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A NEW SPATIO-TEMPORAL MATCHING ALGORITHM FOR 3D-PARTICLE TRACKING VELOCIMETRY

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ABSTRACT

3D-Particle Tracking Velocimetry (PTV) is one of the most flexible techniques for flow measurement, which allows the determination of three-dimensional velocity fields. Research activities in this field performed by the Institute of Geodesy and Photogrammetry at ETH Zurich for more than a decade have reached a status of an operational and reliable measurement method used in hydrodynamics and space applications. The method is based on the visualization of a flow with small, neutrally buoyant particles and recording of particle image sequences with 3-4 CCD cameras. In cooperation with the Institute of Hydromechanics and Water Resources Management at ETH Zurich further progress has been achieved in the improvement of the existing hard- and software solutions. Regarding the algorithmic aspect of the method a new spatio-temporal matching method was developed, implemented and tested on different data sets. In former implementations the determination of the 3D particle positions was separated from the tracking of each particle, while the new method uses a combination of image and object space based information to establish spatio-temporal correspondences between particle positions of consecutive time steps. A system based on 4 CCD cameras is capable of tracking up to 1000 particles at video frequency (25Hz) with a relative accuracy of the velocity vectors of approximately 1:4000 of the field of view. The latest developments of the algorithmic aspects of 3D PTV are described and some examples of the successful application of the method are given in this paper.

INTRODUCTION

The 3D PTV is a technique for the determination of 3D velocity fields in flows. The existing 3D PTV solution developed at the Institute of Geodesy and Photogrammetry applying a object space based tracking algorithm should be improved in a way that the redundant information in image and object space is exploited more efficiently. The use of image and object space based information in combination with a prediction of the particle motion was thought to lead to enhanced results in the velocity field determination. The most important result to be

expected from this work is a substantial increase of the tracking rate in 3D PTV. This is of importance mainly in the context of a Lagrangian analysis of particle trajectories, which can be considered the actual domain of the technique. Long trajectories are an absolute requisite for a Lagrangian flow analysis, as integral time and length scales can only be determined if long correlation lengths have been recorded. In addition, the number of simultaneous trajectories should be large enough to form a sufficient basis for a statistical analysis. Due to interruptions of particle trajectories caused by unsolved ambiguities the number of long trajectories decreases exponentially with the trajectory length. Very long trajectories over hundred and more time instances can so far only be determined if the probability of ambiguities is reduced by a low seeding density, thus concurrently reducing the spatial resolution of the system and the basis for a statistical analysis. A reduction of the trajectory interruptions due to unsolved ambiguities can multiply the yield of long trajectories and thus the usefulness of the results of 3D PTV, which further enlarges the application potential of the technique.

Within the framework of a research project of the Swiss National Science Foundation a new spatio-temporal matching algorithm was developed and implemented. The technique has reached a status of an operational and reliable measurement tool used in hydrodynamics and space applications (Becker et al, 1995, Maas et al, 1997, Willneff and Maas, 2000).

NOMENCLATURE

X_O, Y_O, Z_O :	3D coordinates of projective center O
X_i, Y_i, Z_i :	3D coordinates of object point P_i
x_i, y_i :	Image coordinates of P_i
c :	Principle distance of the camera
x_h, y_h :	Coordinates of the principle point
ω, φ, κ :	Rotation angles of the direction of the optical axis
D_{rot} :	Rotation matrix as function of the rotation angles ω, φ, κ
a_{ij} :	Elements of 3x3 rotation matrix
t_i :	Time step of image sequence

Δt :	Temporal resolution
\vec{x}_p :	Particle position vector
\vec{u}_p :	Particle velocity vector
\vec{a}_p :	Particle acceleration vector
p_{3d} :	Average number of established particles
l_{3d} :	Average number of established links
eff_{3d} :	Tracking efficiency
g_{3d} :	Gain of the new method with respect to the previous method

HARDWARE SETUP

The hardware setup of a 3D PTV system consists of an image acquisition system with up to four CCD cameras including a data storage device, an illumination facility and tracer particles to seed and visualize the flow. Whether high-grade components or off-the-shell products come into operation is depending on the experiment requirements as well as on the budget.

The acquisition system used at the ETH Zurich was upgraded from offline to online image digitization. In the previous system, the image sequences were firstly recorded on analogue videotapes and digitized afterwards, while in the new system two frame grabbers (Matrox Genesis) are used to provide online digitization and storage.

