Search for cosmic ray point sources and anisotropy measurement with the L3+C experiment

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Search for Cosmic Ray Point Sources and Anisotropy Measurement with the L3+C Experiment

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presented by

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Abstract

The precise muon spectrometer of the L3 detector at the LEP accelerator (CERN) is used in the frame of the L3+C experiment as an underground muon telescope. In this work the recorded cosmic muon events are analyzed, in order to examine the presence of deviations from isotropy in the primary flux of cosmic rays. Two aspects are considered: point sources and large scale anisotropy.

The search for point sources includes a survey of the full “visible” sky, a special analysis of some selected bright $\gamma$-sources and a search for periodic signals from Cyg X-3. The point source search is performed on different time scales: 1 day, selected periods, 1 year or 2 years. Muons are selected according to four different energy thresholds ranging from 20 GeV to 100 GeV. The three most relevant candidate signals found in the sky survey are studied in detail. No evidence for signals from point sources is found and upper flux limits of muons are obtained.

Concerning the large scale anisotropy, a harmonic analysis of the variation of the cosmic ray flux along the equatorial component of the sky is performed with the same muon energy cuts as applied to the point source search. Sensitivities of the order of $10^{-4}$ on the relative variations are reached. Indications for significant variations of the cosmic ray flux as a function of the direction to the Sun position (solar anisotropy) are found with the lowest muon energy cut (20 GeV). Analogies between our result and a recent measurement of the solar anisotropy made by the GRAND experiment at 100 MeV energy are observed. No significant correlation is found between the muon flux and the right ascension of the arrival direction (sideral anisotropy).
Riassunto

Il preciso spettrometro di muoni del rivelatore di L3 situato presso l’acceleratore LEP (CERN), è utilizzato nel contesto dell’esperimento L3+C come telescopio sotterraneo di muoni. Nel presente lavoro gli eventi di muoni cosmici, che sono stati registrati, sono analizzati, al fine di ricercare la presenza di anisotropie nel flusso primario dei raggi cosmici. Due aspetti sono presi in considerazione: le sorgenti puntiformi e l’anisotropia su larga scala.


Per quanto riguarda l’anisotropia su larga scala, si esegue un’analisi armonica delle variazioni del flusso di raggi cosmici lungo la componente equatoriale del cielo, ponendo gli stessi limiti sull’energia, utilizzati nella ricerca di sorgenti puntiformi. Si ottengono delle sensibilità dell’ordine di $10^{-4}$ sulle variazioni relative. Utilizzando il limite sull’energia più basso (20 GeV), si riscontrano delle variazioni significative del flusso di raggi cosmici in funzione della direzione rispetto alla posizione del sole (anisotropia solare). Si osservano delle analogie tra questo nostro risultato e una recente misurazione dell’anisotropia solare effettuata dall’esperimento GRAND ad un’energia di 100 MeV. Non è stata trovata per contro alcuna correlazione significativa fra il flusso di muoni e l’ascensione retta della direzione di provenienza (anisotropia siderale).
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Chapter 1

Introduction

1.1 The L3+C Experiment

The properties of the precise muon spectrometer of the L3 experiment [1], which has been originally conceived to measure the particles produced by the electron-positron collisions of the Large Electron Positron Collider (LEP) at CERN in Geneva, offers also an original opportunity for the measurement of cosmic ray muons. For this purpose, an additional timing detector device and an independent trigger and read-out system had to be installed, originating a new underground cosmic ray experiment called L3+C [2] [3] (or L3+Cosmics). The detector is placed under 30 meters of overburden (about 70 meters of water equivalent) of very well known composition. This is enough to screen the detector from the electromagnetic and the hadronic component of the cosmic rays. Muons above the energy threshold of about 15 GeV trigger the detector. The error of the momentum measurement is of a few percent at 45 GeV and increases nearly linearly with energy. The angular uncertainty of the muon direction is dominated by the multiple scattering in the overburden and is better than 1° above 50 GeV. In the last year (2000) of data-taking an air shower scintillator array has been installed on the roof of the hall above the L3 experiment, allowing to measure the air shower size (figure 1.1). This offers a unique opportunity to measure simultaneously the primary energy of the showers, the multiplicity and the momentum distribution of the associated muons which hit the muon spectrometer. The main interest of such measurement is its sensitivity to the controversial primary nuclear composition around $10^{15}$ eV.

The main goal of the L3+C experiment is the measurement of the momentum spectrum of cosmic ray muons between 20 and 2000 GeV (including its angular dependence and charge ratio) with an uncertainty of a few percent. The main interest of this measurement is that it allows to set the normalization of the calculated atmospheric muon neutrino spectrum above 10 GeV. Among other topics are the shadowing effect of the Moon, exotic events, as well as the point source search and the anisotropy measurement. These last two physics topics are the subjects of the work presented here.
Figure 1.1: The final configuration of the experiment consists of two parts: the underground muon spectrometer and an air shower scintillator array which is installed on the roof of the hall above the L3 experiment.

1.2 Goals of this work

When analyzing the arrival direction of cosmic rays a basic question arises. Are the primary cosmic rays which hit the atmosphere isotropic or are there preferred directions causing inhomogeneities? Our main goal is to use the L3+C underground detector as a muon telescope and to measure the deviations of the primary fluxes from isotropy, taking advantage of the pointing precision of the muon detector and of the momentum measurement which allows to set different energy thresholds. Two aspects of deviation are considered here: large scale anisotropy and point sources.

Already from experimental results obtained in the 1930s (just after the discovery of the cosmic rays) it was known that the primary cosmic rays are very isotropic (see for instance [4]). The magnitude of the large scale anisotropy is below the permil level. This difficult measurement needs therefore a good understanding of the performance of the detector (in particular its stability in efficiency), as well as the environmental (atmospheric) effects. This represents a challenge which involved many experiment in the past, whose results and interpretations were often controversial. The degree of anisotropy of the primary cosmic rays is an observable property, which, together with the primary spectrum and the primary composition, represents one of the pieces of the mosaic which allows to get a picture of the production and propagation mechanism of cosmic rays. The importance of the anisotropy measurement in constraining the possible propagation models is pointed out in [5].

The method applied in this work for the large scale anisotropy measurement (chapter
5) is based on the fact that a detector located on Earth, scans the sky along the equatorial component (Right Ascension). Our goal is finally to perform a measurement of the first harmonics of the flux variations along this component. A precise knowledge of the angular dependent efficiency is not important in this context. What matters is indeed the stability of the detection efficiency. Therefore a careful data selection and a performance analysis with emphasis on the stability of the detector efficiency is carried out (chapter 4). In the measurement of the anisotropy, we can take advantage of the large amount of events registered by L3+C (in total 12 milliard), so that a very small statistical uncertainty can be reached. The precise livetime counter allows to take into account correctly all the interruptions of the data-taking. The main handicap of our measurement with respect to other experiments, is a relatively short data-taking period, which does not cover all the seasons of the year. The consequences of this point are discussed.

The experience gained with the analysis of the large scale anisotropy and through the study of the stability and of the data selection is also of benefit for the point source search performed in chapter 6. The same ideas of the sky’s scanning used for the large scale anisotropy is also the basis of the method to determine the background muon flux, which is needed in the search for signals from point sources. A survey of the whole “visible” sky is performed on different time scales and for different energy thresholds. An additional special analysis in the direction of 10 well known γ-sources is carried out.

The experience gained with the analysis of the large scale anisotropy and through the study of the stability and of the data selection is also of benefit for the point source search performed in chapter 6. The same ideas of the sky’s scanning used for the large scale anisotropy is also the basis of the method to determine the background muon flux, which is needed in the search for signals from point sources. A survey of the whole “visible” sky is performed on different time scales and for different energy thresholds. An additional special analysis in the direction of 10 well known γ-sources is carried out.

The existence of signals from a particular direction can only be explained by a source of neutral primary cosmic rays. In fact, the galactic magnetic fields govern and randomizes the trajectories of charged cosmic rays. In section 2.6.4 we discuss the expected fluxes of muons produced in showers induced by photons of the brightest known continuous γ-sources. As a consequence of the small number of muons in γ-induced showers, it is expected that we are probably not sensitive to a continuous signal from these sources. However the detection of burst signals (e.g. from Gamma Ray Bursts) are not excluded [6]. In this context the L3+C experiment has two advantages. Firstly, it is sensitive to an unexplored energy region (∼ 10^2 GeV) in the gap between the satellite γ-ray experiments and the Čerenkov detector measurements. Secondly, compared to Čerenkov detectors that operate only during moonless clear nights, L3+C has an almost continuous data-taking and a much larger field of view.

Periodic burst signals from the direction of the γ-ray source Cyg X-3 were reported in the 80’s by underground muon detectors [7]. Such signals could be detected also by L3+C. Unfortunately it seems that in the last years Cyg X-3 calmed down: signals have no more been observed by underground detectors.

One should also not completely exclude the possibility that exotic neutral particles could be involved [8], [9]. An eventual observation of a muon signal from a particular direction could be induced in fact by a source of these particles and represent a signature of their existence.

The present work is completed with a phenomenological overview of cosmic rays (chapter 2), where emphasis is put on the subjects relevant to this work. In addition a description of the experimental set-up, the read-out electronics and the data processing through the event reconstruction programs (chapter 3) are described.
Appendix A contains a description of the Monte Carlo simulations used for the analysis. They include shower simulations of proton and \( \gamma \) induced showers, a full detector simulation as well as the tracking of the muons in the molasse. A collection of the yearly results of the anisotropy analysis is present in appendix B.
Chapter 2

Phenomenological overview of Cosmic Rays

2.1 Primary composition and spectrum

Primary composition

Cosmic rays consist of particles (called primaries) such as nuclei, electrons, photons, neutrons and neutrinos which cross the space outside the Earth’s atmosphere. Neutrons reaching the Earth origin mainly from the Sun. They could come from other (neighbouring) stars before decaying only if their energy exceeds $\sim 10^{14}$ eV. Some antiprotons are also detected and are interpreted as being generated from the interaction of cosmic-ray protons with the interstellar matter. Other exotic particles may be present. Even if no experimental evidence has been found till now, several hypothesis about their existence have been made (e.g. [8], [9]).

Figure 2.1 shows the measured relative abundances of the primary nuclei at 1 TeV/nucleus; a compilation of results from balloon- and satellite-experiments. The values are compared to the abundances of the elements in the solar system. Protons are the most frequent, followed by helium nuclei. Among the heavy elements there is a relevant part of iron. Interesting is the large relative abundance in the cosmic rays of the group Li, Be, B ($Z = 3-5$), which is explained by the fact that these elements are produced by spallation of heavier nuclei when traveling in the interstellar space. Knowing their abundances, one can estimate the quantity of matter $X$ traversed by cosmic rays, commonly called “grammage”. At 1 GeV/nucleon a value of $X = 7 - 9$ g cm$^{-2}$ has been found [11]. For higher energies this value decreases and its relation with the energy has been fitted with a power law with negative exponent $\beta$ between 0.3 and 0.6.

$$X \propto E^{-\beta} \quad (2.1)$$

Measurements of the abundances of radioactive secondary nuclei like $^{10}$Be, $^{26}$Al and $^{36}$Cl give information on the mean age of cosmic rays. Garcia et al. [12] found a value
Figure 2.1: Relative abundances of the primary cosmic nuclei compared with the solar abundances of the elements H to Ni [10].
Primary composition and spectrum

of $17 \ (^{\pm 24}_{8}) \cdot 10^6$ years for cosmic rays in the energy range 30-150 MeV/nucleon together with a mean interstellar density $<n_H>$ of $0.18 \ (^{+0.18}_{-0.11})$ nucleon/cm$^3$.

Primary spectrum

The differential all-particle primary spectrum in the energy range $10^{11} - 3 \cdot 10^{15} \text{ eV}$ can be well fitted with a power law with a negative exponent $\gamma \approx 2.7$

$$\frac{dI(E)}{dE} = I_0 \cdot E^{-\gamma}. \quad (2.2)$$

If the energy $E$ is expressed in unit of GeV, $I_0$ is approximately $3 \text{ cm}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^{-1}$. The number of particles with Energy larger than a given Energy $E_0$ is evaluated with the integral spectrum

$$I(E > E_0) = \int_{E_0}^{\infty} dE \ I_0 \cdot E^{-\gamma} = \tilde{I}_0 E_0^{-(\gamma-1)} \quad (2.3)$$

with $\tilde{I}_0 = \frac{I_0}{\gamma-1}$ expressed in units of cm$^{-2}$ s$^{-1}$ sr$^{-1}$.

The differential primary spectrum is shown in figure 2.2 multiplied by $E^{2.75}$. Below the energy of $3 \cdot 10^{14} \text{ eV}$ it has been measured directly by rocket, balloon and satellite experiments. Above this energy, the measurement has been obtained indirectly via extensive air shower detection.

Below 20 GeV the primary spectrum becomes time dependent, since the solar modulation and Earth’s magnetic field start to influence remarkably the intensity of the cosmic rays, which is anticorrelated with solar activities and dependent on declination and direction.

Around the energy of $3 \cdot 10^{15} \text{ eV}$ (called “knee” region) there is a change of slope in the power law spectrum (2.2) and the exponent becomes $\gamma \approx 3.08$. The chemical composition of the primary nuclei in this energy region is currently an important topic of discussion, because it could give constraints to the different models which have been developed to explain this steepening. The L3+C combined measurements of the extensive air shower array and the underground muon spectrometer can contribute to solve this controversy and the preliminary results are promising [14]. At about $10^{19} \text{ eV}$ (called “ankle” region), where the extragalactic cosmic rays are expected to be able to penetrate the galactic magnetic field (see section 2.3.1), the primary spectrum becomes again flatter.

Greisen-Zatsepin Kuzmin cut-off

Above an energy of $\sim 5 \cdot 10^{19} \text{ eV}$ (Greisen-Zatsepin Kuzmin cut-off) [15], [16] the protons start to interact with the cosmic background photons (microwave and infrared radiation) with the reaction

$$p + \gamma \rightarrow n + \pi^+, \ p + \pi^0.$$
Figure 2.2: Differential primary all-particles spectrum multiplied by $E^{2.75}$, where $E$ is the particle Energy. [13]

The mean free path of the protons decreases at these energies down to a few Mpc [17]. This seems to be in contradiction with the flattening of the spectrum in the “ankle” region. How this can be explained is still a challenging question.

For gamma rays a similar cut-off exists already at $\sim 10^{14}$ eV [15] (figure 2.3) at which the universe becomes opaque, since they produce an electron-positron pair interacting with the cosmic background photons $\gamma + \text{b.g.}$:

$$\gamma + \text{b.g.} \rightarrow e^+ + e^-$$

When considering gamma rays coming from very distant extra-galactic objects, the opacity becomes important for much lower energies ($\sim 20$ GeV for objects with cosmological redshift $z = 1$ [18]).

2.2 Air showers

2.2.1 Air showers development and components

When a high energy primary particle enters the atmosphere, it interacts with the air causing the production of secondary particles. These can decay or interact again with other air nuclei. The result of these reactions is an air shower. This cascade develops, till the energy of the secondary particles falls below the threshold for particle production. Its
energy will then gradually be lost mainly by ionization or by other radiative processes. If an air shower reaches the ground it is called extensive air shower.

Air showers can be divided into three components:

- The **hadronic component** is produced by the interaction of the primary or secondary hadrons with air nuclei.

- The **electromagnetic component** is the one containing the largest number of particles and is composed by high energetic photons, electrons and positrons. This component is generated mainly by the decay of neutral pions into two photons. These generate a quickly developing electromagnetic shower via electron-positron pair production, which produce new high energetic photons via Bremsstrahlung.

- The **muonic component** is the most penetrating part of the air shower and therefore called hard component. Muons are created by the decay of the charged mesons of
the hadronic component, mainly by pions. Below the critical energy of about 500 GeV they lose their energy primarily by ionization, with an energy loss of about 2 MeV g$^{-1}$ cm$^{-2}$. Very high energetic muons can still be measured by experiments at thousand meters underground.

Air showers can be divided into two types depending on the nature of the primary particle whether it is hadronic or photonic. If the primary particle is a photon the muonic component of the shower is strongly reduced or absent and the air shower is called muon poor (c.f. figure 2.4). For instance according to Fassò and Poirier [20], photons with a primary energy of $10^4$ GeV produce an average number of $3.18\pm0.07$ muons which can be measured at sea level accompanied by $121\pm6$ electrons. Indeed a primary proton of the same energy generates an average of $74.6\pm0.8$ muons and $39.3\pm1.8$ electrons detectable at sea level. Photons generate muons mainly via photoproduction of pions followed by $\pi \rightarrow \mu\nu$ decay. For energies larger than 1 TeV muon-pair production and production of heavy quarks (predominantly charm) with subsequent leptonic decay becomes increasingly important [21].

![Figure 2.4: Integral spectra of muons produced by $\gamma$-induced showers at radial distances smaller than 10 km from the shower center at 222 m above sea level, calculated with a Monte Carlo simulation by Fassò and Poirier [22]. Each curve refers to a different primary energy (given in GeV).](image)

**2.2.2 Relation between primary energy and muon energy**

The relation between the ground level energy cut $E_{cut}$ of vertical muons and the corresponding median primary energy $E_{0}^{\text{median}}$ has been calculated by Gaisser [23] and we
report his results in figure 2.5. For the energies of interest for L3+C we have
\[ E_{\text{median}} \approx 12 E_{\text{cut}}. \] (2.4)

![Figure 2.5: Ratio of median primary energy \( E_{\text{median}} \) to muon energy \( E_{\text{cut}} \) for vertical muons of energy \( E_\mu > E_{\text{cut}} \) [23]](image)

2.3 Propagation of Cosmic Rays in space

2.3.1 Galactic magnetic field

Evidence of the presence of galactic magnetic fields in the Milky Way was reported in 1949 with the discovery of the polarization of the light of distant stars, which is explained by the non-isotropic absorption of the light by particles of interstellar dust oriented by the magnetic field [24]. Since then different methods have been developed to study the galactic field of the Milky Way and during the last 25 years also magnetic fields of other galaxies were investigated. Looking at the polarized electron induced synchrotron radio emission of spiral galaxies at resolutions of 0.1 - 3 kpc, it has been well established that they normally possess a large scale magnetic field structure, coherent over scales of at least 1 kpc as reported by Beck [25] in a review on the recent developments on the galactic magnetism. In addition to the mentioned large scale magnetic field (which is known also as mean, or average, or regular magnetic field) there are small scale random fluctuations. These are caused by turbulences of the interstellar medium, which have sizes that don't exceed 100 pc [25]. Before these observations the presence of a large scale magnetic field was deduced from the high isotropy of cosmic rays [24].
A summary of the method used to measure the galactic magnetic field is reported by Heiles [26]. The magnetic field strength can be obtained with the measurement of the Zeeman splitting of the 21 cm hydrogen line of the HI regions\(^1\). Another method is based on the Faraday rotation of the plane of polarization of linearly polarized radio waves. The rotation angle \(\psi\) is proportional to the integral of \(n_eB_\parallel\) along the line of sight (where \(n_e\) is the electron density and \(B_\parallel\) is the component of the total magnetic field along the line of sight) and with the observed wavelength squared \(\lambda^2\). The quantity \(\psi/\lambda^2\), called rotation measure, is therefore sensitive to the mean magnetic field \(\overline{B_\parallel}\). Taking advantage of the significant improvements obtained in recent years of the measurement of linearly polarized radio waves, Faraday rotation measurements of the radiation emitted by nearby pulsars (within 2-3 kpc) give the most confident estimates concerning the large-scale galactic magnetic field near the solar system. In its proximity the average magnetic field is estimated to be 2 \(\mu\)G. Figure 2.6 shows the measured strength of the large-scale magnetic field measured at four different radial distances from the center of the Milky Way [27]. Since the solar system is close to a reversal of the galactic magnetic field, 4-6 \(\mu\)G is proposed by Beck et al. [25] to be a better representative value of the mean strength of the large-scale magnetic field of the Milky-Way. However, a value of 3 \(\mu\)G is commonly

\(^1\)HI regions are clouds of cold neutral hydrogen.
used in the recent literature. Reliable informations on the magnetic field of the Milky-Way at distances larger than 3 kpc are in fact not available and its large scale features are still uncertain. However models based on the general ideas of the large-scale fields of other galaxies and on the local measurements of the Milky Way’s field can be developed. Figure 2.7 shows the field structure of a recent model of the regular magnetic field in the galactic plane made by Staney [28]. The component perpendicular to the galactic plane is supposed to be absent in this model.

![Diagram of the magnetic field structure in the galactic plane](image)

**Figure 2.7:** Example of a model for the structure of the large-scale magnetic field of the Milky-Way in the galactic plane. Shown are the galactic longitude directions and the position of the Sun. The arrows give the direction and strength of the field. [28]

From this picture it becomes clear that the charged primary cosmic rays (with the exception of the extremely high energetic ones) are hold in the Milky-Way by the galactic magnetic field. Their direction is randomized, with the consequence of a very high isotropy. On the other hand the effect of an eventual anisotropy of extra-galactic origin is canceled.

The typical Larmor radius $r_L$ in parsec of a proton with energy $E$ in a magnetic field of $3 \mu$G is given by

$$r_L[\text{pc}] \approx \frac{E}{3 \cdot 10^{15}\text{eV}}.$$  \hspace{1cm} (2.5)

At primary energies where the L3+C muon spectrometer is mostly sensitive (i.e. $10^{11} - 10^{12}$ eV) this corresponds to a Larmor radius of the order of $\sim 10^{-4}$ pc ($\approx 200$ A.U.). Above a few times $10^{18}$ eV (“ankle-region”) protons leaving the Galactic plane perpendicularly could hardly be deflected back again [29].
2.3.2 Propagation and acceleration of cosmic rays

The origin of cosmic rays and their propagation is still controversial.

Acceleration may occur in one shot at the source (for instance through shock acceleration in supernova remanents). Alternative models predict that cosmic rays are accelerated during the propagation in space (Fermi acceleration\(^2\)\(^{[30]}\)). However this kind of acceleration seems to be unable alone to explain the primary power law spectrum at high energies \(^{[31]}\).

Different models of propagation have been proposed, which try to fit with the observational facts: power-law spectrum, primary composition, anisotropy measurement results, the mean age of cosmic rays of the order of \(\sim 10^7\) years (see section 2.1), and the grammage of matter traversed by the cosmic rays.

A nice review about the models of cosmic ray propagation can be found in \^[5\].

The Leaky Box Model has been in the past a very popular model. This model supposes that cosmic rays are homogeneously distributed inside a confinement region (galaxy) with reflecting boundaries and that they have a finite probability of escaping at each encounter with the boundaries.

Nowadays the diffusion models are the ones which collect most consensous. In these models cosmic-ray density gradients are assumed on galactic scale. The irregularities of the galactic magnetic field and the scattering allow cosmic rays to leave the Galaxy following a random walk (diffusion).

2.4 The coordinate systems used to define directions

A direction defined by an observer on the Earth is commonly described by the horizontal coordinate system (figure 2.8). The direction of a vector \(\vec{d}\) is expressed with the coordinates elevation \(h\) and azimuth \(\phi\). The first coordinate represents the angular distance between \(\vec{d}\) and the horizontal plane and the latter the angle between the projection of \(\vec{d}\) on the horizontal plane and the North cardinal point measured eastward. This system is left-handed. Instead of the elevation the complementary angle \(\theta = 90^\circ - h\), which is called zenith angle, is often used. To describe the position of the celestial objects in the sky, it is worthwhile to use the equatorial coordinate system (figure 2.9), which is a spherical system based on the Earth’s axis of rotation. The direction of the vector \(\vec{d}\) is described by the coordinates declination \(\delta\) and right ascension \(\alpha\). The declination is the angle between \(\vec{d}\) and the equatorial plane of the Earth. The right ascension is the angle between the projection of \(\vec{d}\) on the equatorial plane and the vernal point \(\Upsilon\) (i.e. the position of the Sun at the spring equinox). This system is right-handed.

\(^2\)The principle of Fermi acceleration can be explained as follows. Charged cosmic ray particles encounter plasma clouds with random velocities and interact with them with the possibility of changing the velocity direction. The kinetic energy is conserved in the reference frame of the plasma cloud (supposing static magnetic fields). However seen from the galactic reference frame, after the interaction with the plasma, particles can gain or lose energy. Statistically the probability of increasing the kinetic energy is larger, so that particles undergo a positive average acceleration.
If one knows the horizontal coordinates azimuth $[\theta, \phi]$ and needs to transform them into the equatorial coordinates $[\delta, \alpha]$, the first step to do is to rotate the horizontal system into the so called local equatorial system. The local equatorial coordinate system $[\delta, h.a.]$ is also based on the Earth’s axis of rotation but it is fixed with respect to the horizon and it is conventionally left-handed. Its latitude corresponds to the above defined declination $\delta$.

The longitude $h.a.$ (hour angle) is given by the size of the angle projected on the equatorial plane between $\vec{d}$ and the south direction. The two coordinates can be calculated as:

\[
\delta = \arcsin \left( \cos(\lambda) \sin(\theta) \cos(\phi) + \sin(\lambda) \cos(\theta) \right) \quad (2.6)
\]

\[
h.a. = \arg \left[ -\sin(\lambda) \sin(\theta) \cos(\phi) + \cos(\lambda) \cos(\theta), -\sin(\theta) \sin(\phi) \right] \quad (2.7)
\]

where $\lambda$ is the geographical latitude of the observing place. For the L3+C experiment $\lambda = 46^{\circ}15'07''$.

For practical reason we define the variable $h_n$ to be the negative hour angle $h_n = -h.a.$ (2.8)

so that the systems $[\delta, h_n]$ and $[\delta, \alpha]$ have the same orientation (right-handed). The variable $h_n$ will be used extensively in section 5.1.

The hour angle, as well as the right ascension, are usually given in hours ($1$ hour = $15^{\circ}$), even if, when writing the formula, we will express them in radians.

To transform the local equatorial system into the equatorial system one needs to make a parity transformation with respect to the vertical North-South plane followed by a rotation around the Earth’s axis of an angle corresponding to the local sidereal time $t_s$ (that in the following we will call simply sidereal time). The declination given in the two systems is the same. The right ascension is found from the relation

\[
[\alpha] = [t_s - h.a.] = [t_s + h_n],
\]

\[
(2.9)
\]

Figure 2.8: The horizontal coordinate system.
where the square brackets mean modulo $2\pi$. The sidereal time $t_s$ expresses the measure of the phase of the rotation of the Earth’s with respect to the equatorial system, whose period is called sidereal day. Since the number of sidereal days in one year is one more than the number of solar days, one sidereal day correspond to $365\frac{24}{366} = 365\frac{24}{366} = 365.24$ solar days.

Finally we describe the \textit{galactic coordinate system} (figure 2.10), whose coordinates are the galactic latitude $b_{\text{gal}}$ and the galactic longitude $l_{\text{gal}}$. The galactic latitude is $0^\circ$ on the galactic plane of the Milky Way and is defined in such a way that the Earth’s North pole direction has positive latitude. The galactic longitude is defined to be $0^\circ$ in the direction of the galactic center as seen from the solar system. The solar system is rotating around the galactic center in the direction $l_{\text{gal}} = 90^\circ$. 

Figure 2.9: The equatorial coordinate system.

Figure 2.10: The galactic coordinate system.
2.5 Anisotropy of the cosmic ray flux

Taking into account the angular resolution and the duration of the L3+C experiment the direction of the objects outside the solar system can be considered as fix with respect to the equatorial and to the galactic system. Therefore we say that the two systems are fixed with respect to the sky.

The sideral time as well as the coordinate transformations are calculated with the help of the Fortran library *slalib* [32].

A summary of the mentioned coordinate systems is reported in table 2.1. Further informations about positional astronomy can be found in [33].

<table>
<thead>
<tr>
<th>system</th>
<th>longitude</th>
<th>latitude</th>
<th>equator</th>
<th>long. zero</th>
<th>RH/LH</th>
</tr>
</thead>
<tbody>
<tr>
<td>horizontal</td>
<td>azimuth ($\phi$)</td>
<td>elevation($h$)</td>
<td>horizontal</td>
<td>North</td>
<td>Left-hand</td>
</tr>
<tr>
<td>equatorial</td>
<td>right asc. ($\alpha$)</td>
<td>declin.($\delta$)</td>
<td>Earth’s eq.</td>
<td>Vernal Pt. T</td>
<td>Right-h</td>
</tr>
<tr>
<td>local equat.</td>
<td>hour ang. ($h.a.$)</td>
<td>declin.($\delta$)</td>
<td>Earth’s eq.</td>
<td>South</td>
<td>Left-hand</td>
</tr>
<tr>
<td>galactic</td>
<td>gal.long. ($l_{gal}$)</td>
<td>gal.lat.($b_{gal}$)</td>
<td>gal.equat.</td>
<td>gal.center</td>
<td>Right-h</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of the used coordinate systems.

2.5 Anisotropy of the cosmic ray flux

2.5.1 Definitions and harmonical analysis of the anisotropy

Solar and sideral anisotropy

Let’s consider the intensity of the primary cosmic rays as a function of the equatorial coordinates $I(\alpha, \delta)$ averaged on a long time period (preferably $\geq 1$ year). The fact that the function $I(\alpha, \delta)$ is not constant is called *sideral anisotropy* or simply *anisotropy*. If $I(\alpha, \delta)$ is affected by large scale variations, the detection rate of cosmic rays changes depending on which portion of the sky a detector is looking at. The rotation of the Earth allows an experiment on Earth to scan the sky in the right ascension direction and its detection rate will be affected by a modulation with a period of one sideral day.

If the intensity of the primary cosmic rays depends on the direction relative to the Sun, a modulation with the period of one solar day is noticed. This phenomenon is called *solar anisotropy*.

Measure of the anisotropy

The measure of the degree of anisotropy is often defined as [24],[29]

$$\delta_a = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \tag{2.10}$$

where $I_{\text{max}}$ and $I_{\text{min}}$ are the maximum and the minimum intensity, measured in all different directions. It is possible to measure this quantity only if the angular dependence of
the intensity is assumed to be smooth enough. A very sharp peak could result in \( \delta_a \to 1 \),
even if experimental results give a much smaller \( \delta_a \), due to their finite angular resolution.
Therefore the definition given by equation (2.10) has to be understood in an average sense
and should be related with a given resolution.

To express the direction dependence of the sidereal anisotropy we can define a function
\( \delta_{\text{dir}}(\alpha, \delta) \) for a given primary energy \( E_0 \) as the relative difference of the intensity \( I(\alpha, \delta) \)
from the mean intensity \( <I> \)

\[
\delta_{\text{dir}}(\alpha, \delta) = \frac{I(\alpha, \delta) - <I>}{<I>}
\]  

(2.11)

where \( \alpha \) and \( \delta \) are the equatorial coordinates right ascension and declination. As Király
et al. [29] we call \( \delta_{\text{dir}}(\alpha, \delta) \) the anisotropy function.

Harmonical analysis of the anisotropy

Let’s expand the direction dependent intensity \( I(\alpha, \delta) \) in spherical harmonics

\[
I(\alpha, \delta) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \zeta_{\ell m} Y_{\ell m}(\frac{\pi}{2} - \delta, \alpha).
\]  

(2.12)

\( \zeta_{\ell m} \) are the coefficients of the expansion and \( Y_{\ell m} \) are the spherical functions

\[
Y_{\ell m}(\theta, \phi) = \sqrt{\frac{2\ell + 1}{4\pi} \frac{(\ell - m)!}{(\ell + m)!}} P_{\ell m}^m(\cos \theta) e^{im\phi}
\]  

(2.13)

where \( P_{\ell m}^m(x) \) are the associated Legendre functions [34]. The expansion of the corresponding anisotropy function is

\[
\delta_{\text{dir}}(\alpha, \delta) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \epsilon_{\ell m} Y_{\ell m}(\frac{\pi}{2} - \delta, \alpha).
\]  

(2.14)

with

\[
\epsilon_{\ell m} = \frac{\zeta_{\ell m}}{\zeta_{00}} = \frac{\zeta_{\ell m}}{<I>}
\]  

(2.15)

The goal of the large-scale anisotropy analysis is to find the first few terms of the spherical harmonics expansion 2.14 (up to \( \ell \leq k \)). Király et al. [29] define the anisotropy of \( \ell \)-th order to be the maximum absolute value assumed by the \( \ell \)-th term of the spherical harmonics expansion 2.14 of the anisotropy function. If the anisotropy is of dipole type

\[
\delta_{\text{dir}}(\alpha, \delta) = \sum_{m=-1}^{1} \epsilon_{1m} Y_{1m}(\pi - \delta, \alpha)
\]  

(2.16)

the anisotropy of 1st order (\( \ell = 1 \)) corresponds to the anisotropy \( \delta_a \) defined in equation (2.10). An anisotropy of dipole type is called unidirectional [35] (figure 2.11.a) and is
present in the case that there is a uniform net flow of cosmic rays in a given direction. This is equivalent to the situation in which cosmic rays are isotropic in a particular frame and the Earth is moving with respect to that frame, as described by the Compton-Getting Effect, which will be discussed in the next section.

As it will be seen in more details in chapter 5.2, precise results on the large scale anisotropy measurements can be achieved only by scanning in the right ascension direction a band with a fixed declination range, taking advantage of the Earth’s rotation. Otherwise the systematical error dominates on the amplitude of the anisotropy. Therefore we can measure the equatorial component of the anisotropy $\delta_{\text{dir}}(\alpha)$ as a function of the right ascension $\alpha$ for a band with mean declination $\theta$.

$\delta_{\text{dir}}(\alpha) = \frac{I(\alpha) - <I>}{<I>}$ (2.17)

We have to remark that the right ascension is often identified with the sidereal time $t_s$ (see section 5.2) and instead of $\delta_{\text{dir}}(\alpha)$ the analysis of most of the experiments is based on the measure of

$\delta_{\text{dir}}(t_s) = \frac{I(t_s) - <I>}{<I>}. \quad (2.18)$

Information on the large scale anisotropy are obtained extracting the first few terms of the Fourier series expansion of the anisotropy function $\delta_{\text{dir}}(\alpha)$

$\delta_{\text{dir}}(\alpha) = \sum_{m=1}^{\infty} \xi_m \cos(m(\alpha - \phi_m)) \quad (2.19)$

Of course, even if such measurements are performed at very different declinations, they will not be sufficient to describe the full spatial structure of the anisotropy function. In particular the components with $m = 0$ of the spherical harmonics expansion (2.14), which contribute to the polar component, are not detectable.

The interpretation of the results of the Fourier expansion (eq. 2.19) is simplified, if one assumes that the anisotropy function is axially symmetric, for instance with respect to the local interstellar magnetic field direction (see [36], [29]). In this case the spherical harmonic expansion (2.14) of the anisotropy function simplifies to

$\delta_{\text{dir}}(\alpha, \delta) = \sum_{\ell=1}^{\infty} \eta\ell P_{\ell}^0(\cos \psi) \quad (2.20)$

where $\psi$ is the polar angle measured from the axis of symmetry, i.e. the pitch angle in case that the latter corresponds to the direction of the local interstellar magnetic field. If only the second term ($\ell = 2$) of the spherical expansion (2.20) is present we have a bidirectional anisotropy [35] (figure 2.11.b). If $\eta_2 > 0$ the intensity reaches its maximum value $I_{\text{max}}$ in the 2 opposite directions of the symmetry axis and it varies as

$I(\psi) = I_{\text{min}} + (I_{\text{max}} - I_{\text{min}}) \cos^2 \psi. \quad (2.21)$
If \( \eta_2 < 0 \) the maximum intensity is detected on the plane perpendicular to the symmetry axis and

\[
I(\psi) = I_{\text{max}} - (I_{\text{max}} - I_{\text{min}}) \cos^2 \psi.
\] (2.22)

Cutler et al. [36] describe how the coefficients \( \eta_\ell \) of the expansion in Legendre functions (2.20) are related with the amplitudes \( \xi_i \) of the Fourier series (2.19). Let \((\alpha_{\text{sym}}, \delta_{\text{sym}})\) be the direction of the symmetry axis expressed in equatorial coordinates and \( \delta \) the mean declination of the band which is scanned. Taking into account that the pitch angle \( \psi \) is given by

\[
\cos \psi = \sin \bar{\delta} \sin \delta_{\text{sym}} + \cos \bar{\delta} \cos \delta_{\text{sym}} \cos(\alpha - \alpha_{\text{sym}})
\] (2.23)

one can find the coefficients \( C_{m\ell} \) relating \( \xi_m \) and \( \eta_\ell \) through

\[
\xi_m = \sum_{\ell = m}^{\infty} C_{m\ell} (\delta_{\text{sym}}, \bar{\delta}) \eta_\ell.
\] (2.24)

The first few coefficients are given by

\[
\begin{align*}
\xi_1 &= \eta_1 \cdot \cos \delta_{\text{sym}} \cos \bar{\delta} + \eta_2 \cdot 3 \cos \delta_{\text{sym}} \sin \delta_{\text{sym}} \cos \bar{\delta} \sin \bar{\delta} + \ldots \quad (2.25) \\
\xi_2 &= \eta_2 \cdot \frac{3}{4} \cos^2 \delta_{\text{sym}} \cos^2 \bar{\delta} + \ldots \quad (2.26)
\end{align*}
\]

within corrections of order \( \eta_\ell^2 \). Furthermore all harmonics of \( \delta_{\text{dir}}(\alpha) \) should have the same phase \( \phi_i = \alpha_{\text{sym}} \) modulo \( \frac{2\pi}{m} \) if the signs of \( \xi_i \) are chosen in the proper way. The equivalence of the phases of the different harmonics measured at different declinations \( \bar{\delta} \) would therefore support the axial symmetry of the anisotropy function.

We note that in case of unidirectional anisotropy only the amplitude \( \xi_1 \) of the first harmonic of \( \delta_{\text{dir}}(\alpha) \) is different from 0. In this case it is not possible to extract the declination information of the direction of the symmetry axis \( \delta_{\text{sym}} \), even if bands of different declinations are analyzed.
Indeed for the bidirectional anisotropy one can find both first and second harmonic components. In the case that second or higher harmonics are present (as suggested by the experimental results, e.g. [36], [37]) the different amplitudes $\xi_n$ measured at different declinations would in principle allow to extract the declination information of the symmetry axis $\delta_{sym}$ and the coefficients $\eta_n$. Therefore results of experiments performed at distant latitudes are of great interest.

### 2.5.2 Compton-Getting Effect

The first attempts to study the anisotropy of the cosmic rays were already carried out in the 1930s. Compton and Getting [38] made a prediction based on the rotational speed of the Solar System around the center of our galaxy, which was known at that time to be $275 \pm 50$ km/s (in agreement with recent measurements [39]) in a direction with right ascension $20 \text{ h } 40 \text{ min}$ and declination $+47^\circ$.

Assume that the cosmic rays are isotropic in the frame of the center of mass of the Milky Way. Let’s move with velocity $\vec{v}$ with respect to this frame and let’s observe ultrarelativistic cosmic rays under an angle $\theta$ with respect to $\vec{v}$. If we consider non relativistic frame velocities $|\vec{v}| = \beta c \ll c$, the Doppler effect on the Energy $E'$ in the frame with velocity $\vec{v}$ is given by

$$E' = \frac{E}{1 - \beta \cos(\theta)}. \quad (2.27)$$

In addition to that, in the $\vec{v}$ direction, we would of course observe more particles than in the opposite direction. Compton and Getting showed in their paper that the ratio of the flux $\Phi'$ of particles in the moving frame and the flux $\Phi$ in the rest frame is

$$\frac{\Phi'}{\Phi} = \frac{1}{(1 - \beta \cos(\theta))^3}. \quad (2.28)$$

After correction for atmospheric and Earth’s magnetic field effects, they predicted that due to the motion of solar system with respect to the center of the Milky Way, the intensity variation of the cosmic rays measured at sea level should be roughly 0.1%, within a factor of 2. This amplitude, as well as the expected phase (maximum at $\alpha = 20 \text{h } 40 \text{min}$) was in agreement with the measurement of the intensity as a function of sidereal time performed by Hess and Steinmauer [4].

