. The matrix $U$ is orthogonal (unitary) for $\beta = 1(2)$. Then we find for the distribution of the eigenvalues $E_j$: 
\[ P(\{E_j\}) \propto \prod_{i>j} |E_i - E_j|^{\beta} \exp[-\beta \sum_j V(E_j)], \quad (2.16) \]

The term multiplying the exponent is the Jacobian due to the change of variables from matrix elements to eigenvalues. The Jacobian yields an effective repulsion between neighboring levels (see Fig. 2.6 (a)). One distinguishes three (\(\beta = 1, 2, \text{ or } 4\)) fundamental symmetry classes, but here we will only discuss two of them, namely:

(I) Systems with time-reversal symmetry, where the symmetry index \(\beta = 1\) and the elements of \(H\) are real and therefore \(H\) is symmetric. (Gaussian orthogonal ensemble: GOE)

(II) Systems in which time-reversal symmetry is broken by a magnetic field are described by complex matrices and the symmetry index is \(\beta = 2\). (Gaussian unitary ensemble: GUE)

Figure 2.6: (a) Wigner-Dyson distribution for GOE and GUE. The correlated levels repel each other, which shows up as a vanishing \(P(s)\) for small \(s\). In contrast to that, the uncorrelated levels of regular systems are Poissonian distributed and have a high probability for \(s=0\). (b) The expected nearest-neighbor spacings for a non-interacting chaotic system (\(B \neq 0\)) where the single particle energies are degenerate (e.g., spin degenerate). The distribution has two peaks, and is therefore called bimodal distribution.
2.2.3 Nearest-neighbor level spacings distribution

From the correlated eigenenergy distribution functions of the Gaussian ensembles in equation 2.16 the nearest-neighbor spacings distribution (NNS) can be calculated [35]. With $s = \frac{\Delta}{\langle \Delta \rangle}$ the result of these calculations for GOE and GUE are:

\begin{equation}
P_{\text{GUE}}(s) = \frac{32}{\pi^2} s^2 \cdot \exp[-\frac{4}{\pi} s^2] \tag{2.17}
\end{equation}

\begin{equation}
P_{\text{GOE}}(s) = \frac{\pi}{2} s \cdot \exp[-\frac{\pi}{4} s^2] \tag{2.18}
\end{equation}

These are the Wigner distributions, they are depicted in Fig. 2.6 (a). Both distributions obey the normalization conditions $\int_0^{\infty} P(s) ds = 1$, and $\int_0^{\infty} s \cdot P(s) ds = 1$. These distributions vanish for small energies, demonstrating that the eigenvalues tend to repel each other. This is different for the nearest-neighbor spacings distribution in integrable systems, where the single particle levels spacings are distributed according to a Poisson distribution. We plot this Poisson function in Fig. 2.6 (a) for comparison.

Furthermore, the standard deviation $\sigma = \sqrt{<s^2> - <s>^2}$ for $P_{\text{GOE}}(s)$ is $\sigma_{\text{GOE}} = \sqrt{\frac{4}{\pi} - 1} = 0.523$, and for $P_{\text{GUE}}(s)$ it is $\sigma_{\text{GUE}} = \sqrt{\frac{3\pi}{8} - 1} = 0.422$.

2.2.4 Nearest-neighbor level spacings distribution in dots

As mentioned in section 2.1.2, in a simple single particle model the spin degeneracy of each level yields an even-odd structure in the Coulomb blockade peak spacings. At even $N$ one expects all quantum dot levels to be doubly occupied, whereas at odd $N$ the highest occupied level is only occupied with one electron. Even $N$ to odd $N+1$ transitions therefore show an energy spacing of $e^2/C + \Delta_{N+1}$, and odd $N+1$ to even $N+2$ show an energy spacing of $e^2/C$ only. For a chaotic dot, the single particle level spacings $\Delta_{N+1}$ are distributed according to eq. 2.18 or eq. 2.17. Including the spin degeneracy one would therefore expect a bimodal distribution of the nearest-neighbor spacings distribution $D_{\text{GXE}}(s) = \frac{1}{2}[\delta(s) + P_{\text{GXE}}(s)]$ (with $X=O,U$), where the $\delta$-function takes the spin degeneracy into account. This theoretically expected nearest-neighbor spacings distribution is plotted in Fig. 2.6 (b). Furthermore, the standard deviation $\sigma = \sqrt{<s^2> - <s>^2}$ for $D_{\text{GOE}}(s)$ is $\sigma_{\text{GOE}} = 0.62$, and for $D_{\text{GUE}}(s)$ it is $\sigma_{\text{GUE}} = 0.58$.

Now let us think about how the nearest-neighbor spacings distribution of a quantum dot according to non-interacting random matrix theory should look like and compare it to the experimentally observed statistics, which we show in Fig. 2.7:

(I) Since the energy levels are spin-degenerate, the nearest-neighbor spacings distribution $D_{\text{GXE}}(s)$ should be bimodal. However in the experiments [8, 41, 28,
Figure 2.7: In four different experiments the nearest-neighbor spacings distribution in quantum dots was determined. In none of them a bimodal structure was found. Rather, the nearest-neighbor spacings were better described with a Gaussian than with a Wigner-Dyson distribution. On the other hand the fluctuations of the level spacings, i.e. the width of the distribution differ between the different experiments.

[42], no bimodal structure can be detected in the statistics. The data look as if spins were absent.

(II) If the spin degeneracy is lifted, in a constant interaction and spin-resolved random matrix theory [28] one would still expect, that the nearest-neighbor spacings distribution has an asymmetric shape, because one can describe the subspaces of $\uparrow$ and $\downarrow$ by two independent random-matrix ensembles. The resulting level spacing distribution is asymmetric and has a non-zero value at $s = 0$. However, the experimentally observed distributions [8, 41, 28], are symmetric and closer to a Gaussian distribution.
(III) Furthermore, one expects from random matrix theory that the peak spacing fluctuations are comparable to the mean single particle level spacing $\Delta/2$ or $\Delta$, depending on whether spin degeneracy is lifted or not. The measured widths of the spacing fluctuations show substantial variation between different experiments. The experiments of Sivan et al. [8] and Simmel et al [41] found $\sigma(\Delta E) \approx (2 - 3)\Delta$, while Patel et al. [28] found smaller fluctuations, comparable to $\Delta$: $\sigma(\Delta E) \approx (0.6 - 0.7)\Delta$

We conclude, that the experimentally obtained nearest-neighbor spacings distributions are usually more symmetric and better described by a *single Gaussian* with enhanced values of $\sigma$ and do not agree with the predictions of random matrix theory. This has triggered tremendous recent theoretical work, and we will come back to this in section 6.1. We just want to point out here, that the random matrix theory approach for quantum dots can only be justified for non-interacting electrons easily, while real electrons are charged, and therefore interact with each other. These electron-electron interactions inside the dot were proposed to be one possible explanation for the failure of random matrix theory. This means that on the one hand, the description of a quantum system with interactions is needed to understand the energy spectrum of quantum dots realized in the experiments. On the other hand, it is necessary to reduce the electron-electron interactions experimentally to observe the nearest-neighbor spacings distribution predicted by random matrix theory.

### 2.3 Interacting quantum systems

#### 2.3.1 Gas parameter: $r_s$

In an electronic system one of the most significant parameter characterizing the particle interactions is the ratio of interaction energy to kinetic particle energy. A high ratio means a system dominated by interactions; a low ratio indicates that interactions may be only a perturbation to the kinetic particle motion. For the quantum electron liquid this ratio is the Coulomb repulsion energy of two electrons at their average spacing $E_{\text{int}}$, divided by the free electron Fermi energy $E_{\text{kin}}$:

$$r_s = \frac{E_{\text{int}}}{E_{\text{kin}}} = \frac{g}{2} \cdot \frac{r}{a_B^*} = \frac{g}{2a_B^*\sqrt{\pi n_s}}$$

where $g$ is the degeneracy of the electronic states in the system, $a_B^*$ is the effective Bohr radius and $n_s$ is the electron density of the system. Note that in our system (where $g = 2$ because of the spin degeneracy) $r_s$ is exactly the average inter-electron spacing in units of the effective Bohr radius $a_B^*$. A high density electron gas has an excess of kinetic energy and $r_s \ll 1$. In this limit interactions are a perturbation to the kinetic motion, and the microscopic random phase approximation (RPA) theory describes the system properties very well. A
low density electron system has low kinetic energy and \( r_s \gg 1 \). In this limit the electrons tend towards a solid lattice or “Wigner crystal”. At intermediate densities \( r_s \approx 1 \), the particle kinetic and potential energies are comparable and neither can be treated as a perturbation to the other.

Since the gas parameter as defined above describes interactions in two-dimensional systems, it is not clear how appropriate \( r_s \) characterizes interactions in quantum dots. Therefore, only as long as the quantum dot under investigation can be regarded as a two-dimensional system, i.e. large dots with steep potential walls \( r_s \) is a good entity to describe the interactions in this system.

### 2.3.2 Many-body theory

The many-body Schrödinger equation for a system of interacting particles is generally intractable, even for only two particles. In the case of interacting electrons, a \( 1/r \) Coulomb term must be added to the potential energy that expresses the mutual Coulomb repulsion between all pairs of electrons. The Schrödinger equation for \( N \) electrons is now written as:

\[
\left[ -\frac{\hbar^2}{2m^*} \sum_i \nabla_i^2 - \sum_i V(r_i) + \frac{1}{2} \sum_{i,j} \frac{e^2}{|r_i - r_j|} \right] \Psi(r_1, r_2, \ldots, r_N) = E \Psi(r_1, r_2, \ldots, r_N)
\]

(2.20)

where the \( 1/r \) Coulomb term is a function of all \( N \) particle coordinates. This linking of the particle coordinates makes the equation inseparable, and therefore various approximation methods must be used to solve this problem. All approximation methods rely on a treatment of the \( 1/r \) term in a manner that allows separation into \( N \) single-particle equations that can be individually, or at least self-consistently, solved. The \( N \)-particle wavefunction is then described as a product of \( N \) independent one-particle wavefunctions. For very small numbers of electrons in an atomic problem, the \( 1/r \) term can be eliminated and then reintroduced using standard perturbation theory techniques and variational calculations. For larger particle numbers or any solid-state problem, the \( 1/r \) term must be replaced with a mean-field potential \( V(r) \) - the average potential seen by a single-electron due to repulsion by all the other electrons. Many different methods and approximations can be used to formulate the mean field potential, \( V(r) \), including the Thomas-Fermi approximation, the self-consistent Hartree-Fock (HF) approximation, the random phase approximation (RPA) and the density functional (DF) method.

**Hartree-Fock approximation**

Here, the interaction term appears in two components, the direct Coulomb interaction and the exchange interaction. The direct Coulomb term is simply the repulsion between electron pairs integrated over their average spatial distributions. The exchange term results from the antisymmetrization requirement, and
2.4 Quantum dots in strong magnetic fields

The quantum dots investigated in this thesis are defined by AFM Lithography (c.f. section 3.2). One of the advantage of this technique is that the resulting potential walls are rather steep. This will be the subject of chapter 5, where we estimate the steepness of these walls by transport measurements of two different quantum dots in strong magnetic fields at filling factors $2 < \nu < 4$. The energy level spectrum of quantum dots with a parabolic confining potential is well described by the Fock-Darwin model, that we will introduce here. Furthermore we will compare qualitatively the predictions of the Fock-Darwin model with those for a hard-wall potential in strong magnetic fields. In the end of this section a self-consistent picture for quantum dots in strong magnetic fields is introduced and the consequences of this picture on the transport properties of a quantum dot in this magnetic field regime are discussed.

2.4.1 The Fock-Darwin model

For a circular dot with a parabolic confinement, characterized by the confining strength $\omega_0$, the energy spectrum is the well-known Fock-Darwin spectrum [44, 45] (c.f. Fig. 2.8 (a)).

$$E_{N,k} = \hbar (N + k) \sqrt{\omega_0^2 + \frac{1}{4} \omega_C^2 + \frac{1}{2} \hbar (N - k - 1) \omega_C}$$

(2.21)

Here, we have transformed the radial quantum number $m$ and the angular momentum quantum number $\ell$ from the standard representation into the Landau level index $N$ ($N=1,2,3,\ldots$) and the level index $k$ of the state within a Landau level ($k=0,1,2,\ldots$), $k = (m + \frac{|\ell + \ell|}{2})$ and $N = (m + \frac{|\ell - \ell|}{2} + 1)$. [46]

Furthermore, $\omega_C$ denotes the cyclotron frequency, and each level is assumed to be two-fold spin-degenerate.
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Figure 2.8: (a) Energy spectrum of a circular disk with parabolic confining potential, with $\hbar \omega_0 = 1\text{meV}$ - the Fock-Darwin spectrum. (b) Section of the energy spectrum at a filling factor of $2 < \nu < 4$. States belonging to LL(1) (solid lines) reduce their energy as $B$ is increased, while LL(2) states (dashed lines) run upwards in energy.

The spectrum becomes regular for magnetic fields in which only two spin-degenerate Landau levels are occupied. We label the spin-degenerate Landau level $N$ as LL($N$), $N = 1, 2$. Consequently, the spin-resolved filling factor $\nu$ is always in the regime $2 < \nu < 4$. A corresponding section of the Fock-Darwin spectrum is shown in Fig. 2.8 (b). We have chosen typical experimental numbers: a dot radius of $r = 200$ nm, and $\hbar \omega_0 = 1$ meV. As the magnetic field is tuned, the Fermi level varies in zigzag lines, representing the transfer of electrons between the two Landau levels. The energies of LL(1)-states drop as $B$ is increased, while those of LL(2)-states increase. A quasi-periodic level crossing between LL(1)-states and LL(2)-states is obtained. Furthermore, the density of states is identical in both Landau levels. For $\omega_0 < 1/2\omega_C$, the separation between adjacent states with identical $N$ can be estimated as $\Delta E_N = E_{N,k+1} - E_{N,k} \approx \hbar \omega_0^2 \omega_C$, and the period in $B$ is approximated to first order by $\Delta B \approx \left(\frac{\omega_0}{\omega_C}\right)^2 B$, as can be seen from eq. 2.21. Note in particular that $\Delta B$ is larger than the “bulk value”, which corresponds to the magnetic field needed to change the number of magnetic flux quanta through the dot area $A = \pi (200nm)^2$ by one, $\Delta B_{bulk} = \frac{\hbar}{eA} = 33$ mT, while $\Delta B = 56mT$ for $B=6T$. Furthermore, the difference in slope between LL(1)-states and LL(2)-states has an upper limit of $\frac{dE_2}{dB} - \frac{dE_1}{dB} = 2\hbar \omega_C / B$. 28
2.4. Quantum dots in strong magnetic fields

2.4.2 The hard-wall potential

The spectrum of a dot with a hard-wall potential is obtained by numerical calculation of the zeroes of the hypergeometric function \( _1F_1 \), and looks quite different [47]: most strikingly, the density of states at the Fermi level in LL(2) is higher than in LL(1), provided the Fermi energy is not far above \( \frac{3}{2} \hbar \omega_c \) (Fig. 2.9 (c),(d)). Second, \( \Delta B \) is well approximated by \( \Delta B_{\text{bulk}} = \frac{\hbar}{eA} \Delta E_N \), however, depends sensitively on \( N, k \) and the magnetic field.

We note that in the model considerations above, we have neglected spins for clarity. Inclusion of the twofold occupation of each orbital state due to spin is necessary for a quantitative comparison of the models with the experimental data later on, and reduces all the above average energy level separations and magnetic field periods by a factor of two.

![Energy spectra](image)

**Figure 2.9:** Sections of the energy spectra for circular dots with a radius of 200nm and a filling factor \( 2 < \nu < 4 \), with a parabolic confinement, \( \hbar \omega_0 = 1 \) meV (a) and a hard-wall confinement (c). States belonging to LL(1) (thin full lines) reduce their energy as \( B \) is increased, while those states belonging to LL(2) (dashed lines) are running upwards in energy. The bold lines represent the Fermi level when the number of electrons in the dot is constant. In a parabolic dot, the DOS at the Fermi level within LL(1) and LL(2) are identical (a,b), while in a hard-wall dot (c,d), the DOS in LL(2) is much larger than in LL(1). In (b) and (d), the full circles indicate occupied energy levels, while open circles represent empty states.

2.4.3 Charge density model for a quantum dot

A complementary approach to describe a quantum dot in the Quantum Hall regime has been proposed by McEuen et al. [49]. They measured the addition spectrum of a dot in a magnetic field at filling factor \( 1 \leq \nu \leq 2 \) and showed, that
Chapter 2. Theory

Fermi energy oscillates in a sawtooth way as a function of magnetic field. At first, the data were interpreted using a Fock-Darwin picture for non-interacting electrons and a constant interaction term to model the charging effects [48]. A reinterpretation of the data showed that this model was inadequate and that the experiment was described better by a self-consistent model. We will introduce this model here qualitatively. For this we assume that we are in the regime of filling factors $2 \leq \nu \leq 4$ where the electrons occupy the lowest two Landau levels (for the moment we neglect spin splitting, and consider only orbital Landau levels). At the position where the Landau levels cross the Fermi energy they are compressible and can screen the confining potential. The resulting self-consistent potential is schematically drawn in Fig. 2.10. In the center the upper Landau Level (LL(2)) is partially occupied and behaves as a metal. In this region the electrons screen the confinement potential, leaving the self-consistent potential flat. In between the two Landau levels at the Fermi energy $E_F$ is a region of width $a_1$ where only the lowest Landau level is completely occupied and there are no states at the Fermi energy. At these positions, the self-consistent potential rises by an amount that corresponds to the energy spacing between the two Landau levels. Near the edges, where the lower Landau level (LL(1)) is partially occupied, there is screening and the self-consistent potential is flat.

Chklovskii et al. [50] calculated quantitatively the distribution of the electron density in a two-dimensional electron gas near the gate-induced edge in a self-consistent potential and found:

$$n(x) = n_0 \sqrt{\frac{x - l}{x + l}} \quad (2.22)$$

where $n_0$ is the density of the two-dimensional electron gas in the bulk of the sample, $x$ is measured from edge and $l$ is half the depletion length $2l$. This depletion length is almost constant as a function of magnetic field, and $l$ is given by

$$l = \frac{V_g e}{\pi n_0 e} \quad (2.23)$$

where $e$ the dielectric constant and $V_g$ is the voltage applied to the gate defining the boundary of the two-dimensional electron gas. In this model it was assumed that if $V_g = 0$ the electron density (being zero under the gate) reaches its bulk value $n_0$ right at the gate edge. In this situation $l$ is zero. By applying a negative potential to the gate, electrons are repelled from it, leaving a depleted strip of width $2l$ behind. This situation is sketched in Fig. 2.10 (c). In this model it was assumed, that the two-dimensional electron gas and the gate belong to the same plane.

