Numerical modeling of sea ice:  
A multi-layer thermodynamic snow sea-ice model  
and its validation against SHEBA data

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Summary

A large-scale simulation of the Antarctic sea ice cover is performed using a dynamic thermodynamic sea ice model based on granular material rheology. The model is coupled thermodynamically to a slab ocean model and is forced with monthly mean climatological atmospheric fields based on NCEP re-analysis data. The annual cycles of the ice compactness, thickness and areal extent simulated in a control experiment are in reasonable agreement with corresponding observations. The summer ice extent however is too small when compared to observations and the built-up of the ice cover is delayed due to an underestimation of the ice-albedo feedback owing to the fact that the model uses a constant albedo and does not include a snow cover. In a series of sensitivity experiments, the impact of model parameters and the relevance of physical processes is investigated to get estimates for their relative importance.

Taking up the need of a snow cover and more complex thermodynamics in the large-scale model, a multi-layer thermodynamic snow sea-ice model is developed and validated against observations from the SHEBA experiment in the Arctic. The model validation is carried out in the context of SIMIP2, an international effort on the evaluation of the performance of the thermodynamic component of sea ice models. The model is an energy conserving one-dimensional multi-layer thermodynamic model, including penetrating shortwave radiation and a brine parameterization. Model equations are written in terrain-following coordinates which naturally handles the redistribution of internal energy during re-layering.

A careful analysis of the SHEBA data shows that the snow and ice slabs are horizontally very inhomogeneous and measurements of the snow cover at the official mass balance site of the 'SHEBA Column' seem to underestimate the actual snow thickness at the location where internal snow and ice temperature was measured. Moreover, the snow conductivity derived from internal snow and ice temperature profiles is much higher (0.5 Wm$^{-1}$K$^{-1}$) than the value of 0.14 Wm$^{-1}$K$^{-1}$ measured in in-situ samples using a needle probe or the value of 0.31 Wm$^{-1}$K$^{-1}$ suggested in SIMIP2. The ocean heat flux derived from observed ice thickness evolution and internal ice temperature profiles corresponds well with results computed from turbulence measurements. However, it is smaller than the ocean heat flux provided in SIMIP2 which makes a difference for the simulated ice thickness evolution. Taking into account the best guess of the snow cover, ocean heat flux and thermal conductivity of snow determined in this study, the internal temperature profile and thickness evolution is very well simulated. Considering those corrections, a surface energy budget closes with a residual of 1 Wm$^{-2}$ but biannual differences of up to 7 Wm$^{-1}$. The developed thermodynamic sea ice model reproduced the observed evolution of the snow/ice cover during SHEBA realistically and is proposed as a capable tool for accurately modeling sea ice thermodynamics.
Zusammenfassung

Eine grossräumige Simulation des antarktischen Meereises wurde mit einem
dynamisch-thermodynamischen Meereismodell basierend auf der Rheologie gran¬
ularer Materialien durchgeführt. Das Modell ist thermodynamisch an ein Ein¬
Schicht-Ozean Modell gekoppelt und wird mit klimatologischen Monatsmitteln at¬
mosphärischer Felder auf der Grundlage von NCEP Re-analysen angetrieben. Die
in einem Vergleichsexperiment simulierten Jahresgänge von Eiskonzentration, Eis¬
dicke und flächenmässiger Ausdehnung stimmen mit den entsprechenden Beobach¬
tungen einigermaßen überein. Die Ausdehnung im Sommer ist im Vergleich mit den
Beobachtungen zu klein und das Wachstum des Packeises ist wegen einer Unter¬
schätzung der Eis-Albedo Rückkopplung infolge konstanter Albedo und fehlender
Schneedecke im Modell verzögert. Mit einer Reihe von Sensitivitätsexperimenten
wird der Einfluss von Modellparametern und die Bedeutung physikalischer Prozesse
untersucht, um Schätzungen für ihre relative Wichtigkeit zu erhalten.

Aufgrund der Notwendigkeit einer Schneedecke und komplexerer Thermodynamik
im grossräumigen Modell wurde ein thermodynamisches, vielschichtiges Schnee¬
Meereis-Modell entwickelt und mit Beobachtungen vom SHEBA Experiment in der
Arktis validiert. Die Modellvalidierung wird im Rahmen von SIMIP2, einem in¬
ternationalen Projekt zur Evaluierung der Leistung der thermodynamischen Kom¬
ponente von Meereismodellen, durchgeführt. Das Modell ist ein energieerhaltendes,
eindimensionales, thermodynamisches Modell mit vielen Schichten, das eindringende
Solarstrahlung und die Parametrisierung von Salzlösung berücksichtigt. Die Modell¬
gleichungen sind in geländefolgenden Koordinaten geschrieben, was die Neuvertei¬
lung innerer Energie während der Gitteranpassung auf natürliche Weise erledigt.

Eine sorgfältige Analyse der SHEBA Daten zeigt, dass die Schnee- und Eisschichten
horizontal sehr inhomogen sind, und Messungen der Schneedecke am
offiziellen Ort der Massenbilanz für die 'SHEBA-Säule' scheinen die tatsächliche
Schneehöhe an der Stelle, an der Schnee- und Eistemperatur gemessen wurde, zu
unterschätzen. Zudem ist die von vertikalen Profilen der Schnee- und Eistemperatur
abgeleitete thermische Leitfähigkeit von Schnee (0.5 Wm⁻¹K⁻¹) wesentlich höher
als der Wert von 0.14 Wm⁻¹K⁻¹, der vor Ort mit einer Nadel-Sonde gemessen
wurde, oder der von SIMIP2 vorgeschlagene Wert von 0.31 Wm⁻¹K⁻¹. Der von
der Entwicklung der gemessenen Eisdicke und den Eisstemperaturprofilen abgeleitete
Ozeanwärmefluss stimmt gut mit aus Turbulenzmessungen berechneten Ergebnis¬
en überein. Er ist jedoch kleiner als der in SIMIP2 gegebene Ozeanwärmefluss,
was sich an der simulierten Entwicklung der Eisbasis bemerkbar macht. Unter
Berücksichtigung der in dieser Studie getroffenen besten Schätzung von Schneehöhe,
Ozeanwärmefluss und thermischer Leitfähigkeit von Schnee sind die Tempera¬
turverteilung in Schnee und Eis sowie die Entwicklung der Dicke sehr gut simuliert.
Unter Beachtung der vorgenommenen Korrekturen schliesst ein Energiehaushalt
an der Oberfläche mit einem Residuum von 1 Wm⁻², jedoch mit halbjährlichen
Differenzen von bis zu 7 Wm⁻¹. Das entwickelte thermodynamische Meereismodell
bildet die beobachtete Entwicklung der Schnee-/Eisdecke während SHEBA realis¬
tisch nach und wird als leistungsfähiges Werkzeug für die genaue Modellierung von
Meereis-Thermodynamik vorgeschlagen.
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Chapter 1

Introduction
Polar sea ice covers are spatially and temporally highly variable and have a major influence on the climate of high latitudes and ultimately on the global climate. In late winter, sea ice reaches its maximum extent and covers an area of 14 to 16 million square kilometers in the Arctic and 17 to 20 million square kilometers in the Southern Ocean. In the northern hemisphere, seasonal or perennial sea ice covers the Arctic Ocean and peripheral seas. In the southern hemisphere, the sea ice cover consists mainly of seasonal ice which remains relatively thin all around the Antarctic continent and melts completely during the summer, except in some areas along the coast where multi-year ice can exist. The ice pack around Antarctica has an annular shape extending into the Southern Ocean where it has an open boundary to the north whereas the Arctic sea ice cover is mostly bounded by the land masses of the Eurasian and North American continents. A large fraction of the Arctic sea ice consists of thick perennial ice which can outlast several annual cycles.

Due to the presence of land masses, sea ice interactions play an important role in the Arctic basin. This leads to the formation of ridges which increases the ice thickness locally and results in inhomogeneous spatial thickness distribution. In the Southern Ocean there is relatively less interaction between the ice floes which are on average in a free drift. Sea ice motion results from both atmospheric and oceanic drag, i.e. momentum transfer at both surfaces of the snow/ice slab. In the Arctic, the two dominant features of the sea ice drift are: (1) the Beaufort Gyre, a current in the area northwest of the Canadian Archipelago and (2) the Transpolar Driftstream, a current from the Bering Strait across the north pole towards Fram Strait and the Greenland Sea. The velocity field of the Antarctic sea ice mainly reflects the atmospheric flow, i.e. cyclonic motion below the low pressure systems in the area of the Weddell and Ross seas and the sector of the Indian ocean. There is a westward drift along the Antarctic coast or the edges of the ice shelves, and an eastward flow in the region of the polar front close to the ice edge.

During most of the year, sea ice is covered by a snow layer which undergoes large changes during the annual cycle. Snow drift, induced by storms causes a redistribution of snow which leads to erosion in some areas and to accumulation in others. Wind is also responsible for the compaction of the snow pack in some part. Other frequent physical processes in a snow cover are brine wetting, the formation of depth hoar and a general metamorphosis (recrystallization) on the micro-scale. Due to its complex micro-structure and its particular physical properties (grain matrix, low thermal conductivity), the snow cover plays a crucial role for thermodynamic processes in the snow sea-ice system. In the summer, the snow cover atop the sea ice generally melts completely. As soon as surface melt starts, meltwater either percolates down to and through the sea ice slab (in some cases re-freezing happens) or it remains at the ice surface forming melt ponds and puddles (only in the Arctic). If the ice below a pond becomes too thin, the water drains into the ocean. Surface melt associated with brine drainage above the freeboard leads to desalinization of the upper part of the sea ice due to flushing.

Sea ice substantially influences the atmosphere and the ocean in high latitudes. The importance of sea ice and its snow cover in the climate system is manifold. (1) Sea ice acts as an insulating layer at the ocean-atmosphere interface, significantly reducing the ocean-atmosphere heat flux during winter. The sensible heat flux above
sea ice can be two orders of magnitude smaller than over open ocean (e.g. leads or polynyas). At the same time, sea ice reduces the moisture flux from the ocean to the atmosphere leading to reduced surface cooling due to limited evaporation and energy release to the atmosphere when condensation occurs. (2) Due to the latent heat associated with freezing and melting, sea ice acts as a negative thermal reservoir which damps and delays the forcing signal of the annual cycle. (3) Sea ice and snow are highly reflective to shortwave radiation compared to the ocean which absorbs most of the incident solar radiation. The albedo effect is largest when the shortwave radiation is close to its maximum and the snow cover is still present (in early summer). The high reflectivity to solar radiation substantially reduces the amount of energy available at the snow/ice surface which leads to less melting and increased horizontal ice extent at the end of the melting season. Conversely, a small sea ice cover reflects less shortwave radiation and the ocean absorbs more which further reduces the ice extent. (4) When sea ice forms, salt is rejected which increases the density of the ocean surface waters and hence drives convection in the ocean mixed-layer and finally erodes the seasonal pycnocline leading to the production of the ocean bottom water. In the Southern Ocean, the vertical mixing brings up warm below- pycnocline water into the mixed-layer, whereas in the Arctic, convection does not lead to heat entrainment due to the presence of a cold halocline. When sea ice is melting, a layer of relatively fresh water sits on top of the mixed-layer causing stable stratification and thus reduces vertical mixing. If dynamics are involved, the advective transport of sea ice constitutes a freshwater flux from one region to another if the ice cover melts. (5) Finally, sea ice reduces the momentum transfer from the atmosphere to the ocean which can influence the oceanic circulation pattern. Sea ice is also of particular interest because it is sensitive to regional and global climate change and therefore an indicator for trends in climate and its variations. The horizontal extent, the thickness distribution and the compactness of sea ice are variables which can be used to quantify climate changes.

The thickness distribution and the extent of a sea ice cover are governed by thermodynamic and dynamic processes. The atmospheric and oceanic heat fluxes at the top and bottom surface of the snow/ice determine the thermodynamic evolution of the sea ice cover, i.e. the amount of surface and basal melt and basal freezing. The atmospheric forcing at the surface determines the internal temperature distribution and consequently the conductive heat flux and the heat content of the snow/ice slab. The properties and effects of sea ice described above determine the energy exchange at the atmosphere-ice-ocean interfaces at high latitudes. The evolution of the ice base is a result of the sensible heat flux from the ocean to the ice, jointly acting with the conductive heat flux in the ice and the latent energy used for melting or released during ice formation at the ice base. Surface melt occurs if an energy surplus results from the balance of the radiative, turbulent and conductive fluxes at the snow or ice surface. The presence of melt ponds during summer, leads to an increased absorption of solar radiation and enhances surface melting. Energy which is absorbed in leads and areas of open water warms the ocean mixed-layer and melts the ice laterally and at the base. Thermodynamics controls the timing of events and the rate of processes. Most physical processes governing the formation and evolution of a sea ice cover involve thermodynamics including internal feedback processes and interactions.
In order to resolve the internal temperature and salinity distribution within the ice and consequently the conductive heat flux and the internal storage of heat, a multi-layer thermodynamic model is required. Such models are usually computationally intensive. Simulations using simple linear formulations of a sea ice model with a snow cover on top (zero-layer models) yield less realistic results, however, they are computationally efficient and thus convenient for use in coupled global climate simulations. The model developed for this thesis allows to select an arbitrary number of layers in both the snow and ice which qualifies the model both for large-scale applications with reduced complexity and for detailed small-scale simulations of higher accuracy. The model proposed in this work is a flexible tool to simulate sea ice and its snow cover and constitutes an effort to provide a sea ice model with more realistic thermodynamics. It is an instrument to represent sea ice and to investigate the physical processes occurring in sea ice by conducting scenarios and sensitivity and process studies.

Motivated by the high significance of sea ice in the climate system and the prominent role of thermodynamics in sea ice formation and melt, this thesis addresses the physical processes relevant for describing and simulating sea ice. The objectives of the study are (1) the identification and investigation of key variables and processes in numerical sea ice models and their relative importance, and (2) based on the findings, the development of a numerical model to realistically simulate the evolution of a snow/sea ice cover, and (3) to validate this model against observations.

The outline of the thesis is as follows: Chapter 2 presents a large-scale simulation of the sea ice cover of the southern hemisphere. In a sensitivity study, the relative importance of a series of model parameters is investigated and discussed. Chapter 3 examines observational data from the SHEBA project which are used for the validation of the thermodynamic sea ice model. It highlights all important measurements regarding the mass and energy balance of the ice cover at SHEBA but also critically evaluates the measurements. This chapter includes an energy budget of the heat fluxes at the surface and base of the snow/ice slab. Chapter 4 presents the thermodynamic sea ice model and its validation against SHEBA data. The model is also used to examine the consistency of the measurements, i.e. to validate the data which makes the whole study an assimilation process. Each chapter has its own introduction describing the scope of the specific study. The references of all three chapters are jointly summarized in the bibliography at the end of the thesis. The date format is American standard (month/day/year) and the Julian day is defined with respect to 01/01/1997, 12 am, i.e. January 1, 1997, 1 am is Julian day 0.0417. Fluxes towards a considered surface are defined positive; a flux away from the surface has a negative (syntactic) sign, e.g. upward longwave radiation LW$\uparrow = -300$ Wm$^{-2}$. 
Chapter 2

Simulation of the Antarctic Sea Ice Cover: A Sensitivity Study
Abstract

A set of large scale simulations of the Antarctic sea ice cover using a dynamic thermodynamic sea ice model based on granular material rheology is presented. The model is coupled thermodynamically to a slab ocean model taking into account entrainment heat from the deeper ocean and is driven with monthly mean climatological atmospheric fields based on NCEP re-analysis data. The simulated annual cycles and distributions of sea ice concentration, extent and thickness are compared with observations. A series of 40 sensitivity experiments demonstrates the importance of shortwave and longwave radiation components, heat fluxes, ocean mixed-layer properties, drag coefficients and turning angles. In the simulations, sea ice was very sensitive to changes of the atmospheric absorptivity to shortwave radiation over water and to the atmospheric emissivity. Ocean mixed-layer conditions substantially influence sea ice, the mixed-layer depth has an impact on the annual amplitude of areal extent and the onset of growth and the rate of reduction in sea ice area in spring. The entrainment heat from the deeper ocean mainly controls the ice volume and thickness distribution whereas the maximum sea ice extent remains unchanged. The ocean-atmosphere sensible and latent heat fluxes are more important than the ice-atmosphere sensible and latent heat fluxes, while the sensible heat fluxes dominate over the latent heat fluxes. The model results also show that the difference between the wind and ocean current turning angles is the controlling factor for the sea ice characteristics.
2.1 Introduction

The presence of sea ice in the polar regions can influence the climate in various ways. Sea ice is highly reflective and substantially reduces the amount of solar radiation absorbed by the surface. It also acts as an insulator, reducing the amount of heat loss from the relatively warm ocean to the atmosphere, and typically reduces the amount of momentum transfer between the atmosphere and the ocean. Finally, the salt rejected during ice formation and fresh water release during ice melt modifies the characteristics of the surface waters and influences the vertical motion in the ocean and ultimately the formation of deep waters.

In the Southern Ocean, ice formation is possible south of the polar front where the surface waters are cold and relatively fresh. In this region, sea ice is formed during most time of the year along the continent margin, where south-easterly katabatic winds push the ice away from the coast; the ice is then transported northward by the prevailing winds and melts at the edge of the Antarctic Circumpolar Current (ACC) where warmer ocean temperatures are present. The ice formation along the coast combined with ice melt at the ice margin is in part responsible for the formation of deep waters along the continental shelf and an increased stratification at its northern edge. Moreover, the stratification of the upper ocean over most of the Southern Ocean is relatively weak and salt rejection associated with sea ice formation causes the surface waters to convect, bringing heat from beneath the pycnocline into the mixed-layer. This ocean heat flux is a major contributor to the ocean-atmosphere heat flux keeping the sea ice cover relatively thin, with perennial ice only present in certain bays around the Antarctic continent, in the Ross Sea where the dominant winds are blowing ice against the coast and along the East Antarctic Peninsula where cold waters are present below the mixed-layer and the oceanic heat flux is minimal (Martinson, 1990). In the Weddell and Ross Seas, two well developed low pressure systems are present on average, and the resulting sea ice divergence is also partly responsible for the thin ice cover in those areas (estimates of ice loss due to divergence are provided in Gordon and Huber, 1990; Martinson, 1990; McPhee et al., 1996).

In the Southern Ocean, a small change in the stability of the surface waters can result in a system mode change from a situation where a seasonal sea ice cover and coastal deep water formation is present, to a system where no sea ice cover and open ocean deep water formation is present (Gordon, 1991). The Weddell Sea polynya observed in the years 1974 to 1976 (Campbell et al., 1983) is an indication for such a situation. In this case, a slight decrease in the stability of the water column might increase the oceanic heat flux and result in a thinner ice cover. This in turn will increase the ocean-atmosphere heat flux and strengthen the cyclicity of the atmospheric circulation in the region and therefore sea ice divergence and will result in a further thinning of the sea ice cover. The Weddell Sea polynya has been studied through a series of field work experiments (e.g. Gordon, 1978) and numerical studies (e.g. Timmermann et al., 1999; Lemke et al., 1990).

This study presents a seasonal simulation of the Antarctic sea ice using the dynamic-thermodynamic sea ice model of Tremblay and Mysak (1997) coupled to a slab ocean and forced with NCEP re-analysis atmospheric fields. The sea ice model
has been used in studies for the Arctic Ocean, (e.g. Tremblay and Mysak, 1997; Tremblay et al., 1997; Tremblay and Mysak, 1998; Arfeuille et al., 2000). Although the governing equations describing the sea ice behavior are the same in the Arctic and in the Southern Ocean, different conditions prevail in the two systems and so, different aspects of the sea ice model are being evaluated in an Antarctic sea ice simulation. Close to the Antarctic continent, the Coriolis effect is of second order importance as it scales with sea ice thickness. Also, sea ice in the Arctic is confined by continents on its outer boundary whereas the Antarctic sea ice cover is bounded on its interior by a continent and free to move outward. Consequently, the ice interaction term is less critical in a Southern Ocean simulation and thermodynamic processes are dominant. In Antarctica, although the spatial frequency of ridges can attain levels similar to that of the Arctic, the ridge amplitude is usually smaller and drift conditions with little ice interaction are more often present (except in the Weddell and the Ross Seas) due to the mean divergent nature of the wind forcing (Martinson and Wämser, 1990). On the other hand, the water column in Antarctica is only marginally stable and so brine rejection associated with ice formation causes mixing and deeper ocean ventilation and hence maintains a thinner ice cover all year long. The consideration of this process is crucial for a realistic simulation of the Antarctic sea ice cover.

The present study will show that a sea ice model coupled to a simple slab ocean with prescribed deeper ocean temperature and salinity, and calculated oceanic heat flux based on a bulk parameterization (Martinson and Iannuzzi, 1998), can reproduce most of the features observed in the sea ice field. However, a realistic simulation of the Southern Ocean sea ice cover in coupled ice-ocean-atmosphere models is extremely difficult and small departure in ocean heat flux or atmospheric circulation can be amplified leading to a drastically different representation of the Southern Ocean sea ice cover (Cai and Gordon, 1999).

The present work includes a series of sensitivity experiments which focuses largely on thermodynamic aspects but also on some dynamic issues. The sensitivity study of the sea ice model is performed to gain a better understanding of the sea ice response to variations in oceanic and atmospheric forcing, and various sea ice parameters, as used in coupled models. A number of sensitivity studies of the ice-ocean-atmosphere system in both hemispheres has been published in the last years. Holland et al. (1993) presented a comprehensive sensitivity study for the Arctic sea ice cover, investigating the impact of model parameters and physical processes on the ice characteristics. Stössel and Claussen (1993) have studied the effect of using a more sophisticated drag parameterization (where both the skin and form drag are resolved separately) on the seasonal sea ice cover. Fichefet et al. (2000) and Wu et al. (1999) have published two complementary studies on the effect of a snow cover on the ocean heat flux and sea ice characteristics using a coupled ocean-ice model with specified atmospheric forcing (Fichefet et al., 2000) and with an ice-atmosphere model with specified ocean temperature (Wu et al., 1999). Their results show that the use of a more realistic snow thermal conductivity results in a thinner pack ice in better agreement with observations and a strengthening of the Southern Ocean stratification and a weakening of the Antarctic Bottom Water meridional overturning. The effect of the sea ice cover on the deep water formation and deep water characteris-
2.2. Model description

Stössel et al. (1998) investigated the bottom water formation which is strongly coupled to sea ice processes such as brine release due to sea ice formation and that of employing different wind forcing over the Southern Ocean domain.

In Section 2.2, a description of the dynamic-thermodynamic sea ice model is presented and in Section 2.3, the experimental setup and the forcing fields are described. In Section 2.4, simulation results of a control experiment are compared with satellite observations and in-situ ocean heat flux and ice thickness measurements. In Section 2.5, a sensitivity study of the sea ice model to various ocean, ice and atmosphere related parameters is presented. In this section, the sensitivity experiments are divided into two categories: those related to the thermodynamics of sea ice (Section 2.5.1) and those related to the dynamics (Section 2.5.2). The main conclusions drawn from the simulation results are summarized in Section 2.6.

2.2 Model description

For large-scale simulation of the Antarctic sea ice cover forced by monthly averaged wind stress, both the advection and acceleration terms can be neglected in the sea ice momentum balance equation. Under this approximation, the two-dimensional horizontal motion of sea ice can be described by

$$-\rho_i h f k \times \mathbf{u}_i + A(\tau_a - \tau_w) + \nabla \cdot \mathbf{\sigma} - \rho_i h g \nabla H_d = 0, \quad (2.1)$$

where $\rho_i$ is the sea-ice density, $h$ the mean ice thickness, $f$ the Coriolis parameter, $k$ a unit vector normal to the ice surface, $\mathbf{u}_i$ the ice velocity, $A$ the ice concentration (percentage of a grid cell covered by ice), $\tau_a$ the wind shear stress on the top ice surface, $\tau_w$ the ocean drag on the sea-ice flow, $\sigma_{ij}$ the vertically integrated internal ice stress (normal or shear) acting on a plane which is perpendicular to the $i$-axis and in the $j$-direction, $g$ the gravitational acceleration and $H_d$ the sea-surface dynamic height. Following Gray and Morland (1994), the wind stress and water drag are multiplied by the ice concentration to account for the fact that water may be present in a grid cell. The air ($\tau_a$) and water ($\tau_w$) stresses are obtained from a simple quadratic law with constant turning angle (McPhee, 1975),

$$\tau_a = \rho_a C_{da} |\mathbf{u}_a| (u_a^g \cos \theta_a + k \times u_a^g \sin \theta_a), \quad (2.2)$$
$$\tau_w = \rho_w C_{dw} |\mathbf{u}_w| [(u_i - u_w^g) \cos \theta_w + k \times (u_i - u_w^g) \sin \theta_w], \quad (2.3)$$

where $\rho_a$ and $\rho_w$ are the air and water densities, $C_{da}$ and $C_{dw}$ the air and water drag coefficients, $u_a^g$ and $u_w^g$ the geostrophic wind and ocean current and $\theta_a$ and $\theta_w$ the wind and water turning angles. In the above equation for the wind shear stress, the ice speed is considered small compared to the wind speed and is therefore neglected.

Considering the sea ice to behave as a granular material in slow continuous deformation, the internal ice stress $\mathbf{\sigma}$ can be written as follows (Tremblay and Mysak 1997; Flato and Hibler 1992):
\[
\sigma_{ij} = -p \delta_{ij} - \eta \epsilon_{kk} \delta_{ij} + 2\eta \dot{\epsilon}_{ij},
\]  
(2.4)

where
\[
\eta = \min \left( \frac{p \sin \phi}{\sqrt{(\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2}}, \eta_{\text{max}} \right).
\]  
(2.5)

For small deformation \( (\dot{\epsilon}_1, \dot{\epsilon}_2) \), the coefficient of friction is constant \( (\eta = \eta_{\text{max}}) \) and sea ice behaves as a very viscous fluid. In the above equation, the pressure \( p \) is limited to a maximum value \( P_{\text{max}} \), which is a function of the local ice thickness and concentration. This can be parameterized as follows (Hibler, 1979):

\[
P_{\text{max}} = P^* h \exp[-C(1 - A)],
\]  
(2.6)

where \( P^* \) is the ice strength per meter ice thickness and \( C \) is the ice concentration parameter. The resistance of sea ice to compressive load is considered to be a function of its thickness and concentration, and its shear resistance is proportional to the internal ice pressure at a point. In divergent motion, the ice offers no resistance, and the floes drift freely.

The ice strength in this model is a function of both the mean ice thickness \( h \) and ice concentration \( A \). For this reason, a conservation law for each quantity is necessary:

\[
\frac{\partial h}{\partial t} + \nabla \cdot (h \mathbf{u}_i) = S_h + K_h \nabla^2 h,
\]  
(2.7)

\[
\frac{\partial A}{\partial t} + \nabla \cdot (A \mathbf{u}_i) = S_A + K_A \nabla^2 A,
\]  
(2.8)

where \( K_h \) and \( K_A \) are the diffusion coefficients for ice thickness and concentration, and \( S_h \) and \( S_A \) are the thermodynamic source terms which are given by:

\[
S_h = \frac{1}{\rho_i L_f} \begin{cases} 
A (Q_{ia} - Q_{oi}) + (1 - A) Q_{oa}, & T_o = T_{of}, \ Q_{oa} > 0 \\
A (Q_{ia} - Q_{oi}), & \text{otherwise}
\end{cases}
\]  
(2.9)

\[
S_A = \frac{1}{\rho_i L_f} \begin{cases} 
(1 - A) Q_{oa}/h_0, & T_o = T_{of}, \ Q_{oa} > 0 \\
A \rho_i L_f S_h/2h, & S_h < 0
\end{cases}
\]  
(2.10)

where \( L_f \) is the latent heat of fusion, \( Q_{ia} \) and \( Q_{oa} \) the net ice and oceanic heat fluxes to the atmosphere due to longwave \( (Q_{lw-up}, Q_{lw-down}) \), sensible \( (Q_{\text{sens}}) \), latent heating \( (Q_{\text{lat}}) \) and shortwave radiation \( (Q_{sw}) \), \( Q_{oi} \) the sensible heat flux from the water to the ice, \( h_0 \) a fixed demarcation thickness between thin and thick ice (Hibler, 1979), and \( T_o \) and \( T_{of} \) the temperature and freezing point temperature of the ocean. In equation 2.8, the ice concentration is restricted to lie between zero and one using a mechanical sink term. A detailed description of the sea-ice dynamic and thermodynamic models can be found in (Tremblay and Mysak 1997).
Thermodynamically, a simple two-category (ice and no ice, no snow cover) model is used which calculates the mean ice thickness and concentration in a grid cell (Hibler, 1979). In the vertical, the zero-layer model of Semtner (1976) with a linear temperature profile from the ice base to the ice surface is used.

The sea ice model is coupled thermodynamically to a slab ocean model which takes into account entrainment between the mixed-layer and the deeper ocean associated with ice-growth-induced salinization of the surface waters. The parameterization is based on the temperature and salinity difference between the mixed-layer and the deeper ocean, in a way similar to the bulk analysis presented in Martinson and Ianuzzi (1998). More specifically, for a mixed-layer and deeper ocean temperature and salinity $T_{ml}$, $S_{ml}$ and $T_{do}$, $S_{do}$, the formation of 1 kg of ice will release $S_{ml}$ g of salt and drive $S_{ml}/(S_{do} - S_{ml})$ kg of water to convect, bringing $S_{ml} c_p (T_{do} - T_{ml})/(S_{do} - S_{ml})$ of heat into the mixed-layer to effectively melt. The above ratio is calculated from typical mixed-layer and deeper ocean temperature and salinity observations (D. G. Martinson, personal communication, 2000) and is multiplied by the amount of newly formed ice to calculate an effective ocean entrainment heat flux into the mixed-layer. This term is only taken into account after the seasonal pycnocline has disappeared, when deep ocean heat is readily available to warm up the mixed-layer. For simplicity, the salt deficit associated with the seasonal pycnocline is set to a constant and requires approximately 60 cm of new ice formation to be eliminated.

In this model, ice can form over open water even when the bulk temperature of the ocean mixed-layer ($T_{ml}$) is above freezing ($T_f$). In these regions, the ocean-atmosphere heat flux is calculated assuming the skin temperature of the ocean surface is at freezing point. A sensible heat flux between the bulk of the mixed-layer (at $T_{ml}$) and the surface waters (at $T_f$) is then calculated; the ice growth at the surface is calculated from the difference between the ocean-atmosphere heat flux and this sensible heat flux. Model results show that ice formation is possible for mixed-layer temperatures up to 0.3 or 0.4 degrees above freezing point.

### 2.3 Experimental setup and forcing field

The physical domain considered in all simulations includes the entire Southern Ocean (south of 55°S). A Cartesian mesh with a grid resolution of 111 km is used on a polar stereographic projection of the physical domain. Since the meridional extent of the sea ice in Antarctica is relatively small and since the ice is relatively thin, the f-plane approximation is used. In the present study, the model is forced with prescribed climatological monthly mean wind stresses, surface winds (for the calculation of latent and sensible heat fluxes) and air temperatures from the NCEP re-analysis (1968-96). The forcing fields are assumed to represent the mid-month situation. The forcing field at a particular day is calculated using a linear interpolation of the two closest mid-month values. The field variables were interpolated onto the nodes of the Cartesian 1-degree grid by means of a kriging interpolation method. Ocean currents were set to zero in the present study. The boundary conditions for
the ice dynamic equations are zero normal and tangential velocity at a solid bound¬
ary and free outflow (ice pressure equal to zero) at an open boundary (Hibler, 1979).
For the slab ocean, the temperature at open boundaries is specified from monthly
climatologies extracted from a modified data set of the Climatological Atlas of the
World's Oceans (Levitus, 1982). The initial conditions for the ice model, used in all
simulations, are zero ice thickness and concentration. For the ocean, the tempera¬
ture is set to 0°C everywhere. For both the control and the sensitivity experiments,
the model was integrated for a period of 3 years using a 1-day time step to reach a
stable seasonal cycle. Results shown are from the last year of integration.

The output variables of the sea ice model are the mean ice thickness \( h(i,j) \) and
concentration \( A(i,j) \) averaged over a grid cell and the sea ice velocities \( u(i,j) \) defined
on the vertices of the grid (Arakawa C-grid). Sea ice concentration is defined as the
areal fraction of a grid cell covered by ice. Derived quantities are the total ice covered
area \( A_{\text{tot}} \), the effective ice covered area \( A_{\text{eff}} \) and the total ice volume \( V \):

\[
A_{\text{tot}} = \sum A_g(i,j), \quad A_{\text{eff}} = \sum A_g(i,j) A(i,j), \quad V = \sum A_g(i,j) h(i,j),
\]

where \( A_g(i,j) \) is the geographical grid cell area and the summation is done over all
grid cells in which the ice concentration is larger than a specified cut-off value \( \alpha \) (set
to 0.5). Other derived quantities of interest include the areal mean ice concentration
\( \bar{A} \), the mean ice thickness \( \bar{h} \) and the mean effective ice thickness \( \bar{h}_{\text{eff}} \). Mathematically
they are defined as:

\[
\bar{A} = A_{\text{eff}} / A_{\text{tot}}, \quad \bar{h} = V / A_{\text{tot}}, \quad \bar{h}_{\text{eff}} = V / A_{\text{eff}}.
\]

Although the number of grid cells with low ice concentration is high, sensitivity tests
revealed that the above mean quantities are insensitive to the cut-off value \( \alpha \) since
the effective ice area comprised between the 0% and 50% contour lines is very small
(Figure 2.1).

