


Skript zur Vorlesung Anorganische Chemie I

Educational Material**Author(s):**

Nesper, Reinhard Friedrich; [Grütmacher, Hansjörg](#) 

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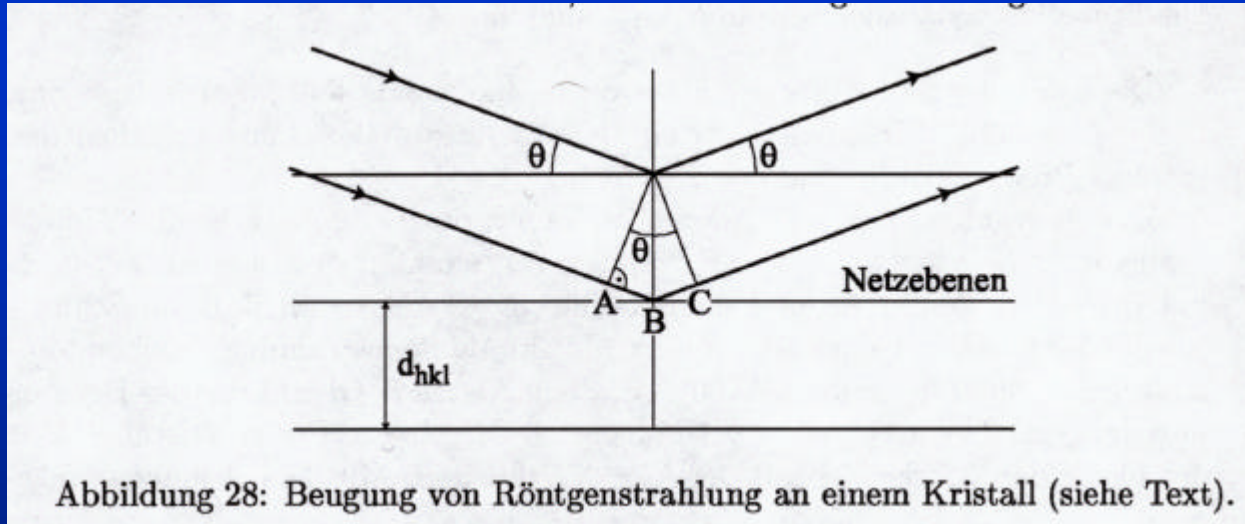
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(X-ray) Diffraction

Some practical aspects of one of the most important tools in solid state sciences

Bragg's Law of Diffraction



constructive interference only, when:

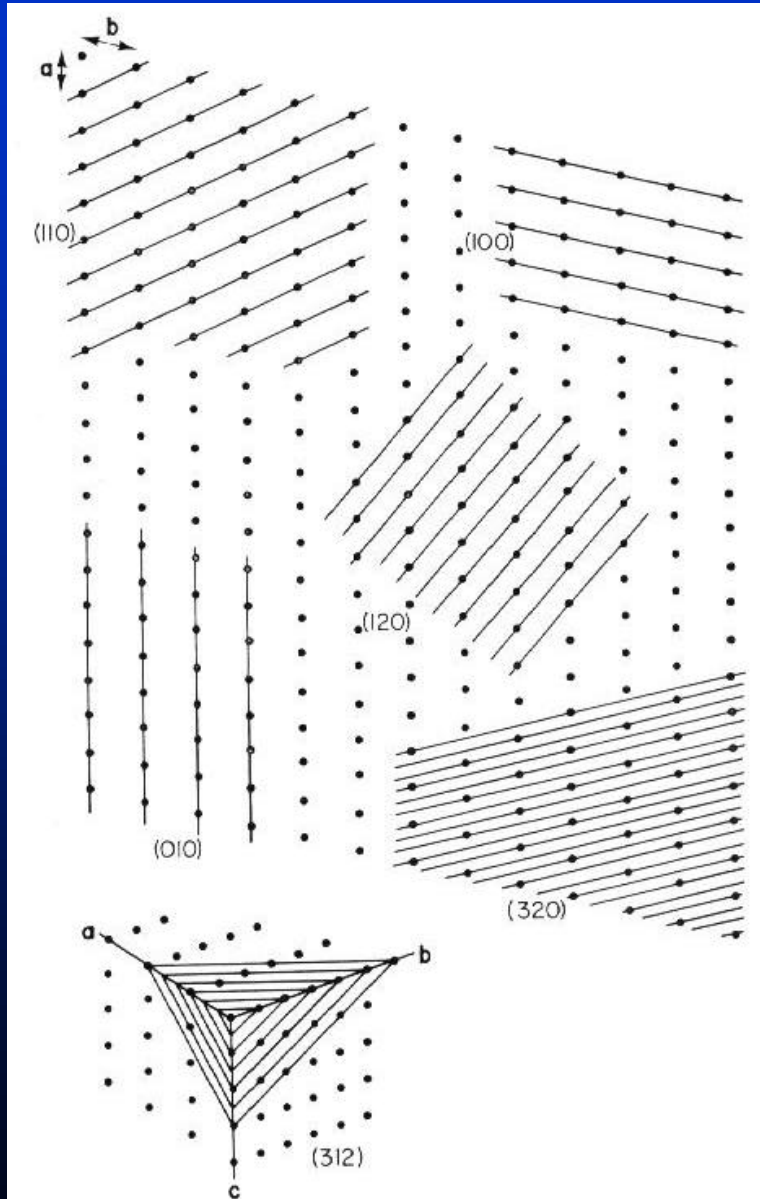
$$\Delta = n \cdot \lambda \quad (\Delta = AB + BC)$$

with:

