Doctoral Thesis

Bubble hydrodynamics in large pools

Author(s):
Kubasch, Jens Holger

Publication Date:
2001

Permanent Link:
https://doi.org/10.3929/ethz-a-004231804

Rights / License:
In Copyright - Non-Commercial Use Permitted

This page was generated automatically upon download from the ETH Zurich Research Collection. For more information please consult the Terms of use.
Bubble Hydrodynamics in Large Pools

A dissertation submitted to the

SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZÜRICH

for the degree of

Doctor of Technical Science

presented by

JENS HOLGER KUBASCH

Dipl.-Ing. (phy.)
born on January 27, 1971 in Offenbach/Main
citizen of Germany

accepted on the recommendation of

Prof. George Yadigaroglu, Zürich, examiner
Prof. Agustín Alonso, Madrid, co-examiner

2001
Acknowledgments

This work was founded by the Paul Scherrer Institute (PSI) in Villigen, Switzerland and by the Swiss Federal Institute of Technology (ETH) in Zürich, Switzerland.

Professor George Yadigaroglu, head of the Nuclear Engineering Laboratory (LKT) of ETH, supervised this work and gave his support to keep the work on track. He left me much freedom, but was always ready to help in an uncomplicated manner.

Professor Prof. Agustín Alonso of Polytechnic University of Madrid followed the work from the beginning and reviewed this thesis. I appreciated his support and expertise.

Salih Güntay, head of the Source Term Section in the Laboratory for Thermohydraulics of PSI, initiated this work. As a member of his group, I received a lot of support from many people of the team. In particular the discussions with him and Dr. Detlef Suckow, who was supervising many technical details of this work, were very helpful. They have shown much confidence in me. This made it possible to follow my own ideas and to make my own experience with running a project. Special thanks are going to the team members Hauke Schütt and Hubert Leute for solving computer problems with me and showing me the way through the administrative jungle at PSI.

Dr. Stephen Lomperski was always ready to listen to my daily problems in the laboratory and we had a lot of fruitful fights. He was reading the thesis first and gave useful hints of how to explain things for somebody from the outside world. The way that Dr. Jonathan Birchley was answering my mathematical questions, made me always think of new things. This was very effective for the work.

Several people have helped me in an uncomplicated and often quick way to set up the experimental facility. In particular the technical support from Max Fehlmann of ETH over the course of more than two years was very valuable. Huseyin Mutlukal and Markus Schriber of PSI assembled the big parts of the facility and they made sure that I never flooded the laboratory because of a leaking pipe.

Beside the interesting work at PSI, I will miss the lunch breaks. They were filled either with discussions about everything but not my work with my colleges Hans-Peter and Jens or (non)competitive hill runs with Steve and Jon.

Last, but not least, many thanks to my parents Horst and Astrid Kubasch. They made it possible to go my way by supporting me in the best manner they can.
## Contents

Nomenclature ........................................ VIII

Abstract ............................................. XI

Zusammenfassung .................................. XIII

1 Introduction ....................................... 1
   1.1 Motivation ..................................... 1
   1.2 Definition of the Problem ..................... 1
   1.3 Literature Survey .............................. 3
      1.3.1 Pool Experiments ......................... 4
   1.4 Objectives and Outline of the Present Work .... 8

2 A Bubble Plume Model ............................ 10
   2.1 Definition of Bubble Plume Parameters ........ 11
   2.2 Simplifications ................................ 11
   2.3 Conservation Equations ....................... 13
   2.4 Solution of the Conservation Equations ....... 15

3 Instruments and Measurement Methods .......... 18
   3.1 Selection of the Instrumentation .............. 18
   3.2 Double Optical Sensor ....................... 23
      3.2.1 Phase Discrimination and Void Fraction Measurements .......... 25
      3.2.2 Bubble Velocity Measurements ............. 30
      3.2.3 Bubble Chord Length Distribution ........ 32
   3.3 Hot-Film Anemometer ......................... 34
      3.3.1 Calibration Tests ......................... 35
      3.3.2 Sensor Sensitivity to Flow Angle .......... 38
      3.3.3 Signal Analysis in Two-Phase Flow ........ 42
      3.3.4 Measuring the Mean Water Velocity in the Pool ....... 45

4 Pool Tests ....................................... 47
   4.1 Experimental Apparatus ....................... 47
   4.2 Experimental Procedure ...................... 49

5 Experimental Results and Discussion .......... 51
   5.1 Test Matrix .................................... 51
   5.2 Flow Patterns and Visualization ............. 53
5.2.1 Bubble Plume Oscillation ........................................ 53
5.2.2 Flow Regime above the Nozzle ................................. 53
5.2.3 Flow Regime in the Bubble Plume .............................. 57
5.3 Bubble Plume Characteristics ...................................... 60
  5.3.1 Void Fraction ..................................................... 60
  5.3.2 Bubble Rise Velocity ........................................... 72
  5.3.3 Water Velocity .................................................. 79
  5.3.4 Relative Phase Velocity ....................................... 87
  5.3.5 Bubble Chord Length Distribution ......................... 92
  5.3.6 Tests with a Pool Depth of 3 m .............................. 95
  5.3.7 Volumetric Air Flow Rate .................................... 97
5.4 Summary of the Experimental Findings ....................... 101

6 Modeling .................................................................. 104
  6.1 Empirical Parameters .............................................. 104
  6.2 Model and Experiment ............................................. 110
  6.3 Conclusion about Modeling ....................................... 112

7 Summary and Conclusions ........................................ 115

References ................................................................ 118

A Testing and Calibrating the DOS ................................. i
  A.1 Theory ...................................................................... i
  A.2 Two Phase Loop ..................................................... iii
  A.3 Test Matrix ........................................................... iv
  A.4 Experimental Results .............................................. v
  A.5 Horizontally and Vertically Oriented DOS ................. x

B Calibrating and Testing the HFA ................................ xv
  B.1 Tow Tank ............................................................... xv
  B.2 Calibration of the HFA ............................................. xvii
  B.3 Orientation Effect .................................................. xviii

C Experimental Facility ............................................... xxi
  C.1 Water Loop and Air Supply .................................. xxi
  C.2 Tank and Traversing System ................................. xxii
  C.3 Process Control and Data Acquisition .................. xxv
D  Preliminary Pool Tests  xxvii
   D.1  Averaging  xxvii
   D.2  Settings for Bubble Rise Velocity Measurements  xxix

E  Tables with Experimental Results  xxxii

F  Momentum flux in the Bubble Plume  xxxv

List of Figures  xxxvi
List of Tables  xxxix
Nomenclature

- $A$ [m$^3$]: cross-section
- $B_c, B_{ub}, B_w$ [1]: Expansion factors
- $b_c, b_{ub}, b_w$ [m]: profile widths
- $d_0$ [m]: nozzle diameter
- $d_b$ [m]: bubble diameter
- $E$ [V]: voltage
- $Fr_m$ [1]: modified Froude number
- $g$ [m/s$^2$]: acceleration due to gravity
- $H$ [m]: pool depth
- $H_0$ [m]: represents hydrostatic pressure at normal conditions ($H_0 = \frac{p_0}{\rho_w g} = 10340$ mm)
- $\dot{j}_a, \dot{j}_w$ [m/s]: volumetric air and water flux
- $M, M_0, M_B$ [N]: total momentum flux, initial momentum flux, momentum flux due to buoyancy
- $n_a, n_C$ [1]: number of HFA sample in air and within the calibration range
- $p, p_0$ [N/m$^3$]: pressure, pressure at normal conditions
- $P_c$ [1/m]: bubble chord length PDF
- $Q_a$ [m$^3$/s]: injected air flow rate (given for normal conditions)
- $Re$ [1]: Reynolds number
- $r = (z, r, r_{\phi})$ [m, m, 1]: position vector and components (cylindrical coordinates)
- $r_{c,e}, r_{c,ub}, r_{c,w}$ [m]: displacements of maximum from the centerline
- $T$ [K]: temperature
- $T_{meas}$ [s]: measuring time
- $t$ [s]: time
- $t_{corr}$ [s]: correction time (modified threshold method)
- $t_{res.}$ [s]: bubble residence time on sensor tip
- $t_{rise}$ [s]: signal rising time
- $u = (u, v, w)$ [m/s]: velocity vector and components
- $U_0$ [m/s]: mean air velocity at the nozzle exit
- $U_{eff}$ [m/s]: velocity reading of the HFA
- $U_{rise}$ [m/s]: bubble rising velocity in stagnant water
- $U_{low}$ [m/s]: velocity of the HFA sensor in the tow tank
- $u_b$ [m/s]: bubble rising velocity in the pool
- $u_w$ [m/s]: axial water velocity
- $u^+_{w}, u^-_{w}$ [m/s]: high and low limit for HFA velocity reading
- $u_{b,c}, u_{w,c}$ [m/s]: centerline profiles: bubble rising and axial water velocity
- $u_r$ [m/s]: relative phase velocity $u_b - u_w$
- $We$ [1]: Weber number
- $\dot{V}_a$ [m$^3$/s]: volumetric air flow rate
- $\dot{V}_w$ [m$^3$/s]: volumetric water flow rate
### Nomenclature

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td></td>
<td>phase identification function</td>
</tr>
<tr>
<td>$y$</td>
<td>$[m]$</td>
<td>bubble chord length</td>
</tr>
<tr>
<td>$z_m$</td>
<td>$[m]$</td>
<td>distance from the virtual origin of the bubble plume (model)</td>
</tr>
<tr>
<td>$z_0$</td>
<td>$[m]$</td>
<td>length of the ZFE</td>
</tr>
<tr>
<td>$z_{0,e}, z_{0,ub}, z_{0,w}$</td>
<td>$[m]$</td>
<td>virtual origins</td>
</tr>
<tr>
<td>$\Delta z_{tip}$</td>
<td>$[m]$</td>
<td>tip distance (DOS)</td>
</tr>
</tbody>
</table>

### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha, \varphi$</td>
<td></td>
<td>yaw and pitch angle (HFA)</td>
</tr>
<tr>
<td>$\beta, \xi$</td>
<td></td>
<td>exponents for power laws</td>
</tr>
<tr>
<td>$\gamma$</td>
<td></td>
<td>momentum amplification factor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$[N/m]$</td>
<td>surface tension ($\sigma_w = 0.072 N/m$)</td>
</tr>
<tr>
<td>$\Delta \tau_{res}$</td>
<td>$[s]$</td>
<td>relative bubble residence time at the DOS</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td></td>
<td>void fraction</td>
</tr>
<tr>
<td>$\epsilon_m$</td>
<td></td>
<td>radial void fraction profile maximum</td>
</tr>
<tr>
<td>$\epsilon_c, \epsilon_{c,0}$</td>
<td>$[1]$</td>
<td>centerline profile, void fraction threshold (void fraction at $z_0$)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$[-]$</td>
<td>parameter to set the void fraction threshold (DOS)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$[1]$</td>
<td>ratio between $b_c$ and $b_w$</td>
</tr>
<tr>
<td>$\Psi$</td>
<td></td>
<td>ratio of bubbles considered to calculate the mean bubble velocity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$[kg/m^3]$</td>
<td>density</td>
</tr>
<tr>
<td>$\rho_{0,0}$</td>
<td>$[kg/m^3]$</td>
<td>air density at normal conditions</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>$[-]$</td>
<td>phase identification function, obtained by the DOS</td>
</tr>
</tbody>
</table>

### Indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>air</td>
</tr>
<tr>
<td>$b$</td>
<td>bubble</td>
</tr>
<tr>
<td>$c$</td>
<td>centerline</td>
</tr>
<tr>
<td>$DOS$</td>
<td>double optical sensor</td>
</tr>
<tr>
<td>$gas$</td>
<td>gas</td>
</tr>
<tr>
<td>$HFA$</td>
<td>hot-film anemometer</td>
</tr>
<tr>
<td>$liq.$</td>
<td>liquid</td>
</tr>
<tr>
<td>$mod.$</td>
<td>modified threshold method</td>
</tr>
<tr>
<td>$thres.$</td>
<td>threshold</td>
</tr>
<tr>
<td>$ub$</td>
<td>bubble velocity</td>
</tr>
<tr>
<td>$us, ds$</td>
<td>up-stream, down-stream</td>
</tr>
<tr>
<td>$w$</td>
<td>water</td>
</tr>
</tbody>
</table>
Abbreviations

- CFD: computer fluid dynamic
- DOS: double optical sensor
- HFA: hot-film anemometer
- LDA: laser Doppler anemometer
- PIV: particle image velocimetry
- VOF: volume of fluid
- ZEF: zone of established flow
- ZFE: zone of flow establishment
- ZSF: zone of surface flow

Operators

\[
\bar{g}^e = \frac{1}{N} \sum_{i=1}^{N} g_i, \quad g = (g_1, \ldots, g_N) \quad \text{ensemble-average}
\]

\[
\langle G \rangle_A = \frac{1}{A} \int_A G(a) \, da \quad \text{cross-sectional average}
\]

\[
\bar{G} = \frac{1}{T} \int_0^T G(t) \, dt \quad \text{time average}
\]
Abstract

This thesis presents extensive and detailed information about two-phase flow in large pools, obtained from local measurements. The data give a better understanding of two-phase flow under pool scrubbing conditions, i.e., injection of high air flow rates through a single nozzle into large water pools. The experiments were carried out in the context of severe accident research for advanced nuclear power plants.

The pool was 1 m in diameter and pool depths up to 3 m were investigated. Air was injected through a single nozzle (diameters of 5, 10, and 20 mm) at the bottom. The flow rate was varied between 0.42 and 3.33 dm³/s. The upper limit was set to minimize interactions between the two-phase flow and the walls, evidenced by oscillation of the entire bubble plume. Collected without strong plume oscillations, the results are also applicable to pools with diameters larger than 1 m.

The local measurements were performed with a double optical sensor and a hot-film anemometer. Void fraction, bubble and water velocity, and bubble chord length distributions were measured. Signal processing and the calibration of the sensors are discussed in the first part of the work.

The two-phase flow area in the pool was separated into a zone of flow establishment (ZFE) close to the nozzle and a zone of established flow (ZEF) further downstream. In the ZFE, either large individual bubbles or jets were observed at the nozzle exit, depending on the air flow rate and the nozzle diameter. The bubble plume in the ZEF is buoyancy driven and the initial momentum of the injected air plays no significant role. This was shown by tests at constant air flow but with different nozzle diameters: The variation of the void fraction and of the bubble velocity along the pool centerline depended only on the air flow rate and was practically unaffected by the nozzle size.

Void fraction, bubble and water velocity radial profiles collected at different elevations provided information about the expansion of the bubble plume in the horizontal direction. The profiles were fitted with Gaussian curves and their maximum and width were determined. The data show that a bubble plume consists of a bubble core and an entrained water flow area that expands further than the bubble core. The bubbles rise in the entrained water flow, and so the bubble velocity is considerably higher than that of individual bubbles in stagnant water. However, the mean relative phase velocity determined from the bubble and water velocity measurements was higher than that of individual bubbles in stagnant water. Furthermore, the relative velocity was also not constant along the pool diameter. The fact that the bubbles rise typically in swarms through the pool and not as single bubbles may explain these observations.

Bubble chord length distributions were fitted with log-normal distributions. The data indicate
that bubble breakup is dominant over bubble coalescence in a bubble plume.

A semi-empirical bubble plume model was used to describe globally the flow. The model considers water entrainment, the relative phase velocity, and the different rates of expansion of the entrained water flow and of the bubble core. The empirical parameters that are required for the model were derived from the experimental data. Similar models are used for large plumes in lakes in relation to venting phenomena. However, under pool scrubbing conditions, the air is injected with higher momentum and the void fraction is higher. The empirical parameters obtained in the present work are comparable to the results from other experiments related to lake venting. For the two-phase flow situations studied here, good agreement was found between the experiments and model predictions. It points out, that the bubble plume model is applicable to describe the bubble hydrodynamics in large pools under pool scrubbing conditions.
Zusammenfassung


Die untersuchte Wasservorlage hatte einen Durchmesser von 1 m und Wassertiefen bis zu 3 m wurden eingestellt. Die Luft wurde durch eine einzelne Düse (Durchmesser 5, 10 und 20 mm) am Boden eingeleitet und Tests mit Durchflussraten zwischen 0.42 und 3.3 d\(\text{m}^3/\text{s}\) wurden durchgeführt. Die maximale Durchflussrate wurde so gewählt, dass Wechselwirkungen zwischen der Zwei-Phasenströmung und den Wänden klein sind. Dies wird durch geringe horizontale Oszillation der gesamten Blasenwolke innerhalb der Wasservorlage angezeigt. Durch die geringe Oszillation der Strömung, sind die Messergebnisse auch anwendbar auf Wasservorlagen die einen grösseren Durchmesser als 1 m haben.


Das Zwei-Phasengebiet in der Wasservorlage wurde in eine Einlaufzone (ZFE), oberhalb der Düse, und eine Zone mit ausgebildeter Strömung (ZEF), die weiter stromabwärts beginnt, aufgeteilt. In Abhängigkeit vom Düsendurchmesser und Luftstrom wurde in der Einlaufzone entweder die Ablösung grosser einzelner Blasen oder die Bildung eines Luftstrahls beobachtet. Die Ausdehnung der Blasenwolke im Bereich der ausgebildeten Strömung ist abhängig vom Auftriebsfluss, bzw. vom eingeleiteten Luftstrom. Keinen Einfluss auf die Entwicklung der Blasenwolke hat der Eintrittsimpuls der Luft am Düsenausgang. Dies wurde durch Tests mit konstantem Luftdurchfluss und verschiedenen Düsendurchmessern gezeigt. Der Verlauf von Gasgehalt und Blasengeschwindigkeit entlang der Mittelachse der Wasservorlage hängt nur vom Luftdurchfluss ab, und praktisch nicht vom Düsendurchmesser.

Radiale Profile des Gasgehalts und der Blasen- und Wassergeschwindigkeiten wurden in verschiedenen Abständen von der Düse gemessen, um die horizontale Ausbreitung der Blasenwolke zu beschreiben. Die Profile wurden mit Gausskurven gefittet um deren Maximum und Breite zu bestimmen. Die Daten zeigen, dass die Blasenwolke aus einem Blasenkern

Blasensehnelängenverteilungen wurden mit logarithmischen Normalverteilungen beschrieben. Die Daten weisen daraufhin, dass in einer Blasenwolke der Blasenzerfall stärker ausgeprägt ist als die Blasenvereinigung.

1 Introduction

1.1 Motivation

Two-phase flows in large pools are of importance for several industrial applications. Typical examples include: (1) venting of steam, non-condensable gases, and aerosol mixtures into water pools in nuclear power plants and chemical reactors, (2) gas stirring of liquid metal ladles, (3) aeration in water purification and waste treatment plants, (4) and producing barriers against crude oil spreading across a water surface. Application-specific experimental and theoretical investigations have been carried out for all of the above examples. For instance: integral mixing phenomena are studied in water columns to design chemical reactors and water circulation induced by air injection into lakes is examined to optimize lake destratification. There is ongoing work to gain a better understand of the application-specific flow phenomena, including more detailed modeling of the two-phase flows with CFD codes. Improved modeling requires more detailed information about the flow phenomena to validate and develop the models.

The background of the present study falls within the context of pool scrubbing experiments and modeling efforts at the Paul Scherrer Institute (PSI). During a hypothetical severe accident in a nuclear power plant, the large pools available in the containment are foreseen to retain radioactive fission products in the form of aerosol particles and gaseous iodine species. Starting in 1988, an experimental programme was carried out at PSI in the POSEIDON facility [19] (Pool Scrubbing Effect on Iodine Decontamination) to assess aerosol and iodine retention models in integral computer codes like BUSCA [66], SPARC [63], and SUPRA [80]. Code computations were compared with data from POSEIDON tests and nine other experimental programs worldwide in the final report for the European Commission research project about source term fission product behaviour for nuclear safety (Berzal et al. [26]). It is shown that the efficiency of aerosol scrubbing is under-predicted by the computer codes. Therefore, Berzal et al. [26] suggested, beside other improvements, a more complete modeling of two-phase flow under pool scrubbing conditions and its verification by experiments.

1.2 Definition of the Problem

The main characteristics of the two-phase flow investigated here are illustrated in figure 1.1. An axially symmetric flow is assumed in the schematic. The zone close to the point of air injection is called the zone of flow establishment (ZFE). The flow regime in this zone depends strongly on the injector design and the air flow rate. Further away from the injector, a so-called bubble plume is produced in the zone of established flow (ZEF). The bubble plume consists of a bubble core and a surrounding entrained water flow with upward movement. Due to water entrainment
from the bulk, the bubble plume expands while it is rising. The expansion of the entrained water flow is wider than that of the bubble core. Gaussian curves are most commonly used to represent the void fraction and water velocity profiles in the bubble plume. The zone near the pool surface is called the zone of surface flow (ZSF), where entrained water is expelled in the radial direction and bubbles are released into the atmosphere.

![Figure 1.1: Schematic diagram of a two-phase flow investigated here.](image)

A distinction is made in figure 1.1 between a bubble plume in a confined and infinite pool, mean a pool with and without walls. The water in a confined pool re-circulates in large eddies in the bulk and this causes oscillation of the entire bubble plume, while the entrained water in an infinite pool flows into the bubble plume radially and the bubble plume is stationary. In most practical cases, the presence of walls cannot be neglected. The bubble plume oscillating in a confined pool causes stronger time-averaged radial expansion of the flow in comparison
to a stationary bubble plume in an infinite pool. Time-averaged parameters can describe the integral behaviour of such an oscillating bubble plume when the averaging time is much longer than the characteristic time scale of the oscillation. If oscillations occur, the time-averaged measurements will show greater widths and smaller centerline velocities and void fractions than one would find with a stationary plume.

However, the injected air flow rate has a stronger effect on bubble plume expansion than the oscillation. For pool scrubbing in nuclear power plants, high flow rates are injected through single or multiple nozzles and generate jets in the ZFE. The uncontrolled air injection produces bubbles of all shapes and sizes in the ZEF. As an example, single nozzles with diameters between 5 and 20 mm were used in the POSEIDON tests. Jets are generated above these nozzles by injecting air flow rates between 1 to 4 dm$^3$/s.

1.3 Literature Survey

Several experiments for different applications were performed in the past to investigate two-phase flow in pools. Three research institutes conducted experiments to understand the venting of steam and non-condensable gases in nuclear power plants: EPRI\textsuperscript{1} [64, 16], CIEMAT\textsuperscript{2} [36] and PSI\textsuperscript{3} [14]. Bubble plumes in lakes were studied by Milgram [57] and Goossens [32] to model lake venting phenomena. The review by Mazumdar [55] summarizes the investigations for gas stirring of liquid metal ladles by studying air injection into water pools. Water/air experiments are applicable for ladle/gas systems because water at room temperature and molten steel at 1750 K have equivalent kinematic viscosities. Several types of bubble columns and multiphase reactors were studied to optimize processes in the chemical industry. Dudukovic [22] recently reviewed work in that area and identified open questions. One main point is the direct scale-up from small reactors that are well instrumented to large units with the help of CFD.

A number of authors have investigated the various flow zones described in figure 1.1. Davidson [17] derived a correlation that relates the initial bubble volume in the ZFE $V_{\text{initial}}$ to the air flow rate $\dot{V}_a$ and to the acceleration due to gravity $g$.

$$V_{\text{initial}} = 1.38 \frac{\dot{V}_a^{1.2}}{g^{0.6}}$$

Correlation (1.1) is very well accepted in the literature and basically every paper dealing with gas injection into liquid refers to Davidson [17].

\textsuperscript{1}Electric Power Research Institute, USA
\textsuperscript{2}Centro de Investigaciones Energeticas Medioambientales y Tecnologicas, Spain
\textsuperscript{3}Paul Scherrer Institute, Switzerland
Hoefele [38] and Zhao [82] described the transition from bubbling to jetting above a nozzle in terms of a modified Froude number $F_{rn}$ and Weber number $W_e$, respectively.

$$F_{rn} = \frac{\rho_{gas} U_0^2}{g (\rho_{liq.} - \rho_{gas}) d_0} \quad W_e = \frac{\rho_{liq} U_0^2 d_0}{\sigma}$$

The variables are the densities for the gas $\rho_{gas}$ and liquid $\rho_{liq.}$, gas velocity at the nozzle $U_0$, nozzle diameter $d_0$, and liquid surface tension $\sigma$.

Experiments to study bubble plumes in the ZEF are listed in table 1.1 and discussed in the following subsection.

Measurements in the ZSF and in the gas flow above the surface were performed by Friedl [29] and Engebretsen [25] to study the interaction between bubble plumes above leaking gas pipelines in the sea and the resultant gas plume in the atmosphere.

1.3.1 Pool Experiments

Detailed information about two-phase flow in a large pool can be obtained from local measurements. Table 1.1 presents a list of experiments where investigators used local measuring techniques to investigate bubble plume expansion in the ZEF. The studies are separated into three groups: (1) Experimental bubble plume studies where the flow is characterized by empirical correlations, (2) an integral bubble plume model that requires empirical parameters was used to study lake venting phenomena, and (3) well-instrumented small-scale experiments were performed. The following paragraphs briefly discuss each experiment.

Large-Scale Bubble Plume Studies

Comprehensive tests with single- and multiple-nozzle injectors were performed by EPRI [64, 16]. Movies were taken with a camera and were analyzed to determine the initial bubble volume and the local bubble sizes and velocities. The initial bubble volume in the ZFE was correlated with a Weber number and a bubble diameter of about 6 mm was found in the ZEF, independent on the air flow rate. Bubble rise velocities between 0.8 and 1 m/s were observed at the pool centerline.

Castello-Branco [8] performed six tests in a large-scale model of a metallurgical ladle. The test matrix consisted of a non-systematic combination of five different nozzle diameters and five air flow rates. Jets were generated in the ZFE for each test and strong bubble plume oscillation was observed in the ZEF due to high turbulence throughout the pool. Under these conditions, asymmetric radial profiles were measured for various flow parameters when averaging the signals from the local sensors over 300 s. The variation of flow parameters along the pool centerline were measured and the length of the ZFE $z_0$ was defined on the basis of
### Table 1.1: Pool experiments listed in chronological publication order (H: pool depth, D: pool diameter ub: bubble rise velocity, e: void fraction, uw: water velocity).

<table>
<thead>
<tr>
<th>Reference</th>
<th>Pool dimensions in [m] and injector</th>
<th>air flow rate in [dm$^3$/s]</th>
<th>flow parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EPRI [64, 16]</strong></td>
<td>$H = 2.4/4.9$, $D = 1.8/3$ various injector systems</td>
<td>0.1 - 4.5</td>
<td>$u_b$, bubble sizes</td>
</tr>
<tr>
<td><strong>Castello-Branco [9, 8]</strong></td>
<td>$H = 1.6$, $D = 2.25$ nozzle: $d_0 = 2 - 10 \text{ mm}$</td>
<td>2.5 - 7.8</td>
<td>$u_b$, e, $u_w$</td>
</tr>
<tr>
<td><strong>Kobus [47]</strong></td>
<td>channel: $H = 4.5/2 \times (8 \times 280)$ $d_0 = 0.5 - 5 \text{ mm}$</td>
<td>0.4 - 5.8</td>
<td>$u_w$</td>
</tr>
<tr>
<td><strong>Goossens [32]</strong></td>
<td>lake: depth 15 - 27 m lab: $H = 2.5$, $D = 2.5 - 4$ porous plug</td>
<td>85 - 133</td>
<td>$u_w$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.03 - 0.09</td>
<td>$u_w$, e</td>
</tr>
<tr>
<td><strong>Fannelop [28]</strong></td>
<td>$H = 10.0 \times (10.5 \times 10.5)$ nozzle: $d = 25 \text{ mm}$</td>
<td>5 - 22</td>
<td>$u_w$</td>
</tr>
<tr>
<td><strong>Milgram [57, 58]</strong></td>
<td>lake: depth 50 m lab: $H = 3.66$, $D = 1.65$ nozzle: $d_0 = 1 \text{ mm}$</td>
<td>24 - 590</td>
<td>$u_w$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2 - 2.2</td>
<td>$u_w$, e</td>
</tr>
<tr>
<td><strong>Hugi [40]</strong></td>
<td>$H = 3.0 \times (3.0 \times 5.8)$ porous plug, nozzle: $d = 1 \text{ mm}$</td>
<td>0.001 - 0.01</td>
<td>$u_w$</td>
</tr>
<tr>
<td><strong>Friedl [29]</strong></td>
<td>$H = 0.66 - 0.85 \times (1 \times 1)$ nozzle: $d = 6 \text{ mm}$</td>
<td>0.39 - 1.49</td>
<td>$u_w$, e</td>
</tr>
<tr>
<td><strong>Tacke [72]</strong></td>
<td>$H = 0.5$, $D = 0.3$ nozzle: $d_0 = 0.5 - 4 \text{ mm}$</td>
<td>0.1 - 2.7</td>
<td>e</td>
</tr>
<tr>
<td><strong>Anagbo [1]</strong></td>
<td>$H = 0.4$, $D = 0.3$ porous plug</td>
<td>0.2 - 1.2</td>
<td>$u_b$, e, $u_w$ in the bulk</td>
</tr>
<tr>
<td><strong>Sheng [81]</strong></td>
<td>$H = 0.76 \times (0.56 \times 0.56)$ nozzle: $d_0 = 4 \text{ mm}$</td>
<td>0.05 - 0.2</td>
<td>$u_b$, e, $u_w$</td>
</tr>
<tr>
<td><strong>Iguchi [41, 42]</strong></td>
<td>$H = 0.4$, $D = 0.2$ nozzle: $d_0 = 1 - 5 \text{ mm}$</td>
<td>0.01 - 0.04</td>
<td>$u_b$, e, $u_w$</td>
</tr>
</tbody>
</table>

void fraction measurements ($e(z_0) = 0.5$). A correlation between $z_0$ and the modified Froude Number $F_{rm}$ was proposed. The change in the flow parameters along the pool centerline was related to $z_0$ to establish dimensionless correlations. The findings correspond well with data from other experiments.
Lake Venting Studies and Bubble Plume Model

The objective of the experiments studying lake venting phenomena was to determine the entrainment coefficient \( \alpha \) that describes the amount of water entrainment and the ratio \( \lambda \) between the widths of the bubble core and entrained water flow. Table 1.2 summarizes the results.

<table>
<thead>
<tr>
<th>Reference</th>
<th>( \alpha ) [1]</th>
<th>( \lambda ) [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kobus [47]</td>
<td>0.06 – 0.08</td>
<td>-</td>
</tr>
<tr>
<td>Goossens [32]</td>
<td>0.06 – 0.07</td>
<td>1 ± 0.4</td>
</tr>
<tr>
<td>Fannelop [28]</td>
<td>0.07 – 0.1</td>
<td>0.65 ± 0.10</td>
</tr>
<tr>
<td>Milgram [57, 58]</td>
<td>0.04 – 0.13</td>
<td>0.8</td>
</tr>
<tr>
<td>Hugi [40]</td>
<td>0.02 – 0.06</td>
<td>0.4 – 0.8</td>
</tr>
<tr>
<td>Friedle [29]</td>
<td>0.07 – 0.08</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 1.2: Empirical parameters for the bubble plume model.

In 1968, Kobus [47] first investigated bubble plume expansion with a small turbine measuring water velocities at different radial and axial positions in a pool. The data show that bubble plume expansion is independent on the nozzle diameter and mainly controlled by the air flow rate. It was found that the entrainment coefficient increases with the air flow rate.

Goossens [32] did extensive work to scale-up laboratory experiments to bubble plumes in lakes. The parameters \( \alpha \) and \( \lambda \) were determined by laboratory experiments to calculate the venting phenomena in lakes. For comparison, measurements in a lake were performed. The data from the lake experiments indicate that the expansion is much lower than predicted by calculations. This finding is explained by higher turbulence content in laboratory experiments due to a higher bubble density.

Milgram [57, 58] analyzed his data and laboratory measurements from Fannelop [28]. A momentum amplification factor that considers the momentum flux carried in the turbulent bubble plume was determined from the data and introduced into the bubble plume model. This improvement resulted in a better agreement between laboratory experiments and lake venting phenomena under realistic conditions.

Hugi [40] injected air through a single nozzle and porous plugs with different pore sizes to study the influence of the injector on the entire bubble plume. Much lower values for \( \alpha \) and \( \lambda \) were found for the nozzle injector in comparison to the porous plugs. This is understandable since a line of bubbles was generated with the nozzle and bubble swarms were produced with porous plugs. Friedl [29] performed, in addition to the measurements in the ZSF, experiments in the ZEF.

The entrainment coefficient \( \alpha \) in table 1.2 shows a wide scatter. Milgram [57] correlated the
data from different experiments with the air flow rate and a Froude number. An increase in $\alpha$ for bubble plumes generated by high-momentum injection is indicated by the correlations, but there is still a wide scatter of the data. The pool size and the injector design may influence the water entrainment.

The parameter $\lambda$ requires the measurement of void fraction and water velocity profiles. Void fraction profiles were only measured by Goossens [32] and Milgram [57, 58] in their laboratory experiments. The quality of the data is quite low because only a few data points were measured in the pools. Other investigators compared velocity measurements with videos of the bubble plume. Not taking into account the extreme values in table 1.2, the most probable value for $\lambda$ is about 0.8.

**Small-Scale Pool Tests**

For all small-scale experiments, measurements were performed with laser Doppler anemometers and impedance sensors to study the water and air flows, except the work carried out by Tacke [72]. Only void fraction was measured in that work with impedance sensors. The experimental results were used to describe the flow with dimensionless correlations and to compare air injection into a water pool with a nitrogen/mercury system.

Anagbo [1] measured axial and radial mean water velocity components in the bulk to specify the re-circulation pattern produced by the large eddies. For the pool size examined by Anagbo [1], the radial flow zone below the surface had a height of about 0.06 m and water was flowing downward in a region 0.03 m away from the pool wall before it was again entrained into the bubble plume. Close to the bubble plume edge, the axial velocity component of the water flow was about five times higher than the radial velocity component. The re-circulation pattern was only slightly dependent on the air flow rate. A sudden increase in the mean bubble size from 4 to 30 mm was observed by increasing the air flow rate from 0.2 to 0.6 $dm^3/s$. This was explained by bubble coalescence above the porous plug.

Sheng [81], in a small-scale experiment measured isotropic turbulence in the two-phase flow region of the bubble plume and determined coefficients for a modified $k - \varepsilon$ model. A two-phase mixture model that includes the modified $k - \varepsilon$ model was used to compute the flow. This resulted in good agreement between experimental data and computations.

Iguchi [41] measured flow parameters along the pool centerline and derived dimensionless correlations for void fraction and bubble frequency. The data from his small-scale experiments were later used by Castello-Branco [9] who found good agreement with large-scale experiments. In a second study, Iguchi [42] investigated bubble plume expansion and suggested a modified drift flux model for describing the flow. The modifications and the applicability of the drift flux model were left open to discussion. In addition to this, a large amount of data is provided by
Iguchi [42] specifying turbulence structures in the bubble plume.