2.4 Control simulation

2.4.1 Ice concentration

In this section, the simulation results are compared with satellite derived data
(passive microwave sensors: Nimbus-5 ESMR, Nimbus-7 SMMR or DMSP SSM/I)
for climatological monthly mean sea ice concentration of the National Snow and
Ice Data Center, Boulder, CO. The data set comprises 18 successive years from
1973 through 1990. In this data set, monthly sea ice concentration was digitized
on a standard 1-degree grid using a cylindrical projection of the Southern Ocean
(cf. NSIDC data description) extending to about 50 degrees south. The Ross and
Fildmer ice shelves as well as many smaller ice shelves in coastal bays are part of
the land mask.

Note that the climatology of sea ice concentration data (derived from satellite) is
averaged over a smaller number of years (1973-90) than the wind forcing climatology
It was judged preferable to keep the entire time span available for the wind forcing climatology in order to perform a sensitivity study on a more representative base case. The parameters and physical constants used in the control simulation are given in Table 2.1.

In Figure 2.1, simulated values of total ice area, $A_{tot}$, and the areal mean sea ice concentration, $\bar{A}$, are compared with observations. Figure 2.2 shows the spatial distribution of simulated and observed sea ice concentration for the months of March (minimum ice extent), June, September (maximum ice extent) and December. In general, the observed seasonality of the sea ice cover is well reproduced in the simulation results. The ice margin for all four seasons (December, March, June, September) is in good agreement with satellite data and the perennial ice is in the correct locations during summer months. In particular, the asymmetry in the sea ice extent between the Weddell and the Bellinghausen-Amundsen Seas, in part due to the advection of thick eastern Antarctic Peninsula ice by the Weddell Gyre, is well simulated. A region of lower ice concentration at the center of the Weddell Gyre (due to ice divergence) is present in accord with observations (Figure 2.2e,f) and the reduced ice extent in the Indian-Pacific sector is also reproduced (Figure 2.2e,f, more clearly visible from thickness distribution Figure 2.3). The calculated total ice area is too small except during maximum sea ice extent in September, where the

<table>
<thead>
<tr>
<th>Exp. #</th>
<th>Variable</th>
<th>Control</th>
<th>Low Value</th>
<th>High Value</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>01, 02</td>
<td>ice albedo</td>
<td>0.75</td>
<td>0.50</td>
<td>0.85</td>
<td>[-]</td>
</tr>
<tr>
<td>03, 04</td>
<td>ocean albedo</td>
<td>0.05</td>
<td>0.01</td>
<td>0.09</td>
<td>[-]</td>
</tr>
<tr>
<td>05, 06</td>
<td>SW$^1$ radiation over ice</td>
<td>0.12</td>
<td>0.00</td>
<td>0.40</td>
<td>[-]</td>
</tr>
<tr>
<td>07, 08</td>
<td>SW$^1$ radiation over water</td>
<td>0.40</td>
<td>0.10</td>
<td>0.60</td>
<td>[-]</td>
</tr>
<tr>
<td>09, 10</td>
<td>atmospheric emissivity</td>
<td>0.85</td>
<td>0.70</td>
<td>0.95</td>
<td>[-]</td>
</tr>
<tr>
<td>11, 12</td>
<td>atmosphere relative humidity</td>
<td>0.80</td>
<td>0.72</td>
<td>0.88</td>
<td>[-]</td>
</tr>
<tr>
<td>13, 14</td>
<td>minimum ice thickness</td>
<td>0.05</td>
<td>0.01</td>
<td>0.50</td>
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<tr>
<td>15, 16</td>
<td>ocean mixed-layer depth</td>
<td>30</td>
<td>10</td>
<td>50</td>
<td>[m]</td>
</tr>
<tr>
<td>17, 18</td>
<td>ocean diffusion coefficient</td>
<td>10$^{11}$</td>
<td>10$^{10}$</td>
<td>10$^{12}$</td>
<td>[m$^2$s$^{-1}$]</td>
</tr>
<tr>
<td>19, 20</td>
<td>overturning (East Ant. Peninsula)</td>
<td>0.1</td>
<td></td>
<td></td>
<td>[-]</td>
</tr>
<tr>
<td>19, 20</td>
<td>overturning (all other sectors)</td>
<td>2.0</td>
<td>1.0</td>
<td>4.0</td>
<td>[-]</td>
</tr>
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<td>21, 22</td>
<td>seasonal pycnocline salt depletion</td>
<td>0.6</td>
<td></td>
<td></td>
<td>[m ice]</td>
</tr>
<tr>
<td>23, 24</td>
<td>LH$^2$ transfer coeff. (ice-atm)</td>
<td>1.0$\times$10$^{-3}$</td>
<td>0.10$\times$10$^{-3}$</td>
<td>10$\times$10$^{-3}$</td>
<td>[-]</td>
</tr>
<tr>
<td>25, 26</td>
<td>SH$^3$ transfer coeff. (ice-atm)</td>
<td>5.0$\times$10$^{-3}$</td>
<td>0.50$\times$10$^{-3}$</td>
<td>50$\times$10$^{-3}$</td>
<td>[-]</td>
</tr>
<tr>
<td>27, 28</td>
<td>SH$^3$ transfer coeff. (ocn-atm)</td>
<td>0.8$\times$10$^{-3}$</td>
<td>0.08$\times$10$^{-3}$</td>
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<td>[-]</td>
</tr>
<tr>
<td>29, 30</td>
<td>SH$^3$ transfer coeff. (ocn-ice)</td>
<td>2.0$\times$10$^{-3}$</td>
<td>0.20$\times$10$^{-3}$</td>
<td>20$\times$10$^{-3}$</td>
<td>[-]</td>
</tr>
<tr>
<td>31, 32</td>
<td>SH$^3$ transfer coeff. (ocn-ocn)</td>
<td>5.0$\times$10$^{-3}$</td>
<td>0.50$\times$10$^{-3}$</td>
<td>50$\times$10$^{-3}$</td>
<td>[-]</td>
</tr>
<tr>
<td>33, 34</td>
<td>water-ice drag coefficient</td>
<td>3.0$\times$10$^{-3}$</td>
<td>1.2$\times$10$^{-3}$</td>
<td>6.0$\times$10$^{-3}$</td>
<td>[-]</td>
</tr>
<tr>
<td>35, 36</td>
<td>air-ice drag coefficient</td>
<td>1.2$\times$10$^{-3}$</td>
<td></td>
<td></td>
<td>[-]</td>
</tr>
<tr>
<td>37, 38</td>
<td>ice yield stress</td>
<td>5$\times$10$^3$</td>
<td>0.5$\times$10$^3$</td>
<td>50$\times$10$^3$</td>
<td>[Nm$^2$]</td>
</tr>
<tr>
<td>39, 40</td>
<td>wind turning angle</td>
<td>0.0</td>
<td>-5.0</td>
<td>5.0</td>
<td>[deg]</td>
</tr>
<tr>
<td></td>
<td>water turning angle</td>
<td>-15.0</td>
<td>-10.0</td>
<td>-20.0</td>
<td>[deg]</td>
</tr>
</tbody>
</table>

$^1$SW = shortwave, $^2$LH = latent heat, $^3$SH = sensible heat.
Figure 2.1: Annual cycle of total and effective ice covered area and mean ice concentration. All dotted lines represent simulated data and solid lines satellite data. A 50% cut-off for ice concentration was applied.

peak has the right magnitude. The build-up of the ice cover in late summer and fall is delayed by one month leading to a smaller ice extent compared with observations (see Figure 2.1). This could be a result of the constant mixed-layer depth used in this model and the uncertainty in the ocean-ice and ocean-atmosphere heat fluxes in the mixed-layer which has a significant influence on the onset of ice growth and the amplitude of areal extent (discussed in more details in Section 2.5). On the other hand, the simulated areal mean sea ice concentration is mostly overestimated with largest errors in the summer.

Part of the differences may be attributed to the different averaging period for the satellite derived sea ice observation and the forcing fields. Also, the observed mean areal coverage may be overestimated, particularly in the transition seasons, when synoptic scale weather conditions can lead to quick growth of a thin ice layer. Finally, the accuracy of satellite observations decreases in coastal areas where sea ice is not clearly distinguished from land ice and in open ocean areas where the sea ice concentration is low. Satellite derived sea ice concentration becomes less accurate as the fraction of thin ice or low concentration increases (NSIDC data documentation, Cavalieri et al. (1992).

The observed total sea ice covered area shows an asymmetric annual cycle with a fast ice reduction in the spring-summer and a slower increase in the fall-winter. The formation of the sea ice cover in the fall starts near the continent and progresses relatively slowly northward due to thermodynamic effects mainly (there is also advection of ice northward but this process is less significant).
Figure 2.2: Simulated sea ice concentration for the control experiment (a, c, e, g), and satellite derived sea ice concentrations (b, d, f, h) for the months of March, June, September and December. The time slices correspond to the end of summer, fall, winter and spring. No cut-off value for concentration applied.
In the spring, the dislocation of the pack ice takes place rapidly allowing the ocean surface to warm up and causing a rapid decay of the sea ice cover (advection of the ice northward is also partly responsible for this process). Compared to observations, the simulated growth of the sea ice cover is delayed and more rapid once started. The use of a constant mixed-layer depth is partly responsible for this feature (cf. Section 2.5). On the other hand, the ice decay in spring is reproduced correctly.

### 2.4.2 Ice thickness

The average modeled thickness for the whole domain in fall and winter (Figure 2.3) ranges from 50 cm to 75 cm, slightly higher than the observed average thickness of 40–60 cm reported by Wadhams et al. (1987) for the Atlantic sector. Perennial ice of about 1.0–1.2 m thickness is present in the Ross, Amundsen-Bellinghausen and Weddell Seas during summer (Figure 2.2a), due to wind induced ice build-up against the coast during the winter. This is in good agreement with measurements of summer sea ice in the Bellinghausen and Amundsen Seas, where mean thicknesses of 1.3 m and above were found (Haas, 1998). During winter, Worby et al. (1996) measured mean first-year ice thickness of 70 cm and higher further away from the coast in those same regions, in good agreement with the simulated results. Measurements of Jeffries and Adolphs (1997), which show a band of very thin ice along the Ross Sea coastline followed by an extended area of 80 cm thick ice on average and thinner ice towards the ice edge in early winter are also well reproduced. In February, when the minimum ice extent is reached, only compact multi-year pack ice remains.

Simulation results show that the sea ice cover in the various sectors around the continent differs substantially in thickness. For instance, the simulated mean sea ice thickness in the Ross, Bellinghausen and Weddell Seas (above 80 cm) exceeds the mean ice thicknesses in the Atlantic and Indian-Pacific sector by approximately 30 and 40 cm, respectively. This is a result of ice growth due to dynamic processes occurring in specific locations, e.g. in the Weddell and Ross Seas (Figure 2.3), from where it is advected eastward by the mean cyclonic wind field. During the growth season the sea ice cover expands northward mainly due to thermodynamic processes, i.e. advection only plays a minor role in this northward expansion. To this end, the monthly advance of the Weddell Sea ice edge simulated by the model (not shown here) shows that within 30 days, the sea ice expands north-eastward much faster (more than 1500 km/month) than advection process alone would predict (780 km/month), i.e. ice forms locally as a result of thermodynamic processes. East of the Antarctic peninsula ice growth is mainly due to dynamic processes (i.e. the dominant winds blow ice against the coast).

Figure 2.5 shows the thickness distribution for the months of May, July, September and November. In May, most of the area is covered by ice of about 30 cm thickness. In July, dominating thickness categories range from 30 to 90 cm, while during maximum extent, the most frequent thickness is at 60 cm with large areas of thicker ice existing particularly in the Ross, Bellinghausen and Weddell Seas and regions in the eastern Atlantic sector (Figure 2.3). At 60 cm, the seasonal pycnocline is removed and ice-growth-induced convection takes place, keeping the ice thickness relatively
stable. The thicker part of the distribution is mainly due to dynamic processes in the Ross and Amundsen-Bellinghausen Seas. In November, the peak is shifted to 90 cm. Histograms also display the multi-year ice regions with thickness above 2 m in the last category.

2.4.3 Ice velocity

Figure 2.4 shows the simulated ice velocity field for the month of September of the control experiment. Ice is mainly driven by winds and consequently, the ice velocity field follows the atmospheric flow pattern. At the end of the summer and during the early growth season (not shown here), ice moves with 15 to 20 cm s$^{-1}$ along the coast westward or slightly offshore due to the katabatic winds. Further offshore, in the Ross and Weddell Seas, ice is advected by the cyclonic wind field. Near the ice edge, the sea ice flow is divergent and the ice drifts freely. This together with and the high wind speed present in this region during winter, account for the relatively high drift speed during the cold season. In the fall, the Ross Gyre is well developed while in the Weddell Sea, the ice is pushed to the northern Weddell Sea by a U-shaped current. Between the areas where katabatic winds are effective and the cyclonic Antarctic Circumpolar Current there is a belt of low velocity, high divergence and thus reduced ice thickness and concentration. Ice speed increases moving away from this area from almost zero to more than 20 cm s$^{-1}$ near the outer ice margin. In early winter, the circulation of the Weddell Gyre is completely developed. In November, the ice breaks up close to the continent and sea ice drifts away from the land and gradually melts as it is moving northward.

![Figure 2.3: Simulated sea ice thickness for the control experiment during the maximum sea ice extent (September). No cut-off value for concentration applied.](image)

![Figure 2.4: Simulated mean ice velocity field for the month of September of the control experiment. Ice speeds are given in [cm s$^{-1}$].](image)
Figure 2.5: Simulated thickness distribution for the month of May, July, September and November in the control experiment. The summer months are not included as most of the ice has disappeared. Thickness is shown in 10 cm categories. The cut-off value for concentration was set to 1%.
2.5 Sensitivity study

In each sensitivity experiment, one parameter was changed from the control simulation. In a few experiments, the range of the chosen value of the considered parameter is unrealistic, i.e. beyond its physical range, then it is more the importance of a physical process which is analyzed. In some cases, the resulting effects are easy to predict, but the experiments were still performed, as the main purpose of these experiments is the quantification of changes rather than a qualitative description of the model response. The discussion below focuses on the extremes of the annual amplitude, i.e. maximum extent in September and minimum in February. A summary of the sensitivity experiments is given in Table 2.1.

The sensitivity experiments can be divided into two main categories, those involving the thermodynamics of sea ice (Exp. 1-32) and those involving the dynamics (Exp. 33-40). The thermodynamic sensitivity study can be further subdivided into experiments dealing with heat fluxes at the ice and ocean-atmosphere interface (Exp. 21-28), and with ocean heat fluxes and its interactions with sea ice (Exp. 29-32). Not all the sensitivity experiments will be discussed here.

2.5.1 Thermodynamic aspects

Ice albedo

The two extreme albedo values chosen in experiments 1 and 2 are representative of a grey ice situation ($\alpha = 0.5$) and conditions when a fresh snow cover is present on the ice ($\alpha = 0.85$). In both cases (no figures shown here), significant changes only occur in summer. In the grey ice case, the volume, total areal extent and mean sea ice thickness ($V/A_{tot}$) are reduced by 75%, 74% and 45% respectively; in the fresh snow covered ice case, they increase by 37%, 23% and 15%. The areal mean sea ice concentrations are only affected from November through April. The low albedo value of 0.5 leads to a 6% decrease in mean sea ice concentration and the 75% loss of total areal coverage occurring mainly concentrated at the edges of the ice pack in spring and summer. The increased albedo of 0.85 raises the mean ice concentration in summer by 4%. In winter, almost no influence on the ice extent has been observed. Generally, the albedo effect on sea ice characteristics is proportional to the change in the ice albedo used in the simulation. Changes in ocean albedo (Exp. 3 and 4; Ctrl: 0.05, low: 0.01, high: 0.09), only had a small influence on the ice characteristics as the range over which the parameter was varied was small.

Atmospheric absorptivity of shortwave radiation

In general, air temperatures and water vapor concentration are higher over open ocean than over sea ice (Andreas and Cash, 1999). Smaller scale (sub-grid) features such as leads also contribute significantly to the water vapor and heat flux into the atmosphere. The model takes this into account using two different parameters for the atmospheric absorptivity to shortwave radiation, one over water and one over ice. The bulk atmospheric absorptivity for a grid cell is given as the weighted average between the two values.
As in the albedo experiments, the effect of the atmospheric absorptivity on the results is obvious because there is a direct response of the sea ice to the net solar radiation available at the surface. Figure 2.6 shows the results for the sensitivity experiment on the atmospheric absorptivity over ocean. A reduced atmospheric absorptivity over ocean leads to a 28% reduction in total ice area and 31% in volume in the summer and around 10% and 8% in winter (Figure 2.6a,c). The mean ice thickness is increased by 2% in winter. On the other hand, an increase in atmospheric absorptivity over ocean leads to a 74% increase in total ice area and 50% increase in volume at the end of summer and a 13% and 8% increase in winter (shortwave radiation is small but present over a large fraction of the Southern Ocean). The mean ice thickness in winter is 4% lower when compared with the control experiment. Results from a varied atmospheric absorption to shortwave radiation over ice (Exp. 5 and 6, not shown here) show the same tendencies but with much smaller magnitude. Changes in the atmospheric absorptivity over ice has a minor impact since in the summer, when the incoming shortwave radiation is higher, the ice extent is small and the ice albedo is high compared to the ocean albedo.

A change in atmospheric absorptivity directly influences the amount of penetrating shortwave energy, and therefore controls the amount of available energy absorbed by the ocean and consequently, the ocean temperatures during the summer. This in turn influences the onset of ice formation in the fall and the volume of ice formed during a yearly cycle (Figure 2.6a). Changes in mean ice thickness are in the range of ±5%, i.e. the increase or decrease in volume in this experiments mainly results in a change in areal extent (Figure 2.6c) as opposed to ice thickness (Figure 2.6b). Figure 2.6d shows the difference in ice thickness between the two sensitivity experiments. The main differences occur at the ice edge with ice further north when the atmospheric absorptivity is reduced.

**Atmospheric emissivity**

In the present simulations, the atmospheric emissivity is used as a bulk parameter to simulate the effect of an enhanced or reduced greenhouse effect in the atmosphere. The extreme values of emissivity (0.70 and 0.95) used in the sensitivity experiments (9 and 10) represent a clear sky condition and an enhanced greenhouse atmosphere, corresponding to 2×CO₂ concentrations.

A decrease in atmospheric emissivity (Exp.9) results in a large ice gain in all seasons (Figure 2.7a). In some regions (e.g. inner Ross, Bellinghausen and inner Weddell Seas), the ice thickness builds up close to 2 m and the surface areas of multi-year ice grows considerably (Figure 2.7c). Also, the ice extent is very large during the entire year (figures not shown here). In September, the total ice covered area has increased by 50% while in February, the areal gain reaches 150% (Figure 2.7c). The mean ice concentration, however, changes little throughout the year. In winter, the mean thickness decreases by almost 10% since the increase in ice area (50%) is larger than that of the ice volume (35%).

With an increased emissivity (Exp. 10) most of the ice during the warm months is lost and ice starts growing at the end of summer on an almost ice free ocean. Winter extent is also considerably reduced (20%). Difference plots for thickness show that an enlarged areal extent coincides with areas of reduced thickness in the inner ice
2.5. Sensitivity study

pack (outer Ross and Amundsen Seas, Atlantic-Indian sector, Figure 2.7d) whereas an increased greenhouse effect leads to a smaller coverage but thicker ice in the same marginal regions. The reduced thickness within the enlarged ice pack seems to be a consequence of an increased heat flux from the deeper ocean into the mixed-layer due to convection (cf. Exp. 19 and 20) which leads to melting at the underside of the ice. This process is effective for a longer period than in the control experiment since the onset of ice growth occurs earlier in the season. In September, both the ice volume and areal extent have grown considerably (35% in V, 49% in A_{tot}), and the most frequent ice thicknesses are only slightly shifted to thinner ice (Figure 2.7b); the mean ice concentration is reduced by 10%.

Figure 2.6: Sensitivity experiments 7-8: Atmospheric absorptivity to shortwave radiation over open water. Annual cycle of simulated total sea ice area, volume and mean concentration (a). September ice thickness distribution (b), February and September sea ice extent (c) and difference plot in ice thickness between the high and low sensitivity parameter value (d). Simulation results for the control, high and low value of the sensitivity parameter are shown as solid, dashed and dotted line respectively; thick lines for total ice area. In (b) the cut-off for concentration is set to 1%. In (c) the ice extent for the control simulation is shown as a dark shaded area for February and light shaded area for September. No cut-off value is applied. In (d) differences are capped at ±50 cm and no cut-off for concentration is applied. Positive contours are shown with lines, negative with shaded areas only.
Minimum ice thickness

In the model, the volume of newly formed ice over open water is calculated from the continuity equation of mass. The minimum ice thickness parameter \( h_0 \) (cf. Eq. 2.10) will determine the thickness of this new ice and, for a given volume of ice newly formed, its surface area will follow.

Effects on ice characteristics appeared to be proportional to the relative variation of the parameter. Decreasing \( h_0 \) from 5 cm in the control experiment to 1 cm slightly reduced the ice thickness (Figure 2.8b) and marginally increased the areal coverage and mean ice concentration (Figure 2.8a). In this case, the volume of ice formed during the cold season stays approximately the same but is spread over a larger area of lower thickness. As a consequence, the mean ice concentration is a bit higher and the area of open water is reduced. An increase in \( h_0 \) to 50 cm reduces the winter total ice area by 15%, the volume by 8% and the mean concentration by 10% and up to 15% in fall (Figure 2.8a,c). Ice thickness is considerably higher in the coastal areas except in the Weddell Sea where the ice is thinner (Figure 2.8d), the mean ice thickness is considerably increased all year long (8% in September, up to 45% in fall). An increased minimum ice thickness results in a strongly decreased mean ice concentration, and since the volume of ice being formed is spread over a smaller area, the ice thickness is larger. Almost no ice with thickness lower than 50 cm is present (Figure 2.8b), since from April to the end of October there is almost no melting which leads to a reduced effective ice thickness.

Ocean mixed-layer depth

The sensitivity of the seasonal sea ice cover to the ocean mixed-layer depth is investigated in experiments 15 and 16. Reducing the ocean mixed-layer depth from 30 m (control experiment) to 10 m results in an earlier freeze-up by about a month (Figure 2.9a). The maximum ice extent is much larger (Figure 2.9c), and the ice starts to grow quickly at the end of March, and decays faster and earlier in spring. During winter, the mean ice thickness is reduced by 30% (almost 20 cm, see Figure 2.9b); this decrease is largest in the multi-year ice category (40 to 50 cm reduction). The faster decay is due to the thinner sea ice cover (Figure 2.9b) and the fact that a thinner mixed-layer gets warmer during spring and summer leading to an increased ice-ocean sensible heat flux. Also, the warmer summer mixed-layer leads to an increase in ocean-atmosphere heat flux. This increased heat flux is responsible for an increased ventilation of the heat content of the mixed-layer in the summer and a cooler mixed-layer temperature at the beginning of the fall; this influences the timing and growth rate of ice formation in fall. In this experiment, the total volume of ice formed during the year remains almost unchanged compared to the control experiment (Figure 2.9a). However, the mean ice thickness is considerably reduced (30% in September) as the total area increases.

A deeper mixed-layer of 50 m results in a gain in sea ice during the warm season with growth in the fall delayed in comparison to the control case. On the other hand, the ice extent is smaller in the fall-winter and the ice is thicker in most regions, particularly the outer Ross and Amundsen Seas (Figure 2.9d). In September, the mean ice thickness is increased by 15%. In Figure 2.9a, we note a lag of one and a half months between the peaks ice extent and volume between the two experiments.
2.5. Sensitivity study

The thickness distribution also shows a shift in the dominant thicknesses by about 40 cm. At the minimum ice extent (February) the situation is opposite: the deep mixed-layer allows for a larger summer ice extent, while in the shallow mixed-layer situation, the ice is almost completely removed. This is due to a reduced ventilation of the mixed-layer's heat content. As a consequence, the deeper mixed-layer stays warmer in fall and sensible heat fluxes become dominant. This explains the smaller winter ice extent and the increased summer areal coverage since the mixed-layer does not warm up that much as in the control experiment. Thus, a deep ocean mixed-layer damps the annual amplitude of ice extent while the shallow layer increases it. Mean ice concentration shows only minor changes in both experiments.

Ocean mixed-layer entrainment heat flux (overturning)

The entrainment heat from the deeper ocean into the mixed-layer associated with ice formation in winter is calculated from observed temperature and salinity characteristics of the water column and varies geographically. In the model, the parameter 'overturning' quantifies the amount of entrainment heat for a given amount of ice formation.

A smaller value of the parameter overturning (Exp. 19) results in a larger summer ice covered area, an increased ice thickness (Figure 2.10a,c) and a warmer mixed-layer surface temperature when compared to the control experiment. In winter, the sea ice extent is only slightly increased but the mean thickness is distinctively higher (25% increase) and the polynya-like feature in the Weddell Sea is almost absent. The September thickness distribution is now dominated by 1 m ice (Figure 2.10b), while the ice volume is increased by 30%. This is a consequence of the lower entrainment heat flux from beneath the mixed-layer; in this case the surface heat lost to the atmosphere is mainly provided by the latent heat released during ice formation.

An increase in entrainment heat flux (Exp. 20) decreases the ice thickness during the whole year with largest impact in summer. Both mean ice concentration and thickness are slightly lower and more evenly distributed. In spring, the ice decay is faster (as there is less ice present; $A_{Nat}$ and $V$ are reduced by 35%), and a lot of ice has disappeared by the end of summer. The increased amount of entrainment heat strongly reduces the ice thickness in winter everywhere. In the Ross, Amundsen and Bellinghausen Seas, local thinning is up to 0.5 m. Figure 2.10d shows the differences in thickness between the two experiments indicating that in the case of an increased entrainment heat flux the thickness is much lower than in experiment 19 in the whole domain. Other noticeable impacts are the changed summer ice extent and the shift in the peaks for maximum ice extent which occur later for lower value of overturning.

Atmospheric sensible and latent heat fluxes

Experiments 21 to 28 investigate changes in latent and sensible heat fluxes from the ice-ocean to the atmosphere. The sea ice areal extent and thickness are not sensitive to changes in the latent and sensible heat transfer coefficients between the ice and the atmosphere, except in the summer, when lower coefficients lead to a slight reduction in sea ice extent. Changes in the coefficient for sensible heat transfer
are more important than that for the latent heat transfer (no figures shown). The mean concentration is insensitive to sensible and latent heat fluxes from the ocean and the ice to the atmosphere except for the ocean-atmosphere sensible heat transfer coefficient.

In contrast to the above, the latent and sensible heat transfer coefficients from the ocean to the atmosphere have a considerable impact on sea ice conditions. In both cases, a reduction in the coefficients leads to a loss in ice volume and areal extent, and an increase, to a gain in ice volume and extent (Figure 2.11a,c, Exp. 27 and 28; Exp. 23 and 24 not shown here). In the case of the latent heat transfer coefficient (Exp. 23 and 24), its increase has a larger effect on sea ice extent and volume than its reduction. Also, its impact in winter is bigger than in summer. Near the ice edge, the areal fraction of open water is large and thus an increased latent heat flux is accompanied by a higher ocean-atmosphere heat loss. The increased evaporation leads to cooler ocean temperatures and extends the sea ice cover northward.

The ice pack is most sensitive to variations in the ocean-atmosphere sensible heat transfer coefficient. The lower value results in an extremely small ice extent and volume in all seasons. In this case, the ice starts to grow as late as early May and by the end of November most of it has already disappeared (Figure 2.11a), as the sensible heat flux from the ocean to the atmosphere, responsible for cooling of the ocean surface layer, is less effective. Also, the mean sea ice concentration is up to 10% lower all year long. An increase of the sensible heat flux coefficients in turn, results in a much larger areal coverage and volume in all seasons. In September, the ice extent increases by 20% and the ice volume by 47% even though the thickness of the inner part is reduced (Figure 2.11d). The sea ice starts to form earlier (in February) with the minimum extent occurring at the end of January. In winter, a higher ocean-atmosphere sensible heat flux leads to a faster cooling of the ocean surface and thus an earlier formation of sea ice. This process is particularly effective where open ocean areas are exposed to very low temperatures, e.g. along the coast of the continent. Apart from the northward extension of the ice edge, the belt of mostly open ocean between the continent and the ice pack (katabatic wind areas) is filled with thick and compact ice (Figure 2.11d). On the other hand, the regions usually covered with thick ice in the Ross, Bellinghausen and especially the Weddell Seas are now covered with significantly thinner ice. During summer, a narrow band of ice persists around the entire continent. A 10% increased mean concentration at the end of winter and in summer is also observed.

Changes in the latent heat transfer coefficient has a moderate impact on the thickness distribution (not shown here) while changes in the sensible heat transfer coefficient has a strong impact. For a reduced sensible heat transfer coefficient, the mean ice thickness in winter is increased; for an increased value, the thickness distribution is similar to the control but the fraction of multi-year ice is much larger (Figure 2.11b). This is apparent from the winter thickness distribution (Figure 2.11d), where perennial ice contributes to a substantial fraction of the sea ice cover.
2.5. Sensitivity study

Ocean-ice sensible heat flux

In the present model, the ocean mixed-layer is allowed to warm up even though ice is present in a grid cell. The transfer of heat between the ocean and the ice is achieved through sensible heating in a similar manner as between the ice and atmosphere. In experiments 29 and 30, the effects of a changed ocean-ice sensible heat flux were examined. This sensible heat flux along with the entrainment heat and the ocean-atmosphere heat flux regulates the ocean mixed-layer temperature and the amount of ice formation. A large sensible heat flux from the ocean to the ice results in ocean temperatures very close to the freezing point (when ice is present) and in a smaller amount of sea ice present.

A reduced sensible heat flux results in a strongly increased ice volume year round, with most impact in summer months (doubled volume) when the ice covered area is also twice that of the control experiment (Figure 2.12a,c). In winter, the total area is slightly reduced, the volume increases by 23% and the mean thickness is almost 30% higher (Figure 2.12b). Also the increase of the sensible heat flux leads to increased volume in fall and winter (up to 40%) but to a loss of around 40% in summer. The increased flux removes up to 55% in spring and summer while the winter extent stays almost unchanged (Figure 2.12c). Also the ice retreat in the spring occurs much faster. An increased flux slightly raises the mean concentration in winter and leads to an earlier decay of the ice cover in spring by more than half a month while a decrease gives the exact opposite effect for timing and a 10% decrease for the areal mean concentration. Both experiments show thickness gain in the same locations in winter when compared to the control experiment (e.g. along the coastline of the Atlantic-Indian sector and the outer Ross and Weddell Seas; Figure 2.12d) with the mean ice thickness in winter increased by 30%. In the case of an increased heat flux, smaller thicknesses are present in the inner Ross, the Amundsen and the inner Weddell Seas. The thickness distributions of the two experiments are similar but differ from the control simulation with a broader range of sea ice thickness and a peak shifted towards higher thicknesses (Figure 2.12b).

Sensible heat flux between the bulk ocean mixed-layer and the skin surface

In this model, the skin temperature of the ocean is assumed to be at freezing point; whether ice forms or not depends on the heat flux between the bulk of the mixed-layer and the surface ocean, and the ocean-atmosphere heat flux. This sensible heat flux coefficient has been changed in experiments 31 and 32. The ice thickness and areal extent remain almost the same with an increased sensible heat transfer coefficient. The mean ice concentration, on the other hand, is a bit lower in winter (Figure 2.13a). A decrease, in turn, does not affect the mean concentration but increases the areal extent and particularly the volume which are higher in all seasons (24% in $A_{tot}$, 70% in $V$ in September, Figure 2.13a,c) and brings forward the onset of ice growth by one month (end of February, as opposed to end of March) while the timing of volume is not changed. In this case, the mean ice thickness is 30% larger in the winter months (Figure 2.13b,d). During the cold season, the relative increase in volume is about twice as big as the relative areal gain.
Figure 2.7: Exp. 9-10: Atmospheric emissivity. Legend as in Figure 2.6.

Figure 2.8: Exp. 13-14: Minimum ice thickness. Legend as in Figure 2.6.
2.5. Sensitivity study

**Figure 2.9**: Exp. 15-16: Ocean mixed-layer depth. Legend as in Figure 2.6.

**Figure 2.10**: Exp. 19-20: Ocean mixed-layer entrainment heat flux. Legend as in Figure 2.6.
Figure 2.11: Exp. 27-28: Ocean-atmosphere sensible heat flux. Legend as in Figure 2.6.

Figure 2.12: Exp. 29-30: Ocean-ice sensible heat flux. Legend as in Figure 2.6.
2.5. Sensitivity study

Total area, volume and concentration

Distribution of effective thickness

Ice extent in February and September

Difference of thickness distribution

Figure 2.13: Exp. 31-32: Sensible heat flux between the bulk ocean mixed-layer and the skin surface. Legend as in Figure 2.6.