$$\sin\theta = (\Delta/2)/d$$

$$n \cdot \lambda = 2d \cdot \sin\theta$$

Diffraction from Lattice Planes

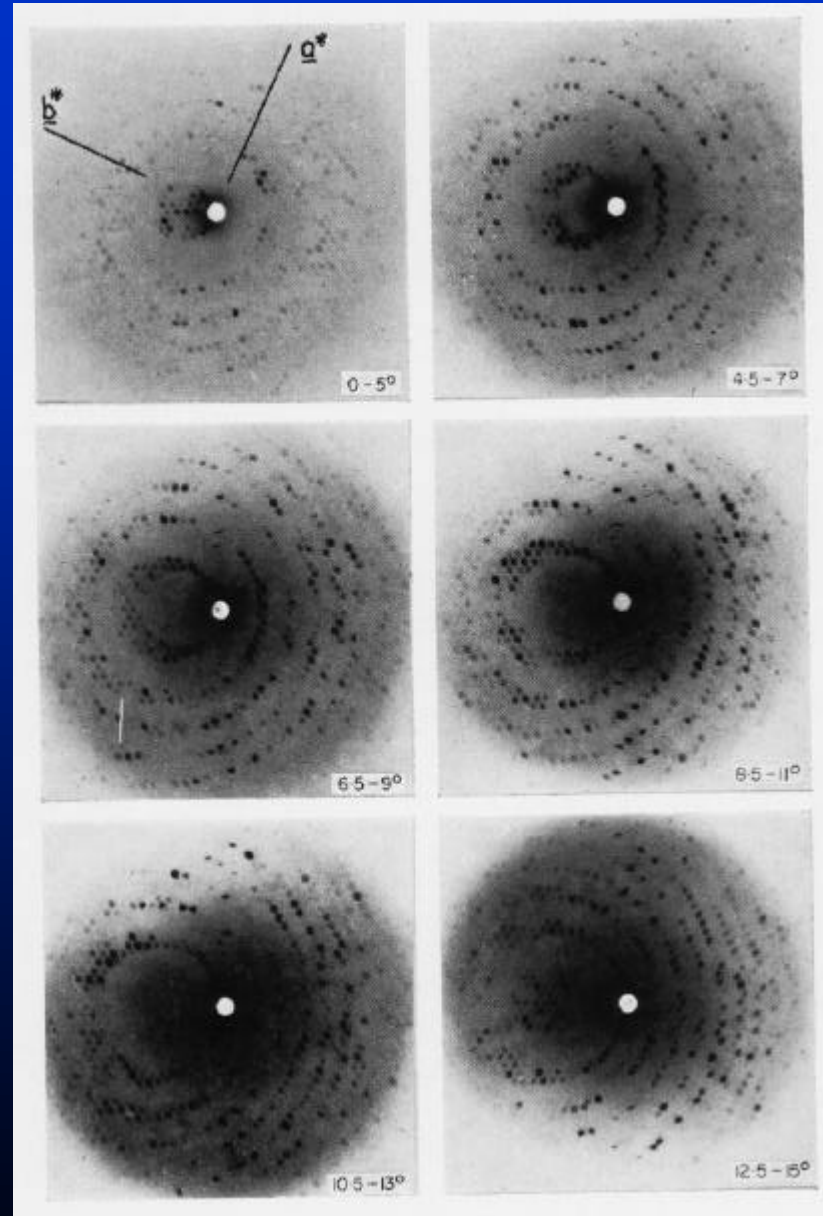
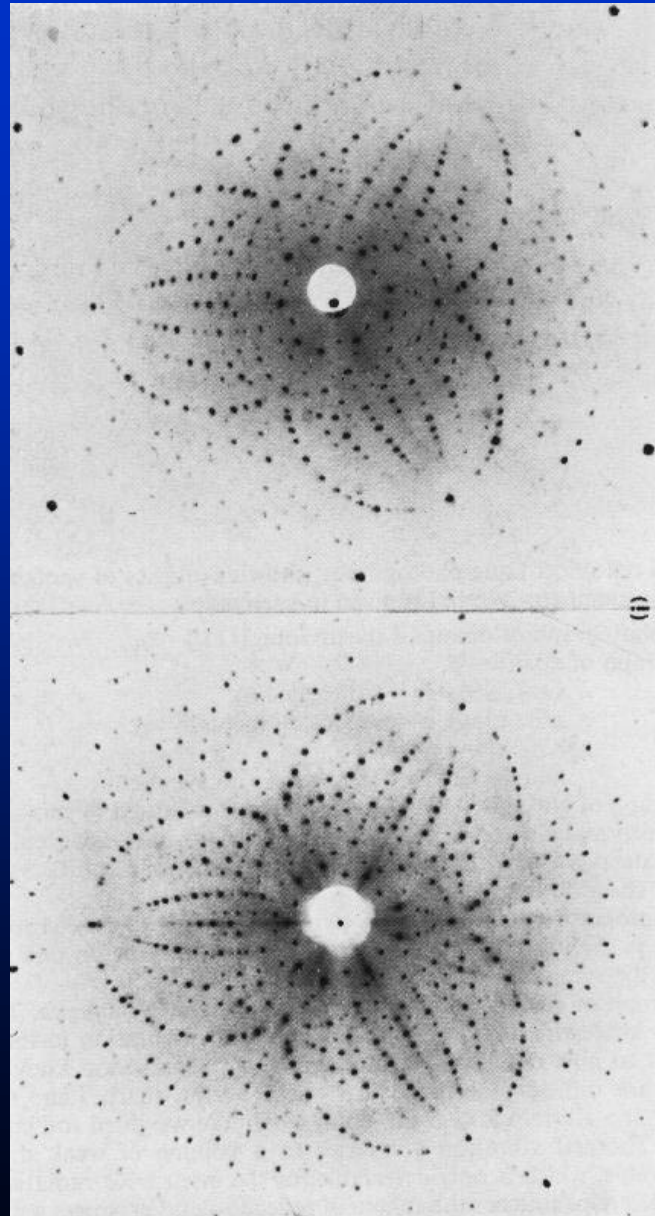


- Each set of planes corresponds to

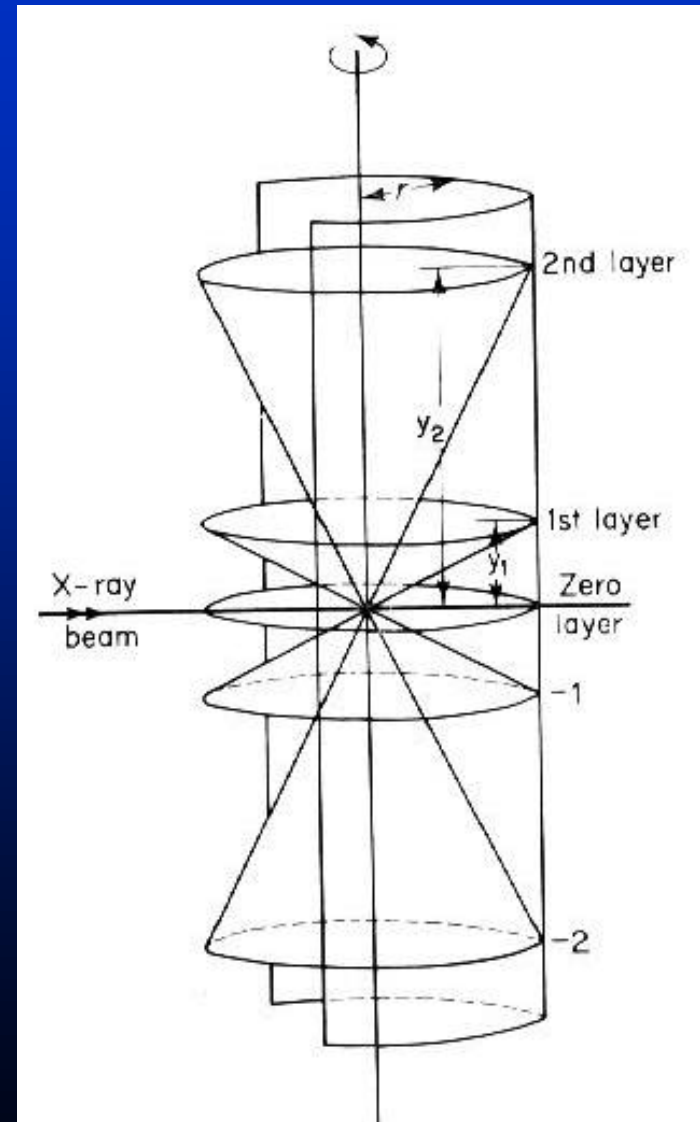
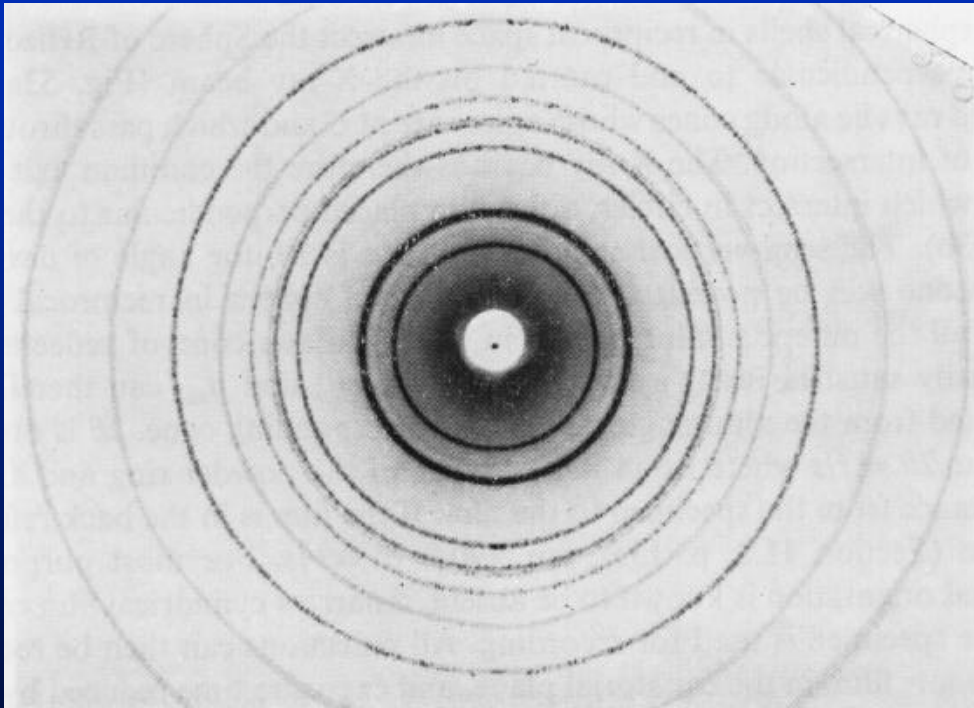
one structure factor

$$S_{hkl}$$

Diffraction from Single Crystals



Diffraction from Powder Crystals



Quadratic Bragg formulas

Tabelle 4: Die quadratischen Braggschen Gleichungen in den 7 Kristallsystemen

Triklin

$$\sin^2 \theta = \frac{\lambda^2}{4} [h^2 a^{*2} + k^2 b^{*2} + l^2 c^{*2} + 2klb^* c^* \cos \alpha^* + 2lhc^* a^* \cos \beta^* + 2hka^* b^* \cos \gamma^*]$$

$$a^* = \frac{1}{V} bc \sin \alpha, \quad \cos \alpha^* = \frac{\cos \beta \cos \gamma - \cos \alpha}{\sin \beta \sin \gamma}$$

$$b^* = \frac{1}{V} ca \sin \beta, \quad \cos \beta^* = \frac{\cos \gamma \cos \alpha - \cos \beta}{\sin \gamma \sin \alpha}$$

$$c^* = \frac{1}{V} ab \sin \gamma, \quad \cos \gamma^* = \frac{\cos \alpha \cos \beta - \cos \gamma}{\sin \alpha \sin \beta}$$

$$V = abc \sqrt{1 + 2 \cos \alpha \cos \beta \cos \gamma - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma}$$

