Power pattern, gain, directivity, effective aperture and stray factor of APRAXOS

Author(s): Fleischer, Daniel

Publication Date: 2003

Permanent Link: https://doi.org/10.3929/ethz-a-004547667

Rights / License: In Copyright - Non-Commercial Use Permitted
Power pattern, gain, directivity, effective aperture and stray factor of APRAXOS

Daniel Fleischer

Gubelstr. 2, CH-8050 Zurich, Switzerland, Daniel.Fleischer@web.de

Created 06.03.2003

Abstract. APRAXOS is a radiotelescope with a parabolic reflector of 5 m in diameter, located at the ETH Zurich. To measure its power pattern, gain, directivity, effective aperture and stray factor we used the sun as radio source. Scanning a field of 14 degrees around the center of the sun, we had enough data to visualize the power pattern of the antenna and to compute the values given above.

Key words. Power pattern, gain, directivity, effective aperture, stray factor.

1. Introduction

An antenna can be defined as the region of transition between a free-space wave and a guided wave (receiving case) or vice versa (transmitting case). In our case we have a receiving antenna attached to the parabolic radiotelescope APRAXOS, located at the ETH Zurich. To characterize an antenna there exist several values. We will take a closer look at values like gain, directivity, effective aperture and stray factor. Therefore we need to have the values beam solid angle and main-lobe solid angle. Other values like HPBW (half-power beam width) and BWFN (beam width between first nulls) are easy to get. To obtain these values we scanned a squarefield of length 14 degrees - the sun right in the middle of it - and measured the electromagnetic radiation at a given frequency-range around $\nu=1.7$ GHz. The way we scanned the squarefield is basically given by figure 1. For further computations we used only the data of the inner square of length 10 degrees.

2. Theory

We want to introduce some basic concepts of radio astronomy. We consider electromagnetic radiation from the sky falling on a flat horizontal area $A$ at the surface of the earth. The infinitesimal power $dW$ from a solid angle $d\Omega$ of the sky incident on a surface of area $dA$ is

$$dW = B \cos \theta \ d\Omega \ dA \ d\nu,$$

where $B$ is the brightness of the sky at position of $d\Omega$ ($W/(m^2\mathrm{Hz \ rad^2})$), $\theta$ is the angle between $d\Omega$ and zenith (rad) and $d\nu$ is the infinitesimal element of bandwidth (Hz). The brightness is one of the fundamental quantities of radio and optical astronomy and is a measure of the power received per unit area per solid angle per unit bandwidth. We define it to be only dependent of the angle $\Omega$ and the frequency $\nu$. Thus, when we want to compute the power $W$ received on an area $A$ for a frequency-interval $[\nu, \nu + \Delta\nu]$, we have to compute

$$W = A \int_\nu^{\nu + \Delta\nu} \int_\Omega B(\Omega, \nu) \cos \theta \ d\Omega \ d\nu. \quad (2)$$

Instead of $B(\Omega, \nu)$ we can also write $B(\theta, \phi, \nu)$, using the already given elevation angle $\theta$ and introducing the azimuth angle $\phi$.

Let us now replace the area $A$ by an antenna. Then we have to use the effective aperture $A_e$ instead of the area $A$ and the power pattern $P_h(\theta, \phi)$ replacing the term $\cos \theta$. $A_e$ is a factor depending on the geometry and the size of the reflector, that takes care, that we can use the same equations as in the case of a flat horizontal area. $P_h$ is di-
mensionless and normalized (in the sense of \( P_n(0, 0) = 1 \)). It is a measure of response of the antenna.