After the discovery of the presence of the galactic magnetic field the interpretation given by Compton and Getting had to be abandoned. However the principles expressed by equations (2.27) and (2.28) are still valid and of interest. They represent the basis for the description of the so called Compton-Getting Effect, which is described in the following paragraph.

Let’s assume isotropic cosmic ray fluxes with an integral primary spectrum according to equation (2.3) in a particular frame. If we move with respect to that frame, according to equations (2.27) and (2.28), we will have a direction dependent integral spectrum $I'$

$$I'(E' > E_0) = \frac{I_0}{(1 - \beta \cos(\theta))^3} \left[ E_0(1 - \beta \cos(\theta)) \right]^{-(\gamma - 1)} \quad (2.29)$$
In first approximation ($\beta \ll 1$) this is equivalent to
\begin{equation}
I'(E' > E_0) = I(E > E_0)[1 + (2 + \gamma)\beta \cos(\theta)],
\end{equation}
leading to a unidirectional anisotropy. In particular one can use equation (2.30) to calculate the effect of the orbital motion of the Earth around the sun, whose speed is 30 km/s. Substituting $\beta = 10^{-4}$ in equation (2.30) one obtains that the fluctuations around the mean of the muon flux due to the Earth’s orbital motion have an amplitude of 0.047%. This causes a corresponding modulation of the detection rate with a period of one solar day. The maximum flux is reached at around 6 a.m. when the vector $\vec{v}$ is pointing in the South direction and reaches its highest elevation.\footnote{An analogous, but larger effect can be seen when looking for meteors. The mean observation rate for similar sky conditions is higher in the second part of the night than in the first.}

\subsection*{2.5.3 Influence of the solar magnetic field on the anisotropy}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2_12.png}
\caption{Schematic view of the heliosphere and its interaction with the interstellar medium [40].}
\end{figure}

The heliosphere is the spatial region around the Sun where the solar wind is blowing and excluding the local interstellar medium (LISM). Since the latter is flowing with respect to the Sun, the heliosphere is expected to deviate from the spherical shape (figure 2.12).
According to recent measurements of the Doppler shifts of the interstellar Fe\II and Mg\II absorption lines made with the Hubble Space Telescope [41] the LISM moves in fact with a velocity of $26\pm1$ km/s with respect to the Sun in the direction of Galactic coordinates $l_{\text{gal}} = 186^\circ \pm 3^\circ$ and $b_{\text{gal}} = -16^\circ \pm 3^\circ$ (in equatorial coordinates $\alpha = 5h02\text{min}$ and $\delta = 15^\circ$). In the downstream region a long tail is expected, in which the solar plasma should extend to very large distances [42].

The solar wind plasma carry out the solar magnetic field, which pervades the full heliosphere. Its strength near the Earth is of the order of $10^2\mu$G [43] and falls approximately with the inverse of the distance from the Sun [36]. The magnetic field is influenced by the solar activity and reverse its polarity at each maximum of the activity, which occurs approximately every 11 years. The polarity of the Sun is said to be positive when the field direction is outward from the Sun in the northern hemisphere and inward in the southern hemisphere [44]. The L3+C data-taking occurred during a maximum of solar activity and the polarity was changing from positive to negative polarity.

A so called neutral sheet rotating with the Sun with a period of $\sim 27$ days separates the hemisphere where the field returns to the Sun from the hemisphere where it emerges from the Sun. At minimal solar activity the neutral sheet is almost flat and parallel to the solar equator with a small spiral wave structure (figure 2.13) whose amplitude is limited to $10^\circ$ of helio-latitudes. At increasing solar activity, towards the polarity reversal of the magnetic field, the neutral sheet can reach very high helio-latitudes and its structure becomes much more complex [45],[43].

![Figure 2.13: Artistic picture of the structure of the neutral sheet. The planetary orbits are also shown. Artist: Werner Heil-1977; Commissioning scientist: John M. Wilcox [40].](image-url)
As already mentioned in section 2.1 the changes of the heliosphere related with the solar activity influences remarkably the absolute intensities of the low energetic primary cosmic ray up to 20 GeV. However the effect of the heliosphere on the large scale anisotropy is important also at higher energies.

**Primary energy** \( \lesssim 10^{11} \text{ eV} \): According to Király et al. [29] below magnetic rigidities\(^4\) of \(10^{11} \text{ V}\) the Larmor radius in the heliosphere is so small that the effects of the galactic anisotropy are completely screened by the heliomagnetic field. The measured anisotropy at these magnetic rigidities has to be explained in terms of diffusion of cosmic rays inside the heliosphere as shown by Hall [44]. Several measurements were performed in this energy-range by neutron monitors (e.g. [46]) and ground level or shallow depth muon detectors (e.g. [47],[48],[49]). The results are strongly dependent on the solar activity and polarity [44] [50] and on the position of the Earth with respect to the neutral sheet [51]. Both sideral anisotropy and solar anisotropy are detected. The latter is present up to energies of \(3 \cdot 10^{11} \text{ eV}\) during the maximum of solar activity [35]. This limit reduces to \(10^{11} \text{ eV}\) during the periods of minimum activity.

**Primary energy range** \(10^{11} – 10^{12} \text{ eV}\): The interpretation of the anisotropy measurements in the energy range \(10^{11}\) to \(10^{12} \text{ eV}\) is the most controversial one, since the effect of the deflecting power of the heliomagnetic field is not well known. According to Jacklyn, deflection from the original direction of cosmic ray protons is expected to be noticeable at least up to magnetic rigidities of \(10^{12} \text{ V}\) [35]. The knowledge of the structure of the heliomagnetic field is not good enough to establish the asymptotic direction outside of the heliosphere. In addition to deflection, particles may be subject to gains or losses of energy in the electric fields associated with the solar wind, resulting in a variation of the intensity of cosmic rays [29].

Also in this energy range the sideral anisotropy presents modulations induced by the heliosphere. An annual variation of the sideral anisotropy which dominates at a magnetic rigidity of \(\sim 10^{12} \text{ V}\) has been observed [37]. This modulation has been seen even up to energies of 10 TeV by the Tibet Air Shower Experiment [52].

A change of phase in the first harmonic in correspondence with a solar field reversal has been reported by Cini-Castagnoli et al. [53] in the observation of underground muon sideral time variations at depths of 60 and 80 m.w.e in London and Turin. This has not been observed at higher energies.

### 2.5.4 Anisotropy outside of the heliosphere

The measured anisotropy above magnetic rigidities of \(\sim 10^{12} \text{ V}\) corresponds to the anisotropy present outside the Heliosphere, since the effect of the latter is strongly reduced and can be neglected.

\(^4\)The magnetic rigidity \(R\) of a given particle is defined as \(R = \frac{p \cdot c}{ze}\), where \(p\) is the particle momentum, \(c\) the speed of light and \(ze\) the particle charge.
Several explanations have been postulated for the origin of the anisotropy outside the heliosphere, first of all the effect of the motion of the solar system with respect to the local interstellar medium (LISM). If cosmic rays would be isotropic with respect to the latter, one would expect a unidirectional anisotropy inferred by the Compton-Getting effect. Király et al. [29] affirm that the interstellar magnetic field is probably frozen into the partially ionized gas of the LISM. In that reference frame the anisotropy could be expected to be dependent on the pitch angle only. In particular cosmic ray particles with small pitch angles have less chance to be reflected by the narrowing of the magnetic field lines and therefore can easily escape. This is know as the loss-cone effect. Following these ideas, Cutler et al. [36], after correcting their anisotropy measurements for the influence of the Compton-Getting effect related to the LISM, develop the interpretation of their data based on the axially symmetric anisotropy function (see equation 2.20).

In case of the presence of a gradient in the cosmic ray density $\rho$, a streaming in the direction of $\vec{B} \times \vec{\nabla} \rho$ is expected, where $\vec{B}$ is the magnetic field [54],[29] (The principle is explained schematically in figure 2.14). If $\rho$ increases in the direction of the center of the galaxy the maximum is expected towards the Galactic North Pol ($\delta = 27^\circ$, $\alpha = 12h51^\prime$).

Figure 2.14: Schematic representation of the streaming in the direction of $\vec{B} \times \vec{\nabla} \rho$. The circles represent the helices projected onto the plane normal to the magnetic field and their width indicate the number of particles involved. Adapted from [54].

**Degree of anisotropy**

It is possible to perform an estimation of the degree of the anisotropy of galactic origin based on the time spent in the galaxy by the cosmic rays, during which they can randomize
Phenomenological overview of Cosmic Rays

their direction. The degree of anisotropy is estimated in [29] as

$$\delta_a \approx \frac{t_{de}}{t_{conf}} \quad (2.31)$$

where $t_{de}$ is the time needed for a neutral particle to leave the galaxy and $t_{conf}$ is the confinement time of the charged particle. Following the same idea Hayakawa [54] express $\delta_a$ as the ratio of the thickness of the galactic disc $L(\sim 10^{21} \text{ cm})$ and the average path length $\lambda_p$. Since the latter is equal to the ratio of the grammage $X$ (defined in section 2.1) and the average density $\rho$ of the medium traversed by the cosmic rays, we get that

$$\delta_a \approx \frac{L}{X/\rho} \quad (2.32)$$

Using the values $X \sim 1 \text{ g/cm}^2$ at 1 TeV and $\rho \sim 3 \cdot 10^{-25} \text{ g/cm}^3$ one gets form equation (2.32) an anisotropy degree of $\delta_a \sim 3 \cdot 10^{-4}$.

Experimental results of the anisotropy at $E_{prim}10^{11} - 10^{14}$ GeV

A recent compilation of the measurements obtained by underground muon experiments and extensive air shower arrays of the first Fourier harmonics of the sidereal time variation of $\delta_{\text{dir}}(t_s)$ (defined in equation 2.18) as a function of the primary energy in the range $10^{11} - 10^{14}$ GeV has been done by Munakata et al. in [55]. This is reported in figure 2.15. The measured amplitudes tends to increase with the primary energy till 1 TeV from about $2 \cdot 10^{-4}$ to nearly $5 \cdot 10^{-4}$ and then they stay almost constant in the energy range $10^{12} - 10^{14}$ eV. From this constancy it could be deduced, taking into account equation (2.32), that the grammage $X$ stays constant in the concerned energy range [29] (i.e. $\beta = 0$ in equation (2.1)).

Concerning higher harmonics, statistically significant positive results have been observed in the second and third harmonics with the measurements of underground muons at the Mayflower Mine detector [36] (median energy $E_m \sim 1.5 \cdot 10^{12}$ eV) and with the EAS array at Mt. Norikura ($E_m \sim 1.5 \cdot 10^{13}$ eV) [37].

Experimental results of the anisotropy at $E_{prim}10^{14} - 10^{18}$ GeV

The amplitude of the first harmonic of the anisotropy function $\delta_{\text{dir}}(\alpha)$ (defined in equation (2.17)) increases in the “knee” region ($10^{15}$ eV $< E < 10^{16}$ eV) and reaches a level of a few percent. In addition the phase changes approximately from 0 hours to 12 hours. A recent compilation in the primary energy range $10^{14} - 10^{18}$ GeV has been made by Clay et al. [56]

It has been reported in some publications that muon-rich extensive air showers (i.e. with a muon content significantly higher than the average for a given primary energy) show significantly larger anisotropies than muon-poor and muon-average extensive air showers (e.g. [57] [58]). This means that heavy primary particles (which produce more muons than light primaries) should present a larger anisotropy than protons. Bressi et
Figure 2.15: Amplitude and phase of the measured first Fourier harmonics of the sidereal time relative variations as a function of the primary energy [55] (for references of the experiments see the original paper).

...al. [59], measuring $\sim 10^4$ muon bundles collected during one month with a ground level muon detector (with estimated primary energy of $\sim 10^{15}$ eV), claim to have measured an amplitude of $(4.9 \pm 1.2)\%$ of the first harmonic of the anisotropy function $\delta_{\text{dir}}(\alpha)$. If this result is correct L3+C could be able to see such an effect analyzing multi-muons. Observations of multi muons at the Poatina muon detector at a depth of 357 m.w.e. however found an amplitude of the first harmonic of only $(0.10 \pm 0.06)\%$ [60].
2.5.5 A new interpretation of the anisotropy measurements below 10 TeV

Recently Nagashima, Fujimoto and Jacklyn [61] proposed a new model (NFJ model) which gives a new interpretation of the sidereal anisotropy at energies below $10^{13}$ eV, based on anisotropy measurements sensitive to different declination intervals performed at locations with different latitudes (in both Southern and Northern hemisphere). These are shown to be compatible with two kinds of anisotropy. One is an anisotropy of galactic origin which consist of a constant deficit flux inside a cone of opening angle $\xi_G \cong 57^\circ$ around the direction with right ascension $\alpha_G = 12$ hours and declination $\delta_G = 20^\circ$. The other is given by a constant excess flux inside a cone with opening angle $\xi_T \cong 68^\circ$ around the direction with right ascension $\alpha_T = 6$ hours and declination $\delta_T = -24^\circ$ (figure 2.16). This anisotropy can be observed up to energies of $10^{12}$ eV. The latter anisotropy is interpreted as to be inferred by the presence of the heliospheric tail, and therefore is called tail-in anisotropy. Nagashima et al. propose that the origin of this excess could be the acceleration of cosmic rays inside the heliomagnetotail or preferential inward diffusion exclusively from the direction of the heliomagnetotail. It has to be pointed out that the direction $(\alpha_T, \delta_T)$ coincide approximately with the direction opposite to the proper motion of the Sun with respect to the neighboring stars ($\alpha = 6.0$ h, $\delta = -29.2^\circ$) and does
not correspond to the direction of motion of the local interstellar medium with respect to the solar system (see section 2.5.3). The tail-in anisotropy is affected by the annual modulation, whose maximum amplitude is reached near the winter solstice, when in its orbital motions around the Sun, the Earth is placed on the side next to the heliomagnetic tail (figure 2.17).

Figure 2.17: Schematic figure showing the direction of the heliomagnetic tail excess with respect to the orbital position of the Earth (according to the NFJ model [61]). The strongest effect on the anisotropy from the heliomagnetic tail is measured near to the winter solstice. The velocity direction of the solar system with respect to the neighboring stars \( v_{\text{star}} \) (proper motion) and the one with respect to the interstellar medium \( v_{\text{LISM}} \) are also represented, including their projection on the ecliptic plane.

The galactic anisotropy, called also loss-cone anisotropy, is observed even at primary energies down to \( \sim 60 \text{ GeV} \), if it is measured in a region out of the influence of the tail-in anisotropy or if the effect of the latter is taken into account. (We want to remark that the axis of the cone \( (\alpha_G, \delta_G) \) is directed toward the galactic North polar region. This could be in favor of a streaming in the direction \( \vec{B} \times \vec{\nabla} \rho \) (see section 2.5.4)).

Following the interpretation of the NFJ model the measurements of the anisotropy analyzed by Nagashima et al. are not compatible with a Compton-Getting effect caused by the motion of the solar system with respect to the local interstellar medium.

In the frame of the NFJ model, the energy range, in which the L3+C experiment is sensitive, gains of importance, in particular the low energetic range (few hundreds of GeV), which in the past didn’t attract the attention of many researchers, due to the difficulties of interpreting the influence of the heliomagnetic field.
2.6 Point sources

2.6.1 Primary particles of point sources

Since the trajectories of the charged cosmic rays are bent by the galactic magnetic field, only neutral primary particles can be observed as originating from point sources. The most natural are the $\gamma$s, that unfortunately generate air showers with a low content of muons (as already mentioned in section 2.2.1). The expected muon flux from known steady $\gamma$-sources will be discussed in section 2.6.4.

Neutrons are expected to be detected exclusively from Sun because of the decay. Neutrinos can be detected by experiments which are sensitive to up-going muons. Exotic neutral particles could play a role if they exist. They are postulated for instance in the theory of Supersymmetry (SUSY). In [9] for example, a quasi-stable neutral hadron containing a very light gluino is proposed as a candidate for the primary particles of cosmic ray events with energies above the Greisen-Zatsepin Kuzmin cut-off.

A neutral particle called Cygnet has been also postulated [8], when in the 80th some groups claimed to have seen an excess of muon rich showers from the direction of Cyg X-3 (see below).

2.6.2 The detection of $\gamma$-sources

$\gamma$-rays are detected up to energies of 20 GeV by satellite experiments. Above these energies statistics becomes too low for the small acceptance of satellite detectors. Higher energies have to be therefore explored from ground on larger surfaces. This is done mainly by means of the detection of Čerenkov light produced by secondary particles (in particular electrons) in air showers. Large ground-based optical telescopes are used for this purpose. Up to the year 2000, Čerenkov telescopes (HEGRA, CANGAROO, Whipple, CAT) were sensitive at primary $\gamma$'s with energies larger than 300 GeV (up to 100 TeV). The energy regime 20-300 GeV was thus completely unexplored. A lower detection threshold requires a much larger area for the collection of Čerenkov light. For this purpose two groups (STACEE and CELESTE) are currently using a solar-power station converted into a huge Čerenkov telescope. Their goal is to reach sensitivities down to energies of 30 GeV.

Čerenkov telescopes are able to operate only during moonless and clear nights and have a narrow field of view. Compared to Čerenkov telescopes, the detection of muons from $\gamma$ induced showers by L3+C is much less sensitive, but has the advantage to operate almost continously and to observe the entire overhead sky. This is of advantage in particular for the detection of burst signals. In this context the MILAGRO experiment [63] is our main competitor. It employs a large water reservoir instrumented with an array of photo-multipliers tubes, which detect the Čerenkov radiation produced by the secondary

In future (~ year 2006), the GLAST satellite telescope and the AMS experiment on the Alpha Space Station will be able to detect $\gamma$-photons up to about 300 GeV.

Particles with energies above 10 TeV can be detected by air shower scintillator arrays. The Tibet-III collaboration reported recently a multi-TeV signal with 4.8 $\sigma$ significance from the Crab pulsar [62].
shower particles. The MILAGRO detector, as well as L3+C, starts to become sensitive at $\gamma$-showers above 100-200 GeV.

### 2.6.3 Main types of cosmic $\gamma$-sources

**Active Galaxies**

Active Galaxies are among the most energetic and distant known objects in the Universe. Their luminosity is concentrated in the center of the galaxy and they are therefore commonly called *Active Galactic Nuclei* (AGNs). Many classes and subclasses of AGNs have been defined, based on the observational properties in the optical and radio wavelengths, rather than on their real nature. Finally about 20 years ago a unified AGN model was developed and widely accepted [64]. The central engine of AGNs is believed to be a super-massive black hole with mass corresponding to $\sim 10^7 - 10^{10}$ solar masses [65]. An accretion disk of hot matter is generated around the black-hole. Through a not well understood mechanism the black hole emits beams of high energy particles along its rotation axis, perpendicular to the galactic plane (figure 2.18). The numerous subclasses of AGN are explained to have origin from geometric effects due to different viewing angles. If a beam is directed to the Earth, high energetic (up to GeV or even TeV) $\gamma$ emission may be observed. In this case the AGNs are called *BLAZARs*.

![Figure 2.18: Artistic view of an Active Galactic Nuclei (AGN). [66]](image-url)
A special class of BLAZARs are the BL Lac objects whose feature is the absence or faintness of emission lines. The most high energetic $\gamma$-signals from AGNs have been detected from BL Lacs. The two BL Lac objects Markarian 501 (Mrk 501) and Markarian 421 (Mrk 421) have been detected by Čerenkov telescopes in the TeV regime.

**Pulsars**

Pulsars are rapidly spinning neutron stars with a strong magnetic field ($\sim 10^{12}$ G). The rotation period varies from a millisecond to a few seconds. With the same extremely regular period, pulsars emit short bursts of electromagnetic radiation in the whole spectrum from radio up to $\gamma$-rays. Even if many open questions about the physics of pulsars are present, the radiation is thought to be emitted mainly by the synchrotron radiation of the electrons accelerated in the electric field induced by the rotating magnetic field.

A particular category of pulsar are so called *plerions* [67] which are filled-center supernova remnants. Some plerions have been observed to present unpulsed TeV $\gamma$-ray emission. The most famous of them is the Crab-pulsar originating form a Supernova observed in the year 1054. In the so called self-Compton model [65], [68] one supposes that the production of TeV $\gamma$-rays arise from the inverse-Compton scattering of the synchrotron emitted $\gamma$-rays and the electrons accelerated in the shock front between the pulsar-wind and the surrounding nebula.

**X-ray binaries**

X-ray binaries consist of a system composed of a star and a neutron star orbiting at a short distance from each other. Matter originating from the companion star is accreted around the neutron star and spirals inward. When the magnetic forces dominate the matter flows along the field lines onto the magnetic poles. At this point so called “hot spots” are produced, which radiate X-rays.

As in the case of AGNs, jets of relativistic particles may be also produced on the rotation axis. These lead to high energetic $\gamma$-emission as observed for instance from the X-ray binaries Cyg X-3 and Her X-1.

**Cyg X-3:** Cygnus X-3 for example is thought to be a binary system composed of a red giant and a neutron star (figure 2.19). The system has an orbital period of 4.8 hours and it is placed at an estimated distance of about 37000 light-years. Taking into account the apparent intensity and the large distance, Cyg X-3 is deduced to be one of the brightest X-ray sources in the Milky Way ($2 \cdot 10^{37}$ erg/s). A dozen of these sources could produce the whole power of the galactic cosmic rays [69].

The first time that Cyg X-3 gained notoriety after its discovery in the 1966 as a simple X-ray source, was in the 1972 when its radiation intensity in the radio wavelengths increased by more than thousand times its normal intensity. During the 80th Cyg X-3 became again a hot topic. The Kiel group [70] observed with an air shower scintillator array and shielded counters an excess of showers from the direction of Cyg X-3 with a
Figure 2.19: Schematic picture of the Cyg X-3 system. The system is thought to be composed by a red giant and a neutron star. Matter from the red giant is accreted around the neutron star. High energetic particles (e.g. protons) are accelerated near the neutron star by an unknown mechanism. The particles emitted in the direction of the red giant, generate shower of secondary particles inside the atmosphere of the giant star. γ-rays or other exotic neutral particles produced in these showers can be detected on Earth when the line of sight of the neutron star is tangent to the atmosphere of the red giant.

significance of 4.4 σ. In addition the signal showed a periodicity of 4.8 hours as the orbital period. They derived a time-averaged integral flux of γ-ray photons of $(7.4 \pm 3.2) \cdot 10^{-14}$ cm$^{-2}$s$^{-1}$ above $2 \cdot 10^{15}$ eV and $(1.1 \pm 0.6) \cdot 10^{-14}$ cm$^{-2}$s$^{-1}$ above $10^{16}$ eV. A contemporaneous observation at Haverah Park reported a similar result [71]. Few years later signals with the period of 4.8 h from the deep underground observations of muons from the direction of Cyg X-3 were reported by the Soudan I collaboration [72] and by the NUSEX collaboration [73]. The Soudan I collaboration estimated muon fluxes from Cyg X-3 up to $10^{-9}$ cm$^{-2}$s$^{-1}$ above 650 GeV in the period of strongest activity [7]. Such high fluxes could have been detected also by the L3+C experiment (c.f. sections 6.2.2 and 6.2.3). Other experiments reported about TeV and PeV signals from Cyg X-3 in the 80th. Fly’s Eye, which detects air fluorescence induced by the air showers, claimed even evidence for signals up to $10^{18}$eV(!) [74]. A similar conclusion has been obtained by the Akeno group [75], too.

Unfortunately in the last decade it seems that Cyg X-3 calmed down. No further signal above the TeV energy regime has been seen by recent experiments.

---

7The lower energy threshold of L3+C has to be taken into account.
Gamma ray bursts

Another type of $\gamma$-ray source that should be mentioned, are so called Gamma Ray Bursts (GRBs).

Discovered by chance in the 1960s by American scientists searching for nuclear tests over the Soviet Union, GRBs represent one of the most challenging puzzle of modern astrophysics. GRBs are powerful bursts of $\gamma$-radiation reaching energies possibly up to 20 GeV [76]. Their duration varies from 1 ms to tens of seconds and even more. Recently, observations of counterparts in the optical spectrum revealed extragalactic origin (e.g. [77]). This implies that the amount of energy released by these objects should be extremely high ($>10^{53}$ ergs). Such energies are thought to be reachable from the merger of compact objects (neutron stars or black holes) or from the collapse of most massive stars (hypernovae). According to some authors GRBs may also originate from supernova jets directed toward the Earth [78].

2.6.4 Estimation of the number of muons expected from known $\gamma$-sources

Let’s write the differential spectrum of a $\gamma$-ray source with a power law spectrum through

$$\frac{dN_\gamma}{dE_\gamma} = F_\gamma \cdot E_\gamma^{-\gamma} \cdot 10^{-12} \text{cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}.$$  \hfill (2.33)

where $\gamma$ is the spectral index and $F_\gamma$ is the differential flux at 1 TeV in units of $10^{-12} \text{cm}^{-2} \text{s}^{-1} \text{TeV}^{-1}$. The energy of the photons $E_\gamma$ is expressed in TeV units.

Looking inside the third EGRET catalog [79] one discovers that the spectral index $\gamma$ of the differential spectrum of different gamma point sources at 100 MeV varies from about 1.6 to 3.2. However 2 is often considered as a typical spectral index, which is applied for the estimations of the flux of muons expected from the known $\gamma$-sources as done by Halzen et al. [80]. They parameterize this flux in units of cm$^{-2}$s$^{-1}$ for vertical muons observed by a detector with an energy threshold $E_{\text{cut}}$ included in the interval between 0.1 TeV and 1 TeV as

$$N_\mu(E_{\text{cut}}) \cong 2 \cdot 10^{-17} \frac{F_\gamma}{(E_{\text{cut}}/\cos \theta)} \ln \left( \frac{E_{\gamma_{\text{max}}}}{E_{\gamma_{\text{min}}}} \right) f,$$  \hfill (2.34)

where $\theta$ is the zenith angle of the direction of observation of the source and $f$ is a correction factor parameterized as

$$f = \left( \frac{E_{\text{cut}}/\cos \theta}{0.04} \right)^{0.53}.$$  \hfill (2.35)

Photons of energies between $E_{\gamma_{\text{min}}} = \frac{10 E_{\text{cut}}}{\cos \theta}$ and $E_{\gamma_{\text{max}}}$ are supposed to contribute to the production of the detected muons. One can note that the highest energy photons contribute remarkably on the measured muon flux.

To check equation (2.34) and to see if this can be applied also for energies down to 20 GeV we simulate about 4 millions of vertical showers in an energy range between 50
2.6 Point sources

GeV and 100 TeV ($\gamma = 2$) with the extensive air shower Monte Carlo simulation program CORSIKA [81] (see appendix A.1). Table 2.2 report the comparison between the results obtained with the Monte Carlo simulation and the results obtained with equation (2.34) for $F_\gamma = 1$. The simulated and calculated fluxes are in reasonable agreement.

<table>
<thead>
<tr>
<th>$\mu$-Energy cut [GeV]</th>
<th>$#\gamma$ $E_{\text{prim}} &gt; 50$ GeV</th>
<th>$#\mu$</th>
<th>$N_{\mu}^{\text{MC}}$ [cm$^{-2}$s$^{-1}$]</th>
<th>$N_{\mu}$ from eq.(2.34) [cm$^{-2}$s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.</td>
<td>3.98$\times$10$^6$</td>
<td>31157</td>
<td>(1.57$\pm$0.01)$\times$10$^{-13}$</td>
<td>2.2$\times$10$^{-13}$</td>
</tr>
<tr>
<td>30.</td>
<td>3.98$\times$10$^6$</td>
<td>18426</td>
<td>(9.26$\pm$0.07)$\times$10$^{-14}$</td>
<td>1.1$\times$10$^{-13}$</td>
</tr>
<tr>
<td>50.</td>
<td>3.98$\times$10$^6$</td>
<td>8739</td>
<td>(4.39$\pm$0.04)$\times$10$^{-14}$</td>
<td>4.8$\times$10$^{-14}$</td>
</tr>
<tr>
<td>100.</td>
<td>3.98$\times$10$^6$</td>
<td>2929</td>
<td>(1.47$\pm$0.03)$\times$10$^{-14}$</td>
<td>1.5$\times$10$^{-14}$</td>
</tr>
<tr>
<td>1000.</td>
<td>3.98$\times$10$^6$</td>
<td>37</td>
<td>(1.9$\pm$0.3)$\times$10$^{-16}$</td>
<td>2.5$\times$10$^{-16}$</td>
</tr>
</tbody>
</table>

Table 2.2: Summary of the results obtained by the Monte Carlo simulation of vertical gamma showers and comparison with fluxes resulting from equation (2.34). For different muon energy thresholds (first column) we report the number of simulated $\gamma$-showers with energy larger than 50 GeV (second column), the number of produced muons in the simulation (third column) and, assuming $F_\gamma = 1$, the flux of muons (with statistical error) obtained from the Monte Carlo simulation (forth column) as well as the flux of muons resulting from equation (2.34).

We calculate the fluxes expected from four bright sources which have been observed by TeV Čerenkov telescopes. In table 2.3 we report the four sources with their differential fluxes $F_\gamma$ measured by the Čerenkov telescopes and the extrapolations of $F_\gamma$ based on the measurements of the EGRET telescope ($E_{\gamma_{\text{max}}} \sim 30$GeV) which are reported in the Third EGRET catalog [79] assuming a spectral index $\gamma = 2$. The extrapolations of EGRET measurements were used some years ago by Halzen et al. [80] for their estimates of the expected muon fluxes. However, we see that the measurements obtained recently by the Čerenkov telescopes at 1 TeV can give results which differ by more than a factor 10. The results of the muon fluxes are reported in table 2.4.

We remark that the Vela pulsar is not in the field of view of the L3+C muon detector, but it is taken into account in the discussion because it is the brightest source seen by the EGRET telescope.

The spectral index $\gamma$ is often smaller than 2 at 100 MeV (see examples in table 2.3) but at 1 TeV the trend is that the spectrum becomes steeper with $\gamma > 2$. Therefore we stress that the calculations of the muon fluxes (based on $\gamma = 2$) are only approximate. An important parameter in equation (2.34) is the higher energy cut off $E_{\gamma_{\text{max}}}$ of the $\gamma$ rays. For the Crab and the Vela pulsar this is not established yet and we set it to 100 TeV. Mrk 421 and Mrk 501 are extragalactic objects and therefore for $\gamma$-rays above 10 TeV we have to take into account the opacity due to the presence of the intergalactic infrared radiation (see section 2.1 and [18]). Thus we set $E_{\gamma_{\text{max}}} = 10$ TeV for these two extragalactic sources.

Unfortunately, L3+C is not able to separate the small muon fluxes resulting in table
Table 2.3: Four bright sources seen by Čerenkov telescopes. Reported are the differential flux $F_\gamma$ at 1 TeV in units of $10^{-12}$cm$^{-2}$s$^{-1}$TeV$^{-1}$, the spectral index $\gamma$ as measured by Čerenkov telescopes (second and third column), and $F_\gamma$ extrapolated by the measurement at 100 MeV reported in the Third Egret Catalog [79] assuming $\gamma = 2$ (fourth column) and the spectral index at 100 MeV [79] (fifth column).

<table>
<thead>
<tr>
<th>Source name</th>
<th>$F_\gamma$ (C)</th>
<th>$\gamma$ (C)</th>
<th>$F_\gamma$ (EGRET)</th>
<th>$\gamma$(EGRET)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crab</td>
<td>20-30 ([82],[83],[84])</td>
<td>2.5-2.6</td>
<td>226.± 5</td>
<td>2.2</td>
</tr>
<tr>
<td>Mrk 421</td>
<td>$\sim$ 30 ([85])</td>
<td>2.9</td>
<td>13.9± 2</td>
<td>1.6</td>
</tr>
<tr>
<td>Mrk 501</td>
<td>50±20 ([86])</td>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vela</td>
<td>$\sim$ 30 ([87])</td>
<td>2.5</td>
<td>834.±11</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 2.4: Muon fluxes expected from four bright sources calculated with equation (2.34) using the $\gamma$-fluxes measured by Čerenkov telescopes.

<table>
<thead>
<tr>
<th>Source name</th>
<th>$\mu$-Energy cut [GeV]</th>
<th>$N_\mu$ [cm$^{-2}$s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrk 421</td>
<td>20</td>
<td>$0.41 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>$0.20 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>$0.81 \cdot 10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>$0.22 \cdot 10^{-12}$</td>
</tr>
<tr>
<td>Mrk 501</td>
<td>20</td>
<td>$0.70 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>$0.35 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>$0.14 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>$0.39 \cdot 10^{-12}$</td>
</tr>
<tr>
<td>Crab</td>
<td>20</td>
<td>$0.54 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>$0.28 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>$0.12 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>$0.37 \cdot 10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>$0.63 \cdot 10^{-14}$</td>
</tr>
<tr>
<td>Vela</td>
<td>20</td>
<td>$0.65 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>$0.33 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>$0.14 \cdot 10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>$0.45 \cdot 10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>$0.76 \cdot 10^{-14}$</td>
</tr>
</tbody>
</table>

2.4 from the background flux as it will become clear in chapter 6. But very bright bursts signals could indeed be detected [6].
2.7 Atmospheric effects on the muon rate

With increasing of air density, it becomes more probable that a charged meson interact with the air-nuclei instead of decaying into muons. Since the air density is a function of the atmospheric pressure $p$ and the temperature $T$, both quantities are expected to influence the muon flux. This is in fact observed in both ground level and underground measurements [88],[89].

2.7.1 Dependence on temperature

The definition of the effective temperature $T_{\text{eff}}$

To describe the temperature dependence of the flux of muons with zenith angle $\theta$, we introduce a new quantity called effective temperature $T_{\text{eff}}(\theta)$, which is a weighted average of the temperature $T(h)$ as a function of the height $h$ [90]:

$$T_{\text{eff}}(\theta) = \frac{\int_{0}^{\infty} dh \ w(h;\theta) T(h)}{\int_{0}^{\infty} dh \ w(h;\theta)} \quad (2.36)$$

The weight function $w(h;\theta)$ depends on the zenith angle $\theta$ and reaches the highest value in the stratosphere where it is most probable that a muon is produced (figure 2.20). The

![Figure 2.20: The weight $w(h;\theta = 0^\circ)$ as a function of the height $h$ used to calculate the effective temperature [90].](image)
weight function $w(h; \theta)$ is given by [90]

$$w(h; \theta) \sim e^{-\frac{X(h, \theta)}{\Lambda_\pi}} - e^{-\frac{X(h, \theta)}{\Lambda_N}}.$$  \hspace{1cm} (2.37)

$\Lambda_\pi = 160 \frac{\text{g}}{\text{cm}^2}$ and $\Lambda_N = 120 \frac{\text{g}}{\text{cm}^2}$ are the attenuation lengths of pion-air and nucleon-air interactions. $X(h; \theta)$ is the slant depth, which is defined to be the “grammage” of air traversed by an incident particle with a zenith angle $\theta$ down to the height $h$. Let’s approximate the density $\rho(h)$ as a function of the height $h$ with the barometric formula for the isothermal model of the atmosphere:

$$\rho(h) = \rho_0 e^{-h/h_0}$$  \hspace{1cm} (2.38)

where $\rho_0$ is the density of air at ground level and $h_0 = 8 \text{ km}$. If we neglect the Earth’s curvature the slant depth can be written as [91]:

$$X(h; \theta) = \frac{X_0}{\cos \theta} \cdot e^{-h/h_0}$$  \hspace{1cm} (2.39)

where $X_0$ is the pressure at ground level.

Relation between the muon flux and the effective temperature

At ground level the relative variation of the muon flux $\frac{\Delta \Phi}{\Phi}$ is predicted to be proportional to the relative variation of the effective temperature $\frac{\Delta T_{\text{eff}}}{T_{\text{eff}}}$, i.e. [92]

$$\frac{\Delta \Phi}{\Phi} = \alpha \frac{\Delta T_{\text{eff}}}{T_{\text{eff}}}$$  \hspace{1cm} (2.40)

where $\alpha$ is the temperature coefficient, which depends on the lower energy threshold $E_{\text{cut}}$ of the muons. Approximately [90]

$$\alpha = \left[ 1 + \frac{70 \text{ GeV}}{E_{\text{cut}} \cos \theta} \right]^{-1}$$  \hspace{1cm} (2.41)

The effective temperature during the L3+C data-taking period

Balloon measurements of MeteoSwiss in Payern (100 km North-East from CERN) are used to get the effective temperature during the L3+C data-taking period (figure 2.21)[93]. The measurements are performed every 12 hours (at 0h and 12h) or every 6 hours (at 0h, 6h, 12h, 18h). Pressure and temperature are measured as a function of altitude.

The effective temperature shows a remarkable seasonal variation, but is rather stable on time scales of few days. No day-night influence is observed.
Figure 2.21: Effective temperature in the vertical direction as a function of the day of the year during the L3+C data-taking. The values are taken from [93] and refers to Balloon measurements of MeteoSwiss in Payern (100 km away from CERN).
2.7.2 Pressure

The relative variation of the muon flux is also proportional to the variation of pressure $\Delta p$

$$\frac{\Delta \Phi}{\Phi} = \beta \cdot \Delta p$$  \hspace{1cm} (2.42)

where $\beta$ is the barometer coefficient. The theoretical prediction and the experimental results of $\beta$ as a function of the energy threshold can be found in [94]. On the contrary of $\alpha$, $\beta$ decreases, with increasing energy threshold. In the energy regime of interest for L3+C ($\gtrsim 20$ GeV) the influence of the pressure on the muon flux is almost negligible since $\beta$ is quite small (i.e. few times $10^{-2}$%/mb).
Chapter 3

Experimental setup

3.1 The L3+C experiment

As already mentioned in the introduction, the L3+C experiment consists of two parts: an air shower array and the muon spectrometer of the L3 experiment at CERN (figure 3.1). The air shower array is placed on the roof of the hall above the L3 experiment and is composed of 50 modules of 0.5 m$^2$ each covering an area of 30 m $\times$ 54 m. Since the

Figure 3.1: Position of the two components of the L3+C experiment: the muon underground detector and the air shower scintillator array.
measurements of the array are not used in the analysis performed here, we refer to [2] for further details.

The description of the L3 detector, of the muon spectrometer and of the special devices added for the dedicated L3+C experiment are presented in the following sections.

3.2 The L3 detector

3.2.1 Location

![Figure 3.2: The LEP accelerator with the four experiments.](image)

L3 is one of the four experiment of the *Large Electron-Positron Collider* (LEP) at CERN in Geneva (figure 3.2). The ground level of the L3 experimental area is at 450 meters over sea level, while the latitude is 46°25' N and the longitude 6°01' E. The center of L3 is placed at about 44.8 m underground under nearly 30 meters of overburden. This depth is large enough to screen the detector from the electromagnetic and hadronic component of cosmic rays and at the same time small enough to detect a large statistics of cosmic muons. The vertical muon energy threshold is about 15 GeV. In the direction
of the access shafts muon with lower energies are detectable.

