Doctoral Thesis

Dynamics of jet-like flow patterns in the neighbourhood of storms, the tropopause and orography

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Dynamics of jet-like flow patterns in the neighbourhood of storms, the tropopause and orography

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presented by
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Abstract

State-of-the-art research in atmospheric science makes use of various methods and strategies. New insight is gained by developing field basic theoretical principles and concepts, by performing model simulations and by deploying field measurements. In this thesis these approaches are harnessed to analyse some atmospheric flow phenomena featuring remarkable wind fields.

Three types of flow settings are considered. First the mesoscale atmospheric response to orographic (sic. mountain-) induced perturbations. A broad spectrum of phenomena and flow regimes has been explored and discussed extensively in previous studies. However little or no attention has been given to situations with a horizontal (sic. lateral) shear in the incident ambient flow field. Such a background shear can modify the character of the phenomena or generate new effects. Here a series of idealised experiments with lateral shear has been performed with the high-resolution, non-hydrostatic, compressible numerical model MC2 (mesoscale compressible community) in order to examine the influence of lateral shear. In particular consideration is given to the modification of the propagation properties of gravity waves and the influence of the shear upon the general character of the flow regime. It is shown that the presence of horizontal shear in the ambient flow influences significantly the flow regime over the orography (e.g. wave or balanced regime or both at the same time at different locations). For instance, the shear can reduce or even inhibit the vertical propagation of atmospheric waves and has – depending upon the Rossby Number – substantial upstream effects. Gravity waves, generated by a flow over an obstacle and propagating laterally, do not penetrate the shear zone, but are reflected and interfere with the incident wave. Depending on the location of the shear zone substantial interference effects can occur. An analytical description of the flow in the quasi-geostrophic limit is derived to complement the results of the nonlinear MC2 flow simulations.

Second consideration is given to the 1999 Christmas storm 'Lothar'. It was unusual with respect to its evolution over the Atlantic and to its extreme wind speeds. Full physics high resolution NWP model simulations and analyses based on the PV perspective and Lagrangian trajectory calculations revealed the distinctiveness of the event. Key factors for Lothar's development were its translation over the Atlantic as a "diabatic Rossby wave" and its mutual interaction with a deep tropopause fold off the European continent to eventually form a narrow PV tower throughout the whole troposphere and generate severe winds. A key question was: "How unusual was this event?" The synoptic setting that preceded the storm was essential for its development and was characterized by a long, rectilinear zonally-aligned jet stream over the North Atlantic. A systematic climatology of such jets aligned along the 2-pvu contour on isentropic surfaces has been computed for the northern hemisphere on the basis of the ECMWF (re)-analysis data sets for Januaries from 1986 to 99 and for December 1999. This climatology serves to establish the geographical distribution and the frequency
of the jets. It is shown that at low levels (potential temperature $\theta \approx 310$ K) the "jet frequency" has two distinctive maxima, over the western Pacific and the western Atlantic. At higher elevations ($\theta > \approx 330$ K) the high frequency region almost girdles the atmosphere at about $35^\circ$ N and is dissected only in the regions of high inter-annual variability in the eastern Pacific and the Atlantic Ocean. The distinctiveness of the 'Lothar' episode was confirmed and no constellation similar to the 'Lothar'-jet has been found. Finally an examination is undertaken of the link of the inter-annual variability of the "jet frequency" to the NAO teleconnection pattern.

Third attention is focussed on the issue that numerical models often overestimate the gravity wave activity over mountains. Evidence is given that a diurnally evolving boundary layer defines an "effective mountain height", and that the lower boundary for an overflow is therefore time-dependent and no longer stationary. Here a theoretical study based on linear wave theory is undertaken of such an unsteady configuration: an air-stream of uniform flow and stratification over two-dimensional diurnally-oscillating sinusoidal and bell-shaped terrain $h(x, t) = h_0(x)(1 - \alpha \sin \omega_0 t)$. The linear wave solutions for the perturbed flow comprise three parts: the corresponding steady state solution and up- and down-stream travelling waves. It is shown that vertical wave energy transport is possible for a wider range of Rossby Numbers than without oscillation and that the vertical energy flux is increased by the factor $1 + \alpha^2/2$. 

Abstract


Zusammenfassung


Die dritte Studie geht von der Beobachtung aus, dass numerische Modelle die Wellenaktivität über Gebirgen oft überschätzen. Es hat sich gezeigt, dass durch die täglichen Schwankungen der Grenzschicht eine "effektive Gebirgshöhe" definiert wird, und dass damit die Untergrenze für Luftpakete bei einer Überströmung nicht mehr stationär sondern zeitabhängig ist. Hier wurde mit linearer Wellentheorie eine Studie einer solchen zeitabhängigen unteren Randbedingung durchgeführt. Luft strömt mit uniformer Geschwindigkeit und Stabilität über eine zweidimensionale Orographie (die einmal sinusförmig ist und einmal einen isolierten Hügel darstellt), welche im Tagesrhythmus oszilliert $h(x,t) = h_0(x)(1 - \alpha \sin \omega_0 t)$. Die lineare Wellenlösung besteht aus drei Teilen: aus der Lösung des entsprechenden stationären Problems und je einer stromauf- und -abwärts wandernden Welle. Es wird gezeigt, dass der vertikale Energietransport dadurch für einen grösseren Bereich der Rossby Zahl möglich ist und quantitativ um den Faktor $1 + \alpha^2/2$ vergrössert wird.
Chapter 1

Introduction

1.1 Preamble

Weather, atmospheric flow phenomena and weather prediction are the subject of considerable social and scientific interest. They have a prominent role in the media and in discussions about climate change or severe weather events (wind storms or floods) and the associated economical damage. This thesis focuses on some aspects of atmospheric flow phenomena related to strong winds.

Atmospheric flow phenomena that are accompanied by strong winds and/or strong variations in wind strength are of considerable scientific and economic interest. The quest to understand such flow patterns prompts questions of the form "What is their cause?", "What determines their location, their strength and their frequency of occurrence?" or "What makes them distinctive or different?". For instance, in 1994 a catalogue of scientific objectives was compiled for the Mesoscale Alpine Programme (MAP), a huge international field campaign with the goal of resolving some of the most outstanding scientific and practical problems in the realm of weather in mountainous regions (MAP Design Proposal, Binder and Schär 1996). Two of five primary scientific objectives were directly related to strong winds\(^1\). Campaigns like MAP for mountain related issues have also been realised for other themes, e.g the Fronts and Atlantic Storm-Track Experiment (FASTEX) focused on atmospheric cyclone depressions forming in the North-Atlantic Ocean and reaching the west coast of Europe (Joly et al. 1997).

\(^{1}\)"... 2a: To improve the understanding and forecasting of the life-cycle of Foehn-related phenomena, including their three-dimensional structure and associated boundary layer processes. 2b: To improve the understanding of three-dimensional gravity wave breaking and associated wave drag in order to improve the parametrization of gravity wave drag effects in numerical weather prediction and climate models. ..."
1.1 Preamble

INTRODUCTION

Damages resulting from strong wind systems can curtail industrial and human activities. For example severe winds associated with the 1999 Christmas storm 'Lothar' in Central Europe caused enormous damage to buildings, forests and resulted in economic loss and even fatalities. Reinsurance companies are particularly interested in quantifying strong events and learning new insight about the dependencies of the losses on the maximum wind speed or on the track of a cyclone. In this context the questions are no longer “How?” or “Why?” but rather “How much?” . For example the reinsurance company Munich Re adjusted its interpolation scheme for loss expectations depending on wind speed after the analysis of the storm series of December 1999 (‘Anathol’, ‘Lothar’ and ‘Martin’) (Munich Re 2002).

As a side remark we mention that the ongoing discussion on global warming and climate change and its impact on the frequency and the strength of severe events makes it even a political issue.

In Europe such high-wind episodes are linked primarily to two classes of flow systems:

Extratropical cyclonic storms. Winter time, when the temperature difference between equatorial regions and the North Pole is largest, is the optimum season for potentially violent depressions to form in the western Atlantic mid-latitudes. When the large scale flow conditions are favourable whole series of storms can hit Europe as in February/March 1990 or in December 1999. The cyclonic (anti-clockwise) circulation around low pressure centres is for the most part geostrophic and can be very strong near fronts where the isobars kink.

Orographic effects. The Alps mark a major obstacle for air flows particularly for flows in the north-south direction. When air impinges on the Alpine ridge an upstream–downstream pressure gradient is established and downslope wind storms (Föhn) can occur. At the same time gravity waves are generated with a possible local flow overturning at high elevations. Föhn is a common phenomenon in the Alps. It is mostly accompanied by heavy precipitation on the upstream slope and by strong, gusty, warm and dry downslope winds. There have been several huge fires during Föhn events and villages have almost burnt down (e.g. 1911 Triesen, Liechtenstein or 1891 Meiringen, Switzerland). Furthermore Föhn winds often cause damage to buildings and forests. Gravity wave activity can be observed via quasi-stationary wave-like cloud patterns. Strong up- and downdrafts in regions with gravity waves can influence aircraft, and gravity wave breaking is referred to as ‘clear air turbulence’ in aviation.

Considerable progress has been gained in recent decades in the understand-
ing of such atmospheric flow phenomena. The approaches have been based upon:

**New concepts.** Theoretical, dynamically-based concepts of the flow have been derived and facilitate the description and the understanding of the flow. The $PV$ perspective makes use of the multifaceted properties of the potential vorticity ($PV$). $PV$ is a combination of the atmospheric stability and the vorticity of the flow and is conserved under adiabatic frictionless conditions. $PV$ generation gives evidence of diabatic processes such as condensation or radiation, and the invertibility principle states that from a given three-dimensional $PV$ distribution (and the surface temperature distribution) the wind field and all other flow variables can be derived. The $PV$ perspective (Hoskins et al. 1985) provides a more integral view of the flow than say analyses on isobaric surfaces. Lagrangian trajectories reveal the history (or the future) of air parcels and contain information about the origin of an air mass and the processes it undergoes within a certain time interval. Lagrangian trajectories provide both a very valuable complement to Eulerian fields and a new perspective of the temporal evolution of flow patterns (Wernli 1997) and Wernli and Davies 1997).

**Numerical models.** Nowadays the deployment of state-of-the-art numerical models allows for a relatively good simulation of atmospheric flows. In recent years computer power has dramatically increased, so that the models can run at higher resolution in larger domains, with explicit computation of smaller scale processes, in non-hydrostatic mode and with better data assimilation. Furthermore thanks to field experiments parameterization schemes are being continually improved. Numerical models can be used for operational forecasts, to reconstruct events of interest or for idealised simulations to better understand cause and effect. The so-called MC2 model was successfully in quasi-operational use during the MAP special observing period (SOP) in autumn 1999 (Benoit et al. 1997 and Benoit et al. 2001), and the same model version was used to perform highly idealised simulations for the study in chapter 2 of this thesis.

**Field experiments.** Many major international field experiments (e.g. MAP (SOP 1999), FASTEX (SOP 1997) etc.) have been conducted and contributed essentially to the present standard of knowledge. On the one hand a field experiment is a synthesis of knowledge of instruments, concepts and models and on the other hand the collected data and experiences feed back positively with them and propels their further development.

Further successful studies need perforce to be specialized building upon these recent advances and deploying the gamut of available data and techniques.
1.2 Aims of this study

The present study is concerned with specific and specialized (esoteric) aspects of wind-related events that influence Europe and in particular the Alpine region. To pursue the objectives the study deploys the range of research approaches mentioned earlier. Use is made of:

- some of the best available operational data sets (ECMWF re-analyses and operational analyses)
- state of the art mesoscale NWP model (MC2, HRM)
- conventional and current conceptual and theoretical ideas of atmospheric flow dynamics (linear wave theory, $\text{PV}$ perspective, Lagrangian trajectories).

The introductory section 1.3 provides a historical overview of orographic air flow studies. Special emphasis is placed on linear theory of time dependent problems and on lateral shear flows.

Chapter 2 investigates the strength and nature of the flow response of a flow with a zone of lateral shear incident upon orography. Particular attention is devoted to the wind and gravity wave features. In contrast to studies of flows and waves with vertical shear, the problem of lateral shear is a largely neglected area of atmospheric research. This is somehow surprising because many, if not most, "interesting" weather conditions are accompanied by flow with lateral shear. A goal was to close this gap and to shed light on different aspects of such horizontal shear flows. One approach was made with the numerical model MC2 which had performed quasi-operational forecasts during the MAP SOP, but had rarely been used for idealised studies. At the same time the capability of the MC2 model of performing idealised simulations could be tested. Even though our simulations were highly idealised with respect of both orography and incident flow, the same MC2 version was used as during MAP. Such simulations do not seek to reproduce flows under real conditions, but rather their simplicity enables the influence of a small number of key parameters only to be studied. In effect they eliminate the "noise" of other minor effects and facilitate the physical interpretation. Similar experiments with different parameter settings reveal a broad spectrum of possible flow patterns and regimes. Parallel to the numerical simulations theoretical considerations are made in order to validate the model (and the theory) in certain regions of the parameter space and to better resolve physical processes. The approach to flow phenomena with lateral shear presented here is quite broad and novel. Not only that a large area of the parameter space has been covered but also because of the integral view with both theoretical and numerical analyses.
However it is mainly concentrated on idealised aspects and the gap between the ideal and the real world will have to be bridged in further studies.

On the 26th of December 1999 the winter storm 'Lothar' swept over western and central Europe and caused serious damage. This extraordinary and rare event attracted our attention and our curiosity. If we know why 'Lothar' developed the way it did and what made it distinctive from other extratropical cyclones, we can learn a lot not only about this particular storm but also about possible future events, and we can give a statement about the frequency of occurrence of such an episode. The dynamics of this highly unusual storm is discussed in chapter 3. Of particular interest is the distinctiveness of 'Lothar' from the standpoint of wind strength at tropopause levels. The second part tries to answer questions like "How unusual was the 'Lothar' event?". To do so extensive use of the $PV$ perspective was made and trajectory calculations provided essential information of the air masses involved in the cyclone's life-cycle. High resolution mesoscale model runs with full physics and a real atmosphere (in contrast to the idealised simulations of chapter 2) were performed to simulate the storm realistically. A novel description is made of the category of upper tropospheric jets which is regarded as accessory to Lothar's fatal evolution.

In chapter 4 an idealised approach is again adopted to study a special type of orographic flow. The influence of the diurnal variations in the planetary boundary layer and in particular its influence upon ameliorating the downslope winds and gravity wave activity is studied with linear wave theory. This can be viewed as an extension of the pioneering work of Long (1953) and Smith (1979a). A time dependency in the lower boundary condition has not been considered before in this context.
1.3 Literature overview\textsuperscript{1}: Theory of gravity waves over mountains.

Interest in the influence of mountains on atmospheric flows is long standing. In the early days observations of mountain induced phenomena could only be made from the ground and attention focussed on smaller-scale terrain. Ley (1894)\textsuperscript{3} introduced the term \textit{lenticularis-cloud} for lens-shaped clouds generated by orographic gravity waves. Later balloons and aircraft made it possible to observe and measure the mountain atmosphere in situ (see Lammert (1920) or Küttner (1939)).

Preliminary insight on mountain waves can be gained from the examination of linearised equations, i.e. by considering small perturbations of a basic state flow. The main advantage of this approach is that the basic system of equations describing the atmospheric state (such as momentum equations, mass and energy conservation etc.) is simplified and can often be solved analytically.

Most of the observed phenomena are linked with so-called \textit{gravity waves} or \textit{buoyancy waves} in the atmosphere over orographic features. Gravity waves can exist because of the restoring nature of the buoyancy force in a stratified fluid. Rayleigh (1883) showed for the first time that the frequency of gravity waves is limited by a frequency $N$ which is determined by the stratification of the fluid. This frequency is now referred to as the Brunt-Väisälä frequency and denoted by

$$N^2 = \frac{g}{\theta_0} \frac{d\theta_0}{dz}$$

where $\theta_0(z)$ is the potential temperature of the horizontally uniform background flow. In the pioneering work of Queney (1936) gravity waves are discussed which are generated by a uniform flow of velocity $U_0$ over a sinusoidal topography in an atmosphere of constant Brunt-Väisälä frequency $N$. For a linearised Boussinesq system he determined the dispersion relation of gravity waves

$$n^2 = \frac{N^2 - u_0^2 k^2}{u_0^2 k^2 - f^2} (k^2 + l^2)$$  \hspace{1cm} (1.1)

that has to be fulfilled by the wave vector $(k, l, n)$. From this we can see that waves can only propagate vertically (i.e. $n^2 > 0$) if $f/u_0 < k < N/u_0$. This is not sufficient to know the sign of $n$. The formulation of the correct 'radiation

\textsuperscript{1}This overview partly follows the introduction to Leutbecher (1998) and the extended theoretical discussions in Sprenger (1999). Very comprehensive summaries of the history and the milestones of scientific research related to mountain waves can be found therein.

\textsuperscript{3}see Wallace and Hobbs (1977) p. 217
condition’ (i.e. energy must radiate upwards) can be expressed for instance as $knw_0 > 0$ as formulated by Smith (1979a).

1.3.1 2-dimensional flow over a mountain ridge

Queney (1947 and 1948) discussed the flow over a smooth infinitely long mountain ridge $h(x) = h_0/(1 + x^2/a^2)$ perpendicular to the background flow ($w_0$ and $N$ constant). He looks at different regimes depending on the width (parameter $a$) of the ridge. A very narrow ridge ($a \ll w_0/N$) generates a potential flow where the perturbation decays rapidly with height and waves do not propagate vertically. For $N \gg f$ there are three possible regimes (see also formula (1.1)): for a small mountain ($a \approx w_0/N$, see Fig. 1.1 top left) the waves are over the crest and in the lee. Due to the relatively steep slopes the vertical acceleration $dw/dt$ must not be neglected (non-hydrostatic regime). With increasing width ($w_0/N \ll a \ll w_0/f$, see Fig. 1.1 top right) the regime becomes hydrostatic non-rotational where the gravity waves are non-dispersive (i.e. the vertical wavenumber $n \approx N/w_0$ is independent of the horizontal wavelength) and the wave activity is concentrated over the mountain. For even wider ridges ($a \approx w_0/f$, see Fig. 1.1 bottom left) the response is determined by rotation (rotational regime) and waves reappear in the lee. At almost the same time Lyra (1943) undertook a similar linear study of mountain flow with constant $w_0$ and $N$. In contrast to Queney the shape of Lyra’s obstacle was a rectangle. The idea behind this was to compose complex orography out of many rectangles. Compared to Queney the lee waves were much stronger because the steep slopes of the rectangle led to non-hydrostatic effects (one of which are lee waves) and therefore this theory is only applicable to very narrow mountains.

Many authors then have applied these models to more complex background flows or orography. Scorer (1949) expanded the Queney-Lyra model by vertical variations of the background flow and the stability. For an isentropic, incompressible non-rotational and frictionless flow he derived a differential equation for the Fourier transform of the vertical velocity

$$\frac{d^2 \tilde{w}}{dz^2} + \left(l^2(z) - k^2\right)\tilde{w} = 0 \tag{1.2}$$

with $l$ defined as

$$l^2(z) = \left(\frac{N}{w_0}\right)^2 - \frac{1}{w_0} \frac{d^2 w_0}{dz^2} \tag{1.3}$$

and referred to as the Scorer parameter. For a piecewise constant Scorer parameter, equation (1.2) can be solved analytically. Scorer applied this to a two layer atmosphere each with constant $l_i$. If $l(z)$ decreases with height ($l_1 > l_2$)
waves with wavenumber $l_1 > k > l_2$ can propagate in the lower layer but are reflected at the interface of the two layers. These so-called \textit{trapped lee waves} can extend very far downstream and resemble much more observed lee waves than the dispersive tail in Queney's solution (Fig. 1.1 bottom left). Scorer triggered a series of studies of different variations of $l(z)$, e.g. $l \sim 1/z$ of Wurtele, $l \sim e^{cz}$ of Palm and Foldvik or numerically solved solutions to more realistic $l^2$ profiles (see the overview of Smith (1979a)).

Another milestone was the study of Eliassen and Palm (1960). They found that for an internal gravity wave – even if the solution for a general flow is not known – the vertical flux of horizontal momentum

$$F(z) \equiv \rho_0 \int_{-\infty}^{\infty} u(x, z) w(x, z) dx = \text{constant}$$

(1.4)

(i.e. independent of the height $z$) and that the vertical wave energy flux is
proportional and opposite to the wind speed $u_0$

$$I(z) = \int_{-\infty}^{\infty} p(x, z) w(x, z) dx = -u_0(z) F. \quad (1.5)$$

In addition to this they showed that in an atmosphere of vertically varying stability and background velocity (cf. the study of Scorer mentioned above) wave energy can be reflected in the vertical depending on the wave length.

A new approach for the description of a flow in a stratified fluid was given by Long (1953). By requiring incompressibility (in contrast to Queney (1948)) he succeeded in finding an analytical solution for the lee-wave-problem without linearisation. The key component of Long’s advance was the reduction of the non-linear system of equation (with some assumptions) to a linear 2nd order differential equation valid for finite perturbations still. Long’s solution includes non-linear processes which makes it possible to describe very steep streamlines and the onset of static instability.

### 1.3.2 Flow over 3-dimensional orography

The horizontal wave patterns resulting from the flow of air over an isolated hill look similar to the water waves behind a ship. Wurtele (1957) studied the 3D-analogue to Lyra (1943), the flow over an infinitely narrow plateau in a non-rotating atmosphere. Sufficiently far from the obstacle the horizontal pattern of the non-hydrostatic waves are V-shaped not unlike ship waves.

Smith (1980) gives an extensive discussion of a three-dimensional flow ($f = 0$) over an isolated smooth mountain ($U$, $N$ constant). The solution for the vertical displacement $\eta(x, y, z)$ can be written formally as a two-dimensional Fourier integral

$$\eta(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{h}(k, l) e^{i n(k, l)z} e^{i(kx + ly)} dk \, dl \quad (1.6)$$

where the vertical wave number $n$ is shown to be

$$n^2 = \frac{k^2 + l^2}{k^2} \left( \frac{N^2}{U^2} - k^2 \right) \quad (1.7)$$

and $\hat{h}(k, l)$ is the two dimensional spatial Fourier transform of the mountain. For the mountain shape he chose

$$h(x, y) = \frac{h_0}{(x^2/a^2 + 1)^{3/2}} \quad ; \quad r = \sqrt{x^2 + y^2} \quad (1.8)$$
where $h_0$ and $a$ are the height and the length scale of the hill. Due to the particularly simple Fourier transform of (1.8) some surface fields can be retrieved analytically as well as the vertical displacement $\eta(x, y, z)$ asymptotically far away ($r/a \gg 1$) and high aloft ($zN/U \gg 1$ and $r/a = O(1)$). On every horizontal level the wave activity is concentrated on a parabola

$$y^2 = \frac{Nzax}{U}$$

(1.9)

with vertex at the origin (over the summit). The parabola (1.9) widens and the phase surfaces tilt upstream and outward with height (see Figs. 1.2 and 1.3).

Figure 1.2: Vertical displacement (in meters) at various levels over an isolated hill (denoted with a dashed circle). For typical atmospheric values of $N = 10^{-2} \text{s}^{-1}$ and $U = 10 \text{ms}^{-1}$ the corresponding heights $z$ are (from top left to bottom right) 390 m, 790 m, 1570 m and 3140 m (Figure from Smith (1980)).

Smith’s theory was extended by Phillips (1984) for elliptic mountains

$$h(x, y) = \frac{h_0}{(r_*^2 + 1)^{3/2}} \quad r_*^2 = \frac{(x/a)^2 + (y/b)^2}{1}$$

(1.10)

with arbitrary orientation of the hill in the flow. Surface pressure and horizontal wind can be expressed as elliptical integrals. Analytical expressions are derived
for the surface pressure drag (also for the transversal component in the case of
the mountain orientation transverse to the wind). Solutions for more complex
mountain shapes or incoming flows can be sought numerically, see for instance

1.3.3 Waves in shear zones

Real mountain flows are normally not uniform, neither horizontally nor vertically
but shear in one or several directions. Synoptic scale systems aloft or features
at the ground (mountains, sea/land etc.) make the flow variable. An example
is given in Fig. 2.1, where an upper level PV streamer induces a horizontal shear
flow over the Alps.

Gravity waves in a zone of vertical shear (i.e. \( dU/dz \neq 0 \)) have often been
discussed (see e.g. the studies of changing Scorer parameter mentioned earlier).
An interesting situation occurs when the mean wind perpendicular to the moun­
tain ridge \( U(z) \) approaches zero at a certain point. The waves can not penetrate
such a critical layer and are absorbed (critical layer absorption, see Smith (1980)
p. 117 and references therein). However, in a temporally changing velocity field
waves can propagate even through levels of zero mean wind (Lott and Teitelbaum
1993b). Gravity waves in directional shear (i.e. the wind direction changes with
height) are studied theoretically by Shutts (1998) and numerically by Shutts and
Gadian (1999). Very little is known yet about waves in horizontal (sic. lateral)
shear (i.e. \( dV/dx \neq 0 \)). Chapter 2 approaches this issue from various theoretical
and idealised standpoints.
1.3 Literature overview: theory of waves over mountains

1.3.4 In-stationary flows

In the vast majority of articles about linear theory of mountain waves the approximation of a steady basic state flow is applied. Exceptions are Bell (1975), Bannon and Zehnder (1985) and Lott and Teitelbaum (1993a, 1993b). Bell (1975) considered a harmonic back-and-forth motion

\[ U(t) = U_0 \cos\omega_0 t \] (1.11)

in a stably stratified fluid over an obstacle. This is a simple representation of a tidal flow over orography. In the general case, waves are generated not only at the driving frequency \( \omega_0 \) but also at all of its harmonics. Only those waves with a frequency less than the ambient Brunt-Väisälä frequency \( N \) can freely propagate. The waves of the \( n^{th} \) harmonic are sensitive to the \( n^{th} \) derivative of the geometry of the mountain \( h(x) \). For a very slowly varying flow the wave pattern is quasi-static and resembles the classic lee wave problem. Bannon and Zehnder (1985) developed the idea further for a Boussinesq rotational flow with a basic state velocity as a sum of an oscillatory and a constant part:

\[ U(t) = U_0 (1 + \Delta \cos\omega_0 t). \] (1.12)

They also looked at the surface pressure drag during the accelerating and decelerating phases. The solutions are consistent with Bell (1975). Lott and Teitelbaum (1993b) assumed a similar time dependency of the background flow as (1.12) but are interested in the propagation properties of the waves rather than in their generation.

In chapter 4 of this thesis a new kind of time dependency will be discussed: the temporal change of the lower boundary condition.
Chapter 2

Lateral shear flow near mesoscale orography

2.1 Introduction

2.1.1 Motivation: PV filament over the Alps

It is at most a truism that when the weather in the Alpine region is “interesting” horizontal shear is an integral feature of the incident flow. One example is shown in Fig. 2.1. In early November 1999 a PV streamer extends from Scandinavia to the Pyrenees and moves eastwards. When approaching the Alps the streamer gets more and more elongated in the north-south-direction and thinner and eventually breaks and ends up as a cut off PV anomaly in the eastern Mediterranean Sea (for the disruption mechanism see Morgenstern and Davies (1999)). During this process on 5 November 1999 the thin filament of high PV extends from northern Poland to the Gulf of Genoa and connects the almost cut-off PV anomaly over Sardinia with the stratospheric “PV-pool” to the north (left panel, ECMWF analysis data). The wind field associated with this PV structure is a northerly flow to the west of the filament, a southerly flow to the east with accompanying heavy precipitation in the Po valley and on the Alpine south-side. In effect there is a lateral shear zone with a zero-velocity-line aligned N-S over the Swiss Alps. Such an upper-level pattern is typical for extreme precipitation episodes on the Alpine south-side as described by Massacand et al. (1998). Likewise the article of Liniger and Davies (2002) uses this particular MAP case to analyse the streamer’s sub-structure with both the LFP (Lagrangian Forward Projection) method and DIAL measurements.

