AFM-defined cavities and antidot arrays from classical chaos to quantum percolation

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AFM-defined Cavities and Antidot Arrays
from Classical Chaos to Quantum Percolation

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## Contents

List of symbols vii
Summary 1
Zusammenfassung 3

1 Introduction 5

2 Basic concepts 7
  2.1 Two-dimensional electron systems (2DES) .................. 7
    2.1.1 Important length scales ........................ 7
    2.1.2 Magnetotransport .............................. 8
  2.2 The Landauer Büttiker formalism ........................... 10
    2.2.1 Quantum point contacts (QPCs) ........................ 10
    2.2.2 Multi terminal resistance ........................... 11
  2.3 Quantum dots .......................................... 12
    2.3.1 Introduction .................................... 12
    2.3.2 Coulomb blockade .................................. 12
    2.3.3 Quantum mechanical level spacing ........................ 14
    2.3.4 Peak shape and line width ............................ 15
  2.4 Quantum interference effects ............................ 17
    2.4.1 The Aharonov–Bohm effect ........................... 17
    2.4.2 Ballistic conductance fluctuations ........................ 18
    2.4.3 Weak localization .................................. 18
  2.5 Antidot lattices ........................................ 18
    2.5.1 Introduction .................................... 18
    2.5.2 Magnetotransport .................................. 19
    2.5.3 Bandstructures and Hofstadter butterfly ................ 24

3 Sample fabrication and experimental setup 27
  3.1 The Ga(Al)As material system ............................ 27
  3.2 Lithography ........................................... 28
    3.2.1 Optical lithography ................................ 28
    3.2.2 AFM-lithography ................................... 29
  3.3 Measurement techniques ................................. 34
    3.3.1 Resistance measurements ............................ 34
    3.3.2 Cryostats ....................................... 35
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>ErAs islands for backgate insulation</td>
<td>37</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>37</td>
</tr>
<tr>
<td>4.2</td>
<td>Wafer</td>
<td>37</td>
</tr>
<tr>
<td>4.3</td>
<td>Effective barrier height</td>
<td>38</td>
</tr>
<tr>
<td>4.4</td>
<td>Charging effects in the ErAs island system</td>
<td>41</td>
</tr>
<tr>
<td>4.5</td>
<td>2DES characterization</td>
<td>44</td>
</tr>
<tr>
<td>4.6</td>
<td>Summary</td>
<td>46</td>
</tr>
<tr>
<td>5</td>
<td>The empty cavity</td>
<td>47</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>47</td>
</tr>
<tr>
<td>5.2</td>
<td>Square cavity</td>
<td>47</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Skipping orbits</td>
<td>47</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Billiard simulation</td>
<td>49</td>
</tr>
<tr>
<td>5.2.3</td>
<td>Measured and calculated magnetoresistance</td>
<td>50</td>
</tr>
<tr>
<td>5.3</td>
<td>Rectangular cavity</td>
<td>53</td>
</tr>
<tr>
<td>5.4</td>
<td>Summary</td>
<td>54</td>
</tr>
<tr>
<td>6</td>
<td>Magnetoresistance in antidot lattices</td>
<td>55</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>55</td>
</tr>
<tr>
<td>6.2</td>
<td>Sample characterization</td>
<td>55</td>
</tr>
<tr>
<td>6.3</td>
<td>Commensurability peaks</td>
<td>57</td>
</tr>
<tr>
<td>6.4</td>
<td>Aharonov–Bohm-type oscillations</td>
<td>65</td>
</tr>
<tr>
<td>6.5</td>
<td>Aharonov–Altshuler–Spivak oscillations</td>
<td>70</td>
</tr>
<tr>
<td>6.6</td>
<td>Peak splitting</td>
<td>70</td>
</tr>
<tr>
<td>6.7</td>
<td>Summary</td>
<td>71</td>
</tr>
<tr>
<td>7</td>
<td>Percolation in a quantum dot network</td>
<td>73</td>
</tr>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>73</td>
</tr>
<tr>
<td>7.2</td>
<td>Percolation</td>
<td>74</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Concepts and terminology</td>
<td>74</td>
</tr>
<tr>
<td>7.2.2</td>
<td>Conduction through percolating systems</td>
<td>75</td>
</tr>
<tr>
<td>7.3</td>
<td>Experimental findings</td>
<td>77</td>
</tr>
<tr>
<td>7.3.1</td>
<td>Sample characterization</td>
<td>77</td>
</tr>
<tr>
<td>7.3.2</td>
<td>The open regime</td>
<td>78</td>
</tr>
<tr>
<td>7.3.3</td>
<td>The Coulomb blockade regime</td>
<td>82</td>
</tr>
<tr>
<td>7.3.4</td>
<td>Local and global properties of the spanning cluster</td>
<td>85</td>
</tr>
<tr>
<td>7.4</td>
<td>Summary</td>
<td>89</td>
</tr>
<tr>
<td>8</td>
<td>Artificial bandstructures</td>
<td>91</td>
</tr>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>91</td>
</tr>
<tr>
<td>8.2</td>
<td>Measurements</td>
<td>93</td>
</tr>
<tr>
<td>8.2.1</td>
<td>Weak potential modulation</td>
<td>93</td>
</tr>
<tr>
<td>8.2.2</td>
<td>The tight-binding limit</td>
<td>94</td>
</tr>
<tr>
<td>8.2.3</td>
<td>Antidot arrays</td>
<td>95</td>
</tr>
<tr>
<td>8.3</td>
<td>Summary</td>
<td>95</td>
</tr>
<tr>
<td>9</td>
<td>Conclusions and outlook</td>
<td>99</td>
</tr>
</tbody>
</table>
Appendices

A: List of Samples ........................................ 103
B: Sample processing ...................................... 105

Bibliography ................................................ 109

Acknowledgements .......................................... 119

Journal Publications ..................................... 121

Curriculum Vitae ........................................... 123
Lists of symbols

<table>
<thead>
<tr>
<th>physical constant</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit charge</td>
<td>-e</td>
</tr>
<tr>
<td>Vacuum dielectric constant</td>
<td>$\epsilon_o$</td>
</tr>
<tr>
<td>Flux quantum</td>
<td>$\phi_o = h/e$</td>
</tr>
<tr>
<td>Planck’s constant</td>
<td>$h = 2\pi\hbar$</td>
</tr>
<tr>
<td>Boltzmann’s constant</td>
<td>$k_B$</td>
</tr>
<tr>
<td>Conductance quantum</td>
<td>$e^2/h$</td>
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</tbody>
</table>

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<thead>
<tr>
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<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2DES</td>
<td>two–dimensional electron system</td>
</tr>
<tr>
<td>AFM</td>
<td>atomic force microscope</td>
</tr>
<tr>
<td>SPM</td>
<td>scanning probe microscope</td>
</tr>
<tr>
<td>CB</td>
<td>Coulomb blockade</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width at half maximum</td>
</tr>
<tr>
<td>HWHM</td>
<td>half width at half maximum</td>
</tr>
<tr>
<td>QPC</td>
<td>quantum point contact</td>
</tr>
<tr>
<td>SdH</td>
<td>Shubnikov de Haas</td>
</tr>
<tr>
<td>MBE</td>
<td>molecular beam epitxy</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>a</td>
<td>antidot lattice constant</td>
</tr>
<tr>
<td>L, W, A</td>
<td>system size (length, width, area)</td>
</tr>
<tr>
<td>tg, bg</td>
<td>topgate, backgate</td>
</tr>
<tr>
<td>B</td>
<td>magnetic field</td>
</tr>
<tr>
<td>$\vec{A}$</td>
<td>vector potential</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>magnetic flux through a loop</td>
</tr>
<tr>
<td>$C_S$</td>
<td>self-capacitance of a quantum dot</td>
</tr>
<tr>
<td>$E_C$</td>
<td>constant interaction energy</td>
</tr>
<tr>
<td>$\alpha_G$</td>
<td>gate lever arm</td>
</tr>
<tr>
<td>$\epsilon_N$</td>
<td>single particle energy of the $N^{th}$ level</td>
</tr>
<tr>
<td>$\Delta_N$</td>
<td>spin degenerate single particle level spacing</td>
</tr>
<tr>
<td>$E_F$</td>
<td>Fermi energy</td>
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<tr>
<td>$k_F$</td>
<td>Fermi wavevector</td>
</tr>
<tr>
<td>$v_F$</td>
<td>Fermi velocity</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td>Fermi wavelength</td>
</tr>
<tr>
<td>Q</td>
<td>charge</td>
</tr>
<tr>
<td>I</td>
<td>current</td>
</tr>
<tr>
<td>V</td>
<td>voltage</td>
</tr>
<tr>
<td>$\rho$</td>
<td>conductance</td>
</tr>
<tr>
<td>$R_{xx}$</td>
<td>longitudinal resistance</td>
</tr>
<tr>
<td>$R_{xy}$</td>
<td>Hall resistance</td>
</tr>
<tr>
<td>$\Gamma_{s,d}$</td>
<td>tunnel coupling</td>
</tr>
<tr>
<td>$\ell_c$</td>
<td>magnetic length</td>
</tr>
<tr>
<td>$\ell_e$</td>
<td>elastic mean free path</td>
</tr>
<tr>
<td>$\ell_\varphi$</td>
<td>phase coherence length</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>transport scattering time</td>
</tr>
<tr>
<td>$\tau_\varphi$</td>
<td>phase coherence time</td>
</tr>
<tr>
<td>$\tau_q$</td>
<td>quantum life time</td>
</tr>
<tr>
<td>$m^*$</td>
<td>effective electron mass in GaAs</td>
</tr>
<tr>
<td>$\mu$</td>
<td>electron mobility</td>
</tr>
<tr>
<td>$n_e$</td>
<td>electron sheet density</td>
</tr>
<tr>
<td>$\nu$</td>
<td>filling factor</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>cyclotron frequency</td>
</tr>
<tr>
<td>$r_c$</td>
<td>cyclotron radius</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
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</tbody>
</table>
Summary

In this thesis the electronic transport properties of semiconductor nanostructures are investigated. In particular we focus on square and rectangular cavities and antidot superlattices, that were fabricated by local anodic oxidation with an atomic force microscope (AFM). GaAs/AlGaAs heterostructures with a high mobility two-dimensional electron system (2DES) 34 nm below the surface, served as the starting material. The wafers were grown by molecular beam epitaxy at the University of California at Santa Barbara by Danny Driscoll and Art Gossard.

Resistance measurements were performed on the square and rectangular cavities with different current and voltage probe configurations in the presence of a perpendicular magnetic field at 1.7 K. Oscillations in the magneto-resistance were observed and explained by commensurability effects of orbits skipping along the sides of the cavities. The good quantitative agreement with calculations based on the Landauer–Büttiker formalism and a classical billiards simulation demonstrates the high quality of AFM-defined nanostructures.

The antidot lattices under study consist of square 20 × 20 antidot arrays enclosed by cavities with openings in the corners for current and voltage probes. In low temperature magnetotransport measurements, commensurability peaks, Aharonov–Bohm-type oscillations, and h/2e oscillation were detected in samples with lattice constants of 150 nm and 300 nm. From this we conclude, that the electronic properties of AFM-defined antidot lattices are comparable to the best arrays fabricated by other techniques. In addition the special top- and backgate tunability of the a=150 nm sample allowed us to do the first investigations of these effects as a function of background mobility and wavefunction symmetry in z-direction at constant electron sheet densities.

The transition from the conducting to the insulating phase towards low electron densities was studied in an array with a lattice constant of a=120 nm at 90 mK. In the open regime, the behavior of the conductivity is well modeled by a classical random resistor network with superimposed quantum fluctuations, while in the tunneling regime, clear Coulomb resonances appeared with on average increasing charging energies towards lower electron sheet densities. These findings were analyzed in the framework of the ‘links, nodes and blobs model’ for percolation proposed by Stanley and Coniglio. The extracted critical exponents are in good agreement with numerical calculations. A strong negative average magneto-resistance in the Coulomb blockade regime was found to be in good quantitative agreement with a theoretical model by Raikh and Glazman based on magnetic field dependent tunneling between individual quantum dots. This type of system could be of interest in the field of quantum information processing, especially if it is pos-
sible to address and tune individual sites of the lattice. The observed microscopic transport phenomena can also be compared with measurements on macroscopic samples of granular materials like metallic or semiconductor nanocrystals.

The observability of an artificial bandstructure is discussed in the weak potential modulation, the tight-binding and the antidot regime.

In addition, layers of ErAs islands were characterized with respect to their insulating and charging properties. The samples consisted of GaAs/AlGaAs heterostructures with a two-dimensional electron system 34 nm below the surface and a conducting layer, the so-called backgate, 1.3 \( \mu \)m below the surface. Twenty layers of ErAs islands with a separation of 25 nm are sandwiched between these two conducting planes and insulate them with respect to each other. This is a consequence of overlapping Schottky barriers surrounding the ErAs islands, and breakdown voltages above \( \pm 7 \) V could be achieved at 1.7 K. From thermal activation measurements an effective Schottky barrier height of 0.6 eV across the island system was determined. From persistent charging effects a Schottky barrier height of 0.58-1.0 eV onto the islands was extracted. In addition persistent photoconductivity was present in one of the two wafers as a consequence of electrons being excited off the islands. Layers of ErAs islands are an alternative to low temperature grown GaAs for insulating two-dimensional electron systems from backgates and could find a number of applications, especially if local charging is feasible, e.g. by nanostructured electrodes.
Zusammenfassung

In der vorliegenden Arbeit werden die elektronischen Transporteigenschaften von Halbleiternanostrukturen experimentell untersucht. Insbesondere handelt es sich hierbei um quadratische und rechteckige Kavitäten und Antidotübergitter, die durch lokale oxidation mit einem Rasterkraftmikroskop (AFM) hergestellt wurden. Als Ausgangsmaterial dienten GaAs/AlGaAs Heterostrukturen mit einem hoch beweglichen zweidimensionalen Elektronensystem 34 nm unter der Oberfläche, die mit Molekularstrahlepitaxie gewachsen wurden.

Widerstandsmessungen an der quadratischen und rechteckigen Kavität wurden in verschiedenen Konfigurationen bei senkrecht angelegtem Magnetfeld und bei Temperaturen von 1.7 K durchgeführt. Es konnten magnetfeldabhängige Oszillationen der Leitfähigkeit beobachtet werden, die mit Kommensurabilitätseffekten der an den Banden reflektierten Elektronenkonfigurationen erklärt werden können. Die gute quantitative Übereinstimmung der Messungen mit Berechnungen basierend auf dem Landauer Büttiker Formalismus und einer klassischen Billiardsimulation, sprechen für die hohe elektronische Qualität AFM-definierter Nanostrukturen.


Anschließend werden Proben mit verschiedenen Parametern hinsichtlich der Beobachtbarkeit einer künstlichen Bandstruktur diskutiert.

Zusätzlich wurden Schichten von ErAs Inseln hinsichtlich ihrer Isolationseigenschaften und Ladungszustände charakterisiert. Die hierzu verwendeten Proben bestehen aus GaAs/AlGaAs Heterostrukturen mit einem zweidimensionalen Elektronensystem 34 nm unter der Oberfläche und einer leitfähigen Schicht, dem sogenannten Backgate, 1,3 µm unter der Oberfläche. Zwischen diesen beiden leitfähigen Ebenen sind 20 Schichten mit ErAs Inseln mit einem Abstand von 25 nm eingewachsen, die das zweidimensionale Elektronensystem von dem Backgate isolieren. Dies geschieht aufgrund von überlappenden Schottkybarrieren, die sich um die in das GaAs eingebetteten ErAs Inseln herum bilden. Bei Temperaturen unter 1.7 K konnten Durchbruchspannungen von über ±7 V erzielt werden. Aus Messungen des thermisch aktivierten Transports wurde eine effektive Schottkybarrierehöhöhe von etwa 0.6 meV für das Inselsystem ermittelt. Durch statisches Aufladen der ErAs Inseln konnte eine Barrierehöhe von 0.58-1.0 meV auf die ErAs Inseln bestimmt werden. In einem der beiden Wafer konnte zusätzlich persistente Photoleitfähigkeit beobachtet werden, die auf eine Verringerung der Elektronenzahl auf den Inseln zurückzuführen ist. Neben ihren Isolationseigenschaften, könnten ErAs Inseln von großem Interesse sein, falls lokale Ladevorgänge, z.B. durch nanostrukturierte Elektroden, möglich sind.
Chapter 1
Introduction

The conductance of a macroscopic piece of metal at room temperature is, to a good approximation, proportional to its cross sectional area and inverse length. For very small conductors at low temperatures this simple relation no longer holds. Instead, ballistic and quantum mechanical effects reflecting the sample geometry, as well as charge quantization in integer multiples of $e = 1.602 \times 10^{-19} \text{C}$ begin to play a dominant role. These phenomena are the focus of interest in the research field of mesoscopic electronics. Historically the first experiments were done on small metal junctions and grains. In the last 15 years a large part of this interest has shifted to the study of semiconductor nanostructures. This change was motivated by two factors: the extreme, almost atomic precision to which semiconductor structures can be fabricated, and secondly the smaller effective electron masses in these materials, that make quantum effects more accessible. The cleanest and most popular material systems today are Ga(Al)As heterostructures. By growing a suitable sequence of GaAs and Al$_x$Ga$_{1-x}$As layers it is possible to confine the conduction electrons to their ground mode in z-direction, while they are free to propagate in the xy-plane. Clean two-dimensional electron systems (2DESs) of this type exhibit the quantum Hall effect in a perpendicular magnetic field at low temperatures. By introducing additional lateral confinement, one-dimensional quantum wires and zero-dimensional quantum dots can be fabricated. The high carrier mobilities bring the samples into the deeply ballistic regime, where the electrons are primarily scattered off the edges of the artificially defined nanostructures. In this thesis electronic transport through square arrays of insulating islands, so-called antidots, in 2DESs are studied at low temperatures ($< 5\text{K}$). Systems of this type have attracted considerable interest in several respects. By applying a perpendicular magnetic field, classical phase space can be tuned from regular to chaotic. This yields direct experimental insight into the correspondence between classical and quantum chaotic systems. Another objective is to induce an artificial two-dimensional band structure with parameters and symmetries that can be controlled at will. This could open a window on the fractal splitting of the crystal bands as a function of magnetic field according to Hofstadter’s famous prediction. A long term goal is to fabricate a quantum dot lattice with sites that can be addressed and manipulated independently. These structures could find applications in the emerging field of quantum information processing. All these topics have in
common, that antidot arrays with very short lattice constants and high regularity are desirable. In the following work we investigate AFM-lithography as a means of achieving these specifications and interpret the resulting electronic effects. In addition a new type of wafer containing ErAs islands for backgate insulation is characterized with respect to the charging properties and Schottky barrier heights of the island layers. The chapters of this thesis are organized as follows:

- **Basic concepts** presents a short overview of some of the most important concepts in mesoscopic electronic transport. Although good introductory texts exist [1, 2, 3], this chapter was added for completeness and referencing. A summary of the most important previous experimental and theoretical work on antidot lattices is also given.

- **Sample fabrication and experimental setup** contains a detailed description of AFM-lithography with special emphasis on antidot lattices. Wafer processing as well as the electronic measurement configurations and cryogenic equipment are described.

- **ErAs islands for backgate insulation** The Schottky barrier heights and charging properties of ErAs islands embedded between the 2DES and a backgate electrode are studied. This new type of wafer design was used to fabricate antidot lattices with top- and backgate tunability.

- **The empty cavity** Measurements on a plain square and rectangular cavity are presented and modeled with a classical billiard simulation and the Landauer–Büttiker formalism.

- **Magnetoresistance in antidot lattices** is studied in samples with different lattice constants. Commensurability effects, Aharonov–Bohm type oscillations, $h/2e$ oscillations, and a novel peak-splitting effect are investigated as a function of electron sheet density, wavefunction symmetry and background mobility.

- **Percolation in a quantum dot network** Electronic transport through a multiply connected multi-terminal quantum dot network is probed. Percolation models are applied to explain the conductivity as a function of electron sheet density. A strong positive magnetoconductance effect in the Coulomb blockade regime is in good quantitative agreement with a theoretical model by Raikh and Glazman.

- **Artificial bandstructures** are discussed in terms of samples with weak potential modulations, in the tight-binding limit and in the antidot case. Experimental requirements as well as possible signatures are considered.
Chapter 2
Basic concepts

2.1 Two-dimensional electron systems (2DES)

A two-dimensional electron system (2DES) is a sheet of electrons that is so thin, that all charge carriers are confined to their quantum mechanical ground mode in the z-direction perpendicular to the conducting plain. These systems are usually realized by growing a sequence of different semiconductor layers on top of each other, so that the z-component of the conduction band only falls below the Fermi energy within a narrow region, forming a quantum well. Details concerning the wafers used in this thesis and calculations of the conduction band profiles and wavefunction components in z-direction will be discussed in chapters 3 and 4.

The density of states in a 2DES is constant and independent of its electron sheet density, \( n_e \):

\[
\rho = \frac{m^*}{\pi \hbar^2} \tag{2.1}
\]

where \( m^* \) is the effective electron mass and spin degeneracy has been taken into account. The Fermi energy measured from the bottom of the conduction band is given by:

\[
E_F = \frac{\pi \hbar^2}{m^* n_e} \tag{2.2}
\]

2.1.1 Important length scales

Since electron motion is restricted to two-dimensions, practically all important length scales apply to the conducting xy-plain, the only notable exception being the width of the quantum well. It is also important to keep in mind, that electronic conduction is completely dominated by Fermi surface properties, due to the Pauli-principle \(^1\).

One of the most important length scales characterizing mesoscopic systems is the Fermi wavelength \( \lambda_F \):

\[
\lambda_F = \sqrt{\frac{2\pi}{n_e}} \tag{2.3}
\]

\(^1\) for \( k_BT << E_F \).
In an Ga(Al)As heterostructure with a 2DES density of \( n_e \approx 3 \times 10^{15} \text{m}^{-2} \), \( \lambda_F \approx 40 \text{nm} \). If the size of a nanostructure approaches this order of magnitude, effects from the quantum mechanical confinement energy will become important. On the other hand, in a typical metal such as Cu or Ag, the Fermi wavelength is of the order of a few Angstrom. For this reason quantum mechanical contributions can usually be neglected in metallic nanostructures.