This literature survey shows that many experiments and theoretical studies were performed in the past to characterize bubble formation above a single nozzle (see also the book from Cliff [13]). Much less literature is available that describes the bubble plume. The well instrumented small-scale experiments provide detailed information about flow phenomena. The experiments that study lake venting phenomena have the disadvantage that practically no data are available about the bubbly core. This is understandable because the mean void fraction is very low ($\leq 0.02$). The bubble plume is typically modeled using information about the entrained water flow and some basic assumptions for the bubble flow. The experiments that describe the flow with empirical correlations provide a lot of data that are important for pool scrubbing conditions, that is high air flow rates injected through a nozzle into a pool, but both studies have their limitations. Water velocity and void fraction were not measured in the EPRI experiments [64, 16] and the tests by Castello-Branco [8, 9] are difficult to interpret because of the strong bubble plume oscillations.

1.4 Objectives and Outline of the Present Work

A comprehensive experimental data base is not available to improve the modeling of bubble plumes under pool scrubbing conditions. Therefore a sophisticated large-scale test facility was build at PSI to carry out a test series and to create a data base to be used for model improvement and validation. The pool is 1 m in diameter and the maximum depth is 4.4 m. Local void fraction, bubble and water velocities, and bubble chord lengths were measured with double optical sensors and a hot-film anemometer within the pool. The bubble plume model used for the description of lake venting phenomena was selected to provide an understanding of the flow parameters involved in the problem.

The bubble plume model is presented in chapter 2. Conservation equations and required empirical parameters are derived. The 2D model considers water entrainment into the bubble plume, relative phase velocity, and the different bubble core and entrained water flow expansion. So far, the model was used to compute lake venting phenomena and in the present work it is applied to the much more chaotic flow behaviour that exist when high air flow rates are injected through a single nozzle into a pool.

The description of the experimental part of the work starts in chapter 3. First, an overview of various local measurement techniques for two-phase flows is presented to justify the selection of the hot-film anemometer and the double optical sensor. Subsequently, the signal processing for these sensors is explained. Chapter 4 describes the experimental setup.

Experimental data from the pool tests are presented in chapter 5. They describe the bubble
core expansion and entrained water flow and the variation in void fraction and phase velocities along the pool centerline. Furthermore, bubble chord length distributions are discussed. In chapter 6, the experimental data are used to determine the empirical parameters for the bubble plume model and to compare model predictions with the experiment. The findings are compared with information from the literature. Finally, conclusion are drawn in the last chapter and suggestions are made for future work.
2 A Bubble Plume Model

The main objective of the theoretical part in the present work is the integral description of the bubble plume in the ZEF. There is no attempt to model detailed flow phenomena, e.g. turbulence structures in the pool, since the model should be usable in system codes which compute the sequence of events in case of a severe accident in a nuclear plant. The computing time is limited in these codes and usually more detailed modeling requires additional computing efforts. Semi-empirical models are a good choice for the flow description in these codes since they consider the main physical phenomena and cost practically no computing time. On the other hand, such models need empirical information which has to be provided by experimental data.

A standard semi-empirical model for two-phase flow is the drift flux model. Kataoka & Ishii [45] and Iguchi [42] used it to describe two-phase flow in a pool. Both investigators derived correlations for the pool void fraction and the drift velocity by integrating void fraction and velocity profiles over the pool diameter, but these correlations can only be employed when the bubbles are distributed over the whole pool diameter.

The main problem in the present work is to describe bubble plume expansion. In the experiments reported here, the bubbles will never occupy the entire pool diameter. A bubble plume model is selected that describes the horizontal expansion of the flow, and the development of the water velocity and void fraction along the pool centerline.

The bubble plume model used here is widely used for the computation of lake venting phenomena (see table 1.1). It follows the idea from Taylor [75] that bubble plumes are comparable to single phase plumes. Fannelop [27] and Hugi [40] have summarized the various papers about bubble plume models and they also give an overview of different model extensions. Some of these extensions, like destratification/mixing phenomena discussed by Schladow [69] and Leitch [52], may also be applicable in the nuclear field but such a theoretical study is not the topic of the present investigation.

The subject of this chapter is the formulation of the bubble plume model. Subsequently, (1) the parameters of a time-averaged axially symmetric bubble plume are defined, (2) the model simplifications are listed, (3) the integral conservation equations are derived for air and water continuity and momentum of the mixture, and (4) the empirical parameters needed to close the set of conservation equations are identified. An approximate solution for the resulting set of conservation equations is presented at the end.
2.1 Definition of Bubble Plume Parameters

The important parameters for a bubble plume in a pool are shown in figure 2.1. Air is released from a single nozzle at a water depth $H_0$ and the injector is considered as a point source. The flow above the nozzle is dominated by the initial momentum of the air, the air expansion, and the break up of the initial air jet into bubbles. This part of the flow, the ZFE, is relatively small compared to the ZEF which is the subject of this chapter. The bubble plume in the ZEF is driven by buoyancy and consists of a bubble core and a wider expanded entrained water flow with substantially upward velocity. The time-averaged radial profiles of the axial water velocity component $u_w(r, z)$ and void fraction $\epsilon(r, z)$ in a cross-section can be expressed by Gaussian curves (time-averaging operators are omitted in this chapter for simplification):

$$u_w(r, z) = u_{w,m}(z) \exp\left(-\frac{r^2}{b_w(z)^2}\right)$$  \hspace{1cm} (2.1)
$$\epsilon(r, z) = \epsilon_m(z) \exp\left(-\frac{r^2}{b_c(z)^2}\right) = \epsilon_m(z) \exp\left(-\frac{r^2}{\lambda b_w(z)^2}\right)$$  \hspace{1cm} (2.2)

where $r$ and $z$ represent the radial and axial coordinates. The centerline values of the profiles are denoted by $u_{w,m}$ and $\epsilon_m$, respectively. The radius of the entrained water flow $b_w$ and the bubble core radius $b_c$ are named width in the present work because this nomenclature is common in the literature. Their ratio

$$\lambda = \frac{b_c}{b_w}$$  \hspace{1cm} (2.3)

expresses the proportion between the lateral diffusion of the bubbles and the lateral diffusion of momentum, which Engebretsen [25] considers as the turbulent Schmidt number.

The entrained water flow expands usually more widely than the bubble core ($\lambda \leq 1$). Therefore, $b_w$ scales the width of the entire bubble plume. The virtual origin $z_{0,w}$ of the bubble plume can be calculated by extrapolating the functional relationship $b_w(z)$, determined in the ZEF, to the point where $b_w = 0$. The virtual origin $z_{0,b}$ can be found by extrapolating $b_c(z)$.

2.2 Simplifications

The following simplifications and assumptions are made to model the axially symmetric bubble plume:

1. The mean flow is stationary and it is expressed in terms of time-averaged quantities.

2. The bulk water is stagnant and no bubble plume oscillation is taken into account (infinite pool). This supposes also that the radial water velocities in the plume are much smaller than the axial components and they are neglected.
Figure 2.1: Characteristic parameters to describe an axially symmetric bubble plume consisting of a bubble core and entrained water flow.

3. No mass transfer is taken into account across the interface between the air bubbles and the water.

4. The water is the continuous phase and is modeled as incompressible.
5. Both phases have the same constant temperature and it is assumed that the bubbles expand isothermally while rising.

6. The total pressure at pool elevation $z$ is the sum of the ambient pressure and the pressure difference in the two-phase mixture between $z$ and the pool surface. This pressure difference is assumed to be equal to the hydrostatic pressure of water only between $z$ and the pool surface. Furthermore, any cross-sectional pressure gradient in the bubble plume is neglected.

7. The ratio $\lambda$ between $b_r$ and $b_w$ is assumed to be constant.

8. The bubble drag is taken into account by introducing a constant relative phase velocity between water flow and rising bubbles:

$$ u_r = u_b - u_w \quad . \quad (2.4) $$

9. The radial void fraction and axial water velocity profiles are assumed to be self-similar and they can be scaled by the widths and the maxima of the Gaussian curves.

10. The amount of water entrainment from the bulk water into the plume is proportional to the product $b_w(z) u_{w,m}(z)$. The factor of proportionality $\alpha$ is called entrainment coefficient (entrainment hypothesis) and it is supposed to be constant over the whole pool depth.

11. The momentum transport with the fluctuation part of the axial water velocity component is proportional to the mean axial velocity. The factor of proportionality is called the momentum amplification factor $\gamma$ and it is assumed to be constant.

### 2.3 Conservation Equations

The conservation equations for air and water mass and momentum of the mixture are derived for an axially symmetric bubble plume. The equations are time-averaged and all parameters are considered to be time-averaged as well. Cross-sectional averages are calculated by integrating the Gaussian curves from $r = 0$ to $\infty$ and $\varphi = 0$ to $2\pi$. The origin of the cylindrical coordinate system is the virtual origin of the bubble plume. Its depth is denoted as $H_v = H + z_{0,w}$ and the distance measured from this point is $z_m$.

The air conservation equation can be written as:

$$ 2\pi \int_0^\infty [u_w(r,z_m) + u_r] \epsilon(r,z_m) r dr = \dot{V}_{w,m} \frac{H_v + H_0}{H_v + H_0 - z_m} \quad . \quad (2.5) $$
On the left hand side of (2.5), the air flow rate at elevation $z$ is calculated by integrating the local volumetric air flux $j_a = u_w \epsilon$ over the plume cross-section. The local bubble rise velocity is approximated by the sum of the axial water velocity component and the constant relative phase velocity between the phases (assumption 8). The volumetric air flow rate $\dot{V}_{a,m}$ injected at the virtual origin is computed at the prevailing local pressure at this point. No mass transfer is taken into account and isothermal air expansion takes place while the bubbles are rising (assumptions 3 and 5). In (2.5), the height $H_0 = 10.34 \, m$ stands for the atmospheric pressure $p_0 = 101.3 \, kPa$ at the pool surface. It is given by $\frac{p_0}{p_w g}$, where $p_w = 998.2 \, kg/m^3$ is the density of the water and $g = 9.81 \, m/s^2$ is the acceleration due to gravity.

The entrainment hypothesis is used to formulate the water conservation equation:

$$\frac{d}{dz_m} \left( 2 \pi \int_0^\infty u_w (r, z_m) [1 - \epsilon (r, z_m)] r \, dr \right) = \alpha \cdot 2 \pi b_w (z_m) u_{w,m} (z_m). \quad (2.6)$$

The integral in (2.6) represents the water flow at elevation $z$, calculated from the local volumetric water flux $j_w = u_w (1 - \epsilon)$. The increase in water flow rate in the height $dz_m$ must be balanced by water entrainment from the bulk. The amount of water entrainment is quantified on the right hand side of (2.6) by using the entrainment hypothesis, suggested by Morton [60]

$$\frac{d\dot{V}_w}{dz_m} = - \lim_{r \to \infty} (2 \pi r v_w) = \alpha \cdot 2 \pi L U, \quad (2.7)$$

where it is supposed that the water flow, which is flowing with the radial velocity component $v_w$ from the bulk water into the bubble plume, is represented by a characteristic length scale $L$, a characteristic velocity $U$, and the entrainment coefficient $\alpha$. By assuming a Gaussian curve for the axial water velocity profile, these two characteristic parameters can be replaced by the width of the plume $b_w$ and the maximum axial water velocity component $u_{w,m}$ (assumption 10).

The conservation equation for the momentum expresses the rate of change of momentum in the plume which is driven by the buoyancy of the bubbles present in the cross section:

$$\frac{d}{dz_m} \gamma 2 \pi \rho_w \int_0^\infty u_w^2 (r, z_m) [1 - \epsilon (r, z_m)] r \, dr = \rho_w g 2 \pi \int_0^\infty \epsilon (r, z_m) r \, dr. \quad (2.8)$$

The right hand side of (2.8) represents the buoyancy and the integral on the left hand side is the mean momentum flux multiplied by the momentum amplification factor $\gamma$ (assumption 11). The terms including the air density are neglected on both sides of the equation since the water density is at least two orders of magnitude larger than the air density. Therefore the portion of air momentum flux $\rho_a u_a^2 \epsilon$ is ignored on the left hand side and the density difference $\rho_w - \rho_a$ in the buoyancy term on the right hand side is approximated by the water density $\rho_w$. 
Now, the conservation equations are integrated over the bubble plume cross-section by inserting the radial profiles \( u_w(r, z) \) (2.1) and \( \epsilon(r, z) \) (2.2). This yields the following set of coupled conservation equations.

- **Air conservation equation:**
  \[
  \pi \lambda^2 b_w^2(z_m) \epsilon_m(z_m) \left( \frac{u_{w,m}(z_m)}{1 + \lambda^2} + u_r \right) = \frac{\dot{V}_{a,m}}{H_v + H_0} \frac{H_v + H_0}{H_v + H_0 - z_m} \tag{2.9}
  \]

- **Water conservation equation:**
  \[
  \frac{d}{dz_m} \left( b_w^2(z_m) u_{w,m}(z_m) \left( 1 - \frac{\lambda^2 \epsilon_m(z_m)}{1 + \lambda^2} \right) \right) = \alpha \cdot 2u_{w,m}(z_m) b_w(z_m) \tag{2.10}
  \]

- **Conservation equation for the momentum:**
  \[
  \frac{d}{dz_m} \left( b_w^2(z_m) u_{w,m}^2(z_m) \left( \frac{1}{2} - \frac{\lambda^2 \epsilon_m(z_m)}{1 + 2\lambda^2} \right) \right) = g\lambda^2 \epsilon_m(z_m) b_w^2(z_m) \tag{2.11}
  \]

The three dependent variables are \( b_w(z_m), \epsilon_m(z_m), \) and \( u_{w,m}(z_m) \). Four parameters have to be determined to close the set of differential equations: (1) the ratio of the widths \( \lambda \) (equation 2.3), (2) the relative phase velocity between the phases \( u_r \) (equation 2.4), (3) the momentum amplification factor \( \gamma \), and (4) the entrainment coefficient \( \alpha \) defined by rewriting (2.7):

\[
\alpha = \frac{d\dot{V}_w}{dz_m} \frac{1}{2\pi u_{w,m} b_w} \tag{2.12}
\]

To compare the model predictions with the experiment, the distance \( z_{0,w} \) between the nozzle and the virtual origin of the bubble must be known. Experiments are required to determine the parameter for solving the set of differential equations.

### 2.4 Solution of the Conservation Equations

In order to find a solution for the three dependent variables \( \epsilon_m, u_{w,m}, \) and \( b_w \), the set of conservation equations has to be solved. This can either be done with numerical methods, or approximate solutions can be obtained by using further simplifications. Friedl [29] derived an approximate solution of the conservation equations.

The centerline void fraction \( \epsilon_m \) can be easily expressed as a function of the other dependent variables \( u_{w,m} \) and \( b_w \) by rewriting (2.9):

\[
\epsilon_m(z_m) = \left( \frac{H_v + H_0}{H_v + H_0 - z_m} \right) \frac{\dot{V}_{a,m}}{\pi \lambda^2 b_w^2(z_m) \left( \frac{u_{w,m}(z_m)}{1 + \lambda^2} + u_r \right)} \tag{2.13}
\]
The remaining conservation equations (2.10) and (2.11) determining $u_{w,m}(z_m)$ and $b_w(z_m)$ are simplified by applying the Boussinesq approximation. The intention is to neglect small density differences except in the buoyancy terms, which eliminates the terms containing $\epsilon_m$ on the left hand side of (2.10) and (2.11). The approximation results in an overestimate of the entrained water flow rate

$$b_w^2 u_{w,m} \left(1 - \frac{\lambda^2 \epsilon_m}{1 + \lambda^2} \right)$$

and the momentum flux

$$b_w^2 u_{w,m} \left(\frac{1}{2} - \frac{\lambda^2 \epsilon_m}{1 + 2 \lambda^2} \right).$$

It is useful to estimate the size of the terms containing $\epsilon_m$ to determine when they can be neglected. Therefore, $\lambda$ is assumed to be 0.8, which is a typical measured value. If the Boussinesq approximation is valid, the following inequalities are expected for (2.10) and (2.11), respectively:

$$\frac{\lambda^2}{1 + \lambda^2} \epsilon_m = 0.4 \epsilon_m \ll 1$$

$$\frac{\lambda^2}{1 + 2 \lambda^2} \epsilon_m = 0.35 \epsilon_m \ll \frac{1}{2}$$

It is seen that both terms depend linearly on the void fraction. Water flow rate and momentum flux are overestimated by 10% for void fractions of 0.22 and 0.13, respectively. Both void fractions are comparable high void fractions. In the pool, they are expected only close to the injector or for extreme high air flow rates. This suggests that the Boussinesq approximation is acceptable for the modeling of the ZEF, especially when the void fraction is low.

Using the Boussinesq approximation and inserting (2.13) into (2.11), one obtains two coupled differential equations for the two variables $b_w$ and $u_{w,m}$.

$$\frac{d}{dz_m} \left(b_w^2 (z_m) u_{w,m} (z_m) \right) = \alpha \cdot 2 u_{w,m} (z_m) b_w (z_m)$$

$$\frac{d}{dz_m} \left(b_w^2 (z_m) u_{w,m}^2 (z_m) \right) = \frac{2 (1 + \lambda^2) g \dot{V}_{a,m}}{\pi \gamma (u_{w,m} (z_m) + (1 + \lambda^2) u_r) \left(H_v + H_0 - z_m\right)} \left(H_v + H_0 - z_m\right)$$

Friedl [29] derived an approximate solution for $b_w$ and $u_{w,m}$ by transforming the two differential equations in a dimensionless form. An asymptotic expansion to the second order is used to solve the set of differential equations. The result is a dimensionless description of bubble plume expansion and of the variation in the axial water velocity component along the centerline. After a backward transformation, one obtains the result in physical units:
\[ b_w(z_m) \approx \frac{6}{5} \alpha z + \frac{6}{110} \left( \frac{24}{25} \frac{\pi \gamma \alpha^2}{(1 + \lambda^2) g V_{a,m}} \right)^{\frac{1}{3}} \alpha \left( 1 + \lambda^2 \right) u_r z^{\frac{3}{5}} , \quad (2.20) \]

\[ u_{w,m}(z_m) \approx \left( \frac{24}{25} \frac{\pi \gamma \alpha^2}{(1 + \lambda^2) g V_{a,m}} \right)^{\frac{1}{3}} z^{-\frac{1}{3}} - \frac{7}{22} \left( 1 + \lambda^2 \right) u_r \]
\[ + \frac{13}{121} \left( \frac{24}{25} \frac{\pi \gamma \alpha^2}{(1 + \lambda^2) g V_{a,m}} \right)^{\frac{1}{3}} \left( \left( 1 + \lambda^2 \right) u_r \right)^2 z^{\frac{1}{5}} . \quad (2.21) \]

Equations (2.20) and (2.21) contain only the terms from the approximate solution that contribute more than 5 percent to the sum for the case of interest here. Other terms appearing in the asymptotic expansion solution are neglected. The magnitude estimation for the different terms was done by inserting into (2.20) and (2.21) typical values that have been determined from the experiments in present work. Therefore the above approximation is only valid for bubble plumes in pools with depths between 2 and 3 m and for air flow rates up to 5 dv_m/s.

The dimensionless form of the approximate solution was compared by Friedl [29] with a fourth-order Runge-Kutta method solution. The approximate solution, with the reduced number of terms, agreed very well with the numerical solution for the conditions in laboratory experiments (pool depth of a few meter), but the up scaling to lake venting phenomena (depths between 100 and 200 m) was difficult and caused various problems. The objective here is to compute bubble plumes in pools with depths of a few meter and therefore equations (2.13), (2.20), and (2.21) can provide a solution to compute the variation in the maximum void fraction, the width of the plume, and the maximum water velocity along the pool centerline. The empirical parameters of the model derived here will be determined in chapter 6 using the experimental data obtained in the present work. Furthermore, the model predictions are compared with the experiment.
3 Instruments and Measurement Methods

3.1 Selection of the Instrumentation

To determine the empirical parameters $\lambda$, $\alpha$, and $u_r$ for the bubble plume model, it is necessary to measure the radial profiles of the void fraction and of the bubble and water velocities at different pool elevations. Various instruments are available to measure these local two-phase flow parameters. Table 3.1 gives a short overview of non-intrusive and intrusive local two-phase flow instruments. Flow parameters that can be measured and references are listed for each instrument. The measurement methods and their application in a pool are discussed in detail in a longer report [50] that was written at the beginning of this project to select the instruments. The following paragraphs summarize this report and some additional measurement methods that were examined later.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Flow Parameters</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Non-intrusive instruments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ultrasonic Doppler method</td>
<td>Bubble and water velocity</td>
<td>[74, 84]</td>
</tr>
<tr>
<td>Laser Doppler anemometer</td>
<td>Water velocity</td>
<td>[2, 71]</td>
</tr>
<tr>
<td>Particle image velocimetry</td>
<td>Bubble and water velocity</td>
<td>[10, 34]</td>
</tr>
<tr>
<td>X- and $\gamma$-ray tomography</td>
<td>Void fraction and flow imaging</td>
<td>[56, 78]</td>
</tr>
<tr>
<td>Capacitance tomography</td>
<td>Void fraction and flow imaging</td>
<td>[54, 46]</td>
</tr>
<tr>
<td><strong>Intrusive instruments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wire-mesh tomography</td>
<td>Void fraction and flow imaging</td>
<td>[68, 65]</td>
</tr>
<tr>
<td>Optical / impedance sensor</td>
<td>Bubble velocity and void fraction</td>
<td>[59, 61, 7]</td>
</tr>
<tr>
<td>Electromagnetic velocity meter</td>
<td>Water velocity</td>
<td>[21, 62]</td>
</tr>
<tr>
<td>Hot-film / wire anemometer</td>
<td>Water velocity and void fraction</td>
<td>[4, 53, 3]</td>
</tr>
</tbody>
</table>

Table 3.1: Instruments for local two-phase flow measurements.

An ultrasonic velocity profile monitor (UVP) was developed by Takeda [74] and was used by Zhou [84] to investigate developed water/air bubbly flow in a vertical, rectangular channel. The UVP is installed outside the flow field. It emits a pulse of an ultrasonic signal into the fluid. The pulse is repeated with a certain frequency. A sensor detects the echo from the surface of either microparticles suspended in the fluid or bubbles. The fluid and bubble velocities are measured from the Doppler shift of the ultrasonic frequency and the position is determined by the time delay between the front of the initial pulse and reception of its echo. Collecting a certain number of events at different radial positions permits the measurement of mean velocity profiles. The instrument is characterized by the maximum measurement depth $R_{max}$ and the maximum detectable velocity $u_{max}$. The application of the UVP is limited. Takeda [74] derived
a limiting condition by relating $R_{max}$ and $u_{max}$ to the ultrasonic frequency $f_0$ and the sound velocity $c$: $u_{max}R_{max} \leq c^2/8f_0$. This shows that $u_{max}$ decreases as $R_{max}$ increases for a fixed ultrasonic frequency. For the pool tests, the maximum detectable velocity at the centerline ($R_{max} = 0.5 \text{ m}, f_0 = 4 \text{ MHz}$) would be $0.14 \text{ mm/s}$. This velocity is three orders of magnitude smaller than the expected fastest phase velocities of about $1.5 \text{ m/s}$ in the pool centerline, and therefore an UVP is not applicable for the tests.

The basis of laser Doppler anemometer (LDA) measurements is light scattering in the interference pattern that is formed at the intersection of two laser beams. Seeding particles are added to the fluid, and when such a particle passes through the beam crossing volume it scatters light and produces a characteristic signal. The frequency of the amplitude oscillations of this signal is proportional to the particle velocity component normal to the interference pattern. The LDA technique is often used for fluid velocity measurements in single-phase flows. In two-phase flows, the presence of bubbles raises two problems: (1) Bubbles passing through the interference pattern generate signals that can be interpreted as seeding particle signals and (2) some seeding particles in the interference pattern might not be detected because of bubbles intersecting the laser beam, or scattered light from a seeding particle may be reflected away by bubbles before it is detected. These problems are discussed Boerner [2] and Suzanne [71]. The conclusion from both studies is that laser Doppler anemometers are useful only for measurements in two-phase flows with void fractions $\leq 0.02$. At such low void fractions, disturbances by bubbles are limited enough to allow detection of a sufficient number of seeding particles in a reasonable measuring time. For example, Boerner [2] needed a measuring time of $1800 \text{ s}$ (1200 good seeding particles) to obtain the mean water velocity at a void fraction of 0.017. Void fractions much higher than 0.02 are expected during the pool tests here and therefore LDA would not be suitable for water velocity measurements within the bubble plume.

Particle image velocimetry (PIV) is a straightforward method for measuring velocities in a flow field. A laser system illuminates seeding particles in the fluid and a digital camera takes images at a known framing rate. Subsequently, the images are processed to determine the distance traveled by individual seeding particles between one frame and the next. In this way, a 2D velocity vector plot can be established for the flow field. In two-phase flows, again the problem is the distinction between scattered light from seeding particles and bubbles. An image analysis technique is presented by Hassan [34] and it was used for studying bubble detachment from an orifice in a small water box. A PIV system was used by Chen [10] to investigate two-phase flow in a bubble column. The data indicate that problems appear for velocity measurements at void fractions higher than 0.1 because of too much light scattering on bubble surfaces. A PIV system for two-phase flows was recently developed by DANTEC [77]. The system uses fluorescent seeding particles, a filter in front of the digital camera, and a sophisticated signal
processing technique to separate seeding particles from bubbles. This system was not available at the beginning of the present work. At any rate, there are still some open questions regarding such a system for a large flow field and for void fractions above 0.1 (shadowing of the flow field behind bubbles). The use of a PIV system for phase velocity measurements in the pool would have required further extensive development of this measurement method, which was not subject of the present work.

X-ray and γ-ray attenuation techniques are used to detect void fraction by sending a beam through a flow and measuring the reduced intensity of the emerging beam. Water scatters and absorbs energy more than air and so the emerging beam increases in intensity with void fraction. Teyssedout [78] developed a γ-ray absorption technique that allows to measure axial void fraction profiles along a pipe by determining the cross-sectional void fraction at different elevations. Mewes [56] utilized the attenuation technique to build a tomographic reconstruction of the void fraction field. He measured void fractions along different chords in the cross-section of a bubble column and developed a reconstruction algorithm to visualize the void fraction distribution in the cross-section. The main problem with using X-ray and γ-ray attenuation techniques for pool experiments is the strength of the required radiation sources. The diameter in the present tests is 1 m, which requires a radiation source of 1 mSv. Installing such a source in the lab was not possible due to serious problems with the radiation protection.

Capacitance tomography is based on the different permittivities of water and air (the ratio is 81:1 for water at 20°C). Several electrodes are mounted along the circumference of the measurement plane to measure capacitance along different chord lengths. These data are processed with a reconstruction algorithm to determine void fraction distributions. The critical parameter for this measuring instrument is the ratio between electrode length and flow field diameter, because it controls the fringing effect. This is when the electric field lines travel outside the area between the electrodes. Sensors described by Lowe [54] and Klug [46] for pipe flow experiments are typically 1–2 diameters in length. If this technique was used in the pool (\(D = 1\) m) the electrode height would have been between one and two meters. The sensor provides an average void fraction over the sensor length and this spatial resolution is far too poor for this study.

An alternative to the non-intrusive capacitance tomography is electrode-mesh tomography, which is an intrusive instrument since the sensor is immersed in the flow. Prasser [65] presented an electrode-mesh sensor that was made of two planes of thin, parallel wires (\(d_{wire} = 0.1 \) mm) to measure the void distribution over the cross-section of a vertical pipe (\(D = 51.2 \) mm). The two planes, each with 16 parallel wires, were placed in the pipe with an axial distance of 1.5 mm and perpendicular to each other. Looking from the top, a mesh with 256 intersections was thus created in the pipe cross-section. Computer-controlled electronics were developed to
measure the instantaneous conductivity of the two-phase flow at the intersections. Reinecke [68] describes a more sophisticated sensor design with three sets of parallel wires to study the flow in a horizontal pipe of 59 mm diameter. The conductivity between the wires was measured in that experiment and these data were processed with a reconstruction algorithm to visualize void fraction distributions. There are two known problems with electrode-mesh tomography. The first is that the conductivity measurement between two wires is influenced by neighboring wires, the so-called cross-talk problem. The second problem is wire vibration in the flow, which changes the wire distance and affects the conductivity. Scaling up the electrode-mesh sensor for the pool tests would have increased wire vibration because wires would have been quite long. Furthermore, the arrangement and the spatial distance of the wires in the pool has to be studied in order to find the optimum setup for the mesh to minimize cross-talk. The application of this technique to measurements in a pool would have required to further develop the instrument.

Impedance and optical sensors are often used for local void fraction and bubble rise velocity measurements in two-phase flows. Impedance sensors detect the change of either resistivity or capacitance that depends on the fluid phase at the sensor tip. The change of refractive index between air and water is used by optical sensors to distinguish between phases at the sensor tip location. The first impedance sensor was developed by Neal & Bankhoff [61] in 1963, and Miller & Mitchie [59] built the first optical sensor in 1970. Different impedance and optical sensor designs and the corresponding signal processing are described in a review by Cartellier [7]. Neither measurement method is limited by void fraction because the sensor tip is directly located at the measurement point and the surrounding fluid flow not disturb phase detection. But of course, the sensor disturbs the flow and therefore the sensor tip should be as small as possible. There are no restrictions to the use of an impedance or optical sensor in the pool, and there is a great amount of previous experience with these instruments, which is of help in selecting a sensor and developing signal processing software.

Electromagnetic liquid velocity sensors are based on the induction law: The flow of an electrically conducting fluid across a magnetic field generates a voltage proportional to the flow velocity and the magnetic field strength. To use such a sensor, the fluid must have a reasonably high electric conductivity. The principles for measurements in water flows are described by Dobson [21]. Ohnuki [62] used a two-component sensor to measure mean value and variation in water velocity in a large pipe (\(D = 0.48 \text{ m}\)) where air and water were injected. Compared to other intrusive measuring instruments, the tip of electromagnetic velocity sensors is large. For example, the sensor used by Ohnuki [62] had a cylindrical tip of 9 mm in diameter while typical impedance or optical sensor tips are \(\leq 200 \mu\text{m}\) in diameter. Signals from electromagnetic velocity sensors are averaged over periods between 0.05 to 10 s, indicating a bandwidth of not more than 20 Hz. The sensor cannot distinguish between water and bubble signals be-
cause clear bubble signals cannot be identified in the large measuring volume over an averaging period. Nevertheless, the advantage of electromagnetic velocity sensors is that they can measure along two or even three orthogonal axis. If the flow field is large compared to the sensor dimension and velocity variations of interest are slow, the electromagnetic velocity sensor can provide valuable data. Because of the large sensor size and the low time resolution (no phase separation), local water velocity measurements with electromagnetic velocity sensors are difficult to interpret in bubble plumes and, furthermore, Ohnuki [62] recommended validation with a second local instrument.

Hot-film and hot-wire anemometers (HFA/HWA) measure local fluid velocities. The measurement principle utilizes the change in heat transfer rate from a small heated element (thin film or wire) with fluid velocity. HFA/HWA are often used for measurements in air. Use in water is made possible by coating the heated element with an electrical insulator such as quartz. The fluid must be clean and temperature gradients should be avoided during measurements. It is in particular required for measurements in water because the overheat ratio between sensor and fluid in water is 5 to 10 times lower than in air. Bruun [4] shows that this results in water to a much higher sensitivity to temperature changes. In two-phase flows, the change in the phase present at the sensor from water to air can be clearly detected because the heat transfer decreases rapidly when a bubble hits the sensor tip. First experiments were conducted by Bremhorst [3] to study the interaction between air bubbles in water and a hot-wire anemometer. The findings are used by various authors to develop signal processing algorithms for the distinction between bubbles and water. For example, algorithms based on signal derivation and thresholds, are presented in Liu [53] and Ellingsen [24]. Like optical sensors, HFA/HWA work at any void fraction and they were used in various experiments for measuring local water velocities in pipes. The application of a HFA/HWA in a pool is possible when the water is conditioned carefully during the tests to avoid sensor pollution and temperature fluctuations.

The overview of the various instruments used for measuring local two-phase flow parameters leads to the conclusion that either an optical or an impedance sensor coupled with a HFA/HWA are the best choices for determining the required flow parameters for the bubble plume model. After comparing different sensors, a double optical sensor and a hot-film anemometer with wedge shaped sensor have been purchased from RBI and DANTEC, respectively. Sections 3.2 and 3.3 describe these sensors and the signal processing that was developed.
3.2 Double Optical Sensor

In water/air two-phase flows, optical sensors detect bubbles as they pass through the sensor tip. Sensors with two tips (double optical sensors, DOS) measure also the bubble rise velocity by determining the time of flight as a bubble moves from one tip to the other. Figure 3.1 shows schematically the working principle of an optical sensor with one tip. The change of refractive index between air and water is used to identify the phase at the sapphire tip. Light is guided from a light source to the tip by a glass fibre. In water, most of the light leaves the tip, while in air, it is reflected back into the fibre. The light returning from the tip is routed to a light detector by a semi-transparent mirror system. An increase in backscattered light increases the detector voltage indicating the presence of air at the tip, while a lower voltage corresponds to water. Light source, light detector, and semi-transparent mirror system are integrated in one device, the optical amplifier.

A custom-made DOS and two optical amplifiers were purchased from RBI [67] for the pool tests. Figure 3.2 shows a picture of the sensor and specifies the tip dimensions. The amplifier outputs are sent to an AD-converter for further signal processing.

For technical reasons, the DOS was horizontally oriented in the pool. In this configuration, the sensor is perpendicular to the flow direction of the bubbles, as illustrated by the sketch in figure 3.2. Commonly, phase detection sensors are oriented vertically. Preliminary tests described in appendix A.5 have proven, however, that the sensor orientation is practically not affecting the phase detection and so the sensor orientation in the pool has no influence on the measurements.