The sensors define the spatial and temporal resolution. The length of the recorded digital image sequences is nowadays restricted by the storage device capabilities. The data rate for a 60 Hz full-frame camera with a resolution of 640 x 480 pixels is about 19 MB/sec, and hence in an experiment which lasts for 1 minute four cameras deliver a total amount of about 4.5 GB image data. Hardware setups for 3D PTV used at ETH Zurich are described in (Maas, 1992b, Virant, 1996 and Stür, 1999). A possible hardware design is shown in Figure 1.

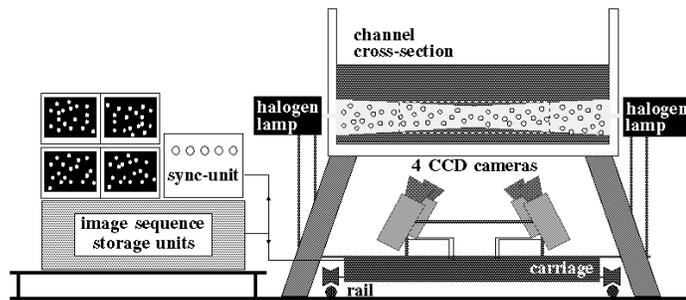


Figure 1: PTV hardware components

MATHEMATIC MODEL

The fundamental equation of photogrammetric 3D particle coordinate determination is the collinearity equation, which states that object point $P_i (X_i, Y_i, Z_i)$, camera projective centre O and image point $P'_i (x_i, y_i)$ lie on a straight line (Figure 2).

This mathematical formulation (eq. 1), which contains three coordinates X_O, Y_O, Z_O of the projective center O and three angles ω, φ, κ describing the direction of the optical axis, applies to a pinhole camera model not considering any distortions:

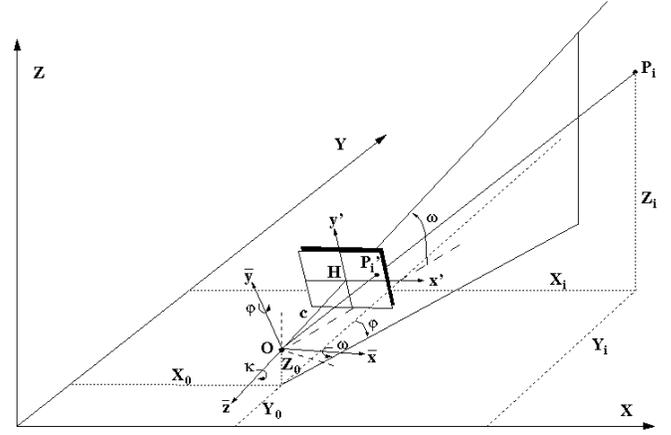


Figure 2: Collinearity equation

$$\begin{aligned} x_i &= x_h - c \cdot \frac{a_{11}(X_i - X_0) + a_{21}(Y_i - Y_0) + a_{31}(Z_i - Z_0)}{a_{13}(X_i - X_0) + a_{23}(Y_i - Y_0) + a_{33}(Z_i - Z_0)} \\ y_i &= y_h - c \cdot \frac{a_{12}(X_i - X_0) + a_{22}(Y_i - Y_0) + a_{32}(Z_i - Z_0)}{a_{13}(X_i - X_0) + a_{23}(Y_i - Y_0) + a_{33}(Z_i - Z_0)} \end{aligned} \quad (1)$$

$$\text{with: } D_{rot} = D(\omega, \varphi, \kappa) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

This mathematical formulation has to be extended to meet the physical realities. The effects caused by lens distortion and analogue to digital conversion are usually compensated by a set of additional parameters. The lens distortion is compensated by a set of 5 additional parameters modelling radial and decentering distortion (Brown, 1971). The compensation of electronic effects, generally the unknown difference between the pixel rate of a camera and the clock rate of a frame grabber, is often performed with an affine transformation using two parameters (El-Hakim, 1986, Grün and Beyer, 1986). The complete set of parameters has to be determined in a system calibration before, during or after the image acquisition of an actual measurement task. Based on this calibration facility, particle tracking with micron-accuracy becomes possible.

Due to the multimedia geometry (the particles are observed through air, glass and water) the optical path is broken twice. With the knowledge of the camera calibration data and the interfaces between the different optical media including their refractive indices this effect can be strictly modeled.