Furthermore in this model it was found that the width of the incompressible strip separating LL(1) and LL(2) is given by
2.4. Quantum dots in strong magnetic fields

Figure 2.10: (a) Self-consistent picture of interacting electrons in a quantum dot where only two Landau levels (electron spin is neglected) are occupied. Sketch of the density distribution \( n(x) \) and the corresponding potential with the upwards bent Landau levels. (b) Top view of the quantum dot. Grey areas indicate the metallic regions, while the white ring indicates the incompressible region of width \( a_1 \) formed in the dot. (c) The two-dimensional electron gas (2DEG) formed at the edge of an electrostatic gate at potential \(-V_g\). The depleted region \( 2l \) depends on \( V_g \). The gray bright area indicates the semiconductor host material with a high dielectric constant \( \epsilon \).

\[
a_1 = \sqrt{\frac{2\hbar \omega_c \epsilon}{\pi^2 e^2 dn/dx \mid x=x_1}}
\]  

(2.24)

where \( x_1 \) is the center of the incompressible strip separating LL(1) and LL(2). From 2.24 it follows, that the width of the strip depends strongly on the initial slope in the density distribution. Electrostatic considerations have established a relation between the potential steepness and the depletion length [51].
2.4.4 Transport through a quantum dot in high magnetic fields

If the spin splitting can be neglected in the charge density model a quantum dot in the regime of filling factors $2 \leq \nu \leq 4$ can be described as a metallic ring located concentrically around a metallic disc. Electrons added to the dot are added to one of these two metallic regions. If the insulating strip is wide enough, tunneling between the two metallic regions is minimal; they will effectively act as two independent electron gases. The charge is separately quantized on each LL. Not only is the total number $N$ of electrons in the dot an integer, but also the numbers of electrons $n_1$ in LL(1) and $n_2$ in LL(2) are integers. Effectively, we have a two-dot, or dot-in-dot model of the system. This schematic picture of a quantum dot in high magnetic fields is is plotted in Fig. 2.10 (b).

As the magnetic field $B$ increases, the electrostatic potential of LL(2) rises and that of the first LL(1) drops. Therefore, the energy for adding an electron to the first Landau level $\mu_1(n_1, n_2)$ and hence the peak position, decreases with increasing $B$. This continues until it becomes energetically favorable for an electron to move from the second to the first Landau level. This electron redistribution causes the electrostatic potential of the first Landau level to jump from $\mu_1(n_1, n_2)$ to $\mu_1(N_1 + 1, n_2)$, while the total number of electrons in the dot $N = n_1 + n_2$ is constant. The energy difference for this charge rearrangement $\mu_1(n_1, n_2) - \mu_1(n_1 + 1, n_2)$ is equal to the interaction energy between LL(1) and LL(2) minus the single particle energy in LL(1).

The peak heights can be explained similarly. The peak amplitude for adding the $n$th electron is strongly suppressed at $B$ fields where it is energetically favorable to add the electron to the LL(2), since the coupling of the inner disk to the leads is strongly suppressed. This corresponds to the magnetic field range where the peak position (if observable) is rising. On the other hand, if it is energetically favorable to add an electron to the outer LL(1), the peak amplitude is high since the outer ring couples well to the leads. The energetic position of the corresponding Coulomb peak drops with increasing magnetic fields.
Chapter 3

Sample fabrication and measurement setup

In this chapter, we explain how we fabricate and measure quantum dots. In the first section we introduce the gallium-arsenide/aluminium-gallium-arsenide (=GaAs/Al\textsubscript{x}Ga\textsubscript{(1-x)}As) material system, and explain how very high quality two-dimensional electron gas can be realized. For the definition of nanostructures in this material system, we use a rather new technique which was invented in our group and is called local oxidation with an atomic force microscope (AFM) or AFM Lithography. This technique and its properties will be the subject of section 3.2. The tunability of these nanostructures can be enhanced by an additional top gate, as will be shown in section 3.3. In the last section the measurement setup is discussed.

3.1 The two-dimensional electron gas

Semiconductor nanostructures have become model systems for the investigation of electrical conduction on short length scales. This development was made possible by the availability of semiconducting materials of high purity and crystalline perfection. Such materials can be structured to contain a thin layer of highly mobile electrons - a two-dimensional electron gas. These systems combine desirable properties, e.g. a low and tunable electron density, and therefore large Fermi wavelength ($\lambda_F \approx 30\text{nm} - 40\text{nm}$). Such a two-dimensional electron gas can be realized for example in the GaAs/Al\textsubscript{x}Ga\textsubscript{(1-x)}As material system. All experiments presented here have been performed on heterostructures of this type.

3.1.1 The Ga[Al]As system

The GaAs/Al\textsubscript{x}Ga\textsubscript{(1-x)}As ($x = 0.3$) heterostructures (Ga[Al]As) discussed here are grown by molecular beam epitaxy. The heterostructure consists of several superimposed layers of GaAs and Al\textsubscript{x}Ga\textsubscript{(1-x)}As as shown in Fig. 3.1. The two-
dimensional electron gas forms at the interface between the GaAs substrate and an undoped $Al_xGa_{(1-x)}As$ spacer layer. An n-type dopant is added, either as a single layer (δ-doping) or mixed with a layer of $Al_xGa_{(1-x)}As$ (modulation doping) [52].

A second layer of undoped $Al_xGa_{(1-x)}As$ and a GaAs cap layer (to prevent oxidation of the $Al_xGa_{(1-x)}As$) completes the heterostructure. By varying the thickness of the spacer layer, the properties of the two-dimensional electron gas such as mobility and electron density, can be tuned over a wide range.

Figure 3.1: Layer structure of the sample used in our work. The concentration of aluminum is $x = 0.3$.

Figure 3.2: Simulation of the band-structure for the sample drawn in Fig. 3.1. The simulation was done with a self consistent Poisson-Schrödinger solver [53]

Fig. 3.1 shows the layer sequence of the heterostructure. It has a $Al_xGa_{(1-x)}As$ layer which holds a δ-donor layer located 17nm below the surface, resulting in a two-dimensional electron gas 34nm below the surface. The conduction electrons supplied by the donors are confined in a narrow potential well (c.f. Fig. 3.2), which is nearly triangular and is formed by the repulsive barrier due to the conduction band offset between GaAs and $Al_xGa_{(1-x)}As$ of approximately 0.3 eV and by the attractive electrostatic potential due to the positively charged donors in the doping layer. The motion of the electrons in the potential well is quantized perpendicular to the interface, but is free parallel to the interface. This results in the formation of two-dimensional subbands in the well. Usually, only the subband associated with the lowest discrete confinement level is populated. Because of the close match in lattice parameters between GaAs and $Al_xGa_{(1-x)}As$ (≈ 0.7% mismatch), the interface is only slightly strained and can be essentially defect free. Combined with the spatial separation of the two-dimensional electron
gas from the ionized donors, the few defects lead to a very low elastic scattering rate. The resulting high electron mobility $\mu_e \approx 10^5 - 10^6 \text{cm}^2/\text{Vs}$ and long mean free path $l_e \approx 1 - 10 \mu\text{m}$ allow the fabrication of nanostructures, i.e. quantum dots, in which electrons travel ballistically and scatter predominantly at the device boundaries.

The high quality samples used in this thesis were grown by W. Wegscheider and M. Bichler. These shallow heterostructures have a large sheet density of $n_s = 4 - 6.3 \cdot 10^{15} \text{m}^{-2}$ depending on the applied top gate voltage (see section 3.3.2). This high electron density enables us to define quantum dots with reduced electron-electron interactions. The mobility is $92 \text{m}^2/\text{Vs}$ at a temperature of $T=0.1$ K. Furthermore, it has turned out that these shallow heterostructures are crucial for the AFM lithography technique which we discuss in the next section.

### 3.2 AFM Lithography

All the devices presented in this thesis have been defined by a novel lithography technique called AFM Lithography. The principle of this technique is local oxidation (LO) of semiconductor (e.g. Ga[Al]As heterostructures) or metal surfaces with an atomic force microscope, and it is explained in the first part of this section. In the second part, we will discuss the advantage of this patterning technique that are especially important for the study of quantum dots, namely, small depletion lengths and high specularity of the boundary scattering.

#### 3.2.1 AFM setup

Anodic oxidation is used in industrial processes to oxidize large areas of semiconductor or metal surfaces [54, 55]. The general principle of local oxidation with an AFM is essentially identical (Fig.3.3). Here, the water film forming under ambient conditions on top of the substrate provides the electrolyte. A conductive AFM tip acts as cathode, while the chip to be nanostructured is grounded. However, the electrochemical processes are possibly quite different from those taking place in conventional anodic oxidation. For a discussion of this issue, see Refs. [56, 57, 58, 59]. The AFM we use is a commercial device (Accurex II, Topometrix), equipped with a standard, conductive (boron-doped) Si cantilever for non-contact operation, with a spring constant of $20 \text{N/m}$. The tips are not sharpened. The coarse lateral structure of the sample is pre-patterned by optical lithography. A Hall bar is defined by wet chemical etching, and Ni-AuGe Ohmic contacts are alloyed into the heterostructure. At the end the patterned sample is glued and bonded into a chip carrier. Hence, the oxidation can be controlled in situ, by measuring the resistance of the AFM induced barriers. A more detailed description of the sample preparation can be found in the thesis of Ryan Held who pioneered this technique in our group [60].
Figure 3.3: (a): Scheme for conventional anodic oxidation: a negative voltage applied between cathode and anode (material A), immersed in an electrolyte (e.g., water), leads to oxidation of the anode surface, as indicated by the reaction equation. (b): Scheme for local oxidation on a Ga[Al]As heterostructures with an AFM. Here, the voltage is applied between the AFM tip and the electron gas, resulting in oxidation of the GaAs cap layer, and depletion of the electron gas underneath. The electrolyte is formed by the water film present under ambient conditions.

The AFM resides in a sealed chamber with a nitrogen atmosphere whose humidity (typically 40% to 50%) is controlled via a feedback loop. The pattern to be exposed is aligned with respect to the writing field, and with a home-made software.

### 3.2.2 Local oxidation of Ga[Al]As heterostructures

Ishii et al. [61] were the first ones to oxidize directly the GaAs cap layer of a Ga[Al]As heterostructure. They observed an increase in the resistance in the two-dimensional electron gas below the oxidized area. Recently, Held et al. [62] demonstrated that the two-dimensional electron gas can actually be depleted by local oxidation of the cap layer, provided the electron gas is sufficiently close to the surface, i.e. the distance between surface and two-dimensional electron gas is smaller than $\approx 50\,nm$.

The pattern to be exposed is aligned with respect to the writing field. With a home-made software, the tip is negatively biased (typical voltages vary between -20V and -27V) and scanned across the sample. The tip-sample interaction is kept constant with a standard AFM feedback loop for tapping mode operation [63]. Typical scan speeds are or the order of 1$\mu m/s$. As a result, the sample surface oxidizes in close vicinity to the AFM tip, yielding oxide lines of typically 100nm in width and 8nm to 10nm in height, which reflects the increased volume of the oxides as compared to GaAs (Fig. 3.4). Laterally insulating lines allow
3.2. AFM Lithography

Figure 3.4: Surface topography (left) and height profile (right) of the heterostructure surface after patterning of two oxide lines, which appear as bright lines. The lines have a typical width of 100nm and a height of 8 - 10nm. In this sample, the oxide lines separate the electron gas into a quantum wire with a lithographic width of 150nm, and two planar gate electrodes, pg1 and pg2.

Lateral gate voltages to be applied in the different insulating regions of the two-dimensional electron gas. This is what we call in-plane gates. Such in-plane gate structures have been first fabricated by bombardment with focused ions introduced by Wieck et al. [64]. The big advantage of the lateral gates is that they can be combined with top gates. Hence, by scanning the biased tip over the surface, it is possible to define such in-plane gate structures, by directly oxidizing (and therefore depleting the two-dimensional electron gas underneath) the GaAs surface locally.

Depletion of the two-dimensional electron gas by surface oxidation

We find that the two-dimensional electron gas is depleted underneath the oxide lines in shallow HEMT structures for temperatures below 77K, again provided that the distance between two-dimensional electron gas and sample surface is smaller than about 50nm. The underlying mechanism can be understood in a simple picture: as the cap layer is oxidized, the semiconductor surface gets closer to the two-dimensional electron gas, while the surface area, and thus the number of surface states, is slightly increased. In these shallow two-dimensional electron gases it is well known that only ≈ 10% of the donor electrons from the doping layer go into the two-dimensional electron gas, while the remaining 90% fill the surface states. A small reduction of the distance between surface and the two-dimensional electron gas, changes the internal electric fields and can lead to depletion. By selectively removing the semiconductor oxide with a HCl dip etch, we have found that the presence of the oxide is irrelevant for the depletion. From this point of view, LO is very similar to a carefully tuned, shallow chemical etch of the cap layer [65]. Simulations with a 1-dimensional Poisson-Schroedinger solver confirm this picture ([60]).
3.2.3 Depletion length and specularity

For many experiments it is essential to transfer the lithographic pattern with high accuracy and without loss of electron mobility into the two-dimensional electron gas. Hence, the lateral depletion length $l$ and the specularity of the scattering at the boundaries are key parameters of the lithographic technique.

The depletion length of the oxide lines has been determined by Held et al. [66], by fabricating a set of quantum wires of different lithographic widths. Their electronic width was determined from magnetoresistivity measurements and by using the fact, that the magneto-oscillations are not periodic in $1/B$ as in a homogeneous two-dimensional electron gas, but rather contain information on the lateral electrostatic confinement as well. Using a model by Berggren et al. [67] the measured number of occupied one-dimensional subbands can be fitted as a function of $1/B$, and by this the electronic wire width $w_e$ can be determined. Out of this analysis they find that the depletion length is $l=(15 \pm 5)$nm, for oxide lines with a height of 8nm-10nm and with the top gate grounded. The quantum wires investigated in this analysis were defined in heterostructures with the two-dimensional electron gas 37nm below the surface. The depletion length depends on the height of the oxide lines and on the layer sequence in the heterostructures. We found in samples with the two-dimensional electron gas 34nm below the surface that the depletion length for oxide lines of 18nm height is $\approx 45$nm. This is just an estimate of the depletion length from the lithographic width of the quantum point contacts of $l \approx 90$nm and the fact that we had to apply $+100$ mV at the top gate to open the point contacts. However, it is clear that the closer the two-dimensional electron gas and the oxidized region are, the larger are the depletion lengths.

Furthermore the scattering at the boundaries of structures defined by LO is very specular.[60] The specularity was found to be comparable to specularities in wires defined by top gates, but significantly larger than in wires written by ion beam lithography [68].

Local oxidation on semiconductor heterostructures is a powerful tool for fabricating tunable nanostructures. Besides the extremely high quality of the electronic confinement in terms of lateral depletion length and specularity, there are other advantages [60, 69], namely easy definition of self-aligned gate structures, modification and repair of the oxidized patterns, simple performance of exposure tests, and alignment of nanostructured top gates with respect to nanostructured in-plane gates with an accuracy in the nm range.
3.3 Combination of in-plane gates with top gates

In order to enhance the tunability of an in-plane gate nanostructure, we cover the whole nanostructure with a homogeneous Ti/Au top gate electrode. Fig. 3.5 shows the current-voltage characteristics of the two-dimensional electron gas across the oxide line for various top gate voltages $V_{tg}$, which tune the height of the tunnel barrier with a lever arm of $dE/dV_{tg}= 0.025$ eV/V, as estimated from the top gate voltage needed to deplete the unpatterned two-dimensional electron gas. This tunability is used in the experiment to adjust the conductance through the quantum point contacts of the quantum dots.

![Figure 3.5: Current-voltage characteristics of a tunable tunnel barrier, patterned by writing an oxide line which is covered by a top gate electrode (inset). The breakdown voltage can be tuned with the top gate voltage $V_{tg}$. For $V_{tg} \geq 500mV$, the tunnel barrier is removed (see text).](image)

The breakdown voltage in Fig. 3.5 vanishes for $V_{tg} = 500mV$, hence we estimate the barrier height to 12.5 meV, a value consistent with temperature dependent data. Thus, operation of devices at liquid nitrogen temperature is possible, but not at room temperature for the experimental parameters used. Note that in Fig. 3.5, the current remains zero for positive bias voltages if $V_{tg} \leq -220mV$. In this regime, the voltage between top gate and source depletes the two-dimensional electron gas on this side.

Furthermore, the top-gate that covers the whole nanostructure, provides screening and therefore reduces the e-e- interactions, which is a very important parameter in the system, as pointed out in section 2.3.1.

3.3.1 Effect of the top gate on the gas parameter

In section 2.3.1, we introduced the gas parameter $r_s = \frac{E_{tal}}{E_{kin}}$ which is a measure for the e-e-interaction of a system. Now let us estimate in a simple picture how
the top gate influences $r_s$. This is illustrated in Fig. 3.6:

![Figure 3.6: Sketch of the screening situation. Two neighboring electrons produce image charges in the top gate, which is $d = 34\text{nm}$ away from the two-dimensional electron gas.](image)

The electron gas lies $d = 34\text{nm}$ below the surface, which is covered by the top gate. In a two-dimensional electron gas with a sheet density of $n_s = 5.9 \cdot 10^{15}\text{m}^{-2}$ the average separation between two electrons is $r = \frac{1}{\sqrt{\pi n_s}} = 7.3\text{nm}$. In the top gate image charges build up, reducing the effective Coulomb energy between the electrons in the following way:

$$E_{\text{int}}^{\text{scr}} = \frac{e^2}{4\pi \epsilon \epsilon_0} \left( \frac{1}{r} - \frac{1}{2d} - \frac{1}{\sqrt{r^2 + 4d^2}} \right)$$

(3.1)

Therefore we estimate the screened gas parameter of the order of $r_s^{\text{scr}} = E_{\text{int}}^{\text{scr}}/E_{\text{kin}} = 0.56$, which is significantly smaller than the unscreened gas parameter $r_s = 0.72$. In the following we will use the unscreened gas parameter, but are aware of the fact that the e-e interactions are further reduced due to screening by the top gate.

### 3.3.2 Sheet density as a function of top gate voltage

Since the nanostructures discussed here have been measured with voltages applied to the top gate, we have to consider the effect of these top gate voltages on the sheet electron density. We determined this density in different ways and compare the results in Fig. 3.7. We measured the density by Hall effect and by the Shubnikov-de Haas (SdH) oscillations [70]. In addition we estimated it in a parallel plate capacitor model and by a numerical simulation of the bandstructure.