We assume the radiation to be unpolarized, so we additionally have to use a factor of \( 1/2 \), because we can only measure one certain polarization direction. Finally, with an antenna we receive the power

\[
W = \frac{1}{2} A_e \int_{\nu}^{\nu + \Delta \nu} \int_{\Omega} B(\theta, \phi, \nu) P_n(\theta, \phi) \, d\Omega \, d\nu. \quad (3)
\]

Just for completeness let us introduce another important quantity in radio astronomy. The flux density \( S \) (W/(m²Hz)) of a given source is defined by

\[
S = \int_{\text{source}} B(\theta, \phi) \, d\Omega. \quad (4)
\]

3. Measured values

The power pattern \( P_n \) of the antenna of APRAXOS is one of the main results and is given in figures 2 (rectangular coordinates) and 4 (spherical coordinates). To measure the power pattern we made the following assumptions:

- the sun is the only radio source around the scanfield,
- the sun is a point source,
- the sun is infinitely far away from the telescope.

The power pattern consists of a number of lobes. The lobe with the largest maximum is called main lobe. The figures also clearly show some minor lobes. Their existence is due to interference effects, just like rings of interferometers for example. Out of the figures we get two important values for characterizing the antenna: HPBW (half-power beam width) and BWFN (beam width between first nulls). Another way to describe the characteristics of the power pattern is in terms of solid angle. We define the beam solid angle \( \Omega_A \) (rad²)

\[
\Omega_A = \int_{4\pi} P_n(\theta, \phi) \, d\Omega. \quad (5)
\]
Fig. 4. Power pattern in spherical coordinates.

Analogous we define the main-beam solid angle $\Omega_M$ (rad$^2$)

$$\Omega_M = \int_{\text{main lobe}} P_A(\theta, \phi) \, d\Omega.$$  \hspace{2cm} (6)

Now we can define the directivity $D$ of the antenna

$$D = \frac{4\pi}{\Omega_A} = \frac{U_{\text{max}}}{U_{\text{avg}}}.$$  \hspace{2cm} (7)

where $U_{\text{max}}$ is the maximum radiation intensity and $U_{\text{avg}}$ the average one.

Directive gain $D(\theta, \phi)$ is defined through

$$D(\theta, \phi) = DP_n(\theta, \phi).$$  \hspace{2cm} (8)

So we can define $D_{\text{main}}$, respectively $D_{\text{minor}}$ as the directive gain in the maximum of the appropriate lobes.

To compute the effective aperture $A_e$ we use

$$A_e = \frac{\lambda^2}{\Omega_A},$$  \hspace{2cm} (9)

where $\lambda$ is the wavelength according to the frequency we used. Our last value of interest is the stray factor $\varepsilon_m$ and is defined through

$$\varepsilon_m = \frac{\Omega_m}{\Omega_A},$$  \hspace{2cm} (10)

where $\Omega_m = \Omega_A - \Omega_M$ is called minor-lobes solid angle.

Fig. 5. Magnification of the minor lobes in figure 4.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>1.70 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Integration time</td>
<td>100 ms</td>
</tr>
<tr>
<td>Scanfieldwidth</td>
<td>7°</td>
</tr>
<tr>
<td>Scanning speed</td>
<td>0.2° /s</td>
</tr>
</tbody>
</table>

Table 1. Settings.

4. Methods for measuring

The voltage induced in the antenna is received by a system of instruments, that amplify the signal. Finally the signal is digitally received by a PC, that is also responsible for steering the telescope. It takes about 3 hours to scan the whole scanfield. During this time the sun moves a non-neglectable way on the horizon. So we have to take care, that the centre of the scanfield is always incident on the sun. This is done by an automatic tracking program via the PC. The coordinates of the scanfield therefore are always relative to the moving centre. This movement induces a general problem, that in our case we can luckily neglect. The relative coordinates to the centre, let us call them $\Delta \phi$ and $\Delta \phi$ are computed as difference of the actual position of the telescope and the actual position of the sun, both in spherical coordinates. Now if the sun would be close to zenith the lower part of the scanfield would be very much wider than the upper one. The movement of the centre makes it even worse, because we get some time-dependencies on the geometry of the scanfield.
If we visualize the data in rectangular coordinates or in spherical coordinates with the centre fixed to a certain direction, we can expect some unwanted distortions of the power pattern. At first one could even try to explain the asymmetry of figure 4 with this fact. But this is not the case. There must be other reasons for the asymmetry, like little asymmetries in the reflector or the antenna itself. In this period of the year, when we did our measurements, the sun was below 30°, so there is only a very small difference between the upper and lower part of the scanfield, speaking of less than 10% in width. For our measurements we used the settings given in table 1.

