Observation of Taurus A with the 5 m radio-telescope of ETH in Zurich

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Observation of Taurus A with the 5m Radio-Telescope of ETH in Zurich

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1 Crab Nebula

In Chinese annals of the year 1054AD we can read of a “guest star”. This star was far brighter than Venus. After three weeks the “guest star” vanished. We can still see some remnants of this event. On picture 1 we see the Crab Nebula taken with the VLT-telescope. This Nebula is some 6500 light-years away.

![Crab Nebula taken with VLT](image)

Figure 1: Crab Nebula taken with VLT

1.1 Supernova

Today we know that this “guest star” was in fact a supernova, the explosion of a star. On picture 1 we see some remnants of this supernova. In the center of the nebula is a pulsar which isn’t visible on picture 1. The supernova which produced the Crab Nebula is believed to be of Type II explosion from a massive (10 – 15 $M_\odot$) star. The iron-rich core of the star collapses inward under the influence of its own gravity. Energy will be released in form of neutrinos. The outer layers of the star also fall inward under gravity, providing the energy to power the nuclear reactions that generate the supernova’s electromagnetic radiation. The outer layers of the star are blasted into space. The expanding shell of gas begins to glow as it collides
with atoms in the interstellar medium. Under the high pressure in the core protons and electrons combine to form neutrons. The collapse stops when the neutron pressure is equal to gravitation pressure. The neutron pressure is a consequence of the Pauli exclusion principle. This new extremely compact object is called neutron star. Conservation of angular momentum leads to the fast rotation of neutron stars. It seems safe to say that every star possesses some magnetic field. These magnetic fields are normally quiet low. The magnetic field is bonded to the star’s gases, so that during the collapse the field becomes concentrated. This means for a neutron star that the magnetic field increases by a factor of $10^{10}$. The rotation of a neutron star combined with the strong magnetic field builds an object called pulsar.

![Figure 2: Pulsar in the Crab Nebula](image)

In the year 1968/1969 the pulsar PSR B0531+21 could be identified with the central star of the Crab Nebula. On picture 2 we can see an X-ray image taken by the Chandra satellite. The rotation period of this pulsar is 33.2ms. Before the discovery of rapidly rotating pulsars astronomers believed that every star ends in a white dwarf. It was assumed that dying stars somehow manage to eject enough matter so that their corpses do not exceed the Chandrasekhar limit. The Chandrasekhar limit gives a limit for the mass whether a star ends in a white dwarf or in a neutron star or even a black
hole. This limit is at $1.5M_\odot$. The pulsar in the Crab Nebula could not be a white dwarf. The density of a white dwarf is not big enough to allow rotation periods of 33ms. Centrifugal-force would tear apart such a star. From the existence of the neutron star we can approximate the progenitor star’s mass of the Crab Nebula as $10 - 15M_\odot$.

1.2 Model for the Crab Nebula

The radio source Taurus A was identified with the Crab Nebula by J.G. Bolton. When relativistic charged particles are accelerated they loose energy in form of synchrotron radiation. This synchrotron radiation we can measure with our radio telescopes. In picture 2 we see the crab pulsar with its jets. The jets are along the axis of the magnetic poles. The magnetic field lines originate and end in the magnetic poles. Charged particles can’t cross these field lines because of lorentz-force. Therefore we have this torus-like structure of captured charged particles. In the direction of the jets there are no field lines to be crossed. Charged particles will be accelerated so that they can escape from the pulsar. This flux of charged accelerated particles builds the jets. The particles are bounded on spirals around the field lines, so that they emit synchrotron radiation. The energy for all these processes comes from the pulsar. Conservation of energy leads to a slowdown of the pulsar’s rotation rate. In general the rotation axis of a pulsar is not in a line with the jets. When the Earth’s orbit and the direction of the jet cross we can measure a pulse. The time interval between two pulses determines the rotation rate. The crab pulsar changed its rotation rate from 33Hz to 29Hz since it had been first measured.