During the present work, the signal processing technique to calculate void fractions and bubble velocities with the double optical sensor was developed using methods from the literature. The methods and the related parameter settings, such as the threshold adjustment for the phase discrimination, are described in sections 3.2.1 and 3.2.2. To test the signal processing techniques and the parameter settings, calibration tests were performed in a pipe. Details about these calibration tests are presented in appendix A. A method to measure bubble chord length distributions is described in section 3.2.3.
Instruments and Measurement Methods

**Figure 3.1:** Working principle of the optical sensor.

**Figure 3.2:** Double optical sensor from RBI.
3.2.1 Phase Discrimination and Void Fraction Measurements

The instantaneous phase at the position \( r \) of the upstream sensor tip is determined from the amplifier output \( E_{op}(t) \) by setting a threshold to distinguish between water and air. The information is translated into the phase identification function \( X(r,t) \):

\[
X(r,t) = \begin{cases} 
1 & \text{air at position } r \text{ at time } t \\
0 & \text{water at position } r \text{ at time } t 
\end{cases}
\]  

(3.1)

The mean local void fraction at position \( \bar{\varepsilon}(r) \), defined as the ratio between the time \( t_{air} \) when air is present at the sensor tip and the total measuring time \( T_{meas} \), is obtained from the phase identification function with:

\[
\bar{\varepsilon}(r) = \frac{t_{air}}{T_{meas}} = \frac{1}{T_{meas}} \int_0^{T_{meas}} X(r,t) \, dt
\]

(3.2)

For the pool tests, the 12 bit AD-converter acquires the signal from the optical amplifier using a sampling rate of 50 ksamples/s in a range between \( \pm 10 \, V \). A typical bubble signal is shown in figure 3.3. The bubble arrival time \( t_{a} \) at the sensor tip and the bubble detachment time \( t_{d} \) from the sensor tip are determined first from the signal by using a threshold \( E_{\text{threshold}} \). The first digital sample that is above the threshold and the next sample that is below the threshold provide \( t_{a} \) and \( t_{d} \), respectively. The period between these two points is the bubble residence time at the sensor tip \( t_{\text{res}} = t_{d} - t_{a} \). During the measuring time \( T_{meas} \), \( N \) bubbles will hit the upstream sensor tip and a data set \( \Sigma_{us} \) is created containing all bubble arrival and detachment times.

\[
\Sigma_{us} = \{(t_{a,1}, t_{d,1}), \ldots, (t_{a,i}, t_{d,i}), \ldots, (t_{a,N}, t_{d,N})\}
\]

(3.3)

The mean void fraction is calculated from the bubble residence times at the upstream tip \( \Sigma_{us} \):

\[
\bar{\varepsilon}(r) = \frac{t_{air}}{T_{meas}} = \frac{\sum_{i=1}^{N} t_{d,i} - t_{a,i}}{T_{meas}} = \frac{\sum_{i=1}^{N} t_{\text{res},i}}{T_{meas}}
\]

(3.4)

The advantage of using bubble arrival and detachment times for signal processing, instead of using the raw data \( E_{op}(t_{i}) \), is that relatively few numbers need to be stored on the computer. It is unnecessary to store the entire digitized signal. During the pool tests, the raw data file was deleted directly after \( \Sigma_{us} \) was created, which reduced the amount of stored data by a factor more than 1000, depending on the total number of bubbles stored in \( \Sigma_{us} \). Note, however, that this method requires a careful setting of the threshold \( E_{\text{threshold}} \) because the information in the
raw data files with the rising and falling edges is lost, so that the data cannot be re-analyzed, for example with new threshold settings.

Figure 3.4 demonstrates the method used to determine $E_{\text{threshold}}$. At first, 20000 signal samples around the water and air signal levels are acquired with the AD-converter. Two probability distributions for the signals $P(E_{op})$ are established from these samples to determine the most probable water and air signals, which represent water level $E_w$ and air level $E_a$, respectively. The threshold is calculated by adding the fraction $\Lambda$ of the signal span $E_a - E_w$ to the water level:

$$E_{\text{threshold}} = E_w + \Lambda (E_a - E_w) .$$  (3.5)

The method permits to detect and set $E_{\text{threshold}}$ automatically, which allows to compensate a slightly sensor shifting ($\leq 0.5V/day$) during a pool test.

It is seen in figure 3.4 that 20000 signal samples are sufficient to determine $E_w$ and $E_a$. The signal distribution around the water level has practically the shape of a Gaussian curve, which indicates white noise generated by the optical amplifier. The signal distribution of the air level is wider because the signal rises not always exactly to the same level when the sensor is surrounded by a bubble and the distribution includes partially signals from the rising and falling
The setting of $\Lambda$, and therefore the setting of $E_{\text{threshold}}$, affects the void fraction measurement because it changes slightly the bubble arrival and detachment times (see figure 3.3). A higher threshold setting increases $t_a$ because the signals do not rise instantaneously from water to air level as a bubble hits the sensor tip. The bubble detachment time $t_d$ decreases by increasing the threshold. The result is that, for increasing $\Lambda$, the residence time of each bubble and the mean void fraction calculated with (3.4) decreases.

The signal rising and falling edges are related by Cartellier [5] to de-wetting and wetting phenomena at the sensor tip. They affect the void fraction measurements when the dimensions of the sensor tip are in the range of the bubble size. The radius of the sensor tips used for the pool tests is 0.08 mm (see figure 3.2), which is much smaller than the bubbles in the pool (mean bubble diameter of about 6 mm). Thus, in the present case, sensor de-wetting and wetting has no practical relevance for the void fraction measurements.

The threshold method works for most bubble signals, but there are two chances for signal misinterpretation: Pre-signals in front of a bubble signal and electronic noise in the rising or falling edge. The raw data signal $E_{\text{op}}(t)$ in figure 3.5 shows one example for each error. The
Instruments and Measurement Methods

The appearance of pre-signals was investigated by Cartellier [6] and Schmitt [70] by comparing sensor signals with high speed videos. It is shown by Cartellier [6] that the pre-signals are caused by sensor tip imperfections. A modified threshold method was introduced by Schmitt [70] to avoid signal misinterpretations (noise and pre-signals) by merging together two bubbles, when the period between bubble detachment time $t_{d,i}$ and arrival time $t_{a,i+1}$ of the following bubble is less than a correction time $t_{corr}$.

The difference between simple and modified threshold methods is illustrated in figure 3.5 by showing the phase identification functions $X(t)$ and $X_{mod}(t)$, respectively. Errors caused by noise in the falling edge and the pre-signal are corrected with the modified threshold method because the periods $t_{a,2} - t_{d,1}$ and $t_{a,4} - t_{d,3}$ are shorter than $t_{corr}$. Four bubbles are counted with the simple threshold method and two bubbles are counted with the modified threshold method.

![Figure 3.5: Sensor signal demonstrating the difference between the simple and modified threshold method.](image)

Using the modified threshold method for re-analyzing a complete data set $\Sigma_{us}$ means that

$$\text{if} \quad (t_{a,i+1} - t_{d,i} \leq t_{corr}) \quad \text{then}$$

$$\ldots, (t_{a,i}, t_{d,i}), (t_{a,i+1}, t_{d,i+1}), \ldots \quad \text{is replaced by} \quad \ldots, (t_{a,i}, t_{d,i+1}), \ldots$$
When $M$ bubble pairs are merged together in a data set $\Sigma_{us}$ with $N$ bubbles, then a modified data set $\Sigma_{us,mod}$ with $N - M$ bubbles can be created:

$$\Sigma_{us,mod} = \{(t_{a,1}, t_{d,1}), \ldots, (t_{a,j}, t_{d,j})\} \text{ with } J = N - M \quad \text{(3.6)}$$

Void fraction is again determined with (3.4). The void fraction increases when $\Sigma_{us,mod}$ is used instead of $\Sigma_{us}$, because the periods $t_{a,i+1} - t_{d,i}$ are added to the residence times when two bubbles are merged together. The difference between void fraction measurements with $\Sigma_{us,mod}$ and $\Sigma_{us}$ is small since the total residence time of the bubbles is much larger than the times that are added to the total bubble residence time when bubbles are merged together. Correction with the modified threshold method has a stronger effect on the measurement of the total bubble number.

For the pool tests, $\Lambda$ and $t_{corr}$ were adjusted on the basis of calibration tests in a vertical pipe (see appendix A). In total, 12 tests for different air and water flow rates were performed to measure void fraction profiles with two sensors in the test sections at two elevations ($z_b, z_c$) along the pipe. The profiles were integrated over the pipe cross section to calculate the integral void fractions $\langle \epsilon_{DOS} \rangle_A$. For comparison, the integral void fractions $\langle \epsilon_{pressure} \rangle_V$ were determined from differential pressure measurements corrected by taking into account the pressure loss due to friction. As a further check, the drift flux model was used to calculate the void fractions $\langle \epsilon_{drift} \rangle_A$.

The calibration tests show that the threshold should be near the water level, so $\Lambda$ was set to 0.05. The correction time $t_{corr}$ was set to 0.2 ms. By looking at the signal and the corresponding phase identification function, it is seen that mainly signal misinterpretations due to signal noise are corrected with the modified threshold method. The dependency of void fraction measurements on the setting of $\Lambda$ and $t_{corr}$ is discussed in appendix A in more detail.

Figure 3.6 presents the results of calibration tests with the optimum settings for $\Lambda$ and $t_{corr}$. The accuracy of local void fraction measurements is within $\pm 10\%$ of those from the pressure measurements and the calculations with the drift flux model.

Optimum threshold settings obtained from various experiments with optical sensors are presented by Cartellier [7]. These settings are highly scattered ($\Lambda = 0.1 - 0.5$) and calibration is recommended for each sensor. Usually, void fraction measurements with local sensors (optical/impedance sensors) were compared to integral measurement methods (pressure drop/gamma attenuation). Mean deviations from $\pm 2\%$ up to $-30/ + 40\%$ are cited by Cartellier [7]. Typically, the deviations are in the region of $\pm 15\%$. Comparing this value with the result from the calibration above leads to the conclusion that the sensor and the phase separation method used for the pool tests correspond to the state-of-the-art for void fraction measurements in two-phase flows.
3.2.2 Bubble Velocity Measurements

The bubble rise velocity, the axial velocity component of the bubble movement in the pool, is measured with the DOS by determining the time of flight of a bubble passing through the upstream and downstream sensor tips (separation distance: $z_{\text{tip}} = 0.750 \pm 0.038 \text{ mm}$). The sketch in figure 3.2 illustrates a bubble approaching the DOS and figure 3.7 shows a typical example of sensor signals from a bubble that was detected by both tips. The bubble arrival times at the upstream $t_{a,us}$ and downstream $t_{a,ds}$ sensors are used to determine the time of flight $\Delta t$ and to calculate the bubble rise velocity:

$$u_b = \frac{z_{\text{tip}}}{t_{a,ds} - t_{a,us}} = \frac{z_{\text{tip}}}{\Delta t}. \quad (3.7)$$

The method to measure the velocity of each individual bubble is hereafter referred to as the time of flight method (TOF).

The ensemble-average bubble rise velocity $\bar{u}_b$ during $T_{\text{meas}}$ is determined from the bubble arrival times that are stored in the data sets $\Sigma_{\text{mod,us}}$ and $\Sigma_{\text{mod,ds}}$ which are established at the upstream and downstream tip, respectively. The trouble is that not every bubble detected by the upstream
tip can be linked to a signal from the downstream tip. This has two causes:

- The bubble residence times $t_{res,us}$ and $t_{res,ds}$ for up- and downstream tips may be different because the tips pierce the bubble along different chords. This can be caused by random bubble motion or a significant velocity component in the horizontal plane.

- The bubble may be strongly deflected by the upstream tip and a much shorter chord travels through the downstream tip, or the bubble may completely miss the second tip.

To link bubble signals in pairs, the data sets $\Sigma_{mod,us}$ and $\Sigma_{mod,ds}$ are analyzed in the following fashion: For each bubble arrival time at the upstream tip, the following bubble arrival time on the downstream tip is used to calculate $\Delta t$. Two conditions must be met, however, to retain $\Delta t$ for calculating the ensemble-average bubble rise velocity. First, it is required that $\Delta t$ lies between $\Delta t_{min}$ and $\Delta t_{max}$, which means that the bubble rises within a prescribed velocity range. If this condition is fulfilled, the relative difference between the bubble residence times $|\Delta t_{res}|$,

$$|\Delta t_{res}| = \left| \frac{t_{res,us} - t_{res,ds}}{t_{res,us}} \right|.$$ (3.8)
must be below a certain limit.

If both conditions are satisfied, the measured \( \Delta t \) is included in the calculation of the ensemble-average bubble rise velocity.

\[
u_b^c = \frac{z_{up}}{K} \sum_{i=1}^{K} \frac{1}{\Delta t_i}.
\] (3.9)

The number of successfully linked bubble signals \( K \) and the total number of detected bubbles at the upstream sensor \( J_{us} \) are used to calculate the percentage of linked bubble signals:

\[
\Psi = \frac{K}{J_{us}} \times 100\%
\] (3.10)

For the pool tests, bubbles within the velocity range of 0.19 to 2.88 m/s (\( \Delta t_{min} = 0.26 \, ms \), \( \Delta t_{max} = 4.00 \, ms \)) were considered for the calculation of the average bubble rise velocity and \( |\Delta \tau_{res}| \) was limited to 0.2. Bubble velocity probability density functions (PDF) and distributions of the relative difference between bubble residence times were obtained to verify the settings. The results from these preliminary tests are discussed in appendix D.2.

In addition, the settings for \( \Delta t_{min}, \Delta t_{max}, \) and \( \Delta \tau_{res} \) were also verified by the calibration tests in the vertical pipe. The volumetric air flow rate \( \dot{V}_{a}^{DOS} \) was calculated from the local void fraction and bubble rise velocity measurements with:

\[
\dot{V}_{a}^{DOS} = A_{pipe} \langle j_a^{DOS} \rangle_A = A_{pipe} \langle \bar{u}_b^c \rangle_A
\] (3.11)

Considering the pressure at the cross-section used, the measurements are compared with the air flow controller setting at the inlet of the pipe \( \dot{V}_{a}^{AFC} \). Again, the 12 calibration tests that are described in appendix A and the measurements at two pipe elevations are used for comparison. Figure 3.8 shows the results. A deviation of ±15 % between the two independent measurements of the air flow rate is found for most tests.

A comparison between high speed video recordings and sensor signals could have been helpful for testing the TOF method in more detail. This was not done during the work. Nevertheless, the calibration tests have shown that the deviation between the local and the integral measurements are in the range of ±15 %, which corresponds again to the state-of-the-art for the DOS (Cartellier[7]).

3.2.3 Bubble Chord Length Distribution

The chord length \( y \) of an individual bubble traverse can be obtained from the up- and downstream bubble signals linked together. Considering a bubble moving in the vertical direction, the
Figure 3.8: Volumetric air flow rate determined with the DOS compared to the volumetric air flow meter measurements.

chord length is calculated with the bubble rise velocity and the residence time at the upstream sensor tip:

\[ y = u_b \cdot t_{res,us} \quad . \]  

(3.12)

During \( T_{meas} \), an ensemble of \( K \) chord lengths is determined by analyzing data sets \( \Sigma_{mod,us} \) and \( \Sigma_{mod,ds} \). The corresponding bubble chord length probability density function (PDF) is established by dividing the interval \([0, y_{max}]\) into \( m \) equal bins. The bin centers are defined by:

\[ y_i = y_{max} - (i + 1/2) \Delta y, \quad (0 \leq i \leq m - 1) \]

(3.13)

where

\[ \Delta y = \frac{y_{max}}{m} \quad . \]

(3.14)

The measured frequency \( p \) of having a chord length between \( y_i \) and \( y_{i+1} \) is given by:

\[ p (y_i < y < y_{i+1}) = \frac{n_i}{K} \]

(3.15)

where \( n_i \) is the number of chord length observations between \( y_i \) and \( y_{i+1} \).
3.3 Hot-Film Anemometer

A hot-film anemometer (HFA) is used to measure the mean axial water velocity $u_w$ in the pool. The measurement method is based on convective heat transfer from a small heated film element into the flow. Velocity changes affect the heat transfer rate, which is measured and translated into a velocity reading with a calibration equation.

The HFA system was purchased from DANTEC. The sensor electronics consist of a main unit type 55M01 and a Wheatstone bridge type 55M10 [76]. Figure 3.9 shows a picture of the wedge-shaped film sensors 55R32 used for the pool tests and a sketch to define the sensor fixed coordinate system with the corresponding velocity components. The HFA works in the constant temperature mode, which means that the electronics keep the film at constant temperature by adjusting the current through the film element as the heat transfer changes. For the measurements, it was found that the optimum temperature difference between the film element and the water is 14 K. Lower temperatures increase the sensitivity of the sensor to temperature fluctuations in the flow and higher temperatures caused local burn-outs at the film element and damage to the sensor. The system frequency response was adjusted to 10 kHz and the output signal of the electronics was acquired with an AD-converter with a sampling rate of 20 kSamples/s.

![Figure 3.9: Wedge-shaped film sensor.](image)

It was necessary to frequently calibrate the HFA for the pool tests. The calibration tests were performed in a tow tank. Calibration equations either based on heat transfer correlations or empirical equations can be selected to specify the functional relationship between the velocity and HFA signal. In section 3.3.1, the data from calibration tests are presented and two calibration
equations are compared to find the best agreement between calibration data and curve fits for the calibration equations.

Heat transfer from the film element depends upon both the velocity and the direction of the flow. This means that a given flow speed can produce different velocity readings for different flow directions in relation to the sensing element. The velocity reading of the HFA is therefore an effective velocity that includes directional sensor sensitivity. Ideally, the HFA should be sensitive to a single velocity component, but since this is not the case and each flow component contributes to the velocity reading, it is necessary to study the sensor sensitivity to flow angle. Tests were carried out in the tow tank to measure the effective velocity for different sensor orientations and different flow speeds. The results are discussed in section 3.3.2.

In two-phase flow, it is necessary to remove the bubble signals from the HFA recordings before the mean water velocity can be calculated. Section 3.3.3 explains the phase separation method used for the pool tests to identify whether the film element is surrounded by either water or air.

Finally, the signal processing for the pool tests is presented in section 3.3.4. It considers the sensor calibration and the phase separation method.

### 3.3.1 Calibration Tests

Figure 3.10 illustrates the water-filled tow tank for the HFA calibration tests. The sensor is towed with a defined acceleration through the tank filled with stagnant water while the HFA signal $E_{HFA}(t)$ is acquired. The tow velocity $U_{low}(t)$ is calculated from the acceleration and then correlated with $E_{HFA}(t)$ to establish the calibration data set $U_{tow}(E_{HFA})$.

The instantaneous water velocity is related to the HFA sensor signal $E_{HFA}(t)$ without any averaging. The small sensor measures the instantaneous local velocity at this tip; thus, the calibration obtained in the two tank is applicable to any laminar or turbulent flow. The calibration tests were performed with the sensor axis $z_{HFA}$ aligned with the tow direction. This means that the effective velocity reading $U_{eff}$ of the HFA is equivalent to $U_{low}$. The experimental setup permits the calibration in the velocity range of 0.08 to 2.42 m/s.

Bruun [4] discusses various calibration equations that are used in the literature to fit the calibration data $U_{eff}(E_{HFA})$. Two calibration equations are tested here to determine the smallest deviation between data and curve fits.

The most common calibration equation is based on the heat transfer correlation for thin wires (Kings Law):

$$U_{eff} = \left( \frac{E_{HFA}^2 - A}{B} \right)^{1/n}$$  \hspace{1cm} (3.16)

$A$, $B$ and $n$ are the calibration coefficients.
A fourth-order polynomial is proposed by George [30] as an alternative to Kings Law:

$$U_{eff} = \sum_{i=0}^{N=4} A_i \left( E_{HFA}^2 \right)^i .$$

Figure 3.11 shows the calibration data for one sensor during the period of the pool tests together with curve fits for Kings Law and polynomial fits which are plotted by thin and thick lines, respectively. The plots illustrate that polynomial fits cover the calibration data better than Kings Law within the whole velocity range. The related normalized standard deviation $\Delta E^2$ is calculated to specify the deviation between calibration data and curve fits for either (3.16) or (3.17):

$$\Delta E^2 = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( 1 - \frac{E_{fit}^2}{E_{data}^2} \right)^2} .$$

The values for $\Delta E^2$ are listed in the legend of the graph in figure 3.11. They confirm the observation that the polynomial fits cover the calibration data better than Kings Law. The finding is in accordance with the results made by Bruun [4], where polynomial or exponential equations with more than two terms are recommended instead of Kings Law for a more accurate description of the calibration data. For the pool tests, the fourth-order polynomial is used to translate the HFA signal into a velocity reading.

The reproducibility of the HFA measurements during the period of the pool tests is demonstrated by comparing the three calibration curves in figure 3.11 with the mean curve $U_{eff}^c \left( E_{HFA}^2 \right)$, calculated from all calibration curves. In figure 3.12, absolute and relative devia-
Figure 3.11: HFA calibration data and comparison between Kings Law and fourth-order polynomial.

Tions of the calibration curves from $U_{eff}^c$ are plotted as a function of $U_{eff}$. The graphs indicate that the reproducibility is in the range of ±6% for velocities higher than 0.3 m/s. For very low velocities, it increases up to 20%, which corresponds to a maximum difference of 0.02 m/s.

HFA measurements at low velocities require great care because they are also affected by small temperature fluctuations in the water. Water used in the tow tank and the pool tests was held at a steady temperature with an accuracy of ±0.1 K, which is the technical limit. The sensitivity of the HFA to temperature fluctuations can be reduced by increasing the temperature difference between the film element and the water, but this was also pushed to the limit by setting it to 14 K, as explained above.

Reproducible and constant results are achieved thanks to the precise adjustment of the water temperature in the tow tank. The same water conditioning loop keeps the water temperature constant during the pool tests. This makes it possible to perform long term measurements over some days without re-calibrating the HFA and without using a temperature correction for the calibration equation.
3.3.2 Sensor Sensitivity to Flow Angle

The HFA is intended to measure a single flow component, which, for the pool tests is the axial water velocity component $u_w$. However, the convective heat transfer from the film element is also affected by the radial and swirling velocity components $v_w$ and $w_w$ in the pool. Tests with different sensor orientations were performed in the tow tank to study the influence of $v_w$ and $w_w$ upon $u_w$ and to measure the corresponding effective velocities $U_{eff}$ with the HFA. The results are used to compare $U_{eff}$ with $u_w$ and to specify the measurement error that is caused by the sensitivity of the sensor to flow angle when $U_{eff}$ is interpreted as $u_w$ in the pool. The main findings are discussed here, while the detailed results are presented in appendix B.

The HFA sensor orientation in the cylindrical pool is illustrated in figure 3.13. The sensor is placed in the pool so that the coordinate system $r$ and the sensor coordinate system $r_{HFA}$ (figure 3.9) are related as follows:

\[ z = z_{HFA}, \quad r = x_{HFA}, \quad r_{\varphi} = y_{HFA}, \]  
\[ u = u_{HFA}, \quad v = v_{HFA}, \quad w = w_{HFA}. \]  

These relations are given by equations (3.19) and (3.20).
Yaw angle $\alpha$ and pitch angle $\varphi$ identify the direction of $u_w$ with respect to the sensor.

![Wedge shaped film sensor](image)

**Figure 3.13:** Velocity components seen by the HFA sensor in the pool.

Figure 3.14 illustrates the tests with different sensor orientations in the tow tank. The sensor was moved with a tow velocity $U_{\text{tow}}$ and the angles $\alpha$ and $\varphi$ were varied to simulate different velocity vectors $u_w$ in the pool. The sensor orientation was changed by setting one angle to $0^\circ$ and varying the other angle between $0^\circ$ and $90^\circ$ in steps of $15^\circ$. The velocity vector $u_w$ in the pool is represented by $U_{\text{tow}}$ and the magnitude of the axial velocity component $u_w$ is calculated with the cosine of $\alpha$ and $\varphi$, respectively. The maximum tow velocity in these tests was limited to $1.2 \text{ m/s}$.

The results from the tests are shown in figure 3.15. The HFA readings $U_{\text{eff}}$ are plotted as a function of the yaw and pitch angle for five different tow velocities. In addition, curves $u_w$ calculated using the cosine law, are also plotted. $U_{\text{eff}}$ decreases by rotating the sensor, as expected, and follows fairly closely the cosine law. The difference between $U_{\text{eff}}$ and $u_w$ is smaller for the yaw-angle effect than it is for the pitch-angle effect. In both cases $U_{\text{eff}}$ is not dropping down to zero when the sensor is positioned perpendicularly to the tow direction because the velocity components $v_w$ or $w_w$ produce still some sensor cooling and this is measured and translated into a velocity reading by the HFA.

The graph for the pitch-angle effect indicates that $U_{\text{eff}}$ is increasing between $30^\circ$ and $45^\circ$ instead of decreasing. This finding was reproducible for two wedge-shaped film sensors of the same type. An explanation for this behaviour could not be found but is probable due to the particular construction of the wedge enhancing heat transfer in this range. Moreover, at very low velocity, the variation in $U_{\text{eff}}$ with $\varphi$ is rather flat, as would have been expected at the limit.
Figure 3.14: Investigation of the orientation effect in the tow tank. Left and right hand sketches: Sensor element in horizontal and vertical position, respectively.

for a cylindrical wire sensor (Bruun [4]).

The relative deviation $\Delta_u$ between $u_w$ and $U_{eff}$ is calculated to quantify the measurement error when the velocity reading of the HFA is interpreted as the axial water velocity in the pool:

$$\Delta_u = \frac{u_w - U_{eff}}{u_w} = \left(1 - \frac{U_{eff}}{U_{low} \cos \omega_{sensor}}\right) 100.$$  (3.21)

The values for $\Delta_u$ from the tow tank tests are plotted in figure 3.16. At an angle of zero, the sensor is aligned with the tow direction and $\Delta_u$ is zero because $U_{eff} = u_w$ (as in the calibration tests). For yaw and pitch angles smaller than 40° the measurement error is within about ±10 % and ±15 %, respectively. When $u_w$ decreases further by rotating the sensor, the difference between $U_{eff}$ and $u_w$ is larger. $\Delta_u$ is infinite when the sensor is positioned perpendicular to the tow direction since $u_w$ is zero in this configuration but $U_{eff}$ is not zero.

A quantitative description of sensor cooling for different orientations is difficult because the heat transfer from the film element can only be approximated by the cooling of a plate. But the design of the film supports play also an important role for the velocity field and the cooling of the film element. The latter cannot be considered by a simple approach to calculate sensor cooling.

The main flow direction in the pool is along the axial velocity component $u_w$, but any radial and swirling velocity components, $v_w$ and $w_w$, respectively, will also affect the measurement.
Figure 3.15: Measured $U_{eff}$ and computed $u_w$ in the tow tank.

The measurement error depends on the yaw and pitch angle which can be expressed by the velocity ratios:

$$\alpha = \arctan \left( \frac{u_w}{v_w} \right), \quad \varphi = \arctan \left( \frac{u_w}{v_w} \right) . \quad (3.22)$$

Anagbo [1] used a two-component LDA to measure $u_w$ and $v_w$ independently in the bulk of a pool. The data demonstrate that the ratio between the velocity components is about 0.2 at the edge of a bubble plume, which corresponds to a yaw angle of $11^\circ$. For HFA measurements, this denotes that the velocity reading $U_{eff}$ over-predicts $u_w$ by less than 5% in the worst case. Moreover, CFD calculations also show that the water flows mainly upward in the bubble plume and radial and swirling velocity components are one order of magnitude smaller than axial velocity component. Thus, according to the tow tank calibration (figure 3.15) there is little effect on the measurements from radial and swirling flow components. Only at low velocities, there is an important effect from swirling flow and steady rotation of the flow in the pool, but strong swirling velocity components are not expected.

Additional tests were performed in the present work to further study the effect of sensor orientation within the pool. Water velocity profiles were measured for two different sensor orientations. The main axis of the sensor was oriented parallel to the pool axis ($z = z_{HFA}$) and
In summary, the HFA reading $U_{eff}$ is interpreted in the pool tests as the axial water velocity component $u_w$. The tests in the tow tank with different sensor orientations demonstrate that the sensor signal is not adversely affected by radial and swirling velocities ($v_w, w_w$) when they are small in relation to $u_w$, as expected in the pool.

### 3.3.3 Signal Analysis in Two-Phase Flow

A phase separation method for the HFA signal is needed in two-phase flows to distinguish between water and bubble passages. Bremhorst [3] studied the interaction between bubbles and wire sensors and related the findings to the sensor signals. A typical HFA sensor signal in the...
bubble plume is shown in figure 3.17. The known signal characteristics for wire sensors can be also identified for the wedge-shaped sensor. The signal associated with a direct bubble hit is described here. A direct hit is when the bubble surrounds the film element completely while passing. Other bubble-sensor interactions, such as partial or glancing hits, are discussed by Bremhorst [3], but the main signal characteristics do not differ much from the direct hit.

Figure 3.17: HFA output signal from wedge-shaped sensor and the phase separation method to identify water signals.

Before the bubble is first penetrated by the film element, the sensor signal increases because the water around the bubble rises faster than the surrounding water further away from the bubble. The signal declines fast when the bubble front hits the film element due to the reduced heat transfer in air. The signal falls until the whole film element is dry. The signal peak observed after the film element again enters the water is caused by the bubble wake.

Tests in the literature with wire sensors have shown that a water film may be formed between the prongs holding the wire while the sensor is surrounded by the bubble and the sensor is dry. This water film may break and some water may hit the wire producing a small peak. The same peaks are observed in the wedge-shaped sensor signal and they can be explained in a similar way.
Various phase separation methods based on signal thresholds and slopes are suggested in the literature. They are summarized by Bruun [4]. A combined threshold/slope method proposed by Liu [53] is used for the pool tests. The method identifies different signal characteristics with an algorithm consisting of eight conditions. For the description of the algorithm, the parameters listed in table 3.2 are needed.

<table>
<thead>
<tr>
<th>threshold</th>
<th>$E_T$</th>
<th>slope threshold</th>
<th>$E_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_i$</td>
<td></td>
<td>$E_b =</td>
<td>P_b</td>
</tr>
<tr>
<td>$E_f$</td>
<td></td>
<td>$E_f =</td>
<td>P_f</td>
</tr>
</tbody>
</table>

Table 3.2: Nomenclature for phase separation method.

The phase separation method is described by Liu [53] as follows: If $E_i > E_T$, and if any of the following eight conditions is satisfied, $E_i$ is in water and otherwise in air.

(a) If at least two water samples lie between two consecutive bubbles:

1. $E_b < E_S$ and $E_f < E_S$ (remain in water)
2. $E_b < E_S$ and $E_f > E_S$ (entering bubble)
3. $E_{i+1} > E_T$ and $P_b > E_S$ and $E_f < E_S$ (entering water)
4. $E_f < 0$ and $E_b < E_S$ and $E_f < 1.5E_S$ and $E_{i+2} < E_{i+1}$ and $|E_{i+2} - E_{i+1}| > E_S$ (overshooting due to bubble approaching)
5. $E_{i-1} < E_T$ and $E_{i+1} > E_T$ and $P_f > 0$ and $P_b > E_S$ and $E_f < 1.5E_S$ (overshooting due to bubble detaching)

(b) If only one water sample lies between two consecutive bubbles:

6. $P_b > 2S$ and $P_f > E_S$
7. $P_b > S$ and $P_f > 2E_S$
8. $E_{i-1} < E_T$ and $E_{i+1} < E_T$

The algorithm takes into account the main signal characteristics, but not the water film break signals mentioned above. Such a short signal interruption during a bubble passage is comparable to signal misinterpretations by an optical sensor due to noise or pre-signals. The modified threshold method used for processing the DOS signal is used also here to eliminate very brief signal interruption (film break effect). The correction time $t_{corr}$ is set again to 0.2 ms.
Liu [53] discusses no method for adjusting $E_T$ and $E_S$. The influence of the parameters on the average HFA reading $U_{eff}$ was tested in order to find an adequate setting. HFA signals were acquired in the pool over a period of 300 s and then processed with a range of values for $E_T$ and $E_S$. These measurements are compared to $u_{w,ref}$ obtained with the reference parameters $E_{S,ref}$ and $E_{T,ref}$. The results for two tests are illustrated in figure 3.18. The plots show that $u_w$ is practically independent of the parameter settings over a wide range. For the pool tests, the reference parameters $E_{S,ref}$ and $E_{T,ref}$ given in figure 3.18 are used to analyze the HFA signal.

![Figure 3.18: Dependency of the calculated mean water velocity on the signal and slope thresholds.](image)

As an example, the HFA signal, illustrated in figure 3.17, was treated using the method described above. The bubble passages and the water signals are clearly identified, allowing proper calculation of the mean water velocity.

### 3.3.4 Measuring the Mean Water Velocity in the Pool

The following procedure is used to determine the mean water velocity in the pool: First, the bubble passages are detected in the HFA signal and then the velocity readings within the calibration range are identified. This is done with two functions:
The digitized phase identification function $X_{HFA,i}$ that is established with the phase separation method from the HFA signal.

$$X_{HFA,i} = \begin{cases} 
1 & \text{if } E_i \text{ is in air} \\
0 & \text{if } E_i \text{ is in water}
\end{cases} \quad (3.23)$$

The function $C_i$ to check whether the data sample $E_i$ represents a voltage signal within the calibration range.

$$C_i = \begin{cases} 
1 & \text{if } E_i \text{ corresponds to a velocity within 0.08 and 2.42 m/s} \\
0 & \text{if not}
\end{cases} \quad (3.24)$$

Functions (3.23) and (3.24) permit to identify the water signals that are within the calibration range. The data samples $E_i$ are translated into a velocity reading $u_{w,i}$ by using the calibration curve (3.17) and a mean is calculated with:

$$u_{w} = \frac{1}{N - n_a - n_c} \sum_{i=1}^{N} u_{w,i} (1 - X_{HFA,i}) C_i . \quad (3.25)$$

$N$ is the total number of samples, $n_a$ is the number of samples in air, and $n_c$ is the number of samples out of the calibration range.

Water velocities smaller than the lower limit of the calibration range (0.08 m/s) are expected in the pool. When the HFA signal drops temporarily below that limit, the velocity can be anything between 0.08 and 0 m/s. Considering this uncertainty, the true velocity must lie between the following limits:

$$\bar{u}_w^- = 0.08m/s \frac{n_c}{N - n_a} + \bar{u}_w \left( \frac{N - n_a - n_c}{N - n_a} \right) \quad (3.26)$$

$$\bar{u}_w^+ = 0m/s \frac{n_c}{N - n_a} + \bar{u}_w \left( \frac{N - n_a - n_c}{N - n_a} \right) \quad (3.27)$$

The first weighting factor in (3.26) and (3.27) represents the fraction of water signals $(N - n_a)$ that are out of the calibration range and the second weighting factor stands for the water signals within the calibration range.
4 Pool Tests

4.1 Experimental Apparatus

The schematic diagram of the facility for the pool tests is shown in figure 4.1. Tests are performed in a tank that is connected to a water loop and a regulated air supply. A PC is used for the data acquisition and control of various parameters and functions of the facility.

The tank is 1 m in diameter and pool depths $H$ between 1.25 and 4.4 m can be obtained by moving the nozzle mounted on a sliding platform. The pool depth is defined by the distance between the nozzle exit and the upper end of water overflow (see figure 4.1). Air is injected upward through the single nozzle in the centre of the pool. Water is fed into the tank at the bottom and collected by an overflow section at the top. The two-phase flow can be observed through windows along one side of the tank (see figure 4.2). A traversing system positions three
local sensors in a plane \((z, r)\) parallel to the observation windows. Figure 4.3 shows a picture of the sensors within the pool. The sensor positions are limited because of various installations within the tank. The maximum radial sensor position \(r_{\text{max}}\) is 340 mm and the minimum distance between the pool surface and the sensor tips is 1 m, which means that \(z_{\text{max}} = H - 1\ m\). A detailed drawing of the tank and the installations inside is shown in appendix C.2.

The tank is filled with demineralized water. During the tests, water temperature and purity are carefully controlled in the system because the local sensors are sensitive to temperature fluctuations and contamination of the sensor tips. The water flow is driven by a pump operating at a constant flow rate. A heater and a cooler keep the water temperature constant at 20±0.1°C. A set of two filters (1 and 5 \(\mu\)m) is used for cleaning the water.
The air flow is adjusted by two mass flow controllers. They operate in different flow ranges\textsuperscript{4}: 5 to 100 $l_n/min$ and 30 to 300 $l_n/min$ (0.08 to 1.67 $dm^3/s$ and 0.5 to 5 $dm^3/s$). Any flow rate within these ranges can be set with an accuracy of ±2\%. The mass flow controllers are supplied with dry and filtered air at room temperature.