2.5.2 Dynamic aspects

Drag coefficients

In experiments 33 and 34, the influence of the air-ice $C_{da}$ and water-ice $C_{dw}$ drag coefficients on the sea ice drift and sea ice cover have been examined. Since the ratio of the two coefficients ($C_{da} : C_{dw}$) is the key factor influencing the sea ice dynamics (Martinson and Wämser, 1990), sensitivity experiments on this ratio have been performed. To modify this ratio, the air-ice drag coefficient was kept constant and the water-ice drag coefficient was changed. In the control simulation, a ratio 1:2.5 has been used as suggested by Martinson and Wämser (1990).

A 1:1 ratio, (i.e. a reduced water drag coefficient, Exp. 33) leads to an almost complete removal of multi-year ice in summer and a nearly unchanged winter ice extent (Figure 2.14a,c). Also, the ice extent is much smaller in spring, the rate of decay is much higher and the ice volume and ice area are reduced in all seasons with larger changes in the summer (50% for $A_{tot}$ and $V$) than in the winter (22% for $V$, no change for $A_{tot}$). Since thermodynamic conditions are not modified in these experiments, differences from the control simulation are direct effects of the ice dynamics and indirect thermodynamic effects through the coupling with the dynamics. During the decay of the ice pack, ice floes are freely drifting with increased velocity and thus travel over larger distances. In spring, the ice is advected northward more efficiently into warmer waters where melting is more effective. In winter, the ice compactness is high and ice interaction reduces drift speed.
An 1:5 ratio, results in a doubled area of perennial ice in summer, unchanged ice extent but slightly different shape and position of the ice cover edge in winter. The mean ice concentration only shows minor changes during ice decay and in the summer months (Figure 2.14a). The strongly delayed melting and break-up of the ice in spring is due to the larger ice thickness of almost 20% during that season. The peaks in the winter ice thickness distributions are clearly different (50 cm in the peak thickness) in the two experiments (Figure 2.14b). This is also shown in Figure 2.14d where differences in thickness between the two experiments are above 0.5 m in large areas. This is explained by the increased or reduced ice advection to lower latitudes due to a modified water-ice drag.

**Ice yield stress**

The ice yield stress is defined as the compressive strength of a 1 m thick ice cover at 100% concentration. Once this critical value is reached, the pack ice fails and ridging starts. It has a strong influence on the spatial distribution of ice concentration and on thickness distribution of the sea ice cover (Exp. 35 and 36). The most noticeable impact of a reduced ice yield stress (Exp. 35) is the appearance of polynya-type areas with reduced ice thickness and concentration in the Ross and Weddell Seas. The mean concentration is not changed significantly but the spatial variation in ice concentration is larger than in the control case (not shown here). In summer, the area covered by sea ice is reduced by 50% and the ice volume strongly increases, (more than 200%, Figure 2.15a,c). As a result, the mean ice thickness exceeds 4 m in the late fall and in the summer. This is due to the fact that ice is more easily deformable and becomes thicker due to ridging in the winter. In winter, the volume is increased by 18% but the area remains almost the same. The thickness distribution looks similar to the control case except that the fraction of multi-year ice now comprising of very thick ice of up to a few meters is almost doubled (Figure 2.15b). Reducing the ice strength allows for easier ridging and results in a more divergent flow in the center of the Weddell and Ross Gyres. These processes, divergence and ridging, happen in two different locations, close to the ice edge and within the inner ice pack, leading to large areas of open water. The open ocean areas refreeze quickly which explains both the additional amount of ice volume formed in winter, and the increased ice thickness. As a consequence, the volume of perennial ice in summer is large as well as the deviations of the mean ice concentration from the control simulation in winter.

In experiment 36, an increased ice yield stress leads to evenly distributed thickness and concentration fields with low gradients and, thickness in regions with usually highest values are considerably reduced. Figure 2.15d shows the areas where most thickness change occurs, i.e. the centers of the Ross and Weddell Gyres and east of the peninsula. These are the regions where either divergence or persistent winds blowing ice against the land play an important role. The tendency towards the formation of polynya-like features is now suppressed. This is because the ice now provides more resistance to compressive load and thus prevents large divergence at the centers of gyres. On the contrary to experiment 35, open water areas are small and therefore no additional volume is formed during winter.
2.5. Sensitivity study

Wind and ocean current turning angles

The wind and water turning angles ($\theta_a, \theta_w$) account for deviations between the surface air flow and water current and the geostrophic flow (or current) outside the planetary boundary layer. In this study, the model is forced with surface wind stresses and consequently the wind turning angle is set to zero. For the ocean a value of $\theta_w = -15^\circ$ is used based on measurements by Martinson and Wämsler (1990).

The wind and water turning angles have been changed in experiments 37 through 40. The simulation results show that the sea ice characteristics are influenced by the difference between the wind and the water turning angle rather than by their individual values. The growth of the ice cover in fall and winter as well as the ice maximum and minimum extent are not affected by different turning angles (Figure 2.16a,c). As a consequence of a reduced difference in turning angles ($\theta_a - \theta_w = -10^\circ$), more ice remains during the melting period while an increased difference ($\theta_a - \theta_w = -20^\circ$) removes the ice earlier and faster. The latter situation is a result of a more divergent ice flow which brings more ice into warmer lower latitudes where it melts faster. This is confirmed by the difference plot for thickness (Figure 2.16d) which shows increased thickness along the coast from the peninsula throughout the whole Atlantic-Indian sectors and almost to the Ross Sea where the ice is forced more towards the Antarctic continent where it piles up and fills the usually open water areas along the coast. Also, in the outer Weddell Sea ice advection into warmer waters is increased; this explains the additional ice which will be subject to a quick melt in the spring in this region. On the other hand, it is reduced in the central area of the Weddell Gyre. A reduced difference between the turning angles leads to higher thickness in the central region of the gyre and reduced thickness at the periphery. The thickness distribution is hardly changed in both cases (Figure 2.16b).
Figure 2.14: Exp. 33-34: Drag coefficients. Legend as in Figure 2.6.

Figure 2.15: Sensitivity experiments 35-36: Ice yield stress. Legend as in Figure 2.6.
2.6 Conclusions

A simulation of the Antarctic sea ice cover and an investigation of the model’s sensitivity to selected physical parameters used in a control simulation are presented. The model is coupled thermodynamically to a simple mixed-layer ocean as well to a simplified atmosphere. Results indicate that the model is capable of simulating the important characteristics of the Antarctic sea ice. The model reproduces the observational data for the growth and decay of a sea ice cover in an annual cycle. The ice thickness, ice edge position, areal coverage, average concentration as well as their spatial and temporal distribution are in reasonable agreement with observations. Model outputs for thickness and volume are also compared to measurements, but due to sparse data they have to be considered preliminary. There are still some irregularities such as a delayed onset of ice growth in the austral fall followed by a too fast spread out. Also, the summer sea ice extent is underestimated.

The sensitivity experiments include the impact of short and longwave radiation, sensible and latent heat fluxes, ocean mixed-layer parameters, drag coefficients and turning angles. In the model experiments, sea ice is highly sensitive to changes of the atmospheric absorptivity to shortwave radiation over water and to the atmospheric emissivity. Also, the ocean mixed-layer depth, the entrainment heat from beneath the mixed-layer or the amount of sensible heat transferred from the ocean to the ice or
within the mixed-layer critically influence the sea ice characteristics. The importance of ice-atmosphere and ocean-atmosphere sensible heat fluxes clearly dominates that of the latent heat fluxes, the previous being most active at the ocean-atmosphere interface. Variations in the drag coefficients show significant changes in ice extent during ice decay in spring and in thickness distribution in winter. The amount of perennial ice in summer is slightly dependent on winter ice volume (cf. Exp. 31-32) but is not strongly related to the maximum extent in winter (cf. Exp. 15-16). The mixed-layer depth particularly dominates the amplitude of sea ice extent during the annual cycle. The quantity of entrainment heat from the deeper ocean does not shift the onset of growth or the areal extent but strongly affects the spatial and especially the thickness distribution instead.

The temperature of the mixed-layer ocean at the end of summer seems to be a controlling factor for winter ice extent (cf. Exp. 9-10). The ice characteristics react very sensitively to an altered mixed-layer depth. While ice is forming, convection in a shallow layer is very effective and the entraining heat suppresses or even prevents ice growth for some time. Otherwise heat loss to the atmosphere, i.e. cooling of the thin mixed-layer, is effective and also the seasonal pycnocline will be eroded early. This leads to a fast and wide ice growth, which happens in experiment 15 at the end of March. The fast decay is due to the fact that ice thickness is reduced, break-up occurs more easily and the shallow mixed-layer warms up quicker. A deep mixed-layer exhibits the opposite behavior; convection is more inertial and cooling takes much longer. Therefore, ice formation starts later and the margin of freezing temperature at the ocean surface is shifted to higher latitudes which limits the ice extent. The overturning parameter, representing the ocean mixed-layer entrainment heat flux, has major impact on winter ice thickness and volume and also on the ice extent in the spring-summer. It is responsible for ice melting at the underside of the ice cover and thus responsible for a strong shift of the maximum of thickness distribution. It even may prevent ice formation when set to a large value, or it allows for a quicker and more extended ice growth when set low or ineffective. Overturining only acts during ice growth when salt is rejected and therefore only influences physical processes during the build-up of an ice cover, but indirectly all year round since it substantially affects the ice thickness. The amount of ice outlasting summer also depends on how much ice was present in the preceding winter. The entrainment heat thus can delay or accelerate ice formation in the corresponding season depending on its value (Exp. 19-20).

Validation of model results is possible for most ice characteristics except for sea ice thickness for which data in the Antarctic region are sparse. As demonstrated by the experiments, the ocean and the atmosphere being the adjacent media, fundamentally control sea ice and thus it might be useful to commit further refinements to the simplified atmosphere and ocean models. Such improvements as well as some enhancements of the thermodynamic section of ice the model is feasible and scheduled for the near future.
Chapter 3

The Surface Energy Balance of the Arctic from SHEBA Data
Abstract

The Sea Ice Model Intercomparison Project, Part 2, Thermodynamics (SIMIP2) uses observations of the year-long SHEBA field experiment as a set of forcing data for validation of thermodynamic sea ice models. To judge the consistency of the SIMIP2/SHEBA data, a heat budget of the Arctic was done. A detailed evaluation of observations shows that snow depth and ice thickness measurements have to be aligned with the measured temperature profiles to get correct estimates of the ocean heat flux and the conductive heat flux in the snow and ice. The heat budget of surface fluxes shows a mean energy deficit of 7 Wm\(^{-2}\) during winter and a mean surplus of 5 Wm\(^{-2}\) during summer. The total annual net balance gives a residual of 1 Wm\(^{-2}\). The winter difference is a consequence of an imbalance of the net atmospheric and the conductive heat flux, in the summer it is a discrepancy of the net atmospheric and flux and the energy of melt. Results from the heat budget analysis helped to provide an error estimate on various atmospheric heat fluxes, the conductive heat flux and the ocean heat flux. These error bars are useful to quantify the ability of thermodynamic sea ice or coupled atmosphere-ocean models to reproduce the observed snow and ice characteristics. It was found that individual measurements of the snow and ice thickness or even means of several gauges are not representative for the locations where temperature profiles in the snow and ice were measured. Various adjustments to the snow and ice thickness were necessary. Observed skin temperatures and changes of the vertical temperature gradient in the snow suggest that the snow cover at the reference heat and mass balance site was thicker than most adjacent measurements indicate. An analysis of the conductive heat flux at both sides of the snow-ice interface suggests an effective conductivity of the snow of 0.5 Wm\(^{-1}\)K\(^{-1}\).
3.1 Introduction

Sea ice and its seasonal snow cover have a large effect on the heat exchange between the oceans and the atmosphere at high latitudes. Due to its high albedo, insulating properties and impact on the stratification of the ocean mixed-layer during formation and melt, sea ice is a major factor determining the energy budget of the atmosphere-ice-ocean system. The thickness distribution of sea ice and its thermal and physical properties play a key role in the complex heat budget at the atmosphere-ocean interface. Sea ice characteristics change drastically during the annual cycle. In winter, the almost contiguous and compact sea ice is generally covered by snow while in summer the bare ice surface can be heavily ponded and the ice cover breaks up to a fragmented collection of freely drifting floes.

As the energy balance of the Arctic is of relevance for the global climate, it was intensively measured and analyzed in the past (e.g. Untersteiner, 1961; Untersteiner, 1964; Fletcher, 1965; Aagaard and Greisman, 1975; Maykut, 1978; Maykut, 1986; Lindsay, 1998; Jordan et al., 1999). During the Arctic Ice Dynamics Joint Experiment (AIDJEX) in 1975/76, data were collected and energy fluxes estimated. Results of the heat budget and mass balance during AIDJEX are discussed in Maykut (1982) and Maykut and McPhee (1995). There were also modeling efforts for simulating and computing the energy fluxes, atmospheric conditions and ice characteristics, e.g. by Maykut and Untersteiner (1971) and Ebert and Curry (1993). Perovich et al. (1997) present findings of a 15-months single point record of ice temperature profiles and mass balance observations at a multi-year ice floe in the Beaufort sea collected during 1993/94.

A more recent effort for understanding the physical processes in the Arctic was the Surface HEat Budget of the Arctic Ocean (SHEBA) field experiment in 1997/98. The main goals of SHEBA were (1) the investigation of the energy budget of the atmosphere-ice-ocean system in high northern latitudes during a full annual cycle, (2) to get detailed insight in complex interactive mechanisms such as the ice-albedo or cloud-radiation feedbacks, and (3) to contribute to an improved treatment of sea ice in large-scale climate models. To this end, a Lagrangian single column observing system was established where measurements from the deep ocean to the high atmosphere were collected. This vertical transect is referred to as the 'SHEBA Column'. A general overview and summary of the SHEBA project is given by Perovich et al. (1999) and Uttal et al. (2002). Results of the observed atmospheric radiative and turbulent fluxes are presented by Intrieri et al. (2002) and Persson et al. (2002). A series of studies describes the evolution of the snow and ice characteristics over the annual cycle (e.g. Perovich et al., 2001; Perovich and Elder, 2001; Perovich et al., 2002; Sturm et al., 2002). In an investigation of McPhee et al. (1998), observations regarding the ocean mixed-layer and sea ice properties of the SHEBA drift are compared to previous measurements and differences and changes are analyzed. Perovich and Elder (2002) estimate monthly averages of the ocean heat flux at SHEBA from observations of thickness changes at the ice base and internal ice temperatures. A detailed description of the measurement set-up including the obtained data regarding snow and ice physics and their optical properties can be found in Perovich et al. (1999).
Most of the past studies present the various components of the surface heat balance as a time series of average monthly values of the heat fluxes or the annual total energy for each component. Provided that those budgets are closing, they give only little or no information on short time scales and obscure all events of high frequency forcing such as synoptic-scale weather patterns. The focus of the present study is on the energy budget at the snow or ice surface and the ice base as the SHEBA observations allow for a high temporal resolution of the heat budget and its components. However, there is also an emphasis on the mass balance of sea ice and its snow cover and their thermodynamic evolution. The energy budget presented in this study is done in the context of the Sea Ice Model Intercomparison Project, Part 2, Thermodynamics (SIMIP2) to obtain an observational base for comparing and validating model results. It addresses the issue of the coherence of the given time series used to force thermodynamic sea ice models and whether the budget of involved fluxes is closing or not.

Measurements for an energy budget are generally independent observations compiled in different media with different instruments and measurement techniques which involves some error. Even the precision of a single measuring device can be problematic when systematic errors are involved. For instance, there is a significant uncertainty in the accuracy of snow fall measurements and also in the areal representativeness of a point measurement of snow cover thickness. It was noticed that under-catch due to wind effects leads to significant errors in measurements of solid precipitation (Yang, 1999). Uncorrected measurements of snow precipitation generally underestimate the accumulation at the surface in the Arctic (Walsh et al., 1998). As a result, the snow cover measured at SHEBA will be carefully examined and adjusted based on the analysis of observed internal and surface temperatures of both snow and ice.

There is also some vagueness in the thermal conductivity of snow and consequently the magnitude of heat conduction in a snow cover. The thermal conductivity of snow has been determined in numerous studies and investigations in the field and in the laboratory using both direct and indirect measurement methods (Sturm et al., 1997). A general problem is that in all these efforts to derive the thermal conductivity of snow it is given only as a function of snow density although other physical properties of the snow cover such as its temperature and the bonding of individual grains determine the pure thermal conduction while the grain size and shape plays an important role for the effective conductivity. Also conditions in the atmospheric surface layer like wind speed and humidity are crucial characteristics as they force wind pumping and the advective and diffusive moisture flux in the snow cover. The review of Sturm et al. (1997) shows that there is a considerable scatter in measured data of the conductivity owing to the micro-structure of the snow and it was found that differences in these micro-physical properties can result in an order-of-magnitude range in conductivity of snow at a given density. This implies that density is not the only controlling parameter for snow conductivity. However, the range of thermal conductivity of snow measured at SHEBA was very small. The mean of 194 locations was 0.14 Wm$^{-1}$K$^{-1}$ with a standard deviation of 0.03 Wm$^{-1}$K$^{-1}$ (Sturm et al., 2001). To account for the ambiguity in the snow conductivity (the low SHEBA value of 0.14 Wm$^{-1}$K$^{-1}$ and the commonly used value
of 0.31 Wm$^{-1}$K$^{-1}$) which was also detected in the SHEBA observations (Sturm et al., 2002), this work thoroughly addresses the determination of the thermal conductivity and the conductive flux in the snow cover in contrast to direct measurements at the SHEBA ice floe. In the present study, the conductivity of snow is evaluated indirectly from observed internal temperatures and compared to values of direct measurements.

The outline of this chapter is as follows: In Section 3.2, the relevant measurements and data used in the energy budget will be presented followed by a presentation of results in Section 3.3 which is divided into several subsections: In Section 3.3.1 the observations are analyzed, Sections 3.3.2 and 3.3.3 evaluate the measured ice and snow slabs, respectively, and Section 3.3.4 investigates the thermal conductivity of snow. The heat budget at the surface and base of the sea ice cover and problems associated with the closing of the budget will be presented in Sections 3.3.5 and 3.3.6. Finally, Section 3.4 summarizes the most important findings and conclusions.

### 3.2 SHEBA data

The SHEBA field experiment covered a complete seasonal cycle from October 1997 through October 1998 in the Beaufort and Chukchi seas of the Arctic Ocean. SHEBA data are available through the Joint Office of Science Support, University Corporation for Atmospheric Research (JOSS/UCAR)¹. Atmospheric data were measured at the meteorological towers of the SHEBA Project Office (SPO) and the Atmospheric Surface Flux Group (ASFG) and at various locations above the ground using Portable Automated Mesonet (PAM) stations (Persson et al., 2002).

Most data used in this study are taken from an integrated data set which was compiled and processed for the ongoing Sea Ice Model Intercomparison Project, Part 2: Thermodynamics (SIMIP2)². The atmospheric part of the SIMIP2 data set comprises wind speed, air temperature and humidity measured at a level of 10 m, shortwave and longwave radiation, and the precipitation rate. The albedo provided for SIMIP2 was measured at a point close to the SHEBA heat balance site Pittsburgh on an undeformed multi-year ice floe with a snow cover during winter and a bare ice surface without melt ponds in the summer (Perovich et al., 1999). Ocean heat flux data of the SIMIP2 data set was derived from ice characteristics such as internal ice temperature and basal melt rates (Perovich and Elder, 2002). The SIMIP2 data were linearly interpolated to a time step of 1 hour and run from October 31, 1997 through October 8, 1998, i.e. they nearly form a complete annual cycle.

In addition to the SIMIP2 data set, upward longwave radiation and atmospheric turbulent heat fluxes (from high frequency eddy correlation measurements by sonic anemometers and calculated for the 10 m level from observed vertical profiles of the air temperature and relative humidity from the SHEBA tower) were taken from an integrated atmospheric data set³ (Beesley et al., 2000; Duynkerke and de Roode, 2001) which contains hourly averaged surface meteorological and flux data collected

¹http://www.joss.ucar.edu/cgi-bin/codiac/projs7SHEBA
²http://acsys.seos.uvic.ca/acsys/simip2/
³http://www.atmos.washington.edu/%7Eroode/SHEBA.html
by the ASFG. The turbulent heat fluxes from the integrated data set are used for a comparison with the sensible heat \( F_{\text{sh}} \) and latent heat \( F_{\text{lh}} \) calculated from the SIMIP2 10 m atmospheric data using simple bulk formulations,

\[
F_{\text{sh}} = \rho_a c_{pa} C_{\text{sh}} |u_a| (T_a - T_{\text{surf}}) \\
F_{\text{lh}} = \rho_a L_s C_{\text{lh}} |u_a| (q_a - q_{\text{surf}}) ,
\]

where \( \rho_a, c_{pa}, u_a, T_a \) and \( q_a \) are the density, heat capacity, velocity, temperature and specific humidity of air, respectively, \( T_{\text{surf}} \) is the surface temperature, \( L_s \) is the specific latent heat of sublimation, \( C_{\text{sh}} \) and \( C_{\text{lh}} \) are the sensible and latent heat transfer coefficients and \( q_{\text{surf}} \) is the specific humidity at the snow/ice surface. The specific humidity at the snow/ice surface, \( q_{\text{surf}} \), was computed according to \( q_{\text{surf}} = 0.622 \frac{e_s}{P_{\text{surf}} - 0.378 e_s} \), where \( P_{\text{surf}} \) is the air pressure at the surface and \( e_s \) is the saturation vapor pressure which is a function of the surface temperature calculated after Murry (1966). The three available variants of the turbulent heat fluxes show generally a similar pattern although in some periods they differ in magnitude likely owing to the different parameterizations and measuring techniques. It is known that the eddy-correlation measurements of the turbulent heat fluxes are not very accurate (cf. documentation of the ASFG integrated data set). Therefore, and for consistency reasons, this study uses the turbulent fluxes computed from the SIMIP2 data.

Skin temperatures of the snow and ice were taken from the PAM stations at the SHEBA locations Atlanta, Baltimore and Florida\(^4\), (Persson et al., 2002). The skin temperatures are based on radiation measurements made close to the surface (Claffey et al., 1999). In this study, skin temperatures are used for the computation of the turbulent heat fluxes, for an alternative determination of the snow thickness and for comparison with measured near-surface air temperatures. Finally, turbulence measurements of the upper part of the ocean mixed layer derived from eddy correlation\(^5\) (McPhee, 2002) have been used to calculate the ocean heat flux which is compared with the one provided by SIMIP2.

The annual cycle of the observations is shown in Figure 3.1. The downward long-wave radiation is mainly a function of the air temperature and is therefore similar to the pattern of the air temperature. The albedo is at 0.84 during the cold season, decreases slightly when the snow surface gets warmer and drops rapidly when the surface gets wet at the beginning of June. The specific humidity is generally below 1 g kg\(^{-1}\) during winter and increases concomitantly with the air temperature. The sensible heat flux mainly follows the wind speed while the summer peak of the latent heat flux follows the time evolution of the specific humidity indicating an upward moisture flux.

The internal snow and ice temperature was measured at nine heat balance sites (Perovich et al., 1999) six of which were used in the present study (Figure 3.2). The six sites were located around the SHEBA camp and represent a variety of ice types: Baltimore (first-year ice), Pittsburgh (snow covered multi-year ice), Quebec 1 and 2 (young ice and multi-year ice with little snow), Seattle (ponded ice) and Tuk (consolidated ridge). Measurements were performed over an annual cycle using a

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\(^4\)http://www.joss.ucar.edu/cgi-bin/codiac/dss?13.121
\(^5\)http://www.joss.ucar.edu/cgi-bin/codiac/dss?13.529
line of YSI thermistors that extends about a meter below the ice base down into the ocean mixed layer and a few centimeters to decimeters into the air above the snow or ice surface (Perovich and Elder, 2001). The vertical resolution (given by the spacing between the thermistors) is 10 cm, and the length of a given thermistor string was 1 m. To cover the vertical snow and ice extent, several rods were attached to each other. At some sites (Pittsburgh and Quebec), two overlapping strings with a vertical shift of 5 cm were used to increase the resolution just above and below the snow-ice interface. In both cases, this caused problems as there were biases in the instruments (possibly due to different calibration) resulting in zig-zag shaped temperature profiles with amplitudes up to several tens of a degree. These wiggles are particularly problematic when temperature gradients are calculated from the profiles. The accuracy of the thermistors was 0.1°C.

Locations where the snow and ice thickness was measured during the SHEBA experiment are called mass balance sites. There were more than 100 mass balance sites deployed on a variety of ice types at SHEBA, ranging from thin first-year ice to thick multi-year ice. For snow and ice thickness measurements, typically three gauges were set up around the thermistor string forming a triangular cluster with distances of about one meter from the thermistor line (Perovich and Elder, 2001). In some cases there were many thickness gauges in the vicinity of a thermistor string. In this study, the three closest gauges to the thermistor line were used, except for Seattle and Tuk where six and five gauges, respectively, have been taken into account. Snow depth was measured with snow stakes which were frozen into the ice. Ice thickness was measured using hot wire gauges which were placed adjacent to the stakes. Accuracies of the stakes and wire measurements are about 1 cm. A more detailed description of instruments and measurement methods can be found in (Perovich and Elder, 2001). Since snow cover heterogeneity is present over spatial scales of a few meters (Sturm et al., 2002), only a few thickness gauges were close enough to the thermistor strings to be representative. The temperature was measured automatically with a frequency of one hour whereas the thickness data were usually collected once every 1-2 weeks during winter, and once every two days during the summer. The temperature measurements at the various heat balance sites started in late October 1997 and ran until mid September 1998 which gives a record length of almost 11 months. Perovich and Elder (2001) present a summary of the mass balance measurements and an analysis of the temporal evolution of the internal ice temperatures at various sites; a comprehensive description of the instrumental setup including the data sets is provided by Perovich et al. (1999).

The individual sites at the SHEBA Ice Station were exposed to relatively uniform forcing; incident radiation showed little spatial variability and surface air temperatures were within 1-2°C at the various locations within several km of distance (Claffey et al., 1999; Perovich and Elder, 2001). The spatial heterogeneity of the snow and sea ice cover on the other hand is much more variable (see Perovich et al. (2001) and Perovich et al. (2002) for a detailed description).
Figure 3.1: Annual cycle of (a) air temperature at 10 m (solid) and 2.5 m (dotted), absolute values of (b) downward (solid) and upward (dotted) longwave radiation, (c) downward (solid) and upward (dotted) shortwave radiation and albedo (dashed), (d) specific humidity of air at 10 m (solid) and 2.5 m (dotted), (e) wind speed at 10 m (solid) and 2.5 m (dotted), and (f) sensible and latent heat flux: computed from vertical SHEBA tower profiles of air temperature and relative humidity at 10 m (dashed), eddy-correlation (dotted) and bulk parameterization using 10 m SIMIP2 atmospheric data (solid). All variables are smoothed using a 7-days running mean.
3.3 Results and discussion

3.3.1 SHEBA data analysis

The sites where skin temperatures were measured are not identical with the locations of thermistor strings. As observed skin temperature will be used for deriving the snow thickness at the sites of the thermistor strings (see Section 3.3.3), it is averaged to fill data gaps in the time series of individual sites. Measured skin temperatures at the three considered locations show only little scatter, therefore the mean is considered representative for the heat balance sites (e.g. Pittsburgh) where
it was not measured. SIMIP2 proposes a constant snow density of 330 kg m\(^{-3}\) which is used to convert observed precipitation rates into snow thickness. This is a gross approximation and ignores processes like surface snow sublimation, drifting snow, compaction and the fact that the fresh snow density is about 100 kg m\(^{-3}\). Assuming a snow density of 330 kg m\(^{-3}\) results in a relatively thin snow cover compared to the stake measurements. To account for the mismatch, measured precipitation rates are increased by 50\% as proposed in SIMIP2.

In this study, observations from Pittsburgh (the official mass balance site for the SHEBA Column and SIMIP2) are examined in details. There were five mass balance gauges at Pittsburgh, however two of them were operational for very short time, and only the three covering the whole measuring campaign (gauges no. 53, 69 and 71) are considered. The initial, maximum, minimum and final snow and ice thicknesses for the selected gauges and the individual mean snow thickness are listed in Table 3.1. The bottom line of the table is the best estimate of the snow/ice conditions at the location of the temperature measurement (see Sections 3.3.2 and 3.3.3). The snow thickness increases during winter to reach a maximum in mid May, just a few days before surface melt started on May 29 prompted by rainfall (Perovich et al., 1999). Surface melt of the ice began when the snow cover disappeared in mid June, whereas the maximum ice thickness was reached at the beginning of June.

During the annual cycle, measurements made at different locations showed the same general evolution although substantial variations are present on horizontal scales of just a few meters. This has direct implications for the interpretation of temperature profiles (Perovich and Elder, 2001; Sturm et al., 2001). On average, the total ablation of the ice surface at SHEBA ranged from 0.5 to 1 m (Perovich et al., 2001), and the snow cover disappeared completely during the melt season. Considering all measurement sites at SHEBA, the average snow cover thickness was 34 cm with a mean bulk density of 320 kg m\(^{-3}\) (Sturm et al., 2001), thus its total melting corresponds to an average surface ablation of 11 cm water equivalent. At Pittsburgh, the mean winter snow thickness measured at gauge no. 53, 69 and 71 differs by up to 12 cm with a maximum daily difference between any two locations of 30 cm. Maximum differences in measured ice draft from the same three gauges amount up to 50 cm. When comparing snow thickness measurements from different mass balance sites (several hundreds of meters to a few kilometers apart), differences of 32 cm in mean winter snow thickness are observed with maximum daily differences of 70 cm (location Tuk). Maximum differences of up to 80 cm in ice draft measurements are observed between different sites (Figure 3.2).

Therefore, local snow and ice thickness measurements are strictly local and cannot be considered representative of the mean snow or ice thickness even on scales of a few meters. Since the mass balance gauges are typically a few meters away from the thermistor lines, any individual thickness measurement or the average thickness measurement may not be representative of the thickness at the thermistor line. This question is not only important for the computation of the energy budget (e.g. calculation of conductive heat flux), but also for the validation of thermodynamic sea ice models against SHEBA data as proposed by SIMIP2. This is clearly seen at all sites including Pittsburgh (Figure 3.2) where the individual stake measurements are plotted together with the internal temperature profile evolution.
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Table 3.1: Summary statistics of snow and ice thickness measurements (in meters) at the Pittsburgh location. $H_{ini}$, $H_{max}$, $H_{fin}$ are the initial, maximum and final snow thickness, respectively. $H_{avg}$ is the mean winter snow thickness and its standard deviation, $h_{ini}$, $h_{max}$ and $h_{min}$ are the initial, maximum and minimum ice thickness, respectively, $gB$ is the total basal ice growth, $m_b$ is the total basal melt and $m_s$ is the total ice melt at the surface. The initial thickness is that of October 31, 1997. In all cases the minimum snow thickness $H_{min}$ was zero and the final ice thickness $h_{fin}$ equal to $h_{min}$. 'Avg' is the mean of the three gauges and 'BE' is this study's best estimate for the snow thickness and the ice draft (see end of this section).

<table>
<thead>
<tr>
<th>gauge</th>
<th>$H_{ini}$</th>
<th>$H_{max}$</th>
<th>$H_{fin}$</th>
<th>$H_{avg}$</th>
<th>$h_{ini}$</th>
<th>$h_{max}$</th>
<th>$h_{min}$</th>
<th>$gB$</th>
<th>$m_b$</th>
<th>$m_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>053</td>
<td>0.05</td>
<td>0.33</td>
<td>0.26</td>
<td>0.22 ±0.07</td>
<td>2.26</td>
<td>2.78</td>
<td>1.58</td>
<td>0.52</td>
<td>0.43</td>
<td>0.77</td>
</tr>
<tr>
<td>069</td>
<td>0.06</td>
<td>0.45</td>
<td>0.12</td>
<td>0.26 ±0.11</td>
<td>1.70</td>
<td>2.40</td>
<td>1.37</td>
<td>0.70</td>
<td>0.43</td>
<td>0.60</td>
</tr>
<tr>
<td>071</td>
<td>0.05</td>
<td>0.24</td>
<td>0.07</td>
<td>0.14 ±0.06</td>
<td>2.13</td>
<td>2.78</td>
<td>1.68</td>
<td>0.65</td>
<td>0.34</td>
<td>0.76</td>
</tr>
<tr>
<td>Avg</td>
<td>0.05</td>
<td>0.31</td>
<td>0.15</td>
<td>0.21 ±0.07</td>
<td>2.03</td>
<td>2.65</td>
<td>1.54</td>
<td>0.62</td>
<td>0.40</td>
<td>0.71</td>
</tr>
<tr>
<td>BE</td>
<td>0.14</td>
<td>0.45</td>
<td>0.15</td>
<td>0.31 ±0.11</td>
<td>1.84</td>
<td>2.54</td>
<td>1.50</td>
<td>0.70</td>
<td>0.43</td>
<td>0.60</td>
</tr>
</tbody>
</table>

3.3.2 Ice thickness evolution

A possible way to determine the best fitting thickness measurement is an analysis of the temperature profiles, similar to what Perovich and Elder (2001) propose. This method works well for defined sharp changes in gradient at the snow/ice surface and ice base but gives ambiguous results if large and sudden changes in surface forcing occur and the internal temperature profile is still responding to the change, and during summer when the air temperature is close to 0°C resulting in almost isothermal snow or ice. Another limitation is the vertical resolution of the thermistors which is 10 cm. In the lowest part of relatively thick multi-year ice, the vertical temperature gradient is nearly constant most of the time and the level of the base is defined by the ocean freezing temperature. At Pittsburgh and also at some other locations, all individual ice thickness curves have very similar shape indicating that the local differences in snow and ice thickness do not noticeably affect the temporal evolution of the basal ice thickness. Accordingly, the ice thickness at the location of the thermistor line can be found by applying a constant shift to an observed basal evolution until a best match with the level of ocean freezing temperature is obtained. The accuracy of such a method is ±5 cm in accord with the thermistor line resolution. The thickness curve obtained following this procedure is shown in Figure 3.3 (thick white line) and is considered more accurate than the determination of the ice base from changes of the temperature gradient at the ocean-ice interface for the reason mentioned above. Its key characteristics are given at the end of Table 3.1 (referred to as the 'best estimate', BE).