### 3.2.2 The L3 coordinate system

The L3 coordinate system \((x, y, z)\) is defined as follows. The origin is in the center of the L3 detector, where the electron beam and the positron beam of the LEP accelerator collide. The \(x\)-axis is horizontal and directed towards the center of LEP (figure 3.3). The \(z\)-axis is tangent to the LEP circumference (beam axis) and it is inclined by 0.796° with respect to the horizontal plane. The \(y\)-axis is perpendicular to the \(xz\)-plane and oriented upwards. For practical reasons we define also an horizontal coordinate system \((x', y', z')\) applying a rotation of an angle \(\beta = 0.796°\) of the L3 coordinate system around the \(x - axis\).

\[
\begin{align*}
x' &= x \\
y' &= y \cos \beta - z \sin \beta \\
z' &= y \sin \beta + z \cos \beta
\end{align*}
\] (3.1)
3.2.3 The L3 detector devices

The L3 detector [1] (figure 3.4) is designed to measure with high accuracy hadrons, muons, electrons (including anti-particles) and high energy photons arising from the electron-positron collisions of the LEP beam. The detector is composed of many devices (sub-detectors) which are surrounded by a huge octagonally shaped solenoidal magnet of 12 meters in diameter and 12 meters in length. A magnetic field of 0.5 Tesla is present along the z-direction, allowing the measurement of the charge and the momentum from the curvature of the trajectories of the charged particle. The main detector components encountered inside the magnet cavity starting from the center are the following:

- The Silicon strip Microvertex Detector (SMD), whose main purpose is to measure short-lived particles.
- The Time Expansion Chamber (TEC) which is a tracking device for charged particles.
- The Electromagnetic Calorimeter (ECAL) made of 12000 Bismuth Germanium Oxide (BGO) crystals, suited to measure the development of the electromagnetic showers generated by the electrons, positrons and γ’s.

Figure 3.4: Schematic layout of the L3 detector. On the top the t0-scintillators added for the L3+C experiment are also drawn.
3.2 The L3 detector

- The Hadron Calorimeter (HCAL): a sampling calorimeter made of Uranium absorber plates intercalated with proportional wire chambers layers.

- The high precision Muon Chambers (MUCH) barrel which are described in more details in the next section.

Only the muon chamber barrel is used for the L3+C experiment, however the effect of the central detectors (especially the HCAL) on the cosmic muons, concerning the energy loss and the multiple scattering, has to be taken into account.

### 3.2.4 The muon chambers

For the construction of the L3 detector a large emphasis has been given to the muon chamber device. Among the four LEP experiments, L3 is the one with the largest muon detector. The consequent high precision of the angular and momentum measurement, together with the relative high acceptance, is of advantage for the study of cosmic muons.

The muon detector installed inside the magnet cavity is composed of two Ferris Wheels, called Master and Slave (figure 3.5). Each Ferris wheel is subdivided in eight octants. The octants are numbered as in figure 3.6 from 0 to 7. For each octant we define a local coordinate system \((x_l, y_l, z_l)\) so that

\[
\begin{align*}
    x_l &= \cos \left( 2\pi \frac{(n - 2)}{8} \right) x + \sin \left( 2\pi \frac{(n - 2)}{8} \right) y \\
    y_l &= -\sin \left( 2\pi \frac{(n - 2)}{8} \right) x + \cos \left( 2\pi \frac{(n - 2)}{8} \right) y \\
    z_l &= z
\end{align*}
\]  

\[(3.2)\]
where \( n \) is the octant number and \((x, y, z)\) the L3 coordinate system. In this way, the \( x_l \)-axis is parallel and the \( y_l \)-axis is perpendicular to the outer face of the concerned octant.

Figure 3.6: Front view of the L3 detector showing the layout of the muon chambers: the eight octants (numbered from 0 to 7) and the three P-chamber layers. The Z-chambers are marked in dark black color.

Three layers of drift chambers (P-chambers) in each octant (figure 3.7) measure the track position of the muons in the magnetic bending plane (xy-plane).

The P-chambers are divided into cells, of 101.5 mm width. In the middle of the cells there is the anode plane. It is composed of wires parallel to the \( z_l \)-axis which are placed at a distance of \( \Delta x_l = 4.5 \) mm from each other (figure 3.8). The 3 external wires are called guard wires. Otherwise the so called field wires are alternated with the sense wires. In the MI and MO chambers there are 16 and in the MM chambers 24 sense wires per cell. The electrons which rise from the ionization induced by a muon passing through the cell, drift to the sense wires, pushed by the electric field inside the cell. In the proximity
3.2 The L3 detector

![Diagram of the L3 detector layers]

Figure 3.7: Front view of an octant with the 3 layers of P-chambers and the 4 layers of Z-chambers. The internal layer of the P-chambers is marked with MI, the central layer with MM and the outer layer with MO. The MM and MO layer are divided in two chambers. The MI layer is composed of one chamber only. The Z-chamber layers are noted starting from the center with MII, MIO, MOI and MOO.

of the sense wire an avalanche is generated. This produces a signal, which is transmitted from the sense wire to the read-out electronic. From the precise measurement of the drift time, the distance of the muon track from the sense wire is obtained. The single wire resolution is around 200 μm, depending on the distance of the track from the sense wire.

The wall of the cells are formed by the cathode wires, which are called mesh wires. They are placed at a distance of Δx_l = 2.5 mm from each other.

The principle of the momentum measurement can be explained as follows: if the muon track is assumed to be circular (i.e. neglecting the interactions with the matter), the momentum component \( p_\perp \) (in GeV units) perpendicular to the magnetic field direction \((xy\)-plane\) results to be

\[
p_\perp \approx \frac{0.3 \cdot B \cdot L^2}{8S}.
\]

where \( B = 0.5 \) T is the magnetic field, \( L = 2.9 \) m is the distance between the MI and the MO chambers and \( S \) the sagitta (in meters) (figure 3.9).

On the two faces of the MI and MO P-chamber layers, additional drift chambers (Z-chambers) are installed in order to measure the \( z \)-coordinates of the tracks. The Z-chambers are formed by two layers of cells as shown in figure 3.10. The Z-chamber layers are noted with MII, MIO, MOI and MOO as shown in figure 3.7.
3.3 The t0-detector

For the measurement of the drift time of the electrons, one needs to have the information of the time $t_0$ at which the muons cross the muon chambers. For the muons detected by the L3 experiment, since they are a product of the collisions of the LEP beam, this information is obtained from the beam crossing signal. To obtain a precise $t_0$ information of the cosmic muons measured with the L3+C experiment, an additional detector (t0-detector) is installed above the magnet of the L3 detector. The t0-detector is composed of a scintillator system, whose elementary unit are scintillator tiles of size 25 cm $\times$ 25 cm $\times$ 2 cm (figure 3.11). 16 tiles form a so called cassette whose surface is 1 m $\times$ 1 m (figure 3.12). 6 cassettes are grouped into modules of size 3 m $\times$ 2 m (figure 3.13). 12 modules are installed on octant 1 (side 1) and octant 2 (side 2). Above octant 3 (side 3)
3.4 The L3+C trigger-, readout- and data acquisition system

The trigger requirement of the L3+C experiment is of course very different from L3. Therefore a complete independent readout and trigger system has been designed for the L3+C experiment. In this way the two experiments can run at the same time. The signals originating from the muon chambers are split in two in the so called Cosmic Personality Cards (CPC), after being amplified and passing through discriminators (figure 3.14).

---

Figure 3.10: Schematic side view of the cell configuration of a z-chamber (adapted from [96]) (units in cm)

Figure 3.11: A single t0-scintillator tile of size 25 cm × 25 cm.

Additional 10 modules\(^1\) are placed. The three layers cover a total area of 202 m\(^2\).

Two photomultipliers (PM) are mounted on each scintillator module. Wavelength shifting fibers are glued into grooves engraved on the scintillator tile surface and transport the light signal to the photomultipliers. If two coincident pulses above a given threshold are seen in both PM’s inside a time window of 15 ns the signal is accepted. The effect of the thermal noise inside the PM’s is therefore reduced, so that the operation with very low light amplitudes is possible.

---

\(^{1}\)One of the 10 modules of side 3 contains only 4 cassettes instead of 6
The CPC modules retain the same functionality as the old original Muon Personality Cards (MPC) (which had the task to send the TDC information to the L3 muon trigger). In addition they contain the TDC chips, that digitize the drift signals for the L3+C experiment, as well as a majority logic which provides the input to the L3+C trigger module. A majority signal is fired basically every time that the number of wire-hits produced in one P-chamber during a given time interval exceeds a programmable threshold. The time interval is set equal to the maximum possible drift time.

Also the signals from the t0-scintillators are treated by the CPC modules. One majority signal contains in this case a simple “OR” of all scintillator hits.

The majority signals from the CPCs are sent to the L3+C timing and trigger module (CTT). 12 trigger classes are defined according to table 3.1. In the data analysis presented in this work only the events of trigger class 1 are considered. Some trigger classes are forseen for the study of the trigger efficiency. Other trigger classes identify events which
3.4 The L3+C trigger-, readout- and data acquisition system

Figure 3.14: Simplified scheme showing the read-out of the muon chambers by the L3 and L3+C experiment. The pulses from the sense wires are first amplified and then pass through discriminators. The signals are split just after the discriminators and are treated independently by the L3 and the L3+C electronics. The t0-signal of L3+C originates from the t0-scintillators, while the t0-signal of L3 is obtained from the beam crossing signal.

normally can be reconstructed only with the cross-octant version of the reconstruction program (see section 3.5), which is currently under development. The trigger classes 10-12 are suited mainly to search for eventual exotic events.

So called NIMROD modules collect the informations from the CPCs. Whenever a trigger signal is sent to the NIMROD by the CTT module, the event information is made available for storage.

The data-taking is organized in runs of typically 10-20 minutes lengths.

A Real Time Clock running on a 5MHz temperature-stabilized crystal is synchronized with the GPS time every minute. This generates a precise time information which is stored for each event with precision of 1 μs.

A precise livetime counter is included in the data acquisition system. The livetime is however stored in units of 0.8388608 seconds only. For each event the livetime available since the beginning of the corresponding run is rounded to the lowest unit and stored. Most of the downtime of the system is due to the maintenance work as well as the time between the stops and starts of the runs. During one run the average dead time is only about 2%.

For a cross check of the livetime precision a clock signal from the GPS is sent every second to the cosmic trigger. If the clock signal is sent when the system is alive, the clock signal is registered. A good agreement is found between the number of clock signals and the livetime measured by the livetime counter (see table 4.7 in section 4.5).
### Experimental setup

<table>
<thead>
<tr>
<th>Class</th>
<th>1999 run</th>
<th>2000 run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>Triplet in any octant</td>
<td>Triplet in any octant</td>
</tr>
<tr>
<td></td>
<td>with a scintillator hit</td>
<td>with a scintillator hit</td>
</tr>
<tr>
<td>Class 2</td>
<td>Triplet in octant 0 or 4</td>
<td>Triplet in octant 0 or 4</td>
</tr>
<tr>
<td></td>
<td>without scintillator hit</td>
<td>without scintillator hit</td>
</tr>
<tr>
<td>Class 3</td>
<td>Triplet in octants 1,2 or 3</td>
<td>Triplet in octants 1,2 or 3</td>
</tr>
<tr>
<td></td>
<td>without scintillator hit</td>
<td>without scintillator hit</td>
</tr>
<tr>
<td>Class 4</td>
<td>Triplet in octants 5,6 or 7</td>
<td>Triplet in octants 5,6 or 7</td>
</tr>
<tr>
<td></td>
<td>without scintillator hit</td>
<td>without scintillator hit</td>
</tr>
<tr>
<td>Class 5</td>
<td>Three singlets in adjacent octants</td>
<td>Three singlets in adjacent octants</td>
</tr>
<tr>
<td></td>
<td>with a scintillator hit</td>
<td>with a scintillator hit</td>
</tr>
<tr>
<td>Class 6</td>
<td>Two doublets</td>
<td>Two doublets</td>
</tr>
<tr>
<td></td>
<td>with a scintillator hit</td>
<td>with a scintillator hit</td>
</tr>
<tr>
<td>Class 7</td>
<td>Doublet and two singlets</td>
<td>Doublet and two singlets</td>
</tr>
<tr>
<td></td>
<td>with a scintillator hit</td>
<td>with a scintillator hit</td>
</tr>
<tr>
<td>Class 8</td>
<td>Doublet and singlet</td>
<td>Two or more P-chamber planes</td>
</tr>
<tr>
<td></td>
<td>with a scintillator hit</td>
<td></td>
</tr>
<tr>
<td>Class 9</td>
<td>Doublet with scintillator hit</td>
<td>Three or more P-chamber planes</td>
</tr>
<tr>
<td>Class 10</td>
<td>Triplet and at least one singlet</td>
<td>Triplet and at least one singlet</td>
</tr>
<tr>
<td>Class 11</td>
<td>Five P-chambers</td>
<td>Five P-chamber planes</td>
</tr>
<tr>
<td>Class 12</td>
<td>Six or more P-chambers</td>
<td>Six or more P-chamber planes</td>
</tr>
</tbody>
</table>

Table 3.1: The 12 trigger classes. Triplets, doublets and singlets in the online context are group of majority signals from three, two or one chamber layers of one octant.

The detector status is continuously monitored. Informations as the high voltage status, the discriminator levels, the magnet current and the scintillator rates are stored in the online database.

### 3.5 Event reconstruction

The raw-data obtained with the L3+C muon spectrometer are processed by a complex computer program called REL3C, whose main task is to reconstruct the direction and the momentum of the muon tracks. A scanning program (SCL3C), mainly used for debugging purposes, includes a graphical interface, which allows to get a picture of the reconstructed event.

We give here a general overview of the main steps accomplished by the version (404) of the reconstruction program used for the analysis performed in this work.

**Determination of the hit position:** Taking into account the time information of the t0-detector and the TDC counts associated to a particular wire, the drift time of the
electrons is calculated for each wire. The position at which the drift electrons emerged (called hit position) projected onto the plane of measurements (xy-plane for P chambers, yz-plane for Z chambers) is estimated with the help of a cell map [97], which takes into account the electron trajectory in the magnetic and electric fields. In the P chambers, two hit positions have to be assigned to each hit sense wire (left-right ambiguity). The ambiguity can be normally resolved only in an advanced stage of the reconstruction.

The problem of the ambiguity in the Z-chambers is reduced by the fact that one cell layer is shifted by half a cell size in the z-direction with respect to the second cell layer (see picture 3.10).

**Pattern recognition:** Groups of at least 6 quasi-aligned hit positions [98] in one P-chamber are recognized in the pattern recognition phase [95] and form so called P-segments. Normally two ambiguous P-segments have to be considered (figure 3.15).

Z-segments are indeed formed by 3 or 4 quasi-aligned hit positions\(^2\) either in the MOO/MOI or in the MIO/MII Z-chamber pairs (figure 3.16).

![Figure 3.15: Left-right ambiguity](image)

Segment matching: After the P-segments and the Z-segments of one octant are found, the reconstruction program tries to combine them in a 3-dimensional muon sub-track. The best sub-track candidates are selected considering as quality criteria the number of P-segments and Z-segments, and the \(\chi^2\) over the number of degrees of freedom of the circle fit, applied to the hit positions belonging to the concerned P-segments. The left-right ambiguity is usually resolved at this stage.

\(^2\)In some particular cases, less than 3 hit positions are allowed for a Z-segment, if the direction is confirmed by other hits in the other Z-chamber pair of the same octant.[98]
Figure 3.16: Example of a Z-segment composed of 4 hit positions in the MOI and MOO Z-chambers of a lower octant.

For the classification of the sub-tracks, the following terminology is used:

- Sub-tracks with 3 P-segments are called *P-triplets*.
- Sub-tracks with 2 P-segments are called *P-doublets*.
- Sub-tracks with 2 Z-segments are called *Z-doublets*.
- Sub-tracks with 1 Z-segment are called *Z-singlets*.

**Corrections and swimfit:** Once the sub-tracks have been selected, one applies some corrections, which use the information of the 3-dimensional position of the hits as well as the associated scintillator signal. This corrections include the updated time of flight of the muon, the time of propagation delays of the signal in the sense wires and the corrections for the muon chamber alignment and for the wire sag. In addition, in order to reject noise signals, hits which are too far away from the fitted sub-track are dropped.

All track fits performed up to this point, assume that the trajectory of the muon is perfectly helicoidal. In order to take into account the energy loss, the multiple scattering of the muon and the inhomogeneities of the magnetic field, a so called *swimfit* [99] is applied at this stage.

**Matching of sub-tracks:** Two sub-tracks of different octants may be generated by the same muon. In order to take the decision, whether or not two sub-tracks match to a single muon track, a matching $\chi^2$ is calculated, taking into account the differences in the direction, position and momenta of the two sub-tracks (described by 5 variables) and
the corresponding covariance matrices\(^3\). The ambiguities which are still present at these stage, are resolved choosing the tracks with the lowest matching \(\chi^2\). Two sub-tracks are considered to match if their matching \(\chi^2\) is below a given threshold.

For two matched sub-tracks, the combined momentum and direction backtracked to a reference point is calculated and stored. The reference point is chosen to be the intercept of the track with the boundary of the so called LEP3 volume\(^4\).

Sub-tracks which are not matched with other sub-tracks are also stored as tracks.

**Backtracking to the surface:** Of course, to perform the physical analysis, one wants to know the momentum and direction at surface of the muons, which produce the reconstructed tracks down in the L3+C muon spectrometer, together with their uncertainties. These are obtained by means of the software package GEANE [99], which evaluates the momentum loss, the bending in the magnetic field and the uncertainty in the direction arising from the multiple scattering of the muons. The geometry (including the access shafts) and the chemical composition of the rocks above the L3 detector are taken into account (c.f. figure A.4 in the appendix). The backtracking is considered successful, if it falls inside a circle of radius 111.3 m above the L3 detector.

**Remarks about new versions of the reconstruction program:** A new version of the reconstruction program tries to fit all segments of one track simultaneously, even if they belong to different sub-tracks. This allows a much better momentum resolution at very high energies (larger than few hundreds GeV). This version is currently used for the raw-data processing.

Another version (called cross-octant), which is able to reconstruct tracks from segments belonging to adjacent octants) is currently tested and improved. This version allows to increase the acceptance of the L3+C muon spectrometer: this is of advantage in particular to increase the statistics of multi-muon events or to search for exotic events.

---

\(^3\)The mean values of the elements of the covariance matrices are parameterized as function of the momentum [100].

\(^4\)The LEP3 volume is an ideal prism containing the whole magnet and the t0-scintillators. The beam axis crosses perpendicularly the basis of the prism, whose shape is a regular octagon. The two basis are placed at \(z = -870\) cm and \(z = 870\) cm. The lateral faces are parallel to the muon chamber layers and are placed at a distance of 933 cm from the beam axis.
Chapter 4
Detector performance

4.1 Data selection

4.1.1 Event selection

Selection of single muons

The performance of the reconstruction of the events depends on how the muons cross geometrically the detector. The best momentum and angular resolution is achieved in the case that a muon track is composed by two P-triplets and two Z-doublets (defined in section 3.5) as in the example of figure 4.1. These represent our “golden” events. To select these events we define a so called “strong selection” or selection of TYPE A, which requires:

- Events of trigger class 1.
- Exactly one track (successfully backtracked to surface).
- 2 P-triplets (matched).
- 2 Z-doublets (matched).
- The swimfit of each sub-track gives $\chi^2/n.d.f. < 10$. ($n.d.f.$: “number of degrees of freedom”).
- The track passes near a hitted $t_0$-scintillator module ($2\sigma$ tolerance).
- Sub-tracks don’t intercept both Master and Slave Ferris wheels\(^1\).
- $\text{var}(\varphi) < 10^{-3} \text{ rad}^2$.

\(^1\)The relative positions of the Master and Slave Ferris wheels are not known precisely enough.
4.1 Data selection

Figure 4.1: “Golden Event” in the L3 detector. It is composed by two P-triplets and two Z-doublets.

The angular variance $\text{var}(\varphi)$ is extracted from the variances of the zenith and azimuth angles $\theta$ and $\phi$, as obtained by the backtracking to surface.

$$\text{var}(\varphi) = \text{var}(\theta) + \text{var}(\phi)\sin^2\theta$$

(4.1)

The rate of the “golden” events is unfortunately very low (~7 Hz for muons with surface energy larger than 30 GeV). If the quality of the track reconstruction and in particular of the momentum resolution is far more important than high statistics, it is good to use this strong selection. This is the case for the momentum spectrum analysis.

For the main subjects of this work however high statistics is much more important than the momentum resolution. For this reason a compromise between track quality and statistics has to be found. In doing that we should however not forget that a too low requirement on the track quality could lead to confuse a muon track with noise, which would affect seriously the stability of the detector response.

Therefore we define a so called “weak selection” or selection of **TYPE B**, which requires:

- Events of trigger class 1.
- Exactly one track (successfully backtracked to surface).
- **At least one sub-track must be a P-triplet and Z-doublet**, whose swimfit gives $\chi^2/n.d.f < 10$.

- Track passes near a hit scintillator module ($2\sigma$ tolerance).

- Sub-tracks don't cross both Master and Slave Ferris wheels.

- $\text{var}(<\varphi) < 10^{-3} \text{ rad}^2$.

  The selection of type B is used for the performance analysis described in this chapter, if not otherwise specified, and for the anisotropy analysis of single muons (see chapter 5).

  For the point sources search (see chapter 6) the stability of the detector response is less important than for the anisotropy analysis. For this search we therefore define the following selection (**TYPE C**) in which our requirements concentrate on the angular resolution only.

  - Exactly one track (successfully backtracked to surface).

  - $\text{var}(<\varphi) < 10^{-3} \text{ rad}^2$.

  - Track pass near a hit scintillator module ($2\sigma$ tolerance).

**Selection of multi-muons**

We describe in the following paragraphs the selection applied for the analysis of the multi-muon anisotropy, reported in section 5.8.3.

It happens relatively often (~1.5% of the triggered events) that two sub-tracks produced by a single muon are not matched by the reconstruction program and are considered as two tracks. This represents a problem, since the percentage of double muons is only about 0.1%. To overcome that problem, for the selection of multi-muon with at least $n$ muons, we require that the number of sub-tracks reconstructed in one event is at least $N_s = 2n - 1$.

Since we are interested in muons belonging to the same shower, another requirement is the parallelism. We therefore select only tracks which are inside a cone of 7.5° from the average direction of the muons, following the procedure shown in the diagram of figure 4.2. To determine the average direction we use only tracks which follow some minimal requirements on the quality (called quality $H$). For at least one sub-track of the tracks of quality $H$ we require:

  - It is a z-doublet.

  - The swimfit gives $\chi^2/n.d.f < 20$.

  - After the reconstruction phase (following the segment matching phase) in which the dropping of the hits is performed (see reconstruction description in section 3.5), the number of hits in at least two P-segments must be larger or equal to 6 (each) and the number of hits in the Z-chambers must be at least 4.
Figure 4.2: Diagram showing the procedure of the multi-muon selection.
4.1.2 Selection of time intervals

During the data-taking many problems can occur, that affect the quality of the detection and in particular the stability of the efficiency. Therefore a careful selection of the time intervals of the data-taking to be used for the analysis, is of primary importance.

Selection of type 1.a: A first possibility for the interval selection, is to use the informations available on the database of the online monitoring, to select which runs have to be used [101]. For the selected runs, the following is required:

1. Runs must be longer than 2 seconds and contain more than 1000 events.
2. The trigger rate must be smaller than 600 Hz.
3. The rate of the scintillator signals is not too high. This selection criterion is applied by means of a cut on a particular online parameter, which has been introduced for the monitoring of the scintillator rate.
4. The voltage of the wires in the muon chambers is at nominal value.
5. The discriminator levels in the muon chambers are at nominal value.
6. The voltage of the photomultipliers connected to the scintillators is at nominal value.
7. The magnet current is at nominal value.
8. The database information is available and it is not corrupted.

This selection criteria alone are however noted to be insufficient to always guarantee a good quality of the runs. The failures of the selection based on the online monitoring can be explained by the following reasons.

- The informations are written in the online database with a variable delay of tens of seconds. Therefore the information present in the database is not very precise in time.
- Not all variables can be taken into account by the monitoring.
- Short time problems are overlooked or taken too seriously.

When analyzing the rates of reconstructed events according to a particular event selection, it is discovered that some runs contain an abnormal low rate of selected events. Sometimes a comparison with the online logbook allows to discover the problems that affect the runs with the abnormal rate. In this case the runs are rejected by hand. However for few runs (out of $\sim 10^5$) no particular reason has been found for the abnormal low rate.
4.1 Data selection

Selection of type 1.b: For the anisotropy analysis that is discussed in chapter 5 a good stability of the detector efficiency, which is reflected in the event rate, is absolutely necessary, at least on the time scale of one day. Since the run list obtained with the selection of type 1.a, can not guarantee such stability, we perform on that list a further selection based on the deviation of the rate from its average.

For each run \(i\) obtained from the 1.a selection and starting at time \(t_i\), we register the corresponding livetime \(l_i\) and the number of events \(n_i\) with energy larger than 30 GeV, selected according to the event selection of type B (slightly modified). We then calculate the rate \(R_i\)

\[
R_i = \frac{n_i}{l_i}
\]

and the “running average” rate \(\bar{R}_i\) on the time interval \(T_i = [t_i - 2 \text{ days}, t_i + 2 \text{ days}]\).

\[
\bar{R}_i = \frac{\sum_{\{i|t_i \in T_i\}} n_i}{\sum_{\{i|t_i \in T_i\}} l_i}
\]

We can then determine the \(\chi^2\) for the run \(i\) as

\[
\chi^2_i = \frac{(R_i - \bar{R}_i)^2}{n_i}\]

For the selection of type 1.b we require \(\chi^2_i < 4\). Figure 4.3 shows the running average rate \(\bar{R}_i\) and the mean rate of the excluded runs as a function of the day.\(^2\)

In table 4.1 we present, together with the full statistics, the yearly statistics that is available when applying both selection 1.a and 1.b. It shows that during the year 2000 we loose a larger part of the runs than in the year 1999. The main reason is the high scintillator rate induced by the synchrotron radiation produced by the LEP accelerator. In fact during the year 2000, the last year of LEP activity, the beam energy reached the highest values.

Selection of type 2.a: Looking at the table 4.1 it is clear that the available data are strongly reduced when applying the selections 1.a or 1.b. In particular in the year 2000, statistics is reduced by a factor 4. This is not good, specially for the subjects for which high statistics is very important, like the point sources search. A problem of the selections 1.a and 1.b is that complete runs (typical length 10-20 minutes), during which a problem lasting for a small time interval appeared, have to be removed.

A solution to improve that situation has been proposed in [103]. The idea is to apply a selection of the livetime intervals based on the event rates and the trigger rates. The proposed duration of the livetime intervals is one livetime unit (0.8388608 seconds, see section 3.4). The daily distributions of the number of reconstructed events and the number of triggers per livetime units are studied for that purpose. The distributions in

\(^2\)The variation of the rate along the year will be discussed later in section 4.3.1.
Figure 4.3: The central smooth curve shows the “Running average” $\bar{R}$ of the event rate as a function of the day number from the beginning of the year 1999 (left) and 2000 (right) for muons with energy larger than 30 GeV according to selection B (slightly modified). The single dots represent the mean event rate of the runs discarded by the $\chi^2$-cut.

The example of figure 4.4 represent the rates during one day, in which the LEP accelerator was off. They are nicely fitted with a Gaussian function, whose root of mean squared (RMS) is the square root of the mean value (as expected from statistical fluctuations).

In the event distribution of figure 4.5 we see a bump outside the Gaussian distribution, which is induced by a problem in the high voltage of the muon chambers. For a day in which the LEP accelerator reached the maximal energy we get the distributions of figure 4.6, which shows deviations form the Gaussian distribution also for the trigger rate distribution.

The idea for the selection of type 2.a is to cut all livetime intervals which appear in the queues of the two distributions. For that purpose the central part of the two distributions of every day are fitted with a Gaussian distribution and the mean values $\mu_{\text{trigger}}$ and $\mu_{\text{events}}$ are stored, in order to be used as reference rates.

To reduce the relative statistical fluctuations around the mean value, we increase the interval length to 30 livetime units. For the selection, we allow fluctuations up to 2.5 $\sigma$. 

---

**Selection 2.b**

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<thead>
<tr>
<th>day of year</th>
<th>rate [Hz]</th>
<th>day of year</th>
<th>rate [Hz]</th>
</tr>
</thead>
<tbody>
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<td>200</td>
<td>30</td>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>250</td>
<td>35</td>
<td>250</td>
<td>35</td>
</tr>
<tr>
<td>300</td>
<td>40</td>
<td>300</td>
<td>40</td>
</tr>
</tbody>
</table>

- Running average
- Discarded runs
4.1 Data selection

Figure 4.4: Distribution of the number of triggers and of reconstructed events in intervals with length of one livetime unit (0.8388608 seconds) during one day in which the LEP accelerator is off. No detector problems are present during this day.

Figure 4.5: Distribution of the number of triggers and of reconstructed events in intervals with length of one livetime unit, during one day in which in the P-chambers appeared to have a high voltage problems. (LEP off).
Table 4.1: Statistics obtained by the time interval selection 1.a and 1.b compared with the full statistics. (1999 data include only the period starting from 18th July). [102]

<table>
<thead>
<tr>
<th>Selection Type</th>
<th>1999</th>
<th>2000</th>
<th>1999+2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of runs</td>
<td>1.a</td>
<td>11739</td>
<td>47.4 %</td>
</tr>
<tr>
<td></td>
<td>1.b</td>
<td>10544</td>
<td>42.6 %</td>
</tr>
<tr>
<td></td>
<td>all runs</td>
<td>24756</td>
<td>100.0 %</td>
</tr>
<tr>
<td>Number of triggered events</td>
<td>1.a</td>
<td>1.918·10⁹</td>
<td>62.6 %</td>
</tr>
<tr>
<td></td>
<td>1.b</td>
<td>1.690·10⁹</td>
<td>55.2 %</td>
</tr>
<tr>
<td></td>
<td>all runs</td>
<td>3.061·10⁹</td>
<td>100.0 %</td>
</tr>
<tr>
<td>Livetime [s]</td>
<td>1.a</td>
<td>4.363·10⁶</td>
<td>63.4 %</td>
</tr>
<tr>
<td></td>
<td>1.b</td>
<td>3.838·10⁶</td>
<td>55.8 %</td>
</tr>
<tr>
<td></td>
<td>all runs</td>
<td>6.877·10⁶</td>
<td>100.0 %</td>
</tr>
</tbody>
</table>

Figure 4.6: Distribution of the number of triggers and of reconstructed events in intervals with length of one livetime unit, during one day in which LEP reached for some time the maximal beam energy.
4.1 Data selection

Therefore we require \(^3\)

\[
|n_{\text{trigger}} - 30\mu_{\text{trigger}}| < 2.5 \cdot \sqrt{30\mu_{\text{trigger}}}
\]

(4.5)

\[
|n_{\text{events}} - 30\mu_{\text{events}}| < 2.5 \cdot \sqrt{30\mu_{\text{events}}}
\]

(4.6)

where \(n_{\text{trigger}}\) is the number of triggers in the interval of 30 livetime units and \(n_{\text{events}}\) is the number of reconstructed events in the same interval. The last interval of one run, which does not cover completely 30 livetime units, is accepted also only if its rates are within a 2.5 \(\sigma\) tolerance interval, but we require in addition that the previous interval is accepted. This in order to take into account the increase of the statistical uncertainty \(\sigma\) in the rate.

If the Gaussian fit of the rate distributions results to be bad for a particular day, because there are serious problems affecting the data acquisition for a large part of the day, all events of that days are rejected.

Selection of type 2.b: It has been noted that not all intervals during which the magnet is off are rejected by the selection 2.a. Since this influence very seriously our results (c.f. figure 6.9), we introduce a new selection (2.b), which rejects from the intervals passing the selection 2.a, the runs with the magnet switched off. It is noted that, when the magnet is off, since the muon tracks are straight, the number of tracks with high reconstructed momentum increase remarkably. Therefore an additional check is added, where we monitor that the ratio of reconstructed tracks with momenta larger than 50 GeV/c is stable.

Selection of type 2.c: Another problem not completely removed by the selections of type 2.a and 2.b is the LEP induced noise in the scintillators (c.f. figure 6.19, section 6.2.1). To take into account that, in the selection of type 2.c, in addition to the requirements of the selection 2.b, we apply a cut on the average number of scintillator hits per reconstructed event present in the useful time window of each event. We require that this average number, measured on the intervals of 30 livetime units, is smaller than 1.6 (figure 4.7).

In table 4.2 we can compare the percentage of accepted livetime intervals applying selections 2.b and 2.c. For the year 1999 there is practically no difference. For the year 2000, the intervals rejected by selection 2.c are almost the double than for selection 2.b. However compared to selections 1.a and 1.b, selection 2.c gives more than the double of statistics in the year 2000.

\(^3\)During the days in which the trigger settings were changed, the requirement on the trigger rate given by equation 4.5 is omitted. This happened in particular when the LEP beam energy corresponded to the Z-resonance. Only the trigger class 1 was then required, in order to simplify the study on the momentum resolution and on the efficiency obtained from the muons produced by the Z-decay.
Figure 4.7: Distribution of the average number of scintillator hits per reconstructed events present in the useful time window, obtained for intervals of 30 live-time units. The three plots refer to the data with interval selection 2.b of the year 1999 (upper left), of the year 2000 (upper right) and of the last 11 days of data-taking of the year 2000, during which the LEP accelerator was switched off (lower plot). The hatched line shows the cut on the average number of scintillator hits of 1.6, set in the selection 2.c.
4.2 Efficiency of the muon detector

### 4.2.1 Scintillator efficiency

The efficiency of the t0-scintillators of the L3+C experiment has been studied with help of the set $X$ of tracks, which contain at least one P-segment, which crosses a wire plane of a P-chamber [104]. In this case, the $t_0$ time can be extracted even neglecting the information obtained from the t0-scintillators. This is achieved, fitting the P-segment and using the $t_0$ time as a free parameter. In fact if a wrong $t_0$ time is used, the P-segment hit positions are observed to be not aligned (figure 4.8). The $t_0$ time obtained from the fit of the P-segments, which cross a wire plane, is used in this study to reconstruct all tracks of the set $X$ (including the event of trigger classes which doesn’t require a hit from the t0-scintillators). To get the efficiency of the t0-scintillators, one checks how often the t0-scintillators crossed by the backtracked reconstructed track, are hit at the correct time.

The efficiency for the different modules, obtained from the data at the end of the year 1999, are shown in figure 4.9. We obtain an average efficiency of more than 96 %. The efficiency map for the end of the year 1999 and the year 2000 is shown in figure 4.10. We note that at the end of the year 2000, there are two neighboring t0-scintillator-cassettes

<table>
<thead>
<tr>
<th>Selection Type</th>
<th>Year 1999</th>
<th>Year 2000</th>
<th>Total 1999+2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.b</td>
<td>81.4 %</td>
<td>75.0 %</td>
<td>76.9 %</td>
</tr>
<tr>
<td>2.c</td>
<td>81.6 %</td>
<td>50.6 %</td>
<td>60.0 %</td>
</tr>
</tbody>
</table>

Table 4.2: Percentage of livetime intervals accepted by the selections of type 2.b and 2.c.
Figure 4.9: Efficiency of the scintillator modules at the end of 1999 \[2\].

Figure 4.10: Efficiency map of the scintillators at the end of 1999 (left) and 2000 (right) on the \(xz\)-plane (L3 coordinates). From left to right one find on the map first side 3 of the scintillator layers, then side 2, where one can observe a white cross (corresponding to the space between the modules introduced in order to allow the access for maintenance work), and finally the side 1. \[105\]
above the upper octant (located next to position $x \approx 1m$, $z \approx 0.5m$) whose efficiency dropped down below 50%. We will report again about this cassettes, when discussing the stability of the detector (section 4.3.2).

4.2.2 Efficiency of the muon chambers

The efficiency of the cells of each muon chamber layer has been studied applying following idea [106]. The tracks are reconstructed neglecting the read-out information from the muon chamber layer $L$ under study. Once the intersection with the layer $L$ of a so reconstructed track is established, it is checked if the track is registered also by the layer $L$.

An example of the results, which concerns the P-chambers in the octant 2 and 6 of the master Ferris wheel is reported in the plots of figure 4.11. Represented is the efficiency as a function of the progressive number of the half-cells (i.e. the portion of space enclosed by a sense and a mesh wire plane). The wires of some half-cells (dead half-cells) are broken or disconnected. Their high voltage may also be constantly smaller than the nominal value, with the consequence that the concerned half-cells have a lowered efficiency (“ill cells”). The efficiency of those cells has been noted to be unstable. In particular a strong correlation with the changes of pressure and temperature has been noted [107].

The problematic half-cells according to the online database are marked with an hatched area on the plots of figure 4.11. Comparing the online database informations with the measured efficiency, one can note that the situation is not perfectly described by the online database (see for instance the “ill” half-cells 48 and 49 in the MO chambers of octant 6).

After these observations, it has been decided that, in order to increase the stability of our data-set, all signals from those cells having less than 80% of efficiency, have to be ignored by the standard reconstruction program version (version 404) used for the analysis performed in this work, even if they are considered as normal in the database. For that purpose an efficiency map of all cells is established for the end of both year 1999 and 2000. The efficiency map of the end of a year is used for the full year, since it is assumed that the efficiency of each cell reaches his minimum at the end of the year.

In picture 4.12 we report, as a function of the time, the percent of cells that are active according to the online database.
Figure 4.11: Efficiency of the muon p-chambers in the octants 2 (above) and 6 (below) of the master Ferris wheel at the end of the year 1999. On the horizontal axis we report the progressive number of the half-cells (i.e. the portion of space enclosed by a sense and a mesh wire plane). Half-cells near to a wire plane, that according to the online database is dead or has a lowered high voltage, are marked with a double hatched area. In case that a mesh wire plane is dead, also the second next half-cells (marked with an horizontal hatched area) are strongly affected by a reduced efficiency (because of the distorted electric field). [105]
Figure 4.12: Percentage of active cells in the p-chambers during 1999 and 2000.
4.3 Stability of the muon detector

We note that part of the stability analysis described in this section has been first performed with an earlier version (402) of the reconstruction program than the standard version used otherwise in this work (version 404). For every plot and analysis which refers to the version 402, an explicit notice appears.

4.3.1 Variation of muon detection rate

An important point in the study of the stability of the efficiency of the L3+C muon spectrometer is the variation of the muon detection rate.

A first important observation in this context, is that the data taken before the 15th of July 1999 (day 196) have a very unstable rate. This can be seen on the figure 4.13. The plot represents the daily average rate of the events with energy larger than 20 GeV selected according to the event selection of type B and the interval selection 1.a. This plot is obtained with data processed with version 402 of the reconstruction program. After

Figure 4.13: Average of the rate of events with energy larger than 20 GeV (selection type B + 1.a) for all days of the year 1999, processed with version 402 of the reconstruction program. Error bars represent the statistical error. The rates before the 15th of July (day number 196) are very unstable.
4.3 Stability of the muon detector

the day 196 the rate is seen to be higher and much more stable than before.

This observation can be related to the fact that, during the first 2 months of data-taking (up to the 15th of July 1999), there were improvements and works performed on the detector and on the hardware. The main interventions which lead to the stability of the muon detection rate, seem to concern the improvement of the magnetic shielding of the photomultipliers connected to the t0-scintillators. In fact the last intervention of this kind, was performed exactly on the 15th of July. The data taken before this date are discarded for the analysis and has been even not processed by the version 404 (standard) of the reconstruction program.

After the 15th of July, in the plot of figure 4.13, one can also observe an abrupt change of rate occurring at the day 233 (21th August). This is in coincidence with a change in the number of active cells (see figure 4.12 and table 4.3).

The daily average detection rate of muon tracks reconstructed by the version 404 of the reconstruction program which fulfills the requirements of event selection B and interval selection 1.b, is shown on the plot of figure 4.14 for both years of data-taking. To produce the plot for the year 2000 we discard all tracks crossing the 2 scintillators cassettes mentioned in section 4.2.1, which lost more than half of efficiency in the year 2000. The reason is that the efficiency of these scintillators is noted to be unstable (see section 4.3.2).

As explained in the section 4.2.2, in the standard version of the reconstruction program, all signals coming from the cells of the P-chambers which are dead or inefficient at the end of the year, are ignored by the reconstruction program. Therefore the abrupt change at the day 233 in figure 4.13 is not observed anymore in figure 4.14 and the data efficiency is expected to be more stable than in version 402.

