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an old question and a new answer

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Abstract

We consider a closed economy where a risk neutral bank competes with a competitive bond market. Firms can finance a risky project either by a bank credit or by issuing a bond which is directly sold to risk averse investors who also can hold safe deposits at the bank. We show that a monopolistic bank tends to allocate more capital to lower quality projects but there are some interesting qualifications. If the asymmetric information concerns only the success probability, then we observe adverse selection while if it concerns only the expected return, bad types are driven out of the market.

Keywords: Credit market, bond market, risk aversion, adverse selection.

JEL classification: D82, G21.

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1 Introduction

There is a large literature on the role of banks or general financial intermediaries which tries to justify their existence by pointing out, for example, their ability of qualitative asset transformation (liquidity transformation, maturity transformation etc.). Despite of this special service offered by banks capital markets have gained increasing importance in the course of the last century and have become a strong competitor for banks on both sides of their balance sheets: on the asset side because firms can obtain finance not only from a bank but also on the capital market by issuing bonds or equity, and on the liability side because more and more consumers invest their savings at capital markets which offer profitable alternatives to the traditional bank deposit. Thus, any study of the role and functioning of banks should consider the interdependence in the network between investors (consumers), banks and capital markets.

In this paper our focus is on the role of banks in project financing. More than 10 years ago an empirical study by James (1987) found that financial markets react positively on the announcement that a quoted firm has obtained a bank loan. More recent studies summarized in James (2000) confirm this finding which seems to be paradox since in practice direct debt is less expensive than bank loans so that there is the stylized fact that only those firms get finance via bank credits which cannot issue direct debt (because they cannot credibly communicate their credit worthiness in a context of asymmetric information). This argument is often quoted as an answer to the question why banks tend to accumulate bad risks and against this background there seems to be no reason for bank loans to serve as a positive signal on the stock market.

What is usually thought to make bank credits more expensive than direct debt is the fact that banks use costly monitoring technologies in order to induce borrowers to spend enough effort for turning the project into a successful one. Many papers make this monitoring activity the defining characteristic of a bank: if an investor decides to monitor he becomes a bank (Besanko and Kanatas (1993), Diamond (1991), Holmstrom and Tirole (1997), Rajan (1992)). In a multiperiod model Diamond (1991) finds that firms with a good reputation of repaying their debt obtain direct financing while those with lower credit ratings have to rely
on bank loans. In a different approach Holmstrom and Tirole (1997) show that only highly leveraged firms are financed by banks. Both papers therefore indicate that banks tend to finance lower quality firms, where lower quality may mean less successful as in Diamond (1991) or less well capitalized as in Holmstrom and Tirole (1997). However, it is questionable whether these findings really explain the observed risk accumulation of banks. In practice banks can do without monitoring whenever it is profitable and offer the same terms of contract to firms seeking credit that are offered by capital markets. In this sense banks even have a comparative advantage over capital markets since they can serve good risks as well as bad risks with their monitoring technology. Moreover, nowadays there are good reasons to doubt that banks distinguish themselves by their monitoring or screening technology. In practice rating agencies play an important role in screening firms and the ratings of bond issuing firms are public information.

Recently, the relationship banking aspect of financial intermediation has been emphasized. In Bolton and Freixas (2000) the distinguishing role of the bank is to help firms through times of financial distress. Since this is costly, again only the riskier firms rely on bank loans while the safer ones obtain direct finance on the bond market. While relationship banking is certainly an important aspect we will offer a completely different answer to the old question why banks choose their credit policy such that they may end up financing lower quality projects. Our results are driven by the risk aversion of investors and the fact that a bank which competes with the bond market on both sides of its balance sheet has to take the investors’ risk aversion into account. To our knowledge the literature on bank versus capital market financing has always assumed that investors are risk neutral which seems to be difficult to justify.

In order to work out precisely the consequences of our approach we completely neglect the monitoring role of banks and choose a setting with asymmetric information prior to contracting (firms have a given project type prior to contracting) rather than of asymmetric information evolving after contracting (firms choose the type of project after contracting). Most models that study the choice of financing source on the part of firms consider the latter moral hazard type of asymmetric information. However, we believe that asymmetric information prior to contracting is an issue at least as important as asymmetric information after
contracting. Typically there are good and bad project ideas and it is not only a matter of effort to turn an unsuccessful project into a successful one.

In our model a monopolistic bank faces competition from a bond market. Hence we combine a principal-agent situation with a competitive market. Risk neutral firms seek finance for projects and can obtain credit from banks or issue bonds. There are two types of firms which differ in the quality of their project. Only the proportions of the two types of firms are common knowledge while the type of a single firm is its private information. Following Hellmann und Stiglitz (2000) we measure project quality along two dimensions, namely the expected return and the success probability of the projects. Risk averse consumers can invest their capital in safe bank deposits or in risky bonds. The risk neutral bank sets the credit volume and the interest rates on deposits and credit such as to maximize expected profits. In doing so the bank anticipates the equilibrium on the bond market. The firms in our model have no capital. Hence, collateral cannot serve as a screening device as in Bester (1985) or Besanko and Thakor (1987) and the bank can only sort firms by setting the interest rate so high that one type of firm leaves the market.

We find that generically there will always be a mix of financing source in equilibrium, i.e. the bond market does not dry up (cf. Besanko and Kanatas (1993)). Moreover, our results confirm the stylized fact that banks tend to finance lower quality projects. For example, the credit volume is increasing in the risk of the financed projects and decreasing in their expected return. However, interestingly, we also obtain different results depending on the dimension in which we measure project quality. If firms only differ in the success probability of their projects there will be a pooling equilibrium if the proportion of high risk projects in the economy is small and a screening equilibrium if the proportion is large. In the latter case the low risks are driven out of the market. This is the classic adverse selection effect. If, on the contrary, firms only differ in the expected return of their projects, then there is pooling for small proportions of the “good” project (high expected return) while there is screening if its proportion in the economy is large. If there is screening we observe a positive selection effect, meaning that the low quality project is driven out of the market.

Our paper is organized as follows. In section 2 we set up our model.
section 3 the profit maximizing contract for a monopolistic bank is determined. We briefly contrast our results with the case of a competitive bank in section 4. Finally, we conclude the paper in section 5. All proofs are in the appendix.

2 The Model

There are three types of agents in our economy, investors, firms and a bank. There are two periods, \( t = 0 \) and \( t = 1 \) and there is only one good (capital) in each period. Firms seek finance for a risky project either at a bank or on the bond market and we explicitly allow for a mix of financing source. The investment into the project has to be made in \( t = 0 \) while returns are realized in \( t = 1 \). Investors are the only agents who possess funds in our economy. Thus, if the bank wants to give a credit to a firm it has to raise funds from the investors. The precise characteristics of the agents are as follows.

**Investors:** There is a continuum of identical investors. Hence we consider a representative investor who has an endowment of 1 unit (of capital) in \( t = 0 \) and nothing in \( t = 1 \). The investor is risk-averse and his preferences for period 1 consumption\(^1\) are represented by the von Neumann-Morgenstern utility function \( u(x) = \ln(x) \) for \( x > 0 \).\(^2\) If there is no bank and no bond market, in period 1 the investor consumes his period 0 endowment, i.e. we assume that there is a storage technology and that capital is non-perishable.\(^3\)

**Firms:** Firms are owned and run by managers who aim to maximize firms’ expected period 1 profits, i.e. we assume managers to be risk-neutral. We can think of the firm as a start-up. The manager has a project idea but no financial resources to realize the project which requires a fixed investment of 1 unit in period 0. If the firm gets finance the manager will only engage in the project if the expected return covers his opportunity cost \( K > 0 \). This participation

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\(^1\) Period 0 consumption is already completed.

\(^2\) We choose this specific utility function so that we can explicitly compute the equilibria of our economy which allows us to make qualitative as well as quantitative statements.

\(^3\) This assumption could easily be relaxed to allow for a positive depreciation rate without any qualitative change of our results.
constraint may, for example, reflect his opportunities to work as an employee for some other firm instead of starting his own business.

There is a continuum of firms or managers which we distinguish by the quality of their projects. The total mass of firms equals the total mass of investors so that the existing funds in the economy exactly equal the capital needed in order to realize all projects. Project quality is measured along two dimensions, namely expected return and risk as reflected by the project’s success probability (cf. Hellmann und Stiglitz (2000)). If $\theta = (\mu, \sigma), \mu \in \mathbb{R}_+, \sigma > 1$, is the type of the firm, then the return of the project is $Q(\theta)$ with probability $p(\theta) = 1/\sigma$ and 0 with probability $1 - p(\theta)$, and the expected return is $p(\theta)Q(\theta) = \mu$. The distribution of project returns is independent across all projects. There is asymmetric information concerning the type of a firm which is only known to the manager. Since the firm has no financial resources collateral cannot serve as a screening device between different projects as in Bester (1985). Managers are not subject to moral hazard, though, i.e. they cannot choose the quality of their project. Project returns are not publicly observable unless the firm does not repay its debt and is declared bankrupt. Thus, ex post verification of types is only possible in case of bankruptcy. We will come back to this point when we discuss credit contracts.

We will assume that there are two types of firms in the economy, $\theta^1$ and $\theta^2$, with corresponding proportions $\lambda > 0$ and $1 - \lambda > 0$ that are common knowledge among all agents in the economy.

**Bank:** The bank has no financial resources in either period and has to obtain funds from investors in order to lend money to a firm. The bank is risk-neutral and aims to maximize its expected period 1 profit from lending to firms and borrowing from investors.

There are 3 assets that can be traded in our economy, a deposit contract, a credit contract and a bond. There is restricted participation in these markets and there are short selling constraints and buying floors which are different for the different agents in our economy. In our simple model these constraints cannot be obtained endogenously but they are exogenously justified.
**Deposit Contract:** A deposit contract is characterized by a guaranteed payment of $r^D \geq 0$ per unit in period 1 which is independent of the state of the world that is realized, i.e. independent of the failure of any firm’s project. Hence, $r^D - 1$ is the interest rate on deposits. Only the bank and the investor are allowed to trade in the deposit contract and the bank is restricted to go short, while the investor is restricted to go long in this contract.

