Efficient indoor radio channel modeling based on integral geometry

Author(s):
Hansen, Jan Carsten

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Efficient Indoor Radio Channel Modeling Based on Integral Geometry

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presented by
JAN HANSEN
Dipl.-Phys., Universität Freiburg, Germany
born 1 April 1973
citizen of
Bochum, Germany

accepted on the recommendation of
Prof. Dr. P. E. Leuthold, examiner
Prof. Dr. R. Mathar, co-examiner
Prof. Dr. H. Bölcskei, co-examiner

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Abstract

This thesis presents a novel concept of stochastic radio channel modeling. It subdivides radio channel modeling into two tasks: the description of wave propagation within given deterministic surroundings and the characterization of the key geometrical properties of these surroundings. Based on this approach, an indoor radio channel model in the frequency range of 2.5 to 60 GHz is developed. The model is completely analytical. It describes the indoor radio channel in dependence on a set of key parameters: the frequency, the antenna characteristics, the pathloss exponent, an average reflection coefficient of the investigated environment, the spatial distribution of the transmitter and the receiver and their minimum distance to each other, and the volume and surface area of the domain in which transmitter and receiver are located. For a system designer, these quantities are easily accessible; only minimum knowledge about the investigated building and the communication system is required. For the derivation of the channel model, geometrical methods are combined with ray optical principles. For three different communication systems — WLANs with fixed and random position of the access point, and Ad-Hoc networks with no hierarchy — the mean pathloss (MPI) between two communicating users is calculated in dependence on the above-mentioned parameters. The MPI itself is important; if it is, for instance, used to describe the power of unintentionally received signals, it models the average interference power level within a wireless network. It is furthermore essential for the derivation of the mean and the standard deviation of the large-scale distribution of the pathloss. Since the pathloss is known to be lognormal distributed, its mean and standard deviation suffice for a complete characterization of its statistics. Geometrical arguments are presented which demonstrate why the lognormal distribution provides a suitable description. In addition to the large-scale distribution of the pathloss, the power delay profile (PDP) is derived for a single room environment. It is proven that the PDP in such simple surroundings is exponential. An analytical expression for the delay spread is extracted which is a suitable upper bound for the delay spread in more complicated environments.

The combination of the different results yields a complete description of the frequency selective, Rayleigh fading indoor radio channel. The underlying distribution functions — lognormal for the large-scale variations
of the received power and exponential for the PDP — have been long known on an empirical basis. In this thesis, they are presented in analytical form in dependence on a few, easily accessible key parameters. The derived channel model is verified against simulated and measured data and implemented as a MATLAB code.
Kurzfassung

Die vorliegende Arbeit behandelt ein neuartiges Konzept zur stochastischen Kanalmodellierung. Das vorgeschlagene Konzept trennt die Kanalmodellierung in zwei Teile: Zunächst muss die Wellenausbreitung in hinreichend genau determinierten Umgebungen beschrieben werden. Hierarchisch werden die für die Wellenausbreitung wichtigen geometrischen Eigenschaften bestimmt.


Auszserdem wird das Verzögerungs-Leistungsdichteprofil (Power Delay
Profile, PDP) innerhalb eines einzelnen Raumes hergeleitet. Die Herleitung ergibt eine Exponentialfunktion. Automatisch resultiert daraus ein analytischer Ausdruck für die Verzögerungsspreizung (Delay Spread, DS), der als obere Schranke für den DS von komplizierteren Umgebungen verwendet werden kann.

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Chapter 1

Introduction

1.1 Motivation of the Work

When Shannon’s paper about the capacity of the band-limited communication channel [128] rang in the rise of modern communications in 1949, Maxwell’s equations, which were published in 1864 [99] and still form the solid foundation of wave propagation, celebrated their 85th anniversary. Today, both digital communications and field theory are well-established disciplines in electrical engineering. The methodology of these two branches is as divergent as it can possibly be. Wave propagation problems are boundary value problems, whose solutions determine a field distribution for a given scenario of sources and objects. The investigated problems are strictly deterministic. Communication engineering, on the other hand, has always been dealing with stochastic systems. Since the central approach is the detection of noisy signals, the application of probabilistic concepts is required.

The recent advances of wireless communications have forced the two disciplines to get in touch. Radio channel modeling emerged at the interface between modern communications and wave propagation and consequently comprises the various methods of both disciplines. The almost universal application of wireless devices made it necessary to rethink the traditional solution strategies of Maxwell’s equations. Their rigorous solution is restricted to a far too limited number of cases; their approximate solution, in particular in the far field and at high frequencies, and the corresponding development of simulation tools has undergone a rapid development. In contrast, the radio channel is understood as a system that distorts a signal and may randomly cause errors in transmitted bit streams.

\(^1\)The cited paper, *A Dynamical Theory of the Electromagnetic Field*, contains only an abstract which was read to the Royal Society. The entire work was published in the following year in the *Philosophical Transactions* [100].
For a communication engineer, it is much easier to regard the channel as a box with stochastic properties. Codes and algorithms for data transmission are, for instance, designed to maximize throughput, or to minimize the probability of a transmission error; their performance is often tested within radio channels described by a particular distribution function.

Currently existing models are situated anywhere between a purely deterministic view, derived from Maxwell’s equations, and a purely stochastic one that describes the random distortion of a transmitted signal. The stochastic models, which are characterized by particular distribution functions, allow a very efficient calculation of this distortion. It is remarkable that many of these distributions are derived from the Gaussian distribution. This finding, which simply implies that the stochastic radio channel obeys a law of large numbers, is very important. The Gaussian bell curve depends on its mean and standard deviation only, and these average quantities are robust. Experiments have indeed revealed some kind of robustness. Data obtained from measurement campaigns in specific environments can represent the radio channel behavior in similar scenarios, where similarity is very vaguely defined in terms of, e.g., building sizes and materials.

The main problem of channel modeling is the dependence of the channel on a seemingly huge amount of parameters. A problem of this kind occurs, however, in many fields of science. Statistical Mechanics, for instance, exploits the statistical properties of a system with huge degrees of freedom; it advocates a distinction between the microscopic world, which is a system that can adopt a vast amount of different states, and the macroscopic world, which can be characterized by a few, characteristic observables.

The microscopic world of channel modeling comprises all details needed to solve Maxwell’s equations, i.e., the location and the dielectric properties of sources and objects. The macroscopic world is described by average quantities such as the mean received power, its standard deviation, and the delay spread. The link between the two 'worlds' is based on two premises. The first is well-established and concerned with the simplification of the solution of Maxwell’s equations. The second is a little harder to define. Intuition and all experimental evidence show the tremendous impact of the environment on the radio channel. The robustness of the channel
against small changes in the environment indicates that the geometrical quantities, on which the channel depends, are robust, too. Hence, these must exist in the macroscopic world, and channel models can be developed in dependence on these robust geometrical quantities. Models designed that way will be much faster and easier to handle than those defined in the microscopic world; they contain geometric parameters, so that a single model of this type is applicable to a wide range of different environments.

It is, however, not at all obvious which macroscopic geometric quantities are suitable for channel modeling, nor how they could actually be derived. Such a derivation is the aim of this thesis. A relation between the microscopic and the macroscopic world is established with the aid of an integral expression which combines a deterministic wave propagation model with a stochastic description of the environment and the possible positions of transmitter ($T_x$) and receiver ($R_x$). Subsequently, this integral is applied to design a model for the indoor radio channel. It is demonstrated in simulations how the channel statistics depends on the amount of side information that is available about the geometry of the environment. The simulations show that a reduction of the knowledge about the environment to only a few key parameters is sufficient to determine the basic robust quantities which are for instance required for link or system level simulations. Such a reduction then allows an analytical approach to the stochastic radio channel. This approach is based on concepts which have been derived in Integral Geometry. Integral Geometry is a branch of Mathematics located at the interface between Probability Theory and Geometry. It provides precisely those methods required to study wave propagation for random configurations of $T_x$ and $R_x$ in environments about which only little side information is available. With the aid of the Integral Geometric methods, an indoor channel model is derived on the basis of a well-known empirical pathloss formula and of ray optical concepts. The channel model completely characterizes the Rayleigh fading indoor radio channel in a large frequency range of 2.5 to 60 GHz (Gigahertz). It provides analytical expressions for the distribution of the pathloss, its mean and standard deviation, the PDP (power delay profile) and the delay spread. The approximate solutions require only a minimum of input parameters: the characterization of the wave propagation by the pathloss exponent, a description of the antenna properties by a related
exponent, the wavelength, the minimum distance between and the spatial
distribution of $T_x$ and $R_x$, and the dimensions of the environment in which
$T_x$ and $R_x$ are located. The resulting channel model is both simple to ap-
ply and flexible, since it relies on analytical expressions and is independent
from measured data.

1.2 State of Research

Channel modeling provides insight into wave propagation within only
roughly determined environments. The corresponding parameter set re-
fects the sophistication of the wireless systems for which it is developed.
Due to the tremendous recent interest in MIMO (multiple-input multiple-
output) systems, measurement campaigns are at present carried out with
antenna arrays, which allow the angular resolution of the received sig-
nal. Most common is a statistical evaluation of the measured data, as
for instance done in [4, 113, 148, 142]. The aim of these studies is to
find (joint) distributions of important channel parameters and their cor-
relations. More theoretical work is [7, 1] where parametric models are
derived from previously measured data. A study which includes an in-
vestigation of physical wave propagation is [135]. The above studies were
carried out in outdoor environments. For indoor surroundings, the MIMO
radio channel is treated in [141, 82]. These studies focus on the derivation
of the channel properties via its correlation matrices. With the research
on MIMO channels, the investigation of the channel capacity has received
attention; related studies, also for indoor / microcell environments, are
[107, 86, 8]. Further recent measurements deal with the new topic of the
Ultra-Wide-Band radio channel [29, 23].

Measurements are only one possibility to assess the channel. Ray trac-
ing studies have gained increasing attention. For these studies, an envi-
ronment is constructed on a computer, and numerous ray tracing simu-
lations are run for this setup. Subsequently, channel parameters can be
statistically evaluated. Whereas the necessary idealistic description of the
surroundings is a disadvantage, those models are generally statistically
more reliable. For the indoor radio channel, a ray tracing based model is
derived in [151, 51, 50], outdoor environments are treated in [25, 138, 34].

Measurements and ray tracing studies are quite opposite strategies in
order to tackle the same problem. For measurements, the impact of the environment cannot be parameterized explicitly and is thus only empirically available; ray tracing requires its complete definition. Outdoors, a third approach is common. The surroundings are geometrically characterized in a very simplified way as in [71], where the power spectrum of the fading signal is calculated based on the assumption of a uniform angular distribution of received scattered power. Current investigations in this field include [73, 106, 70, 115]. Even though very suitable for a statistical application, the drawback of these models are their very heuristic assumptions about the geometrical characterization of the environment and, partly, about the description of wave propagation. The model described in [44] works without any side information about wave propagation. It captures all potentially existing features of the MIMO channel, without, however, relating them to a concrete, existing environment.

A more thorough approach with regard to the fundamentals of wave propagation is [27], which generalizes the well-known Walfish-Bertoni model for the outdoor radio channel [140]. In this model, the surroundings are less drastically simplified; wave propagation is treated on a theoretically much more consistent basis. However, it already requires some computer power for its evaluation.

All approaches, whether based on measurements, on ray tracing simulations, or on a geometrical approach, determine the radio channel as a function of a particular geometry of the surroundings. Obviously, the structure of the surroundings has a fundamental impact on the channel. However, all approaches have in common that this impact seems only unsatisfactorily characterized. Measurements are most realistic, but can hardly be used to study the impact of geometrical key quantities on the radio channel systematically and explicitly. In ray tracing studies, the environment seems overdetermined. The creation of surroundings is always site-specific and gives hence no direct insight either into how a geometry, and particularly a slight variation of it, may influence the channel. In the third case, the geometrically motivated models, this aspect is handled in the most appealing way; it is assumed that a very rough characterization of the surroundings suffices to yield a suitable picture of the channel. Still, the approach is intuitive, and doubts always remain whether the chosen parameters, which differ greatly from model to model, are precisely the
ones required. This fact has, to the knowledge of the author, only once been directly addressed. In [61], an indoor channel model is presented which is parameterized by geometrical key parameters. The model is site-specific, and hence similar to the one of Walfish-Bertoni. However, it is recognized that the geometry of the environment is a quantity that may be separately treated from wave propagation; particularly the random positions of the $T_x$ and the $R_x$ are described by integrals over geometric objects such as rooms or parts of floors, and they are analytically solved. The entire resulting model, however, must still be numerically evaluated.

1.3 Outline of the Thesis

In the following chapter, wave propagation and wave propagation models are discussed. It begins with a review of deterministic wave propagation and a brief outline of ray optical methods and corresponding propagation mechanisms such as reflection and diffraction. These methods are the basis for a raytracer which was implemented in the course of the thesis. In the same chapter, the view is broadened towards semi-deterministic and stochastic wave propagation models. Thus, a deeper insight into the methods of channel modeling is given, as far-reaching as necessary for an understanding of the concept of this thesis. The overview is restricted to indoor propagation.

Chapter 3 covers all geometrical methods required for the analytical derivations presented in this thesis. Together with a brief introduction to the foundations of the mathematical techniques, the main relations which hold for densities of points in convex bodies are stated. These results are trivial for a single point, but they get involved if two points and in particular different spatial distributions of two points are considered. Since the mathematical content of this chapter is intended to remain as intelligible as possible, only a short introduction into the background of the used methods is given. Instead of rigorous derivations, explanations are motivated geometrically. More precise insight can be obtained from the references which include the common literature of the field.

In Chapter 4, an integral expression is presented which characterizes the interplay between deterministic wave propagation, the geometry of the surroundings, and the spatial distribution of $T_x$ and $R_x$. The integral
expression allows to understand how the properties of a specific communication system (e.g., random locations of a user and an access point) influence the resulting stochastic properties of the radio channel. With the aid of the integral expression, different existing channel models are classified. Furthermore, three different spatial distributions of $T_x$ and $R_x$ are defined which are investigated in the subsequent chapter.

A numerical solution of this integral is performed in Chapter 5. Joint densities of different channel parameters such as amplitudes or AoD (angle of departure) are examined. The aim of the study is to demonstrate the impact of the environment on the resulting channel statistics; furthermore, it intends to clarify what impact the individual deterministic wave propagation mechanisms have in a given situation. As the side information about an environment decreases, only some key geometrical quantities begin to dominate the radio channel. The simulations pave the way for an establishment of macroscopic parameters that can suitably characterize the stochastic radio channel.

The analytical derivation of these robust, macroscopic quantities is carried out in Chapter 6, which can be regarded as the main chapter of this thesis. The distribution of the pathloss is investigated, and closed form expressions for their means and standard deviations are derived for the several network types defined in Chapter 4. The results are discussed and compared to each other. Furthermore, the PDP is calculated, and an upper bound for the delay spread is extracted.

The verification of the derived expressions is given in Chapter 7. The verification is performed both by ray tracing and by measurements at 2.4, 5.25, and 60 GHz. Ray tracing has the advantage that the validity of the analytical results can be systematically studied with high statistical accuracy; it lacks, of course, the completely realistic description of the radio channel. The measurements were performed at different locations within two different buildings. The distribution of the pathloss, its mean and standard deviation, as well as the delay spread are evaluated, discussed and compared to the analytically obtained data. A conclusion and an outlook are given in the last Chapter 8. Several appendices contain mathematical details as well as a brief introduction to the software tools that were developed in the course of this thesis.
1. Introduction
Chapter 2

Wave Propagation and Wave Propagation Models

The deterministic description of wave propagation has long been a classic discipline in electrical engineering, and sophisticated methods have been developed to solve Maxwell’s equations for arbitrary distributions of sources and materials. These methods have not only found application in the description of wave propagation, but also, for instance, in antenna and circuit design. In the first section of this chapter, a brief glance over some fundamental aspects is given.

With the rise of digital wireless communications, a new type of problem evolved: Wireless communication devices have become a mass product and are supposed to operate at high data rates and under rigid power constraints at cm- and mm-wavelengths anywhere within man-made environments. In order to achieve these goals, insights about the radio channel have become indispensable. Methods which can approximate Maxwell’s equations in the interesting environments have been developed; they mainly rely on a ray-optical description of the field and are outlined in the second section.

Wave propagation models are not necessarily based on deterministic expressions. Many empirical models have been developed that incorporate only some or even no information about the properties of the electromagnetic field; these models rely on particular stochastic properties, or they are derived from experimental data. They are outlined in the last section.

2.1 Fundamentals of Field Theory

In principle, Maxwell’s equations are the basis for the calculation of any field problem. For a time-harmonic electromagnetic field and in linear,
homogeneous, and isotropic media, they can be written as [13]

\[ \nabla \times \mathbf{E}(\mathbf{r}) + j \omega \mu \mathbf{H}(\mathbf{r}) = -\mathbf{M}(\mathbf{r}), \quad (2.1) \]
\[ \nabla \times \mathbf{H}(\mathbf{r}) - j \omega \epsilon \mathbf{E}(\mathbf{r}) = \mathbf{J}(\mathbf{r}), \quad (2.2) \]
\[ \nabla \cdot \mathbf{E}(\mathbf{r}) = \frac{q_e(\mathbf{r})}{\epsilon}, \quad (2.3) \]
\[ \nabla \cdot \mathbf{H}(\mathbf{r}) = \frac{q_m(\mathbf{r})}{\mu} \quad (2.4) \]

where \( \nabla \) is the nabla operator, and \( \times \) the vector product in three dimensions, where \( \cdot \) denotes the scalar product, \( \mathbf{E} \) and \( \mathbf{H} \) denote the electric and the magnetic field vector, \( \omega \) its frequency, \( \epsilon \) and \( \mu \) the permittivity and permeability, \( \mathbf{J} \) and \( \mathbf{M} \) the electric and magnetic current densities, and \( q_e \) and \( q_m \) the electric and the magnetic charge density. Whereas the electric charge and current densities may be either real or virtual, no real magnetic charges or current densities exist. They are introduced for reasons of symmetry and simplify the solution of Maxwell’s equations. The magnetic charges can be regarded as the origin of a magnetic current as used, for instance, in Huygens’s principle, which is outlined below. All quantities are functions of a location \( \mathbf{r} \) in space. For the sake of brevity, these arguments are subsequently omitted.

Eqs. (2.1) - (2.4) represent a set of coupled differential equations. For their solution, it is common to introduce the vector potentials \( \mathbf{A} \) and \( \mathbf{F} \). The introduction of \( \mathbf{A} \) requires \( q_m = 0 \), i.e., \( \mathbf{A} \) is a quantity that arises due to the existence of electric charges only. Analogously, an equation for \( \mathbf{F} \) is obtained from \( q_e = 0 \) which implies that \( \mathbf{F} \) describes effects caused by magnetic charge densities. Since Maxwell’s equations are linear, the resulting fields can simply be superposed, so that the solution of the entire field problem is obtained.

Setting \( q_m = 0 \), (2.4) reads \( \nabla \cdot \mathbf{H} = 0 \), and there exists a vector \( \mathbf{A} \) such that

\[ \mathbf{H}_A = \frac{1}{\mu} \nabla \times \mathbf{A}. \quad (2.5) \]

The absence of magnetic charges, i.e., \( q_m = 0 \), implies \( \mathbf{M} = 0 \). The substitution of (2.5) in the Maxwell equation (2.1) and the application of (2.2) results in [13]

\[ \nabla^2 \mathbf{A} + \omega^2 \mu \epsilon \mathbf{A} = -\mu \mathbf{J}. \quad (2.6) \]
Eq. (2.6) represents three (one for each vector component) inhomogeneous differential equations for \( \mathbf{A} \). If the same procedure is applied to (2.3) with \( q_e = 0 \), a vector potential \( \mathbf{F} \) is obtained for which the equation

\[
\nabla^2 \mathbf{F} + \omega^2 \mu \epsilon \mathbf{F} = -\epsilon \mathbf{M}
\]

holds, which is very similar to (2.6).

Once the vector potentials \( \mathbf{A} \) and \( \mathbf{F} \) are obtained, one can calculate the corresponding electric and magnetic fields by

\[
\begin{align*}
\mathbf{E}_A & = \frac{1}{\mu} \nabla \times \mathbf{A}, \\
\mathbf{E}_F & = -\frac{1}{\epsilon} \nabla \times \mathbf{F}, \\
\mathbf{H}_F & = -\frac{1}{j \omega \mu} \nabla \times \mathbf{E}_F.
\end{align*}
\]

The total fields are then given by the superposition of the fields generated from the potentials \( \mathbf{A} \) and \( \mathbf{F} \), namely

\[
\begin{align*}
\mathbf{E} & = \mathbf{E}_A + \mathbf{E}_F, \\
\mathbf{H} & = \mathbf{H}_A + \mathbf{H}_F.
\end{align*}
\]

In many cases, the solution of the two equations for the vector potentials \( \mathbf{A} \) and \( \mathbf{F} \), (2.6) and (2.7), together with the calculation of the entire field by (2.8) - (2.11), can simplify the solution of the electromagnetic field problem given by Maxwell’s equations (2.1)-(2.4) significantly; the advantage gained is a higher symmetry of the latter equations.

The solutions of the well-known differential equations (2.6) and (2.7) for homogeneous space are given by the following integrals [13]:

\[
\begin{align*}
\mathbf{A} & = \frac{\mu}{4\pi} \int \int \int_D J(r') \frac{\exp(-jk|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} dV_3, \\
\mathbf{F} & = \frac{\epsilon}{4\pi} \int \int \int_D M(r') \frac{\exp(-jk|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|} dV_3.
\end{align*}
\]

In these equations, \( k \) is the absolute value of the wave vector and given by \( k^2 = \omega^2 \mu \epsilon \). The vector potentials are evaluated at any point \( \mathbf{r} \) in space.
The integration domain $\mathcal{D}$ comprises all points $r'$ in the $n = 3$ dimensional volume $V_n$ in which the electric or magnetic current densities exist, and $||.||$ denotes the Euclidean distance.

The expressions (2.14) and (2.15) remind, at least for surface currents, of Huygens's principle, which states [13] that 'each point on a primary wavefront can be considered to be a new source of a secondary spherical wave and that a secondary wavefront can be constructed as the envelope of these secondary spherical waves'. In (2.14) and (2.15), the vector potentials $A$ and $F$ at $r$ are constructed by the superposition of spherical waves of all currents $J$ or $M$, respectively, present in $\mathcal{D}$. Huygens's principle, originally developed for scalar problems, can indeed be reformulated for the vector valued electromagnetic field. The fields outside an imaginary closed surface are thereby obtained by placing suitable electric and magnetic current densities over the closed surface, so that the boundary conditions are satisfied ('surface equivalence theorem'). This strategy relies on the uniqueness of the solution of partial differential equations. The uniqueness of the solution of an electromagnetic field problem can be guaranteed when 'the sources within the region plus the tangential components of the electric field over the boundary, or the tangential components of the magnetic field over the boundary, or the former over part of the boundary and the latter over the rest of the boundary' are known [13].

The surface equivalent theorem can be applied to a problem like the one in Fig. 2.1. A closed surface $\partial \mathcal{D}$ is chosen. Within it, there are sources represented by current densities $J^{(0)}$ and $M^{(0)}$. The sources radiate fields...
\( E^{(0)} \) and \( H^{(0)} \) in all directions. If only the field outside \( \partial \mathcal{D} \) is of interest, the volume equivalence theorem states that the sources \( J^{(0)} \) and \( M^{(0)} \) can be removed and replaced by current densities \( J_{\partial \mathcal{D}} \) and \( M_{\partial \mathcal{D}} \) on \( \partial \mathcal{D} \). These current densities need to produce the field \( E^{(0)} \) and \( H^{(0)} \) only outside \( \partial \mathcal{D} \); the field inside \( \partial \mathcal{D} \), which is not of interest, can take any value. If the field inside \( \partial \mathcal{D} \) is assumed to be 0, the volume equivalence theorem states that the current densities on \( \partial \mathcal{D} \) which produce the field \( E^{(0)} \) and \( H^{(0)} \) outside \( \partial \mathcal{D} \) can be obtained by [13]

\[
J_{\partial \mathcal{D}} = \hat{n} \times H^{(0)}, \\
M_{\partial \mathcal{D}} = -\hat{n} \times E^{(0)}
\]  

(2.16)

(2.17)

where \( \hat{n} \) is the vector that is normal to the boundary and directed towards the region in which the field shall be determined. Once \( J_{\partial \mathcal{D}} \) and \( M_{\partial \mathcal{D}} \) are calculated, one can obtain the entire field using (2.14), (2.15), and (2.8) to (2.13).

Even though it appears to be a circular strategy to replace a field by a current density which is again used to calculate this field, the volume equivalence principle has found wide application. The fields derived from this principle using the integrals (2.14) and (2.15) are exact down to phase information. The field inside \( \partial \mathcal{D} \), and hence the material and its distribution, are irrelevant. Although they can be inhomogeneous, and although the solution of Maxwell’s equations would accordingly require more involved methods, Huygens’s principle thus allows to remove the inhomogeneities so that relations like (2.14) and (2.15) can be employed.

### 2.2 Ray Optics

A drawback of Maxwell’s equations is the difficulty to find solutions to the boundary conditions for complicated, real scenarios. Typically, the boundary conditions must be specified between free space and a scatterer, or between free space and a reflector or a diffracting object. If the wavelength is small compared to the distances between these obstacles, one can simplify the problem by considering sub-areas which comprise only a single object, or the \( T_x \) or the \( R_x \). Maxwell’s equations can be solved in the far field and interactions characterized in terms of reflection, diffraction, and
scattering at particular objects. Field problems can then be described by a concatenation of free space propagation and interaction with obstacles.

This approach simplifies the calculation of field problems, since solutions for canonical objects can be derived separately. The assumptions that objects are in the far field and that the wavelength is small compared to the dimensions of the surroundings are essential for the derivation of the desired expressions. A comparison with more exact methods shows, however, that their accuracy is surprisingly high even if the far field assumption and the condition of the small wavelength-to-object ratio is not fulfilled [57, 72, 120].

The derivation of ray optical methods is based on a series expansion of the electromagnetic field. If only the lowest order of this expansion is considered, the GO (Geometrical Optics) expressions are obtained. Their drawback is obvious, since ray optics results in zero field strengths in the shadow of objects. Diffracted rays were first introduced within the framework of the GTD (Geometrical Theory of Diffraction). This theory still has a shortcoming in that the diffracted fields can become singular in the transition regions around the shadow boundaries. The occurrence of the singularities was finally avoided in the succeeding UTD (Uniform Geometrical Theory of Diffraction) [103].

2.2.1 Ray Optical Construction of the High Frequency Field

The ray optical construction of a high frequency field is obtained from the wave equations (2.1) to (2.4) in source-free regions where \( J = M = 0 \) and \( q_e = q_m = 0 \). The field is expanded in a series as

\[
E(r, \omega) \sim \exp(-j k \Psi(r)) \sum_{\nu=0}^{\infty} \frac{E_{\nu}(r)}{(j \omega)^{\nu}}, \quad (2.18)
\]

\[
H(r, \omega) \sim \exp(-j k \Psi(r)) \sum_{\nu=0}^{\infty} \frac{H_{\nu}(r)}{(j \omega)^{\nu}}, \quad (2.19)
\]

where \( \sim \) denotes proportionality. The function \( \Psi \) is the so-called phase function which determines the surfaces of constant phase of the wave [103]. The expansion allows an evaluation of Maxwell’s equations for each \( \nu \). If the wavelength tends towards 0, only the first, \( \nu = 0 \) term remains, which
is called the GO field. Even though there is ‘no a-priori knowledge that the expressions (2.18) and (2.19) are valid whether as a convergent or asymptotic series’ [103], they are the main premise for many ray optically based tools. They have found widespread application, and their reliability with regard to field prediction has been amply documented (e.g. [74, 9, 94, 33, 129]).

The substitution of (2.18) and (2.19) in Maxwell’s equations and their evaluation with regard to \( \nu \) yields, for \( \nu = 0 \), the set of equations

\[
\begin{align*}
\sqrt{\frac{\mu}{\epsilon}} (\nabla \Psi \times E_0) &= H_0 \\
\sqrt{\frac{\epsilon}{\mu}} (\nabla \Psi \times H_0) &= -E_0 \\
\nabla \Psi \cdot E_0 &= 0 \\
\nabla \Psi \cdot H_0 &= 0 \\
|\nabla \Psi|^2 &= 1 \\
\text{Re}[E \times H^*] &= \sqrt{\frac{\epsilon}{\mu}} |E_0|^2 (\nabla \Psi) \\
E_0(s) &= E_0(0) \exp \left[ -\frac{1}{2} \int_0^s \nabla^2 \Psi \, ds' \right],
\end{align*}
\]

from which various properties of the GO field can be derived. In these equations, \( \text{Re}[\cdot] \) denotes the real part of a complex number, \(|\cdot|\) the absolute value and * conjugation.

Eqs. (2.20)-(2.23) state the orthogonality of the field components \( E_0, H_0 \), and of \( \nabla \Psi \). The latter, \( \nabla \Psi \), is normalized (2.24) and can be identified as proportional to the wave vector \( k \), which, in free space, determines the propagation direction of the wave and is known to be perpendicular to both \( E_0 \) and \( H_0 \). The vector \( \nabla \Psi \) thus gives the direction of the power flow (2.25) and is important for the local parameterization of the curve along which the ray propagates. If such a curve \( r(s) \subset \mathbb{R}^3 \) is parameterized by its arclength \( s \), (2.26) describes the evolution of a field which has field strength \( E_0 \) at \( s = 0 \). This equation will prove useful later on. The instantaneous direction of the power flow \( \nabla \Psi \) is the tangent vector \( t \) of the curve \( r(s) \). The tangent vector is defined by \( t = \frac{dr}{ds} \), and it thus holds that

\[
\nabla \Psi = \frac{dr}{ds}.
\]
Using (2.24) and the chain rule of differentiation, one obtains
\[ \frac{d^2 \mathbf{r}}{ds^2} = 0, \] (2.28)
and hence
\[ \mathbf{r}(s) = C_1 s + C_2 \] (2.29)
where \( C_\nu, \nu = 1, 2, \) are constants. Thus, GO rays in a homogeneous medium are straight lines. The homogeneity of the medium is essential for this statement; in inhomogeneous media, the ray trajectories are usually curved.

Eq. (2.26) demonstrates that \( t = \nabla \Psi \) is also involved in the evolution of the amplitude with regard to \( s \). In GO terms, rays propagate in so-called ray tubes. In the direction of propagation, the surface of the ray tube is the constant phase surface of the ray. Its shape is locally characterized by its Gaussian curvature \( G_C(0) = \frac{1}{\rho_1 \rho_2} \) at a reference point \( s = 0 \). The values \( \rho_1 \) and \( \rho_2 \) are the principle radii of curvature of the surface measured on the central ray. They provide a local characterization for the curvature of the surface [22]. At a distance \( s \), one has \( G_C(s) = \frac{1}{(\rho_1 + s)(\rho_2 + s)} \).

Using an argument based on the equation of continuity, or, casually spoken, on the conservation of energy, the fraction \( \frac{G_C(s)}{G_C(0)} \) turns out to obey a relation very similar to (2.26) [103], namely
\[ \frac{G_C(s)}{G_C(0)} = \exp \left[ -\frac{1}{2} \int_0^s \nabla^2 \Psi ds' \right]. \] (2.30)
Substituting the definition of \( G_C \), the GO field is given by
\[ E(s) = E(0) \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}} \exp -jks, \] (2.31)
\[ H(s) = H(0) \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}} \exp -jks \] (2.32)
where
\[ E(0) = E_0 \exp -j k \Psi(0), \] (2.33)
\[ H(0) = H_0 \exp -j k \Psi(0), \] (2.34)
and where \( E(s) \) and \( H(s) \) are locally related by (2.20). Eqs. (2.31) and (2.32) show that the GO field is determined by the radii of curvature of
the surface of constant phase. At the points \( s = -\rho_\nu, \nu = 1, 2 \), the field is infinite. These singularities are called caustics. If the surface of constant phase lies on a sphere, a caustic has a well-known physical explanation. For a sphere, it holds that \( \rho_1 = \rho_2 = \rho \). If the reference point is moved into the caustic, one has \( \rho = -s \), and hence

\[
E(s) = \frac{C_0}{s} \exp(-jks). \tag{2.35}
\]

The usual expression for a spherical wave is obtained. The value \( C_0 \) is an excitation factor for the wave, as it is obtained when the equations for the vector potentials, (2.14) and (2.15), are solved in the far field of a point source in free space [13].

In the case of a plane wave, the constant phase surfaces at the selected reference point are planar. Hence, \( \rho_\nu \to \infty, \nu = 1, 2 \), and (2.31) reduces to

\[
E(s) = E(0) \exp(-jks), \tag{2.36}
\]

which demonstrates that plane wave has no caustics.

In more general cases, a caustic may change the phase of the GO field by \( \pi \) [103].

### 2.2.2 Reflection and Transmission

The GO approach allows to determine the direction of propagation and the polarization of a reflected wave with an arbitrary wavefront on a smooth, but not necessarily plane surface. Essential for this approach is the perfect conductivity of the surface, which can be expressed by the equation

\[
\hat{n} \times E(r) = 0, \tag{2.37}
\]

where \( \hat{n} \) is the normal vector of the surface. If the field is decomposed into an incident field \( E^i \) and a reflected field \( E^r \), and if the resulting expression is substituted into the series expansion (2.18), it follows that

\[
\Psi_i(r) = \Psi_r(r) \tag{2.38}
\]

\[
\hat{n} \times E^i_\nu(r) = -\hat{n} \times E^r_\nu(r) \text{ for all } \nu. \tag{2.39}
\]

From (2.38), one can deduce the relation

\[
\theta_i = \theta_r \tag{2.40}
\]
where $\theta_i$ and $\theta_r$ are the angles of the incident and of the reflected wave at the surface. This is Snell’s law of reflection, derived for a perfectly conducting surface.

Using Snell’s law and several geometrical considerations, the direction of propagation $\hat{s}^r$ of the reflected ray in dependence on the corresponding vector $\hat{s}^i$ of the incident wave can be derived. Since the incident and the reflected wave, together with $\hat{n}$ and the tangent vectors of the surface at the point of reflection, form two congruent triangles along the surface (Fig. 2.2), one has

$$\hat{s}^r = \hat{s}^i - 2(\hat{n} \cdot \hat{s}^i)\hat{n}. \quad (2.41)$$

The polarization of the reflected wave with regard to the plane of incidence can also be obtained from $\hat{s}^i$ and $\hat{n}$, or from $\hat{s}^r$ and $\hat{n}$, respectively. Purely geometrical considerations yield \cite{103}

$$\hat{e}^i_\parallel = \frac{\hat{s}^i \times (\hat{n} \times \hat{s}^i)}{||\hat{s}^i \times (\hat{n} \times \hat{s}^i)||}, \quad (2.42)$$

$$\hat{e}^r_\parallel = \frac{\hat{s}^r \times (\hat{n} \times \hat{s}^r)}{||\hat{s}^r \times (\hat{n} \times \hat{s}^r)||}, \quad (2.43)$$

$$(\hat{n} \cdot \hat{s}^i)\hat{e}^i_\perp = \hat{n} \times \hat{e}^i_\parallel, \quad (2.44)$$

$$(\hat{n} \cdot \hat{s}^r)\hat{e}^r_\perp = \hat{n} \times \hat{e}^r_\parallel, \quad (2.45)$$

where $\hat{e}^i_\parallel$ and $\hat{e}^r_\parallel$ are the unit vectors which point into the direction of the parallel component of the incident and of the reflected field, respectively, and where $\hat{e}^i_\perp$ and $\hat{e}^r_\perp$ represent the corresponding unit vectors for the perpendicular component. Snell’s law of reflection (2.40) gives $\hat{e}^i_\perp = \hat{e}^r_\perp$. By (2.41) and (2.42)-(2.45), the reflected path of the wave and its polarization
with regard to the plane of incidence is completely specified. The remaining task is to find the reflection coefficients $R_{\parallel}$ and $R_{\perp}$, the former for the component that is parallel, the latter for that perpendicular to the plane of incidence. As the GO assumes perfectly conducting surfaces (2.37), the result is very intuitive and not of great applicability for most real cases. It holds that $|R_{\parallel}|$ and $|R_{\perp}|$ are 1, since there are no reflection losses at a perfectly conducting surface [103]. Except for perfect conductors, these coefficients can also be applied in case of real metals.