3.3.3 Snow thickness evolution

The determination of the local snow surface is more difficult since the individual snow thickness measurements show very different time evolutions. In fact, discontinuities in the vertical temperature gradient at the snow surface are not always
Figure 3.3: Measured temperature and snow/ice thickness evolution of selected gauges (solid: no. 71, dotted: no. 69, dashed: no. 53) at the Pittsburgh location. The length of the thermistor string defines the vertical range of temperature measurements. The snow-ice interface is at $z=0$, as soon as surface melt starts, the ice surface is ablated below $z=0$. The bold line represents the shifted ice base.

The available data provides several possibilities to determine the snow surface at the location of the thermistor string. The first option is simply to use the mean snow thickness of gauges no. 53, 69, 71. An alternative way to determine the snow thickness is to use the measured skin temperature: Each temperature of a measured temperature profile in the snow corresponds exactly to a given level in the snow slab provided the profile is strictly monotonically increasing or decreasing. Therefore, a measured skin temperature can be used to determine a corresponding level in the snow pack by interpolating between the levels of the two thermistors adjacent to the given skin temperature. The derived level defines the snow surface provided the prescribed skin temperature is within the temperature range of the profile which has to be monotonic to get a unique result. In this method, the snow surface temperature is assumed to be equal to the skin temperature. This method was applied to the temperature profiles excluding those which are not monotonic to avoid ambiguous values for the level of the snow surface. The derived snow surface was smoothed using...
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A 7-days running mean. Vice versa, a measured level of the snow surface can be used to determine the corresponding temperature of the profile by linearly interpolating between two thermistors to get the temperature at the given depth. The derived temperature is then considered as the surface temperature.

Another option is to consider the gradient of the temperature profiles in the snow cover using an algorithm to determine the snow-air interface. In this method, the level of the most pronounced change of the temperature gradient was defined as the snow surface. Again, non-monotonic profiles are excluded, missing values interpolated and the resulting curve smoothed using a 7-days running mean. Finally, the change of the temperature gradient has been used again as an indication for the snow surface but in a visual analysis of sets of temperature profiles. To this end, groups of 200 subsequent temperature profiles were plotted and the level where most profiles show a jump in the gradient was taken as the snow-air interface. Despite the fact that this is a subjective method, the pattern of a collection of profiles gives more confidence in the determination of the change of gradient than an automated analysis of a single profile and generally makes it easy to exclude ambiguities caused by individual profiles or short series of profiles in transition. In Figure 3.4, the snow surfaces derived from the various methods described above are shown and compared to the snow depth obtained from the SIMIP2 precipitation rates including the proposed correction of a factor 1.5.

Based on the findings of the alternative methods, the snow thickness measured at gauge no. 69 is judged the most appropriate, particularly for the relatively thick snow layer at the end of the winter. As a consequence of the comparison of the measured skin temperature and the surface temperature derived from the mean snow thickness from the gauges, the snow cover depth is corrected as follows: considering the information provided by the alternative methods, the measurements at gauge no. 69 are increased between Julian days 300 and 480 and adopted afterwards. The estimated surface (Figure 3.4) still includes errors in the order of the grid resolution but is assumed to be more realistic than the average of the gauges which was identified as too low. From now on, the corrected snow thickness is used in the present study. Its key characteristics are given at the end of Table 3.1 (referred to as the 'best estimate', BE). Interestingly, the mean of BE (31 cm) corresponds closely with the mean snow cover from all measurements determined at SHEBA (34 cm, Sturm et al. (2001), see Section 3.3.1).

As a thick snow cover lasts longer than a thin snow layer under melting conditions, the snow cover thickness determines the onset of melt of the ice surface. Such a temporal shift of the onset influences the total amount of surface melt of the ice slab during the summer season. This can be seen from Figure 3.4 where the difference between the surface ice melt during summer measured at gauges no. 69 and the mean of the three gauges is 11 cm.

The skin temperature and the gradient methods fail when snow and ice temperatures approach the freezing point or when the gradient of the observed temperature profiles is small and errors can be larger than the thermistor resolution resulting in an uncertainty regarding the derived snow thickness. Therefore, the given results can only be considered as estimates. A small spacing of the thermistors and high accuracy is of advantage and increases the precision of the methods. The analyses become
Figure 3.4: Snow thickness and ice surface at the Pittsburgh site derived with various methods: stepwise visual (bold dots) and automated gradient analysis (dotted), skin temperature (solid), mean of gauges no. 53, 69 and 71 (bold solid), measured precipitation rates (dashed). The white lines are the best estimate of the snow thickness and the ice surface. The vertical line shows the observed onset of melt.

problematic for near-isothermal conditions. Nevertheless, all alternative methods for finding the snow surface yield a thickness which is significantly higher than the mean of the three gauges. Applying the method described above, the mean snow thickness of gauges no. 53, 69, 71 is used to derive a surface temperature (Figure 3.5).

The resulting surface temperature is significantly higher than the corresponding radiometer derived skin temperature and thus must still be within the snow pack. Differences are largest (up to 7°C) during the three cold spells in winter, denoting that this snow depth substantially underestimates the true snow thickness, i.e. the profile was truncated at a level above which the snow temperature gradient further continues. This truncation, or more general, the setting of the snow depth has consequences for the resulting surface temperature and the calculated conductive heat flux, which will be discussed in Section 3.3.4. For comparison, also the 10 m air temperature and the temperature measured by the uppermost thermistor of the Pittsburgh rod (45 cm above the initial snow-ice interface) are included in Figure 3.5. The latter is considered as a near-surface air temperature as the maximum snow thickness of all three gauges is apparently lower than 45 cm. The skin temperature was also computed using the measured upward longwave radiation \( T_{\text{skin}} = (LW_{\uparrow} (\epsilon \sigma)^{-1})^{1/4} \). As it is very similar to the one measured at the PAM stations (cf. Section 3.2), the curve is not shown in the figure.
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3.3.4 Surface conductive heat flux

As a dry snow cover is built up of a complex matrix of ice crystals, it is a mixture of such potentially multiply altered crystals and air which contains some water vapor. In contrast to conduction in a solid body, in a snow layer the following processes are involved: conduction, convection, heat transport due to diffusion of water vapor as a consequence of ventilation and thermal radiative exchange. The resulting total heat transport is often referred to as the effective heat conduction which can be much higher than a value of the same sample determined in a laboratory environment. An increase of the effective conductivity of snow is related to a corresponding increase in density. A comprehensive review and discussion of most existing parameterizations relating density and thermal conductivity of snow is provided by Sturm et al. (1997). In situ needle-probe measurements of the snow conductivity at SHEBA gave an average value of 0.14 Wm\(^{-1}\)K\(^{-1}\) (Sturm et al., 2001; Sturm et al., 2002). This is a factor 2 lower than the commonly used value of 0.31 Wm\(^{-1}\)K\(^{-1}\) (also proposed in SIMIP2) which corresponds to a bulk density of about 330 kg m\(^{-3}\) and is possibly due to thick layers of hoar in the SHEBA samples. Nevertheless, there are indications that the effective conductivity was rather 0.31 Wm\(^{-1}\)K\(^{-1}\) than the needle-probe value (Persson et al., 2002; Sturm et al., 2001; Sturm et al., 2002).

According to the desired precision, the conductive heat flux at the snow surface or the ice base (needed in Section 3.3.6) can be determined either by considering the whole snow or ice layer (single-layer models) or just the uppermost part in the
snow and the lowest part at the ice base (multi-layer models) and calculating the
temperature gradient with a linear or higher order difference scheme (Figure 3.6).
The signal of forcing variations at the surface is smoothed and damped with depth
as there are temporary changes of heat storage within the snow and ice layers. As
a consequence, the conductive heat flux varies with depth. The storage of internal
energy in the ice during non-stationary situations (transition periods) leads to non-
linear temperature profiles. As a result, a bulk gradient over the whole ice thickness
yields just a low precision approximation of the conductive heat flux at a bound-
ary. Therefore, heat conduction determined from a single-layer model is discarded
whereas it is computed at the surface and base of the snow and ice slabs (Figure 3.6)
using $K_s = 0.31 \text{ Wm}^{-1}\text{K}^{-1}$ and $K_i = 2.03 \text{ Wm}^{-1}\text{K}^{-1}$. In the snow, it was computed
within 10 cm next to the surface (which is defined by the best estimate snow cover)
and to the interface, in the ice within 30 cm next to the interface and to the ice base.
In both cases the temperature gradient was approximated with a first order linear
difference. The results shown in Figure 3.6 give evidence that during winter there
is a large discrepancy between the calculated conductive heat flux in the snow just
above the snow-ice interface and the heat conduction in the ice just below the inter-
face. Due to the storage of heat in the ice slab in spring (Figure 3.3), the conductive
flux at the ice surface has an opposite sign than at the base, i.e. heat is conducted
from above and below towards the inner part of the ice layer.

![Figure 3.6: Conductive heat flux (absolute values) at various levels of the snow/ice
slab: snow surface (solid), snow base (dotted), ice surface (dash-dotted) and ice base
(dashed). Time series are smoothed using a 7-days running mean.](image)

There is a general problem with the conductive heat flux. In winter, heat conduc-
tion in the snow, calculated from the observed temperature profiles is much smaller
than the computed conductive flux anywhere in the ice slab at a given time. This
means that there is a discontinuity of the conductive heat flux at the snow-ice interface when the selected thermal conductivities of snow and ice are involved. The same observation is valid for all other SHEBA sites where temperature profiles in snow and ice were measured.

A possibility to estimate the heat conductivity in the snow is given by the continuity of the heat flux at the snow-ice interface, \(-K_s \left( \frac{dT_s}{dz_s} \right) = -K_i \left( \frac{dT_i}{dz_i} \right)\). To reduce the effects of external forcing such as penetrating shortwave radiation only the winter and spring periods with no or low solar radiation are considered. At the six presented heat/mass balance locations (Figure 3.2) temperature gradients on both sides of the snow-ice interface are approximated with second order asymmetric differences (in some cases first order linear fit due to special circumstances) provided a given conductivity of ice (2.03 Wm\(^{-1}\)K\(^{-1}\)). The sites Quebec 2 and Tuk were excluded since at Quebec 2 the thin snow cover (5-10 cm) prevents to determine a reliable temperature gradient and at Tuk there are only two thermistors above the snow-ice interface which is even not well defined in the temperature profiles owing to large temperature gradients near the ice surface in winter. In addition, the most realistic snow cover measured at Tuk (solid line in Figure 3.2f) is very thin as well. At the four other sites (Baltimore, Pittsburg, Quebec 1 and Seattle), the derived snow conductivity ranges from 0.41-0.55 Wm\(^{-1}\)K\(^{-1}\) (Figure 3.7).

**Figure 3.7:** Thermal conductivity of snow determined from approximated temperature gradients on both sides of the snow-ice interface: (a) Baltimore, (b) Pittsburgh, (c) Quebec 1, (d) Seattle. The time step is one hour, the ablation period is not considered. The thick line is a 7-days running mean, the horizontal line represents the mean. A value of 2.03 Wm\(^{-1}\)K\(^{-1}\) was used for the conductivity of ice. Only values in the range 0.1-1.0 Wm\(^{-1}\)K\(^{-1}\) were considered in the calculation.
There is some scatter in the derived conductivity which mainly results from small inaccuracies of individual thermistors and snow/ice thickness. The method of approximating the temperature gradient was selected according to the situation of the measured temperature profiles. At Baltimore, both gradients are computed using a second order asymmetric difference where in the ice the three successive thermistors below the interface are considered. The overlapping of two thermistor lines on both sides of the interface at Pittsburgh lead to some wiggles in the temperature profile due to instrument bias in that interval (cf. Section 3.2). Therefore, the calculation of temperature gradients is handled as follows: In the snow, a second order asymmetric difference was used involving three adjacent but non-equidistant sensors including the one at the interface, whereas in the ice, a linear temperature gradient between the interface and the second sensor in the ice (leaping the first) is used. This gives a relatively good approximation of the temperature gradient in the upper part of the ice and avoids the effect of the zig-zag in the temperature profiles. As the snow-ice interface is not represented by a sensor at Quebec 1, second order asymmetric differences using the three thermistors above and below the interface respectively, are applied for computing the temperature gradients. Seattle is a kind of ideal case because both gradients can be determined right at the interface using second order asymmetric differences including the thermistor at the snow-ice interface in both cases. A calculation of the thermal conductivity using linearly approximated temperature gradients at Quebec 2 yields a mean of about 0.44 Wm\(^{-1}\)K\(^{-1}\) despite considerable scatter of results. This value is well in the range of the four presented sites; their mean is 0.49 Wm\(^{-1}\)K\(^{-1}\). The continuity of the conductive heat flux at the snow-ice interface requires an increase of the initial thermal conductivity of snow (0.31 Wm\(^{-1}\)K\(^{-1}\)) as demonstrated in the results.

### 3.3.5 Surface energy budget

A complete energy budget at SHEBA can be made from measurements of the radiative fluxes (including the albedo), the atmospheric turbulent heat fluxes, the conductive heat flux (inferred from internal temperature profiles) at the snow or ice surface and the energy of melt (inferred from changes in snow/ice thickness). The surface energy budget can be written as (fluxes towards the considered surface are defined positive, fluxes away from the surface negative)

\[
(1 - \alpha)(1 - I_0) \, SW_\downarrow + LW_\downarrow + LW_\uparrow + SH + LH + C_s + M_s = 0
\]

where \(\alpha\) is the albedo, \(I_0\) is the fraction of shortwave radiation penetrating into the interior of the snow or ice (0.15), \(SW_\downarrow\) is the shortwave radiation, \(LW_\downarrow\) and \(LW_\uparrow\) are the downward and upward longwave radiation, \(SH\) and \(LH\) are the sensible and latent heat flux, respectively, \(C_s\) is the effective subsurface conduction including radiative fluxes, moisture diffusion and advective processes and \(M_s\) is the energy of melt at the surface. The salinity of the melting surface of multi-year sea ice is assumed to be equal to zero due to meltwater flushing, therefore the effective specific latent heat of fusion at the surface is considered constant (i.e. independent of salinity) in the computation of the energy of melt. In this equation, the turbulent heat fluxes are not directly measured. They are computed from basic atmospheric variables.
such as air temperature, wind speed and humidity (Section 3.2) and the measured skin temperature. In the budget of surface fluxes (Figure 3.8), the albedo and the radiative fluxes are measured directly while the turbulent fluxes, the energy of melt and the conductive heat flux in the snow and ice are inferred from other measured quantities. The accuracy of the energy of melt and the turbulent heat fluxes depends on the measured ablation rate at the surface and the reliability of the turbulence parameterization, respectively. During surface ablation, the peaks present in the measured ice surface of gauge no. 69 (cf. Figure 3.4) were removed since there was no surface ice accretion. The improved slope yields a better representation of the energy of melt. The conductive heat flux in the budget is computed using the snow thermal conductivity determined at the Pittsburgh site (0.55 Wm$^{-1}$K$^{-1}$) and the best estimate of the snow thickness and the ice surface. Its accuracy depends mainly on the thermal conductivity and on the level of the derived snow surface since the temperature gradient is computed in a layer below the defined surface.

A reason for biases in the total net balance is the resolution of the thermistors in the snow and ice which prevents to calculate a very precise conductive heat flux right at the surface. Also, there is some uncertainty regarding the magnitude of the energy of melt since it is computed from melt rates of the snow and ice slabs which depend

![Figure 3.8: Measured components of the SHEBA surface energy budget. The conductive heat flux at the surface (pink) and the energy of melt (cyan) are computed using the best estimate snow and ice thickness and the measured internal temperature at the Pittsburgh site. Net shortwave (blue) and net longwave (orange) radiation, sensible and latent heat flux computed with bulk parameterizations (green, black), sum of atmospheric surface fluxes, i.e. net radiative plus turbulent fluxes (violet) and total of all fluxes (red). The vertical lines mark the defined divide of summer and winter (April 1, dashed) and the onset of melt (May 29, dotted). Fluxes are smoothed using a 7-days running mean.](image-url)
on the specified snow and ice thickness (Figure 3.4). Consequently the derived energy of melt may not be consistent with the prescribed forcing fluxes. Such uncertainties may explain the imbalance during summer to some extent. The spatial variability of the snow cover and the inhomogeneity of the ice pack which both impact on the internal snow and ice temperature are likely to result in horizontal temperature gradients in such areas. This spatial variability in temperature and the resulting heat transport (e.g. from a relatively warm hummock to a cold snow-filled depression of a former melt pond) might be another explanation why the winter time conductive heat flux is too small compared to the atmospheric net flux. However, a test using a two-dimensional heat conduction model showed that this effect is negligible.

The overall mean error of about 1 Wm\(^{-2}\) is small but it is the result of a relatively high energy deficit in winter and surplus in summer which even are of different origin. Further, the balance of atmospheric and conductive heat flux is only valid for an infinitesimally thin surface slab. As the forcing signal penetrates with some delay and is also attenuated, the calculation of the conductive heat flux over a certain vertical extent involves some error, i.e. the computed heat conduction will always underestimate the true value. Due to the resolution of the spatial sensors, the thermistor string is certainly missing some of the high frequency changes of the surface forcing.

At equilibrium, the sum of the atmospheric surface fluxes is balanced by the surface conductive flux. For the observed fluxes, this is not the case at the Pittsburgh location where conduction at the surface during winter only partly balances the atmospheric net flux. In winter, the conductive heat flux follows the pattern of the net atmospheric flux which is dominated by the net longwave radiation while solar radiation is absent. The imbalance in winter (until April 1, when net shortwave radiation becomes significant) results from a discrepancy of the net atmospheric flux and surface conduction and amounts to a mean energy deficit of 7 Wm\(^{-2}\) for the surface. During summer, a mean surplus of 5 Wm\(^{-2}\) is observed which principally results from an imbalance of the atmospheric net flux and the energy of melt at the surface (e.g. in early June, Julian days 510-560). In the summer, the conductive heat flux is close to zero or even slightly downward while the net atmospheric flux is supposed to be equal to the energy used for melting.

The three peaks in the conductive heat flux (Figure 3.6) in early winter (around Julian days 360, 380, 410) are due to the three episodes of very low air temperatures, which lead to correspondingly low downward longwave radiation (Ohmura, 2001), ultimately resulting in low temperatures at the snow surface and in the interior (Figures 3.1 and 3.5). Figure 3.8 shows how the conductive heat flux in winter is controlled by the net longwave radiation. The net longwave radiation has large variability and is negative throughout the whole year indicating heat loss from the surface to the atmosphere. The net shortwave radiation is modestly growing with a sudden increase in early June which is the time when the snow cover disappeared and the albedo drops to a lower value. When the surface temperature is at the melting point, the energy surplus is used for melting; this heat flux becomes the major counter-balance of the net solar flux during summer. The turbulent heat fluxes are small and play a minor role in the budget. The latent heat flux is close to zero except during the onset of the melt season when it has a peak indicating moisture
3.3. Results and discussion

transport from the ice to the atmosphere. Figures 3.8 and 3.5 display the coupling of net longwave and sensible heat fluxes through the surface temperature during the cold season: heat loss due to an upward net infrared flux is associated with a downward sensible heat flux towards the snow surface while it is generally upward during summer when surface melt occurs. Means over the given record length for all components of the energy budget are given in Table 3.2, most time series comprise of about 11 months and can be considered as quasi-annual means.

<table>
<thead>
<tr>
<th>Table 3.2: Components of the surface energy budget [Wm$^{-2}$]: shortwave down, up, penetrating and net (SW↓, SW†, SW$p$, SW*), longwave down, up and net (LW↓, LW†, LW*), sensible and latent heat flux (SH, LH), sum of the atmospheric fluxes (Q), conductive heat flux at the surface ($C_b$), energy of melt ($M_b$) and the total net flux ($Q^*$). All values are means of the unsmoothed time series. The second line gives the corresponding annual total energy (for 365 days) in $[10^8 \times J m^{-2} yr^{-1}]$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SW↓</td>
</tr>
<tr>
<td>101.6</td>
</tr>
<tr>
<td>32.0</td>
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3.3.6 Basal energy budget

The budget at the ice base involves the ocean heat flux ($F_{ocn}$), the conductive heat flux into the ice ($C_b$) and the energy of melt ($M_b$). All three components of the budget are derived indirectly from the internal ice temperature profiles and the growth and melt rates at the ice base. The ocean heat flux used in the basal energy budget, on the other hand, is calculated as a residual to follow SIMIP2 and the approach of Perovich and Elder (2002). There are also some observations of the ocean heat flux at SHEBA from measured covariances of temperature and vertical velocity component in the upper ocean. These will be compared with the heat flux residual below, (fluxes towards the considered surface are defined positive)

$$F_{ocn} + C_b + M_b = 0,$$

For calculating the energy of melt, an expression for the effective specific latent heat (Bitz and Lipscomb, 1999) is used which accounts for the dependence of both the specific heat and the latent heat on temperature and salinity. The freezing temperature of a brine solution is given by its salinity times the negative of an empirical constant ($\mu = 0.054°C ppt^{-1}$). A value of 33 ppt was used to specify the salinity of the upper ocean at SHEBA, defining a constant freezing temperature of ocean water at -1.8°C which is a close approximation of the ice base temperature throughout the whole year (-1.4 to -1.8°C). A linear approximation of a measured salinity profile yields a value of 3.2 ppt at the ice base, which results in a temperature and salinity dependent effective specific latent heat of fusion of 0.91 $L_0$, where $L_0$ is the specific latent heat of freshwater ice. The conductive heat flux at the ice base is determined from a linear approximation of the internal temperature gradient within 30 cm above the ice base (see Section 3.3.4). Heat consumed or released for
melting or freezing is computed from the rate of change in thickness at the ice base and the effective specific latent heat of fusion. Due to the varying sampling frequency (2 days - 2 weeks) for the basal growth and melt rates, the derived energy of melt is dictated by this temporal resolution in contrast to the hourly values of the conductive heat flux. The basal conductive heat flux, the energy of melt and the ocean heat flux are shown in Figure 3.9, their means over the given record length are $-8.5 \text{ Wm}^{-2}$, $2.8 \text{ Wm}^{-2}$ and $5.7 \text{ Wm}^{-2}$, respectively. The mean of the ocean heat flux provided in SIMIP2 (Figure 3.11) amounts to $8.2 \text{ Wm}^{-2}$ and would result in an imbalance in the basal energy budget if considered.

The basal energy budget closes exactly, as the ocean heat flux is derived from the other two components. The ocean heat flux calculated as a residual, however, will be compared to measured ocean heat flux (McPhee, 2002) later in this section. Nevertheless, there may still be errors in the computed conductive heat flux and particularly in the energy of melt as it follows from infrequent thickness measurements of the basal ice surface. Although a 7-days running mean was applied to the energy of melt, there is variability with dominant periods of 1-2 weeks as a result of the changes in the basal rate of growth or melt according to the sampling frequency. In this calculation, the ocean heat flux follows the pattern of the energy of melt shifted in magnitude by the amount of the conductive heat flux. Hence, the ocean heat flux is negative in some intervals which does not happen in reality. It might be the result of a missing flux, e.g. horizontal advection in the ocean.

To get a more representative estimate of the ocean heat flux at SHEBA, it was determined at other five mass balance sites where internal ice temperature was

![Figure 3.9: Components of the basal energy budget at the Pittsburgh site: Conductive heat flux (dotted), energy of freeze/melt (dashed) and ocean heat flux (solid). Time series were smoothed using a 7-days running mean.](image)
3.3. Results and discussion

Figure 3.10: Ocean heat flux as derived from internal ice temperatures and growth/melt rates at the ice base from six SHEBA sites: Baltimore (black), Pittsburgh (blue), Quebec 1 (cyan), Quebec 2 (green), Seattle (orange) and Tuk (pink). The red line is an average of all sites; dotted lines are excluded from the calculation of the mean. Data were smoothed with a 30-days running mean.

measured (Figure 3.10). Intervals of an individual site where the ocean heat flux drastically departs from the ensemble (dotted lines in Figure 3.10) were excluded from the computation of the average.

Despite some differences between the various locations (e.g. ice type, internal temperature distribution), the residual ocean heat fluxes of the individual energy budgets at the ice base show very similar patterns. From October to mid January the ocean heat flux is almost zero. The low ocean heat fluxes are due to the stratification of the mixed layer associated with the previous summer melt and freshwater release. This period is followed by two peaks; one around early February and another in mid March. At most combined SHEBA heat and mass balance sites the observed basal growth rate shows a decrease or even a turns into a melt rate for a short time in late March. The increase in the ocean heat flux in early February and the peak from mid to late March are due to storm activity (cf. wind speed in Figure 3.1e), mixing parts of the upper ocean, and probably also due to a change in bathymetry (sea floor topography), as the ship was drifting into a region of shallow water (75°N, 160°W) during the mentioned period (Perovich et al., 1999; Perovich and Elder, 2002). Beginning in May, the ocean heat flux starts to increase and reaches a maximum of about 20 Wm$^{-2}$ in early August before it drops relatively fast to about 2 Wm$^{-1}$ in early October. The big peak in summer is due to insolation which partly penetrates the ice, open water and leads and eventually warms the ocean. While the ice concentration decreases, large amounts of shortwave radiation is absorbed by areas of open water which get stratified and warm up. This leads to lateral melt which fur-
ther prevents vertical mixing. When a storm passes by, mixing occurs which warms up the ocean mixed layer. The change in the basal growth rate around Julian day 450 (Figure 3.3) is a clear indication for the increased ocean heat flux at that time (Figure 3.10) since no particularly warm temperature signal is propagating from the top, which could influence the gradient and thus the conductive flux at the ice base.

In Figure 3.11 the mean ocean flux from the six heat balance sites is compared to ocean heat flux observations derived from turbulence measurements (McPhee, 2002) and to the data provided in SIMIP2 (Perovich and Elder, 2002). The gaps in the observational data were interpolated and the resulting time series was smoothed with a 30-days running mean. In May and June fouling of the sensors caused by jellyfish results in very low turbulent heat flux measurements (M.G. McPhee, personal communication, 2002). Nevertheless, the peaks in February/March and August are clearly captured by the observations derived from eddy-correlation.

![Figure 3.11: Ocean heat flux derived from turbulence measurements (dotted, thin solid). Time series are smoothed using a 30-days running mean. The thick solid curve is the averaged ocean heat flux derived from the growth/melt rates at six SHEBA locations presented in Figure 3.10. The dashed line represents the ocean heat flux as provided in SIMIP2 (no smoothing applied).](image)

The determination of the ocean heat flux from ice thickness and internal ice temperature was also applied in other investigations and studies. In an earlier experiment McPhee and Untersteiner (1982) derived the ocean heat flux from the temperature gradient and growth rate of sea ice. They found values for the ocean heat flux of less than 2 Wm$^{-2}$ from March to May northwest of Spitsbergen in the Arctic Ocean (drifting ice station FRAM1, 1979). Wettlaufer et al. (1990) found in the Arctic Ocean a range of 0 to 37 Wm$^{-2}$ for the oceanic heat flux, whereas Perovich et al. (1989) found even larger values (7-128 Wm$^{-2}$) in the marginal ice zone in December.
These values are about an order of magnitude larger than ocean fluxes computed in many ice-ocean models. According to Wettlaufer (1991), the oceanic heat flux is an important component of the sea ice energy and mass balance but often horizontally very inhomogeneous. For an unmanned 15-months measuring deployment in the Beaufort sea in 1993/94, Perovich et al. (1997) determined an annually averaged heat flux of 4 Wm$^{-2}$ with a maximum average summer peak of 9 Wm$^{-2}$. They found the ocean heat flux during winter to be very close to zero and they also computed it as a residual of the basal energy budget. The findings of the present study are in agreement with both of the mentioned results. The ocean heat flux measured during AIDJEX (Maykut and McPhee, 1995) was by a factor 3 higher than the average of 4 Wm$^{-2}$ determined in the Beaufort sea. All these results show, that the ocean heat flux is very variable on different horizontal and temporal scales. The oceanic flux derived from SHEBA data, however, are within the range of previous observations and the result of near-zero values during winter is confirmed by several studies.

3.4 Conclusions

Measurements from the SHEBA field experiment are used for an energy budget of the snow/ice system. SHEBA data were compiled in an integrated data set which comprises forcing and initialization data for the Sea Ice Model Intercomparison Project, Part 2, Thermodynamics (SIMIP2). The goal of the project is to provide an evaluation data base for validating and comparing thermodynamic sea ice models. Observations of the heat fluxes at the snow or ice surface can be used to verify the surface energy balance. The presented energy budget was done to evaluate the consistency of the SHEBA/SIMIP2 data, to investigate if the budget of heat fluxes at the selected levels of the 'SHEBA Column' (a vertical cylinder from the ocean to the atmosphere as a Lagrangian drifter) is closing and consequently to get an estimate for errors in the observations. As the SHEBA Column is strictly one-dimensional, the energy budget must be of a specific point. In this study, the SHEBA heat and mass balance site 'Pittsburgh' was selected involving the measured temporal evolution of the snow and ice characteristics at that place.

Some of the components of the surface and basal energy budget, such as the turbulent heat fluxes, the ocean heat flux and the conductive heat flux must be calculated from other observations due to the lack of measurements. Snow and ice thickness was measured at many locations on the SHEBA ice floe representing a variety of ice types but not exactly at the same place as temperature profiles in the snow and ice. As a consequence, there is some ambiguity in the definition of the snow surface and ice base right at the locations of the ice temperature observation. The levels of the snow surface and the ice base had to be aligned with information from the measurement of the internal and skin temperature of the snow. The resulting mean snow cover depth at Pittsburgh is rather 30 cm than the average of about 20 cm obtained from measurements of three thickness gauges deployed within a distance of a few meters around the Pittsburgh heat balance site.

Doing an energy budget at a point requires certain observations at exactly that point. These variables are the snow and ice characteristics, the albedo and the ocean
heat flux. It is only locally valid and specific for that location. The use of components in the budget that are averages of a certain area nearby or around the site, likely lead to errors and mismatch of the involved fluxes. This is particularly valid for the conductive heat flux in the snow and ice and the ocean heat flux which varies considerably from place to place.

The performed surface energy budget shows the relative importance of all constituents at a time and reveals periods of temporary imbalances. There is an energy deficit of 7 Wm\(^{-2}\) during winter and a surplus of 5 Wm\(^{-2}\) during summer. The winter difference is a consequence of an imbalance of the net atmospheric flux and the conductive heat flux at the surface which is too small. The summer difference results from an imbalance of the net atmospheric flux and the energy of melt which is determined from observed melt rates. The melt rates, however, are computed from changes in the observed evolution of the snow and ice surface and the ice base.

It turned out that the conductive heat flux in the snow is a problematic component which causes major differences in the closure of the budget. Measured temperature profiles in the snow and ice are used to compute temperature gradients from which conductive fluxes are determined. The conductive flux in the snow is only about half in magnitude compared to heat conduction in the ice assuming thermal conductivities of 0.31 Wm\(^{-1}\)K\(^{-1}\) and 2.03 Wm\(^{-1}\)K\(^{-1}\) for snow and ice, respectively. Since the temperature gradients and the conductivity of the ice are given, the discontinuity of the conductive flux at the snow-ice interface can be eliminated by adjusting the thermal conductivity of snow. To minimize the discontinuity, a value of about 0.55 Wm\(^{-1}\)K\(^{-1}\) is necessary. Such a value is high in comparison to commonly used snow conductivities but significantly reduces the discrepancy between the conductive and the net atmospheric flux in the energy budget. In addition to Pittsburgh, the thermal conductivity of snow was derived at three other sites. The determined values were in a range from 0.41 to 0.55 Wm\(^{-1}\)K\(^{-1}\) with a mean of 0.49 Wm\(^{-1}\)K\(^{-1}\).

Differences in ice type and spatial variability of the snow and ice thickness can result in horizontal temperature gradients which are not considered in the present one-dimensional energy budget. Such gradients can become significant close to leads, hummocks, ridges or depressions of former melt ponds that were later filled with snow, but they are not large enough to explain the small vertical heat conduction in the snow as a result of horizontal heat loss. Heat conduction in the snow is a combined process of pure conduction and heat transfer by non-conductive processes such as radiative heat transfer, convection and moisture diffusion. Such processes result in an effective heat conduction which can be larger than assumed.