**Credit Contract (Private Debt):** A credit contract delivers a return of $r^C \geq 0$ in period 1 if the debtor has enough assets to fulfill his obligations. Hence, $r^C - 1$ is the interest rate on credit. If the debtor does not fulfill his obligation to repay the credit he has to declare himself bankrupt at no cost and the bankrupt’s assets, if any, go to the creditor. Thus, there is limited liability on the part of the debtor. Under such a contractual arrangement the creditor has no incentive to declare himself bankrupt if he has enough funds to repay his debt. Hence, if $r^C$ does not exceed the project’s return in the good state a credit contract will always deliver $r^C$ or 0 depending on whether the project was successful or not. Observe that, as already mentioned above, ex post verification of returns is not possible since there is no way to force the firm into bankruptcy unless it does not repay its credit. Hence, a standard debt contract (cf. Gale and Hellwig (1985)) is the only feasible contractual arrangement under limited liability and there is no way to screen the firms using this instrument without driving one firm out of the market.

We restrict the bank to go long and the firms to go short in a credit contract while the investor is not allowed to trade in this contract.

**Bond Contract (Public Debt):** A bond contract is the same contractual arrangement as a credit contract except for two differences. Firstly, the bank is not allowed to trade in bonds, while the firms are again restricted to go short and the investor is restricted to go long in the bond contract. Secondly, a bond is junior to a credit contract with respect to the repayment obligation, i.e. if the firm is bankrupt the bank credit has to be repaid first and the remaining assets, if any, go to the bond holder. However, this difference in the seniority structure between a credit and a bond contract is irrelevant here since in case of a project
failure the firm has no assets at all and it will invest in the project only if it is able to repay its debt on the credit contract and on the bond whenever the project is successful. The latter is due to the manager’s opportunity cost as we will see below. The uncertain return of the bond contract in period 1 is denoted by \( r^B \geq 0 \), i.e. \( r^B - 1 \) is the interest the investor receives for 1 unit of bond if the project is successful.

We consider a situation where there is a monopolistic bank which sets the interest rate on deposits as well as on credit and which sets the credit volume \( C \), where \( 0 \leq C \leq 1 \). In our model the bank does not set the deposit volume it is willing to accept which can again be exogenously justified.\(^4\) All other agents act as price takers, i.e. the investor takes \( r^D \) and \( r^B \) as given and chooses the optimal portfolio of deposits and bonds and firms take as given \( r^C \) and \( r^B \) and the credit volume \( C \) and choose the optimal financing strategies for their projects. We will now have a look at the optimization problems of the different agents in detail.

**Optimization problem of the firm:** Let the firm be of type \( \theta \). Given \( C, 0 \leq C \leq 1, r^C \geq 0, r^B \geq 0 \), the firm chooses whether to participate or not and whether to accept the bank’s offer or rather obtain all finance for the project on the bond market. If \( C \geq 0 \) is the amount of credit the firm borrows from the bank and \( B \geq 0 \) is the amount of bonds it sells on the bond market, then its expected period 1 profits are

\[
\Pi = \begin{cases} 
  p(\theta) \max\{0, Q(\theta) - Cr^C - Br^B\}, & \text{if } C + B \geq 1 \\
  0, & \text{else.}
\end{cases}
\]

and the firm will only participate if the expected profits cover its opportunity cost, i.e. if \( \Pi \geq K \). Hence, the profit maximizing choice of the firm is

\[
C^\theta(C, r^C, r^B) = \begin{cases} 
  C, & \text{if } r^B \geq r^C \\
  0, & \text{else}
\end{cases}, \quad B^\theta(C, r^C, r^B) = 1 - C^\theta(C, r^C, r^B)
\]

and the firm participates if and only if

\(^4\)We have in mind standard savings deposits here.
\[ p(\theta) \left( Q(\theta) - C^\theta(C, r^C, r^B) r^C - B^\theta(C, r^C, r^B) r^B \right) \geq K. \]

Hence, we see that the only way the bank can separate the firms is by offering a credit contract which is only accepted by one type of firm.

**Optimization problem of the investor:** The investor can invest in deposits, giving a riskless return of \( r^D \) in period 1, and in a bond, giving a return of \( r^B \) in period 1 unless the firm that issued the bond is bankrupt. If \( r^D < 1 \), the investor will not invest into deposits but rather store his money until period 1.\(^6\) Hence, the bank will never offer \( r^D < 1 \) and we will only consider the case \( r^D \geq 1 \) in the following. The investor (knowing the firm’s participation constraint) correctly anticipates that bankruptcy can only occur in case of a failure of the project.\(^7\) We assume that the investor cannot diversify the risk at the bond market, i.e. there is no “pooling security” as in Bisin and Gottardi (1999) and investors cannot build a pooling bond by themselves, which is justified if pooling on a small scale is too costly. Hence, risk pooling is provided by the bank in our model while the investor buys bonds from one firm only. If both types of firms offer a bond, then chance determines whether the investor obtains a type \( \theta_1 \) or a type \( \theta_2 \) bond.

Therefore, given his belief \( \beta \) about the proportion of type \( \theta_1 \) in the pool of firms offering a bond contract, and taking as given \( r^D \geq 1 \) and \( r^B \geq 0 \) the investor maximizes

\(^5\)To be precise, the maximizer of the expected profit function is not unique if expected profits are never positive for the given \( (C, r^C, r^B) \). However, in this case, the firm will not participate so there is no harm in selecting a particular maximizer then. Also, we assume that the firm accepts the bank’s offer in case of indifference, i.e. whenever \( r^B = r^C \). Introducing another tie breaking rule would not change our result qualitatively as long as each firm accepts the bank’s offer with a positive probability if \( r^C = r^B \).

\(^6\)Recall that we assumed the depreciation rate to be 0.

\(^7\)Recall from the optimization problem of the firm that it will only participate if expected profits exceed \( K > 0 \), hence in equilibrium there is no bankruptcy in case of a success of the project.
\[ U(\beta, r^B, r^D) := \beta \left[(1 - p(\theta^1))u(Dr^D) + p(\theta^1)u(Dr^D + Br^B)\right] \]
\[ + (1 - \beta) \left[(1 - p(\theta^2))u(Dr^D) + p(\theta^2)u(Dr^D + Br^B)\right] \]
\[ = (1 - \bar{p}(\beta))u(Dr^D) + \bar{p}(\beta)u(Dr^D + Br^B) \]

s.t. \( D, B \geq 0 \) and \( D + B \leq 1 \),

where \( \bar{p}(\beta) = \beta p(\theta^1) + (1 - \beta)p(\theta^2) \). Since \( u(x) = \ln(x) \) is strictly monotone the investor optimally chooses \( B = 1 - D \). From the first order conditions which are necessary and sufficient given the strict concavity of \( u \) we obtain

\[ D(\beta, r^B, r^D) = \begin{cases} 
(1 - \bar{p}(\beta)) \frac{r^B}{r^B - r^D}, & \text{if } \bar{p}(\beta)r^B \geq r^D \\
1, & \text{else}
\end{cases} \]

Hence, the investor always invests in the riskless deposit contract and he also invests in the bond unless its expected return falls short of the deposit return.

**Admissible contracts for the bank:** The bank is restricted to choose \((C, r^C, r^D)\) from a set of admissible contracts which can naturally be of two types, pooling or screening. While both types obtain finance at the same conditions in a pooling contract, a screening contract separates both types so that only one type obtains finance while the other does not participate at the conditions of the contract. As we have seen before, the latter is the only possibility to separate the firms. A contract is admissible if it induces an equilibrium in our economy, i.e. in particular, if the bond market is cleared. The bank anticipates this equilibrium and chooses a contract such as to maximize its expected period 1 profits.
Definition 2.1 \((C, r^C, r^D)\) with \(0 \leq C \leq 1\) and \(r^C \geq 0, r^D \geq 1\), is an admissible pooling contract if there exists \(r^B \geq 0\) and a belief \(\beta\) such that

\[
\begin{align*}
(i) & \quad C^{\theta_i}(C, r^C, r^B) = C \text{ for } i = 1, 2, \\
(ii) & \quad p(\theta_i) \left( Q(\theta_i) - Cr^C - (1 - C)r^B \right) \geq K \text{ for } i = 1, 2, \\
(iii) & \quad \beta = \lambda, \\
(iv) & \quad 1 - D(\beta, r^B, r^D) = 1 - C.
\end{align*}
\]

Hence, \((C, r^C, r^D)\) is an admissible pooling contract if both types of firms participate and accept the contract (conditions (i) and (ii)), if the consumer has correct beliefs concerning the proportion of type \(\theta^i\) firms on the bond market (condition (iii)) and if there is market clearing on the bond market (condition (iv)). Observe that the market clearing condition on the bond market implies that \(C = D(\beta, r^B, r^D)\), i.e. the bank obtains enough funds in order to give credit to the firms in period 0. The bank’s expected profit at an admissible pooling contract \((C, r^C, r^D)\) is given by

\[
\Gamma^P(C, r^C, r^D) = C(\bar{p}(\lambda)r^C - r^D).
\]

As we have discussed before, in a screening contract one type of firm is driven out of the market:

Definition 2.2 \((C, r^C, r^D)\) with \(0 \leq C \leq 1\) and \(r^C \geq 0, r^D \geq 1\), is an admissible screening contract if for some \(i \in \{1, 2\}\) there exists \(r^B \geq 0\) and a belief \(\beta\) such that

\[
\begin{align*}
(i) & \quad C^{\theta_i}(C, r^C, r^B) = C, \\
(ii) & \quad p(\theta_i) \left( Q(\theta_i) - C r^C - (1 - C)r^B \right) \geq K > p(\theta_j) \left( Q(\theta_j) - C r^C - (1 - C)r^B \right) \text{ for } j \neq i, \\
(iii) & \quad \beta = \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{else} \end{cases}
\end{align*}
\]
Thus, \((C, r^C, r^D)\) is an admissible screening contract if it is only accepted by the firm of type \(i\), while type \(j\)’s participation constrained is violated (conditions (i) and (ii)), if the investor has correct beliefs concerning the proportion of type \(\theta_1\) firms on the bond market (condition (iii)) and if there is market clearing on the bond market (condition (iv)). Again the market clearing condition on the bond market implies that the bank gets enough deposits in order to give credit to firms in period 0. The bank’s expected profit at an admissible screening contract \((C, r^C, r^D)\) is given by

\[
\Gamma^S(C, r^C, r^D) = \left[ \beta \lambda p(\theta_1) + (1 - \beta)(1 - \lambda)p(\theta_2) \right] Cr^C - D(\beta, r^B, r^D)r^D, 
\]

where \(\beta\) is the corresponding correct belief of the investor.

