Doctoral Thesis

Finite-difference time-domain modeling of ground-penetrating radar antenna systems

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Finite-Difference Time-Domain Modeling of Ground-Penetrating Radar Antenna Systems

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH
for the degree of

Doctor of Natural Sciences

presented by

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Zusammenfassung


Zunächst überprüfe ich die Gültigkeit des Algorithmus durch Vergleich von Eingangsimpedanzen modellierter und reeller Bow-Tie-Antennen mit verschiedenen Spreizwinkeln. Anschließend untersuche ich die Strahlungscharakteristiken von An-


ABSTRACT

tions demonstrate the variations of near- and intermediate-field radiation patterns generated by realistic antennas. Commonly used asymptotic far-field patterns of infinitesimal electric dipoles are only poor approximations for the illumination of the shallow subsurface by GPR antennas.

I then apply the simulation tool to an investigation of GPR antennas located above realistic earth models that incorporate stochastic models of topographic roughness of the air–soil interface and small-scale subsurface heterogeneities of the electrical material properties permittivity and conductivity. The heterogeneities are characterized by scale-invariant fractal fluctuations of the parameters. The resulting energy radiation patterns and input impedances reveal that topographic roughness affects the coupling of the antenna to the ground, whereas heterogeneities in the subsurface influence antenna radiation through scattering and absorption along the propagation path.

Finally, I investigate characteristics of resistively loaded antennas located in free space and above various half-spaces. For this purpose, I use a subcell algorithm that enables thin material sheets to be FDTD modeled. Three basic types of antenna panels are considered: perfectly electrical conductors (PEC’s), panels with a constant finite conductivity, and panels with optimized conductivity profiles of the so-called Wu-King type. To cover a broad range of antenna designs, the antenna geometries are chosen to be quasi-linear bow-tie antennas with a flare angle of 5° and planar bow-tie antennas with a flare angle of 90°. The resultant input impedances, radiation patterns, radiated pulse shapes, and surface-charge distributions on the antenna panels demonstrate that antennas with Wu-King conductivity profiles are remarkably insensitive to the operating environment. Furthermore, the quality of the radiated signal is superior to those of the PEC and constant finite-conductivity antennas, primarily due to the effective suppression of antenna ringing; the current distribution of Wu-King-type antennas resembles that of a quasi-infinitesimal electric dipole. Antennas with broad planar panels are found to be more sensitive to the presence of a dielectric half-space than those with quasi-linear shapes, mostly because of the more pronounced capacitive coupling that affects the planar structures. A drawback of the Wu-King-loaded antennas is the loss in radiation efficiency. The maximum energy fluxes radiated into the ground from Wu-King antennas are found to be one-order-of-magnitude lower than from antennas with PEC terminals.
List of Acronyms

ABC .................................................. absorbing boundary condition
BE .................................................... boundary element
FD ..................................................... finite difference
FDTD ................................................... finite-difference time-domain
FE ..................................................... finite element
FI ....................................................... finite integral
GPML ................................................. generalized perfectly matched layer
GPR ..................................................... ground-penetrating radar
MoM .................................................... method of moments
PEC ..................................................... perfect electrical conductor
PML ..................................................... perfectly matched layer
UPML .................................................. uniaxial perfectly matched layer
Chapter 1

Introduction

1.1 General remarks and historical aspects

Ground-penetrating radar (GPR) is a remote-sensing technique that employs electromagnetic waves for investigating the subsurface. As indicated by its name, the frequency range of GPR lies within the radar band, roughly between 10 MHz and 5 GHz. GPR surveys can be conducted from the earth’s surface, between boreholes and the earth’s surface, within the ground between boreholes, and from above the ground using aircraft or helicopters. For common surface-to-surface GPR applications, a transmitter antenna placed on or near the air–soil interface sends electromagnetic pulses into the ground. These pulses are partially reflected at changes in electrical material properties in the shallow subsurface. The reflected electromagnetic signals are recorded by a nearby receiver antenna ready for processing and display. Surface GPR is effectively an electromagnetic echo-sounding technique.

The first reported GPR system is documented in a German patent by Leimbach and Löwy (1910a). It consisted of an array of borehole antennas (Figure 1.1). This technique aimed at measuring the radiated amplitudes in order to outline the extent of electrically conductive material. In the same year, Leimbach and Löwy (1910b) also filed a patent for a surface GPR technique for the detection of subsurface reflecting interfaces (Figure 1.2).

In both original GPR techniques, the antennas operated in a continuous-wave mode. Techniques published in subsequent years were based on continuous-wave excitation and such wave phenomena as interference, absorption, reflection, and diffraction of the radiated waves. The first technique based on pulsed excitation of transmitter antennas was presented in a patent by Hülsenbeck (1926). Since then, most GPR systems and techniques have used electromagnetic pulses (e.g., Daniels,
Figure 1.1: Sketch of the first "GPR system" comprising an array of borehole antennas. This is a copy of the original sketch in the Leimbach and Löwy (1910a) German patent.

Such pulse techniques were extensively developed for probing permafrost, fresh water and salt deposits. Interest in GPR received a boost in the early 1970's because of its applications in the Apollo lunar expeditions. The major reason for choosing GPR was that it could be applied remotely from spacecrafts and other vehicles without direct contact to the ground.

GPR has become a widely used remote sensing technique in the earth, environmental, and civil engineering sciences. Moreover, it has been adopted for use in a variety of other fields, such as hydrology, archeology, and criminology. Applications of this technique are now multifold, ranging from geological surveying to forensic investigations (e.g., GPR 1994–2002). Many of these applications require special antenna designs and experimental configurations for the data acquisition. Common surface GPR antenna designs range from wire dipoles to planar bow-tie and three-dimensional antennas. Shielding the antennas as well as resistively loading are design features that affect the character of the radiated signal. In addition the

Figure 1.2: Sketch of the first surface GPR antenna system for detecting a subsurface reflecting interface. This is a copy of the original sketch in the (Leimbach and Löwy, 1910b) German patent.
radiative properties of antennas may be influenced by nearby dielectric interfaces, such as the earth’s surface. In close proximity to the ground, typical antenna characteristics (e.g., input impedance and radiation patterns) undergo significant changes compared to those of the antenna in free space. Moreover, typical GPR surveys environmental conditions are generally neither ideal nor uniform, such that GPR antennas have to cope with changing soil conditions, topographic roughness of the air–soil interface, and surface reflectors.

An inherent problem in designing surface GPR antennas is the near impossibility of determining antenna characteristics through laboratory measurements. Laboratory experiments for designing radar antennas are usually performed under free-space conditions; they cannot account for complications associated with the earth’s surface or heterogeneities in the shallow subsurface. Analytical solutions are only available for unrealistic idealized conditions, such as infinitesimal dipole sources located on or above homogeneous dielectric half-spaces. Numerical modeling alone offers the means to analyze the radiative properties of GPR antennas under realistic conditions, to understand better the generation and recording of GPR data, and to improve data quality.

1.2 Numerical modeling

Since the 1960’s, computational power has been growing rapidly, thus fostering the development of advanced numerical methods for a wide variety of scientific disciplines. In computational electromagnetics, there are numerous methods and approaches for solving Maxwell’s equations or equations related to Maxwell theory. Many of these methods can be classified as either domain or boundary techniques. Both technique classes involve a series expansion of a function \( f \), where \( f \) typically is a vector field (Hafner, 1999):

\[
f = \sum_{k=1}^{K} A_k f_k. \tag{1.1}
\]

In this formalism, \( f_k \) denotes the basis functions and \( A_k \) the coefficients. For a domain technique, the computational domain \( D \) is discretized and the solution of the field equations in \( D \) have to be approximated numerically. Along a given boundary \( \partial D \), the series expansion (1.1) has to fulfill analytically the corresponding boundary conditions. For a boundary technique, on the other hand, the boundary \( \partial D \) of the domain \( D \) is discretized and the boundary conditions are solved numerically, whereas the expansion of \( f \) has to fulfill analytically the given field equations in \( D \). Some
well-known examples of domain techniques are the finite-element (FE) method, the method of moments (MoM), and the finite-difference (FD) method. An example of a boundary technique is the boundary element (BE) method.

In the FE method, the model domain is divided into several subdomains, the so-called finite elements (e.g., Silvester and Ferrari, 1996). In each element, the field function \( f \) is approximated by a set of linear basis functions \( f_k \). The most common geometrical shapes for the finite elements are triangular elements in 2D and tetrahedral elements in 3D, which allow for highly accurate approximations of complex curved surfaces and arbitrary volumes. Like other pure domain techniques, the FE method does not explicitly account for open infinite domains; it requires the use of absorbing boundary conditions (ABC’s) at the model edges. Conversely, in the BE method, it is the boundary of the model domain that is discretized in subdomains (finite elements), such that problems with ABC’s do not occur (Hafner, 1999).

The MoM involves a series expansion of the currents (Harrington, 1993), rather than of the electromagnetic fields, which is employed in most other methods. The fields are then calculated by integration of the current expressions. For this reason, integral formulations and Green’s functions are commonly used to obtain solutions for the fields. Since the MoM involves expanding the currents and not the fields, it is advantageous for open domains: the fields may extend to infinity while the currents are restricted to a finite domain. Although it is a domain technique in general, the MoM becomes a boundary technique when only perfect electrical conductors (PEC’s) are involved. In this case, only surface currents are present in the model, so that the discretized domain "collapses" to the boundary of the model (Hafner, 1999). A popular implementation of the MoM is the Numerical Electromagnetics Code, better known as NEC (e.g., Balanis, 1997). This code is designed for analyzing the interaction of electromagnetic waves with arbitrary structures consisting of conducting wires and surfaces.

The FD method is amongst the oldest and most popular numerical techniques. It is a domain technique that involves the discretization of the differential operators in Maxwell’s equations or the derived differential equations. A similar technique is the finite-integral (FI) method, which discretizes the integral form of Maxwell’s equations and yields codes identical to those of the FD method (Hafner, 1999). Originally, the FD method was mostly implemented in the frequency domain, but is now widely used in the time domain because of the increased memory available in modern computers. In the finite-difference time-domain (FDTD) method of Yee
CHAPTER 1. INTRODUCTION

Figure 1.3: Discretization scheme of the Yee algorithm (Yee, 1966). The six components of the electromagnetic field are discretized in a staggered grid and referenced by the spatial indices $i$, $j$, and $k$ in the $x$, $y$, and $z$ directions, respectively. In addition to the spatial staggering, the components of the magnetic field are also offset in time from those of the electric field by a half time-step.

(1966), the differential forms of Maxwell’s equations are discretized in a staggered grid, where the electric and magnetic field components are offset both in space and time by half discretization intervals. This results in a discretization of the model space known as Yee cells (Figure 1.3). As explained in Section 1.4, this arrangement automatically fulfills boundary conditions for PEC’s. Field components throughout the discretized model space are obtained by update equations that express the field components of the new time-steps in terms of field components computed during previous time-steps.

In contrast to other methods mentioned above, the FDTD method does not require complicated mesh generation and administration, construction of large systems of linear equations, or the formulation of potentially complex integral equations and model-specific Green’s functions (Taflove and Hagness, 2000). Therefore, relatively little analytical work is needed and simple versions of the method are quite easy to program. Tradeoffs, on the other hand, include the need for a large amount of random access memory, ABC’s to model open “infinite”spaces (Figure 1.4) and problems of grid design for complicated non-orthogonal geometrical structures (e.g., Jürgens et al., 1992).

With increasingly powerful computers, the FDTD method has become ever more popular due to its versatility and conceptually simple approach. Since Yee’s seminal publication in 1966, a very large number of FDTD-related studies has been published. Figure 1.5 illustrates this development from 1966 to 1996. The FDTD method has been applied to a wide variety of electromagnetic problems, ranging from simple radiation and scattering investigations to the design of high-speed electronic circuits and quantum-optical systems such as lasers. As an example of military
Figure 1.4: Snapshots displaying the $E_x$ component of an electromagnetic field in vertical slices of the computational volume without (top) and with (bottom) ABC's. Without ABC’s, the waves radiated from an infinitesimal electric dipole are reflected at the boundary of the volume and the wavefronts are clearly deformed. Conversely, with ABC’s, no reflections are visible and the propagation of the wavefronts is hardly affected by the boundaries.

Figure 1.5: Development of FDTD-related publications since the key paper of Yee (1966). Adapted from Taflove (1998).
technology applications, Figure 1.6 shows an impinging electromagnetic wave on a radar-guided missile, and Figure 1.7 shows an example from the broad range of bioelectromagnetic applications with growing social relevance. It displays the specific absorption rate of electromagnetic waves within a human head as a result of a cell phone being placed near an ear at an angle of 30°. The head model is based on magnetic resonance imaging. Taflove (1998) and Taflove and Hagness (2000) provide detailed overviews of the state-of-the-art FDTD method with diverse applications in computational electrodynamics.

Figure 1.7: Specific absorption rate of the electromagnetic field radiated from a cell phone adjacent to a human head. Adapted from Taflove (1998).
Because of its flexibility and simplicity, the FDTD method is favored for the modeling of antennas in general and GPR antennas in particular. Complex antenna designs and the simulation of highly heterogeneous environments can be accommodated in the FDTD method. Moreover, the basic output data of FDTD modeling (i.e., the electromagnetic field components for all time-steps of the simulation) in combination with common visualization software deliver vivid illustrations of the physics governing the propagation of electromagnetic waves. It is for these reasons that I have chosen the FDTD method for the work presented in this thesis.

1.3 Absorbing boundary conditions

As outlined above, the FD method requires special treatment of the model-space boundaries. To update the field components at a certain point in the FD lattice, knowledge of the neighboring field components is required. This is not fulfilled at the model boundaries. If the model space is simply truncated by, for example, Dirichlet boundary conditions, the waves at the boundary are totally reflected. To simulate an "open" domain, ABC's that effectively reduce unwanted reflections from the boundary of the model space have to be implemented. Such boundary conditions can basically be classified as either analytical or material ABC's (Taflove, 1998).

Analytical ABC's are based on differential equations that permit the wave to propagate only in one direction. These one-way wave equations and associated discretized one-way operators are the basis of, for example, Engquist and Majda's (1977) and Mur's (1981) ABC's. There exist a number of other approaches for determining analytical ABC's and diverse grid termination techniques using various differential operators. Many of these ABC's only perform well under a narrow range of angles of incidence (usually around normal incidence) of the impinging waves. The success of material ABC's during the last decade has notably reduced the importance and popularity of analytical ABC's.

The concept of material ABC's is to surround the computational domain by absorbing material. Similar to analytical ABC's, early material ABC's only performed well for impinging waves close to normal incidence. In the seminal work of Bérenger (1994), a new type of material ABC's was introduced, referred to as the perfectly matched layer (PML). The PML matches the impedance of the impinging waves for any frequency and any angle of incidence. In theory (i.e., in the "continuous world"), no reflections occur at the boundary of the model space. In practice, however, because of the non-continuous discretized nature of the model space, some
boundary reflections are always present. This reflected energy can, in general, be minimized by a suitable choice of the PML parameters. The original PML technique (Bérenger, 1994, 1996) was developed exclusively for lossless media and propagating waves, but has been improved and extended by many authors. A material ABC that also accommodates lossy media and evanescent waves is the generalized perfectly matched layer (GPML) technique presented by Fang and Wu (1996).

PML's are pure mathematical constructs that are not physically realizable. The original PML technique, for example, is based on a split-field formulation of Maxwell's equations, whereas other PML approaches are based on more compact stretched-coordinate formulations or combinations of both formulations, as is the case for the GPML technique (cf. Appendix of Chapter 2).

A more physical approach is the uniaxial PML (UPML), which does not involve the splitting of fields. In this formulation (e.g., Gedney, 1996), the absorbing material is a uniaxial anisotropic material involving permittivity and permeability tensors. Interpretations of PML's as physically realizable media with characteristics similar to the original PML and attempts to physically realize a UPML have been reported (Taflove, 1998; Taflove and Hagness, 2000).

1.4 FDTD antenna modeling

Although the FDTD method has been used to solve electromagnetic scattering problems since the 1980's, the first studies of antenna radiation modeling were published only in the early 1990's. In their pioneering work, Maloney et al. (1990) used an FDTD approach to simulate cylindrical and conical monopole antennas fed by a transmission line. Katz et al. (1991) modeled various radiators such as waveguides, flared horns and parabolic reflectors, and Tirkas and Balanis (1992) modeled monopoles, a waveguide aperture antenna, and a pyramidal horn antenna.

Early GPR-related studies were carried out by Moghaddam et al. (1991) using an infinitesimal electric dipole transmitter in a 2D FDTD grid. Fully 3D simulations of common GPR antennas have been published by Bourgeois and Smith (1996, 1998), Roberts and Daniels (1997), and Holliger and Bergmann (1998). Bourgeois and Smith (1996, 1998) modeled a GPR antenna system designed for detecting buried pipes and land mines (Figure 1.8). The antenna models consisted of bow-tie antennas and wire-dipoles fed by 1D transmission lines. Both antenna types were shielded and the bow-tie antennas were loaded at the outer edges with resistors connected to the metal shield. Roberts and Daniels (1997) simulated the near-
Figure 1.8: Snapshots of the electromagnetic field radiated from a land mine sensor for (a) soil alone, (b) sensor centered over the target, (c) sensor centered over the right edge of the target. From Bourgeois and Smith (1998).
field radiation of unshielded bow-tie antennas fed by an explicitly discretized 3D coaxial transmission line, and Holliger and Bergmann (1998) modeled GPR radiation patterns using a FDTD subcell model for the implementation of wire-dipole antennas (Figure 1.9). A common feature of all previous antenna modeling approaches is that they use the PEC approximation for the metal parts of the GPR antenna system.

The PEC approximation is the simplest and most efficient way to simulate good conductors such as metals. In this approach, the conductivity $\sigma$ of the metallic conductors is assumed to be infinitely high. Therefore, the electric field components parallel to the metal structures have to vanish in order not to produce infinitely high current densities. This is accomplished by simply setting to zero the corresponding electric field components. Similarly, the normal component of the magnetic field vanishes at a PEC surface (e.g., Jackson, 1975). Treatment of PEC’s in the FDTD lattice introduced by Yee (1966) is simple. Ordering the field components as shown in Figure 1.3 automatically fulfills the PEC boundary conditions. For example, if a perfectly conducting surface lies in the $x$-$y$ plane, the tangential electric field components $E_x$ and $E_y$ are set to zero. Thus, the normal component $H_z$ of the magnetic field automatically vanishes, since it only depends on the $E_x$ and $E_y$ components in the FDTD update equations outlined in Chapter 2.

For this thesis, I have developed a 3D FDTD simulation tool for the modeling of GPR antenna systems. To investigate the effects of antenna design on the radiation
characteristics of such systems, a wide variety of antenna designs can be accommodated by this simulation software. The basic antenna types are wire dipoles and bow-tie antennas with arbitrary flare angles. All antennas are fed by a 1D transmission line model, which transports the excitation voltage pulse to the antenna terminal, where it is radiated.\footnote{In a true GPR antenna system, the transmission line is usually replaced by a generator unit located on the input terminals.} The effect of shielding can be simulated by computationally surrounding the antenna with a metal box that can be filled with absorbing material to suppress resonances of the antenna-shield system. To reduce further antenna ringing (i.e., reflections of currents at the edges of the antenna arms) the antennas can be resistively loaded. With the implementation of subcell algorithms for the modeling of thin material sheets (Maloney and Smith, 1992a), special resistivity profiles (Wu and King, 1965) can be obtained that practically eliminate antenna end reflections.

For a closer look at GPR antenna design, the effects of the air-soil interface and varying conditions in the shallow subsurface have to be considered. In general, the characteristics of antennas radiating in close proximity to the earth differ fundamentally from those of antennas radiating in free space. For detailed investigations of the influence of the earth on the antenna characteristics, the simulation tool allows a wide variety of earth models with different permittivities and electrical conductivities to be modeled. To enhance the degree of realism of the models, topographic roughness of the air-soil interface as well as fluctuations of the electrical material parameters of the ground can be accommodated.

1.5 Goals of this thesis

This thesis pursues two closely related goals. One goal is the development of a flexible, versatile and accurate FDTD simulation tool for the modeling of GPR antenna systems. The second goal is the investigation of antenna-earth interactions.

The FDTD simulation tool incorporates state-of-the-art GPML absorbing boundary conditions (Fang and Wu, 1996) and, for an increase in accuracy, the possibility of employing local grid-refinements within the model space. To account for geometrical structures much smaller than the grid size of the model space, subcell algorithms have been added to the core FDTD algorithm. The basic aspects of this simulation tool are presented in Chapter 2, which includes descriptions of how bowtie antennas are modeled and how the antenna terminals are fed by a transmission
line. This simulation tool is tested for different antenna designs and for a variety of half-space models. Update equations for the GPML boundary conditions, as well as the principles of the subgridding algorithm are presented in the Appendices to Chapter 2.

The influence of variable surface and subsurface conditions on GPR antenna radiation is investigated in Chapter 3. The heterogeneous half-space models are characterized by fractal fluctuations of the topographic relief, permittivity and electrical conductivity.

A combined study of antenna design and ground effects relevant to GPR surveying is presented in Chapter 4. In this chapter, the influence of resistive loading is tested on quasi-linear wire dipoles and planar bow-tie antennas with a large flare angle. Resistive loading is modeled using a subcell algorithm for the modeling of thin resistive sheets (Maloney and Smith, 1992a).

Chapters 2 and 3 have been published or accepted for publication in *Geophysics*. Chapter 4 is currently under review in the same journal.
Chapter 2

A Finite-Difference Time-Domain Simulation Tool for Ground-Penetrating Radar Antennas

Bernhard Lampe, Klaus Holliger, and Alan G. Green

*Geophysics, 68, 971–987 (2003)*

2.1 Abstract

The generation and recording of electromagnetic waves by typical ground-penetrating radar (GPR) systems are complex phenomena. To investigate the characteristics of typical GPR antennas operating in diverse environments, we have developed a versatile and efficient simulation tool. It is based on a finite-difference time-domain (FDTD) approximation of Maxwell's equations that lets one simulate the radiation characteristics of a wide variety of typical surface GPR antenna systems. The accuracy of the algorithm is benchmarked and validated with respect to laboratory measurements for comparable antenna systems. Computed radiation patterns demonstrate that the illumination of the subsurface in the near- to intermediate-field range varies significantly according to how the antenna is designed. Our models show the effects of varying the shapes of the antennas, adding
shielding (metal box with and without absorbing material and with and without resistive loading), adding a receiver antenna, and changing the soil conditions. Given the flexibility of this modeling software, we anticipate that it will be helpful in designing GPR surveys and new GPR systems with arbitrary planar structures. It will also be useful in the interpretation of certain GPR data sets.

2.2 Introduction

A detailed physical understanding of electromagnetic (EM) waves radiated from antennas placed on the earth's surface is critical for the further development of ground-penetrating radar (GPR) acquisition systems and for correct interpretations of GPR data. To date, most common GPR processing and imaging techniques are adopted from corresponding methods developed in reflection seismology. Although there are many similarities between GPR and seismic reflection data, several key assumptions of reflection seismic processing and imaging, such as isotropic source radiation and plane-wave far-field approximations, are not valid in most GPR applications. Of particular importance are the largely unexplored effects that the complexly structured transmitter and receiver antennas have on recorded GPR data. This highlights the importance of improving our knowledge of the near- and intermediate-field radiation characteristics of standard GPR antenna systems. Numerical modeling studies that simulate the radiation characteristics of a wide variety of GPR acquisition systems are an appropriate means to achieve this goal.

By synthesizing transient waveforms for infinitesimal dipole elements from exact steady-state equations and then superposing the solutions in a weighted fashion, Arcone (1995) was one of the first to simulate realistic far-field radiation patterns for finite-length GPR antennas. Subsequently, Bourgeois and Smith (1996) and Roberts and Daniels (1997) presented finite-difference time-domain (FDTD) approximations for the radiation of GPR bow-tie antennas fed by transmission lines. Holliger and Bergmann (1998) introduced an efficient FDTD approach for modeling the near-field radiation of a GPR antenna using a center-fed, thin-wire antenna model. These studies achieved reasonable agreement between modeled and measured antenna characteristics. However, they were limited to rather specific GPR antenna systems.

In this contribution, we extend and complement the earlier work by presenting a highly versatile numerical tool that efficiently and accurately simulates a wide variety of surface GPR antenna systems. The antenna types range from quasi thin-wire
antennas to bow-tie antennas with arbitrary flare angles. Extensions to antennas with arbitrary planar structures, bow-tie antennas with continuous resistive loading, and thin-wire antennas can readily be accommodated by the scheme. We begin by describing pertinent aspects of the algorithm and testing its validity in the far-field (for which analytic solutions exist) and with respect to laboratory measurements.

