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Author(s):

Hens, Thorsten; Pilgrim, Beate

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Thorsten Hens and Beate Pilgrim

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Thorsten Hens and Beate Pilgrim*

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*Institute for Empirical Research in Economics, University of Zurich,
Switzerland; thens@iew.unizh.ch*

Reuters AG, Frankfurt, Germany; beate.pilgrim@reuters.com

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1 Introduction

A well known paradox in international trade theory is the so-called *transfer paradox* (cf. Leontief (1936)). This paradox is said to occur if some country donates some of its resources to some other country and yet the donor benefits while the recipient is worse off.

A well known paradox in financial economics is the so-called *sunspot paradox*. This paradox is said to occur if some exogenous event has no direct influence on the economic fundamentals and yet the endogenous equilibrium allocation depends on it.

The purpose of this note is to show that for a large class of economies these two paradoxes are equivalent. Hence, this note provides an interesting link between two strands of the literature which have so far been developed in isolation.

The class of economies we consider are economies with two agents (resp. two countries) whose utility functions are concave transformations of additively separable functions. The case of two countries is the canonical international trade model and additive separability of utility functions is a commonly used assumption in applied general equilibrium theory.

As an application of the linkage between the transfer paradox and the sunspot paradox we contribute to an open question in the sunspot literature: What is the relation of the sunspot paradox to the uniqueness of spot market equilibria? Moreover, using this linkage some new results on the transfer paradox can be expected by application of the sunspot literature.

2 Model

We first outline the sunspot model. The transfer paradox will then be embedded in the sunspot model by some new interpretation of the sunspot states.

There are two periods. In the second period, one of $s = 1, \dots, S$ states of the world occurs. In the first period assets are traded. Consumption only takes place in the second period. There are $l = 1, \dots, L$ commodities in each state. Since our results hold for the case of two agents, even though it is not necessary for the general definitions, we will outright restrict attention to two agents $i = 1, 2$. States are called *sunspot states* because the agents' characteristics within the states, i.e. the agents' endowments $\omega^i \in X^i$ and their utility functions $u^i : X^i \rightarrow \mathbb{R}$, do not depend on them. X^i is a closed convex subset of \mathbb{R}_+^L which denotes agent i 's consumption set. In the sunspot literature the agents' characteristics $[(u^i, \omega^i)_{i=1, \dots, I}]$ are called the *economic*

fundamentals.

Throughout this note we make the

Assumption 1 (Additive Separability) *Both agents' von Neumann-Morgenstern utility functions u^i are additively separable, i.e. $u^i(x_1^i, \dots, x_L^i) = \sum_{l=1}^L g_l^i(x_l^i)$ for all $x^i \in X^i$, where the functions g_l^i , $l = 1, \dots, L$ are assumed to be twice continuously differentiable, strictly increasing and concave. Moreover, we assume that for every agent i at least $L - 1$ of the functions g_l^i are strictly concave and that for all commodities l there is some i for which g_l^i is strictly concave.*

Note that the assumptions on the functions g_l^i guarantee strict quasi-concavity of the function u^i . The class of utility functions covered by this assumption is quite large and it includes all utility functions that are commonly used in applied general equilibrium theory. In particular, the case of Cobb-Douglas utilities defined on $X^i = \{x \in \mathbb{R}_{++}^L \mid u^i(x) \geq u^i(\omega^i)\}$, for $\omega^i \in \mathbb{R}_{++}^L$, is covered by these assumptions, since then $g_l^i(x_l^i) = \alpha_l^i \ln(x_l^i)$ for some $0 < \alpha_l^i < 1$, $l = 1, \dots, L$. From a theoretical point of view the importance of this assumption is that it implies that all goods are normal. An open conjecture is whether the assumption of normal goods is also sufficient for our analysis.

Moreover we assume that

Assumption 2 (Expected Utility) *For both agents, $i = 1, 2$, the expected utility functions are given by*

$$U^i(x^i(1), \dots, x^i(S)) = \sum_{s=1}^S \pi(s) h^i(u^i(x^i(s))), x^i \in X^i,$$

where the h^i are twice continuously differentiable, strictly increasing and strictly concave functions.

Hence what matters for agents' asset demand is the composition $h^i \circ u^i$, which are concave transformations of additively separable functions. Note that Assumptions 1 and 2 together are sufficient to guarantee strict quasi-concavity of the function U^i .

In the first period agents can trade $j = 1, \dots, J$ real assets with payoffs $A^j(s) \in \mathbb{R}^L$ if state s occurs. We denote asset prices by $q \in \mathbb{R}^J$. Agent i 's portfolio of assets is denoted by $\theta^i \in \mathbb{R}^J$.