The two drops of rate around the days 280 and 300 of the year 1999, can be well correlated with a steep decrease in the effective temperature $T_{\text{eff}}$ (c.f. figure 2.21). However a clear relation between muon detection rate and $T_{\text{eff}}$ has not been found, in particular in the year 2000. In fact from the day 100 to the day 170 of that year, the muon rate tends to decrease, while $T_{\text{eff}}$ is growing, in contradiction to what expected. For a more detailed study on this point see [108].

4.3.2 Stability of the acceptance distribution

To perform the data analysis related with the main physics topics of this work (anisotropy and point sources), it is very important to analyze the data separately in periods, in which we have a good stability in the efficiency. In the discussion about the method of the two analysis in chapter 5 and 6, it will become clear that this concerns in particular the direction dependent relative efficiency. A first attempt to choose the periods with a good stability, is done in table 4.3, where we consider the main changes that should be able to affect the detector efficiency. These include the changes of the number of active cells in the P-chambers (visible in figure 4.12), and the interventions on the t0 scintillators (e.g. magnet shielding). We define also a particular period of the year 2000 which includes the last 11 days of data-taking, in which the LEP accelerator was switched off, so that low
Figure 4.14: Average of the rate of events with energy larger than 20 GeV (selection type B + 1.b) as a function of the day for the year 1999 (starting from the 18th of July) and for the year 2000. Error bars represent the statistical error. Data are processed with the reconstruction program version 404 (standard version used for this work).
4.3 Stability of the muon detector

<table>
<thead>
<tr>
<th>Period Nr.</th>
<th>Begin</th>
<th>End</th>
<th>First Run</th>
<th>Last Run</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1999</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>03 May 13:45</td>
<td>13 May 00:40</td>
<td>10001</td>
<td>11256</td>
<td>P-chambers</td>
</tr>
<tr>
<td>2</td>
<td>13 May 00:45</td>
<td>19 May 21:20</td>
<td>11257</td>
<td>14296</td>
<td>P-chambers</td>
</tr>
<tr>
<td>3</td>
<td>19 May 21:25</td>
<td>28 May 12:00</td>
<td>14299</td>
<td>17986</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>4</td>
<td>28 May 12:00</td>
<td>01 Jun 11:30</td>
<td>17987</td>
<td>19380</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>5</td>
<td>01 Jun 11:30</td>
<td>06 Jun 14:55</td>
<td>19380</td>
<td>21406</td>
<td>P-chambers</td>
</tr>
<tr>
<td>6</td>
<td>06 Jun 15:00</td>
<td>08 Jun 09:45</td>
<td>21408</td>
<td>21866</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>7</td>
<td>08 Jun 09:45</td>
<td>24 Jun 12:56</td>
<td>21867</td>
<td>28033</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>8</td>
<td>24 Jun 12:56</td>
<td>30 Jun 12:45</td>
<td>28034</td>
<td>29624</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>9</td>
<td>30 Jun 12:45</td>
<td>01 Jul 15:16</td>
<td>29625</td>
<td>29958</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>10</td>
<td>01 Jul 15:16</td>
<td>10 Jul 12:40</td>
<td>29939</td>
<td>32647</td>
<td>P-chambers</td>
</tr>
<tr>
<td>11</td>
<td>10 Jul 12:45</td>
<td>15 Jul 11:25</td>
<td>32650</td>
<td>34351</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>12</td>
<td>15 Jul 11:25</td>
<td>16 Jul 21:50</td>
<td>34352</td>
<td>34756</td>
<td>P-chambers</td>
</tr>
<tr>
<td>13</td>
<td>16 Jul 21:55</td>
<td>17 Jul 23:45</td>
<td>34692</td>
<td>35143</td>
<td>P-chambers</td>
</tr>
<tr>
<td>14</td>
<td>17 Jul 23:50</td>
<td>21 Aug 14:20</td>
<td>35146</td>
<td>43964</td>
<td>P-chambers</td>
</tr>
<tr>
<td>15</td>
<td>21 Aug 14:25</td>
<td>02 Sep 08:25</td>
<td>43965</td>
<td>47168</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>16</td>
<td>02 Sep 08:25</td>
<td>09 Nov 12:34</td>
<td>47169</td>
<td>60092</td>
<td>t0-scint.</td>
</tr>
<tr>
<td>Year 2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>31 Mar 00:00</td>
<td>02 May 12:40</td>
<td>100978</td>
<td>104403</td>
<td>P-chambers</td>
</tr>
<tr>
<td>52</td>
<td>05 May 13:40</td>
<td>29 May 17:20</td>
<td>104890</td>
<td>107578</td>
<td>P-chambers</td>
</tr>
<tr>
<td>53</td>
<td>29 May 18:25</td>
<td>18 Jul 07:00</td>
<td>108102</td>
<td>113977</td>
<td>P-chambers</td>
</tr>
<tr>
<td>54</td>
<td>18 Jul 08:05</td>
<td>31 Aug 18:55</td>
<td>114181</td>
<td>119671</td>
<td>P-chambers</td>
</tr>
<tr>
<td>55</td>
<td>31 Aug 20:00</td>
<td>21 Oct 09:05</td>
<td>119690</td>
<td>124976</td>
<td>P-chambers</td>
</tr>
<tr>
<td>56</td>
<td>21 Oct 10:10</td>
<td>02 Nov 07:30</td>
<td>125044</td>
<td>126438</td>
<td>P-chambers</td>
</tr>
<tr>
<td>57</td>
<td>02 Nov 08:30</td>
<td>13 Nov 07:57</td>
<td>126452</td>
<td>127837</td>
<td>LEP off</td>
</tr>
</tbody>
</table>

Table 4.3: Definition of periods between the main changes which are expected to affect the efficiency of the detector. The changes taken into account are the variation of the number of active cells in the P-chambers (marked in the table with “P-chambers”) and the intervention on the t0-scintillators (marked with “t0-scint.”). We define also a particular period (57), in which data were taken when the LEP accelerator was switched off.

noise was present in the t0 scintillators. The period of the year 1999 are numbered from 1 to 16 and the ones of 2000 from 51 to 57.

We want to recall, that the changes in the number of active cells in the P-chambers were very important only when we started the analysis with the data processed with version 402 of the reconstruction program. In the standard version (404) these changes should in principle have any effect on the observed detector response, since, as already mentioned, the signal coming from the inactive or inefficient cells at the end of the year are ignored for the whole year.

Since it has been noted is section 4.3.1 that the event rate is very unstable before the 15th of July, the periods from 1 to 11 can be discarded. In the first 2 days after this date, we note twice a change in the number of active cells. Therefore a reasonable analysis
Year 1999  Energy cut: 30 GeV

![Direction distribution](image)

Figure 4.15: Arrival direction distribution $A$ in $(\sin \theta \sin \phi, \sin \theta \cos \phi)$ for detected muons with an energy larger than 30 GeV and fulfilling the requirements of the criteria of the event selection B and the interval selection 1.a. The center of the plot corresponds to the zenith; the vertical direction coincide with the South-North axis and the horizontal direction with the West-East axis. The beam axis is shown by a hatched line. Bin size: $\Delta x_A = \Delta y_A = \frac{1}{50}$

requiring stability can start only at the 18th of July with period 14.

In the description of the methods of the anisotropy analysis and of the point sources search, we will underline the importance of having stable angular distributions. To check that, we will analyze in detail the relative variations in the distribution $A$ of the arrival direction of the muons and their impact point distribution $I$ near the L3 detector.

We describe the distribution $A$ with a 2-dimensional histogram in $x_A = \sin \theta \sin \phi$ and $y_A = \sin \theta \cos \phi$ (figure 4.15), where $\theta$ and $\phi$ are the zenith and azimuth angles. The coordinate system $(x_A,y_A)$ represents the projection of the celestial half sphere above the horizon on the horizontal plane. The $y_A$-direction corresponds to the North cardinal point, while the $x_A$-direction coincide with the East. One can notice a strong increase
of tracks in two lateral bands parallel to the beam axis (shown on the plot by a hatched line). This increase is caused by those tracks whose direction projected on the $xy$-plane (L3 coordinates) are at 20-25$^\circ$ from the vertical. These tracks can be well reconstructed in two of the upper octants. In fact at present, the cell-map (see section 3.5) describes correctly the hit position in the p-chambers only for muon sub-tracks with an inclination below 25$^\circ$ with respect to the sense wire plane. For sub-tracks above this inclination the $\chi^2$ resulting from the swim$\bar{t}$ is much larger than normal and most of the corresponding events are therefore rejected by the event selection B used in the plot.

To analyze the impact point distribution $I$ of the muon tracks on the L3 detector we consider the upper intersection point $P$ of the muon tracks with the border of the LEP3 volume. We remind that this is an ideal octagonal prism surrounding the L3 detector which has been defined in section 3.5. We describe the impact point on the LEP3 volume with the coordinates

$$
\begin{align*}
  x_I &= \frac{z}{\sqrt{x^2 + y^2 + z^2}} \\
  y_I &= \frac{x}{\sqrt{x^2 + y^2 + z^2}}
\end{align*}
$$

where $(x, y, z)$ are the L3 coordinates (defined in section 3.2.2) of the point $P$. The coordinates $(x_I, y_I)$ represent the projection on the $yz$-plane of the unit vector originating from the center of the L3 detector in the direction of the point $P$. The distribution $I$ in $(x_I, y_I)$ is shown in figure 4.16. Since the t0-scintillators are next to the border of the LEP3 volume, one can clearly distinguish the 3 layers of t0-scintillators.

Both distributions $A$ and $I$ are described here by histograms containing $100 \times 100$ bins.

We want now to check if the two distributions $A$ and $I$ are stable inside the periods defined in table 4.3. To do that we split the periods in two half-periods containing an equal number of runs. For each of the two half-periods we establish the two histograms $A$ and $I$ and we note their bin contents with $n_{ij,1}^A$ and $n_{ij,1}^I$ for the first half-period, and $n_{ij,2}^A$ and $n_{ij,2}^I$ for the second $(i = 1, 2, ..., 100$ $ j = 1, ..., 100)$. We normalize the distributions $A$ and $I$ to 1 defining

$$
\begin{align*}
  x_{ij,1}^A &= \frac{n_{ij,1}^A}{N_1} & x_{ij,1}^I &= \frac{n_{ij,1}^I}{N_1} \\
  x_{ij,2}^A &= \frac{n_{ij,2}^A}{N_2} & x_{ij,2}^I &= \frac{n_{ij,2}^I}{N_2}
\end{align*}
$$

where $N_1$ and $N_2$ represent the number of events analyzed in the first and second half-periods.

$$
\begin{align*}
  N_1 &= \sum_{i,j} n_{ij,1}^A = \sum_{i,j} n_{ij,1}^I \\
  N_2 &= \sum_{i,j} n_{ij,2}^A = \sum_{i,j} n_{ij,2}^I.
\end{align*}
$$
Figure 4.16: Impact point distribution $I$ in $(x_I, y_I)$ for detected muons with an energy larger than 30 GeV and fulfilling the requirements of the criteria of the event selection B and the interval selection 1.a. Starting from above one can distinguish the side 1 of the t0-scintillator layer, then the side 2 and on the bottom one finds the side 3.

For non empty bins, we then calculate the asymmetries

$$a^A_{ij} = \frac{x^A_{ij,2} - x^A_{ij,1}}{x^A_{ij,1} + x^A_{ij,2}},$$

$$a^I_{ij} = \frac{x^I_{ij,2} - x^I_{ij,1}}{x^I_{ij,1} + x^I_{ij,2}}.$$  \tag{4.10}

The standard deviation of the asymmetries are obtained from

$$\langle \sigma^A_{ij} \rangle^2 = \frac{n^A_{ij,1}}{(N_1)^2} \left( \frac{d}{dx^A_{ij,1}} a^A_{ij} \right)^2 + \frac{n^A_{ij,2}}{(N_2)^2} \left( \frac{d}{dx^A_{ij,2}} a^A_{ij} \right)^2,$$

$$\langle \sigma^I_{ij} \rangle^2 = \frac{n^I_{ij,1}}{(N_1)^2} \left( \frac{d}{dx^I_{ij,1}} a^I_{ij} \right)^2 + \frac{n^I_{ij,2}}{(N_2)^2} \left( \frac{d}{dx^I_{ij,2}} a^I_{ij} \right)^2$$  \tag{4.11}
which gives

\[
\sigma_{ij}^A = \frac{2}{(x_{ij,1}^A + x_{ij,2}^A)^2} \sqrt{(x_{ij,2}^A)^2 \frac{n_{ij,1}^A}{(N_1)^2} + (x_{ij,1}^A)^2 \frac{n_{ij,2}^A}{(N_2)^2}}
\]

\[
\sigma_{ij}^I = \frac{2}{(x_{ij,1}^I + x_{ij,2}^I)^2} \sqrt{(x_{ij,2}^I)^2 \frac{n_{ij,1}^I}{(N_1)^2} + (x_{ij,1}^I)^2 \frac{n_{ij,2}^I}{(N_2)^2}}
\]

(4.12)

We divide further the asymmetries by their standard deviation

\[
s_{ij}^A \equiv \frac{a_{ij}^A}{\sigma_{ij}^A} \quad s_{ij}^I \equiv \frac{a_{ij}^I}{\sigma_{ij}^I}
\]

(4.13)

The resulting quantities \(s_{ij}^A\) and \(s_{ij}^I\), in absence of systematical variations in the \(A\) and \(I\) distributions, are expected to fluctuate randomly around 0 according to a Gaussian distribution with standard deviation \(\sigma = 1\) (figure 4.17).

![Figure 4.17: Example of distribution of \(s_{ij}^I\) fitted with a Gaussian distribution.](image)

**Stability in the year 1999**

To achieve our goal, to analyze if there are significative differences in the distributions \(A\) and \(I\) between the first half part and the second half part of the periods of table 4.3,
we produce the maps of the quantities $s_{ij}^I$ and $s_{ij}^A$ as in figure 4.18 (for period 14). In this figure one can note that there are systematical variations in the distribution $I$ on

![Figure 4.18: Period 14: Maps of $s_{ij}^I$ and $s_{ij}^A$ showing the variations in the impact point distribution $I$ on the L3 detector (LEP3 volume) and in the direction distribution $A$. Black points show a relative increase of muon tracks after the 4th August, while white points indicate a relative decrease. Muon tracks with an energy larger than 30 GeV are used. (Selection B + 1.b).](image)

side 3. These have been established to be at the percent level. To identify approximately the date at which the change occurs, we split the period 14 in several smaller parts and we compare them. In such a way we discover that the change occurs around the 4th of August 1999 ($\pm 2$ days). No clear explanation has been found on the online logbook about this change. An additional problem during the period 14 has been noted on side 2 after the 15th of August. Therefore it is decided to discard the last few days of this period from our analysis.

Another important change ($\sim 5\%$) is observed in the period 16, which is seen in both $A$ and $I$ distribution (figure 4.19). It seems to occur in coincidence with a hardware intervention on the NIMROD’s which was performed on the 21/22th of September. We discover that the problems come from few cells of the P-chambers of octant 3 on the master Ferris wheel. This can be seen in figure 4.20, where we report the distribution of the intersection points in the local $x_I$ coordinate (defined by equation 3.2) of the reconstructed muon tracks with the P-chambers MI and MO of octant 3.
4.3 Stability of the muon detector

Figure 4.19: Period 16: Maps of $s_{ij}^I$ and $s_{ij}^A$ showing the changes in the impact point distribution $I$ on the L3 detector (LEP3 volume) and in the direction distribution $A$ occurring around the 22th September 1999. Black points show a relative increase of muon tracks, while white points indicate a relative decrease. Muon tracks with an energy larger than 30 GeV are used. (Selection B + 1.b).

Figure 4.20: Distribution of the intersection points of the reconstructed muon tracks with the P-chambers MI and MO of octant 3 in the local $x_I$ coordinate (defined by equation 3.2).
Stability in the year 2000

Strong instabilities (in the rate and in the distributions $A$ and $I$) has been noted in the first week of the period 51, which has been immediately discarded when performing the analysis with reconstruction version 402.

When analyzing with version 404 the variations in the distributions $A$ and $I$ of the different periods, we note strong variations between the periods 51 and 52 (figure 4.21). As discussed before, we don’t expect to see such variations from period to period when version 404 is used, because in this version the influence of the changes of the number of active cells should be eliminated. Therefore we investigate further at this change and we discover that this doesn’t appear exactly at the time limit between period 51 and 52, but few days later ($\sim$ 9th May $\pm$ 2 days). Also in this case no obvious explanation has been found in the online logbook.

Another discovery made with our stability analysis is the loss of efficiency of two neighboring cassettes of the t0-scintillators, which has already been mentioned in the comment of picture 4.10. This can be well observed in figure 4.22, comparing the $A$ distribution of the periods 53 and 54. To understand when this happen, we divide the periods 53, 54 and the next ones in smaller periods and compare the impact point distributions $A$ with

![Figure 4.21: Maps of $s^I_{ij}$ and $s^A_{ij}$ showing the changes from period 51 to period 52 in the impact point distribution $I$ on the L3 detector (LEP3 volume) and in the direction distribution $A$. Black points show a relative increase of muon tracks, while white points indicate a relative decrease. Muon tracks with an energy larger than 30 GeV are used. (Selection B + 1.b).]
4.3 Stability of the muon detector

Changes from period 53 to 54

Figure 4.22: Maps of $s^I_{ij}$ and $s^A_{ij}$ showing the variations from period 53 to period 54 in the impact point distribution $I$ on the L3 detector (LEP3 volume) and in the direction distribution $A$. A loss of efficiency of two cassettes of t0-scintillators becomes evident (in white). Muon tracks with an energy larger than 30 GeV are used. (Selection B + 1.b).

each other. It is noted that the loss of efficiency is gradual. It starts from period 54 and continues up to the end of the data-taking.

With the stability analysis of the distributions $A$ and $I$ we observe also the effect of the noise induced by the LEP accelerator in the t0-scintillators, when in the year 2000, the highest beam energies are reached. Figure 4.23 shows the differences in the year 2000 of the two distributions between the time intervals when the LEP experiments are taking data (LEP status: Physics) and all the other time intervals. When the LEP status is ‘Physics’ (coincident with the high background in the t0-scintillators) the side 3 is affected by a decrease of efficiency $^4$. This has been evaluated to be around 5%. Side 3 is in the outer part of the LEP circumference, where we could expect more synchrotron radiation from the LEP accelerator.

The same effect is also evident when comparing the $A$ and $I$ distributions from period 56 and 57, when using the interval selection 2.b (figure 4.24, left). In the latter period in fact, data are taken with the LEP accelerator switched off. An increase of the mean efficiency is then noted on side 3 during this period. No relevant difference is observed (figure 4.24, right) indeed when we apply the cut on the average number of t0-scintillator hits per reconstructed events (selection 2.c).

$^4$A decrease of efficiency related with the high background in the t0-scintillators has been noted also in the event rate (see e.g. [103]).
Figure 4.23: $s_{ij}^I$ and $s_{ij}^A$ maps showing the difference in the distributions $A$ and $I$ between the time intervals when the LEP experiments are taking data (noise in the t0-scintillators) and all the other time intervals. On the side 3 (white part) less muon tracks are observed when LEP experiments are taking data. Muon tracks (selection B) with an energy larger than 30 GeV are used. The interval selection corresponds to 1.a without the cut on the the rate of the signals coming from the t0-scintillators.

Figure 4.24: $s_{ij}^I$ maps showing the difference of the impact point distribution $I$ from period 56 to 57 with interval selections 2.b (left) and 2.c (right). Event selection C and an energy cut of 30 GeV are applied. Black points show a relative increase of muon tracks.
4.3 Stability of the muon detector

Consequences on the analysis.

After our observations about the stability, we conclude that

- The LEP induced noise in the t0-scintillators affects the stability of the efficiency of the L3+C muon detector. Therefore, it is worthwhile whenever possible to discard the time intervals affected by this problem.

- We see that two cassettes of the t0-detector loose continuously efficiency starting from middle of July 2000. Therefore all events whose backtracking intercept these cassettes, are discarded from the physics analysis starting form period 54 (later called period 74). In the anisotropy analysis, since the stability is of absolute importance, we even discard these tracks for the whole year 2000 for more safety.

- We observe some changes in the impact point distribution $I$ and in the direction distribution $A$, that are not taken into account in the period definition of table 4.3. The reason for these changes has not been clearly understood up to now and no obvious relation with the informations written in the online logbook has been found. Since these changes are not continuous and we can establish approximately the time when they happen, we take them into account in our further analysis, redefining the “stable” periods.

New definition of “stable” periods

After the observations discussed in the previous paragraph we redefine the periods which we will consider as “stable”, in the sense that the angular distributions stay almost constant inside them. These periods are listed in table 4.4.

The table 4.5 shows the livetime available in each “stable” period for the interval selections 1.b, 2.b and 2.c.

For every “stable” period, we check that the distributions $I$ and $A$ of the first and the last half-periods are compatible, looking the corresponding $s^{I}_{ij}$ and $s^{A}_{ij}$ maps. No significative difference is noted applying as before the event selection B with an energy cut of 30 GeV and the interval selection 1.b.

An additional check has been performed. For every day-interval analyzed in the point sources search (see chapter 6) the compatibility of the distributions $I$ and $A$ is checked. To do that we represent both distributions with 2 dimensional histograms with only $10 \times 10$ bins. We apply the event selection C, the interval selection 2.c and an energy cut of 30 GeV. We then calculate a $\chi^2$ for the compatibility of the two distributions summing $s^{I}_{ij}$ and $s^{A}_{ij}$ for all the bins in which they are defined.

$$\chi_{\text{det}}^2 = \sum_{i,j} s^{I}_{ij}$$

Some time limits of the old version of the list (table 4.3) are kept even if no difference in the impact point distribution $I$ and in the direction distribution $A$ is noted, when using version 404 of the reconstruction program. This avoids to have too long periods.
### Table 4.4: "Stable" periods.

<table>
<thead>
<tr>
<th>Period Nr.</th>
<th>Begin</th>
<th>End</th>
<th>First Run</th>
<th>Last Run</th>
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<td>04 Aug 10:30</td>
<td>35150</td>
<td>39988</td>
</tr>
<tr>
<td>42</td>
<td>04 Aug 12:57</td>
<td>15 Aug 16:30</td>
<td>40018</td>
<td>42433</td>
</tr>
<tr>
<td>43</td>
<td>21 Aug 14:25</td>
<td>02 Sep 08:25</td>
<td>43970</td>
<td>47102</td>
</tr>
<tr>
<td>44</td>
<td>02 Sep 08:25</td>
<td>21 Sep 13:30</td>
<td>47186</td>
<td>51321</td>
</tr>
<tr>
<td>45</td>
<td>24 Sep 02:05</td>
<td>10 Oct 00:00</td>
<td>52119</td>
<td>55274</td>
</tr>
<tr>
<td>46</td>
<td>12 Oct 00:00</td>
<td>09 Nov 12:30</td>
<td>55275</td>
<td>60087</td>
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<table>
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<tr>
<td>75</td>
</tr>
<tr>
<td>76</td>
</tr>
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<td>77</td>
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</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>Interval selection</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1.b</td>
</tr>
<tr>
<td></td>
<td>[days]</td>
</tr>
<tr>
<td>41</td>
<td>6.96</td>
</tr>
<tr>
<td>42</td>
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<td>44</td>
<td>7.7</td>
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<td>45</td>
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<td>46</td>
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<td>Tot.1999</td>
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<td>71</td>
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<td>72</td>
<td>3.20</td>
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<td>73</td>
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<td>74</td>
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</tr>
<tr>
<td>76</td>
<td>2.07</td>
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<tr>
<td>77</td>
<td>8.95</td>
</tr>
<tr>
<td>Tot.2000</td>
<td>43.38</td>
</tr>
<tr>
<td>1999+2000</td>
<td>87.15</td>
</tr>
</tbody>
</table>

Table 4.5: Livetime available in the different “stable” periods for the interval selection a.2, b.1 and b.2.
4.4 Performance of the detector versus data selection

\[ \chi^2_{\text{dir}} = \sum_{i,j} s_{ij}^4. \] (4.14)

For the impact point distribution \( I \) the number of degrees of freedom (n.d.f.) (i.e. the number of bins which are not empty in the histogram representing the \( I \) distribution) is 71, while for the direction distribution \( A \) we have 86 degrees of freedom. The plots of \( \chi^2_{\text{det}}/n.d.f. \) as a function of the day is shown in figure 4.25, while \( \chi^2_{\text{dir}}/n.d.f. \) is represented in the plot of figure 4.26.

In the year 2000 we note few days with a very high \( \chi^2_{\text{det}} \) and \( \chi^2_{\text{dir}} \), in particular the days number 127 (6th May) and 230 (17th Aug). Looking inside the online database, we notice, that during almost all the day number 127, the high voltage is not at nominal value in the Z-chambers. A large part of the day 230 was also affected by high voltage problems. All the affected runs are excluded by the interval selection 1.a and 1.b. For the selection 2.a, 2.b and 2.c indeed almost half of the time-intervals are accepted. These are two example where the interval selections 2, which are used in the point sources search, fail. Therefore we keep a particular eye on that days, when performing the daily point sources search in chapter 6. We anticipate however that no particular excesses during these days are found.

In the year 1999 our attention is caught by the instability in the \( \chi^2 \) which appears around the day 263 (20th of September) in coincidence with the change in the distribution shown on figure 4.19. The situation stays quite bad for approximately 20 days. For this reason in table 4.4 we introduce a change of period at the 10th October (from period 45 to 46), in coincidence with a shutdown of 4 days.

4.4 Performance of the detector versus data selection

The idea of this section is to give an impression about the performance of the detector obtained with the different data selections, in order to understand our choices for the data selection criteria exposed in section 4.1.1.

We will first compare the rates of the selected muon data events. Then we will put our attention on the momentum resolution and the angular resolution, comparing the reconstructed muon track with the generated one in the Monte Carlo simulation (presented in the appendix A). Of course one has to be aware that the Monte Carlo simulation is not describing perfectly the reality. In particular we remark that the noise is only partially simulated. The single wire resolution of the P-chambers has been noted to be worse in the data than in the Monte Carlo simulation. In addition the description of the molasse profile in the simulation has been noted to contain an error. We believe that we can get however a rough estimation of the performance, allowing a comparison between the selections.
The idea is to check the compatibility of the impact point distribution $I$ of each day, with the one of the whole “stable” period in which the days are included. For the analysis we require muon tracks with energy larger than 30 GeV and fulfilling the event selection C. We apply in addition the interval selection 2.c.

Figure 4.25: $\chi^2_{\text{det}} / n.d.f.$ as a function of the day for the year 1999 (left) and 2000 (right).

Figure 4.26: $\chi^2_{\text{dir}} / n.d.f.$ as a function of the day for the year 1999 (left) and 2000 (right).
4.4 Performance of the detector versus data selection

4.4.1 Rates

In table 4.6 we present the typical event rates that we get with the event selections A, B and C and with different energy cuts. Using the selection C for the point source search, we can gain more than 50% of statistics than if selection B is used.

Indeed with event selection A we loose nearly 80% of statistics compared with selection B, too much for our physics analysis.

<table>
<thead>
<tr>
<th>Energy cut [GeV]</th>
<th>Event selection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>20</td>
<td>9 Hz</td>
</tr>
<tr>
<td>30</td>
<td>7 Hz</td>
</tr>
<tr>
<td>50</td>
<td>3 Hz</td>
</tr>
<tr>
<td>100</td>
<td>0.8 Hz</td>
</tr>
</tbody>
</table>

Table 4.6: Typical event rate for the event selections A, B and C and for different energy cuts. (end 1999).

4.4.2 Momentum Resolution

In equation 3.3 we show that, for an ideal helical muon track, the momentum component perpendicular to the magnetic field $p_\perp$ is inversely proportional to the sagitta $S$.

Since the uncertainty in the measurement of the sagitta $\Delta S$ is momentum independent, to study the momentum resolution it is convenient to analyze the error on the inverse of the momentum of the muons $\Delta (1/p)$. In fact the inverse momentum, is also expected to have a constant error, if we neglect the interactions of the muon with the matter which causes its trajectory to deviate from the helical shape, as the multiple scattering and the energy loss. This kind of interactions dominate the relative error only for low energies ($\lesssim 40$ GeV). The momentum error $\Delta p$ can then be extracted from

$$\frac{\Delta p}{p} = p \Delta \left(\frac{1}{p}\right)$$

(4.15)

For the events simulated in the Monte Carlo simulation, we calculate the difference between the inverse momentum at surface $1/p_{\text{reco}}$ of the reconstructed and backtracked track and the inverse momentum $1/p_{\text{gen}}$ of the generated muon. The distributions of such difference for the event selections A, B and C is shown in figure 4.27 for muons with reconstructed momentum larger than 50 GeV/c. The central part of the distribution is fitted with a Gaussian. The standard deviation $\sigma$, the mean value and the normalization factor of the fitted Gaussian are reported in the figure. As expected the event selection A gives the best momentum resolution, but the difference with the other selections is not so strong. The selection C gives a relative higher number of events in the queues of the distribution, because of the P-doublets.
Detector performance

Figure 4.27: Momentum resolution for event selections A, B and C extracted from the Monte Carlo simulation. We plot here the histogram showing the distribution of the difference of the inverse of the momentum $1/p_{\text{Reco}}$ of the reconstructed track and the inverse of the momentum $1/p_{\text{Gen}}$ of the generated track for muons with $p_{\text{Gen}} > 50$ GeV/c. The mean value and standard deviation $\sigma$ refer to the Gaussian fitted with the central part of the histogram. Muons are generated with momenta larger than 20 GeV/c.
From the Gaussian fit of the distribution made with the selection B, we get $\sigma = 7.3 \cdot 10^{-4}$ (and a systematical shift of $-2.3 \cdot 10^{-4}$). This $\sigma$ correspond to a momentum resolution $\Delta p$ of 3.3 % at 45.6 GeV/c. At this energy the L3+C experiment has the opportunity to extract the momentum resolution, analyzing the muons originating from the decay of the Z boson produced by the electron-positron collision in the LEP accelerator at a beam energy of 45.6 GeV (Z-resonance). With this analysis a resolution of 4.6% has been obtained [2] for a selection similar to selection B.

4.4.3 Angular Resolution

Angular resolution extracted from Monte Carlo simulation

To get an estimation of the angular resolution (including the effect of the multiple scattering of the muons in the rock and the detector resolution itself), we compare the direction of the muon generated at surface by the Monte Carlo simulation with the direction of the corresponding reconstructed tracks. Their angular distance $<$ is plotted for the different event selection criteria defined in section 4.1.1 in the figures 4.28-4.30. For the analysis we use muon tracks generated with an energy larger than 20 GeV whose reconstructed track has a “measured” energy larger than 30 GeV. In figure 4.31 we show the $<$-distribution obtained from events with an angular variance $\text{var}(\langle \rangle) > 10^{-3}$ (see definition in section 4.1.1).

From the mean value of the angular distance $<$, it is observed that there is not a large difference between the three selections A, B and C. From the plot 4.31 however the importance of setting a cut on the variance of $\text{var}(\langle \rangle)$ is evident. No other criteria, which can be extracted from the informations available from the simulated data processed by the reconstruction program have been found to improve so remarkably the angular resolution, without loosing too much statistics. A similar conclusion has been obtained also from the analysis of the dimuon data [109].

We remark that the cut on the angular variance rejects especially muons of low energy, so that the momentum spectrum of the data sample used in our analysis has not the same shape as the real one. In addition for higher energies ($\gtrsim 100$ GeV), the cut of 0.001 is probably not low enough to fully profit from the available angular resolution. For future analyses concerning the point source search, we suggest therefore to study the possibility to set a momentum dependent cut on the angular variance.

Moon shadow

The moon shadow can be observed with the L3+C data through the missing muon events from its direction. This represents a good check for the precision of our direction measurement. Figure 4.32 plots the significance of the deficit of muon events with energy $E_{\mu}$ larger than 100 GeV in units of standard deviations $\sigma$, as a function of the deflection from the position of the moon. The abscissa represents the direction of the magnetic deflection of the primary as a consequence of the Earth’s magnetic field, assuming that the primary is a proton. For each event, such direction is calculated as a function of the direction
Angular resolution (MC) - Energy cut: 30 GeV

Figure 4.28: Event selection A: Angular distance between the direction of the generated muons and the direction of the reconstructed muon track in the Monte Carlo simulation for energy larger than 30 GeV.

Angular resolution (MC) - Energy cut: 30 GeV

Figure 4.29: Event selection B: Angular distance between the direction of the generated muons and the direction of the reconstructed muon track in the Monte Carlo simulation for energy larger than 30 GeV.

Angular resolution (MC) - Energy cut: 30 GeV

Figure 4.30: Event selection C: Angular distance between the direction of the generated muons and the direction of the reconstructed muon track in the Monte Carlo simulation for energy larger than 30 GeV.

Angular resolution (MC) - Energy cut: 30 GeV

Figure 4.31: For events with \( \text{var}(<) > 0.001 \): Angular distance between the direction of the generated muons and the direction of the reconstructed muon track in the Monte Carlo simulation for energy larger than 30 GeV.
Deficit of muons from the Moon direction ($E_\mu > 100$ GeV)

Figure 4.32: Moon shadow: Significance of the deficit of muons events ($E_\mu > 100$ GeV) in units of standard deviations $\sigma$, as a function of the deflection from the position of the moon. The abscissa represents the direction of the magnetic deflection of the primary as consequence of the Earth’s magnetic field, assuming that the primary is a proton. The ordinate represent the direction perpendicular to the magnetic deflection. The circle in the center of the plot shows the moon position. [110]
and momentum of the muons. The ordinate represent the direction perpendicular to the calculated magnetic deflection.

A shift of the shadow in the direction of the magnetic deflection is clearly visible. Since negatively charged particles are deflected in the opposite direction than protons (figure 4.33), the absence of a deficit on the negative part of the abscissa allows to set a limit on the anti-proton flux in the TeV region.

![Diagram](image)

Figure 4.33: The Earth-moon system as a spectrometer: The moon-shadow originating from protons is shifted westward, while the moon-shadow originating from anti-proton is shifted eastward. [111]

The fact that we see the moon shadow, shows that we are able to reconstruct correctly the direction of the muon tracks.

### 4.5 The precision of the livetime measurement

As mentioned in section 3.4 the livetime is registered in units of 0.8388608 seconds. To get a more precise livetime for a given run we proceed as follows. Let be $N$ the total number of triggered events and $L$ the number of livetime units in a given run (rounded at the lowest unit). If $n_c$ is the last event number before the livetime unit changed from
4.5 The precision of the livetime measurement

$L - 1$ to $L$, we estimate the livetime of the run as

$$ L = \frac{N}{n_c} \cdot 0.8388608 \text{ [s]} \quad (4.16) $$

Runs shorter than two seconds or with less than 1000 events are not used.

The consistency between the livetime $L$ and the number of the clock signals (see section 3.4) can be checked in table 4.7.

<table>
<thead>
<tr>
<th>Period</th>
<th>Number of clock signals</th>
<th>Livetime $L$ [s]</th>
</tr>
</thead>
<tbody>
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<td>798997</td>
</tr>
<tr>
<td>42</td>
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<td>77</td>
<td>843052</td>
<td>842702</td>
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</table>

Table 4.7: Livetime and number of clock signals obtained during the different “stable” periods with time interval selection type 2.c.
Chapter 5

Analysis of the large scale anisotropy

5.1 Relation between primary anisotropy and muon angular distribution

As already mentioned in section 2.5.1, a cosmic ray detector on Earth is able, due to the Earth’s rotation, to scan the sky in the right ascension direction. We try here to develop a mathematical description on that point, to which we will refer also in chapter 6.

Let \( \Phi_i(\delta, \alpha, E_0) \) be the direction dependent differential spectrum of the primary particles of type \( i \) expressed in units of \( \text{m}^{-2}\text{sr}^{-1}\text{s}^{-1}\text{GeV}^{-1} \) (\( i = \text{p,He,Fe,}\gamma,\ldots \)). We define the anisotropy function \( \delta_i^{\text{dir}}(\delta, \alpha, E_0) \) for the primary particles of type \( i \) as

\[
\delta_i^{\text{dir}}(\delta, \alpha, E_0) = \frac{\Phi_i(\delta, \alpha, E_0)}{\Phi_i(E_0)} - 1
\]  

(5.1)

where \( \Phi_i \) is the mean differential spectrum of the corresponding primary type

\[
\Phi_i(E_0) = \frac{1}{4\pi} \int_{-\pi/2}^{\pi/2} d\delta \cos(\delta) \int_0^{2\pi} d\alpha \Phi_i(\delta, \alpha, E_0).
\]  

(5.2)

We try now to describe the relation between the number of detected muons and the flux of the primary particles as a function of direction expressed in equatorial coordinates. Since this depends on the sideral time \( t_s \) we have to know when the detector is on. Let’s describe this by the function \( L(t) \)

\[
L(t) = \begin{cases} 
1 & : \text{if at time } t \text{ we accept data} \\
0 & : \text{if at time } t \text{ we don’t accept data}
\end{cases}
\]  

(5.3)

The livetime distribution \( \mathcal{L}(t_s) \) as a function of the sideral time is expressed as

\[
\mathcal{L}(t_s) = \int dt \ L(t) \ \delta_{\text{Dirac}}(T_s(t) - t_s)
\]  

(5.4)
5.1 Relation between primary anisotropy and muon angular distribution

where $T_s(t)$ is the function giving the sidereal time for a given time $t$ and $\delta_{\text{Dirac}}()$ is the Dirac’s $\delta$-function. $L(t_s) \, dt_s$ corresponds to the livetime seconds in which the detector was on in the sidereal time interval $[t_s, t_s + dt_s]$.

Let’s express with $N_{\mu}(\delta, \alpha; E_{\text{cut}})$ the expected number of measured muons per steradian with an energy at surface greater than $E_{\text{cut}}$ in the direction $(\delta, \alpha)$. $N_{\mu}$ can be written as the sum of the contributions of the different particle types $i$ as

$$N_{\mu}(\delta, \alpha; E_{\text{cut}}) = \sum_i N_{\mu,i}(\delta, \alpha; E_{\text{cut}})$$  \hspace{1cm} (5.5)

where

$$N_{\mu,i}(\delta, \alpha; E_{\text{cut}}) = \int_0^\infty \int_0^{2\pi} dE_0 \, L(t_s) \Phi_i(\delta, \alpha, E_0) \cdot \mathcal{A}_i(\delta, \alpha, t_s, E_0) \cdot A_i(\delta, \alpha, E_{\text{cut}}).$$  \hspace{1cm} (5.6)

$A_i(\delta, h_n, E_0; E_{\text{cut}})$ is the efficiency distribution telling us how we are sensitive to the primary particle $i$ with energy $E_0$ when measuring the muons with energy at surface larger than $E_{\text{cut}}$. $A_i$ has to be understood as:

$$A_i(\delta, h_n, E_0; E_{\text{cut}}) = \int_{E_{\text{cut}}}^\infty dE_\mu \, A(\delta, h_n, E_\mu) \cdot n_{\mu,i}(E_\mu, \theta(\delta, h_n)).$$  \hspace{1cm} (5.7)

where $A(\delta, h_n, E_\mu)$ is the effective area of detection for a muon with Energy $E_\mu$ and direction described by the declination $\delta$ and the negative hour angle $h_n$, while $n_{\mu,i}(E_0, E_\mu, \theta) \cdot dE_\mu$ is the expected number of muons generated by a primary particle $i$ with Energy $E_0$ in the energy interval $[E_\mu, E_\mu + dE_\mu]$ when the zenith angle is $\theta$.

In the equation (5.6) the negative hour angle $h_n$ has been substituted according to equation (2.9). Since here we are interested in large scale fluctuations, we neglect the angular resolution.