We use the new simulation software to determine the near- and intermediate-field radiation characteristics of various realistic GPR antenna systems located above different half-space models. For the typical earth scientist or engineer interested in investigating the shallow subsurface, it is important to understand these characteristics for diverse geological environments. In an attempt to provide some of this information, we use our new software tool to produce snapshots of the radiated electric fields and amplitude radiation patterns for unshielded antennas with different flare angles and antennas shielded by an isolated empty metallic box, by a metallic box containing different types of absorbing dielectric material and by an empty metallic box connected to the antenna by loading resistors. We also examine the influence of placing a receiver antenna close to the transmitter antenna. Finally, we analyze the responses of some of these antenna configurations to changing subsurface conditions (see also Compton et al., 1987; Arcone, 1995; Roberts and Daniels, 1997).

2.3 Methodology

2.3.1 FDTD scheme

Our numerical approach is based on an $O(2, 2)$-accurate FDTD approximation of Maxwell's equations in three dimensions. The finite-difference grid is staggered both in space and in time according to the original algorithm proposed by Yee (1966). In 3D Cartesian coordinates, this results in the following FDTD update equations for the $x$ component of the electric ($E$) and magnetic ($H$) fields:

\[
E_x^{n+1}(i+\frac{1}{2},j,k) = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_x^{n-1}(i+\frac{1}{2},j,k) + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)} (E_x^{n}(i+\frac{1}{2},j,k) - E_x^{n}(i+\frac{1}{2},j-\frac{1}{2},k)) \\
\times \left[ \frac{H_{z}^{n}(i+\frac{1}{2},j+\frac{1}{2},k)}{\Delta y} - H_{z}^{n}(i+\frac{1}{2},j-\frac{1}{2},k) - \frac{H_{y}^{n}(i+\frac{1}{2},j,k+\frac{1}{2}) - H_{y}^{n}(i+\frac{1}{2},j,k-\frac{1}{2})}{\Delta z} \right],
\]  

(2.1)
\[ H_{x(i,j+1/2,k+1)}^{n+1/2} = H_{x(i,j+1/2,k+1)}^n + \frac{\Delta t}{\mu} \times \left[ \frac{E_{y(i,j+1/2,k+1)}^n - E_{y(i,j+1/2,k)}^n}{\Delta z} - \frac{E_{z(i,j+1,k+1/2)}^n - E_{z(i,j+1,k)}^n}{\Delta y} \right], \quad (2.2) \]

where \( \Delta x, \Delta y, \Delta z \) are the spatial increments; \( i, j, k \) are the corresponding spatial indices; \( \Delta t \) is the time discretization interval; \( n \) is the corresponding time index; \( \epsilon \) is the permittivity; \( \mu \) is the magnetic permeability; and \( \sigma \) is the electrical conductivity. Surficial geological materials are generally assumed to be nonmagnetic, such that \( \mu = \mu_0 \) for our simulations. In the notation of equations (2.1) and (2.2), \( E_x^n(i+1/2,j,k) \) denotes the \( x \) component of the electric field at time \( n\Delta t \) and location \( (x = (i + 1/2)\Delta x, y = j\Delta y, z = k\Delta z) \) of the modeled physical volume. There are analogous expressions for the \( y \) and \( z \) components of the \( E \)- and \( H \)-fields.

Numerical stability of the FDTD scheme is ensured when \( \Delta x, \Delta y, \Delta z, \) and \( \Delta t \) are chosen to satisfy the Courant stability criterion:

\[ \Delta t = C_n \frac{1}{c \sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}}, \quad (2.3) \]

where \( c \) is the maximum propagation velocity present in the grid and the coefficient \( C_n \leq 1 \) is the Courant number (e.g., Taflove and Hagness, 2000). The accumulation of numerical dispersion errors needs to be controlled by choosing a sufficiently fine grid spacing with respect to the smallest relevant wavelength present in the source signal. For our \( O(2,2) \)-accurate FDTD scheme, a sampling density of about ten gridpoints per minimum wavelength is generally considered appropriate Bergmann et al. (1996, 1998).

The computational volume is surrounded by a general perfectly matched layer (GPML), a highly effective absorbing boundary condition proposed by Fang and Wu (1996). This boundary condition achieves near-perfect absorption of propagating and evanescent waves of any frequency and any angle of incidence in both lossless and lossy media. The implementation of these absorbing boundaries allows us to simulate a quasi-infinite model space (i.e., without notable reflections from the model edges). The update equations for the electric and magnetic fields in the GPML are given in the Appendix, together with suitable parameters for the absorbing boundary condition.

### 2.3.2 Subgridding

The simulation of GPR antenna systems involves the accurate discretization of small, intricate antenna structures. This will usually require a finer grid spacing than that
needed by the stability and grid dispersion criteria discussed above. Moreover, parts of the model space may be characterized by unusually high permittivities and correspondingly small wavelengths. Using a uniformly fine grid to accommodate such local features is computationally inefficient and may severely limit the model size and degree of realism that can be considered. This problem can be alleviated by using subgridding techniques, which allow local refinements of the spatial discretization Chevalier et al. (1997); Okoniewski et al. (1997); Robertsson and Holliger (1997).

The subgridding algorithm implemented here is adopted from that proposed by Chevalier et al. (1997). It permits dielectric and/or conducting material to traverse the boundaries between the coarse and the fine grids such that it can be used virtually anywhere in the model space. The algorithm requires an odd-integer refinement factor of the grid spacing (i.e., ratio of coarse to fine grid spacing). We implement this algorithm for a refinement factor of three. The resulting finely discretized local grid is anchored in the coarse main grid by colocated electric and magnetic fields. Figure 2.1 shows a local grid embedded in a slice of the main grid in the \( x-y \) plane. Upper- and lowercase letters denote the electromagnetic components in the main and local grid, respectively. The location of the local grid is chosen so that only tangential magnetic field components lie on its boundary. This facilitates the treatment of the boundary conditions when nonmagnetic structures cross its boundaries.

The electric and magnetic fields of the main and local grids are updated separately by applying Yee's 1966 algorithm for each grid. The field components along the boundaries must be treated in a different manner. The colocated \( H \)-field components of the main grid are interpolated in time with a second-order Taylor series expansion to give the corresponding components of the local grid for every time-step (i.e., \( n - \frac{1}{6}, n + \frac{1}{6}, n + \frac{3}{6} \)). The non-colocated local-grid \( H \)-field components on the interface are then calculated from the colocated \( H \)-field components via linear spatial interpolation.

A refinement of the colocated main-grid \( H \)-field components involves replacing them by the corresponding local-grid components at the end of each cycle. To minimize artifacts resulting from mismatched field components along the boundary between the two grid regions, field components in the local grid adjacent to the boundary are set equal to a weighted superposition of the solutions for the main and local grids. Further details on this approach are provided in the Appendix.
2.3.3 Antenna model

In the following, we consider a basic bow-tie antenna that may be unshielded or shielded with a metal casing (Figure 2.2). The metal parts of the antennas and shielding are simulated by perfect electrical conductors, such that all tangential electric field components along the metal surfaces are set to zero. The antenna panels are approximated with a stair-stepped representation of the slanting edges. The antenna is fed by a 1D transmission line Maloney et al. (1994) attached to the antenna terminals. This approximation faithfully simulates coaxial cables or parallel wire transmission lines. Assuming only transverse EM modes propagating along the transmission line, the finite-difference approximations for the current \( I \) and voltage \( V \) are given by

\[
I_{n+1}^{(k' + \frac{1}{2})} = I_{n-1}^{(k' + \frac{1}{2})} - \frac{1}{Z_0} \frac{c \Delta t}{\Delta z'} \left[ V_{n+1}^{(k')} - V_{n-1}^{(k')} \right], \tag{2.4}
\]

\[
V_{n+1}^{(k')} = V_{n}^{(k')} - \frac{c \Delta t}{Z_0} \left[ I_{n+1}^{(k' + \frac{1}{2})} - I_{n-1}^{(k' - \frac{1}{2})} \right], \tag{2.5}
\]

where \( \Delta z' \) is the spatial discretization interval along the cable, \( k' \) is the corresponding index, and \( Z_0 \) is the characteristic impedance of the transmission line. The phase velocity \( c \) along the transmission line is assumed to correspond to the speed of light (i.e., \( c = 1/\sqrt{\epsilon_0 \mu_0} \)). An absorbing boundary condition is implemented at the upper end of the transmission line to prevent voltage pulse reflections from running back to the antenna. For this purpose, we choose the spatial discretization of the cable to be \( \Delta z' = 2c \Delta t \), and we enforce the following equalities:

\[
I_{n+\frac{1}{2}}^{(\frac{1}{2})} = I_{n-\frac{1}{2}}^{(\frac{1}{2})}, \tag{2.6}
\]

\[
V_{n+1}^{(0)} = V_{n-1}^{(1)}. \tag{2.6}
\]

To connect the transmission line to the antenna terminals, the update equation for the current in equation (2.4) at the lower end of the cable is modified according to Ampère's law:

\[
I_{n+\frac{1}{2}}^{(\frac{1}{2})} = \left[ H_{z(i_s+\frac{1}{2},j_s+\frac{1}{2},k_s)}^{n+\frac{1}{2}} - H_{z(i_s+\frac{1}{2},j_s+\frac{1}{2},k_s-1)}^{n+\frac{1}{2}} \right] \Delta z + \left[ H_{y(i_s+\frac{1}{2},j_s+\frac{1}{2},k_s+\frac{1}{2})}^{n+\frac{1}{2}} - H_{y(i_s+\frac{1}{2},j_s+\frac{1}{2},k_s)}^{n+\frac{1}{2}} \right] \Delta y. \tag{2.7}
\]

The indices \( i_s, j_s, \) and \( k_s \) denote the cell centered between the terminals of the antenna. The relationship between the voltage \( V_{k_{max}} \) at the last point of the transmission line and the electric field between the terminals of the antenna is given
by

\[ E_x^n (i_s + \frac{1}{2}, j_s, k_s) = -\frac{V_{E_0}^{(k_{\text{max}})}}{d}, \tag{2.8} \]

where \( d = 2\Delta x \) is the spacing between the antenna terminals. The point \( k_{\text{max}}' \) of the 1D transmission line is thus colocated with the point \((i_s + \frac{1}{2}, j_s, k_s) \) of the 3D grid. The antenna is excited by feeding a Gaussian voltage pulse

\[ V_g(t) = V_0 e^{-\frac{1}{2} ((t - \frac{5}{2}t_g)/\tau_p)^2} \tag{2.9} \]

into the transmission line. The characteristic time \( \tau_p \) of the pulse must be small enough to resolve the antenna geometry. This implies that the frequency spectrum of the excitation pulse has to be relatively constant in the frequency range defined by the dimensions of the antenna. Conversely, the duration of the pulse \( t_g \) must be long enough to ensure a smooth beginning and ending of the excitation, thus avoiding numerical artifacts. For most of our simulations, we use \( \tau_p = 0.25 \text{ ns} \) and \( t_g = 2.36 \text{ ns} \). These values represent approximately the characteristics of a 400-MHz antenna. The amplitude scaling factor \( V_0 \) is set to unity.

To reduce antenna ringing in the presence of shielding, the antenna model can be resistively loaded by connecting the corners of the antenna to the shielding by resistors. For the simulation of the resistors, a lumped current density term \( J_l \) is added to Maxwell's curl equation for the magnetic field Taflove and Hagness (2000):

\[ \nabla \times H = \sigma E + \frac{\partial D}{\partial t} + J_l, \tag{2.10} \]

where \( D \) is the displacement current. With the resistor \( R \) aligned in the \( x \)-direction, the current density is given by

\[ j^{n-\frac{1}{2}} = \frac{\Delta x (E_x^n + E_x^{n-1})}{R \Delta y \Delta z}. \tag{2.11} \]

Inserting (2.11) into (2.10) yields the modified update equation for the electric field component \( E_x \) at the location of the resistor cell:

\[ E_x^n (i+\frac{1}{2}, j, k) = \frac{2\pi R \Delta y \Delta z + \Delta t (\sigma R \Delta y \Delta z + \Delta x)}{2\pi R \Delta y \Delta z + \Delta t (\sigma R \Delta y \Delta z + \Delta x)} E_x^{n-1} (i+\frac{1}{2}, j, k) \]

\[ + \frac{2\Delta t R \Delta y \Delta z}{2\pi R \Delta y \Delta z + \Delta t (\sigma R \Delta y \Delta z + \Delta x)} \]

\[ \times \left[ \frac{H^{n-\frac{1}{2}}_{x(i+\frac{1}{2}, j, k)} - H^{n-\frac{1}{2}}_{x(i+\frac{1}{2}, j-\frac{1}{2}, k)}}{\Delta y} - \frac{H^{n-\frac{1}{2}}_{y(i+\frac{1}{2}, j, k)} - H^{n-\frac{1}{2}}_{y(i+\frac{1}{2}, j, k+\frac{1}{2})}}{\Delta z} \right]. \tag{2.12} \]
2.3.4 Validation of algorithm

Figure 2.3 shows the simulated amplitude radiation pattern of the electric-field $E_\theta$ component ($E$-plane) for an infinitesimal dipole in free space compared with the analytic far-field radiation pattern for the same frequency. For a distance equal to about 4.5 wavelengths, the numerical radiation pattern is in excellent agreement with the analytic solution. To benchmark further the reliability and accuracy of our antenna simulation algorithm, we compare our results with laboratory measurements of antenna input impedance Brown and Woodward (1952). The input impedance of an antenna is determined by recording the incident voltage $V_i(t)$ and the reflected voltage $V_r(t)$ in the transmission line. The corresponding reflection factor in the frequency domain is then given by

$$\rho(\omega) = \frac{V_r(\omega)}{V_i(\omega)} e^{2i\kappa r'},$$  \hspace{1cm} (2.13)

where $\kappa$ is the wavenumber, $\omega$ is the angular frequency, and $r'$ is the distance between the recording point and the antenna terminals. Based on equation (2.13), the complex impedance $Z$ of the antenna can be calculated as

$$Z = Z_0 \frac{1 + \rho(\omega)}{1 - \rho(\omega)},$$  \hspace{1cm} (2.14)

where $Z_0$ is the characteristic impedance of the transmission line.

In Figure 2.4, the computed real part (resistance) and the complex part (reactance) of $Z$ for bow-tie antennas with flare angles ranging from 5° to 90° are compared with the corresponding laboratory-measured data. All computations are carried out for $Z_0 = 200 \ \Omega$. The lengths of the antenna panels range from 19 to 23 cm, depending on the flare angle. The main- and local-grid spacings are 1 cm and 0.33 cm, respectively, and the Courant number is 0.7 in all models of this section. For ease of comparison with the original monopole antenna measurements of Brown and Woodward (1952), our values are scaled with respect to the maximum value of each experimental curve and are plotted with respect to the length of the antenna panel in electrical degrees $A = 360^\circ \times (l \ \omega/2\pi \ c)$, where $l$ is the geometrical length of the antenna panel (Figure 2.2).

Figure 2.4 shows that the antenna input impedances become increasingly broadband with increasing flare angle. Conversely, with decreasing flare angle, the peaks and troughs in the input impedance curves become more pronounced as a result of frequency tuning in the antenna. This is because the electrical length of an antenna (i.e., the path travelled by the current expressed in wavelengths) becomes increasingly well defined as the flare angle decreases, such that the bow-tie antenna begins
to resemble a wire antenna. Bow-tie antennas with flare angles smaller than 10° are practically equivalent to the thin-wire antenna model of Holliger and Bergmann (1998).

The most significant mismatches in Figure 2.4 are the systematically higher frequencies of the peaks and troughs in the simulated input impedance data relative to the laboratory-measured values (see also Leat et al., 1998). We attribute this effect to the observation that real antennas tend to build up capacitances at their ends. This causes the electrical length of a real antenna to be longer than that of its simulated equivalent, which in turn results in the simulated tuning peaks occurring at higher frequencies than the laboratory-measured ones. Consequently, reducing the size of the local grid would not eliminate this mismatch. As expected, this effect decreases with increasing bandwidth of the input impedance (i.e., with increasing flare angle). For broadband antennas with flare angles larger than about 40°, the agreements between the simulated impedance and laboratory-measured data are excellent.

2.4 Simulations

Although our antenna simulations use relatively fine local grids, diffractions caused by the staircasing approximation of the antenna geometry are evident at small flare angles. This effect is illustrated in the left diagram of Figure 2.5, which shows a snapshot of the $E_x$ component in the $x - y$ plane for an antenna (light green staircasing areas) with a flare angle of 10° (reds and yellows represent diffractions generated by the edges of the staircase offsets). Reducing the grid spacing by a factor of two does not significantly reduce these staircasing effects. Nevertheless, even for very small flare angles (e.g., 5° and 10°) of the bow-tie antenna model, surprisingly good matches between the modeled and measured data are achieved (Figure 2.4). For the 90° antenna (right diagram, Figure 2.5), wavefronts reflected at the ends of the panels travel back to the center of the antenna, but staircasing effects are not visible. The results of these benchmark tests demonstrate the accuracy and overall robustness of our algorithm.

In the following, we show snapshots of the electric field and corresponding radiation patterns (i.e., the absolute values of the time-averaged tangential components $E_\theta$ and $E_\phi$) for a selection of GPR antennas located slightly above various half-space earth models; all antennas are located one grid cell above the air–soil interface. For reference, we show the appropriate far-field radiation patterns of infinitesimal hori-
zontal electric dipoles calculated using equations (46a)-(46d) of Smith (1984) on all figures. By including the far-field radiation patterns in Figures 2.7–2.20, it is not our intention to demonstrate for each case the limitations of the far-field approximations; since we are mostly dealing with near- or intermediate-field examples, it is not surprising that the far-field equations are inadequate. Rather, the far-field radiation patterns provide familiar baselines with which to compare the results of our simulations and to see the genesis of nulls and lobes. The far-field radiation patterns of infinitesimal dipoles are familiar to many interpreters of GPR data. An overview of the different parameters used in the simulations of Figures 2.7–2.20 is given in Table 1.

2.4.1 Infinitesimal electric dipole

Figure 2.6 shows snapshots of the $E_x$ component parallel (E-plane) and perpendicular (H-plane) to an infinitesimal electric dipole, together with the corresponding $E$-field radiation patterns (see also Arcone, 1995). The half-space beneath the dipole has a permittivity of $\varepsilon = 5\varepsilon_0$ and a conductivity of zero. The snapshots display spherical waves in the upper and lower half-space and head waves in the lower half-space. Radiation patterns of the tangential electric field components in the E-plane ($E_\theta$) and H-plane ($E_\phi$) are simulated at a radius of 1 m from a point directly below the dipole on the interface. The general shapes of the numerical and analytical far-field radiation patterns for the infinitesimal electric dipole are similar. For example, the ratios between the maximum amplitudes in the E- and H-planes are comparable. However, the side lobes of the numerical radiation pattern in the E-plane are much less pronounced than those of the asymptotic far-field pattern. In the H-plane, the amplitude maximum in the numerical radiation pattern is located significantly beyond the critical angle of 26.6°, in accordance with the computations of Holliger and Bergmann (1998). This illustrates that even in the hypothetical case of an infinitesimal electric dipole antenna, the commonly used asymptotic far-field patterns may be rather poor reference models for the illumination of the shallow subsurface by typical GPR measurements.

2.4.2 Increasing antenna flare angle

Figures 2.7 and 2.8 show snapshots of the $E_x$ component and corresponding radiation patterns transmitted by unshielded bow-tie antennas with 5° and 60° flare angles, respectively. Our simulations for a 90° flare angle are similar to those shown
for the 60° flare angle. In the snapshots of Figures 2.7 and 2.8, the wavefronts in the upper half-space (air) have already left the computational volume. In the lower medium, parts of the head waves are present at a depth of about 50 cm near the left and right borders of the diagrams. A simple comparison of these snapshots with those computed for the infinitesimal dipole (Figure 2.6) illustrates the significant effect that the GPR antenna system has on the radiated EM field and thus on the illumination of the subsurface. For both flare angles, ringing is clearly visible; multiple reflections at the ends of the antennas generate the observed multiple wavefronts. Ringing is most pronounced in the E-plane snapshots of Figure 2.7 (5° flare angle).

Differences between the radiation patterns of the two antennas are small. In the H-plane patterns, the amplitude maxima are ~ 10° farther from the critical angle than the maxima of the numerical radiation patterns for the infinitesimal dipole (Figure 2.6). This implies that when acquiring GPR data in the common perpendicular-broadside mode (i.e., side-by-side parallel transmitter and receiver antennas oriented perpendicular to the direction of the recording profile), a larger subsurface volume is illuminated than predicted by the asymptotic far-field radiation patterns or even by the numerical simulations of an infinitesimal electric dipole. Moreover, there are virtually no side lobes present in the E-planes.

2.4.3 Adding a simple shield

Next, we add a rectangular metal shielding box to the basic bow-tie antenna model. The shielding box is 60 cm long and 38 cm wide and is either 26 cm (Figure 2.9) or 13 cm (Figure 2.10) high. The height of the shielding box has a noticeable effect on the radiation characteristics. In Figure 2.9, the 26 cm height of the box is approximately equal to the quarter-wavelength response of a corresponding tuned half-wavelength dipole antenna in free space. The traces in the upper parts of Figures 2.9 and 2.10 represent the intensities of the electric fields in the lower half-spaces beneath the centers of the antennas (x = 0). The main difference between these two traces is a pronounced third peak at about 40 cm depth in Figure 2.9. This is the result of constructive interference of the direct and reflected downgoing parts of the EM wavefield, resulting in increased reverberations.

The corresponding subsurface radiation patterns are much narrower than for the unshielded case (compare Figures 2.9 and 2.10 with Figure 2.8), particularly in the H-plane, where only a single lobe is observed. In the standard perpendicular-broadside acquisition mode, a shielded antenna thus illuminates a significantly smaller volume of the subsurface than a corresponding unshielded antenna. More-
over, in contrast to the radiation patterns of the unshielded bow-tie antennas in Figures 2.7 and 2.8, the maximum amplitude in the $E'$-plane is effectively equal to the maximum amplitude in the $H$-plane.

### 2.4.4 Filling the shield with absorbing material

Pronounced effects on the radiation patterns are observed when the shielding box is filled with absorbing dielectric material (see also Izzat et al., 1996). In Figures 2.11—2.13, results are displayed for absorbing material with a constant conductivity of $\sigma = 20$ mS/m and permittivities $1.5\epsilon_0$, $3\epsilon_0$, and $20\epsilon_0$. In all cases, the $E$-plane radiation patterns are broadened substantially, to be even broader than the corresponding $H$-plane radiation patterns. This implies that for such GPR antenna systems, the so-called parallel-endfire mode (i.e., parallel transmitter and receiver antennas aligned one after another and oriented parallel to the direction of the recording profile) would provide a slightly more favorable illumination of the subsurface than the standard perpendicular-broadside acquisition mode. By increasing the permittivity of the absorbing material, the ratio of lower to upper half-space maximum amplitude decreases. With $\epsilon = 1.5\epsilon_0$ in the shielding, this ratio is 11 and 10.5 in the $E$- and $H$-planes (Figure 2.11), respectively; with $\epsilon = 20\epsilon_0$, it is 2.8 in both planes (Figure 2.13). The maximum radiation transmitted into the lower half-space does not follow a simple trend. For a half-space permittivity of $5\epsilon_0$ and shielding material with a conductivity of $\sigma = 20$ mS/m, the maximum radiation transmitted into the subsurface is obtained by setting the permittivity of the shielding material to $\sim 3\epsilon_0$ (Figure 2.12).

Filling the shield with lossless dielectric material results in higher maximum radiation levels transmitted into both half-spaces (Figure 2.14) than those transmitted by shielded antennas filled with absorbing material ($\sigma > 0$ mS/m). If the permittivity of the material in the shielding is lower than that of the ground, the conductivity of the filling material has a strong effect on the radiation patterns. This is demonstrated by comparing Figure 2.14 (shielding material with $\epsilon = 3\epsilon_0$, $\sigma = 0$ mS/m) with Figure 2.12 (shielding material with $\epsilon = 3\epsilon_0$, $\sigma = 20$ mS/m). For shielding material with a higher permittivity than that of the subsurface, our simulations show that the effect of conductivity is less pronounced.
2.4.5 Adding loading resistors

Figure 2.15 shows radiation patterns after the 13-cm shielding box without absorbing material is connected to the antenna panels with 200-Ω resistors. The special resistor cells [equation (2.12)] are connected to each corner of the antenna. The antenna is placed over a lossless half-space with $\epsilon = 5\epsilon_0$. Relative to the corresponding unloaded antenna model (Figure 2.10), the loaded antenna model yields lower amplitudes in the upper and lower half-spaces as well as slightly less ringing in the lower half-space (compare the intensities shown by the traces displayed adjacent to the upper-left diagrams of Figures 2.15 and 2.10).