All equilibria we consider in this setting are special cases of competitive equilibria, which are defined in

Definition 1 (Competitive Equilibrium) *A competitive equilibrium is an allocation $(x^*, \theta^*), i=1,2$, and a price system (p^*, q^*) such that*

1. *For both agents $i = 1, 2$:*

$$\begin{aligned} (x^*, \theta^*) &\in \operatorname{argmax}_{x^i \in X^i, \theta^i \in \mathbb{R}^J} \sum_{s=1}^S \pi(s) h^i(u^i(x^i(s))) \\ \text{s.t. } q^* \cdot \theta^i &\leq 0, p^*(s) \cdot x^i(s) \leq p^*(s) \cdot \omega^i + p^*(s) \cdot A(s)\theta^i \text{ for all } s = 1, \dots, S. \end{aligned}$$

2. $x^1(s) + x^2(s) = \omega^1 + \omega^2$ for all $s = 1, \dots, S$.

3. $\theta^1 + \theta^2 = 0$.

Remark 1 *To simplify the exposition when analyzing competitive equilibrium allocations and Pareto-efficient allocations we restrict attention to interior allocations, i.e. to allocations x^i in the interior of \mathbb{R}_+^L , $i=1,2$. A sufficient assumption guaranteeing the interiority of allocations is to impose that the functions h^i and g_i^i satisfy the Inada condition according to which the marginal utility tends to infinity at the boundary of the consumption set \mathbb{R}_+^L .*

Note that a competitive equilibrium consists of S spot market equilibria (one for each spot market economy with endowments $\hat{\omega}^i(s) = \omega^i + A(s)\theta^i$) together with an asset market equilibrium by which the ex-post endowments of the spot markets are generated. It will be convenient to introduce the spot-market economy of the economic fundamentals as a point of reference. To abbreviate notations we therefore let this economy be the spot market economy in the spot $s = 0$.

In the sunspot literature agents transfer commodity bundles across sunspot states by trading assets. In the international trade literature one thinks of transfers of commodities arising from donations. The *transfer paradox* is said to occur if some agent donates some of his resources to the other agent and yet the donor's utility increases while the recipients utility decreases. In this statement the utility comparison is done across the competitive equilibria of the economy before and after the donation. In the standard case of the transfer paradox, the transfer was considered to be a transfer of a non-negative amount of commodities (Leontief (1936)). In order to make the equivalence to the sunspot model more obvious we consider a slightly more general definition of the transfer paradox which only requires that the donated commodities have non-negative value in the competitive equilibrium after the transfer. As Geanakoplos and Heal (1983) have already shown this generalization is innocuous.

Taking care of potentially multiple equilibria the transfer paradox is defined as in

Definition 2 (Transfer Paradox) *Given an economy with fundamentals $[(u^i, \omega^i)_{i=1,2}]$ the transfer paradox occurs if and only if there exists some transfer of endowments (from agent 2 to agent 1), $\Delta\omega(z)$, such that for the economy $[u^1, u^2, \omega^1 + \Delta\omega(z), \omega^2 - \Delta\omega(z)]$ there exists an equilibrium $(x^{*1}(z), x^{*2}(z), p^*(z))$ with $p^*(z) \cdot \Delta\omega(z) \geq 0$ so that $u^1(x^{*1}(z)) < u^1(x^{*1}(0))$ for some equilibrium $(x^{*1}(0), x^{*2}(0), p^*(0))$ of the economic fundamentals, $s = 0$.*

We will show that the occurrence of the transfer paradox is a necessary condition for sunspots to matter. To show a converse of this claim we consider the following stronger notion of the transfer paradox.

Definition 3 (Strong Transfer Paradox) *Given an economy with fundamentals $[(u^i, \omega^i)_{i=1,2}]$ the strong transfer paradox occurs if and only if there exist some transfers of endowments (from agent 2 to agent 1), $\Delta\omega(z)$ and $\Delta\omega(\tilde{s})$ such that for the economies $[u^1, u^2, \omega^1 + \Delta\omega(s), \omega^2 - \Delta\omega(s)]$, $s = z, \tilde{s}$*

1. *there are some equilibria $(x^{*1}(z), x^{*2}(z), p^*(z)), (x^{*1}(\tilde{s}), x^{*2}(\tilde{s}), p^*(\tilde{s}))$ with $p^*(z) \cdot \Delta\omega(z) \geq 0$ and $p^*(\tilde{s}) \cdot \Delta\omega(\tilde{s}) \leq 0$ and*
2. *it holds that $u^1(x^{*1}(z)) < u^1(x^{*1}(\tilde{s})) < u^1(x^{*1}(0))$ for some equilibrium $(x^{*1}(0), x^{*2}(0), p^*(0))$ of the economic fundamentals $s = 0$.*