That we can still see the remains of the supernova of 1054 in the optical spectrum is also a consequence of the pulsar. In picture 1 we see the ionized ejecta of the supernova building these filaments. The ejecta was blasted into the interstellar space by the supernova explosion. In the center is the pulsar. Connecting the two components is the Crab synchrotron nebula. The synchrotron nebula from the pulsar is confined by an expanding ejecta surrounding the Crab. This skin covers the filaments and can be understood as the cooling region behind a radiative shock. The pressure of the Crab synchrotron nebula is responsible for pushing on and accelerating the expanding complex of filaments. The filaments are concentrated in a band surrounding the same axis as the X-ray torus which is visible in picture 2. The difference between the expansion velocity of the outer part of the nebula and the free expansion velocity at that radius assuming an explosion date of 1054AD is the shock velocity.
2 Radio Astronomy

2.1 History

In 1888 Heinrich Hertz produced radio waves of a few centimeters long. At that time it was also known that visible light included only a small range of wavelengths. But it lasted until 1931 to detect radio frequency radiation of extraterrestrial sources.

Karl G. Jansky worked as a radio engineer at the Bell Telephone Laboratories. In 1931, he was assigned to study radio frequency interference from thunderstorms in order to help Bell design an antenna that would minimize static (noise produced by unmodulated radio frequency radiation) when beaming radio-telephone signals across the ocean. With the antenna Jansky built he found some static he couldn’t identify with thunderstorms. He observed that the radiation peaked about 4 minutes earlier each day. If it would have been the sun the period should be exactly a day. The rotation period of Earth with respect to the stars (sideral day) is about 4 minutes shorter than a solar day. Jansky therefore concluded that the source of this radiation must be much farther away than the sun. He identified the source as the Milky Way. This was the beginning of a new window to space. Since 1931 several radio sources have been detected.

Figure 3: The first radio-telescope built by Karl G. Jansky
2.2 Atmospheric windows

That we are able to see the stars in visible light and to detect radio sources from Earth has to do with the Earth’s atmosphere. There are only two spectral windows where electromagnetic waves can pass the atmosphere. We call these windows optical window and radio window. The optical window is limited by the absorption of Ozone $O_3$ and the absorption of water steam $H_2O$. The radio window extends from about 1cm to 10m wavelength. The upper limit comes from reflections at ionosphere due to the electron density. The short-wavelength limit comes from the absorption of water steam and is therefore a function of cloud cover etc.

![Transmissivity of a standard atmosphere](image.png)

Figure 4: Transisstance of a standard atmosphere

In figure 4 we can see that the wavelength range of the optical window is much smaller then the radio window. The borders of these windows can vary from place to place.

2.3 The radio telescope of ETH in Zurich

A radio telescope consists in general of an antenna and a receiver. The parabola antenna in Zurich is 5m in diameter. The antenna positions are controlled separately from a computer and can be indicated by azimuth and
elevation. The scheme of the system in Zurich is shown in figure 5. The incoming radio waves are collected in the parabola mirror and focused on the feed. The induced voltage from there is guided to the preamplifier (3). It is installed in the focuspack near the feed. To calibrate measurements a noise source is also installed.

![Diagram of system scheme]

Figure 5: Predecessor of Apraxos

In (2) we can switch over to measure instead of the antenna the noise source. In our case the calibration will be done by measuring the sun for which daily flux measurements are available. We bridged switch (2) to decrease the system temperature. The local-oscillator (4) produces oscillates in an known frequency $\nu_0$. We couple now the signal of the antenna $\nu_a$ and of the local-oscillator $\nu_0$ in the mixer (5). The frequency of the product is $\nu_{mix} = |\nu_a - \nu_0|$. It is easy to build a local-oscillator with variable frequency so that we can hold $\nu_{mix}$ constant. We transform all the incoming signals into the constant frequency $\nu_{mix}$ by varying the frequency $\nu_0$. Now the signal of the frequency $\nu_{mix} = const$ will be filtered in (6) and sent to the main amplifier (7). In (8) the signal is detected. In other words the oscillation is converted into a voltage of constant sign. Integration (9) of the signal over a certain time allows to smooth disturbances. We have to care that the time interval is not too large otherwise we have a loss of information. The
analog/digital-converter (10) finally transforms the detected voltage or the signal into an input for the computer (11).