The validation of thermodynamic sea ice models requires data from a point measurement although such an observation is not representative for large-scale heat exchange which usually is an average of contributions made by different ice types and thicknesses. Three-dimensional sea ice models have to take into account that the ice pack consists of many different ice categories of a wide range of thickness. The thermodynamic evolution of sea ice and the exchange of heat at the atmosphere-ice-ocean interface depend on the snow and ice thickness and consequently the heat exchange varies locally with the snow and ice properties.
Chapter 4

Validation of a Multi-Layer Thermodynamic Snow Sea-Ice Model against SHEBA Data
Abstract

A multi-layer thermodynamic snow sea ice model was tested against observational data from the SHEBA experiment. The model uses a coordinate transformation which maps the thickness of the snow and ice slabs onto unity intervals and thus enables automatic re-layering of the snow and ice layers while conserving energy. The model is run for 1-year (Oct. 97 – Oct. 98) using observed downward radiation, air temperature, humidity and wind speed for computing turbulent heat fluxes as forcing fields. A detailed analysis of the measured data addresses problems associated with the high horizontal variability of snow and ice thickness and identifies which observations are required for a precise model validation. Simulation results of various snow cover scenarios are compared to each other and to observations. Modeled results agree well with the observed ice thickness evolution and internal ice temperature, except for summer, where the simulated ice thickness is lower than observed. Sensitivity experiments considering the heat conductivity of snow and the ocean-ice sensible heat flux evaluate possible reasons for disagreement between simulations and measurements. This model validation is done as part of the ongoing Sea Ice Model Intercomparison Project, Part 2, Thermodynamics, (SIMIP2).
4.1 Introduction

The sea ice cover in polar regions has an important influence on the high latitude surface energy budget, and consequently on the high latitude and global climate (Curry et al., 1995). Its high reflectivity as well as insulating properties substantially modify the heat balance and heat exchange at the ocean-atmosphere boundary. The ice pack affects the amount of momentum transfer between the atmosphere and the ocean and influences the mixing processes of the ocean surface waters due to salt rejection during ice formation and freshwater release during ice melt. Finally, the ice thickness affects the growth and melt rates because it influences the ocean-atmosphere heat flux.

The albedo and insulating effects of the sea ice are further enhanced if a snow cover is present (e.g. Fichefet and Maqueda, 1997; Fichefet and Maqueda, 1999). The thickness of the snow cover significantly controls the thermodynamic evolution of the sea ice cover. The spatial distribution of snow and ice thickness can be very inhomogeneous and is for snow a function of location and ice type (Adolphs, 1999; Iacozza and Barber, 1999). Investigations of the snow cover on Arctic and Antarctic sea ice are presented by Warren et al. (1999) and Massom et al. (1997), respectively. Also other studies address the importance of the snow cover and its influence on sea ice (e.g. Ledley, 1991; Wu et al., 1999). A comprehensive review is given by (Massom et al., 2001).

Many previous thermodynamic sea ice model studies employ the zero-layer or three-level model of Semtner (1976) or some modifications of it. The zero-layer thermodynamics permit to compute the ice surface temperature and derive the rate of thickness change. The multi-layer model of Maykut and Untersteiner (1971) computes the temporal variation of the internal temperature within the snow and ice layers, including internal heating due to penetrating shortwave radiation and heat storage in brine pockets. Their model also considers the heat conductivity and specific heat capacity as a function of temperature and salinity, however does not take into account the dependency of the effective specific latent heat of fusion on the temperature and salinity which determines the amount of the energy of melt. Variations of those parameterizations are used by Ebert and Curry (1993) and Flato and Brown (1996), who also parameterize the surface albedo instead of prescribing it as an external forcing parameter. Bitz and Lipscomb (1999) introduce an energy-conserving thermodynamic sea ice model in which the internal temperature evolution is coupled to the internal brine pocket melting. In this model, basal growth and surface and basal melt are controlled by the energy of melt, which depends on the salinity and temperature of the ice. A recent study of Bitz et al. (2001) demonstrates the importance of multi-layer thermodynamics in contrast to zero-layer models, which has substantial consequences for the ice thickness, the heat content of the ice and for the mixed-layer ocean properties.

To simulate sea ice-atmosphere and ocean-atmosphere heat exchange in a realistic way, a model that properly resolves the time-variation of the internal ice temperature profile associated with relatively fast changes in forcing (Hanesiak et al., 1999) is required. Such a model also allows the consideration of variable material properties as a function of local temperature or salinity. A multi-layer model can handle the
storage of heat in the snow and ice slabs and can resolve temperature patterns caused by atmospheric variations of synoptic time scale such as fronts or snow fall. Ukita and Martinson (2001) found that the computed surface temperature in a thermodynamic sea ice model is sensitive to the forcing frequency. That is, the layer thickness in the model must be thin enough that a thermal equilibrium can be reached in a time step relative to the thermal conduction in the snow or ice. In their model, the number of layers required to satisfy these conditions is recomputed every time step, resulting in an adjustment of the grid and a redistribution of energy.

In this study, a multi-layer thermodynamic model that provides a realistic representation of the internal temperature distribution and consequently the conductive and surface heat fluxes is presented. The basal and surface heat fluxes control the vertical evolution of snow and ice layers and hence must be known precisely for a correct thickness simulation. A coordinate transformation which is novel for sea ice models enables to specify an arbitrary number of layers within the snow and ice and easily handles the relayering and conservation of energy through an advection term which appears in the transformed energy equation. The variation with temperature of the material properties of sea ice such as the thermal conductivity, the heat capacity and the latent heat of fusion is included to account for brine pockets and the associated internal storage of heat.

This study is part of an international effort, the Sea Ice Model Intercomparison Project, Part 2, Thermodynamics (SIMIP2)\(^1\), on the evaluation of the performance of thermodynamic sea ice models. SIMIP2 is a joint initiative of the World Climate Research Program (WCRP) Arctic Climate SYstem Study / Climate and Cryosphere (ACSYS/CliC) Numerical Experimentation Group, and the Global Energy and Water Cycle EXperiment (GEWEX) Cloud System Study, Working Group on Polar Clouds. It aims at a comprehensive intercomparison of various existing thermodynamic sea ice models and their validation against a set of observational data from the SHEBA (Surface HEat Balance of the Arctic Ocean) project. The goal of SIMIP2 is to improve the formulation of physical processes in climate models such as the thermodynamic evolution of sea ice and its snow cover.

In Section 4.2, the thermodynamic snow sea-ice model used in this study is presented, including a description of the numerical scheme. An overview of the SHEBA forcing data is given in Section 4.3. Section 4.4 presents a discussion of the simulation results and a comparison with SHEBA/SIMIP2 data. The main conclusions drawn from the simulation results are summarized in Section 4.5.

### 4.2 Snow sea-ice thermodynamic model

The model is an energy conserving one-dimensional multi-layer thermodynamic snow sea-ice model. A one-dimensional energy balance equation including penetrating shortwave radiation is used to describe the temperature evolution in snow and ice layers. Brine pockets in the ice are parameterized with approximations of temperature and salinity dependent heat capacity, thermal conductivity and latent heat.

\(^1\)http://acsys.seos.uvic.ca/acsys/simip2/
of fusion. Equations are written in terrain-following coordinates to have automatic relayering. The model allows for a variable number of layers in the snow and ice slabs and a proper representation of the internal temperature profile under fast changing atmospheric conditions. Lateral thermodynamic effects and ice dynamics such as deformation and advection are not considered. Therefore, the model is a Lagrangian model describing the thermodynamic evolution of a vertical line at a point on a sea ice floe. Fluxes towards a surface are defined positive. A list of variables, parameters and physical constants used in the equations below is given in Appendix A, a more detailed derivation of the following equations can be found in Appendix B.

\[ \frac{\partial h}{\partial t} = S_h \]  
\[ \frac{\partial s}{\partial t} = \begin{cases} p \cdot (\rho_w/\rho_s) & \text{if freezing } (T < T_f(S)) \\ p \cdot (\rho_w/\rho_b) + (F_{\text{net}} + F_{cs})/(\rho L_f) & \text{if melting } (T = T_f(S)) \end{cases} \]  
\[ \frac{\partial b}{\partial t} = \frac{F_{\text{ecn}} + F_{cb}}{\rho L_f} \]  

Figure 4.1: Snow sea-ice model schematic including layer and heat flux definitions. s and b are the level of the surface and base, respectively. Subscripts 's' and 'i' denote snow and ice. \( h_s, h_i \) are the snow and ice thickness (positive numbers) respectively. A typical temperature profile and a corresponding conductive heat flux \( F \) are also included.
the temperature in Celsius, \( T_f(S) \) is the freezing temperature of snow or sea ice, \( F_{\text{net}} \) is the atmospheric net heat flux at the surface including the turbulent heat and moisture fluxes (\( F_{\text{sh}} \) and \( F_{\text{lh}} \)), \( F_{\text{cs}} \) and \( F_{\text{cb}} \) are the conductive heat flux at the surface and base, respectively, \( F_{\text{ocn}} \) is the ocean heat flux, \( \rho \) is the density of snow or ice and \( L_f \) is the effective specific latent heat of fusion. \( T_f(S) \) is a function of the salinity \( S \) which is given in parts per thousand [ppt] and is defined by

\[
T_f(S) = -\mu S ,
\]

(4.5)

where \( \mu \) is an empirical constant equal to 0.054°C ppt\(^{-1} \) and \( T_f \) is given in Celsius.

\[
F_{\text{net}} = \varepsilon (F_{\text{lw}} - \sigma(T_{\text{surf}} + T_0)^4) + (1 - \alpha)(1 - I_0) F_{\text{sw}} + F_{\text{sh}} + F_{\text{lh}} \quad (4.6)
\]

\[
F_{\text{sh}} = \rho_a c_{pa} C_{sh} |u_a| (T_a - T_{\text{surf}}) \quad (4.7)
\]

\[
F_{\text{lh}} = \rho_a L_s C_{lh} |u_a| (q_a - q_{\text{surf}}) \quad (4.8)
\]

where \( \varepsilon \) is the surface emissivity, \( F_{\text{lw}} \) is the downward longwave radiation, \( \sigma \) is the Stefan-Boltzmann constant, \( T_{\text{surf}} \) and \( T_0 \) are the surface and the freezing temperature of freshwater in Kelvin, \( F_{\text{sh}} \) and \( F_{\text{lh}} \) are the sensible and latent heat flux, \( \rho_a, c_{pa}, u_a, T_a \) and \( q_a \) are the density, heat capacity, velocity, temperature and specific humidity of air, \( L_s \) is the specific latent heat of sublimation, \( C_{sh} \) and \( C_{lh} \) are the sensible and latent heat transfer coefficients. The specific humidity at the snow/ice surface (\( q_{\text{surf}} \)) is assumed to be at saturation and is computed according to

\[
q_{\text{surf}} = \frac{0.622 \varepsilon_s}{P_{\text{surf}} - 0.378 \varepsilon_s} ,
\]

(4.9)

where \( P_{\text{surf}} \) is the air pressure at the surface and \( \varepsilon_s \) is the saturation vapor pressure which is a function of the surface temperature calculated after Murry (1966).

As we do not consider dynamics and horizontal diffusion, the continuity equation reduces to the simple balance of the local rate of change of thickness and the thermodynamic source term \( S_h \). Reduction in snow or ice thickness at the surface are calculated from the difference between the atmospheric net flux and the conductive heat flux. Thickness changes at the ice underside result from the difference between the ocean-ice sensible heat flux and the conductive heat flux at the ice base. With the brine pocket scheme enabled, internal melt enlarges the cavities but does not reduce the total thickness. If the brine pocket parameterization is disabled, melting of internal layers is possible if penetrating shortwave radiation causes internal warming above freezing temperature. In that case, internal melt reduces the ice thickness.

A scale analysis of the three-dimensional heat conduction equation (Appendix C) shows that for large-scale simulations the horizontal diffusion terms can be neglected as a result of the extremely large aspect ratio of the polar ocean's sea ice cover. Such a simplification is generally possible except close to the ice edge or leads where horizontal temperature gradients may cause lateral heat flux. The internal temperature is described by the one-dimensional heat conduction equation in snow and ice,
\[ \frac{\partial T}{\partial t} = \frac{K}{\rho c_p} \frac{\partial^2 T}{\partial z^2} + Q_r \]  

(4.10)

where \( K \) is the thermal conductivity, \( Q_r = R/(\rho c_p) \) is a radiative source term, where \( c_p \) is the specific heat capacity of snow and ice and \( R \), the absorbed energy per unit volume, is defined as:

\[
R = \left\{ \begin{array}{ll}
  F_{ps} \kappa_s e^{-\kappa_s(h_s - z)}, & z > 0 \\
  F_{pi} \kappa_i e^{-\kappa_i(-z)}, & z < 0
\end{array} \right.
\]  

(4.11)

\[ R = \left\{ \begin{array}{ll}
  F_{ps} = F_{sw}(1 - \alpha)I_0, & \text{at snow or ice surface} \\
  F_{pi} = F_{sw} e^{-\kappa_i h_s}, & \text{at snow–ice interface}
\end{array} \right.
\]  

(4.12)

\( \kappa \) is the extinction coefficient for snow or ice, \( F_{ps} \) and \( F_{pi} \) are the shortwave radiation penetrating the snow or ice surface according to Beer's law, \( F_{sw} \) is the incident solar radiation, \( \alpha \) is the albedo and \( I_0 \) is the fraction of solar radiation penetrating the surface. At the surface, the incident shortwave radiation \( F_{sw} \) is split into three parts: (1) a reflected part \( \alpha F_{sw} \), (2) a fraction absorbed directly at the surface \((1 - \alpha)(1 - I_0)F_{sw}\), and (3) a transmitted part penetrating into the snow or ice \((1 - \alpha)I_0F_{sw}\). In this model, the penetrating shortwave radiation leads to internal heating or melt if snow or ice are at the freezing temperature.

### 4.2.2 Boundary conditions

At the ice base, the temperature \((T)\) is set to the ocean freezing temperature:

\[ T(z = b_i, t) = T_f(S) = -\mu S_0 \]  

(4.13)

where \( S_0 \) denotes the ocean salinity. At the snow-ice interface, the temperature and the conductive heat flux on the ice side \((z = 0^-)\) and on the snow side \((z = 0^+)\) of the interface are equal:

\[ T_s(z = 0^+, t) = T_i(z = 0^-, t) \quad \text{and} \quad -K_s \frac{\partial T_s}{\partial z} \bigg|_{z=0^+} = -K_i \frac{\partial T_i}{\partial z} \bigg|_{z=0^-} \]  

(4.14)

At the surface, the conductive heat flux in the snow (or in the ice if snow is not present), is equal to the atmospheric net heat flux. If this results in a surface temperature \( T_{surf} \) which is above the freezing temperature, then \( T_{surf} \) is set to \( T_f \) and the residual net heat imbalance is used for melting:

\[ -K \frac{\partial T}{\partial z} \bigg|_{z=s} = F_{net} \quad \text{if freezing} \ (T_{surf} < T_f(S)) \]

\[ T(z = s, t) = T_f(S) \]  

(4.15)

\[ T(z = s, t) = T_f(S) \quad \text{if melting} \ (T_{surf} = T_f(S) \text{ and } F_{net} > 0) \]
4.2.3 Coordinate transformation

Terrain-following coordinates were first used for meteorological modeling by Phillips (1957) and later in an ice sheet model by Jenssen (1977). The advantage of the coordinate transformation is the easy handling of a given number of layers while the selected number of grid points in the snow and ice layers remains fixed. The governing equations are rewritten in terrain-following coordinates. To this end, a new coordinate system is introduced with the base and surface positioned at \( \tilde{z} = 0 \) and \( \tilde{z} = 1 \) respectively (Figure 4.2). The original coordinate \( z \) is positive upward and the snow-ice interface intersects at \( z=0 \).

\[
\tilde{z} = \frac{z - b}{s - b} \tag{4.16}
\]

where \( s \) and \( b \) are equal to \( s_s \) and 0 for the snow layer and 0 and \( b_i \) for the ice layer.

![Figure 4.2: Coordinate transformation of the snow and ice components.](image)

Using the chain rule of differentiation, Eq. 4.10 can be written in terms of the transformed coordinate (Eq.4.16); transformed variables are marked with tilde:

\[
\frac{\partial \tilde{T}}{\partial t} + \tilde{w} \frac{\partial \tilde{T}}{\partial \tilde{z}} = \frac{K}{\rho c_p} \frac{1}{h^2} \frac{\partial^2 \tilde{T}}{\partial \tilde{z}^2} + \frac{\bar{R}}{\rho c_p} \tag{4.17}
\]

where \( h = s - b \) and

\[
\tilde{w} = \frac{\partial \tilde{z}}{\partial t} = - \left( \frac{1 - \tilde{z}}{h} \right) \frac{\partial b}{\partial t} - \frac{\tilde{z} \partial s}{h \partial t} \tag{4.18}
\]

The continuity equation (Eq. 4.1) remains the same in transformed coordinates since it is not a function of \( \tilde{w} \). Adding Eq. 4.17 multiplied with \( h \) and Eq. 4.1 multiplied with \( \tilde{T} \) yields the transformed energy equation in flux form:

\[
\left( h \frac{\partial \tilde{T}}{\partial t} + \tilde{T} \frac{\partial h}{\partial t} \right) + \tilde{w} h \frac{\partial \tilde{T}}{\partial \tilde{z}} = \frac{K}{\rho c_p} \frac{1}{h^2} \frac{\partial^2 (h \tilde{T})}{\partial \tilde{z}^2} + \frac{h \bar{R}}{\rho c_p} + \tilde{T} S_h \tag{4.19}
\]
As \( h \) is only a function of time, it is taken into the argument of the differential operators. Differentiating Eq. 4.18 with respect to \( \tilde{z} \),

\[
\frac{\partial h}{\partial t} = - \frac{\partial (h\tilde{w})}{\partial \tilde{z}},
\]

Eq. 4.19 can be written as

\[
\frac{\partial (h\tilde{T})}{\partial t} + \frac{\partial (\tilde{w}h\tilde{T})}{\partial \tilde{z}} - \tilde{T} \frac{\partial (h\tilde{w})}{\partial \tilde{z}} = \frac{K}{\rho c_p h^2} \frac{\partial^2 (h\tilde{T})}{\partial \tilde{z}^2} + \frac{h\tilde{R}}{\rho c_p} - \tilde{T} \frac{\partial (h\tilde{w})}{\partial \tilde{z}}
\]

Substituting \( \tilde{\theta} = h\tilde{T} \) (heat content), leads to the final form of the transformed energy equations for snow and ice:

\[
\frac{\partial \tilde{\theta}_s}{\partial t} = \frac{K_s}{\rho_s c_p h_s^2} \frac{\partial^2 \tilde{\theta}_s}{\partial \tilde{z}^2} - \frac{\partial (\tilde{w}\tilde{\theta}_s)}{\partial \tilde{z}} + \frac{h_s\tilde{R}}{\rho_s c_p}, \quad 0 < \tilde{z} < 1
\]

\[
\frac{\partial \tilde{\theta}_i}{\partial t} = \frac{K_i}{\rho_i c_p h_i^2} \frac{\partial^2 \tilde{\theta}_i}{\partial \tilde{z}^2} - \frac{\partial (\tilde{w}\tilde{\theta}_i)}{\partial \tilde{z}} + \frac{h_i\tilde{R}}{\rho_i c_p}, \quad 0 < \tilde{z} < 1
\]

The transformation introduces an additional advection term in the heat conduction equation which naturally takes into account the energy transport from one layer to the next when the slab thickness is changing and re-layering is necessary. The grid adjusts to the changes in snow and ice thickness while energy is automatically redistributed. The boundaries of the snow and ice slabs are always exactly represented by a grid point where the corresponding temperature value can be computed (Figure 4.2). In the new coordinate system, computations can easily be performed in the same numerical domain.

Finally, the boundary conditions (Eqs. 4.13-4.15) can be rewritten in terms of \( \tilde{\theta} \) and the transformed coordinate \( \tilde{z} \) as

\[
\tilde{\theta}(\tilde{z}_i = 0, t) = \theta_i(S) = -\mu S_o h_i
\]

\[
h_i \tilde{\theta}_s(\tilde{z}_n = 0, t) = h_s \tilde{\theta}_s(\tilde{z}_i = 1, t) \quad \frac{K_s}{h_s^2} \frac{\partial \tilde{\theta}_s}{\partial \tilde{z}} \bigg|_{\tilde{z}_n = 0} = \frac{K_i}{h_i^2} \frac{\partial \tilde{\theta}_i}{\partial \tilde{z}} \bigg|_{\tilde{z}_i = 1}
\]

\[
-\frac{K}{h} \frac{\partial \tilde{\theta}}{\partial \tilde{z}} \bigg|_{\tilde{z}_i = 1} = h F_{net} \quad \text{if freezing (} \theta_{surf} < \theta_i(S) \text{)}
\]

\[
\tilde{\theta}(\tilde{z} = 1, t) = \theta_i(S) \quad \text{if melting (} \theta_{surf} = \theta_i(S) \text{ and } h F_{net} > 0 \text{)}
\]
Eq. 4.14 has been multiplied with the product of the snow and ice thicknesses \((h_s h_i)\) to maintain equality. The radiative source term (Eq. 4.11) is transformed by substituting \(z\) (derived from Eq. 4.16) in Eq. 4.11:

\[
\tilde{R} = \begin{cases} 
F_{ps} \kappa_s e^{-\kappa_s (1-\xi_s)h_s}, & z > 0 \\
F_{pi} \kappa_i e^{-\kappa_i (1-\xi_i)h_i}, & z < 0 
\end{cases} 
\]  
(4.27)

### 4.2.4 Brine pocket parameterization

The matrix of sea ice is assumed to consist of freshwater ice which is infused by a complex system of cavities of various shapes such as cracks, fractures and pockets. Below the freeboard, these cavities are usually filled with a brine solution; above they can contain air, brine or low salinity meltwater originating from the ice surface. The temperature of brine pockets is at freezing and is given by \(T_b = -\mu S_b\), where \(S_b\) is the salinity of the salt water solution in the pocket (grams of salt per kg of water), \(\mu\) is defined as before and \(T_b\) is in Celsius. At equilibrium, \(T_b\) must be at the freezing point for the given salinity, if not the brine pocket will grow or reduce its size thereby adjusting its salinity in such a way that \(T_b\) is equal to the ice temperature \(T_i\) (Schwerdtfeger, 1963). Since \(T_i = T_b\) it follows that \(T_i = -\mu S_b\). The mass fraction of brine \(M_b/(M_b + M_i) = f_b\) can therefore be written as \(-\mu S_i/T_i\).

Brine pockets are parameterized in an implicit way, i.e. their impact is taken into account by approximating the bulk thermal conductivity and bulk heat capacity of an ice layer which are functions of temperature and salinity. Such functions were first introduced by Untersteiner (1961):

\[
K(S, T) = K_0 + \frac{\beta S}{T} \quad \text{(4.28)} \\
c_p(S, T) = c_0 + \frac{\gamma S}{T^2} \quad \text{(4.29)}
\]

where \(K_0\) and \(c_0\) are the thermal conductivity and specific heat capacity of freshwater ice, respectively, \(\beta\) is a constant equal to 0.1172 Wm\(^{-1}\)ppt\(^{-1}\), \(c_p\) is the specific heat capacity of the ice-brine mixture and \(\gamma\) is a constant equal to 1.8\times10^4 J °Ckg\(^{-1}\)ppt\(^{-1}\).

The unit of the temperature \(T\) is Celsius.

In a later work, Ono (1967) showed that the temperature and salinity dependency of the heat capacity in Eq. 4.29 can be derived from first principles, expressing the constant \(\gamma\) in terms of the constant \(\mu\) from the definition of the salinity dependent freezing temperature and the latent heat of fusion of freshwater ice at 0°C: \(\gamma = \mu L_0\) (Bitz and Lipscomb, 1999). Following further the approach of Bitz and Lipscomb (1999), the amount of energy required to melt a unit mass of ice of a given temperature and salinity depends on the effective specific latent heat of fusion which is defined in Eq. 4.30:

\[
L_i(S, T) = c_0(T_f - T) + L_0 \left(1 + \frac{\mu S}{T}\right) . \quad (4.30)
\]
The first term on the right hand side is the specific heat, i.e. the energy needed to bring the ice of temperature $T$ to the freezing temperature $T_f$, the second term is the latent heat, i.e. the amount of energy required to melt a unit mass of sea ice of the salinity $S$. The term $-\mu S/T$ is the mass fraction of brine and thus the term $(1 + \mu S/T)$ represents the amount of unmelted ice per unit mass of sea ice. For an ocean freezing temperature of -1.8°C and a salinity of sea ice of 3.2 ppt, $L_0$ is reduced by about 10% and consequently the growth or melt rate at the base of the sea ice is 10% larger than the one of freshwater ice. The dependency of the material properties on temperature and salinity is shown in Appendix D. The model provides several options for specifying salinity profiles.

4.2.5 Numerical scheme

For integrating the heat conduction equation, a second order forward-in-time, centered-in-space finite-difference numerical scheme, is applied. Fluxes and the vertical velocities are defined on the nodes of the grid, temperature and salinity are located at the center of the grid and hence represent the mean quantity of a grid cell. As the temperature is defined on the centers of the main grid, it is also determined at additional grid points located at the ice base, the snow-ice interface or the ice surface and the snow surface. The chosen grid and the corresponding system of equations is shown in Appendix E. The advection term arising from the coordinate transformation, i.e. the vertical grid velocity which accounts for the redistribution of energy from one layer to another, is evaluated explicitly as well as the snow or ice thickness or their rate of change. That is, the variables of the continuity equation are stepped explicitly, whereas the diffusion term involving the temperature is treated implicitly including the extra grid points. Outgoing longwave radiation, being a function of the surface temperature, is linearized about the previous time step surface temperature and then evaluated implicitly. The conductive heat flux at boundaries is evaluated from the surface and internal temperatures using a second order left or right difference scheme.

The model provides two options for including turbulent heat fluxes, they can either be specified from measurements or calculated from standard bulk formulations using atmospheric data such as wind speed, air temperature and specific humidity. If bulk formulations for the computation of the turbulent heat fluxes are used, the sensible heat flux is evaluated implicitly whereas the calculation of the saturation vapor pressure in the equation of the latent heat flux is based on the temperature of the previous time step.

If any internal temperature exceeds the freezing temperature $T_f$, the amount of melt in that layer is computed from the excess heat in that layer: $\Delta h_{pc_p}\Delta T/(\rho L_f)^{-1}$. This applies only for a model configuration with disabled brine pocket parameterization where internal melt can happen. In the brine pocket mode, internal heating leads to enlargement of the brine pockets and an associated reduction of their salinity while the freezing temperature of the ice (cf. Eq.4.5) is never exactly reached until all ice has melted.
4.3 SHEBA forcing and evaluation data

The forcing and snow and ice observations were collected during the SHEBA field experiment in the Beaufort and Chukchi seas of the Arctic Ocean from October 1997 through October 1998. They include the downward radiative fluxes, the snow and ice albedo, 10 m wind speed, air temperature and humidity, the precipitation rate (mm/day water equivalent), the ocean heat flux, the snow and ice thickness and internal temperature. A set of atmospheric and oceanic SHEBA data was compiled and processed for SIMIP2 ranging from October 31, 1997 through October 8, 1998, i.e. nearly covers a complete annual cycle. The SIMIP2 forcing fields are provided with a temporal resolution of one hour. The precipitation rate is specified as millimeter snow water equivalent and was increased by 50% with respect to the observed, since it was found to be too low compared to snow stake measurements when converted to snow depth involving the density of snow (330 kg m$^{-3}$ in SIMIP2). Atmospheric data were collected at the meteorological towers of the SHEBA Project Office (SPO) and the Atmospheric Surface Flux Group (ASFG) (Beesley et al., 2000; Duynkerke and de Roode, 2001; Persson et al., 2002). Air temperature, velocity and humidity were measured at a level of 10 m above ground. Shortwave and longwave radiation as well as the precipitation rate were measured close to the surface. The snow or ice albedo was measured at a point nearby the SHEBA location Pittsburgh and therefore is judged representative of the local conditions at Pittsburgh. The ocean heat fluxes provided in SIMIP2 (Perovich and Elder, 2002) were estimated from the growth rate and the temperature gradient at the ice base at the Pittsburgh site. As no surface temperatures are provided in the SIMIP2 data set, measured skin temperatures (Persson et al., 2002) from SHEBA are used for comparison with simulated surface temperatures. The various time series of snow depth used in this study were obtained from stake measurements and from different analyses of the measured temperature profiles in the snow which are described in Chapter 3.

The thermodynamic evolution of sea ice was measured at several locations at SHEBA representing a variety of ice types. At these heat balance sites, a vertical line of thermistors measured temperature profiles within the snow and ice and in the air above and the upper ocean below. Snow and ice thickness was measured with snow stakes and hot wire gauges at several mass balance sites, three of which were typically arranged in a distance of about one to a few meters around a thermistor string. The sampling frequency for the temperature data was one hour whereas the thickness data were collected once every 1-2 weeks in winter and about every two days during the summer. Most temperature measurements ran from October 1997 through September 1998, providing a record length of almost 11 months. A more comprehensive documentation of the heat and mass balance observations at SHEBA is given in Chapter 3. Further detailed descriptions of the observational data, instruments and methods can be found in Perovich et al. (1999), Perovich and Elder (2001) and Sturm et al. (2002).

In this study, data from the SHEBA site Pittsburgh is used as validation data as it is the official heat and mass balance site for the 'SHEBA Column' and for the Sea Ice Model Intercomparison Project, Part 2, Thermodynamics (SIMIP2). The Pittsburgh site was on an undeformed multi-year ice floe. The internal snow and ice
temperature record and the snow and ice thickness measurements from the Pittsburgh gauges no. 53, 69 and 71 (three closest gauges to the thermistor string) are shown in Figure 4.3. As the snow and ice thickness were not measured at the place of the thermistor line, the observed snow and ice thicknesses are not exactly representative of the site of the rod. Observations illustrate that both the snow and the ice thickness vary considerably on small horizontal scales. The large differences between the individual snow and ice thickness measurements give evidence for their spatial and temporal variability (Figure 4.3). This situation is typical for all SHEBA locations where snow and ice temperature was measured along with thickness. Chapter 3 explained why the mean of the thickness gauges is not a good approximation for the snow thickness, and it described the methods used to obtain more realistic estimates for the snow and ice thickness at the location of the thermistor line: The ice base was determined by applying a constant shift to a given observed ice thickness time series such that the ice base would be at the ocean freezing point temperature. This is possible since the time evolution of the individual basal ice measurements close to a thermistor string were very similar (Figure 4.3). Regarding the level of the snow surface, the observations from gauge no. 69 are taken, applying a correction to the winter period, which is based on results of the various alternative methods for deriving the snow thickness from the temperature profiles (Chapter 3). The best estimate for the ice base at Pittsburgh is also included in Figure 4.3.

![Figure 4.3: Measured temperature and snow/ice thickness evolution from gauges no. 53 (dashed), 69 (dotted), 71 (solid) at the mass balance site Pittsburgh. The length of the thermistor string defines the vertical range of measurements. The snow-ice interface is at z=0 during the cold period, as soon as surface melt starts, the ice surface is ablated below z=0. The thick lines denote the best estimates for the snow surface and the ice base as explained in the text.](image-url)
4.4 Results and discussion

In this section, model results are presented and compared with SHEBA observations. SIMIP2 considers a simple and strictly one-dimensional case, representing a point on a snow/ice slab, neglecting any horizontal variability such as inhomogeneity of the snow depth or ice thickness due to dynamic processes like deformation or ridging. Also the lateral effects of leads and melt ponds, such as horizontal heat conduction, are not taken into account. Therefore, the following experiments can be considered as a point model applied to a horizontally homogeneous slab of potentially snow covered sea ice, i.e., a uniform ice floe acting as a Lagrangian drifter. The ignored effects of ice dynamics and horizontal inhomogeneities near ice edges are small at the Pittsburgh site and therefore no relevant errors are introduced and this is a meaningful approach. Based on the data presented in Section 4.3, the focus is not only on comparing several experiments with different sets of prescribed forcing variables to the reference case but also on conferring them to each other to get more insight in the relative importance of model parameters or physical processes. After presenting the SIMIP2 control experiment (Section 4.4.1), the impact of the ocean heat flux will be investigated (Section 4.4.2), followed by some experiments which evaluate the effects of various prescribed snow covers (Section 4.4.3). Section 4.4.4 elaborates on the thermal conductivity of snow while Section 4.4.5 presents some consistency tests and the model’s sensitivity to the brine pocket parameterization.

The model is forced with specified hourly downward shortwave and longwave radiation and turbulent heat fluxes which are calculated from basic meteorological variables such as 10 m wind speed, air temperature and humidity, from the SIMIP2 data using a standard bulk formulation (Eqs. 4.7 and 4.8). Further prescribed quantities are the depth of the snow cover, the ocean heat flux at the ice underside and the albedo to properly constrain the forcing. The snow cover can be specified from any measured or derived evolution of the snow thickness, or from observed precipitation rates which are converted to a snow depth accounting for the (specified) density of snow. In all cases, the snow cover is specified only as long as the surface is below freezing temperature. As soon as melting is present, surface ablation is computed from the energy surplus in a given time step. If the snow thickness is prescribed using precipitation rates, snow depth can increase even though the surface temperature is at freezing. All model runs are performed with 10 layers in the ice and 10 layers in the snow and with a time step of one hour.