The right panel of Fig. 2.1 is a zoom over the Alps at the same time instant
2.1 Introduction

LATERAL SHEAR NEAR OROGRAPHY

Figure 2.1: MAP IOP 15, 5 November 1999 00 UTC: Left panel: PV and u/v at 320 K (from ECMWF analyses). The long arrows emphasise the wind direction in the western and in the eastern Alps, respectively. Right panel: Pressure and wind vectors at 10 km for the same time instant. The bold line is the 1500 m height contour (from an MC2 simulation).

(derived from an MC2 forecast) at 10 km altitude. Two low pressure systems are located on either side of the orography, the cyclonic circulation around them (yielding flow parallel to the orography in one section), and also evident are the shear zone over the Alps and the stagnation point over the crest of the ridge (in the Gotthard area). This is a typical flow pattern for a shear zone over and normal to a mountain ridge. It will be described in more detail in section 2.2.

2.1.2 Overview

In this chapter various aspects of horizontal shear flows are tackled and discussed with different approaches. Essentially it is in two parts:

First, starting from the real case mentioned above, an idealised shear flow over a horizontally extended ridge is simulated with the MC2 model. In fact, a series of similar experiments revealed the sensitivity of the quasi-steady state upon the Rossby number. The model results are analysed focusing mainly upon the wind velocity and the pressure fields and also the gravity wave patterns. Quasi-geostrophic theory provides an instrument to construct an analytical solution which is compared with the corresponding MC2 simulation.

The second part is somewhat detached from the motivating example, but follows up the problem of a lateral shear flow near a three-dimensional orographic feature, namely near an isolated circular hill. Here the shear zone is on one (or both) side of the mountain. Again MC2 simulations for different Rossby numbers were performed. Here the main interest is centred on buoyancy waves.
and their interaction with the shear zone at various locations. Finally with the help of shallow water theory a statement is given to the behaviour of the wave in the shear zone. Some quasi-geostrophic analytical solutions are presented and conclude the discussion.

2.1.3 MC2 model description

The MC2 model is the Mesoscale Compressible Community Model of the RPN (Recherche en Prévision Numérique) in Quebec, Canada. The model is based on the full-elastic non-hydrostatic model of Tanguay et al. (1990). The model solves the full set of Euler equations in a limited-area Cartesian domain, using a semi-implicit and semi-Lagrangian integration scheme. It is a versatile modelling tool allowing excellent simulations over a wide spectrum of scales (from supersonic waves to large scale synoptic storms). It includes an extensive physics library for the parameterization of the most important physical processes in the atmosphere and at the surface (Kong and Yau microphysics scheme, TKE, force restore, soil moisture scheme, radiation, shallow convection). A general description of the model is given in Benoit et al. (1997). For the simulations of this study the model version 4.7 has been used, the same version as for the quasi-operational use during the special observing period of the Mesoscale Alpine Programme (MAP) from September to November 1999. Results of the use of MC2 during MAP are given in Benoit et al. (2001). For the idealised simulations presented here all components of the physics package have been switched off, i.e. all runs are dry, without convection or radiation schemes etc., but the non-hydrostatic momentum equations are retained.

The MC2 uses terrain-following Gal-Chen coordinates in the vertical. At low levels they are essentially parallel to $\sigma$-coordinates but flatten out at higher levels and eventually become horizontal at the model top (cf. $\sigma$-surfaces). The size of the simulation domain, the initial conditions and the boundary conditions differ for the various experiments and are described in the corresponding sections.

Unless mentioned otherwise the following attributes hold for all experiments. They take place on an $f$-plane and their horizontal resolution is set to $\Delta x = 5$ km (in the $x$- and $y$-direction). In the vertical there are equidistant model layers every 300 m up to 18 km (total 60 layers). For this version of the model periodic lateral boundary conditions were not an option and a relaxation zone was appended around the actual region of interest. The horizontal extension of the relaxation zone is 25 grid points. In the relaxation zone the primary variables are slowly attenuated towards the values at the boundaries in accord with a specified attenuation function (see Benoit et al. (1997)). In the vertical a sponge zone is placed beneath the model lid. The depth of this layer must be sufficiently
large in order to avoid unphysical reflections at the lid. The gravity waves in our experiments have vertical wave lengths \( \lambda \approx 2\pi U/N \) of the order of 6 km, and hence the sponge must be at least this deep. In these simulations it is set to 25 model layers (=7.5 km).

Except for the validation experiments the time step is \( \Delta t = 60 \) seconds consistent with the Courant-Friedrichs-Levy-criterion

\[
\frac{|c\Delta x|}{\Delta t} < 1 \tag{2.1}
\]

which is a necessary condition for stability of the numerical scheme. Here \( \Delta x \) is the horizontal grid point spacing, the velocity \( c = \max \{|u + c_G|\} \) where \( u \) is the advective velocity and \( c_G \) the horizontal group speed of gravity waves.

The primary fields are the three velocity components \((u, v, w)\), temperature, surface pressure and the orographic height. Secondary variables such as potential temperature, pressure, potential vorticity etc. are calculated from the primary fields.

### 2.2 Shear flow over a mountain ridge

The goal of this section is to analyse the general flow field and some special properties of a series of idealised numerical experiments with the MC2 model for a shear zone across and normal to a long mountain ridge (cf. the real case described above).

#### 2.2.1 Initial conditions

Fig. 2.2 displays the initial conditions for all experiments of this series. The size of the integration domain (excluding the relaxation zone) is 200 \times 200 grid points. The ridge has a Gaussian shape ("Witch of Agnesi") in the pseudo northerly y-direction

\[
h(x) = h_0 e^{-\left(\frac{y - y_0}{a}\right)^2} \tag{2.2}
\]

On the pseudo western \((x < x_l)\) and eastern \((x > x_r)\) edges the ridge decays like a Gaussian in the x-direction

\[
h(x, y) = h_0 e^{-\left(\frac{y - y_0}{a}\right)^2 - \left(\frac{x - x_{l,r}}{a}\right)^2} \tag{2.3}
\]
Figure 2.2: Initial conditions for a shear flow over a ridge. Top left: orography (contour interval 100 m) and wind vectors. Top right: surface pressure (hPa) along the x-axis in accord with the geostrophic balance. Bottom left: one dimensional depiction of the lateral wind shear. Bottom right: "Witch of Agnesi" shaped ridge and vertical temperature profile (spacing 0.5°C). See text for a detailed description.

$h_0$ is the maximum mountain height, $y_0 = 100$ grid points (500 km) and $x_l = 30$ and $x_r = 170$ grid points respectively. The mountain width is $a = 50$ km. The wind is constant to the west ($v = -V_0$) and to the east ($v = +V_0$) of the domain and changes linearly in the central region from $x_0 - a_s/2$ to $x_0 + a_s/2$ ($a_s$=width of shear zone, usually set to 200 km) according to

$$v(x) = \frac{2V_0(x - x_0)}{a_s} \quad (2.4)$$

(see top left and bottom left). The wind field is the same at all levels. In order to maintain such a velocity field on an $f$-plane the geostrophic approximation for the horizontal pressure gradient

$$\frac{\partial p}{\partial x} = f \cdot v(x) \cdot \rho \quad (2.5)$$
is applied (top right). The model atmosphere is stably stratified with an imposed uniform Brunt-Väisälä frequency of $N = 10^{-2}\text{s}^{-1}$ and is in hydrostatic equilibrium.

The foregoing fields serve as the initial conditions for the model simulations and also as the stationary lateral boundary conditions towards which the fields in the interior are relaxed.

### 2.2.2 Parameter space

The nature of the orographic flow response will be studied for different settings of the parameters $V_0$, $h_0$, $f$, $N$, $a$, $a_s$. Consideration will be focussed on key flow regimes. We reduce our study to the parameter space $(R_0, \mathcal{F})$, where $R_0$ is the Rossby number and $\mathcal{F}$ the inverse Froude number.

The Rossby number is defined as

$$R_0 = \frac{U}{fL} \quad (2.6)$$

where $U$ is a characteristic velocity, $f$ the Coriolis parameter and $L$ a characteristic length. $R_0$ can thus be viewed as the non-dimensional ratio of the inertial force ($\sim U^2/L$) and the Coriolis force ($\sim fU$) or equivalently as the ratio of the Coriolis time scale $1/f$ and the advective time scale $L/U$. With the nomenclature of section 2.2.1 it is $R_0 = V_0/a_f$.

The inverse Froude number is based upon the mountain height

$$\mathcal{F} = \frac{NH}{U} \quad (2.7)$$

where $N$ is the Brunt-Väisälä frequency and $H$ a characteristic height. $\mathcal{F}$ can be regarded as the ratio of the energy required to lift an air parcel in a uniformly stratified environment over the height $H$ to the kinetic energy available from the incident flow. In the present notation $\mathcal{F} = Nh_0/V_0$, and $\mathcal{F}$ can also be referred to as non-dimensional mountain height.

The overflow of an infinite ridge can occur in several flow regimes depending upon the pair $(R_0, \mathcal{F})$. Their main features are mentioned here$^1$.

---

$^1$The following discussion follows Davies (1997a).
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2.2 Shear flow over a ridge

Low topography: the $\mathcal{F} \to 0$ limit

This is the regime of linear wave theory as it is used in chapter 4. A complete overview is given by Smith (1979a), for rotational flows by Gill (1982) and also in the literature overview from page 6. We therefore do not present a detailed discussion here. From formula (1.1) it can be inferred that the condition for $R_0$ for the existence and the vertical propagation of gravity waves is

$$1 < R_0 < \frac{N}{f}. \quad (2.8)$$

For all other values of $R_0$ in the $f$-plane limit the perturbation decays in the vertical. Section 1.3.1 and Fig. 1.1 provide a short discussion of the flow response depending upon the mountain width (or equivalently upon $R_0$). The corresponding values of $R_0$ in the series in Fig. 1.1 are (from top left to bottom right) $R_0 = 100, 10, 1, 0.1$. The magnitude of $R_0$ within the wave regime determines the direction of the wave energy radiation. For both limits $R_0 \downarrow 1$ and $R_0 \uparrow N/f$ the energy ray path is horizontal whereas in between energy is radiated away almost vertically. A comprehensive explanation can be found in Sprenger (1999).

All numerical experiments in this section take place in the limit of low orography. For completeness finite mountain heights are also mentioned.

High topography of intermediate width: $\mathcal{F}$ finite, $R_0 > 1$

The flow in this region of the parameter space is essentially hydrostatic for $\mathcal{F} \to 0$. The finite mountain height is responsible for non-linear effects.

An increase in $\mathcal{F}$ for a given $R_0$ primarily results in an increase in the wave amplitude until the wave overtops (breaks) at an elevated altitude at $\mathcal{F} \sim 0.8$ (depending on the mountain shape and the upstream wind profile). Wave breaking develops a surface layer with enhanced downslope and lee-side winds with a large increase in the pressure drag (partly similar to a hydraulic jump).

A further increase in $\mathcal{F}$ to about 1.0–1.3 leads to a retardation of the flow upstream of the crest until the point where flow reversal and upstream blocking occurs. This is explicable by recalling that the inverse Froude number $\mathcal{F}$ is the ratio of the required potential energy of an air parcel to flow over the ridge to the available kinetic energy. The depth of the blocked flow can be estimated as $H_B \sim H - U/N$ and the upstream extension of the blocked low level region is of the order of the Rossby radius of deformation $NH/f$. 

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2.2 Shear flow over a ridge

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High topography of large width: $\mathcal{F}$ finite, $R_0 \ll 1$

The flow for $R_0 \ll 1$ is essentially quasi-geostrophic, similar as in the $\mathcal{F} \to 0$ limit. The amplitude of the evanescent flow can be shown to be proportional to $(1 - R_0\mathcal{F})^{-1}$ with isentropes fusing onto the crest as $R_0\mathcal{F} \to 1$.

2.2.3 Validation and control experiments

In order to validate the numerical model for our configuration, some initial test runs were performed.

First the growth of instabilities in the shear zone was tested. To do so the model was run without mountain but only with the horizontally sheared wind field. The result is that no substantial instability was observed to grow and perturb the basic flow.

Secondly dynamic similarity was checked. After a run with a given set of $(V_0, N, h_0, a, a_s, f)$ the simulation was repeated with doubled values of $V_0$, $N$ and $f$. This leaves the non-dimensional key parameters $R_0$ and $\mathcal{F}$ unaffected. A third experiment was to validate the model's horizontal scaling in the absence of rotation ($f = 0$). The horizontal lengths $a$ and $a_s$ were doubled as well as the horizontal grid spacing $\Delta x$ – in order to resolve the shear zone and the ridge equally – and the length of the time step $\Delta t$, too. After the same number of time steps (3600) the output fields were compared with the control experiment. In both tests the experiments yielded almost identical results.

2.2.4 Variations of $R_0$: the general flow pattern

Considering a flow impinging on an orographic feature, it is obvious that it is retarded on the upstream side of the obstacle and accelerated on the downstream side (lee). Associated with this is high pressure upstream and low pressure downstream (the strength of this effects depends on the actual flow parameters such as $R_0$ and $\mathcal{F}$). Note that for the constellation of Fig. 2.2 there are in effect two lee-side regions and two upstream regions, respectively, one of each on either side of the ridge. Due to the associated pressure gradient along the ridge (supported by vortex stretching near the slopes, see later in Fig. 2.5) the incident flow is deflected to the left (opposed to the Coriolis force), and a stagnation point must be located in the centre of the domain for reasons of symmetry (on an $f$-plane, point-symmetry with respect to the centre of the orography is expected). The resulting pattern is depicted schematically in Fig. 2.3.
A set of experiments (in total 12) has been performed for a constant inverse Froude number $F = 0.5$ but for a range of Rossby numbers ($R_0 = \infty, 20, 10, 4, 2.5, 2, 1.33, 1, 0.67, 0.4, 0.26$ and $0.2$). This is achieved by keeping $V_0 = 10 \text{ ms}^{-1}$ and $a = 50 \text{ km}$ constant but changing the Coriolis parameter from 0 to $10^{-3} \text{ s}^{-1}$. The simulations reached a quasi steady state after 2.5 days (3600 time steps = 3600 minutes).

In the succeeding sub-sections we consider sequentially the sensitivity to the value of $R_0$ of the velocity and the pressure fields.

**The velocity field**

Fig. 2.4 displays the resulting wind speed fields and the wind vectors at the ground (i.e. at the lowest model level) for each simulation. Common features of all simulations are – in agreement with the schematic shown in Fig. 2.3 – the stagnation point in the centre of the domain, the acceleration in the lee and the wind turning to yield a cyclonic flow on either side of the ridge. However the distinctness of these features varies across the parameter space from $R_0 = \infty$ to $R_0 = 0.2$.

For the simulations with $R_0 \approx 2$ we recognise quite similar flow fields: a large X-like zone of weak winds in the centre of the domain (including the stagnation point and the upstream deceleration) and a pronounced lee acceleration to about 1.5 times the background velocity. In the immediate vicinity of the ridge the flow is almost purely in the direction parallel to the ridge over nearly the entire width $a_x$ of the shear zone. As $R_0 \to 2$ the area with cyclonic flow narrows slightly. The apex of the cyclonic vortex remains located at the same spot, namely at the border of the domain, in all these simulations. This might be a surprise since the influence of the mountain should depend upon the Rossby radius of deformation which is proportional to $R_0$ in our case of constant $F$ and mountain.
2.2 Shear flow over a ridge

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Figure 2.4: Flow at ground (at the lowest model level) for variations in $R_0$. Shown is the wind speed $(u^2 + v^2)^{1/2}$ (grey shadings, contour interval 2 ms$^{-1}$) and the wind vectors. The orography is denoted with dashed contours (200 m and 400 m height isolines). The size of the domain is 200×200 grid points = 1000 km×1000 km. See text for discussion.
width $a$. But for these large Rossby numbers the influence of the orography reaches further upstream than the length of the domain and therefore the vortex centre is forced by the boundary conditions to lie at the boundary.

This is no longer the case for $R_0 \approx 1$. The centre of the curved flow approaches more and more the mountain ridge and eventually almost merges with the stagnation point for $R_0 = 0.2$. The zone with along-ridge flow narrows substantially. In this regime, the Coriolis effect is of particular importance, so that the horizontal background pressure gradient to maintain geostrophic balance prevails by far over the supplementary upstream-downstream pressure asymmetry. Thus the smaller $R_0$, the more the flow keeps the direction of the initial wind field. For the low $R_0$ cases the velocity field (and other fields, too) become more and more symmetric not only with respect to the stagnation point but also axisymmetric with respect to the “zero line” (the line where $v = 0$ in the middle of the shear zone). In addition to the argument above, the streamlines of a quasi-geostrophic flow over a hill have – unlike high $R_0$ flows – an upstream-downstream symmetry (see Fig. 1.1 bottom right on page 8).

A quasi-geostrophic streamline over the ridge curves cyclonically on the slopes and anticyclonically over the crest (as to observe in Fig. 2.4 bottom right). This is caused by the effect of vortex stretching and shrinking due to the orography (see Fig. 2.5). The isentropic potential vorticity

\[ IPV = -g(f + \xi) \frac{\partial \theta}{\partial p} \]  

is constant in an adiabatic flow. $\xi$ is the vertical component of the relative vorticity. Upstream of the ridge $\xi$ is zero. As an air parcel rises over the ridge it experiences a weaker stratification at the slopes ($|\partial \theta / \partial p|$ decreases). In order to keep the potential vorticity constant, the relative vorticity $\xi$ must increase ($\xi > 0$, cyclonic). Over the crest the stratification is increased and the flow gains negative relative vorticity. A comparison of the numerical solution for a small $R_0$ with an analytical solution of the quasi-geostrophic setting will be undertaken in
2.2 Shear flow over a ridge

section 2.2.5.

Figure 2.6: Vertical cross-sections in y-direction (from S to N) at $x = 100$ through the entire domain for $R_0 = 2$ and $R_0 = 0.2$. Shown is the wind component parallel to the ridge ($x$-direction). Negative values (into plane) with solid lines, positive values (out of plane) are dashed. The contour spacing is $1 \text{ ms}^{-1}$. The zero-line is omitted. The corresponding pictures of the entire $R_0$ series are in Appendix A.

The flow patterns mentioned above are present throughout a substantial depth of the model atmosphere (see Fig. 2.6). For all flows with high Rossby number the cyclonic flow is evident up to a height above 8 km (recall that the sponge layer starts at 10.5 km). The magnitude of the flow component parallel to the ridge decreases with height, but its maximum gets closer to the ridge axis at high levels. For the highest $R_0$ ($>10$) the flow at intermediate height is remarkably disturbed by buoyancy wave activity (see Appendix A). The horizontal velocities associated with three-dimensional gravity waves are so large that the wave structure is evident even in the pseudo-zonal wind component. Although for low $R_0$ flows the parallel component is generally weaker and more confined to the ridge, the decrease with height is comparable to high $R_0$ cases.

The velocity fields $\vec{v}_{\text{steady}}$ depicted in Fig. 2.4 can be split up into the initial state flow $\vec{v}_{\text{init}}$ (defined in Fig. 2.2 left panels) and its change or residual $\vec{v}_{\text{res}} = \vec{v}_{\text{steady}} - \vec{v}_{\text{init}}$. The residual flow field is shown in Fig. 2.7 for the $R_0 = 2$ and the $R_0 = 0.2$ simulations. In essence it is an orography bound anticyclone centred at the stagnation point. This circulation is a superposition of the effects of vortex shrinking (see Fig. 2.5) and the pressure gradients on either side of the ridge. The maximum residual wind vector decreases steadily from more than $10 \text{ ms}^{-1}$ for $R_0 > 5$ to $5.5 \text{ ms}^{-1}$ for $R_0 = 0.2$ (recall that the maximum background velocity is $10 \text{ ms}^{-1}$). Again, for small Rossby numbers the anticyclone is detached from the N and S boundaries and is limited to the immediate vicinity of the ridge. In the limit of an infinitely long ridge, the additional wind is simply a flow parallel to the ridge on either side of it with decreasing amplitude for increasing distance from the crest (cf. the analytical solution of the quasi-geostrophic flow in section 2.2.5).
Figure 2.7: Residual flow $\vec{v}_{\text{res}} \equiv \vec{v}_{\text{steady}} - \vec{v}_{\text{init}}$ for $R_0 = 2$ and the $R_0 = 0.2$. The maximum wind vectors are 8.7 m s$^{-1}$ for $R_0 = 2$ and 5.5 m s$^{-1}$ for $R_0 = 0.2$, respectively (same scale). The domain and orography are the same as in Fig. 2.4. The corresponding pictures of the entire $R_0$ series are in Appendix A.

The pressure field

The reduced surface pressure field of the non-rotational flow (Fig. 2.8, left panel) has two distinctive features: the up- and downstream pressure asymmetry across the terrain and the low pressure in the centre of the cyclonic flow near the N and S boundaries. Recall that the initial pressure distribution was uniform (1000 hPa) for the $R_0 \to \infty$. flow. The pressure field for $R_0 = 2$ also has the two low pressure centres which can be explained partly by the geostrophic boundary conditions. A detailed discussion of this depression will be given in the next sub-section. In the middle of the domain the pressure has increased compared to the initial state. If we split the pressure field in analogy to the wind field in the previous section into the initial geostrophic pressure field and its change, we obtain a high pressure system over the orography which generates the geostrophic component of the anticyclonic residual wind. The same is true for the case with $R_0 = 0.2$, but as mentioned earlier, the extra high pressure system is only a relatively
small perturbation to the background pressure distribution (the entire series of simulations is shown in Appendix A).

**Balanced flow around vortices**

If a flow around a vortex is time-independent, horizontal, axis-symmetric (no radial flow, no downstream variations) and frictionless, it is referred to as *cyclostrophic*. The low pressure centres on either side of the mountain ridge in our simulations seem to fulfill these conditions. A fluid particle experiences a three-way balance between the centrifugal force, the pressure gradient force and the Coriolis force (cf. e.g. Vandenbrouck et al. (2000)):

$$-r_0 \omega^2 = -\frac{1}{\rho} \frac{dp}{dr} + r_0 f \omega$$

(2.10)

where $r$ is the radial distance from the centre. This explains why even the simulation without Coriolis force ($R_0 = \infty$, see Fig. 2.8 left panel) has two low pressure centres. The angular velocity $\omega$ is positive for an anti-clockwise rotation. Fig. 2.9 is a schematic representation of equation (2.10) for both signs of $\omega$. Note that for an equal velocity $|v|$ the magnitudes of the Coriolis and the centrifugal force also remain the same in the two cases but they are differently orientated. Consequently the pressure gradient in an anticyclone is much weaker. The quantitative appropriateness of the balance (2.10) is checked with some of our experiments.

The angular velocity $\omega$ of the cyclonic flow was determined along the line $x = 100$ from the centre of the depression ($y = 0$) towards the ridge. At these locations, the tangential velocity is $u$ and $\omega = u/r_0$, where $r_0 = y \cdot 5$ km is the distance from the apex. First the experiment with $R_0 = \infty$ ($f = 0$, see Fig. 2.10) was considered. The pressure term (solid) is compared with the centrifugal term.
2.2 Shear flow over a ridge

Figure 2.10: Pressure gradient force $dp/dy$ (solid line, $p = 1$) and centrifugal force $r_0\omega^2$ (dashed) at ground from the low pressure centre (at $x = 100$) radially outwards along the $y$-axis for the simulation without Coriolis force ($R_0 = \infty$). (dashed). The amplitudes are comparable and the location of the maximum matches very well. In this case equation (2.10) is a reasonably good description of the flow around the vortex. The full three-way balance was examined for the two experiments with $R_0 = 2$ ($f = 10^{-4}\text{s}^{-1}$, see Fig. 2.11 upper panels) and $R_0 = 0.67$ ($f = 3 \cdot 10^{-4}\text{s}^{-1}$, lower panels), respectively. In the first case the pressure term

prevails slightly over the sum of the centrifugal and Coriolis terms at the ground (left panel). At an elevated level $z = 3000\text{m}$ the three-way balance is perfect. In general the pressure force decreases (relative to the other terms whose ratio is ±constant) with increasing altitude. The signal of the surface depression which is a direct consequence of the orography weakens with height. For the experiment with $R_0 = 0.67$ the flow is geostrophic to a very good approximation. The small Rossby number makes the centrifugal term almost irrelevant compared to the

Figure 2.11: Same as Fig. 2.10 but for the simulations with $R_0 = 2$ ($f = 10^{-4}\text{s}^{-1}$, upper panels) and $R_0 = 0.67$ ($f = 3 \cdot 10^{-4}\text{s}^{-1}$, lower panels). Pressure gradient force (solid), centrifugal force (dashed), Coriolis force (dotted), and the sum of the centrifugal and Coriolis terms (dash-dotted). Left panels are at the ground ($z = 0$), the right panels at $z = 3000\text{m}$.
Coriolis force. Note that the location of the maximum pressure gradient and the highest angular velocity moves towards the mountain for increasing $f$ in qualitative agreement with the decreasing Rossby radius of deformation. The small deviations from cyclostrophic balance are presumably a consequence of the preconditions which are not fulfilled exactly by the simulated flow.

The low pressure centre is expected to deepen for higher angular velocities. In our setting $\omega$ is determined by the background velocity $V_0$ and by the curvature of the flow, i.e. by the width $a_s$ of the shear zone. Consequently the larger $V_0$ and the smaller $a_s$ are, the deeper the low pressure centre will be.

![Figure 2.12: Anticyclonic flow around a high pressure centre. Drawn are the surface pressure (colour) and the wind vectors. This experiment has a reversed background wind field, i.e. southerly wind to the west and northerly wind to the east of the shear zone, and $R_0 = 2$.](image)

Fig. 2.12 is an example of an anticyclonic flow and a high pressure centre ($R_0 = 2$). As argued in Fig. 2.9 the pressure gradient in the anticyclone is very weak. This experiment has been made with the same initial and boundary conditions as described in section 2.2.1 but with a reversed velocity field $v(x) \rightarrow -v(x)$.

### 2.2.5 Analytical solution for a quasi-geostrophic lateral shear flow over a mountain ridge

The quasi-geostrophic setting with $R_0 = 0.2$ is well suited to compare with the analytical solution obtained from the isentropic framework developed by Schär and Davies (1988). In this framework the lower boundary condition and the governing set of equations are fully linear and for some special mountain geometries it can be solved analytically.

In this section we follow the mathematical derivation of the system of equations of Schär and Davies (1988), adjust it to our shear flow problem and solve it with the standard method. The goal is to demonstrate the qualitative similarity between the flow fields of the simulation and the analytical solution.
Mathematical derivation

In a \((x,y,\vartheta,t)\)-coordinate system the equations for an adiabatic flow of an incompressible, Boussinesq and inviscid fluid on an \(f\)-plane take the following form

\[
\begin{align*}
\mathcal{D}u - fv &= -M_x, \\
\mathcal{D}v + fu &= M_y, \\
\mathcal{D}M_{\vartheta\vartheta} + M_{\vartheta}(\nabla_h \cdot \mathbf{v}) &= 0
\end{align*}
\]

with \(\mathcal{D}\) denoting the total derivative on an isentropic surface. \(M\) is a form of the \textit{Montgomery streamfunction} defined via the hydrostatic equation

\[
\frac{\partial M}{\partial \vartheta} = -\frac{gz}{\Theta}.
\]

The vertical coordinate \(\vartheta\) is measured from a reference isentrope \(\Theta\) here set to the surface isentrope, i.e. \(\vartheta = \Theta - \theta\).