It can be clearly seen that the measured density is not a linear function of the top gate. When comparing the slope $\frac{dn_s}{dV_{tg}}$ calculated in the parallel plate capacitor model, we find that it is larger than the experimentally measured slope. The much more dramatic reduction in the slope at voltages above $V_{tg} = 75\text{mV}$ implies that another effect occurs in that regime. The occupation of a second subband in the confinement along the $z$-axis can be ruled out since the Hall effect measurement should show the combined density of the two subbands and therefore increase further for high top gate voltages. We used a bandstructure simulation program by G. Snider [53] to calculate the electron density as a function of top gate voltage.
3.4. Experimental setup

Experimental resolution of individual peaks in the tunneling density of states requires that the smearing of the Fermi surface is smaller than the lowest energy scale separating adjacent peaks. The two relevant energy scales in the quantum dot spectrum are the charging energy $E_c > 1K$ and the energy spacing between single particle levels $\Delta \approx 100mK$. All electrical measurements were therefore performed at subkelvin temperatures using a $\text{He}_3/\text{He}$-dilution refrigerator [72] with a base temperature of $90mK$ to allow resolution of the individual zero-dimensional quantum states. A 0-15 Tesla superconducting magnet provided a continuously tunable magnetic field perpendicular to the plane of the two-dimensional electron gas. A more detailed description of the system can be found in Ref. [60].

In the Coulomb blockade tunneling regime, the resistance of the device is $R \geq 100k\Omega$ on a Coulomb blockade tunneling peak and $R \geq 10G\Omega$ between Coulomb blockade peaks. The average ohmic contact resistance was $1 - 2k\Omega$. Therefore, a two-terminal voltage bias configuration was the normal measurement mode. Fig. 3.8 shows a schematic measurement configuration.

The dc source-drain bias was generated by a Yokogawa (7651) voltage source. In order not to broaden the Coulomb blockade resonances, the dc-voltage bias should be as small as possible, i.e. $V_{\text{bias}} < kT$. The current was amplified through
Chapter 3. Sample fabrication and measurement setup

Figure 3.8: Schematic measurement circuit. A dc voltage is applied between source and drain. Voltages ($V_{qpc}$) are applied to the point contact gates to “fine”-tune the dot in the CB regime. The current is measured as a function of the in-plane gate voltage $V_{gl}$ or $V_{gll}$. Not shown in this scheme is the additional top gate, that covers the whole sample, where also voltages are applied. The black box is a symbol for the low pass filter, that filters the HF-noise.

Figure 3.9: Example of a Coulomb blockade resonance (o), that is only thermally broadened and the fit (line) for the corresponding lineshape. The FWHM of this trace is 3.5kT in energy.

an Ithaco current preamplifier. The voltage signal was fed directly to a HP voltmeter (34491A) and the data from the HP was collected via the instrument.
3.4. Experimental setup

GPIB computer interfaces. All signals were passed through a special low pass filter built into all leads running to and from the sample. With this setup we are able to measure with a current resolution of 500fA.

By applying gate voltages to the different point contact in-plane gates with different Yokogawa voltage sources, the quantum dot can be tuned in the Coulomb blockade regime. Discrete “zero”-bias Coulomb blockade conductance peaks with lineshapes that closely fit equation 2.11, were generally found in the low coupling regime where the conductance in the Coulomb resonances is \( g_{max} \leq 0.2e^2/h \). If not stated otherwise, the measurements presented here were performed in this weak coupling regime, where the resonance width is only thermally broadened. The electron temperature can be determined by fitting a single Coulomb blockade resonance to the lineshape expected for coherent single-level transport, see equation 2.11. In Fig. 3.9 we show such a fit. We determine an electron temperature of \( T_e = 100 - 120mK \), depending on the cooldown cycle.
Chapter 4

Quantum dots defined by AFM Lithography

In this section we report how we realize quantum dots in a Ga[Al]As heterostructure by LO with an AFM. Furthermore we explain in this chapter how we characterize the dot and extract the important parameters. At the end of the chapter we present a table where characteristics of the quantum dots studied here are listed.

In recent years, a variety of fabrication methods for tunable semiconductor quantum dots has been reported, each having its particular strengths and weaknesses. The most common scheme consists of patterning metallic top gate electrodes by electron beam lithography. By applying negative voltages to these gates, the quantum dot is induced in the electron gas. Its shape and electron density as well as its coupling to the leads can be tuned over wide ranges. The disadvantage of this method is that due to the large lateral depletion lengths of the order of 100nm, the dot shape deviates significantly from the top gate pattern. Furthermore, changing the number of electrons inside the dot changes its shape as well, which is reflected experimentally in gate-voltage dependent capacitances. Patterning the dot and in-plane gate electrodes by wet chemical etching results in quite similar advantages and disadvantages [73, 74]. Other schemes of fabricating quantum dots are focused laser beam-induced doping [75], and focused Ga ion beam implantation in combination with top gates [76]. Here, the lateral depletion is smaller. However, it is known that scattering at edges defined by such implantation techniques is highly diffusive [68], thus reducing, or even destroying, ballistic transport through the dot. With the properties of nanostructures defined by AFM Lithography, like small depletion lengths and highly specular scattering it is possible to overcome some of the problems mentioned above.
4.1 Fabrication

Fig. 4.1 shows the layout of a single electron transistor defined by LO with an AFM as explained in section 3.2.1. The sample was later covered with a top gate. Applying voltages to the top gate changes not only the occupation number of the dot, but also its coupling to the leads, as we will discuss now.

![Diagram showing the layout of a single electron transistor](image)

Figure 4.1: (a) Scanning force micrograph of the oxide pattern that defines the quantum dot RO (see text), which is coupled to source and drain via small gaps in the oxide lines. The dot potential can be tuned using the electrodes IPG1 or IPG2. The picture shows the sample before the top gate metallization. The dashed line indicates the line of the cross section scheme in (b). The oxide lines deplete the electron gas underneath and separate the dot from IPG1 and IPG2. The sample is homogeneously covered by a Ti/Au top gate.

4.1.1 Tuning the point contacts

Usually, in lateral semiconductor quantum dots the tunnel barriers are defined by tunable quantum point contacts. This is also the case in the quantum dots described here.

A quantum point contact [77, 78] is a short and narrow constriction in a two-dimensional electron gas, with a width of the order of the Fermi wavelength $\lambda_F$. The conductance of a quantum point contact is quantized as a function of the channel width in units of $2e^2/h$. In order to observe Coulomb blockade in a semiconductor quantum dot, both point contacts have to be pinched off. This means that the electronic width $W$ of each quantum point contact has to be smaller than $\lambda_F/2$ in order that no mode can propagate through the constriction.
In terms of resistance this means that in a quantum dot the tunnel resistance $R_T$ of each quantum point contact has to be larger than $R_Q/2 = h/2e^2 = 12.9k\Omega$.

The quantum point contacts in our quantum dots are the narrow constrictions separating the dot from source and drain. They are indicated by a white circle in Fig. 4.2 (a). As explained in section 3.3, one advantage of in-plane gate defined nanostructures is that a top gate can be added. We observe that the oxide lines still have good isolation properties after the evaporation of the top gate. In addition, the I-V characteristics across the oxide lines can be tuned by the top-gate voltages. Therefore, also the conductance through the quantum dot can be tuned by this top gate. In fact the dependence on the top gate voltage is of course much stronger than that on the point contact gate voltage. In other words, in the experiment the coarse tuning is made via the top gate, while the fine tuning is done by the lateral point contact gates.

![Quantum dot diagram](image)

**Figure 4.2:** The quantum point contact of the quantum dot are marked by a circle in (a). (b) Conductance $G$ of a quantum dot as a function of point contact gate voltage $V_{IPC1}$, taken at a temperature of $T = 4.2K$. The different traces were taken for stepwise (10mV) increasing top gate voltage from $-150mV$ (bottom trace) to $100mV$ (uppermost trace).

In this section we study two different realizations of such quantum dots, which are RCL and RO. In the first type the dot is separated from source and drain via two highly insulating tunnel barriers when the top gate is grounded. The tunnel barriers are defined by a gap of about $20nm$ in the two oxide lines that separate the dot from source and drain. By applying positive voltages to the top gate, the tunnel barriers can be opened. We call this dot in the following RCL. In the second type the dot is coupled to source and drain via open quantum point contacts when the top gate is grounded. This is achieved by keeping the gaps in the oxide lines at $\approx 50nm$ (Fig. 4.1). Here, negative top gate voltages close the
4.1. Fabrication

quantum point contacts. We call this quantum dot in the following RO.

Fig. 4.2 (b) shows the conductance traces (where we subtracted a constant background conductance), of the type RO quantum dot with the top-gate voltage increasing from $-150\text{mV}$ (bottom trace) to $100\text{mV}$ (uppermost trace). One observes that the conductance increases in steps of about $0.5 \cdot G_0$ ($G_0 = 2e^2/h$).

The conductance of two point contacts in series in zero magnetic field is given by [79]:

$$G_{\text{series}} = \frac{1}{2} [N \cdot G_0 + \frac{(2e^2)}{h} T_d]$$

(4.1)

where $N$ is the number of quantum channels in the point contacts and $T_d$ is the probability for direct transmission through the two point contacts. An analysis of our data shows that $T_d$ in our quantum dot is less than 0.1, as one would expect in devices with sufficiently large point contact separation $L \gg W$ where $W$ is the point contact width [79]. This means that the series conductance of the two point contacts of the quantum dot can be approximately calculated by ohmic addition of the individual resistivities of the two point contacts, as we observe it in Fig. 4.2 (b) The gradual increase of the conductance step for $N = 2$ can be explained by an increase of the background conductance when the top gate voltage is increased. Hence, the dot can be tuned into pinch-off with both the top gate and the IPG1, as stated above.

4.1.2 Coulomb blockade oscillations in quantum dots defined by local oxidation

Once the quantum point contacts are pinched-off, clear Coulomb blockade of the electron transport is observed. Furthermore, by applying voltages to the in-plane gates, the occupation number of the dot can be tuned over wide ranges by leaving its shape basically unchanged. In dot RCL a current through it can be detected for rather high top gate voltages of $360\text{mV} \leq V_{tg} \leq 410\text{mV}$. In this regime the tuning range for IPG2 corresponds to about 100 electrons (Fig. 4.4 (c)), while the tuning range of IPG1 is only about 20 electrons (Fig. 4.4 (b)), which reflects the different influence of these gates on the tunnel barriers.

In Fig. 4.3, we show the conductance through the RO dot. In addition to the Coulomb oscillations, one observes well-known superimposed transmission resonances [80]. In the case of this open dot, we have to apply $V_{tg} = -130\text{mV}$ to close the quantum point contacts. It is obvious that, in this case, the Coulomb period increases with $V_{IPG2}$. In contrast, we observe in dot RCL, where we apply a positive top gate voltage, an approximately constant period of about $4.2\text{mV}$ over wide ranges of $V_{IPG2}$. This different behavior is visualized in the inset of Fig. 4.3, where the Coulomb peak separation as a function of $V_{IPG2}$ for both dot types is shown. We interpret the constant peak separation in dot RCL (open circles) as a consequence of the huge carrier density due to the positive top gate
Figure 4.3: Conductance $\sigma$ through the RO dot. Transmission resonances are observed superimposed on the Coulomb oscillations. In the inset we plot the periods of the conductance oscillations $\delta V_g$ as a function of $V_{IPG2}$ obtained for the different dot types. The open circles represent the period of the peaks in the RCL dot, which remains almost constant over the whole voltage range. For comparison, the periods in $V_{IPG2}$, measured in dot RO (under negative top gate voltages) are represented by the full circles. Here, the period increases significantly with increasing $V_{IPG2}$, since the capacitance between dot and gate changes.

Voltage. Therefore, the electric fields from the in-plane gates are screened very effectively and the dot keeps its shape over the whole $V_{IPG2}$ range. This is not the case in the RO dot (full circles), where a negative top gate voltage is applied, which reduces the electron density in the system, especially the voltage difference between the IPG2 and the top gate gets high.

4.2 Characterization

Now let us discuss how the quantum dots are characterized. As an example, we explain this for dot RCL:

In a first step we look at the capacitances of the different gates to the dot by measuring the period of the Coulomb blockade oscillations as a function of the respective gates. In Fig. 4.4, conductance measurements on dot RCL are shown and in table 4.1 the extracted period in the different gate voltages and the corresponding capacitances $C_g = e/\Delta V_g$ are listed.

The Coulomb charging energy of the quantum dot can directly be determined by measuring the Coulomb gap in the I-V traces. From a set of I-V measurements
4.2. Characterization

Figure 4.4: Conductance $\sigma$ through the RCL dot as a function of the top gate voltage (a), the voltage applied to IPG1 (b) and IPG2 (c).

<table>
<thead>
<tr>
<th>gate</th>
<th>$\Delta V_{\text{gate}}$</th>
<th>$C_{\text{gate}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPG1</td>
<td>4.5mV</td>
<td>$C_{\text{IPG1}}=36\text{aF}$</td>
</tr>
<tr>
<td>IPG2</td>
<td>4.2mV</td>
<td>$C_{\text{IPG2}}=38\text{aF}$</td>
</tr>
<tr>
<td>tg</td>
<td>0.28mV</td>
<td>$C_{\text{tg}}=580\text{aF}$</td>
</tr>
</tbody>
</table>

Table 4.1: List of the periods for the different gates and the corresponding capacitances $C_{\text{g}} = e/\Delta V_{\text{g}}$ for dot RCL.

over several Coulomb oscillation periods, we find a Coulomb gap of $E_c = e/C = 150\mu\text{eV}$ in dot RCL, which means that the self capacitance of the dot is $C = e/E_c = 1070\text{aF}$. Knowing the two capacitances $C$ and $C_{\text{g}}$ we can calculate the leverarm of the system (c.f. eq. 2.5) and find $\alpha = \frac{C_{\text{g}}}{C} = 0.035$.

The capacitances of source and drain cannot be determined in the simple way sketched in the introduction, because of the high capacitance of the top gate to the dot. But we estimate the capacitances by $C_s \approx C_d = \frac{1}{2}(C - C_{\text{tg}} - C_{\text{IPG1}} - C_{\text{IPG2}}) \approx 210\text{aF}$.

The size of the dot can be estimated from the capacitance between top gate and dot $C_{\text{tg}} = 580\text{aF}$ within a parallel plate capacitor. Using the distance
between top gate and the two-dimensional electron gas as the separation between the plates, and a dielectric constant of \( \varepsilon = 13 \), we find an electronic dot area of \( A = 1.7 \cdot 10^{-13} \text{m}^2 = (414 \text{nm})^2 \). The lithographic size of the dot is \( A_{\text{litho}} = 410 \text{nm} \cdot 390 \text{nm} \). The fact that the size determined in the above model is not smaller indicates that the lateral depletion length is probably even less than 20nm at the oxide lines, if about 400mV are applied to the top gate. This is in agreement with the investigations on the depletion lengths (c.f. section 3.2.3) of nanostructures defined by local oxidation. Hence, the size and shape of our quantum dot resemble much more closely the lithographic pattern than dots defined solely by top gates.

Another important energy in a quantum dot system is the spin degenerate single particle energy spacing \( \Delta \). Using the definition of \( \Delta \) given in equation 2.8, we find \( \Delta = 42 \mu \text{eV} \) for dot RCL.

<table>
<thead>
<tr>
<th>property</th>
<th>RCL</th>
<th>RS</th>
<th>stadium (*)</th>
<th>qring</th>
</tr>
</thead>
<tbody>
<tr>
<td>lithographic A</td>
<td>410nm - 390nm</td>
<td>240nm - 360nm</td>
<td>( \approx \pi 620 \text{nm} \cdot 155 \text{nm} )</td>
<td>(515nm)^2</td>
</tr>
<tr>
<td>( \Delta V_{\text{eg}} )</td>
<td>0.28mV</td>
<td>-</td>
<td>0.504mV</td>
<td>0.28mV</td>
</tr>
<tr>
<td>( C_{\text{eg}} )</td>
<td>580aF</td>
<td>-</td>
<td>320aF</td>
<td>580aF</td>
</tr>
<tr>
<td>electronic A</td>
<td>(414nm)^2</td>
<td>(190nm)^2</td>
<td>(311nm)^2</td>
<td>(414nm)^2</td>
</tr>
<tr>
<td>sheet density ( n_a )</td>
<td>5.9 \cdot 10^{15} m^{-2}</td>
<td>5.9 \cdot 10^{15} m^{-2}</td>
<td>5.0 \cdot 10^{15} m^{-2}</td>
<td>5.5 \cdot 10^{15} m^{-2}</td>
</tr>
<tr>
<td>red. ( n_s ) in dot</td>
<td>5.1 \cdot 10^{15} m^{-2}</td>
<td>4.5 \cdot 10^{15} m^{-2}</td>
<td>5.8 \cdot 10^{15} m^{-2}</td>
<td>4.5 \cdot 10^{15} m^{-2}</td>
</tr>
<tr>
<td>number of e-</td>
<td>870</td>
<td>200</td>
<td>370</td>
<td>810</td>
</tr>
<tr>
<td>( E_c )</td>
<td>0.15mV</td>
<td>1.2mV</td>
<td>0.37mV</td>
<td>0.15mV</td>
</tr>
<tr>
<td>( C )</td>
<td>1070aF</td>
<td>130aF</td>
<td>430aF</td>
<td>1070aF</td>
</tr>
<tr>
<td>( \Delta V_{\text{g}} )</td>
<td>4.2mV</td>
<td>10.9mV</td>
<td>[50 ± 10] mV</td>
<td>(7.8 ± 0.5)mV</td>
</tr>
<tr>
<td>( C_{\text{g}} )</td>
<td>38aF</td>
<td>15aF</td>
<td>(2.5 - 4)aF</td>
<td>(18 - 22) aF</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>42\mu eV</td>
<td>200\mu eV</td>
<td>70\mu eV</td>
<td>42\mu eV</td>
</tr>
</tbody>
</table>

Table 4.2: AFM pictures and list of the characteristics of all four types of quantum dots discussed in this thesis. Values where '-' stands, could not be determined for experimental reasons. (*): The stadium dot, was defined in a different heterostructure sample than the other dots, namely a rectangular quantum well structure, therefore it has a different density. Furthermore in this sample the two-dimensional electron gas is 35nm away form the surface.