The mean free path \( l_m \) is the distance over which an electron can travel without being scattered e.g. by an impurity atom or growth dislocation. Scattering off static impurities on a time scale \( \tau_c \) leads to momentum randomization or relaxation within an average time \( \tau_m \). The two times are related by \( \tau_m = \alpha \tau_c \), where \( \alpha \) is a constant between 0 and 1 measuring the ‘effectiveness’ of the collisions. It follows that the mean free path an electron can travel without completely losing its initial momentum is:

\[
l_m = v_F \tau_m = \frac{\hbar \sqrt{2\pi n}}{m^* \tau_m}
\]  

(2.4)

where \( v_F \) is the Fermi velocity.

The phase coherence of an electron is only maintained during a finite time \( \tau_\phi \). It can be destroyed by inelastic scattering e.g. from phonons, from impurities with internal degrees of freedom or from e–e interactions, not however by simple elastic scattering off static impurities. The corresponding phase coherence length \( l_\phi \) is:

\[
l_\phi = \begin{cases} 
v_F \tau_\phi, & \tau_\phi < \tau_m \text{classical transport regime} \\
\sqrt{D \tau_\phi}, & \tau_\phi \gg \tau_m \text{quantum transport regime}
\end{cases}
\]

(2.5)

(2.6)

(2.7)

with the diffusion constant \( D \) for a two-dimensional system:

\[
D = \frac{1}{2} v_F^2 \tau_m
\]

(2.8)

Quantum mechanical interference effects are observed when the length scales of a mesoscopic system become comparable to the phase coherence length. Electron–phonon and electron–electron scattering decrease towards lower temperatures and \( l_\phi \) can approach several microns in high mobility samples below 1 K.

### 2.1.2 Magnetotransport

A standard way of characterizing 2DESs is to measure the longitudinal and Hall conductivities in a Hall bar geometry (Fig. 2.1). In the classical Drude picture, the longitudinal resistivity is independent of magnetic field and the Hall resistance shows a linear increase. The electron sheet density \( n_e \) and mobility \( \mu \) can then be extracted using the relations:

\[
\sigma_{xx} = \frac{L}{W} \frac{I}{V_{xx}} = e n_e \mu, \quad \mu = \frac{e \tau_m}{m^*}
\]

(2.9)

This approach is only valid for low magnetic fields, when electrons are scattered before completing a cyclotron orbit. But at low temperatures, high magnetic
fields and in very pure samples, quantum mechanical effects have to be taken into consideration as well.

The Hamiltonian of an electron moving in a perpendicular magnetic field can be written as:

\[ H = \frac{1}{2m^*}(\vec{p} - e\vec{A})^2 + V(\vec{r}) \] (2.10)

with eigenvalues:

\[ E_n = (n + \frac{1}{2})\hbar\omega_c, \quad n = 0, 1, 2, \ldots \] (2.11)

where \( \omega_c = eB/m^* \) is the cyclotron frequency. This means, that the magnetic field causes the homogeneous density of states to condense onto a series of evenly spaced \( \delta \)-peaks, the so-called Landau levels. As the magnetic field is increased, the energy spacings widen and the number of (spin-split) Landau levels below the Fermi energy, often referred to as the filling factor \( \nu \), decreases. Because the total number of states must remain constant, the Landau levels are highly degenerate:

\[ N_L = \frac{\hbar\omega_c m^*}{2\pi\hbar^2} = \frac{eB}{\hbar} \] (2.12)

In a real sample the Landau levels will be broadened due to scattering off impurities and thermal excitations.

If the Fermi energy \( E_F \) lies exactly between two Landau levels, \( \nu \) is integer and there will be a minimum in the density of states at the Fermi surface. As a consequence the conductivity \( \sigma_{xx} \) and the resistivity \( \rho_{xx} \) will show a minimum at these magnetic fields as well\(^2\). The resulting oscillations in the longitudinal magnetoresistance are called the Shubnikov–de Haas effect (SdH). By measuring the magnetic field values of two consecutive SdH minima and making use of equations 2.1 2.11 it is possible to determine the electron sheet density:

\[ n_e = \frac{2e}{\hbar} \frac{1}{1/B_1 - 1/B_2} \] (2.13)

At sufficiently high magnetic fields \( \rho_{xx} \) goes to zero between Landau levels and the Hall resistance exhibits well defined plateaus at precisely:

\[ \rho_{xy} = \frac{\hbar}{e^2 \nu}, \nu = 1, 2, 3, \ldots \] (2.14)

This phenomena is called the integer quantum hall effect (IQH) \([4]\) and is common to all high quality two-dimensional electron systems regardless of the specific sample properties. In this regime, current is only carried by one dimensional channels at the edges of the sample. These so-called edge states are formed by Landau levels below the Fermi energy, that intersect the Fermi energy as the electron density goes to zero at the rims of the 2DES. The number of edge states is equal to the filling factor \( \nu \). Current transport in these channels is dissipationless, unless a Landau level coincides with the Fermi energy of the bulk 2DES. In this case backscattering between counterpropagating edge states on opposite sides of the sample is possible and a resistance peak in \( \rho_{xx} \) is observed. A simple description of the values of the quantized Hall resistance can be given in the framework of the Landauer–Büttiker formalism.

\(^2\)\(\sigma_{xx} \propto \rho_{xx} \) under these conditions, see e.g. \([1]\)
2.2 The Landauer Büttiker formalism

2.2.1 Quantum point contacts (QPCs)

If the conductance through a narrow constriction in a 2DES is measured as a function of electron density or width of the quantum point contact (QPC), distinct steps in the conductance in units of $2e^2/h$ appear (Fig. 2.2). This observation can be understood by employing the Landauer formula for conduction:

$$G = \frac{2e^2}{h} \sum_{n,m=1}^{N} |t_{nm}|^2 = \frac{2e^2}{h} \text{Tr}(t^\dagger t)$$

(2.15)

where $t$ is the transmission matrix for $N$ modes. In this picture conduction results from transmission, and every ideal one-dimensional channel carries a conductance quantum $2e^2/h$, the factor of 2 arising from spin degeneracy. If the electronic width of the QPC is $W_0$ and $\lambda_F$ is the Fermi wavelength, the number of modes in the constriction is $N_{\text{trans}} = 2W_0/\lambda_F = 2\pi W_0/k_F$. Completely open channels have a transmission close to 1, while for a partially pinched off mode, transmission is exponentially suppressed. This leads to the observed conductance staircase (Fig.2.2):

$$G = \frac{2e^2}{h} N_{\text{trans}},$$

(2.16)

where $N_{\text{trans}}$ is integer. The rounding of the steps is due to tunneling from partially pinched off channels and temperature smearing. A small plateau around $0.7\times2e^2/h$ is often observed and has been termed the O.7 feature. The exact mechanisms leading to this effect are still under debate.
CHAPTER 2. BASIC CONCEPTS

2.2.2 Multi terminal resistance

The two-terminal expression for the conductance introduced by Landauer was extended to multi-terminal configurations by Büttiker [7]. The total transmission from reservoir $\alpha$ to reservoir $\beta$ can then be written as:

$$T_{\alpha \rightarrow \beta} = \sum_{n=1}^{N_{\alpha}} \sum_{m=1}^{N_{\beta}} |t_{\beta\alpha,mn}|^2.$$  \hspace{1cm} (2.17)

and the total current $I_{\alpha}$ through lead $\alpha$ is given by:

$$I_{\alpha} = \frac{2e}{h} ((N_{\alpha} - R_{\alpha})\mu_{\alpha} - \sum_{\beta(\beta\neq\alpha)} T_{\beta\rightarrow\alpha}\mu_{\beta}).$$  \hspace{1cm} (2.18)

where $R_{\alpha} \equiv T_{\alpha\rightarrow\alpha}/N_{\alpha}$ is the incoming current reflected back into lead $\alpha$ and $N_{\alpha}$ denotes the number of modes. The individual terminals are connected to reservoirs by perfect (ballistic) leads and every reservoir has an electro chemical potential $\mu_{i} = eV_{i}$. Confining ourselves to four terminals, the four-terminal resistance reads:

$$R_{mn,kl} \equiv \frac{V_{mn}}{I_{kl}}$$  \hspace{1cm} (2.19)

where the current flows from reservoir k to l and the voltage drop is measured between reservoirs m and n. Setting $I_{m} = -I_{n} = I$ leads to the expression [7]:

$$R_{mn,kl} = \frac{h}{e^2} \left( \frac{T_{km}T_{ln} - T_{kn}T_{lm}}{D} \right)$$  \hspace{1cm} (2.20)

where

$$D = \frac{h}{e^2} (\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21})S$$

$$S = T_{12} + T_{14} + T_{32} + T_{34} = T_{21} + T_{41} + T_{23} + T_{43}$$
2.3. QUANTUM DOTS

\[
\begin{align*}
\alpha_{11} & = \frac{e^2}{\hbar} \left( - \sum_{i \neq 1} T_{i1} - \frac{(T_{14} + T_{12})(T_{41} + T_{21})}{S} \right) \\
\alpha_{12} & = \frac{e^2}{\hbar} \frac{(T_{12} + T_{34})(T_{14} + T_{32})}{S} \\
\alpha_{21} & = \frac{e^2}{\hbar} \frac{(T_{21} + T_{43})(T_{23} + T_{41})}{S} \\
\alpha_{22} & = \frac{e^2}{\hbar} \left( - \sum_{i \neq 2} T_{i2} - \frac{(T_{21} + T_{23})(T_{32} + T_{12})}{S} \right)
\end{align*}
\] (2.21)

Once the transmission matrix for the entire system has been found, e.g. by doing a classical billiard simulation, the resistance for all current and voltage configurations can be calculated.

2.3 Quantum dots

2.3.1 Introduction

Quantum dots are structures in which charge carriers are confined in all three spatial dimensions. The volume is typically small enough to make charge quantization and the quantum mechanical confinement energy important, which has also led to the term ‘artificial atom’ [8, 9]. Semiconducting and metallic quantum dots have been fabricated by chemical synthesis, molecular beam epitaxy, and by employing various lithographic techniques; their electronic structure can be studied by optical and capacitance spectroscopy as well as in electronic transport experiments, see e.g. [10, 11, 12]. Here we will focus on laterally defined quantum dots on two-dimensional electron systems. In these structures a small electron lake is coupled to macroscopic current and voltage leads by QPCs. The energy spectra can then be mapped out by recording the conductance as a function of the electrochemical potential in the dot. Details concerning sample fabrication and the low temperature electronic measurement setup used in this thesis will be discussed in chapter 3.

2.3.2 Coulomb blockade

Consider a small metallic grain coupled to a source and a drain electrode across tunnel barriers [13]. If the grain size is decreased, the charging energy $E_C$, that is needed to add an extra electron to the grain, will increase. When $E_C$ exceeds the thermal excitation energy $k_BT$, current transport through the particle can be inhibited leading to Coulomb blockade. In order to gain a more quantitative understanding of this effect, we can bring a conducting plate or tip, often called a ‘gate’, close to the grain (Fig. 2.3 a). The total capacitance of the grain is than given by the sum of its self capacitance $C_0$, and its capacitance with respect to the gate $C_g$, source $C_S$, drain $C_D$, and possibly other conductors in its vicinity:
By applying a voltage $V_g$ to the gate, the equilibrium number of electrons on the grain, $N$, can be tuned to $Ne - V_g C_g$. This leads to a total electrostatic energy:

$$U(N) = \left(\frac{Ne - C_g V_g}{2C}\right)^2$$ (2.23)

It follows that the electrochemical potential, defined as the energy that is needed to add the $N$th electron to the system, can be written as:

$$\mu_{\text{grain}}(N) = U(N) - U(N - 1) = \frac{e^2}{C}(N - \frac{1}{2}) - e \frac{C_g}{C} V_g.$$ (2.24)

When the electrochemical potential of the source electrode $\mu_S$ lies above the unoccupied state $\mu_{\text{grain}}(N + 1)$ in the grain, it is favorable for an electron in the source to tunnel onto the grain. If in turn $\mu_{\text{grain}}(N + 1)$ is higher than $\mu_D$, the electron will subsequently tunnel into the drain electrode and the cycle can repeat. This leads to the condition $\mu_S \geq \mu_{\text{grain}}(N + 1) \geq \mu_D$ for current transport by sequential tunneling. Otherwise conduction through the system is blocked. It is immediately clear from eq. 2.24, that this condition can be fulfilled for any number of electrons on the grain, if a suitable gate voltage is applied. Choosing $\mu_S \simeq \mu_D$ there will be a series of sharp conduction peaks as a function of $V_g$ (Fig. 2.3 b). The change in electrochemical potential needed to increase the electron number by one is called the \textit{addition energy} $\Delta \mu$:

$$\Delta \mu = \mu(N + 1) - \mu(N) = \frac{e^2}{C} \text{ for } V_g \text{ const.}$$ (2.25)

The addition energy is independent of the electron number for constant $C_i$ and has to be compensated by a change in gate voltage $\Delta V_g$, when going from one

---

**Figure 2.3**: Coulomb blockade: (a) Schematic of a metal grain with tunnel coupling to source and drain. The electron number on the grain can be tuned by applying a voltage to the gate. (b) Schematic showing current resonances as a function of gate voltage. The peaks are evenly spaced if the capacitances $C_i$ remain constant.
2.3. QUANTUM DOTS

Coulomb peak to the next. This imposes the condition \( \mu(N, V_g) = \mu(N + 1, V_g + \Delta V_g) \) for electron transport at constant \( \mu_S \approx \mu_D \), leading to:

\[
\Delta V_g = \frac{1}{e\alpha} \frac{e^2}{C}
\]  

(2.26)

where \( \alpha \equiv C_{pg}/C \) is called the lever arm.

2.3.3 Quantum mechanical level spacing

In semiconductors the Fermi energy \( E_F \) and the effective mass \( m^* \) of the charge carriers are typically much lower than in metals. As a consequence, contributions from the quantum mechanical confinement energy are larger, if the metal grain is replaced by a semiconductor quantum dot. In the constant interaction model [14] this is incorporated by simply adding the quantum mechanical single particle energy spacing \( \Delta E = E_N - E_{N-1} \) to the constant Coulomb term for the addition energy of the \( N \)th electron:

\[
\Delta \mu(N) = \frac{e^2}{C} + \Delta E_N
\]

(2.27)

If \( \Delta E \ll k_B T \), transport is mediated by a single state and the quantum Coulomb blockade regime is reached. A schematic of a conductivity measurement through a quantum dot as a function of gate and bias voltage is shown in Fig. 2.4. Transport is blocked in parameter regions that are shaded grey and the electron number is fixed within such a Coulomb diamond. At the edges of a diamond the electron number can fluctuate by one and conduction sets in. The intersects of the edges with the \( V_g \) axis mark the gate voltages at which Coulomb peaks are observed. The bias voltage maximum \( V_{\text{max}}(N-1) \) at the tip of the N-1th Coulomb diamond directly corresponds to the addition energy for the Nth electron:

\[
eV_{\text{max}}(N-1) = \Delta \mu(N)
\]

(2.28)

This is immediately clear from Fig. 2.4 since \( \mu_s - \mu_d \) can be at most \( \Delta \mu \) without enclosing at least one level. The gate lever arm is defined by \( \alpha = \Delta V_g/V_{\text{max}} \). By subtracting the Coulomb charging energy, it is then possible to reconstruct the single particle energy spectrum from a measurement of the Coulomb peaks as a function of gate voltage. For large dots with steep walls, the average single-particle level spacing is comparable to the reciprocal of the dot area \( A_{\text{dot}} \) times the density of states in the free 2DES:

\[
\Delta E_N \approx \frac{2\pi\hbar^2}{m^* A_{\text{dot}}}
\]

(2.29)

The constant interaction model produces useful results in many cases despite the fact that changes in the background potential and capacitance are completely neglected.

More precise agreement can be achieved with Hartree–Fock calculations. However, numerical solutions are only feasible for small quantum dots containing less than 50 electrons. In some cases analytical solutions can even be found for large
quantum dots if they have pronounced symmetries [15], but in most cases random background scatterers, the coupling to the leads and other perturbations will lead to chaotic motion and only statistical predictions can be made. According to the Bohigas–Giannoni–Schmit conjecture [16], the quantal fluctuations of such classically chaotic systems can be described by random matrix theory (RMT). This approach was introduced by Wigner and Dyson [17, 18] to explain resonances of compound nuclei and has found many applications in quantum physics [19]. The essence of RMT is to choose the most random ensemble of Hamiltonians compatible with the symmetries of the system [20]. A comprehensive review of the resulting statistical distributions relevant for quantum dots can be found in [21].

Figure 2.4: Schematic of Coulomb diamonds: Current is blocked in shaded regions, where the electron number on the dot is fixed. The alignment of the electrochemical potentials of the states in the dot with respect to source and drain is illustrated in resonance and in Coulomb blockade.

2.3.4 Peak shape and line width

Just like in optical spectroscopy on real atoms, the resonances measured by Coulomb blockade spectroscopy have a finite line width. Several effects contribute to peak broadening:

- Source drain bias: A finite bias voltage window has to be applied in order to have a measurable current (typically 2-50 µV). This also means that the
Nth state will be in resonance while \( \mu_S > \mu_N > \mu_D \) and the peak will have a finite width in \( V_g \).

- **Tunnel broadening** corresponds to the natural line width of an atom. The coupling to source, drain and other terminals, leads to a finite lifetime \( \tau_{\text{nat}} \) of the state in the quantum dot and hence to a finite line width \( \Gamma_{\text{nat}} \) given by the uncertainty relation:

\[
\Gamma_{\text{nat}} = \frac{h}{\tau_{\text{nat}}} \quad (2.30)
\]

The larger the dot, the longer an electron needs to traverse it and ‘knock’ at one of the leads. The lower the transmission of the leads, the higher the probability that the electron will be reflected and remain inside the dot. Therefore the escape time will be a function of the dot area or, equivalently, of the inverse average single particle energy spacing \( \Delta E \) (see eq. 2.29) and of the inverse of the lead conductances \( G = G_S + G_D \). This yields the relation [22]:

\[
\Gamma_{\text{nat}} = \frac{h}{8\pi^2 e^2} G \Delta E \quad (2.31)
\]

- **Thermal broadening** is caused by a widening of the Fermi distribution in the leads and by thermal excitations within the dot. Starting from perfect delta peaks, Beenacker [14] calculated the temperature dependent line shape for dots in the classical and Coulomb blockade regimes.

Classical regime \((\Delta E \ll k_B T \ll e^2/C)\), multi level transport:

\[
G^i = \frac{1}{2} G_i^\infty \cosh^{-2}\left(\frac{\alpha(V_g^i - V_g)}{2.5 k_B T}\right) \quad (2.32)
\]

Quantum regime \((k_B T \ll \Delta E \ll e^2/C)\), single level transport:

\[
G^i = \frac{e^2}{4k_B T} \left(\frac{1}{\Gamma_S^i} + \frac{1}{\Gamma_D^i}\right)^{-1} \cosh^{-2}\left(\frac{\alpha(V_g^i - V_g)}{2k_B T}\right) \quad (2.33)
\]

where \( G_i^\infty \) is the high temperature \((k_B T \gg e^2/C)\) conductance through the dot, \( V_g^i \) is the gate voltage value for the \( i^{\text{th}} \) peak and \( \Gamma_S^i \) and \( \Gamma_D^i \) are the transmissions of the leads.

In the classical regime, the peak height is temperature independent and the full width half maximum (FWHM) is 4.35 \( k_B T \). For dots in quantum Coulomb blockade, the maximum peak height increases with decreasing temperature and the FWHM reduces to 3.5 \( k_B T \).

Depending on the details of the experimental situation either of these effects can dominate. At a typical dilution refrigerator temperature of 100 mK, closed dots usually show thermal broadening, while for open dots tunnel broadening dominates.
2.4 Quantum interference effects

Quantum mechanical interference effects are a manifestation of the wave nature of particles at mesoscopic length scales. Therefore all the phenomena described below have close analogs in wave optics. In order to observe quantum interference in the transport properties of mesoscopic conductors, the phase coherence length has to be larger than the geometrical length scales dominating the system.

2.4.1 The Aharonov–Bohm effect

The Aharonov–Bohm effect [23] is based on the quantum mechanical phase change of a charged particle by a vector potential along its path of propagation. If, for example, the wave-packet of a single electron is split into two parts and later rejoined, it will self-interfere. The observed pattern will then depend on the difference in distance and vector potential along the two paths. It is also known from classical electrodynamics, that the integrated vector potential around a closed loop is equal to the magnetic flux enclosed by the loop. From this it can be shown, that the interference pattern will be periodic in the number of magnetic flux quanta enclosed by the two paths of the wavefunction, if the magnetic field B does not act on the particle itself. The relative phase of the two parts of the wavefunction is:

\[ \Delta \phi = \Delta_{\text{path}} - 2\pi \frac{\Phi(B)}{\Phi_0} \] (2.34)

where \( \Delta_{\text{path}} \) denotes the geometrical phase difference, \( \Phi(0) \) is the flux quantum, and \( \Phi(B) \) is the field dependent flux through the loop. This effect was first observed in a solid state device by Webb et al.[24] in a small metal ring and later verified in many other experiments.

Historically, a related effect with half the periodicity of the Aharonov–Bohm oscillations was observed first in thin metal cylinders by Sharvin and Sharvin [25]. These \( h/2e \) or Aharonov–Altshuler–Spivak (AAS) oscillations [26] arise from self-interference between time reversed paths and persist even in macroscopic samples.

![Figure 2.5: (a) Aharonov–Bohm geometry: wavefunction passing a ring through both arms. The self-interference pattern oscillates periodically as a function of \( \Phi(B) \). (b) Superposition of two time reversed paths. This leads to AAS-oscillations as a function of enclosed flux.](image-url)
2.4.2 Ballistic conductance fluctuations

Ballistic conductance fluctuations arise from reflections of the wavefunction off the edges of a nanostructure or from random scatters. Just like speckle patterns in classical optics, this effect is only observed if the phase coherence length exceeds the dimensions of the system. In a ‘clean’ nanostructure without scatterers, the variations in resistance as a function of electron density or magnetic field, will reflect the bound states of the system. In the case of random scatters, the conductivity is predicted to fluctuate on the order of the resistance quantum $2e^2/h$ for $T \to 0$, leading to the term universal conductance fluctuations (UCF).