It is helpful to use dimensionless notation. We define a horizontal length scale \(a\), a vertical scale \(af/N\) and a velocity scale \(V_0\) to give

\[
\hat{x} = \frac{x}{a}, \quad \hat{y} = \frac{y}{a}, \quad \varepsilon \equiv \frac{a_s}{a}, \quad \hat{u} = \frac{u}{V_0}, \quad \hat{v} = \frac{v}{V_0}.
\]

The appropriate scales for the potential temperature and the Montgomery streamfunction are given by

\[
\hat{\vartheta} = \frac{\vartheta}{Naf\Theta}, \quad \hat{M} = \frac{1}{aV_0f}.
\]

Here the symbol (\(^\sim\)) signifies a dimensionless variable. With the appropriate conservation relation for the (dimensionless) quasi-geostrophic potential vorticity and in the quasi-geostrophic limit the set (2.11)-(2.14) is replaced by the following equations (for details see Schär and Davies (1988))

\[
\begin{align*}
\hat{u} &= -\hat{M}_{\hat{\vartheta}}, \\
\hat{v} &= \hat{M}_{\hat{x}}, \\
\hat{\varepsilon} &= -R_0\hat{M}_{\hat{\vartheta}}, \\
\hat{M}_{\hat{x}\hat{x}} + \hat{M}_{\hat{y}\hat{y}} + \hat{M}_{\hat{\vartheta}\hat{\vartheta}} &= \text{const.}
\end{align*}
\]

where \(R_0\) is the Rossby number. The background flow shown in Fig. 2.13 is in
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Figure 2.13: Wind profile for the flow over the mountain ridge. The parameter $\epsilon \equiv a_s/a$ denotes the width of the shear zone normalised to the width of the mountain.

dimensionless notation

\[
\hat{v}_0(\hat{x}) = \begin{cases} 
-1, & \hat{x} < -\frac{\xi}{2} \\
\frac{2\hat{x}}{\xi}, & -\frac{\xi}{2} \leq \hat{x} \leq \frac{\xi}{2} \\
1, & \hat{x} > \frac{\xi}{2} 
\end{cases}
\]  
(2.22)

With (2.19) and (2.20) we find the Montgomery streamfunction $M_0$ for the mean flow

\[
\hat{M}_0(\hat{x}, \hat{y}, \hat{\theta}) = -R_0^{-1} \frac{\hat{\theta}^2}{2} + \begin{cases} 
-\hat{x}, & \hat{x} < -\frac{\xi}{2} \\
\frac{\hat{x}^2}{\epsilon} + \frac{\xi}{4}, & -\frac{\xi}{2} \leq \hat{x} \leq \frac{\xi}{2} \\
\hat{x}, & \hat{x} > \frac{\xi}{2} 
\end{cases}
\]  
(2.23)

Then the equation for the perturbation fields $\hat{z}'(\hat{x}, \hat{y}, \hat{\theta})$ and $\hat{M}'(\hat{x}, \hat{y}, \hat{\theta})$ is

\[
\hat{z}'_{xx} + \hat{z}'_{\hat{y}\hat{y}} + \frac{\hat{z}'_{\hat{\theta}\hat{\theta}}}{\hat{\theta}^2} = 0
\]  
(2.25)

and the surface boundary condition is

\[
\hat{z}'(\hat{x}, \hat{y}, \hat{\theta} = 0) = \hat{\eta}(\hat{x}, \hat{y})
\]  
(2.26)

where $\hat{\eta}$ is the dimensionless orography. The maximum mountain height is $\hat{\eta}_m = \eta_m N/a f = R_0 F$ with $F$ the inverse Froude number.

We set for convenience

\[
\hat{\eta}(\hat{x}, \hat{y}) = R_0 F (\hat{y}^2 + 1)^{-1},
\]  
(2.27)

i.e. the ridge is infinitely long in the x-direction. The shape $\sim 1/y^2$ across the ridge is not exactly the same as for the numerical simulations but allows for an analytical solution.

\[\text{even though the constant in (2.21) is different in the regions with and without shear (2/\epsilon and 0, respectively), the Montgomery streamfunction (2.23) and its partial derivatives are continuous at the shear edges and (2.25) is valid in the entire domain.}\]
We eventually can solve the differential equation (2.25) for the heights \( \hat{\bar{z}}(\hat{x}, \hat{y}, \hat{\vartheta}) \) of the isentropes and derive the associated Montgomery streamfunction with (2.20) and (2.21)

\[
\hat{\bar{z}}(\hat{x}, \hat{y}, \hat{\vartheta}) = \hat{\vartheta} + R_0 \mathcal{F}(\hat{\vartheta} + 1)(\hat{y}^2 + (\hat{\vartheta} + 1)^2)^{-1} \\
\hat{M}(\hat{x}, \hat{y}, \hat{\vartheta}) = -R_0^{-1} \frac{\hat{\vartheta}^2}{2} - \frac{\mathcal{F}}{2} \ln(\hat{y}^2 + (\hat{\vartheta} + 1)^2) + \left\{ \begin{array}{ll} 
-\hat{\bar{x}}, & \hat{x} < -\frac{\xi}{2} \\
\frac{\hat{x}^2}{\epsilon} + \frac{\xi}{4}, & -\frac{\xi}{2} \leq \hat{x} \leq \frac{\xi}{2} \\
\frac{\xi}{2}, & \hat{x} > \frac{\xi}{2} 
\end{array} \right.
\]

(2.28)

(2.29)

This result means that we now know the height of an isentrope \( \hat{\vartheta} \) at any location \((\hat{x}, \hat{y})\) or equivalently the displacement field of all streamlines. Considering (2.29) and keeping in mind that the wind is obtained via the relations (2.18) and (2.19), we see that \( u \) is determined by the "mountain term" \( \ln(\hat{y}^2 + \ldots) \) only and \( v \) by the shear.

### Comparison with the MC2 simulation

In this subsection a few qualitative comparisons are made between the analytic solution (2.29) and the quasi-geostrophic MC2 simulation.

The flow parameters for the simulation in dimensionless notation according to (2.15) and (2.16) are the following: \( R_0 = 0.2, \mathcal{F} = 0.5, \epsilon = 4 \) and \( \hat{V}_0 = 1 \). The full size of the integration domain is 20 x 20. In the vertical \( \hat{z} = 1 \) corresponds to \( z = 5000 \text{ m} \).

In Fig. 2.14 the horizontal (isentropic) flow fields of both the analytic solution and the simulation (upper panels) are shown as well as the vertical profile of potential temperature (lower panels). The turning of the flow on either side of the ridge and the wind direction over the ridge are equally represented on either picture. The maximum wind speed is almost the same, \( \hat{v}_{\text{max}} = 1.03 \) and 1.13, respectively. The vertical potential temperature profile is quite similar, too. The isentropes are symmetric with respect to the crest and their displacement decays with height. Note that the 'Witch of Agnesi' of the numerical simulation is somewhat narrower (and therefore steeper) than the \( 1/\hat{y}^2 \) shape on the left panel and that the decay of the perturbation with height appears to be slightly weaker.

From the momentum equations (2.11) and (2.12) the ageostrophic wind can be calculated (see Schär and Davies (1988))

\[
\hat{\bar{v}}_{\text{ag}} = R_0 \hat{D}_g(k \wedge \hat{v}_g).
\]

(2.30)
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**Figure 2.14:** Comparison of the analytic solution (left panels) and the MC2 simulation (right panels). The depicted domains are the same for the left and the right pictures. Upper panels: streamlines and $u/v$ at the surface (at $\hat{y} = 0$ and at the lowest model level, respectively). The dashed contours denote the ridge. It is at the same position in the analytical case but is infinite in the $\hat{x}$-direction. Lower panels: vertical profile of potential temperature from $\hat{z} = 0$ to 1 which corresponds to $z = 0$ to 5000 m.

The quasi-geostrophically induced ageostrophic flow field is shown in Fig. 2.15 (upper panels). Qualitatively they both show acceleration over the ridge and a cyclonic structure centered over the shear zone over the crest. This is opposed to the anticyclone mentioned on page 24. Quantitatively the effect is small ($max(\hat{v}_{ag}) \sim 0.1$) and even weaker by about a factor of 2 in the analytical case. The field on the lower panels of Fig. 2.15 finally is the residual wind (wind minus mean background flow). As seen on page 24 this flow is essentially parallel to the ridge in the quasi-geostrophic limit. The 'noise' in the right panel could be an effect of the finite ridge length in the MC2 simulation. The maximum wind vector in the analytical case is again about half the value of the simulation which is $max(\hat{v}_{res}) \sim 0.5$. 

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Figure 2.15: Comparison of the analytic solution (left panels) and the MC2 simulation (right panels). The domains are the same for the left and the right pictures. Upper panels: quasi-geostrophic induced ageostrophic flow field. Lower panels: residual wind $\vec{v}_{\text{res}} = \vec{v}_{\text{qg}} - \vec{v}_0$ as introduced on page 24.

Overall the flow field derived from the analytical formulae is very similar to the MC2 simulation. On the one hand this validates the capability of the MC2 model for idealised settings and on the other hand it is a nice and interesting application of the isentropic framework (analytical considerations of a quasi-geostrophic shear flow over an isolated mountain are presented in section 2.3.6).

2.2.6 Variations of $R_0$: buoyancy waves

When air flows over an obstacle, buoyancy (or gravity) waves can be generated. It has been described above how they depend on the geometry of the lower boundary, the stability of the atmosphere and the background flow. An air parcel that is advected along an isentrope (adiabatically) undergoes an undulation when passing a region of buoyancy waves. A measure for such waves can therefore be
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the vertical velocity \( w = \frac{Dz}{Dt} = \frac{\partial z}{\partial t} + (\vec{v} \cdot \vec{V})z \) or a variable containing some information about its vertical change. Mass conservation implies \( \vec{V} \cdot (\rho \vec{v}) = 0 \) and for \( \rho \equiv \rho(z) \) the horizontal divergence is

\[
\vec{\nabla}_h \cdot \vec{v}_h \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial}{\partial z} (\rho w).
\]

For all settings of the simulations presented here non-hydrostatic effects are effectively negligible \((U/aN \sim 10^{-2} \ll 1)\) and the horizontal divergence (or simply divergence) is a suitable indicator for the presence of gravity waves. There are updrafts below a region of positive divergence and downdrafts above it. Thus for the undulating isentropes there is positive (negative) divergence near local maxima (minima) (in \( z \)) of an isentrope.

![Figure 2.16: Evidence of buoyancy waves (horizontal divergence field) at \( z = 5 \text{km} \) for the \( R_0 = 2 \) simulation. The contour interval is \( 5 \cdot 10^{-5} \text{s}^{-1} \). Dashed contour lines denote negative values, solid contours positive values. The domain and orography are the same as in Fig. 2.4. The entire \( R_0 \) series is in Appendix A.](image)

Fig. 2.16 is a horizontal cross-section through the entire domain at the altitude \( z = 5\text{km} \). The divergence pattern is very typical for all simulations with high \( R_0 \). Waves are generated on both sides of the shear zone, where the wind component \( v_\perp \) normal to the ridge is large enough to produce a local Rossby number \( R_0(x,y) \equiv v_\perp(x,y)/af \) greater than unity. We deduced earlier that in a central region near the ridge almost only parallel wind occurs. The normal component is therefore close to zero, the associated local Rossby number \( \ll 1 \), and no vertically propagating gravity waves are generated. Fig. 2.17 displays the local Rossby number \( R_0(x,y = 100) \) along the crest of the ridge.

![Figure 2.17: Local Rossby number along the crest of the ridge (\( y = 100 \)): \( R_0(x,y) = v_\perp(x,y)/af \). For \( x \approx 120 \) it is less than unity, and no wave activity is expected aloft (as shown in Fig. 2.16).](image)
The parabolic zones trailing from the edges of the ridge are described by Smith (1980) as the transition between the undisturbed region to the sides of the ridge and the decaying two-dimensional disturbance behind the ridge. The corresponding patterns for the other high-\(R_0\) experiments are qualitatively very similar. For low Rossby numbers no gravity waves (or only very weak disturbances) are present.

Vertical cross-sections across the ridge (i.e. normal to it) are shown in Fig. 2.18 at \(x = 140\) which is sufficiently far from the centre line for the possible generation of gravity waves. In the wave regime (\(R_0 > 1\), left panel) gravity waves are generated and identified via the divergence field. They propagate almost vertically, but only up to about 4000-5000 m. Analogue vertical cross-section at different \(x\)-positions show, that the height at which the wave is attenuated for the most part depends quite strongly upon \(x\) (see later discussions). The quasi-geostrophic case (\(R_0 \ll 1\), right panel) is simply a potential-like flow over the obstacle.

It is difficult to see from the horizontal and the vertical cross-sections how the gravity waves are distributed three-dimensionally. A three-dimensional representation of the horizontal divergence is depicted in Fig. 2.19 for the case \(R_0 = 2\). The iso-surfaces of positive and negative divergence are evident over the orography. Individual divergence iso-surfaces are tilted from the horizontal and lie about parallel above each other. The whole structure is tilted slightly downstream. As noticed above waves do not propagate to high altitudes in central regions. They do so only near the edge of the ridge, far away from the stagnation point.
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Figure 2.19: Three dimensional distribution of the divergence field for the $R_0 = 2$ simulation from three different views. Shown are divergence iso-surfaces of $\pm 4 \cdot 10^{-5} \text{s}^{-1}$ coloured with vertical velocity $w$. Red colours denote updrafts, blue colours downdrafts, respectively. Wind vectors and the wind speed on the lowest model level are plotted on the ground. The vertical coordinate is model levels, therefore the orography does not appear. The ridge must be thought to extend in the middle from W to E. The arrows in the upper panel denote the viewing directions for the middle and the lower panels.
2.3 Shear flow over an isolated hill

As before, an analysis of the general pattern of a flow over orography and its interaction with a nearby shear zone is to be undertaken in this section. The orography is isolated and the wind field is uniform over the hill and is sheared on
2.3 Shear flow over an isolated hill  \( \text{LATERAL SHEAR NEAR OROGRAPHY} \)

the sides (see Fig. 2.21). This is no longer directly motivated by the example of a PV streamer over the Alps shown in the introduction to this chapter. However, if one edge of such a streamer is near an isolated orographic feature, the conditions are similar to the ones of these experiments. On one side of the streamer edge (in front of it or behind it) the wind is more or less uniform and on the other side (within the streamer) it is sheared. As long as we are in the wave regime \( (R_0 \text{ sufficiently large}) \) it is not of particular importance on which side of the hill the shear zone lies (cf. the “broken symmetry” in the quasi-geostrophic limit discussed in section 2.3.6). The main interest here is to study the propagation of the gravity waves on well directed rays and their interaction with the shear zone.

2.3.1 Initial conditions

The general settings to initiate the experiments of this section are shown in Fig. 2.21. The geometry of the circular bell shaped hill is

\[
h(x, y) = h_0 e^{-\left(\frac{x-x_0}{a}\right)^2 - \left(\frac{y-y_0}{a}\right)^2}.
\]

As mentioned above the wind profile is somewhat different from the one in the previous chapter. Initially the wind is purely in S–N direction. In the centre of the domain the southerly wind is uniform of strength \( V_0 = 10 \text{ ms}^{-1} \) and decays linearly in the shear zones (analoguely to equation (2.4)) towards the E and W boundaries of the domain.

\textbf{Figure 2.21:} Initial conditions for a shear flow near an isolated mountain. Left: orography and wind vectors, middle: surface pressure (hPa) along the x-axis according to the geostrophic balance, right: one-dimensional lateral wind shear on both sides. See text for a detailed description.

The position \( x_0 \) of the mountain top is not the same in all experiments, neither
is the distance $|x_0 - x_2|$ from the mountain to the eastern shear zone. For some experiments the western shear zone is omitted. The size of the integration domain (without relaxation zone) is $150 \times 200$ grid points, in the case of one shear zone only it is $140 \times 200$. The width $a_s$ of the shear zone is $200$ km (half of which is within the domain), i.e. $x_2$ is $100$ km $= 20$ grid points from the right boundary. The $y$-position of the mountain is at $y_0 = 50$ grid points.

2.3.2 Parameter space

As in the discussion of a flow over an infinite ridge in 2.2.2 we distinguish between different flow regimes in the $(R_0, F)$ parameter space. $R_0$ and $F$ are defined as in equations (2.6) and (2.7) with $a =$ half width of the (circular) mountain.

Low topography: the $F \to 0$ limit

Again this regime is characterised well by linear theory. As mentioned in the literature overview (section 1.3.2, p.9) Smith (1980) pioneered in this field. The solution procedure included a double spatial Fourier decomposition of the terrain, solving the wave problem in Fourier space and a Fourier synthesis to obtain the real-space solution. For $f = 0$ he showed that the wave activity was concentrated on the parabole

$$y^2 = \frac{N_{2ax}}{U}$$

(2.32)

with the vertex over the summit (see Figs. 1.2 and 1.3). This result remains qualitatively true for $R_0 \gg 1$ but the parabola gets somewhat asymmetric in a rotating fluid. For $R_0 \geq 1$ a mountain-bound anticyclone becomes evident.

High topography of intermediate width: $F$ finite, $R_0 \gg 1$

As $F$ increases the overflow is made more and more difficult (recall the “energy-argument” in the discussion of equation 2.7). The limited lateral width of the mountain inhibits the wave amplitude and the occurrence of wave breaking for $F \leq 1$. Instead of upstream blocking the flow starts to split and flow around the hill. Various studies had the goal of predicting where the borderline between overflow, flow splitting and wave breaking lies. The “energy argument” first proposed by Sheppard (1956) would imply that the critical non-dimensional mountain height was $F = 1$. But this can not be the whole story, because it makes no distinction between different mountain shapes and their associated different pressure fields. Smith (1989) presented a regime diagram for a non-rotational three-dimensional flow for different horizontal aspect ratios $r$ (ratio width/length of the hill) and
2.3 Shear flow over an isolated hill  LATERAL SHEAR NEAR OROGRAPHY

\( \mathcal{F} \) (denoted as "H" in Fig. 2.22, left panel). Schär (1995) sketched a similar regime diagram for circular hills \((r = 1)\) and rotational flows (right panel). Very

![Regime Diagram](image)

Figure 2.22: Left: Regime diagram for hydrostatic flows over mountains. The parameter \( r \) is a measure for the mountain shape (horizontal aspect ratio), \( H \) is the non-dimensional mountain height (\( \equiv \mathcal{F} \)). Solid lines: linear theory estimates of flow stagnation aloft (A) and at the windward slope (B). Flow splitting and wave breaking modify the whole flow situation, therefore the region where both phenomena occur is uncertain and denoted by dashed lines and question marks. From Smith (1989). Right: Regime diagram for a circular mountain with rotation (according to Trüb (1993), figure taken from Schär (1995)).

stable stratification (or a very high mountain: \( \mathcal{F} \gg 1 \)) acts to suppress vertical displacement of air parcels and induces a symmetrical and almost horizontal flow around the hill. However, Peng et al. (1995) state that for sufficiently high \((\mathcal{F} > 2)\) and small scale mountains \((L < 50 \text{ km})\) the Coriolis effects must not be neglected even for large Rossby numbers.

**High topography of large width: \( \mathcal{F} \) finite, \( R_0 \ll 1 \)**

The flow is basically quasi-geostrophic. Superimposed to the incident flow is a mountain-bound anticyclone with increasing amplitude for increasing \( R_0 \mathcal{F} \). For \( \mathcal{F} > 1 \) a Taylor cap forms, a conical region over the summit with closed streamlines. See Schär and Davies (1988) and section 2.3.6 for an analytical approach to this issue.

**2.3.3 Variations of \( R_0 \): wave patterns**

Fig. 2.23 shows the horizontal divergence field at \( z = 5 \text{ km} \) for a series of simulations of the type described in Fig. 2.21 for different Rossby numbers. The inverse
2.3 Shear flow over an isolated hill

Figure 2.23: Horizontal divergence (in units of $10^{-5}\text{s}^{-1}$, dashed contour lines denote negative values, solid contours positive values) and $v$ (contour interval=2 ms$^{-1}$) at $z = 5\text{ km}$ for five simulations with different Rossby numbers. The bottom right panel is a control simulation with $R_0 = 2$ but without shear and is to be compared with the top right panel. See text for discussion.

Figure 2.24: Same as Fig. 2.23 but at $z = 10\text{ km}$.
Froude number $\mathcal{F} = 0.5$ is the same for all experiments. The flow is unidirectional from the south impinging on a circular hill. On both sides of the mountain the wind decays linearly in a shear zone. Control runs similar to the ones mentioned in section 2.2.3 have been performed. The results of the control run without shear zone are integrated into this discussion for reasons of comparison.

For the experiments with $R_0 > \approx 1$ the typical parabole of high wave activity as derived theoretically by Smith (1980) are evident. Without Coriolis force ($R_0 = \infty$) the divergence pattern is perfectly symmetric. For decreasing Rossby numbers the flow field and the divergence pattern become more and more asymmetric. The downstream end of the shear zone on the right hand side of the hill is displaced by some grid points to the right, and the left branch of the parabola features a larger wave amplitude. For the smallest $R_0$ of our series the wave activity is no longer concentrated on two parabola arms but its distribution rather resembles lee-waves.

When a substantial wave signal reaches to the edge of a shear zone, the velocity field in the shear zone is not strongly perturbed by the interaction with gravity waves. It can be observed that the waves do not penetrate the shear zone but decay quickly in the shear direction. This can be seen from a comparison of the diagrams for $R_0 = 2$ with and without shear zone (top and bottom right). Whereas the divergence patterns are nearly identical in the middle of the domain where the background flow is uniform, they differ strongly where the flow shears in one simulation.

More insight into the behaviour of gravity waves in a lateral shear zone is given by horizontal cross-sections at a higher altitude $z = 10 \text{ km}$ (Fig. 2.24). In general the differences for varying $R_0$ are qualitatively the same as before. Consistent with Smith’s formula (2.32) the parabole widen at higher elevations and thus the angle of incidence upon the shear zone become smaller (measured to the normal). Furthermore the wave amplitude at the shear edge is larger than at lower levels. We now see clearly that the wave is not simply damped in the shear zone but reflected by or within the shear zone. The local wave component in the $x$-direction changes its sign: $(k, l, n) \rightarrow (-k, l, n)$. Near the plane of reflection the wave is amplified because the incident and the reflected wave interfere constructively with each other. A few questions arise at this point: How large can interference effects become? What does the reflected wave look like in three dimensions? Can we explain the reflection? Specific aspects of these questions are discussed in the following sub-sections.
2.3.4 Interference effects

Fig. 2.25 displays some results of three experiments with the same flow parameter settings \((R_0 = 2, \mathcal{F} = 0.5)\) but with the shear zone at different distances from the mountain. The left panel is a control simulation without shear, in the middle panel the edge of the shear zone is one half-width \(a\) of the mountain away from the summit (i.e. \(\hat{s} = 1\), where \(\hat{s}\) is the non-dimensionalised distance), and in the last experiment the shear begins right at the mountain top (\(\hat{s} = 0\)).

![Image of divergence and divergence at a different location](image)

*Figure 2.25: Divergence (in units of \(10^{-5}\) s\(^{-1}\), dashed contour lines denote negative values, solid contours positive values) and \(v\) at \(z = 6.5\) km for three simulations with different shear zone locations. See text for discussion.*

Strong effects of constructive interference can occur where the phases of the incoming and the reflected wave are (almost) equal and their individual amplitudes are large. Since the wave path lies on the parabola the phase lines are long lens-shaped objects crossing the parabola at a small angle (cf. the lines of equal divergence). For this distribution of wave activity it is impossible to find a reflection plane parallel to the flow which projects one arm of the parabola onto the other. But constructive interference is possible for single maxima or minima at a given level. The right panel of Fig. 2.25 is an example: the elevation \((z = 6.5\) km\) and the location of shear edge are chosen such that the original location of the second divergence maximum downstream (at \(y \approx 60\)) and its “mirror image” coincide. The amplification factor of the divergence is \(\sim 1.8\) to 2.

A three-dimensional visualisation of the wave is helpful to inter-relate the foregoing two-dimensional pictures. Fig. 2.26 gives an impression of the spatial distribution of the gravity wave for settings with and without reflection at a shear zone. For the case where no shear disturbs the wave (upper two panels) the “boomerang-shaped” structures are placed above each other, their surfaces
Figure 2.26: 3D representations of the experiments portrayed in Fig. 2.25. The surfaces coloured with red-brown and blue are iso-surfaces of horizontal divergence (cf. Fig. 2.19) of the values $\pm 3.5 \cdot 10^{-5}$ s$^{-1}$. In the three rows the shear zone location is $\hat{s} = \infty$, 1 and 0, respectively. The left column shows the whole structure approximately from the upstream direction. The divergence field on the 2nd model level is plotted on the horizontal cross-section. The same quantities are shown in the right column but from a different viewing angle and with a horizontal cut on the 31st model level ($\sim 9300$ m).
slope outwards and their downstream ends point downwards. A horizontal cut through this structure (top right) reveals the same wave pattern as discussed in the previous section.

The corresponding pictures for the setting with lateral shear on the right hand side are shown in the middle ($s = 1$) and the lower ($s = 0$) panels. The iso-surfaces of divergence are again placed above each other and their ends point downwards. The slope of the surface in the right branch is different now (middle row). In the bottom row the two branches are no longer identifiable.

The right column provides some more understanding of interference effects. In the lower two diagrams we see several localised regions of large divergence which are separated from the "boomerang-like" structures and are located at regions where no strong wave activity is expected (when compared with upper panels). These isolated pieces are presumably regions where the wave has been amplified by constructive interference and where the divergence has reached an absolute value greater than $3.5 \cdot 10^{-5} \text{s}^{-1}$.

### 2.3.5 Solution of the shallow water equations with lateral shear

This section tries to explain analytically why the wave does not propagate through the shear zone but is attenuated strongly within a short distance. Waves in the simple shallow water model are not directly comparable with the gravity waves in a stratified fluid. However, in both cases a profound modification of the wave component in the shear direction is responsible for the attenuation of the wave in the shear zone. But the reflection mechanism can not be explained with this model.

The shallow water equations are with $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y}$

\[
\begin{align*}
\frac{D u}{Dt} - f v &= -g \frac{\partial h}{\partial x} \\
\frac{D v}{Dt} + f u &= -g \frac{\partial h}{\partial y} + f \Lambda y \\
\frac{D h}{Dt} &= -h \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)
\end{align*}
\]

For the study of wave-solutions it is customary to linearise the equations using $h = H + h', u = U + u', v = V + v'$ (see also Fig. 2.27). Assuming an external
surface force balances $fV$ and $fU$ we obtain:

\[
\frac{Du'}{Dt} - f v' = -g \frac{\partial h'}{\partial x} \quad (2.36)
\]

\[
\frac{Dv'}{Dt} + f u' = -g \frac{\partial h'}{\partial y} \quad (2.37)
\]

\[
\frac{Dh'}{Dt} = -H \left( \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) \quad (2.38)
\]

The mean fluid depth $H = \text{constant}$ and the advection operator in the x-direction is defined as $\mathcal{L} \equiv \frac{\partial}{\partial t} + U \frac{\partial}{\partial x}$.

