Optical Zone and Single Pulse Centration in Corneal Refractive Laser Surgery

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Dedicated to my parents, Rita and Peter Büeler-Pugneth
Abstract

Many factors limit the outcome of corneal refractive treatments. Technical deficiencies as well as undesired physiological effects reduce the predictability of the correction. Exact knowledge of how the different parameters interact is necessary to significantly enhance the predictability of the procedure. Decentration is considered to be among the most important disturbance variables in the treatment of higher-order optical irregularities with scanning beam laser systems.

Natural eye movements and improper fixation can not be avoided during the ablation process. Two types of alignment errors can be distinguished in scanning beam refractive surgery. Static alignment errors are characterized by constant translations and rotations of the optical zone relative to the eye. They result from inaccuracies in the initial centration of the correction. The second type of alignment error refers to dislocations of single laser pulses from their ideal overlap positions in the corneal bed. The main reason for such dislocations are intra-operative eye movements. Active eye tracking helps to avoid gross dislocations by registering and compensating ocular movements. The process of measuring the eye position and adjusting the laser deflection mirrors requires some time, which can lead to incomplete compensation of the eye movements, leaving so called movement artefacts. The final outcome of the procedure is defined by the interaction between undesired factors such as movement artefacts, and a number of defined system parameters such as the laser spot size or the type and magnitude of the irregularity to be treated. The knowledge of how these parameters interact is important for both the future optimization of design parameters as well as the definition of tolerance limits for undesired factors (static zone translation, eye tracker latency).
The main goal of this thesis was to address various aspects of alignment in refractive surgery and to evaluate the influence of alignment errors on the final outcome of the treatment. It was further intended to elaborate methodologies for future parameter optimization by providing an insight into the interaction of different laser and treatment parameters. Beside static centration errors a special focus was put on the effect of intra-operative eye movements on the correction of different types of higher-order irregularities. Computer simulations of the scanning beam ablation process were performed to investigate the effect of incomplete movement compensation due to eye tracker latency.

The question on the ideal ocular centration axis for refractive procedures was discussed. The line of sight was shown to have a number of advantages over other ocular reference axes.

The effect of optical zone translation and rotation on image quality was studied theoretically and based on measured wavefront data by evaluating the post-operative RMS wavefront error and the Strehl intensity ratio. The results were published in peer-reviewed journals [1,2]. Computer simulations indicated tolerable zone translations of up to 200 microns and in-plane rotations up to 5 degrees to achieve the level of the best 10% of eyes in an uncorrected population (7-mm pupil). A translatory precision of 70 microns or better and rotations smaller than 1 degree were found to be tolerable to achieve a nearly diffraction limited optical performance in 95% of the measured normal eyes (7-mm pupil).

The measurement of the eye position and the subsequent adjustment of the laser beam requires some time. As the eye can move again during this period, latency must be kept as small as possible to reduce movement artefacts. A study was therefore focused on the effect of laser pulse displacement due to eye tracker latency. The results were published in a peer-reviewed journal [3]. Reduction of spot diameter was shown to make the correction more susceptible to eye movement induced error. Changing the laser spot diameter from 1000 to 250 microns was found to be beneficial only when eye movement is neutralized with a tracking system with a latency below 4 ms. Analyses of this type will be of special importance in the future, as current developments towards solid state and femtosecond lasers in refractive surgery consider spot sizes significantly below 0.5 mm.
The requirements on the accuracy of spot placement might even further increase when treatment of small corneal irregularities such as steep central islands is attempted. The methodology presented in this thesis will enable appropriate parameter optimization for these cases.

Furthermore, the parallax error associated with localizing corneal positions by tracking the subjacent entrance pupil center was quantified. The results were published in a peer-reviewed journal [4]. The tracking error can amount to 30% (or more for eye trackers mounted closer than 500 mm to the eye) of the detected lateral shift. Thus, if the eye tracker registers a lateral shift of the entrance pupil of 0.2 mm away from the tracking reference axis, the point of interest located on the cornea would essentially be 0.26 mm away from this reference axis. A laser pulse fired at that moment would be systematically displaced by 60 microns. Compensation of the parallax error can be considered mandatory in the future as correction of corneal irregularities beyond 6th order is attempted. A method for conservative compensation of this effect was presented. More accurate compensation could be achieved by using custom data of the patient's eye such as ocular length and anterior chamber depth.

Precise static and dynamic alignment was shown to be a basic requirement for the success of measurement and treatment of optical aberrations. Consideration of alignment aspects in laser system design will gain further importance in the future as the spatial dimensions of the structures to be corrected become smaller.


Zusammenfassung


Bedeutung.