We also tried measurements at 1.50, 1.55, 1.60, 1.65, 1.75 and 1.80 GHz, but 1.70 GHz proved to be the best frequency. Especially below 1.60 GHz we had to deal with a lot of scattering in the dataset. We made 3 series with 1.70 GHz, that delivered beautiful results. For the first measures we directly used the PC connected to the telescope. But for further measures we connected this PC via the internet, using VNC (Virtual Network Computing). This proved to work properly and was a very elegant way to do some measures also on weekends.

5. Methods for visualizing and computing

To visualize the power pattern we measured about 90,000 datapoints per scan. The data includes a lot of noise caused by thermal radiation of buildings near the antenna or mobile telephones for example. To get the smooth surface of figure 2 we used the following method. The surface is spanned by points whose coordinates projected to the z-y-plane form a lattice. This lattice is given by all points of the form $(n_x \cdot 0.1^\circ, n_y \cdot 0.1^\circ, z)$, $n_x, n_y \in \mathbb{Z} \cap [-50...50]$. So we had to compute reasonable z-values for about 10,000 points. We decided to use the following method. Let us take a certain lattice point and put a circular environment around it of radius $0.2^\circ$. This environment now includes all points of the original dataset, that are within a range of $0.2^\circ$ of the lattice point (respective its projections). So we set the z-value of the lattice point to the average value of the z-values of the original dataset in the environment. This method proved to be satisfying and delivered a smooth surface. The computation and plotting of the surface was done by the computer algebra program Maple and took about 8 hours of computation time for every plot.

6. Results

In table 2 we present the results of our measurement. They are average values of the 3 measures we did. The error terms are the standard deviations of the 3 values. The expectation value for $\Omega_A$ uses the formula $\Omega_A = (4/3) \text{HPBW}$ (RA86). For $D_{\text{main}} \approx n(d\lambda/\lambda)^2$ we used the efficiency factor $n = 0.518$ (DB02), the diameter $d = 5m$ and the wavelength $\lambda$ corresponding to the frequency we used. HPBW can be approximated by $60^\circ \lambda/d$ and BWFN by 2-HPBW.

### Table 2: Results

<table>
<thead>
<tr>
<th>HPBW/°</th>
<th>Horizontal</th>
<th>Vertical</th>
<th>Expectation value</th>
<th>Deviation/°</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.92±0.03</td>
<td>1.56±0.06</td>
<td>2.1</td>
<td>8.6</td>
<td>26.2</td>
</tr>
</tbody>
</table>

7. Conclusion

The method for smoothing the surface of the power pattern we already presented above was at first just an attempt. But as it proved to deliver good results, we kept it. An improvement would for example be to weight the points of the original data set corresponding to their distance to the lattice point, when we compute the average value of the z-coordinates. Another idea for better results would be to receive more data with a lower amount of integration time. The resolution of the possible positions of the telescope is 14 bit. Especially for the very slow elevation movement, when scanning the field, this resulted in a very uncontinuous movement. A higher resolution would solve this.

A very good idea for totally symmetric scanfields - we mentioned this topic above - would be the following. As usual we start the position tracking program to take care, that our object is right in the centre of the scanfield. But when we compute the relative coordinates $\Delta \theta$ and $\Delta \phi$ we pretend the centre to be on the equator of a new spherical coordinate system. The scanfield now lies one half on the upper and one half on the lower part of the sphere and therefore it is symmetric. For objects close to zenith this would be a great improvement. Nevertheless did we obtain beautiful results, that fully satisfied our needs for determining the values of our interest.
References


DB03, Dominique Buser, *Observation of Taurus A with a 5 m radio-telescope of ETH in Zurich*, ETH Zurich, 2002