If \((C, r^C, r^D)\) is an admissible pooling (screening) contract and if \(r^B \geq 0\) is a corresponding equilibrium interest factor on the bond market (see Definitions 2.1 and 2.2), then we will call \(r^B\) an interest factor on bonds supporting the pooling (screening) contract \((C, r^C, r^D)\). We will show below that the interest factor \(r^B\) supporting an admissible pooling or an admissible screening contract is uniquely determined if the bank’s profit at this contract is nonnegative. However, in general, it is possible that \((C, r^C, r^D)\) is an admissible pooling as well as an admissible screening contract, i.e. that there are two possible equilibria on the bond market each corresponding to a different belief of the investor. In this case there exist \(r^B, \hat{r}^B\) with \(r^B \neq \hat{r}^B\), such that \(r^B\) supports \((C, r^C, r^D)\) as a pooling contract and \(\hat{r}^B\) supports \((C, r^C, r^D)\) as a screening contract as in the following example.

**Example 2.1** Let \(\theta_1 = (11, 4)\) and \(\theta_2 = (11, 4/3)\), i.e. both projects have the same expected return \(\mu = 11\) but project 1 is riskier than project 2 since \(p(\theta_1) = 1/4 < p(\theta_2) = 3/4\). Moreover, let \(\lambda = 0.5\) and \(K = 1\). Then \((C, r^C, r^D)\) with \(C = 20/37\), \(r^C = 40/3\) and \(r^D = 1\) is an admissible pooling and an admissible screening contract: \(r^B = r^C\) supports \((C, r^C, r^D)\) as a pooling contract and the
interest rate factor $\hat{r}^B = 38$ supports $(C, r^C, r^D)$ as a screening contract. We observe that $\Gamma^P(C, r^C, r^D) = 340/111 > \Gamma^S(C, r^C, r^D) = 29/222$.

We will see later under which conditions the multiplicity of equilibria can be excluded.

In the following we make some simple observations concerning the characteristics of admissible pooling and screening contracts.

**Lemma 2.1** Let $(C, r^C, r^D)$ be an admissible pooling, respectively screening contract, such that the bank’s profits are nonnegative, i.e. $\Gamma^P(C, r^C, r^D) \geq 0$, respectively $\Gamma^S(C, r^C, r^D) \geq 0$. Let $\beta$ be the corresponding belief and $r^B$ the corresponding interest rate factor on the bond market supporting $(C, r^C, r^D)$. Then it is true that

$$C > 0, \quad r^B \geq r^C \quad \text{and} \quad \bar{p}(\beta)r^C \geq r^D,$$

where $\bar{p}(\beta) = \beta p(\theta^1) + (1 - \beta)p(\theta^2)$.

The next lemma shows that the interest factor on bonds supporting an admissible contract as a pooling, respectively screening contract, is uniquely determined if the bank’s profit is nonnegative.

**Lemma 2.2** Let $(C, r^C, r^D)$ be an admissible pooling, respectively screening contract, such that $\Gamma^P(C, r^C, r^D) \geq 0$, respectively $\Gamma^S(C, r^C, r^D) \geq 0$. Let $r^B$ and $\hat{r}^B$ be two interest factors on bonds, both supporting the contract $(C, r^C, r^D)$ as a pooling, respectively screening contract. Then $r^B = \hat{r}^B$. 


3 The Profit Maximizing Contract for a Monopolistic Bank

In this section we study the contract which is chosen by a monopolistic bank that maximizes its expected profits. We start by observing that for a profit maximizing contract \((C, r^C, r^D)\) it must be true that \(r^B = r^C\), where \(r^B\) is the interest rate factor on the bond market supporting \((C, r^C, r^D)\). This result is rather intuitive because if \(r^B > r^C\) then there is some room left for the bank to increase its interest on credit if it simultaneously increases \(C\) which together increases the bank’s profit. We state this fact in the following lemma.

**Lemma 3.1** Let \((C, r^C, r^D)\) be an admissible pooling (screening) contract and let \(r^B\) support \((C, r^C, r^D)\) as a pooling (screening) contract. Moreover, let \(\Gamma^P(C, r^C, r^D) \geq 0\) \((\Gamma^S(C, r^C, r^D) \geq 0\). If \(r^B > r^C\), there exists an admissible pooling (screening) contract \((\hat{C}, \hat{r}^C, \hat{r}^D)\) such that

\[
\Gamma^P(\hat{C}, \hat{r}^C, \hat{r}^D) > \Gamma^P(C, r^C, r^D) \quad (\Gamma^S(\hat{C}, \hat{r}^C, \hat{r}^D) > \Gamma^S(C, r^C, r^D)).
\]

In order to determine the profit maximizing contract for the bank we will proceed in two steps. First we will determine the best pooling and the best screening contract ignoring the possibility of multiple equilibria on the bond market. Afterwards we will give conditions under which there is a unique equilibrium on the bond market and we will analyse whether the best pooling or the best screening contract gives the bank higher expected profits. The profit maximizing pooling contract solves

\[
\max_{C, r^C, r^D} \Gamma^P(C, r^C, r^D)
\]

s.t. \((C, r^C, r^D)\) is an admissible pooling contract.

We can restrict to those admissible pooling contracts that give the bank non-negative profits.\(^8\) Then, by Lemma 2.1 and Lemma 3.1 we know that \(r^B = r^C\)

---

\(^8\)If all admissible contracts give negative expected profits the bank will not participate and hence receive zero profits.
if $r^B$ supports the profit maximizing pooling contract $(C,r^C,r^D)$. Hence, the profit maximizing pooling contract for the bank is a solution to the following optimization problem:

$$\text{Max}_{r^C,r^D} F^P(r^C,r^D) := (1 - \bar{p}(\lambda)) \frac{r^C}{r^C - r^D} (\bar{p}(\lambda)r^C - r^D)$$

s.t. $\bar{p}(\lambda)r^C \geq r^D \geq 1$ and

$$r^C \leq \min_{i=1,2} \{\sigma^i(\mu^i - K)\}$$

We get the following result.

**Theorem 3.1 (Profit Maximizing Pooling Contract)** Let $\theta^1 = (\mu^1, \sigma^1)$, $\theta^2 = (\mu^2, \sigma^2)$. Then, there exists a solution to the bank’s optimization problem $(P)$ if and only if

$$\bar{p}(\lambda) \min_{i=1,2} \{\sigma^i(\mu^i - K)\} \geq 1.$$ 

If this condition is satisfied the solution to $(P)$ is given by $(\hat{r}^C, \hat{r}^D)$ with

$$\hat{r}^C = \min_{i=1,2} \{\sigma^i(\mu^i - K)\}, \hat{r}^D = 1,$$

and the profit maximizing pooling contract is given by $(\hat{C}, \hat{r}^C, \hat{r}^D)$, where $\hat{C} = (1 - \bar{p}(\lambda)) \frac{\hat{r}^C}{\hat{r}^C - 1}$. There is trade on the bond market, i.e. $\hat{C} < 1$, if and only if $\bar{p}(\lambda)\hat{r}^C > 1$.

We see that in the profit maximizing pooling contract all rents from the low risk firm but not all rents from the high risk firm are absorbed. However, in general, not all rents go to the bank but they are partly taken by the investor since $\hat{C} = 1$ if and only if $\bar{p}(\lambda) \min_{i=1,2} \{\sigma^i(\mu^i - K)\} = 1$. Hence, generically, the bank’s preference for high interest rates on credit will lead to the emergence of a bond market which is rather intuitive: Given the preferences of the investor the bond market will close down only if the interest rate on this market is very low. Since the interest rate on bonds is positively correlated with the interest rate on credit (in fact they are the same if the contract is chosen optimally) the bank has
to put up with the coexistence of the bond market if it wants to raise interest rates above a certain level.

We will now determine the profit maximizing screening contract. The bank solves

\[
\max_{C, r^C, r^D} \Gamma^S(C, r^C, r^D)
\]

s.t. \((C, r^C, r^D)\) is an admissible screening contract.

Let \(\theta^1 = (\mu^1, \sigma^1), \theta^2 = (\mu^2, \sigma^2)\) be such that

\[\sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K).\]

If \(\theta^1, \theta^2\) do not satisfy this condition even after renumbering, there does not exist an admissible screening contract since condition (ii) in Definition 2.2 is always violated. Hence, w.l.o.g. we will assume that the type 2 firm will drop out of the market if there is screening. This implies \(\beta = 1\) for the correct belief of the investor.

Again we can restrict our search for a profit maximizing screening contract to those contracts that give nonnegative expected profits to the bank. Hence, by appealing to Lemma 2.1 and Lemma 3.1 we find the profit maximizing screening contract as a solution to the following optimization problem:

\[
\max_{r^C, r^D} F^S(r^C, r^D) := (1 - p(\theta^1)) \frac{r^C r^C}{r^C - r^D} (p(\theta^1)r^C - r^D) - (1 - \lambda)p(\theta^1)r^C
\]

s.t. \(p(\theta^1)r^C \geq r^D \geq 1\) and

\[\sigma^1(\mu^1 - K) \geq r^C > \sigma^2(\mu^2 - K).\]

If the value of the objective function at a solution to (S) is negative the bank does not choose a screening contract. Otherwise, the solution to (S) gives the profit maximizing screening contract. Similar to the case of a pooling contract we obtain the following result.
Theorem 3.2 (Profit Maximizing Screening Contract) Let \( \theta^1 = (\mu^1, \sigma^1) \), \( \theta^2 = (\mu^2, \sigma^2) \) be such that \( \sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K) \). Then there exists a solution to (S) if and only if \( \mu^1 - K \geq 1 \). If this condition is satisfied and the solution \((\hat{r}^C, \hat{r}^D)\) to (S) satisfies \( F^S(\hat{r}^C, \hat{r}^D) \geq 0 \), then \( \lambda > p(\theta^1), \mu^1 - K > 1 \) and

\[ \hat{r}^C = \sigma^1(\mu^1 - K), \quad \hat{r}^D = 1, \]

and the profit maximizing screening contract is given by \((\hat{C}, \hat{r}^C, \hat{r}^D)\) with

\[ \hat{C} = \frac{1}{\lambda} \left( (1 - p(\theta^1)) - \frac{\hat{r}^C}{\hat{r}^C - 1} - (1 - \lambda) \right) < 1. \]

As we have already seen in the case of the best pooling contract all rents from the financed firm are absorbed. Again the bank uses its monopoly power to leave the investor with a zero interest rate on deposits. We observe that the best screening contract for the bank will always (not only generically as in the case of pooling) induce some trade on the bond market.