Im Weiteren wurde die Frage nach der idealen Referenzachse für refraktive Behandlungen diskutiert. Es wurde gezeigt, dass die Line-of-Sight, welche das vom Patientenauge fixierte Objekt mit der Mitte der Eintrittspupille verbindet, zahlreiche Vorteile gegenüber anderen Referenzachsen am Auge aufweist.

Der Effekt von translatorischen und rotatorischen Fehlausrichtungen der Behandlungszone auf die post-operative Bildqualität wurde theoretisch und basierend auf gemessenen Wellenfrontdaten durch Auswertung des RMS Wellenfront-Fehlers und der retinalen Strehl-Ratio untersucht. Die Resultate wurden in peer-reviewed Journals publiziert [1,2]. Computersimulationen deuteten auf tolerierbare Translationen in der Höhe von 200 Mikrometern und Rotationen um die Längsachse bis zu 5 Grad hin, wenn die Güte der besten 10% aller Augen in einer unkorrigierten Population erreicht werden soll (7mm Pupillendurchmesser). Eine translatorische Präzision von mindestens 70 Mikrometern und eine rotatorische Genauigkeit von 1 Grad oder besser muss gewährleistet sein, um eine näherungsweise beugungs begrenzte Optik bei 95% der vermessen normalen Augen zu erzeugen (7mm Pupillendurchmesser).

Die intra-operative Messung der Augenposition und die entsprechende Nachführung des Laserstrahls durch Umlenkspiegel erfordern Zeit. Da sich das Auge in dieser Zeitspanne weiterbewegen kann, muss die Latenz so klein als möglich gehalten werden, um die Bewegungsartefakte auf ein Minimum zu reduzieren. Eine Untersuchung konzentrierte sich deshalb auf den Effekt von dezentrierten Laserpulsen.


Es wurde gezeigt, dass präzise statische und dynamische Zentrierung eine Grundvoraussetzung für den Erfolg von Messung und Behandlung von optischen Aber...
rationen darstellt. Die Berücksichtigung von Zentrierungsaspekten in der refraktiven Chirurgie wird in Zukunft an Bedeutung gewinnen, wenn die räumlichen Ausdehnungen der zu korrigierenden Hornhautirregularitäten kleiner werden.


Chapter 1

Motivation

The term "corneal refractive laser surgery" describes all the procedures to surgically reshape the cornea by means of a laser to improve the optical quality of the eye. Three basic classifications can be introduced to structure modern refractive laser surgery:

A Surgical procedures: PRK / LASIK
B Laser technology: Broad beam / Scanning beam
C Type of correction: Classical / Customized

A Surgical procedures
The PRK procedure which was introduced in the late 1980s is characterized by the fact that laser radiation is directly applied to the corneal surface after removal of the epithelial layer. It was recognized that due to the irreversible destruction of the Bowman's layer higher corrections were associated with increased epithelial regrowth reducing the predictability of the treatment. Furthermore, the procedure is causing discomfort to the patient due to central corneal abrasion leading to tearing, photophobia and blurred vision. This is why in the early 1990s a new procedure, the LASIK, was introduced. Prior to laser irradiation a thin corneal flap is cut with a microkeratome and lifted to expose the stromal bed. Reshaping of the cornea is done in the stromal bed and finally the flap is replaced to promote healing.
B Laser technology

Early treatments were performed with so called broad beam lasers. A static laser beam of large diameter (typically 6.0 mm) was applied to the cornea. By placing expanding concentric circular or elliptical apertures into the optical path laser pulses of different diameters were produced enabling ablation of step-like spherical and cylindrical shapes from the cornea. As the maintenance of a homogeneous energy distribution over large diameter pulses proved to be difficult treatment zones of broad beam lasers were limited. This is the main reason why in the late 1990s so called scanning or flying spot lasers were introduced. These systems work with a small laser beam of currently 1.0 mm in diameter which moves around on the cornea. A large number of single laser pulses, mostly with Gaussian energy distribution, is overlapped to ablate the desired structure. No technical limitations confine the diameter of the optical zone.

C Type of correction

Classical corrections as performed since the late 1980s are meant to correct basic optical errors such as sphere and cylinder. The corresponding ablation profiles are basic in shape and can be ablated using either broad or scanning beam technology. The ablation depths are in the order of 20 to 200 microns and the profiles can be expressed by means of 2nd order polynomials. In the late 1990s first attempts were made to perform so called customized corrections of finer and more complicated irregularities featuring ablation depths in the order of 1 to 10 microns. These irregularities which can be measured with corneal topographers or wavefront sensors are often mathematically expressed by means of higher-order polynomials such as Zernike or Taylor expansions.