2.4.6 Influence of a nearby receiver antenna

To assess the effects of a nearby receiver antenna on the transmitted radiation pattern, we computed the responses of models comprising unshielded and shielded transmitter bow-tie antennas (both with 60° flare angle) with corresponding unshielded and shielded receiver antennas juxtaposed in the perpendicular-broadside mode. The receiver antennas were placed 0.55 m from the transmitter antennas, a typical separation for a 400-MHz GPR system. The results (only shown for the unshielded antenna in Figure 2.16) show that the receiver antennas have only minor effects on the radiation patterns transmitted into the subsurface.

2.4.7 Changing the soil conditions

Figure 2.17 illustrates radiation patterns transmitted from an unshielded 60° antenna over a half-space with relatively high permittivity ($\epsilon = 25\epsilon_0$). The recording distance for the radiation patterns is now set to 45 cm, which would correspond to an electrical length of 1 m in a half-space with $\epsilon = 5\epsilon_0$. Relative to the snapshots of Figure 2.8, the higher ground permittivity results in more closely spaced wavefronts, decreased maximum amplitudes of radiation propagating in the upper half-space and increased maximum amplitudes in the lower half-space. The radiation patterns in both planes are much broader than the far-field radiation patterns of an infinitesimal electric dipole. Hence, the increased directivity in the high-permittivity half-space is markedly less pronounced than predicted for the infinitesimal electric dipole (compare with Figure 2.8).

Figure 2.18 shows the response of an antenna with the 13-cm-high shielding box situated above the high-permittivity half-space ($\epsilon = 25\epsilon_0$). Relative to the results for the unshielded antenna, ringing is much more pronounced in the shielded
antenna snapshots (compare with Figure 2.17). It is noteworthy that the degree of
directivity for the shielded antenna does not change significantly in moving from a
low permittivity ($\epsilon = 5\epsilon_0$) to a high-permittivity ($\epsilon = 25\epsilon_0$) environment (compare
with Figure 2.10).

The effects of increasing the conductivity in the ground to 15 mS/m while leav¬
ing the permittivity constant ($\epsilon = 5\epsilon_0$) are shown for the unshielded and shielded
antennas in Figures 2.19 and 2.20, respectively. For these situations, the ratios of
displacement to conduction currents in the ground are reduced significantly. Be¬
cause of the resulting damping, the maximum amplitudes of the unshielded antenna
radiation patterns are now in the upper half-spaces (lower diagram of Figure 2.19); note that the wavefronts in the upper half-space have left the computational volume
in the upper diagrams of this figure. Adding a 13-cm-heigh shield reduces the am¬
plitude of radiation propagating in the air (Figure 2.20), such that the proportion
of energy radiating in the ground significantly increases.

2.5 Conclusions

We have developed a GPR antenna simulation tool based on a FDTD approxima¬
tion of Maxwell’s equations in Cartesian coordinates. The algorithm is flexible and
accurate. Subgridding allows for the numerically efficient modeling of small an¬
tenna structures and high-permittivity materials. Because of its modular structure,
additional antenna designs with planar structures can readily be added to the al¬
gorithm. We have simulated the radiation characteristics of surface GPR systems
with antenna types ranging from quasi-thin-wire antennas to bow-tie antennas with
arbitrary flare angles. Shielding was achieved by superimposing a metal box imme¬
diately above the antenna. Antenna damping was accounted for by filling the shield
with absorbing material or by connecting the antenna to the shield with loading
resistors. The effects that these features have on the radiative properties of the
tested GPR systems and thus on the illumination of the subsurface were illustrated
for various half-space models.

Our results demonstrate that the commonly used asymptotic far-field radiation
patterns for a horizontal electric dipole overlying a dielectric half-space may be poor
representations of radiation patterns generated by common GPR antennas in the
intermediate-field regime.

Unshielded bow-tie antennas with small flare angles are characterized by more
ringing than those with large flare angles. For unshielded bow-tie antennas, the
common perpendicular-broadside acquisition mode is likely to produce optimum results under most soil conditions.

Radiation patterns generated by antennas with simple shields have higher amplitudes in the subsurface than those without shields. Filling the shield with absorbing material markedly changes the radiation patterns. Resistive loading of the antenna system results in less ringing and in lower amplitudes of the radiated EM field. The conductivity of material filling the shield has a strong effect on the radiation patterns if its permittivity is lower than that of the ground.

Based on these results, we suggest that our new simulation software offers the possibility to assist in planning GPR surveys and designing novel GPR antenna systems and to improve our understanding of the effects of GPR antenna systems on recorded data.

2.6 Acknowledgments

This work was supported by grants from ETH Zurich and the Swiss National Science Foundation. We thank Carsten Aulbert (formerly TU Braunschweig) for useful hints with regard to the implementation of the GPML absorbing boundary conditions, Ueli Meier and Marc Lambert for their contributions to the simulation tool, and the Geophysics associate and assistant editors (Steve Arcone and José Carcione, respectively) for constructive comments on the manuscript. ETH Geophysics Publication 1232.

2.7 Appendix

2.7.1 Absorbing Boundary Condition

The generalized perfectly matched layer (GPML) absorbing boundary condition of Fang and Wu (1996) uses Bérenger’s 1994; 1996 original split-field formulation of Maxwell’s equations in stretched coordinates. In the following, we show the discrete GPML equations in the form they have been implemented in our simulation tool. Also shown are the formulations for the coordinate stretching variables $s_i$ and the electric and magnetic conductivities $\sigma_i$ and $\sigma_i^*$ of the layer, where $i = x$, $y$, and $z$.

The time-domain Maxwell’s equations in the stretched coordinates for the split-field subcomponents $E_{xy}$ and $E_{xz}$ are

$$\frac{\partial H_z}{s_{yo}(y) \partial y} = \epsilon \frac{\partial}{\partial t} E_{xy} + (\sigma_0 + \sigma_y) E_{xy} + \frac{\sigma_0 \sigma_y}{\epsilon} E_{xy}^*, \quad (2.15)$$
CHAPTER 2. FDTD SIMULATION TOOL FOR GPR ANTENNAS

\[-\frac{\partial H_y}{s_y(z)\partial z} = \epsilon \frac{\partial}{\partial t} E_{zz} + (\sigma_0 + \sigma_z)E_{xx} + \frac{\sigma_0 \sigma_z}{\epsilon} E^{I}_{zz},\]  

(2.16)

and for the \(H_x\) subcomponents \(H_{xy}\) and \(H_{xz}\)

\[-\frac{\partial E_y}{s_y(y)\partial y} = \frac{\mu}{s_y(y)} \frac{\partial}{\partial t} H_{xy} + (\sigma_0^* + \sigma_y^*)H_{xy} + \frac{\sigma_0^* \sigma_y^*}{\mu} H^{I}_{xy},\]  

(2.17)

\[-\frac{\partial E_y}{s_z(z)\partial z} = \frac{\mu}{s_z(z)} \frac{\partial}{\partial t} H_{xz} + (\sigma_0^* + \sigma_z^*)H_{xz} + \frac{\sigma_0^* \sigma_z^*}{\mu} H^{I}_{xz},\]  

(2.18)

where the magnetic conductivities are obtained from the matching conditions

\[\sigma_x^* = \frac{\sigma_x \mu}{\epsilon}, \sigma_y^* = \frac{\sigma_y \mu}{\epsilon}, \sigma_z^* = \frac{\sigma_z \mu}{\epsilon}.\]  

(2.19)

The values \(E_{xy}^I, E_{xz}^I, H_{xy}^I,\) and \(H_{xz}^I\) are the time-integrated expressions of the corresponding field components. For example,

\[E_{xy}^I = \frac{E_{xy}}{\imath \omega}.\]  

(2.20)

The discretized form of equation (2.20) is then

\[E_{xy}^{n+\frac{1}{2}}(i+\frac{1}{2},j,k) = E_{xy}^{n-\frac{3}{2}}(i+\frac{1}{2},j,k) + \Delta t E_{xy}^{n-1}(i+\frac{1}{2},j,k),\]  

(2.21)

Together with equation (2.21), the FDTD discretization of equation (2.15) yields:

\[E_{xy}^n(i+\frac{1}{2},j,k) = \frac{2e-(\sigma_0+\sigma_x)\Delta t}{2e+(\sigma_0+\sigma_x)\Delta t} E_{xy}^{n-1}(i+\frac{1}{2},j,k) + \frac{2\Delta t}{s_y(y)\Delta y(2e+(\sigma_0+\sigma_y)\Delta t)} \left| \bigg[H_{zx}^{n-\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) + H_{zy}^{n-\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) - H_{zx}^{n-\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k) - H_{zy}^{n-\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k) \right] \]  

\[-\frac{2\sigma_0 \sigma_x \Delta t}{e(2e+(\sigma_0+\sigma_y)\Delta t)} E_{xy}^I(i+\frac{1}{2},j,k).\]  

(2.22)

From equation (2.16) we obtain the update equation for \(E_{xz}\):

\[E_{xz}^n(i+\frac{1}{2},j,k) = \frac{2e-(\sigma_0+\sigma_z)\Delta t}{2e+(\sigma_0+\sigma_z)\Delta t} E_{xz}^{n-1}(i+\frac{1}{2},j,k) - \frac{2\Delta t}{s_z(z)\Delta z(2e+(\sigma_0+\sigma_z)\Delta t)} \left| \bigg[H_{yx}^{n-\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) + H_{yz}^{n-\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) - H_{yx}^{n-\frac{1}{2}}(i+\frac{1}{2},j,k-\frac{1}{2}) - H_{yz}^{n-\frac{1}{2}}(i+\frac{1}{2},j,k-\frac{1}{2}) \right] \]  

\[-\frac{2\sigma_0 \sigma_z \Delta t}{e(2e+(\sigma_0+\sigma_z)\Delta t)} E_{xz}^I(i+\frac{1}{2},j,k).\]  

(2.23)

The magnetic subcomponents \(H_{xy}\) and \(H_{xz}\) are derived from equations (2.17) and (2.18), in which the magnetic conductivity \(\sigma_0^*\) of our modeled materials is set to zero:
The update equations for the remaining eight subcomponents of the EM field in the GPML are derived in the same manner. For the $x$ component, for example, the parameters of the coordinate stretching variables and the conductivity profiles are

\[
s_{x_0}(x) = 1 + s_m \left( \frac{x}{\delta} \right)^p,
\]

\[
\sigma_x(x) = \sigma_m \sin^2 \left( \frac{\pi x}{2\delta} \right).
\]

The parameters $s_m$ and $\sigma_m$, the thickness $\delta$ of the layer, and the exponent $p$ in equation (2.26) determine the absorbing performance of the GPML. The method per se does not provide any guidance as to the choice of these free parameters. For our suite of models, we achieved uniformly excellent absorption (errors from boundary reflections generally <5%) with the following values: $s_m = 4$, $\sigma_m = 0.07$, $\delta = 25$, and $p = 2$.

### 2.7.2 Subgridding Algorithm

We found the efficacy of the subgridding algorithm proposed by Chevalier et al. (1997) and adopted in this paper to be dependent on quite minor implementation details. For example, updating the magnetic field before the electric field resulted in the scheme becoming unstable within less than 1000 time-steps. In the following, we clarify the updating process for the EM field in the computational volume with (i) the main grid only, and (ii) the main grid and the local grid.

For the main grid only, we used the following procedure:

1) Obtain $E^n$ according to equation (2.1).

2a) Excite the $E$-field at the antenna feed [equation (2.8)].
2b) Set to zero the tangential $E$-field components within the metal parts of the antenna.

3) Obtain $H^{n+\frac{1}{2}}$ according to equation (2.2).

4) Return to step 1 at time step $n + 1$.

For the local grid, the colocated magnetic field components on the interface are interpolated at every local time-step by a second-order Taylor series expansion of the corresponding main-grid components at time-steps $n + \frac{1}{2}$, $n - \frac{1}{2}$, and $n - \frac{3}{2}$:

$$h^{n-\frac{1}{2}+\frac{q}{2}} = H^{n-\frac{1}{2}} + \frac{H^{n+\frac{1}{2}} - H^{n-\frac{3}{2}}}{2} \frac{q}{3} + \frac{H^{n+\frac{1}{2}} + H^{n-\frac{3}{2}} - 2H^{n-\frac{1}{2}}}{2} \left(\frac{q}{3}\right)^2, \quad q = 1, 2, 3. \tag{2.28}$$

Moreover, closest to these colocated field components, each local-grid solution for the magnetic field one local cell from the interface [e.g., $h_x(1.5, 1, ki)$ in Figure 2.1] is weighted for itself and for the average value of two neighboring field components: the colocated $H$ field on the interface and the local-grid component two local cells into the grid, i.e., $h_x(1.5, 0, ki)$ and $h_x(1.5, 2, ki)$ in Figure 2.1. Referring to this figure, we have, for example, for the $h_x$ component at the local-grid location $(1.5, 1, ki)$ at every local time-step,

$$h_x(1.5, 1, ki) = w_1 h_x(1.5, 1, ki) + (1 - w_1) \frac{h_x(1.5, 0, ki) + h_x(1.5, 2, ki)}{2}. \tag{2.29}$$

Following the original work of Chevalier et al. (1997), we used their numerically determined value 0.95 for the weight $w_1$.

The main- and local-grid solutions for the electric field at colocated gridpoints closest to the interface are weighted in a similar manner. For example, we have for the $z$ component of the electric field in Figure 2.1, index $(i, j, k)$ and $(1.5, 1.5, ki)$:

$$E_z^n(i, j, k) = w_2 E_z^n(i, j, k) + (1 - w_2) e_z^n(1.5, 1.5, ki), \tag{2.30}$$

$$e_z^n(1.5, 1.5, ki) = (1 - w_2) E_z^n(i, j, k) + w_2 e_z^n(1.5, 1.5, ki). \tag{2.31}$$

After Chevalier et al. (1997), the weight $w_2$ is chosen to be 0.8.

For the local grid comprising the antenna, we used the following procedure:

1) Obtain $E^n$ with the main-grid update equations.

2) Obtain $H^{n+\frac{1}{2}}$ with the main-grid update equations.

3) Obtain $e^{n-\frac{1}{2}}$ with the local-grid update equations.

4a) Excite the local $E$-field at the antenna location.
4b) Set to zero the tangential $E$-field components within the antenna panels in the local grid.

5) Obtain $h^{n-\frac{1}{6}}$ with the local-grid update equations.

6) Time interpolate the colocated $H$-fields on the interface at a time step $n - \frac{1}{6}$ [equation (2.28)].

7) Linear spatially interpolate the non-colocated $H$-fields on the interface.

8) Weight the non-colocated magnetic field components closest to the boundary according to equation (2.29).

9) Repeat steps 3, 4a and 4b at time step $n$.

10) Repeat steps 5, 6, 7 and 8 at time step $n + \frac{1}{6}$.

11) Weight the colocated $E$-field components closest to the interface according to equations (2.30) and (2.31).

12) Repeat steps 3, 4a and 4b at time step $n + \frac{2}{6}$.

13) Repeat steps 5, 6, 7 and 8 at time step $n + \frac{3}{6}$.

14) Replace all colocated main-grid $H$-field components by the corresponding field components in the local grid.

15) Set to zero the tangential $E$-field components within the antenna panels in the main grid.

16) Return to step 1 at time step $n + 1$. 
### Table 2.1: Model parameters, Figures 2.7–2.20 (T = transmitter, R = receiver).

<table>
<thead>
<tr>
<th>Figure</th>
<th>Antennas Type</th>
<th>Length [cm]</th>
<th>Flare Angle</th>
<th>ϵ/ϵ₀</th>
<th>σ [mS/m]</th>
<th>Height of Shield [cm]</th>
<th>Resistors Ω</th>
<th>Material in Shield ϵ/ϵ₀</th>
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Figure 2.1: Cross-section through the staggered main and local grid in the $x$-$y$ plane. The region bounded by thin solid lines represents the local grid. Upper- and lowercase letters denote field components in the main and local grids, respectively. Large symbols along the boundary and inside the local grid denote colocated gridpoints. Marked coordinates [(1.5, 0, $k_1$) etc.] are referred to in the Appendix.

Figure 2.2: Sketch of the basic (a) unshielded and (b) shielded bow-tie antenna models.
Figure 2.3: The amplitude radiation pattern for the tangential component $E_\theta$ of the electric field in the $E$-plane for an infinitesimal dipole in free space (dotted line) and the corresponding asymptotic radiation pattern for the far-field [equations (46a)-(46d) in Smith, 1984] with $\epsilon_1 = \epsilon_2 = \epsilon_0$ (solid line).

Figure 2.4: Comparison of numerical simulations (solid lines) and laboratory measurements (dashed lines) of the resistance (real part of the input impedance) and reactance (complex part) of unshielded bow-tie antennas with flare angles ranging from $5^\circ$ to $90^\circ$. The length of the triangular antenna panel is $A = 360^\circ \times (l \omega/2\pi c)$ in electrical degrees, where $l$ is the geometrical length of the antenna panel shown in Figure 2.2. The laboratory data were digitized from Brown and Woodward (1952).
Figure 2.5: Snapshots of the electric-field $E_x$ component radiated from bow-tie antennas with 10° (left) and 90° (right) flare angles. For the 10° antenna, diffractions resulting from representing the flanks of the antenna by staircase offsets are clearly visible.
Figure 2.6: (Top) $E$-plane (left) and $H$-plane (right) snapshots of the electric field $E_x$ component radiated from an infinitesimal electric dipole located one grid cell above an interface between air and lossless soil ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0 \text{ mS/m}$). The dipole is oriented parallel to the $x$-axis. Spherical waves in the upper (1) and lower half-space (2) and head waves (3) in the lower half-space are visible. (Bottom) Amplitude radiation patterns (blue) for the tangential components of the electric field, $E_\theta$ (left) and $E_\phi$ (right), in the $E$- and $H$-planes, respectively. Recording distance is 1 m. Red lines show corresponding asymptotic radiation patterns for far-field [equations (46a)–(46d) in Smith, 1984]. Data are normalized with respect to the maximum amplitude.
Figure 2.7: (Top) $E$-plane (left) and $H$-plane (right) snapshots of the electric field $E_x$ component radiated from an unshielded bow-tie antenna located one grid cell above an interface between air and lossless soil ($\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The antenna (location identified by the short arrow) is 46 cm long, has a flare angle of $5^\circ$, and is oriented parallel to the $x$-axis. (Bottom) Amplitude radiation patterns (blue) for the tangential components of the electric field, $E_\theta$ (left) and $E_\phi$ (right), in the $E$- and $H$-planes, respectively. Recording distance is 1 m. Red lines show the corresponding asymptotic radiation patterns for the far-field [equations (46a)-(46d) in Smith, 1984]. Data are normalized with respect to the maximum amplitude. Values in bold type equal maximum amplitude of radiation pattern divided by maximum amplitude of $E_\phi$ radiation pattern in Figure 2.8.
Figure 2.8: As for Figure 2.7, except the antenna is 52 cm long with a flare angle of 60°. Values in bold type are maximum amplitude of radiation pattern divided by maximum amplitude of $E_\phi$ radiation pattern.
Figure 2.9: As for Figure 2.8, with the addition of shielding. The shield consists of an empty 60-cm-long, 38-cm-wide, and 26-cm-high metal box. Values in bold type are maximum amplitude of radiation pattern divided by maximum amplitude of $E_\phi$ radiation pattern in Figure 2.8.
Figure 2.10: As for Figure 2.9, except the metal shield is only 13 cm high.
Figure 2.11: As for Figure 2.10, except the metal shield contains absorbing material ($\varepsilon = 1.5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 20 \text{ mS/m}$).
Figure 2.12: As for Figure 2.11, except the permittivity of the absorbing material increases to $\varepsilon = 3\varepsilon_0$. 
Figure 2.13: As for Figure 2.12, except the permittivity of the absorbing material increases to $\epsilon = 20\epsilon_0$.

Figure 2.14: As for Figure 2.12, except the conductivity of the absorbing material is set to zero.
Figure 2.15: As for Figure 2.10, except the empty metal shield is connected to the antenna corners by 200-Ω resistors.
Figure 2.16: As for Figure 2.8, but with a receiver antenna of the same dimensions located 55 cm from the feed of the transmitter antenna in a perpendicular-broadside configuration. Values in bold type are maximum amplitude of radiation pattern divided by maximum amplitude of $E_\phi$ radiation pattern in Figure 2.8.
Figure 2.17: As for Figure 2.8, but with the permittivity of the subsurface increased to $\epsilon = 25\varepsilon_0$. Values in bold type are maximum amplitude of radiation pattern divided by maximum amplitude of $E_\phi$ radiation pattern in Figure 2.8.
Figure 2.18: As for Figure 2.10, but with the permittivity of the subsurface increased to \(\varepsilon = 25\varepsilon_0\).
Figure 2.19: As for Figure 2.8, but with the conductivity of the subsurface increased to $\sigma = 15 \text{ mS/m}$. Values in bold type are maximum amplitude of radiation pattern divided by maximum amplitude of $E_\phi$ radiation pattern in Figure 2.8.
Figure 2.20: As for Figure 2.10, but with the conductivity of the subsurface increased to $\sigma = 15 \text{ mS/m}$. 
Chapter 3

Effects of Fractal Fluctuations in Topographic Relief, Permittivity and Conductivity on Ground-Penetrating Radar Antenna Radiation

Bernhard Lampe and Klaus Holliger

*Geophysics*, in press

3.1 Abstract

Typical ground-penetrating radar (GPR) transmitters and receivers are dipole-type antennas. These antennas have pronounced directivity properties and exhibit strong coupling to interfaces across which there are changes in electric material properties. Antenna coupling to the surface of idealized half-space models has been the subject of intense research for several decades. In contrast, the behavior of antennas in the vicinity of interfaces with realistic topographic fluctuations and/or subsurface heterogeneities has been largely unexplored. To explore this issue, we simulate the responses of a typical surface GPR antenna system located on a suite of realistic fractal earth models using the finite-difference time-domain (FDTD) method.
The models are characterized by topographic roughness of the air-soil interface and small-scale heterogeneous distributions of permittivity and conductivity in the subsurface. Synthetic radiation patterns and input impedance values of the simulated GPR antenna system demonstrate that topographic roughness affects significantly the coupling of the antenna to the ground, whereas heterogeneities in the subsurface influence predominantly the antenna radiation through scattering and absorption along the propagation path.

3.2 Introduction

A typical setup for acquiring surface ground-penetrating radar (GPR) data consists of two closely spaced dipole-type antennas that are moved along profiles on the ground. Depending on the antenna system used, frequencies range from approximately 10 MHz to more than 1 GHz. The corresponding wavelengths range from about 10 m to less than 1 cm in typical surficial material. Surface GPR can therefore be regarded as an "echo-sounding" technique using high-frequency electromagnetic waves. Due to its versatility and high resolution, the GPR technique is widely used throughout the earth, environmental, and civil engineering sciences. Moreover, GPR has established itself as a key remote sensing method in a variety of other fields, such as archeology and criminology.

Understanding the radiation characteristics of GPR antennas located on the earth's surface is vital to improving data acquisition, processing, and interpretation strategies (e.g., Lehmann et al., 2000; Lampe et al., 2003). So far, the generation and propagation of electromagnetic waves in GPR surveys are understood only under simplified and idealized conditions, such as infinitesimal dipole transmitters and receivers located on homogeneous or layered dielectric half-spaces (e.g., Annan et al., 1975; Dai and Young, 1997). Asymptotic far-field expressions for the radiation of horizontal electric dipoles (Engheta et al., 1982; Smith, 1984) are commonly used to estimate the radiation characteristics of GPR antennas. These simplified analyses indicate that the height of antennas above the ground and large-scale variations of subsurface electric material properties have pronounced effects on the radiation patterns and thus on the illumination of the shallow subsurface. It is, however, not known how adequate such idealized approaches are for more complex earth models characterized by realistic fluctuations of surface topography and/or small-scale heterogeneous distributions of subsurface physical properties. In particular, it is not clear whether such complications only increase the amount of clutter (i.e., incoher-
ent backscattered energy) in the recorded GPR data or whether they fundamentally affect the coupling of the antenna to the ground and therefore the illumination of the subsurface.