2.4 General relations

The infinitesimal power \( dW \) from a solid angle \( d\Omega \) of the sky incident on a surface of area \( dA \) is

\[
dW = B \cos \theta d\Omega d\nu
\]

(1)

where \( dW \) = infinitesimal power, \([\text{watts}]\); \( B \) = brightness of sky at position of \( d\Omega \), \([\text{watts m}^{-2}\text{cps}^{-1}\text{rad}^{-2}]\); \( d\Omega \) = infinitesimal solid angle of sky \( (= \sin \theta d\theta d\Phi) \), \([\text{rad}^2]\); \( \theta \) = angle between \( d\Omega \) and zenith, \([\text{rad}]\); \( dA \) = infinitesimal area of surface, \([\text{m}^2]\); \( d\nu \) = infinitesimal element of bandwidth, \([\text{cps}]\).

The brightness \( B \) is a measure of the power received per unit area per unit solid angle per unit bandwidth. If \( dW \) is independent of the position of \( dA \) on the surface the integration over the entire surface \( A \) yields

\[
dW = AB \cos \theta d\Omega d\nu
\]

(2)

By integrating formula (1) we obtain the power \( W \) received over a bandwidth \( \Delta \nu \) from a solid angle \( \Omega \) of the sky.

\[
W = A \int_{\nu}^{\nu + \Delta \nu} \int_{\Omega} B \cos \theta d\Omega d\nu
\]

(3)

The spectral power \( dw \) is the power per unit bandwidth

\[
dw = \frac{dW}{d\nu} = B \cos \theta d\Omega dA
\]

(4)

\( dw \) = spectral power, \([\text{watts} \ \text{cps}^{-1}]\).

If \( dw \) is independent of the position of \( dA \) on the surface \( A \), the spectral power received by the entire surface is then

\[
dw = AB \cos \theta d\Omega
\]

(5)

Integrating (5), one can obtain the spectral power from a solid angle \( \Omega \) of the sky.

\[
w = A \int_{\Omega} \int B \cos \theta d\Omega
\]

(6)
The sky brightness is in general a function of angle. Therefore we write $B(\Theta, \Phi)$. Thus (6) becomes

$$w = A \int \int B(\Theta, \Phi) \cos \Theta d\Omega$$  \hspace{1cm} (7)

In the following figure we see the situation for radio astronomy.

![Figure 6: Antenna lobes (simulation)](image)

The area $A$ must be replaced by the flat horizontal surface of a receiving antenna with the power pattern of the antenna directed toward the zenith ($\Theta = 0^\circ$). We call this area effective aperture $A_e$ of the antenna. The effective aperture is not equal the physical aperture of an antenna, it is somewhat less for antennas of large aperture. The power pattern $P_n(\Theta, \Phi)$ is a measure for the response of the antenna to radiation as a function of the angles $\Theta$ and $\Phi$. The maximum value is unity and it replaces the factor
\( \cos \Theta \) of (7). Replacing \( A \) and \( \cos \Theta \) by \( A_e \) and \( P_n(\Theta, \Phi) \) in (7) we have

\[
w = \frac{1}{2} A_e \int \int B(\Theta, \Phi) P_n(\Theta, \Phi) d\Omega \tag{8}
\]

\( P_n(\Theta, \Phi) \) = normalized power pattern of antenna, [dimensionless]; \( A_e \) = effective aperture, [m²].

For an isotropic antenna is \( P_n(\Theta, \Phi) = 1 \) at all angles. Integration of \( P_n(\Theta, \Phi) \) over \( d\Omega \) leads to

\[
\int \int P_n(\Theta, \Phi) d\Omega = \int d\Omega = 2\pi \tag{9}
\]

On the other hand we have

\[
\int \int \cos \Theta d\Omega = \int_0^{2\pi} \int_0^{\pi/2} \cos \Theta \sin \Theta d\Theta d\Phi = \pi \tag{10}
\]

To obtain the same result from (7) and (8) we need the factor \( \frac{1}{2} \) in formula (8). For constant brightness \( B_c \) (8) can be written

\[
w = \frac{1}{2} A_e B_c \int \int P_n(\Theta, \Phi) d\Omega \tag{11}
\]

By evaluation of the integral in (11) over a solid angle of \( 4\pi \) we get the beam area or beam solid angle \( \Omega_A \) of the antenna.

\[
\Omega_A = \int \int P_n(\Theta, \Phi) d\Omega \tag{12}
\]

\( \Omega_A \) = beam area, [rad²].