2.4.3 Weak localization

Weak localization [27, 28] arises from interference between time reversed paths and leads to an increase of the longitudinal resistance in the presence of even the smallest amount of disorder. In one and two-dimensional systems this gives rise to a completely insulating state in the limit of infinite sample size and a diverging phase coherence length. The same is true for a three-dimensional sample above a certain disorder concentration. As a consequence the resistance will increase and the weak localization effect grows stronger as the temperature is lowered. In two dimensions this leads to a logarithmic decrease in conductivity:

$$\delta \sigma_{WL} = -\frac{2e^2}{h} \ln \frac{l_c}{l_p}$$  \hspace{1cm} (2.35)

Time reversal symmetry can be broken by applying a perpendicular magnetic field, which suppresses weak localization. The magnetoresistance peak around $B=0$ T in the case of randomly distributed loops in two dimensions then takes on the form:

$$\delta \sigma(B) = \frac{e^2}{2\pi^2 \hbar} [\Psi(\frac{1}{2} + \frac{\tau_B}{2\tau_c})\Psi(\frac{1}{2} + \frac{\tau_B}{2\tau_e}) \ln(\frac{\tau_c}{\tau_e})]$$  \hspace{1cm} (2.36)

where $\Psi$ is the digamma function, $\tau_B = \hbar/2eDB$ with the diffusion constant $D$ from eq. 2.8, and system size $\gg \tau_c \gg \tau_e$. This relation can be used to extract the phase coherence length or mean localization radius of the wave functions in a 2DES from a magnetoresistance trace. The AAS-oscillations described above constitute a special case of weak localization around a well defined path.

2.5 Antidot lattices

2.5.1 Introduction

An insulating island surrounded by a 2DES is commonly called an ‘antidot’, since it can be viewed as the topological inverse of an isolated puddle of electrons forming a quantum dot [30], for reviews see [31, 10, 32]. Periodic arrays of antidots have been fabricated by a variety of techniques, most of them based on the Ga(Al)As material system. Typically a thin layer of resist is spun onto a predefined mesa or a Hall bar, that is then illuminated with an electron beam microscope in the desired
pattern. After developing the resist, the periodic potential modulation can be induced by wet chemical etching [33], evaporating a metal film onto the developed resist layer and later applying a topgate voltage [34], or by ion irradiation [35]. An alternative method employing nano-indentation on InAs with an AFM tip has been successfully demonstrated by Cortes-Rosa et al. [36].

Best results have been achieved with modulated topgates and by wet chemical etching, the lattice constants typically being > 200 nm. In a high mobility 2DES, the mean free path is much larger than the lattice constant and the antidots act as the primary scatterers in the system. In the following we will attempt to give a short summary of some of the main experimental observations and theoretical ideas in this field which now encompasses a considerable amount of literature. Since this thesis concentrates on magnetotransport measurements, we will focus on this aspect, however noting that optical investigations have also been conducted, see e.g. [37]. After discussing classical and quantum mechanical features in the magnetoresistance, we turn to the question of artificial bandstructures and the observability of Hofstadter’s famous fractal band splitting spectrum [38].

2.5.2 Magnetotransport

At temperatures below about 20 K, so-called commensurability peaks in the magnetoresistance of electronically well defined square antidot lattices have been observed [33, 43] (Fig. 2.7). Weiss et al. [33] explained this effect using an electron pinball model. The resistance peaks when the magnetic field reaches a value at which the electrons describe cyclotron orbits encircling 1, 2, 4, 9, ... antidots. According to Weiss et al., pinning leads to fewer electrons that can participate in current transport. More detailed numerical investigations were done by Fleischmann et al. [39] in smooth antidot potentials modeled by:

\[ U(x, y) = U_0[\cos(2\pi x)\cos(2\pi y)]^{2\beta}, \]

where \( \beta \) is an integer characterizing the potential steepness, and \( V_0 \) controls
2.5. ANTIDOT LATTICES

Figure 2.7: Calculations of the longitudinal magnetoresistance through antidot lattices, taken from [39]. (a),(b) Model antidot potentials with different steepnesses that were used in the numerical simulations below. (c) Chaotic trajectories and regular orbits encircling 1, 2, 4, 9, ... antidots in the steep potential (a), at different values of the perpendicular magnetic field. (d) Orbits around 1 and 4 antidots in the smoother potential (b). A large unperturbed orbit (dashed line) is confined to an orbit around 4 antidots by deflection. This explains the shift of the commensurability peak around 4 antidots to lower magnetic fields. (e),(f) The simulated magnetoresistance for potentials (a) and (b) respectively (solid curves) is compared with measurements by Weiss et al. [33] (dashed curves).

the ratio between antidot diameter \(d\) and lattice constant \(a\). Trajectories for an ensemble of electrons were calculated by integrating the classical equations of motion for a charged point particle in the above antidot potential, subject to a perpendicular magnetic field:

\[
\begin{align*}
\dot{x} &= v_x, \\
\dot{v}_x &= 2\sqrt{2}(B/B_0)v_x - \partial U/\partial x \\
\dot{y} &= v_y, \\
\dot{v}_y &= -2\sqrt{2}(B/B_0)v_y - \partial U/\partial y
\end{align*}
\]

(2.38)
where $B_0 = \hbar k_F/ea$ and $k_F$ is the Fermi wavevector. Finally the frequency dependent conductivity was determined with the Kubo formula [45]:

$$
\sigma_{ij}(\omega) = \frac{ne^2}{k_B T} \int_0^\infty dt C_{ij} e^{i\omega t},
$$

(2.39)

where $n$ is the electron density, $k_B$ is the Boltzmann constant, $T$ is the temperature, $\omega$ is the applied current frequency and $C_{ij} = \langle v_i(t)v_j(0) \rangle$ is the velocity correlation function averaged over phase space, that can be obtained by numerically integrating equation 2.38.

The simulations revealed, that periodic orbits occupy volumes in phase space that are too small to account for the observed increase in resistance. Instead, quasi pinned orbits and non-linear resonances play a crucial role. In this model all features observed in magnetoresistance traces above 4 K are well explained, including the experimentally observed shift of the commensurability peak around four antidots to lower magnetic fields. Classical calculations by Ando [41], as well as a fully quantum mechanical treatment by Silberbauer [46] led to qualitatively similar results. Experiments on triangular [47], rectangular [48] and disordered arrays [49] confirm the interpretation, that the observed sequence of commensurability peaks reflects the geometry of the antidot lattice.

At temperatures below 1.7 K, fluctuations of quantum mechanical origin appear, that are superimposed on the commensurability peaks. Weiss et al. [42] observed a smooth transition from the Shubnikov–de Haas effect to a $B$-periodic
2.5. ANTIDOT LATTICES

Figure 2.9: Aharonov–Bohm type oscillations: experiment. (a) Magnetoresistance in an antidot lattice and an unstructured region of the 2DES at 4.2 K and 0.4 K. (b) \( \rho_{xx} (T = 0.4 K) - \rho_{xx} (T = 4.2 K) \). On the main commensurability peak the Shubnikov–de Haas minima (marked by triangles) change from a 1/B to a B periodic behaviour with \( \Delta B = 105 \text{ mT} \) (from [42]). (c) Four-terminal magnetoresistance through a \( 9 \times 9 \) antidot array with \( a=240 \text{ nm} \) at 30 mK (from [43]). (d) Fourier transform in different magnetic field regions of the data shown in (c). Peaks corresponding to magnetic flux quanta through an orbit around 1 and 4 antidots appear at magnetic fields close to the respective commensurability peak.

oscillation on the main commensurability peak at 0.4 K (Fig. 2.9). B-periodic oscillations superimposed on the 1\(^{st}\) and 4\(^{th}\) commensurability peak were also detected by Schuster et al. [43] in a \( 9 \times 9 \) antidot array at 30 mK (Fig. 2.9). In both cases the period in B corresponds to about a flux quantum through a cyclotron orbit.

This effect can be explained in the framework of periodic orbit theory developed by Gutzwiller [50, 51, 52, 53, 54], where stable classical orbits lead to a modulation in the quantum mechanical density of states. In the simplest case, the periodic orbit encircling one antidot is dominant. Around this magnetic field the Lorenz force is balanced by the electrostatic repulsion from the antidot wall and the cyclotron radius remains approximately constant if the magnetic field is increased. In addition, the phase matching condition:

\[
S(E, B) = 2\pi \hbar [n + 1/4\alpha]
\]  

(2.40)

has to be fulfilled, where \( n \) counts the number of wavefunction nodes along the periodic orbit and \( \alpha \) is the Maslov index. As a consequence, the density of states
and the magnetoresistance oscillate quasi-periodically with

\[ \Delta B = B_n - B_{n-1} = 2\pi\hbar/(\partial S/\partial B). \]

A quantum mechanical calculation of the magnetoresistance by Uryu and Ando [44] also shows a fine Aharonov–Bohm type oscillation on the main commensurability peak, that is attributed to the existence or stability of a periodic orbit around one antidot (Fig.2.10). In addition Uryu and Ando found the Aharonov–Bohm type oscillation to be more pronounced when a larger number of random background scatterers were included. This was interpreted as a suppression of oscillations from more complex periodic orbits. Other authors have proposed a Hofstadter-type beating of the bandstructure as the source of the observed oscillations [55].

In hexagonal lattices, h/2e oscillations i.e. oscillations with twice the frequency of the Aharonov–Bohm effect were measured around B=0 T [47]. This feature is much weaker or absent in square lattices and is probably related to the Aharonov–Altschuler–Spivak oscillations observed in small metal cylinders [25]. Antidot lattices on ultra high-mobility 2DES at temperatures < 100 mK have also made it possible to study commensurability effects in the fractional quantum Hall regime [56].
2.5.3 Bandstructures and Hofstadter butterfly

In the case of a perfectly symmetric lattice, Bloch’s theorem applies and scattering off the antidots vanishes. An artificial bandstructure forms that is determined by the shape and symmetry of the antidot potential. Indications for this effect have been observed by Albrecht et al. [57, 58] for weak potential modulations, and a number of theoretical results have been published [46, 32]. An example of the complex and overlapping bandstructure at $B = 0$ T calculated by Silberbauer [46] for two different sets of lattice parameters is shown in Fig. 2.11.

An interesting objective is to induce a bandstructure in a parameter range suitable for observing the fractal splitting of the crystal bands as a function of magnetic field as predicted by Hofstadter [38]. A theoretical calculation of the fractal spectrum in an ideal band can be seen in Fig. 2.12. The same type of splitting should also be present in Landau bands, if the antidots are replaced by a weak potential modulation. Indications for this splitting have been observed by Schlösser et al. [58] and by Albrecht et al. [34].

Figure 2.11: Calculated bandstructures of antidot lattices along two symmetry lines of the reciprocal unit cell for $\beta = 4$, $a = 300$ nm (a) and $\beta = 1$, $a = 50$ nm (b) and for $V_0 = 25$ meV at $B = 0$, taken from [46].
Figure 2.12: So-called Hofstadter butterfly spectrum for a band with homogeneous dispersion in the magnetic field range from 0 to 1 flux quantum through a unit cell.
Chapter 3

Sample fabrication and experimental setup

3.1 The Ga(Al)As material system

Two-dimensional electron systems have been observed in a variety of semiconductor structures, one of the first and most prominent being the metal-oxide-silicon field effect transistor (MOSFET). For most applications semiconductor layers with few to no defects are desirable, so that nano-structures based on the material will only be influenced by their lithographic shape. In addition a spherical (or in two-dimensions a circular) Fermi surface in a single conduction band is advantageous, to make the electronic properties of the 2DES isotropic, and to allow a more straightforward quantum mechanical interpretation of the observed phenomena. These requirements are, at present, best met by the Ga(Al)As material system.

Layers of GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$ can be grown on top of each other with almost atomic precision by molecular beam epitaxy (MBE) (see 3.1 (a)). The small lattice mismatch between GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$ ensures almost no growth dislocations from strain relaxation. During the MBE process, the atomic constituents of the two alloys are evaporated from effusion-cells onto a heated, rotating substrate in ultra high vacuum, and the flux of the different kinds of atoms onto the substrate can be controlled by shutters. A schematic of a typical layer sequence is shown in Fig.3.1 (b), where the Si atoms serve as electron donors and a 2DES has formed at the GaAs / $\text{Al}_x\text{Ga}_{1-x}\text{As}$ interface. Figure 3.1 (c) shows a schematic of the valence and the conduction band profiles in z-direction. The conduction band only falls below the Fermi energy in a triangular well, that is so narrow, that only the lowest electronic subband in z-direction is occupied, while the electrons are free to move in the xy-plane. All nano-structures presented in this thesis are based on this type of wafer.

When a metal is evaporated onto the surface of a semiconductor, the junction can be either rectifying or Ohmic. In order to do electronic transport measurements, the 2DES has to be contacted with Ohmic junctions. This can be done by evaporating Au/Ge/Ni pads with an eutectic ratio (Au:Ge=88:12) onto the surface of the wafer and annealing at 430-450°C for 20-60 s (see appendix A or [59] for details).
3.2 Lithography

The nano-structures in this thesis were fabricated using two lithography steps: a) optical lithography for structure sizes down to \( \sim 2 \mu \text{m} \), and b) AFM-lithography for patterning down to \( \sim 10 \text{nm} \).

3.2.1 Optical lithography

Optical lithography was used to define mesas or Hall bars and the Ohmic contact pads. A typical mesa consists of a \( 20 \times 20 \mu \text{m} \) square with contact arms leading out to circular Ohmic contact pads with a diameter of \( \sim 0.3 \text{mm} \) (see Fig. 3.5 (a)). Outside of the mesa and the Ohmic contact pads, the 2DES was removed by wet chemically etching the surface to a depth of about 40-100 nm with a \( \text{H}_2\text{O}:\text{H}_2\text{SO}_4:\text{H}_2\text{O}_2 \) concentration of 100:3:1. The standard procedure for optical lithography is illustrated in Fig. 3.2. After cleaning the wafer with acetone and ethanol in an ultrasonic bath, a thin resist layer is spun onto the surface and baked out. Then the resist layer is exposed to ultra violet light through a mask with the desired Hall bar or mesa structure. Positioning is done with a commercial Karl Suss MJB3 mask aligner. Depending on whether a positive or negative resist is
used, the illuminated or the covered resist areas are removed during development (the case of positive resist is shown). After burning off remnants of the resist in the developed areas with an oxygen plasma etch, the surface is wet chemically etched to define a mesa or Hall bar. For the Ohmic contacts, the entire process is repeated with an appropriate mask. Instead of etching, a Au/Ge/Ni eutectic is evaporated onto the surface and subsequently removed from the non-developed areas by a lift-off in acetone. This procedure works best with resists that form an undercut. For a detailed list of the process steps and parameters for positive and negative resist see appendix B.

Figure 3.2: Schematic of the optical lithography process used to define the mesa and to apply the Ohmic contacts (Au/Ge/Ni evaporation). This process was available with positive and negative resist.

3.2.2 AFM-lithography

Artificial nanostructures are usually fabricated by exposing a thin PMMA resist layer on a sample surface with e-beam lithography. In the case of antidot lattices, the actual potential modulation can then be introduced by etching [33], ion irradiation [35] or by evaporating a top gate onto the developed resist layer [34]. In these cases, the feature size is limited by the proximity effect caused by electron back-scattering off the GaAs surface.

Lithography based on scanning probe microscopes (SPMs) is not subject to this limitation and, at least in principle, patterning right down to the atomic scale is possible. Examples for SPM nanostructuring include scratching into a resist layer [60], directly indenting the surface of an InAs heterostructure [36], material deposition from the SPM-tip onto the substrate [61], or arranging single atoms with an SPM tip [62]. Our approach is to use an AFM with a conducting tip (boron doped Si or TiN) as a tool for local anodic oxidation [63, 64]. This technique was pioneered by Dagata’s group at NIST [65, 66] and has been demonstrated
3.2. LITHOGRAPHY

on a wide variety of substrates [67]. The first to observe a resistance increase in the 2DES of a GaAs heterostructure beneath an oxide line were Ishii and co-workers [68]. Subsequently our group demonstrated, that a 2DES in a GaAs heterostructure less than 50 nm below the surface can be totally depleted this way. Electronically functional quantum wires and quantum dots [69, 70, 71, 15, 72] have been fabricated this way in our group. The very smooth as well as steep potential walls induced by this method seem ideally suited for the realization of especially small devices. Recently other groups have also succeeded in fabricating functioning nanostructures using AFM-lithography [73, 74, 75, 76]. This procedure has the advantage, that no further treatment or processing of the GaAs surface is needed and the exact position of the nanostructure can be chosen after scanning and evaluating the surface immediately before writing.

A schematic of our nano-lithography setup is shown in Fig. 3.3. A commercial Digital Instruments 3100 AFM with a metrology head is operated in a closed chamber under a controlled atmosphere. The temperature was constant at about 23 °C and the humidity was adjusted to values between 30% and 50% by controlling the flow of moisturized nitrogen into the AFM chamber. A conducting tip at the end of a cantilever arm can be scanned across the surface with the xy-piezo tube, while the height is controlled by the z-piezo. Capacitive sensors are used to compensate for long term drifts and creep. For lithography the AFM is operated in

Figure 3.3: Schematic of our AFM-lithography setup. Local anodic oxidation is performed by applying a suitable voltage to the conducting tip of a Digital Instruments Metrology AFM. The tip is positioned in x, y, and z-direction with piezos, and capacitive sensors are used to compensate long term drifts and creep. The blowup of the tip shows the thin water film needed for the oxidation process.
tapping mode, which means that the cantilever is excited to its resonance frequency of about 300 KHz by the tapping-piezo. The amplitude of the oscillation can then be measured with a photo detector. Keeping the amplitude constant during scanning by adjusting the z-piezo extension then allows one to deduce the height profile of the surface, by transforming the applied piezo-voltages into length scales.

Very local oxidation processes can be performed if a negative voltage between -5 and -30 V is applied to the AFM tip, while the chip that is to be nanostructured remains grounded. An important prerequisite is a thin water film that naturally forms on the sample surface under ambient conditions and serves as an electrolyte (see blowup Fig. 3.3). In addition the setpoint of the tip has to be reduced, which is equivalent to an increase in force with which the oscillating tip is pressed down onto the GaAs surface. In general the oxide height increases if larger negative voltages are applied and/or the setpoint is reduced, but the exact relation depends sensitively on the chemical and electronic properties of the tip and on the preparation of the GaAs surface. It should be noted, that strong variations were even found between different batches of nominally identical tips. A humidity above $\sim 35\%$ was also necessary, which probably indicates that the thin water film does not completely wet the surface at lower humidity. The highest aspect ratio for oxide height to width was achieved if lithography was performed at the lowest humidity at which oxidation was stable. This is probably due to a focusing effect of the electric field lines between tip and surface by a water meniscus (see blowup Fig. 3.3). The lower the humidity, the smaller the diameter of the water meniscus and hence the stronger the focusing effect. This could also be the reason for better results achieved with tips that were coated with a conducting layer like TiN, despite the larger tip radius. The higher electronic conductivity should also lead to a stronger bunching of the electric field lines at the tip. In addition TiN coated tips were usually more stable than the uncoated ones over time, which is especially important if arrays with several hundreds of antidots are to be written. This can be attributed to a partial oxidation of the uncoated silicon tips during the writing process, leading to a degradation of the electronic properties.

The wafer with the mesa was oxygen plasma etched for $\sim 3$ min prior to AFM-lithography. This procedure can cause a small decrease in 2DES mobility, but ensures optimal surface smoothness by burning off organic contamination primarily from remaining photo-resist. In some cases an HCl dipetch was also performed if inorganic contaminants like CaCO$_3$ were present.

After placing the sample on a grounded metal chuck under the scanning head, the AFM-chamber is closed. The humidity is adjusted to the desired value (typically 35-45\%, depending on the desired oxide profile) by controlling the flow of moisturized nitrogen through a water bubbler into the chamber. After reaching the targeted parameters, we waited for another 6-12 h in order to let the system stabilize. This is particularly important if constant writing conditions are to be maintained over a longer period of time, e.g. 0.5-1 h as is the case for a 20 $\times$ 20 antidot lattice.

The actual writing parameters were then determined on a testpad outside the mesa, by slowly reducing the setpoint and the tip voltage until the appropriate oxide profile was achieved. Writing a single antidot in an array consists of the
3.2. LITHOGRAPHY

following steps:

- First the AFM-tip has to be scanned to the coordinates of the antidot. This should be done at a low speed (typically 50-300 $\mu$m/s), so as not to disturb the equilibrium between water film and tip too much and to make the position relaxation time of the tip after stopping as short as possible. On the other hand speeds should not be chosen too slow in order to keep lithography time as short as possible and to prevent parameters from changing significantly.

- The tip is held at the antidot coordinates for a short time (typically $\sim 1$ s) in order for the positioning sensors to adjust and for the water meniscus to equilibrate.

- The oxidation voltage is applied (-15 to -35 V) through an RC-circuit, that is used to ramp the voltage over a time constant of about 0.1 s.

- After about 0.1 s the setpoint is reduced and the surface is oxidized (typically for about 3 s). A current flow between tip and sample can be detected.

- The tip is withdrawn to the original setpoint and the voltage is turned off again. The cycle is repeated for the next antidot.

Figure 3.4: Electron sheet density as a function of etch depth for a standard GaAs heterostructure with the interface 34 nm below the surface. The simulation was done with a Poisson-Schrödinger solver [77]. Inset: AFM-images and height profiles of oxide dots before and after removing the oxide with an HCl dip-etch.

Lines are written in the same way, with the only difference, that the tip is scanned during oxidation with a velocity of 100-200 $\mu$m/s and different values for
the setpoint, oxidation voltage and humidity were often chosen. A list of structures and lithography parameters is given in appendix C.

As can be seen in the inset of Fig. 3.4, removing oxide dots with an HCl dip etch leads to pits, with approximately inverted profiles. This suggests, that AFM-lithography is comparable to a controlled shallow wet-chemical etch of the GaAs cap layers. With the help of a Poisson-Schrödinger solver [77], the electron sheet density in the 2DES can be calculated as a function of etch depth (Fig. 3.4 (b)). For a GaAs heterostructure with the GaAs/GaAlAs interface 34 nm below the surface and a layer sequence that is the same for all wafers used in this thesis, the electron sheet density goes to zero below about 11 nm. This value is in good agreement with experimental observations of the minimum oxide height needed to achieve insulation, and allows us to estimate the potential profiles of nano-structures, if lateral screening is neglected (see also chapter 5).

An overview of a typical sample from the nano-structured mesa to the mounting on a cryostat probe is shown in Fig. 3.5.