We now come back to the problem of multiplicity of equilibria on the bond market. Obviously, the profit maximizing screening contract \((\hat{C}, \hat{r}^C, \hat{r}^D)\) can never be an admissible pooling contract: If \( \hat{r}^B \) is the interest rate factor on bonds supporting \((\hat{C}, \hat{r}^C, \hat{r}^D)\) as a screening contract, then any \( r^B \) supporting this contract as a pooling contract would have to satisfy \( r^B = \hat{r}^B \) since the firms accept \( \hat{C} > 0 \) only if \( r^B \geq r^C \). However, \( r^B > \hat{r}^C = \hat{r}^B \) is impossible since the participation constraints of both firms are violated then. On the contrary, Example 2.1 shows that the profit maximizing pooling contract can be an admissible screening contract.\(^9\) If the bank proposes this contract it may turn out that the resulting equilibrium on the bond market leads to screening and gives the bank a lower (maybe even negative) profit than the best pooling contract. In this case it is difficult to predict the bank’s behavior. Under a pessimistic attitude the bank will never propose a pooling contract if this may result in a screening equilibrium giving the bank a lower profit than the pooling equilibrium. On the contrary, if the bank is optimistic, it will act as if there were no multiplicity of equilibria believing that the pooling equilibrium will arise.

\(^9\)Observe that the contract in Example 2.1 is indeed the profit maximizing pooling contract for the given parameters.
Since it is not clear whether one should assume an optimistic or pessimistic attitude on the part of the bank we look for conditions on the parameters of our model under which the multiplicity of equilibria is ruled out. The following lemma provides such conditions.

**Lemma 3.2** Let \( \theta^1 = (\mu^1, \sigma^1) \) and \( \theta^2 = (\mu^2, \sigma^2) \) be given with \( \sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K) \). Let \( (\hat{C}, \hat{r}^C, \hat{r}^D) \) be a profit maximizing pooling contract such that \( \Gamma_P(\hat{C}, \hat{r}^C, \hat{r}^D) \geq 0 \). If

\[
\sigma^1 = \sigma^2 \quad \text{or} \quad \sigma^1 \leq 2\sigma^2 \frac{\mu^2 - K}{\mu^2 - K + 1},
\]

then \( (\hat{C}, \hat{r}^C, \hat{r}^D) \) is not an admissible screening contract.

The conditions in (1) are not only sufficient but also necessary to rule out the multiplicity of equilibria for all \( \lambda \) for which the profit maximizing contract pooling contract gives the bank a nonnegative profit. This is shown by the next lemma (see also Example 2.1 where both conditions in (1) are violated).

**Lemma 3.3** Let \( \theta^1 = (\mu^1, \sigma^1) \) and \( \theta^2 = (\mu^2, \sigma^2) \) be given with \( \sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K) \) and let

\[
\sigma^1 \neq \sigma^2 \quad \text{and} \quad \sigma^1 > 2\sigma^2 \frac{\mu^2 - K}{\mu^2 - K + 1}.
\]

If for some \( \lambda > 0 \) there exists a profit maximizing pooling contract \( (C, r^C, r^D) \) with \( \Gamma_P(C, r^C, r^D) \geq 0 \), then there exists \( \bar{\lambda} > 0 \) such that the corresponding profit maximizing pooling contract \( (\hat{C}, \hat{r}^C, \hat{r}^D) \) yields nonnegative profits and is also an admissible screening contract.

In the following we will rule out multiple equilibria by assuming that one of the conditions in (1) is fulfilled.

If the equilibrium on the bond market is unique, then the best contract for the bank is either the profit maximizing pooling or the profit maximizing
screening contract depending on which contract yields the higher profit. Let 
\( \theta^1 = (\mu^1, \sigma^1), \theta^2 = (\mu^2, \sigma^2) \) be given with 
\( \sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K) \) and let 
\( p_1 = 1/\sigma^1 \) and \( p_2 = 1/\sigma^2 \). From Theorem 3.1 and Theorem 3.2 we recall that 
\[
\tilde{\Gamma}^P(\lambda) = (1 - \bar{p}(\lambda)) \frac{\sigma^2(\mu^2 - K)}{\sigma^2(\mu^2 - K)} - 1 \left( \bar{p}(\lambda) \sigma^2(\mu^2 - K) - 1 \right)
\]
is the profit from the best pooling contract whenever 
\( \tilde{\Gamma}^P(\lambda) \geq 0 \) and 
\[
\tilde{\Gamma}^S(\lambda) = (1 - p_1) \frac{\sigma^1(\mu^1 - K)}{\sigma^1(\mu^1 - K) - 1} \left( \mu^1 - K - 1 \right) - (1 - \lambda)(\mu^1 - K)
\]
is the profit from the best screening contract whenever 
\( \tilde{\Gamma}^S(\lambda) \geq 0 \). In our analysis of the profit maximizing contract we will assume that 
\( \mu^1 - K > 1 \) and \( \mu^2 - K \geq 1 \). The latter assumption guarantees that for any \( \lambda \) there exists an admissible contract yielding a nonnegative profit for the bank while the first assumption guarantees that the choice of contract is nontrivial in the sense that there exists \( \lambda \) such that a profit as well as a screening contract yields nonnegative profits.

**Theorem 3.3 (Profit Maximizing Contract)** Let \( \theta^1 = (\mu^1, \sigma^1) \) and \( \theta^2 = (\mu^2, \sigma^2) \) be given with \( \mu^1 - K > 1, \mu^2 - K \geq 1 \) and \( \sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K) \). Moreover, let 
\[
\sigma^1 = \sigma^2 \quad \text{or} \quad \sigma^1 \leq 2\sigma^2 \frac{\mu^2 - K}{\mu^2 - K + 1}.
\]
Then there exists \( 0 < \lambda^* < 1 \) such that the unique maximizer of the bank’s profits 
is a pooling contract for \( \lambda < \lambda^* \) and it is a screening contract for \( \lambda > \lambda^* \). If 
\( \lambda = \lambda^* \), then the optimal pooling contract yields the same expected profit as the 
optimal screening contract.

The optimal contracts are the contracts defined in Theorem 3.1 and Theorem 3.2, respectively.
The theorem shows that a screening contract is optimal for the bank whenever the proportion of the type 1 project is large enough. In this case type 2 drops out and only project 1 obtains finance. However, it is a priori not clear whether it is the better or lower “quality” project that obtains finance, i.e. whether we have a positive or an adverse selection effect. From our assumption that \( \sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K) \) we see that there is a trade-off between the success probability and the expected return that determines which project is driven out of the market. Let us consider two interesting extreme cases.

**The case** \( \mu^1 > \mu^2, \sigma^1 = \sigma^2 \): In this case the distribution of returns of project 1 dominates the one of project 2 in the sense of first order stochastic dominance and Theorem 3.3 shows that there is a positive selection effect: We observe screening if the proportion of good projects (high expected return) is large enough in which case only the good projects obtain finance.

**The case** \( \mu^1 = \mu^2, \sigma^1 > \sigma^2 \): In this case the returns of project 2 are a mean preserving spread of those of project 1, i.e. the distribution of returns of project 2 dominates the one of project 1 in the sense of second order stochastic dominance. If the second condition in (2) is satisfied from Theorem 3.3 it follows that there is an adverse selection effect if the proportion of high risk types is large since only the latter obtain finance then while the low risks drop out of the market.

Despite of the first case where we observe a positive selection effect, our results suggest that there is a strong tendency on the part of the bank to finance “lower quality” projects. To see this consider the case where \( \mu^1 < \mu^2 \). Then, if the risk of project 1, \( \sigma^1 \), is not too different from the risk of project 2 (so that \( \sigma^1(\mu^1 - K) < \sigma^2(\mu^2 - K) \)), our analysis shows that for a large proportion of type 1 projects there is pooling, i.e. both projects obtain finance. Only if the proportion of project 1 is small, it is driven out of the market and a positive selection effect provides for only project 2 being financed. On the other hand, if \( \mu^1 < \mu^2 \) and if the risk of project 1 is large relative to \( \sigma^2 \) (so that \( \sigma^1(\mu^1 - K) > \sigma^2(\mu^2 - K) \)),
then project 1 always obtains finance. Moreover, if the proportion of project 1 is large, then we have an adverse selection effect and the “higher quality” project 2 is driven out of the market.

There are more incidences of the fact that the bank allocates more capital to lower quality projects which immediately follow from Theorem 3.3 and the characteristics of the optimal screening and pooling contracts. Let \( \theta_1 \) and \( \theta_2 \) be as in the statement of Theorem 3.3. Then we obtain the following comparative statics results.\(^{10}\)

(i) In the region where pooling is the optimal choice for the bank (\( \lambda < \lambda^* \)), the credit volume is increasing in the risk of project 1 and it is decreasing in the expected return of project 2.\(^{11}\) Moreover, the credit volume is increasing in the proportion of the project with the higher risk, i.e. if \( \sigma_1 > \sigma_2 \), then the credit volume is increasing in \( \lambda \), and if \( \sigma_1 < \sigma_2 \), then the credit volume is increasing in \( 1 - \lambda \).

(ii) In the region where screening is the optimal choice for the bank (\( \lambda > \lambda^* \)), the credit volume is increasing in the risk of the financed project and decreasing in its expected return. Also, the credit volume is increasing in the proportion of the financed project. Hence, if the financed project is the riskier one, then the bank allocates more capital to this project if its proportion in the economy increases.