Of greater use are reflection and transmission coefficients for non-perfectly conducting surfaces. In almost all cases, the Fresnel reflection and transmission coefficients are used. These coefficients are derived from Maxwell’s equations for a plane wave that impinges on the boundaries between two infinite half planes. The dielectric properties of the two half planes are expressed by their permittivity $\varepsilon^{(\nu)} = \varepsilon_0 \varepsilon_r^{(\nu)}$ for $\nu = 1, 2$, where $\varepsilon_0$ is the permittivity of free space and $\varepsilon_r^{(\nu)}$ the relative permittivity of the medium $\nu$. For almost all dielectric materials of interest in indoor communications, the relative permeability that describes the magnetic properties of the material can be set to one [149]. The Fresnel reflection coefficients for parallel and perpendicular polarized electromagnetic fields $R_{\parallel}$ and $R_{\perp}$ are then [43]

\[
R_{\parallel}(\theta_i, \varepsilon_r^{(1)}, \varepsilon_r^{(2)}) = \frac{\varepsilon_r^{(2)} \cos(\theta_i) - \sqrt{\varepsilon_r^{(1)} \varepsilon_r^{(2)} - \varepsilon_r^{(1)} \varepsilon_r^{(2)} \sin(\theta_i)^2}}{\varepsilon_r^{(2)} \cos(\theta_i) + \sqrt{\varepsilon_r^{(1)} \varepsilon_r^{(2)} - \varepsilon_r^{(1)} \varepsilon_r^{(2)} \sin(\theta_i)^2}},
\]

\[
R_{\perp}(\theta_i, \varepsilon_r^{(1)}, \varepsilon_r^{(2)}) = \frac{\varepsilon_r^{(1)} \cos(\theta_i) - \sqrt{\varepsilon_r^{(1)} \varepsilon_r^{(2)} - \varepsilon_r^{(1)} \varepsilon_r^{(2)} \sin(\theta_i)^2}}{\varepsilon_r^{(1)} \cos(\theta_i) + \sqrt{\varepsilon_r^{(1)} \varepsilon_r^{(2)} - \varepsilon_r^{(1)} \varepsilon_r^{(2)} \sin(\theta_i)^2}}. (2.46)
\]

The transmission coefficients $T_{\parallel}$ and $T_{\perp}$ are [43]

\[
T_{\parallel}(\theta_i, \varepsilon_r^{(1)}, \varepsilon_r^{(2)}) = \frac{2\varepsilon_r^{(1)} \varepsilon_r^{(2)} \cos(\theta_i)}{\varepsilon_r^{(2)} \cos(\theta_i) + \sqrt{\varepsilon_r^{(1)} \varepsilon_r^{(2)} - \varepsilon_r^{(1)} \varepsilon_r^{(2)} \sin(\theta_i)^2}},
\]

\[
T_{\perp}(\theta_i, \varepsilon_r^{(1)}, \varepsilon_r^{(2)}) = \frac{2\varepsilon_r^{(1)} \cos(\theta_i)}{\varepsilon_r^{(1)} \cos(\theta_i) + \sqrt{\varepsilon_r^{(1)} \varepsilon_r^{(2)} - \varepsilon_r^{(1)} \varepsilon_r^{(2)} \sin(\theta_i)^2}}. (2.47)
\]

If a wave is transmitted through a wall, multiple transmission and reflection occurs. It can be derived from a concatenation of several plane layers
of dielectric materials [13, 43]. Publications on the applicability of multiple reflection models exist for GHz frequencies [124, 15] and indicate that these are able to roughly describe reflection measurements at the examined surfaces.

Since most walls in buildings consist of lossy material (c.f. dielectric constants in [149]), the attenuation caused by wall transmission increases with increasing wall thickness-to-wavelength ratio. Whereas at 5 GHz (6 cm wavelength), the wall transmission is still of considerable importance [83], the field at 60 GHz (0.5 cm wavelength) is already significantly attenuated [131]. A useful comparative study of wave propagation within single rooms and across floors of buildings for 17 and 60 GHz is [15].

### 2.2.3 Diffraction and Scattering

When Maxwell’s equations are solved as a boundary value problem, the entire field existing in the specified region is calculated at once. The field’s different localized interactions with objects are not particularly specified. A ‘scatterer’ in this context denotes any body that interacts with a previously undisturbed field. Typical methods in field theory, such as Huygens’s principle described in Section 2.1, provide an exact treatment of both diffraction and scattering, without, however, distinguishing between them.

The high frequency approximation allows, in contrast, to treat objects that interact with the electromagnetic field separately, and the terms diffraction and scattering are given a concrete meaning. Diffraction and scattering are phenomena that cannot be explained by GO, since GO fields are zero behind shadow boundaries; but the GTD and the UTD are extensions which allow to describe the propagation of the field into the region behind wedges [103]. In contrast to reflection, diffraction is not a specular phenomenon; the diffracting edge can be regarded as a point source with a radiation pattern that depends on the geometry of the wedge as well as on the angle of incidence of the impinging wave. In two dimensions, the wedge itself is the point-like source; in three dimensions, the wedge is a line, and the correct location of the source on the wedge requires some numerical effort. Although the implementation of wedge diffraction in a ray tracing tool is not difficult, it is nevertheless not straightforward either.

At frequencies above about 2 GHz, diffraction is not a dominant carrier
of energy [83]; still, in the transition region between LOS (line-of-sight) and NLOS (non-line-of-sight), and in all NLOS regions where no power can be transmitted even by multiple reflections, diffraction becomes significant [9]. In simple indoor environments above 40 GHz, however, diffraction may be entirely neglected [130].

The concept of diffraction as wave propagation around (sharp) wedges fails to work for such objects as cylinders, where no diffracting wedge is present. In this case, the UTD introduces so-called creeping waves [103], which help describe the field in the shadow zone behind structures without wedges. Apart from the UTD, scattering problems have been solved for canonical objects such as finite and infinite cylinders in fields radiated from various types of sources [16]. Most of these solutions are available as infinite series, and the field depends strongly on the wavelength-to-object ratio, on the scattering angle, and on the shape of the scatterer.

Of practical interest, too, is the scattering at rough surfaces. According to the Fraunhofer criterion, a surface is rough when [43]

\[ \sigma_{Fra} > \frac{\lambda}{32 \cos \theta_i} \]  

(2.48)

where \( \lambda \) is the wavelength and \( \sigma_{Fra} \) the standard deviation of the surface roughness. For an incidence normal to the surface, one has \( \theta_i = 0 \), and the expression on the right hand side is at its maximum.

For lower GHz frequencies, wavelengths are of the order of several cm, and rough surface scattering can be neglected for most wall types. At higher frequencies, when wavelengths are below 1 cm (at 60 GHz, the wavelength is 5 mm), there may be walls for which the assumption of specular wave reflection is no longer exactly valid. The power density in the specular direction is reduced due to the partial cancellation of the statistically varying phases. Up to a factor of 4 in (2.48) (Rayleigh criterion), i.e., up to \( \frac{\lambda}{8 \cos \theta_i} \), reflection can be described by Fresnel reflection coefficients which are modified by a negative exponential that depends on the surface roughness, on \( \theta_i \) and \( \lambda \) [87]. The specular component is thus more and more attenuated. If \( \sigma_{Fra} \) is higher than the bound given by the Rayleigh criterion, numerical methods must be applied [146].

For the practical purpose of indoor wave propagation, scattering at furniture and human bodies is of interest. If furniture is not metallic and if it does not obstruct the main propagation paths, its influence is, due to
the ray optical behavior of the field, not really significant [90, 89]. The same holds for a human body the impact of which is, however, also high when main paths are blocked [96, 24].

2.2.4 Antenna Characteristics

In signal processing, one is commonly concerned with scalar, complex amplitudes, not with the six dimensional electromagnetic field. The antennas of a communication system, which convert an input voltage into an electromagnetic field at the $T_x$, and the field into an output voltage at the $R_x$, can thus actually be regarded as part of the radio channel.

An antenna can be characterized by its pattern and its gain. The pattern is given by

$$C(\Omega) = \frac{\mathbf{E}(\mathbf{r}, \Omega) \exp j k r}{|\mathbf{E}(\mathbf{r}, \Omega)|_{max} \biggr|_{r=const. \rightarrow \infty}}.$$  \hspace{1cm} (2.49)

It is defined on the unit sphere, the directions of which are denoted by $\Omega$, but far away from the antenna ($||\mathbf{r}|| = r = const. \rightarrow \infty$), and normalized to be lower than or equal 1. It has two components which correspond to vertical and horizontal polarization, and which depend on the coordinate system in which the antenna is oriented. The phase factor $\exp j kr$, and particularly its sign, is chosen such that it cancels the phase factor $\exp -j kr$ of an irradiated spherical wave, and that it is thus independent from the distance $r$ between $T_x$ and $R_x$ [43].

The gain of an antenna is defined as the quotient of the (with regard to $\Omega$) highest irradiated spatial power density $S_{max}(\mathbf{r}, \Omega)$ and the power density of the fictitious isotropic antenna $S_{iso}(\mathbf{r})$ [43] if both antennas irradiate the same power,

$$G = \frac{S_{max}(\mathbf{r}, \Omega)}{S_{iso}(\mathbf{r})} \biggr|_{r=const. \rightarrow \infty}.$$  \hspace{1cm} (2.50)

Using the definition of the gain and the antenna pattern, scalar amplitudes can now replace the vector valued field quantities as follows. The transmitted field’s ray optical decomposition into the direct path between $T_x$ and $R_x$ and into a countable number of transmitted, reflected, diffracted, and scattered rays allows an assignment of a virtual point source to each direction of an impinging ray as the origin of this ray. If the $R_x$
is located at the origin, the corresponding received amplitude $\alpha_\nu$ of the point sources $\nu$, which are located at $r_\nu$, is

$$
\alpha_\nu = \frac{\lambda \sqrt{G_{Rx} G_{Tx}}}{4\pi ||r_\nu||} C_{Tx}(\Omega_{\nu, Rx}) S_\nu C_{Rx}(\Omega_{\nu, Tx}) \exp -j \frac{2\pi}{\lambda} ||r_\nu||
$$

(2.51)

where $G_{Rx}$ and $G_{Tx}$ are the gains, and $C_{Rx}(\Omega_{\nu, Rx})$ and $C_{Tx}(\Omega_{\nu, Tx})$ the radiation patterns at the angle of arrival/departure $\Omega_{\nu, Rx}$ and $\Omega_{\nu, Tx}$ of $Rx$ and $Tx$. The quantity $S_\nu$ is a $2 \times 2$ matrix that describes all interactions of the field components of the $\nu$th ray with objects, such as reflections and transmissions [43]. The factor $\frac{\lambda}{4\pi ||r_\nu||}$ covers the free space loss. Eq. (2.51) is normalized such that $|\alpha|^2$ gives the pathloss, which is independent from the transmitted power. The entire received amplitude $\alpha$ is then the sum of all complex amplitudes that are received.

The impact of the polarization and of the pattern of the transmitting and the receiving antenna can be significant. Some years ago, these effects were investigated in a rather straightforward manner [95, 75, 143]. With the overwhelming attention that the MIMO channel has recently received, the topic has gained new significance in communications research. The related recent work was already mentioned in Subsection 1.2.

### 2.3 Wave Propagation Models

The growth of wireless communications has incited the development of a large amount of different modeling approaches, some of which are more, and some less tightly knitted to insights into deterministic wave propagation. At a first glance, it seems very difficult to describe their differences and similarities in a systematic way. All models trade off the balance between complexity and exactness on the one hand, and simplicity on the other. Complexity refers to the description of both the wave propagation and the environment. The two are intertwined, since the rather complicated exact methods require a precise specification of the environment, which is often difficult to realize. If the environment is not modeled exactly, wave propagation is described in more empirical or even in purely statistical terms; these models are usually much simpler to apply.
2.3.1 Deterministic Models

Most deterministic models referring to the radio channel rely on a field description based on the UTD [10, 74, 88] or even on GO only [150, 122, 94, 32]. Numerous studies on the accuracy of these methods have been performed. Ray optics has been successfully applied in frequencies from 900 MHz (Megahertz) up to more than 60 GHz [65, 150, 94, 129]. The number of rays needed to obtain a sufficiently exact description of the field is found to depend very much on the entire received power. In LOS scenarios with a strong direct path, a ray reflected more than twice can already be neglected; for more complicated setups, diffraction and higher orders of reflection should be taken into account [32, 10]. The importance of diffracted rays depends on the frequency; at high frequencies, diffracted rays generally contribute less to the total received field (cf. Subsection 2.2.3).

Several approaches to calculate the ray optical field have evolved. The method of images is widely used to construct the path of reflected rays [102, 127]. Since in complicated environments, an exact construction of the location of all image sources is time-consuming, ray launching methods have been developed. The latter launch rays, track their propagation paths, and neglect them when their amplitude decreases below a predefined level [68, 40]. Strictly deterministic methods are not always applied. Since the determination of the correct dielectric properties of the wall materials is quite difficult, empirically determined parameters are sometimes used, for example in ray tracing based on a trained neural network [144].

All deterministic models require the description of the environment to be as precise as possible, which includes information about the position of $T_x$ and $R_x$. Often, the position of, e.g., the $T_x$ is fixed, and the received field is calculated within a specified area. Corresponding results can be used in field prediction and network planning. The numerical efforts can become intractable, however, if both $T_x$ and $R_x$ are randomly located. Additionally, the design of an investigated environment on the computer is usually time-consuming.
2.3.2 Semi-Deterministic Models

Many models result from large measurement campaigns. For practical purposes, the characterization of the environments in which the campaigns are carried out is usually rough. It usually consists of photographs or a description of the details which are considered to influence the wave propagation most significantly. Typical parameters are building height, street width or, within buildings, room size and wall material. This strategy relies on the assumption that geometrically similar scenarios are similar for wave propagation behavior as well, and that the results obtained from a particular campaign are hence valid for a wider set of environments.

Semi-deterministic models try to incorporate the significant parameters describing the environment as free variables; many of them describe MPI (mean pathloss), which is an average quantity and thus robust enough to obey an indifferent description. The models are based on intuitive assumptions about the field, such as inverse distance relations for the received power, or a particular, angle independent power loss for each wall transmission.

The empirical power-to-distance law for the received power $P_{R_x}$ at a given distance $r$ between $T_x$ and $R_x$ is

$$P_{R_x} = \frac{1}{r^q},$$

or

$$P_{R_x} = 10\log_{10}k - 10q\log_{10}r,$$

logarithmically (2.53)

where $\log_{10}$ is the logarithm to the base 10. The parameters $k$ and $q$ are empirical. Eq. (2.52) is the simplest pathloss model and widely used [118, 31, 45, 91]. The value of $q$ is in the range of about 1.5 to 5, where the lowest values are valid for LOS in buildings, and the highest for heavily obstructed scenarios such as transmission through several floors. The model covers the special case of free space propagation which is obtained for $q = 2$.

There exist several modified versions of this model. Written in the form

$$P_{R_x} = 10\log_{10}k - 10q\log_{10}r - F,$$

an empirical attenuation factor $F$ may describe the attenuation for each floor that is transmitted; a similar model,

$$P_{R_x} = 10\log_{10}k - 20\log_{10}r - F - \kappa'r,$$

(2.55)
observes that in-building pathloss obeys free space pathloss plus an additional loss factor $F$ for each transmitted floor plus an exponential increase with distance modeled by $\kappa'$ [118]. A similar model is the multi-wall-model, which contributes a constant attenuation for each wall and floor transmission [31].

The Walfish-Bertoni model [140] and its recently published successor [27], designed for outdoor scenarios, are exemplarily mentioned here as models which include simplified deterministic information about the environment. Buildings are modeled as half screens which are at equal distance. Again, wave propagation expressions are not evaluated exactly, but only approximated. Still, derived expressions require numerical evaluation. The earlier paper has found considerable interest and demonstrates that the simplification of geometrical parameters of the channel may still yield agreeable modeling results. As can be seen in Subsection 1.2, most geometrically based models have been designed for the outdoor case. An approach for indoor communications is [61]; in this paper, however, stronger emphasis is put on the statistical modeling, particularly on the geometry on the environment.

### 2.3.3 Stochastic Models

Communication engineers not involved in the study of wave propagation tend to regard the radio channel as a linear system. The input is the transmitted signal, its output is an amplitude at the receiving antenna. The system is characterized by its impulse response $h(t, \tau)$, where $t$ denotes the variation of the radio channel over time, and where $\tau$ describes the delay of signals due to their propagation through the channel.

The origin of this view is the well-known article of Bello, published in 1963 in order to characterize the tropospheric channel [14]. In this paper, the impulse response $h(t, \tau)$ was considered to be a so-called gain function that operates on an incoming signal $z_{LT V}(t)$, and subsequently delays it by $\tau$. Constraining himself to purely stochastic impulse responses with mean value zero, Bello then investigated the autocorrelation function $R_h$ of $h$, and its Fourier transforms. Since $R_h$ depends on two variables, there exist three Fourier transforms. Bello related the transformed autocorrelation functions to the Fourier transforms of the impulse response $h$ and obtained, apart from $R_h$, three additional, equivalent autocorrelation
functions. These functions depend on the Fourier pair of $\tau$, which is the frequency of the system’s transfer function, and on the Fourier pair of $t$, which is the Doppler frequency of the time variant system.

This approach allows to derive channel characteristics rigorously. For the mobile radio channel, the constraints of the channel to be WSS (wide sense stationary) and US (uncorrelated scattering) can be imposed on $R_h$ and on its Fourier transforms [37]. In this case, the channel correlation functions are invariant under a translation in time, and the gain functions for different path delays are uncorrelated. For such a system, the delay spread and the coherence time can be defined, which are important in order to determine whether the channel is frequency selective and fast fading [37, 116].

Fleury extended this model into the angular domain [38]. Many publications rely on the terminology developed therein, and accordingly investigate functions such as the PDP of time variant transfer functions by statistical approaches [63, 137, 79, 39].

There are many other statistical studies that work without a theoretical embedding. In these models, interesting quantities such as the amplitude are extracted from measurements, and their behavior is described on the basis of specific statistical assumptions. Distribution functions which match the experimental findings are fitted to the data. Corresponding parameters are tabulated and can be regarded as a valid approximation for environments similar to that in which the study was performed. Typical studies of the indoor channel include [104, 58, 141, 82] for 1-5 GHz, [2, 63, 76, 15] for about 10 to 40 GHz, and [15, 85, 67] for 60 GHz.

Statistics about amplitude variations are usually divided into large-scale and small-scale variations. In case of small-scale variations, $T_x$ or $R_x$ are moved over a small area in which one can assume that only the phases of the field vary, and not the absolute value of the amplitudes of the individual impinging waves. Large-scale variations include variations of the amplitude. In [58], the small-scale variations were recorded within the region of 1-2 m for a wavelength of about 30 cm (900-1300 MHz), and steps in antenna separation of at least 5 m were assumed to be large-scale variations.

On large scales, the absolute value of the received amplitude, $|\alpha|$, is
lognormal distributed ([59] and further references therein), i.e.,

$$P_{\text{logn}}(|\alpha|) = \frac{1}{\sqrt{2\pi}\sigma_{\text{logn}}|\alpha|} \exp\left(-\frac{(\ln|\alpha| - \mu_{\text{logn}})^2}{2\sigma_{\text{logn}}^2}\right), \quad |\alpha| \geq 0 \quad (2.56)$$

where $\ln$ is the natural logarithm. A lognormal distribution is a Gaussian on the logarithmic scale with mean $\mu_{\text{logn}}$ and standard deviation $\sigma_{\text{logn}}$. This distribution has been applied in two different contexts. It has been used to describe the fluctuations around a mean value obtained for all measurements at a particular $r$ [119, 58, 121]; in [43], however, it is noted that the distribution can also be used to describe the statistics of all received amplitudes, independently from $r$.

Variations of the amplitude within a small region follow, with few exceptions, Rayleigh or Ricean distributions [59, 45]. Their densities, $P_{Ra}$ and $P_{Ri}$, are

$$P_{Ra}(|\alpha|) = \frac{|\alpha|}{\sigma_{Ra}^2} \exp\left(-\frac{|\alpha|^2}{2\sigma_{Ra}^2}\right), \quad |\alpha| > 0 \quad \text{and}$$

$$P_{Ri}(|\alpha|) = \frac{|\alpha|}{\sigma_{Ri}^2} \exp\left(-\frac{|\alpha|^2 + A_{Ri}^2}{2\sigma_{Ri}^2}\right) I_0\left(\frac{A_{Ri}|\alpha|}{\sigma_{Ri}^2}\right), \quad |\alpha| > 0, A_{Ri} \geq 0. \quad (2.58)$$

The parameters $\sigma_{Ra}$ and $\sigma_{Ri}$ denote the standard deviations of the Gaussian distributions from which the densities can be derived [116]. The function $I_0$ denotes a zeroth order modified Bessel function of the first kind. The Rayleigh distribution is a special case of a Ricean for $A_{Ri} = 0$ where $A_{Ri}$ characterizes the peak amplitude of a dominant, deterministic contribution to the signal. Correspondingly, a Ricean describes the small-scale fading for LOS situations, where the Rayleigh distribution works for scenarios without a strong component. Further modifications of these distributions exist. A Rayleigh distribution for more than one strong component was derived in [35]; the Nakagami m-distribution [116] contains both Rayleigh and Ricean distributions as special cases.

The ratio between the deterministic (or dominant) component $A_{Ri}$ and the stochastic contributions is defined as the so-called k-factor,

$$k_f = 10 \log_{10} \frac{P_{\text{LOS}}}{2\sigma_k^2}, \quad (2.59)$$
where $\sigma_k^2$ is the power of the scattered components and $P_{LOS} = A_{Ri}^2$ the LOS power [45].

If the channel is frequency selective, the received signal includes multiple versions of the transmitted signal, which are attenuated and delayed in time. The decay of the received power over the delay, which can be more rigorously defined as the PDP within the framework of the WSSUS (wide sense stationary uncorrelated scattering) model, is reported to be exponential within a wide frequency range [63, 85, 54], i.e.,

$$P_{Rx}(\tau) = P_{Rx}(\tau = 0) \exp \left( -\frac{\tau}{\sigma_T} \right). \tag{2.60}$$

The quantity $P_{Rx}(\tau = 0)$ describes the large-scale fluctuations given by (2.56), and $\sigma_T$ is the so-called delay spread. The delay spread has its origin in Bello’s paper [14], and has received considerable attention in literature [117, 20, 60, 58, 75, 101]. A related topic concerns the statistics of arrival times. An obvious choice is a Poisson distribution, which works, however, rather badly [59]. Modified Poisson distributions include those with additional empirical parameters [134, 41] as well as inhomogeneous [80] and double Poisson processes [121].

As already mentioned in Section 1.2, more recent studies extend these probability densities to the directional and the MIMO channel. Reported angular distributions are Laplacians for both the indoor and outdoor azimuth [63, 133, 113]. Related work about the spatial characteristics of the radio channel includes [83, 78, 47]. Results about spatial correlations are cited in [59, 111, 77, 145, 136, 141, 82]. The channel capacity of the MIMO channel is addressed in [8, 21, 44, 110, 86, 82].

All statistical models characterize the apparent randomness of the field directly; they can be considered most successful in communication engineering. However, this approach has prevented the establishment of other than empirical knowledge about the radio channel. Means and standard deviations of the obtained distributions are still only available from measurement campaigns or simulation studies. In most cases, their relation with the surrounding environment is not explained, and hence no expressions of general validity are available.
Chapter 3

Geometric Probabilities and Integral Geometry

3.1 Overview and Mathematical Preliminaries

Some of the key results given in this thesis are derived using methods from Integral Geometry and from Geometric Probability. The origin of Geometrical Probability is usually assigned to Buffon’s clean tile problem. In the 18th century, the French scientist Buffon calculated the probability of a randomly tossed coin to hit or not to hit the intersection between regularly shaped tiles which cover the floor of a room [132]. A similar, more widely known task is Buffon’s needle problem for which instead of the coin a needle, and instead of the tiles a set of parallel lines is assumed; here, the probability of the needle to intersect one of the lines is calculated. Problems of this kind inspired mathematicians to look at this transition region between geometry and probability theory in more detail. This field encompasses today a large number of topics which find, for instance, application in stereology or spatial statistics.

According to [126], Integral Geometry ’is concerned with the study, computation, and application of invariant measures on sets of geometric objects. It has its roots in some questions on geometric probabilities’, but provides a more rigorous approach, as required for the definition of the term ’randomness’. The latter depends on a set of elements on which a particular measure is defined. A randomly located line on a plane can be described by its direction and by its distance from a chosen origin; alternatively, one could choose its direction and the distance between the origin and its intersection point with an $x$-axis of a given coordinate system. Starting with these different densities, different probabilities could,
for instance, be derived for Buffon's needle problem. A more famous example is Bertrand's Paradox: A 'random chord', which arises when a line is randomly dropped on a circle of unit radius, has a length greater than $\sqrt{3}$ with probability $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{4}$ depending on how the randomness of the dropped line is defined [81].

A unique definition of measures in geometric probability requires some kind of constraints. A very natural choice of constraint are invariance properties of the measure. Two very common measures are volume and surface; these are invariant against motions, i.e., their value is independent from the choice of the location of the origin and the orientation of the respective coordinate system. Motion invariance depends on a distance measure. In this thesis, the usual Euclidean distance is (naturally) used; the $\mathbb{R}^n$ is then a Euclidean space denoted by $\mathbb{E}^n$.

Whereas the motion invariant measures for single points are easily established (see below in Subsection 3.2.1), measures of more complicated structures such as linear subspaces of $\mathbb{E}^n$ are difficult to obtain from straightforward operations. However, the geometric properties of precisely those measures are a powerful tool to solve the geometrical problems that arise in stochastic channel modeling.

Common to all approaches to integral geometry is the need of group theoretical methods, since a motion is a group operation. In [123], a standard book in integral geometry, such a group theoretical approach is combined with differential geometry, since the points of the group of motions constitute a differentiable manifold. For each linear subspace of $\mathbb{E}^n$, for which a motion invariant density shall be derived, there exists a subgroup of the group of motions under which the subspace is invariant: A line in $\mathbb{E}^3$ is itself invariant against translations along its own axis; a plane in $\mathbb{E}^3$ is invariant against translations along two axes, and against rotations about its perpendicular. This subgroup, together with the group operations themselves, constitutes a set of submanifolds. These submanifolds can be written as the solution (kernel) of a system of linear (differential) forms. For the group of motions, these linear forms can be constructed such that they are themselves motion invariant. These motion invariant linear forms then uniquely define, up to a constant factor, the motion invariant density of this subspace. This can be explained by the fact that each submanifold, which is the solution of the system of linear
forms, uniquely defines a position and orientation of the linear subspace which is identified by the corresponding subgroup. This general approach has the advantage that it works not only for motions, but also for any other group operation. It allows to prove that in general cases, densities of linear subspaces may not exist for certain group operations.

Another, more recent approach to invariant measures is based on measure theory [125, 126]. Other standard books in this and in closely related fields are [81, 132, 6]; important tutorial articles include [108, 109, 93, 11].

3.2 Densities of Points and Lines

3.2.1 The Density of a Point

The Lebesgue measure [5] is motion invariant and hence a natural candidate as a measure for points. An (unnormalized) measure for a point in an arbitrary domain $\mathcal{D}$ is thus

$$P(\mathcal{D}) = \int_\mathcal{D} d\mathcal{P} = V_n,$$

which is simply the $n$-dimensional volume $V_n$ of $\mathcal{D}$. This measure is unique up to a constant factor [123]. The differential $d\mathcal{P}$ is then the density of this point. This statement is mathematically rough; references to the more stringent framework was given in Section 3.1.

Since $P$ is motion invariant, one can define an arbitrary origin $O$ and transform $d\mathcal{P}$ into spherical coordinates; the point has then the distance $r(\Omega)$ in direction $\Omega$ to $O$, and one can write

$$\int_\mathcal{D} r(\Omega)^{n-1} drd\Omega = \frac{1}{n} \int_{\mathcal{U}_{n-1}} R^n(\Omega)d\Omega = V_n$$

where $R(\Omega)$ is the distance from $O$ to the boundary of $\mathcal{D}$, and where the integral over $\mathcal{U}_{n-1}$ denotes the integral over the unit sphere in $n$ dimensions (Fig. 3.1).

A similar relation, not on $\mathcal{D}$ but on $\mathcal{U}_{n-1}$, is

$$\int_{\mathcal{U}_{n-1}} d\Omega = O_{n-1}$$

(3.3)
where $O_{n-1}$ is the surface of the $n$-dimensional unit sphere. The latter is given by

$$O_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)}$$  \hspace{1cm} (3.4)$$

where $\Gamma$ denotes the $\Gamma$-function [17]. For $n = 3$ one has $O_2 = 4\pi$, which is the surface of the unit sphere, $O_1 = 2\pi$, which is the perimeter of a circle, and $O_0 = 2$.

Eqs. (3.2) and (3.3) are simple and fundamental relations that show the relation between certain integrals over the density of a point with geometrically fundamental expressions. If $f(r)$ is a function of the distance $r$, one can assume that integrals of the type $\int_{D} f(r)d\mathcal{P}$ can be reduced to or bounded by expressions of the type (3.2) and (3.3), and thus related to the volume and the surface of $D$. Whereas the point can be a freely moving user of a communication system, the origin $O$ can represent a fixed access point. To bound relations between their average distances by simple geometric quantities is the strategy on which some of the main results of this thesis are based.
3.2.2 The Density of Two Points and the Density of Lines

Analogously to Subsection 3.2.1, the measure of two points independently distributed within a domain $D$ is

$$P(D \times D) = \int_{D \times D} dP_1 dP_2. \quad (3.5)$$

In the case of two points, not their distance to the origin is of interest, but their distance to each other; the choice of a fixed origin becomes obsolete. The key to tackling this problem is the transformation of the density of two points, $dP_1 dP_2$, into the densities of two points on a line $L$, and into the density of this line, $dL$. The derivation of $dL$, its rigid treatment, and the proof that $dL$ is a well and uniquely defined measure require techniques from differential geometry and group theory already mentioned above. Their geometrical interpretation is briefly outlined in the following (Fig. 3.2). The perpendicular of a line $L$ to an origin $O$ defines a distance $\hat{r} = ||r_0||$ of $L$ to $O$. The line segment $\overline{Or_0}$ is in the plane $F$, which is also perpendicular to $L$ and thus uniquely defined. The point $r_0$ where $L$ intersects with $F$ can be described on $F$ in cylindrical coordinates by its distance $\hat{r}$ from $O$ and by an angle $\phi_L$ relative to a fixed axis of $F$, here $x$. Furthermore, $L$ has a direction which can be described by the pair of angles $(\psi_L, \theta_L)$. The density of lines is the volume element of $F$ at $r_0$, combined with the volume element of the unit sphere, $dL = \hat{r} d\hat{r} d\phi_L \cdot \sin \theta_L d\theta_L d\psi_L$. This density is motion invariant; it does not depend on the location of $O$.

Using $dL$, the density of the location of two points $r_1$ and $r_2$ in $E^n$ can be transformed into the density of these points on $L$, multiplied with $dL$; in equivalence to the transformation of the location of a single point from cartesian to spherical coordinates, the expression must be weighted with the square of the distance of the points, so that one obtains $dP_1 dP_2 = |t_2 - t_1|^{n-1} dt_1 dt_2 dL$, where $t_1$ and $t_2$ are the coordinates of the points relative to $r_0$ on $L$.

Assume now that $f(r)$, similar as in Subsection 3.2.1, is a function of the distance between the points. If $I_{q,n}$ denotes the integral over $f(r)$ and
3. Geometric Probabilities and Integral Geometry

The density of the two points, normalized by $V_n^2$, then

$$I_{q,n} = \frac{1}{V_n^2} \int_{D} \int_{D} f(r) dP_1 dP_2$$

$$= \frac{1}{V_n^2} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} \int_{t_1 \in \mathcal{D} \cap \mathcal{L}} \int_{t_2 \in \mathcal{D} \cap \mathcal{L}} f(r) r^{n-1} dt_1 dt_2 d\mathcal{L} \quad (3.6)$$
where \( r = |t_2 - t_1| \), and where \( \mathcal{L} \cap \mathcal{D} \neq \emptyset \) describes the set of all lines which intersect the domain \( \mathcal{D} \). The symbol \( \emptyset \) denotes the empty set. The line segment \( \{t_\nu | t_\nu \in \mathcal{D} \cap \mathcal{L}\} \) arises from the intersection of \( \mathcal{D} \) with \( \mathcal{L} \) and defines the integration domain for \( t_1 \) and \( t_2 \).

The integral over \( d\mathcal{L} \) is now an integral over all lines that intersect \( \mathcal{D} \). Assume that \( \mathcal{D} \) is convex, and that \( f(r) \) is of the form \( f(r) = \frac{1}{r^q} \) for a \( q \) that is to be specified. The integration of (3.6) over all \( t_1, t_2 \in \mathcal{D} \) can then be easily performed. The integration domain on the line is thereby bounded by real numbers, which are denoted by \( \sigma_a \) and \( \sigma_b \):

\[
I_{q,n} = \frac{1}{V_n^2} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} \int_{t_1 \in \mathcal{D} \cap \mathcal{L}} \int_{t_2 \in \mathcal{D} \cap \mathcal{L}} |t_2 - t_1|^{n-q-1} dt_1 dt_2 d\mathcal{L},
\]

\[
= \frac{1}{V_n^2} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} \int_{\sigma_a}^{\sigma_b} \left( \int_{\sigma_a}^{t_2} (t_2 - t_1)^{n-q-1} dt_1 \right)
+ \left( \int_{t_2}^{\sigma_b} (t_1 - t_2)^{n-q-1} dt_1 \right) dt_2
\]

\[
= \frac{1}{(n-q)V_n^2} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} \int_{\sigma_a}^{\sigma_b} (t_2 - \sigma_a)^{n-q} - (\sigma_b - t_2)^{n-q} dt_2 \quad \text{if } q \neq n
\]

\[
= \frac{2}{(n-q)(n-q+1)V_n^2} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} \sigma^{n-q+1} d\mathcal{L} \quad \text{if } q \neq n + 1.
\]

(3.7)

In this expression, \( \sigma \) is the length of the chord \( \{\mathcal{L} \cap \mathcal{D}\} \) which arises from the integration over the entire section of \( \mathcal{L} \) that intersects \( \mathcal{D} \) (Fig. 3.2). The values \( q \neq n, q \neq n + 1 \) are obviously excluded, since in these cases the integrand is of the form \( \ln \sigma \). Noticeably, the integral diverges for any \( q \geq n \), since in the second line there is a negative exponent, and \( \sigma = 0 \) holds whenever \( \mathcal{L} \) touches \( \mathcal{D} \) at its edge.

The application of the density of lines has considerable advantages, because like in the case of (3.2) and (3.3), particular integrals over \( d\mathcal{L} \) are
related to the volume and the surface of the domain $\mathcal{D}$ [123]:

\[
\int_{L \cap D \neq \emptyset} dL = \frac{O_{n-2}}{(n-1)O_0} A_{n-1} \tag{3.8}
\]

\[
\int_{L \cap D \neq \emptyset} \sigma dL = \frac{O_{n-1}}{O_0} V_n, \tag{3.9}
\]

\[
\int_{L \cap D \neq \emptyset} \sigma^{n+1} dL = \frac{n(n+1)}{2} V_n^2, \tag{3.10}
\]

where $A_{n-1}$ is the surface area of the convex body $D$.

This triple of equations reveals again that certain integrals over the density of lines over a (convex) body do not depend on the particular shape of the body, but only on the integral quantities of volume and surface area. In two dimensions, these three integrals can already be found in [30]. Their geometrical meaning is quite simple if one considers that for $n = 2$, $dL = d\hat{\rho}d\phi_L$ where $\phi_L$ is the direction of the perpendicular of the line to the origin, and $\hat{\rho}$ its length. The integration of $\int \sigma d\hat{\rho}$ is equivalent to the summation of all infinitesimally thin stripes that cover $D$ in direction of $\phi_L$; this summation results in the two dimensional volume $V_2$ of $D$, and in the rotation of this surface area over all possible angles in the factor $\frac{O_{n-1}}{O_0}$. Eq. (3.10) is even more straightforward. It is simply the density transformation (3.7) for $q = 0$. The first equation, (3.8), which is also known as Cauchy's surface area formula [123, 126], is a little more involved. In principle, the expression $\int_{L \cap D \neq \emptyset} dL$ results, for a fixed direction of $L$, in a projection of $D$ onto the hyperplane that is perpendicular to these $L$; the integral over all angles of these projections results in an expression that is proportional to the surface area of $D$.

### 3.2.3 A Relation between two Specific Distributions of two Points

There is a fundamental relation between the uniform distribution of two points in a convex domain and the uniform distribution of one point in the domain and of the other on its boundary. This relation will be employed in the following to derive the mean of a function of the distance of these two points. The following example shall explain the origin of this relation.

Assume that $P_{\text{rect}}$ is the unnormalized measure of a point in a rectangle of sidelengths $a, b$. If the point is uniformly distributed and has
coordinates $x^{(\nu)}$, then the measure of the point being anywhere in the rectangle is

$$P_{\text{rect.}} = \int_0^a dx^{(1)} \int_0^b dx^{(2)}. \quad (3.11)$$

Assume now that the width of the rectangle is parameterized by the variable $s \in \mathbb{R}^+$. Then,

$$P_{\text{rect.},s} = \int_0^a dx^{(1)} \int_0^s dx^{(2)} \quad (3.12)$$

depends on $s$ and, by definition of the derivative of a function,

$$\frac{\partial}{\partial s} P_{\text{rect.},s} = \lim_{\Delta s \to 0} \frac{1}{\Delta s} \int_0^a dx^{(1)} \int_s^{s+\Delta s} dx^{(2)} \quad (3.13)$$

$$= \int_0^a dx^{(1)} \quad (3.14)$$

$$= P_{\partial \text{rect.},s} \quad (3.15)$$

is the measure of the point being distributed on the upper edge of the rectangle. Hence, there is an intrinsic relation between the distribution of a point within a domain and the distribution of the point on one of its boundaries. The relation simply makes use of the fact that the partial derivative of a multivariate measure reduces the range of the respective coordinate to a point. A simple geometrical explanation is given in Fig. 3.3. It argues that the measure of point 2 to be on $\partial D$ is actually the difference between two measures of the points being in $D$ and the points being in an inflated version of $D$. The inflation of the domain $D$ by an increment $\Delta s$ results in a slightly larger measure. The subsequent subtraction of the measure of two points in the inner domain from the measure of the inflated domain eliminates, in a probabilistic sense, the possibility of the two points to be simultaneously in the inner domain; the two remaining events are that one is in the inner and one in the small stripe which is the difference between the two domains, and that both are in this stripe. For infinitely small $\Delta s$ the latter event is negligible, and with proper normalization, it follows that one obtains the measure for one point to be in the domain and one on its boundary.

A relation of this kind was first mentioned by Crofton in 1885 (Eq. (68) in [30]) and is called Crofton’s Formula on mean values [98]. However, Crofton’s Formula gives no information about the distribution of the
point on the boundary, which is by no means automatically uniform. The
distribution is obviously uniform, if \( \mathcal{D} \) is a sphere; if not, the increase of
\( \mathcal{D} \) by \( \Delta s \) means that the body is inflated in a particular direction; only if
\( \mathcal{D} \) is uniformly inflated within each direction, it is equally likely to have
the point on a section of \( \partial \mathcal{D} \) of particular length.