The complete instrumentation of the water loop and the air supply are presented in appendix C.1.

### 4.2 Experimental Procedure

To run the pool tests automatically, the LabVIEW\textsuperscript{5} program POOLexp was developed. Figure 4.4 shows the flowchart for a pool test and the functions controlled by POOLexp. A test starts with the adjustment of the experimental boundary conditions, i.e. the initial positioning of the nozzle and the sensors, setting the air flow rate, and adjusting water flow rate and pool temperature. Subsequently, the measurements begin. The sensors are positioned at the first measurement point, and the double optical sensor and hot-film anemometer signals are acquired.

\textsuperscript{4}$l_n/min$: normal liters per minute. $T_{ref} = 273K$, $p_{ref} = 101325Pa$

\textsuperscript{5}trade mark from National Instruments, www.ni.com
with the AD-converter and stored in raw data files. Subsequently, the data files for the double optical sensor are pre-processed before deleting them. The measurement procedure is repeated until all measurements (about 200 data points for each test) are completed. At the end of a test, the result files for the double optical sensor created by the pre-processing and the raw data files obtained from hot-film anemometer signals are transferred to a backup system.

To realize a continuous acquisition of the sensor signals, the AD-converter operates in the so-called stream to disk mode, which means that the data are directly stored on the hard disk during acquisition. The advantage is that the amount of data is limited by the hard disk space and not by the computer memory. For example, it is possible to continuously acquire sensor signals over a period of 3600 s with a sampling rate of 100 ksamples/s (about 700 MByte data).

The complete concept for the process control and the data acquisition is explained in detail in appendix C.3.

Figure 4.4: Experimental procedure for the pool tests.
5 Experimental Results and Discussion

The test matrix is specified at the beginning of this chapter. Subsequently, the flow patterns within the pool are discussed and images for different flow conditions are presented. Detailed information about the bubble plume characteristics is produced by the DOS and HFA measurements. The measurements describe the bubble plume expansion and the variation of various flow parameters as a function of the distance from the nozzle. The main conclusions obtained from the experimental results are drawn at the end of the chapter.

5.1 Test Matrix

The experimental test matrix is shown in table 5.1. The nozzles \( (d_0 = 5, 9.9, 20 \text{ mm}) \), previously used for the pool scrubbing tests in the POSEIDON facility [19], were installed in the new experimental apparatus for air injection. Limits for the volumetric air flow rate \( \dot{Q}_a \) (given for normal conditions) and the pool depth \( H \) were considered and set to minimize bubble plume oscillations. Video recordings have demonstrated that the bubble plume oscillates strongly for parameters exceeding those listed in table 5.1 (see section 5.2). Oscillations indicate interactions between the flow and the wall, which should be prevented so that the experimental findings are applicable to pools with diameters larger than 1 m.

The mean air velocity at the nozzle exit \( U_0 \) is given in table 5.1 and is defined as:

\[
U_0 = \dot{\bar{V}}_{a,0} \frac{4}{\pi d_0^2},
\]

where \( \dot{\bar{V}}_{a,0} \) is the volumetric air flow rate at the nozzle exit

\[
\dot{\bar{V}}_{a,0} = \dot{Q}_a \left(1 + \frac{H}{H_0}\right)^{-1} \frac{T_{\text{pool}}}{273K}
\]

calculated by considering the hydrostatic pressure and the pool temperature \( T_{\text{pool}} \). The atmospheric pressure is expressed by a water column \( H_0 \).

<table>
<thead>
<tr>
<th>( \dot{Q}_a [\text{dm}^3/s, (l_a/\text{min})] )</th>
<th>( d_0 = 5 \text{ mm} )</th>
<th>( d_0 = 9.9 \text{ mm} )</th>
<th>( d_0 = 20 \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H = 2 \text{ m} )</td>
<td>( H = 2 \text{ m} )</td>
<td>( H = 3 \text{ m} )</td>
<td>( H = 2 \text{ m} )</td>
</tr>
<tr>
<td>0.42, (25)</td>
<td>19.1 m/s</td>
<td>4.8 m/s</td>
<td>4.4 m/s</td>
</tr>
<tr>
<td>0.83, (50)</td>
<td>38.2</td>
<td>9.5</td>
<td>8.8</td>
</tr>
<tr>
<td>1.67, (100)</td>
<td>76.3</td>
<td>19.1</td>
<td>17.7</td>
</tr>
<tr>
<td>3.33, (200)</td>
<td>152.7</td>
<td>38.2</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Table 5.1: Mean velocities at the nozzle exit for the pool tests.
A volumetric water flow rate of 6 m³/h was circulated through the pool to adjust the temperature T_{pool} at 293.0 ± 0.1 K. This results in an average velocity of 2.1 mm/s upwards in the pool.

Measurements were performed with the DOS and HFA along the pool diameter (radial profiles) at selected distances from the nozzle (see table 5.2) and along the pool centerline (centerline profiles). The HFA was used only in tests with the 9.9 mm nozzle and at a pool depth of 2 m.

Table 5.2: Distances z from the nozzle for the radial profiles for tests with pool depths of 2 and 3 m.

<table>
<thead>
<tr>
<th>H</th>
<th>z [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 m</td>
<td>250, 500, 750, 1000</td>
</tr>
<tr>
<td>3 m</td>
<td>250, 500, 750, 1000, 1250, 1500, 1750, 2000</td>
</tr>
</tbody>
</table>

The DOS and HFA signals were acquired at each point of measurement over a period of 600 s, in order to obtain steady mean void fractions, bubble velocities, and water velocities. Preliminary tests were performed to set the optimum measurement time (see appendix D.1). Notation indicating time and ensemble average will be omitted in this chapter for simplification.

Table 5.3 lists the test identification used in the following sections. The first number indicates the air flow rate Q_a in l_n/min and the second number is the nozzle diameter d_0 in mm. Either the number for Q_a or d_0 is replaced by the letter X to specify a test sub-series later in the text and graphs. Sensor positions within the pool are denoted by (z, r). The axial position z and the radial position r, both measured from the nozzle exit, are given in mm.

<table>
<thead>
<tr>
<th>Q_a[d m³/s, (l_n/min)]</th>
<th>d_0 = 5 mm</th>
<th>d_0 = 9.9 mm</th>
<th>d_0 = 20 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H = 2 m</td>
<td>H = 2 m</td>
<td>H = 2 m</td>
</tr>
<tr>
<td></td>
<td>H = 3 m</td>
<td>H = 3 m</td>
<td>H = 3 m</td>
</tr>
<tr>
<td>0.42, 25</td>
<td>T25.5</td>
<td>T25.10*</td>
<td>T25.20</td>
</tr>
<tr>
<td>0.83, 50</td>
<td>T50.5</td>
<td>T50.10*</td>
<td>T50.20</td>
</tr>
<tr>
<td>1.67, 100</td>
<td>T100.5</td>
<td>T100.10</td>
<td>T100.20</td>
</tr>
<tr>
<td>3.33, 200</td>
<td>T200.5</td>
<td>T200.10</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Identification of the pool tests.
5.2 Flow Patterns and Visualization

Preliminary video recordings were taken through the observation windows to visualize the bubble plume oscillation in the pool, the flow formation in the ZFE, and the flow pattern in the ZEF. The video camera recorded 25 frames per second with the shutter set to 1 ms. The still images, shown here, provide a good qualitative understanding of the two-phase flow and the flow pattern.

5.2.1 Bubble Plume Oscillation

Observations of the bubble plume by eye and the video recordings demonstrate that the bubble plume oscillation depends mainly on the aspect ratio between pool depth and diameter. The finding is in agreement with another visual study by Chen [11] and results from the CFD calculations by Delnoij [20]. Figure 5.1 illustrates the two modes proposed by Chen [11] to explain bubble plume oscillations at different aspect ratios. A so-called cooling-tower or Gulf-Stream mode is established when the aspect ratio is less than or equal to unity. In these configurations, the bubble plume is stabilized by one large eddy that encloses the plume. The water is flowing upward in the middle and downward near the wall. The staggered-eddies mode occurs at greater aspect ratios and causes bubble plume oscillation. A row of eddies at distinct pool elevations is formed, that pushes the bubble plume temporarily to different sides of the pool. The eddies rotate in the swirling direction and move up and down in the pool. The bubble plume oscillates with an amplitude that increases with air flow. The two-phase flow becomes completely unstable and chaotic when the bubble plume expands outwards to the wall. In such extreme cases, bubbles may move downward in the pool because of strong water re-circulation.

The test matrix was established on the basis of the preliminary video recordings. Limits for $V_{a,0}$ and $H$ were set to minimize bubble plume oscillations during the tests. Tests with aspect ratios equal to unity could not be performed because the minimum distance between the sensor tips and the pool surface at the water overflow is 1 m. With a pool depth of 1 m the DOS and HFA cannot be positioned above the nozzle.

5.2.2 Flow Regime above the Nozzle

Figures 5.2 and 5.3 present photographs of the ZFE for T50.10 and T200.10. The detachment of individual large bubbles from the nozzle is observed for T50.10. The initial bubbles break up into bubble swarms containing bubbles of all shapes and sizes. In particular, large distorted bubbles occupying a large volume fraction of the flow were observed away from the injector. Further increase in $V_{a,0}$ produces an air jet and large formations of air above the nozzle. An example for such a flow pattern is illustrated in figure 5.3. The high relative velocity between
the air and the water produces disturbances at the interface which are indicated by ripples at the jet interface. This causes bubble break-up, water droplet entrainment into the air jet, and the dispersion of small bubbles into the surrounding water (Zhao [82]).
Figure 5.2: Photograph of the ZFE: T50.10.

Figure 5.3: Photograph of the ZFE: T200.10.
In the literature, various attempts can be found to predict the transition from the bubbling to the jetting regime in the ZFE, but there is considerable disagreement, because the general phenomenon of bubble formation is shown to be surprisingly complicated (Wallis [79]). A condition for air jet formation above the nozzle is given by Wallis [79] and depends on the mean air velocity through the nozzle exit:

\[ U_0 = 1.25 \left[ \frac{4\sigma}{9 g (\rho_{\text{liq}} - \rho_{\text{gas}}) d_0^2} \right]^{0.5} \left[ \frac{g (\rho_{\text{liq}} - \rho_{\text{gas}})^{0.25}}{\rho_{\text{gas}}^2} \right] \]  

(5.3)

Here \( \sigma \) is the surface tension of the liquid and \( \rho_{\text{liq}} \) and \( \rho_{\text{gas}} \) the liquid and gas densities, respectively. The right hand side of (5.3) specifies a critical velocity \( U_{0,\text{crit}} \). Higher velocities correspond to the jetting regime and lower velocities to the bubbling regime. The values of \( U_{0,\text{crit}} \) that are related to the test conditions in the present work are:

\[ U_{0,\text{crit}} (d_0 = 5\, \text{mm}) = 6.38\, \text{m/s} \]
\[ U_{0,\text{crit}} (d_0 = 9.9\, \text{mm}) = 3.19\, \text{m/s} \]
\[ U_{0,\text{crit}} (d_0 = 20\, \text{mm}) = 1.59\, \text{m/s} \]  

(5.4)

The comparison with values for \( U_0 \), listed in table 5.1, shows that all tests fall in the jetting regime.

Zhao [82] derived an alternative transition criterion, based on a combined Kelvin-Helmholtz and Rayleigh-Taylor instability approach, that describes the velocity and growth rate of disturbances at the interface. The transition from bubbling to jetting is connected to a critical Weber number:

\[ We_{\text{crit}} = \frac{\rho_{\text{liq}} U_0^2 d_0}{\sigma} \geq 10.5 \sqrt{\frac{\rho_{\text{liq}}}{\rho_{\text{gas}}}} \]  

(5.5)

All tests in table 5.1 fulfill condition (5.5), which means that they belong to the jetting regime.

Hoefele [38] proposed a third transition criterion. The modified Froude number

\[ Fr_m = \frac{\rho_{\text{gas}} U_0^2}{g (\rho_{\text{liq}} - \rho_{\text{gas}}) d_0} \]  

(5.6)

and the density ratio between the gas and liquid were used to establish a jet behaviour diagram. Video recordings were made from gas injection (air, helium, argon) into pools of liquid (mercury, water) to set up the diagram. The test points for the present work lie either in the bubbling regime or the transition regime between bubbling and jetting.

None of the previously discussed transition criteria distinguishes between the two different flow patterns shown in figures 5.2 and 5.3. After studying the literature, it can be concluded that defining a transition criterion is rather complex. The problems can be summarized as follows:
• It is difficult to define a transition point between bubbling and jetting in experiments. The discrimination between bubbling and jetting is commonly performed by studying high speed video recordings [38, 51] and so the evaluation of the flow regime depends largely upon the judgement of the observer. In the transition regime between bubbling and jetting, this is particularly susceptible to subjective factors.

• In the theoretical analysis, the change of the driving force that controls bubble formation should be described. In the bubbling regime, the surface tension holding the bubble on the circumference of the nozzle plays the significant role, while in the jetting regime, the momentum of the gas injection and the acceleration of the surrounding liquid are more important. There was nothing found in the literature that considers the change of the driving force to derive a transition criterion between bubbling and jetting.

In the literature, there is no universal criterion for the transition between bubbling and jetting illustrated in figures 5.2 and 5.3 based on gas flow rate, nozzle diameter and fluid properties. No new attempt is made here to find such a criterion because it is not the main objective of the work and the required measuring techniques (e.g. high speed video camera and PIV system) were not available.

5.2.3 Flow Regime in the Bubble Plume

The main objective of the work is the investigation of the two-phase flow in the ZEF under pool scrubbing conditions. As was stated for the POSEIDON tests [19], the flow pattern, related to pool scrubbing, is comparable to the chaotic churn turbulent flow pattern in pipes. It means that the bubble ensemble in the pool is frothy and disordered like the bubble ensemble in pipes (Taitel [73]).

Figures 5.4 and 5.5 illustrate the two-phase flow in the pool, one meter away from the nozzle. The test conditions correspond to the same two tests discussed above. An ensemble of large and small bubbles can be identified in both images. Surprisingly large bubbles are observed for T50.10, which means that the break-up of the bubble formations seen in figure 5.2 is not complete at a distance of 1 m from the injector. The bubble density for T200.10 is much higher than for T50.10 and large bubble formations are found that rise through the pool with a velocity well above the rise velocity of individual large bubbles in stagnant water.

The two photographs in figures 5.4 and 5.5 indicate a chaotic flow regime where coalescence and bubble break-up occurs. The wide spectrum of bubble sizes results in different relative velocities between the bubbles and this causes coalescence when the bubbles catch up with each other. The large bubbles in the bubble plume are not stable and they break-up into smaller ones.
Nevertheless, the zone further away from the injector is referred to as ZEF (zone of established flow) from now on because the disequilibrium between bubble break-up and coalescence is much smaller there than in the ZFE (zone of flow establishment). The distance from the nozzle where the change between the ZFE and ZEF takes place is later defined on the basis of centerline void fraction profiles.

Video recordings have been taken for tests with different nozzle sizes. A dependency of the flow pattern in the ZEF on nozzle diameter $d_0$ is not observed. The change in the initial momentum at the nozzle exit by changing $d_0$ in the range of 5 to 20 mm has no significant influence on bubble formation in the ZEF.

The flow patterns illustrated in the POSEIDON report [19] for tests under pool scrubbing conditions are similar to the flow patterns shown here. Therefore, the test conditions in the present work are comparable to pool scrubbing tests with regard to the flow pattern. It means an uncontrolled and chaotic air injection in the ZFE and an ensemble of frothy and disordered bubbles in the ZEF. The flow pattern above the nozzle changes from single bubble to an air jet injection with increasing $\dot{V}_{a,0}$, but a transition criterion describing the phenomena was not found. The video images in the ZEF have shown that only the bubble density is increasing with $\dot{V}_{a,0}$ but no considerable differences in the flow pattern were visually observed with regard to bubble sizes and shapes.
Figure 5.4: Photograph of the ZEF: T50.10.

Figure 5.5: Photograph of the ZEF: T200.10.
5.3 Bubble Plume Characteristics

Detailed bubble plume characteristics were obtained from the DOS and HFA measurements. The experimental results for the tests at a pool depth of 2 m are discussed first in separate sections for the void fraction, bubble rise and axial water velocity, relative phase velocity and bubble chord length distribution data. The bubble plume expansion (bubble core and entrained water flow) and the variation in the flow parameters with distance from the nozzle are illustrated by the data. Dimensionless correlations are derived to characterize the flow and to compare the data with previous experiments described in the literature. Subsequently, the tests at a pool depth of 3 m are presented to discuss the extrapolation of the experimental findings to pools with larger depths. An error analysis is made at the end by computing the volumetric air flow injected into the pool with that computed from the DOS measurements.

The various parameters that are discussed in the following subsection are illustrated in figure 2.1.

5.3.1 Void Fraction

Radial void fraction profiles in a bubble plume are commonly described by Gaussian curves. Figure 5.6 shows void fraction profiles for T100.10 and the corresponding Gaussian curve fits, defined by:

$$\epsilon (r, z) = \epsilon_m (z) \exp \left( -\frac{(r - r_{c,e} (z))^2}{b_e^2 (z)} \right).$$

(5.7)

The maximum void fraction $\epsilon_m$, the bubble core width $b_e$, and the displacement from the pool centerline $r_{c,e}$ are listed together with the statistical errors $\Delta \epsilon_m$, $\Delta b_e$, and $\Delta r_{c,e}$ for each profile in figure 5.6. All parameters are determined by a standard fitting procedure. The statistical errors given or plotted in the following graphs as error bars consider the scatter of the measurement points. The numerical values that specify the Gaussian curve fits for all tests listed in table 5.1 can be found in the appendix E.4.

Comparisons between experimental data and curve fits show that the radial void fraction profiles are very well represented by Gaussian curves. The profiles are symmetric and the displacements of the maxima from the pool centerline are small. The largest displacement for pool tests is 34.3 mm, which is less than 4 % in relation to the pool diameter. For most tests, displacements smaller than 2 % of the pool diameter were measured. Good symmetry and the centering of the profiles were obtained because the bubble plume oscillated very little and the void fraction was measured over a period of 600 s, which is long compared to any oscillation frequency.
Figure 5.6: Radial void fraction profiles for T100.10.
Experimental Results

The self-similarity of the void fraction profiles is made evident in figure (5.7). The characteristic profile parameters $b_e$ and $e_m$ are used to derive the dimensionless form of the radial void fraction profiles, defined by:

$$
\frac{\epsilon(r, z)}{e_m(z)} = \exp\left(-\frac{r^2}{b_e^2(z)}\right).
$$

Figure 5.7 shows the results for T100.10 and TX.10 at $z = 500\ mm$. The graphs prove the self-similarity of the profiles at different distances from the nozzle and for different air flow rates.

Figure 5.7: Self-similarity of the void fraction profiles.

Various experimental studies [9, 29, 41, 72, 58, 32] have selected the same type of curve fit to describe the radial profiles. Symmetry and centering problems were, however reported, in particular by Castello-Branco [9], for large-scale tests with high air flow rates. They are caused by strong bubble plume oscillations in the pool and the measurement time of 300 s was, apparently, insufficient to obtain steady mean values.

Bubble plume oscillation and the parameters derived from the radial void fraction profiles are discussed below in detail. The measurements describe bubble core expansion and the decrease in void fraction with increasing distance from the nozzle is studied.
Bubble Plume Oscillation

Bubble plume oscillations can never be totally avoided in a finite pool. Figure 5.8 shows the time dependent void fraction measured at position (250, 0) for T200.10 and T25.10. The signal of the DOS was continuously acquired over a period of 4800 s and divided into time intervals $\Delta t$ of 8 s. The void fraction in each interval was determined. The data in figure 5.8 demonstrate that the mean void fraction and the amplitude of the fluctuation increase with $\dot{Q}_a$. A characteristic frequency was, however, not found by computing the power spectrum. The flow is too complex to obtain any quantitative information from these data.

![Figure 5.8: Variation of the void fraction in time at a given position for two different air flow rates.](image)

Bubble Core Expansion

Figure 5.9 shows the bubble core widths $b_c$ versus $z$ for all tests at a pool depth of 2 m. The data for individual tests are fitted with a linear function

$$b_c (z) = B_c z + b_{0,c} = B_c (z - z_{0,c}) \quad \text{with} \quad z_{0,c} = \frac{b_{0,c}}{B_c}$$

(5.9)
Figure 5.9: Bubble core widths versus distance from the nozzle.
to specify the expansion factor \( B_e = (db_e/dz) \) and the virtual origin \( z_{0,e} \) of the bubble core.

The measurements show that bubble core expansion depends on \( V_{a,0} \) while \( d_0 \) plays no role. The linear fits for T200.X and T100.X describe the measurement points better than the fits for T50.X and T25.X. The observation can be explained by a transition from a flow regime where the entire bubble plume behaves more like a jet to one where the bubbles rise in a limited column within the pool. The radial void fraction profiles for T25.20 are shown in figure 5.10 as an example for the bubble-column regime.

The linear fits in figure 5.9 approximate bubble core expansion quite well in the jet-like regime while the deviations between fits and measurements are stronger in the bubble-column regime. In fact, it becomes difficult to measure the smaller expansion factors, still observed in the bubble-column regime, because the void fraction further away from the injector varies along the radius only between 0.03 and zero. Henderson [35] proposed top-hat profiles as an alternative to Gaussian curves. However, top-hat profiles would give an imperfect description of the profiles because a smooth decrease in void fraction with distance from the pool center was measured also for the tests with low air flow rates. In any case, the standard deviation of the Gaussian curve is a good measure for bubble core expansion and the linear fits represent the increase in \( b_e \) with \( z \) quite well.

The parameters \( B_e \) and \( z_{0,e} \) are plotted in figure 5.11 against \( V_{a,0} \). The errors \( \Delta B_e \) and \( \Delta z_{0,e} \) are obtained from the fitting procedure for the linear functions and the uncertainty of the virtual origin is calculated by the error propagation method:

\[
\Delta z_{0,e} = \sqrt{\left( \frac{\Delta b_{0,e}}{B_e} \right)^2 + \left( \frac{b_{0,e} \Delta B_e}{B_e^2} \right)^2}.
\]  

(5.10)

The data for \( B_e \) and \( z_{0,e} \) at constant air flow rates show no systematic dependence on \( d_0 \). This emphasizes again that \( d_0 \) is not affecting the flow characteristics at larger distances from the injector. For this reason, tests with different nozzles are considered as repetitions of tests with the same \( V_{a,0} \) and average values are calculated to compensate the scatter of the data. The resulting plots are illustrated as dotted lines in figure 5.11. The correlations for \( B_e \left( \bar{V}_{a,0} \right) \) and \( z_{0,e} \left( \bar{V}_{a,0} \right) \) are determined from the average values to permit numerical calculations.

\[
B_e = 0.082 V_{a,0}^{0.3}, \quad \bar{V}_{a,0} = \left[ \frac{dm^3}{s} \right]
\]

(5.11)

\[
z_{0,e} = -200 mm \left( 1 + 0.14 \bar{V}_{a,0}^{2} \right), \quad \bar{V}_{a,0} = \left[ \frac{dm^3}{s} \right]
\]

(5.12)

The expansion factor \( B_e \) increases with \( \bar{V}_{a,0} \), indicating that the bubble core expands more at higher air flow rates. The phenomenon is well specified by (5.11).
Figure 5.10: Radial void fraction profiles for T25.20.
**Experimental Results**

The virtual origin of the bubble core is located below the actual nozzle position. The average value of $z_{0,e}$ for T25.X is $-450 \text{ mm}$ but the scatter of the three data points is high and the error bands are wide. This reflects the difficulty of fitting the data $b_e(z)$ with linear functions in the bubble-column regime. The virtual origins found for the remaining tests are very well represented by an asymptotic value of $-200 \text{ mm}$. The empirical correlation (5.12) is established by setting the exponent to $-2$ and the factor to $-200 \text{ mm}$. It considers the rapid increase in $z_{0,e}$ with $\dot{V}_{a,0}$ and the leveling off to the asymptotic value. At low air flow rates the validation of (5.12) is questionable.

**Void Fraction along the Centerline**

The decrease in void fraction along the pool centerline is shown in figure 5.12. The graphs include the maximum void fractions $\epsilon_m$, determined from the radial profiles, and the centerline void fraction profiles $\epsilon_c$. For measuring the centerline profiles, the minimum distance of the DOS from the nozzle was set to $50 \text{ mm}$ for most tests. This was increased to $100 \text{ mm}$ for tests with high air injection velocities $U_0$ to prevent sensor damage.

The plots in figure 5.12 illustrate that the measurements for high air flow rates show less
Figure 5.12: Centerline void fraction profiles and radial profile maxima.
scatter than tests with low air flow rates. It demonstrates again the difference between the jet-like and bubble-column flow regimes. A more distinctive void fraction maximum can be determined in the jet-like flow regime in comparison to the bubble-column flow regime. Nevertheless, a characteristic decrease in void fraction with $\dot{V}_{a,0}$ is observed in all tests.

The void fraction close to the nozzle increase with $\dot{V}_{a,0}$ and decrease with increasing $d_0$ when $\dot{V}_{a,0}$ is constant. Further away from the injector, the nozzle size does not affect the void fraction. This fact can be used to identify the length of the ZFE $z_0$ and the change to the ZEF, respectively. The length $z_0$ is set where the void fraction of the centerline profiles matches a void fraction threshold $\epsilon_{c,0}$. Tacke [72], Iguchi [41] and Castello-Branco [9] proposed that $\epsilon_{c,0}$ be equal to 0.5. The argument was that this value characterizes the change between the churn turbulent and the bubbly flow regimes in pipes based on the approach for the maximum allowable packing of spherical bubbles in a cubic lattice (Dukler [23]). Experiments in pipes show that the change takes place at a void fraction between 0.3 and 0.25, which is explainable by random bubble motion. Using again the approach for maximum allowable packing of spherical bubbles and considering free spacing, i.e. half the bubble radius, results in a void fraction of about 0.25 (Dukler [23]).

The measurements in the present work, which are the most comprehensive in comparison to the previous experiments, demonstrate that setting $\epsilon_{c,0}$ equal to 0.5 is too rough, because the void fraction variation at this distance depends strongly on the nozzle size. Reducing $\epsilon_{c,0}$ to 0.25 results in much lesser dependency on $d_0$ and therefore the lower value of $\epsilon_{c,0} = 0.25$ was adopted for defining the change between ZFE and ZEF. Justifying the void fraction threshold with the argument that this change corresponds to the transition between churn turbulent and bubbly flow in pipes is doubtful because the flow here is more complex. The reason for setting $\epsilon_{c,0} = 0.25$ is that it matches the experimental observations.

The decrease in void fraction with distance from the nozzle is correlated in the previous studies with a dimensionless power law:

$$\frac{\epsilon_c}{\epsilon_{c,0}} = \left( \frac{z}{z_0} \right)^{-\xi} \quad (5.13)$$

Figure 5.13 illustrates the centerline void fraction profiles for TX.10 and the same profiles plotted against the dimensionless distance from the injector. The values for $z_0$ listed in the legend are determined from the plots, and the exponents $\xi$ are obtained by fitting the data $\epsilon (z/z_0)$. TX.5 and TX.20 and are analyzed with the same method and the results are given in table 5.4. The power function in the graph at the right hand side of figure 5.13 is plotted with a mean exponent $\xi = 1.15 \pm 0.18$ calculated using all tests.

The linear trends in the profiles for void fractions smaller than 0.25 in the logarithmic graph demonstrate very well that the variation in the centerline void fraction profiles can be repre-
Experimental Results

Figure 5.13: Centerline void fraction profiles TX.10 as a function of \( z \) and \( z/z_0 \).

<table>
<thead>
<tr>
<th>( z_0 ) [mm]</th>
<th>( \xi )</th>
<th>( z_0 ) [mm]</th>
<th>( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T25.5</td>
<td>108</td>
<td>T25.20</td>
<td>91</td>
</tr>
<tr>
<td>T50.5</td>
<td>136</td>
<td>T50.20</td>
<td>125</td>
</tr>
<tr>
<td>T100.5</td>
<td>191</td>
<td>T100.20</td>
<td>161</td>
</tr>
<tr>
<td>T200.5</td>
<td>297</td>
<td>T200.20</td>
<td>250</td>
</tr>
</tbody>
</table>

Table 5.4: Values of \( z_0 \) and \( \xi \) for TX.5 and TX.20.

sent by a power law. Scaling the axial positions with \( z_0 \) and describing the decrease with an exponent equal to 1.15 results in good agreement between the measurements and (5.13). The values for \( \xi \) in the present work correspond to the findings made by Tacke [72] and Castello-Branco [9] (mean value: 1.22) and Iguchi [41] (scatter: 0.98-1.3).

The increase in \( z_0 \) with increasing \( \hat{V}_{a,0} \) is very well illustrated by the graph at the left hand side of figure 5.13. In preceding studies, \( z_0 \) was correlated with the modified Froude number (5.6) by the following functional relationship:

\[
\frac{z_0}{d_0} = CFr_m^{1.15} \\
(5.14)
\]
Slightly different values of the parameters were found for small-scale tests performed by Tacke [72] and Iguchi [41] \((C = 5, \beta = 0.3)\) and large scale tests by Castello-Branco [9] \((C = 6.8, \beta = 0.27)\). The correlations are valid for \(Fr_m\) between 1 and \(1 \cdot 10^3\). The high values for \(Fr_m\) were reached by injecting air at low flow rates (about \(0.1 \, dm^3/s\)) with nozzles 0.5 mm in diameter (see also Table 1.1).

The experimental data for \(z_0/d_0\) against \(Fr_m\) and the correlation for the present work \((C = 13.2, \beta = 0.23)\) are compared in Figure 5.14 to the previously derived correlations. The previously derived correlations are modified to take into account the different void fraction threshold \(\epsilon_{c0}\). The change in \(\epsilon_{c0}\) from 0.5 to 0.25 corresponds to an increase in \(z_0\) and this must be considered in (5.14). The distance from the nozzle where the void fraction is equal to 0.25 can be calculated by using (5.13) and the mean exponent \(\xi = 1.2\) that is obtained from the previous tests:

\[
\frac{0.25}{0.5} = \left( \frac{z(\epsilon_c = 0.25)}{z_0(\epsilon_{c0} = 0.5)} \right)^{-1.2} \rightarrow z(\epsilon_c = 0.25) = 1.77 \cdot z_0(\epsilon_{c0} = 0.5) . \quad (5.15)
\]

Replacing \(z_0\) in (5.14) with \(z(\epsilon_c = 0.25)\) by using (5.15) shows that \(C\) increases by a factor of 1.77 when changing \(\epsilon_{c0}\). The corresponding parameter combinations for small and large-scale tests are \((C = 8.9, \beta = 0.3)\) and \((C = 6.8, \beta = 0.27)\), respectively.

Figure 5.14 indicates that the present tests cover a wide range of \(Fr_m\) that includes buoyancy \((Fr_m < 1)\) and inertia \((Fr_m > 1)\) controlled air injection. The derived correlation for both kinds of air injection agrees well with the results of previous tests, where air injection was only controlled by inertia. In the present tests bigger nozzles are used to inject much higher air flow rates in contrast to the small-scale tests performed by Tacke [72] and Iguchi [41]. Castello-Branco [9] studied injection of high air flow rates into large pools but no systematic tests were performed, i.e. tests with the same air flow rate but different nozzle diameter. The present work is novel in that nozzle size and air flow rate are varied systematically and bubble plume oscillation is limited.

The dependency of \(\dot{V}_{a,0}\) and \(d_0\) on the variation in the centerline void fraction profiles can be shown by summarizing the correlations derived before. Centerline void fraction profiles are specified by the combination of (5.13) and (5.14) with the parameters \(C = 13.2\) and \(\beta = 0.23\):

\[
\epsilon_c(z) = 0.25 \left( \frac{z}{z_0} \right)^{-1.15} = 4.9Fr_m^{0.26} \left( \frac{z}{d_0} \right)^{-1.15} \quad z \geq z_0 . \quad (5.16)
\]

A relationship for \(z_0\) can be determined by inserting (5.6) into (5.14) and translating \(U_0\) back into \(\dot{V}_{a,0}\) with (5.1):

\[
z_0 = 14.8 \left( \frac{\rho_a}{g (\rho_w - \rho_a)} \right)^{0.23} \dot{V}_{a,0}^{-0.46} d_0^{-0.15} . \quad (5.17)
\]
Equation (5.17) demonstrates that varying $V_{a,0}$ affects $z_0$ more than a change in $d_0$. The idea was that $z_0$ should be independent on the nozzle size but the measurements of $z_0$, listed in figure 5.13 and table 5.4, have already shown that the condition is not entirely fulfilled. However, the data indicate a clear trend. The void fraction criterion $e_{v,0} = 0.25$ to specify the change between the ZFE and ZEF is simple but a more sophisticated transition criterion was not found in the literature. By looking again at figures 5.2 and 5.3 that show photographs of the flow in the ZFE and recalling the previously explained problems of describing the flow in the ZFE, it becomes clear that it is even more difficult to identify the change to the ZEF better than that achieved by the void fraction criterion.

5.3.2 Bubble Rise Velocity

Radial bubble rise velocity profiles are shown in figure 5.15. The measurement points of each profile are fitted by a Gaussian curve with an offset of 0.19 m/s:

$$u_b(r, z) = 0.19 m/s + u_{b,0}(z) \exp \left( -\frac{(r - r_{cubb}(z))^2}{b_{ubb}^2(z)} \right)$$ (5.18)
to determine the amplitude of the exponential term $u_{b,0}$, the width of the bubble rise velocity profiles $b_{ub}$, the displacement from the pool centerline $r_{c,ub}$, and the corresponding statistical errors $\Delta u_{b,m}$, $\Delta b_{ub}$, and $\Delta r_{c,ub}$. The values for all tests are listed in table E.5. The maximum bubble rise velocity in the center $u_{b,m}$ is given by:

$$u_{b,m} = 0.19 \text{m/s} + u_{b,0} .$$

The offset for the Gaussian curve is the minimal bubble velocity that can be measured by the DOS. It is also the rise velocity of bubbles, 1 $mm$ in diameter, in stagnant water (Haberman & Morton [33]).

Each measurement point represents an ensemble-averaged bubble rise velocity over all detected bubbles during the measurement time of 600 s. In practice, a few thousand bubbles were considered to calculate the average at the pool centerline while, less than 50 bubbles were detected at the edge of the bubble core. Void fraction is a measure for the probability of detecting a bubble.

The bubble rise velocity profiles have maxima at the pool centerline and the velocity decreases along the radius. The Gaussian curves permit to identify the maxima and the displacements of the maximum from the pool centerline. The displacements $r_{c,ub}$ are smaller than 4% in relation to the pool diameter for all tests. The parameter $b_{ub}$ describes quite well the expansion of the velocity profiles. Nevertheless, the profiles do not have the distinctive bell shape of a Gaussian curve. A gaussian curve has points of inflexion at $r = \pm b_{ub}/\sqrt{2}$, which are not seen in the measurements themselves.