To address these questions, we simulate and analyze GPR antenna radiation patterns positioned above a variety of realistic earth models characterized by small-scale fluctuations of the topography of the surface and/or the permittivity and electrical conductivity of the subsurface.

3.3 Antenna model

Simulations are performed using a GPR antenna simulation tool based on a finite-difference time-domain (FDTD) solution of Maxwell’s equations in three dimensions (Lampe et al., 2003). The computational volume is surrounded by highly efficient absorbing boundary conditions known as generalized perfectly matched layers (Fang and Wu, 1996). Our antenna model consists of an unshielded bow-tie antenna with a flare angle of 60° and a length of 52 cm (Figure 3.1). These dimensions correspond to a broadband dipole-type GPR antenna with a dominant operating frequency of $\sim 400$ MHz. The antenna is modeled as a perfect electrical conductor by setting to zero the tangential electric field components within the bow-tie metal panels. The GPR antenna system is excited by a compact Gaussian voltage pulse fed into a one-dimensional transmission line model (Maloney et al., 1994) connected to the antenna terminals.

3.4 Earth models

3.4.1 Fractals

In GPR antenna modeling, the ground is usually represented as an idealized half-space with a flat air–soil interface and one or more homogeneous layers (e.g., Dai and Young, 1997). For realistic earth models, however, topographic roughness as well as subsurface heterogeneities need to be considered (Robertsson and Holliger, 1997). Topographic variations and fluctuations of subsurface material parameters are universally scale-invariant, or fractal (e.g., Turcotte, 1997). This implies that similar features occur over a wide range of scales. The inherent fractal nature of geological phenomena is for example illustrated by the fact that a picture of an outcrop needs to be scaled with an object of well-defined size, such as a person, a hammer, or a coin. Without such scaling, it is generally impossible to tell whether
CHAPTER 3. EFFECTS OF FRACTAL FLUCTUATIONS ON GPR

the picture was taken at the centimeter, meter, or even kilometer scale. A description of the small-scale, "random" features of geophysical earth models, such as the roughness of the air-soil interface and/or the distribution of the electrical material parameters in the subsurface, in terms of stochastic fractals is therefore appropriate and realistic. Such fractal fluctuations can be simulated by taking the square root of the desired power spectrum, randomizing the phase, and taking the inverse Fourier transform. This procedure provides a space-domain representation of fractal heterogeneities with a Gaussian probability density function. For our purposes, we use the so-called von Kármán autocovariance model (Goff and Jordan, 1988), which is suitable for characterizing band-limited, scale-invariant media. The corresponding power spectrum decays as

$$P(k) \propto \frac{1}{(1 + k^2a^2)^{\nu + E}}, \quad (3.1)$$

where $k$ is the spatial wavenumber, $a$ is the correlation length, $\nu$ is the Hurst number, and $E$ is the Euclidean dimension. For $ka << 1$, equation (3.1) corresponds to a white spectrum, whereas for $ka >> 1$, it emulates the power-law behavior of typical fractal phenomena (Mandelbrot, 1983, p. 254). The correlation length $a$ thus corresponds roughly to the "visual scale" (i.e., the scale of the spatially coherent "blobs") of a random fractal model. Features that are of the order of the correlation length or smaller are scale-invariant or fractal, whereas features that are significantly larger are spatially uncorrelated, i.e., "purely random" and not fractal. For $ka >> 1$, the fractal dimension $D$ of the corresponding stochastic process is given by

$$D = E + 1 - \nu, \quad (3.2)$$

where $0 \leq \nu \leq 1$. For a given Euclidian dimension $E$, the "roughness" and complexity of a fractal stochastic process thus increases with a decreasing Hurst number. As examples, for $E = 1$, $D = 1$ (i.e., $\nu = 1$) corresponds to a smooth curve with a continuous derivative, whereas $D = 2$ (i.e., $\nu = 0$) corresponds to a very rough "surface-filling" curve.

3.4.2 Topographic fluctuations and subsurface heterogeneities

Empirical evidence and theoretical models suggest that topographic fluctuations are governed by Brownian walk statistics (Turcotte, 1997, pp. 163–178). For $E = 1$, the power spectrum of a Brownian walk process is characterized by a spectral exponent
\( \beta \) of 2, such that the power spectrum decays as \( 1/k^2 \). From equation (3.1), the spectral exponent is obtained as

\[
\beta = 2\nu + 1. \tag{3.3}
\]

It follows that the Hurst number of a Brownian walk process is \( \nu = 0.5 \). With this value of \( \nu \) and an Euclidian dimension of \( E = 2 \), equation (3.2) gives a fractal dimension of \( D = 2.5 \) for our self-affine topography models.

There is widespread evidence from well-log data (i.e., \( E = 1 \)) that the power spectra of the distributions of virtually all petrophysical parameters in the crust scale uniformly as \( \sim 1/k \) (Hardy and Beier, 1994, p. 87, Leonardi and Kümpel, 1998, Everett and Weiss, 2002, Kelkar and Perez, 2002, p. 76). This corresponds to a Hurst number \( \nu \) close to 0 (equation (3.3)). For our three-dimensional (\( E = 3 \)) models of permittivity and conductivity distributions, we choose \( \nu = 0.1 \) and, correspondingly, \( D = 3.9 \) (equation (3.2)).

The parameterization of our stochastic models can be summarized as

\[
s(\mathbf{r}) = s_0 + \Delta s(\mathbf{r}), \tag{3.4}
\]

where \( \mathbf{r} \) denotes a specific location on the finite-difference grid and where \( s_0 \) (either constant elevation \( h \), constant permittivity \( \epsilon \), or constant conductivity \( \sigma \)) and \( \Delta s \) (either \( \Delta h \), \( \Delta \epsilon \), or \( \Delta \sigma \)) are the deterministic background value and stochastic component of a model parameter, respectively. For the topographic roughness and permittivity, we use a zero-mean Gaussian distribution for the stochastic component. This approach is consistent with empirical evidence for topographic fluctuations (Turcotte, 1997, pp. 163–178). The nature of permittivity distributions in the shallow subsurface is not well documented. Nevertheless, given the rather narrow range of permittivity values of common surficial rocks and soils (Davis and Annan, 1989), a Gaussian probability density function can be regarded as a reasonable first-order approximation.

The electrical conductivity in the crust can vary over several orders of magnitude. Empirical evidence suggests that the distribution of conductivity in the subsurface follows a lognormal probability density function (Bahr, 1997). A lognormally distributed random field \( \Delta s_{\text{ln}} \) is obtained from a Gaussian-distributed stochastic process \( \Delta s_{\text{Gauss}} \) through the following transformation (Turcotte, 1997, pp. 28–37):

\[
\Delta s_{\text{ln}} = e^{\Delta s_{\text{Gauss}}}. \tag{3.5}
\]

The lognormal distribution is characterized by a coefficient of variation

\[
c_v = \frac{s_{\text{ln}}}{\Delta s_{\text{ln}}} = \sqrt{e^{c^2_{\text{Gauss}}} - 1}, \tag{3.6}
\]
where $\sigma_{\text{Gauss}}$ is the standard deviation of the underlying Gaussian process and $\sigma_{\text{ln}}$ and $\bar{A} s_{\text{m}}$ are the standard deviation and mean value of the lognormal distribution, respectively.

Histograms of permittivity and conductivity for a typical subsurface model are displayed in Figure 3.2. Since most surficial geological materials are largely nonmagnetic, we assumed magnetic permeability of $\mu = \mu_0$ for all our simulations, with $\mu_0$ denoting the vacuum value. Figure 3.3 shows a vertical slice through one particular realization of a stochastic model characterized by fractal fluctuations of topography, subsurface permittivity, and conductivity (the pattern of conductivity variations is the same as that presented for the permittivity variations, only the scaling is different). In our numerical simulations, we have tested various realizations of the stochastic components of our models (corresponding to different seed numbers for the random number generator). In the following, however, we present simulations for one particular stochastic realization (Figure 3.3), since we found that the various realizations qualitatively lead to consistent results and conclusions.

### 3.5 Results

#### 3.5.1 Homogeneous half-space

The dimensions of subsurface models presented in this contribution are $1.7 \times 1.7 \times 1.7$ m. Figures 3.4a and 3.4b show $E$- and $H$-plane views of the electric field $E_x$ component propagating into a homogeneous dielectric half-space ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m) from an antenna placed 1 cm above a flat air–soil interface. In both snapshots, the wavefronts in the upper half-space (air) have already left the computational volume, whereas the wavefronts in the lower medium show reverberations from the undamped antenna. Parts of the head waves are present at a depth of about 50 cm near the left and right model edges. Radiation patterns corresponding to the intensity of energy flow (i.e., the radial component $S_r$ of the time-averaged Poynting vector $<\mathbf{S}> = <\mathbf{E} \times \mathbf{H}>$) at a distance of 70 cm from a point on the interface directly below the central gap of the antenna are shown in Figures 3.4c and 3.4d. In this and all other radiation patterns, the data are normalized with respect to the maximum amplitude, and the values in bold type are the ratios of the maximum amplitudes of the respective radiation patterns to the maximum of the $H$-plane radiation pattern for the lossless dielectric half-space (i.e., Figure 3.4d). The radiation patterns for this reference model (Figures 3.4c and 3.4d) are displayed as gray dashed lines in all corresponding figures for the various lossless subsurface
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models.

Snapshots and radiation patterns for a homogeneous half-space with the same permittivity as Figure 3.4, but with constant conductivity of 10 mS/m, are presented in Figure 3.5. Note the markedly reduced amplitudes compared to the lossless model. The reference radiation patterns of Figures 3.5c and 3.5d are displayed as gray dashed lines in all relevant figures for the various conductive half-space models.

The height of the antenna above the ground has an effect on the input impedance of the antenna. This is demonstrated in Figure 3.6, in which the input resistance and input reactance (i.e., the real and complex parts of the input impedance) are plotted as functions of frequency for various antenna heights above a flat half-space. Although the overall shapes of the curves are similar, the main peaks and troughs are shifted to higher frequencies as the antenna height is increased (i.e., the optimal excitation frequency range has shifted to higher values). Moreover, for these antenna heights, the maximum values of the input impedances increase with increasing antenna height as a response to the higher wave impedance of free space compared to the dielectric ground (see also Bhattacharyya, 1963). The antenna performance, such as transmitted and radiated power, depends critically on the input impedance for which the antenna system is designed (Balanis, 1997, pp. 73-77). Although the transmitter output and the receiver input of common GPR systems are generally designed to match input impedances of a certain range, impedance changes of the above magnitude might nevertheless affect the performance of GPR antenna systems.

Figure 3.7 displays the normalized maximum of the energy flow in the lower half-space as a function of antenna height over the subsurface (corresponding to the maximum values of the energy radiation patterns). These curves show that for the first few centimeters above the ground, the maximum values of these radiation patterns in the lower half-space increase with increasing height. Above a certain height (approximately 5 cm for the E'-plane and 2 cm for the H-plane), the maximum value of the energy radiated into the lower half-space starts to decrease. Note that Figure 3.7 only shows the maximum values of the energy flux, which is not representative of the total energy radiated into the half-space. Comparable results have been obtained by Smith (1984), Wensink et al. (1990), and Turner (1994). Following Turner (1994), we interpret this observation to be due to the fact that for small elevations above the interface, the directivity of the antenna is increased whereas its coupling to the interface is not yet significantly diminished (see also Smith, 1984).
3.5.2 Topographic roughness

For the next simulation, the flat air–soil interface of the homogeneous half-space model is replaced by random topographic fluctuations with a standard deviation of 2.91 cm (Figure 3.8). The resulting snapshots and radiation patterns reveal distortions in both planes with respect to the reference patterns for the homogeneous half-space.

We have carried out a number of simulations for models with differing topographies. For most of these models, the maximum energy in both the $E$- and $H$-planes has higher values than the reference radiation patterns for the homogeneous lossless half-space model. Based on the results illustrated in Figure 3.7, we interpret this as being mainly because the average antenna height in the presence of topographic roughness is slightly larger than that used for our reference flat-surface model.

Figure 3.9 displays input impedances for models whose topographic roughness results in average antenna heights of 1.8 and 3.4 cm. For comparison, the impedance of the same antenna placed 1 cm above the corresponding reference model with planar interface is also shown. As for Figure 3.6, a marked shift of the curves toward higher frequencies as well as an increase in the peak values is observed for increasing antenna heights. Indeed, the impedance values in Figures 3.6 and 3.9 are quite similar, albeit not identical, which indicates that the impedance is not invariant with surface roughness. We therefore interpret the effect of the topographic roughness on the antenna characteristics as a combination of scattering and changes in coupling. We find that the overall changes of the radiated energy for the different topographic models are comparable to those observed for models with similar antenna heights above a flat air–soil interface. Moreover, by correlating the asymmetric deformations of the radiation patterns for different realizations of the models with the structure of the corresponding topographies, we infer that these deformations are primarily due to scattering. Our results thus indicate that the maximum values of the radiation patterns as well as the input impedance are strongly influenced by the average height of the antenna, whereas the asymmetric deformations of the radiation patterns observed for these simulations, such as the $H$-plane radiation pattern in Figure 3.8, are primarily a consequence of laterally varying scattering.

3.5.3 Permittivity fluctuations

The effects of subsurface permittivity fluctuations on the energy radiated from GPR antennas are shown in Figure 3.10. The standard deviations of the permittivity fluctuations are 2.5% (Figures 3.10a and 3.10b), 5% (Figures 3.10c and 3.10d), and
10% (Figures 3.10e–3.10j) with respect to the deterministic background permittivity of \(5\varepsilon_0\). To highlight the influence of the permittivity fluctuations, models shown in Figure 3.10 have flat surfaces and zero conductivities.

Permittivity fluctuations of 2.5% produce only small changes in the radiation patterns (Figures 3.10a and 3.10b). For a constant correlation length (Figures 3.10a–3.10f), the distortions of the radiation patterns become progressively more pronounced with increasing permittivity fluctuations.

Figures 3.10e–3.10j show radiation patterns for permittivity fluctuations with a standard deviation of 10% and varying correlation lengths \(a\) of 1, 0.1, and 5 m. Small-scale distortions of the radiation patterns are most noticeable for the correlation length \(a = 0.1\) m (compare Figures 3.10g and 3.10h with Figures 3.10e, 3.10f, 3.10i, and 3.10j). For the 400-MHz antenna and a background permittivity of \(5\varepsilon_0\), a correlation length of 0.1 m corresponds roughly to the case \(\kappa a \approx 1\) (\(\kappa\) is the wavenumber of the electromagnetic waves), where maximum scattering can be expected (Wu and Aki, 1988). Note that a correlation length of 5 m exceeds the dimensions of our model space, such that the corresponding "random" medium can be regarded as a pure, non-bandlimited fractal.

An important result of our simulations is that the input impedance is not significantly affected by the permittivity fluctuations (Figure 3.11); for the model with high permittivity fluctuations (standard deviation of 10%), neither the input resistance nor reactance show substantial deviations from the corresponding impedance curves for the homogeneous half-space reference model. These observations suggest that the coupling of the antenna to the interface is not markedly affected by stochastic fluctuations of subsurface permittivity. This is consistent with results obtained by Slob and Fokkema (2002) for a perfectly conducting wire antenna located over a layered half-space. As a consequence, we assume that the observed radiation pattern perturbations shown in Figure 3.10 are primarily from scattering effects along the wave propagation path.

### 3.5.4 Topographic roughness and permittivity fluctuations

Snapshots and radiation patterns for a GPR antenna located on an inhomogeneous half-space with topographic roughness and permittivity fluctuations are displayed in Figure 3.12. The amplitudes and shapes of the radiation patterns in the upper half-space are similar to those observed for the model characterized by topographic roughness only (Figure 3.8). In the lower half-space, the effects of the topographic roughness and permittivity fluctuations superpose, such that the \(H\)-plane radiation
pattern is dominated by the effect of increased average antenna height (Figure 3.8) and the $E$-plane radiation pattern is influenced approximately equally by the two phenomena. This is in general agreement with our observations with regard to the height of the antenna above a flat half-space, which indicate that the variability of the radiated energy in the $H$-plane differs from that in the $E$-plane (Figure 3.7). The small-scale variations of the radiation patterns clearly show the "fingerprint" of scattering as a result of the permittivity fluctuations (Figures 3.10e and 3.10f).

### 3.5.5 Conductivity fluctuations

For the radiation patterns in Figure 3.13, the lossless dielectric half-space model is replaced by a lossy one with a uniform permittivity of $\epsilon = 5\varepsilon_0$ and the stochastic conductivity distribution shown in Figure 3.2. This model has a mean conductivity of 10 mS/m, a coefficient of variation of 0.75 (equation (3.6)), and minimum and maximum conductivities of 1 and 115 mS/m, respectively. The radiation patterns in this illustration are now compared to those of the second reference model, which corresponds to a homogeneous conductive half-space with $\epsilon = 5\varepsilon_0$ and $\sigma = 10$ mS/m (Figure 3.5). Finite conductivities primarily affect the radiated amplitudes. In the GPR wave-propagation regime ($\frac{\omega}{c} \ll 1$, where $\omega$ is the angular frequency of the georadar signal), the attenuation coefficient can be expressed as

$$\alpha \approx \frac{1}{2} \sigma \sqrt{\frac{\mu}{\epsilon}},$$

such that the damping of electromagnetic waves depends linearly on the conductivity. By comparison, fluctuations of velocity ($v \approx \frac{1}{\sqrt{\mu}}$) are to a first-order approximation independent of $\sigma$. Scattering effects from conductivity fluctuations are, therefore, expected to be small. However, the lognormal distribution of conductivity results in small parts of the model being outside the GPR regime. In these regions, $\sigma$ causes both attenuation and scattering. With this in mind, the smooth larger-scale distortions of the radiation patterns in Figure 3.13 are interpreted as being due to laterally variable absorption in the inhomogeneous subsurface, whereas the smaller scale perturbations are likely the result of minor scattering.

Figure 3.14 shows input impedances as a function of frequency for antennas located above a half-space with constant conductivity and a half-space with stochastically varying conductivity. For comparison, the input impedance for the lossless reference model is also plotted. Differences between the results for the conductive (or lossy) models are insignificant. Moreover, the only significant deviations of these results from the curve for the lossless model occur at low frequencies, where
the electromagnetic energy travels in a diffusive rather than in a wave-propagation mode. This, however, does not affect the antenna characteristics in the operating frequency range. These results are consistent with our interpretation that subsurface variabilities of permittivity and conductivity primarily affect the antenna radiation by scattering and attenuation but do not change significantly the coupling of the GPR antenna to the ground.

### 3.5.6 Conductivity and permittivity fluctuations

Figure 3.15 shows the influence of combined permittivity and conductivity fluctuations (deterministic background values are $5\varepsilon_0$ and 10 mS/m with standard deviations $0.5\varepsilon_0$ and 6.2 mS/m for permittivity and conductivity, respectively). Note that the spatial distributions of permittivity and conductivity are perfectly correlated (Figure 3.3). Comparison of Figure 3.15 with Figures 3.10e and 3.10f suggests that small-scale perturbations in the radiation patterns are governed mainly by the permittivity fluctuations, whereas the conductivity has a substantial modulating effect on the larger scale variations.

### 3.5.7 Conductivity, permittivity, and topography

Finally, Figure 3.16 shows the results for the realistic case of a combination of all three stochastic parameters. Fluctuations in topography, permittivity, and conductivity are described in Figure 3.3. The effects of each component of fluctuation are shown in Figures 3.8, 3.10, and 3.13. Conductivity fluctuations are primarily responsible for the overall attenuation of energy radiation into the lower half-space, whereas distortions of the shape of the radiation patterns seem to be dominated by topographic roughness and permittivity fluctuations.

### 3.6 Conclusions

We have investigated the effects of topographic roughness and stochastic subsurface permittivity and conductivity heterogeneities on the radiation characteristics of a typical GPR antenna. Topographic roughness has a significant effect on the overall shape of the radiation patterns, which we quantify by the time-averaged Poynting vector. The maximum energy radiated into the lower half-spaces is generally higher for interfaces with topographic roughness. This is likely because topography not only causes scattering but also affects the average height of the antenna above the
ground and thus its coupling to the interface. Scattering that results from permittivity fluctuations in the subsurface also perturbs the radiation patterns. Such perturbations increase with increasing magnitude of the permittivity fluctuations. Scattering effects are particularly pronounced for a correlation length of 0.1 m, which corresponds approximately to the case $\kappa a \approx 1$, where maximum scattering can be expected. The primary influence of conductivity fluctuations on antenna radiation is caused by laterally variable attenuation, whereas scattering effects seem to be of rather minor importance. Our study indicates that neither permittivity nor conductivity fluctuations alter significantly the coupling of the GPR antenna to the ground. Combinations of the various stochastic model components do not point to a pronounced dominance of one or the other parameter. Rather, the radiation patterns seem to reflect a complex superposition of the individual contributions. This investigation indicates that plausible stochastic fluctuations of the topography of the air-soil interface and the electrical material parameters in the subsurface may cause GPR data to differ substantially from those expected for a corresponding idealized half-space model.

3.7 Acknowledgments

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Figure 3.1: Sketch of the simulated bow-tie antenna fed by a transmission line.

Figure 3.2: Histograms of simulated permittivity (left) and conductivity (right) heterogeneities. Permittivity has a Gaussian distribution with $5\varepsilon_0$ mean value and $0.5\varepsilon_0$ standard deviation. Conductivity is lognormally distributed with 10 mS/m mean value and 6.2 mS/m standard deviation. The coefficient of variation (equation (3.6)) for the lognormal conductivity distribution is $c_v = 0.75$.

Figure 3.3: Vertical slices through a 3D fractal model in the $E$-plane (left) and $H$-plane (right), characterized by scale-invariant fluctuations of topography, permittivity, and conductivity (not displayed). Mean values and standard deviations of permittivity and conductivity are as for Figure 3.2. Standard deviation and maximum variations of topography are 2.91 cm and ±9 cm, respectively. Correlation length $\alpha$ is 1 m for all fluctuations.
Figure 3.4: (a) $E$-plane and (b) $H$-plane snapshots of the electric field $E_x$ component radiated from a bow-tie antenna located 1 cm above a flat-surface homogeneous half-space ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). (c) and (d) show radiation patterns corresponding to the radial component of the Poynting vector in the $E$- and $H$-planes, respectively. Recording distance is 70 cm. Values in bold type = (maximum amplitude of radiation pattern) / (maximum amplitude of $H$-plane radiation pattern).
Figure 3.5: As for Figure 3.4, but with a conductive half-space ($\epsilon = 5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 10$ mS/m). The radiation patterns are normalized with respect to the maximum amplitude in Figure 3.4. Values in bold type = (maximum amplitude of radiation pattern) / (maximum amplitude of $H$-plane radiation pattern in Figure 3.4).
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Figure 3.6: Input impedances of a bow-tie antenna for various heights $h$ above a flat-surface homogeneous half-space ($\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m).

Figure 3.7: Normalized maximum amplitudes in $E$-plane (left) and $H$-plane (right) for a bow-tie antenna located at various heights above a flat-surface homogeneous half-space ($\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m).
Figure 3.8: As for Figure 3.4, but with the antenna located above an air-soil interface with random topographic fluctuations (see topography in Figure 3.3). Arrow indicates center of the antenna. Black lines in (c) and (d) correspond to the radiation patterns for the model in (a) and (b). Gray dashed lines show radiation patterns for a flat-surface homogeneous half-space. Values in bold type = (maximum amplitude of radiation pattern) / (maximum amplitude of H-plane radiation pattern in Figure 3.4).
Figure 3.9: Input impedances of a bow-tie antenna for two average heights $\bar{h}$ (1.8 and 3.4 cm) above a homogeneous half-space ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m) with topographic roughness. For comparison, the input impedance of the antenna above a flat surface with $h = 1$ cm is also plotted.
Figure 3.10: Radiation patterns corresponding to the radial component of the Poynting vector in the E-plane (left) and H-plane (right) for a half-space with random permittivity fluctuations and a flat air-soil interface (black lines). Standard deviations in percent of the background permittivity and correlation lengths of the random fluctuations are (a) and (b) 2.5% and 1 m, (c) and (d) 5% and 1 m, (e) and (f) 10% and 1 m, (g) and (h) 10% and 0.1 m, and (i) and (j) 10% and 5 m. Recording distance is 70 cm. Gray dashed lines show the radiation patterns for a homogeneous half-space with a flat air-soil interface. Values in bold type = (maximum amplitude of radiation pattern) / (maximum amplitude of H-plane radiation pattern in Figure 3.4).
Figure 3.11: Input impedances of a bow-tie antenna located above half-spaces with a flat surface ($h = 1$ cm) and either a constant permittivity (solid line) or the varying permittivities shown in Figures 3.2 and 3.3 (dotted line).
Figure 3.12: As for Figure 3.8, but with random permittivity fluctuations with a standard deviation of 10% with respect to the background permittivity. Model is shown in Figure 3.3.
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Figure 3.13: Radiation patterns corresponding to the radial component of the Poynting vector in the E-plane (left) and H-plane (right) for a half-space with random conductivity fluctuations (Figure 3.2) and a flat air-soil interface (black lines). Correlation length is 1 m. Recording distance is 70 cm. Gray dashed lines show the radiation patterns for a homogeneous conductive half-space (Figure 3.5). Values in bold type = (maximum amplitude of radiation pattern) / (maximum amplitude of H-plane radiation pattern in Figure 3.4).