For any discrete source, the integral of the brightness over the source yields the total source flux density.

\[
S = \int \int B(\Theta, \Phi) d\Omega \tag{13}
\]

\( S \) = flux density of source, [Jan or wattsm⁻²cps⁻¹].

Most radio sources have a flux density of the order of \( 10^{-26} \text{ Jan} \). The quantity \( 10^{-26} \text{ wattm}^{-2} \text{cps}^{-1} \) or \( 10^{-26} \text{ Jan} \) is called a flux unit. If a radio source is
observed by an antenna, the measured flux density $S_0$ is less than the flux density of the source $S$.

$$S_0 = \int \int_{source} B(\Theta, \Phi) P_n(\Theta, \Phi) d\Omega$$  \hspace{1cm} (14)$$

The antenna pattern $P_n(\Theta, \Phi)$ is like a weighting function. For sources of small extent we have $P_n \approx 1$. Then the measured flux density and the real flux density are almost equal.

### 2.5 Rayleigh-Jeans Law

*Planck’s radiation law* allows to calculate the brightness of the radiation from a blackbody radiator at a temperature $T$ and frequency $\nu$.

$$B = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$ \hspace{1cm} (15)$$

$h =$Planck’s constant; $\nu =$frequency, [cps]; $c =$velocity of light; $k =$Boltzmann’s constant; $T =$temperature, [$^\circ$ K].

In the radio part of spectrum is $h\nu << kT$ so that we may use the approximation

$$e^{h\nu/kT} - 1 \approx 1 + \frac{h\nu}{kT} - 1 = \frac{h\nu}{kT}$$ \hspace{1cm} (16)$$

Use of relation (16) in formula (15) leads to

$$B = \frac{2h\nu^3}{c^2} \frac{kT}{h\nu} = \frac{2\nu^2 kT}{c^2} = \frac{2kT}{\lambda^2}$$ \hspace{1cm} (17)$$

$\lambda =$wavelength, [m].

This equation is the *Rayleigh-Jeans radiation law*.

### 2.6 Antenna Temperature

According to equation (8) we have

$$w = \frac{1}{2} A_e \int_{\Omega} B(\Theta, \Phi) P_n(\Theta, \Phi) d\Omega$$ \hspace{1cm} (18)$$

For an antenna inside of a blackbody enclosure at a temperature $T$ we have a constant brightness $B_c$. The Rayleigh-Jeans law allows to calculate the brightness in equation (18). Thus we have

$$B(\Theta, \Phi) = B_c = \frac{2kT}{\lambda^2}$$ \hspace{1cm} (19)$$
Introducing (19) into (18) and use of equation (12) yields

\[ w = \frac{1}{2} A_c \frac{2kT}{\lambda^2} \int \int P_n(\Theta, \Phi) d\Omega = \frac{kT}{\lambda^2} A_c \Omega_A \]  

(20)

The effective aperture \( A_c \) and the beam solid angle afford the equation

\[ \lambda^2 = A_c \Omega_A \]  

(21)

This relation follows from reflections on the electric field intensity in the aperture and the radiated field. It will not be shown here. From (20) and (21) we see that

\[ w = kT \]  

(22)

Equation (22) is also true for a resistor of resistance \( R \) and temperature \( T \). In our case \( T \) is not the temperature of the antenna structure itself. The temperature of the radiation resistance is determined by the temperature of the emitting region which the antenna “sees” through its directional pattern. It is the temperature of the region within the antenna beam which determines the temperature of the radiation resistance. The temperature of the antenna radiation resistance is called the *antenna temperature*. Thus from (18) and (22) the received power per unit bandwidth is given by

\[ w = \frac{1}{2} A_c \int \int B(\Theta, \Phi) P_n(\Theta, \Phi) d\Omega = kT_A \]  

(23)

Use of (14) leads to

\[ S_0 = \frac{2kT_A}{A_c} \]  

(24)

### 2.7 Gain of a parabola antenna

The *directivity* is defined as the ratio of the maximum radiation intensity to the average radiation intensity.

\[ D = \frac{U(\Theta, \Phi)_{max}}{U_{avg}} \]  

(25)

\( U(\Theta, \Phi)_{max} \) = maximum radiation intensity, [watts rad\(^{-2}\)]; \( U_{avg} \) = average radiation intensity, [watts rad\(^{-2}\)].