A decrease in bubble rise velocity $u_{b,m}$ with increasing distance from the nozzle is seen in figure 5.15. The rise velocity is always higher than that of bubbles in stagnant water, even 1 m away from the injector. The bubbles rise, however, within an entrained water flow, as shown later by the water velocity measurements. The bubble rise velocity at the edge of the bubble core, where the water velocity is close to zero, is also higher than the bubble rise velocity in stagnant water because of slight bubble plume oscillations. Indeed, faster bubbles, which were closer to the bubble core center, were detected at the edge of the bubble plume driven by oscillations and they were considered to calculate the average velocity.

The bubble rise velocity measurements in the pool are analyzed below in a similar manner to the void fraction measurements. The expansion of the profiles, measured by $b_{ub}$, is described first and then the decrease in bubble rise velocity with increasing distance from the nozzle is analyzed with centerline profiles. A dimensionless correlation is derived from the centerline bubble rise velocity profiles.
Figure 5.15: Radial bubble rise velocity profiles for T100.10
Expansion of the Bubble Rise Velocity Profiles

The widths of the bubble rise velocity profiles $b_{ub}$ were plotted against $z$ for individual tests and then fitted with linear functions

$$b_{ub}(z) = B_{ub}z + b_{0,ub} = B_{ub}(z - z_{0,ub}) \quad \text{and} \quad z_{0,ub} = \frac{b_{0,ub}}{B_{ub}}$$

(5.20)

to determine the expansion factor $B_{ub}$ and the virtual origin $z_{0,ub}$. The values for $B_{ub}$ and $z_{0,ub}$ are plotted against $V_{a,0}$ in figure 5.16. The errors $\Delta B_{ub}$ are determined by the fitting procedure and the error propagation is used to calculate $\Delta z_{0,ub}$:

$$\Delta z_{0,ub} = \sqrt{\left(\frac{\Delta b_{0,ub}}{B_{ub}}\right)^2 + \left(\frac{b_{0,ub} \Delta B_{ub}}{B_{ub}^2}\right)^2}.$$  

(5.21)

The plots in figure 5.16 illustrate that the expansion of bubble rise velocity profiles is influenced only by $V_{a,0}$ and not by $d_0$. Average values for tests with the same air flow rate but different nozzle sizes are calculated to derive correlations for $B_{ub}(\dot{V}_{a,0})$ and $z_{0,ub}(\dot{V}_{a,0})$.

![Figure 5.16](image)

Figure 5.16: Expansion factor and virtual origin to calculate $b_{ub}$ with (5.20).

Figure 5.16 shows that $B_{ub}$ increases with $\dot{V}_{a,0}$. Comparison between the correlations $B_{ub}(\dot{V}_{a,0})$ in figure 5.16 and the expansion factor for the bubble core (5.11) shows that $B_{ub}$...
Experimental Results

is about twice as large as Be for all tests. The virtual origin $z_{0,ub}$ is below the actual position of the nozzle for all tests. For T25.X, the averaged value for $z_{0,ub}$ is at about $-800 \text{ mm}$ but the scatter of the data is high. The remaining tests are well represented by an asymptotic value equal to $-300 \text{ mm}$. The same trend is observed in figure 5.11 for the virtual origin of the bubble core $z_{0,e}$, where a lower value is found for T25.X ($-450 \text{ mm}$) and the asymptotic value is $-200 \text{ mm}$. The findings point out that $b_{ub}$ is significantly larger than $b_e$ for all tests and at any distance from the nozzle. This reflects the fact that the bubble rise velocity profiles are flatter than the void fraction profiles.

Bubble Rise Velocity along the Centerline

The decrease in bubble rise velocity with increasing $z$ is shown in figure 5.17. The graphs include the centerline profiles $u_{b,c}$ and the radial profile maxima $u_{b,m}$. The measurements demonstrate that, close to the injector, the velocity depends on $\dot{V}_{a,0}$ and $d_0$, while further away only $\dot{V}_{a,0}$ affects the decrease in velocity. The centerline profiles confirm the separation into the ZFE and ZEF.

The bubble rise velocity in the ZFE increases for smaller nozzle sizes when $\dot{V}_{a,0}$ is constant because the initial momentum at the nozzle exit increases. In addition, the variation in velocity with $z$ close to the injector depends on $\dot{V}_{a,0}$. For T25.X, the velocity decreases monotonically because large bubbles detach from the nozzle and break up into swarms of smaller bubbles, which causes a dissipation of the initial momentum generated at the nozzle exit. For higher air flow rates, the centerline profiles show peaks near the nozzle exit. This indicates acceleration of bubbles in the region of the air jet above the nozzle and deceleration as they move away from the air jet.

The measurements with different nozzle sizes and the same air flow rate show that the initial momentum at the nozzle exit plays no significant role in the ZEF. Kobus [47] explained the fact by studying the momentum flux in the bubble plume. The total momentum flux $M$ is given by the sum of the initial momentum flux at the nozzle exit $M_0$ and the momentum flux generated by the change in buoyancy $M_B$. The momentum flux $M_B$ increases because of the decompression of the bubbles as they rise through the pool:

$$M(z) = M_0 + M_B(z) \simeq \dot{m}_a U_0 - \frac{\dot{m}_a}{\langle u_b \rangle_A} \frac{P_0}{\rho_{a,0}} \ln \left( 1 - \frac{z}{H_0 + H} \right).$$

(5.22)

The variables in (5.22) represent the air mass flow rate $\dot{m}_a$, the cross-sectional average bubble rise velocity at a certain distance from the injector $\langle u_b \rangle_A$, the air density under normal conditions $\rho_{a,0}$, and the reference pressure $P_0$. The ratio between $M_0$ and $M_B$ can be used to analyze the
Figure 5.17: Centerline bubble rise velocity profiles and radial profile maxima.
Experimental Results

different contributions of the two terms on \( M \):

\[
\frac{M_0}{M_B(z)} \simeq U_0 \langle u_b \rangle A \frac{\rho_{a,0}}{p_0} \frac{H_0 + H}{z} .
\]  
(5.23)

A detailed derivation of (5.22) and (5.23) is given in appendix F.

The terms in (5.23) can be approximated for the present tests by \( \rho_{a,0}/p_0 \simeq 1.2 \cdot 10^{-4} \text{ s}^2/\text{m}^2 \), \( \langle u_b \rangle_A \simeq 1 \text{ m/s} \), and \( H_0 + H \simeq 12 \text{ m} \). Inserting these estimates into (5.23) gives:

\[
\frac{M_0}{M_B(z)} \simeq \left( 1.5 \cdot 10^{-4} s \right) U_0 z^{-1} .
\]  
(5.24)

For buoyancy controlled flow, we may assume that \( M_B \) is ten times higher than \( M_0 \). This condition is fulfilled in the present work (\( U_{0,max} = 152.7 \text{ m/s} \)) at distances from the nozzle greater than 0.23 m. The estimation proves that the influence from initial momentum, which varies with nozzle size for constant \( \dot{V}_{a,0} \), is negligible far from the injector. The result is that void fraction and bubble rise velocity in the ZEF are influenced chiefly by buoyancy forces and so these parameters depend only upon the bubble density in the bubble plume, which means on \( \dot{V}_{a,0} \).

Castello-Branco [9] derived a dimensionless correlation to describe the relationship between \( u_{b,c} \) and \( z \) in the ZEF for different air flow rates:

\[
\frac{u_{b,c}}{g^{0.4} \dot{V}_{a,0}^{0.2}} = 1.60 \left( \frac{z}{z_0 (\epsilon_{c,0} = 0.5)} \right)^{-2.04} + 1.82 \left( \frac{z}{z_0 (\epsilon_{c,0} = 0.5)} \right)^{-0.08} .
\]  
(5.25)

The length of the ZFE \( z_0 \) was set where \( \epsilon_c = 0.5 \) and the values for \( z_0 \) in (5.25) must be modified according to (5.15) to compare the correlation with the present work:

\[
\frac{u_{b,c}}{g^{0.4} \dot{V}_{a,0}^{0.2}} = 0.5 \left( \frac{z}{z_0 (\epsilon_{c,0} = 0.25)} \right)^{-2.04} + 1.74 \left( \frac{z}{z_0 (\epsilon_{c,0} = 0.25)} \right)^{-0.08} .
\]  
(5.26)

The centerline bubble rise velocity profiles in the present work are analyzed in a fashion similar to that of Castello-Branco [9]. Figure 5.18 shows the dimensionless centerline profiles for TX.10 and the derived correlation for the present work together with (5.26). To establish the present correlation, the length \( z_0 \) is calculated with (5.17) for individual tests and the dimensionless velocity profiles are fitted with an equation in the form of (5.25) for \( z/z_0 \geq 1 \). The exponents \(-2.04\) and \(-0.08\) are set constant to \(-2\) and \(-0.1\), respectively and the two factors are determined for all tests individually. The resulting correlation with the averaged factors is:

\[
\frac{u_{b,c}}{g^{0.4} \dot{V}_{a,0}^{0.2}} = 0.32 \left( \frac{z}{z_0 (\epsilon_{c,0} = 0.25)} \right)^{-2} + 1.87 \left( \frac{z}{z_0 (\epsilon_{c,0} = 0.25)} \right)^{-0.1} .
\]  
(5.27)
Figure 5.18: Dimensionless centerline bubble rise velocity profiles and correlations (5.26) and (5.27).

Figure 5.18 illustrates that (5.27) agrees well with (5.26) and with the experimental data in the present work. The first term in (5.27) is large only at small distances from \( z_0 \) while further away only the second term is important. The terms represent an empirical measure of the contributions of the initial momentum and the buoyancy, respectively. The weighting of the second term is about equal in (5.26) and (5.27) while the weighting of the first term is higher in (5.26) than in (5.27). This is due to the fact that (5.26) was also derived from velocity measurements closer to the nozzle, where the contribution of the initial momentum is higher. But the experimental validation by Castello-Branco [9] in this region was quite poor with only one point of measurement per test. The present data show that the application of the correlation closer to the nozzle than \( z/z_0 = 1 \) is problematic. Nevertheless, the variation in bubble rise velocity profiles with \( z \) in the ZEF is very well characterized with either (5.25) or (5.27).

5.3.3 Water Velocity

Figure 5.19 shows radial profiles of the axial water velocity component \( u_w \) for T100.10. The velocities are given in terms of a range between \( u_w^+ \) and \( u_w^- \) covering the uncertainty in measur-
Experimental Results

ing instantaneous water velocities below 0.08 m/s, which is the lower limit of the calibration of the HFA (see section 3.3.4). Gaussian curve fits for each data set are also plotted:

\[ u_+^w (r, z) = 0.08 m/s + u_{w,0}^w (z) \exp \left( -\frac{(r - r_{c,w}^+ (z))^2}{(b_{w}^+ (z))^2} \right), \quad u_{w,m}^- = 0.08 m/s + u_{w,0}^- \]  \hspace{1cm} (5.28)

\[ u_-^w (r, z) = u_{w,m}^- (z) \exp \left( -\frac{(r - r_{c,w}^- (z))^2}{(b_{w}^- (z))^2} \right). \hspace{1cm} (5.29) \]

From the data in figure 5.19 it is noted that the uncertainty in velocity, specified by \( u_+^w - u_-^w \), increases with the radius \( r \). Near the pool centerline, the measured velocity rarely drops below 0.08 m/s and so \( u_+^w \) and \( u_-^w \) as defined by (3.26) and (3.27) are nearly identical. But further from the pool centerline, where the average velocity is lower, the instantaneous velocity drops more often below 0.08 m/s. As the fraction of measurements below 0.08 m/s increases, so does the uncertainty in measuring the water velocity. In the limit where the entire signal is below 0.08 m/s, \( u_+^w \) and \( u_-^w \) approach 0.08 and 0 m/s, respectively.

An offset equal to the lower limit of the HFA calibration range is added to the exponential term in (5.28) since it represents the minimum value for \( u_+^w \). The minimum value for \( u_-^w \) is equal to zero and so no offset is added to (5.29). Figure 5.19 lists for both \( u_+^w \) and \( u_-^w \) the Gaussian curve maximum \( u_{w,m}^- \), the width \( b_w^- \), the displacement of the maximum from the pool centerline \( r_{c,w}^- \), and the statistical errors.

The uncertainty in the water velocity measurements plays virtually no role in determining \( u_{w,m}^- \) and \( r_{c,w}^- \). The widths \( b_w^- \) are somewhat larger than \( b_w^+ \). The true value is in the range between \( b_w^- \) and \( b_w^+ \). However, the difference between both values is smaller than 10 % for all profiles measured during the tests. The lower limit for the width \( b_w^- \) and the maximum \( u_{w,m}^- \) are used for the further analysis of the water velocity measurements since it is more realistic to assume that the water velocity goes to zero at the edge of the bubble plume. The label '-' is omitted from now on for simplification. The numerical values \( b_w, u_{w,m}, \) and \( r_{c,w} \) for all tests performed with the HFA (TX.10) are listed in table E.3.

A dimensionless form of the water velocity profiles can be expressed by:

\[ \frac{u_w (r, z)}{u_{w,m} (z)} = \exp \left( -\frac{r^2}{b_w^2 (z)} \right) \]  \hspace{1cm} (5.30)

Figure 5.20 shows the dimensionless profiles for T100.10 and TX.10 at \( z = 500 \) mm. The plots confirm the self-similarity of water velocity profiles.

Figure 5.19 shows that the volumetric flow rate of the entrained water \( \dot{V}_w \) in the bubble plume increases with distance from the nozzle. This volumetric flow rate is proportional to the bubble
Figure 5.19: Radial profiles of the axial water velocity profiles for T100.10.
plume cross section and the mean water velocity, which scale with $b_w^2$ and $u_{w,m}$, respectively. As the distance from the nozzle increases, the width $b_w$ increases significantly while $u_{w,m}$ decreases only slightly. This points out that $\bar{V}_w$ increases with distance from the nozzle. Quantitative analysis of $\bar{V}_w$ as a function of $z$ appears later in chapter 6. In this section, expansion of the entrained water flow and decrease in $u_{w,m}$ with distance from the nozzle are described with empirical correlations. These correlations are formed in a fashion similar to that described for the void fraction and bubble rise velocity measurements.

### Entrained Water Flow Expansion

The increase in $b_w$ with $z$ is illustrated in figure 5.21. The data set for each test is fitted with a linear function

$$b_w (z) = B_w z + b_{0,w} = B_w (z - z_{0,w}) \quad \text{with} \quad z_{0,w} = -\frac{b_{0,w}}{B_w} \tag{5.31}$$

in order to determine the expansion factor $B_w$ and the virtual origin $z_{0,w}$ of the entrained water flow. Figure 5.22 shows the values for $B_w$ and $z_{0,w}$ plotted against $\bar{V}_{a,0}$. The error $\Delta B_w$ is
Experimental Results

determined by the fitting procedure and $\Delta z_{0,w}$ is calculated with:

$$\Delta z_{0,w} = \sqrt{\left(\frac{\Delta b_{0,w}}{B_w}\right)^2 + \left(\frac{b_{0,w} \Delta B_w}{B_w^2}\right)^2}.\quad (5.32)$$

Figure 5.22 shows that the expansion factor $B_w$ increases with $\dot{V}_{a,0}$ and $z_{0,w}$ is below the actual position of the nozzle for all tests. The same trend was observed for the parameters $B_e$ and $z_{0,e}$, which specify the bubble core expansion (see figure 5.11).

The functional relationship $B_w (\dot{V}_{a,0})$ can be described with:

$$B_w = 0.128 \dot{V}_{a,0}^{-0.3}, \quad \dot{V}_{a,0} = \left[\frac{dm^3}{s}\right].\quad (5.33)$$

Comparison between the correlations (5.33) and (5.11) shows that the entrained water flow expands by a factor 1.551 more than the bubble core (0.121 versus 0.078).

The variation in the virtual origin $z_{0,w}$ with $\dot{V}_{a,0}$ is difficult to describe due to the uncertainties in determining $b_w$ from the radial velocity profiles. The correlation

$$z_{0,w} = -45mm \left(1 + 0.37 \dot{V}_{a,0}^{-2}\right), \quad \dot{V}_{a,0} = \left[\frac{dm^3}{s}\right].\quad (5.34)$$

plotted in figure 5.22 is established with the same approach as the correlation for $z_{0,e} (\dot{V}_{a,0})$ (5.12). The decrease in $z_{0,w}$ with $\dot{V}_{a,0}$ is described by $\dot{V}_{a,0}^{-2}$ and an asymptotic value equal to $-45 \text{ mm}$ is set for $z_{0,w}$, instead of $-200 \text{ mm}$ for $z_{0,e}$.

The fact that $B_w$ and $z_{0,w}$ are larger than the corresponding parameters for the bubble core expansion indicates that the entrained water flow expands more than the bubble core. This observation is in agreement with various other experimental findings described in the literature [32, 28, 57, 40] (see table 1.2). It is discussed further in chapter 6.
**Experimental Results**

![Graph](image-url)

**Figure 5.21:** Width of the entrained water flow versus distance from the nozzle.

![Graph](image-url)

**Figure 5.22:** Expansion factor and virtual origin of the entrained water flow.
**Water Velocity along the Centerline**

The centerline water velocity profiles $u_{w,c}$ and the radial profile maxima $u_{w,m}$ are plotted in figure 5.23. There is a decrease in $u_{w,c}$ with increasing $z$, which is first strong but then very gradual, and an increase in $u_{w,c}$ with $V_{a,0}$. The variation in $u_{w,c}$ with $z$ indicates again a separation into an initial region where the flow depends upon the initial momentum flux $M_0$ and a region further from the nozzle where the flow is controlled the buoyancy flux $M_B$.

![Figure 5.23: Centerline water velocity profiles and radial profile maxima.](image)

The functional relationship between $u_{w,c}$ and $z$ for tests with different air flow rates was characterized by Castello-Branco [9] with the following dimensionless correlation:

$$
\frac{u_{w,c}}{\rho^{0.4} V_{a,0}^{0.2}} = 0.26 \left( \frac{z}{z_0(\epsilon_{c,0} = 0.25)} \right)^{-2.30} + 1.05 \left( \frac{z}{z_0(\epsilon_{c,0} = 0.25)} \right)^{-0.08} . \quad (5.35)
$$

Like the correlation for the centerline bubble rise velocity profiles (5.26), the first term in (5.35) is more important near $z_0$, where the flow is still influenced by $M_0$. The second term represents the variation in $u_{w,c}$ with $z$ at larger distances from $z_0$, where the flow is affected by the increase in $M_B$. 
For the present tests, figure 5.24 shows the measured centerline water velocities profiles in dimensionless form together with the correlation derived from these data

\[ \frac{u_{w,c}}{g^{0.41}V_{a,0}^{0.2}} = 0.49 \left( \frac{z}{z_0} \right)^{-2} + 0.89 \left( \frac{z}{z_0} \right)^{-0.1} \]  

(5.36)

and (5.35). Equation (5.36) has the same form as (5.35) with first and second exponent changed to $-2$ and $-0.1$, respectively. To find the factors of the first and second terms, a best fit to measurements at distance $z/z_0 > 1$ was made for each test. The factors from all tests are averaged to derive (5.36).

Figure 5.24: Dimensionless centerline water velocity profiles and correlations (5.35) and (5.36).

The plots in figure 5.24 show that (5.36) and (5.35) are not in good agreement. This suggests that higher water velocities, for a given $V_{a,0}$, were found by Castello-Branco [9] than in the present work. The discrepancy of about 20% is explainable by the methods of analyzing the signals from the velocity sensors. In the present work, $u_{w,c}$ represents the time-averaged water velocity, measured by the HFA. Castello-Branco [9] used a small turbine to measure the instantaneous water velocity. The time-average and a maximum velocity were determined from the velocity readings of the turbine. The maximum velocity was obtained by averaging peaks from
the reading and this measurement was about 20% higher than the time-averaged velocity from the same reading. Correlation (5.35) was established from the maximum velocity in contrast to (5.36), which is derived from the average velocity given by the HFA.

5.3.4 Relative Phase Velocity

The bubble and water velocity measurements have shown that the bubbles rise within an entrained water flow. As a first guess, it is assumed that the relative phase velocity \( u_r = u_b - u_w \) is near the bubble rise velocity in stagnant pure water \( U_{rise} \).

Bubbles rise in stagnant liquid was investigated in various studies over the past 50 years. The textbooks from Cliff et al. [13] and Wallis [79] summarize the work. A description of bubble rise velocity for small bubbles \((d_b < 1 \text{ mm})\) is given by a modified Stokes Law, which considers the non-rigid surface of spherical bubbles:

\[
U_{rise} = \frac{d_b^2 g (\rho_{aq} - \rho_{gas})}{12 \mu_{aq}} .
\]

(5.37)

The experiments from Haberman & Morton [33] show that the rise velocity of air bubbles in water \((1 \text{ mm} < d_b \leq 10 \text{ mm})\) is in the range of 0.2 to 0.35 m/s. For larger bubbles \((d > 10 \text{ mm})\), effects of surface tension and viscosity are negligible and the pressure distribution around the bubble plays the significant role. For such bubbles, which are usually cap-shaped, Davies & Taylor [18] derived a functional relationship between \( U_{rise} \) and the radius at the stagnation point of the bubble, \( R_c \):

\[
U_{rise} = \frac{2}{3} \sqrt{g R_c} .
\]

(5.38)

Collins [15] performed various experiments to study the bubble rise velocity in pools of different sizes and introduced a scale factor \((SF)\) that considers the ratio between bubble and pool diameters. An empirical correlation, based on (5.38), between \( U_{rise} \) and the equivalent bubble diameter \( d_b \) was established from these experiments:

\[
U_{rise} = 0.71 \sqrt{gd_b (SF)} .
\]

(5.39)

Only when \( d_b / D_{pool} < 0.125 \), the scale factor varies from unity and so, for the tests here, \((SF)\) is one. Air bubble rise velocities in water given by Haberman & Morton [33] and Collins [15] are shown in figure 5.25. The dotted line represents the measurements from Haberman & Morton [33] and the straight line describes the monotonic increase in \( U_{rise} \) calculated from (5.39).

For the present tests, the centerline profiles of the relative phase velocity \( u_{r,c} = u_{b,c} - u_{w,c} \) are presented in figure 5.26, together with the relative phase velocity \( u_{r,m} = u_{b,m} - u_{w,m} \), which is

\(^6\text{The present tests are performed with demineralized water that is cleaned by 1 \text{ \mu m} filter.}\)
obtained from the radial profile maxima. It is seen that $u_r$ varies between 0.35 and 0.65 m/s and there is a trend towards increasing relative velocities with $V_{a,0}$.

The mean bubble diameter in the pool is in the range of 4 to 6 mm. Considering only bubbles in this range, a relative phase velocity of about 0.25 m/s is expected in the bubble plume. But large bubbles are also observed and they rise faster than the smaller bubbles. Many small bubbles travel in the wakes of the large bubbles. The phenomenon is called wake acceleration because it increases the average bubble rise velocity of the entire flow. Krishna [49, 48] studied the phenomenon by performing experiments in bubble columns operating in the churn-turbulent regime and by simulating bubble interactions with VOF (volume of fluid) calculations. He included an acceleration factor ($AF$) into (5.39) to predict the rise velocity in bubble columns including swarms of small bubbles following large bubbles:

$$U_{rise} = 0.71 \sqrt{gd_b(SF)(AF)}$$ \hspace{1cm} (5.40)

The correlation for ($AF$) is not applicable to bubble plumes since it is based on the superficial gas velocity in bubble columns and a meaningful superficial gas velocity cannot be defined for an expanding bubble plume. Nevertheless, the same tendency towards higher relative phase velocity with increasing $V_{a,0}$ is observed in both the pool and bubble column experiments. This
Experimental Results

Figure 5.26: Centerline profiles of relative phase velocity.

is reasonable since more large bubbles were observed in both experiments with increasing $\dot{V}_{a,0}$ and this enhances the wake acceleration effect.

Radial profiles of $u_r$ for T100.10 are shown in figure 5.27. The measurements $u_r = u_b - u_w$, determined from the radial bubble and water velocity profiles, are plotted together with the calculations from the Gaussian curve fits (5.18) and (5.29):

$$u_r(r) = 0.19m/s + u_{b,0} \exp \left( \frac{-(r - r_{c,ub})^2}{b_{ub}^2} \right) - u_{w,0} \exp \left( \frac{-(r - r_{c,w})^2}{b_{w}^2} \right).$$

Figure 5.27 shows symmetric peaks in the wings of the radial profiles. The difference between $u_r$ at the peaks and the minimum in the center decreases along the pool centerline with increasing distance from the nozzle. The cause of these maxima is at present not well understood. The radial variation in $u_r$ cannot be explained by a radial variation in bubble size and so it is clear that $u_r$ is not approaching $U_{rise}$. The following example suggests the reason why $u_r$ is not equal to $U_{rise}$. It considers the measuring methods for $u_b$ and $u_w$.

The idealized velocity readings of the HFA and the DOS near a swarm of bubbles are shown in figure 5.28. The velocity readings are established by averaging the sensor signals over a length of time $\Delta t$. A swarm of bubbles rising in the pool induces a water flow in a region larger than the bubble swarm itself. The HFA detects water flow before bubbles reach the sensor. This
Figure 5.27: Radial profiles of relative phase velocity profiles for T100.10.
is represented in a simplified fashion by the velocity increase to 0.1 m/s. As the bubble swarm continues upward, a bubble passes through the HFA. The water velocity is higher due to the presence of this bubble, as shown on the graph by an increase to 0.7 m/s. This is followed by a decrease in water velocity as the swarm travels away from the HFA, which is shown by a reduction in velocity reading. Measuring the bubble rise velocity with the DOS at the same position as the HFA gives a single velocity reading of 1 m/s for the period when the bubble passes through the DOS. The relative phase velocity $u_r$ is calculated by the difference between time-averaged water velocity and the ensemble-averaged bubble rise velocity. For the example in figure 5.28, the relative phase velocity would have been 0.8 m/s.

In reality, the passage of a bubble swarm near a HFA and a DOS is more complex. However, the idealized example in figure 5.28 demonstrates that water flow, induced by a bubble swarm, generates a water velocity reading at the HFA without detecting bubbles with the DOS at the same time. The mean water velocity is obtained from the time average over the whole measurement period, while the mean bubble velocity is the ensemble average from the detected bubbles. Calculating $u_r$ by these measurements is different from $U_{rise}$ of individual bubbles.

The symmetric maxima in the radial profiles of $u_r$ may be a result of a phenomenon more complex than that previously described. For example, the variation in void fraction and water velocity along the pool diameter must be also considered since they represent the probability of detecting bubble swarms near a HFA. An analysis of this problem is not made here since the importance of the various parameters involved in it are not yet clear.
Experimental Results

The uncertainties in bubble and water velocity measurements have a great influence in determining the absolute value of the relative phase velocity. Comparison between DOS measurements in a pipe and the drift flux model have shown that the bubble rise velocity is measured within an error band of ±15% (figure 3.8). Frequent re-calibration of the HFA over the course of the pool tests has demonstrated that the reproducibility of water velocity measurements is ±6% (figure 3.12). These individual errors produce a systematic error for the relative phase velocity measurements in the range of 0.1 to 0.2 m/s. Nevertheless, the tendency towards higher relative velocities with increasing $V_{a,0}$ and the radial profile peaks away from the pool centerline are not explainable by systematic error. Both observations must have physical reasons.

5.3.5 Bubble Chord Length Distribution

The PDFs of bubble chord length $P_c$ were determined with the method explained in section 3.2.3. It was found that they are very well represented by lognormal distributions, which permits to describe the change of $P_c$ in the pool.

Figure 5.29 shows bubble chord length PDFs for T100.10 at four distances from the nozzle. The points plotted in each graph represent the measurements and the continuous line corresponds lognormal distribution:

$$P_c(y) = \frac{1}{\sqrt{2\pi} y^e \ln \sigma_y} \exp \left( -\frac{(\ln y - \ln y^e)^2}{2 (\ln \sigma_y)^2} \right).$$

The mean

$$\ln y^e = \frac{\sum \ln y_i}{K}$$

and the standard deviation

$$\ln \sigma_y = \left[ \frac{\sum (\ln y_i - \ln y^e)^2}{K - 1} \right]^{1/2}$$

were calculated for the ensemble of $K$ chord lengths collected during the measurement time of 600 s.

The change of $P_c$ with increasing distance from the nozzle is described by the variation of $\ln y^e$ and $\ln \sigma_y$. As seen in figure 5.29, the probability of finding large chord lengths decreases with increasing distance from the nozzle. The parameters $\ln y^e$ and $\ln \sigma_y$ decrease with increasing distance from the nozzle, indicating a shift of the maximum towards smaller values of $y$ and the narrowing of the distributions.

The distinction of the flow regimes in the pool, namely the ZFE and ZEF, can be justified by the observed change in $P_c$. The analysis of the video recordings showed that the disequilibrium
Figure 5.29: Bubble chord length PDF at different distances from the nozzle for T100.10 together with lognormal distribution.
Experimental Results

between bubble breakup and coalescence is smaller in the ZEF than in the ZFE. The change in $P_c$ can be associated with a change in bubble breakup and coalescence rate.

Figure 5.30 shows the variation of $ln y^e$ and $ln \sigma_y$ with distance from the nozzle for TX.10. The two parameters decline rapidly close to the nozzle and then more gradually. Consequently, $P_c$ changes considerably with $z$ close to the nozzle and slightly at larger distances from the nozzle. This suggests that more bubble breakup occurs in the ZFE than in ZEF. The large bubble formations, developed above the nozzle, breakup into swarms of small bubbles quickly in the ZFE; lesser change in the bubble size distribution is expressed indirectly by $P_c$ in the ZEF.

Figure 5.30: Lognormal distribution parameters of the chord length distributions versus distance from the nozzle.

The decrease in $ln y^e$ and $ln \sigma_y$ at larger distances from the nozzle denotes that the mean bubble diameter decreases with distance from the nozzle. Due to the decrease in hydrostatic pressure with increasing distance from the nozzle, the presence of equilibrium between bubble breakup and coalescence should increase the mean bubble diameter. Since the opposite is observed, it can be concluded that bubble breakup is dominant over bubble coalescence in the ZEF.

No significant dependency of $ln y^e$ and $ln \sigma_y$ in the ZEF on the air flow is shown by the data
Experimental Results

in figure 5.30. This indicates that the bubble size distribution does not depend strongly on the air flow rate. The finding is consistent with observation of the flow patterns in the pool.

Herringe [37] and Clark & Turton [12] proposed a method to translate chord length measurements into bubble size distributions. An attempt to apply this method to the present measurements was not successful because it was not possible to describe the bubble chord length distributions by a Gamma distribution, which is required to determine the bubble size distribution from the chord length measurements, according to this method.

5.3.6 Tests with a Pool Depth of 3 m

All tests described in the previous sections were conducted with a pool depth of 2 m. Additional tests were carried out with a pool depth \( H \) equal to 3 m (TX.10*) to study the applicability of the previous findings to larger pool depths. Since the pressure at the nozzle is only slightly higher for these tests, it is not expected to greatly influence the results. The air flow rate for TX.10* was limited to 100 \( l_n/min \) because the bubble plume expands to the wall and oscillates strongly for higher air flow rates. Measurements were performed only with the DOS for a range of distances from the nozzle between 250 and 2000 mm (see table 5.2).

The bubble core widths \( b_c \) for TX.10* are plotted in figure 5.31 against \( z \) together with the corresponding points from tests with a pool depth \( H = 2 \) m. The graph includes the linear fits (5.9), which were determined from the tests with \( H = 2 \) m (measurements with \( H = 2 \) m are limited to \( z = 1000 \) mm).

The experimental data for T25.10* and T50.10* are in good agreement with the extrapolated fits, while deviations are observed for T100.10*. The data indicate a wider bubble core expansion for T100.10* in comparison to T100.10. In fact, the test conditions for T100.10* are already problematic because the bubble core reaches the wall close to the pool surface, which means that the bubble plume may already be influenced by the presence of the wall. Excluding T100.10*, the bubble core expansion for \( z > 1000 \) mm can be described by extrapolating the linear relationships between \( b_c \) and \( z \) that were obtained from measurements for \( z \leq 1000 \) mm.

Void fractions, measured along the pool centerline for TX.10*, are plotted against \( z/z_0 \) in figure 5.32 together with correlation (5.16), which was obtained from the test data with \( H = 2 \) m. It is seen that (5.16) is also applicable to tests with \( H = 3 \) m. The decrease in void fraction with distance from the nozzle is predicted rather well over two decades.

Figure (5.33) shows centerline bubble rise velocity profiles in dimensionless form for TX.10* and correlation (5.27). Again, good agreement is found between the data obtained from tests with \( H = 3 \) m and the correlation determined from tests with \( H = 2 \) m.
Figure 5.31: Bubble core width versus distance from the nozzle for TX.10* in comparison with the corresponding tests with a pool depth of 2 m.

Figure 5.32: Dimensionless centerline void fraction profiles for TX.10* in comparison with (5.16).
5.3.7 Volumetric Air Flow Rate

The bubble rise velocity $u_b$ and void fraction $\epsilon$ profiles measured along the pool diameter at elevation $z$ with the DOS can be used to calculate the volumetric air flow rate in the cross-section of the bubble plume $\dot{V}_a^{DOS}(z)$. Comparing it with the injected air flow rate $\dot{Q}_a$ specifies the measurement error of the profile measurements. The method was already used in section 3.2.2 to calibrate the DOS in a pipe.

$$\dot{V}_a^{DOS}(z) = A \left( j_a(r,z) \right)_A = \int_0^{2\pi} \int_0^\infty u_b(r,z) \epsilon(r,z) r dr d\varphi$$ \hspace{1cm} (5.45)

assuming azimuthal symmetry of the profiles. The local volumetric air flux $j_a(r,z)$ can be calculated with the Gaussian curves for the radial void fraction and bubble rise velocity profiles specified by (5.7) and (5.18), respectively. Equation (5.18) describes the bubble velocity profiles by using an offset of 0.19 m/s. Clearly these profiles cannot be integrated to infinity. So to perform the integration, the measurements are represented by Gaussian curves with $u_{b,m}$ and $b_{ub}$ (see (5.19) and (5.18)) with no offset. With this approach and by setting the displacements...
of the maxima from the centerline ($r_{c,e}$ and $r_{c,ub}$) to zero, the radial volumetric air flux profiles are given by:

$$j_a (r, z) = e_m u_{b,m} \exp \left( -r^2 \left( \frac{1}{b_t^2} + \frac{1}{b_{ub}^2} \right) \right).$$  \hspace{1cm} (5.46)$$

Figure 5.34 shows the volumetric air flux profiles for T10.100 that were obtained from the measurements and the approximation with (5.46). Since $r_{c,e}$ and $r_{c,ub}$ are small, the measurements are well represented by the centered Gaussian curve. The variation along the pool diameter is also very well described by using $b_{ub}$ and $b_t$ to calculate the width of the Gaussian curve. Computing (5.45) with (5.46) results in:

$$V_{aDOS} (z) = \pi u_{b,m} e_m \frac{b_{ub}^2 b_t^2}{b_{ub}^2 + b_t^2},$$

where $V_{aDOS}$ is expressed in terms of the widths and the profile maxima.