4.3 Properties of other quantum dots

Beside the quantum dot presented in the preceding chapter we will present here three further quantum dots. They will be described in the next chapters. In
from the properties observed in dot RCL we conclude, that quantum dots fabricated by LO have the following characteristics: (i) They are highly tunable by in-plane as well as by top gate electrodes over wide ranges. (ii) They have a constant capacitance between the tuning gate IPG2 and the dot. The second point indicates that shape deformation is small in these dots. This is an essential property in order to investigate the level spacing statistics in quantum dots and this will be important in chapter 6. Furthermore we have seen, that the transfer of the lithographic pattern to the electron gas is excellent. This is caused by operating the quantum dots with positive top gate voltages and by the small depletion lengths of the oxide lines. In chapter 5 we will discuss the consequences of these short depletion lengths on the transport in high magnetic fields.
Chapter 5

Hard wall quantum dots in strong magnetic fields

As discussed in section 2.4, it is well established that in quantum dots in the regime of filling factors \(2 < \nu < 4\), the conductance shows periodic patterns over wide ranges of magnetic fields and gate voltages. Furthermore, it has been demonstrated that the energy level spectrum of the dot can be reconstructed from such measurements, which yield information on the confining potential.[49] We therefore characterize and compare in this section dot RCL and dot RS by magnetotransport experiments in this regime of filling factors. By comparing the extracted energy spectra with the models described in section 2.4, we find that the potential walls in these quantum dots are steeper than parabolic.

5.1 Experiment

The difference between dot RCL and dot RS are firstly the electronic size (c. f. section 4.3 and Fig. 5.1) and secondly the oxidation depth of the defining oxide lines, which leads to different shapes of the confining potential. In dot RCL the height of the oxide lines is 10nm. Furthermore, the dot was measured with a voltage of 400mV applied to the top gate and therefore the depletion length is expected to be negligibly small. In dot RS, on the other hand, the height of the oxide lines is 20nm which leads to a rather high (for LO) depletion length of 50nm. This is illustrated in Fig. 5.1 (c) and (d). The voltage applied to the top gate in this sample was 200mV.

From measurements in a magnetic field around filling factors \(2 \leq \nu \leq 4\), we are able to estimate the steepness of the confining potential for both dot types. The essential idea is to detect deviations from a parabolic confinement via magnetotransport experiments which are used to map out the dots’ addition spectra, from which the energy levels are reconstructed. For small Landau level filling factors, a parabolic confinement and a hard-wall confinement generate quite different energy spectra. In particular, at a suitable magnetic field, the density of
states (DOS) in the second Landau level will be higher than in the first Landau level in a quantum dot with steep walls (see Fig. 2.9).

![Figure 5.1: Surface topography of the two dots (the small dot RS, (a), and the large dot RCL (b)) under study. Both dots are tuned by a voltage applied to planar gate I. The gates labelled “qpc” are used to adjust the coupling to source and drain. Schematic cross sections through dot RS and dot RCL are shown in (c) and (d), respectively, including the top gate. Sketched are also the oxide lines (white ovals) and the resulting potential profile. The two-dimensional electron gas is 34 nm below the surface. The conductance $G$ of dots RS and RCL at $2 \leq \nu \leq 4$ as a function of magnetic field and $V_I$ are shown in (e) and (f), respectively, in a gray scale plot. The data are taken at a temperature of 90 mK. Both dots show a large conductance (bright areas) only if a LL(1)-state aligns with the Fermi level in the leads (see text).]
Fig. 5.1 (e) and (f) compare the Coulomb blockade measurements for dot RS and dot RCL at high magnetic field. The small dot (Fig. 5.1 (e)) shows structures similar to those reported in earlier experiments [48, 49]: as the magnetic field is changed, the Coulomb blockade resonances move in zigzag lines. Their average separation in gate voltage corresponds to one Coulomb blockade oscillation period. In regions where the levels move downwards in energy, their amplitude is high, while in regions where they move upwards in energy, their amplitude is strongly suppressed. Regions of high conductance occur when a state belonging to LL(1) aligns with the Fermi level in source and drain. As the magnetic field increases, their energy is reduced. On the other hand, states belonging to LL(2) will move upwards in energy as B is increased, leading to their depopulation. Since the LL(2) states are residing in the inner region of the dot (Fig. 2.9 (d)), their coupling to the leads is small, which results in a strongly suppressed peak amplitude.

A similar measurement for dot RCL, Fig. 5.1 (f), reveals a different structure. The Coulomb blockade period at B = 0 in this sample is ΔVf = 4.2 mV, see Fig. 5.3 (a). In contrast to dot RS, the separation between successive stripes of high conductance in Vf - direction does not correspond to one Coulomb blockade period at B = 0, but rather to 3.5 Coulomb blockade periods on average. This striking suppression of Coulomb blockade peaks at high magnetic field, can be seen in Fig. 5.3 where a plunger gate sweep at B = 0T in (a) is compared to a sweep at B ≈ 8T in (b). In the corresponding phase diagram it is not straightforward to see how the conductive LL(1) states are connected by LL(2) states, i.e., how the zigzag lines run. We will discuss this point in detail in section 5.2.3. Furthermore, there is an overlap between adjacent regions of high conductance along the Vf - direction, which we attribute to thermal activation, since in dot RCL, the level separation is comparable to kBT.

5.2 Reconstruction of the energy spectra

In this section, we construct the energy level spectra of the two dots from the measurements of Figs. 5.1 (e) and (f). While this is straightforward for dot RS, it requires a more detailed understanding of the addition spectrum measured for dot RCL, which we gather from activated transport experiments. The spectra obtained will be compared to the Fock-Darwin and the Hard-wall potentials, with the spin splitting included.

5.2.1 Energy spectrum of the small dot RS

In Fig. 5.2 (a), the occupation numbers of LL(1) and LL(2) and the corresponding "phase diagram" are compared to the measurement. Here, a phase is given by (n1, n2), where ni denotes the number of electrons in LL(i). The gaps between
5.2. Reconstruction of the energy spectra

Figure 5.2: (a) Sketched phase diagram of the small dot, as an overlay on the data of Fig. 5.1 (e). Each phase is given by \((n_1, n_2)\), where \(n_i\) denotes the number of electrons in LL(i). If \(n_2\) changes, the conductance remains zero, due to the poor coupling to the leads. A change in \(n_1\) results in a high conductance. (b) Reconstruction of the energy level spectrum. We find the level spacings \(\Delta_{1}^{RS} = 310 \mu\text{eV}, \Delta_{2}^{RS} \approx 400 \mu\text{eV}\). (c) Phase diagram of the large dot, as an overlay on the data of Fig. 5.1 (f). The phases have the same meaning as in (a). See text on how they were determined. The dark, dashed lines indicate excited LL(1) states. The capital letters and the bold circle are explained in Fig. 5.5. (d): Reconstruction of the energy level spectrum. The level spacings \(\Delta_{1}^{RCL} = 160 \mu\text{eV}, \text{ and } \Delta_{2}^{RCL} \approx 60 \mu\text{eV}\) are found.

the zigzag lines in energy (gate voltage) direction correspond to the charging energy \(e^2/C\), plus the separation between the single-particle energy levels inside the dot. In Fig. 5.2 (b) we subtracted \(E_{C}^{RS}\) in (a), and obtain bright lines, which correspond to the magnetic field dependence of adjacent LL(1) states. The slightly alternating separations reflect the Zeeman splitting. An average spacing
between neighboring LL(i) - states of $\Delta_{1}^{RS} = 310 \mu eV$, and $\Delta_{2}^{RS} \approx 400 \mu eV$ is extracted. Note that since the LL(2) states are not directly visible, their position can only be guessed from the point where the bright lines corresponding to the LL(1) states overlap, and $\Delta_{2}^{RS}$ can be no more than a rough estimate.

The Fock-Darwin model in the limit of strong magnetic fields states that $\Delta_{1}^{S} = \Delta_{2}^{S} \approx \frac{\hbar \omega_{c}}{2}$ (the factor of $\frac{1}{2}$ takes the spin degeneracy into account), from which we obtain $\hbar \omega_{0} \approx 2.75 \text{ meV}$, an at first sight reasonable, although large value. Further analysis, however, reveals problems with the interpretation in terms of a Fock-Darwin model:

(i) Since $r = 90 \text{ nm}$, we can calculate $E_{F}$ inside the dot from $E_{F} = \frac{1}{2} m^{*} \omega_{c}^{2} r^{2}$, which would result in $E_{F} = 27 \text{ meV}$, larger than in the two-dimensional electron gas.

(ii) We observe $\Delta B = 75 \text{ mT}$ (c.f. Fig. 5.2 (a) and (b)). Within the model in strong magnetic fields, however, $\Delta B \approx \frac{1}{2} \left(\frac{\omega_{c}}{\hbar}\right)^{2} B \approx 180 \text{ mT}$, is expected.

(iii) The slope of the energy levels, $\frac{dE_{i}}{dB}$ (i denotes the LL index) deviates from the Fock-Darwin values: here, $\frac{dE_{x}}{dB} - \frac{dE_{y}}{dB} \approx 2\hbar \omega_{c}/B$, as observed experimentally in previous work.[48] However, we estimate $\frac{dE_{x}}{dB} - \frac{dE_{y}}{dB} \approx 5\hbar \omega_{c}/B$, which is much to large, even considering the experimental uncertainty in $\frac{dE_{x}}{dB}$.

These inconsistencies can be significantly reduced by assuming a hard-wall potential. Then the observed $\Delta B \approx 75 \text{ mT}$, in reasonable agreement with the $\Delta B_{\text{bulk}} \approx 80 \text{ mT}$ expected for $r = 90 \text{ nm}$. Also, both the energy level separation and $\frac{dE_{x}}{dB} - \frac{dE_{y}}{dB}$ can be much larger than in a parabolic dot.

Hence, although the behavior of dot RS is in qualitative agreement with a parabolic confinement, a quantitative analysis suggests that its confinement resembles a hard-wall potential.

5.2.2 Transport through the large dot RCL

In Fig. 5.3 we plotted Coulomb blockade oscillations in dot RCL as a function of $V_{T}$ at zero magnetic field and at strong magnetic field $B=8.2T$. We find that the separation of neighboring (double) peaks in strong magnetic field is $\Delta V_{T} = 15mV$, and is therefore on average 3.5 times larger than at zero magnetic field.

Why does the period in $V_{T}$ of the conducting regions in dot RCL not correspond to the Coulomb blockade oscillation period at $B=0T$, as it does in dot RS? We explain this by the electrostatic substructure that builds up in the dot at this magnetic field (as discussed in section 2.4.3). Since the magnetic field does not change the size of the dot dramatically, the period in gate voltage should stay roughly constant. But because in this magnetic field regime the dot is split up into two metallic regions of different coupling to the leads, the transfer processes
5.2. Reconstruction of the energy spectra

Figure 5.3: Coulomb blockade oscillations as a function of $V_f$ at (a) $B=0$T and around (b) $B=8.2$T. The average peak spacing at zero magnetic field is $\Delta V_f = 4.2mV$, while in strong magnetic field they are separated by $\Delta V_f = 15mV$.

Figure 5.4: Coulomb diamonds of the large dot, measured over 40 Coulomb blockade periods, in a gray scale plot. For each completely visible Coulomb diamond, two to four diamonds are suppressed. In these regimes of suppressed conductance, the Coulomb blockade oscillations can be observed only under sufficiently large source-drain bias voltages ($|V_{sd}| > 200\mu V$). This effect results in the formation of “meta-diamonds” and corresponds to the energy necessary to carry current via excited LL(1) - states.
into LL(2) cannot be observed (as explained in section 2.4.4). We find, that the conductance is suppressed for 70% of the Coulomb blockade resonances corresponding to a ratio between suppressed peaks and visible peaks of about 2.5:1. The conductance doublets in $V_f$-direction, Fig. 5.1 (f), remain visible at small $|V_{sd}|$. These observations lead us to conclude that the density of LL(2) states at the Fermi level is about 2.5 times the density of LL(1) states. LL(2) states get occupied when aligned with the Fermi energy, but since their coupling to the leads is negligible, transport via these states is not measurable, even at large $V_{sd}$. Rather, transport occurs via excitation of LL(1) states. This is also visible in the Coulomb diamonds, i.e. the conductance as a function of both $V_f$ and the source-drain voltage $V_{sd}$, at $B = 8T$ (Fig. 5.4). The shape of the diamonds with suppressed conductance tends to follow the shape of the nearest diamond in which the conductance outside the Coulomb blockade is not suppressed, which indicates that the activated current flows predominantly via LL(1) states.

Furthermore, at large bias voltages, i.e. for $|V_{sd}| > 200\mu V$, an average Coulomb blockade oscillation period of $\Delta V_f = e/C_f = 3.5$ mV is observed, slightly smaller than $\Delta V_f$ observed at $B=0$.

### 5.2.3 Energy spectrum of the large dot RCL

With this information, we are now able to reconstruct the energy level spectrum of dot RCL by subtracting the $E_c$ from the addition spectrum of Fig. 5.1 (f). The phase diagram is shown in Fig. 5.2 (c). Here, we have determined the phase boundaries due to LL(2) states from the activated transport measurements of Fig. 5.4, concluding that we must cross $\approx 2.5$ LL(2)-states on average between adjacent LL(1)-states. The zigzag lines obtained are again separated by one Coulomb period in $V_f$, and it can be seen that the different structure originates from the fact that most states at the Fermi level couple extremely poorly to the leads.

Performing the same procedure as in section 5.2.3, we reconstruct the energy spectrum of dot RCL, Fig. 5.2 (d). An energy level separation between LL(1) states of $\Delta_{1}^{RCL} = 160 \mu eV$, and $\Delta_{2}^{RCL} = 60 \mu eV$ for LL(2) is found. Hence, the density of LL(2) states at the Fermi level is 2.5 times larger than the density of LL(1) states, which indicates a significant deviation from a Fock-Darwin potential. In addition, note that $\Delta B = 13 mT = \frac{1}{2} \Delta B_{bulk}$. This is expected for a hard-wall dot, since for each additional flux quantum, two LL(1) states are generated. We thus conclude that the energy spectrum of dot RCL deviates even qualitatively from the Fock-Darwin model, but agrees well with a hard-wall confinement.
5.3 Hard walls and the charge density model

The energy spectrum of RCL suggests that certain aspects of the energy spectrum of quantum dots cannot be understood in the above picture. It has rather to be interpreted in terms of a charge-density model, as described in section 2.4.3. This is particularly true when strong magnetic fields are present, [49, 29] and therefore raises the question whether our observations hint towards steep walls also within a charge-density model. To answer this issue, we study the depletion of dot RCL along the vertical arrow in Fig. 5.2 (c), i.e., as we proceed from point A to point E.

![Figure 5.5: Energies $E_1$ and $E_2$ of LL(1) and LL(2) as the large dot is depleted along the arrow A to E in Fig. 5.2 (c), in the single-particle picture (a). Within the charge density model, the dot develops an internal electrostatic structure (b). A compressible ring of LL(1) states at the Fermi level is separated by an incompressible region (white) from the compressible region formed by LL(2) states in the center of the dot, and an additional capacitance forms inside the dot. Here, $w_1$ and $a_1$ denote the width of the outer compressible region and of the incompressible region, respectively. (c): Energies of the core and the ring in the charge density model as the dot is depleted.](image)

For comparison, we first look at the depletion process within the single-particle picture. The corresponding changes in the energies $E_i$ of LL(i) are depicted in Fig. 5.5 (a). At our starting point A, we have removed an electron from LL(1) into the reservoirs. At point B, a LL(2)-electron is transferred into the leads. This, however, does not lead to significant conductance, since the coupling of the LL(2)-states to the leads is poor. In point B, the energy of LL(2) drops by $\Delta_2^{RCL} + E_C$, and that one of LL(1) by just $E_C$. The same process takes place at points...
C and D, with the corresponding energy shifts. At point E, the next LL(1)-electron is removed from the dot. Consequently, 3 electrons have been removed from LL(2) and one from LL(1). In order to repeat this process cyclically, $\Delta_1^{\text{RCL}}$ has to be larger than $\Delta_2^{\text{RCL}}$. At increased source-drain bias voltages, we would start to see activated transport via nearby LL(1)-states at some points between A and E. The value of $V_f$ which such processes require is the smallest activation energy given by the points where the black, dashed lines intersect with the arrow. The separation between the phase boundary parallel to the arrow and the arrow is a measure of the activation energy necessary. These varying activation energies give rise to the observed "meta-diamonds" in Fig. 5.4.

Within a charge-density model, the measured phase diagram gets a different interpretation [49, 50, 81]. For filling factors $2 \leq \nu \leq 4$, LL(1) forms a compressible ring at the dot edge, Fig. 5.5 (b). At the Fermi level, LL(2) is spatially separated from this ring by an incompressible region and forms a compressible disk in the dot center; $n_i$ in Fig. 5.2 (c) now denotes the number of electrons in the outer ring ($i=1$) and in the center disc ($i=2$). Here, we assume that spin splitting does not generate an additional electrostatic structure. The two compressible regions are coupled via an intra-dot capacitance $C_{12}$; charging the outer ring will now change the electrostatic energy of the inner dot by $E_{12} = e^2/C_{12}$. If we perform a gate sweep similar to the one indicated by the arrow in Fig. 5.2 (b), a removal of an electron from the outer ring would change the energy of the ring by $E_{c,1}$, and the one of the inner dot by $E_{12}$.[29] Similarly, removing an electron from the inner dot changes its energy by $E_{c,2}$, and that one of the ring by $E_{12}$, which gives rise to the energy ladder depicted in Fig. 5.5 (c). Within this picture, $E_{c,i}$, $i=1,2$ reflect the single electron charging energies of the two compressible regions, and hence the size, of the outer ring and the inner dot, respectively. Therefore, in this picture we see that, $E_{c,1} > E_{c,2}$ and the area of the ring must be smaller than the area of the inner dot. This means that the width of the outer compressible strip, $w_1$, is small compared to the dot radius.

### 5.4 Estimation of the wall steepness

While we observe clear signatures of a confinement that is steeper than parabolic, it is not straightforward to determine the actual steepness of the walls. We can give no more than an order of magnitude estimate for the edge steepness $\frac{dE}{dx}$. First we consider dot RS. According to Sivan and Imry [82], one can approximate $\Delta_1$ in a hard-wall dot by $\Delta_1 \approx \hbar \omega_C \ell_B / L$, where $\ell_B$ denotes the magnetic length and $L$ the dot diameter. Our measurements of $\Delta_1$ are in good agreement with this expression, which indicates that the level separation is not strongly reduced by a finite wall steepness. We can estimate the steepness at the Fermi level in dot RS from the Fock-Darwin model, despite its shortcomings, to $\frac{dE}{dx} = m^* \omega_0^2 r$, which gives $\frac{dE}{dx} \approx 0.6 \text{ meV/nm}$.
Furthermore, the charge-density model can be quantified in some more detail for dot RCL. Since $E_{12}$ corresponds to the separation between adjacent conductive regions in Fig. 5.2 (c), we estimate $C_{12} \approx 60aF$. We denote the width of the outermost compressible stripe by $w_1$, and the width of the incompressible stripe as $a_1$ (see Fig. 5.5 (b)). The ratio between the area of the center region and the outer ring is about 2.5, since we have 2.5 times more levels in the inner region than in the outer ring. We use the formula for a planar capacitor[81] to estimate the ratio $\frac{w_1}{a_1} \approx \frac{1}{4}\exp(C_{12}/4e\varepsilon_0r) \approx 0.5$, leading to $w_1 \approx 25\text{ nm}$ and $a_1 \approx 50\text{ nm}$. $\hbar\omega_c$ drops completely over the distance $a_1$. Hence, we can estimate $\left.\frac{dE}{dx}\right|_L \approx \frac{\hbar\omega_c}{a_1} \approx 280\mu\text{eV/nm}$. This is the average wall steepness. The edge of dot RCL is thus almost one order of magnitude steeper than in conventional dots, as estimated from self-consistent charge-density functional calculations.[49] Since the lateral depletion length is smaller in dot RCL than in dot RS, one might qualitatively expect that the wall steepness is larger. This, however, is not clearly supported by our data, but the two numbers can not be compared directly since in the case of the small dot we estimated the steepness at the Fermi energy, while in the large dot we estimated the average steepness.