SYSTEM CALIBRATION

In order to establish correspondences between particle images and to compute 3D particle coordinates, the interior and exterior orientations as well as the additional parameters of the cameras have to be determined in a calibration procedure. The calibration is being performed using images of a reference body with discrete targets, which is inserted into the observation volume before or after the experiment. Using the mathematical model of spatial resection, the orientation and calibration

parameters of a camera can be determined from one single image of the calibration reference body under suitable illumination, if the 3D coordinates of the targets are known.

IMAGE ACQUISITION AND INPUT DATA FOR 3D PTV

Input data for 3D PTV processing are image sequences acquired by synchronized cameras. The flow is visualized by particles that appear as bright spots that can be detected automatically from the images. An example of a particle image used for 3D PTV is shown in Figure 3.

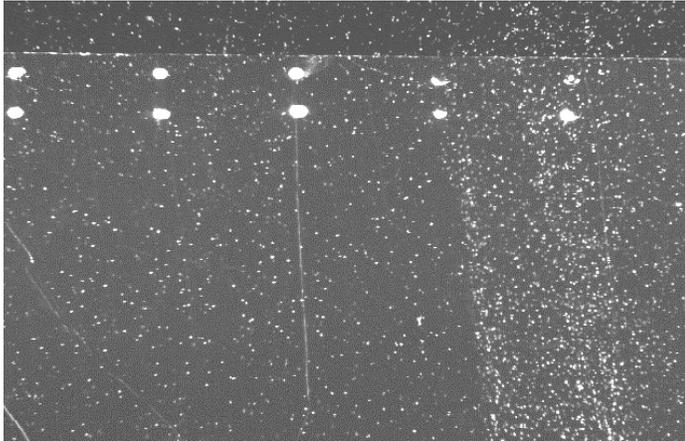


Figure 3: Particle image for 3D PTV

In addition to the synchronous image sequences the camera orientations and additional parameters resulting from the system calibration are required as input data.

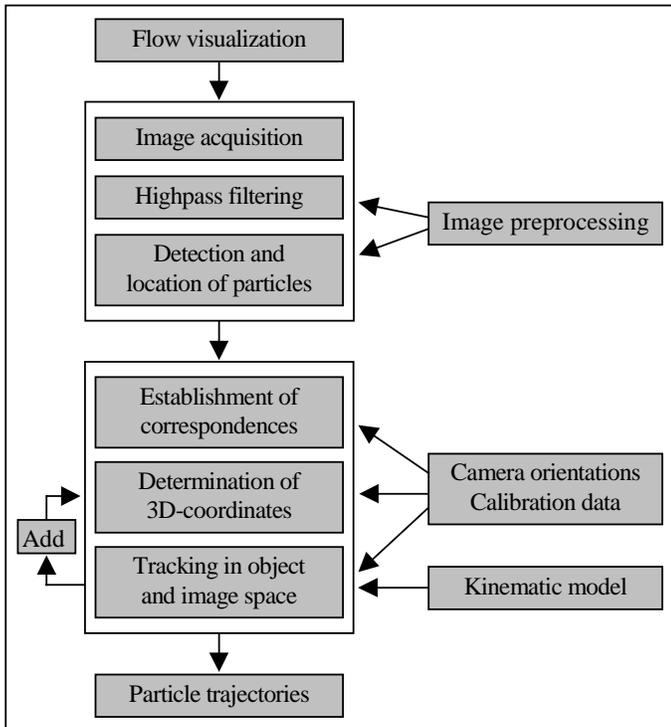


Figure 4: Processing scheme.

Additional particle positions are calculated during the tracking

PROCESSING STEPS

After the acquisition of the image sequences the data is processed in the following steps:

- Image preprocessing by highpass filtering to remove non-uniformities in the background illumination
- Detection of particles in the images by a modified thresholding operator, localization of particles with subpixel accuracy by a centroid operator
- Establishment of stereoscopic correspondences using epipolar constraints. For details see (Maas, 1991, Maas, 1992b and Dold and Maas, 1994)
- Determination of 3-D particle coordinates
- Tracking in object- and image space

The results of the automated processing are trajectories of the particles in the three-dimensional velocity field. The processing scheme of 3D PTV is shown in Figure 4.