Figure 3.5: (a) Schematic illustrating a crosssection through an AFM-defined antidot lattice. (b) Mesa on a GaAs chip with contact arms leading out to Ohmic contacts. The mesa is covered with a TiAu topgate. (c) AFM-image of a mesa (brown) with an AFM-defined structure containing an antidot lattice (bright). The chip is glued onto a chip carrier and the arms of the mesa are wired to the contact pads on the chip carrier (schematic). The Ohmic contact pads are not shown. (d) Picture of a chip carrier mounted in the top loading probe of a cryostat.
3.3 Measurement techniques

3.3.1 Resistance measurements

For high sample impedances $> 100 \, \text{kΩ}$, the resistance was measured with an in-house built current voltage converter (Fig. 3.6). The bias voltage is applied with a programmable DC source (Yokogawa 7651) and scaled down by a fraction of $\pm \frac{1}{20000}$, before it is symmetrically applied to the source and drain contacts. The current through the sample is amplified by a factor of $10^9 \, \text{V/A}$ and the corresponding voltage is read out with an HP-multimeter. Low pass filters were included in order to prevent electronic heating through the leads. With this setup we were able to achieve a current resolution of about 200 fA.

Low impedance samples ($< 100 \, \text{kΩ}$) were measured with a Stanford SRS 830 lock in amplifier (Fig. 3.6 (b)). An AC voltage $U$, typically at a frequency between 10 and 100 Hz, was ‘transformed’ into an AC current by connecting a 10 or 100 MΩ resistor in series. Neglecting the sample resistance, the current in the circuit is then given by $U/10 \, \text{MΩ}$ or $U/100 \, \text{MΩ}$. The AC voltage drop over the sample is detected at the same frequency and, together with the current, yields the resistance. This technique has the advantage, that voltage offsets, and noise at other frequencies are suppressed, but low pass filters can only be used, if the cutoff frequency is above the AC frequency of the lockin. All measurements are computer automated with ‘virtual instruments’ in National Instruments LabVIEW programs.

Figure 3.6: (a) Diagram of an current voltage converter for symmetric bias. This setup was used to measure resistances $> 100 \, \text{kΩ}$. (b) Resistance measurement setup using a lockin amplifier. This setup was used for resistances $< 100 \, \text{kΩ}$.
3.3.2 Cryostats

Low temperature measurements were performed in two cryostats: a conventional $^4$He system with a variable temperature insert (VTI) for measurements between 1.6 K and 300 K, and a dilution refrigerator with a base temperature of about 90 mK.

A schematic of the $^4$He system is shown in Fig. 3.7 a). The outer vacuum chamber and the liquid $^4$He main bath at 4.2 K, insulate the insert from the surroundings at room temperature. The insert itself is thermally decoupled from the $^4$He main bath by the inner vacuum chamber. By opening a needle valve in a tube connecting the main bath with the insert, and simultaneously pumping the insert with a rotary pump, the sample can be cooled to 1.6 K, by evaporative cooling. Higher temperatures can be achieved by applying power to the heating coil, and magnetic fields between 0 and 9 T are available with a superconducting magnet.

The dilution cryostat is illustrated in Fig. 3.7 b). The mixing chamber is surrounded by an outer vacuum chamber, a liquid nitrogen cold shield at 77 K, the $^4$He main bath at 4.2 K, and an inner vacuum chamber. In principle the cooling mechanism in a $^3$He-$^4$He dilution refrigerator is similar to that of a conventional $^4$He cryostat. Instead of pumping the $^4$He vapor over a liquid bath, $^3$He vapor is effectively pumped in a liquid $^4$He atmosphere. In practice a $^3$He-$^4$He mixture is cooled below a critical temperature, so that a $^3$He rich or ‘concentrated’ phase forms, that floats on a $^4$He rich fraction, often referred to as the ‘dilute’ phase. Cooling takes place at the phase boundary, that is located in the mixing chamber. An effect analog to evaporative cooling sets in, if the $^3$He concentration in the dilute phase is reduced. This is done by pumping the He vapor in the still. Since the partial pressure of $^3$He is about 1000 times larger than the partial pressure of $^4$He under these conditions, practically only $^3$He is removed. This leads to an osmotic pressure gradient, that draws $^3$He from the phase boundary in the mixing chamber into the still. The pumped $^3$He is cleaned in a series of oil mist filters, a liquid nitrogen cold trap, and a liquid helium cold trap, before it is pre-cooled in the $^4$He main bath and recondensed on the 1 K pot, which acts as a miniature $^4$He cryostat. After passing the heat exchangers the liquid $^3$He is added to the concentrated phase in the mixing chamber, and the cycle can repeat. The sample is placed below the phase boundary, where the temperature is lowest, and cooling power is highest. Because the cryostats are made of conducting steel, they act as Farady cages and thus shield the sample from electromagnetic radiation. Our dilution refrigerator is fitted with a top loading probe, which allows us to exchange the sample while the He mixture remains condensed in. The system is equipped with a 13 T superconducting magnet.
Figure 3.7: (a) Schematic of a $^4$He cryostat with a base temperature of 1.7 K. The heating coil allows the insert to be heated to any temperature up to 300 K. This layout is called a variable temperature insert (VTI). The superconducting magnet can produce a field of up to 9 T. (b) Schematic of a $^3$He-$^4$He dilution refrigerator with a base temperature of 90 mK. The superconducting magnet can produce a field of up to 13 T. The sample can be changed with a top loading probe, while the $^3$He-$^4$He mixture remains condensed in.
Chapter 4

ErAs islands for backgate insulation

4.1 Introduction

Two-dimensional electron systems (2DESs) with backgate electrodes have many applications in nanostructure physics. If a topgate is added, different properties of the 2DES such as wave function symmetry in z-direction, the electron mobility, or the effective g-factor can be tuned at constant electron density. Usually a GaAs spacer layer grown at low temperatures is used to insulate the 2DES from the backgate [78, 79]. Here we present a novel approach based on overlapping Schottky barriers surrounding nanometer-sized self-assembled ErAs islands. The effective barrier height of the island layers was determined from temperature dependent transport measurements from the 2DES to the backgate electrode and charging effects of the ErAs islands were observed as a function of applied backgate voltage and during illumination with an LED. Finally we characterized the electron sheet density and mobility of the 2DES as a function of top- and backgate voltage.

4.2 Wafer

The two wafers under study were grown by molecular beam epitaxy (MBE) at the University of California at Santa Barbara by Danny Driscoll and Art Gossard. The wafers are nominally identical with the exception of slightly different growth conditions for the ErAs islands. A schematic of the layer sequence and the conduction band profile calculated with a Poisson-Schrödinger solver [77] is shown in Fig. 4.1. The 2DES is located about 40 nm below the surface at an AlGaAs/GaAs interface, separated by 1.3 μm from the metallic backgate (Si doped GaAs). Sandwiched in between are twenty layers of ErAs islands spaced by 25 nm of GaAs. As has been reported previously [80, 81], ErAs spontaneously forms islands when evaporated onto GaAs, the morphology depending on the growth parameters. The equivalent of 1.5 (1) mono layers of Er were deposited for every island layer at roughly 540 °C (500 °C) for wafer A (B). According to [81] this corresponds to an ErAs island size of about 5-6 nm for wafer A and 3-5 nm for wafer B.
Standard Hall bar geometries were defined by wet chemically etching to a depth of about 50 nm and separate pits with a depth of about 1.1 µm were etched for the contacts to the backgate (Fig. 4.2). The Au/Ge/Ni eutecticum for the Ohmic contacts to the 2DES and backgate was applied and annealed simultaneously for 10-20 s at 450 °C. Later a Ti/Au topgate was evaporated over the entire Hall bar using a shadow mask technique.

Figure 4.1: Left: Layer sequence of the MBE-grown wafer. Right: Schematic of the conduction band profile based on a simulation with a Poisson-Schrödinger solver [77] (not to scale!). The barrier height of the ErAs island layers was taken from experiment (see below).

4.3 Effective barrier height

At 1.7 K the 2DES is well insulated from the backgate with resistances in the GΩ range up to breakdown voltages of about ±7 V on the backgate (Fig. 4.3). It is remarkable that very similar breakdown characteristics of 7 to 12 V have been observed in macroscopic ErSi₂ Schottky contacts on n-type GaAs [82]. The topgate has a breakdown voltage of only about 400 mV, which is not surprising since it is only separated by about 40 nm from the 2DES as compared to a distance of 1.3 µm between 2DES and backgate.

In the following we will describe how the effective Schottky barrier height of the
CHAPTER 4. ERAS ISLANDS FOR BACKGATE INSULATION

Figure 4.2: Schematic of a Hall bar with a topgate and a backgate electrode.

Figure 4.3: Current-voltage characteristics: (a) between topgate and 2DES and (b) between backgate and 2DES for wafer A and B.

ErAs island system embedded between the 2DES and backgate can be determined from thermionic emission measurements. Current-voltage sweeps between -50 and +50 mV were done from 1.7 K to room temperature (see insets Fig. 4.4 for raw data), while the topgate was held at 0 V. After the reading on the temperature sensor had stabilized, we waited for another twenty minutes before taking a trace. The I/V characteristic is basically linear within this voltage window, so that the current at a fixed bias voltage $V_{bias} < 50$ mV can be taken as a measure for the slope at $V_{bias} = 0$ mV. A plot showing the exponential increase in current between backgate and 2DES at a constant bias voltage of 25 mV can be seen in Fig. 4.4.

In a simple model, the surfaces of the ErAs islands can be expected to form Schottky contacts with respect to the surrounding GaAs. Electrons are then drawn onto the islands and wide, overlapping space charge regions surround the islands due to the low donor concentration in the GaAs. This makes the island layers appear as a single continuous potential barrier between 2DES and backgate. In this picture the temperature dependence of the current across the barrier can be described by the well known relation for thermionic emission across a Schottky barrier [83]:

\[
I = A T^2 \exp \left( \frac{eV}{k_B T} \right) \exp \left( \frac{q \Phi}{k_B T} \right)
\]

where $A$ is the thermionic emission constant, $T$ is the temperature, $e$ is the electronic charge, $V$ is the voltage across the barrier, $k_B$ is the Boltzmann constant, $q$ is the electronic charge, and $\Phi$ is the barrier height.
Figure 4.4: Temperature dependent current between backgate and 2DES at a constant backgate bias of 25 mV. Insets: I/V-characteristics between backgate and 2DES at different temperatures for wafers A and B.

Figure 4.5: Arrhenius plot of the current between 2DES and backgate at a constant backgate bias of 25 mV. Inset: Illustration of the Schottky barrier minima between ErAs islands, that are predominantly probed by this method.
CHAPTER 4. ERAS ISLANDS FOR BACKGATE INSULATION

\[ J(V_{bias}) = A^*T^2 \exp\left(-\frac{e\Phi_B^*}{k_BT}\right) \left[ \exp\left(\frac{eV_{bias}}{k_BT}\right) - 1 \right] \]

\[ \approx A^*T^2 \exp\left(-\frac{e\Phi_B^*}{k_BT}\right) \frac{eV_{bias}}{k_BT}, \quad \text{for } \frac{eV_{bias}}{k_BT} \text{ small} \]

(4.1)

where \( J \) is the current density and \( A^* \) is the effective Richardson constant. It should be noted, that this analysis also remains valid for two Schottky barriers in series with opposite polarities, as would be the case for a solid layer of Er in GaAs. From an Arrhenius plot it is then possible to determine the effective Schottky barrier height \( \Phi_B \) of the island layers (Fig. 4.5). The obtained values are \( \Phi_B = (540 \pm 20) \text{ meV} \) for wafer A and \( \Phi_B = (650 \pm 20) \text{ meV} \) for wafer B. Our results are in excellent agreement with barrier heights of 640 meV for low and 800 meV for high quality interfaces reported for ErSi\(_2\) diodes on n-type GaAs [82] and with investigations on lithographically defined arrays of tungsten discs embedded in GaAs with barrier heights between 550 meV and 900 meV [84, 85].

This method mainly probes the minima between the ErAs islands (see inset Fig. 4.5), because current flow is dominated by the channels with lowest resistance. A microscopic model of transport based on magnetic polaron hopping has been proposed by Schmitt et al. [81] for in-plane electronic transport. However for current transport perpendicular to the ErAs plains this effect probably plays a negligible role, because of the relatively large inter layer spacing (25nm), as well as the large separation to the non magnetic source and drain electrodes. In the following section we will examine charging effects related to the maximum Schottky barrier height onto the islands.

### 4.4 Charging effects in the ErAs island system

If the backgate is set above or below a certain threshold voltage, persistent charging of the ErAs islands is observed (Figs. 4.6 and 4.7). This effect can be quantified by measuring the Hall electron density in the 2DES at 0 V backgate, then setting the backgate voltage to a higher value and measuring the electron density at 0 V backgate again. A series of such measurements that plot the electron density at 0 V backgate over the previously applied peak backgate voltage is shown in Figs. 4.6 and 4.7. Separate cooldowns were done for sweeps towards positive and negative backgate voltages, in order to bring the sample back into a well defined equilibrium state.

In wafer A onset occurs above 2.8 V and in wafer B above 1.4 V and below -2 V. In all cases the electron sheet density in the 2DES decreases linearly above threshold as a function of the previously applied backgate voltage. A value for the Schottky barrier height from the surrounding GaAs onto the islands, can be obtained by scaling the threshold values with the ratio of the distances 2DES/ErAs layer and backgate/2DES:

\[ \Phi_S = V_S \frac{300 \text{ nm}}{1300 \text{ nm}} \quad \text{for negative backgate voltage} \]
4.4. CHARGING EFFECTS IN THE ERAS ISLAND SYSTEM

Figure 4.6: Electron density at 0 V backgate as a function of a previously applied peak backgate voltage at 1.7 K in the Er A wafer. Inset: Model of an ErAs island with strongly asymmetric Schottky barriers resulting from different overgrowth conditions.

Figure 4.7: Electron density at 0 V backgate as a function of a previously applied peak backgate voltage at 1.7 K in the Er B wafer. Inset: Model of an ErAs island with slightly asymmetric Schottky barriers resulting from different overgrowth conditions.
Figure 4.8: Current between backgate and 2DES as a function of temperature after illumination with a red LED at a backgate bias of 5 mV. The two traces represent measurements taken during separate cooldowns. Upper inset: exponential increase in current at low temperatures. Lower inset: Schottky barrier surrounding an ErAs island before (dashed line) and after illumination (solid line).

\[
\Phi_O = V_O \frac{500\text{nm}}{1300\text{nm}} \quad \text{for positive backgate voltage}
\]

(4.2)

The resulting values for the Schottky barriers onto the ErAs islands from the substrate (backgate) and the overgrowth (2DES) sides are summarized in table 4.1. The discrepancies between the values for \(\Phi_B\) and \(\Phi_{S,O}\) can be explained by different interface qualities and dopant concentrations on the substrate and overgrowth sides of the ErAs islands. The lower Schottky barrier height onto the islands in the B wafer seems to be in contradiction with the higher value obtained for the effective barrier height \(\Phi_B\). However this seeming contradiction can easily be resolved, if one keeps in mind, that the barrier consists of 20 parallel island layers. If different layers have different island morphologies and barrier heights, the layers with the highest Schottky barriers will dominate the effective barrier.

<table>
<thead>
<tr>
<th>sample</th>
<th>(\Phi_S)</th>
<th>(\Phi_O)</th>
</tr>
</thead>
<tbody>
<tr>
<td>wafer A</td>
<td>0.58 eV</td>
<td>1.08 eV</td>
</tr>
<tr>
<td>wafer B</td>
<td>0.51 eV</td>
<td>0.58 eV</td>
</tr>
</tbody>
</table>

Table 4.1: Schottky barrier height onto the islands from the substrate and overgrowth sides.
4.5. 2DES CHARACTERIZATION

height $\Phi_B$, whereas island layers with low Schottky barriers will be charged first, which determines $\Phi_{SO}$.

Analyzing the linear dependence of the electron density on the backgate voltage above threshold in the framework of a capacitance model, we find a distance of about 1.3 $\mu$m. This is equivalent to the distance 2DES-backgate. We therefore propose the following simple picture: At the threshold values the applied potential slope compensates the Schottky barrier and electrons start to be captured onto the islands. If the voltage is increased, additional electrons are trapped onto the islands and produce an area charge corresponding to the applied backgate voltage minus the threshold voltage. These electrons remain on the islands when the backgate is grounded again, leading to a meta stable state with a persistent offset voltage and thus a lower 2DES electron density.

A red light emitting diode (LED) was used to illuminate the samples for several seconds at 1.7 K. After illumination the resistance backgate/2DES is drastically reduced from several G$\Omega$ to about 600 k$\Omega$, in wafer A. In wafer B the resistance is only reduced during illumination to the M$\Omega$ range and then returns to the insulating G$\Omega$ regime after turning off the LED. The persistent photo conductivity in wafer A can be interpreted in terms of a reduced effective barrier height after illumination, resulting from fewer electrons on the ErAs islands. Figure 4.8 plots the current between 2DES and backgate at a constant bias of 5 mV as a function of temperature. A rise in current from the 2DES into the backgate is observed, that peaks at about 17.5 K before dropping back to its equilibrium value before illumination. This behavior is not observed without prior illumination and can be explained by a persistent lowering of the Schottky barriers surrounding the islands. As soon as the thermal energy of the electrons becomes large enough to recharge the islands, the original barrier height is restored. In order to get an estimate of the energy scales involved, we applied an analysis based on thermal activation. From the initial exponential increase on the low temperature side of the peak (upper inset Fig. 4.8), where recombination is not yet dominant, a barrier height of $\Phi_T = (0.3 \pm 0.2)$ meV can be extracted using an Arrhenius plot. The peak maximum at 17.5 K and the half width at half maximum correspond to a thermal energy of $(1.5 \pm 0.4)$ meV. This value is comparable to a calculated Coulomb charging energy of about 1.5 meV for a metallic disk in GaAs with a radius of 50 nm. Charging energies of the same order of magnitude have been reported for tungsten disks with diameters of 50 nm [85]. If the sample is further heated to room temperature, a current/voltage dependence that reflects the equilibrium barrier height is observed.

4.5 2DES characterization

Figure 4.9 plots the electron sheet density and mobility as a function of top- and backgate voltage for wafers A and B. The electron density could be tuned from about 2 to $6.5 \times 10^{15}$ m$^{-2}$, while the mobility varied from 3 to 12 m$^2$/Vs. These values are comparable to high quality 2DESs in AlGaAs heterostructures without backgates, especially if the close proximity of the 2DES to the sample surface (34 nm) is taken into account. From the linear part of the electron density
Figure 4.9: Electron sheet density and mobility as a function of topgate and backgate voltage for wafers A and B. All data taken at 1.7 K.

Figure 4.10: Simulations of the 2DES wave function components in z-direction at a constant electron sheet density for different top- and backgate settings. Inset: Schematic illustrating the distances between topgate, 2DES and backgate.
vs backgate voltage slope, a distance of 1.4 µm between the 2DES and backgate could be calculated using a capacitor model. This agrees well with the separation of 1.3 µm predicted from the growth protocol. According to [80] the high quality crystalline overgrowth can be explained by seeding of GaAs between the ErAs islands. Simulations of the wave function in z-direction with a Poisson-Schrödinger solver [77] show, that the 2DES is tightly pinned to the AlGaAs-GaAs interface and only shifts by about 1-2 nm or 3-5 unit cells of GaAs (Fig. 4.10).

4.6 Summary

We have demonstrated that layers of ErAs islands are an alternative to low temperature grown GaAs for insulating two-dimensional electron systems from backgate electrodes. Breakdown voltages of up to -7 V and over +7 V at 1.7 K were achieved. From thermal activation and charging experiments we extracted the effective barrier heights $\Phi_B$ across the island system and the Schottky barrier heights $\Phi_S$ and $\Phi_O$ onto the ErAs islands from the substrate and overgrowth sides. These results are summarized below:

<table>
<thead>
<tr>
<th>sample</th>
<th>island size (nm)</th>
<th>$\Phi_B$ (meV)</th>
<th>$\Phi_S$ meV</th>
<th>$\Phi_O$ meV</th>
<th>$\Phi_I$ meV</th>
</tr>
</thead>
<tbody>
<tr>
<td>wafer A</td>
<td>5−6</td>
<td>(580 ± 20)</td>
<td>–</td>
<td>1080</td>
<td>(0.3 ± 0.1)</td>
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<tr>
<td>wafer B</td>
<td>3−5</td>
<td>(650 ± 20)</td>
<td>510</td>
<td>580</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 4.2: $\Phi_B$: Effective Schottky barrier height across the island layers. $\Phi_S$: Schottky barrier height onto the islands from the substrate side. $\Phi_O$: Schottky barrier height onto the islands from the overgrowth side. $\Phi_I$: Schottky barrier height after illumination.

The high electronic quality of the two-dimensional electron system indicates good crystalline overgrowth over the ErAs layers, especially if the close proximity to the surface is taken into account. Persistent photoconductivity due to a reduced Schottky barrier height after illumination was observed in wafer A, but was absent in wafer B. This discrepancy probably arises from different morphologies of the ErAs islands as a consequence of the slightly different growth conditions.

Insulating ErAs island layers could be interesting for many applications in nanostructure physics, especially if local charging is feasible. One could envision e.g. manipulating a two-dimensional electron system with a layer of close by ErAs islands that can be charged in a controlled way with structured topgates.
Chapter 5
The empty cavity

5.1 Introduction

In this chapter we study the electronic properties of an empty $3 \times 3 \mu m$ square cavity and a $3 \times 6 \mu m$ rectangular cavity defined by AFM-lithography. Both structures were covered with a TiAu topgate, that allowed us to tune the global electron density from about $1.5 \times 10^{15} m^{-2}$ at -0.2 V to $6 \times 10^{15} m^{-2}$ at 0.5 V topgate. Oscillatory features in the magnetoresistance that appear at low magnetic fields can be interpreted intuitively within a skipping orbits model. A more precise quantitative understanding was gained, by calculating the magnetoresistance within the Landauer–Büttiker formalism. The required magnetic field dependent transmission and reflection coefficients were obtained from a classical billiard simulation. This analysis gives direct insight into the relation between the lithographic geometry and electronic properties of AFM-defined structures on two-dimensional electron systems.