Remark 2 *Note that if the economic fundamentals have at least two equilibria then the transfer paradox occurs. Our definition covers this case because then $\Delta\omega = 0$ is already sufficient to obtain $u^1(x^{*1}(z)) < u^1(x^{*1}(0))$ for the two equilibria $s = 0, z$. Moreover if the economic fundamentals have at least three equilibria then by the same reason the strong transfer paradox occurs.*

3 Main Result

In this section we show that under the maintained assumptions (two agents with utility functions being concave transformations of additively separable functions) the existence of sunspot equilibria is equivalent to the occurrence of the transfer paradox.

To prepare for this result we first define the agents' indirect utility function and their marginal utility of income within each state:

Let

$$v^i(s) = v^i(p(s), b^i(s)) = \max_{x^i \in X^i} \sum_{l=1}^L g_l^i(x_l^i(s)) \quad s.t. \quad p(s) \cdot x^i(s) \leq b^i(s)$$

be the *indirect utility* of agents i in state s . Since the functions $g_l^i, l = 1, \dots, L$ are concave and since at least $L - 1$ of them are strictly concave there is a unique point x^i at which the indirect utility attains its maximum, given that for all commodities the prices $p_l(s), l = 1, \dots, L$ and the income $b^i(s)$ are positive. In our model the income $b^i(s)$ will be given by $p(s) \cdot \omega^i + r(s)$, respectively by $p(s) \cdot (\omega^i + A(s) \cdot \theta^i)$. In the analysis of the sunspot model the agents' *marginal utility of income* will be important

$$\lambda^i(s) = \partial_v h^i(v^i(s)) \partial_b v^i(p(s), b^i(s)).$$

Any competitive equilibrium induces an ordering of the agents' utilities across states, where by Pareto-efficiency the order of agent 1 is inverse to the order of agent 2. The following lemma demonstrates that for both agents the order of the marginal utilities of income are inverse to the order of their (indirect) utilities.

Lemma 1 *Without loss of generality assume that in a competitive equilibrium*

$$v^1(1) \leq v^1(2) \leq \dots \leq v^1(S).$$

Then under Assumption 1 it follows that

$$\lambda^1(1) \geq \lambda^1(2) \geq \dots \geq \lambda^1(S)$$

and that

$$\lambda^2(1) \leq \lambda^2(2) \leq \dots \leq \lambda^2(S).$$

Moreover, if $v^1(\tilde{s}) < v^1(z)$ for some $\tilde{s}, z \in \{1, \dots, S\}$ then the corresponding inequality in the marginal utilities of income is also strict.

Proof

Assume that

$$v^1(\tilde{s}) \leq v^1(z) \quad (\text{resp. that } v^1(\tilde{s}) < v^1(z)) \quad \text{for some } \tilde{s}, z \in \{1, \dots, S\}.$$

Then for some commodity, say $k \in \{1, \dots, L\}$ we must have that

$$x_k^1(\tilde{s}) \leq x_k^1(z) \quad (\text{resp. that } x_k^1(\tilde{s}) < x_k^1(z)).$$

Moreover, Pareto-efficiency within spot markets implies that for all states $s = 1, \dots, S$ the marginal rates of substitution are equal across agents, i.e.

$$\frac{\partial g_m^1(x_m^1(s))}{\partial g_l^1(x_l^1(s))} = \frac{\partial g_m^2(x_m^2(s))}{\partial g_l^2(x_l^2(s))}$$

for any pair of commodities (l, m) . Note that $x_m^2(s) = \omega_m^1 + \omega_m^2 - x_m^1(s)$, $s = 1, \dots, S$. Hence if the functions g_l^i are concave and if for some agent the function g_l^i is strictly concave then it follows that

$$x_l^1(\tilde{s}) \leq x_l^1(z) \quad (\text{resp. that } x_l^1(\tilde{s}) < x_l^1(z)) \quad \text{for all } l = 1, \dots, L.$$

Without loss of generality assume that $l = n$ is the numeraire in all states $s = 1, \dots, S$, where n is chosen such that g_n^1 is strictly concave. Hence we have shown that

$$v^1(1) \leq v^1(2) \leq \dots \leq v^1(S) \quad (\text{with } v^1(\tilde{s}) < v^1(z) \quad \text{for some } \tilde{s}, z)$$

implies for the numeraire that

$$x_n^1(1) \leq x_n^1(2) \leq \dots \leq x_n^1(S) \quad (\text{with } x_n^1(\tilde{s}) < x_n^1(z)) \quad \text{for some } \tilde{s}, z).$$

