Doctoral Thesis

Hubbard model: crossover from one to two dimensions

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Hubbard model: Crossover from one to two Dimensions

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Abstract

In this thesis, we investigate coupled one-dimensional electron systems (in the following called $N$-leg ladders) and the crossover to 2D as the number of coupled chains $N$ goes to infinity (dimensional crossover). The focus will be on the half-filled (i.e., one electron per lattice site) and lightly doped, weakly interacting Hubbard ladders. In this regime, the Hubbard model exhibits a strong competition between antiferromagnetic, umklapp, and Cooper processes and therefore allows for a variety of different phases. For the determination of the physical properties, we use a combination of (one-loop) renormalization group and bosonization techniques.

After an introduction to the physics of the single chain and Luttinger liquids, we discuss in detail the two-leg ladder. For the spinless two-leg ladder, we obtain that a finite interchain hopping $t_\perp > U$ ($U$ is the interaction strength) has the effect of rendering the ladder superconducting. Furthermore, we find a phase where superconductivity coexists with charge density wave correlations. For the spin-1/2 case, we revisit previous results.

We then treat ladders with $N > 2$. First, we study in detail the half-filled case. For a small on-site repulsion $U$, the $N$-leg ladders are equivalent to a (weakly interacting) $N$-band model, characterized by Fermi velocities $v_j$. At half-filling, due to nesting, $v_j = v_{N+1-j}$. Carefully examining the renormalization group equations, we find in the groundstate a decoupling into band pairs $(j, N+1-j)$ (plus a single band for $N$ odd), i.e., the band pair $(j, N+1-j)$ flows at the energy $te^{-ov_j/U} (\alpha \sim 1)$ to a two-leg ladder fixed point and becomes frozen out. As a result, we recover the odd-even effect that is present in Heisenberg spin-ladders: even-leg ladders have a spin-gap, while odd-leg ladders exhibit one gapless spinon-mode [for $N$ odd, the band $(N+1)/2$ behaves as a single chain].

The 2D Hubbard model is expected to be an antiferromagnetic Mott insulator with two gapless magnon-modes. Therefore, the spin-gap present
in even-leg ladders (and the odd-even effect) has to vanish for increasing $N$. In the small-$U$-case, we obtain a double-exponential decrease of the spin-gap as a function of $N$, $E_c \sim t \exp[-a \exp(bN)]$ ($a \ll 1$ and $b \sim 1$). Furthermore, we find an analytical expression for the effective Hamiltonian of the 2D-like antiferromagnetic Mott insulator that appears at energies above $E_c$. Interestingly, the charge-sector is the same as in the two-leg ladder, i.e., there is phase coherence between the bands $j$ and $N+1-j$.

Finally, we investigate the doping away from half-filling. The lightly doped case, we treat as a perturbation of the half-filled low-energy Hamiltonian. Similar as at half-filling, we obtain an odd-even effect: even-leg ladders become a 1D superconductor (Luther-Emery liquid), while odd-leg ladders become a Luttinger liquid. The same phases have been found in numerical treatments of the strongly interacting $t$-$J$ ladders. For increasing doping, phase coherence between all band pairs sets in and the ladders become a 2D-like superconductor, described by an (effective) $d$-wave BCS Hamiltonian. We find, that the origin of the renormalization group instability is a
Zusammenfassung

In der vorliegenden Doktorarbeit untersuchen wir gekoppelte eindimensionale Elektronensysteme (im folgenden N-Bein Leitern genannt) und den Übergang zu 2D wenn die Anzahl gekoppelter Ketten \( N \) nach Unendlich strebt (dimensionaler Übergang). Das Schwerpunkt werden wir auf die halb gefüllten (d.h., ein Elektron pro Gitterplatz) und leicht gedopten, schwach wechselwirkenden Hubbard Leitern legen. In diesem Parameterbereich herrscht im Hubbard-Modell eine starke Konkurrenz zwischen antiferromagnetischen, umklapp und Cooper Prozessen und somit sind verschiedene Phasen möglich. Für die Bestimmung der physikalischen Eigenschaften benützen wir eine Kombination von Renormierungsgruppen und Bosonizierungs-Techniken.
Leitern mit einer ungeraden Anzahl Beinen ein Spin-Mode masselos ist [für \( N \) ungerade verhält sich das Band \((N + 1)/2\) wie eine einzelne Kette].

Das 2D Hubbard-Modell ist (höchstwahrscheinlich) ein antiferromagnetisches Modell.