

# Planar subdivisions

**Author(s):**
Tóth, Csaba Dávid

**Publication Date:**
2002

**Permanent Link:**
https://doi.org/10.3929/ethz-a-004394174

DISS. ETH No. 14628, 2002

# Planar subdivisions

A dissertation submitted to the
Swiss Federal Institute of Technology, ETH Zürich
for the degree of Doctor of Technical Sciences

presented by
Csaba Dávid Tóth
M.Sc.Math. Eötvös Loránd University, Budapest
DEA MDFI. Université de la Méditerranée, Marseille
born in 1974, citizen of Hungary

accepted on the recommendation of
Prof. Dr. Emo Welzl, ETHZ, examiner
Prof. Dr. Günter Rote, FU Berlin, co-examiner

# Summary

Subdivisions of the real Euclidean space and, specifically, planar subdivisions represent a fundamental tool in combinatorial and computational geometry. Many "divide and conquer" techniques exploit geometric structure through use of adequate subdivisions. Applications in range searching, ray tracing, robot motion planning, incremental algorithms, incidence problems, surface approximation algorithms, etc. reduce their running time and space complexity by working on a small number of cells of bounded complexity instead of the complete geometric structure.

This thesis does not intend to give an overview of planar subdivisions, nor of their internal hierarchy. Rather, it exhibits some recent progress on three different, though related, types of planar subdivisions, which are discussed separately in the three chapters to follow.

The three topics can be placed in a common framework of visibility. Two points $p$ and $q$ are mutually visible if the open line segment $pq$, connecting the two points, does not interfere (i.e., intersect or cross) any of the given obstacle.

Chapter 1 considers subdivisions of simple polygonal domains (or preferably, *art galleries*). Our goal is to cover any art gallery with a small number of polygonal domains such that each is visible from one point under a fixed angle $\alpha \in (0°, 360°]$ (called $\alpha$-guard, for obvious reasons) within the art gallery. In such a covering, we say that the guards monitor the art gallery. Guards may be required to be stationed at vertices of the polygon, at most one at each vertex. The optimization problem, that is, finding the minimal number of guards for a given art gallery is known to be NP-hard. There is, however, a minimal number of $\alpha$-guards who can monitor any possible art gallery with a fixed number $n$ of vertices. As a function of $n$, we investigate this minimum and show recent results and, sometimes, the optimal solutions.

In Chapter 2, we are given $n$ disjoint obstacles in the plane. Our task is to illuminate either all boundary points of the obstacles, or all points not covered by the obstacles using point-like light sources. Finding a minimal set of light sources, or even finding the minimal number of necessary light sources is NP-hard. The known reductions to the 3-SAT or the set-cover cover problem are analogous to those for art gallery problems. Here, again, we present a worst case analysis, showing that a certain number of light sources –as a function of the number $n$ of obstacles – suffice for any set of disjoint obstacles of one type (e.g. triangles, line segments). In certain cases, we are able to find the worst case optimal solution. Interestingly enough, this simple illumination problem splits into several slightly different versions once the obstacles in question are

line segments or curves in the plane. One version was considered and solved by J. O'Rourke, we address or show some progress on the others (and solve three of them).

Chapter 3 deals with a given set of disjoint line segments in the plane, too. The plane is partitioned by the *binary space partitioning*, or glass cutting algorithm along line segments. The partitioning segments are not necessarily *visibility segments*; i.e., they may cross some given segments but each such crossing costs one unit. We do not aim at finding the partitioning with minimal cost, but give new upper and lower bounds on the worst case cost of a partitioning as a function of the number $n$ of given line segments and the number $k$ of distinct orientations of the segments.