5.2 Square cavity

5.2.1 Skipping orbits

Figure 5.1 shows the Hall resistance and the longitudinal resistance across a diagonal of the square cavity (see inset) at an electron sheet density of $6 \times 10^{15} m^{-2}$. The longitudinal resistance exhibits a strong decrease, while the Hall resistance has an enhanced Hall slope around $B=0 T$. These features will be analyzed in more detail below. Superimposed are regular oscillations in the resistance with a periodicity of about 85 mT. In a simple model these oscillations can be explained by a commensurability effect between the diameters of classical cyclotron orbits and the side lengths of the cavity (lower inset Fig. 5.1). Whenever the side length is an integer multiple of the cyclotron diameter, electrons are ‘focused’ from one corner to the next. This will lead to a higher Hall resistance, because the electrons will be injected into the ‘Hall corner’ more efficiently, thereby enhancing the Hall voltage. The longitudinal resistance on the other hand shows resistance minima at these fields, which can be attributed to a better coupling between source and drain.

47
Figure 5.1: The two-terminal resistance between corners 2 and 4 shows a pronounced negative magnetoresistance, whereas the Hall resistance exhibits an enhanced slope at low fields. Both have superimposed oscillations that can be identified with skipping orbits, illustrated in the lower inset. The black line is a guide to the eye representing the linear Hall resistance. Inset: AFM-image of the $3 \times 3 \mu m$ cavity.

Figure 5.2: Two-terminal magnetoresistance between corner 2 and 4 as a function of topgate voltage at 1.65 K. The topgate was changed in 100 mV steps.
In particular at non-commensurate magnetic fields, more electrons are scattered around the cavity and exit again through the source contact, which is equal to enhanced reflection or equivalently resistance. The commensurability condition is given by:

\[
L_{\text{side}} = n D_c = n 2 \frac{\hbar k}{e B} \quad \text{with} \quad n = 1, 2, \ldots
\]

\[\Rightarrow B_{\text{com}} = n B_c \quad \text{with} \quad B_c = 2 \frac{\hbar k}{e L_{\text{side}}} \quad (5.1)
\]

where \(L_{\text{side}}\) is the side length of the cavity, \(B_{\text{com}}\) are the commensurate field values, and \(D_c\) is the cyclotron diameter. Taking the lithographic side length of 2.7 µm yields a value of \(B_c \approx 95\) mT which is somewhat larger than the measured value of \(\sim 85\) mT. However this difference can easily be accounted for, if the electrons are injected from the center of the corners and the angle of incidence and reflection off the side walls is smaller than 90°. Both effects shift the value for \(B_c\) to lower fields.

If the electron sheet density is lowered by applying smaller voltages to the top-gate, the oscillations become washed out (Fig. 5.2). At lower electron densities, screening is reduced and the number of random scatters increases, bringing the system closer to the diffusive regime. When the electrons are no longer predominately scattered off the walls of the cavity but instead by random scatters, the skipping orbits are destroyed and geometry effects become less important. It can also be noted, that a temperature increase has a much smaller influence, and leaves the skipping effects unchanged to well above 10 K.

### 5.2.2 Billiard simulation

A shortcoming of the simplified model of skipping orbits presented above, is that only very few trajectories are taken into account. In a more realistic picture, electrons will be injected from the source corner with a wide distribution of angles, leading to very different trajectories. In this section we attempt to gain a more complete understanding by no longer considering single orbits, but instead a large ensemble of possible trajectories.

In ballistic nanostructures the resistance originates from reflection and transmission of electrons entering and leaving the leads (sect. 2.2). The two terminal expression given by Landauer [86] was later generalized by Büttiker [7] to include multiterminal configurations (see eq. 2.21). In order to calculate the magnetoresistance one only needs to find the B-dependent reflection and transmission coefficients. This was done by calculating the trajectories of 6000 electrons at every magnetic field value and noting the corner through which the electron left the cavity. The reflection and transmission coefficients are then given by the fraction of electrons exiting the different leads at a given magnetic field. This approach has already been successfully applied to similar nanostructures by Beenacker and van Houten [87]. We assumed isotropic electron injection from the center of a corner
over an opening angle of 120°. A more precise model would have to consider the number of transmitted modes and their emission characteristics, that also depend on the exact shape of the constriction. However in a real experiment, random scatters and fluctuations of the background potential will also play an important role (see e.g. Fig. 2.2), so that our somewhat arbitrary approximation appears to be of reasonable accuracy. In addition we assume a perfect cavity with specularly reflecting hard walls at T=0 K. Owing to the four-fold rotational symmetry of the problem, it is sufficient to do the simulation for injection from one corner. Figure 5.3 displays the magnetic field dependent transmission and reflection coefficients obtained from a simulation by Philippe Stauffenegger [88]. Inserting these values into equation 2.21:

$$R_{mn,kl} = \frac{\hbar e^2}{4\pi \alpha} \left( \frac{T_{km}T_{ln} - T_{kn}T_{lm}}{D} \right)$$

(5.3)

allows one to calculate the resistance of the square cavity for all current and voltage configurations.

![Diagram showing electron injection and trajectories](image)

Figure 5.3: Upper left: 6000 electrons were injected homogeneously over an angle of 120°, lower left: examples of two trajectories. Right: transmissions as a function of magnetic field determined from the fraction of electrons exiting through the respective terminal.

### 5.2.3 Measured and calculated magnetoresistance

Writing the Landauer–Büttiker expression for the four-terminal resistance and exploiting the symmetries of the square cavity we arrive at:

$$R_{12,43} = \frac{\hbar}{e^2} \frac{T_{41}T_{32} - T_{42}T_{31}}{D} = \frac{\hbar}{e^2} \frac{T_{41}T_{21} - T_{31}^2}{D}$$

(5.4)

The calculated resistance is in good qualitative agreement with measurement, as can be seen in Fig 5.4. The positions of the minima, as well as the damping of the
oscillations with increasing order is well reproduced. It is immediately clear from
the calculated transmissions (Fig. 5.3) that $T_{31}$ dominates for $B \to 0$T leading
to a negative total resistance at $B=0$ T. This can also be understood intuitively:

at zero magnetic field, a ballistic electron injected into the cavity, can traverse it
across the diagonal and be absorbed in the opposite corner. If, for example, the
two upper corners act as source and drain, this will lead to a reversed polarity
of the voltage in the two lower corners and a negative value for the four terminal
resistance. Negative resistances of this type were observed in some traces below
1K (not shown), but in most traces the resistance only showed minima without
a change in sign. The reasons are probably random scatters and non specular
reflections off the cavity walls. In contrast, it is interesting that the damping of the
oscillation amplitude towards higher magnetic fields is mainly a geometrical effect,
since it is also present in the simulation where only perfect specular reflection was
considered.

$$R_{2-terminal} = R_{13,13} = \frac{\hbar}{e^2} \frac{T_{11}T_{33} - T_{13}T_{13}}{D} = \frac{\hbar}{e^2} \frac{R_{11} - T_{31}^2}{D}$$  (5.5)

Obviously, the strong negative magnetoresistance appearing in the measured
resistance (Fig.5.5), is not reproduced by calculation (lower inset Fig.5.5). This is
because the ‘contact resistance’ from coupling into the cavity was neglected. If the
openings are viewed as two QPCs in series, the magnetoresistance through these
constrictions is given by the expression [89]:

![Figure 5.4: Four terminal measurement at 1.6 K (black line) and calculated resistance (red line) based on a billiard simulation and the Landauer–Büttiker formalism (eq. 2.21). Inset: schematic of the measurement geometry.](image-url)
\[
R_{QPC} = \frac{h}{2e^2} \left( \frac{1}{N_{\text{min}}} - \frac{1}{N_{\text{wide}}} \right) \quad (5.6)
\]

with

\[
N_{\text{min}}(B) = \left( k_F l_c / \pi \right) \arcsin \left( W / 2 l_c \right) + \left( W / 2 l_c \right) \times \left[ 1 - \left( W / 2 l_c \right)^2 \right]^{1/2}, \quad \text{if} \quad W < 2 l_c
\]

\[
N_{\text{min}}(B) = k_F l_c / 2, \quad \text{if} \quad W > 2 l_c \quad (5.7)
\]

where \( l_c = \hbar k_F / eB \) is the classical cyclotron radius and \( W \) is the width of the constriction. The resulting magnetoresistance for \( W = 0.21 \, \mu m \) and \( n = 6 \times 10^{15} \, m^{-2} \) is shown in the upper inset of Fig.5.5. The lithographic widths of the openings in the corners of the cavity \( W_l = 0.21 \, \mu m \) are in good agreement with the electronic width of \( W_e \approx 0.2 \, \mu m \), which can be estimated from the value \( B_D \) at which the negative magnetoresistance levels out (about 1.2 T at \( n = 6 \times 10^{15} \, m^{-2} \)). Comparing \( W_e \) with the cyclotron diameter at \( B_D \), yields:

\[
W_e = \frac{2\hbar k_e}{e B_D} \quad (5.8)
\]

If the magnetoresistance from the QPCs and the square cavity are added, reasonable agreement with experiment is achieved.

Figure 5.5: Two terminal measurement across the cavity at 1.6 K (black line) and calculated resistance (red line) based on a billiard simulation and the Landauer–Büttiker formalism (eq. 2.21). The negative magnetoresistance is due to a better coupling into the cavity at higher magnetic fields. Upper left inset: schematic of the measurement geometry. Upper right inset: calculated magnetoresistance from skipping orbits in the square. Lower inset: calculation of the magnetoresistance from coupling into the cavity according to [89].
For the Hall resistance, the Landauer–Büttiker formula reads:

\[ R_{\text{Hall}} = R_{13,24} = \frac{\hbar}{e^2} \frac{T_{21}T_{33} - T_{23}T_{31}}{D} = \frac{\hbar}{e^2} \frac{T_{21}^2 - T_{41}^2}{D} \]  

(5.9)

A comparison with an experimental trace is plotted in Fig. 5.6. Again the positions of the resistance minima are well reproduced by simulation. The enhanced Hall slope could be due to a kind of funneling effect, but a microscopic understanding of the responsible trajectories proved to be difficult. A similar billiard simulation by Beenacker and van Houten [87] also produced a ‘last Hall plateau’ of the type we observe. However experiments by Ford et al. on smaller square geometries, in general showed a quenching of the Hall effect around \( B = 0 \) T [90].

![Figure 5.6: Hall measurement at 1.7 K (black line) and calculated Hall resistance (red line) based on a billiard simulation and the Landauer–Büttiker formalism (eq. 2.21). Inset: schematic of the measurement geometry.](image)

5.3 Rectangular cavity

The rectangular cavity is very similar to the square cavity sample, with the only nominal difference being its dimensions of \( 3 \times 6 \) µm. The Hall resistance and the two-terminal resistance across the long and the short side are presented in Fig. 5.7. The 95 mT oscillation period corresponding to the short side is prominent. The most noteworthy difference as compared to the square cavity, is the asymmetric Hall effect. This demonstrates that different sides of the cavity are probed for different signs of the magnetic field. The skipping orbits down the long sides are strongly suppressed, because the trajectories are twice as long and therefore much more sensitive to random background potential fluctuations. Only very faint
oscillations with a period of $\sim 45 \text{ mT}$ can be seen on the negative magnetic field side.

![Graph showing four-terminal resistances](image)

Figure 5.7: Four-terminal resistances $R_{12,43}$, $R_{14,23}$ and $R_{13,24}$. The main difference as compared to the square cavity is the asymmetry in the Hall resistance. Inset: AFM-image of the $3 \times 6 \mu \text{m}$ cavity.

### 5.4 Summary

We find very good agreement between the lithographic shape and electronic properties of the AFM-defined structures. At low electron sheet densities, the number of random background scatterers increases and the system approaches the diffusive regime. Towards higher electron densities, transport through the cavities becomes ballistic and reflection off the walls is highly specular. Geometrical commensurability effects observed in the magnetoresistance are well reproduced by calculations based on the Landauer–Büttiker formalism and a classical billiard simulation. This can be viewed as a consistency check for the electronic shape of the structures and lends additional support to the steep wall potentials observed in AFM-defined quantum dots [91]. We conclude, that AFM-lithography is a useful tool for fabricating well defined nano-structures of high electronic quality.
Chapter 6

Magnetoresistance in antidot lattices

6.1 Introduction

Magnetotransport experiments on antidot lattices in two-dimensional electron systems (2DES) have attracted considerable interest over the last 10 years (see also section 2.5). Effects observed include classical commensurability peaks around 1, 2, 4, ... antidots and Aharonov–Bohm-type oscillations. Most samples studied so far consist of a 2DES in an AlGaAs heterostructure, that was patterned by e-beam lithography and wet chemical etching. Antidot lattices with a pitch of 200 - 300 nm are typically fabricated with this technique.

In this chapter the magnetoresistance across two finite AFM-defined antidot arrays is studied. The first structure contains a 20 × 20 antidot array with a lattice constant of a=300 nm (Fig.6.1) covered with a TiAu topgate, while the second array has a lattice constant of a=150 nm and can be tuned with a TiAu topgate and a backgate that was grown into the wafer (see chapter 3 for details). After characterizing the two samples, measurements of the classical commensurability peaks are presented. Aharonov–Bohm-type oscillations and Aharonov–Altshuler–Spivak oscillations were detected at 1.7 K and are analyzed in the following sections. Finally we discuss a new peak splitting phenomena observed on the main commensurability peak in the 150 nm sample.

The main focus of this chapter is on how the electronic properties of AFM defined antidot arrays compare to those of samples fabricated by other methods, if new effects appear when the lattice constant is reduced, and on what happens if the background mobility changes or the 2DES wavefunction is shifted in z-direction.

6.2 Sample characterization

300 nm sample: Figure 6.1 shows an AFM image of a 20 × 20 antidot array with a lattice constant of a=300 nm, termed the ‘300 nm sample’. The sample was fabricated from a high quality GaAs/AlGaAs heterostructure containing a 2DES 34 nm below the surface with a 4.2 K mobility of $\mu = 500000 \text{ cm}^2/\text{Vs}$ at an electron
density of $5 \times 10^{11} \text{cm}^{-2}$. The array is enclosed by a square cavity with openings in the corners that serve as current and voltage leads. After lithography the entire structure was covered with a TiAu topgate that allows the global electron density to be tuned between 2 and $6.5 \times 10^{15} \text{m}^{-2}$.

![AFM micrograph](image)

Figure 6.1: (a) AFM micrograph of the $20 \times 20$ antidot array with a lattice constant of $a=300 \text{nm}$ enclosed by a square cavity with openings in the corners that serve as current and voltage leads. Bright regions are oxidized, leading to a depletion of the underlying 2DES. (b) Blow-up of a $2 \times 4$ segment of (a) with a line scan measuring the oxide height. After AFM lithography the entire structure was covered with a TiAu topgate.

**150 nm sample**: An AFM micrograph of the structure referred to as the ‘150 nm sample’ is depicted in Fig. 6.2. The layout is the same as for the 300 nm sample, but with a lattice constant of $a=150 \text{nm}$. Besides being covered with a TiAu topgate, the wafer also contains a conducting layer $1.3 \mu\text{m}$ below the surface, that serves as a backgate. By applying appropriate voltages to the top- and backgate, the electron density can be varied, or the 2DES can be shifted while the electron density is held constant. A simulation of the $z$-component of the 2DES wave function for different gate settings at constant electron density shows a shift of the wavefunction maximum by about 1-2 nm in growth direction, the equivalent of about 3-5 unit cells of GaAs (see Fig. 4.10). A more significant change is observed in the shape of the tail of the wavefunction on the GaAs side of the heterostructure, which could be of importance for weak anti-localization or other spin related effects. From this we conclude, that the change in antidot potential felt by the 2DES when it is pushed towards or away from the interface is small. What primarily changes is the electron mobility. For a more detailed characterization of the wafer see chapter 3.

Quantitative insight into the nature of the induced antidot potential can be gained by comparing local anodic oxidation with an AFM tip to a controlled wet chemical etch of the surface layers of a GaAs heterostructure. The oxide height is practically equivalent to the corresponding etch depth, as can be seen by removing the oxide with an HCl dip-etch. It is therefore possible to approximate the potential landscape induced by AFM-lithography from the oxide profile, by employing a Poisson–Schrödinger solver [77] to simulate the 2D electron density...
as a function of cap layer thickness (inset Fig.6.3(b)). A second contribution arises from the periodic displacement of the TiAu topgate relative to the 2DES due to the height of the oxide dots (Fig.6.3 (a)). This leads to a capacitive potential modulation if a voltage is applied to the topgate. The height profile of an antidot from the 150 nm sample (Fig. 6.2 (b)) converted into a potential modulation for a series of topgate voltages is shown in Fig.6.3 (b). For the regime of topgate voltages studied here (±240 mV) the potential is completely dominated by the contribution from local anodic oxidation and remains essentially constant. The potential slopes are very steep leading to an antidot diameter of about 28 nm for an electron density of $5.1 \times 10^{15}$ m$^{-2}$ and about 34 nm at $3.4 \times 10^{15}$ m$^{-2}$. Possible contributions from Fermi level pinning or strain originating from the topgate metals were not taken into account.

### 6.3 Commensurability peaks

The upper graph in Fig.6.4 shows the longitudinal magnetoresistance across diagonal 1 (corner 1-3) and the Hall resistance of the 300 nm sample at $V_{tg}=100$ mV at 4.2 K. Clear commensurability peaks around 1 and 4 antidots and the corresponding Hall plateaus appear. A closeup of the quenched Hall effect is presented in the inset. The lower graph in Fig.6.4 plots traces of the longitudinal resistance as a function of the topgate voltage. As expected the overall resistance decreases as the topgate voltage and hence the electron density increase. Impurities and background potential fluctuations are screened more effectively at higher electron densities, leading to a longer mean free path between the antidots and, as a consequence, higher order commensurability peaks are resolved. These observations
Figure 6.3: (a) Schematic of a cross-section through a wafer with oxide pillars and a TiAu topgate. (b) Potential modulation of the underlying 2DES at different topgate settings, induced by an antidot with a height profile as shown in Fig. 6.2 (b). Two contributions were taken into account: Capacitive effects from the modulated distance between topgate and 2DES and depletion due to the oxidation process. Inset: electron density as a function of etch depth calculated with a Poisson–Schrodinger solver [77]. At constant density the potential profile remains essentially unaltered as the top and backgate are swept.

are in good agreement with measurements done on ‘infinite’ lattices (few hundred micrometers) on Hall bars [33] and on samples with a similar layout, that were fabricated by e-beam lithography and wet chemical etching [43]. Evidently the magnetoresistance is not significantly influenced by the finite size of the array or the design of the enclosing cavity.

Table 6.1 lists the magnetic fields for commensurability peaks around 1 and 4 antidots and compares them to values calculated with the expression:

$$r_c = \frac{n_k f}{e B} = \frac{\hbar \sqrt{2\pi n e}}{e B}$$

(6.1)

where $r_c$ is the cyclotron radius, and $n$ is the electron density.
Figure 6.4: a=300 nm sample: The upper plot shows the four-terminal magnetoresistance between corners 1 and 3 (Rxx) and Hall resistance (Rxy) at $V_{tg} = 100$ mV and 1.7 K. Inset: Closeup of the so-called quenched or negative Hall effect. Lower plot: Longitudinal magnetoresistance at a series of topgate voltages at 1.7 K.
6.3. COMMENSURABILITY PEAKS

Figure 6.5: a=150 nm sample: Longitudinal and Hall resistance at 9K. Upper plot: sweeps at a constant density of $5.1 \cdot 10^{15} \text{m}^{-2}$ for a series of top- and backgate settings. Lower plot: topgate / backgate series at a constant electron density of $3.75 \cdot 10^{15} \text{m}^{-2}$. 
Figure 6.6: a=150 nm sample: Longitudinal resistance across a diagonal and Hall resistance at 9 K. Upper plot: sweeps at a constant electron density of $3.4 \cdot 10^{15} \text{ m}^{-2}$ for a series of top- and backgate settings. Lower plot: topgate / backgate series at a constant density of $2.6 \cdot 10^{15} \text{ m}^{-2}$. 
Figure 6.7: (a) Longitudinal magnetoresistance across a diagonal in the 150nm sample with illustrations of the definitions used for the plots below. The values were recorded as a function of topgate voltage after readjusting the electron sheet density to a constant value with the backgate. Peak height statistics at a constant electron density are shown in (b) for 5.1 \cdot 10^{15} \text{ m}^{-2}, (c) for 3.75 \cdot 10^{15} \text{ m}^{-2}, (d) for 3.37 \cdot 10^{15} \text{ m}^{-2}, and (e) for 2.62 \cdot 10^{15} \text{ m}^{-2}.
Figure 6.8: a=150 nm sample: (a) Electron sheet density in a lead (unpatterned 2DES) as a function of top- and backgate voltage. The settings at which the magnetoresistance traces shown in Figs. 6.5 and 6.6 were taken are marked by circles. (b) Same plot as in (a) but measuring the electron density in the antidot array. (c) Electron sheet density in a Hall bar made of the same wafer as a function of top- and backgate voltage. The gate settings corresponding to the magnetoresistance traces in the antidot array are marked by circles. (d) Same plot as in (c) but plotting the electron mobility in the Hall bar. (e) Comparison of the electron density in the antidot lattice and the lead for the four constant density sweeps in in Figs. 6.5, 6.6. (f) Density difference between lattice and lead converted to meV.
6.3. COMMENSURABILITY PEAKS

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<th>CP 4</th>
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<td>–</td>
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</tr>
</tbody>
</table>

Table 6.1: Measured and calculated peak positions for commensurability peaks around one and four antidots (CP 1 and CP 4) for the 300 nm sample. The errors are estimates based on the uncertainty in peak position and on the uncertainty in electron sheet density used for the calculations.

For the main commensurability peak, $r_c$ is equal to half the lattice constant $a/2 = 150$ nm in the 300 nm sample. The peak around 4 antidots is shifted to significantly lower fields, where a classical orbit can no longer exist without colliding with the surrounding antidots (see Fig. 2.7). This discrepancy arises from a nonlinear resonance as explained by Fleischmann et al. [39] using numerical simulations of classical electron trajectories in smooth antidot potentials. The experimental values for the position of the commensurability peak around 4 antidots at about 25-35% of the magnetic fields for the main commensurability peak are in good agreement with these simulations.

In the measurements on the 150 nm sample, we focus on top- and backgate sweeps at constant electron density. Again the four-terminal magnetoresistance is taken across diagonal 1 from corner 1 to 3, while the Hall voltage is measured across diagonal 2 between corners 2 and 4. The electron density in the lattice was determined from the position of a well-defined Shubnikov–de Haas minimum at a known filling factor. Constant density sweeps were done by setting the topgate voltage to a fixed value and then adjusting the backgate voltage until the specified electron density was reached. The backgate voltage is not a well defined parameter, because the ErAs islands between 2DES and backgate can become charged over time (see chapter 3). This leads to an effective voltage offset to the backgate. However, if the back gate-voltage is readjusted to give the same electron density at the same topgate voltage, the magnetoresistance was found to be virtually unchanged down to the smallest reproducible fluctuations. It is therefore sensible to define a state of the system in terms of a topgate voltage and electron density.