However, we conclude from this reasoning, that the potential shape at the boundary of an oxide line resembles more a hard-wall potential than a parabolic potential. The deviations from a parabolic confinement become more pronounced as the dot size is increased. The average wall steepness is of the order of 0.3 meV/nm.

5.5 Conclusions

We have shown that the quantum dots studied here have indeed steep confinement walls. This wall steepness is caused by (i) the patterning method (AFM lithography), (ii) the shallow and high density two-dimensional electron gas used and (iii) the top gate, that covers the whole nanostructure.

The option of patterning quantum dots (and other nanostructures) with steep walls by lithographic means is of interest from both a technological as well as from a physical point of view. Steeper confinement is a prerequisite for higher pattern densities. Also, changes in size and shape are reduced as a gate voltage is tuned. This is of particular importance for statistical properties of quantum dots [18] and will be discussed in the next chapter. Another consequence of steep walls is, for example, only a small reduction of the Fermi energy in quantum dots as compared to the bulk value, this is important in order to fabricate nanostructures with reduced interactions, and will also be addressed in the following chapter.
Chapter 6

Spin pairing in chaotic quantum dots

6.1 Earlier theoretical and experimental work

As we already pointed out in section 2.2.4 the predictions of random matrix theory for the nearest-neighbor spacings distribution in quantum dots have not been confirmed, i.e. in the experiments one observes no bimodal structure in the statistics. The data look as if spins were absent.

In the absence of spin degeneracy one would still expect, that the nearest-neighbor spacings distribution has a Wigner-Dyson shape, or that it is at least asymmetric. However, the experimentally observed distributions [8, 41, 28], are symmetric and closer to a Gaussian distribution.

Furthermore, one expects that the peak spacing fluctuations are comparable to the mean single particle spacing, that is $\Delta/2$ or $\Delta$ depending on whether spin degeneracy is lifted or not. The measured width of the spacing fluctuations shows substantial variation between different experiments. The experiments of Sivan et al. [8] and Simmel et al. [41] found $\sigma(\Delta E) \approx 2 - 3\Delta$, while Patel et al. [28] found smaller fluctuations, comparable to $\Delta$: $\sigma(\Delta E) \approx 0.6 - 0.7\Delta$. This disagreement of the predictions of random matrix theory and the experimentally obtained nearest-neighbor spacings distributions have triggered tremendous recent theoretical work. There are several effects which can lead to deviations from simple constant interaction and random matrix theory predictions, which we want to summarize here.

6.1.1 Shape deformations by gate voltage changes

It was pointed out by Vallejos et al. [19] that deviations from a Wigner-Dyson distribution may be due to shape deformation of the dot as the gate voltage changes. At gate voltage $V_g^N$, corresponding to the Nth conductance peak, the shape of the dot is $x_N$. At the degeneracy point $V_g^{N+1}$ of the
6.1. Earlier theoretical and experimental work

next peak, the shape of the dot has changed to $x_{N+1}$. The measured energy spacing (with the constant charging energy $e^2/C$ subtracted) is now given by

$$\Delta E = E_{N+1}(x_{N+1}) - E_N(x_N) = E_{N+1}(x_{N+1}) - E_N(x_{N+1}) + E_N(x_{N+1}) - E_N(x_N),$$

where $E_j(x)$ are the single-particle energies of the dot with shape $x$. Here $\delta_N(x_{N+1}) = E_{N+1}(x_{N+1}) - E_N(x_{N+1})$ denotes the spacing between successive levels in a dot with a fixed shape $x_{N+1}$, while $\delta x_N(N) = E_N(x_{N+1}) - E_N(x_N)$ denotes the parametric variation of the $N$th level as the shape of the dot changes between peaks. The $\delta(x_{N+1})$ are distributed according to random matrix theory. If $\delta x_N(N)$ fluctuates on a scale of the order or larger than the mean single particle level spacing, the measured spacings $\Delta E$ are not Wigner-Dyson distributed. Vallejos et al. showed in this case that the distribution becomes more symmetric and is better described by a Gaussian distribution. Therefore, the shape deformation induced by a variation of the gate voltage is one possible explanation for the symmetric shape of the nearest-neighbor spacings distribution. Within this picture one can also understand that if the fluctuations of $\delta x_N(N)$ are large enough this effect can also be responsible for the absence of bimodal structure in the nearest-neighbor spacings statistics.

6.1.2 Spectral scrambling

The single-particle spectrum is expected to change with the addition of electrons not only because of changes in the dots shape, but also, because of electron-electron interactions that lead to charge rearrangements on the dot, and therefore to a scrambling of the energy spectrum. This has been experimentally tested by Patel et al. [17], who investigated the temperature dependence of Coulomb blockade peak height correlation. In the absence of scrambling, theoretically one would expect that the number of correlated peaks $n_e(T)$ is equal to $k_B T/\Delta$, which is roughly the number of quantum levels that contribute to a Coulomb blockade peak. This peak-to-peak correlations $n_e(T)$ should be sensitive to a changing spectrum. If the spectrum were fixed, the number of correlated peaks $n_e(T)$ would increase approximately linearly with $T$. However, if the addition of each electron changes the spectrum, then $n_e(T)$ is expected to saturate at a certain value $m$ that measures the number of added electrons required to scramble the spectrum completely. These expectations are confirmed both experimentally and theoretically. Patel et al. investigated quantum dots with different sizes and found that in the smallest device, saturation occurs at $m=3$-4 already for $T \geq 0.5\Delta$, while for the larger dots, $n_e$ continues to increase with temperature. The same values for $m$ were obtained by Stewart et al. [16] who investigated the correlation between addition and excitation spectra in similar quantum dots.

The size dependence of $m$ suggests that spectral scrambling is reduced in larger dots. This is in agreement with theoretical results [15], where $m \propto g^z$ (with $z=0.5$ or 1) is found.
Chapter 6. Spin pairing in chaotic quantum dots

Since $g \propto \sqrt{N}$ one expects the spectrum of either larger dots or dots with higher electron densities to be less sensitive to the addition of electrons. The interaction parameter in the quantum dots studied in the above experiment was $1.2 < r_s < 1.5$, thus, it should be possible to enhance $m$ by reducing $r_s$ as much as possible.

6.1.3 Electron-electron interactions

Blanter et al. [10] studied the influence of electron-electron interactions (e-e interactions) on the statistics of Coulomb blockade peak spacings. Therefore they used the random phase approximation (RPA) and assumed a disordered dot in the metallic regime, but they claim that their results should also hold in the ballistic quantum Coulomb blockade regime. They account for e-e interaction by introducing the effective single-particle Hamiltonian $H_N$ which depends explicitly on the number of excess electrons $N$. The first empty state is separated from the last filled one by a gap of order $e^2/C$. The Hamiltonian $H_N$ is essentially random for each $N$. They assumed that the statistical properties of its single-particle excited states are the same as that of single electron states in a random potential, in particular they obey random matrix theory. The distance between two adjacent peaks can be decomposed in the following way:

$$E_N = (\mu_N^{N+2} - \mu_N^{N+2}) + (\mu_N^{N+2} - \mu_N^{N+1}) = E_1 + E_2$$ (6.1)

Here $\mu_j^j$ is the energy of single-particle eigenstate $j$ of the Hamiltonian $H_j$. Both quantities $E_1$ and $E_2$ are random. The statistical properties of the latter one are trivial, since it is just the distance between two adjacent single-particle levels of the same random Hamiltonian $H_N$. Hence, they obey random matrix theory; in particular, the average $< E_2 > = \Delta$, while the fluctuations are those predicted by random matrix theory.

The quantity $E_1$ is the shift of the $(N+2)$th single-particle level due to the addition of a new electron to the $(N+1)$th particle system and its average is $< E_1 > \approx e^2/C$, with negligible corrections. Thus, the average spacing between the Coulomb blockade peaks is $\Delta + e^2/C$ in agreement with the constant interaction model. Furthermore they claimed that the fluctuations of the quantity $E_1$ are proportional to $\Delta/\sqrt{g}$, where $g$ is again the ballistic Thouless conductance. This means, that $\sigma(E_1)$ depends on $\Delta$, and on the interaction strength since $r_s^{-1} \propto g$. In this discussion the spin of the electrons was neglected. Spin effects will be included in the next subsection.

RPA breaks down for strong interactions. Sivan et al. [8] used an Anderson model of a disordered dot with electron-electron interactions to calculate numer-

---

\(^1\text{Here } g \text{ is the ballistic Thouless conductance. It is defined in quantum dots as the ratio of the Thouless energy } E_T = h/t_f \text{ to the level spacing. Here, } t_f = L/v_F, \text{ is the time an electron needs to pass through the quantum dot of size } L \text{ - the time of flight. Therefore, the dimensionless conductance is } g = h/(t_f \Delta) \approx h v_F/(L \Delta).\)
6.2. Experiment

The calculations can be done only for a very small number of electrons (much fewer than in the experiments). When $r_s$ increases from zero, the spacing distribution deviates from the Wigner-Dyson distribution and becomes approximately a Gaussian for $r_s \geq 1$. Furthermore, the fluctuations increase with $r_s$, and for $r_s \gg 1$ they find $\sigma(E) = (0.1 - 0.2)e^2/C$.

6.1.4 Spin effects

Including spin in the RPA discussion of Blanter et al. [10] leads to two states with the same energy but different values of spin; their eigenfunctions are identical. If the state state is occupied by an electron, the energy of the $\downarrow$ state is shifted by $\xi$. This shift is the amount of interaction energy the $\downarrow$ electron has to "pay" to occupy the same orbital state as the $\uparrow$ electron occupies. $\xi$ is given by:

$$\langle \xi \rangle = \begin{cases} -\frac{\Delta}{\pi} \sqrt{2} r_s n(\sqrt{2} r_s), & r_s \ll 1 \\ \sim \alpha \Delta, & r_s \geq 1 \end{cases}$$

(6.2)

where $r_s$ is the gas parameter and and $\alpha > 1$ is a constant depending on the interaction strength. It is not simple to find an expression for $\xi$ in the intermediate regime $0 < r_s < 1$. But one can roughly estimate it by knowing, that for $r_s = 0.2$ one finds according to eq. (6.2) $\langle \xi \rangle = 0.11 \Delta$ on the other hand for $r_s = 1$ the value of $\langle \xi \rangle$ has been numerically determined [84] to be $\langle \xi \rangle = 0.6 \Delta$.

In the weak interaction regime ($r_s \ll 1$) it follows, that $\langle \xi \rangle \ll \Delta$. This means that the set of peaks should be split into pairs and the reduced spacing $\Delta E_N - e^2/C$ between two peaks of the same pair, (states $\uparrow$ and $\downarrow$) is of the order of $\langle \xi \rangle$, while that between two different pairs is of the order of $\Delta$. Furthermore, since the eigenfunctions of states states $\uparrow$ and $\downarrow$ are identical, the corresponding peak heights are correlated, and the resulting picture is a set of pairs with small reduced spacings and correlated heights.

On the other hand, for strong interaction $r_s > 1$ the fluctuations in the splitting energy $\langle \xi \rangle$ exceeds $\Delta$, and reordering of energy levels takes place. This leads to a smearing of the peak spacing statistics, and no signature of spin can be seen in it. The spacing fluctuations in this case are enhanced in comparison with the spinless situation, since a new type of ensemble is created, with the spectrum formed by a superposition of two random matrix theory-type set of levels.

6.2 Experiment

From the above discussion it is clear, that shape deformation and e-e interaction can smear out spin effects in the nearest-neighbor spacings statistics and enhance the fluctuations. In order to reduce $r_s$ as much as possible, we chose a heterostructure with a high electron density, further increased by a top gate voltage of $+100 \text{ mV}$ to $n_s = 5.9 \cdot 10^{15} \text{ m}^{-2}$. This results in $r_s = 0.72$, which is
smaller than in all previous experiments; additional screening is provided by the top gate, as mentioned in 3.3.1. The quantum dot we discuss in this chapter is sample RS (c.f. table 4.2).

The measurements were carried out in the weak coupling regime, \( \hbar \Gamma \ll k_B T \ll \Delta \). Here, \( \Gamma \) denotes the coupling of the dot to source and drain. The conductance \( G \) was measured as a function of the voltage \( V_I \) applied to the planar gate \( I \) (see inset in Fig. 6.1 (a)). Magnetic fields \( B \) applied perpendicular to the sample surface and \( V_{II} \) were used as parameters. The observed Coulomb blockade oscillations (Fig. 6.1 (a)) are fitted to a thermally broadened line shape, in order to obtain the positions and amplitudes of the peaks.

![Image](image.png)

**Figure 6.1:** (a) Right inset: AFM picture of dot RS (taken before evaporation of the top gate) of the oxide lines (bright) that define the dot, coupled to source (S) and drain (D) via tunnel barriers, which can be adjusted with the planar gates PC1 and PC2. Gates I and II are used to tune the dot. Main figure: Conductance \( G \) as a function of \( V_I \), showing Coulomb blockade resonances. (b) Linear fit (line) of the peak spacing \( \Delta V_I \) as a function of \( V_I \) (dots). The average peak spacing is almost constant, indicating small shape deformations.

Fig. 6.1 (b) shows the peak spacing \( \Delta V_I \) as a function of \( V_I \). Compared
to conventional dots defined by top gates we find a much smaller variation of the average peak spacing as $V_I$ is tuned, although the fluctuation of individual spacings is 15% of $E_c$. The running average of the spacings as a function of $V_I$ is obtained from a linear fit

$$RA(V_I) = 11.43mV + 6.7 \cdot 10^{-4} \cdot V_I(mV)$$

From the slope we find that the capacitance between the dot and gate I varies by only 3% over the whole scan range, as compared to, for example, a factor of 3 in Ref. [41]. This indicates that tuning gate I or II predominantly changes the energy of the conduction band bottom, while the dot is only slightly deformed.

### 6.2.1 Parametric evolution of neighboring Coulomb blockade peaks as a function of magnetic field

In Fig. 6.2 (a), five consecutive Coulomb blockade peaks are shown as a function of magnetic field in a logarithmic grayscale plot. A pronounced pairwise correlation of both amplitude and peak position is observed (peak b correlates with peak c, and peak d with peak e, respectively) in almost the whole observed magnetic field field regime. We interpret this parametric pair correlation in terms of a model developed by Baranger et al. [83]. The constant interaction model is used to subtract $E_c$ from the peak spacings. The remaining individual energy separations equal $\Delta/2$ on average and reflect the fluctuating level separations inside the quantum dot, which consist of two parts. We assume that two paired peaks belong to the same spatial wave function, labelled by $i$, of opposite spin, and are split by an interaction energy $\xi_i$, while the energy of consecutive states with different orbital wave functions differs by $\Delta_i - \xi_i$. This interpretation is supported by measuring the peak separation of a pair of strongly correlated peaks for $0 \leq B \leq 3T$. We find a linear increase on average. This indicates the Zemann splitting of spin paired states. This will be further discussed in section 6.5.

Since the separations between the two levels of equal spin $\Delta_i - \xi_i$ and the possibly also the separation of a spin pair $\xi_i$, vary as a function of magnetic field, levels may cross and the ground state of the dot can be either a singlet or a triplet state. Higher spin states are expected to be unlikely [10, 11]. At the singlet-triplet transitions, kinks in the parametric peak evolution occur and the pair correlation is interrupted [83]. We can identify such kinks in our data, among other features. Fig. 6.2 (b) shows the amplitudes of peaks c, d and e. The correlation between peaks d and e is very strong around $B=0$. For $0.4T < B < 0.61T$, this correlation is interrupted, while the amplitudes of peaks c and e are correlated instead. In this regime, correlated kinks in the evolution of peaks c and d are observed (Fig. 6.2 (c)). In Fig. 6.2 (d), a possible corresponding scenario for the parametric dependence of energy levels is sketched: (left) two avoided crossings occur between level pair $i$ and level pair $i+1$. This leads to the position of peaks c, d, and e as
Figure 6.2: (a) Logarithmic grayscale plot of parametric variations in a magnetic field $B$ for 5 consecutive Coulomb blockade peaks. A pair correlation in peak position and amplitude is observed, which is interrupted in certain ranges of $B$, for example in the region between the dashed lines.

(b) Parametric amplitudes for peaks c,d, and e, offset by $0.2 \frac{e^2}{h}$ each. The correlation between peak d and e is lost in $0.4T < B < 0.6T$, and e correlates with c instead.

(c) The corresponding position of the peak maxima. The traces are offset for clarity. At magnetic fields labelled by 1 and 3, kinks in the peak position occur, while the separation between peak d and e jumps across the region of suppressed amplitude from $\xi_{de}$ to $\xi_{de}'$.