The average radiation intensity is given by the total power \( W \) radiated
divided by $4\pi$, and the total power is equal to the radiation intensity $U(\Theta, \Phi)$ integrated over $4\pi$. Hence
\[
D = \frac{U(\Theta, \Phi)_{\text{max}}}{W/4\pi} = \frac{4\pi}{\int \int \frac{U(\Theta, \Phi)}{U(\Theta, \Phi)_{\text{max}}} d\Omega}
\]  
(26)

The poynting vector is proportional to the radiation intensity so that
\[
\frac{U(\Theta, \Phi)}{U(\Theta, \Phi)_{\text{max}}} = \frac{P(\Theta, \Phi)}{P(\Theta, \Phi)_{\text{max}}} = P_n(\Theta, \Phi)
\]
(27)

With the use of (27) and (12) relation (26) can be expressed as
\[
D = \frac{4\pi}{\int \int P_n(\Theta, \Phi) d\Omega} = \frac{4\pi}{\Omega_A}
\]
(28)

From (21) and (28) we have
\[
D = \frac{4\pi}{\lambda^2} A_e
\]
(29)

If ohmic losses are not negligible we have to distinguish between the actual effective aperture (including the effect of ohmic losses) and an effective aperture based entirely on the pattern (losses neglected). Thus, we may write
\[
A_e = k_0 A_{e_p}
\]
(30)

$k_0 =$ohmic-loss factor, dimensionless ($k_0 \leq 1$); $A_{e_p} =$effective aperture as determined entirely by pattern, $[m^2]$.

Then the directivity is given by
\[
D = \frac{4\pi}{\lambda^2} A_{e_p}
\]
(31)

The gain is defined as
\[
G = Dk_0 = k_0 \frac{4\pi}{\lambda^2} A_{e_p}
\]
(32)

We calculate now the gain of the radio telescope in Zurich. The telescope has a diameter of $d = 5m$ and an ohmic-loss factor of about $k_0 \approx 0.5$.

\[
G = k_0 \frac{4\pi}{\lambda^2} \frac{d^2 \Pi}{4} = k_0 \left( \frac{\pi d}{\lambda} \right)^2
\]
(33)
The gain expressed in dezibels (33) becomes

\[
G(d, \lambda, k_0) = 10 \log \left[ k_0 \left( \frac{\pi d}{\lambda} \right)^2 \right]
\]  

(34)

Figure 7: Gain of the parabola antenna in Zurich

2.8 Theoretical flux of Taurus A

In figure 7 we see that the gain of the antenna increases with higher frequencies. Therefore one could assume that the highest frequency should be used for our measurements. But in general the flux of a source also depends on the frequency. According to the Rayleigh-Jeans law the flux density \(S\) of an object at a wavelength \(\lambda\) in the radio spectrum is given by

\[
S = \int \int B(\Theta, \Phi) d\Omega = \frac{2k}{\lambda^2} \int \int T d\Omega
\]

(35)

\(T\) = equivalent blackbody temperature, \([\degree K]\).

Uniform temperature over the source leads to

\[
S = \frac{2k}{\lambda^2} T \Omega_s
\]

(36)
\( \Omega_S = \text{source solid angle}. \) We have the proportionality

\[
S \propto \lambda^n
\]  
(37)

If the temperature of a source is constant with wavelength the spectral index \( n \) is equal to \(-2\). This variation in the flux is typical for the thermal radiation from a blackbody. Nonthermal sources have a positive index. In a more accurate estimate we may use the relation [6]

\[
\log(S[\text{Jy}]) = a + b \cdot \log(\nu[\text{MHz}]) + c \cdot \log^2(\nu[\text{MHz}])
\]  
(38)