Figure 3.14: Input impedances of a bow-tie antenna located above half-spaces with a flat surface \( (h = 1 \text{ cm}) \) and either constant conductivity \( (0 \text{ or } 10 \text{ mS/m}) \) or the varying conductivities shown in Figure 3.2.
Figure 3.15: As for Figure 3.13, but with random permittivity fluctuations as shown in Figures 3.2 and 3.3.

Figure 3.16: As for Figure 3.15, but with the addition of random topographic fluctuations as shown in Figure 3.3.
Chapter 4

Resistively Loaded Antennas for Ground-Penetrating Radar: A Modeling Approach

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submitted to Geophysics

4.1 Abstract

The design of surface ground-penetrating radar antennas is inherently difficult, primarily because the presence of the air-soil interface greatly complicates both analytic and laboratory-based approaches aimed at characterizing the antennas. Versatile numerical simulation techniques capable of describing the key physical principles governing GPR antenna radiation offer new solutions to this problem. We use a finite-difference time-domain (FDTD) solution of Maxwell's equations in three dimensions to explore the radiation characteristics of various bow-tie antennas (including quasi-linear antennas) operating in different environments. The antenna panels are either modeled as having an infinite conductivity (i.e., a perfect electrical conductor (PEC)), a constant finite conductivity, or a Wu-King finite-conductivity profile. Finite conductivities are accommodated through a subcell extension of the classical FDTD approach, with the model space surrounded by highly efficient generalized perfectly matched layer (GPML) absorbing boundary conditions. Input
impedances, radiated waveforms, and radiation patterns of bow-tie antennas with Wu-King conductivity profiles are largely invariant when placed in free space or above diverse half-space earth models. By comparison, antennas with PEC or constant finite-conductivity panels have variable characteristics that depend somewhat on their operating environment, with planar broadband bow-tie antennas exhibiting a higher degree of sensitivity to their environment than quasi-linear ones. Antennas with constant finite-conductivity panels are considerably more robust (i.e., less sensitive to their environment) than their PEC analogs, because the loss resistance is increased and the effective electrical length of the antenna becomes shorter when the antenna panels are resistively loaded. For the extreme case of Wu-King conductivity profiles, the current in the antenna panels approaches that of a quasi-infinitesimal electric dipole. This is shown by the surface-charge distributions on the various antennas and by the corresponding energy radiation patterns. Unfortunately, the favourable characteristics of the latter antennas are counter-balanced by markedly lower radiation efficiency. For the antenna designs considered in this study, we found that the peak energy radiated into earth models from bow-tie antennas with Wu-King conductivity profiles is about one order-of-magnitude lower than for antennas with PEC terminals.

4.2 Introduction

In contrast to conventional microwave antenna systems, which are designed for the transmission or reception of high-frequency electromagnetic waves in nearly homogeneous free-space conditions, ground-penetrating radar (GPR) antennas deployed on or near the earth's surface have to cope with varying surface conditions and highly heterogeneous subsurface soils and rocks. Such environments may have a strong influence on the radiation properties and performance of GPR antennas (e.g., Annan et al., 1975; Engheta et al., 1982; Smith, 1984; Arcone, 1995; Radzevicius et al., 2003; Lampe et al., 2003). Ideally, a well-designed GPR antenna should (i) have stable antenna characteristics for a wide variety of acquisition conditions, (ii) radiate a consistent broadband pulse, and (iii) be characterized by high transmitter efficiency.

For efficiency, the antenna should be minimally resistive. Undamped antennas, however, tend to be distinguished by multiple reflections from the antenna ends. This results in a "ringy" emitted/received signal that interferes with primary reflections from the shallow subsurface. This problem can be reduced by loading the GPR
antennas with resistors. A key study of optimized resistive loading was presented by Wu and King (1965). To eliminate internal reflections, they proposed increasing the resistivity of the antenna arms towards their ends. Such Wu-King designs have been successfully applied to the optimization of several types of antennas (TEM horns—Kanda (1983); V-antennas—Esselle and Stuchly (1991); parallel-plate waveguides—Maloney and Smith (1992b); conical monopoles—Maloney and Smith (1993a); cylindrical monopoles—Maloney and Smith (1993b); bow-tie antennas—Shlager et al. (1994)). All of these studies, as well as the original work of Wu and King (1965), were conducted on the basis of antennas operating in free space. Since antenna characteristics are affected by the presence of dielectric interfaces, it is not clear how applicable these results are to surface-based GPR investigations.

The experimental characterization of GPR antenna systems located at or near the earth’s surface is inherently difficult (e.g., Annan et al., 1975; Wensink et al., 1990). Consequently, realistic numerical simulations of such systems are of considerable importance. In this study, we use a finite-difference time-domain (FDTD) modeling approach to explore and compare the characteristics of various realizations of bow-tie antennas. The accuracy of the modeling approach is benchmarked via a comparison of modeled input impedance data with laboratory-measured values. We then provide details about the resistive loading optimization process. Finally, we investigate input impedances, radiated waveforms, and radiation patterns for various antenna realizations operating under diverse conditions, including varying degrees of subsurface moisture and topographic roughness of the air-soil interface.

4.3 Methodology

4.3.1 Modeling approach

Our antenna simulation tool is based on a FDTD solution of Maxwell’s equations in 3D Cartesian coordinates (Lampe et al., 2003). The computational volume is surrounded by highly efficient absorbing boundary conditions known as generalized perfectly matched layer (GPML; Fang and Wu, 1996). We only consider bow-tie antennas excited by a compact Gaussian voltage pulse fed into a one-dimensional transmission line model (Maloney et al., 1994) connected to the antenna input terminals (Figure 4.1).

In a previous study (Lampe et al., 2003), we demonstrated the accuracy and versatility of our simulation tool for undamped antennas modeled as perfect electrical conductors (PEC’s; the electric field components parallel to the metal surfaces
of the antenna panels were set to zero). In the present study, we extend our algorithm to accommodate finite electrical conductivities in the antennas using the subcell method described by Maloney and Smith (1992a). This algorithm allows us to model material sheets much thinner than the grid spacing. These sheets may have arbitrary conductivity distributions, making them suitable for modeling realistic resistively loaded GPR antenna systems.

To benchmark the accuracy of the modified algorithm, we model the impedances (i.e., resistances and reactances) of bow-tie antennas with a constant conductivity of \(10^7\) S/m, which corresponds approximately to the conductivity of copper. Figure 4.2 shows a comparison of these numerical results with corresponding laboratory measurements (Brown and Woodward, 1952) for bow-tie antennas with flare angles of 5°, 10°, 40°, and 90°. The impedances are plotted as a function of the length of the antenna panel in electrical degrees \(A\) (i.e., the ratio of antenna panel length \(l\) to wavelength \(\lambda\) multiplied by 360°: \(A = l/\lambda \times 360°\); \(A\) is effectively a measure of frequency). We scale the modeled impedances to the laboratory-measured ones, which we multiply by a factor of two, because they refer to monopole antennas. Lampe et al. (2003) compare the same laboratory values to the impedances computed for a suite of bow-tie antennas distinguished by infinite conductivities (i.e., the PEC approach).

Overall, the agreement between the observed and modeled input impedances in Figure 4.2 is excellent, demonstrating the validity of our modeling approach. The antenna input impedances become increasingly broadband with increasing flare angle. Peaks and troughs in the input impedance curves become more pronounced with decreasing flare angle as a consequence of frequency tuning within the antenna. This results from the electrical length of the antenna (i.e., the path travelled by the current expressed in terms of wavelengths) becoming increasingly well defined as the flare angle decreases, such that the bow-tie antenna begins to resemble a linear wire antenna. Moreover, with decreasing flare angle, the numerical results tend to be systematically shifted towards higher frequencies compared to the laboratory-measured values. This can be explained by capacitances building up at the ends of the real antennas (Lampe et al., 2003), effectively increasing their electrical length.

In addition to bow-tie antennas, wire-dipole antennas are quite common in GPR applications. PEC versions of such antennas can be accommodated by the FDTD method using subcell algorithms (Umashankar et al., 1987; Holliger and Bergmann, 1998). These algorithms cannot be readily extended to account for the finite conductivities of antenna wires. Fortunately, bow-tie antennas with small flare angles
approximate well the characteristics of wire-dipole antennas. This is demonstrated in Figure 4.3, in which modeled impedance curves for a PEC wire-dipole antenna with a radius of 2 mm and bow-tie antennas (PEC and constant conductivity of $10^7$ S/m) with a flare angle of $5^\circ$ are compared with the relevant laboratory measurements (Brown and Woodward, 1952). The small differences between the impedances for the PEC antenna models and those for the thin-sheet approximation with a conductivity of $10^7$ S/m (e.g., as for copper) demonstrate that the metal parts of the antennas, as long as they are not resistively loaded, are approximated well by the PEC approach. Conversely, thin-sheet antennas made of copper are good approximations for idealized PEC antennas. Overall, the mismatch between the modeled results for the wire-dipole and the $5^\circ$-bow-tie antennas is rather small, comparable in magnitude and character to the mismatch between the modeled and laboratory-measured values. In the following, we use bow-tie antennas with a flare angle of $5^\circ$ as an approximation for wire-dipole antennas.

4.3.2 Wu-King resistive loading of antennas

Wu-King resistive loading of antennas is characterized by a resistance distribution of the form (Shlager et al., 1994):

$$R(r/l) = R(l/2) \frac{r/l}{1 - r/l},$$

where $R$ is the resistance per square unit, $r$ is the distance from the antenna input terminal, $l$ is the length of the antenna panel, and $R(l/2)$ is the resistance at $r = l/2$. In this paper, $r$ and $l$ are measured with respect to the long axis of the triangular bow-tie antennas (Figure 4.1). The corresponding conductivity profile is then defined as:

$$\sigma(r/l = 0) = \sigma_0 \quad \text{and} \quad \sigma(r/l > 0) = \sigma(1/2) \frac{1 - r/l}{r/l},$$

where $\sigma(1/2)$ is the reciprocal of the product of $R(1/2) \Delta V$, and $\Delta V$ is the volume of a conducting-sheet grid cell. Thus, the conductivity profile for the discrete antenna models not only depends on the antenna length, but also on the discretization (i.e., on the grid size of the model). To avoid infinitely high conductivities at the input terminals of the discrete antenna models, the conductivity at $r = 0$ is fixed to a specified value $\sigma_0$. The resistance/conductivity profile of a Wu-King-loaded discrete antenna model is fully constrained by the choice of $\sigma_0$ and $\sigma(1/2)$ (see equation 4.2).

In Figure 4.4, a discretized Wu-King conductivity profile for a bow-tie antenna panel with a length of 26 grid cells is compared with the corresponding continuous
profile. Depending on the length of the antenna panel, a high value of $\sigma_0$ may lead to significant conductivity steps near $r = 0$, resulting in undesired reflections within the antenna panel. Conversely, a low value of $\sigma_0$ can lead to pronounced reflections at the antenna input terminals. The parameter $\sigma(1/2)$ affects the rate of decay of the conductivity profile. For an effective reduction of end reflections, this parameter should be chosen to ensure that the resulting conductivity curve approaches small values at the outer edges of the panels.

A well-known drawback of Wu-King conductivity profiles is the tradeoff between ringing suppression and antenna radiation efficiency (e.g., Wu and King, 1965; Maloney and Smith, 1993a). Finding an appropriate Wu-King profile for a given antenna is, therefore, an experimental optimization problem that in part can be addressed through numerical modeling studies.

4.4 Results

In the following, we investigate the responses of bow-tie antennas with flare angles of 5° and 90°. As illustrated in Figure 4.3, the 5°-bow-tie antenna is a reasonable approximation for a wire-dipole antenna. For both bow-tie antennas, we consider three different types of conductivity distribution in the antenna panels: PEC, constant finite-conductivity (or constant resistive loading), and Wu-King-type conductivity profiles. The latter two types represent common resistive loading employed by practical designers and manufacturers of GPR antennas (P. Annan, personal communication, 2003). We first consider the responses of these antennas in free space and then evaluate their performances for a variety of earth models.

4.4.1 Antenna characteristics in free space

Figure 4.5 illustrates some key effects of the three basic types of conductivity distribution in the antenna panels for a bow-tie antenna with a flare angle of 90° and length $l = 26$ cm. Shown are the incident and reflected voltage pulses in the transmission line for PEC, constant finite-conductivity loading with $\sigma = 5000$ S/m, and Wu-King loading with $\sigma_0 = 10^4$ S/m and $\sigma(1/2) = 200$ S/m (see equation 4.2). Peaks at $\sim 1.2$ and $\sim 3.8$ ns represent the incident voltage pulse and its reflection at the input terminals, respectively. Those at $\sim 6.5$ ns correspond to reflections from the edges of the antenna panels, which should be minimal for a well-designed antenna system. For the PEC antenna, the reflection from the antenna input terminal is comparatively small. In contrast, constant finite-conductivity and Wu-King
loading of the antenna panels result in relatively strong reflections at the antenna input terminals because of relatively large mismatches between the transmission line and the antenna impedances. For all antennas, this reflection could be substantially reduced by appropriately adjusting the characteristic impedance of the transmission line with respect to that of the antenna. This is, however, not an objective of this study. Compared to the PEC antenna, constant finite-conductivity loading of the panels reduces notably the reflections from the edges of the panels, whereas Wu-King loading effectively eliminates internal reflections.

Figure 4.6 displays the input impedances for the investigated antennas. For both flare angles, Wu-King antennas are more broadband than PEC antennas or antennas with constant finite-conductivity loading, with the magnitudes of the resistance and reactance converging rapidly, approaching constant values.

Figures 4.7 and 4.8 compare the waveforms and corresponding amplitude spectra of the tangential component of the electric field radiated by the different antennas. The recordings were simulated at a distance of 70 cm below the antenna terminals. As expected from Figures 4.5 and 4.6, the pulses radiated by the Wu-King antennas are much more broadband (lower diagrams of Figures 4.7 and 4.8) and much less distorted (upper diagrams of Figures 4.7 and 4.8) than those radiated from the other two antenna types. In agreement with analytic expressions for dipole radiation (e.g., Smith, 1997), this pulse shape corresponds to the derivative of the induced Gaussian source waveform (Figure 4.5).

The spectra of pulses radiated from the Wu-King antennas are notably broader and have only one maximum at higher frequencies than the dominant peaks of the spectra of pulses radiated from the other two antenna types. This indicates that, although the physical dimensions of all antennas are identical, the effective electrical lengths of the Wu-King antennas and corresponding pulses are much shorter than those of the PEC and constant finite-conductivity antennas. The spectra of pulses emitted by the PEC and constantly loaded antennas have 1–2 subsidiary maxima as a result of resonances within the antennas. These maxima are more pronounced for the 5°-antennas (Figure 4.7), which have a marginally narrower bandwidth than the 90°-antennas (Figures 4.8). For the 90°-antennas, the spectra for the PEC and the constant finite-conductivity antennas are very similar, both being moderately broadband.

Unfortunately, the increase in bandwidth and reduction of ringing in the Wu-King antennas are accompanied by a pronounced decrease in antenna efficiency and thus in lower levels of radiated energy. The peak amplitudes for the Wu-King
antennas are only 30–40% of those for the corresponding PEC antennas (upper diagrams of Figures 4.7 and 4.8). For the constant finite-conductivity antennas, the peak amplitudes are reduced to 45–60% of their PEC analogs.

4.4.2 Antenna characteristics for a homogeneous half-space model

In the following, we explore the radiation characteristics of antennas located on a nonconductive homogeneous half-space earth model with permittivity $\epsilon = 5\epsilon_0$. These material parameters are representative of a dry sandy soil. For this and other half-space models considered in this study, the magnetic permeability $\mu$ is set to its free-space value $\mu_0$.

Figure 4.6 compares the input impedances for the half-space earth model with those for free space. For the PEC antenna panels (Figure 4.6a), the input impedances for both flare angles are noticeably affected by the presence of the half-space. The peaks and troughs in the input impedance curves are shifted towards lower frequencies (i.e., lower values in electrical degrees) relative to the free-space responses. The reason for this is that the antennas become electrically longer as they "sense" the higher permittivity and effective smaller wavelengths in the underlying half-space. Moreover, the maximum values of the input impedance curves are reduced due to the lower wave impedance of the dielectric half-space. The overall shapes of these input impedance curves are, however, similar to their free-space analogs.

The antennas with uniform resistive loading (Figure 4.6b) reveal a somewhat different behavior. Differences between the input impedance curves for the half-space and free-space models are much smaller for the constant finite-conductivity antenna than for the corresponding PEC antenna. Both the resistances and reactances show less variability with frequency and the antennas become more broadband. A reason for this is that the loss resistance of the antenna is not dependent on the outside local environment, but the radiation resistance does (the resistive part of the input impedance consists of radiation and the loss resistances (e.g., Balanis, 1997). Resistive loading only increases the loss resistance of the input impedance. As a consequence, the ratio of voltage to current at the terminal is mainly governed by the loss resistance, such that changes in the radiation resistance are less obvious in the input impedance curves for constant finite-conductivity antennas than for PEC antennas.

Input impedance curves for the Wu-King-loaded antennas are little affected by the presence of the dielectric half-space. Differences between the half-space and
free-space curves are hardly noticeable for the resistances and very small for the reactances. Moreover, the impedance curves for both flare angles are surprisingly similar.

Figures 4.9 and 4.10 show the waveforms and amplitude spectra of the tangential component of the electric fields radiated into the lower half-space at a distance of 70 cm below the antenna terminals. The pulses radiated from the PEC and constantly loaded antennas are characterized by complex multicyclical waveforms and bandlimited amplitude spectra similar to their free-space analogs (Figures 4.7 and 4.8). As for antennas in free space, the peak amplitudes for Wu-King antennas over the dielectric half-space are 30–40% of those for the corresponding PEC antennas (upper diagrams of Figures 4.9 and 4.10). For the constant finite-conductivity antennas, the peak amplitudes are reduced to 60–65% of their PEC analogs. These figures provide an indication of the reduction in penetration depth of resistively loaded antennas relative to PEC antennas. However, given that all practical GPR antennas are resistively loaded, these estimated reductions in amplitude are overly pessimistic.

The dominant maxima of the spectra of the radiated waveforms occur at slightly lower frequencies than those observed in the free-space spectra. In contrast, the pulses radiated from the Wu-King-loaded antennas and the corresponding spectra are not markedly affected by the presence of the half-space; the compact shapes of the pulses and the broadband nature of the spectra in Figures 4.9 and 4.10 are practically identical to those in Figures 4.7 and 4.8. Although Wu-King loading was originally developed for a free-space radiation environment, this antenna optimization procedure seems to be equally well suited for half-space applications.

### 4.4.3 Antenna characteristics for variable and inhomogeneous half-space models

To explore the performance of the different antenna types in the presence of variable subsurface conditions, we have placed them above half-spaces with material properties representing dry and wet soils. In our simulations, the dry soil has the same electrical material parameters as the half-space considered above ($\varepsilon = 5\varepsilon_0$, $\sigma = 0$ mS/m), whereas the wet soil has a permittivity $\varepsilon = 18\varepsilon_0$ and a conductivity $\sigma = 100$ mS/m. Unexpectedly, Figure 4.11 shows that the input impedance curves for all three antenna types are only slightly influenced by the very different subsurface conditions, with the curves for the Wu-King loading having the smallest variations with respect to the local environment and the curves for
the PEC antennas having the largest variations. The variations are greater for the 90°-antennas than for the 5°-antennas.

Finally, we place the antennas above a homogeneous half-space model ($\epsilon = 5\epsilon_0$, $\sigma = 0$ mS/m) characterized by fractal topographic roughness along the air-soil interface. The topographic models have a fractal dimension of 2.5, an isotropic correlation length of 1 m, and a standard deviation of 2 cm. Maximum differences in the elevations are ±6 cm. Details on the character and generation of such fractal surfaces are given in Lampe and Holliger (2003). Figure 4.12 demonstrates that the input impedances for the PEC antennas are more sensitive to topographic roughness than the others. Input impedance curves for the PEC antennas are characterized by small variations in the amplitudes and frequencies of the maxima and minima. Again, the relative magnitudes of these effects are a little more pronounced for the 90°-PEC antenna than for the corresponding 5°-antenna. In an earlier study of PEC antennas, Lampe and Holliger (2003) interpreted these differences in input impedance as being primarily due to the increased average height of the antennas above the ground as a result of the topographic roughness. Input impedance curves for the other two antennas are little affected by topographic roughness, with the curves for the Wu-King antennas being the least influenced, consistent with the observation that the performance of the Wu-King antennas is not greatly dependent on the presence or not of the half-space.

4.5 Discussion

4.5.1 Surface-charge distributions

The above observations suggest that the current distribution (and thus the surface-charge distribution) on the 90°-bow-tie PEC antenna is more severely affected by the presence of the half-space than currents on the 5°-bow-tie PEC antenna. This interpretation is supported by Figures 4.13 and 4.14, which compare snapshots of the surface-charge distributions on all three antenna types situated above a half-space and within free space at a fixed point in time (9.6 ns). The surface-charge distribution on the 90°-antenna in free space (Figure 4.14) has a complicated pattern resulting from diverse current paths involving multiple reflections of charges at the edges of the antenna. Surface-charge distributions on the same antenna located above a dielectric half-space are much more organized, resembling a dipole-type charge distribution with relatively homogeneous loading of the two antenna panels. For the 5°-PEC antenna (Figure 4.13), the surface-charge distributions for both
recording environments are relatively homogeneous. At the same point in time, the polarity of the surface-charge distribution is reversed for the two cases. Such a difference can be partially explained by the different effective electrical lengths of an antenna in close proximity to a dielectric half-space and the same antenna in free space (the current along the antenna close to the dielectric half-space is slowed down to an intermediate value between the velocity of electromagnetic waves in free space and that in the dielectric half-space (e.g., Rutledge and Muha, 1982; Compton et al., 1987)). In addition, there is a capacitive effect that arises from the interaction of the antenna with the dielectric half-space. This capacitive effect contributes to a slower exchange of charges on the antenna panels. Clearly, such capacitive effects tend to be more pronounced for planar antennas than for quasi-linear ones.

For the constant finite-conductivity antennas ($\sigma = 5000 \, \text{S/m}$), the surface-charge distribution on the 90°-antenna is again more severely affected by the presence of the half-space than that on the 5°-antenna. This finding is consistent with the information contained in the corresponding impedance curves of Figure 4.6b. Finally, the surface-charge distributions on the Wu-King antennas are similar for both recording environments, which is again consistent with the similarity of the impedance curves in Figure 4.6c. For both flare angles of the Wu-King antennas, the snapshots of the surface-charge distributions reveal a pronounced accumulation of charges towards the antenna edges. This charge accumulation is essentially stationary in time, such that reflections are not generated.

A likely reason for the remarkable robustness of antennas with Wu-King loading is the rapid decay of currents on the antennas with increasing distance from the feed point. As a consequence, changes of current distribution induced by interactions of the antennas with the ground are too small in magnitude and spatial extent to have noticeable effects on the waveform of the radiated signal or on the input impedance.