From this we get for a non-rotating flow ($f = 0$) with a basic state aligned in the x-direction ($V = 0$) but with a variation in the even flow direction ($U = U(y)$).

\[
\mathcal{L}u' + v' \frac{\partial U}{\partial y} = -g \frac{\partial h'}{\partial x} \quad (2.39)
\]

\[
\mathcal{L}v' = -g \frac{\partial h'}{\partial y} \quad (2.40)
\]

\[
\mathcal{L}h' = -H \left( \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) \quad (2.41)
\]

This set of equations for $(u', v', h')$ can be reduced to a single equation for $h'$ by the following sequence of operations: Take $\mathcal{L}(2.41)$ and substitute $\mathcal{L}u'_x$ via equation (2.39).

\[
\mathcal{L}^2 h' + H \frac{\partial}{\partial x} \left( -v' U_y - g h'_x \right) = -H \mathcal{L} v_y' \quad (2.42)
\]

\[
\Rightarrow \quad \left( \mathcal{L}^2 - g H \frac{\partial^2}{\partial x^2} \right) h' = H \frac{\partial}{\partial x} (v' U_y) - H \mathcal{L} v_y' = 2H U_y v'_x + g H h'^y \quad (2.43)
\]

Similar as before we substitute $\mathcal{L}v'_y$ via equation (2.40) and simplify to get

\[
(\mathcal{L}^2 - g H \nabla^2) h' = 2H U_y v'_x \quad (2.44)
\]
Take \( \mathcal{L}(2.44) \)

\[
\mathcal{L}(2.44) = 2HU_y \mathcal{L}h'_x = -2(gH)U_y h'_{xy} \tag{2.45}
\]

This leads to a single differential equation for \( h'(x, y, t) \) of the form

\[
\mathcal{L}(\mathcal{L}^2 - gH \nabla^2) h' = -2gHU_y h'_{xy} \tag{2.46}
\]

Consider a plane wave (see Fig. 2.27) with fixed wave number \( k \) in x-direction of the form

\[
h'(x, y, t) = F(y) e^{i(kx - \omega t)}. \tag{2.47}
\]

Insertion into (2.46) yields

\[
i(Uk - \omega)[- (Uk - \omega)^2 - gH(-k^2 + \frac{\partial^2}{\partial y^2})]F = -2gHU_y ik \frac{\partial F}{\partial y} \tag{2.48}
\]

\[
\Leftrightarrow \left[ \frac{\partial^2}{\partial y^2} - \frac{2U_y k}{Uk - \omega} \frac{\partial}{\partial y} + \frac{(Uk - \omega)^2}{gH} - k^2 \right] F = 0 \tag{2.49}
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{wind_profile.png}
\caption{Wind profile: constant shear \( U(y) = \Lambda y \).}
\end{figure}

For a time-independent state \( \omega = 0 \) with linear shear \( U(y) = \Lambda y \) equation (2.49) becomes

\[
\Leftrightarrow \left[ \frac{\partial^2}{\partial y^2} - \frac{2\Lambda k}{\Lambda y k} \frac{\partial}{\partial y} + \frac{(\Lambda k)^2}{gH} - k^2 \right] F = 0 \tag{2.50}
\]

\[
\Leftrightarrow \left[ \frac{\partial^2}{\partial y^2} - \frac{2}{y} \frac{\partial}{\partial y} + k^2(\gamma^2 y^2 - 1) \right] F = 0 \tag{2.51}
\]

where \( \gamma^2 \equiv \frac{\Lambda k}{gH} \). Equation (2.51) is a differential equation for the complex amplitude \( F(y) \) of the plane wave (2.47) in a region of shear \( \Lambda \) in y-direction.
To eliminate the partial derivative $\frac{\partial^2}{\partial y^2}$ in (2.51), we look for an equivalent differential equation in $\chi(y)$ without this term. We make the Ansatz:

$$F(y) = A(y)\chi(y) \quad (2.52)$$

and insert this into (2.51). We get

$$A\chi_{yy} + 2\chi_y \left( A_y - \frac{A}{y} \right) + \chi \left( A_{yy} + \frac{2A_y}{y} + A\gamma^2 y^2 - 1 \right) = 0 \quad (2.53)$$

To eliminate the coefficient of $\chi_y$ set

$$A(y) = y$$

and hence (2.53) is simplified for $y \neq 0$ to

$$\left[ \frac{\partial^2}{\partial y^2} + k^2(\gamma^2 y^2 - 1) + \frac{2}{y^2} \right] \chi = \left[ \frac{\partial^2}{\partial y^2} + l^2 \right] \chi = 0 \quad (2.54)$$

with

$$l^2(y) \equiv k^2 \left( \gamma^2 y^2 - 1 \right) + \frac{2}{y^2 k^2} \quad (2.55)$$

Now that we have a pseudo wave-like component in the $y$-direction, we can also define a group speed in this direction $v_g(y) \equiv \partial \omega / \partial l$.

If the wave number $l(y)$ changes very little on the spatial scale $L \equiv 2\pi/l$ the solution of the differential equation (2.54) can be locally approximated by

$$\chi(y) = C \cdot e^{il(y)y} \quad (2.56)$$

and the plane wave (2.47) is modified in the presence of the shear to

$$h'(x, y, t) = C \cdot y \cdot e^{i(l(y)y + kx - \omega t)} \quad (2.57)$$

If

$$y < y_1 = \sqrt{\frac{k + \sqrt{k^2 - 8\gamma^2}}{2\gamma^2 k}} \to \frac{1}{\gamma} \quad (k \to \infty) \quad (2.58)$$
2.3 Shear flow over an isolated hill

Figure 2.29: 3D-illustration of (2.57) with the wind profile of Fig. 2.28. A plane wave $h'(x, y, t) = e^{i(kx + ly - \omega t)}$ is excited in a shallow water system. The background flow $U = Ay$ is built up in the system in the $x$-direction and increases linearly from the left to the right (increasing $y$). The $z$-axis denotes the (normalised) amplitude of the wave. The steady state of the wave in this new environment includes an additional wave component in $y$-direction (according to the relation (2.55)) which varies with $y$. This is shown in the picture. While the wave number $k$ (or the wave length $2\pi/k$) in the $x$-direction remains constant, the wave length in the shear direction ($y$) becomes larger for $y \to 0$. Therefore the wave crests of the time-independent solution bend. The amplitude decreases linearly in $y$ for decreasing $y$ until the critical value $y_1$ is reached (see 2.58). For even smaller $y$ the amplitude decays exponentially.

the term on the right hand side in equation (2.55) is less than zero and therefore $l(y)$ purely imaginary and $h'(x, y, t)$ (2.57) is damped in $y$-direction (see Fig. 2.29).

With the shallow water model alone the reflection of internal gravity waves on a lateral shear zone can not be explained. However, the calculations give an idea of why waves do not penetrate through such a zone.
2.3.6 Analytical solution for a quasi-geostrophic lateral shear flow over an isolated mountain

The final aspect of a lateral shear flow over an isolated mountain is the application of the isentropic framework from section 2.2.5 to the setting of an isolated mountain. The results of this section are not validated by experimental data but are nevertheless worth considering to complete the manifold discussion.

The differential equation for the height of the isentropes (2.25) and the lower boundary condition (2.26) are the same for the present problem as before. The mean background wind \( \bar{v}_0 \), the corresponding Montgomery streamfunction \( \bar{M}_0 \) and the mountain profile \( \bar{\eta} \) have to be adopted to the present situation.

Shear zone on the right hand side of the mountain

The normalisation to non-dimensional parameters is done as in (2.15) and (2.16). The length \( a \) is the half width of the circular hill. The wind profile in Fig. 2.30

\[
\hat{v}_0(\hat{x}) = \begin{cases} 
1 & , \hat{x} \leq \hat{s} - \frac{\epsilon}{2} \\
\frac{1}{2(\hat{x} - \hat{s})^2} & , \hat{x} > \hat{s} - \frac{\epsilon}{2}
\end{cases}
\]  

(2.59)

and the associated Montgomery streamfunction of the mean flow is

\[
\hat{M}_0(\hat{x}, \hat{y}, \hat{\vartheta}) = -R_0^{-1} \frac{\hat{\vartheta}^2}{2} + \begin{cases} 
\frac{\hat{x}}{\epsilon} & , \hat{x} \leq \hat{s} - \frac{\epsilon}{2} \\
-\frac{(\hat{x} - \hat{s})^2}{\epsilon} + \hat{s} - \frac{\epsilon}{4} & , \hat{x} > \hat{s} - \frac{\epsilon}{2}
\end{cases}
\]  

(2.60)

The geometry of the hill (surface isentrope) is set to

\[
\hat{z}'(\hat{x}, \hat{y}, \hat{\vartheta} = 0) = R_0 \mathcal{F}(\hat{x}^2 + \hat{y}^2 + 1)^{-3/2}.
\]  

(2.61)

Now (2.25) can be solved analytically and yields
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\[
\begin{align*}
\ddot{\xi}(\hat{x}, \hat{y}, \hat{\theta}) &= \hat{\theta} + R_0 \mathcal{F}(\hat{\theta} + 1) \left( \hat{x}^2 + \hat{y}^2 + (\hat{\theta} + 1)^2 \right)^{-\frac{3}{2}} \\
\dot{M}(\hat{x}, \hat{y}, \hat{\theta}) &= -R_0^{-1} \frac{\hat{\theta}^2}{2} + \mathcal{F} \left( \hat{x}^2 + \hat{y}^2 + (\hat{\theta} + 1)^2 \right)^{-\frac{1}{2}} \\
&+ \left\{ \begin{array}{ll}
\hat{x} & , \hat{x} \leq \hat{s} - \frac{\epsilon}{2} \\
\frac{-\sqrt{3}}{2} \hat{x} + \hat{s} - \frac{\epsilon}{4} & , \hat{x} > \hat{s} - \frac{\epsilon}{2}
\end{array} \right.
\end{align*}
\]

(2.62)  (2.63)

Note that in the limit \( \hat{s} \to \infty \), i.e. no shear near the hill, (2.62) and (2.63) are identical to the solution in Schär and Davies (1988) (see Fig. 2.31 top left). The wind is obtained via the relations (2.18) and (2.19). The zonal wind component \( u \) is determined by the mountain only, whereas \( v \) is composed of contributions of the hill and of the background (shear) flow.

Here we present the interaction of the quasi-geostrophic flow over an isolated hill with a shear zone at varying distance \( \hat{s} \) from the mountain. We choose the parameter setting \( R_0 = 0.1, \mathcal{F} = 3.33 \) so that the criterion for Taylor cap formation (in this framework \( \mathcal{F} > 3\sqrt{3}/2 \), cf. Schär and Davies (1988)) is satisfied and physically meaningful solutions exist (i.e. \( R_0 \mathcal{F} < 0.5 \)).

Fig. 2.31 shows the flow fields on the surface isentrope for different values of \( \hat{s} \), from \( \hat{s} = \infty \) (i.e. no shear zone) to \( \hat{s} = 0 \) (shear directly over the summit). In absence of a shear zone (top left) the flow is deviated to the left around the hill by the anticyclonic flow generated by vortex shrinking, and the mountain anticyclone is just strong enough to form a Taylor cap slightly to the right of the summit. The streamlines in an undisturbed shear zone are parallel and their gradient is proportional to the local wind speed. The interaction of both features is displayed in the other panels.

Even when the zero line is relatively far away (\( \hat{s} = 4 \), top middle) the flow disturbance by the mountain generates an \( \dot{M} \)-gradient in the \( \hat{y} \)-direction at the zero line of the shear zone. According to (2.18) this generates a wind component in the \( \hat{x} \)-direction. For \( \hat{y} > 0 \) is \( \partial \dot{M} / \partial \hat{y} < 0 \) and thus \( \dot{u} > 0 \). An air parcel slightly to the left of the zero line is therefore blown over the centre of the shear into the region of reversed flow. It is then advected by the flow into the negative \( \hat{y} \) direction. For \( \hat{y} < 0 \) the signs of \( \dot{M}_y \) and \( \dot{u} \) change and the air parcel is blown back again over the zero line. A closed anticyclonic circulation in the shear zone has been generated. When the shear zone approaches the mountain, the \( \dot{M} \)-gradient in the shear zone is increased and the “shear anticyclone” is strengthened. Finally the anticyclonic features of the mountain and the shear zone merge and amplify as they coincide. When the zero line is exactly over the summit (bottom right) the two anticyclones complement each other ideally and the vortex over the mountain is very strong (same streamline spacing as a measure of the wind speed in all plots).

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Figure 2.31: Montgomery potential and wind vectors at the surface isentrope for different locations $\tilde{s}$ of the zero line. The edge of the shear zone is at $\hat{x} = \tilde{s} - \epsilon$ and $\epsilon = 4$. The hill is indicated by the dashed circle at the origin of the coordinate system. The flow parameters are $R_0 = 0.1$, $\mathcal{F} = 3.33$ and the values for the zero line are $\tilde{s} = \infty$, 4, 3, 2, 1, 0 (from top left to bottom right). The displayed domain is the same for all sub-plots. The streamline spacing is equally set to $\Delta M = 0.31$ in all plots.

Shear zone on the left hand side

Figure 2.32: Same as Fig. 2.30, but shear zone on the left side of the hill (when looking downstream).

When the shear zone is on the left-hand side of the hill (when looking downstream, see Fig. 2.32) the corresponding wind field formulation is

$$\hat{v}_0(\hat{x}) = \begin{cases} 
\frac{1}{2(\tilde{s} - \hat{s})} & , \quad \hat{x} \geq \tilde{s} + \frac{\epsilon}{2} \\
\frac{1}{2\epsilon} & , \quad \hat{x} < \tilde{s} + \frac{\epsilon}{2} 
\end{cases}$$

(2.64)
Figure 2.33: Same as Fig. 2.31 but the shear zone is on the left side of the hill (when looking downstream). Compared with Fig. 2.31 the flow pattern is completely different due to the flow asymmetry around the hill. Same constant streamline spacing as in Fig. 2.31.

The geometry of the hill and the vertical isentrope displacement are identical to (2.61) and (2.62), respectively. The Montgomery streamfunction differs little from (2.63). It is

\[ \hat{M}(\hat{x}, \hat{y}, \hat{\theta}) = -R_0^{-1} \frac{\hat{\theta}^2}{2} + \mathcal{F} \left( \hat{x}^2 + \hat{y}^2 + (\hat{\theta} + 1)^2 \right)^{-1/2} \]

\[ + \left\{ \begin{array}{ll}
\hat{x} \\
\frac{(\hat{x}-\hat{\theta})^2}{\epsilon} + \hat{s} + \frac{\epsilon}{4}, \quad \hat{x} < \hat{s} + \frac{\epsilon}{2}
\end{array} \right. \]

(2.65)

This configuration is a mirror image (with respect to the \( \hat{y} \)-axis) of the former problem and we could naively assume that so is the flow field. But there is no symmetry in the flow! The Coriolis parameter is still positive and therefore the deflection due to vortex shrinking is to the same direction as before (Fig. 2.33 top left, \( \hat{s} = -\infty \)). Now that the shear is on the left hand side the deformation of the flow field in the shear zone is such that \( \partial \hat{M} / \partial \hat{y} < 0 \) for \( \hat{y} > 0 \). To the right of the zero line an air parcel is blown away from the centre of the shear. To the left of the shear centre is \( \hat{u} > 0 \), too and air flows from the left over the
2.4 Summary

Many aspects of lateral shear flows have been discussed and a large variety of phenomena observed and described in this chapter. Motivated by a real case of the MAP IOP 15 we generalised some main features of the problem and performed idealised high resolution NWP model simulations in order to be able to analyse the influence of the lateral shear near orography separated from other effects. Analytical calculations for the quasi-geostrophic limit (small Rossby numbers) revealed great similarity with the model results.

In general the flow regime can differ locally from the "global" regime or various scales can exist simultaneously. The presence of the Coriolis force breaks the symmetry between forward and backward shear (+ or - sign of $\partial v/\partial x$).

A shear flow over a long mountain ridge and its dependency upon the ("global") Rossby number has been studied intensively. In all simulations two vertically extended low pressure systems are evident on each side of the ridge and the cyclonic flow around them dominates the flow pattern. The impact of the orography and the shear zone decreases substantially with decreasing Rossby number. For large enough $R_0$ the two depressions have the character of a meso-$\beta$-scale system where the equilibrium of an air parcel is determined not only by the pressure gradient and the Coriolis force but also by the centrifugal force. The gravity waves in the low-$R_0$ regime behave somewhat surprisingly in the shear zone over the ridge. They are generated if the local Rossby number is larger than approximately unity but they do not propagate to large altitudes due to three-dimensional wave dispersion. A qualitative comparison of an MC2 simulation in the quasi-geostrophic regime and analytically deduced flow fields shows a great similarity not only for the horizontal geostrophic flow but also for ageostrophic components, the residual wind and the vertical structure.

In a second series of simulations a flow impinges on an isolated circular mountain which is uniform immediately over the mountain but is sheared nearby. The properties of orographically induced gravity waves and their interaction with a lateral shear zone have been of special interest here. With the help of a simple
shallow water model we illustrated why a gravity wave can not penetrate through such a shear zone. MC2 simulations showed reflections of the gravity waves and interference with the incident wave. For a proper choice of the position of the shear zone regions with substantial constructive interference could be identified and visualised in a three-dimensional view. The same quasi-geostrophic isentropic framework as before was applied to this problem. There is an interesting interaction of the shear zone and the Taylor cap circulation when the edge of the shear zone is close enough to the summit. The identical setting but with the shear zone on the left hand side of the mountain (when looking downstream) provided striking evidence for left-right asymmetry of the shear position for the general flow field.
Chapter 3

A severe extratropical cyclone and the jet stream

In the morning of December 26 1999 an unusually intensive and violent storm hit western and central Europe. The evolution of the cyclone named ‘Lothar’ has been studied and published in Wernli et al. (2002). Section 3.1 gives an overview of this paper. The entire publication is in Appendix C. One of the most important features of the ‘Lothar’ episode was the steep tropopause “wall” with a long lasting strong and straight jet stream over the Atlantic Ocean. Section 3.2 focuses on this aspect. A climatology of long and rectilinear jets is compiled for 15 winter months in the northern hemisphere and the ‘Lothar’ jet is related to this climatology.

3.1 Dynamical aspects of the life cycle of the winter storm ‘Lothar’ (24-26 December 1999)

This section is essentially a summary of the results of the paper in Appendix C on page 125. Most of the figures which are referred to here do not appear multiply but can be found in the Appendix C.

\footnote{The German Weather Service (DWD) gives names to each North Atlantic cyclone. The storm ‘Lothar’ has also been referred to as the ‘French storm’ or the ‘1999 Boxing Day low’ (Le Blancq and Searson 2000).}
3.1 The winter storm ‘Lothar’

3.1.1 Introduction

‘Lothar’ was one of the most destructive storms in Central Europe for decades. It caused huge damage to forests and buildings and claimed more than 50 lives. The extremely high wind velocities were mainly responsible for the damages. Many stations (especially in the Swiss Middleland) measured the highest wind speeds on record (e.g. 158 km h\(^{-1}\) in Zürich or 208 km h\(^{-1}\) on the mountain Hörnli in northeastern Switzerland). Unlike the central and western Alps, the Alpine south-side was not affected by ‘Lothar’.

On the synoptic scale the dominant weather system in the run-up to the ‘Lothar’ event was a deep low pressure system, ‘Kurt’, in southwestern Norway which had caused very stormy weather in the entire region for several days. In the analysis ‘Lothar’ is drawn as a frontal wave system on the trailing cold front of ‘Kurt’ (Fig. C.1). At 500 hPa ‘Lothar’ was only a small temperature perturbation on 00 UTC 26 Dec 1999 (6 hours prior to its maximum intensity). Twelve hours later (Fig. C.2) ‘Lothar’ is situated over central Germany and is most violent in Switzerland at this time.

The cyclone had some remarkable features. ‘Lothar’ was strong, of a relatively small scale (meso-β), intensified very rapidly when it hit the European continent, and it was rather badly forecast. We do not comment on the latter but try to explain the other features by studying the storm’s life-cycle from its first appearance on 12 UTC 24 Dec 1999 off the coast of Newfoundland until its decay after 18 UTC 26 Dec.

After a few comments on the data sets and the analysis tools used, the life-cycle of ‘Lothar’ is described in two parts: the translation and the intensification phase.

3.1.2 Data and diagnostic tools

Two data sets are used in the present study: analysis data from the operational ECMWF T319L60 assimilation cycle, and output from mesoscale limited-area NWP hindcast simulations. ECMWF analysis data (0.5° spatial resolution, available every 6 hours) was used to analyse the overall life-cycle. In order to be able to study the small-scale cyclone during its rapid intensification phase on a relatively short time scale additional simulations have been performed with the HRM (‘high resolution model’), the new version of the former ‘Europa Modell’ of the DWD (See Majewski (1991) or Lüthi et al. (1996)). HRM output was available every hour from 12 UTC 25 Dec to 12 UTC 26 Dec with a horizontal resolution of 28 km and on 40 vertical levels.
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3.1 The winter storm ‘Lothar’

The $PV$ perspective (Hoskins et al. 1985) and Lagrangian analyses (Wernli and Davies 1997) are the main tools which are used to diagnose the key features of the genesis and rapid intensification of ‘Lothar’. The following aspects of the $PV$-$\theta$-framework are of special importance in this study: a) Potential vorticity ($PV$) is conserved in an adiabatic flow, e.g. near the tropopause, b) latent heat release (e.g. by cloud diabatic processes) generates positive $PV$ anomalies, c) a positive $PV$ anomaly has a cyclonic wind field associated with it, i.e. a column of high $PV$ (‘$PV$ tower’) in the troposphere is a favourable structure for strong cyclonic winds.

Forward and backward trajectory calculations provide information of the origin of air masses involved in the processes.

3.1.3 Translation phase

The jet

The dominant feature in the Atlantic Ocean was an extraordinarily strong ($v > 60 \text{ ms}^{-1}$), long ($L > 7000 \text{ km}$), straight and zonally aligned jet stream which persisted for about 84 hours. (See Fig. 3.1). At its western end the jet structure

![Figure 3.1: Persisting jet over the Atlantic in late December 1999. Shown is $PV$ at 320 K and the wind speed (contours of 60, 70, 80 and 90 ms$^{-1}$).](image)

is sustained by anticyclonic Rossby wave breaking. The very strong $PV$ gra-
3.1 The winter storm ‘Lothar’

SEVERE CYCLONES AND JETS

dient across the tropopause (assumed to be the 2 pvu iso-surface) is reinforced by a band of low $PV$ to the south of the jet axis and accompanied by strong baroclinicity.

‘Lothar’ was ‘born’ in this environment as a shallow cyclone on around 24 Dec 12 UTC in a strong baroclinic zone in the western Atlantic at $\sim$40°N, far south of the jet axis (see Fig. C.3 (a–c)). During its passage over the Atlantic ‘Lothar’ remained shallow with no prominent perturbation at upper levels and was permanently accompanied by strong precipitation (d–f). The low $PV$ band mentioned earlier has been shown to be the outflow region of air parcels which ascended rapidly from low levels where they released latent heat associated with precipitation. During the very rapid translation ‘Lothar’ intensified moderately.

Diabatic Rossby wave

The ongoing precipitation near ‘Lothar’ and the very high translation speed are indications for a process referred to as diabatic Rossby wave (Snyder 1991; Snyder and Lindzen 1991; Parker and Thorpe 1995). The cyclonic circulation around the low level $PV$ anomaly advects warm moist air from the south. Due to the baroclinicity northwards moving air is forced to rise, moisture condensates and latent heat is released to the east of the anomaly. This leads to a positive $PV$ tendency at low levels. The low level $PV$ anomaly is thus permanently regenerated in the direction of the thermal wind. Fig. C.13 provides strong evidence for this mechanism during the translation phase of ‘Lothar’. Backward and forward trajectories also showed, that ‘Lothar’s’ translation was not simply advection in the mean flow. Furthermore in a dry HRM simulation, i.e. where all moist processes were switched off, ‘Lothar’ decays over the Atlantic before it reaches Europe (see e.g. the core pressure evolution in Fig. C.6). This underlines once more the crucial importance of diabatic processes.

3.1.4 Rapid intensification phase

It was rather unusual that the cyclone intensified first when it was over land despite increased surface friction and reduced moisture advection. The HRM simulation provides a high enough spatial and temporal resolution to study the explosive cyclone intensification. In only 19 hours from 25 Dec 12 UTC to 26 Dec 07 UTC the minimum surface pressure dropped by 22 hPa in the HRM run. In measurements it was even 22 hPa in 6 hours! South-north oriented cross-sections across the low pressure centre (Fig. C.8) for both the moist and dry HRM simulations illuminate the process of deepening. As ‘Lothar’ gets underneath the jet axis (associated with a deep and narrow tropopause fold) the low level
PV anomaly couples with the tropopause fold to form a vertically well aligned PV tower. The PV tower formation is illustrated impressively in the three-dimensional representation of Fig. C.7. Within a very short time a narrow fold of stratospheric air penetrates deep into the troposphere towards the growing low level PV anomaly. This is the optimal structure to induce strong cyclonic winds. Fig. 3.2 shows the same 2 pvu iso-surface as Fig. C.7(c) but shaded with wind speed instead of potential temperature. Note the very high speeds (dark grey) around the PV column and in particular at its south-western side.

Backward trajectories from the region where the two PV features almost merge (500–600 hPa in the tower) demonstrates that air parcels were either adiabatically advected from the stratosphere with a adiabatic descent (isentropic downgliding) at the end or that they ascended diabatically with strong latent heat release from the lower troposphere (Fig. C.10). The deep stratospheric intrusion can even be identified as a dry spot over northern France on a water vapour satellite image (Fig. C.9).
3.1 The winter storm ‘Lothar’

**Bottom-up development**

The S-N cross-sections through Lothar’s centre for the dry HRM simulation (Fig. C.8(e-h)) and the corresponding three-dimensional pictures (not shown) indicate not only that the low level system dies without diabatic processes but also that no stratospheric intrusion from upper levels is formed. Since moist processes play a role for the low level anomaly only but not for the tropopause region, this means that the low level system induced the ‘downward pulling’ of the tropopause. This bottom-up development is schematically illustrated in Fig. C.14: If the low level PV anomaly is situated underneath the jet axis and the tropopause is sufficiently low then the cyclonic circulation around the low level system induces a northerly flow at tropopause level to the west (upstream) of it. Due to the strength of the jet and its associated strong baroclinicity a southward advection also means a downgliding on the steeply sloping isentropic surface. The tropopause is pulled down and forms a PV tower with the low level PV anomaly. This may even be a positive feedback mechanism. The PV tower enhances the cyclonic flow, advects more moist air from the south and favours a continuing downward advection.

The translation of this model to the real case has been made in Fig. C.12. The 298 K isentropic surface intersects both the stratospheric intrusion and the low level anomaly. The isentropic wind difference between the dry and the moist HRM simulation at 298 K reveals a cyclonic circulation around ‘Lothar’ and an anticyclonic one ~700 km to the west of it. In between a southward flow advects stratospheric air downwards. A rough estimate of the descent rate is in accord with the values obtained from trajectory calculations.

Only shortly after the rapid intensification phase the system decays (also quite rapidly) when moving inland towards eastern Europe.

### 3.1.5 Conclusions

Lothar was an unusual storm and so was its life-cycle. Several preconditions had to be fulfilled at the right time and at the right place. The presence of an intense and straight upper level jet was of crucial importance not only because of the associated tropopause fold properties but also because of the strong baroclinicity at all levels. Section 3.2 focuses on these features and searches for episodes similar to the jet prior to ‘Lothar’. The diabatic Rossby wave character of ‘Lothar’ in its early phase was necessary for the sustainment and the translation of the low level anomaly. Finally the extremely rapid bottom-up development was only possible because of the perfect ‘cooperation’ of the diabatically generated low level system and the aforementioned properties the tropopause.
3.2 Climatology of long straight zonal jets

In section 3.1 and in Wernli et al. (2002) the evolution of ‘Lothar’ over the Atlantic Ocean and its intensification over western Europe has been discussed extensively. Two key factors that emerged from the analysis were:

a) the importance of diabatic processes and

b) the interplay of a low level system with an exceptionally strong, extended, rectilinear and zonally aligned jet.