For Taurus A the parameters are \( a = 3.915 \pm 0.031 \), \( b = -0.299 \pm 0.009 \), \( c = 0 \). As described in section (1.2) Taurus A is a source for synchrotron radiation and is therefore nonthermal. In figure 8 the flux of Taurus A is shown according to

\[
S(\nu) = 10^{3.915 - 0.299 \log(\nu \times 10^6)}
\]  
(39)

![Flux of Taurus A](image.png)

Figure 8: Theoretical flux of Taurus A

For the antenna in Zurich we can also calculate the antenna temperature in the case of Taurus A. From (24) follows

\[
T_A = \frac{S_0 A_e}{2k} = \frac{S_0 \pi d^2 k_0}{4 \times 2k} = \frac{S_0 \pi d^2 k_0}{8k}
\]  
(40)
Figure 9: Antenna temperature for Taurus A

The analysis of figure 7 to figure 9 let me choose frequencies around 1700MHz to be tested first.

3 Measurements

The radio telescope of ETH stands in the city of Zurich. It is clear that this place is not optimal. Nevertheless it is possible to do some experiments. The detection of Taurus A will be at the limit of the instrument in this surroundings. Some frequencies are no longer applicable because of man made noise (mobile communication is called man made noise by astronomer’s). The first excercise was to find quiet frequencies with a low receiver noise temperature.

3.1 Receiver noise temperature

In formula (22) the noise power per unit bandwidth from an antenna is given by $kT_A$. The noise power from the antenna is then

$$W_{NA} = kT_A \Delta \nu$$

(41)

$W_{NA}$ = antenna noise power, [watts]; $\Delta \nu$ = bandwidth, [cps].

Our system consists of antenna, receiver and the connecting cables. We
have losses in the transmission line and thermal noise from the components of the receiver. The system noise power is the sum of antenna noise power and receiver noise power.

\[ W_{sys} = W_{NA} + W_{NR} = k(T_A + T_{RT}) \Delta \nu \]  

(42)

We define the system noise temperature as

\[ T_{sys} = T_A + T_{RT} \]  

(43)

\( T_{RT} = \) receiver noise temperature (including transmission line), [°K]; \( W_{NR} = \) receiver noise power referred to the antenna terminals, [watts].

The y-factor is the ratio of the system temperature of the sun and the system temperature of the sky.

\[ y = \frac{T_{sys}(sun)}{T_{sys}(sky)} \]  

(44)

According to (43) we have for the y-factor:

\[ y = \frac{T_{RT} + T_A(sun) + T_A(sky)}{T_{RT} + T_A(sky)} \]  

(45)

We isolate \( T_{RT} \) from (45)

\[ T_{RT} = \frac{T_A(sun)}{y - 1} - T_A(sky) \]  

(46)

\( T_A(sun) \) can be calculated by knowing the sun flux and the geometry of the telescope according to formula (40). The y-factor is determined graphically from a calibration measure with the sun. For \( T_A(sky) \) I supposed 20°K.

The values for the daily sun flux are available on the web so that \( T_{RT} \) can be calculated. Beside the y-factor \( T_{RT} \) also helps to judge the different frequencies. Comparing different frequencies let me choose 1675MHz for the measurements. All measurements had been done with this frequency by the intention of cumulate several measurements.

### 3.2 Allen-Variance-Plot

In every measurement we take we have an error \( E \). From Gaussian statistics we know that taking \( N \) measurements minimizes the error by a factor \( 1/\sqrt{N} \). The receiver has a bandwidth of \( \Delta \nu \). Events with a frequency below \( \Delta \nu \) can't be separated by the receiver. The receiver registrates then only one event.
This means that these events are not independant. To have two independant events the time interval must be at least $\Delta t = 1/\Delta \nu$. In other words in a time interval $\tau$ we have $N = \tau \ast \Delta \nu$ independant events. Supposing a Gaussian distribution of these events we obtain the relation:

$$\Delta T = \frac{T_{sys}}{\sqrt{N}} = \frac{T_{sys}}{\sqrt{\tau \Delta \nu}}$$  \hspace{1cm} (47)

In the figure below this relation is shown on a logarithmic scale.