Let’s write $\Phi_i$ as the sum of an isotropic mean component and the perturbation $\Phi_i^\delta$ causing the anisotropy

$$\Phi_i(\delta, \alpha, E_0) = \Phi_i(\alpha) + \Phi_i^\delta(\delta, \alpha, E_0)$$  \hspace{1cm} (5.8)

where according to equations (5.1) and (5.2) the perturbation $\Phi_i^\delta$ can be written as

$$\Phi_i^\delta(\delta, \alpha, E_0) = \Phi_i(\alpha) \cdot \delta_i^\text{dir}(\delta, \alpha, E_0).$$  \hspace{1cm} (5.9)

Substituting the two addends of equation (5.8) separately in the expression (5.6) we get respectively the number of detected muons by the isotropic component $N_{\mu,i}^{\text{iso}}$ and the ones generated by the perturbation $N_{\mu,i}^\delta$ for the particle type $i$ with

$$N_{\mu,i} = N_{\mu,i}^{\text{iso}} + N_{\mu,i}^\delta.$$  \hspace{1cm} (5.10)
If we define \( \tilde{A} \) so that:

\[
\tilde{A}(\delta, h_n; E_{\text{cut}}) = \sum_i \int_0^\infty dE_0 \Phi_i(E_0) \cdot A_i(\delta, h_n, E_0; E_{\text{cut}})
\]  

(5.11)

substituting \( h_n \) according to equation (2.9) we get

\[
N_{\mu, \text{tot}}^{\text{iso}}(\delta, \alpha; E_{\text{cut}}) \equiv \sum_i N_{\mu i}^{\text{iso}}(\delta, \alpha; E_{\text{cut}}) = \int_0^{2\pi} d\tau A(\delta, [\alpha - t_\tau]; E_{\text{cut}}).
\]  

(5.12)

With the measurement of the muon distribution \( N_{\mu}(\delta, \alpha; E_{\text{cut}}) \) for a given muon energy \( E_{\text{cut}} \) it is not possible to get the information about the anisotropy function component \( \delta_{\text{dir}}^i(\delta, \alpha, E_0) \) of every single particle type nor about its dependence from the primary energy \( E_0 \). However we can define a weighted average anisotropy function at which the measurement can be sensible, applying the weights \( N_{\mu i}^{\text{iso}}(\delta, \alpha; E_0) \)

\[
\tilde{\delta}_{\text{dir}}(\delta, \alpha; E_{\text{cut}}) = \sum_i \int_0^\infty dE_0 \frac{\delta_{\text{dir}}^i(\delta, \alpha, E_0) N_{\mu i}^{\text{iso}}(\delta, \alpha; E_0)}{\sum_i \int_0^\infty dE_0 N_{\mu i}^{\text{iso}}(\delta, \alpha; E_0)}
\]  

(5.13)

where of course the largest contribution comes from the protons. \( \tilde{\delta}_{\text{dir}}(\delta, \alpha; E_{\text{cut}}) \) is normally considered to be an estimate of the anisotropy function at the primary energy \( E_{0,\text{median}} \approx 12E_{\text{cut}} \) as given by equation (2.4).

Using the definition of equation (5.13) we find that

\[
\sum_i N_{\mu i}^\delta(\delta, \alpha; E_{\text{cut}}) = \tilde{\delta}_{\text{dir}}(\delta, \alpha; E_{\text{cut}}) \cdot \int_0^{2\pi} d\tau sL(t_\tau) \cdot \tilde{A}(\delta, [\alpha - t_\tau]; E_{\text{cut}})
\]  

(5.14)

so that taking into account equations (5.10) and (5.12) we are able to express \( \tilde{\delta}_{\text{dir}}(\delta, \alpha; E_{\text{cut}}) \) as

\[
\tilde{\delta}_{\text{dir}}(\delta, \alpha; E_{\text{cut}}) = \frac{N_{\mu}(\delta, \alpha; E_{\text{cut}})}{\int_0^{2\pi} d\tau sL(t_\tau) \cdot \tilde{A}(\delta, [\alpha - t_\tau]; E_{\text{cut}})} - 1.
\]  

(5.15)

If we would know precisely \( \tilde{A}(\delta, h_n; E_{\text{cut}}) \) it would be easy to calculate \( \tilde{\delta}_{\text{dir}} \) with the help of equation (5.15) and the measurement of \( N_{\mu}(\delta, \alpha; E_{\text{cut}}) \). If we would try to calculate \( \tilde{A} \) with the Monte Carlo simulation we would never come to a useful precision for the tiny effect that we want to measure. From the plot A.7 reported in the appendix, it is clear that our Monte Carlo simulation is unable to reproduce the true angular distributions with a precision better than 10%, while the expected anisotropy is smaller than 1 permil in the energy range of interest for L3+C. (see chapter 2.5). To achieve the goal of getting informations about the anisotropy we should therefore proceed in another way.
5.1 Relation between primary anisotropy and muon angular distribution

More informations can be obtained using the (acceptance) distribution $\tilde{N}_\mu(\delta, h_n; E_{\text{cut}})$ of the detected muons in local equatorial coordinates, that we can measure experimentally. Since

$$\tilde{N}_\mu(\delta, h_n; E_{\text{cut}}) = \tilde{A}(\delta, h_n; E_{\text{cut}}) \int_0^{2\pi} dt_s (1 + \bar{\delta}^{\text{dir}}(\delta, [t_s + h_n])) \cdot \mathcal{L}(t_s)$$

(5.16)

we can see that in case of isotropy (i.e. $\delta^\text{dir}_i = 0$), $\tilde{A}$ represent the direction dependent detection rate. In fact

$$\tilde{A}(\delta, h_n; E_{\text{cut}}) = \frac{1}{L_t} \tilde{N}_\mu(\delta, h_n; E_{\text{cut}})$$

(5.17)

where $L_t$ is the total livetime

$$L_t = \int_0^{2\pi} dt_s \mathcal{L}(t_s).$$

(5.18)

In the case that the primary cosmic rays are not isotropic ($\bar{\delta}^{\text{dir}}(\alpha, \delta) \neq 0$) some interesting results can be achieved when $\mathcal{L}(t_s)$ is constant, i.e.

$$\mathcal{L}(t_s) = \frac{L_t}{2\pi}$$

(5.19)

Equation (5.16) becomes then

$$\tilde{N}_\mu(\delta, h_n; E_{\text{cut}}) = L_t \cdot \tilde{A}(\delta, h_n; E_{\text{cut}}) \int_0^{2\pi} dt_s (1 + \bar{\delta}^{\text{dir}}(\delta, [t_s + h_n]))$$

(5.20)

Since $\bar{\delta}^{\text{dir}}$ is integrated over the whole period of $t_s$, the integral part of equation (5.20) will be independent from the hour angle $h_n$.

$$\int_0^{2\pi} dt_s (1 + \bar{\delta}^{\text{dir}}(\delta, [t_s + h_n])) = \int_0^{2\pi} dt_s (1 + \bar{\delta}^{\text{dir}}(\delta, t_s))$$

(5.21)

It follows that

$$\tilde{A}(\delta, h_n; E_{\text{cut}}) = \frac{2\pi \bar{N}_\mu(\delta, h_n; E_{\text{cut}})}{L_t \int_0^{2\pi} dt_s (1 + \bar{\delta}^{\text{dir}}(\delta, t_s))}$$

(5.22)

For fixed declinations $\delta_1$ the shapes of the distributions in the hour angle of $\tilde{A}(h_n)$ and $\bar{N}_\mu(h_n)$ are the same, i.e.

$$\frac{\int_0^{2\pi} d h_n \tilde{A}(\delta_1, h_n; E_{\text{cut}})}{\int_0^{2\pi} d h_n \frac{\bar{N}_\mu(\delta_1, h_n; E_{\text{cut}})}{\hat{\bar{N}}}} = \frac{\int_0^{2\pi} d h_n \bar{N}_\mu(\delta_1, h_n; E_{\text{cut}})}{\int_0^{2\pi} d h_n \frac{\tilde{A}(\delta_1, h_n; E_{\text{cut}})}{\hat{\tilde{A}}}}.$$
We note also that in case of constant livetime distribution (eq. 5.19) the measured muon distribution in equatorial coordinates \( N_\mu(\delta, \alpha; E_{\text{cut}}) \) is

\[
N_\mu(\delta, \alpha; E_{\text{cut}}) = (1 + \tilde{\delta}_{\text{dir}}(\delta, \alpha)) \cdot L_t \int_0^{2\pi} dt_s \tilde{A}(\delta, t_s; E_{\text{cut}}).
\]  

(5.24)

The integral part of equation (5.24) doesn’t depend on the right ascension \( \alpha \). Therefore for fixed declinations \( \delta_1 \), the right ascension component of \( 1 + \tilde{\delta}_{\text{dir}}(\delta_1, \alpha) \) has the same shape as the distribution of the measured muons \( N_\mu(\alpha) \), i.e.

\[
\frac{1 + \tilde{\delta}_{\text{dir}}(\delta_1, \alpha)}{2\pi \int_0^{2\pi} d\alpha \left[ 1 + \tilde{\delta}_{\text{dir}}(\delta_1, \alpha) \right]} = \frac{N_\mu(\delta_1, \alpha; E_{\text{cut}})}{2\pi \int_0^{2\pi} d\alpha N_\mu(\delta_1, \alpha; E_{\text{cut})}}.
\]

(5.25)

\section{5.2 Summary of the methods used by other experiments}

From equations (5.23) and (5.25), it becomes clear that a measurement of the amplitude of the anisotropy becomes possible in the right ascension direction even if the deviations from the isotropy are much smaller than the precision that can be achieved in establishing the direction dependent acceptance with Monte Carlo calculations. For the accuracy of this measurement the only factors that matter, are statistics and the stability in time of the direction dependent efficiency. For the declination direction indeed we have not the tools to overcome the obstacles given by the insufficient precision of the knowledge of the efficiency as a function of the direction and in particular of the effective area \( A(\delta, h_n, E_{\mu}) \).

This is true also for all experiments carried out till now. The obtained significative results concern the anisotropy seen in the distribution in right ascension of the cosmic rays regardless of the declination, i.e.

\[
\tilde{\delta}_{\text{dir}}(\alpha) = \frac{\pi/2}{-\pi/2} \int d\delta \left[ \tilde{\delta}_{\text{dir}}(\alpha, \delta) \int_0^{2\pi} d h_n \tilde{A}(\delta, h_n) \right]
\]

(5.26)

Alternatively the data can be analyzed likewise in bands along the right ascension direction included in given declination intervals.

The methods applied can differ slightly from experiment to experiment, but all of them use the principles described above.

One method follows directly from the above discussion. One can try to produce a subset of events in which the livetime distribution \( L(t_s) \) is constant and then use equation (5.25). In the analysis of the data of the MACRO experiment [112] [113] this is achieved
as follows. The sidereal day is divided in several interval of equal length. Once the interval with less exposure time (livetime) is found, from each of the other intervals, events are rejected randomly in order to get the the same exposure time in each of them. Another possibility that go in this direction is to give a weight to each event proportional to the inverse of the livetime available in the corresponding interval.

The most common method used (e.g. [55], [36], [52]) is to assume (as approximation) that \( \mathcal{A}(h_n) \) has the shape of the Dirac’s delta function \( \delta_{\text{Dirac}}(h_n) \). In this case from equation (2.9) follows that the right ascension can be identified with the sidereal time (\( \alpha \equiv s.t. \)). Therefore the right ascension distribution is simply given by the detection rate as a function of the sidereal time; the arrival direction information is not used. This procedure can be justified when the mean of the absolute value of the hour angle of the detected events \( |h:a| = |h_n| \) is much smaller than the scale on which one wants to measure the fluctuations of the anisotropy. For instance the Kamiokande II experiment [55] uses this method claiming that, due to the fact that \( |h:a| = 28^\circ \), the assumption that \( \alpha \equiv s.t. \), has not a large influence on the final results. Some experiments divide the sky in fixed windows with respect to the horizontal reference frame and apply the same method separately to every window. This is of course possible only if the statistics is high enough, so that in every window the statistical error is smaller than the measured effect. The advantages of this procedure are that \( |h:a| \) will be reduced and that the differences of the results in the different windows can be used to estimate the systematical error. In addition bands of different declination ranges can be analyzed independently. For instance the EAS-TOP experiment at the Gran Sasso Laboratory [114] uses five windows: one in the vertical direction (\( \theta < 20^\circ \)) and one for each of the four cardinal points for more inclined tracks. In the air shower array experiment on the Mt. Norikura [37] the difference of two directional (eastward and westward) air showers observations are used even to eliminate the atmospheric effects, assuming that these influence in the same way both eastward and westward observations. The basic idea is represented in figure 5.1. The \( n \)th-harmonic vector \( V^n \) measured in the vertical direction (\( h.a. = 0 \)) is given by the sum of the component \( C^n \) generated by the atmospheric effect and the component \( V^{*n} \) generated by the anisotropy. The harmonic vectors seen by the eastward and westward observations \( E^n \) and \( W^n \) have the same common component \( C^n \) caused by the atmospheric effects, but the anisotropy components \( E^{*n} \) and \( W^{*n} \) are rotated by the angle \( \pm \beta \) which is equal to \( n \) times the mean absolute value of the hour angle measured by one of the two directional air shower observations.

\[
\beta = n \cdot \overline{h.a.} \tag{5.27}
\]

Knowing \( \beta, E^n, W^n \), one can find in a unique way \( C^n \) and the true anisotropy component \( V^{*n} \). Unfortunately with our statistics we are not able to make a measurement with sufficient precision of the different components for which such correction could give a significant improvement. The atmospheric effect will be however included in our estimation of the systematical error, which will be discussed later.

In the subsection 5.3 we will discuss the implications and how to apply the method based on the sidereal time distribution to our dataset and then in subsection 5.4 we develop
an alternative method which includes the arrival direction information of the muons based on the equations (5.23) and (5.15).

5.3 Method based on the sideral time distribution (Method A)

As discussed in section 2.5.1 the goal of the large scale anisotropy study is to perform a harmonical analysis of the anisotropy function $\tilde{\delta}^{\text{dir}}(\alpha)$ and get the amplitudes $\xi_i$ and phases $\varphi_i$ of the low harmonics.

$$\tilde{\delta}^{\text{dir}}(\alpha) = \sum_{m=1}^{\infty} \xi_m \cos(m(\alpha - \varphi_i)) \quad \text{(5.28)}$$

If we use the method based on the identification of the right ascension with the sideral time, we analyze harmonically the mean detection rate of the muons $R(t_s)$ as a function of the sideral time $t_s$.

$$R(t_s) = R_0 + \sum_{m=1}^{\infty} R_m \cos(m(t_s - \varphi_m)) \quad \text{(5.29)}$$

$$\xi_m = \frac{R_m}{R_0} \quad \text{(5.30)}$$

This can be achieved also calculating the Fourier transform of the relative time-varying intensity $\frac{I(t)}{\langle I \rangle}$ of the muons and get an amplitude spectrum as done by Cutler et al. [36]
5.3 Method based on the sideral time distribution

(Method A)

Figure 5.2: Amplitude spectrum near frequency of 1 day$^{-1}$ seen by the Mayflower Mine Detector. Vertical dotted lines indicate from left to right the anti-sideral frequency, solar frequency, and the sideral frequency [36].

(see figure 5.2). Looking at the variations of the amplitude spectrum at the frequencies which don’t have a particular meaning, one can get an estimation of the amplitude of the noise and of the systematical uncertainty of the measurement.

Besides of the sideral frequency and its harmonics ($\nu_s = (1 + 1/365.24)$ day$^{-1}$) also the solar ($\nu_s = 1$ day$^{-1}$), the extended sideral ($\nu_{s*} = (1 + 2/365.24)$ day$^{-1}$) and the anti-sideral frequencies ($\nu_{*} = (1 - 1/365.24)$ day$^{-1}$) are of interest. The latter would be seen with a corresponding side lobe of the same size at the sideral frequency if a real effect at the solar frequency is modulated with an annual frequency (beat effect). If such a signal would be present, its contamination should be subtracted from the measured signal at sideral frequency to find the real sideral effect. On the other side a modulation in the sideral frequency would be seen with signals at the extended sideral frequency and at the solar frequency. A signal at a solar frequency alone could be found in the case the sun influences the detection rate, e.g. indirectly via meteorological effects. Also the orbital motion of the Earth is expected to produce a signal with this frequency as a consequence of the Compton-Getting Effect (see chapter 2.5.2).

When performing the analysis the question arises, whether there is also a signal at the tidal frequency. Since tides act not only on the sea, but also on the atmosphere, one could imagine that they can influence the muon detection rate, due to the same principles generating a dependence of the muon intensity on the pressure and on the temperature. Therefore we had a special look at the period corresponding to the average time length between two transits of the Moon ($T_{\text{moon}} = 1.0350501$ days$^{-1}$). A signal induced by the tides would be seen mainly in the second harmonic, since in the period $T_{\text{moon}}$ appear two

\[ T_{\text{moon}} = \frac{T_e}{T_e - T} \quad \text{where} \quad T_e = 29.530589 \text{ days} \quad [115] \] is the synodic orbital period of the moon
high tides and two low tides.

Since an experiment has a limited data-taking time, it is well known that one expects a corresponding limitation in the spectral resolution (in analogy to Heisenberg’s uncertainty principle). If the experiment spans from a time \( t = 0 \) to \( t = T \) the expected spectral resolution is of the order of

\[
\Delta \nu \sim \frac{1}{T}. \tag{5.31}
\]

Therefore to clearly separate the solar fundamental frequency from the sidereal one we need to have a data-taking period of at least one year. Things improve for higher harmonics: to separate the \( n \)-th solar harmonics from the \( n \)-th sidereal harmonics we need to take data for a minimum of \( 1/n \) years.

Let’s look at this problem in more detail. Imagine that we measure \( f(t) = \frac{I(t)}{<I>} \) with an ideal experiment lasting for infinite time and that \( f(t) \) is a sum of sinusoidal functions (with frequency \( \nu_n \), amplitude \( \xi_n \) and phase \( \varphi_n \)) and noise \( n(t) \)

\[
f(t) = \sum_n \xi_n \cos(2\pi \nu_n(t - \varphi_n)) + n(t) \tag{5.32}
\]

assuming that for the noise \( n(t) \)

\[
\lim_{T \to \infty} \frac{1}{T} \int_0^T dt \ n(t) e^{2\pi i \nu t} = 0. \tag{5.33}
\]

We define the complex amplitude of \( f(t) \) for the frequency \( \nu_n \) as

\[
\hat{\xi}_n = \xi_n e^{2\pi i \nu_n \varphi_n} \tag{5.34}
\]

Performing the Fourier analysis of the function \( f(t) \), we can calculate the complex amplitude \( \hat{\xi}(\nu) \) for the frequency \( \nu \) through

\[
\hat{\xi}(\nu) = \lim_{T \to \infty} \frac{2}{T} \int_0^T dt \ f(t) e^{2\pi i \nu t}. \tag{5.35}
\]

For the frequencies \( \nu = \nu_n \) we get \( \hat{\xi}(\nu_n) = \hat{\xi}_n \), otherwise equation (5.35) vanishes (\( \hat{\xi}(\nu) = 0 \)).

However every experiment is time limited. For an experiment lasting from time \( t = -T/2 \) to \( t = T/2 \) we will find an amplitude spectrum which is the convolution of the ideal amplitude spectrum \(|\hat{\xi}(\nu)|\) with sinc\((\pi T \nu)\).

\[
\left| \frac{2}{T} \int_0^T dt \ \frac{I(t)}{<I>} e^{2\pi i \nu t} \right| = |\hat{\xi}(\nu)| \ast \text{sinc}(\pi T \nu) \tag{5.36}
\]
where
\[
sinc(x) = \frac{\sin(x)}{x}. \tag{5.37}
\]

The situation is more complicated in the L3+C experiment, since the data-taking period is split in two big periods, one in 1999 from time \( t_1 \) to \( t_2 \) and one in 2000 from time \( t_3 \) to \( t_4 \). To get an idea of what we can expect, we calculate the amplitude gotten via the Fourier transform windowed on our data-taking period of the function
\[
f(t) = \sin(2\pi \nu t) \tag{5.38}
\]
which represents an ideal oscillation with sideral frequency.

\[
|\xi(\nu)| = \frac{2}{(t_2 - t_1) + (t_4 - t_3)} \left| \int_{t_1}^{t_2} dt f(t)e^{2\pi i\nu t} + \int_{t_3}^{t_4} dt f(t)e^{2\pi i\nu t} \right|. \tag{5.39}
\]

The result is shown in figure 5.3. On the resulting spectrum unfortunately one finds a large amplitude also at the solar frequency. This unwanted effect is unavoidable if the data-taking doesn’t contain all parts of the year. In particular in the L3+C data-taking the winter period is missing completely. So we have to keep in mind, that if we would find an effect with a sideral period, it will not be possible to completely disentangle it from a periodical effect which follows the solar period.

Figure 5.3: Expected amplitude spectrum from an ideal oscillation with sideral frequency analyzed on the data-taking period of L3+C. Vertical dotted lines indicate from left to right the anti-sideral frequency, solar frequency, and the sideral frequency.
Besides of the fact that experiments are time limited, one has to handle also short time interruptions. In the paper of Cutler et al. [36] this is done as follows. The data-taking is split in $N$ intervals of length $\Delta t$. The fractional muon intensity deviation from the average of the $n$-th interval is calculated as

$$x_n \equiv \frac{\Delta I}{\langle I \rangle} = \frac{I(n\Delta t) - \langle I \rangle}{\langle I \rangle}. $$

The complex amplitude $\xi(\nu)$ for the frequency $\nu$ is then calculated thorough the discrete Fourier transform as

$$\xi(\nu) = \frac{2}{M} \sum_{n=0}^{N-1} x_n e^{2\pi i \nu \Delta t} \quad (5.41)$$

where $M$ is the number of intervals in which data are present.

If equation (5.41) is applied, all intervals containing data will have the same weight, independently from the fraction of the time in which the detector is on. The data-taking of the L3+C experiment is quite irregular. In particular if we apply the selection of the runs based on the database (selection type 1.b, see section 4.1.2), the dataset is quite discontinuous and the percentage of livetime contained in different intervals $\Delta t$ will suffer of strong variations. Nevertheless we can take advantage from the precise livetime counter.

To better take into account the characteristics of L3+C we prefer to use another method which will give a more correct weight to the events contained in each interval. We analyze independently 90 frequencies $\nu_n$ near the solar and the sideral frequency.

$$\nu_n = \nu_{\odot} + \frac{n - 40}{4} (\nu_4 - \nu_{\odot}) \quad (n = 1, 2, ..., 90)$$

$\nu_{40}$ corresponds to the solar frequency, $\nu_{36}$, $\nu_{44}$ and $\nu_{48}$ are respectively the anti-sideral the sideral frequency and the extended sideral frequency. We define the phase corresponding to the solar frequency to be the mean local solar time. The phases corresponding to the other frequencies are defined as follows. There is a time $t_x$ near the autumn equinox in which the solar time is equal to the sideral time. We choose the phases of all frequencies, in order to be all the same at the time $t_x$ in the year 2000.

For every frequency $\nu_n$ we fill two histograms, both having a number of bins $N_{\text{bin}}$ (where normally we set $N_{\text{bin}}=240$). The first histogram contains the event distribution versus the phase of the corresponding frequency. We denote with $e_m$ its bin contents in the phase intervals

$$\frac{2\pi m}{N_{\text{bin}}} < \phi < \frac{2\pi (m + 1)}{N_{\text{bin}}} \quad (5.43)$$

The second one contains the livetime distribution versus the phase and the bin contents for the same intervals (5.43) is noted with $L_m$. To fill the latter histogram we calculate the
5.3 Method based on the sideral time distribution
(Method A)

contribution of each run \(i\) in the following way. We produce a histogram with the event
distribution of the run \(i\) as a function of the phase (which normally fills only few bins) and
we denote its bin contents for the phase intervals (5.43) with \(N_m^i\) \((m = 0, 1, \ldots, N_{\text{bin}} - 1)\).
Then

\[
\mathcal{L}_m = \sum_i \frac{N_m^i L_t^i}{\sum_{k=0}^{N_{\text{bin}}-1} N_k^i}
\]

(5.44)

where \(L_t^i\) is the livetime of the run \(i\). Note that the phases of the solar and the sideral
frequency are given respectively by the local solar time and the local sideral time. In
figure 5.4 the two histograms for the sideral frequency for events with a surface energy
cut of 30 GeV are shown as an example. The data selection is applied as described in
section 4.1 (Type B+1.b).

![Event distribution and livetime distribution for the sideral frequency](image)

Figure 5.4: Event distributions and livetime distributions for the sideral frequency for
muons with a surface energy larger than 30 GeV. Due to the very low degree of anisotropy
of the cosmic rays it is not possible to see by eye any difference in the shape of the two
distributions.

In a further step the bin contents of the histogram, containing the event distribution
is divided by the bin contents of the livetime distribution histogram so that we get a third
histogram containing the mean rate \(R_m\) versus the phase.

\[
R_m = \frac{e_m}{\mathcal{L}_m}
\]

(5.45)

After that we calculate for each bin the fractional muon intensity deviation \(x_m\) from the
average (see figure 5.5)

\[
x_m = \frac{\Delta I}{< I >} = \frac{R_m - < R >}{< R >}.
\]

(5.46)
where

\[ < R > = \frac{\sum_{m=0}^{N_{\text{bin}}-1} e_m}{N_{\text{bin}}-1} \sum_{m=0}^{N_{\text{bin}}-1} L_m \]  

(5.47)

Through the discrete Fourier transform of \( x_m \) we get the complex amplitude \( \xi_k \) of the

![Graph showing fractional intensity deviation vs. sideral time](image)

Figure 5.5: Fractional intensity deviation \( x_m \) from the average as a function of the sideral time for muons with a surface energy larger than 30 GeV. The vertical bars represent the statistical error \( 1/\sqrt{e_m} \). Corrections according to section 5.6 are applied. (\( N_{\text{bin}} = 12 \)).

The \( k \)-th harmonic

\[ \xi_k = \frac{2}{N_{\text{bin}}} \sum_{m=0}^{N_{\text{bin}}-1} x_m e^{2\pi i m k / N_{\text{bin}}} \]  

(5.48)

The real amplitude \( \xi_k \) and the phase \( \varphi_k \) of the \( k \)-th harmonic are then given by

\[ \xi_k = |\xi_k| \quad \varphi_k = \frac{1}{k} \arg(\xi_k). \]  

(5.49)
5.4 Method based on the right ascension distribution
(Method B)

If we take into account the information of the Right Ascension of the arrival direction of the muons we can calculate the anisotropy with help of equations (5.15), where \( \mathcal{A} \) is identified with the acceptance distribution of the muons in the local equatorial coordinates according to equation (5.23). In our analysis we neglect the declination information and we want to measure only \( \tilde{\delta}^{\text{dir}}(\alpha) \) as given in equation (5.26). In this way equation (5.15) after the substitution of \( \mathcal{A} \) according to equation (5.22) becomes

\[
\tilde{\delta}^{\text{dir}}(\alpha) = \frac{N_\mu(\alpha)}{\frac{1}{L_\ell} \int_0^{2\pi} dt_s \mathcal{L}(t_s) \cdot \tilde{N}_\mu(\alpha - t_s)} - 1. \tag{5.50}
\]

\( N_\mu(\alpha) \) is the event distribution in right ascension and \( \tilde{N}_\mu(h_n) \) the event distribution in the negative hour angle \( h_n \) (figure 5.6). Note that the denominator of equation (5.50)

![Figure 5.6: Typical event distribution \( \tilde{N}_\mu(h_n) \) in negative hour angle \( h_n \) and livetime distribution \( \mathcal{L}(t_s) \) for muons with a surface energy larger than 30 GeV obtained from data acquired during 1 day (1st August 1999). The convolution of the two distributions according to equation (5.53) gives the expected distribution under assumption of isotropy (figure 5.7).](image)

represents the expected event distribution in right ascension in case of isotropy \( N_\mu^{\text{iso}}(\alpha) \).

We remind that equation (5.22) used to infer equation (5.50) has been obtained under the assumption that the livetime distribution \( \mathcal{L}(t_s) \) is constant. The livetime distribution corresponding to L3+C data-taking is not perfectly constant. To correct that, one could
give a weight to each event proportional to the inverse of the content of the corresponding livetime bin, when filling the histogram of the event distribution $\bar{N}_\mu(h_n)$. However, the anisotropy is so small and the livetime distribution homogeneous enough on large scale (e.g. see figure 5.4) that we don’t expect that this correction has a large effect. Therefore we omit it.

Of course the direction dependent efficiency of the L3+C muon spectrometer (described in equation (5.7) by $A(\delta, h_n, E_\mu)$) is not perfectly constant in time (see section 4.3.2). So we analyze the data separately in the different “stable” periods defined in table 4.4 applying the selection of the runs of type 1.b and the event selection of type B described in section 4.1. For each period $k$ we get the livetime distribution $L_k(t_s)$ and the event distributions $N_{\mu,k}(\alpha)$ and $\bar{N}_{\mu,k}(h_n)$.

$L_k(t_s)$ is represented by a histogram with bin contents $L_{m,k}$ and bin intervals described by (5.43). We represent the distribution $N_{\mu,k}(\alpha)$ by a histogram with bin contents $N_{m,k}$ and intervals

$$-\pi + \frac{2\pi(m - \frac{1}{2})}{N_{\text{bin}}} < \alpha < -\pi + \frac{2\pi(m + \frac{1}{2})}{N_{\text{bin}}}$$

$$(m = 0, 1, ..., N_{\text{bin}} - 1).$$

and $\bar{N}_{\mu,k}(h_n)$ by a histogram (figure 5.6) with bin contents $\bar{N}_{m,k}$ and intervals

$$-\pi + \frac{2\pi(m - \frac{1}{2})}{N_{\text{bin}}} < h_n < -\pi + \frac{2\pi(m + \frac{1}{2})}{N_{\text{bin}}}$$

$$(m = 0, 1, ..., N_{\text{bin}} - 1).$$

We set the number of bins as $N_{\text{bin}} = 240$.

The expected event distribution $N_{\mu}^{\text{iso}}(\alpha)$ given in the denominator of equation (5.50) is obtained with the convolution of the distribution $\bar{N}_\mu(h_n)$ and the livetime distribution $L(t_s)$ and dividing the result by the total livetime. Therefore we calculate the bin contents of the corresponding histogram through

$$N_{t,k}^{\text{iso}} = \frac{1}{L_t} \cdot \sum_{n=0}^{N_{\text{bin}}-1} L_{n,k} \cdot \bar{N}_{[n+N_{\text{bin}}/2-n],k}$$

where the square brackets mean modulo $N_{\text{bin}}$ and $L_t$ is the total livetime, i.e.

$$L_t = \sum_{m=0}^{N_{\text{bin}}-1} L_m$$

As example we show in figure 5.7 the agreement between the expected and the measured event distributions in right ascension $N_{\mu}^{\text{iso}}(\alpha)$ and $N_{\mu}(\alpha)$ for muons with a surface energy larger than 30 GeV. The two distributions are obtained with data of 1 day only.
5.4 Method based on the right ascension distribution (Method B)

![Event distribution in right ascension](image)

Figure 5.7: Measured and expected event distribution in right ascension for muons with a surface energy larger than 30 GeV detected during 1 day (1st August 1999).

(1st August 1999), so that the statistical fluctuations of the measured distribution around the smooth curve of the expected distribution are still visible.

The distributions \(N_{\mu k}^{\text{iso}}(\alpha)\) and \(N_{\mu k}(\alpha)\) of all “stable periods” are then added together.

\[
N_m^{\text{iso}} = \sum_k N_{m,k}^{\text{iso}} \tag{5.55}
\]

\[
N_m^{\text{ms}} = \sum_k N_{m,k}^{\text{ms}} \tag{5.56}
\]

The bin content for the intervals (5.51) of the histogram corresponding to the anisotropy distribution in right ascension \(\delta_{\text{dir}}(\alpha)\) is then calculated as

\[
x_m = \frac{N_m^{\text{ms}}}{N_m^{\text{iso}}} - 1. \tag{5.57}
\]

In figure 5.8 we report the example of the resulting anisotropy distribution for the energy cut of 30 GeV. After that a harmonic analysis of \(x_m\) is performed as in section 5.3.
Analysis of the large scale anisotropy

Figure 5.8: Anisotropy distribution $\delta_{\text{dir}}(\alpha)$ in right ascension for muons with a surface energy larger than 30 GeV. The vertical bars represent the statistical error. Corrections according to section 5.6 are applied. For graphical reason in the plot the number of bins is reduced from the original 240 to 12.

To get an idea of the systematical error we proceed as in section 5.3 looking at the fluctuations of the amplitude at the frequencies described by equation (5.42). To do that we replace some variables of the distributions used above. The phase $\hat{\phi}$ of the concerned frequency is used in place of the sideral time $t_s$. Instead of the right ascension we use the pseudo right ascension $\tilde{\alpha}$ which is defined as

$$[\tilde{\alpha}] = [\hat{\phi} + h_n].$$

(c.f. equation 2.9). We note that for the solar frequency, the sun position corresponds to $\tilde{\alpha} \approx 12$ hours (within an error of about 15 minutes).

5.5 Declination at which L3+C is sensitive

Any results of the anisotropy analysis is of course in relation with the declination at which the measurement is sensitive. Therefore we report in figure 5.9 the declination distributions of the events selected for the anisotropy analysis of the L3+C data for different lower energy cuts.
Figure 5.9: Distributions of the analyzed events in declination $\delta$ for different lower energy cuts. The distributions are normalized to 1.

5.6 Corrections

5.6.1 Compton-Getting Effect due to Earth’s orbital motion

With the discussion in section 2.5.2 it becomes clear that our analysis has to take into account the Compton-Getting Effect inferred by the Earth’s orbital motion. Considering equation (2.30), this is achieved giving a weight $w$ to each event

$$w = \frac{1}{1 + (2 + \gamma) \frac{16}{c} \cos(\theta)} \quad (5.59)$$

when filling the histogram corresponding to the used event distributions. The Earth’s orbital velocity $\vec{v}$ is calculated with the help of the library slalib [32]. The angle $\theta$ is obtained via the scalar product of the source direction $\vec{d}$ of the muon and the orbital velocity $\vec{v}$

$$\cos(\theta) = -\frac{\vec{v} \cdot \vec{d}}{|\vec{v}|} \quad (5.60)$$
5.6.2 Correction based on the variation of the muon rates

In section 4.3.1 we saw that changes in the efficiency of the L3 muon spectrometer and meteorological effects influence the detection rate. Due to the fact that after applying the selection of the runs according to section 4.1.2 our data are very discontinuous in time, this changes of the detection rate affect not in the same way all the datasets corresponding to a particular sideral time bin. As a consequence our results of the anisotropy analysis may be distorted. No clear relation between the meteorological measurements of temperature and pressure has been found yet in our data [108] and therefore it is not possible to apply corrections based on them in a correct way.

To overcome that problem, we proceed to a smoothing of our data. A “running average” of the detection rate is calculated for every selected run on the interval of time lasting from 12 hours before the run to 12 hours after the run. When filling the histogram corresponding to the livetime distribution, the contents are weighted with a factor proportional to the “running average” of the detection rate. A similar correction is applied also to the data of the Mayflower Mine experiment [36] and of the Yakutsk’s underground muon telescope [51]. Such a correction should strongly reduce the meteorological effects, since the most important parameter on which the muon rates depend, which is the effective temperature $T_{\text{eff}}$, does not change so much during one day (see chapter 2.7 and [93]).

When applying the correction weights the normalization of the histograms may be changed. To avoid that the final result is affected, we correct the normalization of the expected distribution $N_{\mu}^{\text{iso}}(\alpha)$ so that it corresponds to the normalization of the measured distribution $N_{\mu}(\alpha)$. When the method based on the sideral time distribution is used (section 5.3) the normalization is corrected automatically by applying equation 5.46.

5.7 Spectrum and systematical uncertainty

In figure 5.10 we report as an example the amplitude $\xi_n$ of the first harmonics as a function of the 90 frequencies $\nu_n$ (defined in equation 5.42) of the anisotropy function $\delta_{\text{dir}}(\hat{\alpha})$ for a surface energy lower cut of 20 GeV. Method B (section 5.4) is used here. The result for the second harmonics is shown in figure 5.11.

As discussed in the sections 5.3 and 5.4 the idea of analyzing many frequencies besides of the four of interests (anti-sideral $\nu_{36}$, solar $\nu_{40}$, sideral $\nu_{44}$, extended-sideral $\nu_{48}$) is to estimate the combined statistical and systematical uncertainty. To achieve this goal we consider the distribution $D$ of the (real) amplitudes $\xi_n$ for the 86 frequencies $\nu_n$ besides the four of interest (figure 5.12). We expect the complex amplitudes $\xi_n$ of these frequencies to follow a two dimensional Gaussian distribution on the complex plane centered in the origin. Therefore the real amplitudes $\xi_n = |\xi_n|$ should be distributed according to the Rayleigh distribution

$$H(\xi; \sigma) = K|\xi|e^{-\frac{\xi^2}{\sigma^2}}$$

where $K$ is a normalization factor, which is equal to $\frac{1}{\sigma}$ in the case that $H(\xi; \sigma)$ is nor-
Figure 5.10: Amplitude $\xi_n$ of the first harmonic of the relative muon intensity variation as a function of $\dot{\alpha}$ (see equation 5.58) for frequencies near $1 \text{ day}^{-1}$ and for a surface energy lower cut of 20 GeV. Vertical lines indicate from left to right the anti-sideral frequency, the solar frequency, and the sideral frequency. (Obtained with Method B).

In the table 5.1 we report the errors $\sigma_{\text{fit}}$, obtained with the mentioned fit for different energy cuts, for the first three harmonics and for both methods A and B. We compare it with the expected statistical error $\sigma_{\text{stat}}$, which is [36]

$$\sigma_{\text{stat}} = \sqrt{\frac{2}{N}}$$ (5.62)

where $N$ is the number of analyzed events. There is a good agreement between the statistical error $\sigma_{\text{stat}}$ and the one obtained with the fit $\sigma_{\text{fit}}$. We conclude therefore that the systematical error compared with the statistical one is negligible.
Figure 5.11: Amplitude $\xi_n$ of the second harmonic of the relative muon intensity variation as a function of $\dot{\alpha}$ (see equation 5.58) for a surface energy lower cut of 20 GeV. Vertical lines indicate from left to right the double anti-sideral frequency, the double solar frequency, and the double sideral frequency. (Obtained with Method B).
5.7 Spectrum and systematical uncertainty

![Energy cut: 20 GeV](image)

Figure 5.12: Histograms showing the distribution $D$ of the amplitudes $\xi_n$ for $n \neq 36, 40, 44, 48$ of the spectra shown in the figures 5.10 and 5.11. The histograms are fitted with the Rayleigh distribution (equation 5.61).

<table>
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<th>Energy cut [GeV]</th>
<th>$k$-th Harmonic</th>
<th>Number Events</th>
<th>$\sigma_{\text{stat}}$ [%]</th>
<th>$\sigma_{\text{fit}}$ [%] (Method A)</th>
<th>$\sigma_{\text{fit}}$ [%] (Method B)</th>
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<td>0.08</td>
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<td>0.11</td>
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<td>0.08</td>
</tr>
<tr>
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<td></td>
<td></td>
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<td>0.09</td>
</tr>
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<td></td>
<td></td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
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<td></td>
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<td>0.09</td>
</tr>
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<td>0.14</td>
</tr>
<tr>
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<td></td>
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</tr>
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Table 5.1: For different energy cuts and for the first three harmonics, we report the number of events used in the analysis, the statistical uncertainty $\sigma_{\text{stat}}$ of the complex amplitude $\xi_k$ of the $k$-th harmonic and the uncertainty $\sigma_{\text{fit}}$ obtained from the fit with equation 5.61 of the amplitude distributions $D$ obtained with the methods A and B.
5.8 Results

5.8.1 Single muons

In the tables 5.2 - 5.5 we report the results of the amplitude $\xi$, the phase $\varphi$ and $\sigma$ of the four frequency of interests (sideral, solar, anti-sideral and extended-sideral).