Monoklin

$$\sin^2 \theta = \frac{\lambda^2}{4} \left[\frac{h^2}{a^2 \sin^2 \beta} + \frac{k^2}{b^2} + \frac{l^2}{c^2 \sin^2 \beta} - \frac{2hl \cos \beta}{ac \sin^2 \beta} \right]$$

Orthorhombisch

$$\sin^2 \theta = \frac{\lambda^2}{4} \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2} \right]$$

Tetragonal

$$\sin^2 \theta = \frac{\lambda^2}{4a^2} [h^2 + k^2 + \left(\frac{a}{c}\right)^2 l^2]$$

Hexagonal und trigonal

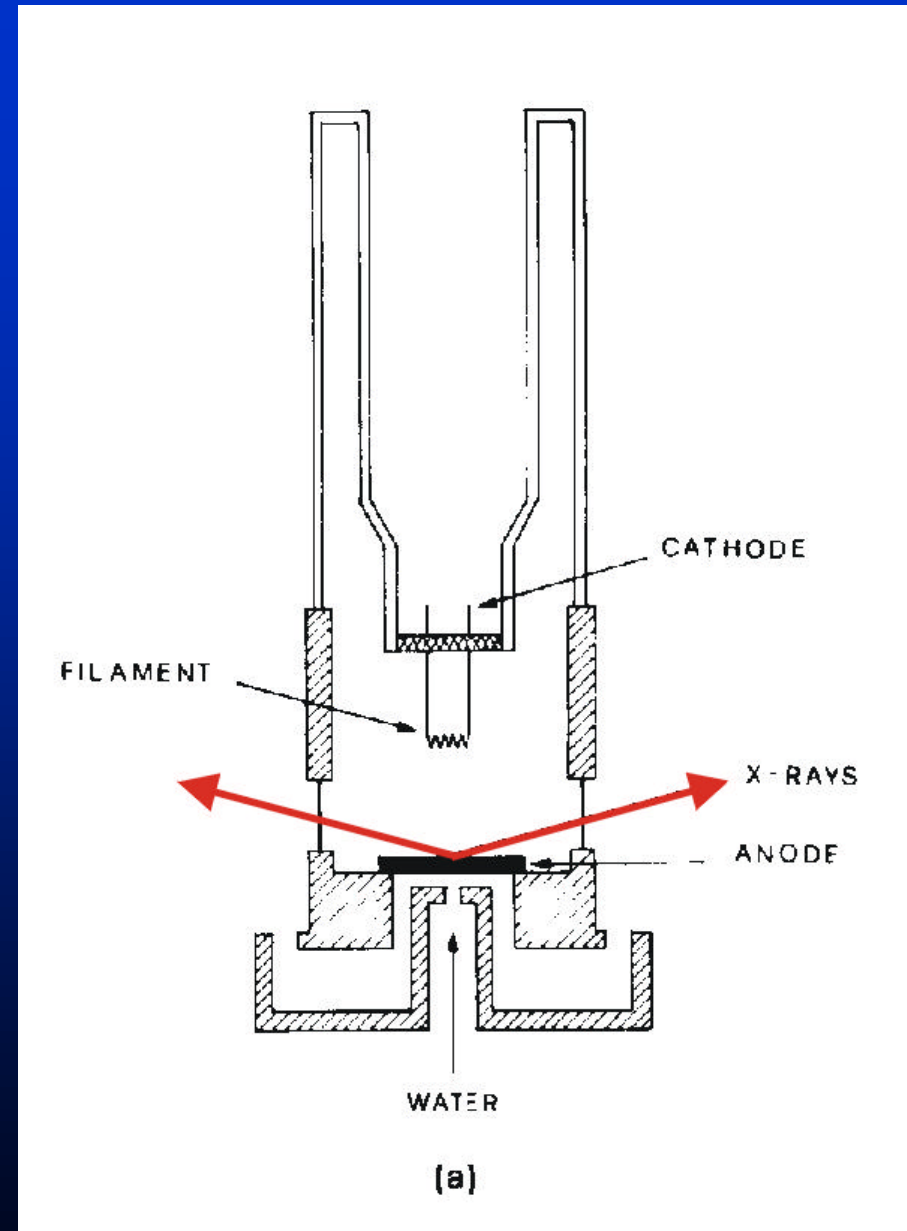
$$\sin^2 \theta = \frac{\lambda^2}{4a^2} \left[\frac{4}{3} (h^2 + k^2 + hk) + \left(\frac{a}{c}\right)^2 l^2 \right]$$

Kubisch

$$\sin^2 \theta = \frac{\lambda^2}{4a^2} [h^2 + k^2 + l^2]$$

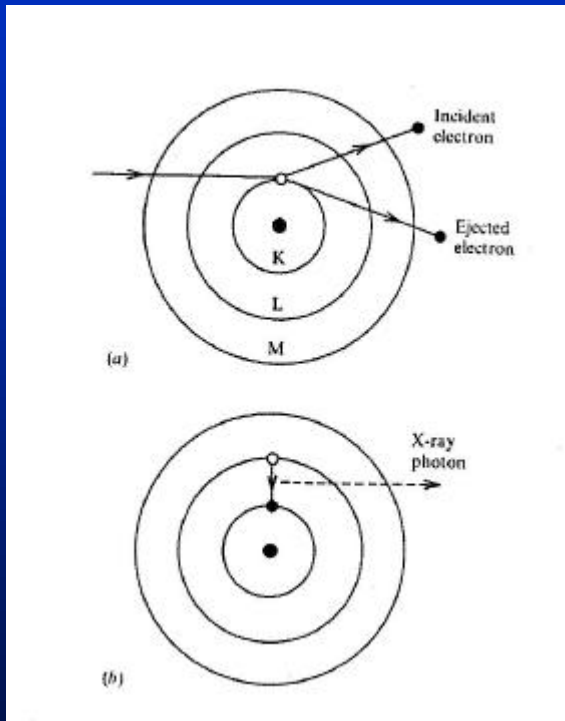
Working Principle of the X-ray tube

- Tungsten wire at 1200-1800°C (about 35mA heating current)
 - High Voltage 20-60 kV
 - max. Power 2.2-3 kW
- Typical operating values for
Cu: 40 kV, 35 mA
Mo: 45 kV, 35 mA



Spectrum of the X-ray tube

Characteristic radiation



Bremstrahlung (white radiation)

$$E_{\max.} = E_0 = e \cdot V_0 \text{ and with } E = (h \cdot c) / \lambda:$$
$$\lambda_{\min} / \text{\AA} = (h \cdot c) / e \cdot V_0 = 12.34 / (V_0 / \text{kV})$$