Due to fine nature of the structures ablation can only be done using scanning beam technology. However, the first generation scanning beam lasers that are currently in use were not originally designed to perform customized treatments. They were developed for classical corrections of sphere and cylinder offering the advantage of larger treatment zones over broad beam lasers. The clinical usefulness of treating higher-order irregularities was recognized for two reasons. On the one hand real clinical indications can exist for such a treatment mostly in eyes that have undergone certain surgical procedures such as corneal transplantation. Such eyes usually suffer from very high irregularities causing visually disabling
effects. Eyes with decentered ablation zones following refractive surgery are another example. On the other hand it was recognized that customized treatments have the potential to become a very important marketing tool for both the industry and the clinicians. It was mainly for the second reason that the idea of customized corrections was pushed forward into the clinics in a very fast way while the necessary scanning beam technology was only adopted and not adjusted to the new requirements of customized treatments. Even more, the exact requirements towards technology were and are still not completely known. This is reflected by the fact that wavefront-guided surgery to treat "normal" irregularities got FDA-approved only in late 2002, whereas topography-guided surgery to treat "high" irregularities with real clinical indications is not yet approved.

**Centration in refraction laser surgery**

Among the many unknowns in customized refractive surgery with scanning beam lasers are the requirements toward centration or alignment. The term centration deals with positioning relative to an intercept or point, whereas alignment relates to orientation with an axis. Yet, in this thesis the term centration will be used in the same way as the term alignment. Centration is considered one of the most important issues in refractive surgery. In order to understand the meaning and the aspects of centration within the process of customized refractive surgery Fig. 1.1 is meant to provide a closer insight.

The process starts with a measurement device that is used to assess the eye's irregularities. The irregularities are passed on to an ablation profile generator that converts them into an appropriate ablation map for the laser. The ablation map is passed on to a shot list generator that calculates the number and the locations of the laser pulses necessary to ablate the profile. Subsequently a laser source emits single pulses to a scanner device that redirects them to their ideal incident positions on the eye. For that purpose the scanner device receives information on the ideal shot coordinates from the shot list generator. The now directed pulses are reshaping the corneal surface. What is finally of interest in any process is the final output which in this case is a measure of visual performance. By comparing the post-operative with the pre-operative visual performance the success of the process can be rated.
Figure 1.1: Process diagram of scanning spot laser surgery. Each subsystem (including the eye) can add to the overall displacement of single laser pulses on the cornea. The output response of each subsystem is determined by the input signal and the chosen design parameters, but also by disturbance effects such as noise and design deficiencies (dashed arrows).

Unfortunately as in any technical system the final output is not only determined by the input signal and the chosen design parameters, but also by undesired effects (disturbance variables) such as noise and design deficiencies (dashed arrows). Misalignment or deceleration are considered to be among the most important disturbance variables in scanning spot refractive surgery. They can either occur as a constant relative misalignment of the eye relative to the scanner device. In this case the entire ablation profile is decentered relative to the ideal location. Additionally, such a static misalignment can be superimposed by displacements of single laser pulses in the corneal bed. As any ablation profile is approximated by overlapping several hundreds or thousands of laser pulses displacement can lead to a bad approximation of the profile. Single pulse displacements can have various
origins as the following analysis should point out. Like any technical system the scanner device has a limited accuracy. The question arises if this accuracy is good enough for the specific application. Intra-operative eye movement by the patient is probably the most important reason for single pulse displacements. During the treatment the patient is not anaesthetised. He is told to fixate a light under the laser during the 30 to 60 seconds of the procedure. Voluntary fixation, however, does not suppress ocular micro-movements such as tremor and drifts. Video eye-tracking is meant to compensate for these movements. A video eye tracker is supplied with a video image of the moving eye. The subsequent determination of the translatory and rotatory movements can be impaired both by the parallax error (due to eccentric viewing of the pupil) and by the limited resolution of the image. The position correction values are passed on to the scanner device where they are merged with the current shot positions from the shot list calculator. The entire process of acquiring a video image, image transfer, image processing and scanning mirror adjustment requires some time. Thus, the quality of the eye movement compensation further depends on the time lag (latency) from the image acquisition in the eye tracker to the corresponding mirror adjustment in the scanner.

Classification
The various aspects of misalignment can be classified in two basic groups (Tab. 1.1).

Static misalignment of the entire optical zone can result from a reference shift between measurement and treatment or an imprecise calibration of the beam scanner relative to the optical axis of the system. The question on the ideal reference axis for refractive treatments will be addressed in Chapter 2. Consistency of the reference axes between measurement and treatment is of utmost importance in the treatment of higher-order irregularities as the amplitude of the measured structures does not only depend on the optics of the eye but also on the chosen reference axis. Chapter 3 will focus on the effect of static translation and rotation of the optical zone.