Hence, summarizing, we find that the higher the risk of a project or the lower its expected return, the more capital it obtains from the bank relative to the bond market. We have seen that the bank’s preference for lower quality projects has several aspects. It concerns not only the credit volume but also the type of contract (pooling or screening) that is chosen. Thus, by and large our results confirm the stylized fact that banks tend to finance lower quality projects. However, there are also some interesting qualifications depending on how we measure the quality of a project. Our findings are natural in a context

\(^{10}\)We have to restrict to local statements since we do not know how the credit volume in the optimal pooling contract compares to the one in the optimal screening contract at the threshold \( \lambda^* \), where we have a change of regimes from pooling to screening.

\(^{11}\)Observe that the credit volume is independent of the expected return of project 1.
where a risk neutral bank has to raise funds from risk averse investors whose
demand for the safe deposit increases with the riskiness of the bond. This effect
is not captured in partial equilibrium models where it is usually assumed that
banks face a totally elastic supply of funds by investors at some exogenously given
interest rate. In our model the supply of funds clearly varies with the type of
the firms seeking finance on the bond and credit market and the bank will take
this into account when setting the credit volume and the interest rates. Hence,
our approach offers a novel explanation for the stylized fact mentioned above and
this is the risk aversion of investors.

4 The Case of a Competitive Bank

In this section we briefly contrast the results obtained for a monopolistic bank
with those obtained for a competitive bank which takes the interest rates as given.
We find that the bond market dries out in this case.

We first reconsider the optimization problems of the different agents in our
economy. The optimization problem of the investor has not changed. The investor
still takes as given $r_B \geq 0$ and $r_D \geq 1$ and his utility maximizing demand for
deposits, given a belief $\beta$ about the proportion of type $\theta_1$ projects in the economy,
is

$$D^I(\beta, r_B, r_D) = \begin{cases} 
(1 - \bar{p}(\beta)) \frac{r_B}{r_B - r_D}, & \text{if } \bar{p}(\beta)r_B \geq r_D, \\
1, & \text{else}
\end{cases}$$

where, as before, $\bar{p}(\beta) = \beta p(\theta_1) + (1 - \beta) p(\theta_2)$.

The firms now only take as given $r_C \geq 0$ and $r_B \geq 0$ and therefore choose

$$C^{\theta_i}(r_C, r_B) = \begin{cases} 
1, & \text{if } r_C < r_B, \\
0, & \text{if } r_C > r_B, \\
\text{arbitrary } \in [0, 1], & \text{if } r_B = r_C.
\end{cases}$$

A firm of type $\theta_i$ will participate if and only if

$$p(\theta_i) \left( Q(\theta_i) - C^{\theta_i}(r_C, r_B)r_C - (1 - C^{\theta_i}(r_C, r_B))r_B \right) \geq K.$$
Finally, the bank takes as given $r^D \geq 1$ and $r^C \geq 0$ and solves

$$\text{Max } \bar{p}(\vartheta)Cr^C - Dr^D$$

s.t. $D \geq C \geq 0$,

where $\vartheta$ is the bank’s belief about the proportion of type $\theta^1$ firms applying for a credit. This optimization problem has a solution if and only if

$$\bar{p}(\vartheta)r^C \leq r^D.$$  

In this case the solution is given by

$$C^B(\vartheta, r^C, r^D) = \begin{cases} 
0, & \text{if } \bar{p}(\vartheta)r^C < r^D \\
\text{arbitrary } \in [0,1], & \text{if } \bar{p}(\vartheta)r^C = r^D
\end{cases}$$

and $D^B(\vartheta, r^C, r^D) = C^B(\vartheta, r^C, r^D)$. As before, we can now define pooling and screening equilibria for this economy.

**Definition 4.1** $(r^C, r^B, r^D)$ with $r^C \geq 0, r^B \geq 0$ and $r^D \geq 1$ is a **pooling equilibrium** if there exist beliefs $\beta$ and $\vartheta$ for the investor and the bank, respectively, such that

(i) $C^{\theta_i}(r^C, r^B) = C^B(\vartheta, r^C, r^D)$ for $i = 1, 2$,

(ii) $p(\theta_i) \left( Q(\theta_i) - C^{\theta_i}(r^C, r^B) r^C - (1 - C^{\theta_i}(r^C, r^B)) r^B \right) \geq K$ for $i = 1, 2$,

(iii) $\beta = \vartheta = \lambda$,

(iv) $D^I(\beta, r^B, r^D) = D^B(\vartheta, r^C, r^D)$.
Definition 4.2 \((r^C, r^B, r^D)\) with \(r^C \geq 0, r^B \geq 0\) and \(r^D \geq 1\) is a screening equilibrium if there exists \(i \in \{1, 2\}\) and beliefs \(\beta\) and \(\vartheta\) for the investor and the bank, respectively, such that

\(\text{(i) } C^{\vartheta}(r^C, r^B) = C^B(\vartheta, r^C, r^D),\)
\(\text{(ii) } p(\vartheta^i) \left( Q(\vartheta^i) - C^{\vartheta^i}(r^C, r^B) r^C - (1 - C^{\vartheta^i}(r^C, r^B)) r^B \right) \geq K + p(\vartheta^j) \left( Q(\vartheta^j) - C^{\vartheta^j}(r^C, r^B) r^C - (1 - C^{\vartheta^j}(r^C, r^B)) r^B \right) \text{ for } j \neq i,\)
\(\text{(iii) } \beta = \vartheta = \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{else} \end{cases},\)
\(\text{(iv) } D^I(\beta, r^B, r^D) = D^B(\vartheta, r^C, r^D),\)
\(\text{(v) } 1 - D^I(\beta, r^B, r^D) = (\beta \lambda + (1 - \beta)(1 - \lambda))(1 - C^{\vartheta^i}(r^C, r^B)),\)
\(\text{(vi) } C^B(\vartheta, r^C, r^D) = (\beta \lambda + (1 - \beta)(1 - \lambda)) C^{\vartheta^i}(r^C, r^B)).\)

One immediately sees that there cannot exist a screening equilibrium since condition (i), (iv) and (v) in Definition 4.2 together with the fact that \(C^B(\vartheta, r^C, r^D) = D^B(\vartheta, r^C, r^D)\) imply that

\[1 - C^{\vartheta^i}(r^C, r^B) = (\beta \lambda + (1 - \beta)(1 - \lambda)) (1 - C^{\vartheta^i}(r^C, r^B)),\]

which is only fulfilled if \(C^{\vartheta^i}(r^C, r^B) = 1\) since \(\beta \lambda + (1 - \beta)(1 - \lambda) < 1\). But then, from condition (i) it follows that \(C^B(\vartheta, r^C, r^D) = 1\) which is not possible given condition (vi). Hence there can only be pooling equilibria and we get the following result, the proof of which is in the appendix.

Theorem 4.1 There exists a pooling equilibrium if and only if

\[\bar{p}(\lambda) \min_{i=1,2} \{\sigma^i(\mu^i - K)\} \geq 1.\]

If this condition is satisfied, then any \((r^C, r^B, r^D)\) with \(r^B = r^C, \bar{p}(\lambda)r^C = r^D \geq 1\) and \(r^C \leq \min_{i=1,2} \{\sigma^i(\mu^i - K)\}\) constitutes a pooling equilibrium. In this case \(D^I(\beta, r^B, r^D) = 1,\) where \(\beta = \lambda.\)
Hence, there is no trade on the bond market and the competitive bank makes zero profit. The resulting interest rates are efficient in the sense that it is not possible to improve the investor without lowering the firms’ profits or vice versa unless the bank makes a negative profit. The extremes of the “efficiency frontier” are the equilibria \((r^C, r^B, r^D)\) such that

- \(r^D = 1, r^C = r^B = 1/\bar{p}(\lambda)\), which maximizes the firms’ profits,
- \(r^C = r^B = \min_{i=1,2}\{\sigma^i(\mu^i - K)\}, r^D = \bar{p}(\lambda)r^C\), which maximizes the investor’s utility.

## 5 Conclusion

We have presented a simple model of a closed economy where a bank competes with a bond market on both sides of its balance sheet: consumers can invest their capital in risky bonds instead of safe deposits and firms can issue bonds instead of obtaining a bank credit. Therefore, when setting the credit volume and the interest rates, a monopolistic bank has to take into account the resulting equilibrium on the bond market. In particular, the bank does not face an infinitely elastic supply of funds at an exogenously given interest rate as it is commonly assumed in the literature that deals with bank versus bond market financing.

With its profit maximizing contract the bank generically leaves room for a bond market since it prefers to set the interest rates on credit so high such that the resulting high bond rates induce the consumer to invest part of his capital in bonds. Obviously, such effects can only be observed in a closed model of the type presented here. Our results offer a new explanation for the fact that banks tend to finance lower quality projects. Different from the literature, which has focused on the monitoring activity of banks, our results are driven by the risk aversion of the investors, a fact that has been ignored so far.

Although many of our findings do not depend on the dimension in which we measure the quality of the projects, there is also an important qualification that we would like to stress: While we observe an adverse selection effect in a screening equilibrium whenever projects only differ in risk, there is a positive selection effect whenever projects only differ in expected return. This is natural...
since the expected profit of a firm depends positively on the expected return and negatively on the success probability. But apart from this qualification, we generally find that the credit volume increases with the risk of the financed projects and decreases with their expected return.

One may wonder why there is a role for the bank in our model. The answer is that the bank has the ability to diversify risk so that it can offer a safe deposit contract to the risk averse investor, who cannot diversify risk on the bond market himself, e.g. due to high transaction costs (which we have not modeled though). Hence, the bank may be interpreted as a mutual fund in our model.

As a final remark observe that by definition in our model there is no credit rationing in equilibrium (cf. Stiglitz and Weiss (1981)): If, given the interest rates for a bank credit and for bonds, a firm is willing to obtain finance its demand is satisfied. Even if the bank does not offer to finance the whole project the remaining funds can be raised on the competitive bond market where interest rates adjust such that demand equals supply.

A Appendix

Proof of Lemma 2.1: Let \((C, r_C, r_D)\) be an admissible contract (screening or pooling) and let \(\beta\) be the corresponding belief of the investor and \(r_B\) be the interest rate factor on bonds supporting \((C, r_C, r_D)\).