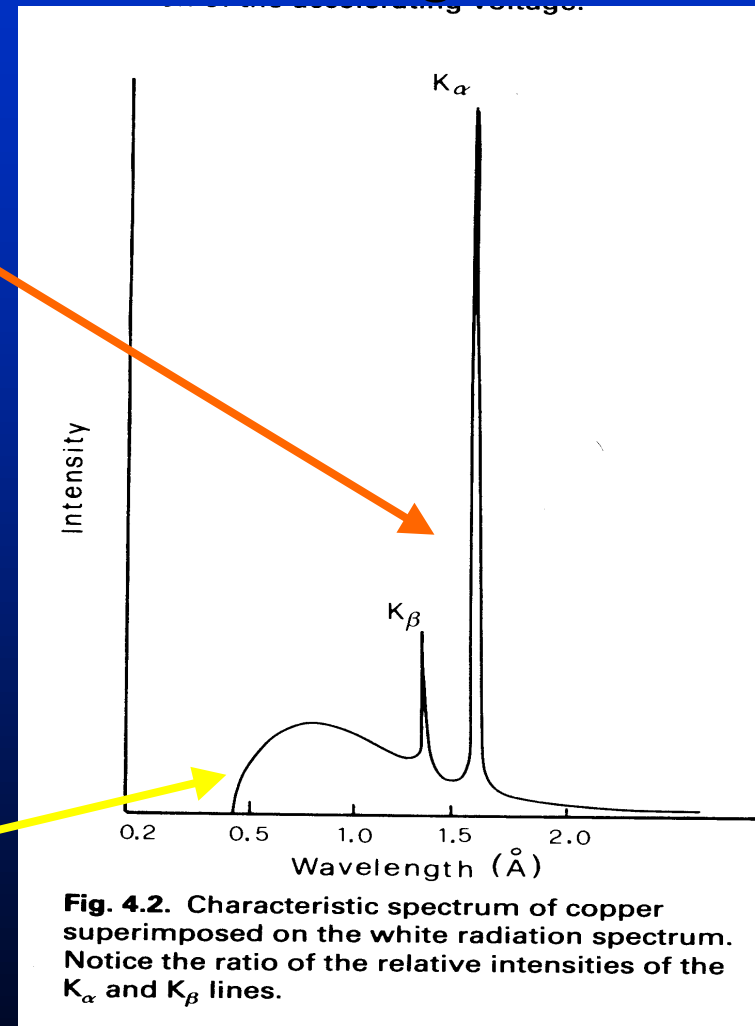
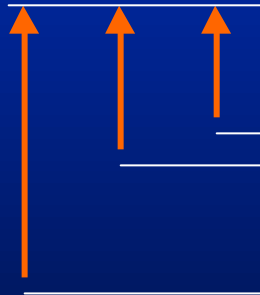


Fig. 4.2. Characteristic spectrum of copper superimposed on the white radiation spectrum. Notice the ratio of the relative intensities of the K α and K β lines.

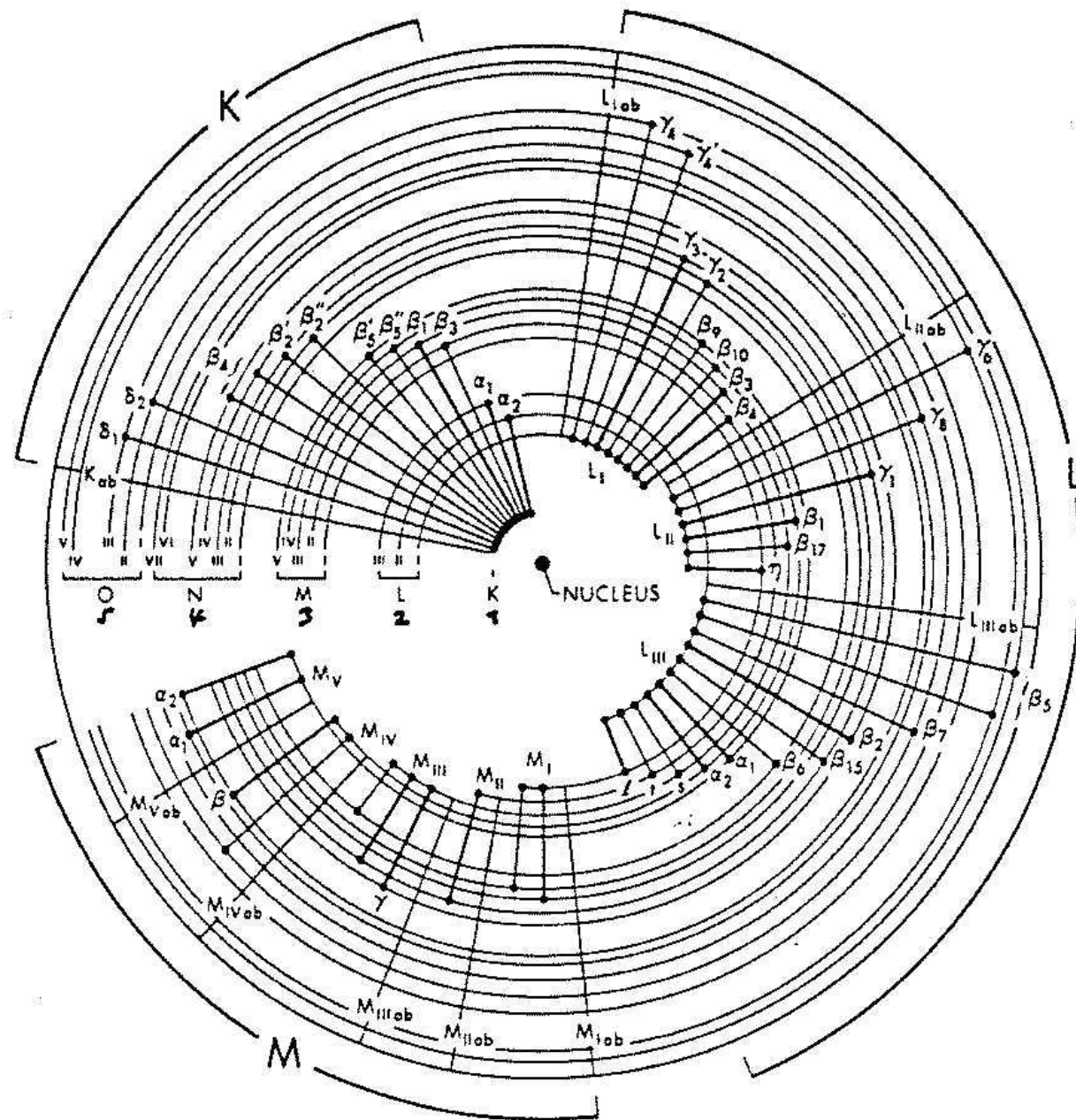
- $n=1,2,3$ (principal quantum number), corresponds to K, L, M... shells
- $l=0, 1, \dots, n-1$ (orbital quantum number)
- $j=|l \pm s|$; $s=1/2$ (spin-orbit coupling)
- $m_j=j, j-1, j-2, \dots, -j$
- Rules: Transition only, when $\Delta l \neq 0$

$K\beta_1$ $K\alpha_2$ $K\alpha_1$



X-ray notation	Quantum numbers				Maximum electron population
	n	l	j	m_j	
K	1	0	$\frac{1}{2}$	$\pm \frac{1}{2}$	2
L_I	2	0	$\frac{1}{2}$	$\pm \frac{1}{2}$	2
L_{II}	2	1	$\frac{1}{2}, \frac{3}{2}$	$\pm \frac{1}{2}, \pm \frac{3}{2}$	2
L_{III}	2	1	$\frac{3}{2}$	$\pm \frac{3}{2}, \pm \frac{1}{2}$	4
M_I	3	0	$\frac{1}{2}$	$\pm \frac{1}{2}$	2
M_{II}	3	1	$\frac{1}{2}, \frac{3}{2}$	$\pm \frac{1}{2}, \pm \frac{3}{2}$	2
M_{III}	3	1	$\frac{3}{2}$	$\pm \frac{3}{2}, \pm \frac{1}{2}$	4
M_{IV}	3	2	$\frac{3}{2}, \frac{5}{2}$	$\pm \frac{3}{2}, \pm \frac{5}{2}$	4
M_V	3	2	$\frac{5}{2}$	$\pm \frac{5}{2}, \pm \frac{3}{2}, \pm \frac{1}{2}$	6
N_I	4	0	$\frac{1}{2}$	$\pm \frac{1}{2}$	2
N_{II}	4	1	$\frac{1}{2}, \frac{3}{2}$	$\pm \frac{1}{2}, \pm \frac{3}{2}$	2
N_{III}	4	1	$\frac{3}{2}$	$\pm \frac{3}{2}, \pm \frac{1}{2}$	4
N_{IV}	4	2	$\frac{3}{2}, \frac{5}{2}$	$\pm \frac{3}{2}, \pm \frac{5}{2}$	4
N_V	4	2	$\frac{5}{2}$	$\pm \frac{5}{2}, \pm \frac{3}{2}, \pm \frac{1}{2}$	6
N_{VI}	4	3	$\frac{5}{2}, \frac{7}{2}$	$\pm \frac{5}{2}, \pm \frac{7}{2}, \pm \frac{3}{2}, \pm \frac{1}{2}$	6
N_{VII}	4	3	$\frac{7}{2}$	$\pm \frac{7}{2}, \pm \frac{5}{2}, \pm \frac{3}{2}, \pm \frac{1}{2}$	8