A much more general treatment of the above relation between the two
types of densities can be found in [12]; related methods are given in [42]
and [98]. In [12], the problem is described in terms of an integral over an
arbitrary manifold \( \mathcal{M} \). The change over time \( t \) of this integral is shown to
be of the form

\[
\frac{d}{dt} \int_{\mathcal{M}} f dV_{\mathcal{M}} = \int_{\mathcal{M}} \frac{\partial f}{\partial t} dV_{\mathcal{M}} + \int_{\partial \mathcal{M}} f \cdot v_{\mathcal{M}} dS_{\mathcal{M}}
\]  

(3.16)

where \( f \) is an arbitrary scalar function on \( \mathcal{M} \). The change of the integral
over \( t \) can be decomposed into the change of \( f \) over \( t \), and the change
of the manifold over \( t \). The latter results in an integral over the surface
\( \partial \mathcal{M} \) of \( \mathcal{M} \), in which the function \( f \) is weighted with the velocity \( v_{\mathcal{M}} \)
of the change of \( \mathcal{M} \) in direction of the normal of the boundary. The proof
of (3.16) requires Stoke’s Theorem and represents not only a generalized
version of Crofton’s formula, but also of the Equation of Continuity [66]
and other fundamental integral relations.

For an application in the present case, \( f \) is a function that depends
on the distance between two points in three dimensions, which represent
the \( T_x \) and the \( R_x \) of a communication system. Thus, the manifold \( \mathcal{M} \)
is six dimensional, as is the integral \( \int_{\mathcal{M}} f dV_{\mathcal{M}} \). According to [12], \( \mathcal{M} \)
is constructed from the cartesian product of the domains \( \mathcal{D}_\nu, \nu = 1, 2, \infty \)
in which each of the points is located. The set of two points in \( \mathbb{E}^n \) is

\[
R + \Delta s
\]

2

1

\( R \times 0 \)

Figure 3.3: The inflation of a domain.
represented by a single point in $\mathbb{E}^{2n}$. The boundary in this case is the union

$$\partial \mathcal{D}_1 \times \mathcal{D}_2 \cup \mathcal{D}_1 \times \partial \mathcal{D}_2$$

(3.17)

where the sign $\times$ in this respect represents the cartesian product between sets.

For the special case of $\mathcal{D}_1 = \mathcal{D}_2 = \mathcal{D}$, this union becomes twice the domain $\mathcal{D} \times \partial \mathcal{D}$. Then, denoting the domain $\mathcal{D} \times \partial \mathcal{D}$ by $\mathcal{M}$ and $\mathcal{D} \times \partial \mathcal{D}$ by $\partial \mathcal{M}$, one arrives at a special case of (3.16) for two interchangeable points,

$$\frac{d}{dt} \int_{\mathcal{M}} f dV_{\mathcal{M}} = 2 \int_{\partial \mathcal{M}} f v_{\mathcal{M}} dS_{\mathcal{M}}.$$  

(3.18)

The right hand side gives the mean of the function $v_{\mathcal{M}} f$ for one point uniformly distributed within $\mathcal{D}$, and one on its boundary. This relation will be the starting point for the derivation of the mean values of $f$. In the above expression, there is no information about the way to parameterize the change of $\mathcal{M}$ over $t$, and hence about the values of $v_{\mathcal{M}}$. A particularly suitable parameterization and the corresponding results are introduced in Subsection 6.3.3.
Chapter 4

A Novel Approach to Stochastic Channel Modeling

4.1 A Fundamental Equation for the Stochastic Radio Channel

In stochastic channel modeling, one is typically concerned with finding the distribution of some quantity $\xi$; it can be, for instance, an amplitude, a path delay, or even a vector combined of several of these. For this purpose, models are designed that are valid in a particular environment, which is in most cases only approximately determined. Suppose that the environment is described by a set $\Gamma_0$, i.e., a set of many specific environments $\gamma$; specific means here that $\gamma$ characterizes the environment precisely enough to determine a field distribution. If the distribution of the positions $r_{Tx}$ and $r_{Rx}$ of $T_x$ and $R_x$ is given, the law of total probability applied to the sought conditional distribution $P(\xi|\Gamma_0)$ yields [112]

$$P(\xi|\Gamma_0) = \sum_{\gamma \in \Gamma_0} \int_{D_{Tx}} \int_{D_{Rx}} P(\xi|r_{Tx},r_{Rx},\gamma)P(r_{Tx},r_{Rx},\gamma)dr_{Rx}dr_{Tx} \quad (4.1)$$

and, for the moments of the distribution,

$$\langle \xi^l|\Gamma_0 \rangle = \sum_{\gamma \in \Gamma_0} \int_{D_{Tx}} \int_{D_{Rx}} \int_{\Xi} \xi^l P(\xi|r_{Tx},r_{Rx},\gamma)P(r_{Tx},r_{Rx},\gamma)d\xi dr_{Tx} dr_{Rx} \quad (4.2)$$

where $\langle \cdot | \cdot \rangle$ is the conditional expectation, $P(r_{Tx},r_{Rx},\gamma)$ the joint density of $r_{Tx}$, $r_{Rx}$, and $\gamma$, $P(\xi|r_{Tx},r_{Rx},\gamma)$ the conditional density of $\xi$ given the former three variables, and $D_{Tx}$ and $D_{Rx}$ are the domains in which $T_x$ and $R_x$ are located. The integration is performed over the volume elements $d\mathbf{r}_\nu$ of either the volume of $T_x$ or of $R_x$; the range of integration for $\xi$ is
At first glance, this equation seems to yield a rather inconvenient description of the radio channel. It provides, however, some fundamental insights into the inherent relation between wave propagation and the channel's stochastic behavior. As outlined in Chapter 2, wave propagation itself can be exactly or approximately described by deterministic models, if the surroundings of $T_x$ and $R_x$ are sufficiently well defined; in (4.1) and (4.2), this description substitutes the conditional distribution $\mathcal{P}(\xi | r_{T_x}, r_{R_x}, \gamma)$ or the function $\xi(r_{T_x}, r_{R_x}, \gamma)$, which is deterministic if $r_{T_x}$, $r_{R_x}$, and $\gamma$ are given. The preciseness of the description of the environment, i.e., the information available about its geometry and its dielectric properties, is given by the cardinality of $\Gamma_0$. If $\Gamma_0$ contains a single element, the environment is deterministic. If the environment is undetermined to some degree, $\Gamma_0$ contains many elements.

Expressions (4.1) and (4.2) thus allow a general characterization of the stochastic radio channel. This characterization relies on two different aspects: The first is the method of field description, which determines the conditional distribution of the quantity $\xi$ with regard to a given location of $T_x$ and $R_x$ and an environment. The second concerns the communication system and the surroundings in which it operates. The geometrical structure of the radio channel becomes obvious here: the possible positions of $T_x$ and $R_x$, and the size and shape of $\mathcal{D}_{T_x}$ and $\mathcal{D}_{R_x}$, depend on the system under investigation.

<table>
<thead>
<tr>
<th>System</th>
<th>Particular Domains for $\mathcal{D}<em>{T_x}$ and $\mathcal{D}</em>{R_x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile Networks, WLANs</td>
<td>$\mathcal{D}<em>{R_x} = \mathcal{D}$, $\mathcal{D}</em>{T_x} = {r_0}$, or $\mathcal{D}<em>{T_x} = \mathcal{D}$, $\mathcal{D}</em>{R_x} = {r_0}$</td>
</tr>
<tr>
<td>WLANs</td>
<td>$\mathcal{D}<em>{R_x} = \mathcal{D}$, $\mathcal{D}</em>{T_x} = \partial \mathcal{D}$, or $\mathcal{D}<em>{T_x} = \mathcal{D}$, $\mathcal{D}</em>{R_x} = \partial \mathcal{D}$</td>
</tr>
<tr>
<td>Ad-Hoc Networks</td>
<td>$\mathcal{D}<em>{R_x} = \mathcal{D}</em>{T_x} = \mathcal{D}$</td>
</tr>
</tbody>
</table>

Table 4.1: $R_x$ and $T_x$ of different wireless systems cover different domains.

The overview in Tab. 4.1 shows how different systems are related to different integrals of the kind of (4.1) and (4.2). In a mobile network, the elements of $\Gamma_0$ describe rather large areas, with $r_{T_x}$ and $r_{R_x}$ being the positions of a base station fixed at $r_0$ and a user randomly distributed in
4.2 Classification of Investigated Scenarios

The evaluation of (4.1) and (4.2) requires a specification of the spatial distribution of $T_x$ and $R_x$, and of the domains $D_{T_x}$ and $D_{R_x}$. Since usually, no information about the location of the user is available, a spatially uniform distribution for the transceiver, which operates either in $T_x$ or $R_x$ mode, is the only suitable choice. The other part of the link is the access point. Its spatial distribution depends on the system under investigation.

In the following, three different systems will distinguished. In all of them, both user and access point will be located within the same convex domain $D$, which specifies the range in which the system operates. The domain can be equal to the cell; in the case of interference studies, it can also cover several cells. The user is always uniformly distributed within $D$; the distribution of the access point, however, specifies a particular system, which is displayed in Fig. 4.1:

1. WLAN with fixed access point at $r_0 \in \{D\}$. The access point will usually be located somewhere at the ceiling (left).
4. A Novel Approach to Stochastic Channel Modeling

<table>
<thead>
<tr>
<th>Model</th>
<th>( \Gamma_0 )</th>
<th>Positions ( r_{Tx} ) and ( r_{Rx} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ray Tracing [10, 25]</td>
<td>contains single element fixed</td>
<td>location of base station</td>
</tr>
<tr>
<td>Walfisch - Bertoni [27, 140]</td>
<td>contains a single element which is completely determined, mobile is fixed in the middle between two buildings</td>
<td></td>
</tr>
<tr>
<td>semi-empirical [45, 106]</td>
<td>contains many arbitrary distribution, defined by, e.g., fixed distance between ( T_x ) and ( R_x ) and by a trajectory of the moving terminal</td>
<td></td>
</tr>
<tr>
<td>empirical model [31, 28, 39]</td>
<td>contains many arbitrary distribution, defined by trajectory and some locations of ( T_x ) and ( R_x )</td>
<td></td>
</tr>
<tr>
<td>WSSUS stochastic channel model [37]</td>
<td>( \Gamma_0 ) has almost arbitrary elements</td>
<td>no information about ( r_{Tx}, r_{Rx} )</td>
</tr>
</tbody>
</table>

Table 4.2: Classification of existing channel models.

2. WLAN with unknown position of the access point. The access point will be randomly distributed at the ceiling or the outer walls. For reasons of symmetry, this corresponds to a distribution of a point on the edge of \( \mathcal{D} \), i.e. \( \partial \mathcal{D} \) (middle).

3. Ad-Hoc network with no hierarchy. The access point is uniformly distributed in \( \mathcal{D} \) (right).

The three different assumptions about the location of the access point are sensible with respect to real network types; they also cover not any, but the three geometrically most succinct cases. It is hard to determine generally whether the investigation of further spatial distributions, for instance Gaussians, would yield significantly different channel statistics. Preliminary simulation results indicate that some channel parameters are more sensitive than others. The mean received power and the k-factor are robust against changes in the spatial distribution of access point and
user [51]; but the conditional distributions of pathlengths depend on this distribution whenever the geometrical parameters which describe the environment are well determined [50].

In the following, the three mentioned, geometrically quite different cases will be investigated in simulations, analytically, and experimentally. Even though the impact of all potential spatial distributions on the channel is not discussed, the results still indicate under which conditions channel parameters are robust against variations of the properties of the environment.

Figure 4.1: Three distributions of user (U) and access point (A).
In the present chapter, (4.1) and (4.2) are evaluated by MC (Monte Carlo) simulations for the downlink of a WLAN at 60 GHz which operates in a single room environment. The dependence of the channel characteristics is systematically investigated for different configurations of antenna polarizations, for different wall materials, and for different room geometries and spatial distributions of $T_x$ and $R_x$. The results are displayed as two dimensional joint probability densities of the received amplitudes, as spatial power densities in the azimuth-elevation plane of the $R_x$, and as two dimensional probability densities of the location of virtual sources. The geometric simplicity of the environment allows then to draw conclusions about how fundamental deterministic properties of wave propagation behave for different distributions of $r_{T_x}$, $r_{R_x}$, and for different $\Gamma_0$. The essence of this chapter is that for a sufficiently large set $\Gamma_0$, the radio channel is to great extent determined by a few geometrical key quantities. In the subsequent chapter, these quantities are then used to derive an analytical model of the channel.

5.1 The Monte Carlo Ray Tracing Tool

A simple ray tracing tool was constructed on the basis of the theory of Section 2.2; a short description of the software is given in Appendix D. The wave propagation was modeled by GO. The tool serves to calculate the received field at an $R_x$ which is in an empty single room with the $T_x$, including full information of the polarization of the waves. The raytracer is
Based on the method of images. Virtual sources are created to calculate the field of reflected waves. Since the room is empty, only the LOS path and the reflections are modeled; diffraction and transmission have a low impact at 60 GHz (c.f. Subsections 2.2.2 and 2.2.3). The information about path lengths and angles is described by the joint probability distribution $P(\{r_\nu\}) = P([r_1; r_1 + \Delta r_1], \ldots, [r_{L^\text{vs}}; r_{L^\text{vs}} + \Delta r_{L^\text{vs}}])$, which measures the event that the virtual sources $\nu$ are located within the intervals $[r_\nu; r_\nu + \Delta r_\nu]$, $1 \leq \nu \leq L^\text{vs}$; $L^\text{vs}$ is the model order, i.e., the maximum number of considered virtual sources. The variable $r_\nu$ denotes the position of a point in three-dimensional space, and $[r_\nu; r_\nu + \Delta r_\nu]$ defines a small cube of sidelengths given by the components of $\Delta r$.

The total number of virtual sources required for a sufficiently exact evaluation of the received field depends on the investigated scenario. Since the power decay in the single room channel is exponential (c.f. Subsection 2.3.3), the main energy components impinge within the first few reflections (cf. Subsection 2.3.1). For the present case, all reflections up to second order are taken into consideration.

Once the positions of the virtual sources are known, the respective relative permittivities $\varepsilon_\tau^{(\nu)}$ are needed in order to calculate the received field and the detected amplitude. As in the case of the location of the virtual sources above (5.1), the joint distribution of the received amplitudes, $P(\{a_\nu\}) = P([a_1; a_1 + \Delta a_1], \ldots, [a_{L^\text{vs}}; a_{L^\text{vs}} + \Delta a_{L^\text{vs}}])$, can be written in terms of the density of the corresponding physical quantities, $\{r_\nu\}$ and $\{\varepsilon_\tau^{(\nu)}\} = (\varepsilon_\tau^{(\nu)}, \ldots, \varepsilon_\tau^{(L^\text{vs},e_{L^\text{vs}}^{(\nu)})}$, with $L^\text{vs},e_{L^\text{vs}}^{(\nu)}$ being the maximum number of dielectric constants required to describe the ray paths:

$$P(\{a_\nu\}) = \frac{1}{Z_0} \int \ldots \int_{D_{L^\text{vs}}} P(\{a_\nu\}|\{r_\nu\}, \{\varepsilon_\tau^{(\nu)}\}) P(\{r_\nu\}) P(\{\varepsilon_\tau^{(\nu)}\}) \, dr_1 \ldots dr_{L^\text{vs}}.$$ (5.2)

Here, the density $P(\{r_\nu\})$ is obtained from (5.1), and the deterministic expression $P(\{a_\nu\}|\{r_\nu\}, \{\varepsilon_\tau^{(\nu)}\})$ follows according to (2.51); the conditional density $P(\{\varepsilon_\tau^{(\nu)}\}|\{r_\nu\})$ describes the probability that particular materials
with dielectric constants \( \{ \varepsilon_r^{(\nu)} \} \) constitute the walls where the paths of the rays from \( \{ \mathbf{r}_\nu \} \) intersect. It contains the history of the propagating waves as they impinge on the \( R_x \), and relates the dielectric properties of the walls to the positions of the virtual sources.

The marginal density \( p(\{ \varepsilon_r^{(\nu)} \}) \) of \( p(\{ \varepsilon_r^{(\nu)} \}| \{ \mathbf{r}_\nu \}) \) describes rooms of different wall materials, and can thus be used to define different propagation scenarios. An old-fashioned office room, for instance, has heavy walls, rather small windows, and may have some walls of lighter material; in contrast, a modern office room may have light-weight walls with a large proportion of glass, and with only a minor amount of heavy materials. By applying specific probability densities, the influence of the wall materials on the statistics of the received amplitudes and on the power spectra can be studied systematically.

It should be noted that (5.2) does not include NLOS situations. NLOS occurs when the direct path is blocked, e.g., by a large piece of furniture or by a human body. The probability that the direct path or multiple paths are blocked is required to simulate such a situation. In order to extend the present model to a NLOS situation, one could simply erase the direct path or artificially attenuate it.

The joint densities of the amplitudes and of the location of the sources is estimated for several scenarios. Each scenario \( \Gamma_0 \) is defined by a probability density for the room size, for the dielectric constants that describe the wall materials, and for the locations of \( T_x \) and \( R_x \).

\( T_x \) and \( R_x \) are located within the same room, which is characterized by its length, width and height; the entire geometry can thus be described by a three dimensional vector \( \mathbf{a^r} \), each component of which is uniformly distributed between a lower and an upper bound. The densities for the location of \( T_x \) and \( R_x \) are also assumed to be uniform. Apart from the last set of simulations, \( T_x \) is always randomly attached anywhere on the ceiling, and \( R_x \) is bounded between a fifth and a half of the room’s height anywhere above the floor. This corresponds to the downlink in a WLAN, where the terminal is somewhere between table and head height, and where the access point is attached somewhere on the ceiling. This kind of system was introduced as one of the three types defined in Section 4.2. Only for the last simulations, the bounds of the spatial distributions are changed in order to explicitly investigate the influence of the distribution of \( T_x \) and
5. Joint Densities of the Radio Channel of Single Rooms: Numerical Results

$R_x$ on the received power.

Assuming the densities for the locations of $T_x$ and $R_x$ and the room size to be mutually independent, formula (5.1) can be written as

$$P(\{r_L\}) = \sum_{a^r, r_{T_x}, r_{R_x}} P(\{r_L\}|r_{T_x}, r_{R_x}, a^r) p(r_{T_x}) p(r_{R_x}) p(a^r),$$  

where $p(r_{T_x})$, $p(r_{R_x})$ and $p(a^r)$ are assumed to be the product of their three independent vector components. The values $a^r$, $r_{T_x}$, and $r_{R_x}$ are now discrete numbers drawn by the random number generator of the computer. A sampled joint probability density can then be obtained from (5.3), if the interval length $\Delta$ introduced for (5.1) is chosen sufficiently small.

The simulation of the amplitudes is performed using (5.2), with $p(\{r_L\}) = \prod \nu p(r_L(\nu))$ being independent from $\{r_L\}$. Thus, for each point of reflection a new realization of the dielectric constant is drawn. Since only the first and second order of reflection are simulated, the event that two reflection points are on exactly the same location and must therefore have the same dielectric constant is impossible for the investigated setups.

The densities for the dielectric constants are discrete and of the form

$$p(\epsilon_r) = \sum_{\nu=1}^{L^\nu} \beta_\nu \delta(\epsilon_r - \epsilon_r(\nu))$$

where $\sum_{\nu=1}^{L^\nu} \beta_\nu = 1$, where $\delta(\cdot)$ is the Delta function, and where $L^\nu$ is the total number of different materials in the rooms.

The antenna of $T_x$ is assumed to have a uniform pattern in the lower half sphere and a zero pattern in the upper one. Its polarization is chosen as linear. The antenna of $R_x$ is also linearly polarized. It has an isotropic pattern, which corresponds to a uniformly random orientation of the terminal in space.

Each figure is based on 2048 realizations of the radio channel. For the calculation of the power densities and the average power, uniformly distributed phases are assumed.
5.2 Simulation Results

5.2.1 Delays and Angles of Departures

An AoD is most easily obtained when the raytracer does not calculate the position of the virtual source, but that of the virtual receiver. This strategy is shown in Fig. 5.1, where the dependence of the locations of the virtual receivers ($vR_x$) on the room geometry is demonstrated for a deterministic scenario. With increasing room size, the distances between the virtual receivers and the path lengths of transmitted rays increase as well, and the azimuth AoDs $\phi$ become close to 0 and 180°. If the room is rectangular, the largest path lengths are obtained when the rays are transmitted into the direction of the largest room dimension.

If the room dimensions and the locations of $T_x$ and $R_x$ are random, the distribution of the path lengths and the AoDs reflect the probability density used to describe the room’s geometry. Fig. 5.2 displays simulation results based on rooms the size of which is uniformly distributed within $4 \text{ m} \times 4 \text{ m} \times 4 \text{ m}$ and $30 \text{ m} \times 8 \text{ m} \times 4 \text{ m}$. The rooms range from very small, cubic ones to large, rectangular office rooms. Fig. 5.2 a) displays the joint density of the path lengths of all rays up to the second order reflection combined with their respective azimuth AoDs. The two long ‘fingers’ at $0^\circ/180^\circ$ indicate long path lengths at these angles and show that for the chosen distribution of sidelengths, many rooms have a long, rectangular shape. The long side is parallel to the $x$-axis, so that the $T_x$
Figure 5.2: Two dimensional densities. a) joint density of azimuths and pathlengths b) azimuths of reflected paths conditioned on the direct path.

transmits all rays that have a long path length into an azimuth close to 0° or 180°. At short path lengths, the azimuth of the transmitted rays is about uniformly distributed; this part of the distribution refers to smaller, more cubic rooms.

Fig. 5.1 shows that the virtual receivers are located on a three dimensional grid. Such a grid structure yields strong correlations among the AoDs. This is demonstrated in Fig. 5.2 b), which shows the density of the azimuth angles of the reflected rays, conditioned on the azimuth angle of the direct path. This plot can be decomposed into a contribution from the
first order floor reflected ray, and a contribution from the wall reflected rays and reflections of higher order. The floor reflected ray has always the same azimuth angle as the direct path, which causes the diagonal that is visible in the plot. The azimuth of a first order wall reflection is identical to the azimuth of the direct path if $T_x$ and $R_x$ are in a line parallel to the wall's normal vector. These cases can be seen as the high peaks in the azimuth-azimuth planes at $(0°, 0°), (±90°, ±90°), (−180°, −180°)$. Two opposing walls contribute almost equally to the density and cause the very regular structure of the density. Most AoDs of higher order reflected rays also behave in a very regular way and can therefore not be distinguished in this pattern. The plot of the azimuth angles is independent from the absolute values of the room size and depends only on the ratio of the sidelengths.

5.2.2 Joint Distribution of Amplitudes and Spatial Power Densities

Dependence on Antenna Polarizations

The influence of the antenna polarizations on the statistics of the amplitudes and of the received power is investigated for a room of fixed size $(8 \text{ m} \times 8 \text{ m} \times 4 \text{ m})$ and of fixed dielectric constant ($\epsilon_r = 6.14 - j0.3$, concrete at 60 GHz [31]). Two configurations are investigated: The linearly polarized $T_x$ and $R_x$ are either both parallel aligned to the floor of the room, or both perpendicular to this configuration. These two setups will be abbreviated as parallel and perpendicular polarization.

In Fig. 5.3, the resulting spatial power densities in the azimuth-elevation plane of the $T_x$ are plotted. Slight differences between the two plots can be recognized. The result for parallel polarization is shown in a). The highest contributions are found at the azimuth angles $0°, ±90°, −180°$. In this case, $T_x$ and $R_x$ are in a line parallel to the normal vectors of two opposing walls. The parallel transmitted field vector is perpendicular to the plane of incidence at the wall and is again received as a parallel component. If $T_x$ and $R_x$ are not aligned parallel to the normal vectors of two opposing walls, the reflection involves both the perpendicular and the parallel component of the field at the wall. The reflection coefficient for parallel polarization in the plane of incidence is generally lower; due to
a stronger attenuation of this component, the reflected field vector is no longer completely parallel polarized to the floor, and the amplitude of the received component decreases.

In Fig. 5.3 b), the power spectrum is shown for perpendicularly polarized antennas. In this plot, all reflections are of about equal strength in the $\phi$ domain. The perpendicularly polarized field vector of the antenna has always perpendicular and parallel components at the surface of the walls; there is hence no azimuth angle that yields stronger reflections.

The elevation range which contributes most to the received power is in both plots between $90^\circ$ and $135^\circ$. This seems surprising, since the shortest
Simulation Results

propagation path, and hence the lowest free space loss, is accounted at an elevation of 180°, when the $T_x$ is directly above the $R_x$. If, however, both are randomly distributed above the entire floor, this event occurs with the lowest probability and cannot contribute to the received power.

**Dependence on the Dielectric Properties of the Room**

In order to investigate the influence of the dielectric constant on the statistics of the received amplitudes, rooms of different wall materials are created. The room size is again fixed at $8 \times 8 \times 4$ m. The following materials are chosen: aerated concrete at $60$ GHz ($\epsilon_r = 2.26 - j0.10$), concrete at $60$ GHz ($\epsilon_r = 6.14 - j0.30$), glass at $60$ GHz ($\epsilon_r = 6.13 - j0.50$), and brick at $24$ GHz ($\epsilon_r = 4 - j1$). The first three values were taken from [31], the fourth from [62]. The values represent typical building materials and cover a relatively wide range of possible dielectric constants. The last value was explicitly chosen because it bridges the gap between the high values of glass and concrete and the low value of aerated concrete.

The impact of the dielectric constant on the statistics of the radio channel is demonstrated in Fig. 5.4. The figure displays the joint densities of the amplitude of the direct path and of the first order reflected amplitude. The plot a) results from simulations in a room completely made of concrete, b) is from a room completely made of aerated concrete; antennas are parallel polarized. The somewhat irregular shapes of the densities give an impression of the complicated interplay between the geometry of the room, the Fresnel reflection coefficients, the dielectric constants, and the polarization of the antennas. This interplay makes an exact prediction of received amplitudes in deterministic scenarios a fairly complicated task. The variation of the dielectric constant results, however, only in a scaling of the reflected amplitudes, i.e., in a scaling of the right hand axis of the displayed plots.

**Dependence on the Room Geometry**

For an investigation of the influence of the room geometry on the spatial power density at the $T_x$, random rooms of sidelengths uniformly distributed between $4 \times 4 \times 4$ m and $30 \times 8 \times 4$ m are simulated. The distribution of the dielectric constants (5.4) is chosen to represent a
modern room type with 50 \% glass, 30 \% aerated concrete and 20 \% concrete. The result of the simulation is shown in Fig. 5.5. Compared to the power densities in Fig. 5.3, the received power is much more concentrated both in the azimuth and in the elevation plane. The rooms investigated in this chapter are, on average, bigger and more rectangular than the structure with equal sidelengths that was investigated in 5.2.2. Similar to the case of the location of the virtual receivers in Fig. 5.2, the directions in which power is transmitted to the receiver is restricted to certain, narrow ranges. The highest fraction of power is transmitted into the 0° and 180°
azimuth, since for the given distributions of room sizes, sidelengths tend to be bigger in this particular direction; the probability that a receiver is in this direction is higher. The examples in Fig. 5.3 and Fig. 5.5 show that an approximate construction of the shape of the spatial power density is possible even if only rough information about the environment around $T_x$ and $R_x$ is given.

The investigations presented in this chapter reveal that some knowledge about the geometry of the environment is crucial, and for approximate statements even sufficient, in order to make predictions about the radio channel. Statistics of AoDs, of path lengths, and of spatial power densities clearly reflect the properties, and in particular the symmetries, of the environment from which they are obtained. Furthermore, two copolarized antenna configurations and the impact of the wall materials on the statistics of the amplitudes were examined. For the former, the geometry of the environment could again explain the results. The differences between the two states of polarization are only visible in rooms with deterministic size; for randomly distributed room sizes, power densities for all antenna configurations look similar and are basically determined by
the geometry of the environment. The complicated structure of the joint
distribution of the amplitudes in dependence on the Fresnel reflection co-
efficients and on the dielectric constants of the wall materials is, again,
only visible in deterministic structures and for unique building materials.
For randomly chosen materials and room sizes, the fine structure of the
joint distribution becomes arbitrary and hence meaningless.

The conclusions drawn in this chapter can easily be extended to more
complex environments. The more complex an environment becomes, the
less impact do particular deterministic properties of wave propagation
have. For a simple channel model, some rough information about geo-
metrical quantities suffices. These quantities need to describe the size
and potentially the shape of the domain in which \( T_x \) and \( R_x \) are located.
Furthermore some approximate knowledge about the wave propagation
behavior within these domains must be included. In the following chap-
ter, such quantities are introduced in terms of a geometrically motivated,
analytical channel model.
Chapter 6

The Robust Quantities of the Indoor Radio Channel: Analytical Results

6.1 Physical Preliminaries

In the previous chapter, it was demonstrated that for a characterization of the fundamental properties of the indoor radio channel, only approximate knowledge about the geometry of the environment and hence also about the wave propagation behavior is required. Based on these findings, an analytical approach to the stochastic radio channel can now be presented. It exploits the fact that no exact, but only approximate expressions are required; these expressions must depend on robust quantities which allow to parameterize an environment and its wave propagation characteristics in a straightforward, at best even an intuitive manner.

The starting point of the derivation is the pathloss formula (2.52),

\[
\langle P_{R_x}\rangle_{r, q, \kappa} = \kappa \frac{1}{r^q},
\]

which is now repeated in the view of (4.2). The model gives the empirical pathloss as a mean over an (undetermined) set of environments, i.e., \( \gamma \in \Gamma_0 \), for a given distance \( r \) between \( T_x \) and \( R_x \). The environment, however complicated it may be, is characterized by the two parameters \( \kappa \) and \( q \). Intuitively, it holds that \( \kappa = \langle P_{R_x}\rangle_{(r = 1 \text{ m}), q, \kappa} \). For the present study of the indoor radio channel, it is assumed that

\[
\kappa = \left( \frac{\lambda}{4\pi} \right)^2.
\]

The constant \( \kappa \) is calculated with the aid of the Friis equation [118] with \( r = 1 \text{ m} \). In indoor scenarios this assumption is justified: the probability
6. The Robust Quantities of the Indoor Radio Channel: Analytical Results

that a path between $T_x$ and $R_x$ is blocked within 1 m distance is low, and hence pathloss curves generally intersect approximately this point. The model (6.1) can thus be rewritten as

$$\langle P_{R_x} \mid r, q, \lambda \rangle = \left( \frac{\lambda}{4\pi} \right)^2 \frac{1}{r^q}. \quad (6.3)$$

It is important to take into consideration that (6.3) is valid only in the far field of an antenna; if the distance $r$ did approach zero, the expression would diverge.

The use of (6.3) is a fundamental step in the development of the channel model. All interactions of the propagating waves with the surroundings are absorbed into the pathloss exponent $q$. The choice of the pathloss exponent decides about the inner structure of the environment, about walls, furniture, the impact of diffraction, scattering, and anything else that determines wave propagation. This strategy seems risky, since the user of the model needs a good intuition in choosing this parameter. But any channel model requires at some point information about the wave propagation behavior within the environment for which it is designed. The chosen relation (6.3) has here two advantages. The first is its simplicity and very intuitive applicability. Many models which characterize wave propagation in more detail, for instance, models which operate on a ray basis and on numbers of transmissions and reflections require more input parameters; they would yield only more site-specific, but probably statistically not more exact results. Secondly, (6.3) is despite of its simplicity of very general form. Many pathloss models which include floor or wall transmissions, and all models which are based on free space propagation, can within limits be derived from relations similar to the given one [49]. Furthermore, the pathloss is, at least at lower GHz frequencies, a very well investigated quantity. It is tabulated for many different cases [118]. For the higher frequencies, additional measurements would be desirable, but these are beyond the scope of this thesis.

6.2 The Distribution of the Received Power

A number of papers deal with the large scale distribution of the pathloss, which was already discussed in Subsection 2.3.3. It is widely assumed that
the pathloss is lognormal distributed around some mean value, which can, for instance, be obtained from (6.1). In the following, this assumption is not proven, but it is justified in a very simple fashion by geometrical reasoning.

Consider first a single room. The received amplitude is always the sum of several impinging waves, one of which may be a strong direct path and others reflected once or several times. The distribution of the amplitude of the direct path follows a $\frac{1}{r}$ relation and depends on the spatial distributions of $T_x$ and $R_x$. For simple spatial distributions, the distribution of $\frac{1}{r}$ can in principle be calculated, but the results are difficult to obtain and rather meaningless [98]. Already for a uniform distribution of two points $r_1$ and $r_2$ in a cube, the distribution of their distance in three dimensions fills an entire page and contains integrals which cannot be analytically solved. Two dimensional distributions are much more easily obtained, since the transformation $r^2 = (x_1^{(1)} - x_1^{(2)})^2 + (x_2^{(1)} - x_2^{(2)})^2$ of the cartesian coordinates of the points requires only a twofold instead of a threefold convolution of the densities of the vector components of $r_1$ and $r_2$. The evaluation of these integrals usually yields smooth density functions, which are, however, always piecewise defined. The basis of such a density transformation is the integral (4.1). Two numerically obtained solutions of (4.1) are shown in Fig. 6.1. For the figures in the upper half, a) and b), two points are uniformly distributed in a cube of sidelength 1, and their inverse distance $\frac{1}{r}$ is calculated for 20,000 realizations; for the figures in the lower half, c) and d), a different distribution, discussed below, is chosen. The plots a) and c) show the histogram on linear scale, and b) and d) on logarithmic scale, together with a fit of a lognormal and a Gaussian distribution for a) and c), and b) and d), respectively. The lognormal distribution fits surprisingly well, even though too much mass is distributed under the tail. The same holds for the —equivalent— Gaussian on dB (dezibel) scale, where large values are too dominant. However, real communication systems always obey a minimum distance $\rho_{min}$ between $T_x$ and $R_x$. This distance eliminates the contributions for large values of $\frac{1}{r}$, so that the resulting distribution appears — particularly if it is based on experimental data — to be even better lognormal distributed. But if the distribution of $\ln \frac{1}{r} = -\ln r$ is lognormal, so is the distribution of $\ln \frac{1}{r^2} = -q\ln r$; on the ln scale, the Gaussian bell curve is simply stretched.
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Figure 6.1: Distribution of the inverse of the $T_x-R_x$ distances. a) linear scale, cube of sidelength 1 b) dB scale, same cube c) two distant cubes, linear scale d) two distant cubes, dB scale.

by a factor $q$.

Consider now $T_x$ and $R_x$ as not located within the same room, i.e., let $\rho_{min} \gg r$. One has then the random variable $\frac{1}{(r+\rho_{min})^q}$, which is on logarithmic scale $\ln \left( \frac{1}{(r+\rho_{min})^q} \right) = -q \left( \ln(\rho_{min}) + \ln(1 + \frac{r}{\rho_{min}}) \right)$. The histogram of this distribution is shown in Fig. 6.1 c) and d), again for 20,000 trails, with both points distributed in cubes of unit sidelength. The cubes have a distance of more than 15 times this length. Note that both curves now fulfill the assumption of $\frac{1}{r^q}$ to be lognormal much better. Both curves have a very similar shape, since the Taylor expansion of $\frac{1}{(r+\rho_{min})^q} \simeq 1 - q \frac{r}{\rho_{min}}$ on linear scale behaves like the expression $-q \left( \ln(\rho_{min}) + \ln(1 + \frac{r}{\rho_{min}}) \right)$.
6.2. The Distribution of the Received Power

on dB scale. A close inspection of the tails of the Gaussian indicates that now, the histogram is a little too low there.

The given results do not prove that the received power is lognormal. They show, however, in a very simple and intuitive manner that a $\frac{1}{r^q}$ relation between two quite arbitrarily distributed points yields a histogram that can be approximated by a lognormal distribution. It does not matter whether the points are distributed in the same or in different domains. The lognormal distribution not only fits locally obtained data, i.e., all experimental data obtained for a particular distance $\rho_{min}$ between $T_x$ and $R_x$; it also works well for globally obtained data, i.e., all measured data within a particular environment, independently from $\rho_{min}$. However, the latter statement requires that the pathloss exponent is constant for the entire environment. Since this is often not the case, the assumption that the received power is lognormal over all distances is not always fulfilled. Typical examples are buildings where some sections have many partitions and others only a few or none. In such cases, one obtains different values of $q$ for the different sections and hence different lognormal distributions. Further explanations are given in Section 7.2 on the basis of experimental data.

One can draw the preliminary conclusion that the two well-known statements about the radio channel, i.e., the validity of a pathloss model of the form $\frac{1}{r^q}$ and the (locally) lognormal distribution of the received power, are redundant. The former is sufficient — for purely geometrical reasons and by the integral expression (4.1) — to generate a distribution which can be approximated by a lognormal. The mathematical proof of this statement is not carried out here and should not be underestimated. The literature about distributions of distance is, due to their application to problems of spatial statistics, vast ([3, 46, 114, 19, 64] and in particular [98]); it shows that the analytical expressions for distributions of distances depend strongly on the shape of the considered object, even though in terms of physics, this is not reasonable. A slight perturbation of a body does not completely alter the properties of the distributions of distances of inner points. For a calculation of precise distributions, however, a slight deviation of the geometry of a body from, for instance, a cube, can cause great difficulties. Hence, the transformation of a piecewise defined density of a distance between to points within a regular body into a lognormal
distribution can possibly cause severe mathematical problems.

### 6.3 Derivation of the Mean Pathloss

If (4.2) is applied to the calculation of the MPI, as required for the simulation of the lognormal distribution of $P_{R_z}$, one obtains

$$
\langle P_{R_z} | q, \lambda \rangle = \int_{D_{T_x}} \int_{D_{R_x}} P_{R_z}(q, \lambda)p(r_{T_x}, r_{R_z} | q, \lambda)dr_{T_x}dr_{R_z}.
$$

The MPI thus depends on the environment characterized by $q$, on the wavelength, and on the domains in which $T_x$ and $R_x$ are located.