TRACKING OF PARTICLES

When a particle is moving in the 3D object space, its trajectory is imaged as a 2D path in image space of the observing camera(s). If corresponding particle images were found in at least two cameras the 3D particle position can be determined. If the temporal assignment over the time is also possible, the trajectory can be reconstructed. Figure 5 shows the trajectory of a particle in an object volume over four time steps and its 2D paths in image space recorded with a two-camera arrangement.

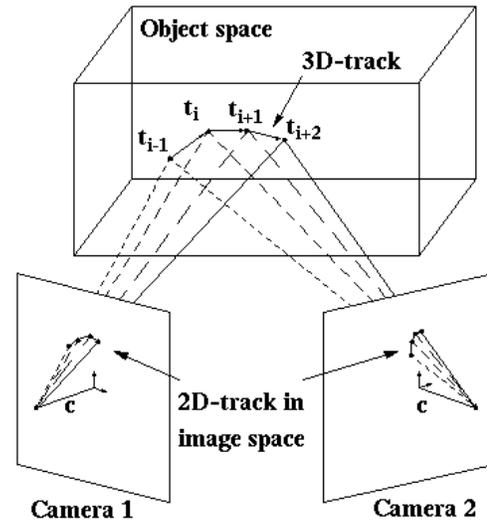


Figure 5: Particle trajectory in image and object space

The goal of the tracking procedure is to select the correct link for the same particle from one time step to the next one. The developed image and object space based tracking technique for spatio-temporal matching uses different criteria to find corresponding particle positions in object space as well as in image space. The following criteria can be used for a reliable and effective assignment:

- The velocity of a particle is limited in all three components of the motion vector
- The Lagrangian acceleration of a particle (the difference of two consecutive velocity vectors of one particle) is limited

- In cases of ambiguities the assignment with the smallest Lagrangian acceleration is the most probable one
- Similarity in brightness, width, height and sum of grey values of the pixel of a particle image in two consecutive time steps

The first criterion defines a three-dimensional search volume, whose size depends on the minimum and the maximum velocity in all three coordinate directions. The limitation of the Lagrangian acceleration defines a conic search area. From three consecutive time steps the difference of the magnitude and direction of predicted and found particle position can be calculated. In the case of ambiguities a quality function is used to get the final assignment as proposed in the third criteria. The fourth criteria could be included in the quality function, however the actual implementation does not take into account the similarities of the particle images. In spite of existing correlation in size and brightness of respective particle images the use of this criteria is not as valuable as expected.

ALGORITHMIC ASPECTS OF SPATIO-TEMPORAL MATCHING

The two most important processing steps of 3D PTV – the establishment of spatial and temporal correspondences between particle images in synchronous image sequences of multiple CCD cameras – which were strictly separated in the existing implementations, have been unified. Former implementations try to reconstruct the trajectories of the particles in two ways:

- The particle positions are determined for each single time step; after that, tracking is performed in 3D space. In the previous implementation at ETH Zurich multi-camera correspondences are established by epipolar line intersection techniques in a first step, then tracking is performed in 3D space. A similar procedure is used in the 3D PTV system of the University of Tokyo (Kasagi and Nishino, 1990).
- Other methods first track the particles in image space. Then the spatial correspondences between the 2D trajectories are established (e.g. Adamczyk and Rimai, 1988 and Netzsch and Jähne, 1993).

In the implementation at ETH Zurich one of the main processing steps is the establishment of multi-camera correspondences by epipolar line intersection techniques (Papantoniou and Dracos, 1989, Papantoniou and Maas, 1990, Maas et al, 1993, Malik et al, 1993). Although the number of ambiguities can be reduced by the use of 3-4 cameras in a suitable configuration they cannot be solved completely. Some detected particle locations still remain unmatched (obviously not all detections in image space can be matched, e.g. when a particle is visible only in one image). Assuming a 4-camera setup a particle at best can be detected in four images which delivers 5 redundant observations (4x2 image coordinate observations minus 3 object space coordinate unknowns). If a particle can be tracked over some consecutive time steps a quite reliable prediction of the particle position in the next time step can be made. The redundant image coordinate observations combined with the prediction for the next particle position should allow the establishment of spatio-temporal

connections even when the velocity field has a high density or the movement of the tracer particles is fast.