Questions that arise are “How unusual was the jet that accompanied ‘Lothar’?” and/or “How often do such jets occur?” The goal of this section is to address these questions by evaluating the climatological frequency and geographical distribution of long and straight jets and to compare them with the one that prevailed for the ‘Lothar’ case.

The jet during ‘Lothar’ was

- **strong**: low tropopause, pronounced baroclinicity on all levels throughout the troposphere.

- **long, rectilinear and persistent** (quasi-stationary): the diabatically generated low level $PV$ anomaly had enough time to get underneath the jet axis.

- **zonal**: permanent advection of warm and moist (subtropical) air.

These properties are not independent, e.g. a persistent and rectilinear jet stream on, say, 315 K, is almost always associated with high wind speeds and a certain length (see e.g. Fig. 3.3). A long, straight and zonally orientated jet is furthermore the only constellation which can be (more or less) stationary during a couple of days due to the absence of propagating Rossby waves.

3.2.1 Model data

A jet climatology has been computed systematically for the northern hemisphere on the basis of the European Centre for Medium Range Weather Forecast

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3.2 Climatology of long straight zonal jets

(ECMWF) analyses for Januaries from 1986 to 1999 and for December 1999. January data have been chosen for two reasons: firstly it is the season when Lothar occurred and secondly jets in the northern hemisphere are generally strongest in January. For the years until 1993 re-analysis fields (ERA15) based on the T106L31 global weather prediction model were used, from 1994 onwards operational analyses with resolutions of T213L31 (94-98), T319L31 (January 99) and T319L60 (December 99). The horizontal resolution of the interpolated fields was 1°, and in the vertical ~25 hPa at tropopause level. Output is available on 00 UTC, 06 UTC, 12 UTC and 18 UTC, i.e. on 124 times per month.

3.2.2 Contour and jet definition

In the extratropics jets are usually situated at tropopause-breaks. The (dynamical) tropopause is taken to be the 2 pvu surface, and the 2 pvu contour on an isentropic surface provides a guide for the potential location of such a jet. Here this quantitative relationship is used, and jets are not identified from the wind field but from the geometry of the 2 pvu contour on isentropic surfaces. To find long, rectilinear and zonal jets, a search is made for long, rectilinear and zonal sections of 2 pvu contour on different isentropic surfaces. In order to cover a reasonable depth of the atmosphere the jet locations were computed on the 280 K to 350 K surfaces at every 5 K interval.

The process of jet-recognition involves two steps: First, using a variant of a streamer identification algorithm (Wernli and Sprenger 2002), the 2 pvu contour is identified on the respective θ-surface (i.e. 124 x 15 x 15 contours) and is approximated as a polygon containing about 2000 points. Thereafter each contour is screened for sections fulfilling the following criteria:

- length $L > L_{\text{min}}$
- rectilinearity: $\frac{L}{\text{direct distance}}^3 \leq R_{\text{jet}}$
- zonality: $\Delta \text{lat}/L < T_{\text{lat}}$

Note that the parameters $L_{\text{min}}$ (minimum jet length), $R_{\text{jet}}$ (jet ratio) and $T_{\text{lat}}$ (tolerance in latitude) can be chosen independently. Every section of the 2 pvu contours satisfying these criteria is defined as jet$^4$. From now on a "jet" is referred to this definition. As all jets are E-W orientated to a large degree, the third criterion has normally not been applied.

$^3$distance along a line from the start point to the end point of the jet with a constant slope in the longitude-latitude frame: $\frac{\Delta \text{lat}}{\Delta \text{lon}} = \frac{\Delta \text{lat}}{\Delta \text{lon}}$ (≠ spherical distance).

$^4$see footnote on page 63
3.2 Climatology of long straight zonal jets

An illustration of such a jet is shown in Fig. 3.3 (left panel). It is an example of a 2 pvu contour on an isentropic surface with a straight section (a “jet”) over the Atlantic Ocean. A comparison of this contour with the PV field on the same isentropic surface (right panel) verifies the correct computation of the contour and the proper position of the jet. Note that the region of high wind speed (bold black contour in the right panel) coincides more or less with the region of the jet as obtained from the 2 pvu contour alone. Climatological computations will be presented for medium long jets $L_{\text{min}} = 2000 \text{ km}$; $R_{\text{jet}} = 1.02$ (see sections 3.2.4 to 3.2.6), for very long jets $L_{\text{min}} = 5000 \text{ km}$ and for extremely long jets $7000 \text{ km}/R_{\text{jet}} = 1.005$ (see section 3.2.7).

Since the term “jet” is normally associated with “high wind speed” some remarks to the comparison between the “jet frequency” and the velocity fields are made in section 3.2.5.

3.2.3 Graphical representation of the “jet frequency” (medium long jets: $L_{\text{min}} = 2000 \text{ km}$)

Each jet is represented by a series of points (given as longitude-latitude-pairs) about 30 km apart. This information is projected onto a $1^\circ \times 1^\circ$ grid with the method of grid point filling (this process is illustrated in Fig. 3.4). When at least one point on the jet gets closer to a grid point than 0.5°, the value of the “jet frequency” at that grid point increases by one. All “jet frequency” plots presented here (except for Fig. 3.3) are superpositions of all jets within a specified
3.2 Climatology of long straight zonal jets

Figure 3.4: Schematic diagram of grid point filling. The open circles denote grid points, the bold black lines represent three jets, and the grey shadings/hatch styles are a measure for the "jet frequency" at each grid point (white=0, hatched=1, light grey=2, dark grey=3). See text for explanations.

It has been noted earlier that the contours and the jets were computed on isentropic surfaces every 5 K from 280 K to 350 K. As can be seen in Fig. 3.5 the 2 pvu isoline slopes downward towards the equator, intersecting isentropes from about 380 K at the equator to 300 K near the poles. Isentropes below 300 K normally do not transect the tropopause and therefore almost no continuous 2 pvu contour around the globe can be found on these levels and consequently no straight sections.

3.2.4 Geographical distribution and amplitude of the "jet frequency" (medium long jets: \( L_{\text{min}} = 2000 \text{ km} \))

It has been noted earlier that the contours and the jets were computed on isentropic surfaces every 5 K from 280 K to 350 K. As can be seen in Fig. 3.5 the 2 pvu isoline slopes downward towards the equator, intersecting isentropes from about 380 K at the equator to 300 K near the poles. Isentropes below 300 K normally do not transect the tropopause and therefore almost no continuous 2 pvu contour around the globe can be found on these levels and consequently no straight sections.

The climatological mean geographical distribution of the (normalized) frequency of medium long jets for all 15 months (Jan 86-99 and Dec 99) is shown in Fig. 3.6 for \( \theta \) from 305 K to 345 K. The dominant pattern on all levels (de-
SEVERE CYCLONES AND JETS  3.2 Climatology of long straight zonal jets

Figure 3.5: Vertical distribution of potential temperature zonal means (solid contours) for January 1993. Isentropes intersect the tropopause (2 pvu isoline, heavy black contour) at \( \sim 320 \) K in mid-latitudes and at \( \sim 350 \) K at 30\(^\circ\)N. The dashed contours are temperature [in K] (from Holton et al. (1995)).

spite quantitative differences) is the three maxima over eastern Asia/western Pacific Ocean (referred to as maximum 1), eastern North America/western Atlantic Ocean (maximum 2) and northern Africa/Middle East (maximum 3). Due to the poleward slope of the tropopause the jets on higher isentropic surfaces are situated further south. With increasing \( \theta \) the maxima shift slightly to the west. In contrast to the maxima 1 and 2 the one over northern Africa is not particularly pronounced for \( \theta < 325 \) K. Above that level its relative amplitude increases rapidly and becomes comparable to the dominant maximum over Japan (see also Table 3.1). The minimum between the maxima 1 and 3 in the Himalaya region almost disappears at 345 K and the two maxima join to form a relatively narrow band of high “jet frequency” which spans two-thirds of the globe at about 30–35\(^\circ\)N. The maximum amplitudes of the three maxima on each level are summarised in Table 3.1.

<table>
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<td>7</td>
<td>11</td>
<td>13</td>
<td>15</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3.1: Approximate amplitude of the three “jet frequency” maxima in Fig. 3.6 (in % as described in section 3.2.2). For discussion see section 3.2.4.

The “jet frequency” minima over the eastern North Pacific and eastern North Atlantic are extremely pronounced. There are rarely long jets in these regions
Figure 3.6: "Jet frequency" of medium long jets for all 15 months from $\theta=305\,\text{K}$ to $\theta=345\,\text{K}$. For the interpretation of the values see section 3.2.3. Here the threshold values for the jet definition are $L_{\text{min}} = 2000\,\text{km}$, $R_{\text{jet}} = 1.02$.

and their meridional variability is very large. This prevents a band of high "jet frequency" from spanning the entire globe. The entire "jet frequency" distribution is a spiral-like pattern beginning in the eastern North Atlantic and ending at about the same longitude but $10^\circ$–$15^\circ$ further north.

For each month and for all 15 months of this climatology the temporally av-
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3.2 **Climatology of long straight zonal jets**

![Diagram showing climatological distributions](image)

**Figure 3.7**: Comparison between the climatological distributions of the “jet frequency” and the mean wind speed (all 15 months). “Jet frequency” and \( VEL = (u^2 + v^2)^{1/2} \) (contours from 30 ms\(^{-1}\) (dashed), spacing 10 ms\(^{-1}\)) on 320 K and 350 K. The corresponding plots for all \( \theta \)-levels can be found in Appendix B.

Averaged velocity field \( VEL = (u^2 + v^2)^{1/2} \), where \( u \) and \( v \) are the zonal and the meridional wind components, respectively, has been computed on the isentropic surfaces and compared with the “jet frequency” distributions (the diagrams of the 15-month-means and of all individual months on all levels are listed in Appendix B). The \( VEL \) fields averaged over all months match reasonably well the corresponding “jet frequency” patterns (see Fig. 3.7). Also for \( VEL \) the western Pacific maximum is the most dominant one, but for this field the maximum over the western Atlantic/North America is evident even on lower isentropes. The “jet frequency” maxima are shifted slightly upstream when compared with the \( VEL \) maxima. Above about 325 K the “jet frequency” and the \( VEL \) maxima in this region dissociate more and more from each other. Unlike for the “jet frequency” fields there is no isolated \( VEL \) maximum in northern Africa. As will be seen in the section about inter-annual variability the congruence of the two fields for individual months is much worse than for the 15 months climatology.

Hoskins and Hodges (2002) inter-compared the fields of a great number of low level and upper level variables which are linked with the Northern hemispheric storm tracks. A spiral of high storm activity around the globe was proposed similar to the “jet frequency” spiral on high levels. The very pronounced maxima in the western North Pacific and in the western North Atlantic reappear in all fields (an example is shown in Fig. 3.8). The maximum over northern Africa and the eastern Mediterranean which is not present in the upper level \( VEL \) mean (but in the “jet frequency” fields), is picked out by some variables which emphasise the smaller synoptic scales, such as \( V_{850} \) and \( \xi_{850} \) (\( VEL \) and relative vorticity at
3.2 Climatology of long straight zonal jets

850 hPa and $\omega_{500}$ (Omega at 500 hPa).

The deviance of the “jet frequency” distribution from other variables characterising jets (particularly from the wind speed $V_E L$) is not a weakness of the present study. The contrary is true. The “jet frequency” contains information of several variables and is customised to look at long and straight jets which cannot be described equally accurate by other fields.

In general the amplitude of the “jet frequency” is smaller at lower levels. Fig. 3.9 shows the integral of the “jet frequency” over all grid points on each $\theta$-level. In the diagram the quantity on the x-axis is defined as

$$\text{“jet area”} = \sum_{\text{all grid points}} \text{(jet frequency)} \times \text{(area represented by grid point)} \ (3.1)$$

This is equivalent to the total area that all jets cover. Now consider the “jet area” average on each $\theta$-level (black open circles). For medium long jets (upper panel) there is about three times more “jet area” on 350 K than on 300 K, but almost 50 times more for the same comparison for very long jets (lower panel). On higher levels the jets are generally longer and therefore the ratio of the “jet area” for medium and very long jets on the same level decreases with increasing $\theta$ (from a factor of 10 at 300 K to a factor less than 2 at 350 K).

3.2.5 Inter-annual variability

The inter-annual variability (i.e. the year-to-year differences between the particular months) is substantial in both the geographical distribution and the amplitude.

As can be seen in Fig. 3.9 the “jet area” varies not only from low to high levels
but also from year to year. The differences are larger at high levels. There is a spectacular systematic difference between the years before and after 1996. From 1997 onwards the “jet-area” is substantially larger than in the Januaries before.

Figure 3.9: Sum of the “jet frequency” over all grid points (“jet area”, see equation (3.1)) for medium long jets $L_{\text{min}} = 2000$ km; $R_{\text{jet}} = 1.02$ (upper panel) and for very long jets $L_{\text{min}} = 5000$ km; $R_{\text{jet}} = 1.005$ (lower panel). Each bar corresponds to the “jet area” of one month at one $\theta$-level (same grey shading for the same month at different levels). The open circles denote the average value at one level.
Since this cannot be explained by different data sources or models, it can be a random accumulation or a real phenomenon. It will be discussed later in section 3.2.7 that the “long jet episodes” show a similar accumulation for the years 97–99 (cf. Table 3.2). It could hence be argued that the lifetime of very long jets has increased rather than their number, especially in the western Pacific. However, we do not speculate about a possible trend but it might be worth investigating this in greater detail.

![Figure 3.10: Year-to-year variability: examples. Upper panels: “jet frequency” on the 320 K isentrope for January 87 and December 99, lower panels: “jet frequency” on the 330 K isentrope for January 92 and January 98. See text for discussion. The corresponding monthly pictures for all months at all levels are in Appendix B.](image)

On “jet frequency” plots of individual months maxima and/or minima can be absent entirely in one month or can be very distinctive in another month. The only feature apparent in every single month and at every level is the “jet
frequency” maximum over eastern Asia/western Pacific. In regions with relatively low “jet frequency” (i.e. eastern Pacific/Atlantic) the variability is generally largest. Fig. 3.10 shows the “jet frequency” distribution for two particular months at the same level but with strongly different patterns. The upper two panels ($\theta=320 \text{K}$) differ substantially over the Atlantic. Whereas almost no jet reached further east than 30°W in Jan 87, numerous jets span the entire Atlantic in Dec 99. A more systematic study of the “jet frequency” in the eastern North Atlantic is made in section 3.2.6. Again both panels show the three maxima and the two other minima despite a generally lower amplitude for Jan 87. The lower two panels are for two individual months on the 330 K isentropic surface. In Jan 98 the “jet frequency” minimum over the Himalaya is absent and the band of high values is coherent over a zonal distance of about 250°, from 20°W to 130°W. Somewhat unusual in Jan 92 are high values equatorward of 20°N near the Atlantic coast of Africa and over southeastern Asia.

![Figure 3.11: Comparison between monthly “jet frequency” distributions and the mean wind speed. Monthly means of the “jet frequency” and average wind speed ($V_{EL}$ from 40 ms$^{-1}$, contour interval 10 ms$^{-1}$) for Jan 1995 at 320 K and Jan 1989 at 335 K. The corresponding pictures for all months at all $\theta$-levels can be found in Appendix B.](image)

As mentioned in section 3.2.4 the “jet frequency” field and the temporal mean of the velocity for individual months differ quite a lot from each other. Apart from the western Pacific where there is always high “jet frequency” and strong winds, the allocation of high “jet frequency” with high wind speed is not straightforward. Although it is often true we can neither say that a high “jet frequency” implies a strong mean wind (see the eastern Mediterranean in the left panel of Fig. 3.11) nor does a high mean wind speed imply many straight jets (see NE Pacific and NE Atlantic in Fig. 3.11, right panel). In general the wind maxima match better the “jet frequency” maxima at low levels.
Massacand (1999) computed northern hemispheric $PV$ distributions at 320 K for the Januaries 1980–1996. The distributions of the $PV$ gradient given therein (Appendix B) partly match the “jet frequency” for the corresponding month presented here. However, there are also differences similar as the ones to the $VEL$ means.

### 3.2.6 Correlations “jet frequency” ↔ NAO index

The North Atlantic Oscillation (NAO) is the dominant mode of winter climate variability in the North Atlantic region ranging from central North America to Europe and even into Northern Asia. It is an index for a large scale seesaw in atmospheric mass between the subtropics and the polar region. The NAO index is defined as the normalized pressure difference between a station on the Azores and one on Iceland. An extended version of the index can be derived for the winter half of the year by using a station in the southwestern part of the Iberian Peninsula (Hurrell 1995). Here we use the NAO index calculated from data for SW Iceland (Reykjavik) and Gibraltar (Jones et al. 1997), see Fig. 3.12, left panel.

The positive NAO index phase connotes a stronger than usual subtropical high pressure centre and a deeper than normal Icelandic low. This is linked to more and stronger winter storms crossing the Atlantic Ocean on a more northerly track and to relatively warm and wet conditions in northern Europe and cold and dry winters in northern Canada and Greenland. On the other hand the negative NAO phase goes along with a weak subtropical high and a weak Icelandic low. The consequences are fewer and weaker winter storms crossing on a more west-east pathway. They bring moist air into the Mediterranean and cold air to northern Europe (for an overview see e.g. http://www.ldeo.columbia.edu/NAO/).

The baroclinicity and storm intensity/frequency over the Atlantic Ocean are closely related to the NAO pattern, and a link can also be expected for the “jet frequency”. Note however that the NAO index is a signal measured at the ground whereas the motion of weather systems and the occurrence of jets happens at higher altitudes. The 15-month time series (Jan 86–99 plus Dec 99) of the NAO index is therefore compared with the corresponding time series of the “jet frequency” in different regions (more precisely, the sum of the “jet frequency” values over all grid points in a box of $10^\circ \times 10^\circ$). The right panel of Fig. 3.12 illustrates the good correlation of the two quantities for a particular region in the eastern North Atlantic. The correlation coefficient has been computed for each $10^\circ \times 10^\circ$ box between the equator and $80^\circ$N. In Fig. 3.13 each $10^\circ \times 10^\circ$ box is coloured with the corresponding correlation coefficient. The correlation is significant on a 95% level if the absolute value of the correlation coefficient exceeds...
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Figure 3.12: Left panel: Monthly means of the NAO index 1985–2000 (grey lines, according to Jones et al. (1997)). The black asterisks denote the 15 values for the Januaries 86–99 and December 99. Right panel: NAO index (solid line) and “jet frequency” (dashed line) in the box 10°–20°W × 40°–50°N. The correlation coefficient between the two time series is 0.703.

0.5. On the composite for the isentropes from 305 K–320 K a dipole pattern can be recognised: a region in the northeastern Atlantic/ northwestern Europe with a strongly positive NAO correlation signal and a region over Greenland where the “jet frequency” correlates negatively. This result seems reasonable. Jets over central and northern Greenland are expected only when the Icelandic
3.2 Climatology of long straight zonal jets

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low is weak (negative NAO phase). On the other hand jets can only reach far to the east into northwestern Europe when the Icelandic low is deep and the pressure gradient over the entire Atlantic is strong. In addition, an analogue procedure of computing correlation coefficients has been made for the frequency of the whole 2 pvu contours (“contour frequency”). Most of the highly (anti- ) correlating regions for the jets also appear in the corresponding picture for the entire contours. This means that it is not the frequency of long jets that is sensitive to the phase of the NAO but rather the (general) location of the contours (i.e. the tropopause). This is confirmed by the corresponding correlation plots (not shown) of very long jets\(^6\) \( (L_{\text{min}} = 5000 \text{ km and } R_{\text{jet}} = 1.005; \) see section 3.2.7). For these jets no regions with significant correlations could be identified. In essence it is not the frequency of long jets that is correlated to the NAO index but the location of the 2 pvu contour.

Above \( \sim 320 \text{ K} \) the correlation signals disappear. There are only a few \( 10^{\circ} \times 10^{\circ} \) boxes with significant correlations, but they are all isolated. Again, below 320 K the different distributions of the “jet frequency” during either NAO phase are evident even “by eye”. The three-month-composites with the lowest monthly NAO index (Jan 87, 96 and 97, see Fig3.12) and the three months with the highest NAO index (Jan 89, 90 and 93) are shown in Fig3.14. The most remarkable features are consistent with the picture drawn for the correlations. First consider the \( \theta = 310 \text{ K} \) level (upper two panels): The diagram of the low NAO composite shows a relatively high “jet density” over Greenland and a large region with little jet activity in the eastern Atlantic. For the high NAO composite the large number of straight Atlantic jets extending to Britain and Scandinavia is evident. The same is also true but somewhat less pronounced on the \( \theta = 320 \text{ K} \) level (lower two panels).

3.2.7 Very long jets

In the previous sections we have looked predominantly at medium long jets (length > 2000 km). The large number of such jets provided a good picture of the general properties of the “jet frequency”. To respond to the question posed in the introduction related to the ‘Lothar’ jet, we now restrict the search to very long jets.

“Jet frequency” : jets longer than 5000 km

In contrast to the previous sections more stringent conditions have been applied to the jets’ length and rectilinearity. We picked out all jets with a minimum length

\(^6\)recall that the minimum length of the jets in the previous sections is ‘only’ 2000 km.
3.2 Climatology of long straight zonal jets

Regions of high “jet frequency” in these diagrams are candidates for a comparison to the ‘Lothar’ jet. However, the depictions do not contain any information about the duration or stationarity of such a jet. Each of the four months shown in Fig. 3.15 includes one of the “top nine” episodes of the longest (spatially and temporally) jets (discussed in the next paragraph). The striking straight line over the Atlantic in Dec 99 is the successfully mapped ‘Lothar’ jet (see also the example in Fig. 3.3). The other panels have their most notable features in eastern
3.2 Climatology of long straight zonal jets

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Figure 3.15: “Jet frequency” of jets longer than 5000 km and with $R_{jet} = 1.005$ on the 315 K isentropic surface.

Asia and in the Pacific Ocean.

The longest jet episodes

The previous sub-sections showed the geographical distributions of monthly mean “jet frequencies”. Here we seek single episodes of extremely long and also long lasting jet events. To this end we apply the following criterion:

Within 24 hours (four output times) at least two jets longer than 7000 km (and $R_{jet} = 1.005$) must exist on either the $\theta=310\,\text{K}$ or the $\theta=315\,\text{K}$ level at a similar geographical location.
The 310 K isentrope is a relatively low level for a closed 2 pvu contour, and the contours at this level are quite strongly undulated. If a very long straight section is present at 310 K it is possible that this is a deep tropopause “wall” as it was the case for the ‘Lothar’ jet. The same is true for the 315 K isentrope but less strict. Table 3.2 summarises the dates, locations, durations and frequencies of the “top nine events” according to the criterion above.

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</tr>
<tr>
<td>VI</td>
<td>19-22 Jan 98</td>
<td>85..170</td>
<td>30-35</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>320K only. E-As./W-Pac.</td>
</tr>
<tr>
<td>VII</td>
<td>15-21 Jan 92</td>
<td>90..180</td>
<td>30-40</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>320K only. E-As./W-Pac.</td>
</tr>
</tbody>
</table>

Table 3.2: “Top nine longest jet events” according to the framed criterion above. The longitude is in °E, the latitude in °N. In the columns “310”, “315” and “320” (θ-levels in K) are the numbers of jets longer than 7000 km which were detected on the corresponding θ-level during the episode.

In this listing the ‘Lothar’ jet (case I) is outstanding for several reasons. It is...

- the case with the **most** detected jets on every single level.
- one of only two cases with 7000 km long jets on **all three levels**, i.e. large congruence of the contour shape on these levels (strong PV gradients on all levels).
- one of only two cases in the **Atlantic** rather than in the Pacific Ocean.
- the **longest lasting** event.

\[\text{the longitude of their start points and end points, respectively, differ only by a few degrees}\]
3.2 Climatology of long straight zonal jets

SEVERE CYCLONES AND JETS
In order for an episode to be fully comparable to the 'Lothar' case, a relatively small-scale low-level cyclone must exist near the jet axis, move underneath it to form a $PV$ tower and induce very strong winds. The nine jet cases were therefore examined for such low-level systems. The geopotential height $Z$, its gradient $|\nabla_p Z|$ and the wind speed at 850 hPa were used as indicators for low level cyclones. Only jet case II turned out to fulfill these criteria. There was a long, straight and strong jet for about two days, the contours are almost congruent from 310 K to 320 K with very strong $PV$ gradients, and a low level $PV$ anomaly is situated almost exactly underneath the jet axis at 176°E/41°N on 06 UTC 10 Jan 1999 (see Fig. 3.16, upper and middle rows). When we look at the two vertical cross sections (two panels in the middle) the similarities to 'Lothar' are obvious (cf. the corresponding diagrams for 'Lothar' on the bottom panels). The deep tropopause fold reaches down to almost 550 hPa and is vertically aligned with the low-level $PV$ anomaly. Its core value is >1.5 pvu at 850 hPa. The induced wind field has a very large horizontal gradient and is very strong to the south of the "PV-tower" (30–40 ms$^{-1}$ at 850 hPa). In the tropopause fold there are (adiabatic) downdrafts of very dry stratospheric air due to the tilt of the isentropes whereas diabatic heating produces updrafts to the north above the low level system. However, in the ‘Lothar’ case (Fig. 3.16 bottom panels) all these features were more extreme, i.e. the low level $PV$ is stronger in amplitude and further extended vertically, the tropopause fold deeper and narrower, the up- and downdrafts stronger, the low level winds more severe etc. Also the life cycle of the low level cyclone could not be proved to be similar to 'Lothar' (slow approach to the jet axis from the south). Nevertheless jet case II is the only case in the 15 investigated months which is “lotharlike” to a certain degree.
3.2.8 Summary

The distribution of the northern hemispheric "jet frequency" (as defined in section 3.2.2) has been evaluated and provided an answer regarding the rarity of the pre-
'Lothar' jet.

In the mean the frequency of medium long jets has three distinctive maxima located at about 120°–140°E, 100°–80°W and 0°–40°E. The first maximum is the most pronounced on all levels and at all times. The latitude depends on the isentropic surface. The inter-annual variability is large, especially in the regions of low "jet frequency" in the eastern North Pacific and the eastern North Atlantic. A correlation between the "jet frequency" in the Atlantic and the NAO index was found for $\theta \leq 320$ K. In comparison with other persistent very long jets (longer than 5000 km or 7000 km) the 'Lothar' jet is remarkable for its duration, depth and location. Among nine extreme jet events which fulfilled a set of criteria, seven were in the Pacific and only one was accompanied with a low level cyclone and therefore comparable with 'Lothar'. Thus the stark inference is that the 'Lothar' jet was a very rare event. No similar jet was detected during the Januaries 86-99. The "jet frequency" field defined in this study is one more variable to describe upper-tropospheric features. In contrast to others it can be related via the knowledge we obtained to low-level cyclones and gains increased significance.
Chapter 4

Stratified flow over topography with time-dependent surface boundary condition

4.1 Introduction and motivation

Theoretical studies of atmospheric flow over orography have focussed primarily on the steady state setting. Consideration has been given to a range of effects including the strength and vertical shear of the incident flow, the geometry of the obstacle, the stratification of the fluid, and the influence of the earth's rotation (see the extensive overview in section 1.3 from page 6). Here a study is undertaken of one particular unsteady configuration: an air-stream of uniform flow and stratification over a two-dimensional and diurnally-oscillating terrain \( h(x, t) = h_0(x)(1 - \alpha \sin(\omega_0 t)) \) for two different types of terrain shapes \( h_0(x) \).