### 4.5.2 Radiation patterns

The reduction in radiation efficiency of Wu-King antennas results in a substantial loss of maximum radiated energy. This is illustrated by the energy radiation patterns for the basic antenna types considered in this study (Figures 4.15-4.18). The radiation patterns are quantified by the radial component of the time-averaged Poynting vector $S = \langle E \times H \rangle$, which is evaluated at a radial distance of 70 cm from a point directly below the input terminals on the air–soil interface. All radiation patterns have been normalized for display purposes. The bold numbers to the lower right of the radiation patterns are the following ratios: maximum amplitude of en-
energy radiated from the constant finite-conductivity or Wu-King antennas divided by maximum amplitude of $H$-plane radiation from the analog PEC antennas. For flare angles of 5° and 90°, the maximum energy fluxes radiated into the lower half-space by the Wu-King antennas are only 6 and 13% of the energy fluxes radiated by the PEC antennas, respectively (Figures 4.17 and 4.18). The corresponding percentages for the constantly loaded antennas are 16 and 37% (Figures 4.15 and 4.16).

The shapes of the radiation patterns for the constantly loaded antennas are very similar to those of the PEC antennas, whereas the radiation patterns for the Wu-King antennas are narrower and hence more directive. Furthermore, the radiation patterns for the Wu-King antennas are essentially independent of the antenna flare angle (compare Figures 4.17 and 4.18). Indeed, the Wu-King radiation patterns are strikingly similar to those of an infinitesimal electric dipole (Figure 4.19). Regardless of the antenna geometry and radiation environment, the effective current distributions (i.e., the current distributions responsible for radiation) on the Wu-King antennas are confined to regions close to the feed point and thus resemble the current distribution of a short electric dipole. It is for this reason that the differences in the impedance curves (Figure 4.6c), radiated waveforms (Figures 4.7–4.10), and radiation patterns (Figure 4.19) for the Wu-King antennas are rather insignificant. This interpretation is also consistent with the surface-charge distributions for the Wu-King antennas (Figures 4.13 and 4.14), which are largely independent of the antenna geometry.

4.6 Conclusions

We investigate the radiative properties of bow-tie GPR antennas with flare angles of 5° and 90° operating under different conditions. The antenna panels either have infinite conductivity (PEC), constant finite-conductivity or Wu-King-type conductivity distributions. The 5°-antenna can be regarded as a good approximation for a linear wire-dipole antenna, whereas the 90°-antenna is representative of a typical broadband planar antenna.

Radiative properties of PEC antennas are observed to be quite sensitive to their operating environment and to the geometry of the antenna panels. Constant finite-conductivity loading of the antenna panels decreases the sensitivity to their operating environment. This effect is more pronounced for the 5°-antenna than for the

\footnote{A comparable behavior is observed for antennas with a very low constant conductivity (not shown).}
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90°-antenna. Wu-King-antenna radiation patterns, by comparison, are remarkably similar for the two antenna-panel geometries and are not greatly influenced by the operating environment. We conclude that the Wu-King antenna loading criteria, originally developed for free space, are equally applicable for surface GPR antennas. Antennas of this type are broadband in nature and thus capable of radiating low-distortion signals that bear a close relation to the input voltage pulse driving the antenna.

Our analysis of surface-charge distributions on the antenna panels offers an explanation for most of the computational results. Decreasing the conductivity of the panels leads to a lowering of the total current on the antenna and a shortening of the effective electrical length of the antenna. As a consequence, the bandwidth of the antenna is increased and the input impedance becomes less sensitive to changes in the material parameters of its surroundings. For a Wu-King conductivity profile, the resulting surface-charge distributions on the antenna panels and the associated radiation patterns begin to approximate those of an infinitesimal electric dipole.

Unfortunately, the favorable characteristics of Wu-King antennas are accompanied by a high loss in radiation efficiency and lower levels in radiated energy. We found that the maximum energy radiated into half-space earth models is about one order-of-magnitude smaller for Wu-King antennas than for corresponding PEC antennas. In practice, standard Wu-King loading is thus probably impractical for most types of GPR antennas. It compromises the need to have robust broadband antennas with high radiation efficiency. For surface GPR environments, defining optimum antenna physical characteristics is a complex and largely empirical task; there are few theoretical guidelines and realistic laboratory measurements are not feasible. We suggest that numerical simulation approaches of the type presented in this study may play a key role in assisting GPR antenna developers in designing optimal antennas.

Finally, it is conceivable that an extensively parallelized version of the forward modeling code presented in this paper will allow us to realistically simulate and quantitatively interpret observed GPR data sets.

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Figure 4.1: Sketch of the simulated bow-tie GPR antenna system.

Figure 4.2: Comparison of observed and simulated input impedances for bow-tie antennas with varying flare angles. Solid lines—simulated values for thin-sheet antenna models with $\sigma = 10^7 \text{ S/m}$; crosses—laboratory-measured values of Brown and Woodward (1952). Lampe et al. (2003) compare the same laboratory-measured values to simulated values based on the PEC model. $A[\text{°}]$ is the length of the antenna panels in electrical degrees.
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Figure 4.3: Comparison of input impedances using different realizations of a bow-tie antenna with a flare-angle of 5° and a wire-dipole antenna. Solid line—PEC approximation; dotted line—thin sheet approximation with $\sigma = 10^7 \text{ S/m}$; dashed line—wire-dipole antenna; crosses—laboratory measurements of Brown and Woodward (1952).

Figure 4.4: Continuous (dashed line) and discretized (solid line) Wu-King conductivity profile. The discretized antenna panel has a length of 26 grid cells. The values used for $\sigma_0$ and $\sigma(1/2)$ are $10^4 \text{ S/m}$ and 200 S/m, respectively.
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Figure 4.5: Comparison of incident and reflected voltage pulses in the transmission line for various realizations of bow-tie antennas with a flare angle of 90°.

Figure 4.6: Comparison of input impedances of bow-tie antennas with flare angles of 5° (dashed line) and 90° (solid lines) for free space and a half-space. Thick lines—antennas located in free space; thin lines—antennas located 1 cm above a dielectric half-space ($\epsilon = 5\epsilon_0$ and $\sigma = 0$ S/m). (a) PEC antennas; (b) antennas with panels of constant 5000 S/m-conductivity; (c) Wu-King-loaded antennas. $A[\circ]$ is the length of the antenna panels in electrical degrees.
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Figure 4.7: Waveforms of the radiated $E_x$ component of the electric field and corresponding amplitude spectra for different realizations of a 5°-bow-tie antenna located in free space. Recording distance is 70 cm below the antenna terminals.

Figure 4.8: As for Figure 4.7, but for a 90°-bow-tie antenna.
Figure 4.9: Waveforms of the radiated $E_z$ component of the electric field and corresponding amplitude spectra for different realizations of a $5^\circ$-bow-tie antenna located 1 cm above a dielectric half-space ($\epsilon = 5\epsilon_0$). Recording distance is 70 cm below the antenna terminals.

Figure 4.10: As for Figure 4.9, but for a $90^\circ$-bow-tie antenna.
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Figure 4.11: Comparison of input impedances of bow-tie antennas with flare angles of 5° (left) and 90° (right) located 1 cm above half-spaces. Black lines—dry sandy soil ($\epsilon = 5\epsilon_0$, $\sigma = 0$ mS/m); blue lines—wet sandy soil ($\epsilon = 18\epsilon_0$, $\sigma = 100$ mS/m). $A[°]$ is the length of the antenna panels in electrical degrees.

Figure 4.12: Comparison of input impedances of bow-tie antennas with flare angles of 5° (left) and 90° (right) located above half-spaces ($\epsilon = 5\epsilon_0$, $\sigma = 0$ mS/m) with (blue lines) and without (black lines) fractal topographic roughness along the surface. Fractal topographic roughness is characterized by a maximum elevation of $\pm 6$ cm and a standard deviation of 2 cm. $A[°]$ is the length of the antenna panels in electrical degrees.
Figure 4.13: Normalized surface-charge distributions for bow-tie antennas with a flare angle of 5° situated in free space or located 1 cm above a half-space ($\varepsilon = 5\varepsilon_0$ and $\sigma = 0$ S/m). Simulation time is 9.6 ns.

Figure 4.14: Normalized surface-charge distributions for bow-tie antennas with a flare angle of 90° situated in free space or located 1 cm above a half-space ($\varepsilon = 5\varepsilon_0$ and $\sigma = 0$ S/m). Simulation time is 9.6 ns.
Figure 4.15: Energy radiation patterns in $E$- and $H$-planes (left and right, respectively) for bow-tie antennas with a 5° flare angle situated 1 cm above a dielectric half-space ($\varepsilon = 5\varepsilon_0$ and $\sigma = 0$ S/m). Recording distance is 70 cm. Gray lines—PEC antenna; black lines—antenna with constant conductivity of 5000 S/m. Values in bold type are maximum amplitudes of radiation pattern of constant finite-conductivity antenna divided by maximum amplitude of $H$-plane radiation pattern for PEC antenna.

Figure 4.16: As for Figure 4.15, but for a bow-tie antenna with a 90° flare angle.
Figure 4.17: Energy radiation patterns in $E$- and $H$-planes (left and right, respectively) for bow-tie antennas with a $5^\circ$ flare angle. Recording distance is 70 cm. Gray lines—PEC antenna; black lines—Wu-King antenna. Values in bold type are maximum amplitudes of radiation pattern of Wu-King antenna divided by maximum amplitude of $H$-plane radiation pattern for PEC antenna.

Figure 4.18: As for Figure 4.17, but for a bow-tie antenna with a $90^\circ$ flare angle.
Figure 4.19: Comparison of energy radiation patterns in $E$- and $H$-planes (left and right, respectively) for three different types of antenna. Black lines—infinitesimal electric dipole; gray solid lines—Wu-King antenna with a $5^\circ$ flare angle; gray dashed lines—Wu-King antenna with a $90^\circ$ flare angle. Recording distance is 70 cm.
Chapter 5

Conclusions and Outlook

For this thesis, a flexible and versatile 3D FDTD simulation tool for GPR antenna systems was developed. It has been used to explore the influence of a variety of design aspects and of diverse earth models on GPR antenna radiation characteristics. This simulation tool allows fine antenna structures or parts of the model space with unusually high permittivities to be modeled by employing subgrids (i.e., grid-refinements). By comparison, subcell techniques can be employed for the modeling of resistive-material sheets and for the accurate calculation of PEC wire structures with diameters much smaller than a grid cell. The model space is surrounded by state-of-the-art PML absorbing boundary conditions. The simulated antenna designs comprise infinitesimal electric dipoles, wire dipoles, and planar bow-tie antennas. All antennas can be shielded and the bow-tie antennas can be loaded with end-resistors or with continuous resistivity profiles along the antenna panels. Feeding of the antennas is achieved through a 1D transmission line model. The earth models consist of half-spaces characterized by a broad range of permittivity and conductivity values. The realism of the half-space models is enhanced by including scale-invariant stochastic distributions of these subsurface electrical material properties and/or small-scale topographic roughness of the air–soil interface.

The various simulations demonstrate that illumination of the subsurface in the near- to intermediate-field range varies significantly according to how the antenna is designed. In particular, optimized resistive loading results in a significant improvement of the overall robustness of the antenna and of the quality of the radiated signal. The coupling of the antenna to the ground is affected by topographic roughness of

\[\mu\] The magnetic permeability is set to its free-space value throughout this work. Recent studies underline that this is an accurate approximation for common geological materials (van Dam et al., 2002). For special purposes, however, variable permittivity could be implemented in the FDTD code in the same way as it has been done for the other material properties.
the air–soil interface, whereas antenna radiation in the subsurface is predominantly influenced by subsurface heterogeneities through scattering and absorption along the propagation path of the waves.

5.1 Frequency-dependent materials

Throughout this work, the electromagnetic material parameters were assumed to be frequency-independent. For most applications concerned with the radar-frequency range and common geological materials, this is a valid assumption (Davis and Annan, 1989; Olhoeft and Capron, 1994), because the bandwidth of GPR is generally limited to 2–3 octaves. However, in special cases, such as high-frequency GPR applications involving porous media with a high water content or salt water environments with high conductivities, substantial dispersion may occur. To account for these situations, it would be desirable to incorporate the frequency-dependence of electrical material properties in the FDTD update-equations.

In frequency-domain finite-difference modeling, the treatment of frequency-dependent material is relatively easy, since the corresponding material properties appear as multiplicative factors in the formalism of Maxwell’s equations. In the time-domain, however, frequency-dependence typically leads to convolutional integrals, which tend to be numerically demanding. Nevertheless, there are FDTD equations that account for dispersion of the electromagnetic properties with a single pole (i.e., with one singularity of the dispersion relation) and even for more complicated dispersion models with multiple poles (e.g., Kunz and Luebbers, 1993; Bergmann et al., 1998). For most relevant dielectric materials, first- and second-order poles describe sufficiently their frequency-dependence. Bourgeois and Smith (1996), for example, present an algorithm for the treatment of permittivity and conductivity frequency-dependence that is based on the Debye relation for complex permittivity (e.g., King and Smith, 1981):

\[ \epsilon(\omega) = \epsilon' (\omega) - i\epsilon'' (\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + i\omega\tau}, \]

where \( \epsilon_\infty \) is the optical permittivity (i.e., for infinitely high frequencies), \( \epsilon_s \) is the static permittivity (i.e., for zero frequency) and \( \tau \) is the relaxation time. Correspondingly, the frequency-dependence of the effective conductivity can be described by

\[ \sigma_e (\omega) = \sigma_0 + \omega \epsilon'' (\omega), \]

where \( \sigma_0 \) denotes the low-frequency component of the conductivity. Bergmann et al. (1998) modeled materials with higher order relaxation mechanisms for permittivity
and conductivity using FDTD schemes of varying temporal and spatial accuracy. These algorithms only account for the frequency-dependence of permittivity and conductivity. However, FDTD algorithms for the modeling of magnetic permeability dispersion can be developed using analogous approaches.

5.2 Parallel computing

A major drawback of FDTD surface GPR modeling is the large amount of random-access memory required. To control numerical dispersion effects, an $O(2,2)$ (i.e., second-order accurate in both time and space) FDTD code requires at least 10 grid points per minimum wavelength. In terms of the Nyquist criterion, the FDTD model space is thus highly oversampled. This leads to large data arrays that have to be stored in the memory of the computer. For simulations involving very higher permittivities and hence very short wavelengths, the very large memory capacities of modern PC’s or workstations are easily exhausted, severely limiting the size and hence the degree of realism of models that can be considered. For example, the 250×250×250 grid-cell simulations carried out for Chapter 2 are in the upper range of feasible model sizes on single workstations.\(^2\)

There have been reports of 3D FDTD applications for objects as large as aircrafts that are illuminated by 1 GHz radar waves (Taflove and Hagness, 2000). Such large-scale modeling requires the use of parallel computer architectures (i.e., multiprocessor computers or computer networks, such as workstation clusters).

The basic concept of parallel computing for the FDTD method is to divide the entire model space into numerous subspaces with each subspace assigned an independent processor of the parallel computer architecture. Assume a global model space with spatial dimensions $N_x, N_y,$ and $N_z$ (in number of grid cells). This model space is divided into $P$ subspaces, where $P$ is equal to the number of processors available. Let $P_x, P_y,$ and $P_z$ be the number of subspaces in the $x$, $y$, and $z$ directions, respectively. The corresponding dimensions of the single subspaces are then $N_x/P_x$, $N_y/P_y$, and $N_z/P_z$. To avoid load imbalances between the processors, the subspaces should, ideally, have the same dimensions. In general, however, the above ratios are non-integer values and the dimensions of the subspaces may be slightly uneven. Simple algorithms can be applied to make sure that each processor is assigned a subspace of similar size (Taflove, 1995).

\(^2\)The calculation times for such models are ~ 5.5 hours on a 750 MHz processor with 2 GB RAM.
Figure 5.1: Illustration of a linear profile and recorded data in the common-offset mode. Adapted from Davis and Annan (1989).

An essential aspect of parallel FDTD algorithms is that at each boundary of a subspace the field components required to update the components on the boundary of the adjacent subspace have to be communicated between the corresponding processors. This interprocessor communication is computationally expensive, but usually this extra load is more than compensated for by the higher speed of the calculations performed via the parallel processing.

Parallel computer architecture should allow very large-scale modeling to be performed. For surface GPR modeling, it could even be used in a rather basic way, without implementing a special parallel-processing algorithm. For example, the FDTD modeling of a simple bistatic (common-offset) radar profile with the antennas moving along the profile (Figure 5.1) is an extremely time consuming task, since for every transmitter-receiver antenna position, a new simulation has to be launched. With a multiprocessor computer, each simulation could be calculated by a single node, using either the whole model space or, for large models, only the relevant parts of the model space, suitably surrounded by PML’s. This suggests that, given a sufficient number of powerful processors, it should be possible to simulate an entire GPR profile in roughly the same time as is currently needed for the conventional non-parallel computation of a single data trace.
5.3 Further applications

The studies presented in Chapters 2, 3, and 4 clearly do not exhaust the potential of the developed FDTD simulation tool. In principle, a wide variety of other applications are feasible. Since this code was mainly developed for the specific modeling of this thesis, many new simulation projects may require adjustments or modifications to be made to the code. In the following, I list some applications of my FDTD modeling tool, which I consider to have particular scientific potential or practical significance for antenna designers and GPR users:

- Design of an "optimal" antenna shield with lossy dielectric and/or magnetic material that eliminates or reduces cavity resonance effects. One might consider, for example, the use of anisotropic material that is being used for the realization of certain material ABC's (cf. Section 1.3).

- Combined design of continuously loaded antennas and shielding with emphasis on producing an undisturbed radiated pulse shape (i.e., a pulse free of internal reflections and other noise).

- Examination of interactions of electronic communication devices, such as mobile phones, with GPR systems. The US Federal Communications Commission has recently ordered restrictions on the use of GPR with respect to certain frequency and power limitations (Olhoeft, 2002).

- Investigation of polarization/depolarization problems due to target responses, for example with crossed dipoles (e.g., Radzevicius and Daniels, 2000) or multicompontent antenna measurements (Lehmann et al., 2000; van der Kruk et al., 2003).
Appendix A

Subcell Algorithm for Thin Resistive Sheets

For the simulation of bow-tie antennas with continuous resistivity distributions along their panels, I have adopted a subcell algorithm presented by Maloney and Smith (1992a). In this algorithm, special cells are defined for the modeling of thin material sheets with the desired resistive properties. In the following, I present the corresponding FDTD update equations in the form they have been implemented in the simulation tool. The basic elements of this algorithm include the following two steps:

i) Define a sheet with thickness $d$ smaller than the dimension of a normal grid cell. The permittivity and conductivity of this sheet are denoted as $\epsilon_s$ and $\sigma_s$, respectively.

ii) Split the electric field perpendicular to the sheet into an inner and an outer component. This is illustrated in Figure A.1, where the sheet lies in the $x$-$y$ plane and the $E_z$ component is split into an outer component $E_{z0}$ referenced by $(i,j,k+\frac{1}{2})$ and an inner component $E_{zi}$ with the spatial indices $(i,j,k^*)$.

The derivation of the FDTD update equations starts from Maxwell's equations in their integral form:

\[
\oint \mathbf{H} \, dl = \oint (\epsilon \frac{\partial}{\partial t} \mathbf{E} + \sigma \mathbf{E}) \, dS \tag{A.1}
\]

\[
\oint \mathbf{E} \, dl = -\oint \mu \frac{\partial}{\partial t} \mathbf{H} \, dS \tag{A.2}
\]

\(^1\text{The discretization of these equations leads to the finite-integral method (cf. Section 1.2).}\)
Using this approach, the update equations of all field components for which the integration path or the area enclosed by the path are contained fully or partly within the shield are modified with respect to the standard update scheme. For the outer component $E_{z0}$, the update equation is of the standard form presented in Section 2.3.1, since the integration contour is not affected by the sheet:

$$E_{z0}^n (i,j,k+\frac{1}{2}) = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_{z0}^{n-1} (i,j,k+\frac{1}{2}) \left. \right|_{(i,j,k)} + \frac{2\Delta t}{2(2\varepsilon + \sigma \Delta t)} \left. \right|_{(i,j,k+\frac{1}{2})}$$

For the inner component $E_{zi}$, the integration path lies completely within the sheet, resulting in the following update equation with the same form as equation (A.3), using $\varepsilon_s$ and $\sigma_s$ instead of $\varepsilon$ and $\sigma$:

$$E_{zi}^n (i,j,k^*) = \frac{2\varepsilon_s - \sigma_s \Delta t}{2\varepsilon_s + \sigma_s \Delta t} E_{zi}^{n-1} (i,j,k^*) \left. \right|_{(i,j,k^*)} + \frac{2\Delta t}{2(2\varepsilon_s + \sigma_s \Delta t)} \left. \right|_{(i,j,k+\frac{1}{2})}$$

For the tangential components $E_x$ and $E_y$, portions of the integration path/area of equation (A.1) are inside the material sheet. Therefore, the update equations are
APPENDIX A. SUBCELL ALGORITHM FOR THIN RESISTIVE SHEETS

as follows:

\[
E^n_x (i+\frac{1}{2}, j,k) = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E^{n-1}_x (i+\frac{1}{2}, j,k) + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)} E^n_x (i+\frac{1}{2}, j,k)
\]

\[
\times \left[ \frac{H^{n-\frac{1}{2}}_z (i+\frac{1}{2}, j+\frac{1}{2}, k) - H^{n-\frac{1}{2}}_z (i+\frac{1}{2}, j-\frac{1}{2}, k)}{\Delta y} - \frac{H^{n-\frac{1}{2}}_y (i+\frac{1}{2}, j,k+\frac{1}{2}) - H^{n-\frac{1}{2}}_y (i+\frac{1}{2}, j,k-\frac{1}{2})}{\Delta z} \right], \tag{A.5}
\]

\[
E^n_y (i, j+\frac{1}{2}, k) = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E^{n-1}_y (i, j+\frac{1}{2}, k) + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t)} E^n_y (i, j+\frac{1}{2}, k)
\]

\[
\times \left[ \frac{H^{n-\frac{1}{2}}_x (i, j+\frac{1}{2}, k+\frac{1}{2}) - H^{n-\frac{1}{2}}_x (i, j+\frac{1}{2}, k-\frac{1}{2})}{\Delta z} - \frac{H^{n-\frac{1}{2}}_z (i+\frac{1}{2}, j+\frac{1}{2}, k) - H^{n-\frac{1}{2}}_z (i-\frac{1}{2}, j+\frac{1}{2}, k)}{\Delta z} \right], \tag{A.6}
\]

with \(\varepsilon\) and \(\sigma\) denoting the average permittivity and conductivity:

\[
\varepsilon = \left(1 - \frac{d}{\Delta z}\right) \varepsilon_0 + \left(\frac{d}{\Delta z}\right) \varepsilon_s, \tag{A.7}
\]

\[
\sigma = \left(\frac{d}{\Delta z}\right) \sigma_s. \tag{A.8}
\]

For the magnetic field, only the update equations for the tangential components \(H_x\) and \(H_y\) differ from the standard FDTD update equations. Involving the components \(E_{zo}\) and \(E_{zi}\), the discretization of equation (A.2) yields:

\[
H^{n+\frac{1}{2}}_x (i, j+\frac{1}{2}, k+\frac{1}{2}) = H^{n-\frac{1}{2}}_x (i, j+\frac{1}{2}, k+\frac{1}{2}) + \frac{\Delta t}{\mu} \left[ \frac{E^n_y (i, j+\frac{1}{2}, k+1) - E^n_y (i, j+\frac{1}{2}, k)}{\Delta z} \right] - \left(1 - \frac{d}{\Delta z}\right) \frac{E^n_{zo} (i, j+1, k+\frac{1}{2}) - E^n_{zo} (i, j, k+\frac{1}{2})}{\Delta y} \right], \tag{A.9}
\]

\[
H^{n+\frac{1}{2}}_y (i+\frac{1}{2}, j, k+\frac{1}{2}) = H^{n-\frac{1}{2}}_y (i+\frac{1}{2}, j, k+\frac{1}{2}) + \frac{\Delta t}{\mu} \left[ \left(1 - \frac{d}{\Delta z}\right) \frac{E^n_{zo} (i+1, j, k+\frac{1}{2}) - E^n_{zo} (i, j, k+\frac{1}{2})}{\Delta x} \right] + \left(\frac{d}{\Delta z}\right) \frac{E^n_{zi} (i++, j, k) - E^n_{zi} (i, j, k++)}{\Delta x} - \frac{E^n_x (i+\frac{1}{2}, j, k+1) - E^n_x (i+\frac{1}{2}, j, k)}{\Delta z} \right]. \tag{A.10}
\]
Appendix B

Subcell Algorithm for Thin Wires

The subcell algorithm used in Chapter 4 for the modeling of PEC wire antennas is adapted from Umashankar et al. (1987). It allows thin wires with diameters much smaller than the dimensions of a normal grid cell to be modeled.