From the first order condition to the maximization problem

$$\max_{x^i \in X^i} u^1(x^1(s)) \quad \text{s.t.} \quad p(s) \cdot x^1(s) \leq b^1(s)$$

we get that $\partial_b v^1(p(s), b^1(s)) = \partial g_n^1(x_n^1(s))$ for all $s = 1, \dots, S$. Since h^1 and g_n^1 are strictly concave and since $x_n^1(s)$ and $v^1(s)$ are increasing (resp. strictly increasing) in s we get that

$$\lambda^1(1) \geq \lambda^1(2) \geq \dots \geq \lambda^1(S) \quad (\text{resp. that } \lambda^1(z) > \lambda^1(\tilde{s})).$$

The claim for $i = 2$ follows analogously from the inverse inequalities

$$x_1^2(1) \geq x_1^2(2) \geq \dots \geq x_1^2(S)$$

and from

$$v^2(1) \geq v^2(2) \geq \dots \geq v^2(S),$$

the latter inequalities being implied by Pareto-efficiency within spot markets. \square

Theorem 1 (Main Result) *Suppose both agents' utility functions are concave transformations of additively separable functions (Assumption 1 and Assumption 2), then*

1. the transfer paradox is a necessary condition for sunspots to matter and
2. the strong transfer paradox is a sufficient condition for sunspots to matter.

Proof

1. To link the transfer paradox to the sunspot economy consider

$$r(s) := \overset{*}{p}(s) \cdot A(s)\overset{*1}{\theta},$$

i.e. the transfer of income from agent 2 to agent 1 as generated by asset trade in some competitive equilibrium.

A necessary condition for optimal portfolio choice is

$$\sum_{s=1}^S \lambda^i(s)\pi(s)r(s) = 0, i = 1, 2,$$

which we call the first-order conditions for asset demand.¹

Without loss of generality assume that

$$v^1(1) \leq v^1(2) \leq \dots \leq v^1(S),$$

which by Pareto-efficiency within spot markets implies

$$v^2(1) \geq v^2(2) \geq \dots \geq v^2(S).$$

Now suppose, the transfer paradox does not occur. Then from Remark 2 it follows that the economic fundamentals must have a unique equilibrium and for all states $s = 1, \dots, S$, $r(s) \geq 0$ is equivalent to $v^1(s) \geq v^1(0)$ and $v^2(s) \leq v^2(0)$. Accordingly $r(s) \leq 0$ is equivalent to $v^1(s) \leq v^1(0)$ and $v^2(s) \geq v^2(0)$. As above $s = 0$ is the index of the reference economy given by the economic fundamentals.

Let \bar{s} be such that $r^1(s) \leq 0$ for all $s \leq \bar{s}$ and $r^1(s) \geq 0$ for all $s > \bar{s}$, then the first-order conditions for asset demand imply that

$$\sum_{s \leq \bar{s}} (\lambda^1(s) - \lambda^2(s))\pi(s)|r(s)| = \sum_{s > \bar{s}} (\lambda^1(s) - \lambda^2(s))\pi(s)|r(s)|.$$

¹This condition follows from $\sum_s \lambda^i(s)\pi(s)p(s)A(s) = \gamma^i q$ together with $q \cdot \theta^i = 0, i=1,2$.

The expected utility functions U^i are invariant with respect to positive affine transformations of the utility functions $h^i \circ u^i$. Hence without loss of generality we can choose the functions h^i such that for the reference economy $s = 0$ we have $\lambda^i(s) = 1$.

Moreover, given this normalization, from the lemma proven above we know that the differences $(\lambda^1(s) - \lambda^2(s))$ are non-negative for $s \leq \bar{s}$ and they are non-positive for $s > \bar{s}$.

Hence, if sunspots did matter, then at least one of these differences together with the corresponding $r(s)$ has to be non-zero, which contradicts the derived equality.

2. Suppose the strong transfer paradox occurs, then there exist transfers indexed by \tilde{s}, z such that

$$r(z) \geq 0, r(\tilde{s}) \leq 0 \quad \text{and for some equilibria} \quad v^1(z) < v^1(\tilde{s}) < v^1(0)$$

where $v^1(0)$ refers to agent 1's utility in an equilibrium of the spot economy $s = 0$.

Given the utility functions u^1, u^2 and given the total endowments $\omega^1 + \omega^2$ consider the set of Pareto-efficient allocations as being parameterized by the income transfers r .