The main issue of the new algorithm is the handling of redundant matching information. Not only the 3D particle coordinates of the single time step but also the detections in image space are used to establish the temporal correspondences. The tracking procedure works in the following way:

- From a 3D particle position \bar{x}_p at time step t_i a position for consecutive time step t_{i+1} is predicted under the assumption of a piecewise constant particle velocity \bar{u}_p (eq. 2). A simple linear extrapolation is possible if the particle of step t_i already has a valid link to the previous time step t_{i-1} , otherwise the position \bar{x}_p from t_i itself is used as search position. The temporal resolution Δt is given by the frame rate of the used camera (e.g. 25Hz frame rate of the European video norm leads to a Δt of 0.04 seconds).

$$\begin{aligned}\bar{x}_p(t_{i+1}) &= \bar{x}_p(t_i) + \bar{u}_p(t_i) \cdot \Delta t \\ &= 2 \cdot \bar{x}_p(t_i) - \bar{x}_p(t_{i-1}) \\ \text{with } \bar{u}_p(t_i) &= \frac{\bar{x}_p(t_i) - \bar{x}_p(t_{i-1})}{\Delta t}\end{aligned}\quad (2)$$

- The search of suitable candidates is performed in image space. This requires the backprojection of the search position into the image space of all cameras. This can easily be done when camera orientations and multimedia geometry are known.
- The search is also performed in the image space of the cameras where the particle of step t_i could not initially be found or matched. Based on a list of possible candidates a search position \bar{x}_p for time step t_{i+2} is predicted. If a valid link to the previous time step t_{i-1} exists the predicted position is calculated under the assumption of constant particle acceleration \bar{a}_p (model of second order, eq. (3)), otherwise again a linear extrapolation is used.

$$\begin{aligned}\bar{x}_p(t_{i+2}) &= \bar{x}_p(t_{i+1}) + \bar{u}_p(t_{i+1}) \cdot \Delta t + \frac{1}{2} \cdot \bar{a}_p(t_{i+1}) \cdot \Delta t^2 \\ &= \frac{1}{2} \cdot (5 \cdot \bar{x}_p(t_{i+1}) - 4 \cdot \bar{x}_p(t_i) + \bar{x}_p(t_{i-1})) \\ \text{with } \bar{a}_p(t_i) &= \frac{\bar{u}_p(t_{i+1}) - \bar{u}_p(t_i)}{\Delta t}\end{aligned}\quad (3)$$

- For each found path a quality criterion is calculated. The parameters for the quality measure are the acceleration and the change of direction of the particle.
- Conflicting particle connections are avoided by the quality criteria.
- If a particle, which already has a valid link to the previous time step t_{i-1} can be connected to a particle from time step t_{i+1} but no candidate in step t_{i+2} is available the quality factor is calculated from the time steps t_{i-1} , t_i and t_{i+1} .

- If still no link could be established with the already calculated 3D particle positions, the remaining unmatched detections in image space from time step t_{i+1} around the predicted location are used to determine an additional particle position (again the reconstructed path has to fulfill the quality criterion).

The number of unsolved ambiguities resulting from the epipolar line intersection techniques can be decreased due to the redundant information and the knowledge of the spatio-temporal procedure. In the case of ambiguities particle trajectories are often interrupted for only a few time steps.