The geometrical character of the integral (6.4) is apparent. In the following, the methods introduced in Chapter 3 are applied to obtain analytical bounds for (6.4). The bounds are derived for an arbitrary dimension $n$ of the environment; finally, $n = 2, 3$ is applied. The three different network types that were introduced in Section 4.2 are separately investigated. Each network type constitutes a particular geometrical problem. The simplest is type one, for which only the user is randomly distributed and the access point is fixed. In a second step, the problem of two randomly distributed users, i.e., an Ad-Hoc network, is tackled. From its solution, the last network type, that of a WLAN with unknown position of the access point, can be derived. The resulting bounds on the MPI do, from an analytical perspective, not look very similar; the solution of the corresponding integrals requires more or less sophisticated strategies. A discussion of the evaluated expressions reveals, though, that the MPI is quite similar; but the variances differ.

In the following, the notation of access point and user or, respectively, of $T_x$ and $R_x$ is changed into indices 1 and 2, which simply denote two points distributed in the domain $\mathcal{D}$. As Maxwell’s equations are reciprocal, these changes are legitimate; the resulting bounds are valid both for uplink and downlink. Note also that the results of Chapter 3 are valid for convex domains only. In the following, the domain $\mathcal{D}$ is therefore convex. This is no drawback for the application of the resulting model to most indoor scenarios.
6.3.1 WLANs Type I

In this subsection, a bound for the MPI for a WLAN with fixed access point is derived. This network is denoted by WLAN type I, as defined in Section 4.2. If one of two points in a domain $\mathcal{D}$ is fixed, inequalities can be derived from the relations (3.2) and (3.3) of Subsection 3.2.1. The position of one point can be chosen as the origin $O$ within a specified coordinate system. Omitting the factor $\left(\frac{\lambda}{4\pi}\right)$, (6.4) then transforms into an expression for the MPI of a WLAN with fixed access point $I^{W_f}_{q,n}$, which reads

$$I^{W_f}_{q,n} = \frac{1}{V_n} \int_{\mathcal{D}\setminus\mathcal{B}_{\rho_{min}}(r_1)} \frac{1}{r^q} d\mathcal{P}.$$  \hspace{1cm} (6.5)

Since the points are uniformly distributed, the integral can be written as an integral over the density of points $d\mathcal{P}$ as introduced in Chapter 3. As this density of points is not normalized, the normalization constant $\tilde{V}_n$ is required; it describes the volume $V_n$, corrected by the volume of the sphere around the fixed access point. This is in the bounds of the integral denoted by the expression $\mathcal{D}\setminus\mathcal{B}_{\rho_{min}}(r_1)$, where $\mathcal{B}_{\rho_{min}}(r_1)$ is a ball of radius $\rho_{min}$ around the position of source 1 at $r_1$. The distribution of $r_2$ is restricted to $\mathcal{D}$ without this ball. This restriction is important, since it ensures that the far field condition in (6.3) is fulfilled. All derived results depend on $\rho_{min}$, and it is shown that this parameter becomes highly important for large $q$.

Eq. (6.5) can be transformed as (c.f. (3.2))

$$I^{W_f}_{q,n} = \frac{1}{\tilde{V}_n} \int_{\mathcal{D}\setminus\mathcal{B}_{\rho_{min}}(r_1)} \frac{1}{r^q} d\mathcal{P} \hspace{1cm} (6.6)$$

$$= \frac{1}{\tilde{V}_n} \int_{\mathcal{U}_{n-1}} \int_{\rho_{min}}^{R} r^{n-1-q} dr d\Omega$$

$$= \frac{1}{(n-q)\tilde{V}_n} \left( \int_{\mathcal{U}_{n-1}} R^{n-q}(\Omega) d\Omega - \rho_{min}^{n-q} \int_{\mathcal{U}_{n-1}} d\Omega \right)$$

$$= \frac{1}{(n-q)\tilde{V}_n} \left( \int_{\mathcal{U}_{n-1}} R^{n-q}(\Omega) d\Omega - \rho_{min}^{n-q} O_{n-1} \right) \text{ if } q \neq n.$$  \hspace{1cm} (6.6)

Strictly speaking, this transformation works only if $\mathcal{B}_{\rho_{min}}(r_1)$ is completely within $\mathcal{D}$, which is in the following the only considered case. As a further
simplification, \( \tilde{V}_n \simeq V_n \) is set. This approximation is legitimate, since, for instance, \( V_3 = V_3 - \frac{4}{3}\pi \rho_{\text{min}}^3 \) in three dimensions. Usually, the size of \( D \) is much larger than \( B_{\rho_{\text{min}}} \), since \( \rho_{\text{min}} \) only has to exclude the near field of the \( T_x \). Already for the extreme case where \( V_3 \) is a cube of 1 m sidelength and \( \rho_{\text{min}} = 0.5 \text{ m} \), the ratio \( \frac{\tilde{V}_n}{V_3} \) is about 0.5. Hence, for larger \( D \), the inaccuracy of the bounds is well below 3 dB.

The first term on the right hand side of (6.6) can now be bounded by Hölder’s inequality. As Hölder’s inequality is crucial in order to bound almost all integrals treated in this thesis, it is presented separately in Appendix A.

For the present case, one sets for \( 0 < q < n \) the exponent \( l = \frac{n}{n - q} \) and obtains \( l' = \frac{n}{q} \). Setting furthermore \( f = R \) and \( g = 1 \), where \( f \) and \( g \) are the integrands as defined in A, one obtains

\[
\int_{\Omega_{n-1}} R^{n-q} d\Omega \leq \left( \int_{\Omega_{n-1}} R^n d\Omega \right)^{1 - \frac{q}{n}} \left( \int_{\Omega_{n-1}} d\Omega \right)^{\frac{q}{n}}
= (nV_n)^{1 - \frac{q}{n}} \rho_{\text{min}}^\frac{q}{n-1} \quad \text{if } q < n \quad (6.7)
\]

where in the last step, (3.2) and (3.3) are used. In combination with the rather simple geometrical relations given in Chapter 3, the application of Hölder’s inequality thus allows to bound the MPI for the given network type by an expression which depends only on the volume of the domain \( D \), on its dimension, and on the pathloss exponent \( q \) that describes the radiation characteristics within \( D \).

The expression (6.4) is continuous in \( q \). For \( q \to n \), one can see from (6.6) and (6.7) that both the nominator and the denominator of \( I_{q,n}^{W,f} \) tend towards 0. Application of the rule of de l’Hopital can hence be used to yield the limit of the entire bound, as obtained from (6.6) and (6.7), for \( q = n \) as

\[
\lim_{q \to n} \frac{1}{(n-q)V_n} \left( (nV_n)^{1 - \frac{q}{n}} \rho_{\text{min}}^\frac{q}{n-1} - \rho_{\text{min}}^{n-q}O_{n-1} \right)
= \frac{O_{n-1}}{V_n} \left( \ln \left( \frac{(qV_n)^{\frac{1}{q}}}{\rho_{\text{min}}O_{n-1}^{\frac{1}{q}}} \right) \right). \quad (6.8)
\]
For $q > n$, the sign of the bound (6.7) is reversed (Appendix A),

$$\int_{\mathcal{U}_{n-1}} R^{n-q}d\Omega \geq (nV_n)^{1-\frac{n}{q}} O_{n-1}^{\frac{n}{q}} \quad \text{if } q > n.$$  \hspace{1cm} (6.9)

But now, the factor $(n-q)$ is also negative, so that the product of the left hand side of (6.9) and of $(n-q)$ is negative. The application of (6.6), (6.7), and (6.9) hence yields also an upper bound for $q > n$, since a negative quantity is replaced by one whose absolute value is actually lower.

The expressions (6.6), (6.7), (6.8) and (6.9) can thus be combined to a single upper bound for all $q$:

$$I_{q,n}^{WF} \lesssim \frac{1}{(n-q)V_n} \left( (nV_n)^{1-\frac{n}{q}} O_{n-1}^{\frac{n}{q}} - \rho_{min}^{n-q} O_{n-1} \right) \quad \text{if } q \neq n,$n

$$I_{q,n}^{WF} \lesssim \frac{O_{n-1}}{V_n} \left( \ln \frac{(qV_n)^{\frac{1}{q}}}{\rho_{min} O_{n-1}^{\frac{1}{q}}} \right) \quad \text{if } q = n.$$  \hspace{1cm} (6.10)

The $\lesssim$ sign denotes that due to the change of $\tilde{V}$ into $V$, the bound is only approximate.

**Discussion:**

The derived inequalities are independent from the chosen origin $O$, i.e., from the absolute location of the fixed access point. As the bound is always an upper bound, it is tightest if $O$ is chosen such that $I_{q,n}^{WF}$ takes the highest value. This is achieved when the distances between $R_x$ and $T_x$ are minimized, hence if $O$ is in the center of $D$. If $O$ is moved to the boundary, the bound becomes less tight. Symmetry considerations yield a simple strategy to improve the bound. Imagine that $O$ is exactly in the middle of a rectangle, as in Fig. 6.2 a). For reasons of symmetry, the MPI in this scenario equals the MPI for a distribution where the user is only in one half of the rectangle; the setup has the same MPI than one in which a source is on the boundary of the rectangle of half the size. But a similar setup is displayed in b); here, the source is placed in a rectangle of half the size of that in a). Since in b), it is exactly in the middle, the bound calculated for the smaller size is tightest. Hence, if the bound for a source on the boundary of a domain is too loose, the domain can be extended to twice its size, so that the source is in the middle of a symmetric, larger
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Figure 6.2: Symmetry considerations for the calculation of the pathloss.

domain as in a). Since the MPI in this larger domain equals that in the smaller one, the bound calculated for the former case is valid for both domains. But the bound for the former case is tightest, since the source is in its center.

The behavior of the bounds is shown in Fig. 6.3, where the MPI and the analytical results are plotted against the pathloss exponent \( q \) in a range from 1.4 to 5.2. The bounds are displayed by spheres, and the simulated values, obtained from 10,000 runs, by triangles. In Fig. 6.3 a), the bounds are investigated within a room of size 10×10 m\(^2\) for \( n = 2 \), and the access point is fixed in the center. The minimum distance \( \rho_{\text{min}} \) is chosen as 1 m and 2 m. The bounds are continuous; the special case \( q = 2 \) in (6.7) fits smoothly into the entire curve. Bounds and simulated means fit — on dB scale — almost exactly, which is a surprising result that indicates the tightness of Hölder's inequality in this case. Differences between the bounds are hardly visible; close inspection reveals that they are below 10%. The bounds are actually slightly lower than the simulated curves, even if they are upper bounds. The reason for this behavior is the exchange of \( \tilde{V}_n \) into \( V_n \), i.e., the normalization of the analytic expression by a factor that is slightly too large. This effect is more apparent for the lower curves, obtained for the larger \( B_{\rho_{\text{min}}}(r_1) \) with \( \rho_{\text{min}} = 2 \) m, where the ratio between \( \tilde{V}_n \) and \( V_n \) is 0.6 dB. For low values of \( q \), the results are very close to each other. For increasing \( q \), the differences in MPI between the cases \( \rho_{\text{min}} = 1 \) m and \( \rho_{\text{min}} = 2 \) m is up to 10 dB.

In Fig. 6.3 b), the results of the same simulation for \( n = 3 \) are displayed. The room has a size of 10 m × 10 m × 4 m, the access point is fixed in the middle of the ceiling. Again, the difference of the curves with regard to \( \rho_{\text{min}} \) is higher for high \( q \). It is interesting to observe that the difference of the simulated means between the two figures is not large, i.e., the two and the three dimensional cases are similar; the height of the structure is
Figure 6.3: Bounds for the MPI (○) compared to simulated means (△).
a) $n = 2$ b) $n = 3$ c) $n = 3$, $\rho_{min} = 2.5$ m for the real and the extended domain.
negligible compared to its floor size. The bounds for low $q$ are still very close to the simulated means; for high $q$, there is a deviation of up to 3 dB. This deviation is low compared to the power loss of 15 and 20 dB, respectively, over the range of $q$. It can be contributed to the location of the access point at the ceiling of the room, i.e., at the boundary of $\mathcal{D}$, which makes the bound less tight.

This effect is demonstrated in Fig. 6.3 c), where the bounds are compared to the simulation for $n = 3$ in a large domain of $40 \text{ m} \times 20 \text{ m} \times 5 \text{ m}$ and for $\rho_{\text{min}} = 2.5 \text{ m}$. The simulated curve (triangles) corresponds to a location of the access point in a corner of the structure. The bounds are obtained for a domain of $40 \text{ m} \times 20 \text{ m} \times 5 \text{ m}$ (upper circles), and for an extended domain of twice the size in all dimensions, i.e., $80 \text{ m} \times 40 \text{ m} \times 10 \text{ m}$ (lower circles). As explained in the context of Fig. 6.2 above, both bounds are upper bounds; since the access point is exactly in the upper corner, one can extend the domain such that the access point is in the center of a new domain. The resulting bound will match the simulated mean better.

For large $q$, the bounds are actually well represented by the second term in (6.10), since the first becomes negligible for high exponents. Simulations show that for $n = 2$, a good approximation with an error of less than a few dB already holds at $q \geq 3$; for $n = 3$, a value of at least 4 is required. The power is then concentrated on the surface of the sphere, defined by the minimum distance $\rho_{\text{min}}$ between $T_x$ and $R_x$; the actual volume of $\mathcal{D}$ only has an impact on MPI because it is also the normalization constant for the mean value.

Due to the different exponents in the expression, the further analytical evaluation of (6.10), i.e., an explicit ratio of the bounds for $n = 2$ and $n = 3$, is complicated. A simple observation is, however, that $\lim_{V_n \to \infty} W_{q,n}^f \to 0$, since the normalization constant $V_n$ is always larger than any factors of $V_n$ in the nominator. For an infinitely large domain $\mathcal{D}$, the MPI tends to 0, since the probability that $T_x$ and $R_x$ are at a very large distance becomes 1.

One can additionally state that the equality of the bound is never achieved. According to Appendix A, equality in Hölder’s inequality holds for two functions $f$ and $g$ if and only if $|f|^l$ is proportional to $|g|^{l'}$. Since $g \equiv 1$ and $f = R$, this case occurs only for $l = 0$, i.e., $l' = 1$. This
corresponds to \( n = q \), where the integral cannot be explicitly calculated and must be evaluated using the rule of de l'Hôpital.

### 6.3.2 Ad-Hoc Networks

The case of an Ad-Hoc network has its geometrical equivalent in Subsection 3.2.2. Two points are uniformly distributed in a domain \( \mathcal{D} \), with the restriction to keep a minimum distance \( \rho_{\text{min}} \) to fulfill the far field condition of (6.3). The integral (6.4) transforms in this case into

\[
I_{q,n}^{AH,1} = \frac{1}{V_n \tilde{V}_n} \int_{\mathcal{D}} \int_{\mathcal{D} \setminus B_{\rho_{\text{min}}}(r_1)} \frac{1}{r^q} d\mathcal{P}_1 d\mathcal{P}_2. \tag{6.11}
\]

The normalization volume \( V_n^2 \) is replaced by \( V_n \tilde{V}_n \), since the \( T_x \) cannot take any position in \( \mathcal{D} \). However, one can use \( \tilde{V}_n \simeq V_n \) for the same reasons as in Subsection 6.3.1.

The transformation of the integral (6.11) requires the more advanced techniques of Subsection 3.2.2. In the Appendix B, (3.7) is applied to (6.11) to yield

\[
I_{q,n}^{AH,1} = \frac{2}{(n-q)(n-q+1)V_n \tilde{V}_n} \left\{ \int_{\mathcal{D} \setminus B_{\rho_{\text{min}}}(r_1)} \sigma^{n-q+1} d\mathcal{L} - \rho_{\text{min}}^{n-q+1} \int_{\mathcal{D} \setminus B_{\rho_{\text{min}}}(r_1)} d\mathcal{L} - (n-q+1) \int_{\mathcal{D} \setminus B_{\rho_{\text{min}}}(r_1)} (\sigma - \rho_{\text{min}}) \rho_{\text{min}}^{n-q} d\mathcal{L} \right\} \tag{6.12}
\]

where \( \sigma \) is the length of the chord that arises when the line \( \mathcal{L} \) intersects the convex domain \( \mathcal{D} \). As opposed to (3.7), however, the integration is now carried out over all chord lengths greater than the radius \( \rho_{\text{min}} \) of the sphere around \( r_1 \). The integral consists hence of three terms. For \( \rho_{\text{min}} = 0 \), only the first term exists and the integral is the same as (3.7). This term increases for \( \rho_{\text{min}} = 0 \) without bound when \( q > n \), since \( \sigma \) may become 0; a negative exponent occurs in the course of the evaluation of the integral. The second and the third term are needed for correction and to keep the bounds finite.

For a further evaluation of the first term, Hölder’s inequality is used (Appendix A). Set \( f = \sigma, g = 1 \), and choose \( l = n-q+1 \). For \( q < n \), one
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has then \( l' = \frac{n-q+1}{n-q} \) and

\[
\int_{L \cap D \neq \emptyset} \sigma dL \leq \left( \int_{L \cap D \neq \emptyset} \sigma^{n-q+1} dL \right)^{-\frac{1}{n-q+1}} \left( \int_{L \cap D \neq \emptyset} 1 dL \right)^{\frac{n-q}{n-q+1}}. \tag{6.13}
\]

For \( q > n \), the same relation holds with reversed sign, so that

\[
\int_{\sigma > \rho_{\text{min}}} \sigma^{n-q+1} dL \geq \left( \int_{\sigma > \rho_{\text{min}}} \sigma dL \right)^{n-q+1} \left( \int_{\sigma > \rho_{\text{min}}} 1 dL \right)^{q-n}
\]

if \( q < n \),

\[
\int_{\sigma > \rho_{\text{min}}} \sigma^{n-q+1} dL \leq \left( \int_{\sigma > \rho_{\text{min}}} \sigma dL \right)^{n-q+1} \left( \int_{\sigma > \rho_{\text{min}}} 1 dL \right)^{q-n}
\]

if \( q > n, q \neq n+1 \). \tag{6.14}

The expressions on the right hand side are of the form (3.8) and (3.9), except that the integration is not carried out over all \( L \cap D \neq \emptyset \), but only over those with \( \sigma > \rho_{\text{min}} \). Whenever the line \( L \) intersects the domain \( D \) only slightly (i.e., when \( \sigma < \rho_{\text{min}} \), which happens particularly at wedges of \( D \)), the line is excluded from the integration domain. The resulting difference can be exactly calculated if the shape of the domain is known. For a two dimensional rectangle with sidelengths \( a, b \), one has, e.g.,

\[
\int_{\sigma < \rho_{\text{min}}} \sigma dL = \rho_{\text{min}}^2. \tag{6.15}
\]

The corresponding expression in three dimensions for a cuboid with height \( c \) is already more involved. Since the considered domains are often sections of entire floors of buildings and \( \rho_{\text{min}} \) is comparatively small (e.g., \( \rho_{\text{min}} \simeq 1 \text{ m} \)), the contribution of (6.15) can be neglected. Hence, the integration is carried out over all chords. In both dimensions, however, the condition

\[
2\rho_{\text{min}} < \min(a, b) \text{ if } n = 2 \quad \text{or} \quad 2\rho_{\text{min}} < \min(a, b, c) \quad \text{if } n = 3 \tag{6.16}
\]

must be fulfilled. This condition is important; if it is neglected, the analytical bounds may yield senseless, e.g., complex, numbers. Even though introduced in this subsection, the condition ensures also the applicability of (6.10) in the previous subsection, because for the validity of (6.10) it is required that the ball with radius \( \rho_{\text{min}} \) lies completely within \( D \).
The substitution of (6.14) in (6.12) yields, together with the application of (3.8) and (3.9), a lower bound for $I_{q,n}^{AH,1}$ if $q < n$. For $n < q < n + 1$, the bound for $\int \sigma^{n-q+1}d\mathcal{L}$ changes to an upper bound, but the factor $\frac{2}{(q-n)(n-q+1)}$ also changes its sign. Together with this change of sign in (6.12), the bound still remains a lower bound, since too much is now subtracted. For $q > n + 1$, the factor $\frac{2}{(n-q)(n-q+1)}$ is positive, the expression $\int \sigma^{n-q+1}d\mathcal{L}$ is upper bounded, and hence the entire expression is an upper bound for $I_{q,n}^{AH,1}$.

Finally, the cases $q = n$ and $q = n + 1$ must be evaluated separately. The denominator $\frac{2}{(n-q)(n-q+1)}$ is 0 in both cases. Substituting (3.8) and (3.9) in (6.12), one obtains a term of the form

$$\left[ \frac{O_{n-1} V_n}{O_0} \right]^{n-q+1} \left[ \frac{O_{n-2}}{(n-1)O_0} A_{n-1} \right] q-n$$

$$- (n - q + 1) \rho_{min}^{n-q} \frac{O_{n-1}}{O_0} V_n + (n - q) \rho_{min}^{n-q+1} \frac{O_{n-2}}{(n-1)O_0} A_{n-1}. \tag{6.17}$$

For $n = q$, it holds

$$\left[ \frac{O_{n-1} V_n}{O_0} \right] - (\rho_{min})^0 \frac{O_{n-1}}{O_0} V_n = 0, \tag{6.18}$$

i.e., the expression $I_{q,n}^{AH,1}$ is a $\frac{0}{0}$ limit. The same is valid for $q = n + 1$, namely

$$\left[ \frac{O_{n-2}}{(n-1)O_0} A_{n-1} \right] + (-1) \rho_{min}^0 \frac{O_{n-2}}{(n-1)O_0} A_{n-1} = 0. \tag{6.19}$$

Hence, the cases $q = n$ and $q = n + 1$ can be obtained by an application of the rule of de l’Hopital. The result is a continuous bound for all $q > 0$. This behavior is not self-evident. Even though the expression (6.11) is a continuous function in $q$, the continuity of the bound cannot be taken for granted.
Collecting all results, one has

\[ I_{q,n}^{AH,1} \geq \frac{2}{(n-q)(n-q+1)V_n^2} \left( \eta_n^{(2)} \frac{(n-q+1)}{(n-q+1)\rho_{min}^{(1)}} \right) - (n-q+1)\rho_{min}^{n-q+1}\eta_n^{(1)} \]

\[ + (n-q)\rho_{min}^{n-q+1}\eta_n^{(1)} - (n-q+1)\rho_{min}^{n-q}\eta_n^{(2)} \]

if \( q < n + 1, q \neq n \),

\[ I_{q,n}^{AH,1} \geq \frac{2}{V_n^2} \left[ \eta_n^{(2)} \left( \ln \frac{\eta_n^{(2)}}{\eta_n^{(1)}} - 1 \right) + \eta_n^{(1)} \rho_{min} \right] \]

if \( q = n \),

\[ I_{q,n}^{AH,1} \leq \frac{2}{(n-q)(n-q+1)V_n^2} \left( \eta_n^{(2)} \frac{(n-q+1)}{(n-q+1)\rho_{min}^{(1)}} \right) - (n-q+1)\rho_{min}^{n-q+1}\eta_n^{(1)} \]

\[ - (n-q+1)\rho_{min}^{n-q}\eta_n^{(2)} + (n-q)\rho_{min}^{n-q+1}\eta_n^{(2)} \]

if \( n + 1 < q \),

\[ I_{q,n}^{AH,1} \sim \frac{2}{V_n^2} \left[ \eta_n^{(1)} \left( \ln \frac{\eta_n^{(1)}}{\eta_n^{(2)}} - 1 \right) + \eta_n^{(2)} \rho_{min} \right] \]

if \( q = n + 1 \). \quad (6.20)

The abbreviations \( \eta_n^{(1)} \) and \( \eta_n^{(2)} \) are introduced as

\[ \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} d\mathcal{L} = \frac{O_n}{(n-1)O_0} A_{n-1} = \eta_n^{(1)} \]

\[ \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} \sigma d\mathcal{L} = \frac{O_{n-1}}{O_0} V_n = \eta_n^{(2)}. \quad (6.22) \]

The \( \simeq \), and \( \lesssim \) and \( \gtrsim \) signs indicate that the expressions are only approximate, as explained above in the context of (6.6) and (6.15). Since the bound is continuous and a lower bound for \( q < n + 1 \) and an upper one for \( q > n + 1 \), it must intersect the exact curve for \( I_{q,n}^{AH,1} \); the bound is tightest for these values of \( q \).

The application of Hölder’s inequality as performed in (6.13) is not the only possibility. Consider now the choice of functions \( f = s^{n-q+1}, \)

\( g = 1 \) and choose \( l \) such that \( (n-q+1)l = (n+1) \); one obtains \( l = \)
(1 - \frac{q}{n+1})^{-1}, l' = \frac{n+1}{q}, and for q < n + 1, it follows that
\[
\int_{\mathcal{L} \cap D \neq \emptyset} \sigma^{n-q+1} d\mathcal{L} \leq \left( \int_{\mathcal{L} \cap D \neq \emptyset} \sigma^{n+1} d\mathcal{L} \right)^{1-\frac{q}{n+1}} \left( \int_{\mathcal{L} \cap D \neq \emptyset} 1 d\mathcal{L} \right)^{\frac{q}{n+1}}.
\]

(6.23)

Thus, an upper bound is obtained that can be evaluated in terms of \(A_{n-1}\) and \(V_n\) using (3.8) and (3.10). The reversed version of Hölder’s inequality can be applied for q > n + 1 to yield
\[
\int_{\mathcal{L} \cap D \neq \emptyset} \sigma^{n-q+1} d\mathcal{L} \geq \left( \int_{\mathcal{L} \cap D \neq \emptyset} \sigma^{n+1} d\mathcal{L} \right)^{1-\frac{q}{n+1}} \left( \int_{\mathcal{L} \cap D \neq \emptyset} 1 d\mathcal{L} \right)^{\frac{q}{n+1}}.
\]

(6.24)

If the case q = n is considered, as in (6.18) and (6.19), one obtains the terms
\[
\left( \frac{n(n+1)}{2} V_n^2 \right)^{1-\frac{q}{n+1}} \left( \frac{O_{n-2}}{O_0(n-1) A_{n-1}} \right)^{\frac{q}{n+1}}
- (n-q+1)\rho_{\min}^{n-q} \left( \frac{O_{n-1}}{O_0} V_n \right) + (n-q)\rho_{\min}^{n-q+1} \left( \frac{O_{n-2}}{O_0(n-1) A_{n-1}} \right).
\]

(6.25)

For n = q, one has
\[
\left( \frac{n(n+1)}{2} V_n^2 \right)^{\frac{1}{n+1}} \left( \frac{O_{n-2}}{O_0(n-1) A_{n-1}} \right)^{\frac{n}{n+1}} - \left( \frac{O_{n-1}}{O_0} V_n \right) \neq 0
\]

(6.26)

for all general cases. Thus, since the denominator \((n-q+1)(n-q)\) is 0, the expression diverges at \(n = q\). Even though the correcting terms for \(\rho_{\min} > 0\) are introduced, the bound is not continuous at this value of q. For q = n + 1, it holds that
\[
\left( \frac{O_{n-2}}{O_0(n-1) A_{n-1}} \right) - \left( \frac{O_{n-2}}{O_0(n-1) A_{n-1}} \right) = 0,
\]

(6.27)

so that a \(\frac{0}{0}\)-limit is obtained, which can be calculated using the rule of de l’Hopital.

Combining these results, one arrives at a second bound \(I_{q,n}^{AH,2}\) of the
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form

\[ I_{q,n}^{AH,2} \lessapprox \frac{2}{(n-q)(n-q+1)V_n^2} \left( \eta_n^{(3)} \frac{1}{n+1} \eta_n^{(1)} \frac{q}{n+1} \right) + (n-q)\rho_{min}^{n-q+1} \eta_n^{(1)} - (n-q+1)\rho_{min}^{n-q}\eta_n^{(2)} \]

if \( q < n \)

\[ I_{q,n}^{AH,2} \gtrapprox \frac{2}{(n-q)(n-q+1)V_n^2} \left( \eta_n^{(3)} \frac{1}{n+1} \eta_n^{(1)} \frac{q}{n+1} \right) + (n-q)\rho_{min}^{n-q+1} \eta_n^{(1)} - (n-q+1)\rho_{min}^{n-q}\eta_n^{(2)} \]

if \( q > n, q \neq n+1 \)

\[ I_{q,n}^{AH,2} \gtrapprox \frac{2}{V_n^2} \frac{\eta_n^{(1)}}{\rho_{min}} \left( \ln \left( \rho_{min} \left( \frac{\eta_n^{(3)}}{\eta_n^{(1)}} \right)^{\frac{1}{n+1}} \right) - 1 \right) + \frac{2}{V_n^2} \frac{\eta_n^{(2)}}{\rho_{min}} \]

if \( q = n+1 \) (6.28)

where \( \eta_n^{(1)} \) and \( \eta_n^{(2)} \) are defined as in (6.21) and (6.22), and where the constant \( \eta_n^{(3)} \) is

\[ \eta_n^{(3)} = \frac{n(n+1)}{2} V_n^2. \] (6.29)

The term for \( q < n \) is an upper bound, since the factor \( \frac{2}{(n-q)(n-q+1)} \) is greater than 0 and the first term is an upper bound. The other two terms are lower bounds. For \( n < q < n+1 \), the factor is negative and the first term in the bound is still an upper bound. For \( q > n+1 \), the factor is positive and the first term is a lower bound. The bound diverges for \( q = n \), but it is continuous for \( q = n+1 \).

**Discussion:**

The derived bounds depend on the volume and on the surface area of the domain \( \mathcal{D} \). The ratio of \( A_{n-1} \) and \( V_n \) determines their tightness. The bounds (6.20) and (6.28) become arbitrarily bad if \( V_n \) becomes arbitrarily flat, since the factor \( V_n \) is always in the denominator and the surface remains greater zero. For \( q > n \), however, the bound diverges for \( \rho_{min} = 0 \) in any case; if \( \rho_{min} > 0 \), the condition (6.16) prohibits that the expressions can be applied to arbitrarily flat domains. As in Subsection 6.3.1, all bounds become 0 for \( V_n \rightarrow \infty \), since in a domain of infinite extent, the
Figure 6.4: Bounds for the MPI (○, □) compared to simulated means. a) $n = 2$ (flat domain, △) b) $n = 3$, for a height of 1 m (△) and 6 m (◇).

The behavior of the bounds is shown in Fig. 6.4; simulations with again 10,000 runs are compared to the analytical bounds for a structure of 10 x 5 m$^2$ floor size, with $\rho_{\text{min}} = 1$ m over a large range of $q$. Domains of 1 m and of 6 m height are investigated. Fig. 6.4 a) shows the results for $n = 2$, Fig. 6.4 b) for $n = 3$. For the former, the simulated curve is
displayed by triangles, the bounds by circles. The height has no impact in two dimensions, so that only one simulated curve results. The smooth bound which starts at low $q$ below the simulated curve is the bound given by (6.20). It is continuous for all $q$ and always within a distance of about 1 dB to the simulated value. Theoretically, it intersects this curve at $q = 3$; however, it only comes very close, but does not intersect. This is due to the exchange of $\tilde{V}_n$ by $V_n$, which effects the last terms of the bound. The other bound (6.28) starts as an upper bound for low $q$ and diverges at $q = 2$. For $q > 2$, it becomes a lower bound and converges to the formerly lower bound. For large $q$, both bounds nearly coincide, since only the last terms with the factor $p_{min}$ contribute to the result. The first term in each expression, which depends on the entire volume of the domain, becomes negligible. The equality of the bound (6.20) with $I_{q,n}^{AH,1}$ is approximately obtained for $q = n + 1$. For this $q$, the sign in Hölder’s inequality is reversed, but the bound itself remains continuous. This $q$ is the value for which Hölder’s inequality achieves equality between $f^{n-q+1} = s^{n-q+1}$ and $g = 1$.

For $n = 3$, two simulated curves result. The upper one (triangles) corresponds to a height of $\mathcal{D}$ of 1 m, the lower (diamonds) to a height of 6 m. Whereas the first domain may describe an Ad-Hoc network that operates within a single room and includes all devices between table and head height, the latter one would extend over a large volume, for instance a high factory hall or two floors of a building. It could also represent a WLAN within a small building, in which the position of the access point is totally unknown. The upper simulated curve is very close to the curve for $n = 2$, which demonstrates the validity of a two dimensional approach if $\mathcal{D}$ is flat. The two dimensional model is very important in this case, because $n = 3$ can, due to condition (6.16), not be applied if $2p_{min}$ is greater than the height of $\mathcal{D}$. For a height of $\mathcal{D}$ of 6 m, the bounds in the picture are shown as squares. They display the same properties as those for $n = 2$, except that they diverge for $n = 3$. They are also fairly tight; in the range up to $q < 2$, the upper bound is quite a good approximation with an error of less than 3 dB, and above that $q$, the other bound can be used. The bounds become worse if $\mathcal{D}$ becomes flatter; in this case, the two dimensional approximation is better. Since for large $q$, the bound and the simulated values will always coincide, a comparison of the bounds for
$n = 2$ and $n = 3$ shows whether the former is a valid approximation.

### 6.3.3 WLANs Type II

WLANs of type II refer to the WLANs with the unknown position of the access point, as defined in Section 4.2. The MPI for this network type can be approximated with the aid of the fundamental relation between the distribution of two points within a domain and the distribution of one of them on the boundary. It was already outlined in Subsection 3.2.3.

One starts from the relation (3.18) for $f = \frac{1}{||r_1 - r_2||^q}$ where $r_1$ and $r_2$ denote the two points in the domain $\mathcal{D}$. A lower bound for the sought expression is again obtained by applying Hölder’s inequality for $l = \infty$ (Appendix A),

\[
\frac{d}{ds} \mathcal{M} = \frac{d}{ds} \int_{\mathcal{M}} \frac{1}{||r_1 - r_2||^q} dV_{\mathcal{M}} = 2 \int_{\partial \mathcal{M}} v_{\mathcal{M}} \frac{1}{||r_1 - r_2||^q} dS_{\mathcal{M}} \leq 2 ||v_{\mathcal{M}}||_\infty \int_{\partial \mathcal{M}} \frac{1}{||r_1 - r_2||^q} dS_{\mathcal{M}}. \quad (6.30)
\]

The parameter $t$ denoting a time in Subsection 3.2.3 has been replaced by the dimensionless parameter $s$, since the change of the manifold $\mathcal{M}$ does here not depend on time, but is merely a mathematical tool. As both points are in the same domain $\mathcal{D}$, the considered manifold is, as explained in the context of (3.18), $\mathcal{M} = \mathcal{D} \times \mathcal{D}$, and $\partial \mathcal{M}$ denotes the boundary $\partial \mathcal{M} = \partial \mathcal{D} \times \mathcal{D}$. The sought expression is $\int_{\partial \mathcal{M}} \frac{1}{||r_1 - r_2||^q} dS_{\mathcal{M}}$, which is the (unnormalized) mean value of the function $f = \frac{1}{||r_1 - r_2||^q}$ for two points, of which one is on the boundary of $\mathcal{D}$ and the other within (c.f. [12]). Note that the points $r_1$ and $r_2$ are combined to a $2n$ dimensional point in the $2n$ dimensional manifold $\mathcal{M}$. In $\mathcal{M}$, the function $f$ is in this manifold not a function of two, but one of a single point.

The key idea that is pursued here is to parameterize the known integral $\int_{\mathcal{M}} \frac{1}{||r_1 - r_2||^q} dV_{\mathcal{M}}$ (c.f. Subsection 6.3.2), such that it can be explicitly derivated by $s$, and then to apply (6.30). Parameterize a manifold $\mathcal{M}_s$ in dependence on a parameter $s$ such that $\mathcal{M} = \mathcal{M}_s|_{s=0}$. Then, for both $r_1$
and \( r_2 \) in spherical coordinates, one can write

\[
\mathcal{M}_s = \int_{U_{n-1}} \int_0^{R(\Omega,s)} \int_{U_{n-1}} \int_0^{R(\Omega,s)} \frac{1}{||r_1 - r_2||^q} r_1^{n-1} dr_1 d\Omega_1 r_2^{n-1} dr_2 d\Omega_2.
\]

Choose now \( R(\Omega, s) = (1 + s)R(\Omega) \). Then, by substitution of \( r_\nu = (1 + s)\tilde{r}_\nu, \nu = 1, 2, \)

\[
\begin{align*}
\mathcal{M}_s & = \int_{U_{n-1}} \int_0^{R(\Omega)(1+s)} \int_{U_{n-1}} \int_0^{R(\Omega)(1+s)} \frac{1}{||\tilde{r}_1 - \tilde{r}_2||^q} \tilde{r}_1^{n-1} d\tilde{r}_1 d\Omega_1 \tilde{r}_2^{n-1} d\tilde{r}_2 d\Omega_2 \\
& = (1 + s)^{2n-q} \int_{U_{n-1}} \int_0^{R(\Omega)} \int_{U_{n-1}} \int_0^{R(\Omega)} \frac{1}{||\tilde{r}_1 - \tilde{r}_2||^q} \tilde{r}_1^{n-1} d\tilde{r}_1 d\Omega_1 \tilde{r}_2^{n-1} d\tilde{r}_2 d\Omega_2 \\
& = (1 + s)^{2n-q} \mathcal{M}_s \bigg|_{s=0},
\end{align*}
\]

so that

\[
\frac{d}{ds} \mathcal{M}_s \bigg|_{s=0} = \frac{d}{ds} \mathcal{M} = (2n - q) \mathcal{M}.
\]

Eq. (6.34) provides a simple expression for the substitution into the left hand side of (6.30).