Smith et al. (2002) proposed that only that part of a mountain is exposed to an incident flow which is above a stagnant layer in the valleys. In effect the problem of an oscillating lower boundary layer constitutes a simple and highly idealised representation of the daily evolution of such planetary boundary layer in mountainous regions.

4.2 Derivation of the wave equation

The model atmosphere is a stably stratified (constant Brunt-Väisälä frequency \( N \)), inviscid, hydrostatic Boussinesq fluid on an \( f \)-plane Cartesian coordinate system \((x, y, z)\). The basic state flow is a uniform and constant zonal velocity
field $U$. The linearised dynamics of a slightly perturbed hydrostatic flow satisfy (see e.g. Bannon and Zehnder (1985))

$$
\mathcal{D}u - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x} \tag{4.1}
$$

$$
\mathcal{D}v + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y} \tag{4.2}
$$

$$
\frac{1}{\rho_0} \frac{\partial p}{\partial z} = \frac{g \rho}{\rho_0} \tag{4.3}
$$

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4.4}
$$

$$
\mathcal{D}_\rho = \frac{\rho_0 N^2}{g} w \tag{4.5}
$$

where $(u, v, w)$, $p$ and $\rho$ are the perturbation velocity, the perturbation pressure and the perturbation density fields. $\rho_0$ is a reference density, $f$ the Coriolis parameter, $N^2 = -(g/\rho_0) d\rho_S/dz$ the square of the buoyancy frequency associated with the stable basic state density stratification $\rho_S$ and $\mathcal{D} \equiv \frac{\partial}{\partial t} + U \frac{\partial}{\partial x}$ is the advection operator.

This system of equations can be condensed to a single partial differential equation for one dependent variable with the following sequence of operations: Take $\partial_x(4.1) + \partial_y(4.2)$ to obtain

$$
\mathcal{D}(u_x + v_y) - f(v_x - v_y) = -\frac{1}{\rho_0} (p_{xx} + p_{yy}) = -\frac{1}{\rho_0} \nabla_h^2 \rho \tag{4.6}
$$

likewise $\partial_x(4.2) - \partial_y(4.1)$ results in

$$
\mathcal{D}(v_x - u_y) + f(u_x + v_y) = 0 \tag{4.7}
$$

Apply $\mathcal{D}$ to (4.6) and use (4.7) and (4.4) to get

$$
(\mathcal{D}^2 + f^2)(u_x + v_y) = -(\mathcal{D}^2 + f^2)w_z = -\frac{1}{\rho_0} \nabla_h^2 \rho. \tag{4.8}
$$

Finally $\partial_z(4.8)$ and substitution using (4.3) and (4.5) yields

$$
(\mathcal{D}^2 + f^2)w_{zz} = \frac{1}{\rho_0} \nabla_h^2 \mathcal{D} p_z = -\frac{1}{\rho_0} \nabla_h^2 \mathcal{D} g \rho = -\nabla_h^2 N^2 w \tag{4.9}
$$

$$
(\mathcal{D}^2 + f^2)w_{zz} + N^2 \nabla_h^2 w = 0 \tag{4.10}
$$

The vertical velocity $w$ and the vertical displacement $\eta$ of an air parcel are
connected via \( w = \mathcal{D}\eta \) and the wave equation (4.10) becomes

\[
(\mathcal{D}^2 + f^2)\eta_{zz} + N^2 \nabla_h^2 \eta = 0 \tag{4.11}
\]

and in two spatial dimensions (\( \frac{\partial}{\partial y} \equiv 0 \))

\[
(\mathcal{D}^2 + f^2)\eta_{zz} + N^2 \eta_{xx} = 0. \tag{4.12}
\]

### 4.3 Boundary conditions

The model atmosphere is vertically unbounded. The upper boundary condition requires that energy radiates upwards, i.e. the phase lines tilt upstream and the phase speed in the system of an air parcel is directed downwards. This implies that the vertical and the horizontal wave numbers are of same sign (see also section 4.5). The lower boundary is stipulated to be a two dimensional orographic feature with a harmonic time dependency. If we assume a diurnally evolving boundary layer (Smith et al. 2002) the boundary of the free atmosphere is represented by “summits” of constant heights and by “valleys” in between with a time-dependent depth (see Fig. 4.1):

\[
h(x, t) = h_0 \cos k_0 x \cdot (1 - \alpha \sin \omega_0 t) + h_0 \alpha \sin \omega_0 t \tag{4.13}
\]

This is a harmonic oscillation of the orography in \( x \) and time plus a time-dependent shift in the \( z \)-direction (but independent of \( z \)). We reduce the problem to the pure oscillation only\(^1\). In particular two cases are considered: a sinusoidal

\[\text{(\textsuperscript{1})later in this chapter solutions are derived for the vertical displacement of the flow over such an orography. To obtain the solution for (4.13) we must simply add the term } h_0 \alpha \sin \omega_0 t \text{ to the full solutions for (4.14).}\]
4.3 Boundary conditions

STRATIFIED FLOW OVER TOPOGRAPHY...

Figure 4.2: Time-dependent orography. The full line corresponds to the mean height $h_0$ of the orography, the dashed lines to $(1 \pm \alpha)h_0$ (here $\alpha = 0.5$) at the times $t = 6\,h$ and $t = 18\,h$. The upper panel is the harmonic orography (4.14) and lower panel the isolated hill (4.15), each with their characteristic widths $a$ and $2\pi/k_0$, respectively.

topography (see Fig. 4.2 top)

$$h(x, t) = h_0 \cos k_0 x \cdot (1 - \alpha \sin \omega_0 t) \quad (4.14)$$

and an isolated bell-shaped hill (see Fig. 4.2 bottom)

$$h(x, t) = \frac{h_0}{1 + (x/a)^2} (1 - \alpha \sin \omega_0 t). \quad (4.15)$$

The natural frequency of the oscillation is $2\pi$/day and the horizontal extension ($a$ or $2\pi/k_0$) of the orography is set to typical values for mountains where explicit computations are performed. The maximum height of the mountain is $h_0(1 + \alpha)$. Linear theory is valid for small orography only. The criterion is that the so-called non-dimensional mountain height$^2$ $F \equiv NH/U < 1$. For larger $F$ the streamlines computed with linear theory cross each other which is obviously unphysical. In the present case the restriction for linearity is therefore more stringent than for a non-oscillating orography:

$$\frac{N h_0 (1 + \alpha)}{U} < 1 \quad (4.16)$$

The lower boundary condition assumes that the flow follows the terrain at the ground, i.e.

$$\eta(x, z = 0, t) = h(x, t). \quad (4.17)$$

$^2$also referred to as inverse Froude number, cf. the definition and discussion in chapter 2 from page 18
4.4 Strategy

The strategy adopted to derive the solution for $\eta(x, y, t)$ from (4.12) is standard (see e.g. Smith (1980) or Bannon and Zehnder (1985)). The flow chart (Fig. 4.3) gives an overview over the procedures. The Fourier transform $\hat{f}(\omega)$ of the function $f(x)$ is defined as

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

(4.18)

First the wave equation (4.12) is Fourier transformed with respect to $x \rightarrow k$ and $t \rightarrow \omega$ to get

$$-(\omega + Uk)^2 + f^2)\hat{\eta}_{zz} - N^2 k^2 \hat{\eta} = 0$$

(4.19)

and finally

$$\frac{\hat{\eta}_{zz}}{\hat{\eta}} = -\frac{N^2 k^2}{(\omega + Uk)^2 + f^2} \equiv -n(k, \omega)^2$$

(4.20)

The solution of (4.20) for $\hat{\eta}(z)$ is simply the exponential function

$$\hat{\eta}(k, z, \omega) = \hat{h}(k, \omega) e^{i n(k, \omega) z}$$

(4.21)

where $\hat{h}(k, \omega)$ is the Fourier transform with respect to $x$ and $t$ of the lower boundary condition (4.17).

Formally the full solution for the vertical displacement $\eta(x, z, t)$ in real space is the inverse Fourier transform of (4.21) with respect to $k$ and $\omega$

$$\eta(x, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i n(k, \omega) z} \cdot \hat{h}(k, \omega) \cdot e^{ikx+i\omega t} d\omega dk$$

(4.22)

4.5 Solution for the harmonic orography

For the case of the harmonic orography (4.14) the double Fourier transform $\hat{h}(k, \omega)$ of the mountain shape is

$$\hat{h}(k, \omega) = \frac{h_0}{2} (\delta(k - k_0) + \delta(k + k_0)) \cdot \left(\delta(\omega) + \frac{i\alpha}{2}(\delta(\omega - \omega_0) - \delta(\omega + \omega_0))\right)$$

(4.23)
where $\delta(\omega)$ is Dirac's $\delta$-function. Insertion into the formal solution (4.22) yields the following three-term expression for the vertical displacement $\eta$

$$
\eta(x,z,t) = h_0 e^{ik_0 x} \cdot e^{iz \left( \frac{N^2 k_0^2}{U^2} \right)^{1/2}} + \frac{i\alpha h_0}{2} e^{-i\omega_0 t} \cdot e^{ik_0 x} \cdot e^{iz \left( \frac{N^2 k_0^2}{(\omega_0 + U k_0)^2} \right)^{1/2}} - \frac{i\alpha h_0}{2} e^{i\omega_0 t} \cdot e^{ik_0 x} \cdot e^{iz \left( \frac{N^2 k_0^2}{(-\omega_0 + U k_0)^2} \right)^{1/2}} \quad (4.24)
$$

$$
\equiv \eta_1(x,z) + \eta_2(x,z,t) + \eta_3(x,z,t).
$$

The signs of the vertical wave number are chosen such that the upper boundary condition is fulfilled i.e. $\text{sign}(k_0) = \text{sign}(n)$. The vertical wave number $n$ is taken
Figure 4.4: Flow over cosine-shaped orography with harmonic time-dependency (visualisation of (4.24)). Upper four panels: Streamlines, lower four panels: lines of equal phase. The horizontal axis is normalised to the horizontal wave length $\lambda_x = 2\pi/k_0$ of the orography, the vertical axis is height (the vertical wave length $\lambda_z = 2\pi U/N$ is indicated on the z-axis). From top left to bottom right: each of the three wave components $\eta_1$, $\eta_2$, $\eta_3$ and the superposition $\eta$ of them. The horizontal arrows indicate the direction of propagation with time of the wave crests and the phase lines, respectively. The bold solid line denotes the lower boundary. Parameters: $k_0 = 2\pi/200$ km, $N = 10^{-2}$ s$^{-1}$, $U = 10$ ms$^{-1}$, $\omega_0 = 2\pi$/day, $h_0 = 400$ m, $\alpha = 0.4$, $f = 10^{-4}$ s$^{-1}$, $t = 18$ h.
4.5 Harmonic orography

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from (4.20). Since \( e^{-ik_0x-inz} \) describes the same wave as \( e^{+ik_0x+inz} \), the term \( \eta_2 \) with \( \text{sign}(\omega_0, k_0, n) = (+ + +) \) implicitly also includes the wave with the signs \((- - -)\) of the wave numbers \((\omega_0, k_0, n)\). For \( \eta_1 \) it is \((0 + +)\) and \((0 - -)\) and for \( \eta_3 \) it is \((- + +)\) and \((+ - -)\). All these relationships are taken into account with a factor 2 in (4.24).

Thus the vertical displacement (4.24) is a superposition of three waves \( \eta_i \). \( \eta_1 \) is the (time-independent) standard solution for an overflow over a cosine-shaped surface without time dependency and with orography height \( h_0 \) (Smith 1980). \( \eta_2 \) and \( \eta_3 \) are the modifications due to the oscillation attributable to "Doppler shifted" waves\(^3\).

### 4.5.1 Properties of the three waves

Fig. 4.4 shows the streamlines (upper four panels) and the phase lines (lower four panels) of the flow field (derived from (4.24)). The full solution (bottom right) is composed of a steady state (top left) and two ‘travelling’ waves. The arrows denote the direction into which the whole pattern travels with time. As a consequence of the oscillation there are regions with a high density of streamlines (i.e. increased wind speed) moving upwards with time along the streamlines of the steady state solution. In Table 4.1 a few mathematical properties of the steady state and the Doppler shifted waves are summarised. The main message is that such a flow can transport energy upwards for a wider range of horizontal wave numbers \( k_0 \) than without oscillation. Fig. 4.5 is a plot of the vertical phase

<table>
<thead>
<tr>
<th>Steady State ( \eta_1 )</th>
<th>Travelling waves ( \eta_2, \eta_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega = U k_0 )</td>
<td>( \omega = U k_0 \pm \omega_0 )</td>
</tr>
<tr>
<td>( n^2 = \frac{N^2 k_0^2}{U^2 k_0^2 - f^2} )</td>
<td>( n^2 = \frac{N^2 k_0^2}{(U k_0 \pm \omega_0)^2 - f^2} )</td>
</tr>
<tr>
<td>( c_{pz} = -\frac{U}{N} \sqrt{U^2 k_0^2 - f^2} )</td>
<td>( c_{pz} = -\frac{U k_0 \pm \omega_0}{N k_0} \sqrt{(U k_0 \pm \omega_0)^2 - f^2} )</td>
</tr>
</tbody>
</table>

Table 4.1: Comparison of the frequency (in the system of a air parcel), the vertical wave number and the vertical phase speed of the three waves of (4.24).

\(^3\)recall the footnote on page 85: the solution for (4.13) is then (4.24) + \( h_0 \alpha \sin \omega_0 t \)
Harmonic orography

\[ R_0 = 1 - \frac{\omega_0}{f} \]

\[ R_0 = 1 \]

\[ R_0 = 1 - \frac{\omega_0}{f} \]

\[ k_{c1} \]

\[ k_{c2} \]

\[ k_{c3} \]

\[ k_0 \quad [10^{-5} \text{ m}^{-1}] \]

\[ c_{pz} \]

\[ 0.01 \]

\[ 0.02 \]

\[ 0.03 \]

\[ \text{[m s}^{-1}] \]

Figure 4.5: Vertical phase speed versus orography wave number. The solid line and the dotted line just below it correspond to the steady state solution, the dash-dotted and the dashed line (and their associated dotted lines) to the travelling waves \( \eta_2 \) and \( \eta_3 \) respectively (see (4.24) and Table 4.1). The parameters \( U, N \) and \( \omega_0 \) are the same as in Fig. 4.4. Solid/dashed/dash-dotted lines: \( f = 10^{-4} \text{s}^{-1} \), dotted lines: \( f = 0 \). Arrows show the values of the associated Rossby number in the rotational case below which each of three waves stops propagating.

speed \( c_{pz} \) versus the horizontal wave number \( k_0 \) determined by the orography. A gravity wave can propagate vertically if

\[ n(k_0)^2 > 0 \Leftrightarrow c_{pz}(k_0) < 0 \tag{4.25} \]

This means that, if the separation \( 2\pi/k_0 \) of two mountain crests is too large, no gravity waves are generated but the perturbation decays with height. Whereas (4.25) is true for the steady state (bold solid line) if \( k > k_{c1} = f/U \Leftrightarrow \text{Rossby number} \ 4. R_0 \equiv U k_0/f > 1 \), the flow of the oscillating problem can transport energy vertically for even smaller Rossby numbers. For the vertical wave number of the term \( \eta_3 \) and therefore for the flow over an oscillating lower boundary the condition for the vertical transport of energy by gravity waves is

\[ R_0 > 1 - \frac{\omega_0}{f} \Leftrightarrow \frac{1}{R_0} < 1 + \frac{\omega_0}{U k_0} \tag{4.26} \]

A more quantitative discussion of the energy transport is provided in section 4.7. Table 4.2 indicates a numerical example for (4.26) for a choice of \( (\omega_0, f, U) \) and the implication for the wave length \( \lambda_x \) of the orography. When the orography is oscillating some energy is radiated vertically for much broader hills than without oscillation.

\[ \text{cf. the definition and the discussion of the } R_0 \text{ in chapter 2 from page 18} \]
4.6 Isolated orography

<table>
<thead>
<tr>
<th>( \omega = 0 )</th>
<th>( \omega \neq 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_0 &gt; 1 - \frac{\omega_0}{f} = 0.27 )</td>
<td>( R_0 &gt; 1 )</td>
</tr>
<tr>
<td>( k_0 = \frac{R_0 f}{U} &gt; 2.7 \cdot 10^{-6} \text{ m}^{-1} )</td>
<td>( k_0 &gt; 10^{-5} \text{ m}^{-1} )</td>
</tr>
<tr>
<td>( \lambda_x = \frac{2\pi}{k_0} &lt; 2300 \text{ km} )</td>
<td>( \lambda_x &lt; 630 \text{ km} )</td>
</tr>
</tbody>
</table>

Table 4.2: Numerical examples for the relation (4.26). The independent parameters are \( \omega_0 = 7.27 \cdot 10^{-5} \text{ s}^{-1} \), \( f = 10^{-4} \text{ s}^{-1} \) and \( U = 10 \text{ ms}^{-1} \).

The frequency of the oscillation \( \omega_0 \) is set to \( 2\pi \) day for all computations presented above. In order to obtain the results of this section, \( \omega_0 \) must not be arbitrary for a given set of \((k_0, f, U)\). The existence of both “travelling waves” is possible if

\[
\omega_0 < U k_0 - f \quad \text{and} \quad \omega_0 < f. \tag{4.27}
\]

The first condition is required in order for the vertical wave number \( n_3 \) of the third term in (4.24) to be real. With the same argument follows in the limit \( k_0 \to 0 \) that \( \omega_0 \) must be less than the Coriolis parameter \( f \).

4.6 Solution for the isolated orography

The problem to solve is the same as for the cosine-shaped orography in section 4.5: the evaluation of the formal solution (4.22). The Fourier transform of the mountain (4.15) is rather simple

\[
\hat{h}(k, \omega) = h_0 a e^{-\alpha|k|} \left( \delta(\omega) + \frac{i\alpha}{2}(\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) \right) \tag{4.28}
\]

Unlike in the previous set-up (4.22) can not be solved fully analytically. The analogue to (4.24) for the vertical displacement \( \eta(x, z, t) \) is

\[
\eta(x, z, t) \equiv \eta_1(x, z) + \eta_2(x, z, t) + \eta_3(x, z, t) \tag{4.29}
\]
with

\[ \eta_1(x, z) = \int_0^\infty e^{-ak} \cdot e^{ikx} \cdot e^{iz\left(\frac{N^2k^2}{U^2} - f^2\right)} \, dk \] (4.30)

\[ \eta_2(x, z, t) = \frac{i\alpha}{2} h_0 a e^{i\omega_0 t} \int_0^\infty e^{-ak} \cdot e^{ikx} \cdot e^{iz\left(\frac{N^2k^2}{U^2} + \frac{1}{U^2} + f^2\right)} \, dk \] (4.31)

\[ \eta_3(x, z, t) = -\frac{i\alpha}{2} h_0 a e^{-i\omega_0 t} \int_0^\infty e^{-ak} \cdot e^{ikx} \cdot e^{-iz\left(\frac{N^2k^2}{U^2} - f^2\right)} \, dk \] (4.32)

In general an analytical expression for the integrals in (4.30) to (4.32) cannot be given and they have to be evaluated numerically. Since the horizontal wave number spectrum of the isolated hill ranges from 0 to \( \infty \) the vertical wave number has a singularity at a certain critical value \( k_c \) and therefore the integrals cannot be integrated exactly. The integrals are split into two parts

\[ \int_0^\infty (\ldots) \, dk \approx \int_0^{k_c - \epsilon} (\ldots) \, dk + \int_{k_c + \epsilon}^\infty (\ldots) \, dk \] (4.33)

each of which can be computed numerically for a very small \( \epsilon \). For the special case \( f = 0 \) the second of the two integrals on the right hand side of (4.33) can be solved analytically\(^5\) for \( \eta_3 \). Evaluation of this integral both analytically and numerically provides a reasonable value for \( \epsilon \). In analogy to Fig. 4.4 the streamlines and the phase lines are shown for the case of an isolated hill in Fig. 4.6.

Again, as for the case of the harmonic orography, the superposition of the three parts (4.30) – (4.32) gives the flow field at a given time. However, unlike the previous case the regions of high streamline density (high wind speed) are bound to the orography and remain at the same spots for all times. Fig. 4.7 shows again the streamlines superimposed with the corresponding field of vertical velocity \( w \). It is the amplitude of the extrema that changes with time (and not their locations). As the orography height increases, the down-slope winds near the surface get more intense, too. This signal is then passed on to higher levels. If we look at the example shown in Fig. 4.7: the amplitude of the minimum of \( w \) at \( z \approx \lambda_z/4 \) (3.1 km) is largest at \( t = 20 \) h. It takes about two hours until the maximum at \( z \approx 3\lambda_z/4 \) (9.4 km) is strongest and another two hours until the signal reaches the minimum at \( z \approx 5\lambda_z/4 \) (15.7 km).

In the case of a continuous \( k \)-spectrum the oscillation frequency \( \omega_0 \) can in

\(^5\) according to Gradshteyn and Ryzhik (2000) p. 307: \[ \int_0^\infty e^{-\gamma k - \beta/4k} \, dk = \sqrt{\beta\gamma} K_1(\sqrt{\beta\gamma}) \] if \( \text{Re}(\beta) \geq 0 \) and \( \text{Re}(\gamma) > 0 \) where \( K_1 \) is a Bessel Function of the 3\textsuperscript{rd} kind (or Hankel's Function). Here substitute \( \gamma = a - ix \) and \( \beta = 4i\omega_0 z/U^2 \).
4.6 Isolated orography

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Figure 4.6: Flow over bell-shaped orography with harmonic time-dependency (visualisation of (4.30) – (4.32)). Upper four panels: Streamlines, lower four panels: lines of equal phase. The $x$-axis is normalised to the mountain half width $a$. The vertical wave length $\lambda_z = 2\pi U/N$ is indicated on the $z$-axis. From top left to bottom right: each of the three wave components $\eta_1, \eta_2, \eta_3$ and their superposition $\eta$. The bold solid line denotes the lower boundary. The contour spacing in the phase line plots of $\eta_2$ and $\eta_3$ is $\sim 7$ times less than in the other plots. Parameters: $a = 100\text{ km}, N = 10^{-2}\text{s}^{-1}$, $U = 20\text{ ms}^{-1}$, $\omega_0 = 2\pi/\text{day}$, $h_0 = 1000\text{ m}, \alpha = 0.5, f = 0, t = 0$. 

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principle take any value. There is always a Fourier component with a sufficiently large wave number $k$ to allow for travelling waves. However, the amplitude of a travelling component is negligible unless the dominant component $k_d \approx 2\pi/a$ excites a gravity wave. Therefore the restriction (4.27) is eased to

$$w_0 < \frac{2\pi U}{a} - f$$

(4.34)

for an isolated mountain. Numerically for the computations above: $\omega_0 = 7.27 \cdot 10^{-5}s^{-1}$, $f = 10^{-4}s^{-1}$, $2\pi U/a = 6.28 \cdot 10^{-4}s^{-1}$.

## 4.7 Vertical energy transport

Lyra (1943) and Queney (1947) solved the steady-state perturbation equations and recognised that this was not sufficient to give a unique solution. The "correct" solution was obtained by introducing a small Rayleigh-friction that damped the waves. A more fundamental and less laborious way to derive the comparable solution was proposed by Eliassen and Palm (1954): They pointed out that gravity waves can transport wave energy vertically and that the equivalent of the Lyra-Queney solution can be achieved by selecting all those solutions which
4.7 Vertical energy transport STRATIFIED FLOW OVER TOPOGRAPHY...

transport energy upwards and rejecting the ones corresponding to a downward energy transport. This so-called "radiation condition" is physically meaningful since the energy source – the mountain – is at the ground. Eliassen and Palm (1960) derived analytical expressions for the vertical energy flux. For each Fourier component $k$ of the orography the vertical energy flux (averaged over one wave length and denoted by a bar) is

$$\bar{p}w = \frac{U \rho_0}{k^2} w_z w_z$$  \hspace{1cm} (4.35)

where $w$ is the vertical velocity. In terms of the Fourier transform $\check{w}$ it is

$$\bar{p}w = \frac{1}{2} U \rho_0 \frac{n}{k} |A|^2.$$  \hspace{1cm} (4.36)

The asterisk denotes the complex conjugate value and $\text{Im}(\cdot)$ is the imaginary part. If a complex valued function $A$ can be found such that the Fourier transform of the vertical velocity can be written as

$$\check{w} = A(k) \cdot e^{inz}$$  \hspace{1cm} (4.37)

then the vertical energy flux (4.36) becomes

$$\bar{p}w = \frac{1}{2} U \rho_0 \frac{n}{k} |A|^2.$$  \hspace{1cm} (4.38)

The vertical velocity of an air parcel is given by the advection along the stream-lines:

$$w(x, z, t) = D \eta(x, z, t)$$  \hspace{1cm} (4.39)

and its spatial Fourier transform

$$\check{w}(k, z, t) = D \check{\eta}(k, z, t) \equiv (\partial_t + ikU)\check{\eta}(k, z, t)$$  \hspace{1cm} (4.40)

Again we split the solution into three parts as in (4.24) and (4.30) – (4.32). The functions $A_i(k)$ for each part become

$$A_1(k) = i h_0 U k W$$  \hspace{1cm} (4.41)

$$A_2(k) = -\frac{h_0 \alpha}{2} (U k + \omega_0) W$$  \hspace{1cm} (4.42)

$$A_3(k) = \frac{h_0 \alpha}{2} (U k - \omega_0) W$$  \hspace{1cm} (4.43)

The coefficient $W$ is the "weight" of the particular Fourier component. For the cosine-shaped orography it is given by $W = \delta(k - k_0)$ and for the isolated mountain $W = ae^{-ak}$. In order to get the total vertical energy flux we must integrate (4.38) over all wave numbers $k$. 96
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For the harmonic orography we simply add all three parts for the wave number $k_0$ according to (4.38) and get

$$\bar{pw} = \frac{1}{2} \rho_0 \sum_{i=1}^{3} \frac{n_i}{k_0} |A_i(k_0)|^2$$

$$= \frac{1}{2} \rho_0 U N h_0^2 \left[ \frac{U^2 k_0^2}{(U^2 k_0^2 - f^2)^{1/2}} + \frac{\alpha^2}{4} \left( \frac{(U k_0 + \omega_0)^2}{((U k_0 + \omega_0)^2 - f^2)^{1/2}} + \frac{(U k_0 - \omega_0)^2}{((U k_0 - \omega_0)^2 - f^2)^{1/2}} \right) \right]$$

(4.44)

In the case of the isolated hill we integrate only over those wave numbers which allow for waves to propagate vertically according to the condition (4.25), i.e. for $k > f/U$ for the first part and for $k > (f - \omega_0)/U$ and $k > (f + \omega_0)/U$, respectively, for the others. By substitution all three integrals can be reduced to the same integral. The sum of all three contributions eventually leads to

$$\bar{pw} = \frac{1}{2} \rho_0 U N h_0^2 a^2 \left( 1 + \frac{\alpha^2}{2} \cosh \frac{2 \omega_0 a}{U} \right) \int_{f}^{\infty} \frac{x^2}{(x^2 - f^2)^{1/2}} e^{-2ax/U} \, dx$$

(4.45)

More meaningful than the net amount $\bar{pw}$ is the ratio $R$ of vertically transported energy with and without oscillation. We divide (4.44) and (4.45), respectively, by their first terms (which correspond to the stationary state values of either case) and find

$$R_{\text{cos}} = 1 + \frac{\alpha^2}{4} \frac{(U^2 k_0^2 - f^2)^{1/2}}{U^2 k_0^2} \left( \frac{(U k_0 + \omega_0)^2}{((U k_0 + \omega_0)^2 - f^2)^{1/2}} + \frac{(U k_0 - \omega_0)^2}{((U k_0 - \omega_0)^2 - f^2)^{1/2}} \right)$$

and

$$R_{\text{isol}} = 1 + \frac{\alpha^2}{2} \cosh \frac{2 \omega_0 a}{U}$$

(4.46)

(4.47)

For large Rossby numbers $R_0$ both $R_{\text{cos}}$ and $R_{\text{isol}}$ converge towards the same value

$$\frac{R \to 1 + \frac{\alpha^2}{2}}{R_0 \to \infty}$$

(4.48)

Fig. (4.8) shows the absolute (in arbitrary units) and the relative vertical energy flux (normalised to the flux of the steady state solution). Coming from high Rossby numbers, the energy decreases at $R_0 \approx 1.72 (= 1 + \omega_0/f)$ when one of the travelling waves can no longer propagate but decays with height. At $R_0 = 1$ the same happens to the steady state contribution and at $R_0 \approx 0.27 (= 1 - \omega_0/f)$ to the second travelling wave. From the right panel it can be seen that each of the two travelling waves contributes with $\alpha^2/4$ times the amount of the non-
4.7 Vertical energy transport STRATIFIED FLOW OVER TOPOGRAPHY...