If \((C, r_C, r_D)\) is an admissible pooling contract, then \(\beta = \lambda\) and \(C > 0\) immediately follows from the market clearing condition and the fact that \(D(\beta, r_B, r_D) > 0\). This implies \(r_B \geq r_C\) from the optimization problem of the firms. If, in addition, \(\Gamma^P(C, r_C, r_D) = C(\bar{p}(\lambda)r_C - r_D) \geq 0\), then it follows that \(\bar{p}(\lambda)r_C \geq r_D\) since \(C > 0\).

If \((C, r_C, r_D)\) is an admissible screening contract, w.l.o.g. let \(\beta = 1\). From the market clearing condition on the bond market (condition (iv) of Definition 2.2) it follows that

\[D(\beta, r_B, r_D) = 1 - \lambda(1 - C).\]

Hence, \(\Gamma^S(C, r_C, r_D) = \lambda C \left(p(\theta_1)r_C - r_D\right) - (1 - \lambda)r_D \geq 0\) immediately implies that \(p(\theta_1)r_C \geq r_D\) and \(C > 0\). As above the latter implies that \(r_B \geq r_C\). Since \(\bar{p}(\beta) = p(\theta_1)\) this proves our lemma.

\[\square\]
Proof of Lemma 2.2: Let \( r^B \) and \( \hat{r}^B \) be two interest factors on bonds supporting the admissible contract \((C, r^C, r^D)\) and let \( \beta \) be the corresponding belief of the investor. Observe that \( \beta \) is independent of \( r^B \) also in the case of a screening contract: independently of \( r^B \) it is always the same firm that is driven out of the market. Since \( \Gamma^P(C, r^C, r^D) \geq 0 \) (\( \Gamma^S(C, r^C, r^D) \geq 0 \)) by Lemma 2.1 it follows that \( C > 0, r^B \geq r^C, \hat{r}^B \geq r^C, \bar{p}(\beta)r^B \geq r^D \) and \( \bar{p}(\beta)\hat{r}^B \geq r^D \). Since \( D(\beta, r^B, r^D) = D(\beta, \hat{r}^B, r^D) \) by the market clearing condition (iv) in Definitions 2.1 and 2.2, it then follows that

\[
D(\beta, r^B, r^D) = (1 - \bar{p}(\beta))\frac{r^B}{r^B - r^D} = (1 - \bar{p}(\beta))\frac{\hat{r}^B}{\hat{r}^B - r^D} = D(\beta, \hat{r}^B, r^D).
\]

Since \( r^D \geq 1 \) this implies \( r^B = \hat{r}^B \).

\[\square\]

Proof of Lemma 3.1: Let \((C, r^C, r^D)\) be an admissible pooling (screening) contract and let \( \beta \) be the corresponding belief of the investor and \( r^B \) the interest rate factor on the bond market supporting \((C, r^C, r^D)\) as a pooling (screening) contract. By Lemma 2.1 we know that \( C > 0, r^B \geq r^C \) and \( \bar{p}(\beta)r^C \geq r^D \) since \( \Gamma^P(C, r^C, r^D) \geq 0 \) (\( \Gamma^S(C, r^C, r^D) \geq 0 \)). Hence \( D(\beta, r^B, r^D) = (1 - \bar{p}(\beta))\frac{r^B}{r^B - r^D} \) follows from the optimization problem of the investor. Now suppose \( r^B > r^C \). Then \( C < 1 \) since \( C = 1 \) would imply \( r^D = \bar{p}(\beta)r^B > \bar{p}(\beta)r^C \geq r^D \) which is impossible. We define

\[
\hat{r}^C = C r^C + (1 - C) r^B.
\]

Then, obviously, \( r^B > \hat{r}^C > r^C \). Let \( \hat{r}^B = \hat{r}^C \). We will now consider separately the cases of a pooling and a screening contract.

Pooling: If \((C, r^C, r^D)\) is an admissible pooling contract, it follows that \( \beta = \lambda \) by condition (iii) of a pooling contract. Then, by definition \( \bar{p}(\lambda)\hat{r}^B > \bar{p}(\lambda)r^C \geq r^D \) which implies

\[
D(\lambda, \hat{r}^B, r^D) = (1 - \bar{p}(\lambda))\frac{\hat{r}^B}{\hat{r}^B - r^D} > (1 - \bar{p}(\lambda))\frac{r^B}{r^B - r^D} = C.
\]

since \((1 - \bar{p}(\lambda))\frac{r^B}{r^B - r^D}\) is decreasing in \( r^B \). Let \( \hat{C} = D(\lambda, \hat{r}^B, r^D) \) and \( \hat{r}^D = r^D \). Then, \( \hat{C} > C \) and we will show that \((\hat{C}, \hat{r}^C, \hat{r}^D)\) is an admissible pooling contract.
supported by \( \hat{r}^B \) and that \( \Gamma^P(\hat{C}, \hat{r}^C, \hat{r}^D) > \Gamma^P(C, r^C, r^D) \). To this end let \( \beta = \lambda \) be the correct belief of the investor. Since \( \hat{r}^B = r^C \) we get \( C^{\theta_1}(\hat{C}, \hat{r}^C, \hat{r}^B) = \hat{C} \) for \( i = 1, 2 \), so that condition (i) of an admissible pooling contract is satisfied. Moreover, for \( i = 1, 2 \),

\[
p(\theta^i) \left( Q(\theta^i) - \hat{C}\hat{r}^C - (1 - \hat{C})\hat{r}^B \right) = p(\theta^i) \left( Q(\theta^i) - \hat{r}^C \right) = p(\theta^i) \left( Q(\theta^i) - C r^C - (1 - C) r^B \right) \geq K,
\]

by definition of \( \hat{r}^C \) so that the participation constrained (ii) is satisfied. Conditions (iii) and (iv) are fulfilled by definition. Finally,

\[
\Gamma^P(\hat{C}, \hat{r}^C, \hat{r}^D) = \hat{C} \left( \hat{p}(\lambda)\hat{r}^C - \hat{r}^D \right) > C \left( \hat{p}(\lambda)r^C - r^D \right) = \Gamma^P(C, r^C, r^D).
\]

**Screening:** We only consider the case where \( \beta = 1 \) is the belief of the investor corresponding to the admissible screening contract \((C, r^C, r^D)\). This implies \( \hat{p}(\beta) = p(\theta^1) \). From \( p(\theta^1)\hat{r}^B > p(\theta^1)r^C \geq r^B \) it follows that \( D(1, \hat{r}^B, r^D) = D(1, r^B, r^D) \) by the same monotonicity argument as above. Also, from condition (iv) it follows that \( D(1, r^B, r^D) = 1 - \lambda(1 - C) > 1 - \lambda \) so that \( D(1, \hat{r}^B, r^D) > 1 - \lambda \). We define

\[
\hat{C} = \frac{D(1, \hat{r}^B, r^D) - (1 - \lambda)}{\lambda}
\]

and will argue that \((\hat{C}, \hat{r}^C, \hat{r}^D)\) with \( \hat{r}^D = r^D \) is an admissible screening contract that strictly increases the bank’s profit over the contract \((C, r^C, r^D)\). To this end let \( \beta = 1 \) be the correct belief of the investor. Since \( \hat{r}^B = \hat{r}^C \) we get \( C^{\theta_1}(\hat{C}, \hat{r}^C, \hat{r}^B) = \hat{C} \), so that condition (i) of an admissible screening contract is satisfied. Moreover,

\[
p(\theta^1) \left( Q(\theta^1) - \hat{C}\hat{r}^C - (1 - \hat{C})\hat{r}^B \right) = p(\theta^1) \left( Q(\theta^1) - \hat{r}^C \right) = p(\theta^1) \left( Q(\theta^1) - C r^C - (1 - C) r^B \right) \geq K
\]

\[
> p(\theta^2) \left( Q(\theta^2) - C r^C - (1 - C) r^B \right) = p(\theta^2) \left( Q(\theta^2) - \hat{r}^C \right) = p(\theta^2) \left( Q(\theta^2) - \hat{C}\hat{r}^C - (1 - \hat{C})\hat{r}^B \right)
\]
by definition of \( \hat{r}^C \) so that condition (ii) is satisfied. Conditions (iii) and (iv) are fulfilled by definition. Finally,

\[
\Gamma^S(C, r^C, r^D) = \lambda p(\theta^1) C^r - D(1, r^B, r^D) r^D
\]

\[
= (D(1, r^B, r^D) - (1 - \lambda)) p(\theta^1) r^C - D(1, r^B, r^D) r^D
\]

\[
= D(1, r^B, r^D) (p(\theta^1) r^C - r^D) - (1 - \lambda) p(\theta^1) r^C
\]

\[
< D(1, \hat{r}^B, \hat{r}^D) (p(\theta^1) r^C - \hat{r}^D) - (1 - \lambda) p(\theta^1) \hat{r}^C
\]

\[
= \Gamma^S(\hat{C}, \hat{r}^C, \hat{r}^D),
\]

where the second equality follows from the market clearing condition on the bond market and the last inequality follows from the fact that \( \hat{r}^C > r^C \) and \( D(1, \hat{r}^B, \hat{r}^D) > 1 - \lambda \).

\[\square\]

**Proof of Theorem 3.1:** The necessity and sufficiency of the condition \( \bar{p}(\lambda) \min_{i=1,2} \{ \sigma^i(\mu^i - K) \} \geq 1 \) immediately follows from the constraints of the optimization problem (P). Now let this condition be satisfied. Then we compute

\[
\partial_{r^C} F^P(r^C, r^D) = \frac{1 - \bar{p}(\lambda)}{(r^C - r^D)^2} (\bar{p}(\lambda)(r^C)^2 - 2\bar{p}(\lambda) r^C r^D + (r^D)^2)
\]

which is easily seen to be positive for all \( r^C \geq 0, r^D \geq 1 \), since \( \bar{p}(\lambda) < 1 \). Also

\[
\partial_{r^D} F^P(r^C, r^D) = -(1 - \bar{p}(\lambda))^2 \frac{(r^C)^2}{(r^C - r^D)^2}
\]

which is negative for all \( r^C \geq 0, r^D \geq 1 \) since \( \bar{p}(\lambda) < 1 \). Hence, the solution to (P) is given by \( \hat{r}^C = \min_{i=1,2} \{ \sigma^i(\mu^i - K) \} \), \( \hat{r}^D = 1 \). Since \( F^P(\hat{r}^C, \hat{r}^D) \) is the maximum profit the bank can achieve with a pooling contract, the optimal pooling contract is given by \( (\hat{C}, \hat{r}^C, \hat{r}^D) \) with \( \hat{C} = (1 - \bar{p}(\lambda)) \frac{\hat{r}^C}{\hat{r}^C - 1} \). Obviously, \( \hat{C} < 1 \) if and only if \( \bar{p}(\lambda) \hat{r}^C > 1 \).

\[\square\]
Proof of Theorem 3.2: The necessity and sufficiency of the condition $(\mu^1 - K) \geq 1$ immediately follows from the constraints of the optimization problem (S). Now let this condition be satisfied.