In the following, the update equations are presented in the form they are implemented in the FDTD simulation tool for a wire-dipole antenna parallel to the x-axis (i.e., its spatial indices are \((i, j_0, k_0)\) with \(j_0\) and \(k_0\) being constants). The derivation employs Faraday's law [equation [A.2]], assuming an \(1/r\) dependence of the circumferential magnetic field components \(H_y\) and \(H_z\) and of the radial electric field components \(E_y\) and \(E_z\) close to the wire, where \(r\) is the distance from the wire center. These field components can be written as (e.g., for \(E_y\)):

\[
E_y(i, j_0 + r_0 < j < j_0 + 1) = E_y(i, j_0 + \frac{1}{2}, k_0) \frac{\Delta y}{2} \frac{1}{r}.
\]  

(B.1)

Using the PEC criteria, the tangential electric field component within the wire \(E_x(i + \frac{1}{2}, j_0, k_0)\) is set to zero. With the positions of the field components involved in Faraday's law contour path shown in Figure B.1, this leads to the following update equations:

\[
H_y^{n+\frac{1}{2}} (i + \frac{1}{2}, j_0, k_0 \pm \frac{1}{2}) = H_y^{n-\frac{1}{2}} (i + \frac{1}{2}, j_0, k_0 \pm \frac{1}{2})
\]

\[
\mp \frac{2\Delta t}{\mu \Delta z \ln(\frac{\Delta z}{r_0})} E_x^n (i + \frac{1}{2}, j_0, k_0 \pm 1) + \frac{\Delta t}{\mu \Delta x} \left( E_z^n (i + 1, j_0, k_0 \pm \frac{1}{2}) - E_z^n (i, j_0, k_0 \pm \frac{1}{2}) \right)
\]

(B.2)

\[
H_z^{n+\frac{1}{2}} (i + \frac{1}{2}, j_0 \pm \frac{1}{2}, k_0) = H_z^{n-\frac{1}{2}} (i + \frac{1}{2}, j_0 \pm \frac{1}{2}, k_0)
\]

\[
\pm \frac{2\Delta t}{\mu \Delta y \ln(\frac{\Delta y}{r_0})} E_x^n (i + \frac{1}{2}, j_0 \pm 1, k_0) + \frac{\Delta t}{\mu \Delta x} \left( E_y^n (i, j_0 \pm \frac{1}{2}, k_0) - E_y^n (i+1, j_0 \pm \frac{1}{2}, k_0) \right)
\]

(B.3)

Note that all other components are updated by the standard FDTD scheme.
Figure B.1: Field components involved in Faraday’s law for the derivation of the update equations for a thin wire. $\otimes$ points into the paper, $\odot$ points out of the paper.
Appendix C

Finite-Difference Modelling of Ground-Penetrating Radar Antenna Radiation

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ABSTRACT

The interaction of high-frequency electromagnetic wavefields with a typical ground-penetrating radar (GPR) antenna is complex and its effects on the recorded data are not well understood. Further distortions of the radiation pattern and pulse shape must be expected to arise in the presence of a shielding of the antenna. To address these issues, we present a three-dimensional finite-difference approximation of Maxwell’s equations that allows to model realistically the near-field radiation characteristic of a typical GPR system.

Key words: Bow-Tie Antenna, Electric Dipole, FDTD-Approximation, Maxwell’s Equations, Modelling, Near-Field Radiation, Shielded Antenna
INTRODUCTION

There are many similarities and analogies between ground-penetrating radar (GPR) and seismic reflection data. One major difference is, however, that the near-field region and the source directivity are much more important for GPR than for seismic reflection data. Yet, all common seismic reflection processing and imaging techniques adopted in GPR are based on isotropic source radiation and/or far-field approximations, which are generally not valid for GPR data. This points to the importance of a better understanding of GPR near-field radiation characteristics. Modelling studies are the appropriate means to achieve this goal. Due to the complexity of the problem, analytical or asymptotic approaches are generally not applicable and flexible numerical solutions of Maxwell’s equations are required to faithfully emulate the key physical aspects of a typical GPR antenna system.

Bourgeois and Smith (1996), Roberts and Daniels (1997), and Holliger and Bergmann (1998) presented finite-difference time-domain (FDTD) algorithms to simulate the interaction of high-frequency electromagnetic fields with GPR antennae. In the following, we adopt an approach similar to that of Bourgeois and Smith (1996) to assess how the near-field radiation in a typical GPR experiment is affected by the finite extent of the antenna as well as by its shielding.

METHODOLOGY

Our model is based on an $O(2, 2)$-accurate FDTD-approximation of Maxwell’s equations. The three-dimensional computational volume enclosing the antenna consists of a regular Yee lattice (Yee, 1966), where the spatial discretisations $\Delta x, \Delta y, \Delta z$ are uniformly chosen to $\Delta r = 7.5$ cm. The thus resulting FDTD update equations for the $x$-components of the magnetic and electric fields of the discretised Maxwell’s curl equations are:

$$
H_x^{n+\frac{1}{2}, j, k+\frac{1}{2}} = H_x^n_{j, k+\frac{1}{2}} + \frac{\Delta t}{\Delta r} \mu_{j, k+\frac{1}{2}} \left[ E_y^n_{j, k+1} - E_y^n_{j, k} - E_z^n_{j, k+\frac{1}{2}} + E_z^n_{j, k+\frac{1}{2}} \right]
$$

(C.1)

$$
E_x^{n+\frac{1}{2}, j, k} = \frac{2\varepsilon - \sigma \Delta t}{2\varepsilon + \sigma \Delta t} E_x^n_{j, k} + \frac{2\Delta t}{(2\varepsilon + \sigma \Delta t) \Delta r} E_x^n_{j, k} \left[ H^n_{j+\frac{1}{2}, k-\frac{1}{2}} - H^n_{j+\frac{1}{2}, k+\frac{1}{2}} - H^n_{y(j+\frac{1}{2}, k+\frac{1}{2})} + H^n_{y(j+\frac{1}{2}, k-\frac{1}{2})} \right].
$$

(C.2)
where \( \varepsilon \) is the permittivity, \( \mu \) the magnetic permeability, and \( \sigma \) the electric conductivity. We use the standard Yee notation, so that e.g. 
\[
(i + \frac{1}{2}, j, k) = ((i + \frac{1}{2})\Delta x, j\Delta y, k\Delta z),
\]
with the spatial indices \( i, j, k \). \( \Delta t \) is the time discretisation interval and \( n \) the corresponding index. Analogous expressions are obtained for the remaining field components. For the given spatial discretisation \( \Delta r = 7.5 \) cm and assuming the maximum phase velocity in the model to be equal to speed of light in vacuum, the Courant stability criterion for the above FDTD-scheme (Bergmann et al., 1996) constrains \( \Delta t \) to be 0.108 ns.

A bow-tie antenna lying in the \( x-y \)-plane (Fig. C.1) is situated in the centre of the computational volume. The interaction of the antenna with the electromagnetic field is simulated by setting to zero the electric field components \( E_x \) and \( E_y \) within the metal panels of the antenna. The length of the antenna is chosen to be \( 20\Delta r = 150 \) cm, and the flare angle is 90°. Care needs to be taken to ensure that all the staggered-grid locations of the corresponding \( E_x \)-and \( E_y \)-components are contained within the discretised antenna panels.

The central slot of the antenna is connected to a coaxial feeder cable. Maloney et al. (1994) have shown that a simple 1D transmission line model is appropriate for transient antenna excitations like the one considered in this study. Assuming the electromagnetic field to be transverse electromagnetic (TEM), the update equations for the voltage \( U \) and current \( I \) in the coaxial cable are given by the following finite-difference approximations (Maloney et al., 1994):

\[
I^{n+\frac{1}{2}}_{(k'+\frac{1}{2})} = I^{n-\frac{1}{2}}_{(k'+\frac{1}{2})} - \frac{1}{Z_0} \frac{v\Delta t}{\Delta z'} \left[ U^n_{(k'+1)} - U^n_{(k')} \right],
\]

\[\text{(C.3)}\]

Figure C.1: Schematic illustration of the modelled GPR system. The bow-tie antenna is surrounded by a metal shielding and fed by a coaxial cable connected to the central slot of the antenna. A Gaussian voltage pulse \( U_g \) is fed into the centre of the cable.
\[ U_{n+1}^{(k')} = U_n^{(k')} - Z_0 \frac{v \Delta t}{\Delta z'} \left[ I_{n+\frac{1}{2}}^{(k'+\frac{1}{2})} - I_{n-\frac{1}{2}}^{(k'-\frac{1}{2})} \right], \tag{C.4} \]

where \( \Delta z' \) is the spatial discretisation interval of the cable model and \( k' \) the corresponding index. \( Z_0 = 50 \, \Omega \) is the characteristic impedance of the transmission line and \( v \) the phase velocity in the line. For simplicity, we have set \( v \) to the speed of light \( c \) in free space. At the upper end of the coaxial cable an absorbing boundary condition prevents reflections of the incident voltage pulse from running back to the antenna. For the absorbing boundary condition we choose the spatial discretisation of the cable to be \( \Delta z' = 2v \Delta t \). Absorption at the upper end of the feeder cable is then achieved by enforcing the following equalities:

\[ I_{n+\frac{1}{2}}^{(k')} = I_n^{(k')}, \tag{C.5} \]

\[ U_{n+\frac{1}{2}}^{(k')} = U_n^{(k')}. \tag{C.6} \]

For the connection of the transmission line to the antenna, the finite-difference update equation for the current value \( I_{(k'_\text{max}+\frac{1}{2})} \) at the lower end of the cable is modified according to Ampère's law:

\[ I_{(k'_\text{max}+\frac{1}{2})}^{n+\frac{1}{2}} = \left[ H_{y(i_s+j_s+k_s+\frac{1}{2})}^{n+\frac{1}{2}} - H_{y(i_s-j_s+k_s+\frac{1}{2})}^{n+\frac{1}{2}} \right] \Delta y + \]

\[ + \left[ H_{x(i_s+j_s-k_s+\frac{1}{2})}^{n+\frac{1}{2}} - H_{x(i_s+j_s+k_s+\frac{1}{2})}^{n+\frac{1}{2}} \right] \Delta x. \tag{C.7} \]

The indices \( i_s, j_s, k_s \) denote the cell in the centre of the slot of the antenna. The relation between the voltage fed into the antenna and the electric field in its slot is:

\[ E_x^{n} (i_s+j_s+k_s) = -\frac{U_{n}^{(k'_\text{max})}}{d}, \tag{C.8} \]

where \( d \) is the width of the slot and \( k_s = k'_\text{max} \). For the following simulations we choose \( d = 2\Delta x \).

The antenna is fed by initiating a Gaussian voltage pulse in the centre of the feeder cable:

\[ U_g(t) = U_0 e^{-\frac{1}{2} \left( \frac{t}{\tau_p} \right)^2}. \tag{C.9} \]

The parameter \( \tau_p \) determines the width of the pulse and is chosen to be 1.59 ns. \( U_0 \) is set to unity. The frequency spectrum of \( U_g(t) \) is given by:

\[ \tilde{U}_g(\nu) = U_0 \tau_p e^{-\frac{1}{2} \left( \frac{2\pi \nu \tau_p}{\tau} \right)^2}, \tag{C.10} \]
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and essentially vanishes for frequencies larger than 300 MHz (Fig. C.2). This conforms with the numerical dispersion characteristics of the used $O(2,2)$-accurate FDTD-approximation of Maxwell’s equations (Bergmann et al., 1996).

In a subsequent step, we construct a metal shielding surrounding the antenna. This shielding is simulated in the same manner as the antenna itself, i.e. by setting to zero the electric field components parallel to the metal surfaces. The box-shaped metal shielding has a vertical extent of 1.50 m and a lateral extent of 2.10 m in both the $x$- and $y$-direction.

NUMERICAL RESULTS

In order to assess the influence of the GPR system on the radiated electromagnetic field, we first modelled the wavefield of an electric dipole source in vacuum ($\epsilon = \epsilon_0$, $\mu = \mu_0$ and $\sigma = 0 \text{ mS/m}$). This electric dipole is realised by feeding a Gaussian excitation pulse (equation C.9) into the $E_x$-component at the source location. A snapshot of the thus resulting electromagnetic wavefield is shown in Fig. C.3. Apart from some numerical noise in the interior of the wave front, the displayed wavefield is undisturbed. In a second model setup, a homogenous half space ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0.01 \text{ mS/m}$) has been implemented in the lower half of the computational volume. The corresponding snapshot for an electric dipole source located in vacuum 15 cm above the half space is shown in Fig. C.4.

Comparing the radiation of the electric dipole with that of the unshielded antenna (Figs. C.5, C.6, and C.7) illustrates that the shape and finite extent of the antenna

![Figure C.2: Spectrum of the Gaussian voltage pulse.](image)

Figure C.2: Spectrum of the Gaussian voltage pulse.
Figure C.3: Snapshot of the $E_x$-component of the electromagnetic field radiated by an electric dipole orientated parallel to the $x$-axis in vacuum ($\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The snapshot is taken in the $x$-$z$-plane.

Figure C.4: Snapshot of the $E_x$-component of the electromagnetic field radiated by an electric dipole orientated parallel to the $x$-axis over a half space ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0.01$ mS/m). The dipole is located 15 cm above the half space in vacuum. The snapshot is taken in the $x$-$z$-plane.

cause prominent deformations of the wavefield. Strong quasi-stationary amplitudes of the radiated electromagnetic field are observed adjacent to the antenna panels. These amplitudes originate from reflections and diffractions of the field at the edges of the antenna, since no resistive loading has been implemented in the model. The radiation characteristics of the shielded antenna (Figs. C.8, C.9, and C.10) exhibit a prominent directionality of the radiated wavefield. Much less energy is radiated in the $-z$-direction (upwards) than in the $+z$-direction (downwards) where the shield-
ing is open. In addition to the reflections and diffractions due to the antenna edges mentioned above, reflections within the shielding box, and diffractions of the radiated fields at the boundaries of the box are now present.

Figure C.5: Snapshot of the $E_x$-component of the electromagnetic field of an unshielded bow-tie antenna in the $x$-$y$-plane in vacuum ($\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m).

Figure C.6: Snapshot of the $E_x$-component of the electromagnetic field of an unshielded bow-tie antenna in the $x$-$z$-plane in vacuum ($\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m).
Figure C.7: Snapshot of the $E_x$-component of the electromagnetic field of an unshielded bow-tie antenna over a half space ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0.01$ mS/m). The antenna is located 15 cm above the half space in vacuum. The snapshot is taken in the $x$-$z$-plane.

Figure C.8: Snapshot of the $E_x$-component of the electromagnetic field of a shielded bow-tie antenna in the $x$-$y$-plane in vacuum ($\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m).
CONCLUSIONS

We have used a three-dimensional FDTD-approximation of Maxwell's equations to simulate the radiation characteristic of a typical GPR system. The results were compared with the reference radiation pattern of an electric dipole. With respect to this reference both the finite extent of the GPR antenna as well as its metal shielding have significant effects on the pulse shape and the nature of the radiation pattern. Since these phenomena cannot be readily assessed otherwise, we believe
that this modelling approach should be useful to address fundamental questions in the design and analysis of GPR systems.

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Appendix D

Numerical Modeling of a Complete Ground-Penetrating Radar System

Bernhard Lampe and Klaus Holliger


ABSTRACT

The generation and recording of electromagnetic waves by typical ground-penetrating radar (GPR) sounding systems is complex and the effects of the antennas on the recorded data are not well understood. To address this problem, we present a versatile and efficient GPR system simulation tool. This algorithm is based on a finite-difference time-domain (FDTD) approximation of Maxwell’s equations and allows us to model realistically the radiation characteristics of a wide variety of typical surface GPR antenna systems. The accuracy of the algorithm is benchmarked and validated with respect to extensive laboratory measurements for comparable antenna systems. Given the flexibility of this GPR modeling software, we anticipate that it will be useful not only for the design and interpretation of
GPR surveys, but also for the design of novel GPR sounding systems.

**Keywords:** absorbers, bow-tie antenna, dielectric half-space, FDTD, GPR, near-field radiation, resistive loading, shielding

**INTRODUCTION**

A detailed physical understanding of the propagation of electromagnetic waves radiated from antennas over a dielectric half-space is critical both for a proper interpretation of observed ground-penetrating radar (GPR) data and for the further development of GPR sounding systems. To date, many common GPR processing and imaging techniques are adopted from corresponding techniques in reflection seismology. Although there are many similarities and analogies between GPR and seismic reflection data, many key assumptions of reflection seismic processing and imaging, such as isotropic source radiation and plane wave far-field approximations, are not valid in GPR applications. In particular, the effects of the transmitting and receiving GPR antennas on the recorded data are largely unexplored. This points to the importance of a better understanding of the near-field and intermediate-field radiation characteristics of generic GPR antenna systems. Numerical modeling studies which emulate the radiation characteristics of type GPR sounding systems are the appropriate means to achieve this goal.

Holliger and Bergmann (1998) published an efficient finite-difference time-domain (FDTD) approach for modeling the near-field radiation of a GPR antenna by using a center-fed thin wire antenna model. Bourgeois and Smith (1996) and Roberts and Daniels (1997) presented FDTD approximations of GPR antenna systems consisting of bow-tie antennas fed by transmission lines. All these studies achieved reasonable agreement between modeled and measured data. In this paper, we extend and complement this earlier work by developing a highly versatile and generic algorithm, which allows for the numerical modeling of a wide variety of common surface GPR antenna systems. We shall first give a detailed description of the pertinent aspects of the algorithm. We shall then test the algorithm's validity with respect to laboratory measurements for corresponding antenna systems. Finally, we shall apply this simulation software to evaluate the radiation characteristics of a selection of canonical models of surface GPR antenna systems located over a dielectric half-space earth model.
METHODOLOGY

FDTD Scheme

Our numerical approach is based on an $O(2,2)$-accurate FDTD-approximation of Maxwell's equations in three dimensions. The finite-difference grid is staggered in both space and time according to the Yee algorithm (Yee, 1966). The basic FDTD update equations for the $x$ component of the magnetic and electric field in Cartesian coordinates are

\[
H_x^{n+\frac{1}{2}}(i,j+\frac{1}{2},k+\frac{1}{2}) = H_x^{n-\frac{1}{2}}(i,j+\frac{1}{2},k+\frac{1}{2}) + \frac{\Delta t}{\mu} \left[ \frac{E_y^n(i,j+\frac{1}{2},k+\frac{1}{2}) - E_y^n(i,j+\frac{1}{2},k)}{\Delta z} \right] \\
\frac{E_x^n(i,j+1,k+\frac{1}{2}) - E_x^n(i,j+\frac{1}{2},k+\frac{1}{2})}{\Delta y}, \tag{D.1}
\]

\[
E_x^{n+1}(i+\frac{1}{2},j,k) = \frac{2c-\sigma\Delta t}{2c+\sigma\Delta t} E_x^n(i+\frac{1}{2},j,k) + \frac{2\Delta t}{(2c+\sigma\Delta t)} \left[ \frac{H_z^{n+\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) - H_z^{n+\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k)}{\Delta y} - \frac{H_y^{n+\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i+\frac{1}{2},j,k+\frac{1}{2})}{\Delta z} \right], \tag{D.2}
\]

where $\Delta x, \Delta y, \Delta z$ are the spatial increments, $i, j, k$ the corresponding spatial indices, $\Delta t$ the time discretization interval, $n$ the corresponding time index, $\epsilon$ the dielectric permittivity, $\mu$ the magnetic permeability, and $\sigma$ the electric conductivity. In this notation, $E_x^n(i+\frac{1}{2},j,k)$ denotes, for example, the $x$ component of the electric field at time $n\Delta t$ and location $x = (i+\frac{1}{2})\Delta x, y = j\Delta y, z = k\Delta z$ of the modeled physical volume. Expressions analogous to Eqs. (D.1) and (D.2) are obtained for the remaining $y$ and $z$ components. Surficial geological materials are generally assumed to be non-magnetic, and hence $\mu$ is uniformly chosen to be $\mu = \mu_0$.

Numerical stability of the FDTD scheme is ensured by the Courant stability criterion (Taflove and Hagness, 2000)

\[
\Delta t = C_n \frac{1}{c \sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}}, \tag{D.3}
\]

where $c$ is the maximum propagation velocity present in the grid and the coefficient $C_n \leq 1$ is the Courant number. We chose a value for $C_n$ in the range of 0.70 to 0.75 for all our calculations. The accumulation of numerical dispersion errors can be controlled by choosing a sufficiently fine grid spacing with respect to the smallest relevant wavelength present in the source signal. For our $O(2,2)$-accurate FDTD-scheme, a minimum sampling density of about ten grid points per minimum wavelength is generally considered to be appropriate (Bergmann et al., 1996).
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The computational volume is surrounded by a general perfectly matched layer (GPML), a highly effective absorbing boundary condition proposed by Fang and Wu (1996). This boundary condition achieves near-perfect absorption of propagating and evanescent waves of any frequency and any angle of incidence in both lossless and lossy media. The implementation of these advanced absorbing boundaries thus enables us to emulate a quasi-infinite model space without any notable reflections from the model edges.

Finally, our algorithm allows for the use of subgrids (Chevalier et al., 1997). This implies that locally a finer grid spacing can be employed, for example to accommodate regions of unusually high dielectric permittivity and/or portions of the FDTD model containing objects with complicated geometries, such as the antenna hardware. The use of subgrids enhances the computational efficiency and reduces the notorious staircasing errors arising from the discretization of antenna geometries with slanting edges (Cangellaris and Wright, 1991; Schneider and Shlager, 1997). The implemented subgrid algorithm allows dielectric and/or conducting material to traverse the boundary between the coarse and the fine grid and can therefore be employed virtually anywhere in the model space. At present, we have implemented and tested this subgrid algorithm for a grid refinement factor of three.

Antenna Model

In the following, we consider a bow-tie-type antenna, which may be loaded by resistors and shielded with a metal casing (Fig. D.1). Both the antenna as well as the shielding are simulated as perfect electric conductors, i.e., all tangential electric field components along the metal surfaces are set to zero. The antenna panels are approximated using a stair-stepped representation of the slanting edges. The antenna is fed by a 1-D transmission line model (Maloney et al., 1994) which is attached to the antenna terminals. Assuming only transverse electromagnetic modes propagating along the parallel wire transmission line, the finite-difference approximation for current $I$ and voltage $U$ are given by

$$
I^{n+\frac{1}{2}}_{(k'+\frac{1}{2})} = I^n_{(k'+\frac{1}{2})} - \frac{1}{Z_0} \frac{c \Delta t}{\Delta z'} \left[ U^n_{(k'+1)} - U^n_{(k')} \right],
$$

$$
U^{n+1}_{(k')} = U^n_{(k')} - Z_0 \frac{c \Delta t}{\Delta z'} \left[ I^{n+\frac{1}{2}}_{(k'+\frac{1}{2})} - I^{n+\frac{1}{2}}_{(k'-\frac{1}{2})} \right],
$$

where $\Delta z'$ is the spatial discretization interval along the cable, $k'$ the corresponding index, and $Z_0$ is the characteristic impedance of the transmission line. The phase velocity $c$ along the transmission line is assumed to correspond to the speed of light,
Figure D.1: Schematic illustration of the basic unshielded (left) and shielded bow-tie antenna models.

i.e. \( c = 1/\sqrt{\varepsilon_0\mu_0} \). An absorbing boundary condition is implemented at the upper end of the transmission line in order to prevent reflections of the incident voltage pulse from running back to the antenna. To this end, we choose the spatial discretisation of the cable to be \( \Delta z' = 2c\Delta t \) and we enforce the following equalities:

\[
I_{n+\frac{1}{2}}^{(1)} = I_{n-\frac{3}{2}}^{(1)},
\]

\[
U_{n+1}^{(0)} = U_{n-1}^{(1)}.
\]

To connect the transmission line to the antenna terminals, the update equation for the current (D.4) at the lower end of the cable is modified according to Ampère’s law:

\[
I_{n+\frac{1}{2}}^{(k'_{\text{max}}+\frac{1}{2})} = \left[H_{z_{is+\frac{1}{2},js+\frac{1}{2},ks}}^{n+\frac{3}{2}} - H_{z_{is-\frac{1}{2},js-\frac{1}{2},ks}}^{n+\frac{3}{2}}\right] \Delta z + \left[H_{y_{is+\frac{1}{2},js,ks-\frac{1}{2}}}^{n+\frac{3}{2}} - H_{y_{is+\frac{1}{2},js,ks+\frac{1}{2}}}^{n+\frac{3}{2}}\right] \Delta y.
\]

The indices \( is, js, \) and \( ks \) denote the cell centered between the terminals of the antenna. The relation between the voltage \( U_{k'_{\text{max}}} \) at the last point of the transmission line and the electric field between the terminals of the antenna is given by

\[
E_{x}^{n}(is+\frac{1}{2},js,ks) = -\frac{U_{n}^{k'_{\text{max}}}}{d},
\]

where \( d = 2\Delta x \) is the spacing between the antenna terminals. The point \( k'_{\text{max}} \) of the 1-D transmission line is colocated with the point \( (is + \frac{1}{2}, js, ks) \) of the 3-D grid. The antenna is excited by feeding a Gaussian voltage pulse

\[
U_g(t) = U_0 e^{-\frac{1}{2}((t-t_g)/\tau_p)^2}
\]

into the transmission line. The parameters \( \tau_p = 0.25 \) ns and \( t_g = 2.36 \) ns denote the width and the length of the pulse, respectively. The width \( \tau_p \) has to be chosen sufficiently small to resolve the antenna geometry. This implies that the wave number
spectrum of the excitation pulse is quasi-white with respect to the dimensions of the antenna. Conversely, the duration of the pulse \( t_g \) must be long enough to ensure a smooth beginning and ending of the excitation in order to avoid numerical artifacts. The amplitude scaling factor \( U_0 \) is set to unity.