(d) Scheme of a possible double anticrossing between spin-paired level $i$ and $i+1$ (left, the black arrows indicate the spin), which could lead to the observed structure in the correlation for peaks c, d and e (right).

sketched in Fig. 6.2 (d), right, corresponding to the difference in energy upon changing the electron number in the dot. Consequently, positions and amplitudes of peaks c and e should be correlated in $0.4T < B < 0.61T$, as observed. Note that this correlation is interrupted around $B=0.5T$, possibly due to the influence of another energy level. We emphasize that this kink structure, i.e. a double-anticrossing between 2 spin pairs, is the dominant one for all peak evolutions. This observation does not depend on the cooldown cycle.
Also, $\xi_{de}$ (c.f. Fig. 6.2 (c)) is not constant over the full range of $B$. While $\xi_{de} \approx 0.05\Delta$ for $B<0.22$T, the positions of peaks d and e are not detectable in $0.22$T$<B<0.32$T, since their amplitudes vanish. As the peaks reappear, $\xi_{de}$ has jumped to $\xi_{de}^* \approx 0.25\Delta$. We speculate that possibly a level crossing has occurred in the regime where the amplitudes are suppressed, and hence for $B<0.22$T, a different level pair is at the Fermi energy than for $B>0.32$T. Although $\xi$ fluctuates as the magnetic field is varied, a systematic change of $\xi$ with magnetic field cannot be clearly detected for $B<1$T, which indicates that Zeemann splitting is smaller than the parametric fluctuations in this regime. From the data of Fig. 6.2, we estimate the average interaction energy to $\bar{\xi} \approx 0.5\Delta$ by averaging over all peaks and magnetic fields. As we mentioned in the preceding section, Baranger et al. have estimated $\bar{\xi} \approx 0.6\Delta$ for $r_s=1$. Hence, our findings can be considered as being in agreement with existing theory, while we are not aware of a theoretical prediction for the fluctuations of $\xi$, $\sigma_\xi = \sqrt{\langle \xi^2 \rangle - \langle \xi \rangle^2}$. From the above phenomenology, we conclude that for dots with stronger shape deformations, and hence more level crossings, or in dots with larger $r_s$ (and thus larger $\bar{\xi}$), the spin pairing is frequently interrupted and difficult to detect. This is possibly the reason why spin pairing has not systematically been observed in earlier experiments [8, 41, 28, 42].

### 6.3 Nearest-neighbor spacings statistics

What is the effect of the observed spin pairing on the nearest-neighbor spacings distribution?

In Fig. 6.3, the measured histograms of the normalized nearest-neighbor spacings distribution (see Appendix 1) for $B=0$T (a) and $B \neq 0$T (b) are shown. Each individual $V_f$-sweep contains 15 Coulomb blockade resonances in the low coupling regime. The ensemble statistics have been obtained by measuring $G(V_f)$, and either by changing the magnetic flux by one flux quantum $\phi_0 = \frac{\hbar}{c}$ through the dot ($B \neq 0$T, GUE), or by stepping $V_{TT}$ in units of one Coulomb blockade period ($B = 0$T, GOE), which corresponds to the autocorrelation voltage for the peak amplitudes [17] in our sample. The total number of peak spacings used is 120 for $B=0$T, and 210 for $B \neq 0$T, respectively. The individual level spacings $s$ in units of $\Delta$ are obtained by using the fit of Fig. 6.1 (b); its expectation value is $\bar{s}=0.5$. Both histograms are asymmetric and show no evident bimodal structure. By including the effect of spin pairing into the statistics, however, we can interpret them as bimodal distributions, modified by:

(i) The $\delta$-function in the non-interacting nearest-neighbor spacings distribution $P(s)$ with the expectation value of $\bar{s}_{\delta}=0$ is shifted to $\bar{s}_{\xi} = \bar{\xi}$ and, as a reasonable assumption [84], broadened according to a Gaussian distribution with the standard deviation $\sigma_{\xi^*}$. Here, $\xi^*$ denotes the interaction energy in units of
Figure 6.3: Measured nearest-neighbor spacings distributions (gray bars) for \( B=0 \) (a) and \( B \neq 0 \) (b). The bold solid curves are the fits to \( P_{\text{int}}^\beta(\xi^*, \sigma_{\xi^*}) \), with the fit results as indicated in the figure (see text). Also drawn are the two components of \( P_{\text{int}}^\beta \), i.e. the Gaussian distribution of separations between spin pairs, and its convolution with the corresponding Wigner surmises. In (c,d) the fitted \( P_{\text{int}}^\beta \) are compared to a single Gaussian fit for (c) \( B \neq 0 \) and for (d) \( B=0 \).

(ii) Since one level of a spin pair \( i \) is shifted upwards in energy by \( \xi_i \), the separation between the upper level of spin pair \( i \) and the lower level of pair \( (i+1) \) is given by \( \Delta_i - \xi_i \). Consequently, \( P^\beta(s) \) is shifted to \( s_{\text{shift}}=1+\xi^* \) and convoluted with the Gaussian distribution function of \( \xi^* \).

Combining these two components, the modified nearest-neighbor spacings distribution reads

\[
P_{\text{int}}^\beta(\xi^*, \sigma_{\xi^*}) = \frac{1}{\sqrt{2\pi\sigma_{\xi^*}}} \left\{ \exp \left( -\frac{(s - \xi^*)^2}{2\sigma_{\xi^*}^2} \right) + \exp \left( -\frac{s^2}{2\sigma_{\xi^*}^2} \right) \right\} \times P^\beta(s + \xi^*)
\]

(6.4)

Here, the “\( \times \)” denotes the convolution. Since \( \Delta \) is determined by the dot size and the material parameters, we can fit \( P_{\text{int}}^\beta(\xi^*, \sigma_{\xi^*}) \) to the measured nearest-neighbor spacings distribution with the two fit parameters \( \xi^* \) and \( \sigma_{\xi^*} \) (Fig. 6.3). We obtain \( \xi^* = 0.65 \) and \( \sigma_{\xi^*} = 0.35 \) for \( B = 0T \), as well as \( \xi^* = 0.53 \) and
6.4. Even and odd nearest-neighbor spacings distribution

\[ \sigma_{\xi} = 0.34 \text{ for } B \neq 0T. \]  
Hence, we find that \( \bar{\sigma} \) is higher for \( B = 0T \) than for \( B \neq 0T \), which is in agreement with the theoretical prediction [83]. The fluctuations of \( \xi \) are found to be independent of whether time inversion symmetry is broken or not, and they do not vary continuously with magnetic field within experimental accuracy. Fits of both datasets to a single Gaussian curve are shown in Fig. 6.3 (c) for \( B \neq 0 \) and (d) for \( B=0 \). Obviously the more asymmetric histogram for the \( B = 0 \) in (a) and (d) is not well described by the symmetric Gaussian form. On the other hand the rather symmetric \( B \neq 0 \) histogram in (b) and (c) is similarly well described by a simple Gaussian curve as by \( P^{\beta}_{\text{int}} \).

We emphasize that the fluctuation of \( \xi^*, \sigma_{\xi^*} \), is an empirical fit parameter that includes not only the fluctuations of the spin splitting, but also other contributions, in particular the Zeemann splitting and states with a total spin larger than \( 1/2 \) (i.e. we have neglected situations in which \( \xi_i > \Delta_i \)). Hence, \( \sigma_{\xi^*} \) can be regarded only as an upper limit for the fluctuations of the interaction energy.

6.4 Even and odd nearest-neighbor spacings distribution

Theoretically, for exchange interaction energies \( \leq 0.5\Delta \), one expects the distribution of the odd nearest-neighbor spacings (ONNS) to differ from the distribution off the even nearest-neighbor spacings (ENNS). Since we found \( \xi = 0.53\Delta \) for \( B \neq 0T \) data, it is interesting to separate the odd and the even nearest-neighbor spacings.

In Fig. 6.4 (a) the statistics of both even and odd nearest-neighbor spacings are shown. The data set of this statistics was taken in the same way as explained before above. Here, each individual \( V_f \)-sweep contains 21 Coulomb blockade resonances in the low coupling regime, the total number of peak spacings used is 340. The histogram is asymmetric and shows no evident bimodal structure. By including the effect of spin pairing into the statistics, however, we can interpret them as bimodal distributions modified by the interaction energy \( \xi \).

In Fig. 6.4 (c) and (d) we separated the distribution off the nearest-neighbor spacings in its even and odd (Fig. 6.4 (b)) parts. The total number of nearest-neighbor spacings is 170 for each histogram. In the ENNS histogram a pronounced maximum of the level spacing is located at \( \Delta V_g = 11mV \), while in the histogram for the ONNS a broad maximum is observed at a higher \( \Delta V_g = 11.4mV \). In Fig. 6.4 (b) we plotted a modified level spectrum of a quantum dot by taking a fluctuating exchange interaction energy \( \xi_i \) into account. Following this picture, one expects that if each statistics contained only level spacings of the same kind, the ENNS would contain only the different exchange interaction energies \( \xi_i \) to add a spin down electron in the same orbital wavefunction \( \psi_i \) as the next lower lying spin up electron is already occupying (see Fig. 6.4 (b)). Correspondingly the ONNS distribution would only contain the addition energies for adding a
spin up electron in a different orbital state $\Delta - \xi$. Therefore the maximum of the ENNS and the ONNS distribution are comparable to the expectation value of the exchange interaction energy $\xi$ and $\Delta - \xi$, respectively. In section 6.2.1 we discussed the observation of level crossings, which means that the ENNS and the ONNS distributions contain both type of addition energies, but since $\xi \approx 0.5\Delta$ one expects either the $\Delta - \xi$ or the $\xi$ addition energy type to dominate the statistic. Hence, the ENNS histogram represents predominantly the interaction energy $\xi_i$ for adding a spin down electron. Therefore the maximum at $\Delta V_g = 11mV$ (this means in dot energy $0.4\Delta$) is an indication for the value of $\xi$. The tail at larger $\Delta V_g$ is caused by events in the ENNS statistics which should be counted in the ONNS distribution (namely the $\Delta - \xi$ for adding a spin up electron). Because
of the random level crossings, these inaccurate events contribute to the ENNS statistics, too. The $\Delta - \xi$ addition energies on the other hand, cause the maxima in the ONNS distribution at $\Delta V_g = 11.4 mV$ (this means in dot energy $0.6\Delta$). The important point is that we observe one maximum in each statistics. They are separated by $\approx 0.2\Delta$. From this analysis one could conclude that the exchange interaction energy is less or of the order of $\xi = 0.5\Delta$. Furthermore it is remarkable that the ONNS distribution is broader than the one for the ENNS. This can be explained by the fact that in the ENNS case, the statistics is broadened due to fluctuations of the $\xi_i$ only, while in the odd case the fluctuations of the addition energies are caused by the fluctuations in $\xi_i$ and in $\Delta_i$ and therefore larger.

### 6.5 Magnetic field dependence of the spin splitting

Fig. 6.5 (a) shows a grayscale plot of the conductance of a strongly correlated Coulomb peak pair as a function of magnetic field up to 2 Tesla. In Fig. 6.5 (b) the corresponding peak separation is plotted. Despite large fluctuations, it clearly tends to increase with $B$. In section 6.2.1 we identified correlated peaks as states with different spin, therefore this observation is consistent with a Zeeman splitting.

![Figure 6.5](image)

Figure 6.5: (a) Grayscale plot of the conductance of a correlated Coulomb peak pair as a function of magnetic field up to 2 Tesla. (b) Peak separation (with the Coulomb charging energy subtracted) in units of dot energy as a function of magnetic field. The peak separation increases linearly with magnetic field, despite large fluctuations.

In a single particle picture the energy separation of electrons with opposite spins increases linearly because of the Zeeman splitting $E_z = g\mu_B B$. Although the monotonic change of the peak separation as the magnetic field is tuned
can be observed in each correlated pair, the amount of change in the magnetic field varies remarkably, which possibly indicates the fluctuation of the effective g-factor. From linear fits we find values of $\frac{\Delta E}{\Delta B}$ from $10\mu eV/T$ to $50\mu eV/T$. These are comparable to the Zeeman splitting one would expect using the g-factor of the two-dimensional electron gas. We conclude that the parametric evolution of correlated Coulomb resonances is consistent with Zeeman splitting, which is another sign that the correlated pair is in fact a spin pair.

6.6 Spin pairing in other dots

Here we investigate the stadium quantum dot shown in Fig. 6.6 (a), with respect to spin pairing. This dot is described in detail in table 4.2. The main difference between this stadium dot, and dot RS are: The heterostructure used to define this dot is not a triangular but a rectangular quantum well with the two-dimensional electron gas 35nm away from the surface and with a sheet density $n_s = 5.0 \cdot 10^{15} m^{-2}$. Therefore we estimate the gas parameter to be $r_s \leq 0.78$. However, measurements in the quantum Hall regime reveal, that filling factor $\nu = 2$ is at 7.8T. Therefore the density inside the dot is reduced to $n_{dot} = 3.8 \cdot 10^{15} m^{-2}$ (and $r_s(n_{dot}) = 0.89$). This is a stronger reduction than in dot RS. Furthermore, the
6.6. Spin pairing in other dots

Figure 6.7: (a) Logarithmic grayscale plot of 9 consecutive Coulomb blockade resonances as a function of magnetic field. Pairing in peak amplitude and position is clearly observable. (b) From the positions of these nine Coulomb blockade peaks, the ground state energy spectrum can be constructed. For this we transfer the gate voltage in dot energy and offset the traces accordingly.

shape of this dot is stadium-like and the point contacts are directly opposite to each other. The dot is larger than RS, therefore the single particle level spacing is smaller than that of dot RS. We estimate that the single particle level spacing is $\Delta \approx 70\mu eV$ in this stadium dot.

In Fig. 6.6 (c) a the conductance as a function of plunger gate voltage $V_g$ is plotted. The peak spacing increases with gate voltage. This is due to the change in leverarm, when the voltage difference between top gate ($V_{tg} = 100mV$) and the plunger gate is reduced (c.f. Fig. 4.3). How strong this effect is, is illustrated in Fig. 6.6 (c). Over the nine Coulomb blockade peaks we observe here $\Delta V_g$ changes form 40mV to 60mV, and this overall increase is not linear.

As in quantum dot RS we see signatures of spin pairing in this sample: Fig. 6.7 (a) shows a grayscale plot of nine successive Coulomb blockade peaks as a function of magnetic field. Obviously there are neighboring traces where position and amplitude have the same evolution as a function of magnetic field. The pairs (1,2), (6,7) and (8,9) correlate over almost the whole magnetic field regime shown in the plot, while the correlation in the pair (3,4) and (5,6) is interrupted.

In this dot the peak spacing is more a quadratic than a linear function of the plunger gate. Therefore, we subtract the charging amount $E_c$ of the peak spacings as a polynom of second order from the gate voltage separation. The ground state energy spectrum is then calculated from the fluctuating part of voltage separation via the gate voltage dependent leverarm and the result is depicted in Fig. 6.7(b).

In this figure we observe that there are several traces having the same evolution as a function of the external parameter $B$, and we interpret this in the same way.
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</table>

Table 6.1: List of the measured mean interaction energies $<\xi_i>$ and their fluctuations $\sigma_{\xi_i}$ extracted from the ground state energy spectrum in Fig. 6.7 (b).

as in section 6.2.1. Spin paired states are indicated in Fig. 6.7 (b), as bold black or gray lines. Furthermore we also observe kinks (e.g. between trace 3 and 4 at $\approx 0.6T$). With this observed data set, we cannot make the same analysis as we did for dot RS. This is because we observe only nine neighboring peaks, and this is not enough for the statistics of the peak spacings.

However, to get quantitative information about the splitting between spin paired states, we measure the interaction energies $\xi_1, \ldots, \xi_7$ where we observe spin pairing as indicated in Fig. 6.7(b). The results are listed in Table 6.1.

The mean value of all ground state energy differences determined from this dataset is $<\Delta E_i> = 29.1 \mu eV$. This can be regarded as a measurement of half the spin degenerate single particle spacing $\Delta/2$ in this system. From this we find $\Delta \approx 58 \mu eV$. This is a little bit smaller than the spin degenerate single particle spacing estimated from the size of the dot. With this, the expectation value for $\xi_i$ is $<\xi_i> = 0.42\Delta$. This is clearly less, than the value determined in section 6.3 in dot RS. Probably being caused by the different way we determine $<\xi_i>$. Namely, here we chose selectively states that are spin paired, and only in regimes where they correlate. Fewer “inaccurate” level spacing contributions (i.e. contribution from crossed levels where $\xi_i > \Delta E_i - \xi_i$) are in the statistics here. Since the $r_s$ value in this sample is comparable, or even larger, than in RS, we would expect, that $<\xi_{stadium}> \approx <\xi_{RS}>$.

In Fig. 6.8 we compare the histograms of the correlated addition energies $\xi_i$ determined from the ground state spectrum in (a) with the uncorrelated addition energies $\Delta E_{nc}$. We find that $\Delta E_{nc}$ has a higher expectation value than $\xi_i$ and fluctuates much more, than the $\xi_i$, also in agreement with what is expected.

We conclude from this, that spin pairing in two quantum dots with different shape and area, defined in different samples (i.e. rectangular and triangular quantum well two-dimensional electron gas) has been observed. The parameter common in both systems was the gas parameter $r_s$. We find that the fluctuations
6.7 Conclusions

Here we have shown, that spin effects are observable. We managed this by using a LO with an AFM to define the quantum dots in heterostructures with a high electron sheet density. The interactions in these rigid quantum dots are much smaller than in earlier experiments. In this way we observe spin pairing as a function of magnetic field. Furthermore we observe that this spin pairing is interrupted by kinks as well as other structures in the parametric evolution of the Coulomb blockade peaks. We have extracted the average interaction energy $\bar{\xi}$ between states of identical spatial wave functions but opposite spin and we explained the measured distributions of nearest-neighbor spacings as being composed of the two branches of a modified, bimodal Wigner-Dyson distribution, which takes $\bar{\xi}$ and its fluctuation into account.

The separation of two spin paired states as a function of magnetic fields up to 3T varies, besides fluctuations, roughly linearly. This is consistent with Zeeman splitting.

Furthermore we observed spin pairing in a quantum dot with a totally different shape defined in another heterostructure. Only the interactions in this quantum dot were comparable to those in the first quantum dot.

From this we conclude that for $r_s \approx 0.7$ the interaction induced spin splitting is $\xi \approx \Delta/2$ and therefore spin effects can be observed in quantum dots.

Figure 6.8: histograms for the (a) interaction energy fluctuations $<\xi_i>$ and the (b) energy differences for the uncorrelated groundstate levels $<\Delta E_{nc}>$ extracted from Fig. 6.7 (b).

of the interaction energy $\xi$ is in both systems comparable to $0.5\Delta$, as one would expect for these interaction strengths.

6.7 Conclusions

Here we have shown, that spin effects are observable. We managed this by using a LO with an AFM to define the quantum dots in heterostructures with a high electron sheet density. The interactions in these rigid quantum dots are much smaller than in earlier experiments. In this way we observe spin pairing as a function of magnetic field. Furthermore we observe that this spin pairing is interrupted by kinks as well as other structures in the parametric evolution of the Coulomb blockade peaks. We have extracted the average interaction energy $\bar{\xi}$ between states of identical spatial wave functions but opposite spin and we explained the measured distributions of nearest-neighbor spacings as being composed of the two branches of a modified, bimodal Wigner-Dyson distribution, which takes $\bar{\xi}$ and its fluctuation into account.

The separation of two spin paired states as a function of magnetic fields up to 3T varies, besides fluctuations, roughly linearly. This is consistent with Zeeman splitting.