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Table 5.2: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the spectral analysis of the anisotropy function $\tilde{\delta}^{dir}(\alpha)$ for the *sideral* frequency for different energy cuts. In the last column we report the statistical error $\sigma_{stat}$.

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Table 5.3: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the spectral analysis of the anisotropy function $\tilde{\delta}^{dir}(\alpha)$ for the *solar* frequency for different energy cuts. In the last column we report the statistical error $\sigma_{stat}$. 
Table 5.4: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the spectral analysis of the anisotropy function $\hat{\delta}^{\text{dir}}(\hat{\alpha})$ for the anti-sideral frequency for different energy cuts. In the last column we report the statistical error $\sigma_{\text{stat}}$.

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<td>0.18</td>
<td>6.04</td>
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<tr>
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<td>19.32</td>
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<tr>
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</tr>
<tr>
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<td>3rd</td>
<td>0.14</td>
<td>7.07</td>
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<td>3.47</td>
</tr>
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Table 5.5: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the spectral analysis of the anisotropy function $\hat{\delta}^{\text{dir}}(\hat{\alpha})$ for the extended-sideral frequency for different energy cuts. In the last column we report the statistical error $\sigma_{\text{stat}}$.

<table>
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<tr>
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<td>2.12</td>
<td>0.37</td>
<td>7.22</td>
<td>0.37</td>
<td>7.22</td>
</tr>
</tbody>
</table>
We represent graphically the results of the sidereal and the solar frequency on the dial plots of the figures 5.13-5.18. The error circles have a radius of 1.52σ and represent the 68.5% confidence level (CL) region. The results at the sidereal frequency are compared with the measurements made at the Mayflower Mine by Cutler et al. [36] in the time period 1978-1983 at 500 m.w.e. depth (which corresponds to an energy cut of ~100 GeV). Their measurements are in agreement with ours at 100 GeV.

It is difficult to interpret our results in terms of the NFJ model introduced in section 2.5.5. The NFJ model predict in fact a deficit of galactic origin at α = 12 hours down to primary energies of 60 GeV, which should concern all our observations even at a muon energy cut of 20 GeV. In fact we see a maximum intensity in the first harmonics near to α = 10 hours. We neither observe any indication from the excess cone in the direction of the heliomagnetic tail at α = 6 hours, but this was not expected to be very effective during our data-taking period (yearly modulation).

With our uncertainties, our results are also compatible with an isotropic distribution \( \hat{\delta}_{\text{dir}}(\alpha) = 0 \), with the exception of the result found for the second harmonic at the solar frequency (see figure 5.17 and highest peek of figure 5.11). At 20 GeV with the method B we find an amplitude \( \hat{\varepsilon}_2 \) of 0.36, which is 4.5 σ away from zero. The probability to find a Rayleigh distributed point at a distance of more than 4.5 σ from the mean value is only \( 4 \cdot 10^{-5} \).
Figure 5.14: Dial plots showing the results of the second harmonic of the anisotropy function $\delta^{\text{dir}}(\alpha)$ at the sideral frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region. The hatched circles report the result of Cutler et al. [36].

In figure 5.19 we report the anisotropy distribution $\delta^{\text{dir}}(\tilde{\alpha})$ for the solar frequency. A clear structure is seen. The fit with the sum of the first two harmonics gives a good representation of what observed: $\chi^2 = 6.6$ for 7 degrees of freedom. The constant function $\delta^{\text{dir}}(\tilde{\alpha}) = 0$ gives $\chi^2 = 28.3$ for 11 of degree of freedom$^2$. The GRAND experiment data (figure 5.20) show a similar structure for the variation of the muon counts versus the local solar time for an energy threshold of 0.1 GeV, which is much lower than ours. The data set covers the period from January 1997 to December 2000. The degree of anisotropy $\delta_\alpha$ is 0.2%, which is more than a factor 5 larger than what we observe. Is this similarity only fortuitous or is there some relation between the 2 results at energies which differs of a factor 200? We leave this as an open question.

The tables B.1-B.4 and plots B.1-B.6 of the yearly results are reported in the appendix B. No statistically significant deviation is found from the results of 1999 and the one of 2000. It is interesting to see that the anisotropy seen from the second harmonic at the solar frequency at low energy is present in both year with almost the same amplitude and phase (plot B.5).

$^2$The probability to find $\chi^2 \geq 28.3$ for 11 degrees of freedom is $1.9 \cdot 10^{-4}$. 
Figure 5.15: Dial plots showing the results of the third harmonic of the anisotropy function $\delta^{\text{dir}}(\alpha)$ at the sideral frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region. The hatched circles report the result of Cutler et al. [36].

Figure 5.16: Dial plots showing the results of the first harmonic of the anisotropy function $\delta^{\text{dir}}(\tilde{\alpha})$ at the solar frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.
Figure 5.17: Dial plots showing the results of the second harmonic of the anisotropy function $\delta \tilde{\text{dir}}(\tilde{\alpha})$ at the solar frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.

Figure 5.18: Dial plots showing the results of the third harmonic of the anisotropy function $\delta \tilde{\text{dir}}(\tilde{\alpha})$ at the solar frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.
Analysis of the large scale anisotropy

Anisotropy function  Solar frequency

\[ \chi^2/\text{ndf} \quad 6.578 / 8 \]
\[ \xi_1 \quad 0.1889 \]
\[ \phi_1 \quad 18.27 \]
\[ \xi_2 \quad 0.3158 \]
\[ \phi_2 \quad 3.160 \]

Figure 5.19: Anisotropy distribution in pseudo-right ascension \( \tilde{\alpha} \) for muons with an energy larger than 20 GeV. The distribution is fitted with the sum of the first two harmonics. The vertical bars represent the statistical errors.

Figure 5.20: Muon counts per 20-minutes interval, versus the local solar time measured at surface by the GRAND experiments for muons with an energy larger than 0.1 GeV. The data-taking period includes 4 years (1997-2000). GRAND is located at 41.7\(^\circ\)N, 36.2\(^\circ\)W. [48].
5.8.2 Results at tidal frequency

In section 5.3 the question arose if the atmospheric tides may have an influence on the detection rate. If this would be the case, the amplitude $\xi_2$ of the second harmonic of the anisotropy function $\delta^{\text{dir}}(\hat{\phi})$ at the frequency $\frac{1}{T_{\text{moon}}}$ (see definition in section 5.3) would be expected to be different from zero. The result for 4 different energy cuts obtained with method A is shown in the dial plot of figure 5.21. No positive result has been found.

Figure 5.21: Dial plot showing the results of the second harmonic of the anisotropy function $\delta^{\text{dir}}(\hat{\phi})$ at the frequency $\frac{1}{T_{\text{moon}}}$ for 4 different energy cuts obtained with method A. The amplitude $\xi_2$ is expected to be different from zero in the case that the atmospheric tides influence the detection rate.
5.8.3 Multi-muons

As discussed at the end of section 2.5.4 there are some experimental indications that the degree of anisotropy increases for muon-rich air-showers. We try therefore to perform an analysis of the anisotropy of multi-muons. We analyze events with at least 3 (resp. 5) parallel sub-tracks, which correspond mainly to dimuons (resp. tri-muons) events (see selection of multi-muons event description in section 4.1.1).

No significant deviation from sideral isotropy has been found. In figure 5.22 the plots of the anisotropy function $\delta_{\text{dir}}(\alpha)$ in right ascension obtained with method B is reported. The corresponding dial plots of the first harmonics are shown in figure 5.23.

![Graphs showing anisotropy distributions for multi-muon events with 3 and 5 subtracks.](image)

Figure 5.22: Anisotropy distributions $\delta_{\text{dir}}(\alpha)$ for multi-muon events. On the left plot we require at least 3 parallel sub-tracks, on the right plot we require at least 5 parallel sub-tracks.

We remind that the cross-octant reconstruction (see section 3.5), which is currently under development, is expected to increase the efficiency of the multi-muon reconstruction. A further analysis with this version would thus reduce the statistical errors.
Figure 5.23: Dial plots of the first and second harmonics for multi-muons with a minimum of 3 parallel sub-tracks and with a minimum of 5 parallel sub-tracks.
Chapter 6

Point source search

6.1 Method

6.1.1 Sky survey

The idea developed in section 5.4 to determine the right ascension distribution for the large scale anisotropy analysis, can be adapted for the survey of the sky for point sources. In fact we can divide all our data in bands with a small fixed declination range and determine the right ascension distribution in each band, applying the same procedure used for the large scale anisotropy analysis. In such a way we are able to generate a map of the region of the sky at which the detector is sensible.

Mathematical description of background and signal

In contrast to the large scale anisotropy analysis, for the point source search the angular resolution cannot be neglected. We define \( \tilde{\Phi}_i(\delta, \alpha, E_0) \) to be the convolution of the direction dependent differential spectrum \( \Phi_i \) defined in section 5.1 with the angular resolution \( R_i \)

\[
\tilde{\Phi}_i(\delta, \alpha, E_0) = \Phi_i(\delta, \alpha, E_0) * R_i(\Delta \phi, E_0; E_{\text{cut}}) \tag{6.1}
\]

The function \( R_i(\Delta \phi, E_0; E_{\text{cut}}) \) represents the expected distribution of the angular difference \( \Delta \phi \) between the direction of the primary particles of type \( i \) with energy \( E_0 \) and the measured direction of the muons generated by them with surface energy larger than \( E_{\text{cut}} \). It includes the effects of the multiple scattering of the muons, the angular reconstruction resolution of the detector and the deviation of the surface direction of the muon from the direction of the primary particle. The dependence of \( R_i \) from the direction \( (\delta, h_n) \) is neglected and its normalization is chosen so that

\[
2\pi \int_0^\pi d(\Delta \phi) \sin(\Delta \phi) R_i(\Delta \phi, E_0; E_{\text{cut}}) = 1 \tag{6.2}
\]

The convolution of a spherical distribution \( f(\theta, \phi) \) with the angular resolution \( R_i(\Delta \phi) \) is calculated as
\[ f \ast R_i = \int_{\pi/2}^{\pi/2} d\theta' \int_0^{2\pi} d\phi' \sin(\theta') f(\theta', \phi') R_i(\angle(\theta, \phi, \theta', \phi')) \]  

(6.3)

where \( \angle(\theta, \phi, \theta', \phi') \) is the angle between the directions \((\theta, \phi)\) and \((\theta', \phi')\), i.e.

\[
\angle(\theta, \phi, \theta', \phi') = \arccos \left( \sin(\theta) \sin(\theta') + \cos(\theta) \cos(\theta') \sin(\phi') + \cos(\theta') \cos(\theta') \cos(\phi') \right).
\]

(6.4)

If in section 5.1 we interpret \( \Phi_i \) as \( \bar{\Phi}_i \), the equations discussed there becomes consistent with the fact that we take into account the angular resolution.

We saw in chapter 5 that the degree of the large scale anisotropy is very small at the energies of interest for L3+C. Therefore in the frame of the point source search analysis we assume that the direction dependent differential spectrum \( \Phi_i \) defined in section 5.1 is the sum of an isotropic part (background) and the contribution of few point sources (signal) with declination \( \delta_k \) and right ascension \( \alpha_k \).

\[
\Phi_i(\delta, \alpha, E_0) = \bar{\Phi}_i(E_0) + \sum_k \phi_{i,k}(E_0) \frac{\delta_{\text{Dirac}}(\delta - \delta_k) \delta_{\text{Dirac}}(\alpha - \alpha_k)}{\cos \delta_k}
\]

(6.5)

i.e. (c.f. equation 5.1)

\[
\delta_i^{\text{dir}}(\delta, \alpha, E_0) = \frac{1}{\bar{\Phi}_i(E_0)} \sum_k \phi_{i,k}(E_0) \frac{\delta_{\text{Dirac}}(\delta - \delta_k) \delta_{\text{Dirac}}(\alpha - \alpha_k)}{\cos \delta_k}
\]

(6.6)

where \( \delta_{\text{Dirac}} \) is the Dirac’s \( \delta \)-function and \( \phi_{i,k}(E_0) \) is the primary spectrum of the \( k \)-th point source for the primary particle \( i \). \( \phi_{i,k}(E_0) \) is of course expected to be different from zero only for neutral primary particles. The convolution of \( \Phi_i \) with the angular resolution \( R_i \) gives

\[
\bar{\Phi}_i(\delta, \alpha, E_0) = \bar{\Phi}_i(E_0) + \sum_k \phi_{i,k}(E_0) R_i(\angle(\delta, \alpha, \delta_k, \alpha_k), E_0; E_{\text{cut}})
\]

(6.7)

and the convolution of \( \delta_i^{\text{dir}}(\delta, \alpha, E_0) \) with \( R_i \)

\[
\bar{\delta}_i^{\text{dir}}(\delta, \alpha, E_0) = \frac{\bar{\Phi}_i(\delta, \alpha, E_0)}{\Phi_i(E_0)} - 1 = \frac{1}{\bar{\Phi}_i(E_0)} \sum_k \phi_{i,k}(E_0) R_i(\angle(\delta, \alpha, \delta_k, \alpha_k), E_0; E_{\text{cut}})
\]

(6.8)

According to equations (5.12) and equation (5.17) the number of muons expected from the isotropic part of the primary flux is

\[
\mathcal{N}_{\mu, \text{iso}}(\delta, \alpha; E_{\text{cut}}) = \int_0^{2\pi} d \tau_\tau L(t_\tau) \cdot \bar{N}_\mu(\delta, [\alpha - \tau_\tau]; E_{\text{cut}})
\]

(6.9)
if we neglect the influence of the point sources on the distribution of the detected muons in local equatorial coordinates $\tilde{N}_\mu(\delta, h_n; E_{\text{cut}})$. The mentioned influence is expected to be negligible when the livetime distribution $\mathcal{L}(t_s)$ is almost homogeneous and the sources are not too close to the polar region. In fact according to equation 5.22 and taking into account the angular resolution, for a homogeneous livetime distribution we find that the influence of the sources can be taken into account multiplying the right side of equation (6.9) with the correction factor $F$

$$F = \frac{2\pi}{\int_0^{2\pi} \left( 1 + \frac{\delta}{\delta_s} \right) \, d\delta}$$  \hspace{1cm} (6.10)

where in analogy to equation (5.13)

$$\tilde{N}_\mu^\text{dir}(\delta, \alpha; E_{\text{cut}}) = \sum_i \tilde{N}_\mu^\text{iso}(\delta, \alpha; E_{\text{cut}}) N_{\mu i}^\text{iso}(\delta, \alpha; E_{\text{cut}}) / \sum_i N_{\mu i}^\text{iso}(\delta, \alpha; E_{\text{cut}}).$$  \hspace{1cm} (6.11)

The correction factor $F$ is function of the declination only and the influence of the sources expressed by $\delta^\text{dir}$ is averaged along the bands with constant declination around $\delta_k$. We note that the consequence of neglecting the factor $F$ is that we tend to slightly overestimate the background of the source direction and therefore to underestimate the significance of an eventual signal.

Finally we remark that the number of muons generated by the point sources is given by (c.f. equation (5.14))

$$N_{\mu i}^\text{tot}(\delta, \alpha; E_{\text{cut}}) = \tilde{N}_\mu^\text{dir}(\delta, \alpha; E_{\text{cut}}) \cdot \int_0^{2\pi} d\delta \, \mathcal{L}(t_s) \cdot \tilde{A}(\delta, [\alpha - t_s]; E_{\text{cut}}).$$  \hspace{1cm} (6.12)

**Correction for time dependent efficiency and atmospheric effects**

Let’s now consider the time dependence of $\tilde{A}$. Inside one “stable” period (as defined in section 4.3.2, table 4.4) we assume, that the relative changes of efficiency and the atmospheric effects, after applying the selection of time intervals of type 2.c (see section 4.1.2), affect homogeneously all directions. Therefore we write $\tilde{A}$ as the product of the rate $R(t)$ of the selected events as a function of time $t$ and a time independent factor $A'$

$$\tilde{A}(\delta, h_n, t; E_{\text{cut}}) = R(t) \cdot \tilde{A}'(\delta, h_n; E_{\text{cut}}).$$  \hspace{1cm} (6.13)

The rate $R(t)$ is in reality influenced by the presence of the sources in the field of view of the detector. This tiny effect is negligible for the point source search unless we have a very strong source that we would discover very easily.\(^2\)

\(^1\)A similar procedure is proposed also by Alexandreas et al. [116]

\(^2\)In the large scale anisotropy analysis we are not allowed to apply the separation as in equation (6.13), because the rate $R(t)$ is strongly related with the effect that we want to measure and it has to be considered as a function of the sidereal time.
6.1 Method

The distribution of the selected events $N(t_s)$ as a function of the sideral time $t_s$ can be written as

$$N(t_s) = \int dt \ R(t) \ L(t) \ \delta_{\text{Dirac}}(T_s(t) - t_s)$$

(6.14)

where $L(t)$ is the same function defined by equation (5.3) and $T_s(t)$ is the the sideral time as a function of the time $t$. Taking into account equations (5.4), (6.13) and (6.14) we can rewrite equation (5.12) as

$$N_{\mu, \text{tot}}(\delta, \alpha; E_{\text{cut}}) = \int_0^{2\pi} dt_s \ N(t_s) \mathcal{A}(\delta, [\alpha - t_s]; E_{\text{cut}})$$

(6.15)

and equation (5.17) as

$$\mathcal{A}(\delta, h_n; E_{\text{cut}}) = \frac{1}{N_t} \tilde{N}_{\mu}(\delta, h_n; E_{\text{cut}})$$

(6.16)

where $N_t$ is the total number of events

$$N_t = \int_0^{2\pi} dt_s \ N(t_s) = \int_0^{2\pi} dh_n \int_{-\pi/2}^{\pi/2} d\delta \cos(\delta) \tilde{N}_{\mu}(\delta, h_n; E_{\text{cut}})$$

(6.17)

so that equation (6.9) becomes

$$N_{\mu, \text{tot}}(\delta, \alpha; E_{\text{cut}}) = \int_0^{2\pi} dt_s \frac{N(t_s)}{N_t} \mathcal{A}(\delta, [\alpha - t_s]; E_{\text{cut}})$$

(6.18)

The latter is the basic equation on which we will base our background calculation.

Analysis with L3+C data

To perform the point source search with L3+C data, we first establish the acceptance distribution $\tilde{N}_{\mu}(\delta, h_n; E_{\text{cut}})$ in equatorial coordinates for each “stable” period defined in table 4.4 (example in figure 6.1 for “stable” period 43). The distribution is represented by a two dimensional histogram $H_a$ which covers all directions with declination $\delta > \delta_{\text{min}}$. The size of the bins (that we will call pixels) of the histogram $H_a$ is noted with $p_s$ and is chosen so that

$$N_\alpha = \frac{2\pi}{p_s} \quad \text{and} \quad N_\delta = \frac{\pi - \delta_{\text{min}}}{p_s}$$

(6.19)

are integer numbers. The bin contents of $H_a$ are noted with $\tilde{N}_{i,j}$ and correspond to the number of selected muon tracks seen in the window (pixel) described by the following

\footnote{We remind that all angles that appear in the equations as well as the sideral time are in radians!}
Figure 6.1: Example of acceptance distribution $\tilde{N}_{\mu}(\delta, |\alpha - t_s|; E_{\text{cut}})$ in local equatorial coordinates for period 43 and $E_{\text{cut}} = 30$ GeV. (Pixel size $p_s = 1^\circ$).

Intervals

$$-\pi - \frac{1}{2} p_s + i p_s < h_n < -\pi - \frac{1}{2} p_s + (i + 1) p_s$$

$$\delta_{\text{min}} + j p_s < \delta < \delta_{\text{min}} + (j + 1) p_s$$

(6.20)

where $h_n$ is the negative hour angle. The shift of half pixel size $\frac{1}{2} p_s$ in the first interval description of (6.20) is applied in order to have $h_n = 0$ in the center of the pixels with content $N_{(N_{\alpha}/2),j}$ for all $j$.

The background muon distribution $N_{\mu,\text{tot}}^{\text{iso}}(\delta, \alpha; E_{\text{cut}})$ is then established for the time interval $I_t$, that we want to analyze. $I_t$ can cover either the full “stable” period used to fill histogram $H_a$ or part of it. $N_{\mu,\text{tot}}^{\text{iso}}(\delta, \alpha; E_{\text{cut}})$ is represented by a two dimensional histogram $H_{bk}$ which also covers all directions of the sky with declination $\delta > \delta_{\text{min}}$ and has the same pixel size $p_s$ of the histogram $H_a$. The bin contents of the histogram $H_{bk}$ are noted with $N_{i,j}^{\text{bk}}$ and correspond to the number of background muon tracks expected
in the window (pixel) with directions described by the intervals

\[ i \, p_s < \alpha < (i + 1) \, p_s \]
\[ \delta_{\min} + j \, p_s < \delta < \delta_{\min} + (j + 1) \, p_s \]  

(6.21)

\( (i = 0, 1, ..., N_\alpha - 1) \) \( (j = 0, 1, ..., N_\delta - 1) \).

To calculate the expected background muon distribution we need to know the event distribution \( \mathcal{N}(t_s) \) in sidereal time \( t_s \), for the time interval \( I_t \). We represent the distribution \( \mathcal{N}(t_s) \) with a one dimensional histogram \( \mathcal{H}_t \) (figure 6.2). The bin contents of the latter is noted with \( \mathcal{N}_i \) and corresponds to the number of selected tracks registered in the sidereal time interval

\[ i \, p_s < t_s < (i + 1) \, p_s \]  

(6.22)

According to equation (6.18), the histogram \( \mathcal{H}_a \) representing the expected background event distribution, is calculated performing the convolution of the acceptance distribution represented by \( \mathcal{H}_a \) and the event distribution in sidereal time given by \( \mathcal{H}_n \), i.e.

\[ N_{i,j}^{bk} = \frac{1}{N_t} \cdot \sum_{n=0}^{N_\alpha-1} \mathcal{N}[n] \cdot \tilde{N}_{[(i+N_\alpha/2-n),j]} \]  

(6.23)
where the square brackets mean modulo $N_\alpha$ and $N_t$ is the total number of events present in the $\tilde{N}$ distribution i.e:

$$N_t = \sum_{i=0}^{N_\alpha-1} \sum_{j=0}^{N_t-1} \tilde{N}_{i,j}. \tag{6.24}$$

As example we show the result for period 43 and $E_{\text{cut}} = 30$ GeV in figure 6.3.

![Figure 6.3: Expected background distribution $N^{i_{\text{iso}}}_{\mu,\text{tot}}(\delta, \alpha; E_{\text{cut}})$ in equatorial coordinates calculated for period 43 and $E_{\text{cut}} = 30$ GeV. (Pixel size $p_s = 1^\circ$).](image)

We produce then another two dimensional histogram $H_{\text{ms}}$ for the time interval $I_t$, whose bin contents $N^{\text{ms}}_{i,j}$ represent the number of selected measured events in the same direction window defined by equation (6.21) (example in figure 6.4). To analyze the excesses of the measured distribution of the muon arrival directions $N^{\text{ms}}_{\mu,\text{tot}}(\delta, \alpha; E_{\text{cut}})$ with respect to the expected background distribution $N^{\text{bk}}_{\mu,\text{tot}}(\delta, \alpha; E_{\text{cut}})$, we calculate the probability $P$ to find a number of events larger or equal to $N_{\text{ms}}$ when the number of background events is $N_{\text{bk}}$. $P$ is given by the Poisson cumulative distribution

$$P = \sum_{k=N_{\text{ms}}}^{\infty} \frac{e^{-N_{\text{bk}}}(N_{\text{bk}})^k}{k!}. \tag{6.25}$$
Equation (6.25) is calculated numerically with the help of the library [117]. We usually represent the probability \( P \) with its logarithm (base 10)

\[
\Lambda = - \log(P)
\]  

(6.26)
as shown on the sky map in figure 6.5.

**Grouping of pixels:** To look for excesses at different angular scales we group several pixels of the histograms \( H_{\text{ms}} \) and \( H_{\text{bk}} \) into larger squared bins (some of them overlapping with each other) with effective side length \( N_p \cdot p_s \). “Effective” means that, in order to preserve the solid angle size of the squared bin at high declinations, the right ascension width of the bin is set to \( N_p / \cos \delta \) (rounded at the next unity) times the pixel size \( p_s \). The content \( (N_{\text{ms}}^{\text{rms}})' \) and \( (N_{\text{bk}}^{\text{rms}})' \) of the grouped pixels of the histograms \( H_{\text{ms}} \) and \( H_{\text{bk}} \) are given by

\[
(N_{k,m}^{\text{rms}})' = \sum_{i=kn_s}^{kn_s+N_p-1} \sum_{j=mn_s}^{mn_s+N_p-1} N_{i,j}^{\text{rms}}
\]  

(6.27)
Figure 6.5: Shown is $-\log(P)$ in all pixel directions for the period 43 and $E_{\text{cut}} = 30$ GeV, where $P$ is the probability to measure $N \geq N_{i,j}^{\text{ms}}$ according to Poisson statistics. (Pixel size $p_s = 1^\circ$).

and

$$(N_{k,m}^{\text{bk}})' = \sum_{i=kn_s}^{kn_s+\Delta n_\alpha-1} \left( \sum_{j=mn_s}^{mn_s+N_p-1} N_{i,j}^{\text{bk}} \right)$$

(6.28)

$$k = 0, 1, \ldots, \left\lfloor \frac{N_\alpha}{n_s} - 1 \right\rfloor \quad m = 0, 1, \ldots, \left\lfloor \frac{N_\delta}{n_s} - N_p \right\rfloor$$

where

$$\Delta n_\alpha = \left\lfloor \frac{N_p}{\cos(\delta)} + \frac{1}{2} \right\rfloor.$$  

(6.29)

We note with $[x]$ the integer part of the real number $x$. The square brackets mean modulo $N_\alpha$. The parameter $n_s$ has been introduced to avoid to analyze too many bins,
which overlap with each other. We set it to

$$n_s = \left\lfloor \frac{N_p - 1}{4} + 1 \right\rfloor$$  \hspace{1cm} (6.30)

so that for $N_p \leq 4$ we have $n_s = 1$, which means that we analyze all possible groupings of pixels in bins for the given effective side length. Otherwise for $N_p > 4$ only a subset of the possible groupings of pixels are used. Also the excesses of the bins $(N_{k,m}^{\text{ms}})'$ of the grouped pixels with respect to the background $(N_{k,m}^{\text{bk}})'$ are analyzed calculating the probability $P$ according to equation (6.25).

**Uncertainties in the background estimation:** When using equation (6.25) we neglect the uncertainty in the estimation of the background $N_{i,j}^{\text{bk}}$. Is this uncertainty really negligible compared to the statistical fluctuations of $N_{i,j}^{\text{ms}}$?

Let’s compare the statistical uncertainty of $N_{i,j}^{\text{bk}}$ and the statistical fluctuations of $N_{i,j}^{\text{ms}}$ for the ideal case where we have a homogeneous event distribution with

$$N_i = \frac{N_t}{N_A}$$  \hspace{1cm} (6.31)

where $N_t$ is the total number of events. Let $\bar{N}_j$ be the total number of events in the band with fixed index $j$, i.e.

$$\bar{N}_j = \sum_{i=0}^{N_a-1} N_{i,j}^{\text{ms}} = \sum_{i=0}^{N_a-1} N_{i,j}^{\text{bk}}.$$  \hspace{1cm} (6.32)

$N_{i,j}^{\text{bk}}$ will be equal to $\frac{N_t}{N_A}$ for all $i$ and $N_{i,j}^{\text{ms}}$ will fluctuate statistically around the same value with a standard deviation of

$$\sigma^{\text{ms}} = \sqrt{\frac{\bar{N}_j}{N_A}}.$$  \hspace{1cm} (6.33)

The statistical uncertainty in the acceptance distribution $H_a$ for the bins $\bar{N}_{i,j}$ is given by

$$\sigma^{\text{acc}} = \sqrt{\bar{N}_{i,j}}.$$  \hspace{1cm} (6.34)

Since $N_{i,j}^{\text{bk}}$ is calculated according to equation (6.23), the statistical uncertainty of $N_{i,j}^{\text{bk}}$ is

$$\sigma^{\text{bk}} = \sqrt{\sum_{i=0}^{N_a-1} \left( \frac{N_i}{N_t} \right)^2 \left( \sqrt{\bar{N}_{i,j}} \right)^2} = \frac{1}{N_A} \sqrt{\sum_{i=0}^{N_a-1} \bar{N}_{i,j}} = \frac{1}{N_A} \sqrt{\bar{N}_j}.$$  \hspace{1cm} (6.35)

Comparing equations (6.33) and (6.35) we see that

$$\sigma^{\text{bk}} = \frac{1}{\sqrt{N_A}} \sigma^{\text{ms}}.$$  \hspace{1cm} (6.36)
Therefore, since we perform our analysis with $N_i \geq 360$, we conclude that for almost homogeneous events distributions $N_i$ in sidereal time we can neglect the statistical uncertainty of the background $N_{i,j}^{bk}$. For inhomogeneous events distributions $N_i$ we have a larger $\sigma^{bk}$ than what resulting from equation (6.36). However values of $\sigma^{bk}$ of the same order of $\sigma^{ms}$ are reached only if the event distribution is very inhomogeneous. In particular to have $\sigma^{bk} = \sigma^{ms}$ one needs that $N_i$ is 0 for all $i$ except one.

To check the influence of the pixel size $p_s$ on the calculation of $N_{i,j}^{bk}$ we establish the background histogram $H_{1}^{bk}$ for period 43 with $p_s = 1^\circ$, whose bin contents are noted with $N_{i,j}^{bk1}$. We then calculate the background histogram $H_{2}^{bk}$ with double resolution (i.e. $p_s = 0.5^\circ$) and we add together the content of four neighboring pixels in order to find independently the number of events (that we note with $N_{i,j}^{bk2}$) corresponding to the pixels of the first histogram $H_{1}^{bk}$. We then look at the relative difference

$$
\Delta = \frac{N_{i,j}^{bk2} - N_{i,j}^{bk1}}{N_{i,j}^{bk2}}
$$

(6.37)

The result is shown in figure 6.6. The root mean squared of the difference is about 0.02%. Having in mind the number of events present in each bin $N_{i,j}^{bk1}$ (figure 6.3) we conclude that this difference is completely negligible.

Systematic uncertainties in $N_{i,j}^{bk}$ may also be present, in particular, if the shape of the acceptance distribution $\tilde{A}(\delta, h_n, t; E_{cut})$ changes with the time, due to direction dependent variations of the efficiency of the detector, so that equation (6.13) can not be applied. To check that the systematic uncertainties are negligible we produce the cumulative distribution $C$ of $-\log(P)$ for all our trials. The cumulative distribution $C$ gives as a function of $\Lambda = -\log(P)$ the number of entries which result to have a probability larger than $P$ to correspond to statistical fluctuations of the background$^4$. This is expected to be $T_t/P$, where $T_t$ is the total number of entries in $C$. The typical result of a Monte Carlo simulation with $T_t = 10^4$ entries for $\sigma^{bk} = 0$ and a number of background event set to $N^{bk} = 88$ is shown in the right plot of the figure 6.7 together with the expected distribution (dotted line). The left plot shows the distribution obtained with the entries of the histogram shown in figure 6.5.

We check the influence of $\sigma^{bk}$ on the cumulative distribution $C$, shifting randomly $N^{bk}$ according to a Gaussian distribution of width $\sigma^{bk}$ in the Monte Carlo simulation (figure 6.8). The cumulative distribution $C$ starts to deviate remarkably for almost all the bins from the ideal distribution which follows the function $f(P) = T_t/P$ between $\sigma^{bk}/\sigma^{ms} = 0.3$ and $\sigma^{bk}/\sigma^{ms} = 0.5$. Since, according to equation (6.36), we expect a much smaller statistical error $\sigma^{bk}$ of the background than $0.3\sigma^{ms}$, the distribution of $-\log(P)$ is expected to deviate remarkably from the ideal one only if large systematical errors in the background estimation are present.

Figure 6.9 shows an example in which during our analysis we found large systematic uncertainties. We analyzed the arrival direction of the muons during a day (25th August

$^4$For examples of publications that present their results with cumulative distributions of $-\log(P)$ see [118] [76]
Figure 6.6: Relative difference between the background events obtained in a window of $1^\circ \times 1^\circ$ with histograms with pixel size $p_s = 1^\circ$ and $p_s = 0.5^\circ$.

1999), in which part of the data were taken with the magnet switched off. The runs taken when the magnet was off, were included in our analysis (selection of type 2.a, see section 4.1.2). In addition the livetime distribution $\mathcal{L}(t_s)$ in sideral time and equation (6.12) were used instead of the event distribution $\mathcal{N}(t_s)$ and equation (6.18). Since the muons were not bent by the magnetic field, the rate of selected muon tracks with reconstructed energy larger than 30 GeV increased. Therefore we found excesses in the directions, in which the L3 muon spectrometer was mostly efficient, when data-taking occurred with the magnet off.

Finally we remark that as consequence of the grouping of the pixels, the analyzed neighboring bins are not perfectly independent. Their correlations may sometimes affect the cumulative distributions, as shown in the figure 6.10 obtained with Monte Carlo simulations. These are performed with the same procedure used to obtain the Monte Carlo result of figure 6.7 with an additional step. The latter consists in adding the results of all possible grouping of 10 consecutive trials.
Point source search

Figure 6.7: Typical cumulative distribution $C$ of $-\log(P)$. The left plot is obtained with a Monte Carlo simulation assuming that the background is known precisely (i.e. $\sigma_{bk} = 0$). The right plot is obtained from the data of the period 43 for $E_{\text{cut}} = 30$ GeV (i.e. with the entries of the histogram shown in figure 6.5) The dotted lines represents the expected distribution ($\propto 1/P$).

Figure 6.8: Monte Carlo simulation of $-\log(P)$ cumulative distribution $C$ with $\sigma_{bk} = 0.3 \sigma_{ms}$ (left) and $\sigma_{bk} = 0.5 \sigma_{ms}$ (right). The dotted line represents the expected distribution for $\sigma_{bk} = 0$. 
6.1 Method

Figure 6.9: Results of the analysis of the arrival direction of the muons during the 25th August 1999. Part of the data included in the analysis are taken with the magnet switched off. Left: $-\log(P)$ distribution in equatorial coordinates (pixel size: $1^\circ \times 1^\circ$). Right: Cumulative distribution of $-\log(P)$.

Figure 6.10: 2 Examples of results from Monte Carlo simulations of $-\log(P)$ cumulative distribution $C$ with correlated entries.
6.1.2 Search for signals from known $\gamma$-sources

The search of muon-signals from point sources with the sky survey has two disadvantages. The first is that the number of analyzed points is quite large, so that an eventual low signal from a point source cannot be distinguished from the statistical fluctuations of the background seen in the other directions. The second is that the point source could not be in the center of the analyzed bins.

Therefore we decide to analyze first separately 10 bright known $\gamma$-sources, which are listed in table 6.1. For each source (whose coordinates are noted with $\delta_S$ and $\alpha_S$) we produce special one-dimensional histograms, which contain the informations about the selected events in a band with a fixed declination range $[\delta_S - \Delta \delta, \delta_S + \Delta \delta]$ around the source. The choice of the width of the band $2\Delta \delta$ is discussed in section 6.1.3.

The acceptance distribution in $h_n$ is represented by the histogram $H_{aS}$ whose bins $\tilde{N}_i^S$ contain the number of events in the window

$$-\pi + (i - 1)p_s < h_n < -\pi + ip_s$$

$$\delta_S - \Delta \delta < \delta < \delta_S + \Delta \delta$$

$$(i = 0, 1, ..., N_\alpha - 1)$$

The measured event distribution in right ascension $\alpha$ is represented by the histogram $H_{msS}$ whose bins $N_i^{msS}$ contain the number of events in the window

<table>
<thead>
<tr>
<th>Source name</th>
<th>Right ascension ($\alpha_S$)</th>
<th>Declination ($\delta_S$)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrk 421</td>
<td>11 h 04 min 27 sec</td>
<td>38° 13'</td>
<td>[85]</td>
</tr>
<tr>
<td>Mrk 501</td>
<td>16 h 53 min 52 sec</td>
<td>39° 46'</td>
<td>[86]</td>
</tr>
<tr>
<td>3-C 273</td>
<td>12 h 29 min 06 sec</td>
<td>2° 03'</td>
<td></td>
</tr>
<tr>
<td>Crab</td>
<td>5 h 34 min 32 sec</td>
<td>22° 01'</td>
<td>[82], [83], [84], [118] *, [120] *, [118] *</td>
</tr>
<tr>
<td>Cyg X-1</td>
<td>19 h 58 min 22 sec</td>
<td>35° 12'</td>
<td>[118] *</td>
</tr>
<tr>
<td>Cyg X-3</td>
<td>20 h 32 min 26 sec</td>
<td>40° 57'</td>
<td>[72], [7], [119] *, [120], [118] *</td>
</tr>
<tr>
<td>Her X-1</td>
<td>16 h 57 min 50 sec</td>
<td>35° 21'</td>
<td>[118], [121]</td>
</tr>
<tr>
<td>Geminga</td>
<td>6 h 33 min 58 sec</td>
<td>17° 52'</td>
<td>[122]</td>
</tr>
<tr>
<td>1ES1426+428</td>
<td>14 h 28 min 32 sec</td>
<td>42° 40'</td>
<td>[123]</td>
</tr>
<tr>
<td>1ES2344+514</td>
<td>23 h 47 min 05 sec</td>
<td>51° 42'</td>
<td>[124]</td>
</tr>
</tbody>
</table>

Table 6.1: Analyzed $\gamma$-sources with references to some measurements on TeV emission. We mark with an asterisc measurements which only set upper limits.
\[-\pi + \frac{1}{2} p_s + (i - 1)p_s < \alpha - \alpha_s < -\pi + \frac{1}{2} p_s + i p_s\]

\[-\Delta \delta < \delta - \delta_s < \Delta \delta\]  
\[(i = 0, 1, \ldots, N_\alpha - 1)\]  

(6.39)

The $\alpha$-interval is chosen so that the analyzed source is exactly in the center of the $(N_\alpha/2)$-th bin.

To calculate the background events $N_{i}^{bk,S}$ in the same window as defined in (6.39) we perform the convolution (in analogy to equation 6.23)

\[N_{i}^{bk,S} = \frac{1}{N_t} \cdot \sum_{n=0}^{N_s-1} N_{[n]} \cdot \tilde{N}_{[i-n+1]}^{S}\]  

(6.40)

where the event distribution $N_{[n]}$ and the total number of selected events $N_t$ are the same as defined in section 6.1.1. The histogram containing the values $N_{i}^{bk,S}$ is noted with $H_{bk}^{S}$.

**Grouping of bins:** The bins around the $\gamma$-sources are grouped together in order to have a window of approximately the same width in declination and right ascension direction. Let be

\[\Delta n_\alpha = \left[ \frac{\Delta \delta}{p_s \cos \delta_s} + \frac{1}{2} \right]\]  

(6.41)

then

\[(N_{i}^{bk,S})' = \sum_{i=\frac{N_s}{2}-\Delta n_\alpha}^{\frac{N_s}{2}+\Delta n_\alpha} N_{i}^{bk,S}\]  

(6.42)

\[(N_{i}^{ms,S})' = \sum_{i=\frac{N_s}{2}-\Delta n_\alpha}^{\frac{N_s}{2}+\Delta n_\alpha} N_{i}^{ms,S}\]  

(6.43)

where $(N_{i}^{bk,S})'$ is the number of background events from the source direction and $(N_{i}^{ms,S})'$ the number of measured events from the source direction in the grouped bins.

**A particular case: Cyg X-3**

As discussed in section 2.6.3, periodical signals from Cyg X-3 were reported in the 80th. Therefore in the analysis of this particular source we add a second dimension (divided into 20 bins) to the histograms $H_n$ (figure 6.11), $H_{ms}^{S}$ and $H_{bk}^{S}$ describing its orbital phase.