![Allen-Variance-Plot for 1675 MHz](image)

Figure 10: Allen-Variance -Plot at 1675MHz.

We can make out a trend to lower variance with raised sampletime as demanded from theorie. The plot helps to choose the sampletime for the measurements.

3.3 On-Off-Source

The first measurements of Taurus A had been done with the on-off-source method. The position of the telescope is for a certain time on the source. Then the computer moves the telescope automatically away from the source. The telescope is now in the off-source mode and measures for the same time the sky without source. Afterwards the telescope moves back to the source and the cycle starts again. In figure 11 we can see the plot of these measurements.
Figure 11: Taurus A measured with the On-Off-Source method at 1675MHz; Date: 24.5.02; Sampletime: 3s; Horizontal deflection angle: 10°

This plot allowed me to determine the time intervals for measurements with the drift-method. The drift-method will be explained later. The sharp peaks in the plot are disturbances which can be eliminated by hand. More problematic are the disturbances in the time interval from 11.00 to 11.30 UTC. It is possible that the telescope pointed towards a communication antenna at that time. Therefore this time interval is not usable for measurements. Between 12.00 and 13.00 UTC the plot is very clear so that this time interval is applicable for our measurements. Knowing the time intervals allowed me to obtain “quiet” directions for the telescope. Taurus A will be measured at these positions.

3.4 Flux of Taurus A

To determine the flux of Taurus A we have to fit the plot. The next figure shows the plot of our data $Y$ versus the time $X$. To calibrate the data we fitted the curve with a polynom $p$ of 9th degree.
Figure 12: Polynomial fit for Taurus A measured with on-off-source method at 1675MHz. The polynom is of 9th degree.

Subtracting the polynom $p$ from the data $Y(X)$ generates the new data $Y^-(X)$.

$$Y^-(X) = Y(X) - p(X)$$

In a further step we take the absolut value of our new data $Y^-(X)$.

$$Y_{end}(X) = |Y^-(X)|$$
Figure 13: Plot of $Y^-$ versus X

Twice the median $M(Y_{end})$ of the plot above is just the amplitude for the flux. To calibrate the data we have determined the receiver-noise temperature with a measurement of the sun. The amplitude in the plot is given in $dB$. The antenna temperature $T_A(tau)$ calculates then as followed:

$$T_A(tau) = T_{RT} \times (10^{2.5M(Y_{end})/10} - 1)$$

Thus we get from our data the following value for $T_A$:

$$T_A = 2.59^\circ K$$

The flux can be obtained by use of formula (24):

$$S = \frac{2kT_A}{A_e} \times FU \approx 729FU$$

The theoretical value for Taurus A from formula (38) is 893$FU$. 

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3.5 Drift of Taurus A

A further observation method is called drift-method. We choose a parking position for the telescope which will be passed by Taurus A in the following hours. We start our measurement 1.5 hour before the meeting time of source and telescope.

![Taurus A at 1675 MHz](image)

Figure 14: Drift of Taurus A at 1675MHz; Date: 22.8.02; Sampletime: 3s; Position: azimuth: 150.71°; elevation: 62.17°

In figure 14 such a measurement is shown. At 6.00 UTC Taurus A passed the position of the telescope. At the left side of the peak we see some disturbances. Without these disturbances the peak would have the shape of a gaussian curve. The quality of such a measurement can be judged with the signal to noise ratio. In this case it is $S/\sigma = 6.7$ which is quiet good. To improve this ratio several measurements at 1675MHz had been done. The cumulation of all these measurements should improve the signal to noise ratio by a factor $\sqrt{N}$. 9 out of 19 drift-measurements are listed in table 1.
<table>
<thead>
<tr>
<th>Date</th>
<th>Time [UTC]</th>
<th>Azimuth</th>
<th>Elevation</th>
<th>$S/\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.05.02</td>
<td>10.30</td>
<td>124.28°</td>
<td>54.30°</td>
<td>4.033</td>
</tr>
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<td>125.54°</td>
<td>54.85°</td>
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<td>14.00</td>
<td>225.97°</td>
<td>58.03°</td>
<td>3.3355</td>
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<tr>
<td>4.07.02</td>
<td>13.00</td>
<td>248.53°</td>
<td>47.49°</td>
<td>2.647</td>
</tr>
<tr>
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<td>13.00</td>
<td>249.52°</td>
<td>46.87°</td>
<td>0.968</td>
</tr>
<tr>
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<td>13.00</td>
<td>250.48°</td>
<td>46.24°</td>
<td>7.2072</td>
</tr>
<tr>
<td>22.08.02</td>
<td>6.00</td>
<td>150.71°</td>
<td>62.17°</td>
<td>6.709</td>
</tr>
<tr>
<td>23.08.02</td>
<td>6.00</td>
<td>152.56°</td>
<td>62.49°</td>
<td>5.5136</td>
</tr>
<tr>
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<td>252.57°</td>
<td>44.84°</td>
<td>2.3395</td>
</tr>
</tbody>
</table>