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Chapter 7

Coulomb blockade in a quantum ring

7.1 The Aharonov-Bohm effect

In 1959 Aharonov and Bohm [85] presented a Gedanken experiment, describing how a magnetic flux affects the interference of a split electron wave. The most essential aspect of this effect is that it unambiguously confirms the physical reality of the magnetic vector potential $\vec{A}$, raising it from a mathematical entity to a true physical quantity. However, despite the absence of a magnetic field all the way along the trajectories, the vector potential $\vec{A}$ associated with the field is not zero everywhere along a closed path, irrespective what gauge is chosen.

Quantum mechanically we know that the vector potential $\vec{A}$ modifies the canonical momentum of an electron with charge $-e$ travelling at a velocity $\vec{v}$ according to

$$\vec{p} = \hbar \vec{k} = m\vec{v} + e\vec{A} \quad (7.1)$$

A change in momentum implies a change in phase acquired by the electron after travelling a certain distance. A magnetic field which may be zero at the location of the actual electron trajectory, still affects the phase of the electron wave via the vector potential $\vec{A}$. The total phase acquired by a propagating electron along some path described by a position variable $l$ immediately follows from the canonical momentum of eq. 7.1

$$\Delta \Phi = \int \vec{k} \cdot d\vec{l} = \frac{1}{\hbar} \int (m\vec{v} + e\vec{A})d\vec{l} = \Delta \Phi_v + \Delta \Phi_A \quad (7.2)$$

i.e. the sum of a contribution due to the velocity $v$ of the particle and one resulting from the vector potential $\vec{A}$. Thus, the phase shift induced by the vector potential $\vec{A}$ on an electron propagating once around a closed loop is found as
7.2. Experiments in mesoscopic quantum rings

\[ \Delta \Phi_A = \frac{\Phi}{\Phi_0} = \frac{e}{\hbar} \int \vec{A} \cdot d\vec{l} = \frac{e}{\hbar} \int \text{rot} \vec{A} \cdot d\vec{S} = \frac{e}{\hbar} B \cdot S = 2\pi \frac{\Phi}{\Phi_0} \]  

(7.3)

with \( \Phi_0 \) being the flux quantum \( \hbar/e \). Since the phase difference equals \( 2\pi \) times the enclosed flux in units of the flux quantum \( \Phi_0 \), any effect resulting from this enclosed flux will show a periodic behavior, with a period of one flux quantum.

In the solid state, the Aharonov-Bohm effect manifests itself as a periodic oscillation in the conductance of a sample as a function of an applied magnetic field \( B \). A well defined periodicity requires that the conducting paths through the sample enclose a constant area \( S \) perpendicular to \( B \). The periodicity of the oscillations is then \( \Delta B = \hbar/eS \), plus possible harmonics, i.e. at \( \hbar/2eS \), the Altshuler-Aronov-Spivak oscillations. These Altshuler-Aronov-Spivak oscillations are caused by trajectories that make one revolution around the ring. They contain a contribution from the time-reversed trajectories, whose phase difference at zero magnetic field does not depend on the details of the sample geometry and is zero. Thus the probability for backscattering has a maximum at \( B = 0 \) and consequently the conductance has a minimum. In a geometry with many rings in series or in parallel the \( \hbar/e \) oscillations average out, but the \( \hbar/2e \) oscillations remain. The first observation of the Aharonov-Bohm effect in the solid state was made by Sharvin and Sharvin [86]. They studied a long metal cylinder. Since this is effectively a many ring geometry only the \( \hbar/2e \) oscillations were observed. Later on, the \( \hbar/e \) oscillations were also found in single metal rings [88] and in a two-dimensional electron gas ring defined in a GaAs-AlGaAs heterostructure [89]. Since then the Aharonov-Bohm effect has been investigated in a number of different experiments [90, 91, 92, 93, 94]

7.2 Experiments in mesoscopic quantum rings

In an experiment by Yacoby et al. [95] it was shown for the first time, that part of the tunneling current through a quantum dot in the Coulomb blockade regime is coherent. In this experiment a quantum dot was inserted in one arm of a Aharonov-Bohm ring, and the ring current showed periodic oscillations when the magnetic field was swept. This experiment was performed in a two-terminal device, where the two-terminal conductance \( G_{\text{ring}} \) is an even function of magnetic field \( B \). Therefore, at \( B=0 \) the conductance of the ring \( G_{\text{ring}} \) can only have a maximum (phase \( \phi = 0 \)) or a minimum (\( \phi = \pi \)) at \( B=0 \). [96]. The effective Aharonov-Bohm phase manifests itself only in the amplitude of the Aharonov-Bohm oscillations [97]. Namely, the more the phase of the ring (at \( B=0 \)) mismatches 0 or \( \pi \), the more the amplitude of the Aharonov-Bohm oscillations is reduced. This was demonstrated by Yacoby et al. [98] in a Aharonov-Bohm ring with a gate on the top of one arm. When the density in one arm was continuously
Chapter 7. Coulomb blockade in a quantum ring

changed, the measured phase at B=0 stayed constant, but the h/e oscillation amplitude decreased, vanished and reappeared, but this time the measured phase was shifted by π. In the regime where the Aharonov-Bohm amplitude vanished only the h/2e component can be detected. In the meantime, this phenomenon has been observed in several experiments on different mesoscopic quantum rings [93, 94].

AFM Lithography offers the possibility to define multiple connected nanostructures in a very simple way. For example a Sinai billiard can be defined by a rectangular cavity (as shown in the previous chapters) with an additional antidot in the center (c.f. Fig 7.1 (a)). Such a structure is investigated in this chapter. As we stated above, Aharonov-Bohm oscillations in mesoscopic rings have been observed in several different experiments. Since the transport through a quantum dot is known to be coherent, it is interesting to study what happens to these Aharonov-Bohm oscillations if the quantum ring is gradually pinched off.

7.3 Experiment

In Fig. 7.1 (a) an AFM picture of the quantum ring is shown. The “ring” is defined by a quantum dot of rectangular shape (410nm-510nm) with an oxide antidot in the center. This antidot has an electronic diameter of d ≈ 200nm. The density inside the dot is estimated to be \( n_s = 4.0 \cdot 10^{15} \text{m}^{-2} \), leading to a Fermi wavelength of \( \lambda_F = 40 \text{nm} \). With an estimated depletion length at the oxide lines of about 35nm, we find that the lateral dimension of the arms of the ring vary from 100nm to 225nm (c.f. Fig. 7.1 (b)). Therefore about 4-6 subbands are occupied in the quantum ring.

By applying a voltage of \( \approx 25 \text{mV} \) to the top gates the point contacts can be tuned into pinch off and clear Coulomb blockade oscillations are observed. A typical Coulomb blockade trace is shown in Fig. 7.1 (c). The single particle level spacing is estimated to be \( \Delta = 42 \mu \text{eV} \), where the area of the dot is estimated from the Coulomb blockade period as a function of the top gate, \( \Delta V_{tg} = 280 \mu \text{eV} \). The ratio between the Coulomb resonances width at half maximum (FWHM) and the period in the gate voltage is: \( 7.9 \text{mV}/2.2 \text{mV} = 3.6 \) and therefore the broadening of the peaks is comparable to \( \Delta \) and we are not unambiguously in the single level transport regime.

7.3.1 Quasi “Aharonov-Bohm” oscillations in the peak amplitude

“Aharonov-Bohm”

The conductance of the Coulomb blockade resonances are strongly modulated, when a magnetic field is applied. In contrast to the quantum dot studied in the previous chapter, this modulation is not random, but quasi-periodic, and in
7.3. Experiment

Figure 7.1: (a) AFM picture of the quantum ring defined by a conventional quantum dot with an antidot in the center. (b) Sketch of the quantum ring. The depletion length (gray) at the oxide lines (black) is estimated to be 35nm - 40nm. (c) Coulomb blockade oscillations as a function of gate voltage $V_{pgu}$.

certain magnetic field regimes it is periodic. This is shown in Fig. 7.2, where we plot in (a) the conductance of 8 neighboring Coulomb blockade traces as a function of magnetic field. The periodicity becomes very pronounced for $0.57T < B < 0.8T$ where the period is $\Delta B = 34mT$. Furthermore for fields $B > 0.22T$ the maxima and minima in the amplitude appear at the same magnetic fields in all 8 observed neighboring peaks.

The clear periodic behavior for $B > 0.57T$ can be classically understood as a bound state around the antidot. The observed period in this regime is 34mT. In a classical picture typical trajectories of electrons in a magnetic field are orbits with the cyclotron radius $r_c = \sqrt{2\pi n_s \hbar / eB}$. Therefore $r_c(0.57T) = 194nm$ and the area enclosed by this trajectory is $A_{rc} = 1.18m^2$. The amount of magnetic field to put an additional flux quantum $\hbar/e$ through this area $A_{rc}$ is $\Delta B = 35mT$.

We thus conclude that the characteristics of the Coulomb blockade peak heights shows strong modulation which are quasi-periodic as a function of mag-
Figure 7.2: (a) Evolution of 8 neighboring Coulomb blockade peaks as a function of magnetic field in a grayscale plot. The peak amplitude is quasi periodically modulated. (b) bottom: Peak height of a single Coulomb blockade resonance as a function of magnetic field. top: Separation $\Delta B$ of two conductance maxima as a function of magnetic field $B$. For $0.57 \, T < B < 0.8\, T$, $\Delta B=34\, mT$ is almost constant.

The modulation of the Coulomb blockade resonances with the magnetic field reflects in fact the modulation of the wavefunction amplitude at the two point contacts.

### 7.3.2 Power spectra

In Fig. 7.3 (a) the peak maxima of five neighboring Coulomb blockade peaks as a function of the magnetic field are shown. The bars separate the magnetic field in four different regimes. In the low field regime I, there is no apparent periodicity. Rather quasi-periodic features, which look different in each trace can be observed there. The power spectrum in Fig. 7.3 (b) shows the average power spectrum of all the traces in regime I. There is a broad peak around $f = 20T^{-1}$ indicating, that there are several typical frequencies (areas) contributing to the observed structure. This regime will be studied more extensively in section 7.3.5.

Before we discuss regime II, we want to look at regime III: Obviously all amplitudes are correlated in this regime. The maxima appear almost at the same position in magnetic field, meaning that they shift only $2.5mT$ between neighboring Coulomb traces. This shift probably reflects the influence of the plunger gate on these states. This shift can also be observed in the position of these Coulomb blockade peaks (see Fig. 7.4 (a)). Furthermore the oscillation of the amplitude in this regime is very strong, we find that $(I_{\text{max}} - I_{\text{min}})/I_{\text{max}} = 0.9$. The period in this regime is $34mT$. This is the Aharonov-Bohm period one would expect for a ring with radius 197nm. This corresponds roughly to the radius which
7.3. Experiment

Figure 7.3: (a) Amplitude of five neighboring Coulomb blockade peaks as a function of magnetic field. For clarity the traces are offset by 10 pA. (b,c,d,e) Power spectra (P) of four different magnetic field regimes: (b) regime I, (c) regime II, (d) regime III, and (e) regime IV. Obviously, the characteristic frequencies shift with magnetic field and become more pronounced.

a typical state running around the antidot (without scattering at the boundary) encloses. The power spectrum of this regime is shown in Fig. 7.3 (d). Obviously there is only one dominant frequency, namely $29T^{-1}$, which corresponds to the observed 34 mT. The other features in the power spectrum are: a broad low frequency oscillation and a weaker peak at $\approx 52T^{-1}$. This corresponds to a period of $\approx 19$ mT. This high frequency peak is caused by the double peak features, observed in some peaks in (a). The cause of these double peaks is probably a contribution of states running twice around the antidot.

If we now look at regime II, we see that it is a mixture of regime I and III. We find that the amplitude modulation is almost periodic, but neighboring traces are not correlated over more than two traces. The periods in this regime can be extracted from the power spectrum: First there is a low frequency oscillation of $(7 \pm 2)T^{-1}$. This is the period one would expect for a ring with radius 95 nm, and therefore this frequency contribution is very probably caused by trajectories around the antidot in the center of the quantum ring. Furthermore there is a
period of \((23 \pm 2)T^{-1}\), with higher harmonics \((46 \pm 2)T^{-1}\) and \((69 \pm 2)T^{-1}\). Since 
\(23T^{-1}\) corresponds to \(43\text{mT}\), this frequency is caused by states bouncing off the
dot walls. This time the enclosed area is \((310\text{nm})^2 = \pi(175\text{nm})^2\). This is slightly
smaller than the dot size. The same characteristics can be observed in regime IV,
but with slightly different oscillation periods.

### 7.3.3 Correlation of peak position and peak amplitude

In Fig. 7.4 (a) the measured ground state spectrum of the quantum ring for
\(0.5T \leq B \leq 0.85T\) is plotted. In this regime the level position “oscillates” with
the same period as the amplitude. Furthermore, the level motion of neighboring
peaks is strongly correlated. From the two-dimensional ring spectrum 7.4 one
expects that the levels evolve in ziczac lines with the period of one flux quantum
through the dot area as we observe it. Level crossings show up in neighboring
traces as kinks which look at each other. In the ground state spectrum we observe
only few features (arrow) of such level crossings. From this, we conclude that the
observed oscillation is not caused by level crossings.

![Graphs showing level oscillations and peak positions](image)

**Figure 7.4**: (a) Ground state energy spectrum after subtraction of \(e^2/C\) for \(0.5T \leq B \leq 0.85T\). Obviously the energy levels evolve in a correlated way and there are
only few possible level crossings (indicated by the arrow) (b) Peak height \(G^i(B)\) and
peak positions \(V^i(B)\) both in arbitrary units of the five resonance levels shown in (a)
in the magnetic field regime \(0.55T \leq B \leq 0.77T\). As the peak amplitudes also the peak
positions oscillate. They have the same period but a different phase.

In the whole discussion we neglected spin degeneracy. As discussed in the
preceding chapter, spin paired states would of course lead to correlated motion
of neighboring Coulomb blockade traces, too. Here, we observe a correlation over five neighboring traces. Therefore, we do not think that spin pairing is the cause of the correlated motion in this magnetic field regime.

On the other hand, in this regime the amplitude of the peak position oscillations is \((38 \pm 3) \mu\text{eV}\), which is roughly the single particle energy. The peak positions of all five peaks oscillate with roughly the same amplitude. In the two-dimensional ring spectrum one expects that this amplitude increases with states of higher energy if only one radial mode is occupied. Although the peak position and amplitudes oscillate with the same frequency, the oscillations are not in phase. The cause of this phase shift is not understood at the moment and further investigations are necessary.

7.3.4 Power spectrum at different dot-lead coupling

In Fig. 7.5 the conductance of the quantum ring as a function of \(B\) and \(V_{\text{p}}\) in four different dot lead coupling regimes is shown. In Fig. 7.5 (a) the quantum ring is in the Coulomb blockade regime and the Coulomb blockade peak heights oscillate as a function of magnetic field as discussed in the previous section. When the point contacts are gradually opened on one hand the Coulomb blockade oscillations vanish as expected, on the other hand the oscillation of \(G(B)\) as a function of \(B\) is still observable. Furthermore in the low field regime it looks as if the oscillation of \(G(B)\) becomes more regular.

In Fig. 7.6 the evolution of the power spectrum as a function of the ring coupling to the leads is depicted. In (a) we show the power spectrum of the same data as in Fig. 7.3 (which is also the data shown in Fig. 7.5 (a)), but this time the power spectrum is taken over the whole magnetic field regime. Obviously, as already mentioned before, the dominant frequency is located at \(f = 29T^{-1}\) and can be identified with a flux enclosed by a typical state in the dot at fields around 0.7T. When the dot lead coupling is enhanced, and therefore the levels in the dot get broadened, this frequency remains dominant, even if the dot is open and the level broadening is larger than the charging energy. On the other hand the \(f = 23T^{-1}\) frequency (corresponding to \(\Delta B = 43\text{mT}\) and an area \(A = \pi(175\text{nm})^2\) in the spectrum vanishes if the point contacts are opened. The most interesting thing happens to the \(f = 7T^{-1}\) frequency: When the leads are opened this frequency becomes more pronounced. In the open dot spectrum (f) it appears together with its higher harmonics at \(f = (6.5, 13, 19, 26)T^{-1}\). We conclude from this, that if the dot becomes open the electron trajectories are more sensitive to the antidot than in the closed system. This is because in the open dot the transport is mediated by many different (broadened) levels in the dot. Since each level corresponds to a typical area, their effect on \(G(B)\) is smeared out when the dot is opened. The only area that cannot be smeared is the area that is enclosed by every state, and this is the antidot area.
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Figure 7.5: Conductance in the $(V_{pgu}, B)$-plane for different dot-lead coupling regimes. The point contacts are gradually opened from (a) to (d).

Figure 7.6: Evolution of the power spectrum as a function of dot lead coupling. The power spectra shown here in (a,b,c,d) correspond to the of the four dataset depicted in Fig. 7.5 (a,b,c,d). (See text)
7.3.5 Peak amplitude at low magnetic fields

In Fig. 7.7 (a) we show the conductance of the quantum ring around zero magnetic field in a grayscale plot. The peak heights show quasi-periodic oscillations as a function of magnetic field.

![Grayscale plot of conductance versus magnetic field with peaks and annotations.](image)

Figure 7.7: (a) top: Sweep across 23 neighboring Coulomb blockade peaks and their evolution (bottom) as a function of magnetic field around $B=0\text{T}$. The feature at $V_{pgu}=30\text{mV}$ is a charge rearrangement. (*) marks the traces where the conductance has a maximum at $B=0$. (b) Peak heights as a function of magnetic field for the four traces (i,j,k,l) marked in (a). Trace (k) has maximum amplitude around zero, while the others have a minimum there. (c) Averaged (see text) conductance around $B = 0\text{T}$ in the open dot. The conductance has a minimum in all traces. This minimum is caused by coherent backscattering of time reversed paths.

As we already mentioned in the section 7.3.2 in this regime there is no characteristic periodicity in $B$ observable. This is because at low field each state has a different characteristic area, and these areas are of course also influenced by the varying magnetic field. The $V_{pgu}$ sweep shown in (a) on top of the grayscale plot, is the conductance trace at $B=0\text{T}$. Obviously neighboring peaks are correlated.
in height, and it looks as if the peaks are modulated also as a function of the plunger gate.

Since we measure the quantum ring in a two-point geometry, it is clear that the measured conductance \( G(B) \) has to be symmetric around \( B \) and therefore \( G(B) \) has a maximum or a minimum at \( B=0 \). On the other hand a weak localization correction \(^{34}\) is expected for the average conductance peak height (see Appendix B). Therefore the existence of the maxima is unusual. To our knowledge, such a maximum in a single Coulomb blockade peak amplitude in a quantum dot with conventional (not a ring) shape has not been observed before. Therefore, it should be studied whether this maximum in the zero magnetoconductance is related to the geometry of this quantum ring.