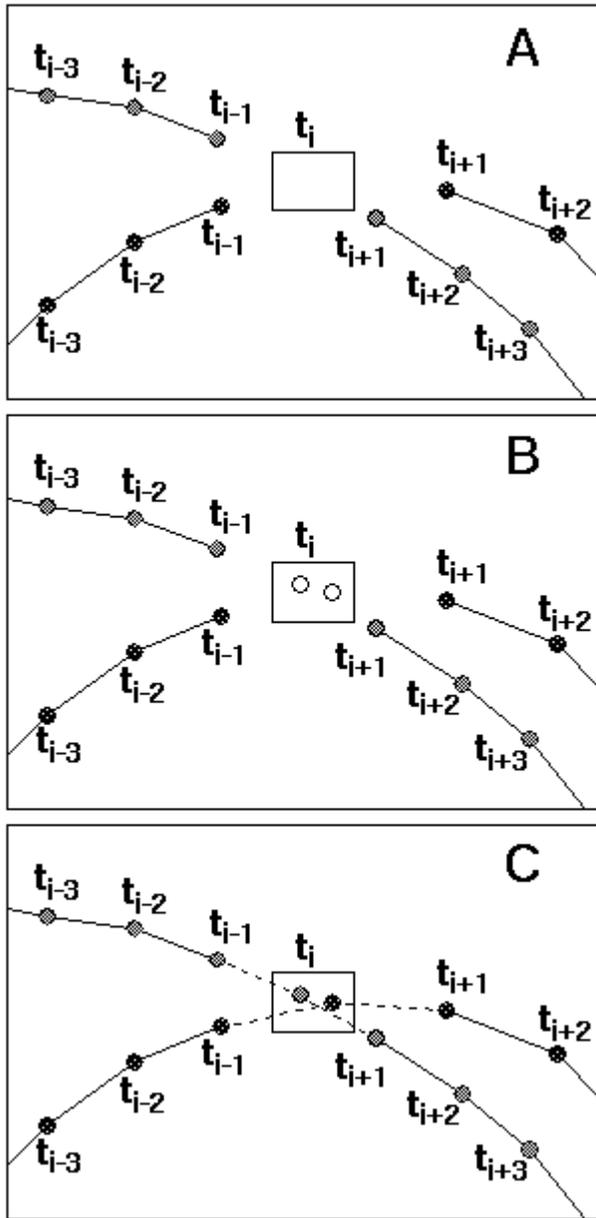


Figure 6: Improvement of spatio-temporal matching. (A) object space based tracking, (B) available data used by the object/image space based tracking method, (C) result of the new object/image space based tracking

With the new algorithm these gaps can be bridged reliably and even the continuation of a trajectory is possible when the redundant information is exploited in a correct way. Figure 6 shows the differences in the use of object/image based information by the tracking algorithms. With the object space based tracking method (A) no particle position in time step t_i could be calculated due to ambiguities in the epipolar line intersection. The available data for the image/object space based tracking are 3D particle positions (filled circles) as well as image coordinates (if correspondence could not be found marked with unfilled circle) is shown in (B). The result of the new tracking algorithm with closed gaps by additionally calculated particle positions from unmatched image coordinates is shown in (C).

Figure 7 shows the result achieved with the previous tracking method.

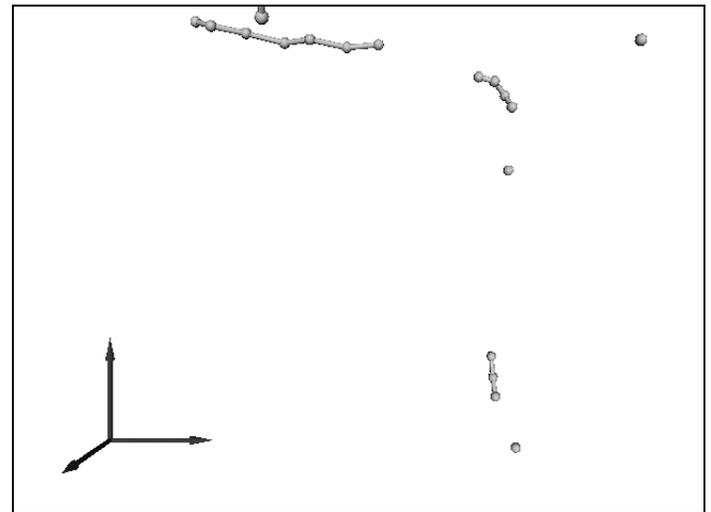


Figure 7: Particle trajectory reconstructed with previous implementation

Figure 8 shows the high potential of the new tracking method.

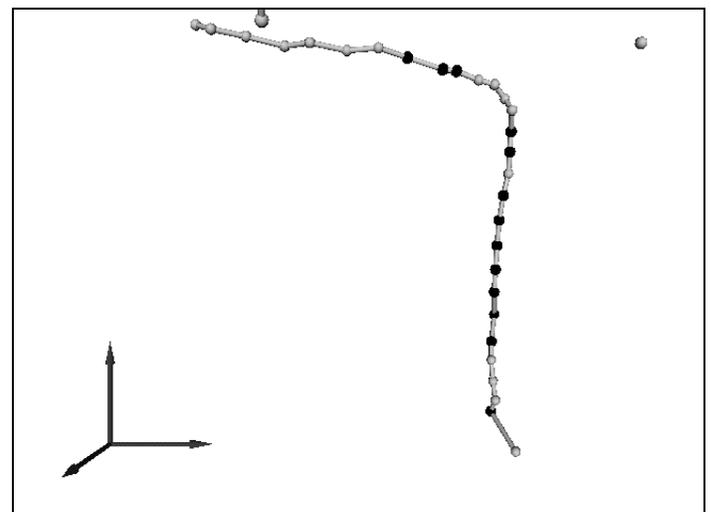


Figure 8: Particle trajectory reconstructed with new spatio-temporal matching algorithm

With the previous implementation the trajectory of the particle could only be reconstructed incompletely (Figure 7), while the new spatio-temporal matching was able to track the particle without gap. The additionally calculated particle positions are marked in black (Figure 8). Of course not every trajectory reconstruction could be improved in that impressive way, but even closing a gap of one or two time steps is a substantial enhancement for the hydrodynamic analysis.