If $\lambda \leq p(\theta^1)$, then

$$F^S(r^C, r^D) \leq (1 - p(\theta^1)) \frac{r^C}{r^C - r^D} (p(\theta^1) r^C - r^D) - (1 - p(\theta^1)) p(\theta^1) r^C$$

$$= - (1 - p(\theta^1))^2 \frac{r^C r^D}{r^C - r^D} < 0$$

for all $r^C, r^D$ satisfying the constraints in (S). Hence, in this case there exists no admissible screening contract giving the bank a nonnegative profit. From now on we assume that $\lambda > p(\theta^1)$. We compute

$$\partial_{r^C} F^S(r^C, r^D) = \frac{1}{(r^C - r^D)^2} \left( p(\theta^1)(r^C)^2 - 2 p(\theta^1) r^C r^D + (r^D)^2 \right) - (1 - \lambda) p(\theta^1)$$

which is easily seen to be positive for all $r^C \geq 0, r^D \geq 1$, if $\lambda > p(\theta^1)$. Also

$$\partial_{r^D} F^P(r^C, r^D) = - (1 - p(\theta^1))^2 \frac{(r^C)^2}{(r^C - r^D)^2}$$

which is negative for all $r^C \geq 0, r^D \geq 1$ since $p(\theta^1) < 1$. Hence, the solution to (S) is given by $\hat{r}^C = \sigma^1 (\mu^1 - K)$, $\hat{r}^D = 1$. Moreover,

$$F^S(\hat{r}^C, \hat{r}^D) = (1 - p(\theta^1)) \frac{\sigma^1 (\mu^1 - K)}{\sigma^1 (\mu^1 - K) - 1} (\mu^1 - K - 1) - (1 - \lambda) (\mu^1 - K) \geq 0$$

implies $\mu^1 - K > 1$. Hence, if $F^S(\hat{r}^C, \hat{r}^D) \geq 0$, the optimal screening contract is given by $(\hat{C}, \hat{r}^C, \hat{r}^D)$ with

$$\hat{C} = \frac{1}{\lambda} \left( (1 - p(\theta^1)) \frac{\hat{r}^C}{\hat{r}^C - 1} - (1 - \lambda) \right).$$

Observe that $(\hat{C}, \hat{r}^C, \hat{r}^D)$ is indeed admissible since $\hat{C} > 0$ follows from $\lambda > p(\theta^1)$. Finally, $\hat{C} < 1$ follows from $\mu^1 - K > 1$. This proves the theorem.

□
Proof of Lemma 3.2: Let \((C, r^C, r^D)\) be an admissible pooling contract with \(\bar{p}(\lambda)r^C \geq r^D\). Let \(r^B\) be the interest rate factor supporting \((C, r^C, r^D)\) as a pooling contract and let \(\hat{r}^B\) be the interest rate factor supporting \((C, r^C, r^D)\) as a screening contract. Define \(p_1 = 1/\sigma^1\) and \(p_2 = 1/\sigma^2\). Then \(\hat{r}^B > r^B\) and
\[
C = (1 - \bar{p}(\lambda)) \frac{r^B}{r^B - r^D} = \frac{1}{\lambda} \left( (1 - p_1) \frac{\hat{r}^B}{\hat{r}^B - r^D} - (1 - \lambda) \right)
\]
by the market clearing conditions in the definition of an admissible pooling and screening contract. This implies
\[
\lambda C < \frac{1 - p_1}{1 - \bar{p}(\lambda)} \left( C - (1 - \lambda) \right)
\]
\[
\Longleftrightarrow 1 - \lambda < C \frac{1 - p_1 - \lambda(1 - \bar{p}(\lambda))}{1 - \bar{p}(\lambda)}.
\]
(3)

Now let \((\hat{C}, \hat{r}^C, \hat{r}^D)\) be a profit maximizing pooling contract with \(\Gamma_p(\hat{C}, \hat{r}^C, \hat{r}^D) \geq 0\). In order to show that \((\hat{C}, \hat{r}^C, \hat{r}^D)\) is not an admissible screening contract it suffices to show that \(\hat{C}\) violates (3). Let \(\alpha = \mu^2 - K\). Since \(\bar{p}(\lambda)r^C \geq \hat{r}^D\) (non-negative profits of the bank) we can apply the argument above. Consider first the case where \(\sigma^1 = \sigma^2\). Then \(p_1 = p_2 = \bar{p}(\lambda)\) for all \(\lambda\) and (3) is given by
\[
1 - \lambda < \hat{C} \frac{(1 - p_1)(1 - \lambda)}{1 - p_1}
\]
which is impossible since \(\hat{C} \leq 1\).

Now consider the case where \(\sigma^1 \neq \sigma^2\) and \(\sigma^1 \leq 2\sigma^2 \frac{\alpha}{\alpha + 1}\). By Theorem 3.2 we know that \(\hat{C} = (1 - \bar{p}(\lambda)) \frac{\alpha}{\alpha - p_2}\). Using this expression (3) is equivalent to
\[
\alpha(p_1 - p_2) \left( \lambda^2 - \lambda \frac{p_2(1 - \alpha)}{\alpha(p_1 - p_2)} + \frac{p_2 - \alpha p_1}{\alpha(p_1 - p_2)} \right) > 0.
\]
(4)

If \(p_1 > p_2\), then (4) is violated for all \(\lambda\) with
\[
\frac{p_2 - \alpha p_1}{\alpha(p_1 - p_2)} \leq \lambda \leq 1.
\]
Since \(\bar{p}(\lambda)r^C \geq \hat{r}^D = 1\) one immediately verifies that (4) is indeed violated for the given \(\lambda\), i.e. \((\hat{C}, \hat{r}^C, \hat{r}^D)\) is not an admissible screening contract.

If \(p_1 < p_2\), then (4) is violated for all \(\lambda \leq 1\). Hence, as in the previous case we have proved that \((\hat{C}, \hat{r}^C, \hat{r}^D)\) is not an admissible screening contract.

\(\square\)
Proof of Lemma 3.3: Define \( p_1 = 1/\sigma^1, p_2 = 1/\sigma^2, \alpha_1 = \mu^1 - K, \alpha_2 = \mu^2 - K. \) Under the assumptions in the statement of the Lemma let \((\hat{C}, \hat{r}^C, \hat{r}^D)\) be a profit maximizing pooling contract for some \( \bar{\lambda} > 0 \) such that \( \Gamma^P(\hat{C}, \hat{r}^C, \hat{r}^D) \geq 0. \) Let \( r^B \) be the interest factor on the bond market supporting \((\hat{C}, \hat{r}^C, \hat{r}^D)\) as a pooling contract. From Theorem 3.1 we know that \( r^B = \hat{r}^C = \alpha_2/p_2, \hat{r}^D = 1, \) and \( \hat{C} = (1 - \bar{p}(\bar{\lambda})) \frac{\alpha_2}{\alpha_2 - p_2}. \) Then, \((\hat{C}, \hat{r}^C, \hat{r}^D)\) is also an admissible screening contract if and only if there exists \( \hat{r}^B > r^B \) such that

\[
\hat{C} = \frac{1}{\alpha} \left( (1 - p_1) \frac{\hat{r}^B}{\hat{r}^B - \hat{r}^D} - (1 - \bar{\lambda}) \right) \tag{5}
\]

\[
\hat{C} \hat{r}^C + (1 - \hat{C}) \hat{r}^B \leq \sigma^1(\mu^1 - K). \tag{6}
\]

From equation (5) it follows that \( \hat{r}^B > r^B \) if and only if

\[
\bar{\lambda} \hat{C} < \frac{1 - p_1}{1 - \bar{p}(\lambda)} \hat{C} - (1 - \bar{\lambda}) \tag{7}
\]

\[\iff \alpha_2(p_1 - p_2) \left( \bar{\lambda}^2 + \bar{\lambda} \frac{p_2(\alpha_2 - 1)}{\alpha_2(p_1 - p_2)} + \frac{p_2 - \alpha_2 p_1}{\alpha_2(p_1 - p_2)} \right) > 0. \tag{8}\]

Also from equation (5) it follows that

\[
\hat{r}^B = \frac{\bar{\lambda} \hat{C} + 1 - \bar{\lambda}}{\lambda \hat{C} + p_1 - \bar{\lambda}} \tag{9}
\]

1. Case: \( p_1 > p_2 \)

Let \((C, r^C, r^D)\) be a profit maximizing pooling contract for some \( \lambda > 0 \) such that \( \Gamma^P(C, r^C, r^D) \geq 0. \) Since \( p_1 > p_2 \) it follows that \( \bar{p}(\lambda) \) is increasing in \( \lambda. \) Hence, for all \( \bar{\lambda} \geq \lambda \) there exists a profit maximizing pooling contract giving the bank a nonnegative profit. One verifies that the inequality in (8) is satisfied for all \( \bar{\lambda} \) under the conditions of the lemma and given that \( p_1 > p_2. \) Moreover, from (9) it follows that \( \hat{r}^B \to \alpha_2/p_2 \) for \( \bar{\lambda} \to 1. \) Hence, \( \hat{r}^B < \alpha_1/p_1 \) if \( \bar{\lambda} \) is close to \( 1 \) since by assumption \( \alpha_2/p_2 < \alpha_1/p_1. \) Choose \( \bar{\lambda} \) such that \( \hat{r}^B < \alpha_1/p_1 \) and let \((\bar{C}, \bar{r}^C, \bar{r}^D)\) be the profit maximizing pooling contract for \( \bar{\lambda}. \) Our arguments show that (5) and (6) are satisfied so that \((\bar{C}, \bar{r}^C, \bar{r}^D)\) is also an admissible screening contract.
2. Case: \( p_1 < p_2 \)

Let \((C, r^C, r^D)\) be a profit maximizing pooling contract for some \( \lambda > 0 \) such that \( \Gamma^P(C, r^C, r^D) \geq 0 \). It is straightforward to see that the inequality in (8) is satisfied for all \( \bar{\lambda} \) such that \( \lambda_1 < \bar{\lambda} < 1 \), where

\[
\lambda_1 = \frac{\alpha_2 p_1 - p_2}{\alpha_2 (p_2 - p_1)}.
\]

Consider first the case where \( \lambda_1 \geq 0 \). Then \( \bar{p}(\bar{\lambda}) \hat{r}^C = \bar{p}(\bar{\lambda}) \alpha_2 / p_2 \geq 1 \) for all \( \bar{\lambda} \), where \((\hat{C}, \hat{r}^C, \hat{r}^D)\) is the corresponding profit maximizing pooling contract. Hence, as above we can choose \( \bar{\lambda} \) close to 1 such that \( \hat{r}^B < \alpha_1 / p_1 \) and the corresponding profit maximizing pooling contract gives nonnegative profits and is an admissible screening contract.