4.4.1 SIMIP2 control experiment

Following the SIMIP2 test plan, the first simulation (Exp. 1, control experiment) is run using initial conditions (October 31, 1997) defined from observational data of snow and ice thickness, internal snow and ice temperature and a salinity profile measured at the SHEBA location Pittsburgh (gauge no. 69, 5 cm and 170 cm respectively). The reference values of model parameters used in the control experiment are given in Appendix A, together with other physical constants, material properties and variables of the model. These initial conditions were used although it was found that these data are not consistent with the observed internal snow and ice tempera-
ture distribution of October 31, 1997 at Pittsburgh (see Section 4.3 and Chapter 3). Section 4.4.2 will describe a second set of simulations using the best estimate of the initial ice thickness of 184 cm (see Section 4.3). The corrected initial ice thickness will be used in all experiments presented after this section.

Hourly data of downward longwave and shortwave radiation, albedo, ocean heat flux, precipitation rates and 10 m wind speed, air temperature and humidity from SIMIP2 (SHEBA) is used as forcing for this control experiment. The set-up of the performed control simulation is identical to the SIMIP2 control experiment, apart from the computation method of the transfer coefficients in the bulk formulations of the turbulent heat fluxes. This study uses constant values (Eqs. 4.7 and 4.8) whereas SIMIP2 proposes a more complex algorithm for calculating the transfer coefficients based on Andreas (1987). The temperature and salinity of the ocean mixed-layer’s surface waters are assumed constant in time; the ocean salinity (33.3 ppt) defines the freezing temperature of the ocean (-1.8°C, Eq. 4.13). The chosen salinity profile in the ice is a linear approximation of the observed profile provided in SIMIP2 with 0.0 ppt at the ice surface to 3.2 ppt at the base. A linear profile is judged to be realistic as Pittsburgh was on a multi-year ice floe which has already undergone at least one melting season, during which meltwater percolated from the surface down into the ice leading to a gradual desalinization of the ice above the freeboard line. The model’s brine pocket parameterization can account for the temperature and salinity dependence of the sea ice material properties. The ocean heat flux is adopted from the SIMIP2 data and the snow cover is prescribed from SIMIP2 precipitation rates given in snow water equivalent per time unit which are converted into snow depth taking into account a bulk snow density of 330 kg m⁻³ as proposed in SIMIP2. Figure 4.4 shows the result of the control experiment and a comparison with the time evolution of the observed internal temperature distribution at the Pittsburgh site. A summary of the simulated ice characteristics is given in Table 4.1, along with the values observed at Pittsburgh for comparison.

Comparing the annual cycle of the simulated (Figure 4.4a) and measured (Figure 4.3) internal temperature and thickness evolution illustrates a reasonable agreement. The three cold spells of the winter 1997/98 (Julian days 355-370, 375-390, 400-415), where the low temperatures penetrate into the ice, are clearly simulated, although the magnitude of the peaks differs depending on the overlying snow cover depth derived from the prescribed precipitation rates. The simulation also captures the relatively low heat content in the inner part of the ice slab just after surface melt started (bulging contours). While the surface is warming, the interior of the ice slab (Figure 4.4a) remains cold (Julian days 500-530) in agreement with the observations (Figure 4.3). This is due to the heat capacity of the ice. In the model, the brine pocket parameterization is mainly responsible for that. In December and January (Julian days 330-390), the simulated internal ice temperature is significantly lower than the observed, which is a consequence of the thin specified snow cover of the control experiment. Low temperatures are propagating from the surface into the ice owing to the relatively small insulating effect of the thin snow cover. In this period, the snow cover derived from the observed precipitation rates disagrees considerably with the individual snow stake measurement at gauge no. 69, e. g. the precipitation data do not show the peak which is present in the stake record. Differences in the
Figure 4.4: (a) Simulated temperature and thickness evolution at Pittsburgh using the initial conditions of the SIMIP2 control experiment. The dashed lines at the surface and base denote the snow and ice thickness measured at gauge no. 69. (b) Difference between simulated and observed (Figure 4.3) internal temperature for Pittsburgh. Red colors denote areas where the simulated temperatures are higher than the observed temperature and blue colors denote areas where they are lower.
snow depth appear firstly because the two measuring devices measure different variables, secondly because they were at different locations and thirdly, errors can also result from the reference snow density of 330 kg m\(^{-3}\) being used in the conversion of precipitation rates to snow depth which is influenced by wind and air temperature.

The onset of snow melt and the melt rate is captured accurately as well as the ice surface ablation of about 60 cm (Table 4.1). The simulated ice thickness remains too small throughout the whole year, the growth rate is too small in winter and the melt rate is too large in spring and summer leading to a gradually increasing discrepancy to observations. There are two possible reasons for the lower ice thickness, firstly the applied ocean heat flux is too large or secondly, the insulating properties of the snow cover are too large assuming that the downward radiative fluxes are accurate, reducing the heat flux from the ocean to the atmosphere which lowers the basal growth rate. The likely reason for the thin ice is the overestimated ocean heat flux as will be shown later in this section.

The difference plot (Figure 4.4b) suggests that the lower half of the ice is modeled too warm by up to 1-2°C during the whole simulation period, however, the simulated temperature at the ice base is defined by the basal boundary condition (ocean freezing temperature) and is therefore correct. The difference appears because the ice thickness evolution measured at gauge no. 69 prescribes the ice base at a too shallow level (see Figure 4.3) and the modeled ice base is even thinner than

Figure 4.5: Simulated evolution of variables from the SIMIP2 control experiment: (a) brine fraction, (b) ice salinity, (c) amount of energy absorbed in the snow and ice, and (d) profiles of penetration shortwave radiation (color of profiles corresponds to Julian day).
the one specified by gauge no. 69. Thus, warmer ice at a given depth in the control simulation is compared to colder ice at the same level in the observations.

During the winter months (Julian days 320-410), simulated temperatures are too low by up to 4°C in the upper part of the ice and in the snow cover and up to 3°C during the melt season as long as surface ablation occurs. The winter difference is a consequence of the thin snow cover calculated from precipitation rates leading to deeper penetration of the cold from the surface down into the ice owing to reduced snow insulation and large temperature gradients. This situation should lead to enhanced basal growth, however, the contrary happens in the simulation, which indicates that the small ice thickness is rather a result of a too large ocean heat flux. The summer difference is presumably an effect of the brine pocket parameterization, which keeps the ice slightly colder than observed at Pittsburgh where it gets almost temperate as soon as melting starts.

To improve the situation, all following experiments use the best estimate (BE) for the ice base (Figure 4.3), which results in a correction of the initial ice thickness from 170 cm in the control experiment to 184 cm. The ice characteristics from a simulation using the best guess initial ice thickness (Exp. 2) are included in Table 4.1.

Figure 4.5 presents the internal snow and ice salinity distribution, the evolution of the brine fraction and the solar radiation which penetrates into the snow and ice. The brine fraction distribution (Figure 4.5a) is a result of the linear salinity profile (Figure 4.5b) and the simulated internal ice temperature distribution (Figure 4.4a). The brine fraction increases with increasing temperature and salinity, the dependency of the brine fraction on temperature and salinity is shown in Appendix D. As a result, the fraction is close to zero in the upper part of the ice during winter and about 0.1 near the ice base \((T=-1.8°C, S=3.2\text{ ppt})\). During summer, when the internal ice temperature is close to the freezing point, the fraction reaches a maximum of about 0.12 almost all over the vertical extent of the ice slab.

| Table 4.1: Summary statistics of observed and simulated ice characteristics at Pittsburgh [m]: initial ice thickness \((h_{\text{ini}})\), maximum thickness before melt starts \((h_{\text{max}})\), minimum thickness at the end of the melt season \((h_{\text{min}})\), basal growth \((g_b)\), surface melt \((m_s)\), basal melt \((m_b)\) and total ablation \((m_t)\). IC = initial conditions. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| experiment                      | \(h_{\text{ini}}\) | \(h_{\text{max}}\) | \(h_{\text{min}}\) | \(g_b\)       | \(m_s\)       | \(m_b\)       | \(m_t\)       |
| observation                     | 1.84            | 2.54            | 1.51            | 0.70           | -0.60          | -0.43          | -1.03          |
| Exp. 1: Ctrl, SIMIP2 IC         | 1.70            | 2.26            | 1.07            | 0.56           | -0.60          | -0.58          | -1.18          |
| Exp. 2: corrected IC            | 1.84            | 2.36            | 1.18            | 0.52           | -0.60          | -0.58          | -1.18          |
| Exp. 3: ocean heat flux         | 1.84            | 2.47            | 1.40            | 0.63           | -0.60          | -0.47          | -1.07          |
| Exp. 4, snow, gauge no. 69      | 1.84            | 2.41            | 1.20            | 0.58           | -0.65          | -0.56          | -1.22          |
| Exp. 5, snow, mean of gauges    | 1.84            | 2.44            | 1.17            | 0.61           | -0.71          | -0.57          | -1.28          |
| Exp. 6, snow, best estimate     | 1.84            | 2.31            | 1.03            | 0.48           | -0.72          | -0.57          | -1.28          |
| Exp. 7, \(K_s = 0.55\)         | 1.84            | 2.42            | 1.15            | 0.58           | -0.72          | -0.55          | -1.27          |
| Exp. 8, brine ignored           | 1.84            | 2.48            | 1.32            | 0.66           | -0.71          | -0.45          | -1.17          |
Figure 4.5c shows the amount of energy absorbed at each level of the snow and ice slabs. The pattern of the daily cycle in the absorbed shortwave radiation can clearly be seen in the figure (high frequency variability). Figure 4.5d presents the corresponding series of profiles of the penetrating solar radiation. During the melt period, the snow and ice surface is moving down; the level of the top of the profiles at the end of the annual cycle indicates the summer surface. At the snow surface, maximum values are close to 300 Wm\(^{-3}\), at the ice surface around 50 Wm\(^{-3}\). As the applied bulk extinction coefficient for snow (10 m\(^{-1}\)) is almost 7 times larger than the bulk extinction coefficient for ice (1.5 m\(^{-1}\)), transmitted solar radiation is effectively absorbed in the upper part of the snow cover. Later in the season, when the net shortwave radiation further increases and the snow cover gets thinner, a certain percentage penetrates even into the ice. In June and July, when shortwave radiation is at its maximum, a small residual part of the penetrating radiation reaches beyond the ice base and is absorbed by the ocean.

### 4.4.2 Ocean heat flux

The heat fluxes at the surface and the base, the snow thickness and the thermal conductivity, density and salinity of the snow/ice slab determine the temperature gradient and the growth and melt rate at the ice base. There might be some error in the measured quantities and the implemented model parameters, however the temperature gradient at the ice base of thick multi-year ice is relatively small and also not very sensitive to small errors in the surface forcing. Therefore, the discrepancy between the observed and simulated evolution of the ice base in the control experiment (Section 4.4.1) may be explained by an overestimation of the ocean heat flux. To identify the influence of the ocean heat flux isolated from surface effects such as induced by the snow cover, an analysis of the ocean heat flux is done before changing the snow cover to evaluate the heat fluxes at the ice underside at the location of the Pittsburgh thermistor string which directly determine the evolution of the ice thickness. The ocean heat flux prescribed in SIMIP2 (Figure 4.6) is similar to the time series of monthly values of the ocean heat flux determined at the Pittsburgh site in an analysis of Perovich and Elder (2002). In Chapter 3, the ocean heat flux was estimated at the various combined heat and mass balance locations at SHEBA in a similar way as in the mentioned study, i.e. it was computed as a residual of the balance of basal heat fluxes including the conductive heat flux and the energy of melt. The mean of the ocean heat fluxes derived for the six SHEBA sites (Baltimore, Pittsburgh, Quebec 1 and 2, Seattle and Tuk) is shown in Figure 4.6, together with the individual ocean heat flux at Pittsburgh and the ocean heat flux specified in SIMIP2. Both the mean ocean heat flux and the individual Pittsburgh ocean heat flux are generally smaller than the ocean heat flux specified in SIMIP2. Exceptions are July and August, where the mean ocean heat flux is significantly larger than the one provided in SIMIP2 whereas the flux at the Pittsburgh location only slightly exceeds the SIMIP2 data for a short time of two weeks around the beginning of August. The average of the ocean heat flux at the Pittsburgh site (Chapter 3) is 5.7 Wm\(^{-2}\) in contrast to the average of 8.2 Wm\(^{-2}\) of the ocean heat flux provided in SIMIP2.
To quantify the influence of the difference in the ocean heat flux, Exp. 3 repeats the control experiment with the ocean heat flux determined at the Pittsburgh site. The result is a significantly improved representation of the ice base (Figure 4.7a), the growth rate which was too low in the control experiment is now almost identical to the observed one, and the simulated maximum ice thickness agrees well with the observed, both in timing and vertical extent (see Table 4.1 for the ice characteristics). The basal melt rate during summer also follows the observed evolution of the ice slab and the simulation terminates with an ice thickness similar to the measured one at the end of the SHEBA experiment in October 1998. Figure 4.7b shows the difference between the simulated and the observed evolution of the thickness and internal temperature evolution. The modeled evolution of the ice base is closer to the observed than in the SIMIP2 control experiment (cf. Figure 4.4b). The fact that this simulation was carried out with the corrected initial ice thickness results in improved agreement between the simulated and observed internal temperature in the lower half of the ice slab (cf. Figure 4.4b). In the following experiments, the estimated ocean heat flux from the Pittsburgh site (set equal to zero where negative, cf. Figure 4.6) will be used, replacing the ocean heat flux provided in SIMIP2.

Figure 4.6: Ocean heat flux as provided in SIMIP2 (dashed), as determined from basal ice characteristics at the Pittsburgh site (thick), mean as derived at six combined heat/mass balance sites at SHEBA (solid).
Figure 4.7: (a) Temperature and thickness evolution simulated with corrected ocean heat flux forcing (derived from internal temperature distribution and growth/melt rates at the ice base measured at Pittsburgh). (b) Difference of thickness and internal temperature evolution between simulation and observation. Dashed lines indicate best estimates of snow and ice thickness.
4.4.3 Snow cover evolution

Investigating the impact of the specified snow cover is significant since it was not measured right at the site of the thermistor lines and thus is one of the main uncertainties in the observations. The application of various methods to indirectly determine the snow surface at the location of the Pittsburgh thermistor string gives results for the snow depth that can differ considerably (Chapter 3). The depth of the snow cover in turn has a substantial impact on the internal snow and ice temperature distribution and thus controls the growth and melt rate at the ice base. To evaluate the effect of the snow cover on the thermodynamic evolution of the sea ice underneath, a series of scenarios is presented showing examples of various snow covers which are either measured or indirectly derived from other observations at the Pittsburgh site or nearby applying different methods. The first case (Exp. 4) uses the snow thickness measurements from a single gauge (no. 69), the second (Exp. 5) uses the mean snow thickness of three gauges (no. 53, 69, 71) which were closest to the thermistor string. The third case (Exp. 6) prescribes the snow depth which was defined as the best estimate described in Chapter 3. Model results of the three simulations are shown in Figure 4.8, together with the best estimate of the snow and ice thickness as a reference. To highlight where the simulated and the observed internal snow and ice temperatures differ, Figure 4.8 rather presents the difference to the observation than just the internal temperature distribution. Key values for the ice characteristics of the snow cover scenarios are summarized in Table 4.1.

The snow cover scenario of the control experiment seems not realistic as the snow cover at SHEBA also grew due to accumulation of drifting snow during periods of storms even if there is no snow fall at that time. On the other hand, it can also be eroded due to the effects of storms. In contrast to the control experiment where every snow fall event builds up the snow thickness leading to a monotonic increase of the snow cover, the selected examples show that a more realistic representation of the snow cover evolution is quite different from the snow cover derived from precipitation rates (Figure 4.4) owing to the effect of blowing snow. Due to its insulating properties, a thick snow cover keeps the interior ice warmer by maintaining a large temperature gradient within the snow pack. In winter, the low conductivity of snow reduces the amount of heat extracted from the ocean and lost to the atmosphere and thus leads to a smaller growth rate at the ice base. This is clearly seen from Figures 4.8a-c where a thicker snow cover generally leads to higher temperatures in the interior of the snow and ice and consequently to a smaller ice thickness.

Exp. 4 demonstrates that a gauge measurement which was not exactly at the location of the thermistor line is not representative for the snow atop the ice at that place (Figure 4.8a). For instance, the large peak in the snow cover at the beginning of January, which coincides with a cold spell, prevents the low air temperatures from propagating into the ice and as a consequence, the ice temperatures in the upper part of the ice slab are too high compared with the observed temperature distribution (Figure 4.3). Exp. 4 uses a direct snow depth measurement but off the site of the thermistor line. The large peak was only observed at this single gauge and the simulated internal ice temperature distribution below the peak in the snow cover does not agree with the corresponding measured temperatures.
Figure 4.8: Difference between simulated and observed temperature evolution at the Pittsburgh site prescribing the snow thickness from (a) an individual thickness gauge (no. 69), (b) the mean of the three gauges no. 53, 69, 71, and (c) best estimate of the snow cover. Dashed lines indicate best estimates of the observed snow and ice thickness.
The mean snow thickness of the three thickness gauges in Exp. 5 gives the thinnest snow cover throughout the whole year resulting in lowest internal ice temperatures and best representation of the ice base (Figure 4.8b). Exp. 5 gives a quite realistic representation of both the internal ice temperature and the evolution of the ice base. However, the representativity of the measured snow cover thickness must be questioned as demonstrated by the results of Chapter 3. The snow cover of Exp. 6 seems too thick during the entire winter season (Figure 4.8c). Due to the insulating effect of the snow cover, the internal ice remains too warm when compared with the observed situation, and consequently the basal growth rate of the ice is substantially reduced. On the other hand, a deep snow cover in spring however (e.g. Exp. 4) has only little impact on the basal growth rate since the snow and ice are already warmer leading to smaller temperature gradients from the ice base to the surface. A thick snow cover during winter has much larger effect on the thickness evolution than in spring and early summer. Owing to its higher albedo relative to bare ice, a thick spring snow cover still has some impact on the time of the onset of the ice surface melt. In all three simulations, the modeled ice thickness is small when compared to observations. The difference increases during the annual cycle and is largest at the end of summer. This is a consequence of the thick snow cover with its strong insulating properties, which reduce the basal growth rate. However, all three experiments use a thermal conductivity of snow of 0.31 Wm$^{-1}$K$^{-1}$ as proposed in SIMIP2, which was found to be too low (Chapter 3). In Section 4.4.4 it will be shown that the best estimate snow cover used in Exp. 6 gives best results when the thermal conductivity of snow is corrected according to the findings of Chapter 3.

4.4.4 Thermal conductivity of snow

According to an analysis of the surface energy budget in Chapter 3, it was found that the thermal conductivity of snow must be increased to 0.55 Wm$^{-1}$K$^{-1}$ (0.14 Wm$^{-1}$K$^{-1}$ from in-situ measurements at SHEBA) to satisfy the continuity of the conductive heat flux at the snow-ice interface at Pittsburgh. Based on this finding, the impact of the thermal conductivity of snow is investigated in detail. In Exp. 7, a thermal conductivity of 0.55 Wm$^{-1}$K$^{-1}$ is tested in the model configuration of Exp. 6, i.e. SIMIP2 control experiment as described in Section 4.4.1 with corrected initial conditions, plus the ocean heat flux correction presented in Section 4.4.2, plus the best estimate snow cover. Simulation results of Exp. 7 (Figure 4.9a) are then compared with the observed internal snow and ice temperature distribution at Pittsburgh (Figure 4.9b).

In Exp. 6 (0.31 Wm$^{-1}$K$^{-1}$), the ice thickness is significantly lower and the internal ice temperatures are higher than those measured at Pittsburgh. During the entire winter season the simulated ice temperatures differ from the observations by about 1-2°C at the ice base to about 5-6°C near the ice surface (Figure 4.8c). The increase of the thermal conductivity of snow in Exp. 7 leads to an enhanced heat transfer from the ice through the snow to the atmosphere during winter and therefore cools the ice slab substantially. The result is an increased temperature gradient due to lower temperatures near the snow-ice interface leading to an increased growth rate at the ice base and thus a realistic representation of the ice thickness during the annual
4.4. Results and discussion

Figure 4.9: (a) Simulated temperature and thickness evolution at the Pittsburgh site using a value for the thermal conductivity of snow of 0.55 Wm$^{-1}$K$^{-1}$. (b) Difference between the simulated and observed internal snow ice temperature evolution.

The simulated internal snow and ice temperature distribution in Exp. 7 matches well with the measured snow and ice temperatures at Pittsburgh, differences are within ±1°C with an exception during the period of snow cover melt where the ice warms a bit too slow (Figure 4.9b).
The thermal conductivity of snow of $0.55 \text{Wm}^{-1}\text{K}^{-1}$, which was inferred from measurements at Pittsburgh (Chapter 3), is an adequate value in terms of representation of the internal ice temperature. The pattern of the ice thickness is well reproduced, only the simulated ice thickness remains a bit too small with a maximum thickness of 12 cm less compared with the observation. Other values of the ice characteristics of Exp. 7 are included in Table 4.1. The underestimation of the ice thickness might result either from the uncertainty in the specification of the snow cover or from some error in the determination of the ocean heat flux, or likely from a combination of both. The simulated ice thickness and internal ice temperature shown in Figure 4.9 are similar to the results obtained in Exp. 6, however the internal snow temperature of Exp. 7 is in better agreement with the observations.

### 4.4.5 Sensitivity and consistency tests

**Surface energy budget**

In the experiments presented above, simulated surface and/or the internal snow and ice temperature is involved in the computation of the outgoing longwave radiation, the conductive heat flux at the surface and base of the snow/ice slab and the turbulent and latent heat fluxes. These components of the modeled surface energy budget are therefore considered simulated as opposed to the shortwave radiation including albedo and the downward longwave radiation which are observed. Figure 4.10 compares the surface energy budget of fluxes which are either directly measured or derived from observations (Figure 4.10a and Chapter 3) with the surface energy budget of model fluxes (Figure 4.10b) which consists both of simulated and prescribed fluxes as explained above. In both budgets, the calculation of the conductive heat flux in the snow is based on a thermal conductivity of snow of $0.55 \text{Wm}^{-1}\text{K}^{-1}$ and the best estimate snow thickness. A constant value for the fraction of shortwave radiation penetrating the surface (0.15) is used both in the observed and the simulated surface energy budget. Since the net atmospheric flux (sum of the radiative and turbulent heat fluxes) and the conductive heat flux plus the energy of melt at the surface is a boundary condition in the model, there is no residual in the simulated budget. The net longwave radiation is lower than the observed indicating a lower simulated surface temperature compared with the observations (Figure 4.11).

Figure 4.10c displays the difference between the measured and the modeled fluxes of the surface energy budget. Since in winter the measured skin temperature is higher than the modeled (Figure 4.11), the surface looses more energy to the atmosphere than in the simulation which might explain why the model slightly underestimates the ice thickness. The simulated and the observed conductive heat fluxes differ considerably until the end of February. Since heat conduction computed for the budget of observed fluxes depends on the prescribed snow thickness which involves some uncertainty, it is subject to errors. The energy of melt is a quantity which was derived indirectly from the measured surface rate of change and is also likely subject to errors. The sum of errors explains the mismatch of fluxes, i.e. differences of the order of $10 \text{Wm}^{-2}$ during biannual periods.
Figure 4.10: Components of the surface energy budget at Pittsburgh: (a) observed, (b) simulated, (c) difference of simulated and observed fluxes. Colors are assigned to fluxes as follows: net shortwave (blue) and net longwave (orange) radiation, sensible (green) and latent heat flux (black), conductive heat flux at the surface (pink), sum of atmospheric surface fluxes, i.e. net radiative plus turbulent fluxes (violet), energy of melt (cyan) and total of all fluxes (red). Fluxes are smoothed using a 7-days running mean.
Surface temperature

The surface temperature simulated in Exp. 7 is compared with the observed skin temperature and the temperature measured by the uppermost sensor of the Pittsburgh thermistor line (45 cm level) which was always in the air and is thus considered as a near-surface temperature (Figure 4.11). In general, the simulated surface temperature agrees well with the measurements. In winter, and here particularly during the cold spells in January and February, the modeled surface temperature is lower than the observed with maximum differences of about 2°C. The discrepancy between model results and measured skin temperatures has direct consequences for the outgoing longwave radiation which is computed from the simulated surface temperature and the surface emissivity. The simulated net infrared balance (Figure 4.10b) is then accordingly lower than the measured (Figure 4.10a). Differences are present particularly when the modeled surface temperature is lower than the observed surface temperature (Figure 4.11). To correct the surface temperature and the associated emitted infrared radiation, the thermal conductivity of snow and possibly also that of ice must be further increased to conduct sufficient energy to the surface. As a result, the growth rate at the ice base would increase as well. Vice versa, the lower thermal conductivity of snow of 0.31 Wm⁻¹K⁻¹ in Exp. 6 yields simulated surface temperatures significantly lower than those simulated in Exp. 7 (results not shown here). Apart from the conductive properties of the snow cover, the surface temperature is naturally determined by the snow and ice thickness. A thin snow cover favors higher surface temperatures, deeper penetration of low temperatures and an increased basal accretion rate due to a larger temperature gradient in the ice during winter.

Figure 4.11: Time evolution of the skin temperature derived from radiometer measurements (solid), the temperature of the uppermost sensor (45 cm level) of the Pittsburgh thermistor line (dashed) and the simulated surface temperature (dotted). Time series are smoothed using a 7-days running mean.
4.4. Results and discussion

Brine pockets

To get an estimate for the influence of brine pockets in the ice, Exp. 7 is repeated with the brine pocket parameterization disabled (Exp. 8). The result is a better representation of the ice base during the cold season but a less realistic distribution of the internal ice temperature in spring and summer (Figures 4.12). The previous finding is a consequence of the interacting effects of the temperature and salinity dependent effective specific latent heat of fusion (Eq. 4.30) and the thermal conductivity (Eq. 4.28) which both decrease with increasing temperature and salinity (Appendix D, Figure D.1e and D.1c). The conditions of -1.8°C and 3.2 ppt at the ice base in Exp. 7 theoretically result in a growth rate 10% larger than without considering the effect of brine; on the other hand, the lower thermal conductivity reduces the conductive heat flux which counteracts the enhanced accretion/melt rate. The less accurate representation of the internal temperature is due to a reduced capability of storing heat during the transition seasons as a consequence of the specific heat capacity of sea ice which increases with increasing temperature and salinity (Appendix D, Figure D.1b). The warmer and the more saline the ice, the more energy is needed to further warm and eventually melt the ice which enlarges the brine pocket fraction. Regarding the evolution of the ice base, the simulation without brine pocket parameterization yields a more realistic ice base but there is some uncertainty in the observed ice base and also in the ocean heat flux which is inferred from observations with error bars. In Exp. 7, the maximum ice thickness is 6 cm smaller than in Exp. 8. The internal temperature distribution is more realistic in Exp. 7, particularly the heat storage in transition seasons represented by the bulge of contours in Figure 4.9a which is less evident in Figure 4.12a. There is about 1 cm of internal melt in Exp. 8 which is added to surface melt.

The implicit parameterization of brine pocket seems to be a reasonable approach, it improves the representation of the internal ice temperature (Exp. 7 and Exp. 8) and yields realistic values of the simulated brine fraction (Figure 4.4) which is a function of local sea ice temperature and salinity (Appendix D, Figure D.1a). However, if the temperature approaches the freezing point $T_f$ (Eq. 4.5) for a given salinity, the formulations of the thermal conductivity and the heat capacity (Eqs. 4.28 and 4.29) approach a singularity, and thus loose their validity in its vicinity. A similar problem arises for the effective specific latent heat of fusion which becomes zero for temperatures approaching the freezing temperature and as a consequence, the rate of change of thickness at the surface (Eq. 4.3) will increase to infinity. This problem only applies for the surface where temperatures reach the freezing point but not to the ice base where the basal ice temperature is below the freezing point due to a given salinity.

Given a non-zero salinity, the presented parameterization of brine in sea ice leads to unphysical values if the temperature is very close to the freezing temperature. Still, this parameterization is considered an appropriate means of describing the impact of brine pockets on the material properties of an ice layer as long as the salinity at the ice surface is zero while surface ice temperatures are at or close to the freezing point. Although Exp. 8 yields quite accurate results, the brine simulation is preferred as it is more realistic and considers the physical effects of brine pockets in sea ice.
Figure 4.12: (a) Simulated temperature and thickness evolution based on Exp. 7 with disabled brine parameterization. The dashed lines denote the best estimate snow and ice thickness. (b) Difference between Exp. 8 and Exp. 7. The dotted lines denote the simulated snow and ice surface from Exp. 7.
4.5 Conclusions

The presented multi-layer thermodynamic snow sea ice model simulates an annual cycle of one-dimensional thermodynamic evolution of a snow-covered multi-year ice floe with good agreement to observations from the SHEBA field experiment from October 1997 through October 1998. The experimental set-up of the simulations corresponds to the control experiment proposed in the context of the Sea Ice Model Intercomparison Project, Part 2, Thermodynamics (SIMIP2) using atmospheric forcing and initial conditions from SHEBA.

An analysis of the observations (Section 4.3) showed that the Arctic sea ice cover has a high horizontal variability, i.e. both the snow and ice layers are inhomogeneous on small lateral scales. Therefore, it is necessary that temperature and thickness are measured at the same place to have a correct representation of the real situation. The evolution of the snow thickness is measured by conducting snow fall measurements or by using ablation stakes. This study uses a reference snow thickness which is a corrected thickness gauge observation explained in Chapter 3.

To perform a successful model validation, a consistent set of observations of a specific site, both atmospheric and oceanic forcing and snow and ice data, is needed. SHEBA is providing such data for a column from the upper ocean through the ice and snow into the atmosphere (part of the 'SHEBA Column'). However, the analysis of the observed data suggests that one specific point is not representative for local sea ice conditions because of the small-scale roughness of the ice and the high spatial variability of the snow cover.

For simulation, a defined control experiment proposed by SIMIP2 was performed. It was found that the provided initial conditions must be corrected to obtain an ice base evolution which is consistent with the observed internal ice temperature. On average, the ocean heat flux provided in SIMIP2 is larger than the ocean heat flux derived from profiles of the internal ice temperature and the ice thickness at the SHEBA location Pittsburgh (Chapter 3). Replacing the SIMIP2 ocean heat flux by the indirectly derived flux, yields an increased ice base in the simulation and thus better representation of the observed situation. In the simulations, the thickness of the snow cover has a strong impact on the amplitude of thermal waves in the ice, influencing the temperature gradient at the ice base and thus affects the ice thickness. For this reason, it is important to have an accurate representation of the internal ice temperature which requires a multi-layer thermodynamic sea ice model which resolves the internal temperature properly. The same applies for the snow layer where the temperature gradient at the surface balances the atmospheric net heat flux. Three variations are used in this study to prescribe the snow thickness: (1) measurements of an individual thickness gauge, (2) the mean of three ablation stake records, and (3) a best estimate based on a gauge measurement and a correction involving findings of various snow cover analyses described in Chapter 3. The snow thickness derived from observed precipitation rates was found to be not realistic.

Simulation results demonstrate the sensitivity of the thermodynamic evolution of a sea ice cover to local snow and ocean surface conditions. In general, the modeled snow sea ice slab is thinner than the measured but still agrees reasonably with observations. Differences are a result of underestimated basal growth during winter and
overestimated basal melt during summer. It is suspected that the prescribed ocean heat flux is slightly too large during the whole year, and particularly in summer.

In the snow cover experiments, it was found that a conductivity of snow of \( K_s = 0.55 \text{ Wm}^{-1}\text{K}^{-1} \) is necessary to obtain the observed temperature distribution in the ice underneath. Such a value has been derived from an analysis of the temperature gradients above and below the snow-ice interface at Pittsburgh (Chapter 3). For improved simulations, the ocean heat flux must be known more accurately as well as the under-ice ocean temperature, and additionally the salinity for the correct determination of the freezing temperature of ocean water.

Both the data analysis as well as the simulations uncover problems associated with data collection and model validation. Model results identify which data are needed for a validation of a thermodynamic sea ice model and also indicate in which accuracy, and thus give suggestions which variables and physical constants should be measured and in which way. Observational data in turn show where the model needs further improvement. The given SHEBA data area unique archive of long-term observations but do not entirely satisfy the prerequisites for an ideal case, which can be used for a high precision validation due to the uncertainty in the evolution of measured snow and ice thickness. Nevertheless they are useful for a detailed evaluation, which provides reliable findings.

As a result of the difficulties with the given measurements and the limited model complexity, the present study may be considered as a model and data assimilation process rather than a one way model validation. This means that the performance of the model was examined using the SHEBA measurements aiming at detecting discrepancies between model results and observational data and hence where the model needs improvement. On the other hand, it was also taken advantage of convincing and plausible results of the simulations and the data analysis to identify possible problems in the observations and in how far the given observations are applicable for a model validation.
Appendix A

Notation summary

Table A.1: Physical parameters and constants used in the model.