Figures 6.5 and 6.6 show magnetoresistance traces at a series of top and backgate settings for four different constant electron densities at 9 K. Like in the 300 nm sample, the resistance decreases and the commensurability features are better resolved at higher electron densities. Within a constant density series the electron mobility increases towards higher topgate voltages. Only at a density of $3.4 \times 10^{15}$ m$^{-2}$ does the mobility increase again at the lowest topgate settings. A possible explanation for the increase in mobility at high topgate voltages is a population of the donor layer with electrons, that leads to improved screening. On the other hand, at very low topgate voltages, all the electrons have left the donor layer and the mobility increases again, because the 2DES is pushed away
from the GaAs/GaAlAs interface, where more scatterers are present. As can be seen in Fig. 6.5 the commensurability peak around 4 antidots is more sensitive to changes in the mobility owing to its longer trajectory. In addition a pronounced weak localization peak around \(B=0\) T appears.

A plot of the electron density in the lead and in the lattice as a function of top- and backgate voltage is shown in Fig. 6.8. Circles mark the values at which traces in the constant density series were taken. By identifying the values for the electron density and topgate settings in the lead with the same settings in an unstructured Hall bar, it is possible to extract the behavior of the background mobility in the wafer during a constant density series. As can be seen in Fig. 6.8 (d), the background mobility in the wafer changes in a way that is consistent with the longitudinal resistance at \(B=0\) T in the lattice. This suggests, that during a constant density sweep the antidot potential remains essentially constant, as can be seen in the simulation in Fig. 6.3, while the background mobility intrinsic to the wafer varies.

Figure 6.8 (e) compares the electron density in the lattice with the electron density in the lead for the four constant density sweeps. During a constant density series, the difference remains practically constant, but as can be seen in Fig. 6.8 (f), the difference in electron density increases from about \(0.35 \times 10^{15} \text{ m}^{-2}\) at the highest density to about \(0.6 \times 10^{15} \text{ m}^{-2}\) at the lowest density. This is somewhat surprising, since the simulation in Fig. 6.3 predicts no saddle points between the antidots. A possible explanation is that we do not measure the maximum electron density between the antidots, but rather an averaged density over the entire cross-sectional area of the potential valley. This would also account for the larger difference at lower densities, where the potential slopes are less steep.

### 6.4 Aharonov–Bohm-type oscillations

Aharonov–Bohm-type oscillations were detected at low electron densities in the 300 nm sample as well as in the 150 nm sample at 1.7 K. As can be seen in Fig. 6.9, there is a smooth transition from the Shubnikov–de Haas to the Aharonov–Bohm-type oscillations superimposed on the main commensurability peak in the 300 nm sample. The period of the Aharonov–Bohm-type oscillations is about 41 mT which corresponds to an area of about \((300 \text{ nm})^2\), or a circle with a diameter of \(a=300\) nm.

Measurements done with different lead configurations on the 150 nm sample exclude an irregularity in a single corner of the lattice as the sole source of the Aharonov–Bohm-type oscillations, indicating that the effect is a lattice property (Fig. 6.10 upper graph). The influence of changes in background mobility can be seen in the difference between a constant density sweep taken at 9 K and at 1.7 K (Fig. 6.10 lower graph). Surprisingly the Aharonov–Bohm-type oscillations are most pronounced at the lowest mobility (see trace highlighted in red Fig. 6.10 lower graph and mobility dependence Fig. 6.7 (e)). This observation is put on a more quantitative basis in Fig. 6.11 (c), where the Fast Fourier Transform (FFT) of the traces plotted in Fig. 6.10 upper graph, is shown. The FFT of the trace at the lowest mobility has the highest and sharpest peak at a frequency corre-
Figure 6.9: (a) a=300 nm sample: Longitudinal magnetoresistance at 1.7 K (black line) and 4.2 K (blue line) and the difference between the two (red line) at a topgate voltage of -100 mV. Minima in the magnetoresistance are numbered. (b),(c) The minima as determined in (a) plotted over B and 1/B. The line fit in (b) yields a period of B=41 mT. The line fit in (c) follows the positions of Shubnikov–de Haas oscillations corresponding to an electron sheet density of $2.9 \cdot 10^{15} \text{m}^{-2}$.
Figure 6.10: a=150 nm sample, upper graph: Aharonov–Bohm-type oscillations are observed from 1.5 T up to 4 T in different configurations with the same gate settings at 1.7 K. The vertical lines are spaced by 0.234 T, corresponding to a flux quantum through a cyclotron orbit with a diameter of 150 nm. Lower graph: Difference between constant density sweeps taken at $2.6 \cdot 10^{15} \text{m}^{-2}$ and at 9 K (Fig. 6.6) and at 1.7 K. The resistance was measured across diagonal 1 from contact 2 to 3. The vertical magenta lines again have a spacing of 0.234 T. A 9 K sweep with a clear commensurability peak around 1 antidot is superimposed (green trace) with short vertical blue lines marking the positions of Shubnikov de–Haas minima.
Figure 6.11: a=150 nm sample: (a) Difference between a trace taken at 1.7 K and 9 K with $V_{tg}=-230.3 \text{ mV}$, $V_{bg}=0.952 \text{ V}$ and an electron density of $2.6 \cdot 10^{15} \text{ m}^{-2}$ (highlighted red in the lower graph of Fig.6.10). (b) The numbered minima marked in (a) plotted as a function of B. A period of $0.23 \text{ T}$ is extracted from the linear fit. (c) Fast Fourier Transform of all traces shown in the lower graph of Fig.6.10. The frequency was transformed into a cyclotron diameter via equation 6.1.
Figure 6.12: a=150 nm sample: (a) Trace across diagonal 1 at 1.7 K with $V_{tg} = -300.0 \text{mV}$, $V_{bg} = +0.5 \text{V}$ and an electron density of $3.1 \cdot 10^{15} \text{m}^{-2}$. (b) The numbered minima marked in (a) plotted as a function of B. A period of 0.92 T is extracted from the linear fit. (c) Fast Fourier Transform of a series of traces at constant density, the frequency is converted to a unit cell length.
sponding to a flux quantum through a circle with a diameter of $a=150\,\text{nm}$. This behavior has been predicted by Ando et al. as a consequence of the suppression of contributions from larger periodic orbits and more complex bandstructure effects due to background scatterers. An analysis of the minima in the trace at the lowest mobility is presented in Fig. 6.11 (a) and (b) and a period of $0.23\,\text{T}$ was found. From this we can identify the periodic orbit around 1 antidot as the origin of the oscillations in accordance with numerical simulations by Uryu and Ando [44]. In contrast, a Hofstadter-type beating would have a periodicity of $0.183\,\text{T}$, corresponding to a flux quantum through a square unit cell.

Another interesting feature is that there is no smooth transition from the Aharonov–Bohm to the Shubnikov–de Haas effect in the 150 nm sample, as observed in the 300 nm sample and reported by Weiss et al. [42]. Instead the Aharonov–Bohm-type oscillations extend up to 4 T and coexist with the Shubnikov–de Haas effect, which could be a consequence of the smaller lattice constant. In an intuitive picture this could mean that the lower number of modes between two antidots in a smaller lattice leads to a more rigid electronic bandstructure.

6.5 Aharonov–Altshuler–Spivak oscillations

A second type of oscillation with approximately half the Aharonov–Bohm period was observed in some traces around $B=0\,\text{T}$ (Fig. 6.12) in the 150 nm sample. This effect has previously been reported for hexagonal lattices [47] and was very sensitive to changes in electron density and position of the wavefunction $z$-component. From the slope in Fig. 6.12 (b), a value of $0.096\,\text{T}$ corresponding to an area of twice the unit cell was extracted. This could point towards a band structure effect connected to the Hofstadter spectrum, but a close relation to the Aharonov–Altshuler–Spivak oscillations observed in small metal cylinders [25] seems more probable.

6.6 Peak splitting

In Fig.6.13 the difference between a 9 K and a 1.7 K series at the highest electron density of $5.1 \times 10^{15} \,\text{m}^{-2}$ is plotted. A peak superimposed on the main commensurability maximum at about 1.5 T splits as the 2DES is shifted. This effect has not been reported previously and could have several possible interpretations:

- **Aharonov–Bohm-type and $h/2e$ oscillations**: The peak splitting could be related to the Aharonov–Bohm-type oscillations or the $h/2e$ oscillations appearing at lower densities, however no real $B$-periodicity over several peaks was observed. In a band structure picture the peak splitting could also be attributed to a disorder induced opening or closing of a band gap.

- **Universal conductance fluctuations**: UCFs are known to change when the disorder configuration is altered. This could be the case, when the 2DES is shifted in $z$-direction at different top- and backgate settings, but two points make this interpretation seem unlikely. The peak splitting feature is about a
factor of 5 larger in amplitude than the magnetic field dependent fluctuations around \(B=0\, \text{T}\) and does not appear to be random in nature. Secondly the feature is present in two different cooldowns, which usually leads to a different set of UCFs.

- **Shubnikov–de Haas oscillations:** Differences in the periodicity of the SdH oscillations in the lattice and leads resulting from small variations in the electron density could be a cause. However, the exact matching of the SdH maxima and minima at higher magnetic fields practically excludes this possibility.

- **Spin related effects:** Spin orbit coupling increases towards higher electron densities. Since not only the electron mobility \(\mu\) but also the wavefunction symmetry in \(z\)-direction changes while the 2DES is shifted, this could be an important factor. Measurements focusing on weak anti-localization below 1 K could yield insight into this interesting possibility.

![1st cooldown](image1.png) ![2nd cooldown](image2.png)

Figure 6.13: Difference between traces taken at 1.7 K and 9 K for a constant density series at \(5.1 \cdot 10^{15}\, \text{m}^{-2}\), for two different cooldowns. The shaded region highlights a peak superimposed on the main commensurability maximum, that splits while the gates are tuned.

### 6.7 Summary

Antidot arrays defined by AFM lithography have an electronic quality comparable to the best lattices fabricated by other techniques [33, 35, 34]. The finite size of the arrays seems to have no significant influence on the magnetotransport properties. As in earlier reports, the classical commensurability peaks are well described by theory [39].
Aharonov–Bohm-type oscillations were detected in both samples. In the a=150 nm lattice a period of 0.234 T was found that matches a flux quantum through a circular orbit with a diameter of 150 nm (0.234 T), which can clearly be distinguished from a flux quantum through a square unit cell (0.183 T). This supports the picture of periodic orbits and makes an interpretation in terms of a Hofstadter-type beating appear unlikely. On the other hand, it is surprising that the oscillations persist up to 4 T, twice the magnetic field of the main commensurability peak, without changing in frequency! The period exactly corresponds to a flux quantum through a circle with the diameter of the lattice constant over the entire field range from 1.5 to over 4 T. Instead of a smooth transition to the Shubnikov–de Haas effect, the two oscillations coexist, which is in marked contrast to earlier reports [42] and observations on the a=300 nm sample. Under these conditions simple periodic orbit theory seems to be inapplicable, because classical orbits would strongly contract, especially if the small antidot diameter of only about 35 nm (from simulation) is compared to the lattice constant of a=150 nm. It is possibly more adequate to think of the electron wavefunctions as being bound to the antidots with concentric states similar to edge states in the quantum Hall regime. The conductance is then modulated by the overlapping states in the valleys that form an extended network. This would also explain the observed constant period of 0.234 T up to very high magnetic fields.

In mobility dependent sweeps we found the most pronounced Aharonov–Bohm-type oscillations at the lowest mobilities. According to Uryu and Ando [44] this can be interpreted as a washing out of more complex band structure effects in the presence of scatterers. This is the first experimental evidence for this somewhat counter intuitive prediction. Aharonov–Altschuler–Spivak oscillations around B=0 T were also present in some traces in the 150 nm sample, but were more sensitive to changes in parameters than the Aharonov–Bohm-type oscillations.

A new peak splitting effect superimposed on the main commensurability peak was observed as the 2DES was shifted in z-direction at high constant electron densities. This could be a spin related phenomena based on changes in the z-component of the wave function as the 2DES is pushed away from the GaAs/AlGaAs interface. Further investigations are needed to gain a better understanding of this interesting feature.
Chapter 7

Percolation in a quantum dot network

7.1 Introduction

In this chapter an especially small antidot lattice is studied at very low electron sheet densities close to pinch-off. In principle there should be a smooth transition from an antidot to a quantum dot array, when the electron density is lowered and the constrictions between neighboring antidots enter the tunneling regime. This corresponds to a cross-over to the tight-binding limit. Systems of this type have been of recent interest in several respects:

- **Quantum information processing:** States in quantum dots have been proposed as a possible realization of qubits, the fundamental units of information used in quantum computing schemes [92, 93]. In order to build a functioning quantum computer a large number of qubits have to be coupled coherently with the possibility of manipulating the individual quantum dots and reading information in and out. High integration is not only important for chip size reduction, but could also help reduce decoherence, one of the main challenges in quantum information processing at present.

- **Quantum dot cellular automata:** Coulomb charging effects in coupled quantum dots have been proposed as a means of realizing nanometer scale circuits for logical operations [94]. This scheme could be scalable to dimensions smaller than conventional field effect transistor technology.

- **Simulation of complex materials:** Materials like the high $T_c$ superconductors could be simulated at larger length scales and with tunable parameters by replacing the atoms with suitable quantum dots [95]. This could yield insight into the band structure and excitations of complex systems.

- **Granular media:** Electronic transport through granular or disordered materials is mediated by hopping or tunneling processes. Systems under study include arrays of metallic nano-crystals [96, 97, 98, 99], layers of semiconductor quantum dots [100, 101], porous silicon [102], and organic molecular
crystals [103]. These systems could have applications in nano-optics and nano-electronics.

- **The metal insulator transition in 2D**: In the presence of even the smallest amount of disorder all 2D electronic systems should be insulators at T=0 K [104]. However, recent observations on Si MOSFETS have suggested that a so-called metallic state might exist at T = 0 K in some materials [105]. A theoretical model by Meir explains this transition in a percolation picture with electron puddles that are incoherently coupled by QPCs [106]. Other works have dealt with quantum percolation in conductor networks [107, 108, 109].

In the tunneling regime the conductance through a QPC is exponentially sensitive to changes in the Fermi wavevector and width of the constriction. This makes the tight-binding band structure extremely sensitive to inhomogeneities in the sample and suggests a percolative description near pinch-off. Possible indications for bandstructure formation will be discussed in chapter 8. After a short introduction to some basic concepts of percolation theory we will discuss the measured data in the framework presented below.

### 7.2 Percolation

#### 7.2.1 Concepts and terminology

The term percolation is derived from the latin word ‘percolare’ meaning ‘to infiltrate’ or ‘to pass through’. This alludes to one of the central problems in the field of percolation: When do two points in a complex system become ‘connected’, enabling e.g. fluid flow or electrical conduction. Historically the first models were presented by P. J. Flory [110] and W. H. Stockmeyer [111] to explain polymerization and gelation in chemistry. The beginning of modern percolation theory is usually attributed to a publication by Broadbent and Hammersly (1957) [112] where a more theoretical description is given and the term percolation is introduced.

Mathematical models usually start with a $d$ dimensional lattice of a given symmetry on which two types of percolation can be defined: ‘site percolation’ and ‘bond percolation’ (Fig. 7.1). In ‘bond percolation’ the sites of a lattice are randomly connected with lines, the so-called ‘bonds’. In ‘site percolation’ the lattice sites are randomly occupied and two sites are connected if they share a common edge (nearest neighbor), not however if they only touch at a corner. Groups of connected sites are called **clusters**. Percolation theory deals with the properties of clusters as a function of the fraction $P$ of occupied bonds or sites. The fraction at which an infinite cluster is formed, that spans the entire (infinite) lattice is called the **percolation threshold** $P_c$. For $n=2$ and square lattices an infinite network forms at $P_c = 0.5$ for bond and at $P_c = 0.592746$ for site percolation.

The length scale characterizing a percolating system is the **correlation length** or **connectedness length** $\xi$. It can be viewed as a measure for the radius of the
clusters predominantly contributing to the mean cluster size. According to [113] \( \xi \) diverges when \( P \) approaches \( P_c \) as:

\[
\xi \propto (P - P_c)^\nu
\] (7.1)

Close to \( P_c \), or when the system is confined to volumes much smaller than \( \xi^d \), no characteristic length scale exists and all relevant functions become power laws, of the type:

\[
property \propto (P - P_c)^{\text{critical exponent}}
\] (7.2)

A list of critical exponents for cubic bond percolation lattices in 1 to 6 dimensions is given in table 7.1.

<table>
<thead>
<tr>
<th>dimension ( d )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
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<tr>
<td>( \beta )</td>
<td>0</td>
<td>5/36</td>
<td>0.45</td>
<td>0.58</td>
<td>0.76</td>
<td>1</td>
</tr>
<tr>
<td>( \beta' )</td>
<td>0</td>
<td>0.5-0.6</td>
<td>0.8-1.0</td>
<td>1.0-1.2</td>
<td>1.02±0.02</td>
<td>1</td>
</tr>
<tr>
<td>( \nu )</td>
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<td>4/3</td>
<td>0.88</td>
<td>0.7</td>
<td>0.6</td>
<td>1/2</td>
</tr>
<tr>
<td>( \zeta_R )</td>
<td>1</td>
<td>1.43±0.02</td>
<td>1.12±0.02</td>
<td>1.05±0.02</td>
<td>1.02±0.02</td>
<td>1</td>
</tr>
<tr>
<td>( \zeta_{\text{min}} )</td>
<td>1</td>
<td>1.49±0.01</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \zeta_B = \nu d - \beta' )</td>
<td>2.16-2.06</td>
<td>1.84-1.64</td>
<td>1.8-1.6</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.1: Critical exponents for different cluster properties in 1 to 6 dimensions. \( \beta \): exponent for the fraction of cites belonging to the IIC [114, 115]. \( \beta' \): backbone order parameter exponent [116, 117]. \( \nu \): exponent for the correlation length [114, 115]. \( \zeta_R \): conductivity exponent [118]. \( \zeta_{\text{min}} \): shortest path exponent [119]. \( \zeta_B \): backbone exponent.

### 7.2.2 Conduction through percolating systems

A physical system with sites, that are randomly connected by electrical resistors is called a random resistor network. In the simplest case all resistors have the
Figure 7.2: (a) Bond percolation cluster: the backbone can be separated into red (cutting) bonds and blue (multiply connected) bonds. The yellow bonds belong to dead ends. (b) Nodes and links model: above $P_c$ clusters or ‘macro molecules’ (links) of the size of the correlation length $\xi$ form a network. Below $P_c$ the network reduces to a single link of magnitude $\xi$. (c) Links, nodes and blobs model: the macro molecules have a substructure consisting of cutting bonds and blobs [see (a)]. (d) Sierpinski carpet with six iterations.

same value and the percolation threshold is reached when enough resistors have been added to make the system conducting. Qualitatively similar problems deal with fluid flow or particle transport. Current through the spanning cluster is only carried by the backbone, while the dangling ends make no contribution (Fig. 7.2 (a)). Three distinct pictures have been proposed to explain transport in a percolating system:

- **Nodes and links:** This picture was introduced by Skal and Shklovskii (1975) [120] and independently by de Gennes (1976) [121]. Above $P_c$ the backbone is viewed as a lattice of points separated by the correlation length $\xi$, that are connected by so-called macro bonds. The macro bonds in turn consist of $L$ links with $L \propto (P - P_c)^\zeta$ and it was argued that $\zeta = 1$. Just below $P_c$ the largest cluster, often termed the incipient infinite cluster (IIC), with linear dimension $\xi$ forms a single macro bond. This model is very easy to handle,
but only gives correct results for \( d > 6 \), when the probability for finding large loops is increasingly small (Fig. 7.2 (b)).

- **Nodes, links, and blobs:** In 1977 Stanley [122] pointed out, that below \( P_c \) the IIC can not only be partitioned into the backbone and the dangling ends (‘yellow’ bonds), but that the backbone itself can be separated into singly connected ‘links’ (‘cutting’ or ‘red’ bonds), and multiply connected (‘blue’) bonds forming ‘blobs’. The links are bottlenecks for current flow and would devide the IIC into two separate clusters if removed (Fig. 7.2 (c)).

- **Sierpinski gasket backbone:** This fractal model of the backbone at \( P_c \) proposed by Gefen et al. (1981) [123] represents the opposite extreme of the ‘links and nodes picture’. All bonds are multiply connected and the backbone is self similar on all length scales (Fig. 7.2 (d)). This model has the great advantage that it can be solved exactly, but in many cases it is an oversimplification of the more intricate cluster structure.

The ‘nodes and links model’ omits the blobs, whereas in the Sierpinski gasket model the links are neglected and only a giant blob is present. Therefore we will focus on the ‘nodes, links, and blobs model’, that is most realistic and can explain the experimentally observed phenomena in our sample best.

Additional considerations have to be taken into account if quantum mechanical interference and localization are included. According to the Anderson result [104], all states are localized in two dimensions at \( T=0 \) and the system is insulating in the presence of even the smallest amount of disorder (section 2.4). This would set the quantum percolation threshold to \( P_{cq} = 1 \), in agreement with some numerical studies [124, 125]. However, other authors report numerical indications for a transition at \( P_{cq} < 1 \) [107, 109]. This field of research is still evolving and more calculations, simulations and experiments are needed to resolve the present controversies.

### 7.3 Experimental findings

#### 7.3.1 Sample characterization

The structure investigated in this chapter has the standard layout discussed previously. An AFM-defined square 20 × 20 antidot array with a lattice constant of 120 nm is enclosed by a cavity with openings in the corners, that serve as current and voltage leads (inset Fig.7.3). After AFM-lithography the entire structure was covered with a TiAu topgate that enabled us to tune the global electron density from 2 to \( 5 \times 10^{15} \) m\(^{-2}\)/Vs. The heterointerface is located 34 nm below the surface and hosts a 2DES with an electron mobility of about 50 m\(^{-2}\)/Vs. At 90 mK clear commensurability peaks around 1 and 4 antidots are visible with superimposed ballistic conductance fluctuations (Fig.7.3) comparable to measurements taken on a similar sample with a larger lattice constant by Schuster et al.[43]. From the magnetoconductance and the oxide height of the antidots, \( \sim 16 \) nm, we conclude, that a true antidot lattice has formed.
7.3. EXPERIMENTAL FINDINGS

As can be seen in Fig. 7.4, the carrier density in the leads is higher by about \(1.5 \times 10^{15} \, \text{m}^{-2}\) as compared to the lattice. The resistance in the leads was measured to be smaller than \(h/2e^2\) down to topgate voltages below -100 mV and can thus be neglected in all measurements discussed in this chapter.