Consider now the case where \( \lambda_1 < 0 \). Then the inequality in (8) is fulfilled for all \( \bar{\lambda} \). Since \( \bar{p}(\lambda) \hat{r}^C = \bar{p}(\lambda) \alpha_2 / p_2 \geq 1 \), \( \bar{p}(\lambda) \) is decreasing in \( \lambda \) (because \( p_1 < p_2 \)) and \( \bar{p}(1) \alpha_2 / p_2 < 1 \) (because \( \lambda_1 < 0 \)), it follows that there exists \( \bar{\lambda} \geq \lambda \) such that \( \bar{p}(\bar{\lambda}) \alpha_2 / p_2 = 1 \). Hence, for the corresponding profit maximizing pooling contract \((\hat{C}, \hat{r}^C, \hat{r}^D)\) it is true that \( \hat{C} = 1 \) and therefore

\[
\hat{C} \hat{r}^C + (1 - \hat{C}) \hat{r}^B = \hat{r}^C = \alpha_2 / p_2 < \alpha_1 / p_1.
\]

Thus, again (5) and (6) are satisfied, i.e. \((\hat{C}, \hat{r}^C, \hat{r}^D)\) is an admissible screening contract.

\( \square \)

**Proof of Theorem 3.3:** Let \( \theta^1 = (\mu^1, \sigma^1) \) and \( \theta^2 = (\mu^2, \sigma^2) \) be given with \( \sigma^1 - K > 1, \mu^2 - K \geq 1 \) and \( \sigma^1 (\mu^1 - K) > \sigma^2 (\mu^2 - K) \). Define \( p_1 = 1 / \sigma^1, p_2 = 1 / \sigma^2, \alpha_1 = \mu^1 - K \) and \( \alpha_2 = \mu^2 - K \). Then

\[
\bar{\Gamma}^P(\lambda) = (1 - \bar{p}(\lambda)) \frac{\alpha_2}{\alpha_2 - p_2} \left( \frac{\bar{p}(\lambda)}{p_2} \alpha_2 - 1 \right) \quad \text{and}
\]

\[
\bar{\Gamma}^S(\lambda) = (1 - p_1) \frac{\alpha_1}{\alpha_1 - p_1} \left( \alpha_1 - 1 \right) - (1 - \lambda) \alpha_1
\]

give the profit at the best pooling and best screening contract, respectively, whenever this profit is nonnegative. Consider the following cases.
(i) Let \( \sigma^1 \neq \sigma^2 \) and \( \sigma^1 \leq 2\sigma^2 \frac{\mu^2 - K}{\mu^2 - K + 1} \). The latter is equivalent to \( 2\alpha_2 p_1 \geq (\alpha_2 + 1)p_2 \). This implies
\[
\frac{\bar{p}(\lambda)}{p_2} \alpha_2 > \frac{\min\{p_1, p_2\}}{p_2} \alpha_2 \geq 1
\]
for all \( \lambda \). Hence, \( \tilde{\Gamma}(\lambda) > 0 \) for all \( 0 < \lambda < 1 \). Since \( \tilde{\Gamma}^S(\lambda) \) is a linearly increasing function in \( \lambda \) while \( \tilde{\Gamma}^P(\lambda) \) is a strictly concave quadratic function in \( \lambda \), the graphs of \( \tilde{\Gamma}^P(\lambda) \) and \( \tilde{\Gamma}^S(\lambda) \) can intersect at most twice. We will now argue that there exists exactly one intersection point \( \lambda^* \) in the interval \([0, 1]\) and we will show that \( 0 < \lambda^* < 1 \). From Theorem 3.2 we know that \( \tilde{\Gamma}^S(\lambda) < 0 \) if \( \lambda \leq p_1 \). Hence, \( \lambda^* > 0 \) since \( \tilde{\Gamma}^P(\lambda) > 0 \) for all \( \lambda \) and there can be at most one intersection point \( \lambda^* > 0 \) since \( \tilde{\Gamma}^P(\lambda) \) is strictly concave and \( \lambda^* > 0 \). It remains to show that \( \lambda^* < 1 \). This follows from the fact that
\[
\lim_{\lambda \to 1} \tilde{\Gamma}^S(\lambda) = (1 - p_1) \frac{\alpha_1}{\alpha_1 - p_1} (\alpha_1 - 1),
\]
\[
\lim_{\lambda \to 1} \tilde{\Gamma}^P(\lambda) = (1 - p_1) \frac{\alpha_2}{\alpha_2 - p_2} \left( \frac{p_1}{p_2} \alpha_2 - 1 \right).
\]
Hence, \( \lim_{\lambda \to 1} \tilde{\Gamma}^S(\lambda) > \lim_{\lambda \to 1} \tilde{\Gamma}^P(\lambda) \) if and only if
\[
\frac{\alpha_1}{\alpha_1 - p_1} (\alpha_1 - 1) > \frac{\alpha_2}{\alpha_2 - p_2} \left( \frac{p_1}{p_2} \alpha_2 - 1 \right).
\] \( \text{(10)} \)

The right hand side of this inequality is easily seen to be increasing in \( \alpha_2 \). Since by assumption \( \sigma^1 \alpha_1 > \sigma^2 \alpha_2 \) it follows that \( \alpha_2 < \alpha_1 p_2 / p_1 \) and therefore \( \text{(10)} \) is satisfied.

(ii) Let \( \sigma^1 = \sigma^2 \) and define \( p = 1/\sigma^1 \). Then \( \alpha_1 > \alpha_2 \) and
\[
\tilde{\Gamma}^P(\lambda) = (1 - p) \frac{\alpha_2}{\alpha_2 - p} (\alpha_2 - 1) \quad \text{and}
\]
\[
\tilde{\Gamma}^S(\lambda) = (1 - p) \frac{\alpha_1}{\alpha_1 - p} (\alpha_1 - 1) - (1 - \lambda) \alpha_1
\]
give the profit of the best pooling and best screening contract, respectively, whenever this profit is nonnegative.
Hence, $\tilde{\Gamma}^P$ is independent of $\lambda$ while $\tilde{\Gamma}^S$ is again strictly increasing in $\lambda$. Since $\alpha_2 \geq 1$, it follows that $\tilde{\Gamma}^P(\lambda) \geq 0$ for all $0 < \lambda < 1$ and we again observe that $\tilde{\Gamma}^S(\lambda) < 0$ for $\lambda \leq p$ and $\lim_{\lambda \to 1} \tilde{\Gamma}^S(\lambda) > \lim_{\lambda \to 1} \tilde{\Gamma}^P(\lambda)$. As before, the latter follows from the fact that

\[
\lim_{\lambda \to 1} \tilde{\Gamma}^S(\lambda) = (1 - p) \frac{\alpha_1}{\alpha_1 - p} (\alpha_1 - 1)
\]

is increasing in $\alpha_1$ and $\alpha_1 > \alpha_2$. Hence, there exists $0 < \lambda^* < 1$ such that a pooling contract is the unique maximizer of the bank’s profits for $\lambda < \lambda^*$ and a screening contract is the unique maximizer for $\lambda > \lambda^*$, while both types of contracts are profit maximizing if $\lambda = \lambda^*$.

\[\blacksquare\]

Proof of Theorem 4.1: Let $(r^C, r^B, r^D)$ be a pooling equilibrium. Then it must be true that $\beta = \vartheta = \lambda$. Since $D^I(\lambda, r^B, r^D) > 0$, from condition (iv) in Definition 4.1 and the definition of $D^B(\cdot) = C^B(\cdot)$ it follows that $\bar{p}(\lambda)r^C = r^D$. From the participation constraint of the firms it follows that $r^C \leq \min_{i=1,2}\{\sigma^i(\mu^i - K)\}$, hence $\bar{p}(\lambda) \min_{i=1,2}\{\sigma^i(\mu^i - K)\} \geq 1$. Also, it is straightforward to show that $r^B \geq r^C$. Suppose by way of contradiction that $r^B > r^C$. Then $\bar{p}(\lambda)r^B > r^D$ which implies $D^I(\lambda, r^B, r^D) < 1$. However, $C^{\theta^i}(r^B, r^C) = 1$ if $r^B > r^C$. This is a contradiction to conditions (i) and (iv) in Definition 4.1. Hence, $r^B = r^C$. If $r^B = r^C$ and $\bar{p}(\lambda)r^C = r^D$, then $C^{\theta^i}(r^C, r^B) = C^B(\lambda, r^C, r^D) = D^I(\lambda, r^B, r^D) = D^B(\lambda, r^C, r^D) = 1$.

Moreover, it is immediate to verify that any $(r^B, r^C, r^D)$ with $r^B = r^C, \bar{p}(\lambda)r^C = r^D \geq 1$ and $r^C \leq \min_{i=1,2}\{\sigma^i(\mu^i - K)\}$ constitutes a pooling equilibrium if $\bar{p}(\lambda) \min_{i=1,2}\{\sigma^i(\mu^i - K)\} \geq 1$.

\[\blacksquare\]

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