Figure 4.8: Vertical energy flux of the flow over the oscillating cosine shaped orography as a function of the Rossby number \( R_0 = k_0 U/f \). Left panel: The solid line is the full solution (4.44), the dashed line denotes the same for the stationary state alone (unspecified units). Right panel: relative vertical energy flux normalized to the steady state (formula (4.46)). The arrows indicate that each of the two travelling waves increases the energy flux by \( \alpha^2/4 \) times the steady state flux.

Since this result is valid for one single Fourier component of the orography, it must also be valid for an arbitrary sum of such Fourier components. Equation (4.47) and the statement in (4.48) reflect this aspect. A plot for the isolated mountain ((4.45) and (4.47)) is shown in Fig. 4.9. For large Rossby numbers the dash-dotted line (ratio of the two other curves) converges at the same value \( 1 + \alpha^2/2 \) as in the case above, i.e. we get the same additional (relative) contribution due to the oscillation as in the case of the harmonic orography. When \( R_0 \) gets smaller more and more components in the wave number spectrum of the hill are no longer in the wave regime (i.e. the Fourier coefficients for the wave numbers in the wave regime become smaller and smaller, see Fig. 4.10) and therefore the total vertical energy transport is decreasing. Unlike in the previous case we have a continuous spectrum and the decrease is not stepwise. The difference between the solid and the black line in Fig. 4.9 only appears to be constant (but is not). The large increase in the relative vertical energy flux for low \( R_0 \) is only due to the vanishing flux for a non-oscillating orography.

Some Rossby numbers for real mountains are for instance (for an assumed incident flow of 10 ms\(^{-1}\) and the respective local Coriolis parameter): Alps (\( R_0 \sim 1 \)), Jura (\( \sim 2.5 \)), Scandinavian Alps (\( \sim 0.8 \)), Rocky Mountains (\( \sim 0.6 \)), Greenland (\( \sim 0.2 \)), Hawaii (\( \sim 3 \)) and Mont Blanc (\( \sim 3.5 \)). However, for high mountains the linearity condition (4.16) is not fulfilled.
Comment on the orography (4.13)

The energy considerations above are true also for the orography with summits of constant height and valleys of varying depth as introduced in (4.13) and Fig. 4.1. The additional term $h_0\alpha\sin \omega_0 t$ is independent of $z$ and does therefore not contribute to the vertical energy flux in (4.35). We can tackle the problem not as an oscillating lower boundary problem but as two steady cases with “effective” mountain heights $h_{\text{max}} = h_0(1 + \alpha)$ and $h_{\text{min}} = h_0(1 - \alpha)$. In the case of a stagnant layer (minimum “effective” mountain height) the amplitude of the mountain waves is reduced by

$$\frac{h_{\text{min}}}{h_{\text{max}}} = \frac{1 - \alpha}{1 + \alpha}. \quad (4.49)$$

According to (4.45) the vertical energy flux is proportional to square of the orography height and is thus smaller by the factor

$$\frac{\overline{p\omega}(h_{\text{min}})}{\overline{p\omega}(h_{\text{max}})} = \left(\frac{1 - \alpha}{1 + \alpha}\right)^2. \quad (4.50)$$

4.8 Summary

The problem of an oscillating lower boundary was examined with linear theory. An increase in the maximum orography height during an oscillation by the
amount $\alpha \cdot h_0$ places a more stringent constraint upon the linearity condition (see (4.16)). Due to the oscillation of the surface boundary the stationary wave solution is supplemented by two “Doppler shifted” waves. One of them, and accordingly the whole flow, is able to generate gravity waves even for Rossby numbers $R_0 < 1$ (see (4.26)). Regions in the flow field with a high streamline density and high wind speed vary with time. For the harmonic orography these areas travel upwards along the phase lines, and the wind extrema over the isolated topography remain in place and keep their sign but their amplitude varies harmonically with time. The most striking analogy between the the two cases is the additional vertical energy transport by the two time-dependent components. It has been shown that both for one single spatial Fourier component and for a continuum the extra vertical energy flux due to the oscillation is $\alpha^2/2$ times the value of the stationary wave (see (4.48)) in the limit of large Rossby numbers.

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Figure 4.10: Fourier spectrum $\hat{h}(k) \propto a \cdot e^{-\alpha k}$ of the isolated mountain for different Rossby numbers $R_0$ ($a = U/R_0 f$, with given $U$ and $f$). Only wave numbers $k \approx 10^{-5} \text{ m}^{-1}$ (without) or $k \approx 2.7 \cdot 10^{-6} \text{ m}^{-1}$ (with oscillation) contribute to the vertical energy transport (vertical dotted lines, cf. Table 4.2). For small $R_0$ the Fourier coefficients in the “wave region” are small and so is the vertical energy flux.
Chapter 5

Valuation, retrospect and outlook

This thesis comprises three main chapters, each of which is relevant in its respective realm of research.

As mentioned in the motivation to Chapter 2 the issue of lateral shear in the vicinity of orographic features and the interaction of orographically induced buoyancy waves with the shear zone has not or very rarely been considered yet. The lack of previous research in this field is the reason why this study is performed from a rather idealised and theoretical point of view. Nevertheless, the range of utilised approaches is broad and spans from idealised NWP simulations to linearised shallow water theory and analytical description of flow fields in the quasi-geostrophic limit. Some of the results are of particular importance. Several features and criteria have to be considered in order to describe and understand properly the existence and the behaviour of atmospheric gravity waves, such as the vertical profile of the background flow, its stratification, the mountain geometry, rotational and non-hydrostatic effects etc. (cf. literature overview in section 1.3). This study adds one more item to this list: lateral shear. It has been shown that the presence of lateral shear influences significantly the capability of waves to reach high altitudes. Furthermore, shear of this kind can also be an impenetrable barrier for horizontally propagating waves. For many real cases this argument is probably not negligible because each jet-like flow is accompanied by zones of lateral shear. The general description of the wind turning on either side of an elongated mountain ridge and its associated low pressure system is novel, too. However, in order to transfer these results to the real world additional work is required. Observational or/and experimental evidence of the large-scale effects explored by this study would complete and validate the proposed concepts, quantify them and give some indication of how close to reality the ideal cases are.

The work about the winter storm ‘Lothar’ and the subsequent climatology of long and straight jets (Chapter 3) had been initiated – like numerous other studies
- directly by the occurrence of this intense storm event. It is based on the results and conclusions of distinguished publications and is a continuation of a series of papers of the Dynamical Meteorology group of ETH Zurich using the tools and methods of the PV-$\theta$-perspective and trajectory calculations. Some rarely discussed mechanisms are suggested to play a key role (e.g. bottom-up cyclone development, diabatic Rossby wave) and are the motivation for further research to be performed. 'Lothar' is a prime example for the statement of Margules (1905) that vertical rather than horizontal motions are crucial for a cyclone's development and that for many real pressure distributions: "... Hier kommt man zu Aufgaben, für welche Energiebetrachtungen alleine nicht ausreichen."

The basic idea behind the jet climatology was a question like “How extraordinary and how rare is such a ‘Lothar’ event?” A question that is interesting not only for scientists but also for insurance companies and last but not least for the public which has become very sensitive to any information about extreme weather events and their alleged/possible connexion to climate change. This study could prove that ‘Lothar’ was rare and special in many aspects and could give some clues that a warmer than normal SST in the Atlantic might have been crucial for its intensification and that a positive trend for ‘Lothar-like’ jet streams could exist. However, quantitative statements are mostly not possible. Furthermore the newly developed variable “jet frequency” is a very specific item in a long series of variables describing jets and storm tracks. In addition to the purely Eulerian nature of most others, the “jet frequency” includes some qualitative information about the potential of rapid cyclone development. It is desirable to generalise and quantify the results of this chapter. Analyses of other extreme events and considerations of longer time series are possible approaches to do so.

The problem of the oscillating two-dimensional-orography in Chapter 4 is the most theoretical study in this thesis and is furthest from any real case. However, it is an interesting piece of fundamental research which makes use of the well developed, popular and successful linear wave theory. The evolution of linear theory happened step by step by extending the existent theory by an additional component/feature. In the present case, the extension consists of the time dependency of the lower boundary condition as it has never been considered before. From the current point of view this does not directly apply to a real atmospheric process, but as indicated in the corresponding sections there is at least a promising starting-point. In any case adjustments to the solutions presented here must be made in order to pursue the link to the real atmosphere any further.
Appendix A

Lateral Shear
Figure A.1: Vertical cross-sections at $x = 100$ through the domain for varying Rossby number $R_0$. Contours of the velocity parallel to the ridge. Negative values (into paper plane) with solid lines, positive values (out of paper plane) are dashed. The contour spacing is 1 ms$^{-1}$. The zero-line is omitted.
Figure A.2: Residual wind $\hat{v}_{\text{res}} = \hat{v}_{\text{steady}} - \hat{v}_{\text{init}}$ at the lowest model level for varying $R_0$. 

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Figure A.3: Reduced surface pressure field for varying $R_0$. Note the different contour spacing (the same within a row).
Figure A.4: $p_{\text{diff}} = p_{\text{steady}} - p_{\text{init}}$ at the lowest model level for varying $R_0$. Note the different contour spacing (the same within a row).
Figure A.5: Horizontal divergence field at $z = 5$ km. Note the different contour spacing in the same lowermost row. Dashed contour lines denote negative values, solid contours positive values.
Figure A.6: Divergence field on vertical cross-sections normal to the ridge at $x = 140$ for varying $R_0$. Note the different contour spacing in the same lowermost row. Dashed contour lines denote negative values, solid contours positive values.
Appendix B

"Jet frequency" distributions

- Figs. B.1 to B.12:
  "Jet frequency" and $\text{VEL} = \sqrt{u^2 + v^2}$. The colourtable is the same for all sub-plots (described in chapter 3 on page 65). The dashed contour is the $30 \text{ms}^{-1}$ isotach, the solid contours are isotachs from $40-90 \text{ms}^{-1}$ with a contour interval of $10 \text{ms}^{-1}$. 
Figure B.1: “Jet frequency” and VEL mean of all 15 months.
Figure B.2: Monthly "jet frequency" and VEL @ θ=300 K
Figure B.3: Monthly “jet frequency” and VEL @ $\theta$=305 K
Figure B.4: Monthly “jet frequency” and VEL @ $\theta=310\,K$
Figure B.5: Monthly “jet frequency” and VEL @ $\theta=315$ K
Figure B.6: Monthly "jet frequency" and VEL @ $\theta=320$ K
Figure B.7: Monthly “jet frequency” and VEL @ θ=325 K
"JET FREQUENCY" DISTRIBUTIONS

Figure B.8: Monthly "jet frequency" and VEL @ $\theta=330 \text{ K}$
Figure B.9: Monthly “jet frequency” and VEL @ $\theta=335\,\text{K}$
Figure B.10: Monthly “jet frequency” and VEL @ $\theta=340$ K
Figure B.11: Monthly “jet frequency” and VEL @ $\theta=345\,\text{K}$
"JET FREQUENCY" DISTRIBUTIONS

Figure B.12: Monthly "jet frequency" and VEL @ $\theta=350$ K
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Appendix C

Dynamical aspects of the life-cycle of the winter storm ‘Lothar’ (24-26 December 1999)¹

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Summary

Dynamical aspects of the life-cycle of the winter storm ‘Lothar’ (24-26 December 1999) are investigated with the aid of ECMWF analysis data and mesoscale model simulations. Neither of these data sets capture the full amplitude of the observed extreme pressure fall and surface wind speeds, but they do help identify a range of key dynamical and physical features that characterise the development of this unusual event. The analysis and interpretation is primarily based upon the evolution of the lower- and upper-level potential vorticity (PV) field complemented by three-dimensional trajectory calculations.

‘Lothar’ originated in the western Atlantic and travelled as a shallow low-level cyclone of moderate intensity towards Europe. This translation took place below and slightly to the south of a very intense upper-level jet and was accompanied by continua-
ous and intense condensational heating that sustained a pronounced positive low-level PV anomaly (not unlike the concept of a ‘diabatic Rossby wave’). No significant PV anomalies were evident at the tropopause level during this early phase of the life cycle. The surface cyclone intensified rapidly when the shallow cyclone approached the jet stream axis. The circulation induced by the diabatically produced low-tropospheric PV anomaly on steeply sloping isentropic surfaces that transect the intense upper-level jet contributed significantly to the rapid formation of a narrow and deep tropopause fold. This stratospheric PV anomaly virtually merged with the diabatically produced ephemeral PV feature to form a vertically aligned tower of positive PV at the time of maximum storm intensity. A sensitivity study with a dry adiabatic hindcast simulation shows no PV tower configuration (and only a very weak surface development) and confirms the primary importance of the cloud diabatic heating for the tropopause fold formation and the rapid ‘bottom-up’ intensification of ‘Lothar’.

A comparison of the anomalously warm Atlantic sea-surface temperatures in December 1999 with the water vapour source regions for the latent heat release that accompanied ‘Lothar’s rapid intensification phase shows a close relationship. This is of importance when discussing the possible implications of climate variability and change on the development of North Atlantic winter storms.

C.1 Introduction

‘Lothar’ was one of the most harmful storms in Central Europe in the last decades. On its way through Europe ‘Lothar’ caused huge damage to buildings and forests in France, Southern Germany, Switzerland and Austria. The official figures of the total amount of windthrown timber in the forests is about 160 million m³, 115 million m³ of which was in France (268% of annual removals), 27 million m³ (69%) in Germany and 12.8 million m³ (280%) in Switzerland. The material cost of the direct damages were of the order of tens of billions of Euros, and more than 50 people were killed in the storm.

The extremely high wind velocities were mainly responsible for the damage. In the Swiss Middleland maximum wind speeds up to 200 km h⁻¹ were measured, for instance 158 km h⁻¹ in Zürich, 147 km h⁻¹ in Basel and 208 km h⁻¹ on the mountain Hörnli in north eastern Switzerland. For many stations it was the highest wind speed ever measured. In the central Alps wind velocities were also very high, e.g. 204 km h⁻¹ on the Jungfraujoch. Unlike in the Swiss Middleland the winds in the Alps were not significantly stronger than during previous serious storm events. The southern side of the Alps was not affected by ‘Lothar’.

³The German Weather Service (DWD) gives names to each North Atlantic cyclone. The storm ‘Lothar’ has also been referred to as the ‘French storm’ or the ‘1999 Boxing Day low’ (Le Blancq and Searson 2000).
Figure C.1: Berliner weather charts for 00 UTC 26 Dec 1999 at: (a) the surface (isobar spacing 5 hPa), (b) 500 hPa, showing the geopotential height (solid lines, contour interval 40 m) and the temperature fields (dashed lines, contour interval 4 K). The box in panel (b) over central Europe shows the domain of the chart shown in Fig. 2.

In Fig. 1 synoptic weather charts are shown at the time before ‘Lothar’ hit the continent and in Fig. 2 some hours after maximum storm intensity, when the centre of the cyclone was situated over central Germany and when the winds were the most violent in Switzerland. On 00 UTC 26 Dec 1999 ‘Lothar’ is some 300 km off the coast of Brittany (Fig. 1(a)). The dominant weather system in central and northern Europe at this time was the deep low pressure system ‘Kurt’ in southwestern Norway which had already caused very stormy weather in the
entire region for several days. In the analysis ‘Lothar’ is drawn as a frontal wave system on the trailing cold front of ‘Kurt’. At the same time on the 500 hPa level (Fig. 1(b)) no signature of a disturbance in the geopotential height field is visible at the location of ‘Lothar’ and only weak perturbations of the temperature and wind fields over southern Britain at the exit of the very strong and straight zonal Atlantic jet with maximum wind speeds of 120 m s\(^{-1}\). Ulbrich et al. (2001) also presented a brief analysis of the large-scale conditions during the development of ‘Lothar’ and emphasised the importance of the extremely high baroclinicity near the cyclone track over the eastern North Atlantic. Twelve hours later (Fig. 2) the centre of ‘Lothar’ has moved over France and is now located over Germany with a minimum pressure value of 974 hPa. Note the extremely large pressure gradients in northern Switzerland, associated with the highest wind velocities and most severe damage at this very moment.

A qualitative comparison with the synoptic description of other severe European winter storms in recent years, for instance the ‘October Storm’ on 15-16 Oct 1987 over southern Britain (Burt and Mansfield 1988) and ‘Vivian’ on 27 Feb 1990 (Schüepp et al. 1994), shows common features and some distinctively different characteristics. All three storms had a comparatively small horizontal
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C.2 Data and diagnostic tools

C.2.1 ECMWF analyses and HRM hindcast simulations

Two data sets are used in the present study: analysis data from the operational ECMWF (European Centre for Medium-range Weather Forecast) T319L60 assimilation cycle, and output from mesoscale limited-area NWP hindcast simulations. The ECMWF data, available every 6 hours and interpolated on a regular longitude/latitude grid with a horizontal resolution of 0.5°, serves to investigate the overall life-cycle of the storm, from its genesis phase in the western North Atlantic to the decay over Central Europe. Its temporal and spatial resolution are however not sufficient to study the details of the rapid intensification phase. To this end hindcast simulations have been performed with the HRM (‘high resolution model’), the new version of the former ‘Europa Modell’ which, until recently, was used by the German Weather Service as their operational hydrostatic NWP model. (See Majewski (1991) or Lüthi et al. (1996) for further information about the model set-up and the physical parameterisations.) In this study the integra-
tion domain was chosen to capture the intensification phase of 'Lothar' after 12 UTC 25 Dec, and the model resolution was set to 0.25° in the horizontal (corresponding to about 28 km) and 40 hybrid model levels in the vertical. Output from the model integrations (a moist control run and a dry sensitivity experiment) has been produced every hour from 12 UTC 25 Dec to 12 UTC 26 Dec. ECMWF analysis fields were used to initialise the HRM hindcast simulations and for the lateral boundary relaxation.

C.2.2 The analysis tools

The PV perspective (Hoskins et al. 1985) and Lagrangian analyses (Wernli and Davies 1997) are the main tools which are used to diagnose the key features of the genesis and rapid intensification of 'Lothar'. The individual characteristics of these tools and their complementarity has been outlined for instance by Davies and Wernli (1997). Here only some specific aspects are mentioned, which are of particular relevance for the present study:

- The equation for the material rate of change of PV \( P = -g \eta \cdot \nabla_p \theta \) can be cast in the following form (in pressure coordinates):

\[
\frac{D P}{D t} = -g \eta \cdot \nabla_p \dot{\theta} - g \nabla_p \theta \cdot (\nabla_p \times F),
\]

where \( \eta = f k + \nabla_p \times v \) is the absolute vorticity, \( \dot{\theta} \) denotes the diabatic potential temperature source and \( F \) the nonconservative force. It indicates (i) that under dry adiabatic conditions (which is a good approximation for the tropopause region in the absence of turbulence) PV is a passive dynamical tracer which for instance directly points to stratospheric intrusions into the troposphere, so-called 'upper-level positive PV-anomalies' (e.g. Kleinschmidt 1950; Bleck and Mattocks 1984; Hoskins et al. 1985; Appenzeller and Davies 1992), and (ii) that latent heat release in saturated ascent regions in the lower and middle troposphere diabatically generates so-called 'low-level positive PV-anomalies' (e.g. Reed et al. 1992; Persson 1995; Stoelinga 1996; Wernli and Davies 1997).

- The vertical alignment of positive upper-level and low-level PV anomalies can lead to the formation of a so-called 'PV-tower' (Hoskins 1990; Rossa et al. 2000), which constitutes a favourable quasi-barotropic vortex structure associated with a strong cyclonic wind field throughout the entire troposphere.

- Backward trajectory calculations and the tracing of physical and dynamical quantities along the trajectories can serve to identify the 'source regions' of
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C.3 Synoptic development

distinct PV or diabatic heating anomalies, and the associated dynamical and physical processes (Wernli and Davies 1997; Rossa et al. 2000).

C.3 Overview of the synoptic-scale development

The life-cycle of 'Lothar' can be divided into three distinct phases: A genesis and quasi-steady translation phase (00 UTC 24 Dec until 18 UTC 25 Dec), a phase of rapid intensification (18 UTC 25 Dec until 06 UTC 26 Dec), and finally the similarly rapid decay. Here we focus mainly on the first two phases and make use of ECMWF analysis fields to get a synoptic-scale overview of the development.

C.3.1 Translation

The first signal of 'Lothar' appeared in the sea level pressure (SLP) field at 00 UTC 24 Dec over the western Atlantic some 1200 km south of Newfoundland. During the following 12 hours the cyclone deepens to a minimum SLP below 1000 hPa within a band of strong precipitation (Fig. C.3(a)). Cyclogenesis takes place on the southern side of an intense baroclinic zone with a meridional temperature difference of more than 20 K over 700 km (Fig. C.3(b)). Consistent with this strong baroclinicity the upper level flow shows maximum wind velocities larger than 80 m s⁻¹ in the region north of the developing cyclone (Fig. C.3(c)).

At the same time instance the PV charts exhibit the following structures: a low level band of anomalously high PV values extends within the precipitation band along the zone of maximum baroclinicity (Fig. C.3(b)). The maximum amplitude of this diabatically produced PV anomaly occurs close to the cyclone centre. The upper level PV structure on the 310 K isentropic surface over the North Atlantic is dominated by the strong gradient associated with the zonally oriented jet (Fig. C.3(c)). On the southern side of the jet slightly negative PV values are present. As verified with a backward trajectory analysis (not shown), this region corresponds to air parcels which underwent rapid ascent and experienced strong latent heat release associated with the precipitation shown in Fig. C.3(a) in the vicinity of 'Lothar'. Note also that no prominent upper-level disturbance (i.e. PV anomaly) can be seen along the jet axis and at the location of the evolving cyclone.

During the following 30 hours 'Lothar' propagates very rapidly towards Europe and intensifies to about 990 hPa minimum SLP (Fig. C.3(d)). Whilst translating over the Atlantic to 20°W the system moves slightly northwards and comes closer to the jet axis. The precipitation signal (Fig. C.3(d)) and the pronounced low-level PV maximum (Fig. C.3(e)) near the core of 'Lothar' point to significant
ongoing diabatic processes that accompany the translation and moderate intensification of the mesoscale cyclone. At upper levels a very weak disturbance is now visible slightly to the north of the low-level vortex (Fig. C.3(f)).

A Lagrangian analysis (not shown) of the positive low-level PV anomalies near the storm’s centre (i.e. the calculation of backward and forward trajectories from the lower tropospheric region where PV > 2 pvu) clearly reveals the continuous regeneration of this PV anomaly by condensational heating. The trajectories’ PV values first increase rapidly when the air parcels enter the region of latent heat
release near the 900 hPa level, and then, during the next 12 hours, they decrease as the air parcels rise rapidly to the mid-troposphere and cross the level of maximum diabatic heating (Stoelinga 1996; Wernli and Davies 1997). Simultaneously another air mass enters the heating region from below, acquires anomalously high PV values (and transiently constitutes the low-level PV anomaly) before also ascending with decreasing PV values to the middle and upper troposphere. This steady regeneration process (which has also been documented in numerical model experiments by Fehlmann and Davies (1999)) and the rapid translation speed of $\sim 30 \text{ m s}^{-1}$ (which is distinctively larger than the ambient 850 hPa flow velocity of $\sim 10 \text{ m s}^{-1}$) are reminiscent of the concept of 'diabatic Rossby waves' (Snyder 1991; Snyder and Lindzen 1991; Parker and Thorpe 1995), as will be discussed further in section 5(b).

C.3.2 Intensification

Close to the European continent the situation changes dramatically. Early on 26 Dec, 'Lothar' hits the European continent and at 06 UTC the storm centre is close to Paris (Fig. C.4(a-c)). The SLP minimum has deepened to 977 hPa and maximum precipitation still occurs close to the centre of the low-level vortex (Fig. C.4(a)). Whereas the low-level baroclinicity is weaker over the continent (compared to the genesis region), the low-level PV anomaly is still very pronounced (Fig. C.4(b)). Inspection of the upper-level structures (Fig. C.4(c)) reveals that the surface cyclone is now located below the left jet-exit region and that an intense PV-wave has developed during the previous 12 hours. A narrow filament of stratospheric PV extends from southern England towards central France and to the northeast of the storm there is a comparatively large region with very low PV values. This area corresponds to the outflow region of the moist ascending trajectories (Wernli 1997) that are responsible for the precipitation over Western Europe. Note also that the upper-level wind speed over the continent is significantly reduced compared to the situation 12 hours earlier (Fig. C.3(f)), and that the tip of the stratospheric filament is vertically aligned with the diabatically produced low-level anomaly. It is at this time and during the following 8 hours that 'Lothar' caused the enormous damage in France, Southern Germany and Switzerland mentioned in the introduction. The preceding intensification phase will be considered in more detail in section 4 with the aid of a mesoscale NWP model hindcast simulation.
C.3 Synoptic development

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Figure C.4: The same as Fig. C.3, but at 06 UTC 26 Dec (left panels) and 18 UTC 26 Dec (right panels).

C.3.3 Decay

Only shortly after the rapid intensification phase the system decays (also quite rapidly) when moving inlands towards eastern Europe. 12 hours after reaching maximum intensity over France the remnant of ‘Lothar’ is situated in southwestern Poland, associated with a weakened surface pressure signal, decreased precipitation (Fig. C.4(d)) and almost no low-level temperature gradient (Fig. C.4(e)). At this stage the advection of moist air is significantly reduced, also due to the increasing distance from the Atlantic. The decaying cyclone is now some 500 km to the north of the jet axis (Fig. C.4(f)), and the previously marked upper-level low PV region east of the depression is less intense due to the reduced intensity of the cloud diabatic processes. More importantly, the weakening low-level cyclonic vortex is now situated below the negative upper-level PV anomaly.
C.4 A closer look at the rapid intensification phase

The above overview of ‘Lothar’s life-cycle based upon ECMWF analysis fields gives a range of indications for the key role of diabatic processes in the storm’s rapid intensification, in particular the presence of a prominent positive low-level PV anomaly associated with intense cloud diabatic heating prior to the rapid intensification, the shallowness of the early cyclone, and the absence of a notable precursor signal at upper-levels.