On the other hand the open quantum ring does not show a maximum in the magnetoconductance. In each trace (out of 80) we see a weak localization dip (see Appendix B). In Fig. 7.7 (c) we plot the conductance around \( B=0 \) \( \delta G = \langle G(B) - G(0) \rangle \), averaged over 80 traces. For \(-0.01T < B < 0.01T\) \( \delta G \) fits well the expected Lorentzian shape. This lineshape is expected in chaotic cavities, which the quantum ring under study (in fact it is a Sinai billiard) is. We determine the crossover field in this way and find \( B_{\sigma} = 6mT \).

In the closed ring each conductance peak is mainly determined by the resonance level at the Fermi energy. We conclude that the observed maxima and minima are caused by the specific transport property of the different levels. In the experiment we find that in some levels the magnetoconductance has a maximum at \( B=0 \). Whether this effect has to do with the special geometry of our dot or not has to be studied. However, averaging \( G(B) \) over the 23 traces, we recover the weak localization dip, and this is in agreement with the theory.

In the following section we discuss a model for a perfect two-dimensional quantum ring. The energy spectrum and the wavefunctions can be determined in this model. From this the evolution of the Coulomb blockade peak heights and positions as a function of magnetic field can be assessed.

### 7.4 Analytical model for a two-dimensional ring

In order to understand the experimental observation on the quantum ring (c.f. 7.3) we discuss an analytical model for a two-dimensional ring. Tan and Inkson [99] proposed an exactly soluble model for electron states in such a ring. They assumed that the radial ring potential has the following form [99]:

\[
V(r) = \frac{a_1}{r^2} + a_2 r^2 - V_o
\]  

(7.4)

where \( V_o = 2\sqrt{a_1a_2} \). It can be verified that the potential has a minimum \( V(r_o) = 0 \) at \( r = r_o = (a_1/a_2)^{1/4} \) where \( r_o \) defines the average radius of the ring.

In the presence of a uniform magnetic field \( B \) perpendicular to the plane of such a two-dimensional ring the Hamiltonian of an electron with effective mass
m* is

\[ H = \frac{\hbar^2}{2m^*} \left[ -\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) - \frac{1}{r^2} \left( \frac{\partial}{\partial \varphi} \right)^2 - i \frac{eB}{\hbar} \left( \frac{\partial}{\partial \varphi} \right) + \frac{e^2B^2}{4\hbar^2r^2} \right] + V(r) \]  

(7.5)

where the vector potential is chosen as \( \vec{A} = \frac{1}{2}Br\varphi \). The energy spectrum \( E_{n,m} \) as a function of magnetic field analytically determined from eq. 7.5 is

\[ E_{n,m} = \left( n + \frac{1}{2} + \frac{M}{2} \right) \hbar \omega - \frac{m}{2} \hbar \omega_c - \frac{m^*}{4} \omega_o^2 r_o^2 \]  

(7.6)

where \( n=0,1,2,... \) and \( m=0,\pm1,\pm2,... \) characterize the radial and the angular momentum, respectively. Furthermore, \( \omega_c = \frac{eB}{m^*} \) is the cyclotron frequency and \( \omega = \sqrt{\omega_c^2 + \omega_o^2} \). \( \lambda = \frac{\hbar}{m^*\omega} \) is the effective magnetic length renormalized by the ring confinement and \( M = \sqrt{m^2 + \frac{2\alpha_1m^*}{\hbar^2}} \).

Fig. 7.8 (a) shows the energy spectrum for a ring with radius \( r_o = 150nm \). The parameters used are \( a_1 = 2.53meV \text{ nm}^2 \), \( a_2 = 0.005meV \text{ nm}^{-2} \) and \( V_o = 225meV \). A two-dimensional ring modelled with these parameters is comparable to the quantum ring studied in this chapter.

Figure 7.8: (a) Energy spectrum for a ring with confining potential \( V(r) \) (see eq. 7.4) for \( a_1 = 2.53meV \text{ nm}^2 \) and \( a_2 = 0.005meV \text{ nm}^{-2} \). In (b) and (c) the radial parts of the wavefunction for \( (n,m)=(1,9) \) and \( (n,m)=(0,14) \) are plotted.

The wavefunctions have the following form:
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\[
\psi_{n,m}(r, \varphi) = \frac{1}{\lambda} \sqrt{\frac{\Gamma[n + M + 1]}{2^{M+1}n!(\Gamma[M+1])^2\pi}} e^{-im\varphi} e^{-\frac{1}{2}(r/\lambda)^2} \left( \frac{r}{\lambda} \right)^M \\
\times \, _1F_1(-n, M + 1, \frac{1}{2}(r/\lambda)^2) \tag{7.7}
\]

Here \(_1F_1\) is the confluent hypergeometric function. In Fig. 7.8 (b) and (c) the radial parts of the wavefunction for the states \((n,m)\) = (1,9) and \((n,m)\) = (0,14) (as indicated in the figure) are depicted. Because of the rotation symmetry of the potential, \(|\psi_{n,m}(r, \varphi)|^2\) does not depend on \(\varphi\).

For a state with angular quantum number \(m\), the Hamiltonian (see eq. 7.5) for the radial motion can be written as

\[
H_m(r) = -\frac{\hbar^2}{2m^2} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + u_m(r) \tag{7.8}
\]

where

\[
u_m(r) = \frac{\hbar^2}{2m^2} \left( \frac{m^2}{r^2} - \frac{eBm}{\hbar} + \frac{e^2B^2}{4\hbar^2}r^2 \right) \tag{7.9}
\]

may be considered as an “effective confinement potential”. It is reasonable to assume that the center of weight of a state with quantum number \(m\) lies at the minimum of this “effective confinement potential”. Therefore \(r_m\) is the radius of a state determined by \(\frac{d\nu_m(r)}{dr} = 0\), from this one finds that \(r_m\) is given by

\[
r_m = \lambda\sqrt{2M} \tag{7.10}
\]

At zero magnetic field the \(m=0\) states have the smallest radius which is equal to the average ring radius \(r_o\). Therefore all the other states are centered in the outer half of the ring. On the other hand, a finite magnetic field shifts the bottom of the subband to the quantum number \(m_o\). This is illustrated in in Fig. 7.9 (a), where \(\nu_m(r)\) is plotted for \(-7 \leq m \leq 6\) and \(B=0.3T\). The minimum of \(\nu_m\), \(\nu_{min}\) and the position of this minimum \(r_m\) depends on the angular momentum and the magnetic field, as plotted in (b) and (c). The state with \(m = m_o = \Phi/\Phi_o = eBr_o^2/2\hbar\) is the state with minimal energy at magnetic field \(B\). States with \(m \leq |m_o|\) have radii \(r_m\) smaller than \(r_o\) (see Fig. 7.9 (c)) and an increasing magnetic field pushes more and more states into the inner side of the ring. Within a uniform magnetic field both the radius and the width of the ring state with quantum number \(m\) are proportional to the effective magnetic length \(\lambda\). Therefore both shrink as the strength of the magnetic field or the radial confinement potential is increased.

Now we want to investigate how the amplitude of a Coulomb blockade resonance depends on magnetic field. For this we assume that only one subband (i.e. \(n=0\)) of the ring is occupied. From measurements of the parametric evolution of
7.4. Analytical model for a two-dimensional ring

Figure 7.9: (a) The “effective confinement potential” for \(-7 \leq m \leq 6\) and \(B=0.3T\). (b) Minima \(u_{\text{min}} = u_m(r_{m,\text{min}})\) as a function of \(m\). (c) The position of these minima \(r_m = r_{m,\text{min}}\) as a function of \(m\).

Coulomb blockade peak positions as a function of magnetic field it is possible to determine such a ring spectrum as depicted in Fig. 7.8(a). One expects a single Coulomb blockade resonance peak to move as a ziczac line as indicated in Fig. 7.10 (a), where each kink reflects a level crossing. The amplitude of the Coulomb blockade resonance is determined by the specific wavefunction of the energy level at the Fermi energy. If only one subband in the quantum ring is occupied all of the wavefunctions have roughly the same shape. On the other hand, the position and the width of each wavefunction depends on \(m\) and \(B\).

This is illustrated in Fig. 7.10 (b) where we plotted the position \(r_m\) of the wavefunction for the sweep indicated in (a). The gray (black) points correspond to the gray (black) lines in (a) and therefore to states \(E_{0,m}(B)\) moving downward (upward) as a function of magnetic field. We find that states moving upward have radii \(r_{m,\text{up}}\) smaller than the radii \(r_{m,\text{down}}\) of states moving downward. The difference of \(r_{m,\text{up}} - r_{m,\text{down}}\) increases with magnetic field. Therefore, in a Coulomb blockade sweep one crosses alternately states lying in the outer side of the ring (gray) and states lying in the inner side of the ring or near \(r_o\). For illustration, we plotted in Fig. 7.10 (c) \(|\Psi_{0,m}(r)|^2\) for two neighboring states in (a), namely \(m=0\) (bold state) and \(m=28\) (dotted state). In this case the shift in position between the two wavefunctions is about 6nm. The wavefunction amplitude at source \(|\Psi(r_s)|^2\) and drain \(|\Psi(r_d)|^2\) is therefore larger for states moving downwards than
Figure 7.10: (a) Part of the energy spectrum shown in Fig. 7.8 (a) in the regime where only one radial subband (n=0) is occupied. Dark and gray lines indicate the states at the Fermi energy if one follows one Coulomb peak. (b) \( r_m \) of the states indicated in (a). (c) \( |\Psi_{0,m}(r)|^2 \) for \( m=28 \) (dotted line in (a)) and \( m=0 \) (bold line in (a)) for states moving upwards. The height of the Coulomb blockade peak \( G_{\text{max}} \) (c.f. eq. 2.12 and 2.13) is determined by the level broadening of source and drain \( h\Gamma_s \), \( h\Gamma_d \). Since \( \Gamma_s \propto |\Psi_s(r_s)|^2 \) and analogously \( \Gamma_d \propto |\Psi_d(r_d)|^2 \) we expect that the Coulomb blockade peak height in a single mode ring oscillates as a function of magnetic field with a period of \( \Delta B = \Phi_0/\pi r_s^2 B \).

If more than one mode in the quantum ring is occupied the spectrum becomes more complex (c.f. Fig. 7.8 (a)). On one hand the states cross with different periods in the energy spectrum. On the other hand the states at the Fermi energy belong to different subbands and have therefore different wavefunctions. The wavefunction amplitude at source and drain depends on \( n,m \) and \( B \). Therefore, the clear periodic oscillation of the Coulomb blockade peak heights as a function of magnetic field is lost in rings with more than one occupied subband.

On the other hand the periodic oscillations in the magnetic field regime III, cannot be explained straightforward within the picture of a quantum ring spectrum because we do not observe the typical level crossings expected.
7.5 Conclusions

Here, we realized a double connected cavity, which in some sense can be regarded as quantum ring. We studied the magnetoconductance as a function of ring-lead coupling. We find that the B-periodicity of the main features is similar. However the remarkable difference between the open and the closed ring is, that the effect of the antidot in the cavity is better observable when the ring is opened, in agreement with what is expected in this geometry.

Furthermore, we find that the amplitudes of the single Coulomb blockade peaks oscillate as a function of B, and that the positions of these peaks also oscillate with the same period. Furthermore it is found that in a certain magnetic field regime (III) the peak position of neighboring traces are mainly correlated. In this regime almost no level crossings are observed. The Coulomb blockade peak oscillations can be understood either in the quantum ring model or in the semiclassical picture of bound states around the antidot. On the other hand, the reason for the correlated peak motion is unclear at the moment.

From the above discussion it is clear, that further experimental investigations require a different design of the ring. Namely, it should much more look like a ring. This means that the device should be designed with narrow arms of constant width, in a way that it can be measured with only one mode occupied. Circular symmetry of this device would of course be a nice advantage. Furthermore, the area of the ring should be small enough in order to have a single particle level spacing in the system that enables spectroscopy of single levels. With AFM Lithography it is possible to realize such a structure.
Chapter 8

Summary

In the first part of this work the fabrication of quantum dots by local oxidation of the surface of a GaAs/AlGaAs heterostructure with an AFM is described. We find that these dots are highly tunable by in-plane as well as by top gate electrodes over wide ranges. Furthermore, it is possible to design such quantum dots with a constant capacitance between the tuning gate and the dot. This leads to small shape deformations in these systems. We see that the transfer of the lithographic pattern to the electron gas is excellent. This is caused by operating the quantum dots with positive top gate voltages and by the small depletion lengths of the oxide lines.

Measurements on such quantum dots in high magnetic field reveal that the dots studied here have indeed steep confinement walls. This wall steepness is caused by the patterning method (AFM lithography), by the shallow and high density two-dimensional electron gas used for the LO and the top gate, that covers the whole nanostructure. This wall steepness reduces the changes in size and shape as a gate voltage is tuned.

Another consequence of steep walls is that the Fermi energy in the quantum dots is not much reduced. In combination with the high-density two-dimensional electron gas that we use as a starting material this leads to high quality quantum dots with large electron densities and therefore reduced electron-electron interactions compared to earlier experiments on semiconductor quantum dots. These interactions are further reduced by the screening of the top gate electrode.

In the second part of this work we show that spin effect are observable in rigid quantum dots with reduced interactions. We find that the parametric evolution of Coulomb blockade peaks shows a pronounced pairwise correlation in both the position and the amplitude in small magnetic fields. We interpret this as spin pairing. Furthermore, we observe that this spin pairing is interrupted by kinks as well as other structures in the parametric evolution of the Coulomb blockade peaks. We explain the measured distributions of nearest neighbor spacings as being composed of the two branches of a modified, bimodal Wigner-Dyson distribution, which takes $\xi$ and its fluctuation into account. We have extracted the
average interaction energy $\bar{\xi}$ between states of identical spatial wave functions but opposite spin. We find that for quantum dots with a value of $r_s \approx 0.7$ the interaction induced spin splitting is $\xi \approx \Delta/2$ and therefore spin effects can be observed in this system.

In the last part of this work, we investigate a double connected geometry, namely, we have fabricated and measured a quantum ring in the Coulomb blockade regime. We studied the magnetoconductance as a function of ring-lead coupling and find, that the B-periodicity of the main features is similar. The remarkable difference between the open and the closed ring is, that the effect of the antidot in the cavity is better observable when the ring is opened, in agreement with what is expected in this geometry.

Furthermore, we find that the amplitudes of the single Coulomb blockade peaks oscillate as a function of magnetic field, and that in a certain magnetic field regime the positions of these peaks also oscillate with the same period and are mainly correlated. Therefore almost no level crossings are observed.

Especially the last experiment leaves many questions unanswered and makes further experimental investigations necessary. It would be interesting to study a different design of the ring. Namely, it should much more look like a ring. This means that the device should be designed with narrow arms of constant widths, in a way that it can be measured with only one mode occupied. Circular symmetry of this device would of course be a nice advantage. Furthermore, the area of the ring should be small enough in order to have a single particle level spacing in the system that enables spectroscopy of single levels. With the AFM Lithography it is possible to realize such a structure.
Appendix A

Finding the normalized peak spacings

The normalized peak spacings were determined in the following way: In a first step we subtract the running average \( RA(V_I) \) (eq. 6.3) and convert the spacing in dot energy in units of the spin degenerate single-particle level spacing \( \Delta_{SD} \). This is called \( \delta_i \), where \( i \) denotes the resonance peak.

\[
\delta_i = \frac{C_i}{C_q} \left( \Delta V_I(V_{i,i}) - RA(V_{i,i}) \right) \Delta_{SD}
\]  

(8.1)

In a second step the experimentally observed distribution \( D_{exp}(\delta_i) \) is normalized:

\[
P(\delta_i) = \frac{D_{exp}(\delta_i)}{\int_0^\infty D_{exp}(\delta_i)d\delta_i}
\]  

(8.2)

The \( \delta_i \) fluctuate around zero, and therefore \( s_i = \delta_i + 0.5 - EW \), where \( EW \) is defined in the following way:

\[
EW = \int_0^\infty d\delta_i \cdot P(\delta_i)d\delta_i
\]  

(8.3)

With \( s \) and \( P(s) \) defined in the above way, they of course fulfill the two normalization conditions:

\[
\int_0^\infty P(s)ds = 1
\]

and

\[
\int_0^\infty s \cdot P(s)ds = 0.5
\]

In this form they can be compared to the theoretically expected distributions.
Appendix B

Ballistic weak localization

Weak localization manifests itself in a phase coherent conductor as a dip in the magnetoconductance around zero magnetic field. This phenomenon can be explained by an enhanced backscattering probability due to the constructive interference of two time reversed electron paths \[70\]. Since a magnetic field will break time reversal symmetry this effect is reduced with increasing magnetic field. In a ballistic cavity the electrons are scattered at the boundaries of the cavity rather than at impurities as in the case of disordered systems. The weak localization effect in open ballistic cavities has been shown to be sensitive to the shape of the cavity (chaotic vs. regular) and can be used to probe quantum chaos. The semiclassical theory \[100\] predicts different line shapes of the averaged magnetoconductance peak near \(B=0\), for chaotic and regular geometries, the difference is caused by the different classical distribution of the effective trajectory areas inside the cavities. For open \((g \gg e^2/h\) ) chaotic cavities the the lineshape is a Lorentzian:

\[
\delta g(B) = < g >^{GUE} - < g(B) > = \frac{R}{2} \frac{1}{1 + \left( \frac{B}{B_{cr}} \right)^2}
\]

(8.4)

where \(R\) is the classical reflection probability and \(B_{cr}\) is the crossover field measuring a typical field required to suppress the weak-localization correction.

Weak localization also occurs in almost closed dots, where the average conductance peak height was found to be smaller for \(B=0\) case than for \(B \neq 0\) \[34\].

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Publications

Fabrication and reshaping of semiconductor nanostructures by AFM lithography
S. Lüscher, R. Held, T. Heinzel, K. Ensslin and W. Wegscheider

In-plane gate single-electron transistor in Ga[Al]As fabricated by scanning probe lithography
S. Lüscher, A. Fuhrer, R. Held, T. Heinzel, K. Ensslin and W. Wegscheider

Coulomb blockade in quantum dots fabricated by AFM Lithography
S. Lüscher, A. Fuhrer, R. Held, T. Heinzel, K. Ensslin and W. Wegscheider

Investigation of spin pairing in a semiconductor quantum dot
S. Lüscher, T. Heinzel, K. Ensslin, W. Wegscheider and M. Bichler

Signatures of spin pairing in chaotic quantum dots
S. Lüscher, T. Heinzel, K. Ensslin, W. Wegscheider and M. Bichler

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R. Held, S. Lüscher, T. Heinzel, K. Ensslin and W. Wegscheider

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