In the following we will distinguish three different regimes in topgate voltage: the open regime above \(-50.8 \, \text{mV}\) with a quasi linear decrease in conductance down to \(-50.8 \, \text{mV}\), where the conductance goes to 0 for the first time (marked by \(V_c\) in Fig. 7.5), the Coulomb blockade regime between \(-50.8 \, \text{mV}\) and about \(-85 \, \text{mV}\) where Coulomb diamonds are observed in the current-bias voltage characteristic, and the insulating regime below \(-85 \, \text{mV}\). The conductances across both diagonals of the array are almost identical over the entire range of topgate voltages aside from superimposed ballistic conductance fluctuations and transmission resonances.

### 7.3.2 The open regime

In the following analysis we consider the conductance of the entire lattice to be dominated by the constrictions between neighboring antidots forming quantum point contacts (QPCs). The area enclosed by four insulating islands can be viewed as a quantum dot or artificial atom with four terminals connecting it to its nearest neighbor quantum dots (see Fig. 7.6). A similar network consisting of cavities
CHAPTER 7. PERCOLATION IN A QUANTUM DOT NETWORK

Figure 7.4: Electron sheet density in the lattice and a lead as determined from Shubnikov–de Haas oscillations, with an estimated error of about $1 \times 10^{14}$ m$^{-2}$. The curves are 3rd order polynomial fits to the data, that were used to convert topgate voltage into electron sheet density (inset formulae).

Figure 7.5: Conductance as a function of topgate voltage across diagonal 1 (A-C) and diagonal 2 (B-C). The percolation threshold $V_c$ at -50.8 mV as well as the insulating, the Coulomb blockade, and the open regime are marked.
with 2-3 orders of magnitude larger areas has been studied by Senz et al. [128]. Assuming perfect symmetry and phase coherence across the entire system, an artificial band structure should form. In this picture resistance is only caused by the coupling into the 4 QPCs in parallel formed by the antidots in the corners of the cavity, leading to a total resistance of $1/4$ of a single constriction. Band gaps should not be present owing to thermal broadening and the complex overlapping bandstructure predicted for open square antidot lattices [32, 46]. If, on the other hand, the lattice is modeled as a network of identical classical resistors and phase coherence is completely neglected, the resistance measured in our geometry was simulated to be about 2.5 times larger than that of a single resistor representing a QPC between two antidots (see Fig. 7.6(b)). Since the experimental situation lies between these two extremes, the measured resistance across the array is probably comparable to the resistance of a single QPC between two antidots.

In practice, sample inhomogeneities due to the fabrication process and stray background charges are always present, leading to random background potential fluctuations. These fluctuations become especially prominent at low electron sheet densities due to reduced screening. This motivates an analysis of our data in terms of a percolation model.

In the open regime, for gate voltages above $-50.8$ mV, the system can be compared to a random resistor network of QPCs. For an open QPC the conductance staircase can be approximated to be linear with respect to the Fermi wavevector $k_f = \sqrt{2\pi n}$, where $n$ is the electron sheet density. This also remains valid for an ensemble of QPCs with different widths if phase coherence is not taken into account. When a QPC enters the tunneling regime, i.e., its conductance drops below $2e^2/h$, we consider it to be closed. In this picture current flow is carried by a cluster of open QPCs connecting two adjacent corners in a square bond percolation lattice (see Fig. 7.6(c)). The critical fraction of bonds (here open QPCs) needed
Figure 7.7: Conductivity across diagonal 1 and 2 as a function of the Fermi wavenumber $k_f$. Inset: Double logarithmic plot of the averaged conductance over $\Delta k_f = k_f - k_{fpc}$ with $k_{fpc} = k_f$ at $P_c$. The slope of the linear fit yields a value of $\zeta_R = 1.2$.

Figure 7.8: Bond percolation conductivity ($\sigma_1 = 1, \sigma_2 = 0$) for the two-dimensional square lattice (from [126]). The heavy line represents calculations based on a renormalization approach by Bernasconi [126]. The dashed line corresponds to the effective medium theory and the vertical bars denote results from Monte Carlo simulations by Kirkpatrick [127].
to form a spanning cluster over an infinite lattice is 0.5 [113]. This value remains
the same for finite lattices due to the self duality of the square bond percolation
problem, though finite size will of course lead to statistical fluctuations around
this value. In our system we expect the threshold value $P_c = 0.5$ to be close to the
critical point $V_c = -50.8 \text{ mV}$.

In the following analysis we will assume \((P - P_c)/P_c \propto \Delta k_f\) where $\Delta k_f = k_f - k_{fpc}$ and $k_{fpc}$ is $k_f$ at $P_c$ and $P$ is the fraction of conducting bonds. This
is a natural choice, because for an open QPC the conductance staircase can be
approximated to be linear with respect to the Fermi wavevector $k_f$, and the QPC
width $W$. The Fermi wavevector was determined from the electron density in the
lattice by extrapolating the relation between topgate voltage and electron density
into the Coulomb blockade regime. Even if the electron density is no longer a well
defined quantity under these conditions, the dependence of the closed fraction of
QPCs on $k_f$ can be extrapolated from the open case. Starting from the percolation
threshold $P_c$ and increasing the fraction of conducting bonds, the conductance is
predicted to show power law scaling [118]:

$$\sigma \propto ((P - P_c)/P_c)^{\zeta_R}$$

According to Straley [129], this relation remains valid for networks with resistance
distributions, as long as the distribution is not anomalous. The qualitative behavior
of the conductance as a function of $k_f$ (Fig. 7.7) closely resembles calculations
by Bernasconi [126], Monte Carlo simulations by Kirkpatrick [127], and the mean
field approximation for a classical random resistor network (Fig. 7.8). The main
difference being the superimposed ballistic conductance fluctuations and transmission resonances. Experimentally, similar results have been obtained by Last and Thouless [130] by punching randomly distributed holes into a sheet of graphite paper. In order to extract the critical exponent, we show a double logarithmic plot of the conductance as a function of $\Delta k_f$ in the inset of Fig. 7.7. A value of $\zeta_R = 1.2$ is found which is in reasonable agreement with the value of $1.43 \pm 0.02$
calculated by Fisch and Harris [118]. However, it must be noted that our result
can only serve as an estimate since it implies a linear relationship between $\Delta k_f$ and
$(P - P_c)/P_c$, which is reasonable, but cannot be verified explicitly by experiment.

### 7.3.3 The Coulomb blockade regime

In order to gain more insight into the electronic properties of the Coulomb blockade
regime, we measured the current as a function of topgate and bias voltage. As
can be seen in Fig. 7.9, blockade is lifted at sufficiently high bias voltages and clear Coulomb diamonds are resolved. Again the data obtained for transport
across both diagonals are qualitatively very similar indicating the homogeneity
of the lattice even at extremely low electron densities. In contrast to analogous
measurements on single quantum dots, overlapping diamonds as well as stretches
in topgate voltage without Coulomb blockade are observed. This indicates the
formation of a network with blockaded regions connected in series and in parallel,
that dominate transport for a given gate voltage setting. As the electron density
is lowered, the clusters of strongly coupled unit cells shrink, and the charging
energies increase. In order to quantify this process, the average of the four contour lines at \( \pm 5 \text{ pA} \) (Fig. 7.9) is taken and plotted with respect to the topgate voltage and the bias voltage converted into cluster size (Fig. 7.10(a)) by employing a plate capacitor model:

\[
E_C = eV_{\text{bias}} = \frac{e^2}{C} = \frac{e^2}{\epsilon \epsilon_0 \frac{A_{\text{dot}}}{d}}
\]

where \( d = 34 \text{ nm} \) is the distance between 2DES and topgate and \( A_{\text{dot}} \) is the area of the dot. The single-particle energy spacing, which gives a contribution of about 10\%, is neglected. In order to relate the geometrical cluster area to the number of quantum dots or unit cells involved, we divide \( A_{\text{dot}} \) by the effective dot area,

Figure 7.9: Current as a function of topgate and bias voltage. White regions correspond to absolute current values above 5 pA. The upper two graphs are overview plots for diagonals 1 and 2. The lower two are blowups showing well defined Coulomb diamonds. All measurements were taken at a He bath temperature of 90 mK at \( B = 0 \).
Figure 7.10: Number of open QPCs (bonds) in a quantum dot with the averaged charging energy as a function of topgate voltage. The minima marked by * reflect the actual dot size best. Inset: Illustration of the ‘links, nodes, and blobs model’ [122, 131]. The blue (multiply connected) and red (cutting) bonds form the backbone, the clusters of blue and yellow (dangling) bonds are called blobs that act as quantum dots. They are linked by the red bonds (see also 7.2 (a)).

Figure 7.11: Double logarithmic plot of the number of bonds vs. $\Delta k_f = k_{fpc} - k_f$, where $k_{fpc}$ is $k_f$ at $V_c$. The slope gives a value of $\zeta_{CB} = -3.0$. 
which we denote by $A_{dot} = \text{(unit cell area - average antidot area)}$. The electronic antidot diameter is estimated to be about 80 nm based on the oxide profile. Strictly speaking this analysis only applies to the bias voltage maxima, i.e., the peaks of the Coulomb diamonds in Fig. 7.9. Therefore the minima in Fig. 7.10(a) marked by * represent the actual dot sizes best. This analysis gives very reasonable values for the number of quantum dots, that are collectively Coulomb blockaded. Close to -50.8 mV, the cluster size is comparable to half the total number dots or unit cells ($\sim 400$) existing in the entire sample. Towards the insulating regime very small clusters down to about 5-10 dots dominate the sample conductance.

As illustrated in Fig. 7.2 (a), the cluster of quantum dots spanning the diagonal of the array can be decomposed into multiply connected blobs of varying size that constitute the blockaded regions, and so-called red or cutting bonds, that form bottle necks carrying the entire current and that would disconnect the two terminals if removed. Close to threshold, the entire cluster forms a giant blob with a charging energy that should equal a quantum dot somewhat smaller than half the size of the entire array. As the electron density is reduced, the number of red bonds increases, and the cluster contains ever smaller blobs in series with growing charging energies. A double logarithmic plot of the averaged dot area as a function of $\Delta k_f$ is shown in Fig. 7.11. The slope gives a critical exponent $\zeta_{CB} \approx -3 \pm 1$. Theoretical calculations of the critical exponent for blob size scaling yield a value of 2.06-2.16 [131], which is of the order of the scaling exponent for the areas of the Coulomb blockaded regions. The exact blob size distribution has been studied theoretically by Gyure et al. [132] and the number of red bonds scales like $\zeta_{red} = -1$ [131] making an overall value for $\zeta_{CB}$ of about $-3$ seem reasonable. However we point out that a precise theoretical understanding of Coulomb blockade scaling that includes effects from clusters connected in series and in parallel is still outstanding. Again, the value for the scaling factor $\zeta_{CB}$ should be viewed as an estimate owing to the uncertainty in $P$, and the averaging in the Coulomb diamond peak height. The fact, that the two exponents $\zeta_R$ and $\zeta_{CB}$ were obtained using the same parameter $k_f$ for $P$ can be viewed as a consistency test.

7.3.4 Local and global properties of the spanning cluster

Three types of measurements were conducted to gain a more detailed understanding of the microscopic nature of transport across the percolation cluster: Four-terminal configurations with non local voltage probes, magnetic field dependent shifts of the Coulomb peaks, and weak localization type measurements to determine phase coherence properties.

The four-terminal measurement setup is illustrated in Fig. 7.12 (a). Current was recorded as a function of topgate and bias voltage between contacts C and D (Fig. 7.12 (b)), while the four-terminal voltage was simultaneously measured between contacts A and B (Fig. 7.12 (c)). A variation in the four-terminal voltage signal by about $\pm 50 \mu V$ as a function of topgate voltage can be eliminated by numerical differentiation with respect to bias voltage. Strong similarities in the differentiated 2- and 4-terminal datasets are apparent (white arrows in Fig. 7.12 (b) and (c)), which indicates, that individual quantum dots have a global impact
on transport through the entire network.

Owing to the percolative nature of the conducting network, blockaded clusters with different sizes and shapes form, that are interconnected by a varying number of terminals. As a consequence, the wave functions in the different dots will have different shapes and symmetries, leading to a unique energy spectrum and magnetic field dependence. This can be exploited to identify individual clusters (or blobs) in the network, by monitoring the energy shift of quantum states across both diagonals as a function of a perpendicular magnetic field. Figure 7.13 shows plots of the current as a function of topgate voltage and magnetic field across diagonals 1 and 2 at a constant bias voltage of 50 µV. Coulomb peaks (current maxima), that show the same magnetic field dependence across both diagonals can be attributed to the same cluster, whereas differing features originate from clusters only participating in transport across one of the two diagonals (see Fig. 7.13). The observation that differing as well as coinciding features are present, suggests that our interpretation in terms of the links nodes and blobs model, with the blobs acting as Coulomb blockaded regions is correct.

Phase coherence across the spanning network was investigated in Coulomb blockade and in the open regime by measuring the conductance as a function of a perpendicular magnetic field at a series of topgate voltages (Fig. 7.14 upper two plots). The magnetic field dependent conductance averaged over traces between topgate voltages from -54 to -70 mV in steps of 0.02 mV in the Coulomb blockade regime and from 30 to 0 mV in steps of 0.25 mV in the open regime are shown in the lower two graphs of Fig. 7.14. Dashed vertical lines mark a flux quantum through the square unit cell (290 mT). The averaged magnetoconductance in the blockaded regime shows an increase with magnetic field by a factor of approximately 3, which is significantly higher than the value of 4/3 predicted by Al Hassid [21] and measured by Folk et al. [133] for single quantum dots. However this behavior is consistent with a strong negative tunneling magnetoresistance effect proposed by Raikh and Glazmann [134] based on wavefunction overlap from neighboring elliptically shaped electron ‘lakes’. They predict a low field magnetoresistance of the form:

$$\frac{\delta R(B)}{R(0)} \approx -\frac{B^2}{B_0^2}$$

(7.5)

where $B_0$ depends on the details of the tunnel barrier, but is typically of the order of $\frac{\hbar}{e} \frac{1}{d_1d_2}$, $d_1$ and $d_2$ being the semiaxis of the electron lakes. A best fit to the magnetoconductance averaged over topgate voltage in the Coulomb blockade regime (Fig. 7.14) yields a value of 2.2 T for $B_0$. This corresponds to an average radius of about 40 nm which is compatible with a single quantum dot confined to a unit cell. Despite the fact, that this analytical result would normally apply to regular elliptical dots, the authors argue, that the low field behavior should be similar for chaotic dots. This is presumably the case in our system. From the radii of the quantum dot wavefunctions, and from the width of the magnetoconductance dip (Fig. 7.14 b) we estimate the lower bound of the phase coherence length to be on the order of $a=120$ nm. Remnants of this effect are still visible in the open regime (Fig. 7.14 d), but weak localization is more prominent. From a fit to the
Figure 7.12:  (a) Schematic illustrating the measurement geometry. Left: two-terminal setup. The current I is measured as a function of bias voltage V. Right: four-terminal configuration. The voltage U is measured as a function of bias voltage V. The two-terminal current measurement and the four-terminal voltage measurement were done simultaneously at a series of topgate voltage settings. (b) Upper panel: Coulomb diamonds in the current signal measured in the two-terminal configuration from contact C to D as in Fig. 7.9. White regions correspond to absolute current values above 5 pA. Lower panel: numerical differentiation $\partial I/\partial V_{\text{bias}}$ of the raw data shown above. (c) Upper panel: Coulomb diamonds in the voltage signal measured in the four terminal geometry. Lower panel: numerical differentiation $\partial U/\partial V_{\text{bias}}$ of the raw data shown above.
7.3. EXPERIMENTAL FINDINGS

Figure 7.13: Upper two plots: current as a function of topgate voltage and magnetic field across diagonals 1 and 2 at a constant bias voltage of 50 µV. The Coulomb peaks (current maxima) shift as the magnetic field is swept perpendicular to the antidot lattice. Lower two plots: numerical differentiation of the data presented in the plots above. Similar features are highlighted with boxes, differing ones with ovals. The common features originate from quantum dots being traversed by the current across both diagonals, whereas differing features can be attributed to dots only participating in transport across one of the two diagonals.
Figure 7.14: Upper two plots: conductance as a function of topgate voltage and magnetic field in the Coulomb blockade regime (left) and the open or antidot regime (right). Lower two plots: magnetoconductance averaged in topgate voltage between −54 and −70 mV in steps of 0.02 mV in the Coulomb blockade regime (left) and between 0 and 30 mV in steps of 0.25 mV in the open regime (right).

The dip between ±5 mT with the relation (see section 2.4):

$$\delta \sigma_{xx}(B) = \frac{e^2}{h} \cdot \frac{2\pi}{3} \left( \frac{L^2}{\Phi_0} \right)^2 \cdot B^2$$

a phase coherence length of 300 nm was extracted for both diagonals. This indicates that the fraction of extended states increases towards higher topgate voltages. Reasons for the short phase coherence length, besides the strong localization of electrons to quantum dots within the individual unit cells, could include electron–electron interactions, charge flickering from nearby quantum dots or instabilities with respect to spontaneous polarization [135].

### 7.4 Summary

We have presented measurements on a multiply connected multi-terminal quantum dot network with tunable coupling. For strong coupling the system behaves like a classical random resistor network with superimposed quantum fluctuations until
charge quantization becomes important in the tunneling regime. Close to threshold, tunnel broadened resonances dominate over Coulomb blockaded regions which should lead to ‘Coulomb quenching’ but finite conductivity at low temperatures in ‘infinite’ systems. For weaker coupling, Coulomb blockade dominates resulting in the theoretically predicted [136, 137, 138] and experimentally observed [139, 140] insulating state for $T \to 0$ with current onset above a bias voltage threshold and hopping transport at elevated temperatures. These findings are compatible with transport properties observed in macroscopic samples consisting of metal and semiconductor colloids [96, 97, 98, 99, 101]. The scaling exponents are in reasonable agreement with random resistor network theory and the links nodes and blobs model of percolation, if the fraction of bonds, $P$, is taken to be proportional to $\Delta k_f$. Our measurements can also serve as an intuitive picture for the formation of the so-called ‘Coulomb gap’ [141], that opens around the Fermi energy as a function of electron localization and Coulomb interactions in granular or disordered materials.

Measurements of the magnetic field dependent shifts of the Coulomb peaks across both diagonals allow us to identify individual blobs in the incipient infinite cluster. Together with observations in a four-terminal configuration, this confirms our model of a multiply connected quantum dot network. A strong negative magnetoresistance around $B=0 \text{T}$ in the Coulomb blockade regime can be attributed to magnetic field dependent tunneling between individual quantum dots, and was found to be in good quantitative agreement with a theoretical model by Raikh and Glazman [134]. From this effect and from the width of the parabolic average magnetoconductance dip, we estimate the lower bound for the phase coherence length to be of the order of $a=120 \text{nm}$ in the Coulomb blockade regime. In the open lattice, standard weak localization behaviour is recovered with phase coherence lengths above $300 \text{nm}$.

More precise values for the critical exponents could be deduced by determining the exact number of open QPCs with scanning probe techniques, or by fabricating samples with individually tunable QPCs. Future efforts could also focus on addressing individual sites of the lattice for quantum information processing applications [94, 92].
Chapter 8

Artificial bandstructures

8.1 Introduction

It has been a long standing goal to induce an artificial band structure by laterally structuring a 2DES. This would make artificial crystals available with tailored electronic properties. One of the first to propose this approach was D. R. Hofstadter in connection with his prediction of a fractal band splitting known as the Hofstadter’s butterfly [38]. This effect is an outflow of the incommensurability between the translational symmetry of a two-dimensional square lattice and the rotational symmetry of the Landau levels induced by a perpendicular magnetic field. By mapping this problem onto the one-dimensional Harper-equation and solving numerically, the bandstructure shown in Fig. 2.12 is obtained. Other motivations are to simulate complex materials like the high Tc superconductors at larger length scales [95], or to have tunable band gaps for optical applications.

The two simplest cases for bandstructure formation are the tight-binding and weak-modulation limits. In the tight-binding limit, bands are formed by overlap between strongly localized wave functions belonging to individual atoms. The resulting bands thus retain many properties of the corresponding atomic states. In the weak-modulation limit, a free electron gas is subject to a weak periodic perturbation. As the potential amplitude is increased, band gaps open. When the potential amplitude becomes comparable to the band width, this approximation breaks down and no simple solution for the bandstructure is available. This situation is comparable to an antidot lattice.

Arrays with lattice constants close to the Fermi wavelength or below are desirable in order to make the typical energies of the band widths and band gaps large enough to be observable in experiment. In particular the following effects have to be taken into consideration:

- **Temperature broadening:** At temperatures $T > 0 \, K$ the fine structure in the density of states is smeared out due to thermal excitations in the lattice and in the leads. In a quantum dot the broadening of an individual Coulomb resonance is $3.5 k_B T$ for single and $4.35 k_B T$ for multi-level transport (see section 2.3.4).

- **Tunnel broadening:** Since AFM-defined arrays are finite - typically $20 \times 20$
in this thesis -, coupling to the leads has to be taken into account. The main influence from the leads is tunnel broadening, which is analogous to the natural line width or lifetime broadening in an atom, a direct consequence of the energy/time uncertainty relation. The longer the electron stays within the cavity, the less the energies of the individual states or the artificial bandstructure will be smeared out (Fig. 8.1 a). In a wave function picture this corresponds to the overlap of the states in the cavity with the states in the leads (Fig. 8.1 b). Three factors determine the electron dwell time: The conductance of the leads, the area of the cavity and the Fermi velocity. This leads to the relation [142]:

\[
\Gamma_{nat} = \frac{h}{8\pi^2e^2}G\Delta E
\]  

(8.1)

where the area and the Fermi velocity are contained in the mean quantum mechanical level spacing \(\Delta E\).