The air flow rate $\dot{Q}_a$, adjusted by the air flow controller, is given for normal conditions. Considering conservation of the air mass flow rate $\dot{m}_a$, the volumetric air flow rate at $z$ is given by the change in the air density from normal conditions $\rho_{a,o}$ to $\rho_a (z)$ for the conditions in the pool at $z$.

$$\dot{Q}_a = \frac{\dot{m}_a}{\rho_{a,o}} (z), \hspace{1cm} \dot{V}_{aAFM} (z) = \frac{\dot{m}_a}{\rho_a (z)}$$  \hspace{1cm} (5.48)$$

Standard conditions are $T_{ref} = 273$ K and $p_{ref} = \rho_w g H_0$. The pool temperature $T_{pool}$ is equal to 293 K for all tests and the total pressure at $z$ can be approximated by the water head ($p (z) = \rho_w g (H_0 + H - z)$). Using the equations above and the ideal gas law gives:

$$V_{aAFM} (z) = \dot{Q}_a \frac{\rho_{a,o}}{\rho_a (z)} = \dot{Q}_a \frac{T_{pool}}{T_0} \frac{H_0}{H_0 + H - z}.$$  \hspace{1cm} (5.49)$$

The uncertainty for $V_{aAFM}$ is that of the adjusted air flow rate ($\pm 2\%$).

Figure 5.35 shows the comparison between $V_{aDOS}$ and $V_{aAFM}$ for all tests. Each data point represents one profile measurement (four/eight per test). Almost every data point is within an error band of $\pm 25\%$. Averaging the deviations between $V_{aDOS}$ and $V_{aAFM}$ yields $-17\%$. The only similar comparison between $V_{aDOS}$ and $V_{aAFM}$ found in the literature was reported by Castello-Branco [8]. He obtained mean errors between $-40$ and $-52\%$, dependent on the distance from the nozzle. This indicates that the present tests are at least more accurate than the experiments of Castello-Branco [8].

The calibration tests of the DOS in a pipe have shown agreement between $V_{aDOS}$ and $V_{aAFM}$ within an error band of $\pm 15\%$ (see figure 3.8), which is state-of-the-art for pipe flow experiments. Taking into account the additional uncertainties in the pool tests when calculating $V_{aDOS}$ (assumptions about the profiles), the error analyses in the pipe and the pool give comparable results.
Figure 5.34: Volumetric air flux profiles: Measurement points and the continuous line represents the calculation with (5.46).
Figure 5.35: Volumetric air flow rate as regulated by the air flow controller and as measured by integrating the DOS profiles.
5.4 Summary of the Experimental Findings

An extensive and detailed database that could be used to improve the modeling of bubble plumes under pool scrubbing conditions was presented in this chapter. Preliminary video recordings have shown that the flow patterns for the present test conditions were similar to those of preceding pool scrubbing experiments. Either large individual bubbles or air jets were formed above the nozzle, depending on the air flow rate and the nozzle diameter. At larger distances from the injector, the flow pattern was comparable with the churn-turbulent flow regime in pipes, meaning that the bubbles were frothy and disordered. Bubble plume oscillation was minimized in the tests by limiting the air flow rate and pool depth, and so the experimental data are also applicable to pools with diameters larger than 1 m.

Detailed information about the bubble plume was obtained by local measurements with a double optical sensor (DOS) and a hot-film anemometer (HFA). Radial and centerline profiles were measured for the void fraction, and the bubble rise and water velocity. The widths of the radial profiles were determined by fitting the experimental data with Gaussian curves. The functional relationships between the widths and the distance from the nozzle were approximated with linear functions. The expansion factors (slopes) and the virtual origins of the bubble core and of the entrained water flow were obtained from the linear functions and then correlated with the air flow rate. The centerline profiles describe the decrease in void fraction, bubble and water velocity with distance from the nozzle. Dimensionless correlations were derived to specify the variations of the parameters along the centerline for different test conditions.

The tests show that the bubble core expands with increasing distance from the nozzle, as indicated by the growing width of the Gaussian curves. The rate of expansion increases with the air flow rate and is independent on the nozzle size. The virtual origin of the bubble core is below the actual nozzle position.

The centerline void fraction profiles show that the nozzle diameter affects the variation in void fraction only close to the nozzle, while it plays no significant role at larger distances. This fact was used to define the distance \( z_0 \) from the nozzle were the zone of flow establishment (ZFÉ) ends and the zone of established flow (ZEF) begins. The experimental data indicate that the influence from the nozzle is small and that this location depends essentially on the air flow rate and is found about where the centerline void fraction is equal to 0.25. This void fraction threshold is used to determine \( z_0 \). By scaling the distance from the nozzle with \( z_0 \), the decrease in void fraction along the pool centerline can be described by a dimensionless function in the form of a power law. A mean exponent of \(-1.15\) results to the best agreement with the experimental data. The finding is in accordance with previous experiments in the literature.

The bubble rise velocity measurements demonstrate that the two-phase flow in the ZEF is
mainly buoyancy driven. This was shown by doing tests at constant air flow rate but with different nozzle sizes. The bubble velocity distribution depends on the air flow rate (bubble density), while the nozzle size (initial momentum of the flow) plays virtual no role. The observation can be proven by estimating the contributions of the initial momentum flux at the nozzle exit and of the momentum flux created by buoyancy to the total momentum flux.

The measurements of the axial water velocity component show that the zone of entrained water flow expands while the velocity at the pool centerline decreases with increasing distance from the nozzle. The combination results in an increase of the volumetric flow rate of the entrained water while the bubble plume rises in the pool, as expected. Comparison of the width of the entrained water flow zone to that of the bubble core shows that the expansion of the entrained water flow zone is significantly greater than bubble core expansion.

The discussion of the relative phase velocity measurements points out at the complexity of two-phase flow in the pool. Video recordings show that large bubbles rise through the pool and capture smaller bubbles in their wakes. The bubble swarm following a large bubble induces water flow in adjacent regions, which is measured by the HFA, while nearby bubbles miss the DOS. Calculating the relative phase velocity from the DOS (gas velocity) and HFA (liquid velocity) readings is not equivalent to the measurement of the rise velocity of individual bubbles in stagnant water. The measurements show that the relative phase velocity varies along the pool diameter. A conclusive explanation for this finding requires measurements that were not possible in this work.

Bubble chord length distributions measured with the DOS can be described by lognormal distributions. The observed variations of the mean chord length and of its variance along the pool centerline further justify the separation criterion used here for the boundary between the ZFE and the ZEF. Close to the nozzle, the mean chord length decreases rapidly with increasing distance from the nozzle and at larger distances only gradually. This shows that more bubble breakup occurs in the ZFE than in the ZEF, as expected. The observed gradual decrease in mean chord length in the ZEF indicates that the mean bubble diameter decreases, which suggests that bubble breakup is more pronounced in this zone of the plume than bubble coalescence.

The comparison between tests with pool depths of 2 m and 3 m demonstrates that extrapolating experimental findings from smaller to larger pool depths is possible; the small changes in hydrostatic pressure did not affect the results considerably. Problems appear when the bubble plume interacts with the wall and starts to oscillate. For tests with \( H = 3 \) m a maximum air flow rate of \( V_{a,0} = 100 \) \( \text{l}_{\text{m}}/\text{min} \) limits operations in the present experimental setup if plume oscillation is to be minimized. With either greater air flow rates or a deeper pool, the interaction of the bubble plume with the pool walls becomes more significant.

Using the radial void fraction and bubble rise velocity profiles to calculate the volumetric air
flux in the pool and compare it with the injected air flow rate shows satisfactory agreement, confirming that the local measurements are state-of-the-art for such flows. The local measurements underpredict the volumetric air flux on the average by $-17\%$, compared to the injected air flow rate (measured with an accuracy of $\pm 2\%$).
6 Modeling

The objective of this chapter is to determine the empirical parameters of the bubble plume model presented in chapter 2 and to test the approximate solution of the conservation equations. The experimental data base is used for this purpose.

6.1 Empirical Parameters

Application of the theory requires specification of five empirical parameters: (1) the ratio $\lambda$ between the widths of the bubble plume and bubble core ($b_w$, $b_c$), (2) the entrainment coefficient $\alpha$, (3) the relative phase velocity $u_r$, (4) the virtual origin of the entrained water flow $z_{0,w}$, and (5) the momentum amplification factor $\gamma$. The relative phase velocity measurements were already discussed in section 5.3.4. The parameters $\lambda$, $\alpha$, and $z_{0,w}$ are deduced from the present experimental data and $\gamma$ is selected on the basis of previous tests, found in the literature.

Width Ratio $\lambda$

The experimental data for $\lambda$ are plotted as a function of the distance from the nozzle in figure 6.1. The error bars are calculated with the statistical errors $\Delta b_e$ and $\Delta b_w$ listed in table E.4 and E.3, respectively. Since:

$$
\Delta \lambda = \lambda \sqrt{\left(\frac{\Delta b_w}{b_w}\right)^2 + \left(\frac{\Delta b_e}{b_e}\right)^2}
$$

The dotted line in figure 6.1 represents the trend of the averaged data points.

In addition, correlation curves are plotted in figure 6.1 for $\lambda$ as a function of $z$. These are derived by using the approach of linear functions for $b_e(z)$ (5.9) and $b_w(z)$ (5.31) to calculate $\lambda$:

$$
\lambda(z) = \frac{b_e(z)}{b_w(z)} = \frac{B_e(z - z_{0,e})}{B_w(z - z_{0,w})}.
$$

Inserting the empirical correlations for $B_e$ (5.11), $z_{0,e}$ (5.12), $B_w$ (5.33), and $z_{0,w}$ (5.34) into (6.2) results in:

$$
\lambda(z) = 0.641 \frac{z + 200mm \left(1 + 0.14\dot{V}_{a,0}\right)}{z + 45mm \left(1 + 0.37\dot{V}_{a,0}\right)} , \quad \dot{V}_{a,0} = \left[\frac{dm^3}{s}\right].
$$

Averaging the experimental data in figure 6.1 demonstrates that $\lambda$ decreases from about 0.85 to 0.7 between $z = 250 mm$ and $1000 mm$. The observation indicates that the entrained water
flow expands faster with distance from the nozzle than the bubble core. In the ZFE, either an air jet or large bubbles are present above the nozzle and $\lambda$ is around unity. At larger distances from the nozzle, the bubble core expands and the large bubbles break-up into smaller ones. Zhou [83] found that a larger fraction of the potential energy of the air flow in the bubble plume is transported into the water when the bubbles are smaller. The result is that the entrained water flow expands faster with distance from the nozzle than the bubble core, which yields a decrease in $\lambda$.

No systematic dependency of $\lambda$ on $\dot{V}_{a,0}$ is indicated by the data in figure 6.1 and the calculation with (6.3) results in only a slight increase of $\lambda$ with $\dot{V}_{a,0}$. Analyzing (6.3) shows that the ratio between the expansion factors $B_c$ and $B_w$, which does not depend on $\dot{V}_{a,0}$, is more important for calculating $\lambda$ than the change in the virtual origins $z_{0,c}$ and $z_{0,w}$, which depends on $\dot{V}_{a,0}$. Therefore, $\lambda$ is practically independent on $\dot{V}_{a,0}$.

The bubble plume model considers no change in $\lambda$ with $z$ and for this reason a mean value must be determined. A value of 0.8 will be used for the modeling and this is justified, in part, by the fact that a change in $\lambda$ of the order of 10% has only a small influence on the results of the model predictions.
The literature review (see table 1.2) has shown that the most probable value for $\lambda$ is 0.8. The present data are in agreement with that value.

**Entrainment Coefficient $\alpha$**

In order to determine the entrainment coefficient $\alpha$, it is necessary to calculate the volumetric water flow rate $\dot{V}_w$ in the bubble plume and the change in $\dot{V}_w$ with $z$. The radial void fraction and water velocity profiles are used to determine $\dot{V}_w$:

$$
\dot{V}_w(z) = A \frac{(1 - \epsilon) \ u_w}{\lambda} = \int_0^{2\pi} \int_0^\infty [1 - \epsilon (r, z)] u_w (r, z) \ r \ d\varphi \ dr . \quad (6.4)
$$

Replacing $\epsilon (r, z)$ and $u_w (r, z)$ in (6.4) with (5.7) and (5.29) and calculating the integral gives:

$$
\dot{V}_w(z) = \pi u_w (z) \ b_w^2 (z) \left(1 - \epsilon_m (z) \frac{1}{1 + (\lambda (z))^2}\right) . \quad (6.5)
$$

The errors for $\dot{V}_w$ are calculated with:

$$
\Delta \dot{V}_w = \sqrt{\dot{V}_w^2 \left[ \left( \frac{\Delta u_w}{u_w} \right)^2 + \left( \frac{\Delta b_w}{b_w} \right)^2 \right] + \left( \frac{\pi u_w b_w \epsilon_m}{1 + \lambda^{-2}} \right)^2 \left[ \left( \frac{\Delta \epsilon_m}{\epsilon_m} \right)^2 + \left( \frac{2\Delta \lambda}{\lambda^3 + \lambda} \right)^2 \right]} . \quad (6.6)
$$

The increase in $\dot{V}_w$ with $z$ is shown in figure 6.2. To calculate $\alpha$, the gradient $B_w = d\dot{V}_w/dz$ must be determined, which is done by fitting the data sets in figure 6.2 with linear functions:

$$
\dot{V}_w(z) = B_{V_w} z + b_{0,V_w} = B_{V_w} (z - z_{0,V_w}) \quad \text{with} \quad z_{0,V_w} = -\frac{b_{0,V_w}}{B_{V_w}} . \quad (6.7)
$$

The entrainment coefficient $\alpha$, defined by (2.12), is derived from the experimental data with:

$$
\alpha(z) = B_{V_w} \frac{1}{2\pi u_{w,m} (z) b_w (z)} . \quad (6.8)
$$

The error for $\alpha$ is calculated with:

$$
\Delta \alpha = \alpha \sqrt{\left( \frac{\Delta B_{V_w}}{B_{V_w}} \right)^2 + \left( \frac{\Delta u_{w,m}}{u_{w,m}} \right)^2 + \left( \frac{\Delta b_w}{b_w} \right)^2} . \quad (6.9)
$$

The functional relationship between $\alpha$ and $z$ is plotted in figure 6.3. It is seen that $\alpha$ varies with distance from the nozzle. The bubble plume model supposes a constant $\alpha$ and so the values for each test are averaged. Under the presumption that $\alpha$ can be approximated by a linear function between $z_i$ and $z_{i+1}$, the mean values at positions midway between data collection
Figure 6.2: Volumetric water flow rate versus z.

Heights are calculated and then averaged by weighting them with their portions of the total height:

$$\alpha^m = \frac{1}{2} \sum_{i=1}^{N-1} \left[ \alpha \left( z_i \right) + \alpha \left( z_{i+1} \right) \right] \left( z_{i+1} - z_i \right) \over \sum_{i=1}^{N-1} \left( z_{i+1} - z_i \right) .$$  \hspace{1cm} (6.10)

For each test, the errors $\Delta\alpha$ calculated with (6.9) are averaged with the same method to determine the mean error $\Delta\alpha^m$. Table 6.1 summarizes the data that are obtained by analyzing the experiments with (6.7) and (6.10).

<table>
<thead>
<tr>
<th></th>
<th>$B_{V_w} \pm \Delta B_{V_w}$ [m$^2$/s]</th>
<th>$z_{0,V_w} \pm \Delta z_{0,V_w}$ [m]</th>
<th>$\alpha^m \pm \Delta\alpha^m$ [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T25.10</td>
<td>0.011 ± 0.001</td>
<td>0.10 ± 0.02</td>
<td>0.078 ± 0.034</td>
</tr>
<tr>
<td>T50.10</td>
<td>0.020 ± 0.002</td>
<td>0.07 ± 0.02</td>
<td>0.090 ± 0.030</td>
</tr>
<tr>
<td>T100.10</td>
<td>0.037 ± 0.005</td>
<td>0.10 ± 0.04</td>
<td>0.109 ± 0.023</td>
</tr>
<tr>
<td>T200.10</td>
<td>0.061 ± 0.003</td>
<td>0.15 ± 0.02</td>
<td>0.138 ± 0.021</td>
</tr>
</tbody>
</table>

Table 6.1: Linear fits for $V_w(z)$ and mean entrainment coefficients.

A straightforward method to deduce $\alpha$ is to use the linear term in (2.20) and the expansion factor $B_{w}$, which is derived by the linear relationship (5.31) between $b_w$ and $z$. Inserting com-
The term $6/5\alpha z$ has the highest weight when calculating $b_w$ as a function of $z$. Thus $b_w$ can be set equal to $6/5\alpha$. Using the empirical correlation for $B_w$ (5.33), $\alpha$ can be expressed as a function of $\dot{V}_{a,0}$:

$$\frac{6}{5} \alpha = B_w = 0.128 \dot{V}_{a,0}^{0.3} \rightarrow \alpha = 0.107 \dot{V}_{a,0}^{0.3}, \quad \dot{V}_{a,0} = \left[ \frac{dm^3}{s} \right].$$ \hspace{1cm} (6.11)

Correlation (6.11) is plotted in figure 6.4 together with the experimental data listed in table 6.4. For comparison, data from the literature are included in the graph (see table 1.2). The data show the trend that $\alpha$ increases with $\dot{V}_{a,0}$.

Deriving a generally applicable correlation between $\alpha$ and $\dot{V}_{a,0}$ with results from different experiments is difficult. Length scales, like the pool depth and diameter, are not the same in different experiments and they may influence water entrainment. Milgram [57] proposed a correlation that relates $\alpha$ with $\dot{V}_{a,0}$ and the void fraction. The correlation cannot be verified with other experiments. Friedl [29] tripled the air flow rate in his tests without finding a significant change in $\alpha$. Hugi [40] found an increase in $\alpha$ by changing $\dot{V}_{a,0}$ only a little. Kobus [47] increased $\dot{V}_{a,0}$ significantly without measuring much higher values for $\alpha$. 

![Figure 6.3: Entrainment coefficient versus distance from the nozzle.](image-url)
The present values for $\alpha$ are higher than in previous tests. The pool diameter here is 1 m, while the previous tests were performed in larger pools. The difference may explain the increase in $\alpha$. The hypothesis is that injecting an air flow into a small pool increases the water entrainment compared to a much large pool, since the velocity of the water re-circulation in the bulk is higher. However, experimental or numerical studies that systematically investigate the change of water entrainment when the pool size is varied were not found in the literature. The problem is left open for discussion.

**Remaining Factors**

The measurements of relative phase velocity $u_r$ were discussed in section 5.3.4. The variation in $u_r$ along the pool diameter, which is demonstrated by the measurements, is not considered by the bubble plume model. The trend towards increasing relative velocities with $V_a$ is small and can be neglected for the modeling. Taking into account the uncertainties in measuring the relative phase velocity, a value of 0.35 $m/s$ is set for the model predictions. The velocity is commonly used when bubble plumes in lakes are modeled.

The origin for the model predictions ($z_m = 0 mm$) is equal to the origin of the entrained
water flow $z_{0,w}$. The position, which is below the actual position of the nozzle, is calculated with correlation (5.34).

The momentum amplification factor $\gamma$ was introduced by Milgram [58]. It specifies the ratio between total momentum flux and the momentum flux in the mean flow of the bubble plume. The ratio depends on bubble density since the passage of the bubbles and their wakes is associated with unsteadiness in the flow. The momentum amplification becomes important when the bubble density is high, as in the present tests. Milgram [58] and Friedl [29] compared experiments with model predictions and they found that the mean flow carries on average 50% of the total momentum flux, which corresponds to a momentum amplification factor of 2. This result is used for the computations here.

### 6.2 Model and Experiment

The approximate solution for the set of equations presented in section 2.4 is used in this section to calculate the expansion of the entrained water flow $b_w(z)$ and the variation in axial water velocity component $u_w$ and of the void fraction $\epsilon$ along the pool centerline.

The maxima of radial profiles $u_{w,m}$ and $\epsilon_m$ appear in the set of conservation equations and in the solution. Only four maxima were measured for each test. A more extensive description of the variation in $u_{w,m}$ and $\epsilon_m$ is given by the centerline profiles $(u_{w,c}, \epsilon_c)$. For this reason, the model predictions are compared with the centerline profiles.

When plotting the experimental data and the model predictions in the same graph, the origins of the different coordinate systems must be taken into account. The axial coordinate for plotting the experimental data is $z$, while the axial coordinate for the model predictions is $z_m$ (see figure 2.1). The origin for the model predictions is the virtual origin of the entrained water flow $z_{0,w}$, which is at $z = -|z_{0,w}|$. In order to plot the experimental data and the model predictions on the scale of the experimental coordinate system, $z_m$ is transformed with

$$z^* = z_m - |z_{0,w}| \quad (6.12)$$

into the coordinate $z^*$ that is equivalent to $z$.

The volumetric air flow rate at the position of the virtual origin is calculated with (see also (5.2)):

$$\dot{V}_{a,m} = \dot{Q}_a \left(1 + \frac{H + z_{0,w}}{H_0}\right)^{-1} \frac{T_{pool}}{273K} \quad (6.13)$$

### Model Predictions and Measurements

The experimental data for the axial water velocity component and the corresponding model predictions are shown in figure 6.5. Good agreement between the experiment and the computations
is found, except for T50.10. The velocity in this experiment is overpredicted by about 0.1 m/s.

![Figure 6.5: Axial water velocity component along the pool centerline: experimental data and computation with (2.21).](image)

The expansion of the entrained water flow is shown in figure 6.6. The model predicts quite well the expansion of entrained water. The correspondence between model and experiment is already indicated in figure 6.4, where the values for \( \alpha \) listed in table 6.1 and correlation (6.11) are compared. Correlation (6.11) is derived by simplifying the model and using the experimental data to find a relationship between \( \alpha \) and \( \dot{V}_{a,0} \). However, the agreement is not a priori obvious because the two procedures to determine \( \alpha \) are different.

The largest deviation between the experimental data and model predictions is found again for T50.10. The reason for the disagreement is that the virtual origin is calculated with correlation (5.34). As seen in figure 5.22, (5.34) overpredicts \( z_{0,w} \) for T50.10 (\( \dot{V}_{a,0} = 0.75 \text{ dm}^3/\text{s} \)) by a factor of two in comparison to the experimental value. The origin of the model would shift further down from the actual nozzle position when considering the experimental value for \( z_{0,w} \). As a result, the plots for T50.10 in figures 6.5 and 6.6 would move to the left, giving better agreement between the experimental data and model predictions. This points out the sensitivity of the model predictions to changes in \( z_{w,0} \).

The variation in the void fraction along the pool centerline can also be predicted with the solution of the bubble plume model discussed here. Figure 6.7 shows the experimental data and the model predictions. The void fraction is overpredicted by the model. The reason is that the
bubble rise velocity $u_b$ in the air conservation equation (2.5) is calculated by adding the relative phase velocity to the water velocity:

$$u_b(r, z) = u_r + u_{w,m} \exp \left( -\frac{r^2}{b_w^2} \right).$$  

(6.14)

The measurements of the radial bubble and water velocity profiles have shown that this model approach is not applicable since the expansion of the bubble rise velocity profiles is greater than expansion of the water velocity profiles ($b_{ub} > b_w$, see figure 5.16 and 5.22). Using (6.14) underpredicts the total volumetric air flux computed by (2.5) by integrating the radial profile. As a result, the void fraction calculated by (2.13) is overpredicted.

### 6.3 Conclusion about Modeling

The purpose was to describe the bubble plume with a simple model. Therefore an integral bubble plume model was introduced, one that has been previously used to study lake venting phenomena. Empirical parameters are needed to close the set of conservation equations and these are determined by the present experimental data. To compute the evolution of the bubble plume, an approximate solution of the model was used.

The entrainment coefficient $\alpha$ is the key parameter for the bubble plume model. An empirical
correlation that relates $\alpha$ to the air flow rate was derived for the present test conditions. Extrapolating this correlation to other air flow rates is doubtful as shown by a comparison with other findings in the literature. In addition to the air flow rate, the pool size may affect the re-circulation of the bulk water and entrainment into the bubble plume. How this influences $\alpha$ is not clear.

The experiments show that $A$ is approximately 0.8, which corresponds to other findings in the literature. Since modest changes in $A$ produce only small changes in the solution to the bubble plume equations, the estimate can be quite approximate but adequate.

The position of the origin $z_{w,0}$ of the model predictions was obtained from a correlation derived from the experimental data. It indicates a position below that of the nozzle and depends on the air flow rate. The measurements show an increase in $z_{w,0}$ from $150 \, mm$ for the test at a low air flow rate to an asymptotic value of $45 \, mm$ for higher air flow rates.

The relative phase velocity $u_r$ was set constant to $0.35 \, m/s$ to calculate the bubble velocity in the air conservation equation as the sum of the water velocity and $u_r$. The experiments have shown that the apparent relative phase velocity, measured by the DOS and HFA, is higher than
0.35 m/s on average and not constant along the pool diameter. The latter finding indicates that the bubble velocity cannot simply be calculated by adding a constant value to the water velocity. The disagreement between void fraction measurements and computations is caused by the assumption of constant relative phase velocity along the pool diameter.

The momentum amplification factor $\gamma$ is set to 2 on the basis of previous studies found in the literature. In these studies, $\gamma$ was determined by comparing model predictions with experiments. In fact, determining the momentum amplification factor directly from experimental data is problematic since the interpretation of turbulence measurements within the bubble plume is difficult. The RMS-values can be calculated from the HFA velocity reading, but their significance is not clear when considering the small-scale turbulence induced by the bubble wakes and the large-scale velocity fluctuations due to bubble plume oscillations. The value of the momentum amplification factor in the present work can also be justified by the good agreement between the model predictions and the experiments.

Comparing the findings here with previous experiments found in the literature is difficult. The test conditions are different (pool size, air flow rate, and injector) and a variety of sensors and data reduction procedures were used to measure the flow parameters. For example, Hugi [40] compared LDA measurements with video recordings of the bubble core to determine $\lambda$. Milgram [58] measured the water velocity at three different locations in a pool cross-section simultaneously to see when the bubble plume is centered and to average only during these times. In the present experiments, radial void fraction and water velocity profiles are measured by time-averaging the signals from local sensors and include any effects due to bubble plume oscillations (although these are expected to be small). In order to compare different experiments and to derive correlations that are applicable to a wide range of flow conditions, the measurement methods should be consistent. This is not the case for the bubble plume experiments listed in table 1.1.

The bubble plume model provides a physical understanding of the flow parameters involved in the problem. Comparing the model predictions for the entrained water flow with the experiment, results in good agreement. This is not surprising since the empirical parameters required for the model are derived from the experimental data.

The new aspect of the present work is that sophisticated two-phase flow measurement techniques were used to set up a detailed data base for bubble plume modeling under pool scrubbing conditions. Information about global parameters such as the bubble core and the entrained water flow widths are also provided by the data base. The bubble plume model and the approximate solution of the set of conservation equations introduced in chapter 2 can be considered as a starting point for further model refinements.
7 Summary and Conclusions

The objective of the present work was to set up a data base that could be used to improve modeling of two-phase flow under pool scrubbing conditions, i.e. the injection of high air flow rates through a single nozzle into a large pool. No detailed information about such two-phase flows was found in the literature (section 1.3). Therefore a sophisticated test facility was build at PSI (chapter 4) to carry out a test series with local measurement techniques. The pool used for the tests was 1 m in diameter and the pool depth could be adjusted up to 4.4 m. The main objective of the work was successfully met.

The two-phase flow area in a large pool can be divided into three zones: The zone of flow establishment (ZFE) close to the nozzle, the zone of established flow (ZEF), and the zone of surface flow (ZSF). The largest zone is the ZEF. The two-phase flow in this zone can be described as a bubble plume that consists of a bubble core and an entrained water flow area. While it is rising, the bubble plume expands in the horizontal direction.

The semi-empirical bubble plume model presented in chapter 2 is widely used to model lake venting phenomena. It is applied here to compute plume behaviour under pool scrubbing conditions. Local measurements are needed to determine the empirical parameters of the set of model conservation equations. These were obtained from the present experiments.

The local measurement techniques used for the tests, a double optical sensor (DOS) and hot-film anemometer (HFA), were selected after a critical literature review of two-phase flow measurement techniques and their possible application to experiments in large pools. Void fraction, bubble and water velocity, and bubble chord length distributions were measured with the DOS and the HFA. The signal processing is presented in chapter 3. The sensors were tested and calibrated extensively before using them in the pool, as well as during the tests. In particular, the sensitivity of the HFA to incoming flow from different directions was studied. It was found that, as mounted, the HFA measured chiefly the axial water velocity component in the pool.

Preliminary video recordings were taken to study bubble plume oscillation in the pool and to illustrate the flow patterns (section 5.2). Bubble plume oscillation depends mainly on the aspect ratio between pool depth and diameter. For a given aspect ratio, the frequency and the amplitude of the oscillation increase with air flow rate. Quantitative information about bubble plume oscillation could not be obtained from the video recordings. To minimize bubble plume oscillation, tests were carried out with aspect ratios 2:1 and 3:1 and with air flow rates limited to 3.3 $dm^3/s$.

Photographs show that the ensemble of bubbles in the ZEF is frothy and disordered, like in the churn-turbulent flow regime in pipes. In the ZFE, either the detachment of individual large
bubbles from the nozzle or jetting was observed. The flow regime depends on the nozzle size and the air flow rate. Various attempts were found in the literature to describe the transition between bubbling and jetting. However, the different transition criteria are not consistent with each other and no good agreement was found with the observations from the present tests.

The local measurements discussed in section 5.3 provide a large amount of detailed information about two-phase flow in pools. Radial and centerline void fraction, bubble rise and water velocity profiles were measured. The data describe the horizontal expansion of the bubble plume (bubble core/entrained water flow) and the decrease of void fraction, bubble rise and water velocity with distance from the nozzle.

The entrained water flow expands more widely than the bubble core. This shows that a shear layer is developed between the upward moving bubbles in the bubble core and the stagnant water in the bulk. Water is entrained continuously into the bubble plume while it is rising, and so the entrained water flow increases with distance from the nozzle. Since the bubbles rise in that entrained water flow, the bubble velocity is considerably larger than that of individual bubbles in stagnant water. However, the relative phase velocity obtained by combining the DOS and HFA velocity readings is different from the rise velocity of individual bubbles in stagnant water. The velocity measurements demonstrate two things: (1) The mean relative velocity increases with the air flow rate, which can be explained by the wake acceleration phenomenon (small bubbles are captured in the wake of faster rising large bubbles) and (2) the relative velocity is not constant along the pool diameter. The latter finding is not completely understood so far.

The centerline void fraction and bubble velocity profiles show that the flow in the ZEF is buoyancy driven. After a short distance from the nozzle, the variation in void fraction and bubble velocity depend only on the air flow rate and not on the nozzle size. The momentum flux in the bubble plume increases due to decompression of the air. This momentum flux is much higher after a short distance from the injector than the initial momentum flux at the injector. The distance $z_0$ from the nozzle where the ZFE ends and the ZFE begins was set at the point where the centerline void fraction is equal to 0.25. Dimensionless correlations to describe the variations in void fraction, bubble and water velocity along the pool centerline were derived by scaling the distance from the nozzle with $z_0$. The correlations are in general agreement with findings from previous experiments in the literature.

Bubble chord length measurements show that considerably more bubble breakup occurs in the ZFE than in ZEF. However, the data indicate that the mean bubble diameter decreases with distance from the nozzle even in the ZEF, which points out that bubble breakup is dominant over bubble coalescence in this zone.

The empirical parameters for the bubble plume model are determined in chapter 6 using the data collected; these are: the entrainment coefficient $\alpha$, the ratio $\lambda$ between the width of the
bubble core and that of the entrained water flow, the virtual origin of the bubble plume \( z_{0,w} \), and the momentum amplification factor \( \gamma \). The values, obtained here for tests under pool scrubbing conditions, are comparable with previous results from other experiments, investigating mainly lake venting phenomena. However, generally applicable correlations that relate the empirical parameters to a wide range of boundary conditions, e.g. the air flow rate, are difficult to obtain using results from different experiments. In particular the pool size may influence water entrainment and this changes the behaviour of the entire bubble plume.

**Recommendations for future work**

Some questions are left open after finishing the present work. The description of the transition between individual bubbling and jetting above the nozzle is not well defined. A combination of a high speed video camera and a PIV system that allows to visualize the flow pattern and to measure the water velocity near the injector simultaneously may provide a better description.

The interpretation of the relative phase velocity measurements presented here is not complete. A better understanding is necessary if the bubble plume model is to be used to describe bubble rise in a pool. The wake acceleration effect, which is important for the mean bubble rise velocity, can be further studied in a separate experiment and with CFD methods. CFD may also be a useful tool to investigate water entrainment into the bubble plume and the effect of pool size. This would reduce the number of costly experiment.

The existing test facility can be used to further investigate two-phase flows in large pools. For example, different kinds of air injectors and extended structures, e.g. a tube bundle, can be installed within the pool to study the flow under these conditions. New local sensors, like phase detection sensors with multiple tips to measure interfacial area concentration and a HFA with more than one heated element to measure more than one water velocity component simultaneously can be used to obtain additional information about bubble plumes. Such, even more detailed, information could be useful in adjusting the parameters used for CFD computations of bubbly plumes and in testing the accuracy of CFD results.
References


A Testing and Calibrating the DOS

For local phase detection, a custom made double optical sensor (DOS) was purchased from RBI [67] and the required signal processing was developed during the present work (see section 3.2). A two-phase-loop (pipe flow) that is connected to the LNX7 facility was build to validate the DOS and the signal processing. The influence of the threshold setting and of the correction time \( t_{corr} \) on phase discrimination is tested by analyzing the same sensor signals with different settings for \( \Lambda \) and \( t_{corr} \). For calibrating the DOS, radial void fraction and bubble rise velocity profiles are measured in the pipe. The integral void fraction in the pipe is determined from radial void fraction profiles. The results are compared with calculations by a two fluid model (pressure drop) and the drift flux model. The volumetric air flux in the two-phase-loop is calculated with the bubble rise velocity and the void fraction, measured by the DOS, and compared to the injected air flow rate.

A.1 Theory

Void Fraction from Pressure Drop Measurements in a Pipe: Two-Fluid Model

The pressure gradient along the pipe \( \Delta p/\Delta z \) is measured in the two-phase-loop and can be connected to the integral void fraction by using a two fluid model. This void fraction is used as a reference to calibrate the DOS.

To find a functional relationship between the void fraction and the pressure gradient, the momentum equation for steady one-dimensional, homogeneous, equilibrium two-phase flow in a pipe can be written as an explicit equation for the pressure gradient (Wallis [79]):

\[
- \frac{dp}{dz} = \rho_m u \frac{du}{dz} + \rho_m g + \frac{P}{A} \tau_w = \left( \frac{dp}{dz} \right)_A + \left( \frac{dp}{dz} \right)_G + \left( \frac{dp}{dz} \right)_{TP} .
\]

(A.1)

In the above equation \( A \) and \( P \) represent the pipe cross-section and the perimeter, \( \tau_w \) is the averaged wall shear stress, and \( \rho_m \) is the mixture density. The three terms on the right hand side can be regarded as the accelerational (\( A \)), gravitational (\( G \)), and frictional components (\( TP \)) of the pressure gradient.