Now we have to distinguish three cases:

Case 1: If $r(z) > 0$

then we know that $b^1(z) > 0$ and therefore there exists $r(\hat{s}) < 0$ sufficiently small such that for some $b^1(\hat{s}) \geq 0$ we get $v^1(\hat{s}) < v^1(z)$ for some equilibrium in \hat{s} .

To construct the sunspot equilibrium consider an economy with the three states $s = \hat{s}, \tilde{s}, z$. In this case the first-order conditions for asset demand become:

$$\lambda^i(\hat{s})\pi(\hat{s})|r(\hat{s})| + \lambda^i(\tilde{s})\pi(\tilde{s})|r(\tilde{s})| = \lambda^i(z)\pi(z)|r(z)|, i = 1, 2.$$

Now choose $\pi(z) < 1$ sufficiently large (and accordingly $\pi(\hat{s}) > 0$ and $\pi(\tilde{s}) > 0$ sufficiently small) such that

$$\pi(\hat{s})|r(\hat{s})| + \pi(\tilde{s})|r(\tilde{s})| < \pi(z)|r(z)|.$$

The expected utility functions U^i are invariant with respect to positive affine transformations of the utility functions $h^i \circ u^i$. Hence without loss of generality, we can choose the functions h^i such that $\lambda^i(z) = 1, i = 1, 2$. Accordingly, from Lemma 1, $\lambda^1(\tilde{s}) < 1$ and we can choose h^1 such that $\lambda^1(\hat{s})$ is sufficiently large to solve the first order condition for $i = 1$. Analogously it follows that $\lambda^2(\hat{s}) < 1$ and we can choose h^2 such that $\lambda^2(\tilde{s})$ is sufficiently large to solve the first order condition for $i = 2$.

To complete the proof choose $A \in \mathbb{R}^{3L \times 2}$ such that²

$$r(s) = p(s) \cdot (A^1(s) - A^2(s)) \quad \text{for } s = \tilde{s}, \hat{s}, z.$$

Finally, note that

$$\sum_s \lambda^1(s) \pi(s) p(s) \cdot A^1(s) = \sum_s \lambda^1(s) \pi(s) p(s) \cdot A^2(s)$$

so that we can choose $q_1 = q_2$. Accordingly we choose $\theta^1 = (1, -1), \theta^2 = (-1, 1)$ so that $q \cdot \theta^i = 0, i = 1, 2$ and $\theta^1 + \theta^2 = 0$. Since we have chosen an economy with two assets, the first-order conditions for asset trade are equivalent to the conditions $\sum_s \lambda^i(s) \pi(s) p(s) A(s) = \gamma^i q$.

Case 2: If $r(z) = 0$ and $r(\tilde{s}) = 0$

then by the strong transfer paradox, even without trading any asset, there is a competitive equilibrium in which sunspots matter.

Case 3: Finally, the case $r(z) = 0$ and $r(\tilde{s}) < 0$

is already covered by the reasoning of the first case if one changes the point of view from agent 1 to agent 2.

□

4 Application

Having established the link between the transfer paradox and the sunspot paradox we now derive some new results on the existence of sunspot equilibria. These results follow by applications of some well-known results from the international trade literature applied to the sunspot paradox.

The issue we want to clarify by these results is the relation of the existence of sunspot equilibria and the uniqueness of spot market equilibria.

²One can even choose A such that assets pay off in a numeraire commodity.

For that purpose the following terminology is quite useful. Ever since Cass and Shell (1983) it is now standard to say that *sunspots matter* if the allocation of the competitive equilibrium depends on the sunspot states, in which case the competitive equilibrium is called a *sunspot equilibrium*. In a sunspot equilibrium sunspots matter because by strict concavity of the expected utility functions sunspot equilibria are not ex-ante Pareto-efficient. A *randomization equilibrium* is a competitive equilibrium in which for some ex-post endowments the equilibrium allocation in every state s is a spot market allocation for the same economy. If, for example, the economic fundamentals allow for multiple equilibria then there is a randomization equilibrium. Mas-Colell (1992) has shown that there can also be randomization equilibria if there are multiple equilibria for *some* distribution of endowments that is attainable via asset trade. In both cases the equilibrium allocation of such a competitive equilibrium is a randomization among the set of equilibria of *some* underlying economy. In randomization equilibria sunspots are a device to coordinate agents' expectations.

The question that arises from these observations is whether sunspot equilibria could be identified with randomization equilibria. This would then make them very similar to publicly correlated equilibria known in the game theoretic literature (Aumann (1974))³.