The phase is calculated following the best quadratic fit of the ephemeris given in [125]. The heliocentric arrival times expressed in reduced Julian Date $JD$ corresponding to
minimum of intensity of the soft X-rays (1-10 keV) is given by

\[ T_n = T_0 + P_0 n + c_0 n^2 \]  \hspace{1cm} (6.44)

\[ T_0 = 40949.39185 \pm 0.00088 \text{ days (JD)} \]
\[ P_0 = 0.199684393 \pm 8.9 \cdot 10^{-8} \text{ days} \]
\[ c_0 = (6.06 \pm 0.17) \cdot 10^{-11} \text{ days} \]

for integer \( n \). Phase 0 corresponds to the minimum of intensity. Therefore solving the equation (6.44) with respect to \( n \), we find the orbital phase of Cyg X-3. This, expressed for geocentric arrival times as a function of the reduced Julian date \( JD \), is found to be

\[ \phi_{\text{CygX3}} = \left( -P_0 + \sqrt{P_0^2 + 4c_0 \left( JD + \frac{t_{\text{corr}}}{86400} - T_0 \right)} \right) \mod 1. \]  \hspace{1cm} (6.45)

The time \( t_{\text{corr}} \) (in seconds) has been introduced to correct for the heliocentric position of the Earth. If we note with \( \vec{x} \) the vector Sun-Earth and with \( \vec{d} \) the unit vector in Cyg X-3
direction
\[ t_{\text{corr}} = \frac{\vec{x} \cdot \vec{d}}{c} \]  
where \( c \) is the speed of light. The vector \( \vec{x} \) is calculated with the help of the Fortran routine’s library \textit{slalib} [32].

### 6.1.3 Ideal bin size

Despite the fact that there are some uncertainties in the angular resolution of the direction of the primary particles of point sources, we have to establish an ideal bin size that we will use for the analysis of the selected point sources described in section 6.1.2 and the upper limits calculations of the sky survey.

When the angular resolution is described by a two-dimensional Gaussian

\[ R(\Delta \vartheta; E_{\text{cut}}) = \frac{\Delta \vartheta}{\sigma^2} \exp \left( -\frac{(\Delta \vartheta)^2}{2\sigma^2} \right) \]  

according to Monte Carlo studies made by Alexandreas et al. [116] the optimal radius \( r_{\text{opt}} \) for a circular bin used to look for signals from point sources can be parameterized by:

\[ r_{\text{opt}} = \left( 1.58 + 0.7e^{-0.88N^{0.36}} \right) \sigma \]  

where \( N \) is the number of background events expected in a circular bin with radius 1 \( \sigma \) around the source. For large \( N (\gtrsim 100) \) \( r_{\text{opt}} \) approaches 1.58 \( \sigma \) and the bin of this size contains roughly 72% of the signal events. Always according to Alexandreas et al. [116] if we use a squared bin (as we do) of the same area as the ideal circular bin the sensitivity of the measurement of a signal will be reduced in a marginal way only (\( \sim 1.5\% \)).

In our analysis however there are many uncertainties about the ideal bin size. The angular resolution gotten by the Monte Carlo simulation in section 4.4.3 is not Gaussian for a given lower muon energy cut \( E_{\text{cut}} \). Furthermore the Monte Carlo (MC) simulations can’t describe perfectly the reality and in particular it doesn’t take into account all the possible effects of the noise hits in the muon chambers and scintillators.

We decide nevertheless to choose the size of the ideal squared bins, so that in the MC detector simulation (see Appendix A), applying the selection of the events used in the point sources search analysis (see chapter 4.1.1), about 70% of the events from a point sources situated in the center of the squared bin, have reconstructed muon direction inside the bin. We analyze the difference between the direction expressed in horizontal coordinates of the generated muon at surface with zenith and azimuth \((\theta_g, \phi_g)\) and the reconstructed direction \((\theta_r, \phi_r)\). After defining

\[ d = \max \left( |\theta_g - \theta_r|, |\phi_g - \phi_r| \sin \theta_g \right) \]  

we choose the half side \( h_b \) of the ideal squared bin so that the ratio \( R \) of selected events with \( d < h_b \) is nearly 70\%. 2\( h_b \) is considered to be the ideal size of the side of the squared bin. The results for different muon energy lower cuts are shown in table 6.2.
<table>
<thead>
<tr>
<th>Energy cut [GeV]</th>
<th>Size of bin $(2h_b)$</th>
<th>$R$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3</td>
<td>72.9</td>
</tr>
<tr>
<td>30</td>
<td>2.4</td>
<td>71.9</td>
</tr>
<tr>
<td>50</td>
<td>1.5</td>
<td>72.4</td>
</tr>
<tr>
<td>100</td>
<td>0.9</td>
<td>73.1</td>
</tr>
</tbody>
</table>

Table 6.2: Size of the squared bin for different muon energy lower cuts, chosen so that according to the MC detector simulation the ratio $R$ of accepted events from a point source is around 70%. We consider here the angular difference between the direction of the generated event at surface and the reconstructed direction.

**Correction for the angle between primary particles and muons**

In table 6.2 we did not take into account the angular difference $\hat{\alpha}_{\text{prim}}$ between the direction of the primary particle and the direction of the muon at surface. To do such a correction we should of course make assumptions about the characteristics of the primary particle generated by the point source and the shape of their spectrum, since this information is not available. We perform such a correction assuming that the primary particle are photons with energies following a spectrum with spectral index 2. We take into account the $\gamma$-shower MC simulation described in appendix A.1.2. The ratios $R$ of events with $d < h_b$ for the same bin sizes used in table 6.2 are shown in the third column of table 6.3. We see that we collect only $\sim 10\%$ less events from the gamma source in the squared bin, in comparison with the results of table 6.2.

<table>
<thead>
<tr>
<th>Energy cut [GeV]</th>
<th>Size of bin [°]</th>
<th>$R$ [%] ($\hat{\alpha}_{\text{prim}}$ corr.)</th>
<th>$R$ [%] ($\hat{\alpha}_{\text{prim}}$ and spectrum corr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3</td>
<td>64.9</td>
<td>69.7</td>
</tr>
<tr>
<td>30</td>
<td>2.4</td>
<td>65.2</td>
<td>69.6</td>
</tr>
<tr>
<td>50</td>
<td>1.5</td>
<td>65.3</td>
<td>68.8</td>
</tr>
<tr>
<td>100</td>
<td>0.9</td>
<td>67.2</td>
<td>70.9</td>
</tr>
</tbody>
</table>

Table 6.3: Ratio $R$ of events with $d < h_b$ after correction for the angular difference $\hat{\alpha}_{\text{prim}}$ between the direction of the primary particle and the direction of the muon at surface (third column) and with additional correction for the shape of the simulated muon spectrum generated by primary $\gamma$ rays with spectral index 2 (forth column).

The spectrum of muons from $\gamma$ induced showers generated in the MC simulation, contains more high energetic muons, which are known to have a better angular resolution, than in proton induced showers (see figure A.3). Our detector simulation is based on proton induced showers. Therefore we apply a correction giving the weight $w$ to the MC
Table 6.4: Bin size used for the upper limit calculation of the sky survey (second column), for which the bin size has to be a multiple of the pixel size $p_s$. The ratio $R$ of events with $d < h_b$ corrected for $\langle s_{\text{prim}} \rangle$ and for the simulated spectrum shape of muons generated in $\gamma$-induced showers is reported in the third column (for $\delta = 0^\circ$).

<table>
<thead>
<tr>
<th>Energy cut [GeV]</th>
<th>Size of bin (2$h_b$)</th>
<th>$R$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>30</td>
<td>2.67</td>
<td>74</td>
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<td>50</td>
<td>1.67</td>
<td>74</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>75</td>
</tr>
</tbody>
</table>

Table 6.4: Bin size used for the upper limit calculation of the sky survey (second column), for which the bin size has to be a multiple of the pixel size $p_s$. The ratio $R$ of events with $d < h_b$ corrected for $\langle s_{\text{prim}} \rangle$ and for the simulated spectrum shape of muons generated in $\gamma$-induced showers is reported in the third column (for $\delta = 0^\circ$).

To calculate the flux upper limits of the sky survey we use maps with a pixel size $p_s = \frac{1^\circ}{2}$ for $E_{\text{cut}} = 20$ GeV and $p_s = \frac{1^\circ}{3}$ degrees for the other energies. We choose the bin size accordingly, as shown in table 6.4. The ratio $R$ given in the table is the one obtained for a declination $\delta = 0^\circ$. Taking into account equation (6.29), we establish the ratio $R$, to be used for the upper flux limit calculations, as a function of the declination.

### 6.1.4 Calculation of upper limits for muon fluxes from point sources

In the following paragraphs we show how we calculate the upper limits for the muon fluxes from point sources.

Knowing the number of background events $N_{\text{bk}}$ and the number of measured events $N_{\text{ms}}$, the first step to accomplish is to calculate the upper limit of the number of signal events. We calculate it with a confidence level (CL) of 90% of the signal $N_u$ solving
numerically the equation [126]

\[
1 - 90\% = \frac{e^{-(N_{bk} + N_{u})} \sum_{n=0}^{N_{mu}} \frac{(N_{bk} + N_{u})^{n}}{n!}}{e^{-N_{bk}} \sum_{n=0}^{N_{mu}} \frac{(N_{bk})^{n}}{n!}}
\] (6.51)

Often the conversion of the number of signal events in the muon flux is calculated averaging the flux along the part of the trajectory of the point source included in the field of view of the detector (e.g. [118]). However the latter flux is difficult to use for comparison between experiments, because this depends on the latitude at which the measurement is performed. To avoid that problem we prefer to estimate the flux (upper limit) of muons that we would get if we assume the point source to be in the zenithal direction. To do that we make the assumption that the zenith angle dependence of the background flux and of the signal flux have the same shape and therefore that signal to background ratio is independent of direction. The upper limit \(\mu\)-flux is so calculated as:

\[
\phi_{\mu > E_{cut}}^{\text{limit, vert}} = \frac{1}{R} \cdot \frac{N_{u}}{N_{bk}} \cdot \phi_{\mu > E_{cut}}^{\text{bk, vert}} \cdot \Omega_{\text{bin}} \cdot F_{m}
\] (6.52)

\(\Omega_{\text{bin}}\) is the space angle corresponding to the search window. \(R\) is the ratio of events which are reconstructed in the squared bin for a point source in the direction of the center of the bin (see tables 6.3 and 6.4). For the background vertical flux \(\phi_{\mu \geq E_{cut}}^{\text{bk, vert}}\) corresponding to the muon energy lower cut \(E_{cut}\) we use the values given in table A.4. Finally \(F_{m}\) is a correction factor reported in table 6.5 which takes into account the momentum resolution.

It corresponds to the ratio between the number of selected MC events with generated energy larger \(E_{cut}\) and the number of selected MC events with reconstructed energy larger than \(E_{cut}\). For low energies the correction factor \(F_{m}\) is negligible.

<table>
<thead>
<tr>
<th>Energy cut [GeV]</th>
<th>Correction factor (F_{m})</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.996</td>
</tr>
<tr>
<td>30</td>
<td>0.989</td>
</tr>
<tr>
<td>50</td>
<td>0.988</td>
</tr>
<tr>
<td>100</td>
<td>0.935</td>
</tr>
</tbody>
</table>

Table 6.5: Correction factor \(F_{m}\) for the momentum resolution.

\(^5\)In the Monte Carlo simulations (see appendix A.1) we find that the zenithal distributions of muons in proton induced showers (with spectral index \(\gamma = 2.7\)) and of the muons produced by primary \(\gamma\)’s (with spectral index \(\gamma = 2\)) are very similar.
6.2 Results

The point source search is performed on different time scales, and applying different cuts on the measured surface energy $E_{\text{cut}}$ of the muons.

**Time scales:** 1 day, 1 “stable period”, 1 year, full dataset

**Energy cut:** 20 GeV, 30 GeV, 50 GeV, 100 GeV

The 1 day period\(^6\) is chosen with the idea to follow the full trajectory in the visible sky of a hypothetical source $S$. The period length in which the latter can be observed depends strongly on the declination $\delta$. If the source $S$ is in the (North) polar region ($\delta \approx 90^\circ$) it can be observed all the day. If it is in the equatorial region ($\delta \approx 0^\circ$) $S$ can be observed for a few hours only. If a remarkable excess is observed in a particular region an analysis on smaller time scales is then performed.

We perform the daily search on the intervals A) from midnight of one day to midnight of the day after and on the intervals B) shifted by 12 hours between noon of one day and noon of the day after. This avoids to miss a signal, if it is emitted near the limit of the time interval analyzed. The daily search of point sources is performed, after we get the acceptance $\bar{N}_\mu(\delta, h_n; E_{\text{cut}})$ for the ’stable period’ in which the analyzed days are included.

To perform the analysis on the periods of 1 year and on the whole dataset, the measured distributions $N_{\mu,\text{tot}}^{\text{ns}}(\delta, \alpha; E_{\text{cut}})$, as well as the background distributions $N_{\mu,\text{tot}}^{\text{iso}}(\delta, \alpha; E_{\text{cut}})$, are established separately for each ’stable period’ and then added together.

The selection criteria applied are described in section 4.1. We use event selection of type C and the selection of time intervals of type 2.c. (with exception of the step 1 analysis of the sky survey where we use the selection 2.b).

6.2.1 Sky survey

6.2.1.1 General remarks

The sky survey is performed searching for excesses of muons from windows of different sizes of grouped pixels.

In a first analysis (step 1) the resolution $p_s = 1^\circ$ (see definition 6.19) is used. The pixel width $N_p$ (see equations (6.27) and (6.28)) of the analyzed windows is set alternatively to 1, 2 and 3, so that we analyze windows of “effective” side length

$$1^\circ \times 1^\circ, \quad 2^\circ \times 2^\circ, \quad 3^\circ \times 3^\circ$$

The minimum declination $\delta_{\text{min}}$ is set to $-21^\circ$, which is the limit at which we can still measure events as shown in figure 6.12.a). In figure 6.12.b) is reported the distribution.

\(^6\)Also the CYGNUS collaboration has performed a daily search for emission from selected point sources with its extensive air shower array in the period from April 1986 to June 1992. [127]. No signal was found.
Figure 6.12: a) Distribution of the analyzed events in declination $\delta$ for an energy lower cut of 30 GeV. b) Same distribution in $\sin \delta$. (Data are integrated over all the right ascension range.)

of the analyzed events in $\sin \delta$. The latter distribution shows the relative efficiency as a function of the declination, since the space angle element $d\Omega$ is proportional to $d(\sin \delta)$

$$
 d\Omega = d\alpha d\delta \cos \delta = d\alpha d(\sin \delta).
$$

(6.53)

In the step 1 analysis we have some interesting observations, which lead to further investigations performed with an improved analysis (step 2). We will show the most interesting results from step 1 and the summary of the final results of step 2.

In step 2 we improve the resolution and we set $p_s = \frac{1}{2}$ for $E_{\text{cut}} = 20$ GeV and $p_s = \frac{1}{3}$ for the other energy cuts, in order to increase the possibilities in choosing the window size and the number of windows that can be analyzed for a given window size. We avoid also to analyze points in which the statistics is too low, so that we set the minimum declination $\delta_{\text{min}}$ to $18^\circ$ ($\sin \delta_{\text{min}} \cong 0.3$) for the daily search and to $0^\circ$ for the search on the other time intervals.

In addition for the daily search we consider the entries in the cumulative $-\log(P)$ histograms only if

- The expected number of background events is larger than 0.1
- The expected number of background events in the last 12 hours of the analyzed period is more than 10% of the one expected from the previous 12 hours.

\footnote{In figure 6.12 b) one can see that the efficiency at $\sin \delta = 0.3$ and $\sin \delta = 0$ are respectively about $\frac{1}{2}$ and $\frac{1}{4}$ of the maximum efficiency.}
6.2 Results

The second condition is introduced to avoid that entries are reported twice with almost the same statistics in the daily intervals A) and B), as in the example shown in figure 6.13. In the step 2 of the analysis we also consider the cut on the average number of scintillator hits per events (see selection of type 2.c in section 4.1.2), which is omitted in step 1 (Type 2.b). Another little difference between the two steps is that, to define the daily intervals, in step 2 we use always the Middle European Summer Time (MEST), indeed in step 1 we use the Geneva’s legal time (i.e. change from MEST to MET at the last Sunday of October).

6.2.1.2 Preliminary observations

Interesting observations of step 1 analysis

The summary of the most relevant excesses found in the step 1 analysis is shown in table 6.6. Reported are two excesses of the daily search (Excess Nr.2 and Nr.3) and one (Excess Nr.1) which is observed in the full data set of the year 1999. The latter has a higher probability \( P \) to be a statistical fluctuation of the background than excesses 2 and 3. However the number of trials performed in the yearly analysis is much lower than in the daily search. The situation is well represented by the figures 6.14 and 6.15.

In the figure 6.14 we report the \(- \log(P)\) cumulative distributions for the yearly search of 1999 performed on \(2^\circ \times 2^\circ\) bins applying an energy cut \(E_{\text{cut}} = 20\ \text{GeV}\). The excess Nr.1 (\(A = - \log(P) = 7.61\)) is clearly separated from the other search trials. The number of analyzed bins is 39600. The yearly search is performed twice (1999 and 2000). The
probability of finding an excess with $\Lambda \geq 7.61$ performing $2 \times 39600$ trials is only about

$$2 \cdot 39600 \cdot 10^{-7.61} \cong 1.9 \cdot 10^{-3} \quad (6.54)$$

In figure 6.15 we show the $-\log(P)$ cumulative distribution for the daily search performed in windows with size of $1^\circ \times 1^\circ$ applying an energy cut $E_{\text{cut}} = 30$ GeV. There is one entry for each analyzed bins and for each analyzed 1 day-interval. The number of search trials is about $2.4 \cdot 10^7$. Due to the high number of trials the daily excesses (Nr.2 and 3) are less significant than the yearly excess (Nr.1). However we think that it is worthwhile to investigate them more carefully.

Concerning excess Nr. 3, we would like to remark that it is found that in a window with the same declination range but with the right ascension interval 5h14' - 5h26' (which does not fit equation 6.29) we find an excess of $-\log(P) = 9.72$ (for $E_{\text{cut}} = 30$ GeV).

**Flux of the 3 most relevant excesses:** We estimate the vertical flux of muons corresponding to the excesses reported in table 6.6. The flux (reported in table 6.7) is calculated as

$$\phi_{\mu > E_{\text{cut}}}^{\text{vert}} = \frac{1}{R} \cdot \frac{N_{\text{ms}} - N_{\text{bk}}}{N_{\text{bk}}} \cdot \phi_{\mu > E_{\text{cut}}}^{\text{bk,vert}} \cdot \Omega_{\text{bin}} \cdot F_m \quad (6.55)$$

using the same ideas and definitions of equation (6.52). For simplicity we make the optimistic assumption, that all events from the source fall inside the analyzed window (i.e. $R = 1$). The obtained fluxes are more than 2 order of magnitude larger than what is expected from the brightest continuous $\gamma$-ray sources as given in the table 2.4 in section 2.6.4.

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Time interval</th>
<th>$E_{\text{cut}}$ [GeV]</th>
<th>Window</th>
<th>Meas. events</th>
<th>Backgr. events</th>
<th>$-\log(P)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1999 (all year)</td>
<td>20</td>
<td>21°-23°</td>
<td>20h44m-20h52m</td>
<td>59651</td>
<td>58328</td>
</tr>
<tr>
<td>2</td>
<td>31.7/1.8.99 (12h-12h)</td>
<td>30</td>
<td>80°-81°</td>
<td>15h44m-16h08m</td>
<td>346</td>
<td>250.0</td>
</tr>
<tr>
<td>3</td>
<td>2/3.11.99 (12h-12h)</td>
<td>20/30</td>
<td>75°-76°</td>
<td>5h12m- 5h28m</td>
<td>385/358</td>
<td>278.5/257.5</td>
</tr>
</tbody>
</table>

Table 6.6: List of the most relevant excesses observed during step 1 analysis. Reported is the time interval of the excess, the energy cut $E_{\text{cut}}$ applied, the position of the window of observation in equatorial coordinates, the number of measured events and of background events in that window, as well as $-\log(P)$ where $P$ is the probability that the excess is caused by statistical fluctuation of the background (see equation 6.25).
6.2 Results

Figure 6.14: $-\log(P)$ cumulative distribution for the complete set of selected data of 1999 with energy cut $E_{\text{cut}} = 20$ GeV in windows with “effective” size $2^\circ \times 2^\circ$ ($p_s = 1^\circ$, $N_p = 2$). (Step 1 analysis)

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Duration</th>
<th>$E_{\text{cut}}$ [GeV]</th>
<th>$\mu$-flux $[10^{-10}\text{cm}^{-2}\text{s}^{-1}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 Year</td>
<td>20</td>
<td>86</td>
</tr>
<tr>
<td>2</td>
<td>1 Day</td>
<td>30</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>1 Day</td>
<td>20</td>
<td>391</td>
</tr>
</tbody>
</table>

Table 6.7: Estimate about the $\mu$-flux corresponding to the excesses of table 6.6 calculated with equation (6.55), assuming for simplicity that all events from the source fall inside the analyzed window (i.e. $R = 1$).

Search for counterparts: We check on the NASA Extragalactic Database (NED)[128], if there are special objects within a cone of $2^\circ$ centered in the bin where we find the 3 excesses reported in table 6.6. We note that it is quite common to find a Quasi Stellar Object (QSO), a X-ray source or a radio source in a $2^\circ$-cone with a random direction. We report in the table 6.8 only the gamma sources or X-ray sources which have been observed also as radio sources. We remark that 3EG J1621+8203 is one of the 271 sources of the
Figure 6.15: $-\log(P)$ cumulative distribution for daily search (step 1) with energy cut $E_{\text{cut}} = 30$ GeV in windows with “effective” size $1^\circ \times 1^\circ$ ($p_s = 1^\circ$, $N_p = 1$). There is one entry per bin and per analyzed 1 day-interval.

3rd Egret catalog [79]. 1ES 1544+820 is a BLAZAR Lacertae type object which is known to be a flaring object having irregular emissions.

Gamma ray bursts (GRB) occurring some years ago are also present in the NED database and are quite common and easy to find in a $2^\circ$-cone with random direction. However GRBs of 1999 and 2000 are currently not listed there. Therefore to look, if the

<table>
<thead>
<tr>
<th>Excess Nr.</th>
<th>Source Name(s)</th>
<th>Type</th>
<th>$\delta$</th>
<th>$\alpha$</th>
<th>$\Delta\theta$</th>
<th>$\varsigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2MASX J2044087+220508 [129]</td>
<td>X-ray</td>
<td>22°05'09''</td>
<td>20h44m08.8s</td>
<td>1.25°</td>
<td>53''</td>
</tr>
<tr>
<td>2</td>
<td>RBS 1524, 1ES 1544+820 [130]</td>
<td>X-ray, BL Lac</td>
<td>81°55'06''</td>
<td>15h40m15.7s</td>
<td>2.5°</td>
<td>1°32'</td>
</tr>
<tr>
<td>2</td>
<td>3EG J1621+8203 [79]</td>
<td>$\gamma$-ray</td>
<td>82°03'</td>
<td>16h21m</td>
<td>0.85°</td>
<td>1°50'</td>
</tr>
<tr>
<td>3</td>
<td>87GB 051013.4+734728</td>
<td>X-ray,AGN</td>
<td>73°51'09''</td>
<td>5h16m31.2s</td>
<td>1.25°</td>
<td>1°40'</td>
</tr>
</tbody>
</table>

Table 6.8: List of X-ray sources with radio counterpart and gamma ray sources within a $2^\circ$-cone around the excesses of table 6.6. $\varsigma$ is the angular distance from the center of the bin in which we find the excess and $\Delta\theta$ is the uncertainty in the position (95% confidence level).
excesses number 2 and 3 are in coincidence with a GRB, we check the BATSE Catalog [131]. The BATSE experiment took data till end of May 2000. There is a GRB on the 3rd of November at 18:42 (UT), but its direction ($\theta = 15^\circ, \alpha = 21h24min$) is not coincident with the excess number 3. The time of the burst is also about 8 hours later than the end of the interval in which we find the excess. In the BATSE catalog we find no other GRB during the same days as the excesses 2 and 3 and no one in the nearby days with direction next to them.

**Daily $-\log(P)$ cumulative distributions:** To have a check that during the day-intervals in which the excesses Nr. 2 and 3 are found, there are not problems causing large systematical errors, we look at the $-\log(P)$ cumulative distributions of the analyzed bins of the corresponding day-intervals (figure 6.16). If we don’t consider the excesses that we are investigating and the correlated bins in their neighborhood (marked with an arrow) the $-\log(P)$ cumulative distribution agrees reasonably with the expected one (dotted line).

![Cumulative trials](image)

Figure 6.16: $-\log(P)$ cumulative distribution for the intervals a) 31 July 1999 12:00 - 1 August 1999 12:00 and b) 2 November 1999 12:00 - 3 November 1999 12:00 for surface energy larger than 30 GeV in windows with “effective” size $1^\circ \times 1^\circ$ ($p_s = 1^\circ, N_p = 1$). The distribution agrees with the expected one (dotted line) with exception of the excesses 2 and 3 reported in table 6.6 and the nearby (correlated) points (marked with an arrow).

**Results with different selection criteria:** Another interesting point is to see how the relevant excesses behave if we apply different selection criteria. Often an abnormal
behaviour of the detector is less evident if high quality events are required. If the observed excesses are caused by such a behaviour the signal to background ratio could decrease for high quality tracks. A comparison for the daily excesses is shown in table 6.9, where we report together with the previous result (selection C + 2.b), the result obtained using the event selection used for the anisotropy analysis (Type B) and the result obtained with the selection of the runs according to the online database (Type 1.a) (see section 4.1). The signal to background ratios are stable.

<table>
<thead>
<tr>
<th>Excess Nr.</th>
<th>$E_{\text{cut}} \ [\text{GeV}]$</th>
<th>Selection</th>
<th>$-\log(P)$</th>
<th>Meas. Events</th>
<th>Back. Events</th>
<th>Ratio signal/back.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>30</td>
<td>C + 2.b</td>
<td>8.25</td>
<td>346</td>
<td>250.0</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B + 2.b</td>
<td>4.29</td>
<td>168</td>
<td>122.25</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B + 1.a</td>
<td>6.78</td>
<td>303</td>
<td>222.29</td>
<td>0.38</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>C + 2.b ¹</td>
<td>8.72</td>
<td>358</td>
<td>257.5</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B + 2.b ²</td>
<td>5.64</td>
<td>185</td>
<td>129.2</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B + 1.a ²</td>
<td>5.90</td>
<td>289</td>
<td>215.9</td>
<td>0.34</td>
</tr>
</tbody>
</table>

1. Time interval based on MET time
2. Time interval based on MEST time

Table 6.9: Daily excesses reported in table 6.6 analyzed with different selections. The type of event selection (B,C) and of the time interval selection (1.a and 2.b) are explained in section 4.1.2.

Table 6.10 shows the behaviour of the excesses Nr.1-3 applying different energy cuts. Only entries with $-\log(P) \geq 3$ are reported, since the computer program used for the analysis does not store the complete information of the excesses with $-\log(P) < 3$. The signal to background ratio are stable with a low tendency to increase with the energy. Of course the significance of the excesses at high energy is lower, due to the decreasing of statistics.

<table>
<thead>
<tr>
<th>Excess Nr.</th>
<th>$E_{\text{cut}} \ [\text{GeV}]$</th>
<th>$-\log(P)$</th>
<th>Meas. Events</th>
<th>Back. Events</th>
<th>Ratio signal/back.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>7.61</td>
<td>59651</td>
<td>58328</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>5.53</td>
<td>47531</td>
<td>46550</td>
<td>0.021</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>8.01</td>
<td>350</td>
<td>254.9</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>8.25</td>
<td>346</td>
<td>250.0</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>5.14</td>
<td>176</td>
<td>124.3</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>9.03</td>
<td>385</td>
<td>278.5</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>8.72</td>
<td>358</td>
<td>257.5</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>5.36</td>
<td>176</td>
<td>123.1</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 6.10: Excesses reported in table 6.6 with different muon energy cuts $E_{\text{cut}}$ (same window size). We report in this table only entries with $-\log(P) \geq 3$. 
6.2 Results

Time development of the relevant excesses: Another relevant question to answer, is if the excesses are spread on the full time interval in which they are observed or if there are bursts in a smaller time interval. The plots in the figures 6.17 and 6.18 show the measured and the background event distribution corresponding to the daily excesses Nr. 2 and 3 in intervals of 1 hour. The background distribution reaches his maximum when the excesses are in a favorable position of observation. Since both excesses are in circumpolar directions they never leave completely the field of view of the L3+C muon spectrometer.

<table>
<thead>
<tr>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (MEST)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0:00</td>
</tr>
<tr>
<td>0:00</td>
</tr>
<tr>
<td>0:00</td>
</tr>
</tbody>
</table>

Figure 6.17: Hour by hour plot of the excess Nr.2. The hatched area represents the expected background events. The circles show the measured events and the error bars their statistical error. The excess is observed in the interval between 31th July 12:00 and 1st August 12:00.

In table 6.11 we report the development of the excess Nr. 1 during the year 1999 and 2000 for the different “stable” periods defined in table 4.4. No burst is observed in correspondence with the excesses Nr. 1-3. The signal to background ratios stay basically constant along the time interval in which the excesses are observed. The signals tend to disappear immediately outside the time intervals corresponding to the excesses. This observation, in particular for the daily excesses Nr.2-3, is in favour of the hypothesis that the excesses are caused by statistical fluctuations, since a shift of the 1-day time interval would reduce strongly the significance of the excesses.
Figure 6.18: Hour by hour plot of the excess Nr.3. The hatched area represents the expected background events. The circles show the measured events and the error bars their statistical error. The excess is observed in the interval between 2nd Nov 12:00 and 3rd Nov 12:00.

Table 6.11: Summary of the measured and background events in the direction of the excess 1 in the different “stable” periods of the year 1999 and 2000.
6.2 Results

Step 2 analysis

On the necessity of the cut on the mean scintillator rate: As mentioned previously, one of the improvement of the step 2 analysis is the introduction of the cut on the average number of scintillator hits per event (interval selection 2.c, see section 4.1.2). The reason for which we are led to apply such a cut becomes evident when looking at the $-\log(P)$ cumulative distributions of the yearly data of large bins in which this cut is omitted. We report such distributions for bins of “effective size” of $8^\circ \times 8^\circ$ ($N_p = 8$, $p_s = 1^\circ$) and an energy cut of 20 GeV in figure 6.19. In such large bins the statistics reaches $\approx 1.5 \cdot 10^6$ and therefore we are sensitive at variations at the permil level. In the plot of the year 2000, when data-taking is often affected by very high background in the scintillators, due to LEP induced noise, we see a strong deviation from the expected distribution (shown by the dotted line).

After applying the cut on the average number of scintillator hits per event, the $-\log(P)$ cumulative distribution for the year 2000 looks “much” better (figure 6.20).

The 3 relevant excesses found in step 1 analysis revisited with step 2 analysis: In table 6.12 we report the results of the step 2 analysis about the previously analyzed excesses. There are not big changes except for a small decrease of the signal of the excess Nr. 3, caused mainly by the change from MET time to MEST when defining the 1-day intervals.
Figure 6.20: $-\log(P)$ cumulative distribution for the year 2000 with cut on the mean scintillator rate (selection type 2.c, see section 4.1.2). ($N_p = 8, p_s = 1^\circ$).

Of course with increased resolution, the number of trials increases, so that the significance of the excesses decreases. This can be seen, looking at the new $-\log(P)$ cumulative distributions (figures 6.21 and 6.22). The excess Nr. 1 is the only one which still keeps some distance from the expected distribution. The excesses Nr. 2 and 3 remain the ones with the highest value of $-\log(P)$ of the daily excesses observed at bin size $1^\circ \times 1^\circ$ and with $E_{\text{cut}} = 30 \text{ GeV}$, but they are now integrated in the expected cumulative distribution (dotted line).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>7.58</td>
<td>59587</td>
<td>58269</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>8.11</td>
<td>345</td>
<td>250.0</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>8.50</td>
<td>379</td>
<td>276.6</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>8.17</td>
<td>352</td>
<td>255.7</td>
</tr>
</tbody>
</table>

Table 6.12: Results of the step 2 analysis for the excesses reported in table 6.6. (Same window size).
6.2 Results

Figure 6.21: $-\log(P)$ cumulative distribution for the available data of 1999 with surface energy cut $E_{\text{cut}} = 20$ GeV in windows with “effective” size $2^\circ \times 2^\circ$ ($p_s = \frac{1^\circ}{2}, N_p = 4$). (Step 2 analysis)

Figure 6.22: $-\log(P)$ cumulative distribution for daily search (step 2) with surface energy cut $E_{\text{cut}} = 30$ GeV in windows with “effective” size $1^\circ \times 1^\circ$ ($p_s = \frac{1^\circ}{3}, N_p = 3$)
Mapping of the excesses: The improved resolution in the step 2 analysis allows to produce the signal map and the $-\log(P)$ map around the 3 relevant excesses found in the step 1 analysis. We report such maps for these 3 excesses in figures 6.23-6.25. In the center of the map we draw a rectangle which shows where the excesses have been found. It seems that no trace of the signal is visible outside the window in which we find the excess.

![Image of a map showing excess Nr. 1 with signal and log(P) values.]

Figure 6.23: $-\log(P)$ and Signal map in equatorial coordinates around the excess Nr. 1. The rectangle in the center of the plots corresponds to the window in which we find the excess.
Figure 6.24: $-\log(P)$ and Signal map in equatorial coordinates around the excess Nr.2. The rectangle in the center of the plots corresponds to the window in which we find the excess.
Figure 6.25: \(-\log(P)\) and Signal map in equatorial coordinates around the excess Nr.3. The rectangle in the center of the plots corresponds to the window in which we find the excess.
Signals from the excesses with different bin sizes: With the improved resolution we can also produce plots, where we report the signal of the excesses Nr.1 (figure 6.26) and Nr.2-3 (figure 6.27) as a function of the “effective” size of the observation window (bin) with the same center as the window in which the excesses are observed. In case of a real signal we would expect to see the signal to increase when enlarging the bin size, because some signal events will of course fall outside the original observation window. For the excess Nr.2 the signal is canceled as soon as we increase the bin size with respect to the original one. The signal from excess Nr. 1 stays nearly constant when increasing the bin size at values larger than the original $2\degree$. The only signal which has a small tendency to increase is the excess Nr. 3, but less than expected if we take into account the angular resolution estimated with the Monte Carlo simulation (see sections 4.4.3 and 6.1.3).

Repeated signal: In the computer programs launched to perform the step 2 analysis, we monitor day by day the windows in which we found the excesses in step 1 analysis. In such a way we discover 2 other signals with $-\log(P) \approx 3$ in the same window as for the excess Nr. 3, occurring on the 27/28th and 29/30th September 1999 in the intervals B (12:00-12:00). The situation is shown in the figure 6.28, where we report the $-\log(P)$ value versus the day number of the year 1999, together with the corresponding cumulative distribution. The probability to find additional two signals with $-\log(P) \approx 3$ from the same direction window of the excess Nr.3 in about $10^2$ days of data-taking is only $\sim 10^{-2}$. 

Figure 6.26: Signal of the excess Nr.1 for different bin sizes ($2h_b$). Error bars represent the statistical uncertainty.
Figure 6.27: Signal of the excesses Nr. 2 and 3 for different bin sizes ($2h_b$). Error bars represent the statistical uncertainty.

Figure 6.28: a) $-\log(P)$ cumulative distribution for daily search in the direction window of the excess number 3 (only year 1999 and intervals B. b) $-\log(P)$ in the direction window of the excess number 3 versus day in 1999 for the day intervals B. Together with the excess Nr.3 ($-\log(P) = 8.17$, 2/3 Nov.) it shows another two excesses with $-\log(P) = 2.90$ (27th/28th Sep.) and $-\log(P) = 3.54$ (29th/30th Sep.).
6.2 Results

One has to note, however, that the first part of both intervals are without data because the magnet has been switched off. The doubt could rise, that after the magnet is switched on it takes some time for the detector to reach a stable situation. However, we notice that the $-\log(P)$ cumulative distributions of both intervals looks normal. We want to remind also that during the last days of September the angular distributions of the accepted events was not very stable (see figures 4.25 and 4.26).

Summary tables of the highest excesses found in step 2 analysis: In table 6.13 we report the summary of the highest excesses found in the yearly search and in the analysis of the full data set as a function of the $-\log(P)$ range for different bin sizes up to $3^\circ$ and for the energy cuts $E_{\text{cut}} = 20, 30, 50$ GeV. The reported trial factor is proportional to the number of analyzed bins. Its dependence from the bin size is related with equation 6.30. The trial factor 1 corresponds to pixel size $p_s = 1^\circ$ and $n_s = 1$ (as in step 1 analysis). A box surrounds the excesses found in step 1 analysis. Likewise we produce table 6.14 for the daily search. Correlated bins are marked with a superscribed number.

Two Checks: To test the results of our analysis, two checks are proposed.

a) Deficits: The first check consists in looking at the deficits of muon events calculating the probability $P$ that such deficits are caused by statistical fluctuations.

$$P_{\text{def}} = \sum_{k=0}^{N_{\text{max}}} \frac{e^{-N_{bk}} (N_{bk})^k}{k!}.$$  \hspace{1cm} (6.56)

where we use the same definitions as in equation 6.25. The expected $-\log(P_{\text{def}})$ cumulative distribution for the deficits should look as the $-\log(P)$ cumulative distribution for the excesses. This has been effectively observed in our analysis (example figure 6.29). We expect that the deficits are caused only by statistical fluctuations of the background. Therefore if we find deficits with a similar value of $P_{\text{def}}$ as the lowest value of $P$ for the observed excesses, it would be a strong indication that the observed excesses are also caused by statistical fluctuation of the background.

The summary of the highest deficits found in the sky survey during step 2 analysis are reported in the tables 6.15 and 6.16 (discussion follows below).

b) Dummy frequency of Earth’s rotation: The second check is to suppose that Earth is rotating with another frequency than the real one. Also in this case any excess can be understood only as statistical fluctuation of the background.

In the tables 6.17 and 6.18 we report the highest excesses found assuming that the Earth’s frequency of rotation is increased of a factor $e = 2.718$. 

Table 6.13: Summary of the number of excesses found in the step 2 analysis with $-\log(P) > 6$ for the full dataset of 1999, of 2000 and of both years together and for three different energy cuts. On the first column the size of the side of the squared bin is reported. On the second column we report the trial factor, i.e. the number of trials with respect to the step 1 analysis for a particular bin size. Then the number of excesses is given as a function of the period and of the probability range (expressed by $-\log(P)$). Empty spaces means that no excess has been found in the given range. The excess which is found in step 1 analysis is surrounded by a small box.
### Table 6.14: Summary of the number of the daily excesses found in the step 2 analysis with \(-\log(P) > 8\) for three different energy cuts. On the first column the size of the side of the squared bin is reported. On the second column we report the trial factor, i.e. the number of trials with respect to the step 1 analysis for a particular bin size. Then the number of excesses is given as a function of the probability-range (expressed by \(-\log(P)\)). Empty spaces means that no excess has been found in the given range. Superscribed numbers show correlated excesses (same day, neighboring bins). Excesses which are found already in step 1 analysis are surrounded by a small box.

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Table 6.15: Summary of the number of **deficits** found in the step 2 analysis with $-\log(P) > 6$ for the full dataset of 1999, of 2000 and of both years. Entries description as in table 6.13. Superscribed numbers show correlated excesses (neighboring bins).
### DEFICITS

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Table 6.16: Summary of the number of the daily **deficits** found with \(-\log(P) > 8\). Entries description as in table 6.14.
Table 6.17: Summary of the number of spurious excesses found using the dummy frequency $\nu_e$ for the rotation of the Earth. Entries description as in table 6.13. Superscribed numbers show correlated excesses (neighboring bins).
Comparing the results of the two checks with the true analysis one does not see big differences between them. In step 2 analysis and in the two checks we find excesses with higher value of $-\log(P)$ than the excesses Nr.2 and Nr.3. We remark that the strongest daily excess found on the 20th April 2000 assuming the wrong frequency of the Earth’s rotation (marked with the superscript 2 in table 6.18), repeats on another day (26th October 1999) in the same window with $-\log(P) = 5.15$ (at 30 GeV cut). This fact shows that we have to be prudent, in taking conclusions from the repetition of the excess Nr.3. The excess Nr.1 remains, between the yearly excesses, the one with the highest value of $-\log(P)$ together with another excess found in the check b) \(^8\).