The accelerational component is neglected in solving the present problem because there is practically no change in the velocity along the axial direction.

The gravitational component can be calculated using the definition of \( \rho_m \):

\[
\left( \frac{dp}{dz} \right)_G = \left( 1 - \epsilon \right) \rho_w + \left( \epsilon \rho_m \right) g .
\]

(A.2)

\(^7\)Large-scale investigation of Natural circulation, condensation and mixing
Appendix

The pressure loss due to friction can be expressed by a two-phase multiplier $\Phi_{LO}^2$ and the pressure gradient $(dp/dz)_{LO}$ for the frictional loss when water flows in the same pipe with the same mass flow rate as the two-phase flow:

$$\left(\frac{dp}{dz}\right)_{TP} = \left(\frac{dp}{dz}\right)_{LO} \Phi_{LO}^2.$$  \hspace{1cm} (A.3)

In order to determine $(dp/dz)_{LO}$, the wall shear stress in a pipe and the friction factor $\lambda$ can be calculated with:

$$\lambda = \frac{8\tau_w}{\rho_w u^2} = 0.3164 \Rightarrow Re^{1/4}, \quad Re = \frac{\rho_w ud_{pipe}}{\eta_w}.$$  \hspace{1cm} (A.4)

The correlation is valid for $Re$ in the range of 2300 to $1 \cdot 10^5$. With (A.4), the pressure loss due to friction can be expressed by:

$$\left(\frac{dp}{dz}\right)_{TP} = \left(\frac{P}{A}\right)_{LO} \Phi_{LO}^2 = \left(\frac{0.3164}{2\frac{d_{pipe}}{m_w}^{3/4}} \rho_w^{3/4} \eta_w^{1/4} u^{7/4}\right) \Phi_{LO}^2.$$  \hspace{1cm} (A.5)

The velocity $u$ is the mean water velocity in the pipe, that represents the mass flow rate of the two-phase flow. It can be replaced by the volumetric water flux $\langle j_w \rangle_A$:

$$\rho_w u A = \rho_w \langle j_w \rangle_A A + \rho_a \langle j_a \rangle_A A \Rightarrow u = \langle j_w \rangle_A + \frac{\rho_a}{\rho_w} \langle j_a \rangle_A \simeq \langle j_w \rangle_A.$$  \hspace{1cm} (A.6)

The two-phase multiplier for turbulent flow can be determined with equation (2.70) in Wallis [79]:

$$\Phi_{LO}^2 = \left[1 + x \left(\frac{\rho_w - \rho_a}{\rho_a}\right) \right] \left[1 + x \left(\frac{\rho_a - \rho_g}{\rho_a}\right) \right]^{-0.25},$$  \hspace{1cm} (A.7)

where $x$ is the quality

$$x = \frac{\langle j_a \rangle_A \rho_a}{\langle j_a \rangle_A \rho_a + \langle j_w \rangle_A \rho_w}.$$  \hspace{1cm} (A.8)

Using the equations above, the pressure gradient in the two-phase loop ($d_{pipe} = 42.6 \ mm$, air/water flow) is given by:

$$-\frac{\Delta p}{\Delta z} = \left[\langle(1 - \langle\epsilon\rangle_A) \rho_w + \langle\epsilon\rangle_A \rho_a\rangle g\right] + K \langle j_w \rangle_A^{7/4} \Phi_{LO}^2.$$  \hspace{1cm} (A.9)

with

$$K = 258 \frac{Pa}{m} \left[\frac{s}{m}\right]^{7/4}.$$  \hspace{1cm} (A.10)

The volumetric flow rates $\langle j_w \rangle_A$ and $\langle j_a \rangle_A$ are known by the test conditions and the pressure gradient along the pipe $\Delta p/\Delta z$ is measured. With these data, the void fraction $\langle\epsilon_{pressure}\rangle_A$ in the cross-section can be determined from (A.9).
Appendix in Air Velocity and Void Fraction from the Drift Flux Model

The drift flux model is a one-dimensional model for the analytical description of two-phase flows. The theoretical framework is derived e.g. by Wallis [79]. In a pipe flow, it takes into account the relative velocity between the phases and the distinct variation in the void fraction and of the bubble rise velocity along the pipe diameter.

The void fraction can be expressed by:

$$\langle \varepsilon_{\text{drift}} \rangle_A = \frac{\langle j_a \rangle_A}{\langle j \rangle_A C_0 + \langle u_{a_j} \rangle_{a,A}}.$$  \hspace{1cm} (A.11)

$\langle j \rangle_A$ is total volumetric flux (average velocity of the mixture over the cross section):

$$\langle j \rangle_A = \langle j_a \rangle_A + \langle j_w \rangle_A$$ \hspace{1cm} (A.12)

$C_0$ is the distribution parameter that describes the variation in the radial velocity and void fraction profiles:

$$C_0 = \frac{\langle ej \rangle_A}{\langle ej \rangle_A \langle j \rangle_A}.$$ \hspace{1cm} (A.13)

$\langle u_{a_j} \rangle_{a,A}$ is the cross-sectional averaged true velocity of the air relative to the average velocity of the mixture:

$$\langle u_{a_j} \rangle_{a,A} = \frac{\langle e (u_a - j) \rangle_A}{\langle e \rangle_A}.$$ \hspace{1cm} (A.14)

Ishii [43] proposed for bubbly flow in pipes the following correlations for $C_0$ and $\langle u_{a_j} \rangle_{a,A}$:

$$C_0 = 1.2 - 0.2 \sqrt{\frac{\rho_a}{\rho_w}}.$$ \hspace{1cm} (A.15)

$$\langle u_{a_j} \rangle_{a,A} = \sqrt{2} \left( \frac{\rho_a \Delta \rho}{\rho_w^2} \right)^{1/4} \left( 1 - \langle e \rangle_A \right)^{7/4}.$$ \hspace{1cm} (A.16)

The void fraction in the two-phase-loop $\langle \varepsilon_{\text{drift}} \rangle_A$ is calculated with (A.11) using the two correlations above and (A.12).

A.2 Two Phase Loop

The two-phase-loop used for the calibration is illustrated in figure A.1. It consists of a vertical plexiglas pipe with two test sections. A mixing chamber for air and water is installed at the bottom. The distance between the mixing chamber and the first test section is 5000 mm, which is long enough to produce developed flow at the location of the DOS. A second test section is installed 500 mm above the first one to test a second DOS simultaneously. At the end of the pipe, an open tank separates water and air, and the water flows back into the water loop.
One DOS is located in each test section. The sensors are moved by stepping motors across the pipe diameter to measure at any radial position. Holes, 1 mm in diameter, were drilled in the pipe wall 250 mm below and above the positions of the sensors to measure the differential pressure. Pressure lines, filled with water, were used to connect these pressure taps to two pressure transmitters, one for each test section (pressure readings $\Delta p_{ch}$ and $\Delta p_{dc}$). A third pressure transmitter is connected to the bottom tap to determine the absolute pressure against atmosphere (pressure reading $p_d$).

### A.3 Test Matrix

Table A.1 lists the calibration tests. The air and water flow rates are given in terms of the volumetric fluxes, calculated under normal conditions. Only tests in the bubbly and churn turbulent flow regime were performed since they represent the flow patterns in the pool.
Appendix v

Re [1] Ja
(\\dot{\langle j_w \rangle}_A \ [m/s], \ Re [1]) 0.04 m/s 0.09 m/s 0.18 m/s 0.26 m/s
0.23, 9798 x
0.35, 14910 x x x
0.65, 27690 x x x
0.92, 39192 x x x

Table A.1: Test matrix for the DOS calibration.

A.4 Experimental Results

Threshold for Phase Discrimination

The threshold $E_{threshold}$ to discriminate between air and water in the DOS signal is calculated with (3.5). The parameter adjusting the threshold is $\Lambda$. It was set for the pool tests to 0.05.

The influence of $\Lambda$ on the measurement of the void fraction was tested in the two-phase loop. The sensor signal was acquired during 60 s in the centerline of the pipe and then analyzed with different values for $\Lambda$ to determine the void fraction. The results for four tests are shown in figure A.2. The graph demonstrates the relative change in $\epsilon$, using $\epsilon (\Lambda = 0.05)$ as a reference.

Increasing $\Lambda$ from 0.05 to 0.25 ($E_{threshold}$ raises by about 1 V) results in a relative void fraction decrease between 3 to 7 %, depending on the total void fraction. It demonstrates that mean bubble residence time on the sensor tip depends little on $E_{threshold}$. The reason is that the rising and falling edges of the bubble signals are quite steep. This points out that the sensor pierces the bubble fast and disturbs the rising bubble slightly.

Modified Threshold Method and Correction Time

The modified threshold method merges two bubbles together, when the period between detachment time for the first bubble and arrival time for the second bubble is less than a correction time $t_{corr}$. For the pool tests, $t_{corr}$ was set to 0.2 ms.

Again, DOS signals, acquired in the two-phase loop for different test conditions, were analyzed with different settings for $t_{corr}$. When $t_{corr}$ is varied between 0 and 1 ms, the relative change in the mean void fraction is less than 0.5 %. The reason for this is that the total bubble residence time is much longer than the times that are added to the total bubble residence time when bubbles are merged together. The modified threshold method affects, however, the measurement of the total number of bubbles $N_b$ stronger.

Figure A.3 shows the ratio of $N_b$, relative to $N_b (t_{corr} = 0.2 ms)$. The test conditions correspond to the test conditions in figure A.2. $N_b$ changes rapidly when $t_{corr} < 0.1 ms$, than gradually. This demonstrates that the modified threshold method corrects mainly signal misin-
Appendix vi

1.3

$z = 5000 \text{ mm}, d_{pipe} = 21.3 \text{ mm}$

- $j_w = 0.92 \text{ m/s}, j_a = 0.26 \text{ m/s}, \varepsilon (\Lambda = 0.05) = 0.235$
- $j_w = 0.65 \text{ m/s}, j_a = 0.18 \text{ m/s}, \varepsilon (\Lambda = 0.05) = 0.219$
- $j_w = 0.35 \text{ m/s}, j_a = 0.08 \text{ m/s}, \varepsilon (\Lambda = 0.05) = 0.162$
- $j_w = 0.23 \text{ m/s}, j_a = 0.04 \text{ m/s}, \varepsilon (\Lambda = 0.05) = 0.112$

Figure A.2: Influence of the threshold setting $\Lambda$ on void fraction measurements.

Interpretations due to noise in the sensor signal and not the appearance of pre-signals in front of a bubble. Signal interruptions shorter than 0.1 ms are often related to noise in the rising edge of the bubble signal while pre-signals cause longer signal interruption.

Two bubbles can be merged together by mistake with the modified threshold method. This situation is very unlikely, however, because two bubbles have to pass the sensor within a very short distance. For example, if two bubbles rise at 1 m/s then the distance between them must ≤ 0.2 mm to be merged together ($t_{corr} = 0.2 \text{ ms}$). The passage of such a bubble pair can never be detected correctly with the sensor, because the presence of the sensor itself will affect the situation. The sensor pierces the first bubble and this slows the bubble down slightly. The second bubble will not slow down immediately and it will touch the first one.
Appendix

Void Fraction

The integral void fraction in a cross-section of the test section in the two-phase loop is determined by integrating radial void fraction profiles, measured with the DOS:

$$\langle \epsilon_DOS \rangle_A = \frac{1}{\pi r_{pipe}^2} \int_{-r_{pipe}}^{r_{pipe}} \int_0^\pi \epsilon(r) r d\phi dr$$  \hspace{1cm} (A.17)

Typical radial void fraction profiles are plotted in figure A.4.

As explained before, the pressure readings of the differential pressure transmitters of the two-phase loop can also be used to determine the pressure. When the pipe is filled with stagnant water, the pressure readings $\Delta p_{eb}$ and $\Delta p_{dc}$ are zero. When water only flows through the pipe, the pressure loss due to friction is measured. Glatt [31] demonstrated that the pressure loss due to friction in the two-phase loop is in accordance with the predictions of (A.4), when $Re < 5 \cdot 10^4$. For the water/air tests listed in table A.1, the pressure readings at the upper test section are plotted in figure A.5.
To determine the pressure gradient on the right hand side of (A.9) from the pressure readings \( \Delta p_{bc} \) or \( \Delta p_{dc} \), the pressure head in the pressure lines \( \rho g \Delta z \) must be considered:

\[
\left( \frac{\Delta p}{\Delta z} \right)_{be/dc} = \rho g \Delta z - \Delta p_{bc/dc} \quad .
\]  

(A.18)

The volumetric void fractions \( \langle \epsilon_{\text{pressure}} \rangle_v \) in the pipe sections \( \Delta z = z_b - z_c \) or \( \Delta z = z_c - z_d \) can be calculated with (A.18) and (A.9). Considering a linear increase of the cross-sectional void fraction \( \langle \epsilon \rangle_A \) along the pipe due to the slight difference in pressure level (ideal gas law), \( \langle \epsilon_{\text{pressure}} \rangle_v \) represents the cross-sectional void fraction at the elevations of the DOS in the middle of the test sections.

The measurements \( \langle \epsilon_{\text{DOS}} \rangle_A \) in the two test sections \( z = 5250 \, mm \) and \( z = 5750 \, mm \) are plotted against \( \langle \epsilon_{\text{pressure}} \rangle_v \) and \( \langle \epsilon_{\text{drift}} \rangle_A \) (A.11) in figure 3.6 for comparison.
Bubble Rise Velocity

The volumetric air flux in the two-phase loop can be calculated from the radial void fraction and bubble rise velocity profiles:

\[
\langle j_a^{DOS} \rangle_A = \frac{1}{\pi r_{pipe}^2} \int_0^{r_{pipe}} \int_{-r_{pipe}}^{r_{pipe}} \epsilon(r) u_b(r) r \, d\phi \, dr .
\]  

(A.19)

Typical radial bubble rise velocity profiles are plotted in figure A.6.

The volumetric air flow rate \( \dot{V}_a^{DOS} \) is determined with \( \langle j_a^{DOS} \rangle_A \) using (3.11). It can be compared with the volumetric air flow rate injected into the pipe \( \dot{V}_a^{AFC} \), which can be obtained from the air flux rate (see table A.1):

\[
\dot{V}_a^{AFC} = A \langle j_a \rangle_A = A \frac{p_{ref} T(z) Q_a}{T_{ref} p(z)} A
\]  

(A.20)

The pressure \( p(z) \) is determined from the pressure readings \( p_d, \Delta p_{bc}, \) and \( \Delta p_{dc} \) considering the hydrostatic pressure in the pressure lines.

A comparison between \( \dot{V}_a^{DOS} \) and \( \dot{V}_a^{AFC} \) is shown in figure 3.8.
Figure A.6: Radial bubble rise velocity profiles in the pipe.

A.5 Horizontally and Vertically Oriented DOS

In addition to calibration, comparison tests were performed with the DOS both oriented horizontally and vertically in a pipe. Such a comparison is desirable because fibre optic probes are usually used with a vertical orientation rather than the horizontal orientation needed for the pool tests. For each orientation, void fraction and number of bubbles per unit time were measured at different air and water flow rates in the pipe. In addition, the sensor performance was tested in detail by studying bubble residence time and the signal rise time as bubbles pass through the sensor.

Figure A.7 shows a sketch of the test facility for sensor orientation tests. The test section in this apparatus is 750 mm away from the mixing chamber. One probe holder, attached to this test section, orients the sensor perpendicularly to the flow direction. The second probe holder, situated in the separation tank, holds the sensor parallel to the flow.

The test matrix to study the orientation effect is shown in figure A.8 where mean volumetric air flux in the pipe is plotted against the mean volumetric water flux. In a first series, tests were performed with a vertically oriented sensor and, in the second series, the same sensor was
horizontally oriented. For these tests, the DOS signal was acquired during 60 s with a sampling rate of 100 ksamples/s. The signals from the same sensor tip were analyzed to compare the two configurations.

As demonstrated in figure A.9, the sensor orientation has very little influence on the void fraction measurements and no effect on the bubble frequency measurements. The comparison between $\epsilon_{\text{vert}}$ and $\epsilon_{\text{hor}}$ indicates that a slightly higher void fraction is measured with the vertically oriented sensor. However, the shift of the data points is within an error margin of $\pm 10\%$, which is in the range of the reproducibility of these tests. No significant trend can be seen by comparing the bubble frequency measurements.

For further testing the sensor performance, the bubble residence time $t_{\text{res}}$ and the signal rise time $t_{\text{rise}}$ were measured for bubbles passing through the sensor. Figure A.10 illustrates the method for measuring $t_{\text{res}}$ and $t_{\text{rise}}$. Upper and lower thresholds were determined by detecting the water level $E_w$ and the air level $E_a$ (see figure 3.4) and using (3.5) to calculate the threshold voltages by setting $\Lambda$ to 0.05 and 0.95, respectively. The bubble residence time is the time interval between the rising and falling edges of the signal. The signal rise time is defined by the period of the rise between the lower and the upper thresholds.

The PDFs of the bubble residence time are shown in figure A.11 for four different test con-
The mean bubble residence times $t_{res}^e$ for the horizontal and vertical orientations are listed in the legend and marked in the graphs. The plots demonstrate that the bubble residence time is practically unaffected by sensor orientation.

The data in figure A.12 show that the bubble rise time for the horizontally oriented sensor is on average the longer than that of the vertically oriented sensor. This indicates that the sensor pierces bubbles faster in the vertical orientation. The bubble rise time is part of the bubble residence time and therefore it does not affect this measurement. The bubble rise times in the signal are added up to calculate the void fraction. Considering the comparison between void fraction and bubble frequency measurements, it can be concluded that the sensor orientation plays virtually no role for void fraction measurements.
Figure A.9: Void fraction and bubble frequency measurements for horizontal and vertical orientations (indices vert. and hor. stand for the sensor orientation).

Figure A.10: Bubble residence time and signal rise time.
Figure A.11: PDFs of the bubble residence time for horizontally and vertically oriented DOS.

Figure A.12: PDFs of the bubble rise signals for horizontally and vertically oriented DOS.
B Calibrating and Testing the HFA

Local water velocities within the pool are measured with a hot-film anemometer (HFA). Bruun [4] recommends to frequently calibrate the sensors when they are used in water because of contamination of the sensor tips. The water temperature and purity used in the calibration and the experiments should be the same. These requirements are realized for the experiments described here by building a tow tank. With this calibration facility, the HFA signal $E_{HFA}$ can be related to a defined tow velocity of the sensor ($E_{HFA}(U_{tow})$). The water in the tow tank is conditioned with the water loop that is used for the pool tests (see figure C.18). In addition to calibrations, the tow tank was used to investigate the directional sensitivity of the sensor. Tests were done in the tow tank with the same tow velocity but different orientations of the sensor, so that the sensor tip was cooled by different effective velocities.

B.1 Tow Tank

Figure B.13 shows the schematic of the tow tank used for HFA calibrations. It consists of an open water tank ($5500 \times 200 \times 190 \text{ mm}$) and a belt drive that is controlled with a stepping motor. The tow tank has a water inlet at one end and an outlet at the other end to condition the water before a calibration test. A platform is attached to the drive belt to hold the HFA sensor and to pull it through the tow tank as the belt rotates. The distance between the submerged sensor tip and the water surface is about $70 \text{ mm}$. For the calibration runs, the sensor tip sits $300 \text{ mm}$ in front of the sensor mounting arm and the distance to both side walls of the tank is $100 \text{ mm}$. With this configuration and some extra margins at both ends of the tow tank, the sensor can run about $4000 \text{ mm}$ along the length of the tow tank for a single calibration test.

Figure B.14 illustrates the various sensor positions within the tow tank to study the influence of effective sensor cooling velocity. In these experiments, the sensor tip is $85 \text{ mm}$ in front of the sensor mounting arm because a smaller sensor support is used. This permits the rotation of the sensor within the tow tank. The angle $\omega_{\text{sensor}}$ between the main axis of the sensor and the direction of movement can be changed in steps of $15^\circ$ from $0^\circ$ to $90^\circ$ with a special sensor mounting arm.

The stepping motor, used for moving the belt drive, is connected to the power drive, controlled by a PC. The setup permits a minimum controlled spinning rate of 2 revolutions per second and a maximum of 22. One revolution of the gear that drives the belt moves the HFA sensor over a distance of $0.11 \text{ m}$. With a gearing of 1:1 between the motor and the gear that drives the belt, the sensor can be moved at speeds between 0.22 to $2.42 \text{ m/s}$. An additional gear is installed to move the sensor slower than $0.22 \text{ m/s}$. Three gear ratios can be selected: 1:1, 2:1 (0.11 to $1.21 \text{ m/s}$), 3:1 (0.07 to $0.81 \text{ m/s}$).
The instantaneous tow velocity $U_{\text{tow}}$ is determined with an increment encoder, which is connected to the belt drive. A disk with 2500 slots spins at the same rate as the belt drive and a light barrage, built with emitter and receiver photo diodes, detects the passage of each slot. In this way, the incremental encoder output signal generates 2500 pulses per rotation. The signal
is acquired with an AD-converter to determine the period $\Delta t_{p,i}$ between two pulses, which represent $1/2500$ of one revolution of the gear that drives the belt (sampling rate AD-converter: $1M\,sample/s$). This measurement is used to calculate the instantaneous tow velocity at the center time $t_i$ of the period $\Delta t_{p,i}$ from:

$$U_{\text{tow}}(t_i) = \frac{0.11m}{2500} \frac{1}{\Delta t_{p,i}(t_i)}.$$  \hspace{1cm} (B.21)

Each pulse in the incremental encoder signal is used to calculate a velocity and a complete tow velocity history $U_{\text{tow}}(t_i)$ is established while the sensor runs along the length of the tow tank. In a second identical run, the HFA signal $E_{\text{HFA}}(t)$ is acquired with the AD-converter and then the two signals are correlated to establish the relation $E_{\text{HFA}}(U_{\text{tow}})$ between HFA signal and sensor velocity.

The accurate design of the belt drive, the computer controlled power drive for the motor, and the precise water conditioning assure the reproducibility of the experiments in the tow tank. Various tow velocity profiles and HFA measurements were performed by Howles [39] to document this reproducibility.

**B.2 Calibration of the HFA**

Figure B.15 shows an example for a tow velocity history $U_{\text{tow}}(t_i)$ and the corresponding HFA signal $E_{\text{HFA}}(t_j)$. Both measurements are performed by accelerating the stepping motor constantly to its maximum speed in about 2 s. To calibrate the HFA, the measurements are correlated between $t_{\text{start}}$ and $t_{\text{end}}$, which represents times close to the controlled minimum and maximum motor spinning rates, respectively.

The correlation between $E_{\text{HFA}}(U_{\text{tow}})$ is established in two steps. At first, the tow velocity history is fitted between $t_{\text{start}}$ and $t_{\text{end}}$ with a linear curve to determine the slope (acceleration of the sensor in the tow tank) and the intercept of $U_{\text{tow}}(t)$ in that range. The linear fit is used in the second step to calculate the tow velocity for any sample of the HFA signal between $t_{\text{start}}$ and $t_{\text{end}}$ (about 38000 data points in figure B.15).

Two calibration data sets are recorded for a complete HFA calibration: the first with a 1:1 gear ratio (analyzed speed range $0.25 - 2.37 \, m/s$) and the second one with a 3:1 gear ratio ($0.08 - 0.25 \, m/s$). These two data sets are put together as one data set that is fitted to determine the calibration equation.

The HFA signals are filtered before each correlation because the HFA is sensitive enough to measure the mechanical vibrations of the sensor tip during the run, caused by moving the belt drive with the stepping motor. A lowpass filter with a cutoff frequency of $10Hz$ is used. The
raw data $E_{HFA}$ and the filtered signal are shown in figure B.15. The filtering reduces the scatter of the HFA signal and a more accurate calibration equation can be determined for $E_{HFA} (U_{tow})$.

### B.3 Orientation Effect

Figure B.16 shows the calibration curves for different yaw-angle $\alpha$ positions of the HFA in the two tank test (see section 3.3.2). The plots were used to determine the data points $U_{eff} (\alpha)$ in figure 3.15. The decrease in $E_{HFA}^2$ obtained by rotating the sensor is very well demonstrated in figure B.16.

The polynomial fit for $\alpha = 0^\circ$ is used to translate $E_{HFA}$ into the velocity reading $U_{eff}$ of the HFA. When the sensor is aligned with the direction of movement ($\alpha = 0^\circ$), $U_{eff}$ is equivalent to the velocity component of the direction of the main sensor axis $u_{HFA}$. In the pool, $u_{HFA}$ is equivalent to the axial water velocity component $u_w$.

As illustrated in figure 3.14, rotating the sensor in the tow tank results in a decrease of $u_w$ as specified by the cosine law. The sensor rotation produces also a reduced sensor cooling which is indicated by the decrease of $E_{HFA}^2$. Taking now the value for $E_{HFA}^2$ and calculating $U_{tow}$ with
the polynomial fit (valid for $\alpha = 0^\circ$) gives the reduced velocity reading $U_{eff}$. This is illustrated graphically by the horizontal and vertical lines in figure B.16 for $U_{tow} = 1 \text{ m/s}$. As seen in figure 3.15, $U_{eff}$ and $u_w$ decrease in about the same way, which means that the HFA measures mainly the axial water velocity component in the pool when the sensor is placed horizontally.

The functional relationship between the $U_{eff}(\varphi)$ (pitch-angle $\varphi$ positions) that is also shown in figure 3.15 was determined in a similar way from the original calibration curves plotted in figure B.17. In this case, the sensor does not follow the cosine law, as expected.
Figure B.16: Tests for different sensor orientations in the tow tank (yaw-angle effect) and corresponding HFA velocity readings for $\alpha = 0$.

Figure B.17: Tests for different sensor orientations in the tow tank (pitch-angle effect).
C Experimental Facility

C.1 Water Loop and Air Supply

Figure C.18: Water loop and air supply connected to the large pool and the tow tank (instrument functions: see table C.2)

Temperatures are measured with type-K thermocouples. The measuring transducers for the thermocouples (Yokogawa Hybrid Recorder, regulators for cooling water and heater) were calibrated with a reference signal at 20°C for compensating the differences between the temperature readings. In this way, the various temperature readings matched in a range of ±0.1°C.
Table C.2: Instruments in the water loop and air supply.

The pressure transmitters were calibrated in a calibration device. The coefficients for linear curve fits which relate the $4 – 20 mA$ sensor signals to the pressures were determined. Accurate resistors (10 Ω) were used to transform the current outputs into voltages, which were measured and translated into mbar by the linear curve fits programmed in the Yokogawa Hybrid Recorder. This measurement chain allows pressure readings within an error margin of ±2% of the reading.

The mass flow controllers that regulate the air flow rate were calibrated by the manufacturer. They control the air flow rates in their working ranges with an accuracy of ±2% of the reading.

C.2 Tank and Traversing System

Detailed drawings of the tank used for the pool tests are shown in figure C.19 and C.20. Filtered water at constant temperature is injected at the tank bottom and an overflow is positioned at the top to extract the water after it has passed through the tank. The water is injected by a vertical pipe 60.3 mm in diameter with a baffle at its end to prevent the development of a jet above the pipe outlet. The overflow is installed within the tank and narrows the pool diameter from 1000 mm to 750 mm at the top 200 mm. The fixed positions of the water injector and overflow make it necessary to move the air injecting nozzle up or down to adjust the injection depth.

The air injection depth $H$ is defined by the distance between the nozzle exit and the pool
The nozzle is mounted on a sliding platform that can be moved up or down by a stepping motor. The nozzle is located at the pool centerline and can be replaced to change the nozzle diameter between 5 and 20 mm. The air supply is connected to the nozzle by a 4 m long flexible nylon tube (ID 9 mm). A horizontal plate, 800 mm in diameter, is mounted on the sliding platform to prevent interactions between the two-phase flow above the nozzle and the pool below the platform. The nozzle exit is 80 mm above this plate. The water injected through the bottom pipe can flow through the ring between the edge of the plate and the pool wall. Any air injection depth between 1.25 m and 4.4 m can be obtained by moving the platform with the nozzle. The accuracy of this adjustment is $\Delta H = \pm 5$ mm. This uncertainty in the air injection
depth is smaller than the observed oscillation of the water surface caused by air injection.

A traversing system is used to position the three local sensors: DOS, HFA and a thermocouple. Figure C.20 shows a top view of this traversing system (cutaway A-A in figure C.19). It consists of a frame and a linear guide attached to the frame. The frame can be moved up or down to change the axial position of the sensors and the linear guide moves the sensors to different radial positions. The three sensor supports are mounted on a small plate on the linear guide, with a horizontal separation of 50 mm. The sensor tips are 330 mm away from the plate.

Support structures and extra hardware within the pool should be kept as small as possible to minimize any unwanted influences on the flow field. The sensor traversing system covers less than 2 % of the entire pool cross-section. Supports such as the upper chain mounting for the frame or the stepping motor for the linear guide are placed as close as possible to the pool wall to minimize flow disturbances. This design limits the range of sensor positions within the pool. The minimum distance between the pool surface and the sensor tips is 1000 mm since the frame can only be moved up to the water overflow. The maximum radial sensor position is $r_{\text{max}} = 340$ mm because of the design for the linear guide.

The traversing system for the local sensors and the nozzle are aligned before each experiment. This is done by moving the double optical sensor 10 mm above the nozzle exit and visually checking the center position. The radial sensor positions are set by moving the small plate with the stepping motor for the linear guide. This horizontal positioning of the local sensors is done with an accuracy of $\Delta r \leq \pm 0.5$ mm. The vertical sensor position is changed by moving the
frame with the attached linear guide up or down. This sets the distance between the nozzle exit and sensor tips with an accuracy of $\Delta z \leq \pm 3 \text{ mm}$. The vertical distance between nozzle exit and sensors can be set more accurately than the pool depth because it is easier to verify this distance with a rule than the distance between the nozzle exit and the water surface at the overflow.

C.3 Process Control and Data Acquisition

A personal computer (PENTIUM I, 166 MHz, WINDOWS NT 4) is used during the pool tests for the following tasks: (1) positioning of the local sensors via stepping motor, (2) setting the air flow rate and (3) acquiring data. The latter task is divided into a high speed data acquisition to acquire the signals from the DOS and HFA and a low speed data acquisition to monitor the process data. Programmes to control the different tasks are developed with the graphical programming language LabVIEW®. The following paragraphs describe the hardware and software for process control and data acquisition.

![Diagram of process control and data acquisition](image)

Figure C.21: Schematic for the process controlling and data acquisition.

The stepper control board is a plug-in card for the PC and runs the stepping motors to set...
the pool depth and to position sensors. The board generates signals to control a power drive unit that supplies the motors with current; the computer controls the number of motor steps, respectively displacements, and the motor speed. The stepping motors operate in the half-step mode and 400 steps are needed per revolution of the motor axis.

Air flow rates are adjusted with the PC by sending set-points via a standard RS-232 interface to a flow computer. The flow computer communicates with air mass flow controllers FC1.2 and FC2.2.

The analog signals from the HFA and DOS are translated into digital data with a 12-bit AD-converter. A shielded connector box is used to interface the analog signals to the AD-converter (National Instruments, PCI-MIO-16E plug in card). The AD-converter can operate at any sampling frequency up to 1.25 $MHz$ Samples/s and input ranges can be adjusted from $\pm 0.05$ to $\pm 10$ V on 8 differential input channels. The settling time for a channel switch during multiple channel data acquisition is 2 $\mu$s. Digital data is stored directly on the computer hard disk during the data acquisition (stream to disk concept). This has the advantage that the amount of acquired data is limited by the free hard disk space and not by the computer memory. For example, continuous data acquisition is possible at 100 $k$Samples/s over a period of 60 min. (about 700 MByte data).

Monitoring process data, like temperatures, flow rates and pressures, require a much slower sampling rate than needed for the signals from the DOS and the HFA. The indicators that provide the process data and the thermocouple within the pool are connected to the Yokogawa Hybrid Recorder (Model HR 2500E), where different inputs, thermocouple signals and voltage signals in different ranges, are transformed into physical values like temperatures, pressures, and flow rates. The Yokogawa Hybrid Recorder sends these data every 2 s to the computer via a GPIB interface.

The signal processing software packages for the DOS and the HFA are developed with LabVIEW. During the experiments, subroutines pre-process raw data acquired by the AD-converter to perform data reduction. The pre-processing creates result files that can be analyzed in detail after the experiments.
D Preliminary Pool Tests

D.1 Averaging

Time averaging is necessary to measure mean flow parameters with the DOS and the HFA. A typical measurement time $T_{meas}$ in pipe experiments is about 60 s. In the pool tests, the bubble plume oscillation must be taken into account in measuring mean values. Preliminary tests in the pool were carried out to determine $T_{meas}$. The signals of the DOS were acquired continuously over a period and then divided into shorter time intervals. The void fraction and bubble rise velocity were measured in each interval to determine the time dependent behaviour of the flow parameters and the development of mean values. The same analysis was performed with the HFA signal to set $T_{meas}$ for the mean water velocity.

Figure D.22 shows the variation in void fraction and bubble rise velocity measurements with time for T100.10 at (250, 0). The signal was acquired over a period of 4800 s and divided into intervals of 5 s. The dotted lines represent the time dependent behaviour of the void fraction and the bubble rise velocity, respectively. The straight lines are the development of the mean values. The variation in the mean void fraction is smaller than ±5 % for an averaging time of

![Figure D.22: Time averaging of void fraction and bubble rise velocity.](image-url)
600 s and a constant bubble rise velocity in a range ±2 % is archived for a time even shorter than this.

The HFA was tested under the same test conditions but the data acquisition time was reduced to 600 s. The total signal was divided into intervals of 1 s. The water velocity and the development of the mean value are plotted in figure D.23 as a function of time. The graph indicates that the mean water velocity is stable within ±2 % after an averaging time of less than 100 s. Nevertheless, $T_{\text{meas}}$ was set to 600 s for the pool tests because water velocity measurements at the edge of the plume were interrupted when the HFA signal was out of the calibration range ($\leq 0.08$ m/s). The averaging time of 600 s was required at the edge of the bubble plume to obtain a sufficient number of HFA signal samples within the calibration range to measure a mean velocity.

The data for T100.10 were discussed here as an example. The same preliminary tests were performed for TX.10. No significant dependence of the averaging time on the air flow rate was observed.
D.2 Settings for Bubble Rise Velocity Measurements

The velocity of an individual bubble is considered and the mean bubble rise velocity $u_b$ in the pool is calculated when two conditions are met (see section 3.2.2):

1. The relative difference between the bubble residence times at the up- and downstream sensor tip $|\Delta \tau_{res}|$ is smaller than 0.2.

2. The bubble rise velocity is in the range of 0.19 to 2.88 m/s.

Bubble residence times PDFs and bubble rise velocity PDFs at one position in the pool are discussed in this section to justify the setting for $|\Delta \tau_{res}|$ and of the velocity range.