The next step is the evaluation of the velocity \( v_M \) explicitly for this parameterization. Use therefore that

\[
\frac{d}{ds} \int_M dV_M = 2 \int_{\partial M} v_M dS_M,
\]

and calculate the derivative of the left hand side of this expression, using
a similar strategy as in (6.32):

\[
\frac{d}{ds} \int_{\mathcal{M}} dV_M \bigg|_{s=0} = \frac{d}{ds} \left( \int_{\mathcal{D}} d\mathcal{P}_1 \int_{\mathcal{D}} d\mathcal{P}_2 \right) \bigg|_{s=0}
\]

\[
= 2 \left( \frac{d}{ds} \int_{\mathcal{D}} d\mathcal{P}_1 \right) \bigg|_{s=0} \left( \int_{\mathcal{D}} d\mathcal{P}_2 \right) \tag{6.37}
\]

\[
= 2 \left( \frac{d}{ds} \int_{\mathcal{U}_{n-1}} \int_{0}^{R(\Omega)(1+s)} r_1^{n-1} dr_1 d\Omega_1 \right) \bigg|_{s=0} \times \left( \int_{\mathcal{D}} d\mathcal{P}_2 \right) \tag{6.38}
\]

\[
= 2 \left( \frac{d}{ds} (1 + s)^n \int_{\mathcal{U}_{n-1}} \int_{0}^{R(\Omega)} r_1^{n-1} dr_1 d\Omega_1 \right) \bigg|_{s=0} \times \left( \int_{\mathcal{D}} d\mathcal{P}_2 \right) \tag{6.39}
\]

\[
\leq 2 \| R(\Omega) \|_{\infty} \left( \int_{\mathcal{U}_{n-1}} R(\Omega)^{n-1} d\Omega \right) \times \left( \int_{\mathcal{D}} d\mathcal{P}_2 \right) \tag{6.40}
\]

\[
= 2 \| R(\Omega) \|_{\infty} \int_{\partial \mathcal{M}} dS_M. \tag{6.41}
\]

The derivation in (6.39) produces a factor \( n \), which is immediately cancelled due to the integration over \( \tilde{r}_1 \), which gives the inverse of \( n \). If \( R \) is then singled out in the integrand in (6.40) and Hölder’s inequality is applied as shown in (6.30), one has in (6.41) an integral over \( \partial \mathcal{D} \times \mathcal{D} \). This expression is identical to the surface integral given by (6.42) and treated in [12]. Comparison with (6.35) gives \( v_M = R \). This term could be expected for geometrical reasoning, since the domain \( \mathcal{D} \) is in each direction \( \Omega \) inflated by a factor of \((1 + s)R\), i.e., it is inflated proportionally to its own length \( R \). Note that the usual strategy of differentiation is based on \( R(\Omega, s) = R(\Omega) + s \) for \( s \to 0 \). In this case, the derivative in (6.39) could be replaced by the commonly known differential quotient, and \( v_M = \text{const.} \) would result.
Finally, the proper normalization constants must be introduced. From the derivation of the integral over $\mathcal{M}$ and over $\partial \mathcal{M}$ in $2n$ dimensional space in Subsection 3.2.3, it follows that

$$\int_{\mathcal{D} \times \mathcal{D}} dV_{\mathcal{M}} = V_n^2, \quad \int_{\partial \mathcal{D} \times \mathcal{D}} dS_{\mathcal{M}} = A_{n-1} V_n. \quad (6.43)$$

Gathering (6.34), (6.40), and (6.43), and with $\langle . \rangle_{\mathcal{D} \mathcal{D}}$ denoting the expectation for two points uniformly in $\mathcal{D}$ and $\langle . \rangle_{\partial \mathcal{D}}$ denoting the expectation for one point on $\partial \mathcal{D}$, it results that

$$\langle \frac{1}{r_q} \rangle_{\partial \mathcal{D}} \geq \frac{V_n^2}{V_n A_{n-1} ||R||_{\infty}} \left( n - \frac{q}{2} \right) \langle \frac{1}{r_q} \rangle_{\mathcal{D} \mathcal{D}} \quad (6.44)$$

where $||R||_{\infty}$ is simply the maximum distance between a point on $\partial \mathcal{D}$ and a point within $\mathcal{D}$. For a cuboid structure, this is the diagonal. Note that the bound becomes arbitrarily bad for $q \to 2n$, as it tends towards 0 and eventually to $-\infty$.

Eq. (6.44) constitutes a trivial relation between the bounds derived in Subsection 6.3.2 and the bounds required in this section. Due to the inequality in (6.44), however, only lower bounds of (6.20) and (6.28) can be substituted in (6.44). But since the factor $||R||_{\infty}$ has high weight, it is possible to work with all bounds from the subsection above. A rigorous proof, which would require a weighting of $||R||_{\infty}$ against the tightness of the bounds (6.20) and (6.28), is omitted; the bounds are investigated in simulations below.

Accordingly, the $\succ$ signs are again used instead of the $\geq$ signs, and one finally has the two bounds $I_{q,n}^{WR,1}$ and $I_{q,n}^{WR,2}$:

$$I_{q,n}^{WR,1} \succ \left( n - \frac{q}{2} \right) \frac{V_n}{A_{n-1} ||R||_{\infty}} I_{q,n}^{AH,1} \quad \text{if } q \neq n, \quad q \neq n + 1,$$

$$I_{q,n}^{WR,1} \succ \frac{1}{V_n A_{n-1} ||R||_{\infty}} \left[ \eta_n^{(2)} \left( \ln \frac{\eta_n^{(2)}}{\eta_n^{(1)}} - n \right) + n \eta_n^{(1)} \rho_{\min} \right] \quad \text{if } q = n,$$

$$I_{q,n}^{WR,1} \succ \frac{n - 1}{V_n A_{n-1} ||R||_{\infty}} \left[ \eta_n^{(1)} \left( \ln \frac{\eta_n^{(1)}}{\rho_{\min}} - 1 \right) + \frac{\eta_n^{(2)}}{\rho_{\min}} \right] \quad \text{if } q = n + 1, \quad (6.45)$$
as derived from $I_{q,n}^{AH,1}$, and

\[
I_{q,n}^{Wr,2} \gtrless \left( n - \frac{q}{2} \right) \frac{V_n}{A_{n-1} ||R||_{\infty}} I_{q,n}^{AH,2} \quad \text{if } q \neq n, q \neq n + 1,
\]

\[
I_{q,n}^{Wr,2} \gtrless \frac{n - 1}{V_n A_{n-1} ||R||_{\infty}} \left( \eta_n^{(1)} \left( \ln \left( \frac{\rho_{\min} \left( \frac{\eta_n^{(1)}}{\eta_n^{(3)}} \right)^{-1/2} - 1 \right) + \frac{\eta_n^{(2)}}{\rho_{\min}} \right) \right)
\]

if $q = n + 1$, \hspace{1cm} (6.46)

as derived from $I_{q,n}^{AH,2}$. Note that now, all bounds are lower bounds, obtained from the rescaling of the bounds in Subsection 6.3.2. A special case are the relations for $n = q$ and $n = q + 1$, which again require the application of the rule of de l'Hôpital.

**Discussion:**

The derived bounds $I_{q,n}^{Wr,1}$ and $I_{q,n}^{Wr,2}$ are plotted in Fig. 6.5. In each part of the figure, they are compared to the simulated mean values for both $n = 2$ and $n = 3$, based on 10,000 runs. The considered domain has a length of 30 m, a height of 20 m, and $\rho_{\min}$ is 5 m. The bounds $I_{q,n}^{Wr,1}$ are marked by squares for $n = 2$ and by ‘+’ for $n = 3$. The curves of $I_{q,n}^{Wr,2}$ are marked by stars for $n = 2$ and by circles for $n = 3$. The simulated curves are marked by diamonds for $n = 2$ and by triangles for $n = 3$.

In part a) of the figure, the bounds are again shown for various $q$, here for a width of $D$ of 10 m. They display a similar behavior as those in Subsection 6.3.2. Even though only $I_{q,n}^{Wr,1}$ (□ and +) could be proven to be a lower bound, both $I_{q,n}^{Wr,1}$ and $I_{q,n}^{Wr,2}$ (★ and ○) are in fact lower bounds, except for $q \to n$. There, $I_{q,n}^{Wr,2}$ diverges and then returns from $-\infty$. For $q \to 2n$, all bounds also drop to $-\infty$. The bounds $I_{q,n}^{Wr,1}$ are not as close to the simulated curve as the bound $I_{q,n}^{Wr,2}$. The bounds for both dimensions are quite close to each other, as are the simulated curves; the reason is the rather large value of $\rho_{\min}$.

In Fig. 6.5 b), the bounds are plotted for $q = 3.2$ with the width of $D$ being increased from 10 to 30 m. The difference between the bounds and the simulated values is again comparably high, but the simulation results and the bounds themselves coincide quite closely. Most striking is the fact that the change in size of $D$ affects the MPI much less than the change in
q. This behavior is actually true for all three network types, as will later be shown by ray tracing simulations. The \( I_{q,3}^{Wr,2} \) does not exist for \( q = 3.2 \); it is too close at the singularity \( q = n \), which is visible also in a). Further discussions on the bounds for the Ad-Hoc network can be found in [53].

6.4 Comparison of the Bounds

One can assume that the derived bounds should behave similar in similar environments, since the impact of the —fix or uniformly distributed— location of an access point may, in the average, not be strong. An analytical comparison is, however, very complicated, since both bounds are composed of several terms most of which cannot be neglected. Instead, a numerical comparison is performed.

In Fig. 6.6, the bounds are in a) displayed for \( q = 1.6 \), in b) for \( q = 3.2 \) and in c) for \( q = 4.5 \). In each part, the bounds for all network types are shown for \( n = 3 \), \( \rho_{min} = 1 \text{ m} \), and for an environment of 20 m length, 4 m height, and a width that varies from 4 to 40 m. The values belonging to the WLAN with fixed access point (\( I_{q,n}^{Wf} \), WLAN type I) are denoted by circles, those which refer to \( I_{q,n}^{W1} \) or \( I_{q,n}^{W2} \) (WLAN type II) by squares, and those which belong to the Ad-Hoc network (\( I_{q,n}^{AH,1} \) or \( I_{q,n}^{AH,2} \)) by triangles. For the first case (low \( q \)), only \( I_{q,n}^{W2} \) is considered for the networks with random position of access point, and for the second and third case (high \( q \)), \( I_{q,n}^{W1} \).

For \( q = 1.6 \), the bounds for the first two types are almost identical. This result is not surprising from a physical, but maybe from a mathematical point of view, since analytically, the respective bounds look quite different. One depends on the volume only, and the other on both volume and surface. The third bound is almost 10 dB lower. However, as this was already observed in comparison with the simulated values (c.f. Fig. 6.5), one can conclude that the mean of this network type is also quite close to the means of the other two. For higher \( q \), the bounds deviate from each other. However, the deviation never exceeds 5 dB for the WLAN with fixed access point and the Ad-Hoc network; the deviation is stronger for \( q = 3.2 \) than for \( q = 4.5 \). For very large \( q \), a deviation proportional to \( \rho_{min} \) should be visible due to the exponents of \( \rho_{min} \) in the last terms in (6.10) and (6.20).
6.5 Complete Characterization of the Lognormal Fading

The bounds derived so far approximate the MPI, i.e., the mean of the square of the received amplitude. This quantity is obtained from averaging
Figure 6.6: Bounds for the MPI for $n = 3$ in dependence on the width $b$ of the domain a) $q = 1.6$ b) $q = 3.2$ c) $q = 4.5$. 
this square of the received amplitude on linear scale. Another aim is, however, the complete analytical characterization of the lognormal fading. The lognormal distribution is a Gaussian on logarithmic scale. Its mean is the mean of the received power in dB, and not on linear scale; the same holds for the standard deviation.

In this section, the already determined bounds are extended to give approximations for the mean and the standard deviation of the lognormal distribution. One can already assume that this extension requires a moment of higher order of \( \frac{1}{\nu^n} \). Fortunately, such a moment is the same function, only with a higher value of the pathloss exponent. However, an additional parameter must be included into this model in order to describe the antenna characteristics. If a suitable expression for the higher moment is found, the remaining task is the straightforward (and well-known) transformation of these moments into the mean and the standard deviation of the lognormal distribution.

For the present case, the lognormal distribution of the amplitude is written as

\[
\mathcal{P}_{\text{logn}}(|\alpha|) = \frac{20}{\ln 10} \frac{1}{\sqrt{2\pi} \sigma_{\text{logn}} |\alpha|} \exp \left( -\frac{(20 \log_{10} |\alpha| - \mu_{\text{logn}})^2}{2\sigma_{\text{logn}}^2} \right).
\] (6.47)

This differs slightly from (2.56). The factor \( \frac{20}{\ln 10} \) is introduced because the density is related to a Gaussian on 'power scale', i.e., with respect to \( 20 \log_{10}(.) \). If it is related to a Gaussian on the scale of the natural logarithm, this factor is dropped.

Written in terms of the received amplitudes \( \alpha = \sum_{\nu} \alpha_{\nu} \), where \( \alpha_{\nu} \) are complex multipath components, one has the empirical finding that

\[
\kappa \left( \frac{1}{r^q} \right) = \langle |\alpha|^2 \rangle = \langle \left| \sum_{\nu} \alpha_{\nu} \right|^2 \rangle,
\] (6.48)

i.e., in the sections above, the second moment of the received amplitude has actually been calculated.

For the derivation of the variance of the received power the fourth moment

\[
\langle |\alpha|^4 \rangle = \langle \left| \sum_{\nu} \alpha_{\nu} \right|^4 \rangle
\] (6.49)

is evaluated. By simply equating (6.48) and (6.49) with the aid of (6.47),
one obtains

$$\mu_{\text{logn}} = 20 \log_{10} e \left( \ln \langle |\alpha|^2 \rangle - \frac{1}{4} \ln \langle |\alpha|^4 \rangle \right) \quad \text{and} \quad (6.50)$$

$$\sigma_{\text{logn}}^2 = (10 \log_{10} e)^2 \left( \ln \langle |\alpha|^4 \rangle - 2 \ln \langle |\alpha|^2 \rangle \right). \quad (6.51)$$

These equations give the mean and the variance of the lognormal distribution in dependence on the second and the fourth moment. The second, (6.48), has been evaluated. In Appendix C, the fourth moment is modeled by

$$\langle |\alpha|^4 \rangle = \kappa^2 \left( \langle \frac{1}{r^{2q}} \rangle + 2 \langle \frac{1}{r^{2q+\tilde{q}}} \rangle \right). \quad (6.52)$$

The result consists of two terms. The first term is caused by the second moment of the received power due to the variation of the distance between $T_x$ and $R_x$. But this is not the only cause for power fluctuations at the $R_x$. The superposition of waves with different phases cause the second term. The correct description of these variations require a characterization of the antenna characteristics. A pencil beam antenna captures only very few waves; variations of the power due to phase variations have minor impact. An antenna with a very broad beam is subject to much higher fluctuations. In the appendix it is demonstrated that this can be described with the aid of a parameter $\tilde{q}$. For the former case, $\tilde{q}$ has very high values, so that the second term of (6.52) is suppressed; for antennas with a broad beam, $\tilde{q}$ has very low values down to 0, so that the phase fluctuations become the dominant source for the variance of the received power.

The advantage of this description is that each term in (6.52) has the form of an expectation of (6.3). Hence, all evaluated bounds can be readily applied, and the lognormal distribution is completely specified. All results that are valid for the derivation of the MPI are applicable to the explicit calculation of (6.50) and (6.51). The different types of bounds yield again several different approximations, which will be investigated in the next chapter.

Discussion:

Even though the calculation of (6.50) and (6.51) is a straightforward application of the bounds for the MPI, the results must be treated with great care. The expression $\langle \frac{1}{r^q} \rangle$ is only approximately known; for the Ad-Hoc
network, it has a singularity for $n = q$, and for the WLAN with unknown position of the access point, it does not exist for $q \geq 2n$. For the calculation of the variance, several values of this type are added and subtracted, and the term $2q + \tilde{q}$, which is required in (6.52), can become quite large.
A brief comparison of the approximations as in Section 6.4 is exemplarily done for the standard deviation, i.e., \((6.51)\). A more detailed investigation is performed later with regard to the simulated and measured channel data in Chapter 7. Values of \(\sigma_{\text{logn}}\) are given in Fig. 6.7 a) with respect to a value of \(q\) of 1.6 and b) \(q = 3.2\), for \(\tilde{q}\) in a large range from 0 to 10 and \(n = 3\). In each part of the figure, the approximation of \(\sigma_{\text{logn}}\) is given by circles for the WLAN with fixed access point, as triangles for that with random access point, and as squares for the Ad-Hoc network. The values are calculated for a domain of 20 m \(\times\) 10 m \(\times\) 4 m with \(\rho_{m\text{in}} = 1\) m.

In Fig. 6.7 a), approximations exist for all network types. The WLAN with fixed access point has the lowest standard deviation. This could be expected, since the fluctuations of the distances are higher if both points can move freely. The difference to the other bounds is quite high (4–5 dB). The similarity shown for the mean values (c.f. Section 6.4) does hence not hold for the variance. The dependence on \(\tilde{q}\) is rather low. It is highest for \(\tilde{q} \in [0; 2]\), and decreases with a lower standard deviation. Note that only one approximation is given for the network types with random position of the access point, even though in principle, two are available, since two bounds exist. However, the bound that has a singularity at \(n = q\) is not applicable here. The first term in \((6.52)\) requires the calculation \(\langle \frac{1}{r_{\tilde{q}}} \rangle\); since \(2q\) is in this case a little higher than \(n\), the expectation is negative and the bound does not exist. Thus, the standard deviations based on this bound are actually not reliable in many cases.

In Fig. 6.7 b), the same bounds are given, but for \(q = 3.2\). A higher value of \(q\) yields a higher standard deviation, since the realizations of \(\frac{1}{r_{\tilde{q}}}\) fluctuate more strongly. The dependence on \(\tilde{q}\) is even lower than for \(q = 1.6\). Note that no bound exists for the WLAN type II with a random position of the access point (noted with N.A. (not applicable) in the figure). The bounds for the MPI of this network type contain the factor \(\langle n - \frac{q}{2} \rangle\) and are not valid for \(q \geq 2n\); consequently, terms of the form \(\langle \frac{1}{r_{\tilde{q}}} \rangle\) cannot exist either. However, the approximations do exist for all \(\tilde{q}\) up to 10. But only the second term of \((6.52)\) depends on both \(q\) and \(\tilde{q}\) and may be smaller than zero. Since this term has a much lower impact due to its higher exponent, it does not disturb the validity of the entire expression.

The dependence of the standard deviation on the dimension \(n\) is low; the approximation of \(\sigma_{\text{logn}}\) for \(n = 2\) yields curves of the same behavior.
which are about 1 dB lower.

6.6 Derivation of an Upper Bound for the Delay Spread

The derivation of the delay spread is based on very simple assumptions, which are investigated in more detail in [54, 55]. Eq. (4.2) is combined with a simplified ray optical approach to calculate the mean of the received power at a particular delay $\tau$; since the MPI will turn out to be a negative exponential, the delay spread is equal to its decay constant and can be extracted from the model.

The key idea is to interpret the delay spread as a quantity that depends mainly on the field strengths and the delay times of the reflected waves; these quantities are basically influenced by the room size and the wall materials, and much less dependent on the absolute position of $T_x$ and $R_x$. Assume that the positions of $R_x$ and $T_x$ are given; their density in (4.2) is then described by Dirac pulses so that the integrals can be readily evaluated. The locations of the virtual sources, which irradiate the reflected waves, are then calculated using the method of images. At zero delay ($\tau = 0$), each virtual source $\nu$ is supposed to transmit a short pulse of width $\tau_{\text{pulse}}$ that needs a delay time $\tau_{\nu}$ until it reaches the $R_x$. The field strength at $R_x$ for a given delay time $\tau$ is finally obtained from the superposition of the field strengths of the total number $N$ of pulses $\nu$ that reach the $R_x$ at this delay. Each pulse is reflected $n_{\nu}$ times at the wall of the chamber, which may have a reflection attenuation $d_E < 1$ that describes the mean attenuation of the electric field. For $\tau_{\text{pulse}} \ll \tau$, the received power $P_{R_x}(\tau)$ per delay time can thus be modeled as the square of the superposition of all attenuated waves $\nu \in \{1, \ldots, N(\tau)\}$, $\tau_{\nu} \simeq \tau$, $r_{\nu} \simeq r$:

$$P_{R_x}(c_0 \tau = r) = \frac{\lambda^2}{8\pi Z_0} \left| \frac{1}{\tau} \sum_{\nu=1}^{N} d_E^{n_{\nu}} E_{\nu} \exp j\varphi_{\nu} \right|^2$$

(6.53)

where $\varphi_{\nu}$ is the phase of the $\nu$th wave, $Z_0$ the free space impedance, $E_{\nu}$ a scalar component of the field strength which is irradiated by the $T_x$, and $c_0$ the velocity of light.

The number of reflections $n_{\nu}$ can be considered as random. For cubic rooms, $n_{\nu}(\tau)$ increases linearly with $\tau$ [54], and it is thus reasonable to
define a random variable $\zeta$ such that

$$n_\nu = \zeta_\nu \tau; \quad (6.54)$$

which has a density $\mathcal{P}(\zeta)$ that depends on the wall-to-wall distances of the chamber only. If one assumes the phases $\varphi_\nu$ to be uniformly distributed, the mean received power per delay time is given by the very general expression [54]

$$\langle P_{R_x}(\tau) \rangle \sim \frac{N(\tau)}{c_0^2 \tau^2} G_P(-2\frac{\tau}{j}d) \quad (6.55)$$

where $G_P(\omega_G) = \int \exp(j\omega_G \zeta) \mathcal{P}(\zeta) d\zeta$ is the characteristic function of $\mathcal{P}$, and $d = \ln(1/d_E)$. This expression states that the slope of a PDP is mainly influenced by three parameters. The first is the number $N$ of rays received within one delay bin, which itself depends on the clutter between $T_x$ and $R_x$ and on the probability that an irradiated wave is backscattered into the room. The second key parameter is the inverse logarithm of the average reflection attenuation $d_E$, and the third is the density $\mathcal{P}$, which depends again on the dimensions of the room.

The number of received rays $N(\tau)$ depends on the environment. In closed structures, the virtual sources that contribute to the received power in one short time bin are located on the surface of a sphere, i.e., $N \sim r^2 \sim c_0^2 \tau^2$. In this case, $N(\tau)$ cancels the free space attenuation, and (6.55) is proportional to $G_P$ only; the power cannot decay more slowly than the slope of this function, since the entire irradiated energy not attenuated due to wall reflections is reflected back into the room. In the case of an open room in which two opposing walls are left out, the virtual sources are located on the edge of a sphere around the room. Thus, $N \sim c_0 \tau$, and the PDP is proportional to $1/(c_0 \tau)G_P$.

In the model, the angle independent reflection coefficient $d_E$ must be derived from the Fresnel reflection coefficients $R_{||}(\theta_\iota, \epsilon_r)$, $R_{\perp}(\theta_\iota, \epsilon_r)$ for parallel and perpendiculary polarized electromagnetic waves incident from an angle $\theta_\iota$ on a surface with complex dielectric constant $\epsilon_r$. As each reflection at an angle $\theta_\iota$ reduces the parallel and the perpendiculary polarized component of the field strength by the absolute values $|R_{||}(\theta_\iota, \epsilon_r)|$ and $|R_{\perp}(\theta_\iota, \epsilon_r)|$, respectively, the geometric mean of these coefficients is a reasonable quantity for an application in an angular independent model if one assumes all reflection angles between $0$ and $90^\circ$ as equally likely. If both
the parallel and the perpendicular component of the field are also equally likely and equally attenuated, a good value of $d_E$ can be calculated using

$$d_E = \frac{1}{2}\left(\langle |R||\langle \epsilon_r \rangle \rangle_{\theta_i} + \langle |R|_{\perp} \langle \epsilon_r \rangle \rangle_{\theta_i}\right), \quad (6.56)$$

with $\langle \cdot \rangle_{\theta_i}$ denoting the mean with regard to $\theta_i$. This mean can be calculated as an arithmetic instead of a geometric mean, since the differences are low and have no impact on the result. This procedure is extremely rough, but it yields very reliable results.

The probability density $\mathcal{P}$ describes the statistics of the number of reflections that the impinging waves have experienced. For a closed cubic chamber, $\mathcal{P}$ may be assumed to be a uniform distribution:

$$\mathcal{P}_u(\zeta) = \begin{cases} \frac{1}{\zeta_{\text{max}} - \zeta_{\text{min}}} & \text{if } \zeta \in [\zeta_{\text{min}}; \zeta_{\text{max}}] \\ 0 & \text{elsewhere.} \end{cases} \quad (6.57)$$

The maximum and the minimum number of reflections per unit delay time in the chamber, $\zeta_{\text{max}}$ and $\zeta_{\text{min}}$, depend only on the chamber’s wall-to-wall distances. For a cubic or almost cubic room with side lengths $a \simeq b \simeq c$, one obtains $\zeta_{\text{max}} = 3c_0/\sqrt{a^2 + b^2 + c^2}$ and $\zeta_{\text{min}} = c_0/\max\{a, b, c\}$. These bounds assume that all virtual sources are active, which is strictly valid only if $T_x$ and $R_x$ have isotropic antenna characteristics. For antennas with a high directivity, this model may not be suitable.

Considering then the mean value theorem of calculus, the evaluation of $G_{\mathcal{P}_u}$ yields

$$G_{\mathcal{P}_u}(-2\tau_d) \sim \exp(-2d\xi_t\tau); \quad (6.58)$$

for the power, one obtains in the mean

$$\langle P_{R_x}(\tau) \rangle = \gamma_0 \exp(-2d\xi_t\tau) \quad (6.59)$$

where $\gamma_0$ is a proportionality constant. For an ideal, very short tunnel with about equal sidelengths $\tilde{a} \simeq \tilde{b}$, one arrives similarly at

$$\langle P_{R_x}(\tau) \rangle = \frac{\tilde{\gamma}_0}{c_0\tau} \exp(-2d\tilde{\xi}_t\tau), \quad (6.60)$$

using a uniform distribution $\mathcal{P}_u$ with bounds $\tilde{\zeta}_{\text{max}} = 2c_0/\sqrt{\tilde{a}^2 + \tilde{b}^2}$ and $\tilde{\zeta}_{\text{min}} = c_0/\max\{\tilde{a}, \tilde{b}\}$. 

Due to the non-linearity of the exponential function, the factor $\xi_\tau$, which contains information about the chamber’s geometry, is not constant; it can be expressed in terms of a cumulant expansion,

$$\xi_\tau = -\frac{1}{2\tau d} \ln G_{\mathcal{P}_u}(\omega_G)|_{\omega_G = -2\frac{\pi}{d}}$$

$$= -\frac{1}{2\tau d} \sum_{\nu=1}^{\infty} \frac{(-2\tau d)^\nu}{\nu!} \kappa_\nu$$

where $\kappa_\nu$ are the cumulants, and it holds that

$$k_1 = \langle \zeta \rangle, \ k_2 = \text{Var}(\zeta)$$

where Var is the variance. For practical purposes, the series can be truncated for $\nu > 2$, so that $\xi_\tau$ is well represented by

$$\xi_\tau = \langle \zeta \rangle - \frac{1}{2} \text{Var}(\zeta) (2d\tau_0)$$

where $\tau_0 = \frac{\tau_{\text{max}}}{2}$ is a good value for the approximation. The quantity $\tau_{\text{max}}$ denotes approximately the maximum expected delay, and its impact on the resulting delay spread is very low.

It has thus been shown that the PDP in a cubic chamber can be modeled by an exponentially decaying function $\exp(-\tau/\sigma_\tau)$ with

$$\sigma_\tau = (2d\xi_\tau)^{-1}.$$ 

The key parameters of this function are the logarithm of the mean reflection attenuation, which actually causes the exponential decay, and the room dimensions, which indicate how often a traveling wave impinges on a wall and is reflected. The first order proportionality of $\xi_\tau$ to the side-length of the room corresponds to the results of [75, 101]; there, simulations demonstrate that the delay spread is proportional to the square root of the floor size. In an open room with two opposing walls missing, the additional radiation loss through the openings is covered by a factor $1/(c_0\tau)$, which leads to higher loss of power. For most indoor cases, however, this additional increase is quite low compared to the exponential decay. This explains why the measured PDPs appear in fact to be exponential. Another consequence is that the delay spread is a very robust quantity. A comparison with measurement results demonstrates later on that the derived model serves as a very good approximation for many propagation
scenarios. Since the application of this model is fairly straightforward, a detailed discussion of the properties of (6.64) is omitted in this section. Further details will be presented in Chapter 7, where the model is compared with ray tracing results and measurements.

6.7 Application of the Model

The result of the present chapter is an analytical expression for the time invariant indoor radio channel of the form

\[ |h(\tau)|^2 = P_{R_x} \exp\left(-\frac{\tau}{\sigma_\tau}\right) \]  

(6.65)

where the random variable \( P_{R_x} \) is approximately lognormal distributed and describes the large scale fading. Since no \( k \)-factor is defined, the small scale fading is Rayleigh distributed. This model is very well-known; the overview of stochastic channel models in Subsection 2.3.3 gives many references to articles which deal with parts of the above models. However, almost all investigations existing so far are empirical; the analytical derivation of (6.65) and even of its individual parts is apparently novel. Since the required derivations are fairly long and since they are based on various approximations and input parameters, this section gives a brief résumé of the application of the model.

A calculation of the key quantities of the stochastic indoor radio channel requires the following steps:

1. Choose a network type: either a WLAN with fixed access point, a WLAN with random position of the access point, or an Ad-Hoc network. Determine the minimum distance \( \rho_{\min} \) between \( T_x \) and \( R_x \), i.e., decide whether user and access point can be in the same cell or whether they may be in adjacent cells. \( R_x \) must be in the far field.

2. Choose the environment in which the system shall operate:
   - Choose its length, width and height.
   - Choose its wave propagation properties in terms of the pathloss exponent \( q \).
3. Calculate the mean and the variance of the lognormal distribution of the received power according to (6.50) and (6.51). These formulae depend on mean values on linear scale; they can be evaluated using (6.10), (6.20), (6.28), (6.45), or (6.46). The formula applied depends on the network type that is considered and, for the network types two and three as defined in Chapter 4, on the two possible bounds. For the calculation of the variance, the specification of a parameter $\tilde{q}$ is required which characterizes the antenna characteristics.

4. Calculate the maximum delay spread according to (6.64).

Once (6.65) is parameterized, the indoor Rayleigh fading radio channel is characterized; now, system or link level simulations can be carried out:

- Draw a random number according to the lognormal distribution. This number specifies the momentarily received power; it corresponds to a particular $T_x, R_x$ configuration in the environment. If the power is high, $T_x$ and $R_x$ are close to each other, or the propagation between them is favorable. If the power is low, they may be far, or the propagation paths are blocked.

- Calculate the PDP with the given delay spread. The bandwidth of the used system now gives taps on which the received power at delay $\tau$ can be calculated with respect to the first path.

- Divide the instantaneous received power into two equal parts. Each part defines now the variance of a zero mean Gaussian distribution of a real and a complex received amplitude. These two distributions characterize the short term fading.

- If the communicating units employ antenna arrays, the model is still applicable under the assumption that the signals at the antennas are uncorrelated but subject to the same large scale fading. For the investigated frequency range of 2.5 to 60 GHz, this would be the case if the antennas are spaced a few wavelengths apart and the array has no more than about 10 wavelengths diameter (cf. Subsection 2.3.3 on small scale and large scale fading).

Eq. (6.65) thus provides a very simple framework within which system simulations for an indoor environment can be carried out. A channel
simulator was implemented with which the described simulation procedure can be easily handled. It is explained in detail in Appendix E. Note, however, that the formulae for the MPI are not only useful to characterize the lognormal distribution function. They can also be applied directly to calculate mean interference power [52].
Chapter 7

Verification of the Analytical Model

In this chapter, the analytically derived results are verified by ray tracing simulations and measurements. The stronger emphasis is on the measurements, since they are a more realistic test for the model. In the context of experimental data, however, statistical accuracy is problematic. Spatially uniformly distributed locations of $T_x$ and $R_x$ require an experimental evaluation of an integral of the forms (4.1) and (4.2) for different domains $\mathcal{D}_{T_x}$ and $\mathcal{D}_{R_x}$. In the case of a WLAN, the measurements are still easy to perform, since one part of the radio link is fixed. For the other two network types, however, both parts of the link are uniformly distributed, so that a reliable experimental investigation requires a great amount of time. The distribution of both $T_x$ and $R_x$ within a given environment results in a vast amount of different configurations. Ray tracing provides a simple method to overcome this problem. A large number of different $T_x$ and $R_x$ combinations can be numerically evaluated, and statistical accuracy is assured. Also, many different frequencies are easily accessible. Hence, the potential inaccuracy of ray tracing due to the idealistic description of the wave propagation is traded off against a higher reliability in statistical terms.

The present chapter is divided into two parts. In the first part, results obtained from ray tracing are presented. In the second, longer part, experimental data is evaluated and discussed. The examined frequencies are 2.4, 5.25, and 60 GHz. The distribution of the pathloss and the MPI obtained from the analytical bounds are examined. Furthermore, the means and standard deviations of the distribution of the pathloss and the delay spread are investigated.
7.1 Verification by Ray Tracing

The ray tracing tool introduced in Section 5.1 is used for simulations at 2.4 and 60 GHz. An Ad-Hoc network and a WLAN with unknown position of the access point are examined. These distributions require a uniformly distributed access point and are thus very impractical for experimental studies. For most of the simulations, either 2048 or 512 realizations of $T_x, R_x$ positions are drawn, the maximum order of reflection is two. For the investigation of the received power, this value is accurate [32]. The proceeding for the delay spread is different. Since the slope of an exponential is investigated, large delays are required. The order of reflection is set to 11; in order to keep the computation time tractable, only single $T_x, R_x$ configurations are calculated; it was ensured that the chosen positions do not influence the slope of the PDP significantly.

7.1.1 Distribution of the Pathloss

The distribution of the pathloss has already been discussed from a geometrical point of view (Section 6.2). It was stated that the distribution of an inverse distance appears lognormal; data which obeys the empirical pathloss formula $\frac{1}{r^q}$ will hence also be lognormal, since on dB scale, the exponent $q$ is only a scaling factor.

In this section, a different, ray tracing approach is taken. The distribution of amplitudes for a WLAN at 60 GHz with the $T_x$ uniformly distributed under the ceiling of a room of $8\,\text{m} \times 8\,\text{m} \times 4\,\text{m}$ size and with the $R_x$ uniformly distributed above the floor of the room, is shown in Fig. 7.1 on dB scale. In a), the histogram of only the direct path is plotted; the direct path is proportional to $\frac{1}{r}$ and thus corresponds clearly to a geometrical distribution as discussed in Section 6.2.

In Fig. 7.1 b), the distribution of the sum of all amplitudes is shown. Again, the result resembles a Gaussian; both its mean and its standard deviation are higher than before. Even though the addition of the amplitudes should result in a Gaussian on linear scale, no strong deviation is found on dB scale, either. This result corresponds to the findings in [97], which proves the asymptotically normal distribution of logarithms of a sum of amplitudes under very general conditions. Note also the similarity of the tail probabilities of Fig. 7.1 b) with those of Fig. 6.1 b).
7.1. Verification by Ray Tracing

Figure 7.1: Distribution of the power at 60 GHz a) direct path only b) superpositions of all paths.

An explanation why the received power seems lognormal distributed can hence also be given by ray tracing. Again, the evidence is no proof, and the deviations of the shown distributions are clearly visible. Better approximations of a Gaussian are actually obtained when both $R_x$ and $T_x$ are uniformly distributed, as in Ad-Hoc networks.
7.1.2 Mean Values and Variances in Ad-Hoc Networks at 2.4 GHz

Simulations are carried out at 2.4 GHz; empty rooms with floor sizes of $30 \times 8$, $30 \times 4$, $20 \times 8$, $20 \times 4$, $12 \times 8$, $8 \times 8$, and $4 \times 4$ m$^2$ are investigated. All rooms are 4 m high. The positions of $T_x$ and $R_x$ are randomly created according to a uniform distribution, the bounds of which are the walls of the room and a height of 0.8 m and 2.0 m above the floor. The building materials are, again, randomly chosen; the distribution corresponds to a rather heavily constructed, old-fashioned office room with 70% concrete, 20% glass, and 10% light concrete building material. The dielectric constants are $\varepsilon_r = 5.5 - j0.10$ for heavy concrete [149], $\varepsilon_r = 5.0 - j0.025$ for glass [139], and $\varepsilon_r = 2.2 - j0.15$ for light concrete [149]. The dielectric constants of the first and the last material are valid for 2.4 GHz, that of glass for 3 GHz. The antennas are isotropic, which corresponds, in the case of linear polarization, to a uniformly distributed orientation of the antennas; this is a realistic assumption for an Ad-Hoc network. Field vectors are transmitted and received either both parallel to the floor of the rooms, or perpendicular to this configuration. However, only the results for the parallel polarization are used in this section; since the results for perpendicular polarization are quite similar, they are omitted. The minimum distance $\rho_{\text{min}}$ is set as $\rho_{\text{min}} = 0.5 \text{ m} \approx 4\lambda$, which is almost the maximum $\rho_{\text{min}}$ allowed for a domain of 1.2 m height. The pathloss model (6.3) is fitted to the result. The obtained pathloss exponents are all between about 1.7 and 1.9. This is realistic, since in environments with a strong direct path and many reflections, the pathloss exponent is known to be lower than 2. The analytical bounds are then calculated using the obtained values for $q$; the volume $V_n$ and the surface area $A_{n-1}$ result from the floor size and the height of the domain in which $R_x$ and $T_x$ can move; as their locations are restricted between 0.8 m and 2.0 m above the floor, this height equals 1.2 m.