\(^8\)One has to keep in mind that the number of trials performed with the step 2 analysis and with the two checks increased of more than a factor 20 with respect to step 1 analysis.
Table 6.18: Summary of the number of spurious daily excesses with $-\log(P) > 8$ found using the dummy frequency $\nu_e$ for rotation of the Earth. Entries description as in table 6.14. Superscribed numbers show correlated excesses (same day, neighboring bins).

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- Energy cut: 20 GeV
- Energy cut: 30 GeV
Conclusion about the preliminary observations: From all observations made till now we conclude that there is no evidence of excesses caused by real signals. The most relevant excesses found in step 1 analysis are probably caused by statistical fluctuations of the background.

6.2.1.3 Summary of the final results

We report a summary of the final results obtained with step 2 analysis. We show the $-\log(\text{P})$ cumulative distributions obtained for the ideal bin size (see table 6.4), for all the energy cuts applied (20, 30, 50, 100 GeV) and for all time scales analyzed. The distribution of the daily search is shown in figure 6.30. The result for the “stable periods” is reported in figure 6.31. With the yearly search we obtain figure 6.32 (1999) and figure 6.33 (2000). The result for both years is reported in figure 6.34. For the latter we report in figure 6.35 the upper flux limit (confidence level 90%) for muons generated by a hypothetical source, calculated with equation 6.52.

No particular excess has been observed. It is noted that there is a tendency of small deviations to the right parts of the $-\log(\text{P})$ cumulative distributions compared to expectation (dotted line) in the yearly search and in the analysis of both years. In the year 1999 the tendency is to have less high excesses than expected. In the year 2000 we have an excess of entries with $-\log(\text{P})$ between 4 and 6. These observations can be thought to be related with the correlation of the entries (c.f. figure 6.10). Concerning the year 2000, some residual effect of the high background in the scintillators due to LEP-induced noise, even after the cut on the mean scintillator rate per events (see selection 2.c in section 4.1.2), may be present and influence the final result. We note that in the analysis of both year of data-taking 0.5 million events are typically contained in one bin of ideal size for an energy cut of 20 GeV. Therefore our sensitivity for excesses reaches nearly the permil level.
Figure 6.30: $-\log(P)$ cumulative distribution for the ideal bin size for different energy cuts. There is one entry for each direction bin and for each analyzed one-day period.
Figure 6.31: $-\log(P)$ cumulative distribution for the ideal bin size for different energy cuts. There is one entry for each direction bin and each “stable period”.
Figure 6.32: 1999 results: $-\log(P)$ cumulative distribution for the ideal bin size for different energy cuts.
Figure 6.33: 2000 results: $-\log(P)$ cumulative distribution for the ideal bin size for different energy cuts.
Figure 6.34: $-\log(P)$ cumulative distribution for the ideal bin size for different energy cuts obtained with the full set of analyzed data.
Figure 6.35: Upper limit maps of muon fluxes from point sources for different energy cuts.
\( (\psi_s = \frac{1}{3}^\circ \text{ but for graphical purposes we draw only one entry per square of } 1^\circ \times 1^\circ). \)
6.2.2 Special sources

For the analysis of the selected point sources reported in table 6.1 we show the results obtained with the same cuts and resolution $p_s$ as used for the step 2 of the sky survey. In table 6.19 we list the results of the analysis of the selected point sources performed for both years of data-taking for the different applied energy cuts. The entries include the number of measured events and of background events together with $-\log(P)$ (the probability $P$ is defined in equation 6.25). The corresponding upper muon flux limit from the concerned source, which is calculated through equation 6.52, is reported in the last column.

In table 6.20 the same informations are listed for the yearly search.

No significant signal is found. The highest excess ($-\log(P) = 1.96$) is compatible with the number of entries in the two mentioned tables which is 120.

The muon fluxes expected from the $\gamma$-induced showers originating from the brightest known continuous $\gamma$-sources are more than two order of magnitudes below the upper limits obtained here.

Also in the daily search and in the analysis of the “stable periods” no significant signal is seen. The outcome of the daily search is shown by the $-\log(P)$-distributions of figure 6.36 for the four energy cuts applied. The results for each selected sources and each analyzed day interval is included in the plotted distributions.

6.2.3 Cyg X-3 versus orbital phase

Finally we report the result about the muon observed from the direction of Cyg X-3 as a function of its orbital phase. In figure 6.37 we show the plots of $-\log(P)$ (the probability $P$ is defined in equation 6.25) versus the orbital phase of Cyg X-3 for the different energy cuts applied in the analysis. Figure 6.38 shows, for the same energy cuts, the upper limit muon flux from Cyg X-3 calculated through equation 6.52, again as a function of its orbital phase.

The most significative excess ($-\log(P) = 2.5$) is noted in the bin corresponding to the phase interval 0.4-0.45 at 30 GeV. At 20 GeV the highest excess is seen in the same phase interval, but this is normal since the two measurements at 20 and 30 GeV are strongly correlated. Taking into account the total number of phase intervals analyzed (4×20), and that there is also an excess with $-\log(P) = 2.3$ at 100 GeV in another phase interval (0-0.05), we don’t think that the excess at 30 GeV can be taken seriously. We conclude therefore that no indications for periodic emission with orbital frequency has been found.
Figure 6.36: Cumulative distribution of $-\log(P)$ for daily analysis of the 10 selected point sources. There is one entry per source and per analyzed 1 day-interval. The two highest excesses at 50 GeV are from Cyg X-3 ($-\log(P) = 4.81$, 20 August 1999) and Geminga ($-\log(P) = 4.31$, 14-15 August 1999).
### Events measured:

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<tr>
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### Energy cut: 100 GeV Window size 0.9° x 0.9°

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Table 6.19: Muon flux upper limits and −log(P) results for selected sources obtained with the full set of analyzed data.
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<tr>
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<td>61089 61289 0.10 0.64(10^{-8})</td>
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<tr>
<td>Crab</td>
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Table 6.20: Muon flux upper limits and \(-\log(P)\) results for selected sources obtained in 1999 and 2000.
Figure 6.37: $-\log(P)$ versus orbital phase for Cyg X-3 for different energy cuts.
\begin{figure}
\centering
\includegraphics[width=\textwidth]{mu-flux_upper_limit_cyg_x_3_CL_90_percent}
\caption{Upper limits of muon fluxes from Cyg X-3 versus orbital phase for different energy cuts.}
\end{figure}
6.2.4 Comparison with results of other experiments

For comparison of our sensitivity with respect to recent results of other muon underground experiments, we select the paper of the MACRO collaboration [118], in which results of a sky survey are also reported. In this paper upper muon flux limits of the order of $10^{-12}$ cm$^{-2}$s$^{-1}$ are given.\(^9\)

Taking into account that the $\gamma$-induced muon flux from point sources at 20 GeV (where we find upper flux limits down to $\sim 2 \cdot 10^{-9}$ cm$^{-2}$s$^{-1}$) is 3 order of magnitude larger than at 1000 GeV (see table 2.4), we conclude that our relative sensitivity is similar to the one of the MACRO experiment. At the energy cuts analyzed in this work, measurements with the sensitivity of L3+C have not been obtained till now.

The muon flux upper limits that we obtain are more than 2 order of magnitudes above the fluxes expected for $\gamma$-induced muons in the direction of the brightest $\gamma$-sources (compare table 2.4 with table 6.19). However, signal of the strength of the one reported in the 80th by the Soudan-1 collaboration [7] from Cyg X-3 (see section 2.6.3) should have been observed with our measurements. Unfortunately, it seems that in the last years Cyg X-3 is less active and no recent observation of signals from Cyg X-3 has been reported by underground muon experiments.

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\(^9\)The MACRO upper muon flux limit is calculated averaging the flux along the part of the trajectory of the point source included in the field of view of the detector. Indeed in our analysis we report the flux to the vertical direction (see section 6.1.4).
Chapter 7

Conclusion and outlook

Anisotropy of cosmic rays

The anisotropy of primary cosmic rays is studied with the L3+C data applying two different methods. Both methods are based on the idea that a fixed detector scans the sky in the right ascension direction thanks to the Earth’s rotation. The first method (conventional one) looks for time variations of the muon detection rates with a period of one day, regardless of the arrival direction of the muons. The second method takes into account also the directional information. A harmonical analysis of the result is performed and the first three harmonics of the anisotropy function at the sideral, solar, anti-sideral and extended sideral frequencies are extracted. A careful data selection allows to reach sensitivities of $10^{-4}$.

No significant deviation from isotropy is observed at the sideral frequency. The results obtained with a muon energy cut of 100 GeV are compatible with other experimental results. Our measurement for a 20 GeV energy cut of the first harmonic doesn’t indeed follow the prediction of the recent NFJ model presented in section 2.5.5.

A significant deviation from isotropy is indeed found for the second harmonics at solar frequency at the lowest energy thresholds (20 and 30 GeV). In this case the structure of the anisotropy function is similar in shape to what has been reported by the GRAND experiment at 0.1 GeV threshold (see figure 5.20), but with a smaller amplitude.

In further analyses, including the tidal frequency and the anisotropy of multi-muons, no significant signal is found.

Comparing L3+C with other experiments, the main handicap is that the data-taking period is too short and is not lasting the whole year. This leads to a spectral resolution which is not able to distinguish completely the main frequencies of interest (solar and sideral). Thus prudence is needed for the interpretation of the results.

Search for point sources

An analysis method for the search of point sources with L3+C data, which includes a precise background calculation, is successfully developed and applied in this work.

A survey of the whole visible sky is performed on 4 time scales (1 day, 1 “stable
Conclusion and outlook

period”, 1 year and 2 years) and with 4 different muon energy cuts (20 GeV, 30 GeV, 50 GeV and 100 GeV). The 3 most relevant excesses are analyzed carefully. One of these excesses is obtained in the yearly search the other two are found in the daily search. Even if the yearly excess keeps some significance and one daily excess repeats two more times on two neighbouring days (with a probability of being statistical fluctuations of the background of the order of $10^{-3}$ each), they are hardly believed to be caused by true signals: it is noted in fact, that a small shift of the solid angle window and of the interval in which the excesses are found, quickly reduces the significance of the signal. The same consideration can be made for a small change of the size of both, angular bins and time intervals. In addition the size of the angular bin, where the 3 excesses are found, is much smaller than the size which we expect to be more sensitive (c.f. section 6.1.3). Thus, no real evidence for signals from point sources can be claimed.

10 selected known gamma sources are analyzed separately with the same energy cuts and time scales as for the sky survey. For Cyg X-3, a particular attention is given for periodical emission at its orbital frequency. No indication for a signal from these particular sources and for periodical signals from Cyg X-3 are found.

The upper muon flux limits that are set with our analysis are more than 2 orders of magnitude higher than what is expected from the direction of the brightest known continuous γ-sources (c.f. section 2.6.4). However signals from Cyg X-3, as the one reported in the 1980th by the Soudan-1 collaboration [7] (see section 2.6.3), could have been observed by the L3+C experiment. (It is a particular feature of this kind of point sources to have a very irregular activity.)

The relative sensitivity of L3+C to muon fluxes from point sources is similar to the one of other important underground muon experiment such as MACRO (see section 6.2.4), which has however a much higher energy threshold. At the muon energies analyzed in this work, the L3+C muon detector can claim to have the highest sensitivities.

Further analysis concerning the point sources could profit of the increased statistics available with the new version of the L3+C reconstruction program, which is able to reconstruct muon tracks, which cross two adjacent muon chamber octants. Such a version is currently tested and improved. Additional clarifications could then be given to what is observed in this work.

Further studies may also try to look for signals from point sources on other time scales than the one analyzed in this work. Time scales of few hours or even less, may hide some burst signals. Good luck to the L3+C colleagues continuing this analysis!
Appendix A

Monte Carlo simulation

A.1 Simulation of extensive air showers

CORSIKA [81],[132] is nowadays one of the most used and known Monte Carlo simulation program of extensive air showers. The program is able to track 50 of the most important elementary particles through the atmosphere. Their interactions, decay, annihilation and secondary particle production in the air can be fully simulated, according to the current experimental knowledge and to the most important theoretical models. Alternatively, for some CPU time consuming processes, there are options which allow to treat the shower development with an analytical approach.

A.1.1 Proton induced showers

Simulation results of proton induced showers made with the CORSIKA program have been presented in [133]. The primary differential spectral index (see equation 2.2) has been set to $\gamma = 2.7$ and the default options have been used for the simulation. This means that below laboratory energies of 80 GeV the hadronic interactions have been treated by the GHEISHA routines [134], otherwise for higher energies the VENUS code [135] has been used. The range of the zenith angle $\theta$ of the simulated primaries has been included in the interval $0.34 \leq \cos \theta \leq 1$.

The resulting momentum spectrum for muons with $\cos \theta > 0.4$ and momentum ranging in the interval $2 \text{ GeV/c} < p_\mu \lesssim 10^3 \text{ GeV/c}$ has been parameterized at the L3+C altitude through (GeV/c units)

$$\frac{dN}{dp_\mu} \sim f_1(p_\mu) = \frac{1}{p_\mu^3} \left[1.922 - 10.17\ell + 18.75\ell^2 - 8.387\ell^3 + 1.107\ell^4\right] \quad (A.1)$$

$$\ell \equiv \log_{10}(p_\mu) \quad (A.2)$$

The distribution in the zenith angle $\theta$ of the arrival direction of the muon could be well described by an affine function in $\cos \theta$.

$$\frac{dN}{d\cos \theta} \sim f_2(\cos \theta; p_\mu) = 1 + a(p_\mu) (1 - \cos \theta) \quad (A.3)$$
where \( a(p_\mu) \) is a momentum dependent coefficient, which has been parameterized as

\[
a(p_\mu) \approx -1.903 + 0.1434 \ln(p_\mu) + 0.0145 \ln(p_\mu)^2. \tag{A.4}
\]

To extend the range of validity for momenta ranging up to \( 10^4 \) GeV/c, the parameterization of \( p_\mu^3 \frac{dN}{dp_\mu} \) has been extended to a polynom of 8th degree [136], so that equation (A.1) is replaced by

\[
\frac{dN}{dp_\mu} = \tilde{f}_1(p_\mu) \equiv \frac{1}{p_\mu^3} \left[-1.472 + 14.42 \cdot \ell - 47.92 \cdot \ell^2 + 80.94 \cdot \ell^3 - 62.84 \cdot \ell^4 + 25.95 \cdot \ell^5 - 6.014 \cdot \ell^6 + 0.7456 \cdot \ell^7 - 0.0387 \cdot \ell^8\right]. \tag{A.5}
\]

Most of the muons detected by L3+C are produced in proton induced showers. Taking further into account that, in the momentum range of interest, the shape of the zenithal and momentum muon distributions of the muon generated by heavy primary nuclei are very similar [133], equations A.5 and A.3 represent a good description of the expected muon background distributions at ground level.

### A.1.2 \( \gamma \)-induced showers

We simulate \( \gamma \)-induced showers with primary energies ranging from 50 GeV to 100 TeV with the CORSIKA program. The main goals are to estimate the angular spread of the muons with respect to the primary direction and to study the number of produced secondary muons above a given energy threshold. A comparison with the results of the simulation of the proton induced shower is also presented.

The primary differential spectrum used in the simulation is described by a power law (equation 2.2) with spectral index \( \gamma = 2 \), according to most observed \( \gamma \)-source spectra. We select the EGS4 option, which enables a full Monte Carlo simulation based on the EGS4 software package [137]. The position, momentum and charge of the muons reaching an altitude of 450 meters above sea level are stored (ground level above L3).

With production (a), we simulate 4 million vertical showers (\( \theta = 0^\circ \)). The result of this simulation can be found in section 2.6.4, where it is used to estimate the flux of vertical secondary muon above a given energy threshold produced by photons originating from \( \gamma \)-sources and to compare it with the theoretical expectations.

In production (b) we simulate 44 million showers generated by \( \gamma \)-particles with a variable zenith angle \( \theta \) ranging from \( 0^\circ \) to \( 60^\circ \), with the main purpose to study the angular spread of the muons with respect to the primary direction. This gives us the possibility to discuss the ideal bin size for the search of point sources (see section 6.1.3). The statistics of the production (b) is shown in table A.1.

The average angle \( \angle_{ps} \) between the primary direction and the muons reaching the ground level at the experimental area is reported in table A.2 as a function of the lower muon energy threshold \( E_{\mu,\text{cut}} \). In figure A.1 we show the \( \angle_{ps} \) distribution for \( E_{\mu,\text{cut}} = 30 \) GeV. Comparing the result with figures 4.28-4.30 in section 4.4.3, it can be noted that the main uncertainty in the primary direction obtained through the measurement of muons with the L3+C detector, is dominated by the uncertainty in the measurement of the muon
Table A.1: Statistics of the Monte Carlo production (b) including the number $N_\gamma$ of generated $\gamma$-showers and the number $N_\mu$ of muons produced with energy larger than $E_\mu;\text{cut}$. The ratio $\frac{N_\mu}{N_\gamma}$ is reported in the last column.

<table>
<thead>
<tr>
<th>$E_\mu;\text{cut}$ [GeV]</th>
<th>$N_\gamma$</th>
<th>$N_\mu \geq 50$ GeV</th>
<th>$N_\mu$</th>
<th>$\frac{N_\mu}{N_\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>44$\times$10$^6$</td>
<td>344835</td>
<td>7.84$\times$10$^{-3}$</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
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<td>2.42$\times$10$^{-3}$</td>
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</tr>
<tr>
<td>100</td>
<td>44$\times$10$^6$</td>
<td>37118</td>
<td>8.44$\times$10$^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure A.1: Distribution of the angular difference $\alpha_\mu$ between the direction of the muons at ground level and the direction of the primary $\gamma$-particles obtained from the Monte Carlo simulation (production (b)) for muons with energy larger than 30 GeV.

direction itself (mainly due to the multiple scattering in the molasse) and not by the angle between the primary particle direction and the muon direction at ground level.

Comparison of the zenithal distribution of the muons produced in the simulation of $\gamma$-induced showers and of proton-induced showers

Looking at the zenithal distribution of the muons produced in the simulated $\gamma$-induced showers (production (b)), we notice that it can again be well approximated through an affine function in $\cos \theta$ (see example in figure A.2), as for proton-induced showers.

If we describe the zenithal distribution with equation A.3 we find that the parameter
Table A.2: Average angular difference $\alpha_{ps}$ between the direction of the muons and the primary direction resulting from the Monte Carlo simulation (production (b)) for 4 different lower energy cuts $E_{\mu,\text{cut}}$.

<table>
<thead>
<tr>
<th>$E_{\mu,\text{cut}}$ [GeV]</th>
<th>$\alpha_{ps}$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.66</td>
</tr>
<tr>
<td>30</td>
<td>0.48</td>
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<tr>
<td>50</td>
<td>0.31</td>
</tr>
<tr>
<td>100</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Figure A.2: Distribution in $\cos \theta$ for the muons of 50 GeV simulated in $\gamma$-induced showers, where $\theta$ is the zenith angle of their arrival direction. The result is fitted with the function $f(\cos \theta) = m(\cos \theta) + k$.

$a(p_\mu)$ can be approximated through

$$a(p_\mu) = -2.431 + 0.424 \ln(p_\mu) - 0.0241(\ln(p_\mu))^2.$$  \hspace{1cm} \text{(A.6)}

In table A.3 we compare the parameter $a(p_\mu)$ that we obtain for proton-induced showers and for $\gamma$-induced showers for 4 different momenta. The results are very similar for both kind of showers.

**Comparison of the momentum spectrum of muons produced in the simulation of $\gamma$-induced showers and of proton-induced showers.**

The shape of the muon momentum spectra of the proton-induced showers and of the $\gamma$-induced showers look very different (figure A.3). This is expected to be caused mainly
A.2 Scheme for the detector simulation

A good detector simulation of the L3+C muon spectrometer is needed to study the acceptance of the detector, in order to be able to calculate the absolute muon flux. In the physics analysis presented in this work the precise knowledge of the absolute flux is of secondary importance and we use the Monte Carlo prediction mainly to understand the performance of the L3+C muon detector (see chapter 4 and section 6.1.3).

<table>
<thead>
<tr>
<th>$p_\mu$ [GeV/c]</th>
<th>$a(p_\mu)$ for proton showers</th>
<th>$a(p_\mu)$ for $\gamma$-showers</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-1.34</td>
<td>-1.38</td>
</tr>
<tr>
<td>30</td>
<td>-1.25</td>
<td>-1.27</td>
</tr>
<tr>
<td>50</td>
<td>-1.12</td>
<td>-1.14</td>
</tr>
<tr>
<td>100</td>
<td>-0.94</td>
<td>-0.99</td>
</tr>
</tbody>
</table>

Table A.3: Coefficient $a(p_\mu)$ which appears in equation (A.3) to describe the angular distribution of muons in proton induced showers and $\gamma$-induced showers.

from the fact that the primary spectral index used to simulate the two kinds of showers is different.

Figure A.3: Momentum spectra for muons with a zenith angle $\theta < 60^\circ$ generated by primary photons with spectral index $\gamma = 2$ as obtained in the Monte Carlo simulation (dashed line) and for muons produced in proton-induced showers with $\gamma = 2.7$ according to equations A.3 and A.5 (continuous line).
The simulation of the L3+C muon detector is performed in four steps.

1. **Generation:** Monte Carlo muon events are generated at ground level and their position, momentum, charge and direction is stored (eventually together with other auxiliary informations as the primary particle type and energy). The generators can be divided into two categories. To the first category belong the generators which perform a **full shower simulation** (e.g. CORSIKA). The second category includes **fast Monte Carlo generators** which generate muons according to given distributions. An example of this kind of generator is L3CGEN [133], which applies the angular and momentum distributions described by equations A.3 and A.5. This generator will be treated in more details in section A.3.

2. **Tracking in the molasse:** The tracking through the molasse is done with the help of the GEANT software package [138]. All main physics processes such as the multiple scattering, secondary particle production (including $\delta$-rays), pair production, energy loss and decay are simulated. The particles are tracked up to a certain energy threshold. The geometry (including the access shafts) and the chemical composition [95] of the material above the L3 detector are taken into account (figure A.4).

3. **Simulation of the detector response:**[133],[139] The hits produced by the generated particle tracks in the different active detectors are simulated. One can choose between an ideal simulation, in which the detector is fully efficient or a realistic simulation, in which the “dead cells” (or the “ill cells”, see section 4.2.2) of the muon chambers are taken into account. In addition some noise hits are generated randomly in the t0-scintillators in the realistic simulation.

   For the ideal simulation a trigger simulation is available.
4. **Reconstruction:** The simulated Monte Carlo events are then reconstructed and backtracked to the ground level in the same way as the data events. (see section 3.5).

**A.3 The ground level generator L3CGEN**

The parameterization of the CORSIKA simulation results for proton-induced showers as described in section A.1.1, are used in the standard single muon generator used for the L3+C simulation, which is called L3CGEN [133]. In the next paragraphs we describe how the charge, momentum, direction and position are assigned to the Monte Carlo muon events in this generator.

**Charge:** The ratio of the number of positive muons to the number of negative muons is known to be about 1.3. No strong correlation between the charge and the momentum or the angle is present. Therefore the charge is assigned randomly to the Monte Carlo muon events, so that the chance probability to be positive is 1.3 times larger than to be negative.

**Momentum:** The application of the distribution described by the function \( \tilde{f}_1(p_\mu) \) (defined in equation A.5) to assign a momentum to the muon events, can be applied directly only if the simulated zenith-angle range is given by \( \cos \theta > 0.4 \). Otherwise a correction which takes into account the angular distribution defined by the function \( f_2(\cos \theta; p_\mu) \) defined in equation A.3 should be applied. The vertical spectrum distribution is given by

\[
\left. \frac{dN}{dp_\mu} \right|_{\text{vert}} \sim f_c(p_\mu) = \frac{f_1(p_\mu)}{\int_{0.4}^1 d \cos \theta \ f_2(\cos \theta; p_\mu)}. \tag{A.7}
\]

The spectrum of the muons with a zenith angle ranging from \( \theta_{\text{min}} \) to \( \theta_{\text{max}} \) is given by

\[
\frac{dN}{dp_\mu} \sim f_c(p_\mu) \cdot \int_{\cos \theta_{\text{min}}}^{\cos \theta_{\text{max}}} d \cos \theta \ f_2(\cos \theta; p_\mu). \tag{A.8}
\]

The momentum is assigned randomly according to the probability distribution given by equation (A.8) in a given momentum range \( p_{\text{min}} < p_\mu < p_{\text{max}} \).

**Direction:** After the assignment of the momentum, the zenith angle \( \theta \) of the muon direction can be assigned according to the probability distribution given by the function \( f_2(\cos \theta; p_\mu) \). The azimuth angle \( \phi \) is indeed assigned according to a flat distribution.
Position - Choice of the surface of generation: A position $P$ at the ground level needs to be assigned to the generated muons. Ideally one would generate homogeneously the muons at ground level inside a circle of radius $r$ around the ground level point $C$ above the center $O$ of the L3 detector (figure A.5). Since the ground level is at a height of $h \cong 44.8$ m above the center of the L3 detector, muons with zenith angle $\theta = 60^\circ$ can reach the L3 detector even if they are at a distance of $\sim 80$m from the point $C$. The radius $r$ should be therefore chosen to be larger than 80 meters, if muons up to this zenith angle are generated. This is very inefficient with respect to computing time since most of the generated muons don’t have the chance to hit the L3 detector. To spare computing time one could apply a preselection, processing the detector simulation only for those tracks having the chance of hitting the L3+C muon detector. This can be achieved considering the projection $Q$ of the points $P$ in the direction $\vec{d} = (-\theta, -\phi)$ assigned to the muon onto the horizontal plane $\alpha$ which includes the center $O$ of the L3 detector (figure A.5). The preselection accepts only the set $T$ of muon tracks, whose projection $Q$ is inside a given surface $S$ around the L3 detector. The same set $T$ can be however easily produced generating homogeneously the points $Q$ on the surface $S$ and projecting them on a straight line in the direction $\vec{d} = (\theta, \phi)$ to the ground level. The position of the points $P$ which we find with this projection is then assigned to the generated muons. The two method to generated the set $T$ are equivalent, but the second one is more straightforward and it is therefore used in the L3CGEN program.

Figure A.5: Schematic figure showing the position assignment to the simulated muon events. The procedure adopted in the L3CGEN program is to generate homogeneously the points $Q$ on the surface $S$ and to project them to the ground level following the arrival direction $\vec{d}$ assigned to the simulated muon.
A.3 The ground level generator L3CGEN

The surface $S$ should be chosen to be as small as possible in order to spare computing time. However one should avoid to exclude too many tracks that could be detected by the muon detector. Therefore a study to determine the surface $S$ is performed. In this study we use a surface $S_1$ (later called $S_1$) much larger than the final surface $S_2$ which is used for the extensive Monte Carlo production. $S_1$ is composed by the set of points whose coordinates $(x', 0, z')$ are included in the intervals $(-12m < x' < 12m$ and $-16m < z' < 16m)$ ($x'$, $z'$ are horizontal coordinates defined in section 3.2.2). We perform a full detector simulation with events generated on the surface $S_1$, generating muons with a momentum larger than 20 GeV/c. We determine the limits $x_{\text{max}}$ and $z_{\text{max}}$ of the surface $S_2$.

\[
|x'| < x_{\text{max}} \\
|z'| < z_{\text{max}}
\]  \hspace{1cm} (A.9)

allowing that in both component $x'$ and $z'$ only $1/00$ of the tracks, which are reconstructed in the detector simulation, are generated outside the surface $S_2$. In total when performing the generation with the surface $S_2$ we miss therefore about $2/00$ of the tracks. Since a strong zenithal dependence is noticed on the distribution of the generated events on surface $S_1$ which are reconstructed (c.f. figure A.6), it is decided to fix the limits $x_{\text{max}}$ and $z_{\text{max}}$ of the surface $S_2$ dependent from the zenith angle $\theta$. This is very important when the angular dependence of the muon flux is studied. The limit of the surface $S_2$ are parameterized through $^1$

\[
|x'| < x_{\text{max}}(\theta) = k(21.258 - 33.15 \cos \theta + 16.15 \cos^2 \theta) m \\
|z'| < z_{\text{max}}(\theta) = k(23.532 - 27.212 \cos \theta + 10.521 \cos^2 \theta) m
\]  \hspace{1cm} (A.10)

Calculation of the livetime.

A calculation of the livetime to be assigned to a set of $N_{\text{gen}}$ generated events is implemented in the program L3CGEN. This is very helpful to study the absolute fluxes or when sets of generated events with different momenta have to be combined together.

The calculation is based on the reference flux of vertical muons at 100 GeV corresponding to

\[
\left. \frac{d^4 N}{dp \, d\Omega \, dt \, dA} \right|_{p=100 \text{ GeV/c}, \theta=0} = 2.8 \cdot 10^{-3} \, \text{m}^{-2} \cdot \text{sr}^{-1} \cdot \text{GeV}^{-1} \cdot \text{s}^{-1}.
\]  \hspace{1cm} (A.11)

\footnote{When changing version of the simulation and reconstruction program, the intervals (A.10) may need to be changed slightly. For safety we set $k = 1.2$ for the Monte Carlo production performed with the most recent versions of the simulation program.}
Figure A.6: Points on the surface $S$ which are reconstructed in the Monte Carlo simulation. The left plot refers to very inclined muons with zenith angle $\theta$ included in the interval $0.4 < \cos \theta < 0.5$. The right plot is made with almost vertical muons $\cos \theta > 0.9$. In the latter plot it is noticed that the points are more concentrated in the central region, as expected.

Taking into account equations (A.3) and (A.7), the livetime is then calculated through

$$t_l = \frac{1}{\int d^4N dp \cos(\theta) \, d\Omega \, dA} N_{\text{gen}} \frac{f_2(1; 100\text{GeV}/c)}{\int_{\cos \theta_{\text{max}}}^{\cos \theta_{\text{min}}} d(\cos \theta) \, f_2(\cos \theta; 100\text{GeV}/c)} \cdot \frac{f_v(100\text{GeV}/c)}{\int_{p_{\text{min}}}^{p_{\text{max}}} dp \, f_v(p)}$$

(A.12)

where

$$\left. \frac{d^4N}{dp \, d\cos(\theta) \, d\Omega \, dA} \right|_{p=100\text{GeV}/c, \theta=0} = 2\pi \left. \frac{d^4N}{dp \, d\Omega \, dA} \right|_{p=100\text{GeV}/c, \theta=0}.$$  

(A.13)

$A_{\text{eff}}$ is the effective area of generation which is calculated as

$$A_{\text{eff}} = \frac{\int_{p_{\text{min}}}^{p_{\text{max}}} dp \int_{\cos \theta_{\text{min}}}^{\cos \theta_{\text{max}}} d(\cos \theta) \, f_2(\cos \theta; p) \, f_v(p)}{\int_{p_{\text{min}}}^{p_{\text{max}}} dp \int_{\cos \theta_{\text{min}}}^{\cos \theta_{\text{max}}} d(\cos \theta) \, f_2(\cos \theta; p) \, f_v(p) \, \frac{1}{A(\theta)}}$$

(A.14)

where $A(\theta)$ is the area of the surface of generation $S_2$ which can be calculated as (c.f. equation A.10)

$$A(\theta) = 4 \, x_{\text{max}}(\theta) \cdot z_{\text{max}}(\theta).$$

(A.15)
A.4 Comparison of the angular distributions obtained with the data and with the Monte Carlo simulation

\(1/A_{\text{eff}}\) corresponds to the average value of \(1/A(\theta)\). \(A_{\text{eff}}\) is therefore calculated inside the L3CGEN program through

\[
\frac{1}{A_{\text{eff}}} = \frac{1}{N_{\text{gen}}} \sum_{\text{all MC Events}} \frac{1}{A(\theta)}. \tag{A.16}
\]

In table A.4 we report the vertical muon fluxes above four given energy thresholds calculated according to equation A.12.

<table>
<thead>
<tr>
<th>Energy threshold [GeV]</th>
<th>(\mu)-Flux ([\text{cm}^{-2} \text{sr}^{-1} \text{s}^{-1}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>(3.36 \cdot 10^{-4})</td>
</tr>
<tr>
<td>30</td>
<td>(1.57 \cdot 10^{-4})</td>
</tr>
<tr>
<td>50</td>
<td>(5.60 \cdot 10^{-5})</td>
</tr>
<tr>
<td>100</td>
<td>(1.23 \cdot 10^{-5})</td>
</tr>
</tbody>
</table>

Table A.4: Vertical muon fluxes for given energy thresholds according to the Monte Carlo generator L3CGEN.

A.4 Comparison of the angular distributions obtained with the data and with the Monte Carlo simulation

Finally we present some plots in which we show the difference between the angular distributions obtained in the data and in the Monte Carlo simulation. The left plot of figure A.7 represents the difference in percent between the impact point distribution \(I\) (defined in section 4.3.2) obtained from the year 2000 data and the corresponding distribution obtained in the real Monte Carlo simulation. The same is shown for the direction distribution \(A\) (see section 4.3.2) in the right plot. Black points represent an excess of events in the data, while white points stay for an excess in the Monte Carlo events. Event selection B is applied and the interval of data-taking are chosen according to selection 1.a (see section 4.1). It is noted that differences up to 20% are present. The main reason which can explain such large differences is that the real detector simulation used in this analysis (version 404) does not take into account all the inefficiencies of the detector with sufficient precision. In the real simulation of the data of the year 2000, it is discovered that some dead cells are not taken into account in the Monte Carlo (see figure A.8). In addition, in the impact point distribution of the data one can also notice an excess of tracks going through the holes between the scintillator modules. This could be an effect due to the noise in the scintillators. Some inefficiencies in the scintillators, which can be noticed in figure 4.10, seems also to be noticeable in figure A.7. The L3+C collaboration is trying to do the best to take into account all these effects. However, it is clear that it is not an
Figure A.7: Difference of the impact point distribution (left) and of the direction distribution (right) between real data events and simulated events during the year 2000. (Selection B+1.a, Energy cut 30 GeV).

Figure A.8: The plot represents the distribution along the z-coordinate (L3 coordinates) of the intercept of the muon tracks with the chambers MI in octant 3. The large discrepancy is due to dead cells of Z-chambers which are not taken into account in real Monte Carlo simulation of the year 2000.

easy task. Precisions below the percent level should be out of reach. The Monte Carlo simulation can therefore not be applied for the anisotropy analysis (see chapter 5).
Appendix B

Yearly results of the anisotropy analysis

<table>
<thead>
<tr>
<th>Energy cut [GeV]</th>
<th>Year</th>
<th>Harmonic</th>
<th>Method A</th>
<th>Method B</th>
<th>σ_{stat} [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\xi_k$ [%]</td>
<td>$\varphi_k$ [°]</td>
<td>$\xi_k$ [%]</td>
</tr>
<tr>
<td>20</td>
<td>1999</td>
<td>1st</td>
<td>0.10</td>
<td>15.24</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2nd</td>
<td>0.11</td>
<td>2.25</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3rd</td>
<td>0.24</td>
<td>7.62</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>1st</td>
<td>0.17</td>
<td>9.54</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
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<td>0.24</td>
<td>6.35</td>
<td>0.11</td>
</tr>
<tr>
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<td></td>
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</tr>
<tr>
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<td>2.36</td>
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</tr>
<tr>
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<td>0.31</td>
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</tr>
<tr>
<td></td>
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<td>1st</td>
<td>0.13</td>
<td>8.05</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>0.26</td>
<td>6.23</td>
<td>0.13</td>
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<td>4.44</td>
<td>0.26</td>
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<tr>
<td>50</td>
<td>1999</td>
<td>1st</td>
<td>0.26</td>
<td>16.85</td>
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</tr>
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<td>6.96</td>
<td>0.14</td>
</tr>
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<td></td>
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<td>1st</td>
<td>0.10</td>
<td>2.04</td>
<td>0.10</td>
</tr>
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<td></td>
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<td>0.43</td>
<td>6.28</td>
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</tr>
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<td>3.57</td>
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<td>17.63</td>
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<td>7.91</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>1st</td>
<td>0.60</td>
<td>1.66</td>
<td>0.67</td>
</tr>
<tr>
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<td></td>
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<td>0.73</td>
<td>6.94</td>
<td>0.02</td>
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<td>3rd</td>
<td>0.27</td>
<td>4.33</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table B.1: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the yearly spectral analysis of the anisotropy function $\delta_{\text{dir}}(\alpha)$ for the sideral frequency for different energy cuts. In the last column we report the statistical error $\sigma_{\text{stat}}$. 
### Table B.2: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the yearly spectral analysis of the anisotropy function $\delta^\text{dir}(\hat{n})$ for the solar frequency for different energy cuts. In the last column we report the statistical error $\sigma_{\text{stat}}$. 

<table>
<thead>
<tr>
<th>Energy cut [GeV]</th>
<th>Year</th>
<th>Harmonic</th>
<th>$k$-th</th>
<th>Method A</th>
<th>Method B</th>
</tr>
</thead>
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<tr>
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<td></td>
<td></td>
<td>$\xi_k$</td>
<td>$\varphi_k$</td>
<td>$\xi_k$</td>
</tr>
<tr>
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Table B.3: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the yearly spectral analysis of the anisotropy function $\hat{\delta}^{\text{dir}}(\vec{\alpha})$ for the anti-sideral frequency for different energy cuts. In the last column we report the statistical error $\sigma_{\text{stat}}$. 
Table B.4: Amplitudes $\xi_k$ and phases $\varphi_k$ of the first three harmonics obtained from the yearly spectral analysis of the anisotropy function $\delta_{\text{dir}}(\hat{n})$ for the extended-sideral frequency for different energy cuts. In the last column we report the statistical error $\sigma_{\text{stat}}$. 

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Figure B.1: Dial plots showing the yearly results of the first harmonic of the anisotropy function $\delta^{\text{dir}}(\alpha)$ at the sideral frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.

Figure B.2: Dial plots showing the yearly results of the second harmonic of the anisotropy function $\delta^{\text{dir}}(\alpha)$ at the sideral frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.
Figure B.3: Dial plots showing the yearly results of the third harmonic of the anisotropy function $\delta^{\text{dir}}(\alpha)$ at the sideral frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.

Figure B.4: Dial plots showing the yearly results of the first harmonic of the anisotropy function $\tilde{\delta}^{\text{dir}}(\tilde{\alpha})$ at the solar frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.
Figure B.5: Dial plots showing the yearly results of the second harmonic of the anisotropy function $\tilde{\delta}^{\text{dir}}(\tilde{\alpha})$ at the solar frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.

Figure B.6: Dial plots showing the yearly results of the third harmonic of the anisotropy function $\delta^{\text{dir}}(\tilde{\alpha})$ at the solar frequency for 4 different energy cuts and for methods A (left) and B (right). The circles represent a 68.5% confidence level region.
Bibliography


[100] Unger M., Private Communication.
[102] Courtesy of Pedro Ladron.

[105] Courtesy of Michael Unger.


[110] Courtesy of Jean François Parriaud and Michel Chemarin.


I would like to thank all my colleagues for the nice time spent together and for their help, support and friendship.

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Curriculum Vitæ

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           of the Institute for Particle Physics at ETH Zurich, under the
           supervision of Prof. Dr. Felicitas Pauss, Prof. Dr. Hans Hofer
           and Dr. Pierre Le Coulter.
           Member of the L3 collaboration and of the L3+C group.