**Validation of Algorithm**

In order to benchmark the reliability and accuracy of our antenna model, we compare our synthetic data with laboratory measurements of the input impedance of corresponding antennas (Brown and Woodward, 1952). The input impedance of an antenna is determined by recording the incident voltage \( U_i(t) \) and the reflected voltage \( U_r(t) \) in the transmission line. The corresponding reflection coefficient in the frequency domain is then given by

\[
\rho(\omega) = \frac{U_r(\omega)}{U_i(\omega)} e^{2ikz},
\]

where \( k \) is the wave number, \( \omega \) the angular frequency, and \( z \) the distance between the recording point and the antenna terminals. Based on Eq. (D.10) the complex impedance \( Z_i \) of the antenna can thus be calculated as

\[
Z_i = Z_0 \frac{1 + \rho(\omega)}{1 - \rho(\omega)},
\]

where \( Z_0 \) is the characteristic impedance of the transmission line. In Fig. D.2 the real part (resistance) and the complex part (reactance) of \( Z_i \) for bow-tie antennas with flare angles ranging from 5° to 90° are compared to the corresponding laboratory-measured data. All computations were carried out for \( Z_0 = 200 \, \Omega \). It should be noted that for all practical intents and purposes, bow-tie antennas with flare angles smaller than 10° are essentially equivalent to the thin wire antenna model of Holliger and Bergmann (1998). For ease of comparison with the original monopole antenna measurements of Brown and Woodward (1952), our normalized values are plotted against the length \( l \) of the triangular antenna panel in electrical degrees, defined by \( A = 360° \times \frac{l \, \omega}{2\pi \, c} \).

As expected, the agreement between the measured and synthetic curves in Fig. D.2 improves with increasing flare angle. For flare angles of 40° and higher, there is virtually perfect agreement between the laboratory measured and the simulated data. Since inaccuracies due to staircasing effects are expected to increase with decreasing flare angle, we have implemented all antennas in subgrids in order to mitigate these effects (Fig. D.3). Nevertheless, diffractions due to the staircasing
Figure D.2: Comparison of laboratory measurements (dash-dotted lines) and numerical simulations (solid lines) of the resistance and reactance of unshielded bow-tie antennas with flare angles ranging from 5° to 90°. \( A \) is the length of the triangular antenna panel in electrical degrees. The laboratory data were digitized from Brown and Woodward (1952).

approximation of the antenna are still evident, for example, in the snapshots of the \( \mathcal{E}_x \) component in the \( x - y \)-plane for a flare angle of 10° as shown in Fig. D.3. It should, however, be noted, that even for very small angles, such as 5° and 10°, a surprisingly good match between measured and modeled data is achieved (Fig. D.2). The results of these benchmark tests underscore the high degree of accuracy and the overall robustness of our algorithm.

SIMULATIONS

Here, we show snapshots of the electric field and corresponding radiation patterns for a selection of canonical GPR antenna models over a homogeneous half-space (\( \sigma = 0 \) mS/m, \( \epsilon = 5\epsilon_0 \)). All radiators are located one grid cell (1 cm) over the air-soil interface. In order to assess the influence of the GPR system on the radiated electromagnetic field, we use the radiation characteristic of an infinitesimal horizontal electric dipole as a reference model.

Fig. D.4 shows snapshots of the \( \mathcal{E}_x \) component parallel (\( E \)-plane) and perpendicular (\( H \)-plane) to the electric dipole together with the corresponding \( E \)-field radiation patterns. The radiation patterns were determined by recording the tangential electric field components in the \( E \)-plane (\( \mathcal{E}_\theta \)) and in the \( H \)-plane (\( \mathcal{E}_\phi \)), respectively. For comparison, we also show the corresponding far-field radiation patterns obtained by the asymptotic expressions published by Smith (1984). There is a vague resemblance in the overall shape of the analytical and numerical radiation patterns. In
Figure D.3: $E$-plane snapshots of the $E_z$ component in the $x - y$-plane. The flare angles are 10° (left) and 90° (right).

In particular, the ratio between the maximum amplitudes in the $E$- and $H$-planes is similar for the analytical and numerical radiation patterns. However, in the $E$-plane, the side lobes of the numerical radiation pattern are much less pronounced compared to the asymptotic far-field pattern, and in the $H$-plane, the amplitude maximum in the numerical radiation pattern is located significantly beyond the critical angle of 26.6°. This illustrates that even in the hypothetical case of the GPR antenna consisting of an infinitesimal electric dipole, the commonly used asymptotic far-field patterns do not represent an adequate reference model for the illumination of the shallow subsurface by GPR measurements. Fig. D.5 shows corresponding snapshots and radiation patterns for an unshielded, 52 cm long bow-tie antenna with a flare angle of 60°. Even a qualitative comparison of the snapshots with those from the infinitesimal dipole (Fig. D.4) illustrates the dramatic effect that the GPR antenna system has on the radiated electromagnetic field and thus on the illumination of the subsurface. In the $H$-plane pattern, we see that, compared to the radiation pattern of the electric dipole, the amplitude maximum has been shifted even further beyond the critical angle. This implies that when acquiring GPR data in the common "parallel-broadside-mode" (i.e. transmitting and receiving antennas both oriented parallel to the structural grain), a much larger subsurface volume is illuminated than predicted by the commonly used asymptotic far-field radiation patterns or even by numerical simulations of an infinitesimal electric dipole. Also note that virtually no side lobes are present in the $E$-plane.

Next, we add a rectangular metal shielding to the basic bow-tie antenna model
Figure D.4: Top: $E$-plane (left) and $H$-plane (right) snapshots of the $E_x$ component of the electromagnetic field radiated from an infinitesimal electric dipole located 1 cm above the interface between air and soil ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The dipole is oriented parallel to the $x$-axis. Bottom: Amplitude radiation patterns for the tangential components of the electric field, $E_\theta$ (left) and $E_\phi$ (right), in the $E$- and $H$-plane, respectively. The dashed lines show the corresponding asymptotic radiation patterns for the far-field (Smith, 1984).
Figure D.5: Top: $E$-plane (left) and $H$-plane (right) snapshots of the $E_z$ component radiated from an unshielded bow-tie antenna located 1 cm above the interface between air and soil ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The antenna is 52 cm long, has a flare angle of 60°, and is oriented parallel to the $x$-axis. Bottom: Amplitude radiation patterns for the tangential components of the electric field, $E_\theta$ (left) and $E_\phi$ (right), in the $E$- and $H$-plane, respectively.
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(Figs. D.6 and D.7). The metal shielding box is 60 cm long and 38 cm wide, and either 26 cm (Fig. D.6) or 13 cm (Fig. D.7) high. The height of the shielding has a pronounced effect on the radiation characteristics. In Fig. D.6, the height of the box corresponds roughly to the quarter wavelength response of a tuned half-wavelength dipole antenna in free space. This antenna design thus aims at ensuring constructive interference for the downgoing part of the electromagnetic wavefield. Consequently, we observe higher amplitudes of the $E$-field component in the half-space but also increased reverberations from the antenna system compared to the 13 cm high shielding (Fig. D.7).

The corresponding radiation patterns are much narrower than in the unshielded case, particularly in the $H$-plane, and hence a significantly smaller volume of the subsurface is illuminated in the standard parallel-broadside acquisition mode than with a corresponding unshielded antenna. Moreover, in marked contrast to the infinitesimal electric dipole and the unshielded bow-tie antenna (Figs. D.4 and D.5), the ratio of the maximum amplitudes in the $E$- and $H$-planes is now almost equal.

The last two figures (Figs. D.8 and D.9) show results for the 13 cm high shielding connected with 200 $\Omega$ resistors (Taflove and Hagness, 2000) to each corner of the antenna (Fig. D.8) and with absorbing material ($\epsilon = 10\epsilon_0$, $\sigma = 20$ mS/m) filling the shielding box (Fig. D.9). Note that in the latter case, the antenna is not resistively loaded. As expected, the model containing the resistors yields smaller amplitudes in the upper half-space and less ringing in the lower half-space. Conversely, filling the metal shielding box with absorbing material, dramatically changes the radiation patterns. In particular, the $E$-plane pattern is broadened substantially, so that the so-called "parallel-endfire-mode" (i.e. transmitting and receiving antennas both oriented parallel to the structural grain and aligned one after another) instead of the standard parallel-broadside acquisition mode is indicated with this GPR antenna system.

CONCLUSIONS

We have developed a 3-D GPR antenna system simulation tool based on a FDTD-approximation of Maxwell's equations. The algorithm is flexible, accurate, and numerically efficient and allows us to realistically simulate the radiation characteristics of a wide variety of common surface GPR systems. Due to its modular structure, additional antenna designs can readily be added to the algorithm. Simulations of canonical GPR antenna systems located on a dielectric half-space illustrate the fun-
Figure D.6: Top: $E$-plane (left) and $H$-plane (right) snapshots of the $E_z$ component radiated from a shielded bow-tie antenna located 1 cm above the interface between air and soil ($\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The antenna is 52 cm long, has a flare angle of 60°, and is oriented parallel to the $x$-axis. The shielding consists of a 60 cm long, 38 cm wide, and 26 cm high metal box. Bottom: Amplitude radiation patterns for the tangential components of the electric field, $E_\theta$ (left) and $E_\phi$ (right), in the $E$- and $H$-plane, respectively.
Figure D.7: Top: E-plane (left) and H-plane (right) snapshots of the $E_z$ component radiated from a shielded bow-tie antenna located 1 cm above the interface between air and soil ($\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The antenna is 52 cm long, has a flare angle of 60°, and is oriented parallel to the x-axis. The shielding consists of a 60 cm long, 38 cm wide, and 13 cm high metal box. Bottom: Amplitude radiation patterns for the tangential components of the electric field, $E_\theta$ (left) and $E_\phi$ (right), in the E- and H-plane, respectively.
Figure D.8: Top: E-plane (left) and H-plane (right) snapshots of the $E_x$ component radiated from a shielded bow-tie antenna located 1 cm above the interface between air and soil ($\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The antenna is 52 cm long, has a flare angle of 60°, and is oriented parallel to the x-axis. The shielding consists of a 60 cm long, 38 cm wide, and 13 cm high metal box. The corners of the antenna are connected to the box by 200 $\Omega$ resistors. Bottom: Amplitude radiation patterns for the tangential components of the electric field, $E_{\theta}$ (left) and $E_\phi$ (right), in the E- and H-plane, respectively.
Figure D.9: Top: $E$-plane (left) and $H$-plane (right) snapshots of the $E_z$ component radiated from a shielded bow-tie antenna located 1 cm above the interface between air and soil ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m). The antenna is 52 cm long, has a flare angle of $60^\circ$, and is oriented parallel to the $x$-axis. The shielding consists of a 60 cm long, 38 cm wide, and 13 cm high metal box. The box is filled with absorbing material ($\epsilon = 10\epsilon_0$, $\mu = \mu_0$, $\sigma = 20$ mS/m). Bottom: Amplitude radiation patterns for the tangential components of the electric field, $E_\theta$ (left) and $E_\phi$ (right), in the $E$- and $H$-plane, respectively.
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Fundamental effects that basic design features of a GPR sounding system have on its radiative properties and thus on the illumination of the subsurface. Our results demonstrate that, even under relatively simple and ideal conditions, the commonly used asymptotic far-field radiation patterns for a horizontal electric dipole over a dielectric half-space neither represent an appropriate reference model nor do they provide adequate guidance as to how the subsurface is illuminated by a GPR system. To a lesser extent, the same is true for numerical solutions of Maxwell's equations for an infinitesimal electric dipole source. The simulation software presented in this paper is therefore a key tool for improving our understanding of the effects of GPR antenna systems on the recorded data, for planning and interpreting GPR surveys, as well as for designing novel GPR antenna systems.

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Appendix E

Effects of Random Heterogeneities and Topographic Fluctuations on Ground-Penetrating Radar Antenna Radiation

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ABSTRACT

Typical ground-penetrating radar (GPR) transmitters and receivers consist of dipole-type antennas. These antennas have pronounced directive properties and exhibit strong coupling to interfaces across which there are changes in electric material properties. Whereas coupling of antennas to smooth interfaces has been the subject of intense research for several decades, the behaviour of antennas in the vicinity of realistic small-scale heterogeneities is largely unexplored. To address this issue, we simulate the responses of a typical surface GPR antenna to a suite of scale-invariant earth models of increasing complexity. Finite-difference time-domain (FDTD) simulations demonstrate that roughness of the air-soil interface has a pronounced effect on radiation patterns. By comparison, small-scale fluctuations
of permittivity only cause relatively minor local distortions of the radiation patterns.

Key words: Antenna Radiation, Dipole-Interface Coupling, FDTD, Fractals, Radiation Pattern, Random Media, Topographic Fluctuations

INTRODUCTION

Understanding the radiation properties of ground-penetrating radar (GPR) antennas located on the earth’s surface is key to improving data acquisition, processing, and interpretation strategies. The propagation of electromagnetic waves in GPR surveys is reasonably well understood under simplified and idealized conditions, such as infinitesimal dipole transmitters and receivers placed on homogeneous or laterally homogeneous layered dielectric half-spaces. Far-field expressions for the radiation of horizontal electric dipoles (Annan et al., 1975; Smith, 1984) are commonly used to estimate the radiation characteristics of GPR antennas. Studies have shown that antenna height above the ground and the electric parameters of the subsurface may have pronounced effects on the radiation patterns and thus on the illumination of the subsurface. It is, however, not clear how adequate such concepts are for more realistic earth models characterized by small-scale fluctuations of surface topography and/or the electric material parameters in the subsurface.

In this contribution, we present results from a numerical study of GPR antenna radiation using earth models containing scale-invariant fluctuations of the topography and permittivity of the subsurface. We concentrate on permittivity fluctuations of the subsurface material parameters, since realistic variations of conductivity can range over several orders of magnitude. This poses some formidable, and yet unresolved problems to subsurface model generation and FDTD modeling.

ANTENNA MODEL

Simulations are performed by a finite-difference time-domain (FDTD) solution of Maxwell’s equations in three dimensions (Lampe and Holliger, 2001). The computational volume is surrounded by highly efficient absorbing boundary conditions known as generalized perfectly matched layers (Fang and Wu, 1996). The antenna model used for our simulations is an unshielded bow-tie antenna with a flare angle of 60° and a length of 52 cm oriented parallel to the x-axis (Fig. E.1). These dimensions correspond roughly to a broadband dipole-type antenna with a dominant operating frequency of 300 MHz. By setting to zero the tangential electric field components
within the bow-tie metal panels, the principal characteristics of the antenna are numerically defined. The antenna is excited by a compact Gaussian voltage pulse that is fed into a one-dimensional transmission line model (Maloney et al., 1994).

**EARTH MODEL**

Realistic subsurface heterogeneities are modeled by multiplying the spectrum of a random distribution by the square root of the desired power spectrum. Inverse Fourier transformation provides a representation of the heterogeneities in the space domain (Holliger, 1996; Holliger, 1997). The chosen power spectrum of the heterogeneities is

\[
P(k) \propto \frac{1}{(1 + k^2 a^2)^{\nu + \frac{E}{2}},}
\]

where \(k\) is the wavenumber, \(a\) the correlation length, \(\nu\) the Hurst number and \(E\) the Euclidean dimension. For \(ka \gg 1\), the above expression (E.1) corresponds to the power-law behavior of typical random fractal phenomena (Holliger, 1996).

The permittivity of the subsurface is defined as \(\bar{\varepsilon} = \varepsilon + \Delta\varepsilon\), where \(\varepsilon\) is the deterministic background permittivity and \(\Delta\varepsilon\) are the random fluctuations. These fluctuations can be modeled by scaling their standard deviation to a certain percentage of the background permittivity. The earth’s topography is modeled by setting \(\nu = 0.5\) and \(E = 2\). For our three-dimensional random subsurface models, we set \(\nu = 0.1\) and \(E = 3\). Our chosen Hurst numbers \(\nu\) are considered to be realistic for these phenomena (Turcotte, 1997). Fig. E.2 shows a vertical slice through an inhomogeneous subsurface model with fractal fluctuations of the topography and permittivity. The arrow indicates the position of the transmitting antenna. The correlation length \(a\) for the topography and permittivity fluctuations is 1 m. Since this corresponds to approximately twice the antenna length, it should result in strong scattering.

**NUMERICAL RESULTS**

Fig. E.3 shows a snapshot of the electric field \(E_x\) component propagating into a homogeneous half-space \((\varepsilon = 5\varepsilon_0, \mu = \mu_0, \sigma = 0 \text{ mS/m})\) from a flat air-soil inter-
Figure E.2: Vertical slice through a 3-D fractal subsurface model with scale-invariant fluctuations of topography and permittivity. The deterministic background permittivity is $5\varepsilon_0$. Standard deviations of topography and permittivity fluctuations are 2.91 cm and 0.5$\varepsilon_0$, respectively.

In both snapshots, the wavefronts in the upper half-space (air) have already left the computational volume, whereas the wavefronts in the lower medium show reverberations due to the undamped antenna. Parts of the head waves are present at a depth of about 50 cm near the left and right borders. This simulation will serve as a reference against which simulations based on the heterogenous models will be compared.

For the snapshots shown in Fig. E.4, the model is modified by replacing the flat air-soil interface by random topographic fluctuations with a standard deviation of 2.91 cm and a maximum deviation of about 9 cm from the mean value (see topography in Fig. E.2). This topographic variation causes clear distortions of the wavefronts. As a consequence, the corresponding radiation patterns (blue lines) shown in Fig. E.5 are considerably deformed with respect to the reference patterns for a flat air-soil interface (red lines). The result of reducing the standard deviation to about 1 cm while keeping the form of the relief constant is shown in Fig. E.6. Here, the distortions are less pronounced compared to those observed for the model with higher elevations (Fig. E.5). Part of the mismatch between the radiation patterns in Figs. E.5 and E.6 is related to differences in average antenna height above the ground, with the higher values being set for the topographically varying models (see Fig. E.2).

The effects of inhomogeneous subsurfaces without topographic variations are
Figure E.3: $E$-plane (top) and $H$-plane (bottom) snapshots of the electric field $E_z$ component radiated from an unshielded bow-tie antenna located 1 cm above a flat air-soil interface with an underlying homogeneous half-space ($\epsilon = 5\epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ mS/m).

shown in Figs. E.7 to E.9. The standard deviations of the permittivity fluctuations are 2.5% (Fig. E.7), 5% (Fig. E.8) and 10% (Fig. E.9) with respect to the deterministic background permittivity of $5\epsilon_0$. The influence of low amplitude inhomogeneities in Fig. E.7 is barely visible. With increasing standard deviation of the fluctuations, the distortions of the radiation patterns become more pronounced, but the overall shape of the radiation patterns remains approximately constant. This suggests that coupling of the antenna to the interface is not greatly influenced by small-scale subsurface heterogeneities and that the observed perturbations of the radiation patterns are due to propagation effects in the heterogeneous subsurface.

Finally, Figs. E.10 and E.11 show snapshots and radiation patterns for a GPR antenna situated on an inhomogeneous half-space with topographic fluctuations (Fig. E.2). The resulting distortions of the radiation patterns are comparable to those observed in Fig. E.4. They are interpreted to be primarily due to scattering from the topographic fluctuations.

**CONCLUSIONS**

Numerical simulations of GPR antenna radiation in the presence of realistic topographic roughness and subsurface heterogeneity have been performed. Topographic roughness may have a significant effect on the overall shape of the radiation pat-
Figure E.4: $E$-plane (top) and $H$-plane (bottom) snapshots of the electric field $E_x$ component radiated from an unshielded bow-tie antenna located on an air-soil interface with random topographic fluctuations (standard deviation of 2.91 cm). Maximum variations of topography are ±9 cm. Arrow indicates center of the antenna.

Figure E.5: Amplitude radiation patterns for the electric field tangential components, $E_\theta$ (top) and $E_\phi$ (bottom), in the $E$- and $H$-planes, respectively, recorded for model in Fig. E.4 (blue lines). Recording distance is 80 cm. Thin red lines show the radiation patterns for a homogeneous half-space with a flat air-soil interface. The data are normalized with respect to the maximum amplitude.
Figure E.6: Amplitude radiation patterns for the electric field tangential components, \( E_\theta \) (top) and \( E_\phi \) (bottom), in the \( E \)- and \( H \)-plane, respectively. Maximum variations of topography are \( \pm3 \) cm (blue lines). Recording distance is 80 cm. Thin red lines show the radiation patterns for a homogeneous half-space with a flat air-soil interface. The data are normalized with respect to the maximum amplitude.

Figure E.7: Amplitude radiation patterns for the electric field tangential components, \( E_\theta \) (top) and \( E_\phi \) (bottom), in the \( E \)- and \( H \)-plane, respectively, for a half-space with random permittivity fluctuations and a flat air-soil interface (blue lines). Standard deviation of the random fluctuations is 2.5% of the background permittivity. Recording distance is 80 cm. Thin red lines show the radiation patterns for a homogeneous half-space with a flat air-soil interface. The data are normalized with respect to the maximum amplitude.
Figure E.8: Amplitude radiation patterns for the electric field tangential components, $E_{\theta}$ (top) and $E_{\phi}$ (bottom), in the $E$- and $H$-plane, respectively, for a half-space with random permittivity fluctuations and a flat air-soil interface (blue lines). Standard deviation of the random fluctuations is 5% of the background permittivity. Recording distance is 80 cm. Thin red lines show the radiation patterns for a homogeneous half-space with a flat air-soil interface. The data are normalized with respect to the maximum amplitude.

Figure E.9: Amplitude radiation patterns for the electric field tangential components, $E_{\theta}$ (top) and $E_{\phi}$ (bottom), in the $E$- and $H$-plane, respectively, for a half-space with random permittivity fluctuations and a flat air-soil interface (blue lines). Standard deviation of the random fluctuations is 10% of the background permittivity. Recording distance is 80 cm. Thin red lines show the radiation patterns for a homogeneous half-space with a flat air-soil interface. The data are normalized with respect to the maximum amplitude.
Figure E.10: $E$-plane (top) and $H$-plane (bottom) snapshots of the electric field $E_x$ component radiated from an unshielded bow-tie antenna located on an air-soil interface with random topographic fluctuations. The half-space has random permittivity fluctuations with a standard deviation of 10% with respect to the background permittivity and topographic relief with maximum variations of ±9 cm. Arrow indicates center of the antenna.

Figure E.11: Amplitude radiation patterns for the electric field tangential components, $E_\theta$ (top) and $E_\phi$ (bottom), in the $E$- and $H$-plane, respectively, recorded for the model in Fig. E.10 (blue lines). Recording distance is 80 cm. Thin red lines show the radiation patterns for a homogeneous half-space with a flat air-soil interface. The data are normalized with respect to the maximum amplitude.
terns; topography not only causes scattering, but it also affects the average height of the antenna above the ground and thus its coupling to the interface. In contrast, permittivity fluctuations cause relatively small perturbations of the radiation patterns. This indicates that coupling of the antenna to local fluctuations in the material properties is likely to be rather insignificant.

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