In Tab. 7.1 and Tab. 7.2, the ray tracing results are compared to the analytical bounds. Tab. 7.1 displays the MPI. The quantity MPI$^{(s)}$ denotes the simulated MPI, MPI$^{(s)}/n = 3$ and MPI$^{(s)}/n = 2$ are the analytical values for the respective value of the dimension, $n$. The two values in the columns that display the analytical quantities correspond, on the left, to the value derived from $I_{q,n}^{AH,1}$ and on the right, to that derived from $I_{q,n}^{AH,2}$. 
The values are given in dB. Note that the pathloss is given, but that the mean received power is actually calculated. The higher values in the table hence correspond to a lower mean received power.

The validity of the bounds is clearly demonstrated. For the seven investigated scenarios, the simulated power is always within the analytically calculated bounds. For \( n = 3 \), both the lower and the upper bound are about 4 to 5 dB above and below the simulated mean. The bounds for \( n = 2 \) are tighter. They differ only by 0 to 3 dB. This confirms the observation in the context of Fig. 6.4, which shows that for a value of \( q \) below 2, the two dimensional bounds are tighter than the three dimensional ones. The total MPI decreases by about 9 dB for decreasing room sizes; this decrease is the same for the lower bound of \( \text{MPI}^{(a)}/n = 3 \) and for both bounds of \( \text{MPI}^{(a)}/n = 2 \). The upper bound of \( \text{MPI}^{(a)}/n = 3 \) does not follow this decrease; it drops by only 5 dB.

In Tab. 7.2, the means and standard deviations of the lognormal distributions are given. The simulated values are denoted by \( \mu_{\text{logn}}^{(s)} \) and \( \sigma_{\text{logn}}^{(s)} \), the analytical approximations for the corresponding value of \( n \) by \( \mu_{\text{logn}}^{(a)}/n = 3 \) and \( \sigma_{\text{logn}}^{(a)}/n = 3 \), and by \( \mu_{\text{logn}}^{(a)}/n = 2 \) and \( \sigma_{\text{logn}}^{(a)}/n = 2 \), respectively; these values are based on (6.50) and (6.51). Again, all values are given in dB, the left hand analytical value in each column is based on \( I_{q,n}^{AH,1} \), and the right hand one on \( I_{q,n}^{AH,2} \). The simulated values are now obtained from a fit of a lognormal distribution to the simulated data on linear scale, and from a fit of a Gaussian distribution on dB scale. Since both values differ slightly, their mean is taken. The empirical parameter \( \tilde{q} \) was set to 0, which is the minimum value and corresponds to an antenna that can absorb the maximum number of multipath components.

The simulated mean in this table differs by about 5 dB from the mean on linear scale in Tab. 7.1, \( \text{MPI}^{(s)} \), for the largest room, and by about 2.5 dB for the smallest one. The reason for this deviation is that \( \mu_{\text{logn}}^{(s)} \) depends both on the second and the fourth moment of the received amplitudes, as given by (6.50). The analytically calculated values are now less tight than in Tab. 7.1. The MPI derived from the bound \( I_{q,n}^{AH,1} \), which is a lower bound for the given \( q \), gives pathlosses for \( n = 3 \) that are up to 10 dB higher than the simulated value. As before, the two dimensional values are much tighter; they deviate only by 3 to 5 dB. In three dimensions, the values derived from the bound \( I_{q,n}^{AH,2} \) do not always exist; for low \( q \), the
value $2q$ required for the derivation of the fourth moment is too close to the singularity of the bound at $q = 3$, so that the values are either negative or complex. In these cases, the entries in the table are marked with N.A. All simulated standard deviations are between 5 and 7 dB. The existing values are quite close to the simulated ones; the standard deviations for the bound $I_{q,n}^{AH,2}$ do not exist for $n = 3$, due to the reasons mentioned. Again, the values derived for the two dimensional case are closer than those for $n = 3$. The accuracy of the standard deviations is within 3 dB; this is surprising, since their derivation requires the addition and subtraction of lower and upper bounds. It is thus impossible to state whether the derived standard deviations, and also the means on logarithmic scale, are lower or upper bounds. Their accuracy depends on the behavior of the bounds for both the second and the fourth moment.
<table>
<thead>
<tr>
<th>room size [m²]</th>
<th>q</th>
<th>MPI&lt;sup&gt;(s)&lt;/sup&gt;</th>
<th>MPI&lt;sup&gt;(a)/n = 3&lt;/sup&gt;</th>
<th>MPI&lt;sup&gt;(a)/n = 2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>30x8</td>
<td>1.8</td>
<td>51.3</td>
<td>56.1</td>
<td>52.2</td>
</tr>
<tr>
<td>30x4</td>
<td>1.7</td>
<td>50.1</td>
<td>54.1</td>
<td>50.1</td>
</tr>
<tr>
<td>20x8</td>
<td>1.9</td>
<td>50.2</td>
<td>54.5</td>
<td>51.0</td>
</tr>
<tr>
<td>20x4</td>
<td>1.8</td>
<td>48.2</td>
<td>52.4</td>
<td>49.2</td>
</tr>
<tr>
<td>12x8</td>
<td>1.9</td>
<td>48.3</td>
<td>52.5</td>
<td>49.3</td>
</tr>
<tr>
<td>8x8</td>
<td>1.9</td>
<td>47.2</td>
<td>51.0</td>
<td>47.9</td>
</tr>
<tr>
<td>4x4</td>
<td>1.9</td>
<td>42.6</td>
<td>46.7</td>
<td>44.1</td>
</tr>
</tbody>
</table>

Table 7.1: Simulated (s) and calculated (a) MPI at 2.4 GHz. The room height is 4 m, the MPI is given in dB.
<table>
<thead>
<tr>
<th>room size [m²]</th>
<th>q</th>
<th>$\mu_{\text{logn}}^{(s)}$</th>
<th>$\mu_{\text{logn}}^{(a)}/n = 3$</th>
<th>$\mu_{\text{logn}}^{(a)}/n = 2$</th>
<th>$\sigma_{\text{logn}}^{(s)}$</th>
<th>$\sigma_{\text{logn}}^{(a)}/n = 3$</th>
<th>$\sigma_{\text{logn}}^{(a)}/n = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30×8</td>
<td>1.8</td>
<td>56.5</td>
<td>67.1 N.A.</td>
<td>59.9 53.7</td>
<td>6.8</td>
<td>9.7 N.A.</td>
<td>8.2 6.3</td>
</tr>
<tr>
<td>30×4</td>
<td>1.7</td>
<td>55.2</td>
<td>64.2 N.A.</td>
<td>58.1 48.7</td>
<td>7.6</td>
<td>9.3 N.A.</td>
<td>8.0 4.9</td>
</tr>
<tr>
<td>20×8</td>
<td>1.9</td>
<td>54.7</td>
<td>64.7 40.8</td>
<td>58.4 51.4</td>
<td>6.8</td>
<td>9.4 N.A.</td>
<td>8.0 5.8</td>
</tr>
<tr>
<td>20×4</td>
<td>1.8</td>
<td>53.1</td>
<td>61.8 N.A.</td>
<td>56.1 47.3</td>
<td>7.1</td>
<td>9.0 N.A.</td>
<td>7.7 4.6</td>
</tr>
<tr>
<td>12×8</td>
<td>1.9</td>
<td>51.1</td>
<td>61.9 42.4</td>
<td>56.1 48.2</td>
<td>6.0</td>
<td>9.3 N.A.</td>
<td>7.7 5.0</td>
</tr>
<tr>
<td>8×8</td>
<td>1.9</td>
<td>50.3</td>
<td>59.8 42.3</td>
<td>54.2 47.3</td>
<td>5.7</td>
<td>8.7 N.A.</td>
<td>7.4 5.0</td>
</tr>
<tr>
<td>4×4</td>
<td>1.9</td>
<td>45.0</td>
<td>53.7 39.2</td>
<td>49.2 40.0</td>
<td>5.2</td>
<td>7.8 N.A.</td>
<td>6.7 2.1</td>
</tr>
</tbody>
</table>

Table 7.2: Simulated (s) and calculated (a) means and standard deviations of the lognormal distribution at 2.4 GHz. The room height is 4 m, the quantities are given in dB.
7.1.3 Means and Variances in WLANs at 60 GHz

In this subsection, a WLAN with unknown position of the access point is investigated. Again, simulations are carried out with respect to rooms of different size. All simulations are performed in the modern room type that was already investigated in Subsection 5.2.2. All rooms have a height of 4 m. The room sizes decrease, as above, from a floor area of 30 m χ 8 m, via 30 m χ 4 m, 20 m χ 8 m, 20 m χ 4 m, 12 m χ 8 m, 8 m χ 8 m down to 4 m χ 4 m. The room of area 8 m χ 8 m was furthermore investigated for several building materials. In the corresponding tables Tab. 7.3 to Tab. 7.5, 8x8 (m) denotes a simulation for the modern room type also used for all other simulations. The entry (o) denotes results which refer to an old-fashioned room type, which is composed of 20% glass, of 10% aerated concrete, and of 70% concrete. The dielectric constants are chosen as ε = 2.26 – j0.10 for aerated concrete, ε = 6.14 – j0.30 for concrete, and ε = 6.13 – j0.50 for glass [31]. The mark (c) denotes a room purely made of concrete, and (a) one made of aerated concrete. The value of q is again 0. The Tx of the WLAN is randomly located under the ceiling, the Rx randomly between 0.8 and 2.0 m above the floor of the room. For the application of the analytical model, ρ_{\text{min}}=2 m is set.

The results for the MPI are given in Tab. 7.3. The values are much higher now, since the frequency is 60 GHz instead of 2.4 GHz. The fitted values of q are similar. The rooms that have the same size but different wall materials can be distinguished by different values of q; q is generally lower if the wall materials of the room are heavier, i.e., if they reflect electromagnetic waves better. The pathlosses confirm the simulation study of Subsection 6.3.3. Both approximations, I_{q,n}^{Wr,1} and I_{q,n}^{Wr,2}, are lower bounds, and most of them are very loose. The two dimensional bound I_{q,n}^{Wr,2} for n = 2 on the right hand side of the column MPI^{(a)}/n = 2 is the tightest in Tab. 7.3, with a difference of up to 4 dB to the simulated value. I_{q,n}^{Wr,1} for n = 3 is a little worse, I_{q,n}^{Wr,1} for n = 2 has in the mean 8 dB deviation, and the worst is I_{q,n}^{Wr,1} for n = 3. The results are worse in Tab. 7.4, which displays the means and standard deviations. The best approximation here is μ_{\text{logn}}^{(a)}/n = 2 based on I_{q,n}^{Wr,1} with about 10 dB difference (left hand side of the column); μ_{\text{logn}}^{(a)}/n = 3 (also left), based on I_{q,n}^{Wr,1}, has about 20 dB difference; the other bounds are not applicable. The standard deviations, on the other hand, are very accurate. For n = 3,
they differ by 2 to 3 dB, for \( n = 2 \), they are actually very well represented.

The rather bad performance of the approximations for the MPI in Tab. 7.3 makes most of the bounds unsuitable for a calculation of the values on logarithmic scale. A possible solution is to replace these bounds by a related approximation in order to achieve a more reliable performance, as shown in Tab. 7.5. In this table, the approximation \( I^{AH,1}_{q,n} \), developed for the Ad-Hoc network, is used for \( n = 2 \) and for \( \rho_{min} = 2 \) m to reproduce the simulated data for the WLAN. The performance of this bound is much better, even though it is actually based on a different spatial distribution for the user and the access point. This is not surprising, since geometrically, these two systems are similar. Even though the approximations presented here are not necessarily better than the best values in Tab. 7.3 and Tab. 7.4, the application of \( I^{AH,1}_{q,n} \) has the advantage that this bound is defined for all \( q \). Hence, no singularities or any other unreliable ranges of \( q \) must be taken into account.

It has become clear that the bounds and approximations tested in this chapter are not equally suitable. The same observation will be made for the other network types as well. In order to give a better overview of all results, a brief summary of the best approximations is presented in Appendix E.
<table>
<thead>
<tr>
<th>room size [m²]</th>
<th>q</th>
<th>$\text{MPI}^{(s)}$</th>
<th>$\text{MPI}^{(a)}/n = 3$</th>
<th>$\text{MPI}^{(a)}/n = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30×8</td>
<td>1.7</td>
<td>82.1</td>
<td>94.1</td>
<td>86.3</td>
</tr>
<tr>
<td>30×4</td>
<td>1.6</td>
<td>81.3</td>
<td>94.7</td>
<td>85.0</td>
</tr>
<tr>
<td>20×8</td>
<td>1.8</td>
<td>81.4</td>
<td>91.7</td>
<td>84.6</td>
</tr>
<tr>
<td>20×4</td>
<td>1.8</td>
<td>80.1</td>
<td>92.9</td>
<td>83.8</td>
</tr>
<tr>
<td>12×8</td>
<td>1.6</td>
<td>79.6</td>
<td>88.1</td>
<td>81.6</td>
</tr>
<tr>
<td>4×4</td>
<td>1.9</td>
<td>76.1</td>
<td>89.7</td>
<td>78.5</td>
</tr>
<tr>
<td>8×8 (m)</td>
<td>1.9</td>
<td>78.6</td>
<td>88.3</td>
<td>81.6</td>
</tr>
<tr>
<td>8×8 (o)</td>
<td>1.6</td>
<td>78.1</td>
<td>86.8</td>
<td>80.3</td>
</tr>
<tr>
<td>8×8 (c)</td>
<td>1.6</td>
<td>75.4</td>
<td>86.6</td>
<td>80.2</td>
</tr>
<tr>
<td>8×8 (a)</td>
<td>1.8</td>
<td>77.7</td>
<td>88.0</td>
<td>81.4</td>
</tr>
</tbody>
</table>

Table 7.3: Simulated (s) and calculated (a) MPI at 60 GHz. The room height is 4 m, the MPI is given in dB.
<table>
<thead>
<tr>
<th>room size [m²]</th>
<th>q</th>
<th>$\mu_{\text{logn}}^{(s)}$</th>
<th>$\mu_{\text{logn}}^{(a)}/n = 3$</th>
<th>$\mu_{\text{logn}}^{(a)}/n = 2$</th>
<th>$\sigma_{\text{logn}}^{(s)}$</th>
<th>$\sigma_{\text{logn}}^{(a)}/n = 3$</th>
<th>$\sigma_{\text{logn}}^{(a)}/n = 2$</th>
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<tr>
<td>30×8</td>
<td>1.7</td>
<td>85.1</td>
<td>103.6</td>
<td>N.A.</td>
<td>94.3</td>
<td>N.A.</td>
<td>6.7</td>
</tr>
<tr>
<td>30×4</td>
<td>1.6</td>
<td>84.8</td>
<td>105.4</td>
<td>N.A.</td>
<td>99.0</td>
<td>N.A.</td>
<td>7.0</td>
</tr>
<tr>
<td>20×8</td>
<td>1.8</td>
<td>83.4</td>
<td>99.9</td>
<td>N.A.</td>
<td>90.8</td>
<td>N.A.</td>
<td>6.3</td>
</tr>
<tr>
<td>20×4</td>
<td>1.8</td>
<td>82.2</td>
<td>102.1</td>
<td>N.A.</td>
<td>95.3</td>
<td>N.A.</td>
<td>6.1</td>
</tr>
<tr>
<td>12×8</td>
<td>1.6</td>
<td>80.8</td>
<td>95.2</td>
<td>N.A.</td>
<td>87.3</td>
<td>N.A.</td>
<td>4.8</td>
</tr>
<tr>
<td>4×4</td>
<td>1.9</td>
<td>76.6</td>
<td>97.3</td>
<td>N.A.</td>
<td>88.3</td>
<td>N.A.</td>
<td>5.0</td>
</tr>
<tr>
<td>8×8 (m)</td>
<td>1.9</td>
<td>79.4</td>
<td>94.7</td>
<td>N.A.</td>
<td>86.1</td>
<td>N.A.</td>
<td>4.6</td>
</tr>
<tr>
<td>8×8 (o)</td>
<td>1.6</td>
<td>79.3</td>
<td>93.3</td>
<td>N.A.</td>
<td>86.2</td>
<td>N.A.</td>
<td>4.9</td>
</tr>
<tr>
<td>8×8 (c)</td>
<td>1.6</td>
<td>76.8</td>
<td>93.3</td>
<td>N.A.</td>
<td>86.1</td>
<td>N.A.</td>
<td>4.9</td>
</tr>
<tr>
<td>8×8 (a)</td>
<td>1.8</td>
<td>78.4</td>
<td>94.5</td>
<td>N.A.</td>
<td>86.4</td>
<td>N.A.</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Table 7.4: Simulated (s) and calculated (a) means and standard deviations of the lognormal distribution at 60 GHz. The room height is 4 m, the quantities are given in dB.
<table>
<thead>
<tr>
<th>room size [m²]</th>
<th>$q$</th>
<th>$\text{MPI}^{(s)}$</th>
<th>$\mu_{\text{log}}^{(s)}$</th>
<th>$\sigma_{\text{log}}^{(s)}$</th>
<th>$\text{MPI}^{(a)}/n = 2$</th>
<th>$\mu_{\text{log}}^{(a)}/n = 2$</th>
<th>$\sigma_{\text{log}}^{(a)}/n = 2$</th>
</tr>
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<tbody>
<tr>
<td>30×8</td>
<td>1.7</td>
<td>82.1</td>
<td>85.1</td>
<td>6.7</td>
<td>83.1</td>
<td>88.5</td>
<td>6.9</td>
</tr>
<tr>
<td>30×4</td>
<td>1.6</td>
<td>81.3</td>
<td>84.8</td>
<td>7.0</td>
<td>83.1</td>
<td>89.5</td>
<td>7.4</td>
</tr>
<tr>
<td>20×8</td>
<td>1.8</td>
<td>81.4</td>
<td>83.4</td>
<td>6.3</td>
<td>82.4</td>
<td>87.4</td>
<td>6.6</td>
</tr>
<tr>
<td>20×4</td>
<td>1.8</td>
<td>80.1</td>
<td>82.2</td>
<td>6.1</td>
<td>82.8</td>
<td>88.6</td>
<td>7.1</td>
</tr>
<tr>
<td>12×8</td>
<td>1.6</td>
<td>79.6</td>
<td>80.8</td>
<td>4.8</td>
<td>80.0</td>
<td>84.5</td>
<td>6.2</td>
</tr>
<tr>
<td>4×4</td>
<td>1.9</td>
<td>76.1</td>
<td>76.6</td>
<td>5.0</td>
<td>82.2</td>
<td>88.3</td>
<td>7.2</td>
</tr>
<tr>
<td>8×8 (m)</td>
<td>1.9</td>
<td>78.6</td>
<td>79.4</td>
<td>4.6</td>
<td>80.9</td>
<td>85.5</td>
<td>6.3</td>
</tr>
<tr>
<td>8×8 (o)</td>
<td>1.6</td>
<td>78.1</td>
<td>79.3</td>
<td>4.9</td>
<td>79.5</td>
<td>83.8</td>
<td>6.2</td>
</tr>
<tr>
<td>8×8 (c)</td>
<td>1.6</td>
<td>75.4</td>
<td>76.8</td>
<td>4.9</td>
<td>79.5</td>
<td>83.3</td>
<td>6.2</td>
</tr>
<tr>
<td>8×8 (a)</td>
<td>1.8</td>
<td>77.7</td>
<td>78.4</td>
<td>3.6</td>
<td>80.5</td>
<td>84.9</td>
<td>6.2</td>
</tr>
</tbody>
</table>

Table 7.5: Simulated $(s)$ and calculated $(a)$ means and standard deviations based on $I_{q,n}^{AH,1}$ for $n = 2$ (60 GHz). The room height is 4 m, the quantities are given in dB.
7.1.4 Investigation of the Delay Spread

The simulations for the delay spread are also performed at 2.4 and 60 GHz. As explained earlier, $T_x$ and $R_x$ are not randomly distributed within a single room, but fixed. Later, in measurements, delay spreads obtained from $T_x$ and $R_x$ on random positions are compared to the analytical formula. The aim of this subsection is to obtain a systematic overview of the properties of formula (6.64).

The evaluation of the ray tracing data is performed as follows: All values of the simulated channel impulse response are summed up within bins, and the resulting data is averaged by a sliding window which shifts the position of the bins by one tenth of a nanosecond for ten steps. The bins have a width of 1 ns (nanosecond) for small rooms (at least two sides 3 m long); for larger rooms, 3 ns are taken in order to suppress fluctuations of the received power per delay time. An exponential curve is fitted to the resulting data, using the slope of the data and its offset as fitting parameters. The slope serves as an estimate for the delay spread $\sigma^{(s)}_\tau$ and is compared with the delay spread $\sigma^{(a)}_\tau$ calculated from (6.64). It should be noted that this procedure differs from the usual way to calculate the delay spread as the second moment of the data, i.e., as the rms delay spread (root-mean-square delay spread) [92]. A calculation of the second moment of the data gives a delay spread which depends on the antenna gain and the $T_x, R_x$ distance; it differs from the second moment of an exponential distribution in that the latter neither considers the appearance of the LOS component after some initial delay time nor its weight due to any antenna gain.

An example of the PDP of a room of $3 \text{ m} \times 5 \text{ m} \times 3 \text{ m}$ is shown in Fig. 7.2. The estimated PDP is given by the fluctuating graph. The estimated slope of the PDP, $\sigma^{(s)}_\tau$, is obtained from the continuous line. The dotted line is the PDP which results if the purely analytically calculated value $\sigma^{(a)}_\tau$ is substituted into an exponential function. The ray tracing simulations thus verify very well the derived result that the PDP is a negative exponential, and also the applicability of the extracted delay spread $\sigma^{(a)}_\tau$.

In a first investigation, closed and cubic rooms of 3 m sidelong are investigated, i.e., the required assumption of a cubic environment with homogeneous wall material is fulfilled. Eight setups are created in which
the wall material, the antenna gains, and the frequencies are varied. Three materials are chosen: wood \((\epsilon_r = 1.8 - j0.3)\), one-year-old concrete \((\epsilon_r = 5.5 - j0.1\) at about 2.5 GHz and \(\epsilon_r = 6 - j0.3\) at 60 GHz), and a mixture of a good conductor and a dielectric \((\epsilon_r = 5 - j100)\); the first two values are taken from [149], the third from [69]; the fourth is chosen freely in order to investigate a situation with rather low attenuation. Three different antenna types are investigated: an isotropic antenna, a Hertzian dipole, and an antenna that radiates isotropically in the azimuth and that has about 6 dB gain in horizontal direction.

The results are given in Table 7.6. They are based on simulations at 2.4 GHz, except for the last where the carrier frequency is 60 GHz. This last simulation was made to show that the slope of the PDP is roughly independent from the frequency. All analytical values agree within 20% with the value of the slope of the exponential function obtained from the simulation. The differences between the PDPs for different wall materials are clearly expressed by the model. For the moderate antenna gains used, the slope of the exponential function does not differ significantly and thus causes no deviation from the model.

In a second step, four non-cubic scenarios are examined. The carrier frequency is 2.4 GHz, and the wall material is concrete. For the non-
Table 7.6: Calculated and simulated delay spreads $\sigma^{(a)}_r$ and $\sigma^{(s)}_r$ for closed cubic rooms.

cubic scenarios, two sidelengths are constant (3 m), whereas the third is stretched between 4 m and 10 m. Thus, non-cubic rooms up to long corridors are simulated. The results are shown in Table 7.7. It turns out that $\sigma^{(a)}_r \gtrsim \sigma^{(s)}_r$; the difference between the two values increases the more the room length is stretched. Stretching the walls violates the assumption of a uniform distribution $p_u$ in (6.54); adjusting the margins of the distribution in (6.57) would improve the model. Still, $\sigma^{(a)}_r$ is surprisingly accurate even in the case of the corridor of 10 m length, and the results of the simulations suggest that the model is quite robust and works well even if the requirement of equal sidelengths of the room is violated.

Table 7.7: Calculated and simulated delay spreads $\sigma^{(a)}_r$ and $\sigma^{(s)}_r$ at 2.4 GHz for non-cubic scenarios.
7.2 Experimental Verification at 5.25 GHz

7.2.1 Measurement Device and Measurement Procedure

Measurements were performed with a preliminary version of the Channel Sounder PropSound built by Elektrobit [36]. The $T_x$ of the sounder operated with 26 dBm transmit power and sent a PN (pseudonoise) sequence of 511 chips length, with each chip of 10 ns duration. The resulting signal-to-noise ratio was about 40 dB. Both $T_x$ and $R_x$ are equipped with a Rubidium clock which gives a clock signal of 10 MHz. At the beginning of a set of measurements, the two clocks are synchronized with a coaxial cable. After synchronization, each clock runs independently. A detailed study about synchronization and phase noise of the Sounder was performed before the measurements were carried out [84]. A picture of the channel sounder is shown in Fig. 7.3. On the left hand side is the $T_x$, which consists of the radio frequency unit and a notebook with the operating software. On the right hand side is the $R_x$ which is also operated by a notebook. Above, there is the radio frequency unit and below, the data storage unit which consists of two personal computers. The measured data is stored on the computers and can be downloaded via the Internet.

The measurements were performed at a carrier frequency of 5.25 GHz. Broadband dipoles at $T_x$ and $R_x$ were used to ensure that the entire bandwidth of the signal could be transmitted and received. The dipoles were manufactured at the laboratory and have a gain of about 1.7 dB. A picture of one of the dipoles is given in Fig. 7.4. The antennas were mounted on trolleys. During each single measurement, the distance between the antennas, which varied from less than 1 m to some 10 m, was recorded. If not otherwise stated, the $T_x$ had a height of 1.3 m, and the $R_x$ was mounted right underneath the ceiling. A single measurement took 2 s, within which the $T_x$ antenna was moved randomly within a circle of about 40 cm diameter; during this time, 30 snapshots of the channel were taken. Each value of the received power is a mean of these snapshots. It was ensured that this procedure yields results which are reproducible with an error of less than about 1-2 dB. The orientation of both dipoles was such that the long, blind axis of the dipoles was parallel to the $y$-axis of a coordinate system given below on the site maps. This orientation guaranteed
that for all measurements with long distances, the maxima of the antenna characteristics pointed to each other. From all data, the pathloss and the rms delay spread were calculated.
7.2.2 Measured Environments and Data Evaluation

Measurements were carried out in two different buildings. In the first, a pre-world war II construction with rather solid inner walls made of brick, two different locations were investigated. Fig. 7.5 and Fig. 7.6 show the respective site maps. The first (Fig. 7.5) is part of the F-floor of the ETF building, which belongs to the Department of Information Technology and Electrical Engineering of ETH Zurich. The floor has several office rooms with an adjacent corridor and a restroom forming its end. The corridor has a length of 15.5 m and is 3.6 m wide, all rooms are 6 m long and vary from 3.4 m to almost 8 m in width. The floor is 3.1 m high. The second location (Fig. 7.6) consists of two rooms in the E-floor (one floor below the F-floor) which are 8.8 m and 3.2 m long, and both 6 m wide and 3.1 m high. The adjacent second building, the ETZ, is a modern building with a very different structure. Measurements were made in the K-floor at the top of the building, where corridors are lined with large metal lockers. The ceiling, which is also covered with thin metal panels, is much lower. A site map of the location is given in Fig. 7.7. Measurements were made in two different corridors. The shorter one is about 20 m long, 2.4 m high, and 2.3 to 4.3 m wide between the lockers. The longer one has the same height, is about 50 m long, and between 2.6 and 4.6 m wide.
In all locations, a regular lattice of points was marked on the floor. On the F-floor, the points had a distance of 2.0 m in the corridor and 1.5 m in the rooms. On the E-floor, they had a distance of 1.5 m in the smaller, and 1.75 m in the larger room. In the ETZ on the K-floor, the distance was 1.4 m. A test measurement in a single room with a much higher density of lattice points had been carried out before to show that the points were sufficiently dense to allow the calculation of the MPI by averaging the values of power obtained from all points. Note that this measurement technique is actually equivalent to experimentally performed numerical integration. Since distributions and their mean values shall be obtained from the measurements, the spatial distribution of the user is simply sampled and evaluated for all sampling points.

On the F-floor, three different $R_x$ positions, all just below the ceiling, were chosen. The first, denoted by $F^{R1}$ in Fig. 7.5, was in the middle of the corridor, the second, $F^{R2}$, at one end of the corridor, and the third, $F^{R3}$, in room F110. For each $R_x$ position, the pathloss was measured for the $T_x$ at all lattice points. This setup corresponds to a WLAN with three different choices for the access point and with randomly distributed users.
Subsequently, the measurements were grouped. One group contains all measurements with the $T_x$ in all rooms ($F^R_{\text{rooms}, \nu = 1, 2, 3}$) except F109, which was not accessible. The next group comprises all measurements with the $T_x$ in the corridor ($F^{R_{\text{corr}}}_{\nu}$), and the third combines all $T_x$ positions ($F^R_{\text{all}}$). There are physical reasons for grouping the results in such a way which will become clear in the discussion below. The total number of measured $T_x$, $R_x$ positions was 129.

On the E-floor, the following setups were measured: For each $T_x$ position, the $R_x$ was located in the middle of each room (1 or 2) right underneath the ceiling. The procedure is thus similar to that in the F-floor and resembles a WLAN with two different choices of access points and randomly distributed users. As the lattice in the larger room had 15 points and that in the smaller room eight, a total of $23 \times 2 = 46$ measurements were carried out, which are denoted by $E^R_{\nu_1}$ for the $R_x$ in room $\nu_1$ and the $T_x$ in room $\nu_2$. In a second measurement, an Ad-Hoc network as defined in Subsection 6.3.2 by a uniform distribution of both $T_x$ and $R_x$, was investigated. The Ad-Hoc network was confined to each of the rooms, and the $R_x$ was mounted 1 m above the floor. Due to the random distribution of $T_x$ and $R_x$, a total of $15 \times 15 = 225$ single measurements resulted in the larger room E112 ($E_1$) and $8 \times 8 = 64$ measurements in the smaller one, E113 ($E_2$).
In the modern building, four different setups were investigated. The first, \((Z_0)\), was within a rather short section of the corridor, in a domain of 7 m length. In this setup, a WLAN with a random distribution of the access point was measured. The \(T_x\) antenna was at a height of 1.2 m randomly distributed on a square lattice with 12 points of 1.4 m distance; the \(R_x\) was also uniformly distributed on this lattice, but at a height of 2.2 m underneath the ceiling, and of 1.2 m at the walls. A total number of 166 measurements resulted.

In the same corridor, a measurement for a WLAN with fixed access point was performed \((Z^{R1})\). The \(R_x\) was again fixed at 2.2 m underneath the ceiling in the middle of a domain of about 20 m length. The \(T_x\) was moved in steps of 1.4 m along 15 points. The same kind of measurements, with the same height of \(T_x\) and \(R_x\), was performed twice in a very long corridor of 50 m length, where 37 points of each 1.4 m distance were measured along a line. In the first of these measurements, the \(R_x\) was at one end \((Z^{R2})\), in the second in the middle of this domain of 50 m length \((Z^{R3})\).

A plot with typical results is shown in Fig. 7.8. As usual, a log-log plot is chosen, since in such a scale, the pathloss model \((6.3)\) yields lines with a slope of \(q\); both axes are scaled by \(20 \log_{10}\). In a), the measurement results of \(FR_2\) are displayed, on the right hand side, those of \(Z_0\). The measurement \(FR_2^{all}\) contains locations of the \(T_x\) in all rooms, i.e., F106, F107, F108, and F110, and the locations of the \(T_x\) in the corridor. In the plot, these different positions are marked with different symbols, which are given in the legend to the plot. Two groups can be distinguished. One group, the circles, belongs to \(T_x\) being in the corridor, and has a rather low slope. The second, with \(T_x\) in the rooms, has different but also roughly constant slope. The slope is given by the lines which are least square fits of the pathloss model \((6.3)\) to the data. The respective pathloss exponents are \(q = 1.4\) for \(FR_2^{corridor}\) and \(q = 4.3\) for \(FR_2^{rooms}\). Two different regions of constant pathloss exponents must be distinguished. These regions are found for all \(R_x\) positions in the F-floor. Wave propagation through the walls of the rooms follows a different \(q\) than wave propagation through the corridor, but \(q\) is still constant. The above-mentioned grouping of the results is a consequence of these findings.

In Fig. 7.8 b), the \(Z_0\) measurement with a random distribution of both
the $T_x$ and the $R_x$ is shown. The result is very typical for all measurements with random distribution of the $R_x$, be it a WLAN or an Ad-Hoc network. The measured pathlosses are much more scattered compared to measurements with fixed access point. This is due to the fact that for each distance $r$, many more configurations are measured, since both $T_x$ and $R_x$ are randomly distributed. Also, the antenna directivity plays an important role. The antennas were oriented such that for long distances, the maxima of the beam patterns were directed towards each other; for short distances, this is the only case if $T_x$ and $R_x$ are in a line parallel to the $x$ axis; if they are in the same distance but parallel to the $y$-axis, the direct path is in the null of their pattern, and the received power drops significantly. The variance of the points at $20 \log_{10}(r) = 0$, which corresponds to the measurements at 1 m distance, is also striking. From free space propagation, a pathloss of 47 dB is expected. The fluctuations already indicate a strong fading in such a close distance to the $T_x$. For the measurements $E_1$ and $E_2$ performed in the old building, pathlosses were also recorded in 0.3 m distance to the $T_x$. These pathlosses were stable within 2 dB and correspond closely to the value expected from free space propagation. A partition into several regions of constant $q$ was not found for these measurements. The environments considered are locations within a single room or a corridor and hence too simple to find regions of different pathlosses $q$.

7.2.3 Distribution of the Pathloss

The experimentally obtained distributions of the pathloss as obtained from $F_{corr}^{R1}$ and from $F_{P110}^{R1}$ are shown in Fig. 7.9 a), b) and c), d), respectively. The linear scale is displayed in a) and c), the dB-scale in b) and d). A log-normal distribution is fitted to the histograms on linear, and a Gaussian to that on dB-scale (continuous lines in the plots). The fits work equally well. The distributions can indeed be approximated by a lognormal, which gives evidence to the considerations in Section 6.2 and Subsection 7.1.1. Their mean, however, is quite different. One can readily conclude that a distribution of the $T_x$ in both the corridor and the room F110 will not yield a single Gaussian distribution, but rather a superposition of two, i.e., a bimodal Gaussian, which is described by their two independent means and variances. The performed measurements indicate that the assumption
of a Gaussian distributed pathloss is hence not valid when the measured statistics is dominated by a particular scenario which has, for instance, much higher amplitudes than all other parts of the investigated environ-
7.2. Experimental Verification at 5.25 GHz

Figure 7.9: Distribution of the pathloss. a) $F^{R1}_{\text{corr}}$ linear scale b) $F^{R1}_{\text{corr}}$ dB scale c) $F^{R1}_{F110}$ linear scale d) $F^{R1}_{F110}$ dB scale.

The Gaussian distribution on dB scale works well for the scenarios that include either the rooms or the corridor for $F^{R1}$ and $F^{R2}$. For $F^{R3}$, the assumption is not valid. For $F^{R3}_{\text{rooms}}$, there is a Gaussian for the $T_x$ in F110 which has a low mean value, and another with a high mean pathloss, for the $T_x$ in F108 and F107. The resulting curve is a bimodal Gaussian. For $F^{R3}_{\text{corr}}$, the same accounts for the two domains in which the $T_x$ is close to the door of F110 and in LOS with the $R_x$, and for the rest of the measurements, which have very low amplitudes. The resulting distribution resembles a Gaussian, but has a large bump at the side of lower pathloss.
and would be better characterized by two Gaussians. In the case of the $F^{R1}$ and $F^{R2}$, the amplitudes are more visibly grouped around a single mean. In the case of the corridor this was explained above, and in the case of the rooms, the measurements show that the mean values of the Gaussians within each room differ by less than 10 dB, and the decay of the mean pathloss for neighboring rooms is rather low. For the scenarios which include all $T_x$ positions, a single Gaussian cannot be found either.

From the measurements carried out on the E-floor, similar conclusions can be drawn. Within single rooms, the mean pathloss can be well described by a Gaussian; if the scenario includes two adjacent rooms, the results can get distorted. In the case of the Ad-Hoc networks, a very neat Gaussian results if the minimum distance between $T_x$ and $R_x$ is $\rho_{min} = 1.5$ m. The inclusion of data measured at $\rho_{min} = 0.3$ m distance leads to a very high, Dirac shaped peak which strongly effects the mean value. However, ray tracing results indicate that this is rather due to the low sampling density of the measurements in the vicinity of the $R_x$. Speaking in terms of numerical integration, the slope of the function $\frac{1}{r^q}$ for small $r$ is so steep that the applied sampling density of $r \approx 1.5$ m is not sufficient for a reliable evaluation of the integral. If a sufficiently high number of measurements was taken in this area, an unimodal Gaussian would most likely be found.

In all measured locations of the ETZ, actually very nice Gaussians were found. All measurements were performed within corridors, i.e., a single room environment. Within such an environment, $q$ is constant, and according to Section 6.2 and Subsection 7.1.1, the resulting distribution can be well approximated by a Gaussian on logarithmic scale.

### 7.2.4 Comparison of the Means and Variances of the Pathloss

In order to apply the derived formulae for the MPI, the power-to-distance law (6.3) for the given frequency of the measurements is fitted to the various subsets of the data in order to obtain the pathloss exponents $q$. Since each subset represents a particular domain in which $T_x$ and $R_x$ are located, the corresponding values for $V_n$ and $A_{n-1}$ can be calculated; they are in fact directly obtained from the height, the length and the width from, for instance, a building map; the values do not need to be very precise, as
the formulae are robust against small variations in the room or building dimensions. For the WLAN measurements, relation (6.10) is applied; for the Ad-Hoc network measurements, relation (6.20) is taken for the bound $I_{\text{Ann}}^{\text{AH},1}$, and (6.28) for the bound $I_{\text{Ann}}^{\text{AH},2}$. For the WLAN measurements with random position of the access point, relation (6.45) is used for $I_{\text{Ann}}^{\text{WR},1}$, and (6.46) for $I_{\text{Ann}}^{\text{WR},2}$. The value of $\bar{q}$ is set to 1. Both $n = 2$ and $n = 3$ are used. In the case of $n = 2$, the condition $2\rho_{\text{min}} < \min(a, b, c)$ (6.16), which is necessary for the application of the formulae for $q > n$, is always fulfilled. If the minimum distance between $T_x$ and $R_x$ exceeds 1.5 m, the formulae cannot be applied for $n = 3$, since the height of both floors is about 3 m and thus $\rho_{\text{min}} < 1.5$ m. In two dimensions, however, the domain of $T_x$ and $R_x$ is approximated as flat, and the maximum of $\rho_{\text{min}}$ depends only on the length and the width of the domain. Whenever $2\rho_{\text{min}}$ is too large for the application at $n = 3$, the value N.A. is stated in the following tables.