<table>
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<th>Variable or constant name</th>
<th>Value</th>
<th>Unit</th>
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<td>[m]</td>
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<td>[J kg$^{-1}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>emissivity of ice, snow</td>
<td>0.99</td>
<td>[-]</td>
</tr>
<tr>
<td>$F_{cs}$</td>
<td>conductive heat flux at the surface</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{cb}$</td>
<td>conductive heat flux at the base</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{lw}$</td>
<td>downward longwave radiation</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{sw}$</td>
<td>downward shortwave radiation</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{ps}$</td>
<td>SW radiation penetrating snow surface</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{pi}$</td>
<td>SW radiation penetrating ice surface</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{sh}$</td>
<td>sensible heat flux at the surface</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{lh}$</td>
<td>latent heat flux at the surface</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$F_{net}$</td>
<td>atmospheric net heat flux at the surface</td>
<td></td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$f_b$</td>
<td>brine fraction</td>
<td></td>
<td>[-]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>empirical constant</td>
<td>$1.8 \times 10^4$</td>
<td>[J °C kg$^{-1}$ ppt$^{-1}$]</td>
</tr>
<tr>
<td>$h$</td>
<td>thickness of snow or ice</td>
<td></td>
<td>[m]</td>
</tr>
<tr>
<td>$h_t$</td>
<td>ice thickness</td>
<td></td>
<td>[m]</td>
</tr>
<tr>
<td>$h_s$</td>
<td>snow thickness</td>
<td></td>
<td>[m]</td>
</tr>
<tr>
<td>$I_0$</td>
<td>fraction of net shortwave radiation penetrating the surface</td>
<td>0.15</td>
<td>[-]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Variable or constant name</td>
<td>Value</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------</td>
<td>-------</td>
<td>------------</td>
</tr>
<tr>
<td>$K_0$</td>
<td>thermal conductivity of freshwater ice</td>
<td>2.03</td>
<td>[Wm$^{-1}$K$^{-1}$]</td>
</tr>
<tr>
<td>$K_s$</td>
<td>thermal conductivity of snow</td>
<td>0.31</td>
<td>[Wm$^{-1}$K$^{-1}$]</td>
</tr>
<tr>
<td>$K_i$</td>
<td>thermal conductivity of sea ice</td>
<td>1.5</td>
<td>[m$^{-1}$]</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>bulk extinction coefficient of ice</td>
<td>10</td>
<td>[m$^{-1}$]</td>
</tr>
<tr>
<td>$L_0$</td>
<td>latent heat of fusion of freshwater ice</td>
<td>$3.34 \times 10^5$</td>
<td>[J kg$^{-1}$]</td>
</tr>
<tr>
<td>$L_f$</td>
<td>latent heat of fusion of sea ice</td>
<td>2.83 $\times 10^6$</td>
<td>[J kg$^{-1}$]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>empirical constant</td>
<td>0.054</td>
<td>[$^\circ$C ppt$^{-1}$]</td>
</tr>
<tr>
<td>$P_{surf}$</td>
<td>air pressure at the surface</td>
<td>1013</td>
<td>[hPa]</td>
</tr>
<tr>
<td>$p$</td>
<td>precipitation rate (water equivalent)</td>
<td>1.28</td>
<td>[mm day$^{-1}$]</td>
</tr>
<tr>
<td>$Q_r$</td>
<td>radiative source term</td>
<td>330</td>
<td>[kg m$^{-3}$]</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>density of freshwater ice</td>
<td>917</td>
<td>[kg m$^{-3}$]</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>density of ocean water at 33 ppt</td>
<td>1025</td>
<td>[kg m$^{-3}$]</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>density of snow</td>
<td>1000</td>
<td>[kg m$^{-3}$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant</td>
<td>5.67 $\times 10^{-8}$</td>
<td>[Wm$^{-2}$K$^{-4}$]</td>
</tr>
<tr>
<td>$S_0$</td>
<td>salinity of ocean water</td>
<td>33</td>
<td>[ppt]</td>
</tr>
<tr>
<td>$S_i$</td>
<td>salinity of newly formed sea ice</td>
<td>3.2</td>
<td>[ppt]</td>
</tr>
<tr>
<td>$S_h$</td>
<td>thermodynamic source term</td>
<td>1.5</td>
<td>[m s$^{-1}$]</td>
</tr>
<tr>
<td>$s_i$</td>
<td>level of ice surface</td>
<td>3.2</td>
<td>[m]</td>
</tr>
<tr>
<td>$s_s$</td>
<td>level of snow surface</td>
<td>0.00</td>
<td>[°C]</td>
</tr>
<tr>
<td>$T_0$</td>
<td>freezing temperature of freshwater</td>
<td>0.00</td>
<td>[°C]</td>
</tr>
<tr>
<td>$T_a$</td>
<td>temperature of air</td>
<td>0.00</td>
<td>[°C]</td>
</tr>
<tr>
<td>$T_f$</td>
<td>freezing temperature of ice/snow/water at a given salinity</td>
<td>0.00</td>
<td>[°C]</td>
</tr>
<tr>
<td>$T_{surf}$</td>
<td>surface temperature of ice/snow</td>
<td>0.00</td>
<td>[°C]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>transformed temperature (heat content)</td>
<td>0.00</td>
<td>[°C m]</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>transformed ice temperature</td>
<td>0.00</td>
<td>[°C m]</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>transformed snow temperature</td>
<td>0.00</td>
<td>[°C m]</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>transformed freezing temperature</td>
<td>0.00</td>
<td>[°C m]</td>
</tr>
<tr>
<td>$\theta_{surf}$</td>
<td>transformed surface temp. of ice/snow</td>
<td>0.00</td>
<td>[°C m]</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
<td>0.00</td>
<td>[s]</td>
</tr>
<tr>
<td>$u_a$</td>
<td>wind speed</td>
<td>0.00</td>
<td>[ms$^{-1}$]</td>
</tr>
<tr>
<td>$w$</td>
<td>vertical velocity component</td>
<td>0.00</td>
<td>[ms$^{-1}$]</td>
</tr>
<tr>
<td>$z$</td>
<td>vertical axis, (positive upward)</td>
<td>0.00</td>
<td>[m]</td>
</tr>
</tbody>
</table>
Appendix B

Derivation of model equations

Transformation of a scalar field:

\[
\frac{\partial \phi}{\partial t} = \frac{\partial \tilde{t}}{\partial t} \frac{\partial \phi}{\partial \tilde{t}} + \frac{\partial \tilde{x}}{\partial \tilde{t}} \frac{\partial \tilde{\phi}}{\partial \tilde{x}} + \frac{\partial \tilde{y}}{\partial \tilde{t}} \frac{\partial \tilde{\phi}}{\partial \tilde{y}} + \frac{\partial \tilde{z}}{\partial \tilde{t}} \frac{\partial \tilde{\phi}}{\partial \tilde{z}} \tag{B.1}
\]

\[
\frac{\partial^2 \phi}{\partial \tilde{z}^2} = \frac{\partial}{\partial \tilde{z}} \left( \frac{\partial \phi}{\partial \tilde{z}} \right) \Rightarrow \frac{\partial \tilde{z}}{\partial \tilde{z}} \frac{\partial^2 \phi}{\partial \tilde{z}^2} = \frac{\partial \tilde{z}}{\partial \tilde{z}} \left( \frac{\partial}{\partial \tilde{z}} \frac{\partial \tilde{\phi}}{\partial \tilde{z}} \right) = \frac{\partial \tilde{z}}{\partial \tilde{z}} \left( \frac{\partial}{\partial \tilde{z}} \frac{\partial \tilde{\phi}}{\partial \tilde{z}} + \frac{\partial \tilde{z}}{\partial \tilde{z}} \frac{\partial^2 \tilde{\phi}}{\partial \tilde{z}^2} \right) \tag{B.2}
\]

Recall transformation equations:

\[
\tilde{z} = \frac{z - b(t)}{s(t) - b(t)} \quad \text{and} \quad \tilde{t} = t \tag{B.3}
\]

\[
z = \tilde{z} [s(t) - b(t)] + b(t) , \tag{B.4}
\]

where \( s \) and \( b \) are the level of the surface and base, respectively, and \( h(t) = s(t) - b(t) \) is the thickness. Apply differential operators to transformation equation:

\[
\frac{\partial \tilde{z}}{\partial t} = \frac{1}{(s-b)^2} \left[ \left( \frac{\partial z}{\partial t} - \frac{\partial b}{\partial t} \right) (s-b) - (z-b) \left( \frac{\partial s}{\partial t} - \frac{\partial b}{\partial t} \right) \right]
\]

\[
= - \frac{1}{s-b} \left[ (1-\tilde{z}) \frac{\partial b}{\partial t} + \tilde{z} \frac{\partial s}{\partial t} \right] = \frac{(1-\tilde{z})}{h} \frac{\partial b}{\partial t} - \frac{\tilde{z}}{h} \frac{\partial s}{\partial t} \tag{B.5}
\]

\[
\frac{\partial \tilde{z}}{\partial z} = \frac{1}{(s-b)^2} \left[ \left( \frac{\partial z}{\partial z} - \frac{\partial b}{\partial z} \right) (s-b) - (z-b) \left( \frac{\partial s}{\partial z} - \frac{\partial b}{\partial z} \right) \right]
\]

\[
= \frac{1}{s-b} = \frac{1}{h} \tag{B.6}
\]
\[
\frac{\partial z}{\partial \tau} = (s - b) = h \\
\frac{\partial z}{\partial t} = 0, \quad \frac{\partial s}{\partial z} = 1, \quad \frac{\partial b}{\partial z} = 0, \quad \frac{\partial \tilde{t}}{\partial t} = 0
\]  
(B.7)

In the snow equation, \( \partial s_s/\partial t \) is negative for surface melt and positive for snowfall. The base of the snow slab is assumed not to change.

In the ice equation, \( \partial s_i/\partial t \) is negative if surface melt occurs. \( \partial b_i/\partial t \) is positive for basal melt and negative for basal growth. For updating the ice thickness, the sign of \( \partial b_i/\partial t \) must be changed.

Transformation of the one-dimensional energy equation (cf. Eq. 4.10):

\[
\frac{\partial \tilde{t}}{\partial t} + \frac{\partial \tilde{T}}{\partial \tau} + \frac{\partial z}{\partial \tau} \frac{\partial \tilde{T}}{\partial z} = \frac{K}{\rho_c h} \frac{\partial \tilde{z}}{\partial \tau} \frac{\partial \tilde{T}}{\partial z} + \frac{\tilde{R}}{\rho_c}
\]  
(B.9)

Substituting all differentials, where \( \tilde{w} = \partial \tilde{z}/\partial \tilde{t} \):

\[
\frac{\partial \tilde{T}}{\partial \tau} + \tilde{w} \frac{\partial \tilde{T}}{\partial z} = \frac{K}{\rho_c h^2} \frac{1}{\partial z} + \frac{\tilde{R}}{\rho_c}
\]  
(B.10)

Differentiate \( \tilde{w} \) with respect to \( \tilde{z} \) (cf. Eq. B.5):

\[
\frac{\partial \tilde{w}}{\partial \tilde{z}} = \frac{1}{h} \left( \frac{\partial s}{\partial t} \frac{\partial b}{\partial t} \right) = -\frac{1}{h} \frac{\partial h}{\partial t}
\]
(B.11)

From Eqs. B.11 and 4.1 it follows

\[
\frac{\partial h}{\partial \tilde{t}} = -\frac{\partial (h\tilde{w})}{\partial \tilde{z}} = S_h
\]
(B.12)

Add the transformed energy equation (Eq. B.10) multiplied with \( h \) and the conservation of mass equation (Eq. 4.1) multiplied with \( \tilde{T} \), considering that \( h \) is only function of \( t \):

\[
\left(h \frac{\partial \tilde{T}}{\partial \tau} + \tilde{z} \frac{\partial h}{\partial t} + \tilde{w} h \frac{\partial \tilde{T}}{\partial z} \right) = \frac{K}{\rho_c h^2} \frac{1}{\partial z} + \frac{\tilde{R}}{\rho_c} + \tilde{T} S_h
\]
(B.13)

Using

\[
\frac{\partial (\tilde{w} h \tilde{T})}{\partial \tilde{z}} = \tilde{w} \frac{\partial (\tilde{w} h)}{\partial \tilde{z}} + \tilde{w} h \frac{\partial \tilde{T}}{\partial \tilde{z}}
\]
(B.14)

and consider Eq. B.12 yields the energy equation in flux form:

\[
\frac{\partial (h \tilde{T})}{\partial \tau} + \frac{\partial (\tilde{w} h \tilde{T})}{\partial \tilde{z}} - \tilde{T} \frac{\partial (\tilde{w} h)}{\partial \tilde{z}} = \frac{K}{\rho_c h^2} \frac{1}{\partial z} + \frac{\tilde{R}}{\rho_c} - \tilde{T} \frac{\partial (\tilde{w} h)}{\partial \tilde{z}}
\]
(B.15)
Substituting $\tilde{\theta} = h\tilde{T}$, the transformed energy equations for snow and ice can be written as:

\[
\frac{\partial \tilde{\theta}_s}{\partial t} = \frac{K_s}{\rho_s c_p h_s^2} \frac{\partial^2 \tilde{\theta}_s}{\partial \tilde{z}^2} - \frac{\partial (\tilde{w} \tilde{\theta}_s)}{\partial \tilde{z}} + \frac{h_s \tilde{R}}{\rho_s c_p} \tag{B.16}
\]

\[
\frac{\partial \tilde{\theta}_i}{\partial t} = \frac{K_i}{\rho_i c_p h_i^2} \frac{\partial^2 \tilde{\theta}_i}{\partial \tilde{z}^2} - \frac{\partial (\tilde{w} \tilde{\theta}_i)}{\partial \tilde{z}} + \frac{h_i \tilde{R}}{\rho_i c_p} \tag{B.17}
\]

Penetrating shortwave radiation below the snow layer:

$F_{pi}$ in Eq. 4.12 is computed as follows:

\[
F_{pi} = F_{ps} - \int_0^{h_s} F_{ps} \kappa_s e^{-\kappa_s (h_s - \tilde{z})} \, d\tilde{z} = F_{ps} - \int_0^{h_s} F_{ps} \kappa_s e^{-\kappa_s h_s e^{-\kappa_s \tilde{z}}} \, d\tilde{z}
\]

\[
= F_{ps} - \left( F_{ps} e^{-\kappa_s h_s} \int_0^{h_s} \kappa_s e^{\kappa_s \tilde{z}} \, d\tilde{z} \right) = F_{ps} - \left( F_{ps} e^{-\kappa_s h_s} \left[ e^\kappa_s h_s \right]_0^{h_s} \right)
\]

\[
= F_{ps} - \left( F_{ps} e^{-\kappa_s h_s} \left[ e^\kappa_s h_s - 1 \right] \right) = F_{ps} - \left( F_{ps} e^{-\kappa_s h_s} e^\kappa_s h_s - F_{ps} e^{-\kappa_s h_s} \right)
\]

\[
= F_{ps} - \left( F_{ps} - F_{ps} e^{-\kappa_s h_s} \right) = F_{ps} e^{-\kappa_s h_s} \tag{B.18}
\]

Salinity profile:

The model provides several options for specifying salinity profiles: vertically and temporally constant, vertically constant but function of ice thickness, linear, prescribed from observations or finally linear above the freeboard line and constant below. The freeboard $h_f$ is the vertical distance from the snow or ice surface to the ocean surface and is given by

\[
h_f = h_i \left( 1 - \frac{\rho_i}{\rho_0} \right) + h_s \left( 1 - \frac{\rho_s}{\rho_0} \right), \tag{B.19}
\]

where $\rho_w$ is the density of ocean water. The level of the ice surface above the water is simply $h_f - h_s$.

If the model is configured with a linear salinity above the water level, the first grid point below the water surface is taken as the level of freeboard.
Appendix C

Scale analysis

Non-dimensionalization of an equation gives typical order of magnitude for each term by introducing appropriate scales for the various quantities involved.

\[ \psi = [\Psi] \psi^* \], where \( \psi^* \) is non-dimensional and of order one in magnitude and \( [\Psi] \) is a characteristic quantity.

Split the three-dimensional energy equation into horizontal (index h) and vertical terms:

\[
\frac{\partial T}{\partial t} + \bar{v}_h \cdot \nabla_h T + w \frac{\partial T}{\partial z} = k \nabla_h^2 T + k \frac{\partial^2 T}{\partial z^2} + \frac{F}{\rho_c \kappa} e^{-\kappa z} \quad (C.1)
\]

\[
\frac{[\Delta T_i]}{[t_a]} \frac{\partial T^*}{\partial t^*} + \frac{[U_h][\Delta T_h]}{[L_h]} \bar{v}_h^* \cdot \nabla_h^* T^* + \frac{[U_v][\Delta T_v]}{[H]} w^* \frac{\partial T^*}{\partial z^*} = 0, \quad 0 \quad (C.2)
\]

In Eq. C.2, numbers give the order of magnitude for ice and for snow respectively. In Tab. C.1, values of the characteristic quantities are listed. From the scale analysis it is concluded that the horizontal diffusion term is negligible, the vertical advection is equal to zero (but will be considered in the coordinate transformation) and the horizontal advection is neglected as it is relatively small and since we have a Lagrangian model.
Table C.1: Scaling variables used in the scale analysis

<table>
<thead>
<tr>
<th>symbol</th>
<th>characteristic variable</th>
<th>ice</th>
<th>snow</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>([t_a])</td>
<td>char. advection time scale (= \frac{L_g}{U_h})</td>
<td>(10^6)</td>
<td>(10^6)</td>
<td>[s]</td>
</tr>
<tr>
<td>([t_i])</td>
<td>char. time scale (=) time step = 1 hour</td>
<td>3600</td>
<td>3600</td>
<td>[s]</td>
</tr>
<tr>
<td>([1])</td>
<td>unity, order of magnitude 1</td>
<td>1</td>
<td>1</td>
<td>[-]</td>
</tr>
<tr>
<td>([H])</td>
<td>char. ice or snow thickness</td>
<td>(1)</td>
<td>(0.1)</td>
<td>[m]</td>
</tr>
<tr>
<td>([L_g])</td>
<td>char. length scale of a grid cell</td>
<td>(10^5)</td>
<td>(10^5)</td>
<td>[m]</td>
</tr>
<tr>
<td>([\Delta T_c])</td>
<td>char. change of temperature in char. advection time ([t_a])</td>
<td>20</td>
<td>30</td>
<td>[K]</td>
</tr>
<tr>
<td>([\Delta T_h])</td>
<td>char. horizontal temperature difference within a grid cell</td>
<td>1</td>
<td>1</td>
<td>[K]</td>
</tr>
<tr>
<td>([\Delta T_v])</td>
<td>char. vertical temperature difference</td>
<td>40</td>
<td>20</td>
<td>[K]</td>
</tr>
<tr>
<td>([\Delta \theta_c])</td>
<td>char. change of heat content in a time step ([H] \cdot [\Delta T])</td>
<td>1</td>
<td>0.4</td>
<td>[K]</td>
</tr>
<tr>
<td>([\Delta \theta_v])</td>
<td>char. vertical difference in heat content ([H] \cdot [\Delta T_v])</td>
<td>40</td>
<td>2</td>
<td>[K]</td>
</tr>
<tr>
<td>([U_h])</td>
<td>char. horizontal snow/ice and ocean speed</td>
<td>(0.1)</td>
<td>(0.1)</td>
<td>[ms(^{-1})]</td>
</tr>
<tr>
<td>([U_v])</td>
<td>char. vertical snow/ice and ocean speed</td>
<td>0</td>
<td>0</td>
<td>[ms(^{-1})]</td>
</tr>
<tr>
<td>([W])</td>
<td>char. vertical advection velocity (growth/melt rate, (\partial h/\partial t))</td>
<td>(2 \times 10^{-7})</td>
<td>(6 \times 10^{-7})</td>
<td>[ms(^{-1})]</td>
</tr>
<tr>
<td>([F])</td>
<td>char. sw radiation penetrating the surface (= \left[F_{sw}\right] \cdot [\tau] \cdot [1 - \alpha]) (= 500 \times 0.15 \times 0.4 \times 0.4 = 30)</td>
<td>10</td>
<td>10</td>
<td>[Wm(^{-2})]</td>
</tr>
</tbody>
</table>

Scaling of the transformed equations (Eq. 4.22, 4.23), (numbers below the braces are computed using ice and snow, respectively):

\[
\frac{[\Delta \theta_c]}{[t_i]} \frac{\partial \theta^*}{\partial t^*} = \frac{k}{[H]^2} \frac{[\Delta \theta_c]}{[1]^2} \frac{1}{(h^*)^2} \frac{\partial^2 \theta^*}{\partial (\bar{z}^*)^2}
\]

\[
= \left(8 \times 10^{-6}, 1 \times 10^{-6}\right) + \left(4 \times 10^{-5}, 9 \times 10^{-6}\right)
\]

(C.3)

\[
- \frac{[W]}{[1]} \frac{\partial (\bar{w}^* \theta^*)}{\partial \bar{z}^*} + \frac{[H][F]}{\rho c_p} \kappa e^{-\kappa [1][H]} h^* F^*
\]

\[
= \left(2 \times 10^{-6}, 5 \times 10^{-6}\right) + \left(8 \times 10^{-6}, 1 \times 10^{-6}\right)
\]

The scale analysis suggests that no term can be neglected.
Appendix D

Material properties

Figure D.1: Temperature and salinity dependency of (a) the brine fraction, (b) the heat capacity, (c) the thermal conductivity, (d) the thermal diffusivity and (e) the latent heat of fusion. Empiric equations relating the thermal conductivity and the density of snow (f).
Appendix E

Discretization

Figure E.1: Numerical grid schematic. The example shows 10 layers in the snow and 10 layers in the ice. The vertical velocity component \( w \) and the heat flux \( Q \) are defined at the nodes of the main grid, the temperature \( T \) and the salinity \( S \) are defined at the center of the grid (staggered grid). Each layer, plus surface, base and interface is represented by a temperature \( T \) and in the ice by a salinity \( S \).
Numerical scheme

Finite difference implicit numerical scheme; the time derivative is discretized with a forward step, the diffusion term second order centered-in-space, and the grid-advection term with a conservative first order upstream scheme. Equations can be written as a linear, inhomogeneous, tridiagonal system of equations:

\[ A_k^{t+1} \tilde{\theta}_k^{t+1} + B_k^{t+1} \tilde{\theta}_{k+1}^{t+1} + C_k^{t+1} \tilde{\theta}_{k+1}^{t+1} = D_k^t \]

\[
\begin{pmatrix}
\vdots \\
A_{k-1} & B_{k-1} & C_{k-1} \\
A_k & B_k & C_k \\
A_{k+1} & B_{k+1} & C_{k+1} \\
\vdots 
\end{pmatrix}
\begin{pmatrix}
\tilde{\theta}_0 \\
\tilde{\theta}_1 \\
\vdots \\
\tilde{\theta}_{k-1} \\
\tilde{\theta}_k \\
\tilde{\theta}_{k+1} \\
\vdots \\
\tilde{\theta}_{N+M} \\
\tilde{\theta}_{N+M+1}
\end{pmatrix} = 
\begin{pmatrix}
D_0 \\
D_1 \\
\vdots \\
D_{k-1} \\
D_k \\
D_{k+1} \\
\vdots \\
D_{N+M} \\
D_{N+M+1}
\end{pmatrix}
\]

The jump conditions \( \tilde{\theta}^+ = \tilde{\theta}^- \) and \( F_c^+ = F_c^- \) at the interface \( (k=N) \) result in an equation with five elements.

\[ P_N^{t+1} \tilde{\theta}_{N-2}^{t+1} + A_N^{t+1} \tilde{\theta}_{N-1}^{t+1} + B_N^{t+1} \tilde{\theta}_N^{t+1} + C_N^{t+1} \tilde{\theta}_{N+1}^{t+1} + Q_N^{t+1} \tilde{\theta}_{N+2}^{t+1} = D_N^t \]

\[
\begin{pmatrix}
\vdots \\
A_{N-2} & B_{N-2} & C_{N-2} \\
A_{N-1} & B_{N-1} & C_{N-1} \\
P_N & A_N & B_N & C_N & Q_N \\
A_{N+1} & B_{N+1} & C_{N+1} \\
A_{N+2} & B_{N+2} & C_{N+2} \\
\vdots 
\end{pmatrix}
\begin{pmatrix}
\vdots \\
\tilde{\theta}_{N-2} \\
\tilde{\theta}_{N-1} \\
\tilde{\theta}_N \\
\tilde{\theta}_{N+1} \\
\tilde{\theta}_{N+2} \\
\vdots 
\end{pmatrix} = 
\begin{pmatrix}
\vdots \\
D_{N-2} \\
D_{N-1} \\
D_N \\
D_{N+1} \\
D_{N+2} \\
\vdots 
\end{pmatrix}
\]

Applying a Gauss elimination of the off-tridiagonal matrix elements at the interface and the surface conserves the tridiagonal system.
Discretization (ice)

Discretization of Eq. 4.22, implicit, the diffusion term with a second order centered difference, the advection term with an energy conserving first order upstream scheme. \( cp, h, R, K \) and \( \bar{w} \) are evaluated explicitly at index \( k, K_i \) and \( cp \) optional iteratively.

\[
\frac{\bar{\theta}_k^{t+1} - \bar{\theta}_k^t}{\Delta t} = -\frac{1}{\rho \, cp} \left( \frac{1}{h_i} \Delta \bar{z}_i \right) \frac{\bar{Q}^{t+1}_k - \bar{Q}^{t+1}_{k+1}}{\Delta \bar{z}_i} + \frac{\bar{w}_k^{t+1} \bar{\theta}_k^{t+1} - \bar{w}_k^t \bar{\theta}_k^{t+1}}{\rho \, cp} \left( \frac{\Delta \bar{z}_i}{\Delta \bar{z}_i} \right) \text{ for } \bar{w}_k > 0 \quad (E.1)
\]

where

\[
\bar{Q}^{t+1}_{k+1} = \frac{K_i}{h_i} \frac{\bar{\theta}_k^{t+1} - \bar{\theta}_k^{t+1}}{\Delta \bar{z}_i} \quad \text{and} \quad \bar{Q}^{t+1}_k = -\frac{K_i}{h_i} \frac{\bar{\theta}_k^{t+1} - \bar{\theta}_k^{t+1}}{\Delta \bar{z}_i} \quad (E.2)
\]

replace \( Q \) into the first term on the RHS of Eq. E.1:

\[
\Rightarrow \frac{\partial \bar{Q}}{\partial z} = -\frac{K_i}{\left( h_i \right)^2} \frac{\bar{\theta}_k^{t+1} - 2\bar{\theta}_k^{t+1} + \bar{\theta}_k^{t+1}}{\left( \Delta \bar{z}_i \right)^2} \quad \text{i.e.} \quad \frac{1}{\rho \, cp} \frac{\partial Q}{\partial z} = K \frac{\partial^2 T}{\partial z^2} \quad (E.3)
\]

\( \bar{w}_k > 0 \):

\[
\frac{\bar{\theta}_k^{t+1} - \bar{\theta}_k^t}{\Delta t} = \frac{K_i}{\rho \, cp \left( h_i \right)^2} \frac{\bar{\theta}_k^{t+1} - 2\bar{\theta}_k^{t+1} + \bar{\theta}_k^{t+1}}{\left( \Delta \bar{z}_i \right)^2} - \frac{\bar{w}_k^{t+1} \bar{\theta}_k^{t+1} - \bar{w}_k^t \bar{\theta}_k^{t+1}}{\rho \, cp} \left( \frac{\Delta \bar{z}_i}{\Delta \bar{z}_i} \right) + \frac{\bar{h}_i}{\rho \, cp} \bar{R}_k^t \quad (E.4)
\]

\( \bar{w}_k < 0 \):

\[
\frac{\bar{\theta}_k^{t+1} - \bar{\theta}_k^t}{\Delta t} = \frac{K_i}{\rho \, cp \left( h_i \right)^2} \frac{\bar{\theta}_k^{t+1} - 2\bar{\theta}_k^{t+1} + \bar{\theta}_k^{t+1}}{\left( \Delta \bar{z}_i \right)^2} - \frac{\bar{w}_k^{t+1} \bar{\theta}_k^{t+1} - \bar{w}_k^t \bar{\theta}_k^{t+1}}{\rho \, cp} \left( \frac{\Delta \bar{z}_i}{\Delta \bar{z}_i} \right) + \frac{\bar{h}_i}{\rho \, cp} \bar{R}_k^t \quad (E.5)
\]

where

\[
\bar{R}_k^t = F_{pi} \kappa_i e^{-\alpha_i \left( 1 - \bar{z}_i \right)} \kappa_i^t \quad (E.6)
\]

ice; \( \bar{w}_k > 0 \):

\[
\frac{\bar{\theta}_k^{t+1} - \bar{\theta}_k^t}{\Delta t} = \frac{\Delta t K_i}{\rho \, cp \left( h_i \right)^2 \left( \Delta \bar{z}_i \right)^2} \left( \bar{\theta}_k^{t+1} - 2\bar{\theta}_k^{t+1} + \bar{\theta}_k^{t+1} \right) - \frac{\Delta t}{\left( \Delta \bar{z}_i \right)^2} \left( \bar{\theta}_k^{t+1} - \bar{\theta}_k^{t+1} \right) + \frac{\bar{h}_i}{\rho \, cp} \bar{R}_k^t \quad (E.7)
\]

ice; \( \bar{w}_k < 0 \):

\[
\frac{\bar{\theta}_k^{t+1} - \bar{\theta}_k^t}{\Delta t} = \frac{\Delta t K_i}{\rho \, cp \left( h_i \right)^2 \left( \Delta \bar{z}_i \right)^2} \left( \bar{\theta}_k^{t+1} - 2\bar{\theta}_k^{t+1} + \bar{\theta}_k^{t+1} \right) - \frac{\Delta t}{\left( \Delta \bar{z}_i \right)^2} \left( \bar{\theta}_k^{t+1} - \bar{\theta}_k^{t+1} \right) + \frac{\bar{h}_i}{\rho \, cp} \bar{R}_k^t \quad (E.8)
\]
Discretization (snow)

Discretization of Eq. 4.23, implicit, the diffusion term with a second order centered difference, the advection term with an energy conserving first order upstream scheme. $c_p$, $h$, $R$, $K$ and $\bar{w}$ are evaluated explicitly at index $k$, $K_1$ and $c_p$ optional iteratively.

\[
\frac{\bar{Q}^{t+1} - \bar{Q}^t}{\Delta t} = -\frac{1}{\rho c_p} \frac{\Delta^2 z}{h_s} \frac{\bar{Q}^{t+1} - \bar{Q}^{t-1}}{\Delta z_s} + \frac{h_s}{\rho c_p} \bar{R}_k \tag{E.9}
\]

where

\[
\bar{Q}^{t+1}_k = \frac{K_s}{h_s} \frac{\bar{Q}^{t+1} - \bar{Q}^t}{\Delta z_s} \quad \text{and} \quad \bar{Q}^{t+1}_{k-1} = -\frac{K_s}{h_s} \frac{\bar{Q}^{t+1} - \bar{Q}^{t-1}}{\Delta z_s} \tag{E.10}
\]

replace $Q$ into the first term on the RHS of Eq. 9:

\[
\Rightarrow \quad \frac{\partial \bar{Q}}{\partial z} = -\frac{K_s}{(h_s)^2} \frac{\bar{Q}^{t+1} - 2\bar{Q}^t + \bar{Q}^{t-1}}{(\Delta z_s)^2} \quad \text{i.e.} \quad -\frac{1}{\rho c_p} \frac{\partial Q}{\partial z} = \frac{K}{\rho c_p} \frac{\partial^2 T}{\partial z^2} \tag{E.11}
\]

$\bar{w}_k > 0$:

\[
\frac{\bar{Q}^{t+1} - \bar{Q}^t}{\Delta t} = \frac{K_s}{\rho c_p (h_s)^2} \frac{\bar{Q}^{t+1} - 2\bar{Q}^t + \bar{Q}^{t-1}}{(\Delta z_s)^2} - \frac{\bar{w}_k^t \bar{Q}^{t+1} - \bar{w}_{k-1}^t \bar{Q}^{t+1}}{\Delta z_s^2} + \frac{h_s}{\rho c_p} \bar{R}_k \tag{E.12}
\]