- Allowed Transitions



Mosley's Law (for multiple electron atoms):

$$1/\lambda = c \cdot (Z - \sigma)^2 \cdot (1/n_1^2 - 1/n_2^2)$$

- Z = atom number
- σ = shielding constant
- n = quantum number

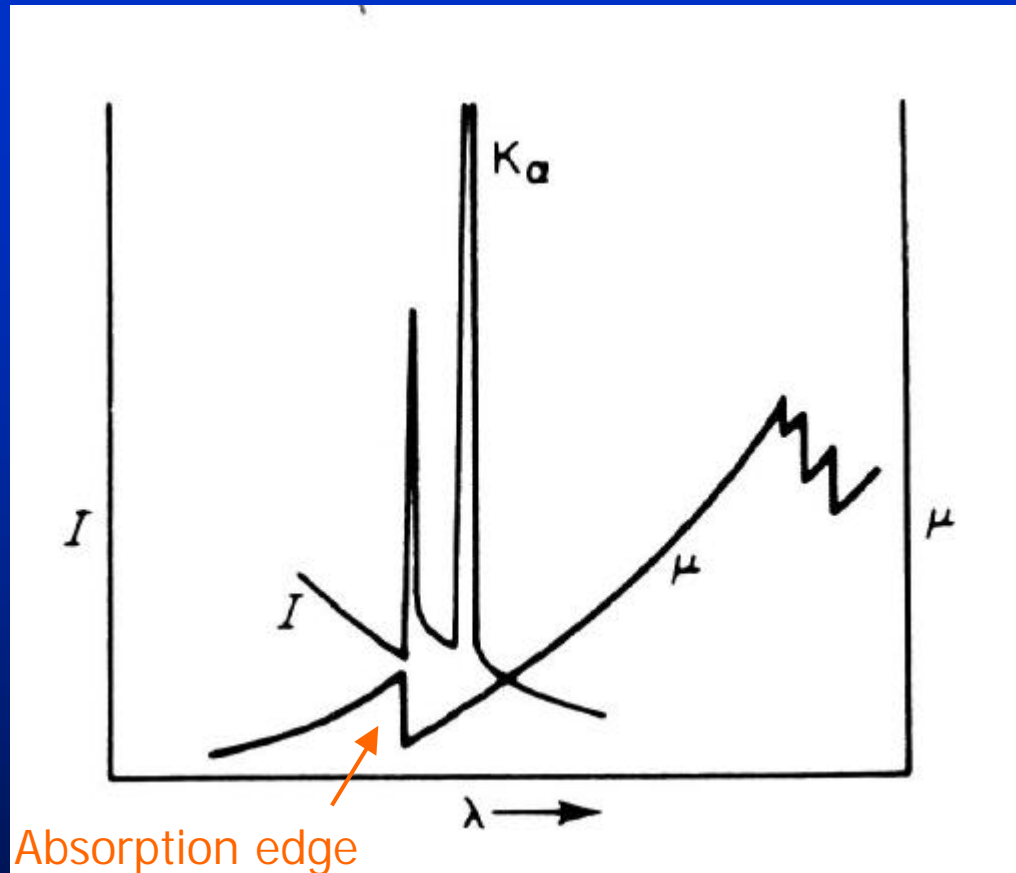
⇒ Decreasing wavelength with increasing Z

Characteristic Wavelengths

in Angstroms (100pm)

Element Symbol	$K\alpha_2$	$K\alpha_1$	$K\beta$	K abs. edge
Cu	1.54433	1.54051	1.39217, 1.38102	1.380
Mo	0.713543	0.70926	0.62099	0.61977
Ag	0.563775	0.559363	0.49701, 0.48701	0.4858
W	0.213813	0.208992	0.17950	0.17837

μ vs. λ



At the absorption edge, the incident X-ray quantum is energetic enough to knock an electron out of the orbital

Monochromatisation of X-rays

- Filters
- Crystal Monochromators

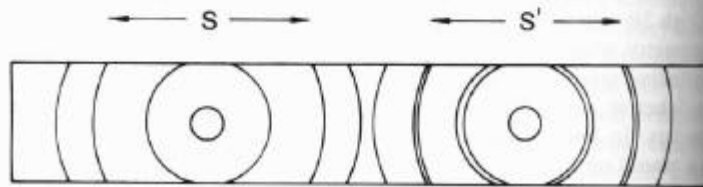
Different Geometries

- Debye-Scherrer
- Bragg-Brentano
- Guinier

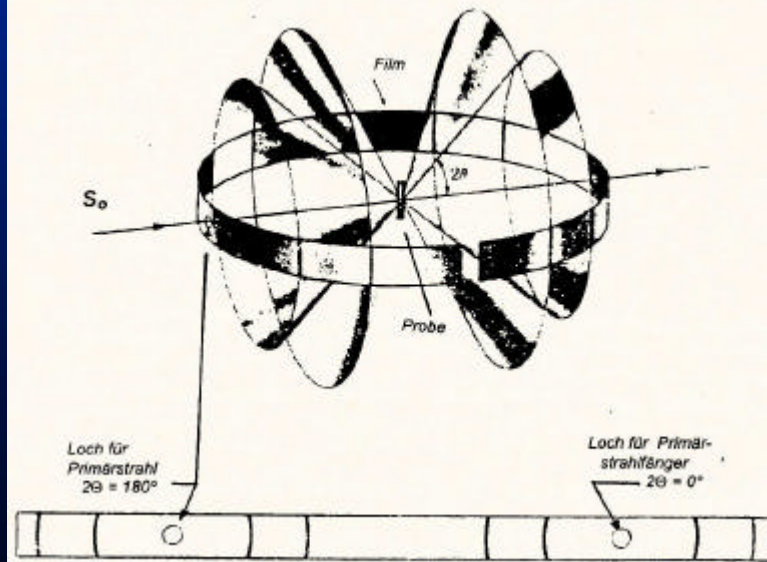
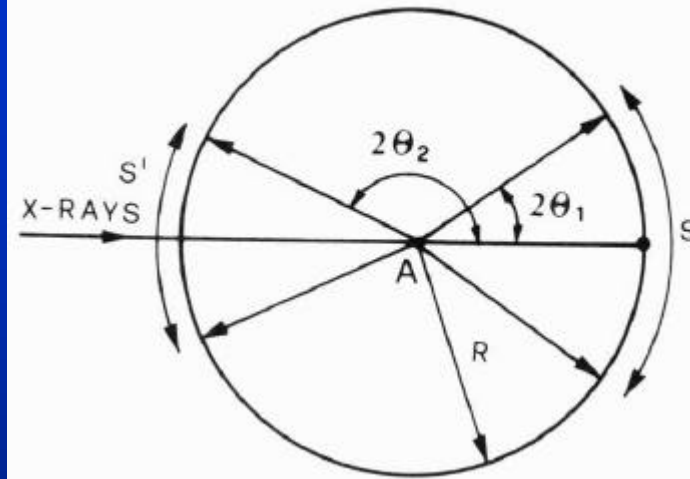
Debye Scherrer



(a)



(b)



Detection of X-rays

- **Film** (Guinier camera, Debye-Scherrer Camera, precession camera)
- **Si(Li) solid state detector** (powder diffractometers)
- **Szintillation counter** (4-circle diffractometer, Stoe powder diffractometer)
- **Position Sensitive Detectors** (Stoe powder diffractometer)
- **Image Plate Detectors** (Stoe IPDS)
- **CCD Detectors** (Bruker SMART system)

Resolution:

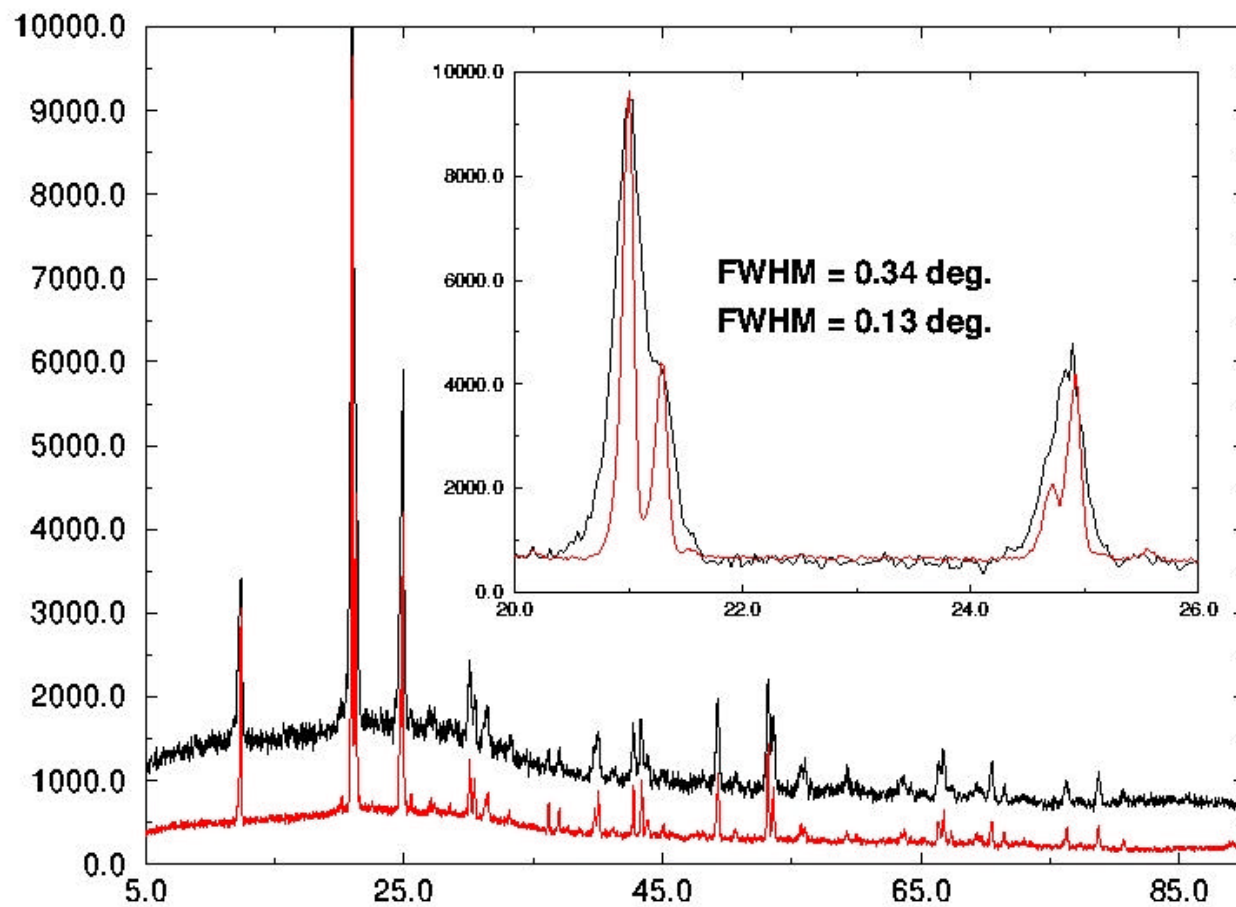


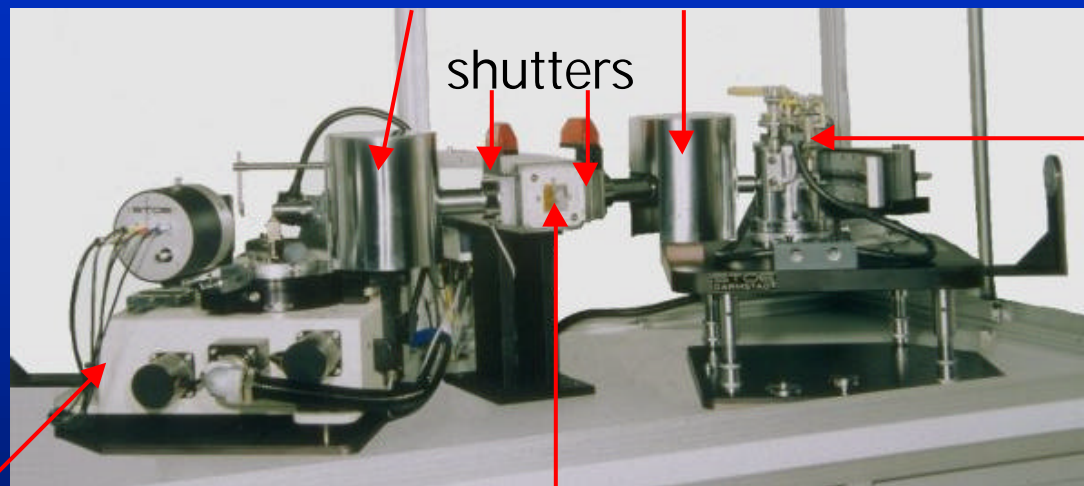
Image plate detectors

- Metal plate with about 18cm diameter, coated with Eu^{2+} doped BaFBr
- X-rays ionize Eu^{2+} to Eu^{3+} and the electrons are trapped in color centers
- Read out process with red laser leads to emission of blue light, when electrons return to ground state
- The blue light is amplified by a photomultiplier and recorded as a pixel image



Setup for a Powder Diffractometer

Ge-monochromators



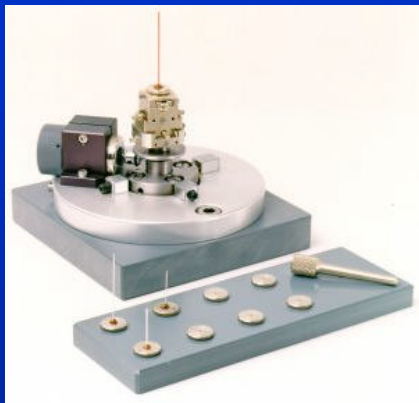
High Temperature Attachment

Goniometer

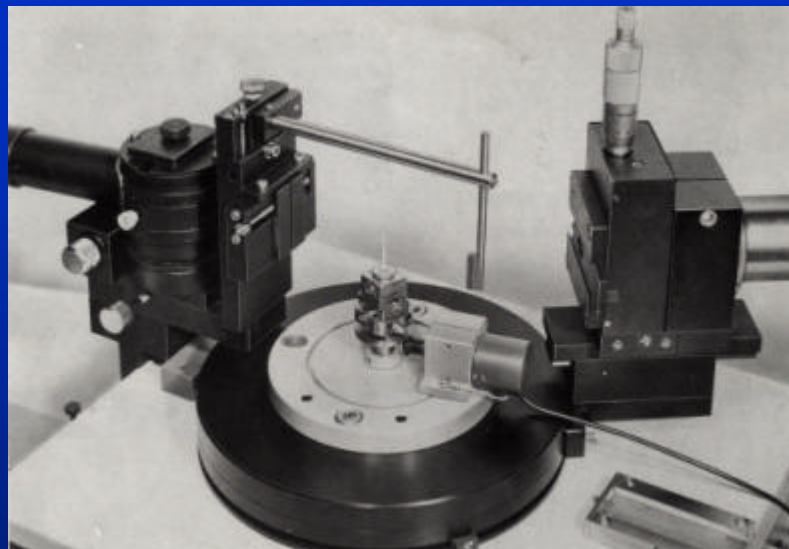
X-ray tube



Different Sample Holders



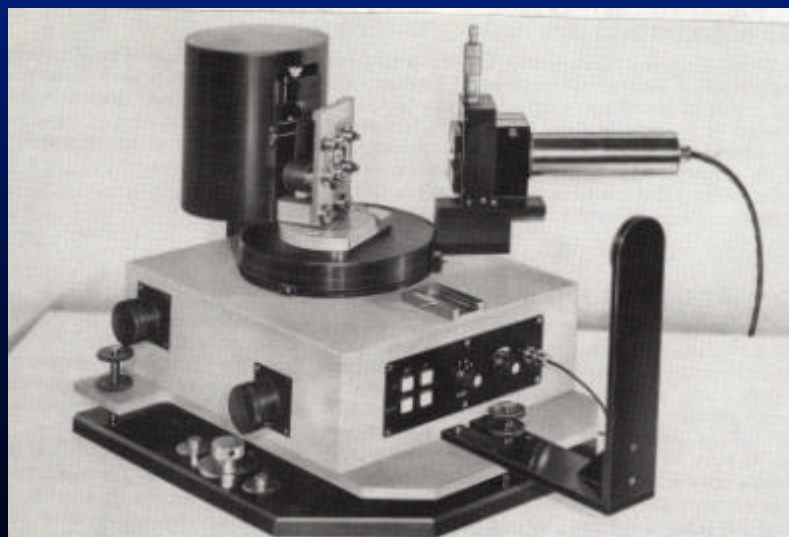
Capillary



Transmission



Reflection



Preparing a sample

Capillary:

For air sensitive samples

Diameter between 0.1 and 1mm, Standard is 0.3 mm

For samples with high absorption 0.1 mm is better suited

Difficulties with soft samples which are not easy to fill in

Transmission sample holder

Good for samples which are not or only moderately air sensitive.