It is obvious that in our setting with sun-independent assets, i.e. when $A(s) = A(1)$, $s = 1, \dots, S$, sunspot equilibria necessarily are randomization equilibria. It is, however, not obvious at all whether with a general asset structure there can also be sunspot equilibria which are different from randomization equilibria. To clarify this point some more definitions are needed.

Definition 4 (Attainable Endowment Distributions) *Given the economic fundamentals $[(u^i, \omega^i)_{i=1,2}]$ and given the asset structure A the endowment distributions $\hat{\omega}^i(s)$, $s = 1, \dots, S$, $i = 1, 2$ is attainable if there exists some competitive equilibrium with $\hat{\omega}^i(s) = \omega^i + A(s)\theta^{*i}$, $s = 1, \dots, S$, $i = 1, 2$.*

Based on the attainability concept we now define the uniqueness concept suggested in Mas-Colell (1992). This condition has later been called *no potential multiplicity* by Gottardi and Kajii (1999).

Definition 5 (Strong Uniqueness) *The economy with the fundamentals $[(u^i, \omega^i)_{i=1,2}]$ satisfy the strong uniqueness property for some asset structure A if the spot market equilibria are unique for every attainable endowment distribution.*

³See Forges and Peck (1995) for relating sunspot equilibria to correlated equilibria in an overlapping generations model.

Remark 3 *In the model of this note markets are intrinsically complete, i.e. Pareto-efficient allocations can be attained even without asset trade. With intrinsically incomplete markets sunspots are known to matter even if the economic fundamentals satisfy the strong uniqueness property (cf. Cass (1989) and the literature that has emerged from it, Guesnerie and Laffont (1988) and Gottardi and Kajii (1999)). In such models it is however not possible to separate the inefficiency arising due to the sunspot paradox from that due to intrinsic incompleteness.*

In our setting with intrinsically complete markets we can derive the following results:

Corollary 1 *Under the maintained assumptions, sunspots do not matter if the strong uniqueness property holds.*

Proof

Suppose sunspots did matter, then from our main result we know that the transfer paradox needs to occur. However, as Trannoy (1986) has shown, this requires to be able to trade to some distribution of endowments (in the Edgeworth-Box) for which there are multiple equilibria, which is a violation of the strong uniqueness property. \square

Corollary 2 *Under the maintained assumptions, in the case of two commodities sunspots do not matter if the economic fundamentals have a unique equilibrium.*

Proof

Suppose sunspots did matter then from our main result we know that the transfer paradox needs to occur. However, as for example Balasko (1978)⁴ has shown, in the case of two commodities this requires to have multiple equilibria for the initial distributions of endowments. \square

Corollary 2 overlaps with a result of Pilgrim (2000) who shows for an economy with an arbitrary number of consumers, two commodities and sun-independent assets that additive separability and non-decreasing relative risk aversion exclude non-trivial sunspot equilibria. From Corollary 2 we can see that in the case of two commodities and sun-independent assets it is not possible to trade from uniqueness to multiplicity. This is because with sun-independent assets sunspots can only matter at distributions of endowments for which there are multiple equilibria.

⁴See also the solution to exercise 15.B.10^C from Mas-Colell, Whinston and Green (1995) that is given in Hara, Segal and Tadelis (1997).

Hens (2000) has claimed that for an economy with two agents and two commodities in which utility functions are concave transformations of Cobb-Douglas utility functions sunspots matter. The Corollaries 1 and 2 both show that this claim is incorrect. Moreover the mistake in Hens (2000) cannot be cured by choosing some other parameter values for the same example⁵ because that example falls into the broad class of economies which are covered by this note. Indeed for Cobb-Douglas economies the equilibrium at the initial distribution of endowments is unique and the strong uniqueness requirement is satisfied for almost all asset structures A .

Corollary 3 shows that as in the case of intrinsically incomplete markets also with intrinsically complete markets sunspots can still matter even if they do not serve as a coordination device among multiple equilibria.