In Tab. 7.8 to Tab. 7.11, the investigated setup, the fitted value of the pathloss exponent $q$, the measured and the analytically calculated MPls, and the means and standard deviations of the lognormal distribution are shown. The tables are divided into the investigated network type and into the results for the MPl and the characteristics of the lognormal distribution. The quantity $\text{MPl}^{(m)}$ denotes the experimentally obtained MPl, which is compared to the analytical values. The experimentally obtained mean of the lognormal distribution is $\mu_{\text{logn}}^{(m)}$. The same tables also contain the standard deviation, where $\sigma_{\text{logn}}^{(m)}$ denotes the measured expressions. The analytical values are here based on (6.50) and (6.51).

The values for the WLAN measurements with fixed access point are given in Tab. 7.8 and Tab. 7.9. The displayed results correspond to scenarios which cover all values of $q$ between 1.4 and 4.3, the measured MPls vary between about 53 dB and almost 84 dB. The analytically calculated values for the linear scale, Tab. 7.8, resemble the measurements very well. Half of the values deviate by 3 dB or less. The strongest deviation is 7.5 dB for $F_{all}^{\text{PI}}$. High pathlosses tend to be met worse than low pathlosses. This is self-evident, since the variance of the pathloss is higher at high $q$, and any value, experimental or analytical, is more sensitive to the input parameters and thus more vulnerable to deviations. As in the simulations, $\text{MPl}^{(a)}/n = 3$ and $\text{MPl}^{(a)}/n = 2$ hardly differ by more than 1 dB. On long floors, the models for $n = 2$ and $n = 3$ are nearly identical. Since the model
for \( n = 2 \) has no restrictions with regard to the height of the domain, it is far more preferable.

In Tab. 7.9, the means and standard deviations of the lognormal distribution are given. As in Section 7.1, the analytical results are less precise on dB scale, since their derivation requires the addition and subtraction of bounds. The deviation between the measured and the calculated means is about 4 dB. Again, the values for \( n = 2 \) and \( n = 3 \) are very similar; in this table, they differ by up to 3 dB. The accuracy of the standard deviation is surprising. In more than 50% of the cases, the mismatch between formula and experiment is equal or below 3 dB; the deviation is systematic.

In some cases, however, the mismatch is quite large: \( E_{R_1}^{T_1+T_2}, E_{R_2}^{T_1+T_2}, F_{rooms}, F_{cor}, F_{all}, F_{all}, \) and \( Z^{R_2} \). For the measurements on the E- and the F-floor, the explanation was given in Subsection 7.2.3. These measurements do not obey a unimodal, but a bimodal Gaussian distribution. The experimentally obtained standard deviation is quite large, since it extends over two neighboring bell curves. The analytical model, however, assumes a unimodal distribution. The large standard deviation of \( Z^{R_2} \) is possibly due to the location of the access point right at one end of the corridor, and not, as in \( Z^{R_3} \), in the middle. Since the model for \( I_{q,n}^{W_r} \) is an upper bound for any position of the access point, the analytical results are similar for both cases, but the measured pathlosses for \( Z^{R_2} \) and \( Z^{R_3} \) differ. The MPI for the former is 6 dB, and its variance 4 dB higher, even though the measurements are performed in the same environment. A possibility to improve the values is to change the dimensions of the domain, as described in Subsection 6.3.1. However, the suggested solution improves the calculated MPI only by 2, and the variances only by 0.5 dB.

Due to the very time-consuming procedure, only three scenarios for the Ad-Hoc network and the WLAN with random position of the access point were investigated by measurements. They are presented in Tab. 7.10 and Tab. 7.11. From the results in Chapter 6 and Section 7.1 one expects the results for the latter to be bad. However, it was demonstrated that due to the physical similarity of the two network types, the approximations could be replaced by the results derived for the Ad-Hoc network. In Tab. 7.10, the means obtained on linear scale are displayed. In the first row, \( Z_0 \), the measurement for the WLAN with random position of the access point, is compared to the bounds \( I_{q,n}^{W_r,1} \) (left part of each row) and \( I_{q,n}^{W_r,2} \) (right part
of each row) for both $n = 3$ and $n = 2$. All bounds are lower bounds, and 
$\text{MPI}^{(a)}/n = 3$ based on $I_{q,n}^{W_r,1}$ deviates most strongly (8 dB difference), 
followed by $\text{MPI}^{(a)}/n = 2$ based on $I_{q,n}^{W_r,1}$ (5 dB), then by $\text{MPI}^{(a)}/n = 3$ 
based on $I_{q,n}^{W_r,2}$ (2 dB); the best result is $\text{MPI}^{(a)}/n = 2$ derived from $I_{q,n}^{W_r,2}$ 
(1 dB difference). The behavior here is the same as in Tab. 7.3. The 
bounds derived for the Ad-Hoc network, $E_1$ and $E_2$, are much tighter. 
For $n = 3$, the lower bound $I_{q,n}^{W_r,1}$ is in fact a lower one for $E_1$ and quite 
close to the measured value for $E_2$. The upper bound $I_{q,n}^{W_r,2}$ is an upper 
bound for $E_1$; it is not applicable for $E_2$, since $q$ is too close to $n = 3$. The 
value of the bound is negative, hence complex on dB scale. For $n = 2$, the 
bounds are tighter, as in the simulations. For $E_1$, both values are a lower 
bound, since $q > n$ (c.f. (6.20) and (6.28)). For $E_2$, the left-hand value 
is an upper bound (since $q > n + 1$), and the value on the right a lower 
bound (since $q > n$). The latter is thus slightly to high. But regarded 
as an approximation, the bounds are very tight. Like in Subsection 7.1.3, 
the MPI for $Z_0$ is also approximated by the bounds $I_{q,n}^{AH,1}$ and $I_{q,n}^{AH,2}$. 
The result is shown in the last row of the table. As in the case of the 
simulations, all analytical values now approximate the measured ones with 
4 dB error or better.

The means and standard deviations of the lognormal distribution are 
compared in Tab. 7.11. The results are again comparable to those of the 
ray tracing simulations. In the first row, the results for the WLAN are 
hardly met. Like in Tab. 7.4, many of the approximations do not exist. 
In particular, none of the bounds for $n = 2$ is applicable, since for $q = 1.7$ 
and $\bar{q} = 1$, one has $2q + \bar{q}$ larger than $2n$, and the corresponding bound 
has dropped to $-\infty$ (Subsection 6.3.3).

The results for the Ad-Hoc network measurements are also similar to 
those in Tab. 7.2. The values for $n = 3$ describe the experimental results 
only very loosely; those for $n = 2$ are much better and within a deviation 
of 4 dB. For $n = 3$, the values based on $I_{q,n}^{AH,2}$ do not exist for $q = 3.4$, 
since $q$ is too close to $n$. The rather high variance of the measurements 
is also well reproduced, except for the result based on $I_{q,n}^{AH,2}$ for $n = 3$. 
If the analytical formulae for the Ad-Hoc network are applied to $Z_0$, the 
results are much better. For $n = 2$, the analytically obtained values are 
within 5 dB of the measured ones, and the standard deviation is well met.
<table>
<thead>
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<th>Location</th>
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<th>( \text{MPI}^{(a)}/n = 3 )</th>
<th>( \text{MPI}^{(a)}/n = 2 )</th>
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<td>56.6</td>
<td>53.9</td>
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Table 7.8: Measured \((m)\) and calculated \((a)\) MPVs at 5.25 GHz, given in dB.
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<th>$\mu_{\text{log}}^{(a)}/n = 2$</th>
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<td>55.5</td>
<td>56.5</td>
<td>5.9</td>
<td>4.4</td>
<td>4.8</td>
</tr>
<tr>
<td>$E_{T2}^R$</td>
<td>1.4</td>
<td>56.6</td>
<td>54.4</td>
<td>55.2</td>
<td>5.4</td>
<td>4.2</td>
<td>4.5</td>
</tr>
<tr>
<td>$F_{\text{corr}}$</td>
<td>4.2</td>
<td>89.0</td>
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<td>89.0</td>
<td>10.0</td>
<td>N.A.</td>
<td>5.9</td>
</tr>
<tr>
<td>$F_{\text{corr}}$</td>
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<td>84.1</td>
<td>N.A.</td>
<td>79.7</td>
<td>7.5</td>
<td>N.A.</td>
<td>5.5</td>
</tr>
<tr>
<td>$F_{\text{rooms}}$</td>
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<td>86.4</td>
<td>N.A.</td>
<td>79.4</td>
<td>7.5</td>
<td>N.A.</td>
<td>4.5</td>
</tr>
<tr>
<td>$F_{\text{rooms}}$</td>
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<td>74.0</td>
<td>68.0</td>
<td>69.7</td>
<td>16.4</td>
<td>7.7</td>
<td>7.7</td>
</tr>
<tr>
<td>$F_{\text{all}}$</td>
<td>3.5</td>
<td>71.8</td>
<td>72.4</td>
<td>72.6</td>
<td>15.3</td>
<td>7.5</td>
<td>8.2</td>
</tr>
<tr>
<td>$F_{\text{all}}$</td>
<td>3.3</td>
<td>73.8</td>
<td>69.2</td>
<td>71.6</td>
<td>15.8</td>
<td>7.2</td>
<td>8.0</td>
</tr>
<tr>
<td>$F_{\text{all}}$</td>
<td>3.9</td>
<td>81.7</td>
<td>72.6</td>
<td>74.4</td>
<td>9.2</td>
<td>8.0</td>
<td>8.5</td>
</tr>
<tr>
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<td>52.2</td>
<td>55.8</td>
<td>6.6</td>
<td>4.3</td>
<td>4.7</td>
</tr>
<tr>
<td>$Z_{T1}^R$</td>
<td>1.6</td>
<td>65.8</td>
<td>57.1</td>
<td>59.7</td>
<td>10.5</td>
<td>4.6</td>
<td>5.4</td>
</tr>
<tr>
<td>$Z_{T1}^R$</td>
<td>1.4</td>
<td>59.4</td>
<td>56.0</td>
<td>58.3</td>
<td>7.1</td>
<td>4.3</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Table 7.9: Measured (m) and calculated (a) means and standard deviations of the lognormal distribution at 5.25 GHz. The values are given in dB.
<table>
<thead>
<tr>
<th>Location</th>
<th>$q$</th>
<th>$\text{MPI}^{(m)}$</th>
<th>$\text{MPI}^{(a)}/n = 3$</th>
<th>$\text{MPI}^{(a)}/n = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_0$</td>
<td>1.7</td>
<td>53.9</td>
<td>62.3</td>
<td>59.5</td>
</tr>
<tr>
<td>$E_1$</td>
<td>2.4</td>
<td>50.9</td>
<td>55.5</td>
<td>45.6</td>
</tr>
<tr>
<td>$E_2$</td>
<td>3.4</td>
<td>48.8</td>
<td>48.5</td>
<td>N.A.</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>1.7</td>
<td>53.9</td>
<td>57.2</td>
<td>50.8</td>
</tr>
</tbody>
</table>

Table 7.10: Measured ($m$) and calculated ($a$) MPVs at 5.25 GHz, given in dB.

<table>
<thead>
<tr>
<th>Loc.</th>
<th>$q$</th>
<th>$\mu_{\text{log}}^{(m)}$</th>
<th>$\mu_{\text{log}}^{(a)}/n = 3$</th>
<th>$\mu_{\text{log}}^{(a)}/n = 2$</th>
<th>$\sigma_{\text{log}}^{(m)}$</th>
<th>$\sigma_{\text{log}}^{(a)}/n = 3$</th>
<th>$\sigma_{\text{log}}^{(a)}/n = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_0$</td>
<td>1.7</td>
<td>53.7</td>
<td>68.6</td>
<td>N.A.</td>
<td>6.5</td>
<td>7.3</td>
<td>N.A.</td>
</tr>
<tr>
<td>$E_1$</td>
<td>2.4</td>
<td>59.3</td>
<td>68.0</td>
<td>48.1</td>
<td>63.1</td>
<td>63.9</td>
<td>8.3</td>
</tr>
<tr>
<td>$E_2$</td>
<td>3.4</td>
<td>59.7</td>
<td>60.2</td>
<td>N.A.</td>
<td>57.7</td>
<td>57.7</td>
<td>12.1</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>1.7</td>
<td>53.7</td>
<td>63.2</td>
<td>N.A.</td>
<td>59.0</td>
<td>48.8</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Table 7.11: Measured ($m$) and calculated ($a$) means and standard deviations of the lognormal distribution at 5.25 GHz, given in dB.
7.2.5 Comparison of the Delay Spread

The derivation of $\sigma^{(a)}_\tau$ is based on a single room environment. It is a loose upper bound for an environment with many rooms and walls, since any inner structure attenuates the received field and decreases $\sigma_\tau$. In order to provide a better explanation of the tightness of the analytical bound, not only the measured mean rms delay spread, $\sigma^{(m)}_\tau$, is given in this subsection, but also the mean value plus one standard deviation, $\bar{\sigma}_\tau^{(m)}$. The maximum of the rms delay spread, which would be the intuitive candidate for a comparison with $\sigma^{(a)}_\tau$, is not used, since it is highly fluctuating and does not have much statistical significance. The dielectric constant used for the calculation of $\sigma^{(a)}_\tau$ is $\varepsilon_r = (5.5 - j0.1)$, which is that of concrete at 5 GHz [149]. The value of $\tau_{max}$ is chosen as 100 ns.

The measured mean rms delay spread varies between about 7 and 25 ns (Tab. 7.12). The analytical values are here in fact a safe upper bound. The match between the theoretical and the experimental values is better if the investigated scenario has less inner partitions. This finding corresponds to the assumptions required for the derivation. For a number of cases, $\sigma^{(a)}_\tau$ coincides well with $\bar{\sigma}_\tau^{(m)}$. However, if the structure of the investigated scenario becomes too complicated or deviates too much from a cube, the analytical value is even larger than this quantity, as the results for $F^{R_{rooms}}_{\nu}$ and $F^{R_{all}}_{\nu}$ for $\nu = 1, 2, 3$ and $Z^{R_{\nu}}, \nu = 2, 3$ indicate. Further details can be found in [48].

All different bounds and approximations have been tested against simulations and measurements in the present chapter. They provide a complete analytical description of the Rayleigh fading stochastic indoor radio channel. The expressions are of varying accuracy. Some are surprisingly exact, some are useless for practical work. In order to simplify the application of the model, a channel simulator has been implemented. This tool relies only on the most suitable approximations; these are, together with a short description of the interface of the simulator, compiled in Appendix E. A summary of the important analytical derivations and the experimental results can be found in [56].
### Table 7.12: Comparison between measured (m) and analytically calculated (a) delay spreads, given in ns (5.25 GHz).

<table>
<thead>
<tr>
<th>Location</th>
<th>$\sigma_{\tau}^{(m)}$</th>
<th>$\sigma_{\tau}^{(m)}$</th>
<th>$\sigma_{\tau}^{(a)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{T1}$</td>
<td>9.3</td>
<td>11.4</td>
<td>12.0</td>
</tr>
<tr>
<td>$E_{T2}$</td>
<td>6.6</td>
<td>7.9</td>
<td>9.0</td>
</tr>
<tr>
<td>$E_{T1+T2}$</td>
<td>11.7</td>
<td>20.3</td>
<td>16.2</td>
</tr>
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<td>13.8</td>
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<td>$F_{corr}$</td>
<td>11.6</td>
<td>17.7</td>
<td>18.3</td>
</tr>
<tr>
<td>$F_{corr}$</td>
<td>9.0</td>
<td>12.3</td>
<td>18.3</td>
</tr>
<tr>
<td>$F_{corr}$</td>
<td>10.0</td>
<td>18.1</td>
<td>18.9</td>
</tr>
<tr>
<td>$F_{rooms}$</td>
<td>12.7</td>
<td>20.1</td>
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</tr>
<tr>
<td>$F_{rooms}$</td>
<td>11.8</td>
<td>20.5</td>
<td>26.3</td>
</tr>
<tr>
<td>$F_{rooms}$</td>
<td>9.5</td>
<td>12.6</td>
<td>26.3</td>
</tr>
<tr>
<td>$F_{all}$</td>
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<td>15.7</td>
<td>27.0</td>
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<tr>
<td>$Z_{R1}$</td>
<td>13.8</td>
<td>18.1</td>
<td>22.8</td>
</tr>
<tr>
<td>$Z_{R2}$</td>
<td>25.5</td>
<td>37.7</td>
<td>53.0</td>
</tr>
<tr>
<td>$Z_{R3}$</td>
<td>15.6</td>
<td>25.0</td>
<td>53.0</td>
</tr>
<tr>
<td>$E_1$</td>
<td>9.4</td>
<td>12.5</td>
<td>12.0</td>
</tr>
<tr>
<td>$E_2$</td>
<td>6.8</td>
<td>8.6</td>
<td>9.0</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>11.8</td>
<td>16.0</td>
<td>22.8</td>
</tr>
</tbody>
</table>
Chapter 8

Conclusion

8.1 Results

A novel approach to stochastic channel modeling is introduced. It divides radio channel modeling into the study of wave propagation for a given communication system in given surroundings, and the characterization of the random locations of the transmitter ($T_x$) and the receiver ($R_x$) of the communication system and the surroundings themselves. This approach is then used to derive an indoor radio channel model in the range of 2.5 to 60 GHz.

The thesis begins with an overview of wave propagation and current modeling strategies. A second introductory part is concerned with the geometrical techniques required for a derivation of the channel model. This derivation basically relies on the properties of point measures in convex bodies. A major result is that the volume and surface of such a body strongly determine its properties.

Subsequently, a novel integral representation of the radio channel is presented. Three classes of integrals are distinguished which each correspond to a particular distribution of the $T_x$ and the $R_x$, and thus to a particular communication system. Each integral poses a particular geometrical problem. The integral allows to characterize existing models with regard to their a-priori knowledge about deterministic properties of the channel.

In Monte Carlo simulations with a ray tracer, the integral is solved for a number of different cases. The impact of the geometry of the environment on the channel is investigated together with the multivariate statistics of particular channel parameters. The influence of the side information about the geometry is studied; it is shown that in an environment that is only roughly determined, the sizes of this environment have the strongest impact on the radio channel due to the undetermined positions of the
distances between $T_x$ and $R_x$.

A clear separation of wave propagation from geometrical aspects allows then to apply the sophisticated geometrical methods introduced before to establish several fundamental relations between the observed wave propagation behavior and the geometric dimensions of the surroundings of $T_x$ and $R_x$. The performed calculations are based on a single empirical input, which is a well-established pathloss formula, and on the ray optical behavior of the high frequency electromagnetic field. The used pathloss formula has — for the system designers interested in stochastic channel models — the advantage that the entire, mostly very complicated inner structure of an investigated building is subsumed to a single parameter, which is the pathloss exponent $q$. This strategy requires, however, some experience in choosing this parameter and its dependence on, for instance, the degree of obstruction, or the amount of furniture in a building. Typical parameters are obtained in this thesis from measurements and simulations; further can be found in literature. On the basis of this pathloss formula, approximations for means and standard deviations of the pathloss are calculated for three different indoor network types. A WLAN with fixed position of the access point and randomly distributed user, a WLAN with random position of the access point, and an Ad-Hoc network, in which both access point and user are randomly distributed. It is demonstrated by geometrical reasoning that the key parameters which are sufficient to characterize the impact of the geometry of the environment on the channel are very simple and easily obtained. Only the volume and the surface area of the domain in which $T_x$ and $R_x$ are located is required. Apart from these two and a modified pathloss exponent which includes the antenna characteristics, only the wavelength and the minimum distance between $T_x$ and $R_x$ are necessary for the complete specification of the distribution of the pathloss. Furthermore, an expression for the power delay profile (PDP) within a single room is derived, and an upper bound for the delay spread is extracted. Now, a complete analytical description of the Rayleigh fading indoor radio channel is given which is highly flexible and independent from experimental data. The description only requires information about the simplest and most accessible parameters of the environment. A building map and a rough notion of the wave propagation properties and the type of communication system is sufficient for a determination of the most
fundamental statistical quantities of the indoor radio channel.

Finally, the model is verified by simulations and measurements in the frequency range of 2.4 to 60 GHz. Both the simulations and the measurements are performed in such a way that the integral representation of the radio channel is implicitly applied and solved for particular cases. All analytically obtained parameters are compared to the measurement and simulation results. Not all bounds are equally suitable; some are too loose or only applicable to specific cases. The investigation shows that models based on $n = 2$ dimensions are generally most advantageous. Furthermore, the best approximations describe all features of the large scale fading very reliably for all network types. The bound for the delay spread is very loose. However, as most systems merely require a safe upper bound, the result is satisfying for practical applications. All features of the bounds that were discussed under purely geometrical considerations in Chapter 6 were also found for the real data in Chapter 7. One can conclude that the radio channel can indeed be studied as an abstract geometrical body, and not only in terms of wave propagation.

8.2 Outlook

The thesis results in the first completely analytical derivation of the stochastic indoor radio channel on the basis of wave propagation properties. It confirms empirical findings which have been established in a rather long period, dating back to the 1990s. These empirical findings have neither provided a complete physical explanation of how deterministic wave propagation in arbitrary surroundings comes to form what is observed as the stochastic radio channel; nor, why the channel is so robust with regard to the geometrical properties of the environment and how these properties can be accessed. The aim of the present thesis is to demonstrate that a mathematical treatment of the radio channel is indeed possible and enables to understand and back up the experimental findings.

The thesis leaves, however, several questions still unanswered and several tasks unfinished. Some shortcomings of the designed model were already mentioned in the final chapters. First of all, the derived quantities are based on a constant pathloss exponent $q$. The measurements demonstrated that this is not always the case. Real environments can be
composed of different regions, each of which must be characterized by a distinct $q$. An extension of the presented model in this regard is straightforward. Since expectation values are additive, the mean pathloss can be calculated for configurations of $T_x$ and $R_x$ in a much larger class of non-convex domains where each of the domains may have distinct properties, such as a particular $q$. One constraint is that the non-convex domain must be convertible in a convex one by a union with a single, disjoint convex set. This is, for instance, possible for a row, but not for an array of rooms [49].

Furthermore, a model more closely related to ray tracing, and thus to physical wave propagation, might be designed. The investigation of the indoor radio channel on the basis of the empirical pathloss formula is actually a general case of a solution based on simplified ray optical assumptions. Since the latter operate with a constant pathloss of $q = 2$, the model could easily be extended to comprise such an approach to the indoor radio channel, as well.

Another drawback of the model is that it is only applicable to the Rayleigh, but not to the Ricean fading channel. The Ricean channel requires a calculation of the k-factor, and thus a strategy that allows the distinction between deterministic and random components. A suitable solution could probably be achieved with the ray tracing approach discussed above. If the MPI is calculated on a single ray basis, strong or LOS components can be distinguished from weak, or random, contributions, and the k-factor statistics becomes accessible. Another possibility is an application of the geometrical methods to the outdoor radio channel. It should be noted, however, that the geometries of outdoor scenarios are much more complicated and variable than those of floors of buildings. The reduction of the outdoor channel to relevant key quantities will, unless it is simply parameterized by the pathloss exponent $q$, prove quite difficult. In its present form, the model furthermore neglects the latest developments in communications towards systems with multiple antennas. These systems exploit the properties of the radio channel much more deeply, so that a simple channel model as it is derived here is not sufficient to capture all required effects. The extension of the integral equation to time variant systems or systems with multiple antennas is in principle straightforward and immediately shows the need for a physical understanding of temporal
or spatial correlations. The numerical investigation of such an equation would, with regard to the level of fast ray optical methods presently available, pose only minor problems. But the extraction of the newly arising key parameters that determine wave propagation would still not be answered.

Many important topics therefore remain open. The model presented in this thesis is suitable for systems which rely on frequency diversity and which use arrays of uncorrelated antennas that are still sufficiently close to experience the same large-scale fading. But particularly those systems have found wide application within the last few years and will spread even more widely in the future. The ongoing discussion about interference and coexistence issues proves that there is a high demand for reliable information about fundamental channel parameters. The techniques for their analytical description, which were introduced in this thesis, may now serve as a starting point for a derivation of more complex channel models that will satisfy the need of the high data rate wireless systems that are still to come.
Appendix A

Hölder’s Inequality

Some of the most important results of this thesis rely on Hölder’s inequality, which is briefly outlined in this appendix. The most common version [5, 18, 26] is the following.

Be $l, l' \in [1, \infty]$ and $\frac{1}{l} + \frac{1}{l'} = 1$. If $\mu_L$ is a Lebesgue measure and the functions $|f|^l$ and $|g|^l$ are integrable on a domain $D$, then

$$\int_D |f \cdot g| \, d\mu_L \leq \sqrt[l]{\int_D |f|^l \, d\mu_L} \sqrt[l']{\int_D |g|^l \, d\mu_L}, \quad (A.1)$$

and equality holds if and only if $C_1|f|^l = C_2|g|^l$ almost everywhere for $C_1$ and $C_2$ constant. The integral $\sqrt[l]{\int_D |f|^l \, d\mu_L}$ denotes the usual $L^l$ norm, abbreviated as $||f||_{L^l}$. The special case $l = \infty$ is defined as

$$||f||_{L^\infty} = \inf_{\mu_L(N) = 0} \sup_{x \in D \setminus N} |f(x)|, \quad (A.2)$$

where $N$ denotes a set of measure 0 [5]. Note that $l = \infty$ implies $l' = 1$ and vice versa.

None of the monographs cited above mention the reversed version of (A.1), which is valid for $l \neq 0 < 1$, and essential for some calculations carried out in this work for large pathloss exponents $q$. In [105], this fact is mentioned, and the reversed inequality is derived for two n-tuples instead of continuous functions. Below, the integral version of Hölder’s inequality is rewritten in the same way for all values of $l \neq 0$.

Consider again the two functions $f$ and $g$ and the numbers $l$ and $l'$ such that $\frac{1}{l} + \frac{1}{l'} = 1$, but now $l < 0$. Introduce then two functions $\tilde{f} = f^{-l'}$, $\tilde{g} = f'^{l'}g''$, and the new exponents $L = -\frac{l}{l'}$ and $L' = \frac{l'}{l'}$. Then, $\frac{1}{L} = -\frac{l}{l'}$, $\frac{1}{L'} = l'$, and hence $\frac{1}{L} + \frac{1}{L'} = l' - \frac{l'}{l} = l'(1 - \frac{1}{l}) = 1$. 

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Use now that

\[
\int_D |\tilde{f} \tilde{g}| \, d\mu_L = \int_D |g|^{l'} \, d\mu_L
\]

\[
\leq \left( \int_D |\tilde{f}|^L \, d\mu_L \right)^{\frac{1}{L}} \left( \int_D |\tilde{g}|^{L'} \, d\mu_L \right)^{\frac{1}{L'}}
\]

\[
= \left( \int_D (|f|^{l'})^{-\frac{1}{l'}} \, d\mu_L \right)^{-\frac{1}{l'}} \left( \int_D |f|^{l'}|g|^{l'} \, d\mu_L \right)^{\frac{1}{l'}}
\]

\[
= \left( \int_D |f|^{l'} \, d\mu_L \right)^{-\frac{1}{l'}} \left( \int_D |fg| \, d\mu_L \right)^{\frac{1}{l'}}.
\]

Between the first and the second line, Hölder’s inequality is applied for $|\tilde{f}|$, $|\tilde{g}|$, and the exponents $L$ and $L'$. This is possible for $L, L' > 1$. Rewriting the first and the last term from above, one then obtains

\[
\left( \int_D |f|^{l'} \, d\mu_L \right)^{-\frac{1}{l'}} \left( \int_D |f \cdot g| \, d\mu_L \right)^{\frac{1}{l'}} \geq \left( \int_D |g|^{l'} \, d\mu_L \right).
\]

This inequality is equivalent to (A.1) with reversed sign. As stated above, it is valid if $l < 0$, which implies $0 < l' < 1$ due to $\frac{1}{l} + \frac{1}{l'} = 1$, and for $L, L' \geq 1$. The condition $L' = \frac{1}{l'} \geq 1$ is trivially fulfilled. The condition $L \geq 1$ requires $|l| \geq l'$. This follows from $\frac{1}{l'} = 1 - \frac{1}{l}$, which is equivalent to $l' = \frac{l}{l-1}$ and also fulfilled for any $l < 0$.

Hence, the reversed version of Hölder’s inequality holds for $l < 0$. The case $0 < l < 1$ is equivalent, since $l < 0$ implies $0 < l' < 1$ and the exponents can be exchanged. As a consequence,

\[
\int_D |f \cdot g| \, d\mu_L \leq \sqrt[l']{\int_D |f|^l \, d\mu_L} \sqrt[l']{\int_D |g|^{l'} \, d\mu_L}, \quad l \geq 1, \quad (A.4)
\]

\[
\int_D |f \cdot g| \, d\mu_L \geq \sqrt[l']{\int_D |f|^l \, d\mu_L} \sqrt[l']{\int_D |g|^{l'} \, d\mu_L}, \quad l < 1, \quad l \neq 0 (A.5)
\]

for any $\frac{1}{l} + \frac{1}{l'} = 1$. 

A. Hölder’s Inequality
Appendix B

Transformation of the Integral over the Density of two Points

The transformation of the integral over two points separated by a minimum distance $\rho_{\text{min}}$ into an integral over the density of lines, $d\mathcal{L}$, and into correcting terms is shown in this appendix. Consider

$$ I_{q,n}^{AH,1} = \frac{1}{V_n \bar{V}_n} \int \int_{\mathcal{D} \setminus B_{\rho_{\text{min}}}(\mathbf{r}_1)} \frac{1}{r^q} d\mathcal{P}_1 d\mathcal{P}_2, \quad (B.1) $$

and use the density transformation (3.6),

$$ I_{q,n}^{AH,1} = \frac{1}{V_n \bar{V}_n} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} \int_{t_1 \in \mathcal{D} \cap \mathcal{L}} \int_{t_2 \in \mathcal{D} \cap \mathcal{L} \setminus B_{\rho_{\text{min}}}(\mathbf{r}_1)} \frac{1}{r^q} r^{n-1} dt_2 dt_1 d\mathcal{L}. \quad (B.2) $$

Let the intersection of the line $\mathcal{L}$ with the boundary of the body $\mathcal{D}$ be denoted by $\sigma_a$ and $\sigma_b$. The quantities are real numbers, since they exist in the local coordinate system of the line. Assume $\sigma_b > \sigma_a$, and let $\sigma = \sigma_b - \sigma_a$ denote the length of the chord that arises when the line intersects $\mathcal{D}$. Now, (B.2) transforms, always excluding $B_{\rho_{\text{min}}}(\mathbf{r}_1)$ from the
integration over \( t_2 \), as

\[
I_{q,n}^{A,H,1} = \frac{1}{V_n V_n} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} d\mathcal{L} \left\{ \int_{\sigma_a}^{\sigma_b - \rho_{\min}} dt_2 \int_{t_2 + \rho_{\min}}^{t_2 - \rho_{\min}} (t_1 - t_2)^{n-1-q} dt_1 \\
+ \int_{\sigma_a + \rho_{\min}}^{\sigma_a} dt_2 \int_{\sigma_a}^{t_2 - \rho_{\min}} (t_2 - t_1)^{n-1-q} dt_1 \right\}
\]

\[
= \frac{1}{(n-q)V_n V_n} \int_{\mathcal{L} \cap \mathcal{D} \neq \emptyset} d\mathcal{L} \int_{\sigma_a}^{\sigma_b - \rho_{\min}} \{(\sigma_b - t_2)^{n-q} - \rho_{\min}^{n-q}\} dt_2 \\
+ \int_{\sigma_a + \rho_{\min}}^{\sigma_b} \{(t_2 - \sigma_a)^{n-q} - \rho_{\min}^{n-q}\} dt_2 \\
= \frac{2}{(n-q)(n-q+1)V_n V_n} \int_{\sigma > \rho_{\min}} \left( \sigma^{n-q+1} - \rho_{\min}^{n-q+1} \\
- (n-q+1)(\sigma - \rho_{\min})\rho_{\min}^{n-q} \right) d\mathcal{L}.
\]

(B.3)

In the last line, the integration is carried out over all lines \( \mathcal{L} \) that are at least as large as \( \rho_{\min} \). Note that this condition depends not only on \( \rho_{\min} \), but also on the orientation of \( \mathcal{L} \), since the cordlength is a function of the orientation.
Appendix C

Derivation of the Fourth Moment of the Received Power

In this appendix it is demonstrated how the fourth moment of the pathloss can be derived as a function which is similar to the empirical pathloss model (6.1). Consider the received amplitudes \( \alpha = \sum_\nu \alpha_\nu \), where \( \alpha_\nu \) are complex multipath components. Then, the fourth moment of the distribution of \( \alpha \) is

\[
\langle |\alpha|^4 \rangle = \left\langle \left| \sum_{\nu_1} \alpha_{\nu_1} \exp(j\varphi_{\nu_1}) \right|^2 \left| \sum_{\nu_3} \alpha_{\nu_3} \exp(j\varphi_{\nu_3}) \right|^2 \right\rangle \tag{C.1}
\]

\[
= \left\langle \left( \sum_{\nu_1} |\alpha_{\nu_1}|^2 + 2 \sum_{\nu_2>\nu_1} |\alpha_{\nu_1}| |\alpha_{\nu_2}| \cos \Delta \varphi_{\nu_1\nu_2} \right) \times \left( \sum_{\nu_3} |\alpha_{\nu_3}|^2 + 2 \sum_{\nu_4>\nu_3} |\alpha_{\nu_3}| |\alpha_{\nu_4}| \cos \Delta \varphi_{\nu_3\nu_4} \right) \right\rangle
\]

\[
= \left\langle \sum_{\nu_1} |\alpha_{\nu_1}|^2 \sum_{\nu_3} |\alpha_{\nu_3}|^2 \right\rangle
+ 2 \left\langle \sum_{\nu_1} |\alpha_{\nu_1}|^2 \sum_{\nu_4>\nu_3} |\alpha_{\nu_3}| |\alpha_{\nu_4}| \cos \Delta \varphi_{\nu_3\nu_4} \right\rangle
+ 2 \left\langle \sum_{\nu_3} |\alpha_{\nu_3}|^2 \sum_{\nu_2>\nu_1} |\alpha_{\nu_1}| |\alpha_{\nu_2}| \cos \Delta \varphi_{\nu_1\nu_2} \right\rangle
+ 4 \left\langle \sum_{\nu_2>\nu_1, \nu_4>\nu_3} |\alpha_{\nu_1}| |\alpha_{\nu_2}| |\alpha_{\nu_3}| |\alpha_{\nu_4}| \cos \Delta \varphi_{\nu_1\nu_2} \cos \Delta \varphi_{\nu_3\nu_4} \right\rangle \tag{C.2}
\]

where \( \alpha = |\alpha| \exp(j\varphi) \) denotes the representation of a complex number by its absolute value and phase, and where \( \Delta \varphi_{\nu_1\nu_2} \) is the phase difference between the two waves \( \nu_1 \) and \( \nu_2 \).

Assuming independence between the statistics of the phases and the
statistics of the amplitudes, which has already been successfully applied in
[54], and using the fact that for uniformly distributed phases, \( \langle \cos \Delta \varphi_{\nu_1 \nu_2} \rangle = 0 \), one obtains

\[
\langle |\alpha|^4 \rangle = \sum_{\nu_1} |\alpha_{\nu_1}|^2 \sum_{\nu_3} |\alpha_{\nu_3}|^2 \\
+ 4 \sum_{\nu_2 > \nu_1} \sum_{\nu_4 > \nu_3} \langle |\alpha_{\nu_1}| |\alpha_{\nu_2}| |\alpha_{\nu_3}| |\alpha_{\nu_4}| \rangle \\
\langle \cos \Delta \varphi_{\nu_1 \nu_2} \cos \Delta \varphi_{\nu_3 \nu_4} \rangle.
\] (C.3)

The interpretation of these results is straightforward: The first term in
(C.3) describes fluctuations of the amplitude itself, the second the fluctuations due to amplitude and phase variations. Since the empirical pathloss model (6.1) states that

\[
\langle E_{\nu_1} E_{\nu_2}^* \rangle = \kappa \frac{1}{r^q},
\] (C.4)

one has for uncorrelated amplitudes

\[
\langle \sum_{\nu_1} |\alpha_{\nu_1}|^2 \sum_{\nu_3} |\alpha_{\nu_3}|^2 \rangle = \langle \sum_{\nu_1} |\alpha_{\nu_1}|^2 \rangle \langle \sum_{\nu_3} |\alpha_{\nu_3}|^2 \rangle = \kappa^2 \frac{1}{r^{2q}}.
\] (C.5)

Now, one can use that

\[
\langle \cos \Delta \varphi_{\nu_1 \nu_2} \cos \Delta \varphi_{\nu_3 \nu_4} \rangle = \langle \cos^2 \Delta \varphi_{\nu_1 \nu_2} \rangle \delta_{\nu_1 \nu_3} \delta_{\nu_2 \nu_4} \\
+ \langle \cos \Delta \varphi_{\nu_1 \nu_2} \cos \Delta \varphi_{\nu_3 \nu_4} \rangle \bigg|_{\nu_1 \neq \nu_3, \nu_2 \neq \nu_4} \\
= \langle \cos^2 \Delta \varphi_{\nu_1 \nu_2} \rangle \delta_{\nu_1 \nu_3} \delta_{\nu_2 \nu_4}
\] (C.6)

for uncorrelated phases with \( \delta_{\nu_1 \nu_2} \) being the Kronecker symbol, and that

\[
\langle \cos^2 \Delta \varphi_{\nu_1 \nu_2} \rangle = \frac{1}{(2\pi)^2} \int_0^{2\pi} \int_0^{2\pi} d\varphi_{\nu_1} d\varphi_{\nu_2} \cos^2(\varphi_{\nu_1} - \varphi_{\nu_2}) = \frac{1}{2}.
\] (C.7)

For the second term in (C.3), it results that

\[
4 \sum_{\nu_2 > \nu_1} \sum_{\nu_4 > \nu_3} \langle |\alpha_{\nu_1}| |\alpha_{\nu_2}| |\alpha_{\nu_3}| |\alpha_{\nu_4}| \rangle \langle \cos \Delta \varphi_{\nu_1 \nu_2} \cos \Delta \varphi_{\nu_3 \nu_4} \rangle \\
= 2 \sum_{\nu_2 > \nu_1} \sum_{\nu_4 > \nu_3} \langle |\alpha_{\nu_1}| |\alpha_{\nu_2}| |\alpha_{\nu_3}| |\alpha_{\nu_4}| \rangle \delta_{\nu_1 \nu_3} \delta_{\nu_2 \nu_4} \\
= 2 \sum_{\nu_2 > \nu_1} \langle (|\alpha_{\nu_1}| |\alpha_{\nu_2}|)^2 \rangle.
\] (C.8)
For uncorrelated amplitudes (note that \( \nu_1 \neq \nu_2 \)), this results in

\[
2 \sum_{\nu_2 > \nu_1} \langle |\alpha_{\nu_1}|^2 \rangle \langle |\alpha_{\nu_2}|^2 \rangle = 2 \sum_{\nu_1 = 1}^{L_{vs}} \langle |\alpha_{\nu_1}|^2 \rangle \sum_{\nu_2 = \nu_1 + 1}^{L_{vs}} \langle |\alpha_{\nu_2}|^2 \rangle 
\]  
(C.9)

where the number of all received components, \( L_{vs} \), clearly has an impact on the variance of the total received power.