The relative difference in the bubble residence times is given by:

$$|\Delta \tau_{res}| = \frac{t_{res,us} - t_{res,ds}}{t_{res,us}}$$  \hspace{1cm} (D.22)

Figure D.24 shows the PDFs for $\Delta \tau_{res}$ for TX.10 at (250, 0). The limit for $|\Delta \tau_{res}|$ is illustrated in the graphs and the percentage of linked bubble signals $\Psi$, defined by (3.10), within the range of $|\Delta \tau_{res}| = 0.2$ is given for each test. Values of $\Delta \tau_{res}$ greater than zero indicate that $t_{res,us} > t_{res,ds}$ and values smaller than zero indicate that $t_{res,us} < t_{res,ds}$.

The bubble residence PDFs are symmetric, with a slight overweight for $\Delta \tau_{res} > 0$. Random bubble motion in different directions in the horizontal plane should result in a perfectly symmetric PDF because the relative difference between the residence times should be randomly distributed. The overweight for $\Delta \tau_{res} > 0$ indicates a measurement error caused by the disturbance of the bubble motion at the upstream sensor tip. The bubble is deflected at the upstream tip and the result is a reduced residence time at the downstream tip. However, the plots in figure D.24 show that the measurement error due to bubble deflection is small.

Between 53 to 60 % of the bubbles are linked together and considered for measuring $u_b$ when $|\Delta \tau_{res}|$ is set to 0.2. The remaining bubbles have a significant velocity component in the horizontal plane due to the random bubble motion or the deflection on the upstream tip is too strong for measuring $u_b$ correctly. The setting of $|\Delta \tau_{res}|$ was varied to test the influence on the measured bubble rise velocity. It was found that the setting of $|\Delta \tau_{res}|$ is practically not affecting the measurement of $u_b$.

The acceptable velocity range is the second condition that must be met in order to consider a single bubble rise velocity to calculate the average. Figure D.25 shows the bubble rise velocity PDFs for TX.10 at (250, 0). The setting of the velocity range for the pool tests is illustrated in the graphs. The total number of bubbles within this range $N_b$ and the measured average $u_b$ is given in the boxes. The bubble velocities vary strongly because the bubble plume wanders. A
Appendix

Figure D.24: Relative change in the bubble residence times for different air flow rates.

DOS at one fixed position in the pool detects fast rising bubbles in the plume center and bubbles that rise in stagnant water at the edge of the plume. Increasing the air flow rate results in more widely spread velocity distributions.

The lower velocity limit selected represents the rise velocity in stagnant water of 1 mm diameter bubbles (see Haberman & Morton [33]). Bubbles larger than 1 mm rise faster than 0.19 m/s while bubbles smaller than 1 mm are very rare in the pool. To consider most of the faster rising bubbles for calculating the ensemble average, the upper limit is set to 2.88 m/s, which corresponds to the time of flight that is as close as possible to 3 m/s. Digitizing of the sensor signals prevents an exact setting. Values higher than 3 m/s and smaller than 0.19 m/s are interpreted as measurement errors and are not taken into account in calculating $u_f$.

Similar methods for calculating the bubble rise velocity are described by Kalkach-Navarro [44] and Suzanne [71], but the selected settings for velocity ranges and maximum differences between bubble residences are not well reported. Kalkach-Navarro [44] allows a maximum $\Delta \tau_{res}$ of 0.3 and sets the velocity range on the basis of the drift flux model. Consequences of these settings are not discussed. Suzanne [71] determines the most probable velocity by cross-correlating the two sensor signals. This measurement is used to define a velocity range and the
sensor signals are re-analyzed to measure individual bubble velocities within this range. The width of the velocity range is not reported. It is mentioned only that about 70% of the bubble signals are linked together.
### E Tables with Experimental Results

<table>
<thead>
<tr>
<th>Test</th>
<th>z [m]</th>
<th>$u_{w,m}$ [m/s]</th>
<th>$b_w$ [mm]</th>
<th>$r_{c,w}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T25.10</td>
<td>250</td>
<td>0.44 ± 0.01</td>
<td>36.1 ± 0.7</td>
<td>-4.8 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.35 ± 0.01</td>
<td>61.7 ± 2.2</td>
<td>-4.4 ± 1.5</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.36 ± 0.01</td>
<td>83.8 ± 2.7</td>
<td>-13.0 ± 1.7</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.38 ± 0.01</td>
<td>96.4 ± 3.5</td>
<td>-6.4 ± 2.2</td>
</tr>
<tr>
<td>T50.10</td>
<td>250</td>
<td>0.54 ± 0.01</td>
<td>48.1 ± 2.2</td>
<td>5.7 ± 1.5</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.44 ± 0.01</td>
<td>75.0 ± 2.7</td>
<td>-6.6 ± 1.8</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.42 ± 0.01</td>
<td>103.3 ± 3.7</td>
<td>-4.4 ± 2.4</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.40 ± 0.01</td>
<td>139.7 ± 5.0</td>
<td>-4.3 ± 3.4</td>
</tr>
<tr>
<td>T100.10</td>
<td>250</td>
<td>0.85 ± 0.02</td>
<td>47.5 ± 1.7</td>
<td>-2.9 ± 1.3</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.60 ± 0.01</td>
<td>82.2 ± 2.3</td>
<td>-1.9 ± 1.5</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.50 ± 0.01</td>
<td>126.5 ± 2.7</td>
<td>2.2 ± 1.8</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.50 ± 0.01</td>
<td>153.9 ± 3.9</td>
<td>-1.5 ± 2.5</td>
</tr>
<tr>
<td>T200.10</td>
<td>250</td>
<td>0.91 ± 0.01</td>
<td>50.3 ± 0.9</td>
<td>-6.3 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.77 ± 0.01</td>
<td>93.0 ± 1.3</td>
<td>-7.3 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.68 ± 0.01</td>
<td>136.2 ± 2.6</td>
<td>-11.9 ± 1.8</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.58 ± 0.01</td>
<td>175.5 ± 4.6</td>
<td>-4.3 ± 3.0</td>
</tr>
</tbody>
</table>

Table E.3: Coefficients of the radial water velocity profiles (Gaussian curves).
<table>
<thead>
<tr>
<th>Test</th>
<th>$z$ [$mm$]</th>
<th>$\epsilon_r$ [1]</th>
<th>$b_r$ [$mm$]</th>
<th>$r_{cc}$ [$mm$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T25.5</td>
<td>250</td>
<td>0.077 ± 0.002</td>
<td>35.7 ± 1.3</td>
<td>-2.8 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.035 ± 0.002</td>
<td>55.0 ± 4.0</td>
<td>3.3 ± 2.7</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.030 ± 0.001</td>
<td>68.1 ± 3.1</td>
<td>-20.3 ± 2.1</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.025 ± 0.001</td>
<td>64.7 ± 3.4</td>
<td>-31.0 ± 2.1</td>
</tr>
<tr>
<td>T50.5</td>
<td>250</td>
<td>0.123 ± 0.004</td>
<td>35.5 ± 1.8</td>
<td>3.3 ± 1.1</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.061 ± 0.002</td>
<td>63.4 ± 2.9</td>
<td>-10.4 ± 1.9</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.040 ± 0.001</td>
<td>75.4 ± 3.1</td>
<td>-12.1 ± 2.0</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.030 ± 0.001</td>
<td>85.4 ± 4.4</td>
<td>-9.8 ± 2.7</td>
</tr>
<tr>
<td>T100.5</td>
<td>250</td>
<td>0.179 ± 0.005</td>
<td>41.8 ± 1.8</td>
<td>-7.5 ± 1.2</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.084 ± 0.002</td>
<td>74.9 ± 2.4</td>
<td>-11.9 ± 1.5</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.057 ± 0.001</td>
<td>91.3 ± 2.4</td>
<td>-14.3 ± 3.3</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.040 ± 0.001</td>
<td>109.9 ± 2.9</td>
<td>-16.5 ± 1.9</td>
</tr>
<tr>
<td>T200.5</td>
<td>250</td>
<td>0.299 ± 0.005</td>
<td>41.6 ± 0.8</td>
<td>-26.7 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.135 ± 0.001</td>
<td>73.6 ± 1.1</td>
<td>-34.3 ± 0.7</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.086 ± 0.001</td>
<td>96.1 ± 2.5</td>
<td>-11.1 ± 1.7</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.057 ± 0.001</td>
<td>123.2 ± 3.6</td>
<td>-25.7 ± 2.4</td>
</tr>
<tr>
<td>T25.10</td>
<td>250</td>
<td>0.082 ± 0.002</td>
<td>32.2 ± 1.2</td>
<td>2.4 ± 0.7</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.042 ± 0.001</td>
<td>47.9 ± 2.7</td>
<td>-4.8 ± 1.78</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.030 ± 0.001</td>
<td>71.2 ± 3.1</td>
<td>-14.3 ± 2.2</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.022 ± 0.001</td>
<td>69.8 ± 2.9</td>
<td>-19.7 ± 1.9</td>
</tr>
<tr>
<td>T50.10</td>
<td>250</td>
<td>0.127 ± 0.003</td>
<td>31.1 ± 1.2</td>
<td>5.0 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.062 ± 0.002</td>
<td>63.6 ± 3.5</td>
<td>-15.8 ± 2.3</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.045 ± 0.001</td>
<td>70.3 ± 2.7</td>
<td>-15.3 ± 1.7</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.029 ± 0.001</td>
<td>85.8 ± 5.8</td>
<td>-3.1 ± 3.9</td>
</tr>
<tr>
<td>T100.10</td>
<td>250</td>
<td>0.193 ± 0.003</td>
<td>40.9 ± 1.1</td>
<td>3.1 ± 0.7</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.089 ± 0.002</td>
<td>67.7 ± 2.0</td>
<td>1.7 ± 1.3</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.057 ± 0.001</td>
<td>89.2 ± 3.0</td>
<td>-7.5 ± 1.9</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.039 ± 0.000</td>
<td>110.5 ± 2.5</td>
<td>2.5 ± 1.6</td>
</tr>
<tr>
<td>T200.10</td>
<td>250</td>
<td>0.250 ± 0.004</td>
<td>52.2 ± 1.4</td>
<td>-3.6 ± 0.9</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.116 ± 0.002</td>
<td>78.8 ± 2.0</td>
<td>-1.3 ± 1.3</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.079 ± 0.001</td>
<td>102.4 ± 2.4</td>
<td>-5.7 ± 1.5</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.055 ± 0.001</td>
<td>127.4 ± 3.8</td>
<td>-1.2 ± 2.6</td>
</tr>
<tr>
<td>T25.20</td>
<td>250</td>
<td>0.057 ± 0.002</td>
<td>37.2 ± 1.5</td>
<td>-1.8 ± 1.0</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.029 ± 0.001</td>
<td>68.5 ± 5.3</td>
<td>0.5 ± 3.5</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.026 ± 0.001</td>
<td>67.4 ± 3.9</td>
<td>-17.2 ± 2.6</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.021 ± 0.001</td>
<td>72.6 ± 4.1</td>
<td>-21.7 ± 2.5</td>
</tr>
<tr>
<td>T50.20</td>
<td>250</td>
<td>0.096 ± 0.004</td>
<td>41.8 ± 2.6</td>
<td>5.3 ± 1.7</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.052 ± 0.002</td>
<td>68.0 ± 3.2</td>
<td>-14.8 ± 2.1</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.033 ± 0.001</td>
<td>90.8 ± 5.4</td>
<td>-12.1 ± 3.5</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.026 ± 0.001</td>
<td>107.9 ± 5.0</td>
<td>-9.0 ± 3.3</td>
</tr>
<tr>
<td>T100.20</td>
<td>250</td>
<td>0.155 ± 0.004</td>
<td>47.1 ± 1.9</td>
<td>5.5 ± 1.3</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.076 ± 0.002</td>
<td>72.8 ± 2.6</td>
<td>1.0 ± 1.7</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.050 ± 0.001</td>
<td>97.5 ± 3.1</td>
<td>-4.8 ± 2.0</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.036 ± 0.001</td>
<td>116.0 ± 3.0</td>
<td>-3.2 ± 2.0</td>
</tr>
<tr>
<td>T200.20</td>
<td>250</td>
<td>0.253 ± 0.003</td>
<td>47.3 ± 1.0</td>
<td>0.0 ± 0.7</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.128 ± 0.001</td>
<td>77.9 ± 1.9</td>
<td>-2.2 ± 1.2</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.080 ± 0.001</td>
<td>101.4 ± 3.1</td>
<td>-8.1 ± 2.0</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.056 ± 0.001</td>
<td>126.8 ± 3.8</td>
<td>-1.5 ± 2.6</td>
</tr>
</tbody>
</table>

Table E.4: Coefficients of the radial void fraction profiles (Gaussian curve).
## Appendix

### Table E.5: Coefficients of the radial bubble rise velocity profiles (Gaussian curve).

<table>
<thead>
<tr>
<th>Test</th>
<th>$z$ [m]</th>
<th>$u_{b,m}$ [m/s]</th>
<th>$b_{ub}$ [mm]</th>
<th>$r_{c,ub}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>T25.5</td>
<td>250</td>
<td>0.93 ± 0.03</td>
<td>146.0 ± 26.9</td>
<td>6.0 ± 9.8</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.85 ± 0.01</td>
<td>154.1 ± 6.1</td>
<td>1.6 ± 3.1</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.85 ± 0.01</td>
<td>167.6 ± 5.6</td>
<td>-24.0 ± 3.2</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.80 ± 0.02</td>
<td>238.9 ± 16.6</td>
<td>-38.3 ± 9.3</td>
</tr>
<tr>
<td>T50.5</td>
<td>250</td>
<td>1.01 ± 0.02</td>
<td>109.4 ± 5.0</td>
<td>-1.1 ± 2.6</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.01 ± 0.01</td>
<td>152.2 ± 5.0</td>
<td>-7.9 ± 2.6</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.96 ± 0.01</td>
<td>192.7 ± 6.3</td>
<td>-10.0 ± 3.6</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.92 ± 0.01</td>
<td>255.1 ± 9.8</td>
<td>11.4 ± 5.8</td>
</tr>
<tr>
<td>T100.5</td>
<td>250</td>
<td>1.34 ± 0.02</td>
<td>121.0 ± 4.8</td>
<td>-7.5 ± 2.2</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.17 ± 0.01</td>
<td>184.2 ± 4.0</td>
<td>-9.5 ± 2.2</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>1.11 ± 0.01</td>
<td>230.3 ± 4.4</td>
<td>-13.9 ± 2.7</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.07 ± 0.01</td>
<td>283.8 ± 6.1</td>
<td>-9.9 ± 3.4</td>
</tr>
<tr>
<td>T200.5</td>
<td>250</td>
<td>1.61 ± 0.02</td>
<td>194.9 ± 2.5</td>
<td>-24.8 ± 1.4</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.36 ± 0.01</td>
<td>207.2 ± 5.8</td>
<td>-27.5 ± 3.1</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>1.28 ± 0.01</td>
<td>247.6 ± 6.3</td>
<td>1.9 ± 3.7</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.24 ± 0.01</td>
<td>303.2 ± 3.5</td>
<td>-31.7 ± 3.1</td>
</tr>
<tr>
<td>T25.10</td>
<td>250</td>
<td>0.94 ± 0.02</td>
<td>91.3 ± 4.0</td>
<td>4.7 ± 2.2</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.86 ± 0.01</td>
<td>139.4 ± 4.3</td>
<td>-5.5 ± 2.3</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.85 ± 0.01</td>
<td>167.5 ± 5.8</td>
<td>-5.8 ± 3.4</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.81 ± 0.01</td>
<td>211.7 ± 11.4</td>
<td>-39.5 ± 5.7</td>
</tr>
<tr>
<td>T50.10</td>
<td>250</td>
<td>1.11 ± 0.02</td>
<td>116.7 ± 5.7</td>
<td>6.8 ± 2.9</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.04 ± 0.01</td>
<td>159.2 ± 5.6</td>
<td>-11.2 ± 2.9</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.99 ± 0.01</td>
<td>204.5 ± 6.6</td>
<td>-13.9 ± 3.3</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.92 ± 0.02</td>
<td>253.2 ± 12.6</td>
<td>9.8 ± 7.8</td>
</tr>
<tr>
<td>T100.10</td>
<td>250</td>
<td>1.34 ± 0.02</td>
<td>115.8 ± 3.8</td>
<td>1.3 ± 2.1</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.19 ± 0.01</td>
<td>176.9 ± 3.5</td>
<td>3.6 ± 1.9</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>1.11 ± 0.01</td>
<td>225.6 ± 5.3</td>
<td>6.3 ± 3.2</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.07 ± 0.01</td>
<td>271.6 ± 5.8</td>
<td>6.2 ± 3.3</td>
</tr>
<tr>
<td>T200.10</td>
<td>250</td>
<td>1.55 ± 0.03</td>
<td>146.4 ± 9.0</td>
<td>-3.5 ± 4.5</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.38 ± 0.01</td>
<td>216.4 ± 3.4</td>
<td>-0.3 ± 1.8</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>1.30 ± 0.01</td>
<td>262.9 ± 4.1</td>
<td>1.5 ± 2.3</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.25 ± 0.01</td>
<td>305.8 ± 5.5</td>
<td>3.6 ± 3.2</td>
</tr>
<tr>
<td>T25.20</td>
<td>250</td>
<td>0.91 ± 0.01</td>
<td>128.5 ± 6.4</td>
<td>5.3 ± 2.5</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.85 ± 0.02</td>
<td>160.6 ± 9.8</td>
<td>-4.0 ± 5.0</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.83 ± 0.01</td>
<td>164.0 ± 7.0</td>
<td>-7.1 ± 4.2</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.79 ± 0.01</td>
<td>225.8 ± 11.7</td>
<td>-14.4 ± 6.6</td>
</tr>
<tr>
<td>T50.20</td>
<td>250</td>
<td>1.09 ± 0.02</td>
<td>138.7 ± 7.6</td>
<td>-4.8 ± 3.5</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.00 ± 0.02</td>
<td>181.2 ± 9.3</td>
<td>-9.6 ± 5.1</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>0.95 ± 0.01</td>
<td>211.3 ± 8.2</td>
<td>-12.3 ± 4.2</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.93 ± 0.01</td>
<td>258.8 ± 8.7</td>
<td>-14.6 ± 4.3</td>
</tr>
<tr>
<td>T100.20</td>
<td>250</td>
<td>1.37 ± 0.01</td>
<td>136.2 ± 4.5</td>
<td>8.5 ± 1.9</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.18 ± 0.01</td>
<td>197.9 ± 5.1</td>
<td>11.0 ± 2.6</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>1.10 ± 0.01</td>
<td>259.6 ± 7.3</td>
<td>5.2 ± 4.0</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.06 ± 0.01</td>
<td>278.5 ± 7.3</td>
<td>5.2 ± 4.1</td>
</tr>
<tr>
<td>T200.20</td>
<td>250</td>
<td>1.59 ± 0.01</td>
<td>127.3 ± 2.8</td>
<td>1.8 ± 1.4</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>1.41 ± 0.01</td>
<td>188.3 ± 3.8</td>
<td>5.3 ± 2.1</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>1.29 ± 0.01</td>
<td>186.8 ± 3.9</td>
<td>3.7 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>1.15 ± 0.01</td>
<td>320.9 ± 6.7</td>
<td>-8.3 ± 3.6</td>
</tr>
</tbody>
</table>
F Momentum flux in the Bubble Plume

The momentum flux at the nozzle exit $M_0$ and the momentum flux generated by the change in buoyancy $M_B$ are considered to estimate the momentum flux in the bubble plume $M$.

\[ M(z) = M_0 + M_B(z) \]  \hspace{1cm} (F.23)

The momentum at the nozzle exit is given by the air mass flow $m_a$ and the velocity at the nozzle exit $U_0$:

\[ M_0 = m_a U_0 \]  \hspace{1cm} (F.24)

$M_B$ increases with distance from the nozzle due to decompression of the bubbles while they rise in the pool:

\[ dM_B(z) = \int_{0}^{\infty} 2\pi \rho w \int_{0}^{r} \rho g \, dr \, dz \]  \hspace{1cm} (F.25)

The integral represents the average void fraction in the cross section that can be approximated with $m_a$ and an average bubble rise velocity $\langle u_b \rangle_A$.

\[ 2\pi \int_{0}^{\infty} \rho g \, dr = A \rho \langle u_b \rangle_A \approx \frac{m_a}{\langle u_b \rangle_A \rho} \]  \hspace{1cm} (F.26)

The air density in a cross-section can be expressed as a function of $z$ with $H_0 = \frac{p_0}{\rho w g}$:

\[ \rho_a = \rho_{a,0} \left(1 + \frac{H - z}{H_0}\right) = \rho_a \rho_{w,0} (H_0 + H - z) \]  \hspace{1cm} (F.27)

Inserting (F.26) and (F.27) into (F.25) gives:

\[ M_B(z) = \frac{p_0}{p_{a,0} \langle u_b \rangle_A} \int_{0}^{z} \frac{1}{H_0 + H - z} \, dz = -\frac{p_0}{p_{a,0} \langle u_b \rangle_A} \frac{m_a}{\langle u_b \rangle_A} \ln \left(1 - \frac{z}{H_0 + H}\right) \]  \hspace{1cm} (F.28)

The total momentum flux along the pool centerline can be estimated with (F.24) and (F.28):

\[ M(z) = m_a U_0 - \frac{p_0}{p_{a,0} \langle u_b \rangle_A} \frac{m_a}{\langle u_b \rangle_A} \ln \left(1 - \frac{z}{H_0 + H}\right) \]  \hspace{1cm} (F.29)

Calculating the ratio between $M_0$ and $M_B$ results in

\[ \frac{M_0}{M_B} \approx U_0 \langle u_b \rangle_A \frac{\rho_{a,0}}{p_0} \frac{H_0 + H}{z} \]  \hspace{1cm} (F.30)

when $\ln \left(1 - \frac{z}{H_0 + H}\right)$ is approximated with $-\frac{z}{H_0 + H}$ (F.30).
List of Figures

1.1 Schematic diagram of a two-phase flow investigated here. .......................... 2
2.1 Characteristic parameters to describe an axially symmetric bubble plume consist¬
ing of a bubble core and entrained water flow. ........................................... 12
3.1 Working principle of the optical sensor. ................................................... 24
3.2 Double optical sensor from RBI. .......................................................... 24
3.3 Typical bubble signal, acquired with the AD-converter. .......................... 26
3.4 Signal distributions around the water and the air level used to determine $E_{\text{threshold}}$. .................................................. 27
3.5 Sensor signal demonstrating the difference between the simple and modified threshold method. .................................................. 28
3.6 Void fraction measurements with the DOS in the pipe compared to pressure measurements and calculations with the drift flux model. ...................... 30
3.7 Relevant times from up- and downstream signals for time of flight method. 31
3.8 Volumetric air flow rate determined with the DOS compared to the volumetric air flow meter measurements. ........................................... 33
3.9 Wedge-shaped film sensor. ................................................................. 34
3.10 Schematic of the tow tank for HFA calibration (top view). .................... 36
3.11 HFA calibration data and comparison between Kings Law and fourth-order polynomial. ..................................................... 37
3.12 Deviation of the calibration curves from the average curve. ................. 38
3.13 Velocity components seen by the HFA sensor in the pool. ...................... 39
3.14 Investigation of the orientation effect in the tow tank. Left and right hand sketches: Sensor element in horizontal and vertical position, respectively. 40
3.15 Measured $U_{\text{eff}}$ and computed $u_w$ in the tow tank. ........................... 41
3.16 Comparison between $U_{\text{eff}}$ and $u_w$. .............................................. 42
3.17 HFA output signal from wedge-shaped sensor and the phase separation method to identify water signals. ........................................ 43
3.18 Dependency of the calculated mean water velocity on the signal and slope thresholds. ..................................................... 45
4.1 Setup for the pool tests. ........................................................................ 47
4.2 Tank with observation windows at the front. ......................................... 48
4.3 Sensors and bubbles. From left to right: Hot-film anemometer, thermocouple and double optical sensor. ................................................ 49
4.4 Experimental procedure for the pool tests. ............................................ 50
5.1 Bubble plume oscillation and aspect ratio. ............................................ 54
5.2 Photograph of the ZFE: T50.10. ......................................................... 55
5.3 Photograph of the ZFE: T200.10. ....................................................... 55
5.4 Photograph of the ZEF: T50.10. .......................................................... 59
5.5 Photograph of the ZEF: T200.10. ....................................................... 59
5.6 Radial void fraction profiles for T100.10. ............................................ 61
5.7 Self-similarity of the void fraction profiles. ........................................... 62
<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.8</td>
<td>Variation of the void fraction in time at a given position for two different air flow rates.</td>
<td>63</td>
</tr>
<tr>
<td>5.9</td>
<td>Bubble core widths versus distance from the nozzle.</td>
<td>64</td>
</tr>
<tr>
<td>5.10</td>
<td>Radial void fraction profiles for T25.20.</td>
<td>66</td>
</tr>
<tr>
<td>5.11</td>
<td>Expansion factor and virtual origin of the bubble core.</td>
<td>67</td>
</tr>
<tr>
<td>5.12</td>
<td>Centerline void fraction profiles and radial profile maxima.</td>
<td>68</td>
</tr>
<tr>
<td>5.13</td>
<td>Centerline void fraction profiles TX.10 as a function of z and $z/z_0$.</td>
<td>70</td>
</tr>
<tr>
<td>5.14</td>
<td>Correlation between dimensionless length of the ZFE and modified Froude number.</td>
<td>72</td>
</tr>
<tr>
<td>5.15</td>
<td>Radial bubble rise velocity profiles for T100.10</td>
<td>74</td>
</tr>
<tr>
<td>5.16</td>
<td>Expansion factor and virtual origin to calculate $b_{ub}$ with (5.20).</td>
<td>75</td>
</tr>
<tr>
<td>5.17</td>
<td>Centerline bubble rise velocity profiles and radial profile maxima.</td>
<td>77</td>
</tr>
<tr>
<td>5.18</td>
<td>Dimensionless centerline bubble rise velocity profiles and correlations (5.26) and (5.27).</td>
<td>79</td>
</tr>
<tr>
<td>5.19</td>
<td>Radial profiles of the axial water velocity profiles for T100.10.</td>
<td>81</td>
</tr>
<tr>
<td>5.20</td>
<td>Self-similarity of the water velocity profiles.</td>
<td>82</td>
</tr>
<tr>
<td>5.21</td>
<td>Width of the entrained water flow versus distance from the nozzle.</td>
<td>84</td>
</tr>
<tr>
<td>5.22</td>
<td>Expansion factor and virtual origin of the entrained water flow.</td>
<td>84</td>
</tr>
<tr>
<td>5.23</td>
<td>Centerline water velocity profiles and radial profile maxima.</td>
<td>85</td>
</tr>
<tr>
<td>5.24</td>
<td>Dimensionless centerline water velocity profiles and correlations (5.35) and (5.36).</td>
<td>86</td>
</tr>
<tr>
<td>5.25</td>
<td>Air bubble rise velocity in pure stagnant water as a function of the bubble size.</td>
<td>88</td>
</tr>
<tr>
<td>5.26</td>
<td>Centerline profiles of relative phase velocity.</td>
<td>89</td>
</tr>
<tr>
<td>5.27</td>
<td>Radial profiles of relative phase velocity profiles for T100.10.</td>
<td>90</td>
</tr>
<tr>
<td>5.28</td>
<td>Water and bubble rise velocity measurements near a bubble swarm.</td>
<td>91</td>
</tr>
<tr>
<td>5.29</td>
<td>Bubble chord length PDF at different distances from the nozzle for T100.10 together with lognormal distribution.</td>
<td>93</td>
</tr>
<tr>
<td>5.30</td>
<td>Lognormal distribution parameters of the chord length distributions versus distance from the nozzle.</td>
<td>94</td>
</tr>
<tr>
<td>5.31</td>
<td>Bubble core width versus distance from the nozzle for TX.10* in comparison with the corresponding tests with a pool depth of 2 m.</td>
<td>96</td>
</tr>
<tr>
<td>5.32</td>
<td>Dimensionless centerline void fraction profiles for TX.10* in comparison with (5.16).</td>
<td>96</td>
</tr>
<tr>
<td>5.33</td>
<td>Dimensionless centerline bubble rise velocity profiles for TX.10* in comparison with (5.27).</td>
<td>97</td>
</tr>
<tr>
<td>5.34</td>
<td>Volumetric air flux profiles: Measurement points and the continuous line represents the calculation with (5.46).</td>
<td>99</td>
</tr>
<tr>
<td>5.35</td>
<td>Volumetric air flow rate as regulated by the air flow controller and as measured by integrating the DOS profiles.</td>
<td>100</td>
</tr>
<tr>
<td>6.1</td>
<td>$\lambda$ versus $z$, experimental data and correlation (6.3).</td>
<td>105</td>
</tr>
<tr>
<td>6.2</td>
<td>Volumetric water flow rate versus $z$.</td>
<td>107</td>
</tr>
<tr>
<td>6.3</td>
<td>Entrainment coefficient versus distance from the nozzle.</td>
<td>108</td>
</tr>
<tr>
<td>6.4</td>
<td>Entrainment coefficient listed in table 6.1 and correlation (6.11).</td>
<td>109</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>6.5</td>
<td>Axial water velocity component along the pool centerline: experimental data and computation with (2.21).</td>
<td>111</td>
</tr>
<tr>
<td>6.6</td>
<td>Expansion of the entrained water flow: experimental data and computation with (2.20).</td>
<td>112</td>
</tr>
<tr>
<td>6.7</td>
<td>Void fraction along the pool centerline: experimental data and computation with (2.13).</td>
<td>113</td>
</tr>
<tr>
<td>A.1</td>
<td>Two-phase Loop.</td>
<td>iv</td>
</tr>
<tr>
<td>A.2</td>
<td>Influence of the threshold setting ( \Lambda ) on void fraction measurements.</td>
<td>vi</td>
</tr>
<tr>
<td>A.3</td>
<td>Influence of the correction time on the measurement of the total number of bubbles.</td>
<td>vii</td>
</tr>
<tr>
<td>A.4</td>
<td>Radial void fraction profiles in the pipe.</td>
<td>viii</td>
</tr>
<tr>
<td>A.5</td>
<td>Differential pressure measurements along the upper test section.</td>
<td>ix</td>
</tr>
<tr>
<td>A.6</td>
<td>Radial bubble rise velocity profiles in the pipe.</td>
<td>x</td>
</tr>
<tr>
<td>A.7</td>
<td>Two-phase loop for DOS orientation tests.</td>
<td>xi</td>
</tr>
<tr>
<td>A.8</td>
<td>Test matrix for DOS orientation tests.</td>
<td>xii</td>
</tr>
<tr>
<td>A.9</td>
<td>Void fraction and bubble frequency measurements for horizontal and vertical orientations (indices ( \text{vert.} ) and ( \text{hor.} ) stand for the sensor orientation).</td>
<td>xiii</td>
</tr>
<tr>
<td>A.10</td>
<td>Bubble residence time and signal rise time.</td>
<td>xiii</td>
</tr>
<tr>
<td>A.11</td>
<td>PDFs of the bubble residence time for horizontally and vertically oriented DOS.</td>
<td>xiv</td>
</tr>
<tr>
<td>A.12</td>
<td>PDFs of the bubble rise signals for horizontally and vertically oriented DOS.</td>
<td>xiv</td>
</tr>
<tr>
<td>B.13</td>
<td>Tow tank (top view) for HFA calibration.</td>
<td>xvi</td>
</tr>
<tr>
<td>B.14</td>
<td>Rotation of the HFA in the tow tank (top view).</td>
<td>xvi</td>
</tr>
<tr>
<td>B.15</td>
<td>Correlated tow velocity profile and HFA signal.</td>
<td>xviii</td>
</tr>
<tr>
<td>B.16</td>
<td>Tests for different sensor orientations in the tow tank (yaw-angle effect) and corresponding HFA velocity readings for ( \alpha = 0 ).</td>
<td>xx</td>
</tr>
<tr>
<td>B.17</td>
<td>Tests for different sensor orientations in the tow tank (pitch-angle effect).</td>
<td>xx</td>
</tr>
<tr>
<td>C.18</td>
<td>Water loop and air supply connected to the large pool and the tow tank (instrument functions: see table C.2).</td>
<td>xxi</td>
</tr>
<tr>
<td>C.19</td>
<td>Installations within the tank.</td>
<td>xxiii</td>
</tr>
<tr>
<td>C.20</td>
<td>Cutaway A-A in figure C.19: Top view of the sensor traversing system.</td>
<td>xxiv</td>
</tr>
<tr>
<td>C.21</td>
<td>Schematic for the process controlling and data acquisition.</td>
<td>xxv</td>
</tr>
<tr>
<td>D.22</td>
<td>Time averaging of void fraction and bubble rise velocity.</td>
<td>xxvii</td>
</tr>
<tr>
<td>D.23</td>
<td>Time averaging of water velocity.</td>
<td>xxviii</td>
</tr>
<tr>
<td>D.24</td>
<td>Relative change in the bubble residence times for different air flow rates.</td>
<td>xxx</td>
</tr>
<tr>
<td>D.25</td>
<td>Bubble rise velocity PDFs for different air flow rates.</td>
<td>xxxi</td>
</tr>
</tbody>
</table>
List of Tables

1.1 Pool experiments listed in chronological publication order ($H$: pool depth, $D$: pool diameter $u_b$: bubble rise velocity, $e$: void fraction, $u_w$: water velocity). 5
1.2 Empirical parameters for the bubble plume model. 6
3.1 Instruments for local two-phase flow measurements. 18
3.2 Nomenclature for phase separation method. 44
5.1 Mean velocities at the nozzle exit for the pool tests. 51
5.2 Distances $z$ from the nozzle for the radial profiles for tests with pool depths of 2 and 3 m. 52
5.3 Identification of the pool tests. 52
5.4 Values of $z_0$ and $\xi$ for TX.5 and TX.20. 70
6.1 Linear fits for $\bar{V}_w(z)$ and mean entrainment coefficients. 107
A.1 Test matrix for the DOS calibration. v
C.2 Instruments in the water loop and air supply. xxii
E.3 Coefficients of the radial water velocity profiles (Gaussian curves). xxxii
E.4 Coefficients of the radial void fraction profiles (Gaussian curve). xxxiii
E.5 Coefficients of the radial bubble rise velocity profiles (Gaussian curve). xxxiv
Curriculum Vitae

Name: Jens Kubasch

Nationality: German

Birth: January 27, 1971, in Offenbach/Main, Germany

Education:
1977 - 1987 Primary and Secondary School in Obertshausen, Germany
1987 - 1990 Grammar School in Offenbach (Technisches Gymnasium)
Division: electrical engineering
1991 - 1996 Study of Physics
at the Technical University of Darmstadt, Germany
Main interests: energy technologies, experimental physics,
and measurement techniques (nuclear and solid-state physics)
1997 - 2001 Dissertation at PSI and ETH-Zürich, Switzerland

Employment:
2001 - Sensirion, the sensor company (Zürich)
Activities: semiconductors and sensors

Military service:
1990 - 1991 Telecommunications division in Kassel, Germany