| Geometric mean of $S/\sigma$ | 2.9324 |

Table 1: List of drift-measurements

The signal to noise ratio shows how much these measurements can vary. In some measurements the disturbances are so strong that it isn’t possible to recognize the passing of Taurus A. I have chosen the best measurements from above to determine the flux of Taurus A. The evaluation could be examined directly in the plots. The calibration was done in the same way as in the on-off-source measurement.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
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<td>Drift</td>
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<td>800</td>
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<td>-</td>
<td>On-Off</td>
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</tr>
</tbody>
</table>

| Arithmetic mean for the flux | 925 |
| Theoretical flux of Taurus A | 893 |

Table 2: Flux determination of Taurus A

3.6 Cumulation of all measurements in table 1

It was the aim to improve the signal to noise ratio by cumulation of several measurements. The disturbances are randomly distributed so that they neutralize. As mentioned before we suppose an improvement of our signal to noise ratio by a factor of $\sqrt{N}$. 

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Figure 15: Taurus A at 1675MHz. Cumulation of all 9 measurements in table 1.

The signal to noise ratio in figure 15 is $S/\sigma(data) = 9.633$. The supposed ratio was:

$$S/\sigma_theorie = \frac{S}{\sigma} \cdot \sqrt{N} = 2.9324 \cdot 3 = 8.7971$$

The slope at left in figure 15 comes from the sun. During the measurements in may the sun was near Taurus A. The sun passed the fix position of the telescope earlier than Taurus A.

4 Results

In the time of my practical Taurus A was only visible during the day. Therefore some measurements couldn’t be used because the disturbances were too strong. Another problem was that the orbit of the sun at the sky passed near the orbit of Taurus A in some measurements. At the left side of figure 15 we can see the consequence. It is possible that this led to an increase of the temperature of the focus pack and therefore an increase of the receiver noise temperature. The figure below shows a measurement which couldn’t be used. We can make out some steps in the curve so that it is impossible to make out the signal of Taurus A. The signal should be around 6.00 UTC.
Figure 16: Drift of Taurus A at 1675MHz; Date: 24.8.02; Sampletime: 3s; Position: azimuth: 154.44°; elevation: 62.78°

The reason for these steps is indeed an increase of the temperature of the preamplifier as shown in figure 17.

Figure 17: Temperature of preamplifier during the drift-measurement of Taurus A at 24.8.02
Figure 17 shows the temperature of the preamplifier versus the time. It was a sunny day and the telescope pointed towards the sun what explains the increase of 5°K during the measurement. All these effects are not helpful to get exact measurements. Nevertheless I think I could prove the existance of Taurus A. If the measurements could be done during the night the result should be even more clear. The measured flux is also acceptable. Comparing the measured flux with the theoretical flux allowes to determine a new ohmic-loss factor $k_0$ for the telescope at 1675MHz.

\[
S_0(\text{theorie}) = \frac{2kT_A(\text{measured})}{A_e} = \frac{2kT_A(\text{measured})}{k_0A_{ep}} \tag{48}
\]

Evaluating the data leads to:

\[k_0(\text{new}) \approx 0.518\]

The results in table 2 had been calculated with the factor $k_0(\text{old}) = 0.5$.

Acknowledgements

At least I would like to thank Christian Monstein for his help and assistance. I never succeeded in asking a question he couldn’t answer.