• **Tamm states:** The broken translational symmetry at the edges of the array also locally modifies the bandstructure leading to so-called Tamm-states [143]. It is therefore possible, that the transport properties probed by the leads do not accurately reflect the properties of the bulk of the array, even if the effect is relatively weak compared to real crystals.

In the following we search for experimental evidence for bandstructure formation in structures with periodic potential modulations induced by AFM-lithography. We start by considering the weak-modulation case in a sample with small oxide dot height. Then we discuss the possibility of band formation in the quantum dot network presented in chapter 8, corresponding to the tight-binding limit. Finally we investigate the intermediate antidot regime.
8.2 Measurements

8.2.1 Weak potential modulation

Figure 8.2 a) shows an AFM-micrograph of a 20 × 20 lattice of oxide dots with \( a = 80 \text{ nm} \) in a square cavity with QPCs in corners A to D. By applying voltages to the in-plane gates PG1 to 4, the conductance through every QPC can be tuned individually. The array is very homogeneous with an average dot height of about 5 nm (Fig. 8.2 b)), which translates into a potential modulation amplitude of \( \sim 5 \text{ meV} \). This is well below the Fermi energy of \( \sim 20 \text{ meV} \).

The idea of the sample was to test for bandstructure formation in the weak-modulation limit. Quantum point contacts were used to keep tunnel broadening smaller than the single-particle energy spacing in a unit cell, that determines the energy scale of the bands. However at the same time it was desirable to keep the coupling as strong as possible, so that the resistance measured across the cavity is noticeably influenced by the bandstructure induced by the lattice and not completely dominated by the QPCs.

Fig. 8.2 c) shows the resistance as a function of topgate voltage in different con-
figurations, while the plunger gate voltages for the QPCs were held constant. Slight similarities are seen e.g. in a common hump around 420 mV, but no clear signatures of an artificial bandstructure are evident. At a topgate voltage of \(+200\) mV there are about \(n_{\text{el}}^2 = 80\) electrons per unit cell with a single particle energy spacing of approximately \(\Delta E \approx 250\) \(\mu\text{eV}\). Thermal broadening is \(\Gamma_{\text{thermal}} \approx 8\) \(\mu\text{eV}\), and tunnel broadening is \(\Gamma_{\text{nat}} \approx 6\) \(\mu\text{eV}\). At a topgate voltage of \(500\) mV, tunnel broadening increases to \(\Gamma_{\text{nat}} \approx 20\) \(\mu\text{eV}\). Since the unpatterned 2DES goes insulating at electron densities below \(1.5 \times 10^{15}\) \(\text{m}^{-2}\), this can be taken as an indication for random background fluctuations with an amplitude of \(\sim 5\) \(\text{meV}\). It is therefore possible, that this random background is stronger than the periodic potential from the oxide dots.

As can be seen in Fig.8.2 d) the resistance is asymmetric with respect to B, which is incompatible with a pure two-terminal measurement between corners A and C. Instead there must be a four-terminal contribution resulting from the split current and voltage leads (not shown in Fig.8.2 a)) close to corners A and C. This in combination with the relatively large background potential fluctuations completely masks the band structure, despite the high quality of of the AFM-defined array. However, indications for an artificial band structure for weak modulation have been reported by Schlösser et al. [58] and Albrecht et al. [57] in high mobility wafers with modulated topgates.

### 8.2.2 The tight-binding limit

At very low electron densities, the constrictions between neighboring antidots enter the tunneling regime and the electron wave functions enclosed by the antidots become increasingly localized. This constitutes a smooth transition to a quantum dot lattice, where every quantum dot is tunnel coupled to its four nearest neighbors.
neighbors. The energy levels of the individual quantum dots combine to form a tight-binding bandstructure.

Figure 8.3 shows the current as a function of bias and topgate voltage across diagonal 1 and 2 in the a=120 nm sample (Fig. 7.9). Dashed vertical lines indicate the number of electrons in a unit cell, if the insulating antidots are not taken into account. Subtracting the insulating islands cuts the number of electrons approximately in half, but also on this scale, no clear signature for a bandstructure is evident. Instead, the Coulomb diamond size and hence the charging energies seem to follow a scaling law typical for percolation transitions.

This is not surprising, since this regime is very sensitive to changes in the tunnel coupling between, and size variations of the individual quantum dots. In addition, capacitive coupling between the individual dots and stray background charges could scramble the bandstructure.

8.2.3 Antidot arrays

Figure 8.4 a), plots the resistance as a function of topgate voltage between 0 and 30 mV across both diagonals of the a=120 nm antidot sample shown in Fig. 7.3 inset. In order to suppress the influence of UCFs, the resistance was averaged with respect to magnetic field between ±290 mT (a flux quantum through the unit cell) at every topgate setting in Fig. 8.4 b). Two maxima in the resistance are observed across both diagonals. The change in electron sheet density between the two peaks corresponds to the addition of ∼ 2.4 electrons to a unit cell, if the insulating antidots are not included. Dividing by two for spin degeneracy, gives a value of 1.2 electrons/unit cell, which reduces to about 1 if the insulating areas of the antidots are taken into account. This makes it tempting to identify the two resistance peaks as precursors for the formation of band gaps and an artificial band structure. However, theoretical calculations predict a complicated overlapping bandstructure for an open antidot lattice (see Fig. 2.11), so that this observation might seem somewhat surprising.

The magnetoresistance as a function of topgate voltage is shown in Figs. 8.4 c) and d). Differences across both diagonals stem from UCFs, that would mask a possible Hofstadter-type splitting of the ‘band’. The temperature broadening Γ_{thermal} ≈ 8 µeV and tunnel broadening Γ_{nat} ≈ 12 µeV are still smaller than the ‘band width’ of 110 µeV.

Very similar features are observed at topgate voltages between 100 and 250 mV (Fig. 8.5).

8.3 Summary

The electronic quality of our samples is probably limited by random potential fluctuations from Si donor atoms, growth defects and stray background charges. This is seen from the electron density at which the unpatterned 2DES pinches off and from changes from cooldown to cooldown also observed for individual quantum dots. Another problem is the growth mode of GaAs during the molecular beam
epitaxy process: Terraces form with a height variation of about ±1 nm on a length scale of a few hundred nanometers called the ‘orange peel effect’. This suggests that limitations are primarily imposed by wafer properties rather than by AFM-lithography.

Weak periodic potentials are probably completely drowned by random background fluctuations in our wafers and therefore no artificial bandstructure was observed. In the tight-binding limit the artificial potential landscape dominates, but because of the low electron densities, screening of defects and irregularities is reduced as well. In addition the tight-binding bandstructure is very sensitive to inhomogeneities, since it is dominated by individual quantum dots in the tunneling regime, rather than by the Fourier components of the entire lattice, like in

![Graphs showing resistance vs. topgate voltage and magnetic field](image)

Figure 8.4: (a) Resistance as a function of topgate voltage across both diagonals in the a=120 nm sample at B = 0 T and T=90 mK. Dashed lines mark resistance maxima, that are present across both diagonals. The difference in electron number was calculated for a unit cell without the insulating antidots. (b) Same as a, but after averaging every resistance value with respect to magnetic field between ±290 mT, which is a flux quantum through a unit cell. (c),(d) Resistance as a function of topgate voltage and magnetic field across diagonal 1 and 2.
the open case. Therefore a percolation-type description as presented in chapter 8 seems more appropriate.

This makes the intermediate antidot regime look most promising as it combines a strong potential modulation with an open lattice. Indeed we might be able to see indications for an artificial band structure in the resistance of an antidot lattice with a lattice constant of 120 nm. However, an exact interpretation is difficult because of the complicated overlapping bandstructure predicted for antidot lattices. Superimposed fluctuations that varied when measuring in different configurations are attributed to UCFs from random scatters. This suggests an interpretation in terms of an artificial band structure with superimposed UCFs.

Figure 8.5: (a) Resistance as a function of topgate voltage across both diagonals in the a=120 nm sample at B = 0 T and T=90 mK. Dashed lines mark resistance maxima, that are present across both diagonals. The difference in electron number was calculated for a unit cell with unstructured 2DES. (b) Same as a, but after averaging every resistance value with respect to magnetic field between 0 and 290 mT, which is a flux quantum through a unit cell. (c),(d) Resistance as a function of topgate voltage and magnetic field across diagonal 1 and 2.
Chapter 9

Conclusions and outlook

Antidot arrays defined by AFM lithography on Ga(Al)As heterostructures have an electronic quality comparable to the best lattices fabricated by other techniques. To the best of our knowledge we were able to achieve the smallest lattice constants ever reported for GaAs heterostructures. The finite size of the arrays, typically 20×20 antidots, seems to have no significant influence on most of the magneto-transport properties. As in earlier reports, the commensurability peaks are well described by classical billiard theories, including the shift of the peak around 4 antidots to lower magnetic fields. Aharonov-Bohm type oscillations were detected in samples with an antidot spacing of a=300 nm and a=150 nm. The energetic resolution in the 150 nm lattice is high enough to clearly identify the period as corresponding to a circular orbit with a diameter of a=150 nm, rather than a flux quantum through the square unit cell. This supports the picture of a modulation in the density of states arising from periodic orbits and makes an interpretation in terms of a Hofstadter–type beating appear unlikely. When varying the electron mobility at constant electron sheet density, we found the most pronounced Aharonov-Bohm-type oscillations at the lowest mobilities. According to Uryu and Ando [44] this can be interpreted as a washing out of more complex band structure effects in the presence of random background scatterers. Aharonov–Altschuler-Spivak type oscillations around B=0 T were present in some traces in the 150 nm and 300 nm sample, but were more sensitive to changes in parameters than the Aharonov-Bohm-type features. A new peak splitting effect superimposed on the main commensurability maximum was observed while sweeping the top- and back-gate at constant high electron densities. This could be a spin related phenomenon connected to a change in symmetry of the z–component of the 2DES wavefunction. More experiments are needed to clarify this interesting issue.

The transition from the conducting to the insulating phase as a function of electron sheet density was studied in a 20 × 20 antidot array with a lattice constant of a=120 nm. In theory there should be a smooth transition from an antidot to a quantum dot array as the electron density is reduced and the constrictions between neighboring antidots enter the tunneling regime. The area enclosed by four antidots can then be viewed as a quantum dot or artificial atom with four terminals connecting it to its nearest neighbor quantum dots. In practice, however, small inhomogeneities inherent to the fabrication process and stray background charges
lead to a percolative transition. For strong coupling above the percolation threshold, the system behaves like a classical random resistor network with superimposed quantum fluctuations. When the lattice enters the tunneling regime charge quantization becomes important and Coulomb blockade is observed. Clusters of quantum dots form collectively blockaded regions, that shrink as the electron density is reduced, leading to on average increasing charging energies and well defined Coulomb diamonds. These findings can be interpreted in terms of the links, nodes and blobs model of percolation proposed by Stanly and Coniglio, if the blobs are identified with the Coulomb blockaded regions. The scaling exponent for the conductivity on the open side and the average charging energy on the Coulomb blockaded side are in reasonable agreement with theoretical calculations, if the fraction of open QPCs is assumed to be proportional to the Fermi wavevector. Very similar observations have been made on macroscopic samples of tunnel coupled semiconductor or metal colloids [101, 100, 97, 98, 99], where the microscopic charging resonances are averaged out. Our measurements can also serve as an intuitive picture for the formation of the so-called Coulomb gap [141], that opens around the Fermi energy as a function of electron localization and Coulomb interactions. A strong increase in average Coulomb blockade peak height with increasing magnetic field around $B=0\,\text{T}$ can be attributed to magnetic field dependent tunneling between individual quantum dots. This delocalization effect is in good quantitative agreement with a theoretical model proposed by Raikh and Glazman [134].

Indications for band structure formation were investigated in the weak-modulation, tight-binding and antidot regime. In the weak-modulation limit no indications for an artificial bandstructure were detected despite a very short lattice constant of 80 nm and high lithographic quality of the array. The potential amplitude, that we estimated to be about 5 meV from a Poisson–Schrödinger simulation [77], is probably completely masked by random background fluctuations having the same order of magnitude. From this we conclude that wafer homogeneity and possibly even properties inherent to the Ga(Al)As material system are the limiting factors, rather than the precision of AFM–lithography. The tight-binding limit was studied in the 120 nm sample. Under these conditions the artificial potential landscape dominates, but because of the low electron densities, screening of defects is reduced as well. In addition the tight-binding bandstructure is very sensitive to inhomogeneities, since it is dominated by the tunnel coupling between individual quantum dots, rather than by the Fourier components of the entire lattice. Therefore a percolation-type description as presented above seems more appropriate. At high electron densities the 120 nm sample constitutes an antidot lattice. Possible signatures for an artificial bandstructure were seen in the electron sheet density dependent resistance across both diagonals of the array. However an exact interpretation is difficult owing to the complex overlapping band structures predicted for antidot lattices. A lead broadened band structure with superimposed universal conductance fluctuations is probably an adequate description.

Future efforts could concentrate on further reducing the lattice period, as well as increasing lattice symmetry and eliminating the random background potential fluctuations. Being able to address and tune individual lattice sites could be of interest for basic explorations in quantum information processing. A fascinat-
ing objective would also be to realize a Kondo–lattice, based on the Kondo–effect recently reported for a single antidot in a perpendicular magnetic field [144]. However this can only be achieved if the fluctuation in electron number in the edge states bound to the different antidots in the lattice is smaller than one. Figure 9.1 shows water surface waves in regular and disordered arrangements of scatterers, that can serve as a possible model for electron wavefunctions in antidot arrays. Local probe studies of antidot lattices could yield experimental insight into this interesting point and might hold surprises relating to the microscopic transport processes and exact nature of wavefunctions in the presence of magnetic fields.

![Figure 9.1: Water surface waves in different arrangements of circular scatterers, from Lindelof et al. [145]](image)

As a ‘by-product’, a new type of backgate insulation as an alternative to low-temperature grown GaAs was investigated. The scheme relies on layers of nanometer sized ErAs islands embedded in GaAs between a conducting backgate layer (Si:GaAs) and the 2DES. Overlapping Schottky barriers surrounding the ErAs islands insulate the 2DES with respect to the backgate up to break down voltages of $\pm 7 \text{ V}$ at 1.7 K. From thermal activation measurements an effective Schottky barrier height across the island system of about 0.6 eV was extracted. If the backgate was set above or below certain threshold voltages, persistent charging of the ErAs islands was observed, which allowed us to determine the Schottky barrier height onto the islands of about 0.6 – 1.0 meV. A strong persistent Schottky barrier reduction after illumination was observed in one wafer, but was absent in another, which indicates the high sensitivity of this effect to the microscopic morphology of the islands. ErAs island layers embedded in GaAs could be of interest for a number of applications especially if local charging, e.g. by a set of microscopic topgates, is feasible.

This thesis shows, that semiconductor nanostructures and antidot lattices in particular, still hold a wealth of interesting physics. Improving sample quality, as well as applying new methods like microwave irradiation or scanning probe techniques can be expected to yield exciting new results.
## Appendix A

### List of Samples

<table>
<thead>
<tr>
<th>name</th>
<th>afm picture</th>
<th>structure</th>
<th>AFM-parameters</th>
<th>measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>square cavity</td>
<td><img src="image1.png" alt="Image" /></td>
<td>$3 \times 3 \mu m$ cavity, 34nm HS with topgate</td>
<td>chapter 5</td>
<td></td>
</tr>
<tr>
<td>rectangular cavity</td>
<td><img src="image2.png" alt="Image" /></td>
<td>$3 \times 6 \mu m$ cavity, 34nm HS with topgate</td>
<td>chapter 5</td>
<td></td>
</tr>
<tr>
<td>300nm</td>
<td><img src="image3.png" alt="Image" /></td>
<td>$20 \times 20$ array, $a=300$ nm 34nm HS with topgate</td>
<td>$27^\circ C$, 40% $V_{ox} = -18$ V / 3 s setpoint = 1.2 → 0.09 V antidot height: 9 nm</td>
<td>chapter 6</td>
</tr>
<tr>
<td>–</td>
<td><img src="image4.png" alt="Image" /></td>
<td>$15 \times 15$ array, $a=300$ nm 34nm HS with topgate and backgate</td>
<td>$27^\circ C$, 38% $V_{ox}= -26$ V / 3 s setpoint = 1.3 → 0.08 V antidot height: 14 nm</td>
<td>–</td>
</tr>
<tr>
<td>name</td>
<td>afm picture</td>
<td>structure</td>
<td>AFM-parameters</td>
<td>measurements</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>-----------</td>
<td>----------------</td>
<td>-------------</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>15 × 15 array, a=300 nm 34 nm HS with topgate and backgate</td>
<td>26.9 °C, 40.5 % aq $V_{ox} = -20 , V / 3 , s$ setpoint = 1.3→0.1 V antidot height: 19 nm</td>
<td>–</td>
</tr>
<tr>
<td>150nm</td>
<td>–</td>
<td>20 × 20 array, a=150 nm 34 nm HS with topgate and backgate</td>
<td>26.6 °C, 41 % aq $V_{ox} = -24 , V / 3 , s$ setpoint = 2.2→0.2 V antidot height: 15 nm</td>
<td>chapter 6</td>
</tr>
<tr>
<td>–</td>
<td>–</td>
<td>20 × 20 array, a=150 nm 34 nm HS with topgate</td>
<td>25.7 °C, 39 % aq $V_{ox} = -17 , V / 3 , s$ setpoint = 1.1→0.08 V antidot height: 10 nm</td>
<td>–</td>
</tr>
<tr>
<td>120nm</td>
<td>–</td>
<td>20 × 20 array, a=120 nm 34 nm HS with topgate</td>
<td>27 °C, 43 % aq $V_{ox} = -18 , V / 3 , s$ setpoint = 2.3→0.2 V antidot height: 14 nm</td>
<td>chapter 7,8</td>
</tr>
<tr>
<td>80nm</td>
<td>–</td>
<td>20 × 20 array, a=80 nm 34 nm HS with topgate</td>
<td>27 °C, 38.6% aq $V_{ox} = -18 , V / 3 , s$ setpoint = 2.8→0.2 V antidot height: 5 nm</td>
<td>chapter 8</td>
</tr>
</tbody>
</table>

Table 9.1: Structures fabricated in the course of this thesis. The AFM-parameters refer to the antidot lattice. Oxide lines were typically written with an oxidation voltage of -20 to -40 V with the same set-points and in the same atmosphere (sometimes with higher humidity) and with a writing speed of 100 to 200 nm/s.
Appendix B

Sample processing

Positive resist (mesa etch)

<table>
<thead>
<tr>
<th>Process</th>
<th>Description</th>
</tr>
</thead>
</table>
| 1. wafer cleaning | • aceton 1 min ultrasound at 90% power  
                     • isopropanol 1 min ultrasound at 90% power  
                     • dry blow with $N_2$  
                     • heat 2 min at 115°C  
                     • HCL-dip for 8 sec |
| 2. lithography       | • spin on one drop of photoresist (Shipley S1805) at 5000 rpm  
                          • soft bake for 2 min at 115°C  
                          • exposure: 3.5 s  
                          • development: Shipley M319 for 45 s  
                          • 3 min rinse in H$_2$O, dry blow with $N_2$  
                          • 3 min oxygen plasma etch |
| 3. mesa etch          | • H$_2$O:H$_2$SO$_4$:H$_2$O$_2$, 100:3:1 etch rate: $1 \text{s} \approx 1 \text{nm}$  
                          • H$_2$O rinse, dry blow with $N_2$  
                          • aceton 1 min ultrasound at 90% power  
                          • isopropanol 1 min ultrasound at 90% power |
### Negative resist (Ohmic contacts)

<table>
<thead>
<tr>
<th>Process</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. wafer cleaning</td>
<td>- aceton 1 min ultrasound at 90% power</td>
</tr>
<tr>
<td></td>
<td>- isopropanol 1 min ultrasound at 90% power</td>
</tr>
<tr>
<td></td>
<td>- dry blow with $N_2$</td>
</tr>
<tr>
<td></td>
<td>- heat 2 min at 115°C</td>
</tr>
<tr>
<td>2. lithography</td>
<td>- spin on one drop of photoresist (Hoechst AZ5214E) at 5000 rpm</td>
</tr>
<tr>
<td></td>
<td>- soft bake for 2 min at 90°C</td>
</tr>
<tr>
<td></td>
<td>- exposure: 3 s (mask with structure)</td>
</tr>
<tr>
<td></td>
<td>- soft bake for 2 min at 115°C</td>
</tr>
<tr>
<td></td>
<td>- exposure: 10 s through empty mask</td>
</tr>
<tr>
<td></td>
<td>- development: Microposit $\sim$ 60 s</td>
</tr>
<tr>
<td></td>
<td>- 1 min rinse in $H_2O$, dry blow with $N_2$</td>
</tr>
<tr>
<td></td>
<td>- 3 min oxygen plasma etch</td>
</tr>
<tr>
<td>3. metal evaporation</td>
<td>- evaporate metal layer sequence for Ohmic contacts onto developed resist</td>
</tr>
<tr>
<td></td>
<td>- lift off in (hot) acetone</td>
</tr>
<tr>
<td></td>
<td>- aceton 5-10 s ultrasound at 10% power</td>
</tr>
<tr>
<td></td>
<td>- isopropanol 5-10 s ultrasound at 10% power</td>
</tr>
<tr>
<td></td>
<td>- dry blow with $N_2$</td>
</tr>
<tr>
<td></td>
<td>- annealing for 15-60 s at 430-450°C in 0.5 bar forming gas</td>
</tr>
</tbody>
</table>
Topgates and Ohmic contacts

<table>
<thead>
<tr>
<th>layer</th>
<th>substrate</th>
<th>Ge</th>
<th>Au</th>
<th>Ge</th>
<th>Au</th>
<th>Ni</th>
<th>Au</th>
</tr>
</thead>
<tbody>
<tr>
<td>thickness (nm)</td>
<td>–</td>
<td>18</td>
<td>50</td>
<td>18</td>
<td>50</td>
<td>40</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 9.2: Metal layer sequence for Ohmic contacts

<table>
<thead>
<tr>
<th>layer</th>
<th>substrate</th>
<th>Ti</th>
<th>Au</th>
</tr>
</thead>
<tbody>
<tr>
<td>thickness (nm)</td>
<td>–</td>
<td>10</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 9.3: Metal layer topgates
Bibliography


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Journal Publications


• A. Dorn, E. Bieri, T. Ihn, K. Ensslin, D. Driscoll and A. C. Gossard *Interplay between random background scatterers and a periodic potential modulation in an antidot lattice*, to be submitted.
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