Sample is placed on a Scotch (Tesa) strip and covered with a second strip.

Be sure, that the sample is only on one(!) side and the second is only for protection.

Reflection sample holder

Only for moderately air sensitive samples

Good for or strongly absorbing samples like for example electrodes or thin films on a substrate

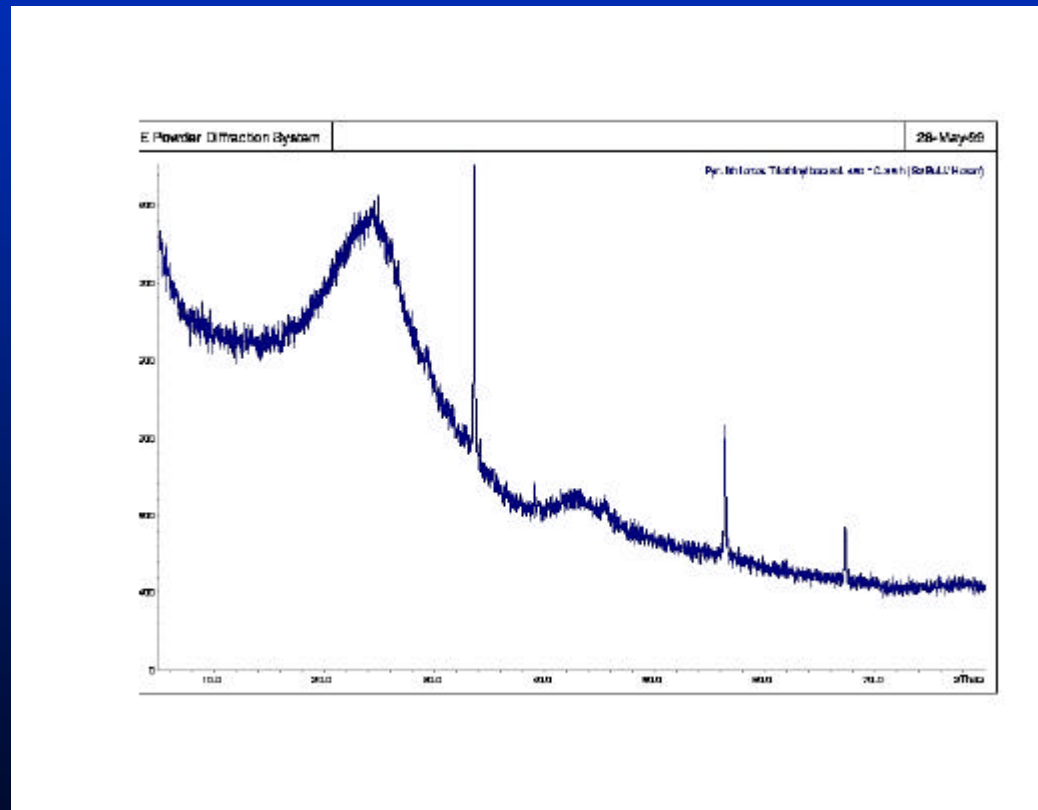
Is used at the moment for in situ electrochemical cell experiments

Cannot be used in connection with the large PSD

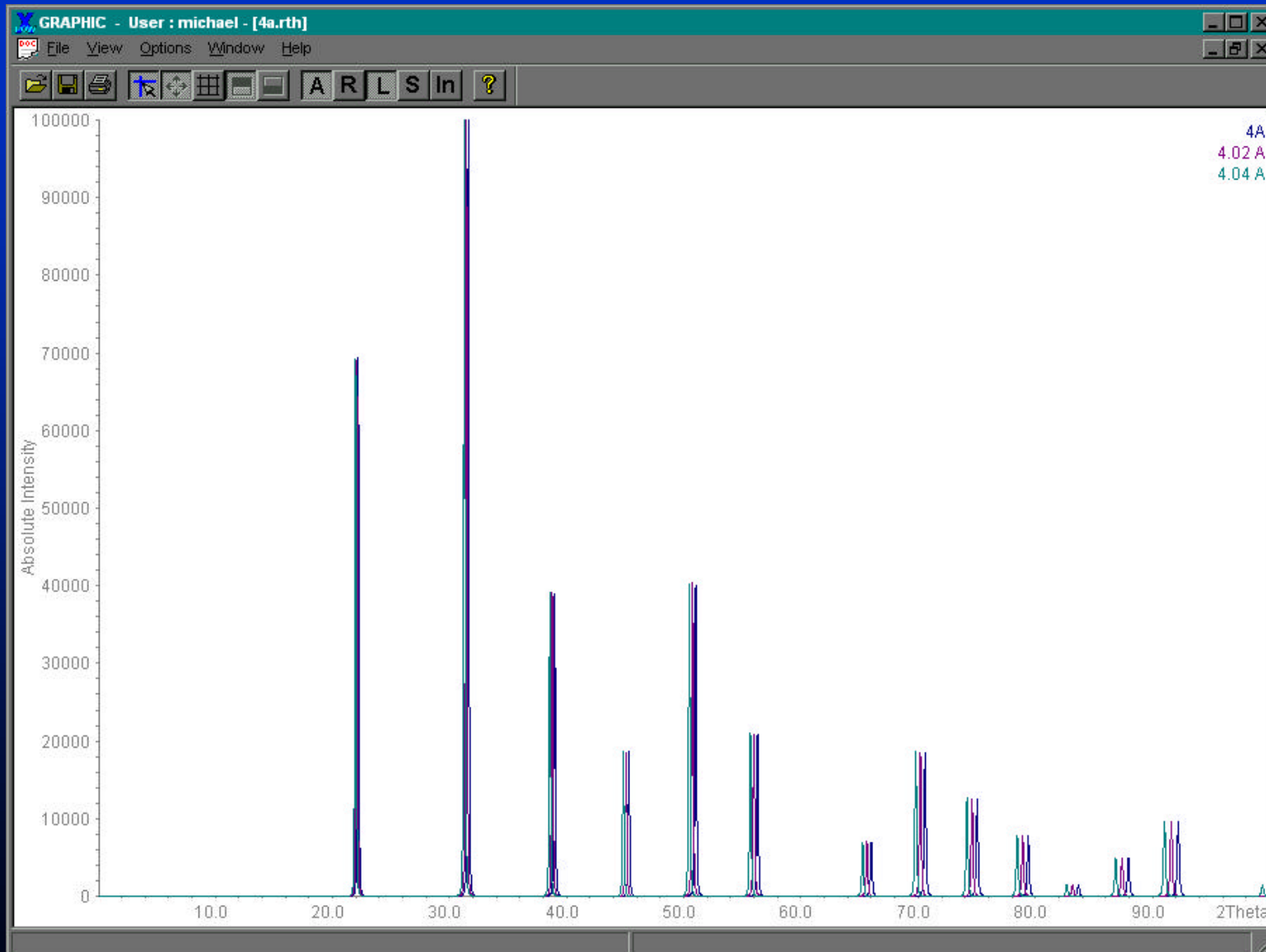
What Information Can We Extract from Diffraction Experiments?