As mentioned above, in order to get a more detailed picture of the rapid intensification phase and to improve the understanding of the underlying dynamical and physical processes, a hindcast simulation with the HRM has been performed, starting at 12 UTC 25 Dec, that is 18 hours prior to maximum storm intensity. The temporal resolution (output every hour) enables the development of the cyclone to be better followed, and the relatively high spatial resolution of 28 km gives more details on the development at smaller scales.

First the accuracy of the control simulation will be briefly assessed, then a three-dimensional visualisation helps to highlight the main ingredients of the developing storm (in terms of PV anomalies) and their evolution, and a Lagrangian analysis sheds light on the history of these anomalies and their vertical interaction. Finally, with the aid of a dry model simulation, the importance of cloud diabatic effects for the formation of and the interaction between the PV anomalies is investigated.

C.4.1 Brief validation of the control simulation

Figure C.5 shows the synoptic situation given by the HRM model at 06 UTC 26 Dec 1999 (18 hours after the start of the simulation), at the time when the cyclone was almost at its maximum intensity. There is generally a good agreement with the ECMWF analysis (compare with Figs. C.4(a-c)). The main differences occur in the sea-level pressure field near the centre of the cyclone, in the amplitude and shape of the positive low-level PV-anomaly over France and in the intensity of the upper-level jet. More specifically, the minimum SLP value is about 4 hPa deeper, and accordingly the SLP gradient south of the storm’s centre is enhanced, which is consistent with the strong near-surface winds given by the HRM (not shown). The region with large low-level PV values over France has more a warm frontal shape, in contrast to the stubby cold frontal character shown by the ECMWF analysis (cf. Fig. C.4(b)). Probably the largest quantitative difference occurs at upper levels, where in the HRM the 80 m s⁻¹ wind velocity contour extends...
C.4 Rapid intensification phase

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Figure C.5: Synoptic situation at 06 UTC 26 Dec 1999 from the HRM simulation (to be compared with the left panels in Fig. C.4). Shown are the same fields as in Figs. C.3 and C.4. The bold line in panel (a) marks the location of the vertical section shown in Fig. C.8(d).

Further into the eastern North Atlantic. This enhanced upper-level jet (compared to the ECMWF) is in accord with the stronger meridional PV gradient on the 310 K surface (Fig. C.5(c)).

Figure C.6 shows the time evolution of the minimum SLP in the centre of the storm as simulated by the HRM, and a comparison with values from ECMWF
and from observations. Qualitatively, ECMWF, HRM and the observations are similar: the lowest SLP core value occurs at 06-07 UTC 26 Dec and all data show a phase of profound deepening during the previous 12 hours, which exceeds the usual criterion for ‘explosive development’ of \(-24 \, \text{hPa}\) per 24 hours (Sanders and Gyakum 1980). Quantitatively however, the models do not capture the really extreme intensity of the observed pressure drop \((-22 \, \text{hPa}\) in 6 hours from 00 to 06 UTC 26 Dec\) with \(-6 \, \text{hPa}\) for ECMWF and \(-11 \, \text{hPa}\) for HRM during the same time period. This is probably due to still relatively coarse resolution of the models compared to the meso-\(\beta\) scale nature of the storm.

Keeping in mind that the adopted model resolution is not sufficient to reproduce the exact intensity of the storm, we conclude that the HRM simulation reasonably well captures the storm’s rapid intensification and its physical/dynamical characteristics as revealed by the ECMWF analyses and surface pressure observations. This permits the use of output from this model simulation for further diagnostic analyses.
C.4.2 A three-dimensional perspective

Three-dimensional snap shots of the evolution of the 2 pvu isosurface are shown for the HRM simulation in Fig. C.7. At the beginning of the intensification phase (Fig. C.7(a)) 'Lothar' appears as a diabatically produced positive low-level PV anomaly below a relatively undisturbed tropopause (as noted earlier when discussing Figs. C.3(e,f)). A moderate and zonally elongated tropopause fold is present further to the west. Six hours later (Fig. C.7(b)) a more localised fold starts to develop in the close vicinity of the slightly more vertically extending low-level vortex, and again six hours later (at 06 UTC 26 Dec, Fig. C.7(c)) this fold penetrates deep into the troposphere and – together with the now vertically elongated low-to-mid level vortex – forms an almost vertically aligned tower of anomalously large PV values. (Also clearly visible is the quasi-stationary spiralling tropopause structure above the slowly decaying depression 'Kurt' near the Scandinavian coast.)

The development towards a vertically aligned PV-tower is also evident from a time series of vertical sections across the centre of the evolving cyclone (see left panels in Fig. C.8). These sections again show the presence of the low-level PV anomaly to the south of the upper-level jet axis at 12 UTC 25 Dec, the subsequent northward migration and vertical extension up to 550 hPa of this anomaly (always within the narrow zone of diabatic heating), and the rapid formation of a very narrow tropopause fold during the 12 hours until 06 UTC 26 Dec as the surface cyclone moves to the north of the jet axis. Finally at 06 UTC 26 Dec the two positive PV anomalies almost merge near the 550 hPa level (Fig. C.8(d)), and a narrow column of high PV values extends from the stratosphere down to the surface. The very small-scale nature of the upper-level anomaly is verified by the METEOSAT water vapour satellite image at 06 UTC 26 Dec (Fig. C.9). It shows a localised and extremely dry patch with a diameter of ~100 km over northern France near 1°E/49°N, very close to the location of the stratospheric intrusion in the HRM simulation.

In agreement with theoretical expectation (cf. section 2) the establishment of this PV-tower corresponds to the time of maximum storm intensity and low-level wind velocities. In contrast to the case reported by Rossa et al. (2000) the present PV-tower has a larger amplitude and a smaller diameter (~200 km compared to ~400 km), indicating a more vigorous associated vorticity (and wind) signal.

Note also from the three-dimensional visualisation (Fig. C.7) the establishment of a convexly shaped high tropopause region to the east of the PV-tower, which corresponds to the outflow region of the moist ascending motion associated with the strong cloud diabatic processes (cf. section 3(b)).
Figure C.7: The 2 pvu iso-surface from the moist HRM simulation at (a) 18 UTC 25 Dec, (b) 00 UTC 26 Dec and (c) 06 UTC 26 Dec (an animation with hourly pictures can be accessed at the following Internet address: http://www.iac.ethz.ch/en/research/cyclone_dynamics.html). The surfaces are coloured with the potential temperature values. Also shown are the 850 hPa horizontal wind vectors (the length of the maximum wind vector corresponds to 43.5 m s$^{-1}$).
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Figure C.8: South-north oriented vertical sections (from 45°N to 55°N) across the centre of ‘Lothar’ for the moist (left panels) and dry (right panels) HRM simulations, at: (a,e) 12 UTC 25 Dec, (b,f) 18 UTC 25 Dec, (c,g) 00 UTC 26 Dec and (d,h) 06 UTC 26 Dec. Shown are the PV field (shaded, in pvu), potential temperature (solid lines, contour interval 5 K), horizontal wind velocity (dashed contours for 80 m s\(^{-1}\)) and, in the left panels, the diabatic heating rate (bold contours for 0.9, 2.7, 4.5 and 6.3 K h\(^{-1}\)). The exact location of the section in panel (d) is marked by the bold line in Fig. C.5(a).
C.4.3 A Lagrangian look at the vertical coupling

The formation of positive upper-level and lower-level PV anomalies, that is the stratospheric intrusion (tropopause fold) and the diabatically induced low-level vortex, and their vertical coupling are the key processes for the dynamical understanding of the rapid intensification phase. The existence of the low-level anomaly prior to the development of the upper-level fold (cf. Figs. C.7(a) and C.8(a)) further hints at the primary role of diabatic processes for this particular event and at the 'bottom-up' character of the cyclone development. Here we briefly visualise the vertical coupling from a Lagrangian perspective.

Figure C.10(a) shows 19 hour backward trajectories from the 500-600 hPa segment of the PV-tower (defined as $\text{PV} \geq 2\text{ pvu}$) at 07 UTC 26 Dec, that is from the region where the upper and lower-level anomalies come together (Fig. C.8(d)). It is evident that two distinct airflows contribute to the formation of this intermediate segment of the cyclonic vortex: a moist and rapidly ascending airflow from the relatively warm boundary layer over the eastern Atlantic (with an averaged potential temperature increase of almost 20 K due to condensational heating), and an essentially stratospheric airstream (as revealed by the fairly constant averaged PV values along the trajectories, cf. Fig. C.10(b)), which travels along the jet-axis and descends prominently during the last 6 hours from about 420 to below 500 hPa (Fig. C.10(c)). This motion is quasi-adiabatic as indicated by the conservation of the trajectories’ mean $\theta$-value (302 ± 3 K) during the 19 hours of integration. Due to the significant descent this air is extremely dry (relative
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Figure C.10: (a) Backward trajectories calculated with winds from the moist HRM simulation from the 500-600 hPa segment of the core region of 'Lothar' (defined as PV≥2 pvu) from 07 UTC 26 Dec to 12 UTC 25 Dec. Individual trajectories are shown in grey, and the black lines indicate the averaged path of the trajectory ensembles S and T. Their 6-hourly position is marked by asterisks, and diamonds show the corresponding location of 'Lothar'. (b,c) Time history of PV (in pvu) and pressure (in hPa) along the trajectory ensembles S and T shown in panel (a).

humidity of about 5-10%) at the time when the two airflows almost merge. The PV values along the ascending branch increase steadily from 0.1 pvu to more than 2 pvu (Fig. C.10(b)) due to cloud condensational heating. This analysis shows again that the narrow tropopause fold (Fig. C.8(d)) has just formed during the 6 hours before the storm attained maximum intensity, and it indicates that the vertical coupling of the stratospheric and diabatically produced ephemeral anomalies
brings together air masses of very different origins and with sharply contrasting humidity values.

Figure C.11 shows vertical sections at 06 UTC 26 Dec across the dry patch in the water vapour satellite image (Fig. C.9). They shed further light on the formation of the dry patch and its link to the trajectory ensemble $S$ (Fig. C.10) which is related to the tropopause folding (Fig. C.7(c)). At this time instance the still descending stratospheric trajectories$^4$ are some 100 km to the southwest of the low-tropospheric positive PV anomaly (Fig. C.11(a)). In the westernmost part of the section the tropopause is significantly folded with relatively moist tropospheric air in the 300-400 hPa layer above the descended stratospheric trajectories (Fig. C.11(b)). Consequently, this region does not appear particularly dry in the satellite image (Fig. C.9). Near 1.5°E there is a deep layer (down to ~450 hPa) of dry stratospheric air (Fig. C.11(b)). Temperature on the 75 mg kg$^{-1}$ isosteric surface can be used as a proxy for the radiance observed by the satellite’s water vapour channel (Ramond et al. 1981). It indicates a very good agreement

$^4$Note that the labels ‘S’ refer to the positions of the backward trajectories with stratospheric origin from the PV tower’s 500-600 hPa segment (which are shown in Fig. C.10) and from the 400-500 hPa segment.
between the satellite image and the model-generated tropopause and humidity structures, both for the black dry patch near 1°E and the white areas near 3.5°E where (almost) saturated air extends up to the comparatively cold tropopause.

C.4.4 Sensitivity experiment without cloud condensational effects

To assess more directly the role of diabatic effects for the final intensification of ‘Lothar’, a dry physics simulation was performed with the HRM, starting again at 12 UTC 25 Dec. The results reveal a striking sensitivity to moist diabatic processes: in the dry run the minimum SLP remains almost constant and does not fall below 990 hPa (uppermost curve in Fig. C.6), the low-level PV anomaly is advected away\(^5\) and not sustained near the centre of the cyclone by continuing latent heat release (Figs. C.8(e-g)), and no narrow tropopause fold evolves over the non-intensifying cyclone (Fig. C.8(h)). The advection of the low-level PV anomaly to the middle troposphere, its slow decay and the absence of the tropopause fold in the dry simulation are confirmed by a three-dimensional visualisation, similar to Fig. C.7 (not shown). The resulting concomitant absence of storm intensification and of a PV-tower configuration confirms the usefulness of the adopted PV-perspective to investigate the storm development, and supports the crucial importance of the diabatic effects for the creation and maintenance of the low-level PV anomaly, for the fold formation, vertical coupling and the particularly intense development of the mesoscale storm.

Finally, a direct comparison is made between the moist and dry simulations in the vicinity of the cyclone in order to investigate qualitatively how the diabatically sustained low-level vortex contributed to the rapid formation of the positive upper-level PV anomaly (i.e. the tropopause fold), once it moved closely to the upper-level jet axis. This comparison is made at 01 UTC 26 Dec, that is at the beginning of the descent of the stratospheric trajectory ensemble (Fig. C.10), on the 298 K isentropic surface, which intersects in the moist simulation both the tropopause (near 50-53°N) and the upper part of positive low-level PV anomaly (near 48.5°N), as shown in Fig. C.12. The 298 K isentropic wind field difference (moist minus dry) reveals two diabatically induced mesoscale circulations of comparable intensity: a cyclonic one associated with the low-level positive PV anomaly, and an anticyclonic one \(~700\) km to the west of ‘Lothar’. The latter corresponds to a

\(^{5}\)Forward trajectory calculations from the initial low-level PV anomaly in the dry simulation show a slight northward movement and associated ascent along the sloping isentropes to mid-tropospheric levels. During the ascent there is significant dispersion of the trajectories and the ensemble does not remain coherent, which strongly reduces the dynamical impact of this air mass with anomalously high PV values. Furthermore, the averaged PV value is reduced by numerical dissipation from 2.5 to 1.0 pvu during the 24 hours after the start of the simulation.
negative PV anomaly (i.e. lower than climatological values) which was produced, as indicated by backward trajectories (not shown), by cloud diabatic processes in the western North Atlantic (cf. the large amplitude precipitation maxima upstream of ‘Lothar’ in Figs. C.3(d) and C.4(a)). Together, the two diabatically induced circulations produce a northerly wind of \( \sim 7 \text{ m s}^{-1} \) near 10°W, in the region of the descending trajectories where the isentropic surface is particularly steep. A rough estimate shows that in the moist simulation this northerly wind leads to a \( \sim 150 \text{ km} \) southward transport of lower stratospheric air parcels during the subsequent six hours which corresponds to a descent of \( \sim 90 \text{ hPa} \) due to the pronounced baroclinicity. This estimate of the magnitude of the isentropic ‘downgliding’ (Hoskins et al. 1985) is in good agreement with the diagnosed descent of stratospheric air shown in Figs. C.10 and C.8(c,d). From Fig. C.12 the impression is that the diabatically induced wind fields associated with upstream precipitation and with ‘Lothar’ itself are about of equal importance. A PV inversion diagnostic would be required to assess the relative importance more quantitatively. In the dry simulation there are no diabatically induced circulations to produce the northerly winds across the jet axis, and consequently there is no tropopause fold formation. To summarise, this analysis provided further evidence for the key role of cloud condensational processes for establishing a vertically co-
C.5 Discussion and further remarks

The investigation of the life-cycle of the European winter storm ‘Lothar’ revealed distinct characteristics which were responsible for its rapid intensification. These features are listed and compared with the findings of earlier studies on cyclogenesis and intense storms. Finally some qualitative consideration is given to the issue of climate variability and extratropical cyclones.

Based upon the combination of diagnostic tools the following key ingredients have been identified for the evolution of ‘Lothar’: 

- the presence of a very intense and straight upper-level jet (and associated tropospheric baroclinicity) which persisted during several days and extended zonally over the entire North Atlantic (see also Ulbrich et al. (2001)),
- the ‘diabatic Rossby wave character’ of ‘Lothar’ in its early phase, that is, the genesis of a shallow cyclone in the western Atlantic to the south of the upper-level jet axis characterised by a pronounced positive low-level PV anomaly in a strongly baroclinic environment, which is steadily regenerated through cloud diabatic processes while travelling rapidly towards Europe (as shown below, the ‘source region’ for the condensational processes was characterised by anomalously warm sea-surface temperatures),
- the extremely rapid ‘bottom-up’ intensification, when the low-level cyclone moved to the northern side of the jet axis and instigated the rapid formation of a tropopause fold which almost merged with the diabatically produced PV anomaly to form a narrow and intense vertically oriented vortex.

C.5.1 Strong upper-level jet

The baroclinicity in the Atlantic sector was exceptional during the development of ‘Lothar’: A qualitative comparison with daily ECMWF analysis data over the central and eastern Atlantic for 11 Januaries from 1984 to 1994 showed that the
jet after Christmas 1999 was the most extreme in terms of intensity, persistence and straight zonal orientation. In this study, we did not investigate the dynamical reasons for the formation and maintenance of this particular jet. Qualitatively it appears from Figs. 3-5 that the jet region was associated with very low (near zero) PV values on the anticyclonic shear side of the jet, which increased the meridional PV gradient. These regions correspond to the outflow region of the major precipitation producing ascending airflows (not shown, see Browning (1990) and Wernli (1997) for a general discussion). It remains a subject for further study to assess the importance of diabatic and other processes for the formation of this particular jet-stream system.

C.5.2 Diabatic Rossby wave character

Idealized studies of convective heating within two-dimensional baroclinic atmospheres revealed the existence of ‘diabatic Rossby waves’, where the diabatic PV tendency plays the role of meridional advection in the classical Rossby wave (see Fig. 6 in Parker and Thorpe (1995)): Positive thermal advection to the east of a low-level positive PV anomaly leads to upward motion and latent heat release (see also Raymond and Jiang (1990)), which is associated with a positive PV tendency at low levels. Consequently, the PV anomaly pattern is steadily regenerated and propagates in the thermal wind direction. As illustrated in west-east oriented vertical sections across the cyclone centre (Fig. C.13) this corresponds nicely to what can been diagnosed during the early phase of ‘Lothar’ (cf. section 3(a)): at 00 UTC 25 Dec the low-level vortex has maximum amplitude near 850 hPa, and the induced southerly wind component (Fig. C.13(a)) is largest some 200 km to the east of this anomaly. The warm air advection associated with the southerly winds leads to saturated ascent along the northward sloping moist isentropes with maximum rising motion (Fig. C.13(b)), diabatic heating (not shown) and strong diabatic PV production (with peak values $> 8 \text{pvu h}^{-1}$, Fig. C.13(b)) again downstream of the low-level vortex. This brief analysis provides evidence for the usefulness of the ‘diabatic Rossby wave’ concept (which has been developed for highly idealised atmospheric flow settings (Snyder 1991; Parker and Thorpe 1995) for the interpretation of the translation phase of ‘Lothar’. However, although the relationship between PV, winds, diabatic heating, and PV generation is qualitatively similar 6 hours before and after the situation shown in Fig. C.13, further investigations will be necessary to acquire a full understanding of the three-dimensional structure of real atmospheric ‘diabatic Rossby waves’.

Mallet et al. (1999) also found some indications of this phenomenon during the early phase of FASTEX IOP17, where (in comparison to the present case) the cyclone developed within a much more complex ambient flow structure. The FASTEX IOP10 cyclone had a similar path to ‘Lothar’ (Joly et al. 1999) and a cursory
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**Figure C.13:** West-east oriented vertical sections at 00 UTC 25 Dec (from ECMWF analysis fields) for the ‘diabatic Rossby wave’ stage of the development at 42.2°N across the centre of ‘Lothar’. Shown are (a) the PV field (shaded, in pvu), potential temperature (thin lines, every 3 K) and two bold contours for 20 and 23 m s⁻¹ southerly wind, and (b) vertical velocity (shaded, in cm s⁻¹, only ascending motion is shown), the 1.5 pvu contour (bold) and the diabatic PV rate (contour interval 2 pvu h⁻¹, negative values are dashed).

Inspection of the ECMWF analyses shows a relatively intense upper-level jet and a shallow low-level cyclone, not unlike the early phase of ‘Lothar’. However, no significant intensification occurred for this FASTEX depression. Probably the closest analogue to the evolution of ‘Lothar’ has been documented by Shapiro et al. (1999; section 7.2). They suggested that a small initial low-level vortex to the south of the axis of an intense North Atlantic upper-level jet was sustained by diabatic processes. As in the present study the vortex travelled rapidly towards Europe and intensified after it passed beneath the upper-level jet axis. But in contrast to ‘Lothar’, in their case the system eventually moved towards Scandinavia and attained a fairly typical synopticscale cyclone structure.

### C.5.3 ‘Bottom-up development’ and diabatic tropopause fold triggering

The rapid formation of a tropopause fold induced by cloud-diabatically produced PV anomalies, the associated explosive cyclone intensification, and the almost complete absence of this intensification in a sensitivity experiment without latent heat release adds a new flavour to the discussion of the role of diabatic processes for (rapid) cyclone development and of the relative importance of low-
level vs. upper-level processes (cf. Davies 1997). As reviewed recently by Mallet et al. (1999) and in more detail by Uccellini (1990) there is not yet a generally accepted consensus view on the effects of latent heat release on cyclone life-cycles (and more important, there is probably a large case-to-case variability when quantitatively assessing these effects). On the one hand, Davis et al. (1993) concluded (based upon three case studies) that the feedback of latent heating onto the interaction of upper-level PV and surface potential temperature anomalies is small. On the other hand, for instance Gyakum (1983) and Kuo et al. (1991) emphasise the nonlinearity of the interaction between baroclinic dynamical and diabatic processes. In all these (and other reported) cases diabatic processes contribute about 30-70% to the amplitude of the surface cyclone deepening (without strongly impacting upon the structure of the systems). This is also in agreement with the adjoint sensitivity study of Langland et al. (1996). In contrast, Cammas et al. (1999) and Pomroy and Thorpe (2000) indicate the possibility that diabatic heating impacts also upon the structural evolution of cyclones, for instance by decoupling upper and lower disturbances which delays their favourable interaction. Clearly the present case of the winter storm ‘Lothar’, where cloud diabatic processes not only amplified but crucially determined the cyclone intensification, falls into a third category. This might be related to the ‘bottom-up’ nature of the storm development, that is to the absence of an initial upper-level precursor disturbance which are typical features of mid-latitude cyclones (Hoskins et al. 1985). As a side remark, note that qualitatively similar types of ‘bottom-up’ developing cyclones can occur in idealised channel model experiments upstream of upper-level induced depressions (Shapiro et al. 1999; Wernli et al. 1999), even without the consideration of diabatic effects.

The general picture of (rapid) cyclone development as described within the PV framework (Hoskins et al. 1985), where an upper-level (stratospheric) positive PV anomaly interacts with low-level positive potential temperature and vorticity anomalies, also holds in the present case. However in most of the documented case studies (e.g. Hoskins and Berrisford 1988; Uccellini 1990; Rossa et al. 2000) and in the idealised simulation of secondary cyclogenesis by Thorncroft and Hoskins (1990) the upper-level anomaly has formed independently from the low-level cyclone and acts as a precursor for cyclogenesis. But in the present case the diabatic heating induces both the lower and upper-level positive PV anomalies, the former directly through diabatic PV production (cf. eq. (1)) and the latter indirectly via the balanced circulation associated with the diabatically produced PV anomalies (cf. Fig. C.12). This second effect is schematically illustrated in Fig. C.14, which can be regarded as a ‘bottom-up version’ of Fig. 21 in Hoskins et al. (1985). It shows how a positive low-level PV anomaly induces perturbations at the trop-
Figure C.14: Schematic illustration of the formation of PV anomalies near the tropopause level associated with the arrival of a positive PV anomaly below an intense upper-level jet region. Grey shading indicates PV values larger than 2 pvu and denotes the diabatically produced PV anomaly in the lower and middle troposphere and the stratospheric part of a steeply sloping isentropic surface, which intersects the low-level vortex. The circulation induced by the low-level vortex (shown by black arrows) leads to northward advection of tropospheric air to the east of the vortex, and to southward and downward advection (indicated by the white arrow) of stratospheric air on the western side.

The downwelling of cold air at the tropopause level and leads to the downward advection of stratospheric air along steeply sloping isentropes slightly upstream of the low-level vortex. Downstream, the wind field induced by the vortex leads to northward isentropic advection of tropospheric air. In turn, the circulation associated with the bottom-up induced upper-level wave may positively feed back on the low-level PV anomaly by advecting potentially moist air from the south towards the storms' center. Note that for the entire mechanism to be effective it is important to have a pronounced low-level vortex close to the jet axis, a relatively low tropopause and a strong baroclinicity (i.e. an intense upper-level jet). These elements are all present at the onset of the rapid intensification of 'Lothar'.

A dry variant of this mechanism has been discussed by Wandishin et al. (2000) within an idealised baroclinic wave development. In their study the surface warm anomaly near the centre of the cyclone induced the cyclonic circulation, instead of the diabatically produced PV anomaly in the present case. In another idealised study of tropopause folding by Ziemianski and Thorpe (2000) the precursor anomaly was a low-level PV anomaly which was produced through dissipative
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Processes associated with surface frontal collapse. Earlier, for the ERICA IOP5 storm, Reed et al. (1993) found indications for an enhanced sinking of stratospheric air near the cloud edge due to latent heat release. Idealised model experiments including latent heat release could be rewarding to study the conditions for ‘bottom-up’ induced tropopause fold formation in more detail.

C.5.4 An impact of warm sea-surface temperatures?

The extreme intensity of the surface wind speeds associated with ‘Lothar’ and the huge damages prompted the public discussion on the relationship of this particular storm to climate change. Here it is not intended to answer this question, but a brief inspection of the sea-surface temperature (SST) field during December 1999 and the track of ‘Lothar’ reveals an interesting relationship. Figure C.15 shows the monthly SST anomaly (relative to a climatological mean) and the ‘source regions’ of the moist trajectories which lead to the continuous latent heat release near the centre of the developing cyclone. These ‘source regions’ have been determined from backward trajectories started every 6 hours back to
00 UTC 24 Dec from the ECMWF grid points where the latent heat release exceeded 10 K (6 h)$^{-1}$. The regions where the backward trajectories were below the 900 hPa level are referred to as water vapour 'source regions'. The figure shows that these regions extend over the entire North Atlantic, slightly to the south of the track of 'Lothar'. The same area (except for its easternmost part) is characterised by anomalously warm SST values. The anomaly has an overall amplitude of about 0.5 K and exceeds 1 K just in the storm's most important water vapour source region. It should also be noted that this mid-Atlantic warm SST patch near 35°W constitutes one of the strongest anomalies in this region during the last 40 years.

This comparison indicates the possibility that anomalously warm SST in the Atlantic contributed to the evolution of 'Lothar' through enhanced surface fluxes of heat and moisture. This could not only be of importance directly for the rapid intensification but in particular for the early translation phase where continuous moisture flux convergence was essential for the sustenance of the low-level positive PV anomaly. Here further investigations are required using (re-)analysis data sets over an extended time period, to systematically identify shallow 'diabatic Rossby wave' type cyclones, their track and evolution and the link with the ambient patterns of SST and atmospheric baroclinicity. Such an analysis could contribute to the understanding of the relationship between climate variability and the category of diabatically triggered 'bottom-up' developing cyclones, for which 'Lothar' was a particularly strong example.

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Bibliography


Bibliography


Bibliography


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- **Climatology of long, straight Jets in the Northern Hemisphere.** EGS XXVII General Assembly. Nice, France, 22-26 April 2002.

- **Dynamical aspects of the life-cycle of the winter storm 'Lothar' (24-26 December 1999).** Seminar at the Deutscher Wetterdienst DWD (Forschung und Entwicklung). Offenbach, Germany, 4 December 2001.


- **Potential Vorticity Structure of a Severe Extratropical Cyclone.** 8th Scientific Assembly of IAMAS. Innsbruck, Austria, 10 - 18 July 2001.

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