Doctoral Thesis

Floer homology and surface diffeomorphisms

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Floer homology and surface diffeomorphisms

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presented by
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Abstract

In this thesis we treat symplectic Floer theory in two dimensions. According to Seidel, this theory assigns a module to each mapping class of a closed connected oriented two-manifold of genus bigger than one. The notion of monotone symplectomorphisms is thereby central.

We compute this module for algebraically finite mapping classes, i.e. classes which do not have pseudo-Anosov components in the sense of Thurston’s theory of surface diffeomorphisms. The Nielsen-Thurston representative of such a class is shown to be monotone. The formula for the Floer homology is obtained from a topological separation of fixed points and a separation mechanism for Floer connecting orbits.

As an example, we consider the geometric monodromy map of an isolated plane curve singularity. We prove a refined version of the classical theorem of A’Campo and Lê which states that the Lefschetz number of such a map vanishes. Our proof uses, besides the A’Campo-Lê Theorem, the theory of splice diagrams which was developed by Eisenbud and Neumann.

This result has two applications in the realm of Floer theory. First, we think of the Milnor fiber of an isolated plane curve singularity as being contained in a closed oriented two-manifold and consider the mapping class which is obtained by extending the geometric monodromy trivially to the ambient space. In this case, our formula for the Floer homology takes a particularly simple form. Second, we show that the Floer homology of the geometric monodromy in itself, as a mapping class of a two-manifold with boundary, vanishes. This confirms a conjecture of Seidel.

In the second part of this thesis, which is independent of the first one, we prove a series of assertions on subgroups of surface groups, i.e. fundamental groups of closed connected two-manifolds.

Our main theorem states that if the rank of a subgroup of a surface group is smaller than the rank of the surface group, then the subgroup is free. This result is related to the Freiheitssatz, which a classical theorem in combinatorial group theory. Our proof uses the classification of compact two-manifolds and a topological lemma of Epstein.

As an application of this theorem, we prove an assertion about homotopy classes of maps from the torus to closed oriented two-manifolds, which is used in the first part of the thesis.