It is obvious that the influence of measurement errors is increasing when a particle position is calculated from fewer observations with a lower redundancy. But if the camera arrangement is suitable, the loss of quality is not substantial and still acceptable compared to an interruption of the particle trajectory. Interpolation is only used for prediction purposes, the calculation of the particle positions to close gaps is strictly based on image space observations.

RESULTS

The result of the 3D PTV processing is a velocity field with a high spatial resolution and a low number of mismatched particles. The application of a purely object space based tracking method leads to a considerable lower number of established connections in combination with a higher error rate. In the case of ambiguities the reliable calculation of additional particle positions from unused detections is only possible if image space based information is used in combination with spatial prediction. Tests with different data sets show the advantages of the combined use of object and image space based information for the spatio-temporal matching and proves the operability of the method for the practical use in different flow measurement tasks. In this paper three different data sets were used to test the new algorithm and to compare the results achieved with the previous implementation. Data set A and B were acquired during the observation of real experiments, while data set C was synthetically generated by simulating a typical PTV hardware setup observing a vortex flow. A visualization of results is shown in Figure 9.



Figure 9: Velocity field (visualization of results)

The data sets were processed with both methods to compare the performance of each implementation. As performance

characteristics served the number of particles and links per time step. The ratio between the number of established links l_{3D} and particles p_{3D} shows the efficiency eff_{3D} of the tracking method (eq.4).

$$eff_{3D} = \frac{l_{3D}}{p_{3D}} \quad (4)$$

Due to the fact that the new method has the capability to add particles links $l_{3D}(add)$, the efficiency $eff_{3D}(rel)$ was also related to the number of particles $p_{3D}(prev)$ derived with the previous method to achieve a better comparability (eq. 5).

$$eff_{3D}(rel) = \frac{l_{3D}(prev) + l_{3D}(add)}{p_{3D}(prev)} = \frac{l_{3D}(new)}{p_{3D}(prev)} \quad (5)$$

A summary of the performance characteristics for comparison of the previous and the new method is listed in Table 1 and Table 2. In all cases the number of particles as well as the efficiency is higher when the data set is processed with the new method.

Data set	Average Number of # (per time step)			
	Particles	Links	Lost links	Added links
A, new	332.6	253.6	78.9	35.0
A, prev	298.3	203.1	95.3	Not possible
B, new	709.1	634.0	75.1	93.0
B, prev	619.7	496.2	123.5	Not possible
C, new	1353.7	1232.9	120.8	86.8
C, prev	1272.7	1091.8	180.8	Not possible

Table 1: Comparison of results achieved with the previous and new method

The ratio between the number of additional links and links established with the previous method was used to quantify the gain g_{3D} of the new method (eq. 6).

$$g_{3D} = \frac{l_{3D}(new) - l_{3D}(prev)}{l_{3D}(prev)} = \frac{l_{3D}(add)}{l_{3D}(prev)} \quad (6)$$

The gain of the new method with respect to the previous one is in the order of 25% for the two data sets from real experiments and 12.9% in the case of the synthetically generated data set.

Data set	Previous method	New method		
	Efficiency eff_{3D} in [%]	Efficiency eff_{3D} in [%]	Efficiency $eff_{3D}(rel)$ in [%]	Gain g_{3D} in [%]
A	68.1	76.2	85.0	+24.8
B	80.1	89.4	102.3	+27.8
C	85.8	91.1	96.9	+12.9

Table 2: Tracking efficiency comparing the previous and the new method

CONCLUSIONS

The paper gives a brief overview about the principles of 3D PTV regarding hardware solution, system calibration and data acquisition. The new spatio-temporal matching method developed at ETH Zurich is described. The actual implementation was tested with simulated and real image sequences and its operability has been shown. The system can be used for manifold measurement tasks in the hydrodynamic flow analysis.

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