$\bar{w}_k < 0$:

\[
\frac{\bar{Q}^{t+1} - \bar{Q}^t}{\Delta t} = \frac{K_s}{\rho c_p (h_s)^2} \frac{\bar{Q}^{t+1} - 2\bar{Q}^t + \bar{Q}^{t-1}}{(\Delta z_s)^2} - \frac{\bar{w}_k^t \bar{Q}^{t+1} - \bar{w}_{k-1}^t \bar{Q}^{t+1}}{\Delta z_s^2} + \frac{h_s}{\rho c_p} \bar{R}_k \tag{E.13}
\]

where

\[
\bar{R}_k = F_{ps} c_n e^{-\nu_n (1-\bar{z}_s^t)} h_s \tag{E.14}
\]

snow; $\bar{w}_k > 0$:

\[
\frac{\bar{Q}^{t+1} - \bar{Q}^t}{\Delta t} = \frac{\Delta t K_s}{\rho c_p (h_s)^2 (\Delta z_s)^2} \left( \bar{Q}^{t+1}_{k+1} - 2\bar{Q}^t_k + \bar{Q}^{t-1}_{k-1} \right) \frac{\Delta^2 z}{C_{1s}} \bar{R}_k \tag{E.15}
\]

snow; $\bar{w}_k < 0$:

\[
\frac{\bar{Q}^{t+1} - \bar{Q}^t}{\Delta t} = \frac{\Delta t K_s}{\rho c_p (h_s)^2 (\Delta z_s)^2} \left( \bar{Q}^{t+1}_{k+1} - 2\bar{Q}^t_k + \bar{Q}^{t-1}_{k-1} \right) \frac{\Delta^2 z}{C_{1s}} \bar{R}_k \tag{E.16}
\]
ice; \( \tilde{w}_k > 0 \):

\[
\begin{aligned}
(A_k) \quad \tilde{\theta}_{k+1}^{t+1} &= (C_1 \tilde{t}_k^{t+1} + C_2 \tilde{w}_k^{t+1} + (-2C_1 - C_2 \tilde{w}_{k+1}^{t+1} - 1) \tilde{\theta}_{k+1}^{t+1} + (C_1 - C_2 \tilde{w}_k^{t+1}) \tilde{\theta}_{k+1}^{t+1} = -\tilde{\theta}_k^{t+1} - C_4 \tilde{R}_k^{t+1} \quad (E.17)
\end{aligned}
\]

ice; \( \tilde{w}_k < 0 \):

\[
\begin{aligned}
(A_k) \quad \tilde{\theta}_{k+1}^{t+1} &= (C_1 \tilde{t}_k^{t+1} + C_2 \tilde{w}_k^{t+1} - 1) \tilde{\theta}_{k+1}^{t+1} + (C_1 - C_2 \tilde{w}_k^{t+1}) \tilde{\theta}_{k+1}^{t+1} = -\tilde{\theta}_k^{t+1} - C_4 \tilde{R}_k^{t+1} \quad (E.18)
\end{aligned}
\]

snow; \( \tilde{w}_k > 0 \):

\[
\begin{aligned}
(A_k) \quad \tilde{\theta}_{k+1}^{t+1} &= (C_1 + C_2 \tilde{w}_{k-1}^{t+1}) \tilde{\theta}_{k+1}^{t+1} + (C_1 - C_2 \tilde{w}_k^{t+1} - 1) \tilde{\theta}_{k+1}^{t+1} + (C_1 - C_2 \tilde{w}_k^{t+1}) \tilde{\theta}_{k+1}^{t+1} = -\tilde{\theta}_k^{t+1} - C_4 \tilde{R}_k^{t+1} \quad (E.19)
\end{aligned}
\]

snow; \( \tilde{w}_k < 0 \):

\[
\begin{aligned}
(A_k) \quad \tilde{\theta}_{k+1}^{t+1} &= (C_1 + C_2 \tilde{w}_k^{t+1}) \tilde{\theta}_{k+1}^{t+1} + (C_1 - C_2 \tilde{w}_k^{t+1} - 1) \tilde{\theta}_{k+1}^{t+1} + (C_1 - C_2 \tilde{w}_k^{t+1}) \tilde{\theta}_{k+1}^{t+1} = -\tilde{\theta}_k^{t+1} - C_4 \tilde{R}_k^{t+1} \quad (E.20)
\end{aligned}
\]

Conductive heat flux at surface and base:
A second order asymmetric difference discretization is used to determine the gradient at the boundary grid point. Coefficients are derived from a Taylor series expansion about the boundary grid points.

snow surface:

\[
h_s F_c \bigg|_{z_s=1} = -K_s \frac{8 \tilde{\theta}_{N+M+1}^{t+1} - 9 \tilde{\theta}_{N+M}^{t+1} + \tilde{\theta}_{N+M+1}^{t+1}}{3 \Delta z_s} \quad (E.21)
\]

snow bottom:

\[
h_s F_c \bigg|_{z_s=0} = -K_s \frac{\tilde{\theta}_{N+2}^{t+1} + 9 \tilde{\theta}_{N+1}^{t+1} - 8 \tilde{\theta}_{N}^{t+1}}{3 \Delta z_s} \quad (E.22)
\]

ice surface:

\[
h_i F_c \bigg|_{z_i=1} = -K_i \frac{8 \tilde{\theta}_{N-1}^{t+1} - 9 \tilde{\theta}_{N-2}^{t+1} + \tilde{\theta}_{N-2}^{t+1}}{3 \Delta z_i} \quad (E.23)
\]

ice bottom:

\[
h_i F_c \bigg|_{z_i=0} = -K_i \frac{\tilde{\theta}_{N-2}^{t+1} + 9 \tilde{\theta}_{N-1}^{t+1} - 8 \tilde{\theta}_{N-2}^{t+1}}{3 \Delta z_i} \quad (E.24)
\]

**Ice base**

\[
k = 0:
\frac{1}{D_0} \tilde{\theta}_0^{t+1} + \frac{0}{C_0} \tilde{\theta}_1^{t+1} = \theta_t - \mu S_0 h_i \quad (E.25)
\]
Ice base adjacent

\( k = 1, \; \tilde{w}_i > 0 : \)

\[
\frac{\theta_i^{t+1} - \theta_i^t}{\Delta t} = \frac{K_i}{\rho_i c_p(h_i)^2} \frac{1}{\Delta \tilde{z}_i} \left[ \frac{\theta_{i+1}^{t+1} - \theta_{i-1}^{t+1}}{\Delta \tilde{z}_i} - \frac{\theta_{i+1}^{t+1} + 9\theta_{i+1}^{t+1} - 8\theta_{i+1}^{t+1}}{3\Delta \tilde{z}_i} \right] - \frac{\tilde{w}_{i+1}^{t+1} - \tilde{w}_i^{t+1} \theta_{i+1}^{t+1}}{\Delta \tilde{z}_i} + \frac{h_i}{\rho_i c_p} \tilde{R}_i^t \tag{E.26}
\]

\[
\theta_i^{t+1} - \theta_i^t = \frac{1}{3} C_{1i} \left( 8\theta_i^{t+1} + 12\theta_{i-1}^{t+1} + 4\theta_{i+1}^{t+1} \right) - C_{2i} \left( \tilde{w}_{i+1}^{t+1} - \tilde{w}_{i-1}^{t+1} \right) + C_{4i} \tilde{R}_i^t \tag{E.27}
\]

\[
\left( \frac{8}{3} C_{1i} + C_{2i} \tilde{w}_i^t \right) \theta_0^{t+1} + \left( -4C_{1i} - C_{2i} \tilde{w}_i^t - 1 \right) \theta_{i-1}^{t+1} + \left( \frac{4}{3} C_{1i} \right) \theta_{i+1}^{t+1} = \frac{-\theta_i^t - C_{4i} \tilde{R}_i^t}{D_i} \tag{E.28}
\]

\( k = 1, \; \tilde{w}_i < 0 : \)

\[
\frac{\theta_i^{t+1} - \theta_i^t}{\Delta t} = \frac{K_i}{\rho_i c_p(h_i)^2} \frac{1}{\Delta \tilde{z}_i} \left[ \frac{\theta_{i+1}^{t+1} - \theta_{i-1}^{t+1}}{\Delta \tilde{z}_i} - \frac{\theta_{i+1}^{t+1} + 9\theta_{i+1}^{t+1} - 8\theta_{i+1}^{t+1}}{3\Delta \tilde{z}_i} \right] - \frac{\tilde{w}_{i+1}^{t+1} - \tilde{w}_i^{t+1} \theta_{i+1}^{t+1}}{\Delta \tilde{z}_i} + \frac{h_i}{\rho_i c_p} \tilde{R}_i^t \tag{E.29}
\]

\[
\theta_i^{t+1} - \theta_i^t = \frac{1}{3} C_{1i} \left( 8\theta_i^{t+1} + 12\theta_{i-1}^{t+1} + 4\theta_{i+1}^{t+1} \right) - C_{2i} \left( \tilde{w}_{i+1}^{t+1} - \tilde{w}_{i-1}^{t+1} \right) + C_{4i} \tilde{R}_i^t \tag{E.30}
\]

\[
\left( \frac{8}{3} C_{1i} \right) \theta_0^{t+1} + \left( -4C_{1i} - C_{2i} \tilde{w}_i^t - 1 \right) \theta_{i-1}^{t+1} + \left( \frac{4}{3} C_{1i} \right) \theta_{i+1}^{t+1} = \frac{-\theta_i^t - C_{4i} \tilde{R}_i^t}{D_i} \tag{E.31}
\]

Interface

\( k = N : \) Jump condition at the interface: To maintain the equality of the jump condition, both sides of the equation have to be expanded by \( h_i \) and \( h_s \) respectively and not only multiplied by the corresponding thickness.

\[
-\frac{K_i}{h_i} \theta_{N-1}^{t+1} - \frac{9\theta_{N-1}^{t+1} + 8\theta_{N+1}^{t+1}}{3h_i \Delta \tilde{z}_i} = -\frac{K_s}{h_s} \frac{-8\theta_{N+1}^{t+1} + 9\theta_{N+1}^{t+1} - \theta_{N+2}^{t+1}}{3h_s \Delta \tilde{z}_s} \tag{E.32}
\]

\( \theta_{N+1}^{t+1} \) is multiply-defined (ice, snow), therefore it is treated as follows using the continuity of temperature at the interface (cf. Eq. 4.14). Introduce the variables

\[
\begin{align*}
    h_s T_{N+1}^{t+1} &:= \tilde{\theta}_{N+1}^{t+1} \\
    h_i T_{N+1}^{t+1} &:= \tilde{\theta}_{N-1}^{t+1} \\
    \tilde{\theta}_{N+1}^{t+1} &= \frac{\theta_{N+1}^{t+1}}{h_i} \\
    \tilde{\theta}_{N-1}^{t+1} &= \frac{\theta_{N-1}^{t+1}}{h_s} \\
    \tilde{\theta}_{N+1}^{t+1} &= \frac{\theta_{N+1}^{t+1}}{h_s} \\
    \tilde{\theta}_{N-1}^{t+1} &= \frac{\theta_{N-1}^{t+1}}{h_i}
\end{align*}
\tag{E.33}
\]

In the jump condition (Eq. E.32) as well as in the interface adjacent equations (Eq. E.42 and E.44) all variables \( \tilde{\theta}_{N+1}^{t+1} \) are written in terms of \( \tilde{\theta}_{N-1}^{t+1} \) and then \( \tilde{\theta}_{N-1}^{t+1} \) is renamed to \( \tilde{\theta}_{N+1}^{t+1} \).
\[ \begin{bmatrix} -\frac{K_1}{3h^2\Delta \tilde{z}_1} & 0^+ & \frac{9K_1}{3h^2\Delta \tilde{z}_1} & 0^+ \\ \frac{8K_1}{3h^2\Delta \tilde{z}_1} & -\frac{8K_1h_s}{3h^2\Delta \tilde{z}_1h_1} & \frac{9K_1}{3h^2\Delta \tilde{z}_2} & 0^+ \\ -\frac{K_2}{3h^2\Delta \tilde{z}_2} & 0^+ & \frac{9K_2}{3h^2\Delta \tilde{z}_2} & 0^+ \\ \frac{8K_2}{3h^2\Delta \tilde{z}_2} & -\frac{8K_2h_s}{3h^2\Delta \tilde{z}_2h_1} & \frac{9K_2}{3h^2\Delta \tilde{z}_3} & 0^+ \\ \end{bmatrix} \begin{bmatrix} \theta_{N+1} \\ \theta_{N+1} \\ \theta_{N+1} \\ \theta_{N+1} \\ \end{bmatrix} + \begin{bmatrix} \frac{K_1}{3h^2\Delta \tilde{z}_1} \\ \frac{8K_1}{3h^2\Delta \tilde{z}_1} \\ -\frac{K_2}{3h^2\Delta \tilde{z}_2} \\ \frac{8K_2}{3h^2\Delta \tilde{z}_2} \\ \end{bmatrix} \begin{bmatrix} \theta_{N-1} \\ \theta_{N+1} \\ \theta_{N+1} \\ \theta_{N+1} \\ \end{bmatrix} + \begin{bmatrix} \frac{9K_1}{3h^2\Delta \tilde{z}_1} \\ \frac{9K_2}{3h^2\Delta \tilde{z}_2} \\ \frac{9K_2}{3h^2\Delta \tilde{z}_3} \\ \end{bmatrix} \begin{bmatrix} \theta_{N+1} \\ \theta_{N+1} \\ \theta_{N+1} \\ \theta_{N+1} \\ \end{bmatrix} = 0 \tag{E.34} \]

\[ P \text{ and } Q \text{ are auxiliary matrix coefficients.} \]

Multiply line \( k=N \) (Eq. E.34) by \( A_{N-1} \) and line \( k=N-1 \) (Eq. E.38 and E.40) by \( P_N \) and subtract:

\[ P_N A_{N-1} \theta_{N-2} + A_N A_{N-1} \theta_{N-1} + B_N A_{N-1} \theta_N + C_N A_{N-1} \theta_{N+1} + Q_N A_{N-1} \theta_{N+2} = D_N A_{N-1} \]

\[ P_N A_{N-1} \theta_{N-2} + P_N B_{N-1} \theta_{N-1} + P_N C_{N-1} \theta_N = P_N D_{N-1} \]

\[ \begin{bmatrix} (A_N A_{N-1} - P_N B_{N-1}) \theta_{N-1} + (B_N A_{N-1} - P_N C_{N-1}) \theta_N + \\ (C_N A_{N-1}) \theta_{N+1} + (Q_N A_{N-1}) \theta_{N+2} = (D_N A_{N-1} - P_N D_{N-1}) \end{bmatrix} \tag{E.35} \]

Multiply line \( k=N \) (Eq. E.35) by \( C_{N+1} \) and line \( k=N+1 \) (Eq. E.42 and E.44) by \( Q_N = Q_N A_{N-1} \) and subtract:

\[ C_{N+1} (A_N A_{N-1} - P_N B_{N-1}) \theta_{N-1} + \\ C_{N+1} (B_N A_{N-1} - P_N C_{N-1}) \theta_N + \\ C_{N+1} C_N A_{N-1} \theta_{N+1} + C_{N+1} Q_N A_{N-1} \theta_{N+2} = C_{N+1} (D_N A_{N-1} - P_N D_{N-1}) \\ (Q_N A_{N-1}) A_{N+1} \theta_N + \\ (Q_N A_{N-1}) B_{N+1} \theta_{N+1} + (Q_N A_{N-1}) C_{N+1} \theta_{N+2} = (Q_N A_{N-1}) D_{N+1} \]

\[ \begin{bmatrix} (C_N A_{N-1}) \theta_{N+1} + (Q_N A_{N-1}) \theta_{N+2} = \tilde{D}_{N} \end{bmatrix} \tag{E.36} \]

**Interface adjacent**

At the snow-ice interface \( (k=N) \), \( \tilde{w}_{N} = 0 \), as \( \partial b_s/\partial t = 0 \) and \( \partial s_i/\partial t = 0 \).
\[ k = N - 1, \bar{w}_{N-1} > 0: \]
\[
\frac{\tilde{\theta}^{t+1}_{N-1} - \tilde{\theta}^{t-1}_{N-1}}{\Delta t} = \frac{K_i}{\rho_i c_p(h_i)^2} \left[ \frac{8\tilde{\theta}_{N-1}^{t+1} - 9\tilde{\theta}_{N-2}^{t+1} + \tilde{\theta}_{N-3}^{t+1}}{3\Delta \tilde{z}_i} - \frac{\tilde{\theta}_{N-1}^{t+1} - \tilde{\theta}_{N-2}^{t+1}}{\Delta \tilde{z}_i} \right] - \frac{\bar{w}_{N-1} \tilde{\theta}_{N-1}^{t+1} - \bar{w}_{N-2} \tilde{\theta}_{N-2}^{t+1}}{\Delta \tilde{z}_i} + \frac{h_i}{\rho_i c_p} \bar{R}_N^{t+1} \quad (E.37)
\]
\[
\left( \frac{4}{3} C_1 + C_2 \bar{w}_{N-1} \right) \frac{\tilde{\theta}_{N-2}^{t+1}}{A_{N-1}} + \left( -4C_1 - C_2 \bar{w}_{N-1} - 1 \right) \frac{\tilde{\theta}_{N-1}^{t+1}}{B_{N-1}} + \left( \frac{8}{3} C_1 \right) \frac{\tilde{\theta}_{N-1}^{t+1}}{C_{N-1}} = -\tilde{\theta}_{N-1}^{t+1} - C_4 \bar{R}_N^{t+1} \quad (E.38)
\]
\[ k = N - 1, \bar{w}_{N-1} < 0: \]
\[
\frac{\tilde{\theta}^{t+1}_{N-1} - \tilde{\theta}^{t-1}_{N-1}}{\Delta t} = \frac{K_i}{\rho_i c_p(h_i)^2} \left[ \frac{8\tilde{\theta}_{N-1}^{t+1} - 9\tilde{\theta}_{N-2}^{t+1} + \tilde{\theta}_{N-3}^{t+1}}{3\Delta \tilde{z}_i} - \frac{\tilde{\theta}_{N-1}^{t+1} - \tilde{\theta}_{N-2}^{t+1}}{\Delta \tilde{z}_i} \right] - \frac{\bar{w}_{N-1} \tilde{\theta}_{N-1}^{t+1} - \bar{w}_{N-2} \tilde{\theta}_{N-2}^{t+1}}{\Delta \tilde{z}_i} + \frac{h_i}{\rho_i c_p} \bar{R}_N^{t+1} \quad (E.39)
\]
\[
\left( \frac{4}{3} C_1 \right) \frac{\tilde{\theta}_{N-2}^{t+1}}{A_{N-1}} + \left( -4C_1 - C_2 \bar{w}_{N-1} - 1 \right) \frac{\tilde{\theta}_{N-1}^{t+1}}{B_{N-1}} + \left( \frac{8}{3} C_1 \right) \frac{\tilde{\theta}_{N-1}^{t+1}}{C_{N-1}} = -\tilde{\theta}_{N-1}^{t+1} - C_4 \bar{R}_N^{t+1} \quad (E.40)
\]
\[ k = N + 1, \bar{w}_{N+1} > 0: \]
\[
\frac{\tilde{\theta}^{t+1}_{N+1} - \tilde{\theta}^{t-1}_{N+1}}{\Delta t} = \frac{K_s}{\rho_s c_p(h_s)^2} \left[ \frac{\bar{w}_{N+2} \tilde{\theta}_{N+2}^{t+1} - \bar{w}_{N+1} \tilde{\theta}_{N+1}^{t+1}}{\Delta \tilde{z}_s} + \frac{h_s}{\rho_s c_p} \bar{R}_{N+1}^{t+1} \right] \quad (E.41)
\]
\[
\left( \frac{8}{3} C_1 + C_2 \bar{w}_{N} \right) \frac{h_s}{h_i} \frac{\tilde{\theta}_{N}^{t+1}}{A_{N+1}} + \left( -4C_1 - C_2 \bar{w}_{N+1} - 1 \right) \frac{\tilde{\theta}_{N+1}^{t+1}}{B_{N+1}} + \left( \frac{4}{3} C_1 \right) \frac{\tilde{\theta}_{N+2}^{t+1}}{C_{N+1}} = -\tilde{\theta}_{N+1}^{t+1} - C_4 \bar{R}_{N+1}^{t+1} \quad (E.42)
\]
\[ k = N + 1, \bar{w}_{N+1} < 0: \]
\[
\frac{\tilde{\theta}^{t+1}_{N+1} - \tilde{\theta}^{t-1}_{N+1}}{\Delta t} = \frac{K_s}{\rho_s c_p(h_s)^2} \left[ \frac{\bar{w}_{N+2} \tilde{\theta}_{N+2}^{t+1} - \bar{w}_{N+1} \tilde{\theta}_{N+1}^{t+1}}{\Delta \tilde{z}_s} + \frac{h_s}{\rho_s c_p} \bar{R}_{N+1}^{t+1} \right] \quad (E.43)
\]
\[
\left( \frac{8}{3} C_1 \right) \frac{h_s}{h_i} \frac{\tilde{\theta}_{N}^{t+1}}{A_{N+1}} + \left( -4C_1 - C_2 \bar{w}_{N+1} - 1 \right) \frac{\tilde{\theta}_{N+1}^{t+1}}{B_{N+1}} + \left( \frac{4}{3} C_1 \right) \frac{\tilde{\theta}_{N+2}^{t+1}}{C_{N+1}} = -\tilde{\theta}_{N+1}^{t+1} - C_4 \bar{R}_{N+1}^{t+1} \quad (E.44)
\]
Surface adjacent

\(k = N + M, \ \bar{w}_{N+M} > 0:\)

\[
\frac{\theta_{N+M}^{t+1} - \theta_{N+M}^t}{\Delta t} = \frac{K_s}{\rho_s c_p (h_s)^2} \frac{1}{\Delta z_s} \left[ \frac{8\theta_{N+M}^{t+1} - 9\theta_{N+M}^{t+1} + \theta_{N+M}^{t+1}}{3\Delta z_s} \right] - \frac{\bar{w}_{N+M}^t \theta_{N+M}^{t+1} - \bar{w}_{N+M}^{t+1} \theta_{N+M}^t}{\Delta z_s} + \frac{h_s}{\rho_s c_p} \bar{R}_{N+M}^t
\]

\(E.45\)

\[
\frac{4}{3} C_1 s + C_2 s \bar{w}_{N+M} + \frac{8}{3} C_1 s \theta_{N+M+1}^{t+1} = \frac{\theta_{N+M+1}^{t+1} - \theta_{N+M+1}^t}{\Delta z_s}
\]

\(AN_{N+M} = B_{N+M}\)

\(E.46\)

\(k = N + M, \ \bar{w}_{N+M} < 0:\)

\[
\frac{\theta_{N+M}^{t+1} - \theta_{N+M}^t}{\Delta t} = \frac{K_s}{\rho_s c_p (h_s)^2} \frac{1}{\Delta z_s} \left[ \frac{8\theta_{N+M}^{t+1} - 9\theta_{N+M}^{t+1} + \theta_{N+M}^{t+1}}{3\Delta z_s} \right] - \frac{\bar{w}_{N+M}^t \theta_{N+M}^{t+1} - \bar{w}_{N+M}^{t+1} \theta_{N+M}^t}{\Delta z_s} + \frac{h_s}{\rho_s c_p} \bar{R}_{N+M}^t
\]

\(E.47\)

\[
\frac{4}{3} C_1 s + C_2 s \bar{w}_{N+M} + \frac{8}{3} C_1 s \theta_{N+M+1}^{t+1} = \frac{\theta_{N+M+1}^{t+1} - \theta_{N+M+1}^t}{\Delta z_s}
\]

\(AN_{N+M} = B_{N+M}\)

\(E.48\)

Surface

Linearization of \((T^{t+1})^4\) about previous time step surface temperature \((T^t)\): Taylor series expansion:

\[
f(x) = \frac{f^0(x_0)}{0!} (x - x_0)^0 + \frac{f^1(x_0)}{1!} (x - x_0)^1 + \frac{f^2(x_0)}{2!} (x - x_0)^2 + \ldots
\]

\[F_{sw}^t = \epsilon \sigma T^4 = \epsilon \sigma (T_0^4 + 4T_0^3(T - T_0)) = -3\epsilon \sigma T_0^4 + 4\epsilon \sigma T_0^3 T \quad \text{where} \quad T_0 = T^t; \ T = T^{t+1}
\]

Atmospheric net heat flux at the surface:

\[
F_{net}^{t+1} = \epsilon (F_{sw}^t + \sigma (-3(T_{surf}^t)^4 + 4(T_{surf}^t)^3 T_{surf}^{t+1}^T) + (1 - \alpha)(1 - \tau) F_{net}^t
\]

\[
+ \rho_a c_p a |u_a|^{t+1} (T_{surf}^{t+1} - T_a^{t+1}) + \rho_a L_a C_{air} |u_a|^{t+1} (\rho_a L_a T_{surf}^t - \theta_{air}^{t+1})
\]

\(E.49\)
$k=N$, (ice surface), flux boundary condition:

\[ -\frac{K_1}{h_i} \frac{\theta_{N-2}^{t+1} - 9\theta_{N-1}^{t+1} + 8\theta_{N}^{t+1}}{3\Delta z_i} = h_i \frac{\theta_{N}^{t+1}}{K_1} (\theta_{N}^{t+1}) \]

\[ \frac{\theta_{N-2}^{t+1} - 9\theta_{N-1}^{t+1} + 8\theta_{N}^{t+1}}{3h_i \Delta z_i} = -h_i \frac{3h_i \Delta z_i}{K_1} \frac{\theta_{N}^{t+1}}{C_0} (\theta_{N}^{t+1}) \]  
\[ (E.50) \]

$k=N$, (ice surface), fixed temperature boundary condition:

\[ \frac{0}{A_N} \frac{\theta_{N-1}^{t+1}}{A_N} + \frac{1}{B_N} \frac{\theta_{N}^{t+1}}{B_N} = \theta_{t} - \mu S_i h_i \]  
\[ (E.51) \]

$k=N$, (surface), turbulent heat fluxes computed using bulk formulation:

\[ \frac{1}{P_N} \frac{\theta_{N-2}^{t+1} - 9\theta_{N-1}^{t+1} + \theta_{N}^{t+1}}{A_N} + \frac{B_N}{B_N} \frac{8 + C5_i (4\varepsilon(T_N^t)^3 + \rho_{\mu} c_{pa} C_{sh} |u_a|^{t+1})}{B_N} \theta_{N}^{t+1} = \]

\[ \frac{-C5_i h_i (\varepsilon F_{lw}^{t+1} - 3\varepsilon(T_N^t)^4 - \rho_{\mu} c_{pa} C_{sh} |u_a|^{t+1} T_{a}^{t+1}) + \rho_{\mu} L_a C_{lh} |u_a|^{t+1} (q_{sat}(T_N^t) - q_{a}^{t+1})}{D_N} \]  
\[ (E.52) \]

$k=N$, (surface), prescribed turbulent heat fluxes from observation:

\[ \frac{1}{P_N} \frac{\theta_{N-2}^{t+1} - 9\theta_{N-1}^{t+1} + \theta_{N}^{t+1}}{A_N} + \frac{B_N}{B_N} \frac{8 + C5_i (4\varepsilon(T_N^t)^3)}{B_N} \theta_{N}^{t+1} = \]

\[ \frac{-C5_i h_i (\varepsilon F_{lw}^{t+1} - 3\varepsilon(T_N^t)^4 + F_{sh}^{t+1} + (1 - \alpha^t)(1 - \tau) F_{sw}^{t} + F_{th}^{t+1})}{D_N} \]  
\[ (E.53) \]

Multiply line $k=N-1$ (Eq. E.38 and E.40) by the off-tridiagonal matrix element $P_N$ and line $k=N$ (Eq. E.50) by $A_{N-1}$ and subtract, cf. Eq. E.54. $\hat{A}_N$, $\hat{B}_N$, $\hat{D}_N$, are the new tridiagonal matrix elements for $k=N$. 

\[ P_N A_{N-1} \hat{\theta}_{N-2} + P_N B_{N-1} \hat{\theta}_{N-1} + P_N C_{N-1} \hat{\theta}_{N} = P_N D_{N-1} \]

\[ (P_N B_{N-1} - A_N A_{N-1}) \hat{\theta}_{N-1} + (P_N C_{N-1} - B_N A_{N-1}) \hat{\theta}_{N} = (P_N D_{N-1} - D_N A_{N-1}) \]  
\[ (E.54) \]
\[ k = N + M + 1, \text{ (snow surface), flux boundary condition:} \]
\[
- \frac{K_s}{h_s} \frac{\bar{\theta}_{N+M+1}^{t+1} - 9 \bar{\theta}_{N+M}^{t} + 8 \bar{\theta}_{N+M-1}^{t+1}}{3 \Delta z_s} = h_s F_{\text{net}}^{t+1} (\bar{\theta}_{N+M+1}^{t+1})
\]
\[ \bar{\theta}_{N+M-1}^{t+1} - 9 \bar{\theta}_{N+M}^{t} + 8 \bar{\theta}_{N+M+1}^{t+1} = -h_s \frac{3 h_s \Delta z_s}{K_s} F_{\text{net}}^{t+1} (\bar{\theta}_{N+M+1}^{t+1}) \quad (E.55) \]

\[ k = N + M + 1, \text{ (snow surface), fixed temperature boundary condition:} \]
\[
0 = \frac{\theta_{N+M+1}^{t+1}}{A_{N+M+1}} + \frac{1}{B_{N+M+1}} \frac{\bar{\theta}_{N+M+1}^{t+1}}{D_{N+M+1}} = \frac{\theta_{t}}{D_{N+M+1}} \quad (E.56)
\]

\[ k = N + M + 1, \text{ (surface), turbulent heat fluxes computed using bulk formulation:} \]
\[
\frac{1}{P_{N+M+1}} \frac{\bar{\theta}_{N+M+1}^{t+1}}{A_{N+M+1}} - \frac{9}{A_{N+M+1}} \bar{\theta}_{N+M}^{t+1} + \left(8 + C5_s \left(4 \epsilon \sigma (T_{N+M+1}^{t})^3 \right) + \rho_a c_{pa} C_{sh} |u_a|^{t+1} \right) \bar{\theta}_{N+M+1}^{t+1} = \]
\[
- C5_s h_s \left( \epsilon \bar{F}_{lw}^{t+1} - 3 \epsilon \sigma (T_{N+M+1}^{t})^3 \right) - \rho_a c_{pa} C_{sh} |u_a|^{t+1} \bar{T}_{t+1}^{t+1} + (1 - \alpha^t)(1 - \tau) \bar{F}_{sw}^{t} + \rho_a L_a C_{lw} |u_a|^{t+1} (\bar{u}_a(T_{N+M+1}^{t}) - \bar{u}_a^{t+1}) \quad (E.57)
\]

\[ k = N + M + 1, \text{ (surface), prescribed turbulent heat fluxes from observation:} \]
\[
\frac{1}{P_{N+M+1}} \frac{\bar{\theta}_{N+M+1}^{t+1}}{A_{N+M+1}} - \frac{9}{A_{N+M+1}} \bar{\theta}_{N+M}^{t+1} + \left(8 + C5_s \left(4 \epsilon \sigma (T_{N+M+1}^{t})^3 \right) \right) \bar{\theta}_{N+M+1}^{t+1} = \]
\[
- C5_s h_s \left( \epsilon \bar{F}_{lw}^{t+1} - 3 \epsilon \sigma (T_{N+M+1}^{t})^3 \right) \bar{F}_{sh}^{t+1} + (1 - \alpha^t)(1 - \tau) \bar{F}_{sw}^{t} + \bar{F}_{lw}^{t+1} \quad (E.58)
\]

Multiply line \( k = N + M \) (Eq. E.46 and E.48) by the off-tridiagonal matrix element \( P_{N+M+1} \) and line \( k = N + M + 1 \) (Eq. E.55) by \( A_{N+M} \) and subtract, cf. Eq. E.59. \( \Lambda_{N+M+1}, \ \bar{B}_{N+M+1}, \ \bar{D}_{N+M+1} \), are the new tridiagonal matrix elements for \( k = N + M + 1 \).

\[
P_{N+M+1} A_{N+M} \bar{\theta}_{N+M-1}^{t+1} + P_{N+M+1} B_{N+M} \bar{\theta}_{N+M}^{t+1} + P_{N+M+1} C_{N+M} \bar{\theta}_{N+M+1}^{t+1} = \]
\[
P_{N+M+1} D_{N+M} \quad \Lambda_{N+M+1}
\]
\[
P_{N+M+1} A_{N+M} \bar{\theta}_{N+M-1}^{t+1} + A_{N+M+1} A_{N+M} \bar{\theta}_{N+M}^{t+1} + B_{N+M+1} A_{N+M} \bar{\theta}_{N+M+1}^{t+1} = \]
\[
D_{N+M+1} A_{N+M} \quad \bar{B}_{N+M+1}
\]
\[
(P_{N+M+1} B_{N+M} - A_{N+M+1} A_{N+M}) \bar{\theta}_{N+M}^{t+1} + (P_{N+M+1} C_{N+M} - B_{N+M+1} A_{N+M}) \bar{\theta}_{N+M+1}^{t+1} = \]
\[
\Lambda_{N+M+1} \quad \bar{D}_{N+M+1} \quad (E.59)
\]
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BIBLIOGRAPHY


Curriculum Vitae

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Education and professional training

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<th>Year</th>
<th>Institution</th>
<th>Location</th>
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<td>06/99 – 05/03</td>
<td>ETH Zürich, Switzerland, Inst. for Atmospheric and Climate Science</td>
<td>Zürich</td>
<td>Doctoral studies in Climatology.</td>
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<tr>
<td>10/95 – 04/99</td>
<td>ETH Zürich, Switzerland, Dept. of Earth Sciences</td>
<td>Zürich</td>
<td>Studies in Climatology, Glaciology, Hydrology; Graduation: Diplom.</td>
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<tr>
<td>10/93 – 09/95</td>
<td>J.W. Goethe-Universität Frankfurt, Germany</td>
<td>Frankfurt</td>
<td>Studies in Geography, Meteorology, Geology; Graduation: Vordiplom.</td>
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<tr>
<td>02/92 – 05/93</td>
<td>C. G. Carus Institut, Chemical Laboratory</td>
<td>Niefern-Oschelbronn</td>
<td>Civil service.</td>
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<td>08/78 – 06/91</td>
<td>Goetheschule, Freie Waldorf-Schule</td>
<td>Pforzheim</td>
<td>Graduation: Abitur.</td>
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Presentations at conferences and workshops

Alliance for Global Sustainability (AGS) 2000, ETH, Zürich: Workshop on Global Climate Change.  
International Glaciological Society (IGS), 2000, UAF, Fairbanks: Symposium on sea ice and its interactions with the ocean, atmosphere and biosphere.  
Alliance for Global Sustainability (AGS), 1999, MIT, Cambridge: Workshop on Global Climate Modeling.