- Determination of known phases
- Crystallinity
- Determination of lattice constants
- Structure solution

Crystalline and Amorphous Phase together:



Effect of a Change of the Lattice Constants

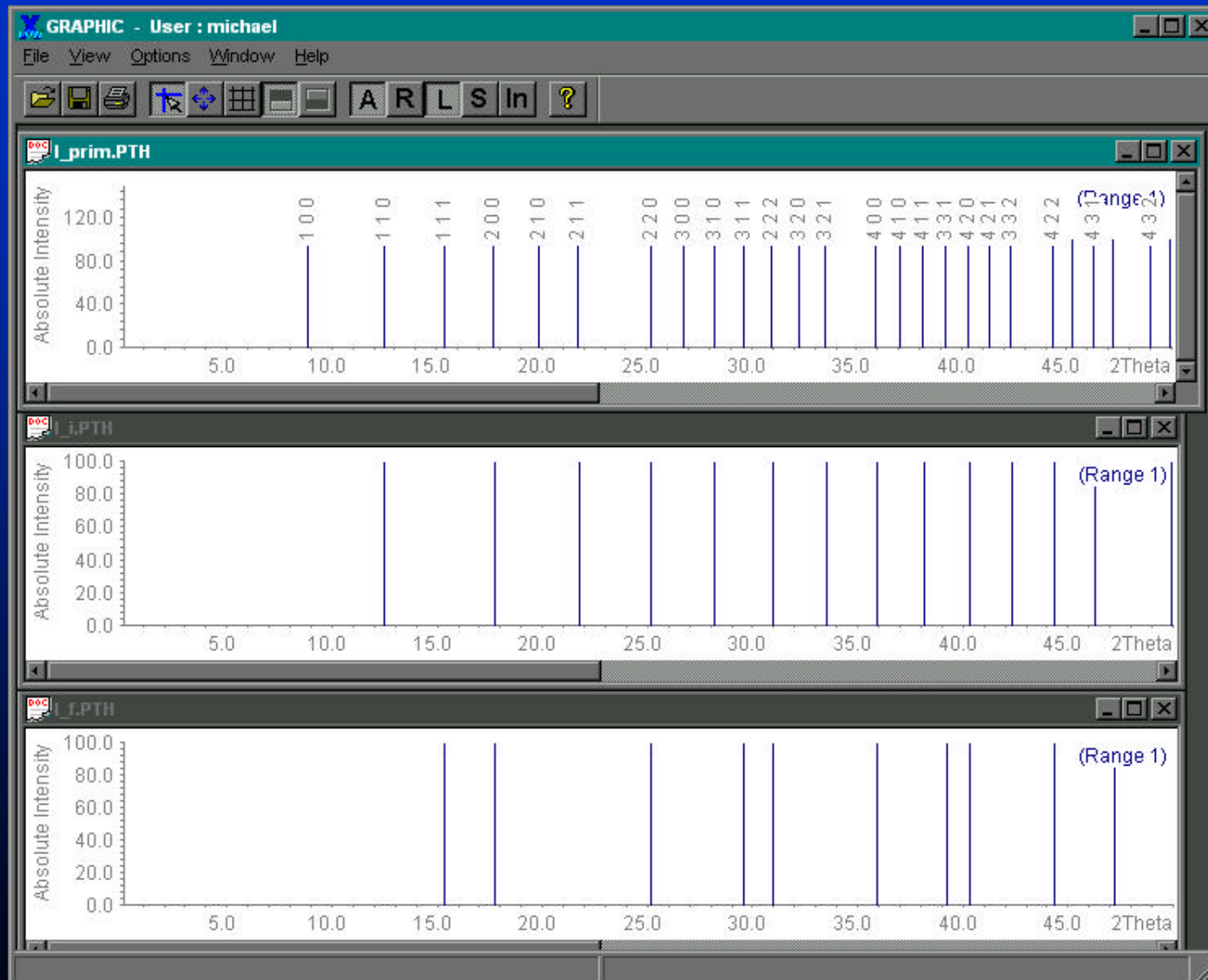


Effect of Centering

P

I

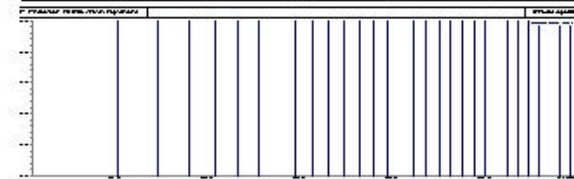
F



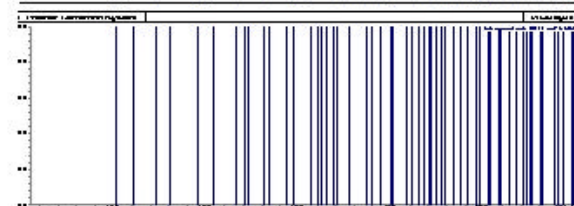
Number of lines changes with symmetry



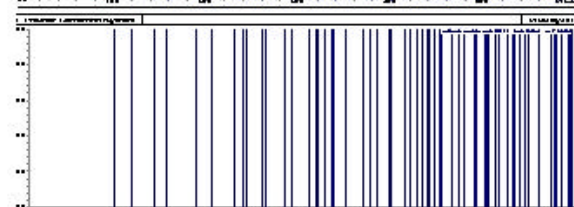
cubic F, $a=8.55$ Å



cubic P, $a=8.55$



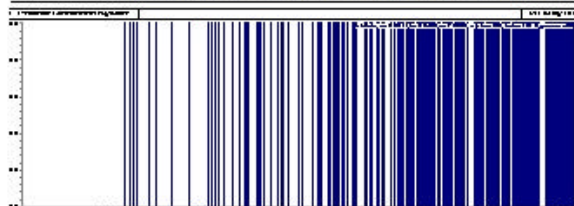
tetragonal, $a=8.55$, $c=7.23$



orthorhombic,
 $a=8.55$, $b=8.54$, $c=7.23$

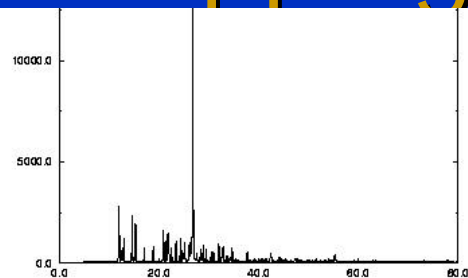


monoclinic,
 $a=8.55$, $b=8.54$, $c=7.23$
 $\beta=112$ deg

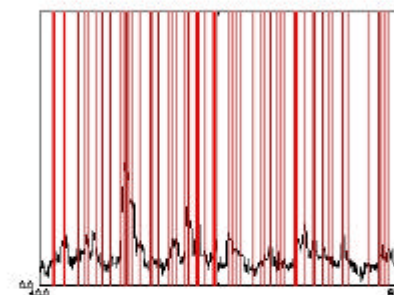
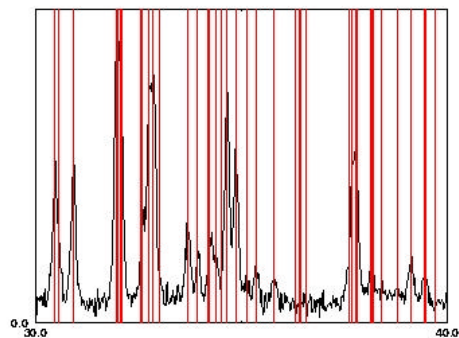
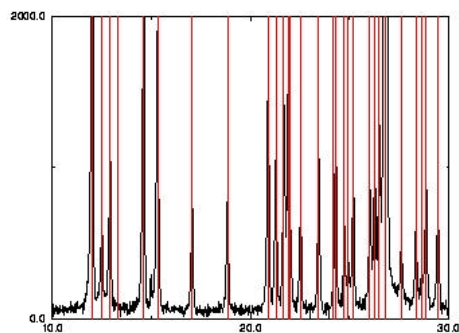


triclinic
 $a=8.55$, $b=8.54$, $c=7.23$
 $\alpha=106$
 $\beta=112$ deg
 $\gamma=60$

Overlapping of Reflections:



triclinic a=8.55, b=8.54, c=7.23 alpha=106 beta=112 deg gamma=60



Databases:

- ICSD (Inorganics, Single Crystal Data, on PC's)
- CSD (Organics, on Wawona)
- METALS (at vsibm1.mpi-stuttgart.mpg.de,
username guest, password guest, metals)

Interaction of Electrons with Matter

Emission of electromagnetic radiation:

Characteristic radiation, discrete energies, $E_c < E_0$

Bremsstrahlung, continuous energie distribution, $E_b \leq E_0$

Luminescence, in the UV or visible Region

Electron emission:

Backscattered electrons (BSE)

Auger electrons

Secondary electron emission (SE)

Effects in the Target:

Electron Absorption (ABS)

Heat