The key idea is to note that

\[
\sum_{\nu_1 = 1}^{L_{vs}} \langle |\alpha_{\nu_1}|^2 \rangle = \kappa \frac{1}{\tau^q} 
\]  
(C.10)

and to assume that the same empirical relation holds for the latter term, i.e.,

\[
\sum_{\nu_2 = \nu_1 + 1}^{L_{vs}} \langle |\alpha_{\nu_1}|^2 \rangle = \kappa \frac{1}{\tau^{q'}} 
\]  
(C.11)

where \( q' \geq q \) ensures a faster convergence of this latter term due to the lower number of multipath components in the sum. Defining the parameter \( \tilde{q} \) for convenience such that \( 2q + \tilde{q} = q + q' \), one has \( \tilde{q} \geq 0 \), and (C.9) hence transforms into

\[
2 \sum_{\nu_2 > \nu_1} \langle |\alpha_{\nu_1}|^2 \rangle \langle |\alpha_{\nu_2}|^2 \rangle = 2\kappa^2 \frac{1}{\tau^{2q+\tilde{q}}}. 
\]  
(C.12)

By combination of (C.3), (C.5), (C.8), and (C.12), the entire formula for the fourth moment can now be written as

\[
\langle |\alpha|^4 \rangle = \kappa^2 \left( \langle \frac{1}{\tau^{2q}} \rangle + 2\langle \frac{1}{\tau^{2q+\tilde{q}}} \rangle \right). 
\]  
(C.13)

The parameter \( \tilde{q} \) is an empirical shaping parameter of the variance. For \( \tilde{q} = 0 \), the sum in (C.11) has the same impact as the sum in (C.10). For high \( \tilde{q} \), the sum (C.11) can be made to contribute only a little, i.e., to suppress the contribution of the phase variations to the variance of the power. Since the variance depends on the antenna pattern [147], \( \tilde{q} \) can be used to parameterize this effect. For an antenna with a very high gain, one has \( \tilde{q} \gg 0 \). All phase fluctuations are suppressed, since only a single beam is captured. The opposite case is \( \tilde{q} \rightarrow 0 \), where the fluctuations due to the superposition of all impinging waves are at their maximum.
C. Derivation of the Fourth Moment of the Received Power
Appendix D

The Ray Tracing Software

The developed raytracer is a numerical implementation of (4.1) and (4.2). The tool aims at both the calculation and the statistical investigation of the electric field within a single room indoor environment.

Its graphical user interface is displayed in Fig. D.1. It is divided into a command window, a section for data storage, one for the general operation of the tool, and one for the definition of the distribution functions and of all deterministic input parameters. The command window is on the top. It shows the status of the raytracer, for instance the number of iterations performed in the course of a field calculation or error messages. Above it, there is a single line for setting the options for the modi for saving data and error handling. Below, there is the section in which the data storage is handled. After the raytracer has run, all data can be exported into ASCII files that can be edited using the 'edit' button. When data is exported, up to three different files are created. One file contains all random data about the room and the antenna positions, another all fixed input parameters, and the third all calculated results such as received complex amplitudes on a single ray basis and their angles of arrival and departure. Since the first and the last file can be extremely large for simulations with many realizations, one can choose in the uppermost command line ('Save Options') which of these files shall be exported. A MATLAB readable version of the data is created when the 'Save File' is saved. When the raytracer is running, automatic backups of the data calculated so far are stored in the 'Backup File'. The backup period can be specified in the 'Save Options'. Since a particular set of input data may be used repeatedly, this data can be saved in the 'Preference File', which is in MATLAB readable format.

In the box below, a calculation can be initiated with 'Calculate', and, if necessary, terminated with 'X'. For statistical investigations that require the investigation of all angles of reflection and all involved material properties, one can include these values into the output data file. This is
optional, since it is not required in most cases and would produce large amounts of data. In the same box, there are the button to enlarge or reduce the size of the interface, and to exit the program. Finally, the latest version of the raytracer has a 'MIMO' button with which, in case of antenna arrays, a singular value decomposition of the obtained channel matrix can be performed.

In the box below, the stochastic and deterministic input parameters can be specified. Since the raytracer can treat uniform linear arrays at both antennas, the number ('Nb') and the spacing ('Space') between the antenna elements must be set. 'Order' denotes the order of reflection. One can choose between stochastic and deterministic positions of the $T_x$ and the $R_x$, and between stochastic and deterministic room sizes. If 'stochastic' is chosen, the number of positions and room sizes to be realized, respectively, must be specified ('Nb pos', 'Nb rooms'). In the deterministic case, the 'Position' of $T_x$ and $R_x$ (top of box) and the 'Roomsize' (underneath) are to be specified. In the random case, bounds of uniform distributions must be given. With regard to the room size, a maximum and minimum size is required ('Max/Min'); both fields have three entries for length, width and height; in the example in Fig. D.1, the bounds for the distribution of the room sizes are equal, which results in a room of fixed size. The specification of the random location of the antennas requires more parameters. The array has a random orientation in space; its mean is specified by the angles 'Phi0' and 'Theta0'. The array can be randomly rotated about this mean within ranges of 'Phimax' and 'Thetamax'. If these ranges reach $2\pi$ or $\pi$, respectively, the array may have any orientation. Underneath, denoted by 'Rel lim up' and 'Rel lim down', the spatially uniform distribution of the middle of the array is defined; for the case of single antennas, this is simply their position. The entries consist of values between 0 and 1 and denote the relative distance of this point from a wall. The three values refer to the $x$, $y$ and $z$ direction of the coordinate system of the room, in which the walls are aligned parallel to these axes. The first value 0.98 in 'Rel lim down' for the $T_x$, for instance, means that the lower bound of the $z$ coordinate of the $T_x$ is 0.98 times the room’s height above the floor. The corresponding value 0.01 for 'Rel lim up' for this coordinate means that the upper bound is 0.01 times the room’s height underneath the ceiling. Hence, the $z$ coordinate of the $T_x$ varies randomly within 1% of the room.
height, just underneath the ceiling. In the example in Fig. D.1, both $T_x$ and $R_x$ are randomly located above the entire floor of the room, just 1\% away from the room’s side walls. The $R_x$ is randomly located between 20\% and 50\% of the room’s height. Hence, the entire scenario corresponds to the downlink of a WLAN with the access point randomly located right underneath the ceiling, and the user anywhere above the floor of the room. The user’s absolute position depends on the specified sidelengths of the room.

Furthermore, an error control is implemented in the raytracer. If senseless values are given, for instance values for the bounds of the distribution of the antenna locations greater than 1, or an antenna array the arms of which are potentially outside the room, a warning will appear in the command window. Also, only those values which are needed for the simulation are actually accessible. If, for instance, the antenna positions are deterministic, all fields related to stochastic quantities will vanish; in the figure, this feature is disabled for demonstrational purposes.

In the lowest box, deterministic wave propagation quantities such as the building materials can be chosen. In the given case, a ’Modern Building Type’ is created. Three different materials, concrete, wood, and glass, and some combinations of them are predefined. The effect of the chosen room type is shown on the right, where the ‘eps-pdf’ is the density function of the building materials. The first value in this box gives the probability of a wave to be reflected at concrete, the second at glass, and the third at wood. The three available dielectric constants represent fairly diverse cases with which many distinct environments can be described. Seven different scenarios are implemented. For the creation of more, or the introduction of other dielectric constants, the source code can be modified quite easily. The feature of variable wall thickness will be implemented in a future version of the raytracer. In the present version, a wall thickness of 0 denotes infinitely thick walls, i.e., only a single reflection is considered. Apart from the Fresnel reflection coefficients, angle independent reflection coefficients for both polarizations can be chosen for further statistical studies (’Refl coeff.’). If they are 0, the Fresnel coefficients are used. The frequency is given in GHz; ’Rho’ denotes the correlation coefficient of the materials on the wall surface. If antenna arrays are used, the spatial correlations between the elements are of interest. These correlations can be extracted
from a joint distribution of the amplitudes of two antenna elements. They require, as can be seen in (4.1), also a joint distribution of the location of the virtual sources and dielectric constants. The former can be calculated by means of geometrical considerations; the latter require the specification of the joint distribution of the wall materials. This is done via the correlation coefficient ‘$\rho$’. Finally, the antenna characteristics of $T_x$ and $R_x$ can be defined as a two dimensional function of the azimuth and elevation angles ‘$\phi$’ and ‘$\theta$’. The coordinate system is set up such that the parallel component, which is the first of the vector, is parallel to the floor of the room. The second is the vertical, which depends on the parallel component and on the direction of the propagating wave. The orientation of the local coordinate system of the antenna can either be relative to the LOS, i.e., it points at the other antenna, or relative to the fixed coordinate system of the room (not displayed in the figure).

The raytracer is self-explaining: if the cursor of the mouse is moved onto a button, a short description of its effect will appear.
Figure D.1: User interface of the raytracer.
D. The Ray Tracing Software
Appendix E

The Channel Simulator Software

E.1 Overview of all Analytical Approximations

The present thesis introduces several analytical approximations of some very important channel parameters. An upper bound for the delay spread, bounds on the mean pathloss, and approximations for the mean and the standard deviation of the lognormal distribution of the large-scale variation of the received power. Apart from the delay spread, which basically depends on the contributions of the reflected waves to the entire field, all quantities are calculated in dependence on the spatial distribution of the transmitter and the receiver. In Section 4.2, three different network types are defined, which correspond to a particular spatial distribution. To each network type, one or several approximations of the mean pathloss and characteristics of the lognormal distribution are developed in Chapter 6. As shown there and in the following Chapter 7, not all of them are equally suitable. For the channel simulator, the best bounds have been picked and are given by reference; they are treated as approximations rather than as bounds. Before these are stated, we give a list of the parameters which are required for the calculation of the channel parameters:

- the frequency
- network type (WLAN with fixed or random position of the access point or Ad-Hoc network)
- the dimension in which the parameters are to be calculated (two or three)
• the length, width and height of the structure which is investigated (required for the calculation of the delay spread; for all other quantities, the surface and the volume of this domain is sufficient)

• the average reflection attenuation (required for the delay spread). This value can in principle be calculated from the dielectric constants of the wall material and the Fresnel reflection coefficients

• the minimum distance between transmitter and receiver

• the pathloss exponent valid with the structure

• the antenna characteristics (e.g., wide or pencil beam)

In the following, the approximations of the channel parameters as used in the simulator are given. Comments state under which conditions the approximation performs usually best.

1. Delay spread (Section 6.6):

   • Use (6.64) for an upper bound. The bound is the tighter the more cubic an environment is, and the less inner partitions it has (Subsection 7.2.5 and in particular Tab. 7.12).

2. Mean pathloss (Section 6.3):

   • general constraint: condition (6.16) must be fulfilled, i.e., the minimum distance between transmitter and receiver must be smaller than one half of the shortest sidelength of the investigated building.

   • For WLANs with fixed access point:
     Apply (6.10). If the height of the building permits, one can apply $n = 3$. If not, use (6.10) for $n = 2$ (Tab. 7.8 and Tab. 7.9).
For Ad-Hoc networks:
Use the bound (6.20) for \( n = 2 \). This bound is better than the bound (6.28), since it does not have a singularity (Fig. 6.4). The bounds for \( n = 2 \) are tighter than for \( n = 3 \); also, no minimum height of the domain is required. If \( q < n \), (6.28) is a reliable upper bound for the mean pathloss (Tab. 7.1, 7.2, 7.10, and 7.11). This bound is, however, not implemented.

For WLANs with random position of the access point:
Use the bound for the Ad-Hoc network. The bounds derived directly for this network type are too loose (Tab. 7.3 - 7.5, 7.10, and 7.11).

3. Characterization of the large-scale fading (6.5):

- To calculate the mean and the standard deviation of the large-scale fading, apply the bounds for the mean pathloss to the calculation of the fourth moment, (6.52), and use this result in (6.50) and (6.51).

For the given approximations, the mean pathloss is very reliably met for all network types. The deviation from the measured mean is about 3 dB. The mean of the lognormal distribution differs a little more. The standard deviations are in most cases very well reproduced. The best performance is achieved for the WLAN with fixed access point, which is probably the most widely used network type. For the other two networks, the approximations exhibit differences up to 5 dB for the mean, and up to 2 dB for the standard deviation. Still, for most channel models, even for those derived from measured data only, these results are highly convincing.

E.2 The Graphical User Interface

The channel simulator software consists of a Graphical User Interface based on MATLAB. It allows an easy handling of the complicated formulae derived in Chapter 6. It aims towards users of the model who are not too familiar with the approximations and the conditions of their validity. Only the most suitable and robust bounds without singularities are
implemented, as they are described above. Hence, there is no choice between different possibilities; all values calculated by the tool are regarded as approximations rather than as bounds. As mentioned in the previous section of this appendix, the approximations for the WLAN with fixed access point are based on (6.10), and those for the Ad-Hoc network and the WLAN with the random position of the access point on $I_{q,n}^{WLAN}$. The dimension $n$ can be chosen; if the condition (6.16) is violated, a warning appears.

The interface, displayed in Fig. E.1, is divided into five sections. On the upper left, one can choose the network types which are WLANs with fixed access point or Ad-Hoc network. The attenuation is required for the calculation of the delay spread and corresponds to the variable $d_E$. Furthermore, the minimum distance $\rho_{min}$, the frequency in GHz, and the dimension $n$ can be chosen. On the right hand side, there are fields for the pathloss exponent $q$, the variable for the antenna pattern $\tilde{q}$, and the length, the width and the height of the structure. On the left in the middle, one can initiate the calculations with the 'Calculate' button, enlarge or scale down the size of the interface, or exit the program. Furthermore, all inserted and calculated data can be saved in a file with a specified filename; the saving of preferences, i.e., of all inserted values, is possible for later reload. Below, one can statistically simulate and save channel impulse responses, which have a statistics according to the one obtained from above. The delay axis must be divided into a number taps which are spaced an inverse bandwidth apart. The window on the lower right shows the current status of the simulator and displays error messages and calculated results.

Any single calculation provides all values required for a complete statistical specification of the indoor Rayleigh fading channel, i.e., the mean pathloss, the mean and standard deviation of the distribution of the power, and the delay spread. The values can then be used for stochastic channel simulations. The channel simulator is self-explaining; if the cursor of the mouse is moved onto a button, a short description of its effect will appear.
Figure E.1: User interface of the channel simulator.
Bibliography


[146] D. Zahn, K. Sarabandi, K. Sabet, and J. Harvey, “Numerical Simulation of Scattering from Rough Surfaces: A Wavelet Based Ap-


Abbreviations

**AoD** angle of departure of a wave, p. 7.

**dB** dezibel, p. 63.

**GHz** $10^9$ Hertz, p. 3.


**LOS** line-of-sight, p. 21.

**MC** Monte Carlo, p. 49.

**MHz** $10^6$ Hertz, p. 24.

**MIMO** multiple-input multiple-output, p. 4.

**MPI** mean pathloss, p. 25.

**N.A.** not applicable, p. 92.

**NLOS** non-line-of-sight, p. 21.

**ns** $10^{-9}$ s, p. 114.

**PDP** power delay profile, p. 3.

**rms delay spread** root-mean-square delay spread, p. 114.

**US** uncorrelated scattering, p. 27.


**WLAN** wireless local area network, p. 45.

**WSS** wide sense stationary, p. 27.

**WSSUS** wide sense stationary uncorrelated scattering, p. 29.
Variables and Constants

$\alpha$ complex amplitude, p. 27.

$\alpha_\nu$ complex amplitude of propagation path $\nu$, p. 23.

$\sigma_r^{(m)}$ upper range of the one-standard deviation confidence level of $\sigma_r^{(m)}$, p. 133.

$\beta_\nu$ probability that a wave propagating in a room is reflected at a material $\nu$, p. 52.

$\Delta$ small interval or displacement of a quantity, p. 52.

$\delta(.)$ Dirac function, p. 52.

$\Delta \alpha_\nu$ small displacement of a complex amplitude $\alpha_\nu$, p. 50.

$\Delta r_\nu$ small displacement of a point $r_\nu$ in all dimensions; characterizes a small volume element, p. 50.

$\Delta \varphi_{\nu_1 \nu_2}$ the difference of the phases of two waves $\nu_1$ and $\nu_2$, p. 145.

$dV_\mathcal{M}$ infinitesimal volume element of the manifold $\mathcal{M}$, p. 40.

$d\mathcal{L}$ the density of a line in $\mathbb{E}^n$, p. 35.

$dr_\nu$ infinitesimal volume element around $r_\nu$, p. 43.

$\epsilon$ permittivity of an arbitrary homogeneous and isotropic medium, p. 10.

$\epsilon^{(\nu)}$ permittivity of an homogeneous and isotropic medium $\nu$, p. 19.

$\epsilon_0$ permittivity of free space, $8.854...\frac{A}{V_m}$, p. 19.

$\epsilon_r^{(\nu)}$ relative permittivity of a medium $\nu$, p. 19.

$\eta_n^{(1)}$ geometrical constant that depends on the surface of a domain of dimension $n$, p. 76.

$\eta_n^{(2)}$ geometrical constant that depends on the volume of a domain of dimension $n$, p. 76.
\[ \eta_n^{(3)} \] geometrical constant that depends on the square of the volume of a domain of dimension \( n \), p. 78.

\( \Gamma \) \( \Gamma \) function, p. 34.

\( \gamma \) completely (with respect to wave propagation) defined environment, p. 43.

\( \Gamma_0 \) arbitrary set of \( \gamma \), p. 43.

\( \gamma_0, \tilde{\gamma}_0 \) proportionality constants, p. 95.

\( \kappa \) wavelength dependent constant of the pathloss law, p. 25.

\( \kappa' \) wavelength dependent constant of the pathloss law, p. 25.

\( \kappa_{uv} \) cumulants, p. 96.

\( \lambda \) wavelength of an electromagnetic wave, p. 21.

\( \mu \) permeability of a medium, p. 10.

\( \mu_L \) Lebesgue measure, p. 141.

\( \mu_{\text{logn}} \) mean value of the lognormal distribution, p. 28.

\[ \mu_{\text{logn}}^{(a)}/n = 2 \] logarithmic mean of the analytically calculated pathloss for \( n = 2 \); this mean is calculated on dB scale and is the mean of the lognormal distribution, p. 105.

\[ \mu_{\text{logn}}^{(a)}/n = 3 \] logarithmic mean of the analytically calculated pathloss for \( n = 3 \); this mean is calculated on dB scale and is the mean of the lognormal distribution, p. 105.

\[ \mu_{\text{logn}}^{(m)} \] logarithmic mean of the measured pathloss; this mean is calculated on dB scale and is the mean of the lognormal distribution, p. 127.

\[ \mu_{\text{logn}}^{(s)} \] logarithmic mean of the simulated pathloss; this mean is calculated on dB scale and is the mean of the lognormal distribution, p. 105.

\( \nu, \nu_i \) integer, used as (alphanumerical) index, p. 14.
\( \omega \)  
angular frequency of an electromagnetic field, p. 10.

\( \Omega \)  
direction of a unit vector, p. 22.

\( \omega_G \)  
argument of the characteristic function \( G_P \), p. 94.

\( \Omega_{\nu, R_x} \)  
direction of the \( \nu \)th impinging wave at the \( R_x \), p. 23.

\( \Omega_{\nu, T_x} \)  
direction of the \( \nu \)th launched wave at the \( T_x \), p. 23.

\( \partial D \)  
boundary of a domain of points, p. 12.

\( A \)  
vector potential existing in a region free of magnetic charges, p. 10.

\( C \)  
antenna directivity, p. 22.

\( C_{R_x} \)  
antenna directivity of the \( R_x \), p. 23.

\( C_{T_x} \)  
antenna directivity of the \( T_x \), p. 23.

\( C_{\nu} \)  
a constant vector in 3d, p. 16.

\( E_A \)  
electric field strength generated by the vector potential \( A \), p. 11.

\( E_F \)  
electric field strength generated by the vector potential \( F \), p. 11.

\( E \)  
electric field strength, p. 10.

\( E^i, E^i_\nu \)  
incident electric field (of series term \( \nu \)), p. 17.

\( E^r, E^r_\nu \)  
reflected electric field (of series term \( \nu \)), p. 17.

\( E^{(0)} \)  
undisturbed electric field, p. 12.

\( E_\nu \)  
\( \nu \)th term in the series expansion of electric field strength that is the basis of the GO, GTD, and the UTD, p. 14.

\( F \)  
vector potential existing in a region free of electric charges, p. 10.

\( H_A \)  
magnetic field strength as generated by the vector potential \( A \), p. 10.
<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$H_F$</td>
<td>magnetic field strength as generated by the vector potential $F$, p. 11.</td>
</tr>
<tr>
<td>$H$</td>
<td>magnetic field strength, p. 10.</td>
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\( r_0 \)  
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\( r_{R_x} \)  
position of the \( R_x \), p. 43.

\( r_{T_x} \)  
position of the \( T_x \), p. 43.

\( r_{\nu} \)  
position of point or object \( \nu \), p. 50.

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\( \Psi_i \)  
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\( \rho, \rho_{\nu} \)  
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\( \rho_{\min} \)  
minimum distance between \( T_x \) and \( R_x \), p. 67.

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power of the scattered components of the field, p. 28.
\(\sigma_{Fa}\) standard deviation of the surface roughness, required for the Fraunhofer criterion, p. 21.

\(\sigma_{Ra}\) parameter of the Rayleigh distribution, p. 28.

\(\sigma_{Ri}\) parameter of the Ricean distribution, p. 28.

\(\sigma_\tau\) delay spread (standard deviation of the exponential distribution), p. 29.

\(\sigma_a\) intersection point of a line \(\mathcal{L}\) with the boundary \(\partial\mathcal{D}\) of a convex domain, p. 37.

\(\sigma_b\) intersection point of a line \(\mathcal{L}\) with the boundary \(\partial\mathcal{D}\) of a convex domain; \(\sigma_b > \sigma_a\), p. 37.

\(\sigma_{\text{logn}}\) standard deviation of the lognormal distribution on dB-scale, p. 28.

\(\sigma^{(a)}_{\text{logn}}/n = 2\) standard deviation of the logarithm of the analytically calculated pathloss for \(n = 2\), p. 105.

\(\sigma^{(a)}_{\text{logn}}/n = 3\) standard deviation of the logarithm of the analytically calculated pathloss for \(n = 3\), p. 105.

\(\sigma^{(m)}_{\text{logn}}\) standard deviation of the logarithm of the measured pathloss; equals the standard deviation of the lognormal distribution, p. 127.

\(\sigma^{(s)}_{\text{logn}}\) standard deviation of the logarithm of the simulated pathloss, p. 105.


\(\tau_0\) constant required for the calculation of the delay spread, p. 96.

\(\tau_{\text{max}}\) maximum delay of impinging waves, p. 96.

\(\tau_{\text{pulse}}\) width of a short pulse, p. 93.

\(\text{MPI}^{(a)}_n = 2\) mean of the analytically calculated pathloss for \(n = 2\); this mean is calculated on linear scale, and subsequently expressed in dB, p. 104.
\( \text{MPI}^{(a)}/n = 3 \) mean of the analytically calculated pathloss for \( n = 3 \); this mean is calculated on linear scale, and subsequently expressed in dB, p. 104.

\( \text{MPI}^{(m)} \) mean of the measured pathloss; this mean is calculated on linear scale, and subsequently expressed in dB, p. 127.

\( \text{MPI}^{(s)} \) mean of the simulated pathloss; this mean is calculated on linear scale, and subsequently expressed in dB, p. 104.

\( \mathcal{P} \) normalized probability density, p. 43.

\( \mathcal{P}_u, \tilde{\mathcal{P}}_u \) densities of uniform distributions, p. 95.

\( \mathcal{P}_\text{Ra} \) density of the Rayleigh distribution, p. 28.

\( \mathcal{P}_\text{Ri} \) density of the Ricean distribution, p. 28.

\( \mathcal{P}_\text{logn} \) density of the lognormal distribution, p. 28.

\( \theta_i \) angle of incidence, p. 17.

\( \theta_r \) angle of reflection, p. 17.

\( \theta_L \) elevation of the line \( \mathcal{L} \), p. 34.

\( \tilde{f} \) a function related to \( f \), p. 141.

\( \tilde{g} \) a function related to \( g \), p. 141.

\( \tilde{\rho} \) distance between the foot of the line \( \mathcal{L} \) and the origin \( O \), p. 35.

\( \tilde{V}_n \) volume of a domain \( \mathcal{D} \) with a ball of radius \( \rho_{\text{min}} \) excluded, p. 67.

\( \tilde{\xi}_r \) factor that constitutes the delay spread for short tunnels, p. 95.

\( \tilde{\zeta}_{\text{max}} \) maximum number of reflections per unit time in a short tunnel of sidelengths \( \hat{a}, \hat{b} \), p. 95.

\( \tilde{\zeta}_{\text{min}} \) minimum number of reflections per unit time in a short tunnel of sidelengths \( \hat{a}, \hat{b} \), p. 95.
\( \tilde{q} \) pathloss exponent required for the calculation of the variance; describes influence of the antenna gain, p. 90.

\( \varphi_\nu \) phase of wave \( \nu \), p. 93.

\( \xi \) arbitrary (possibly vector valued) quantity of the stochastic radio channel, p. 43.

\( \Xi \) set of all \( \xi \), p. 44.

\( \xi_\tau \) factor that constitutes the delay spread for cubic rooms, p. 95.

\( \zeta \) number of reflections per unit time in a room, p. 94.

\( \zeta_{\text{max}} \) maximum number of reflections per unit time in a room of sidelengths \( a, b, c \), p. 95.

\( \zeta_{\text{min}} \) minimum number of reflections per unit time in a room of sidelengths \( a, b, c \), p. 95.

\( \{\alpha_\nu\} \) set of all amplitudes \( \alpha_\nu \), i.e. \( \{\alpha_1, ..., \alpha_{L^v}\} \), p. 50.

\( \{\varepsilon_r^{(\nu)}\} \) set of all dielectric constants \( \varepsilon_r^{(\nu)} \), i.e. \( \{\varepsilon_r^{(1)}, ..., \varepsilon_r^{(L^v, \varepsilon_r^{(\nu)})}\} \), p. 50.

\( \{r_\nu\} \) set of all points (virtual sources) \( r_\nu \), i.e. \( \{r_1, ..., r_{L^v}\} \), p. 50.

\( a, \tilde{a} \) sidelengths, p. 38.

\( A_{R^i} \) amplitude of the deterministic component of a Ricean distributed random variable, p. 28.

\( A_{n-1} \) surface area of a domain in \( n \) dimensions, p. 38.

\( b, \tilde{b} \) sidelengths, p. 38.

\( c \) height of a room or building, p. 74.

\( c_0 \) velocity of light, p. 93.

\( C_\nu \) a scalar constant of index \( \nu \), p. 141.

\( d \) \( \ln d_E \), p. 94.
$d_E$ average reflection attenuation of the electric field, p. 93.

$E_\nu$ scalar field component of wave $\nu$, p. 93.

$F$ floor attenuation factor, p. 25.

$f$ a function with sufficiently nice properties, p. 34.

$G$ antenna gain, p. 22.

$g$ a function with sufficiently nice properties, p. 68.

$G_C$ Gaussian curvature of a surface, p. 16.

$G_{R_x}$ antenna gain of the $R_x$, p. 23.

$G_{T_x}$ antenna gain of the $T_x$, p. 23.

$G_p(.)$ characteristic function of the density $\rho$, p. 94.


$I_0$ 0th order modified Bessel function of the first kind, p. 28.

$I_{q,n}$ measure of two points in $n$ dimensions, weighted with inverse distance law with exponent $q$, p. 35.

$I_{q,n}^{AH,1}$ bound for the Ad-Hoc network, based on (6.20), p. 73.

$I_{q,n}^{AH,2}$ bound for the Ad-Hoc network, based on (6.28), p. 77.

$I_{q,n}^{WF}$ bound for the WLAN with fixed access point, p. 67.

$I_{q,n}^{W_r,1}$ bound for the WLAN with random access point, based on (6.45), p. 84.

$I_{q,n}^{W_r,2}$ bound for the WLAN with random access point, based on (6.46), p. 84.

$k$ the absolute value of the wave vector $k$, p. 11.

$k_f$ k-factor (ratio of the deterministic and the scattered components of the received power), p. 28.

$l$ integer, used as exponent, p. 141.
\( L \) integer, used as exponent, p. 141.
\( l' \) integer, used as exponent, p. 141.
\( L' \) integer, used as exponent, p. 141.
\( L^p \) the \( L^p \) norm, p. 141.
\( L_{r<(v)}^{(v)} \) maximum number of considered materials in a room, p. 52.
\( L_{\nu s,\epsilon_r^{(v)}} \) maximum number of dielectric constants required for a ray-tracing simulation with given model order, p. 50.
\( L_{\nu s} \) maximum number of considered virtual sources (model order), p. 50.
\( n \) dimension of space (two or three), p. 12.
\( N \) number of waves that impinge within a particular delay bin, p. 93.
\( n_{\nu} \) number of reflections that wave \( \nu \) has experienced, p. 93.
\( O \) origin of a coordinate system, p. 33.
\( O_{n-1} \) surface area of a hypersphere in \( n \) dimensions, p. 33.
\( P_{LOS} \) power of the LOS (or the dominant) component of the field, p. 28.
\( P_{Rx} \) received power or pathloss, p. 25.
\( q \) pathloss exponent, p. 25.
\( q' \) \( 2q + q' = q + \tilde{q} \) by definition of \( q' \), p. 147.
\( q_e \) electric charge density, p. 10.
\( q_m \) magnetic charge density, p. 10.
\( r \) distance between \( T_x \) and \( R_x \), p. 22.
\( R(\Omega) \) distance between a point in a domain \( \mathcal{D} \) and the boundary \( \partial \mathcal{D} \) in direction of \( \Omega \), p. 33.
Reflection coefficient for parallel polarized electromagnetic waves, p. 19.

Reflection coefficient for perpendicular polarized electromagnetic waves, p. 19.

Receiver, p. 3.


A one dimensional parameter, p. 15.

The surface of a manifold $\mathcal{M}$, p. 40.

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Maximum irradiated spatial power density of an antenna, p. 22.


Fresnel transmission coefficient for parallel polarized electromagnetic waves, p. 19.

Fresnel transmission coefficient for perpendicular polarized electromagnetic waves, p. 19.

Transmitter, p. 3.

Coordinate of a point $\nu$ on a line $\mathcal{L}$, p. 36.

Velocity of the change of the manifold $\mathcal{M}$ normal to its surface, p. 40.

$n$ dimensional volume of a domain $\mathcal{D}$, p. 12.

Virtual receiver, p. 53.

$x$-axis of a cartesian coordinate system, p. 31.

Coordinates $\nu_1$ (of point or object $\nu$), p. 39.

$y$-axis of a cartesian coordinate system, p. 34.
\( z \) : z-axis of a cartesian coordinate system, p. 34.

\( z_{LTV} \) : signal of a linear time variant system, p. 26.

\( D \) : domain of points, p. 12.

\( D_{Rx} \) : domain of \( R_x \) locations, p. 43.

\( D_{Tx} \) : domain of \( T_x \) locations, p. 43.

\( F \) : a plane perpendicular to \( L \), p. 35.

\( L \) : a line in \( \mathbb{E}^n \), p. 35.

\( \mathcal{M} \) : a manifold as defined in differential geometry, p. 40.

\( \mathcal{M}_s \) : a manifold which depends on a parameter \( s \), p. 81.

\( \mathcal{N} \) : the set of measure 0, p. 141.

\( P_\nu \) : measure of point \( \nu \), p. 35.

\( \mathcal{P} \) : measure, p. 33.

\( U_{n-1} \) : angular domain of a hypersphere in \( n \) dimensions, p. 33.

\( \mathbb{E}^n \) : \( n \) dimensional space of real numbers with Euclidean distance, p. 32.

\( \mathbb{R}^+ \) : space of positive real numbers, without 0, p. 39.

\( \mathbb{R}^n \) : \( n \) dimensional space of real numbers, p. 32.

**Mathematical Symbols**

\((.)*\) : conjugation of a complex quantity, p. 15.

\( \cdot \) : scalar product between two vectors, p. 10.

\( \delta_{\nu_1 \nu_2} \) : Kronecker delta, p. 146.

\( \emptyset \) : the empty set, p. 37.

\( \gg \) : much greater than, p. 147.
\[ \succ \] approximately greater than, p. 72.

\[ \langle . \rangle \] expectation of a quantity, p. 89.

\[ \langle . \rangle_D \] expectation of a function over two uniformly in \( D \) distributed points, p. 84.

\[ \langle . \rangle_{D \partial D} \] expectation of a function over a point uniformly distributed in \( D \) and another on \( \partial D \), p. 84.

\[ \langle . | . \rangle \] conditional expectation of a quantity, p. 43.

\[ \ll \] much lower than, p. 93.

\[ \text{ln} \] natural logarithm, p. 28.

\[ \log_{10} \] logarithm to the base 10, p. 25.

\[ \preccurlyeq \] approximately lower than, p. 69.

\[ \text{Var} \] variance, p. 96.

\[ \nabla \] nabla operator, p. 10.

\[ \sim \] proportional to, p. 14.

\[ \simeq \] about equal, p. 93.

\[ \text{Re}(.) \] real part of an argument or function, p. 15.

\[ \times \] between vectors: vector product in \( \mathbb{R}^3 \), p. 10; between sets: cartesian product, p. 41.

\[ \mathcal{B}_{\rho_{\text{min}}}(r) \] ball of radius \( \rho_{\text{min}} \) around a point \( r \), p. 67.

\[ | . | \] constrained to (for sets), p. 37.

\[ . | . \] for real or complex numbers: absolute value, p. 15.

\[ . | . | \] Euclidean distance, p. 12.

\[ . | . |_{\infty} \] \( L^\infty \) norm, p. 81.
Measurement Environments

$E_{T_x}^{r}$ measurement location on the E-floor in the ETF building, with fixed $R_x$ position and a set of $T_x$ positions; the $T_x$ positions are room 1 (1), room 2 (2), or both rooms (1+2), p. 120.

$E_{\nu}$ measurement with both $T_x$ and $R_x$ randomly located in the room $\nu$ on the E-floor in the ETF building, p. 121.

$F_{T_x}^{r}$ measurement location on the F-floor in the ETF building, with fixed $R_x$ position and a particular set of $T_x$ positions, which include all rooms (all), the corridor (corr) or both, i.e., all positions (all), p. 119.

$Z_{T_x}^{r}$ measurement location on the K-floor in the ETZ building, with fixed $R_x$ position and a set of $T_x$ positions; the $T_x$ positions in the short corridor are denoted by (0), those in the long corridor by (1), p. 121.

$Z_{0}$ measurement on the K-floor in the ETZ building for a WLAN with randomly distributed access point, p. 121.
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Curriculum Vitae

Jan Carsten Hansen, born 1 April 1973 in Bochum, Germany.

Education:
1992 Enrolled for physics at Freiburg University, Germany.
1995 Received B.Sc. (Honors Program) in mathematics/physics from Trent University, Peterborough (ON), Canada.
1996/1997 Research student at the Institute for Chemical Research, Kyoto University, Japan; work in neutron scattering and polymer chemistry.
1998 Diploma in physics of Freiburg University; diploma thesis in theoretical physics at the Freiburg Materials Research Center.
03/2002 Post Diploma in information technology of ETH Zürich.

Professional Experience:
1998-2002 PhD student and research assistant at the Communication Technology Laboratory (CTL) of ETH Zürich (Prof. Leuthold):

- Work in channel modeling and channel measurements for RFIDs and WLANs.
- Research cooperation with several companies.
- Activities in the European projects COST 259 & COST 273.
- Work at the mathematical department of Salzburg University, Austria (Prof. Buchta), in integral geometry and its application to channel modeling.
- PhD thesis.