Corollary 3 *Under the maintained assumptions, even for the case of two commodities, there are sunspot equilibria which are not randomization equilibria.*

Proof

The example we give to prove this corollary is adapted from the Example 15.B.2 in Mas-Colell, Whinston and Green (1995)⁶. There are two commodities and two agents with endowments $[(\omega_1^1, \omega_2^1), (\omega_1^2, \omega_2^2)] = [(2, r), (r, 2)]$. Consumption sets are $X^i = \{x \in \mathbb{R}_{++}^L | u^i(x) \geq u^i(\omega^i)\}$ and utility functions are given by

$$u^1(x^1) = x_1^1 - \frac{1}{8}(x_2^1)^{-8} \quad \text{and} \quad u^2(x^2) = -\frac{1}{8}(x_1^2)^{-8} + x_2^2.$$

Aggregate endowments are $\omega = (2 + r, 2 + r)$ where $r = 2^{\frac{8}{9}} - 2^{\frac{1}{9}} \approx 0.77$. Figure 1 shows the Edgeworth Box of this economy.⁷

The convex line is the set of Pareto-efficient allocations that lie in the interior of the Edgeworth Box. It is given by the function $x_2^1 = \frac{1}{2+r-x_1^1}$. The competitive equilibrium allocations of our example will be constructed out of these interior allocations. In Figure 1 we have also drawn some budget lines indexed by $s = \hat{s}, z, \tilde{s}, 0$ supporting four different Pareto-efficient allocations which are equilibrium allocations in the spot markets once appropriate spot market endowments have been chosen. The sunspot equilibrium we construct exploits the fact that in this example there are three equilibria for the

⁵This possibility is left open by the observation of Barnett and Fisher (2000) who demonstrate that for the specific parameter values chosen in Hens (2000) sunspots do not matter!

⁶See Hara, Segal and Tadelis (1997) for the solution to the original example.

⁷The Figures 1 and 2 have been generated with *MATLAB*[®]. The scripts can be downloaded from the page <http://www.iew.unizh.ch/home/hens>.

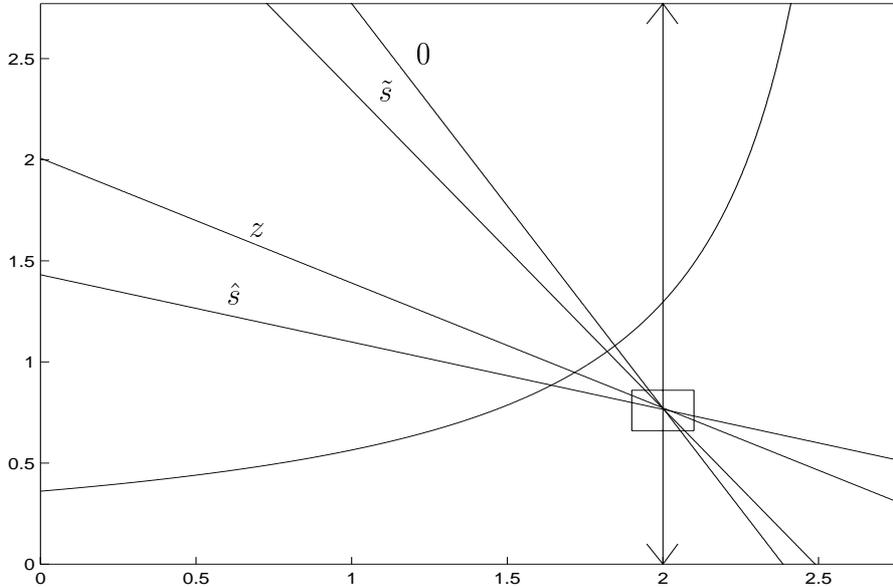


Figure 1: Edgeworth-Box

distribution of endowments $[(\omega_1^1, \omega_2^1), (\omega_1^2, \omega_2^2)] = [(2, r), (r, 2)]$. Taking these endowments as the reference point for the economy $s = 0$, we consider the transfer of endowments as visualized in Figure 2. From the three equilibria at $[(2, r), (r, 2)]$ we have chosen the one with the highest first agent utility to be the equilibrium allocation for the reference situation $s = 0$.

The asset structure A consists of numeraire assets denominated in the second commodity. The vertical line in figure 2 indicates the possible direction of endowment redistributions. With reference to $s = 0$, in the situation z the first agent has received a transfer of the second commodity but his utility decreases. In the situations \hat{s}, \tilde{s} the first agent has donated some of the second commodity and with reference to $s = 0$ his utility decreases. While in \hat{s} it falls to the lowest of the four values, in \tilde{s} it obtains a value between the utility in $s = 0$ and $s = z$. Hence for these transfers the strong transfer paradox occurs and by application of our main result there exists a sunspot equilibrium with spot market endowments given by the intersection of the budget lines \hat{s}, z , and \tilde{s} with the vertical line through the point $(2, 0)$, while the selected equilibrium in reference economy has the budget line 0 .

Although this sunspot equilibrium lies in a neighborhood of a randomization equilibrium it is itself *not* a randomization equilibrium because all spot market endowments differ. \square

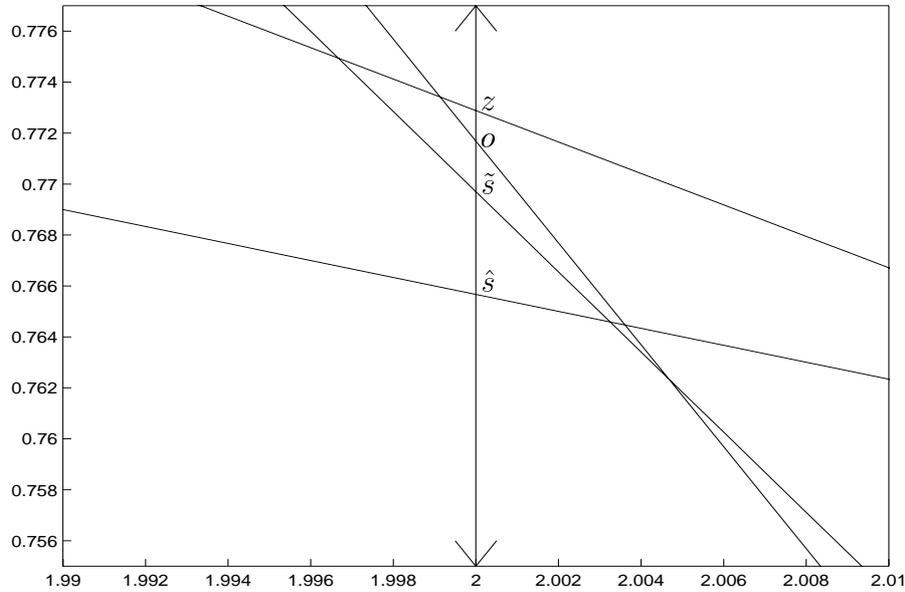


Figure 2: Zoom of the rectangle in Figure 1

References

- Aumann, R. (1974). *Subjectivity and Correlation in Randomized Strategies*, *Journal of Mathematical Economics*, 1(1), 67-96.
- Barnett, R.C. and E.O'N. Fisher (2000). *Comment on: "Do Sunspots Matter When Spot Market Equilibria Are Unique?"*, mimeo, Ohio State University.
- Balasko, Y. (1978). *The Transfer Paradox and the Theory of Regular Economies*, *International Economic Review*, 19(3), 687-694.
- Cass, D. (1989). *Sunspots and Incomplete Financial Markets: The Leading Example*, *The Economics of Imperfect Competition and Employment: Joan Robinson and Beyond*. ed. by G. Feiwel. London: MacMillan.
- Cass, D. and K. Shell (1983). *Do sunspots matter?* *Journal of Political Economy*, 91, 193-227.
- Forges, F. and J. Peck (1995). *Correlated Equilibria and Sunspot Equilibria*, *Economic Theory*, 5(1), 33-50.

- Geanakoplos, J. and G. Heal (1983). A Geometric Explanation of the Transfer Paradox in a Stable Economy Journal of Development Economics, 13, 223-236.*
- Gottardi, P. and A. Kajii (1999). The Structure of Sunspot Equilibria: The Role of Multiplicity Review of Economic Studies, 66, 713-732.*
- Guesnerie, R. and J.J. Laffont (1988). Notes on Sunspot Equilibria in Finite Economies, Volume en l'Honneur d'Edmont Malinvaud, Economica-EHESS. Cambridge MA: MIT Press (English Version).*
- Hara, C., I. Segal and S. Tadelis (1997). Solutions Manual for Microeconomic Theory by Mas-Colell, Whinston and Green Oxford University Press.*
- Hens, Th. (2000). Do Sunspots Matter When Spot Market Equilibria Are Unique? Econometrica, 68, 435-441.*
- Leontief, W. (1936). Note on the Pure Theory of Transfer Explorations in Economics, Taussig Festschrift, Volume 84-92. New York.*
- Mas-Colell, A. (1992). Three Observations on Sunspots and Asset Redundancy. Economic Analysis of Markets and Games: Essays in Honor of Frank Hahn. ed. by P. Dasgupta, D. Gale, O. Hart, and F. Maskin. Cambridge MA: MIT Press.*
- Mas-Colell, A., Whinston, M.D. and J.R. Green (1995). Microeconomic Theory, Oxford University Press.*
- Pilgrim, B. (2000). A Brief Note on Mas-Colell's First Conjecture, Discussion Paper No. 450, University of Bielefeld, Germany.*
- Trannoy, A. (1986). Paradoxe globale des Transfers et Multiplicité des équilibres: deux résultats, Annales D'Economie et de Statistique, 4, 53-61.*

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