Dynamic misalignment refers to the positioning of single laser pulses in the corneal bed. The limited precision of the scanner device as well as intra-operative eye
Table 1.1: Factors impairing the accuracy of alignment can be divided into two basic groups: static misalignment of the entire optical zone, and dynamic misalignment of single laser pulses during the treatment.

movements are the reasons for misalignments of this type. Active eye tracking is intended to compensate for eye movements. Incomplete compensation can occur due to the limited resolution of the tracker, geometrical parallax and latency. Chapter 4 will address the problems of latency and geometrical parallax in detail.

**Interaction of parameters**
The quality of the final output response of the system, i.e. the visual performance of the treated eye, is not only impaired by the magnitude of the static and dynamic misalignment alone. Only the interaction of these undesired factors with a number of chosen system parameters such as the laser spot size and peak fluence or the type and amplitude of the irregularity to be treated finally determines up to what extent the outcome is affected (Fig. 1.2).

It can be assumed for instance, that the same amount of misalignment may be tolerable when correcting gross irregularities, but can be unacceptable when treating finer structures. The exact knowledge of this interaction is important for both the optimization of design parameters as well as the definition of tolerance limits for undesired factors such as static zone translation, or eye tracker latency. So far, no comprehensive study has been published that dealt with those
Figure 1.2: The final output response of the system, i.e. the visual performance of the treated eye, can be impaired by the interaction of undesired factors such as static and dynamic misalignment with a number of chosen system parameters such as the laser spot size or the type of aberration to be treated.

aspects. This thesis is intended to elaborate a methodology for future parameter optimization. It should further provide an insight into the complex interaction of the mentioned parameters.

Worksteps of this thesis
The different worksteps that were performed for this thesis will be shortly pointed out in the following. A main part of the project dealt with the development of methodologies and software programs on the basis of Matlab (Version 6.5.0., The Mathworks Inc.) for the simulation of refractive corrections. No commercially available software was used for any of the simulations. Tools for the following tasks were developed: Programs for 3-dimensional simulation of the scanning spot ablation process including eye movements, eye-tracker latency and other disturbance effects; programs for 3-dimensional ray-tracing through analytical off-axis eye models after simulated refractive corrections; methodology and software for the simulation of post-operative corneal smoothing; programs for data evaluation, optical analysis and parameter optimization. Another part of the project consisted of acquiring measurement data from subjects, mainly measuring wavefront aberrations. Finally, different simulation studies were performed
using the afore mentioned software tools. The results were published in four peer-reviewed publications [1-4] that will be presented in detail in this thesis.


Chapter 2

Centration axes

2.1 Axes of the eye

Corneal surgical procedures require proper centration on the cornea, as decenteration from the reference axis has the potential to introduce new types of optical aberrations (see Chapter 3). Centration, however, is not a simple task due to the lack of direct target points on the corneal surface.

A number of ocular axes can be defined to describe the optical properties of the eye. The points of intersection of these axes with the corneal anterior surface are often used for centering ocular surgery procedures.

Most technical optical systems are rotationally symmetric about one line, the optical axis. The centres of curvature of each refracting or reflecting surface lie on one line. Unfortunately, the human eye is not a centered optical system and does not contain a true optical axis, as the cornea and the lens are slightly decentered and tilted relative to each other. The concept of optical axis can be applied to the eye by defining the optical axis as the line of "best fit" through the centers of curvature of the "best fit" spheres to each surface. However, even if the optics would be perfectly aligned, the fact that the fovea is positioned temporally of the optical axis still renders the eye a non-rotationally symmetric optical system.

Assuming an eye with nearly centered optical elements, the optical axis (OA) passes through the centres of curvature of the refracting surfaces as well as through the corneal vertex V (Fig. 2.1) [1]. The line of sight (LOS) is the
2.1 Axes of the eye

Figure 2.1: Axes of the eye: optical axis (OA), line of sight (LOS), visual axis (VA), line of the coaxially sighted corneal reflex (LCCR), pupillary axis (PA). Defined points in the eye: corneal vertex (V), entrance pupil center (EPC), nodal points (N,N'), corneal center of curvature (CC)

The chief ray joining the fixation point and the center of the entrance pupil (EPC). It is considered the most important axis for describing visual function including refraction procedures, as it defines the center of the light bundle entering the eye. Unfortunately, this axis is not fixed, as the pupil center shifts with changing pupil size [11, 12, 13]. The visual axis (VA) is the line joining the fixation target and the foveal image through the nodal points (N,N'). It would be a convenient reference point for visual functions, as it does not depend on the pupil size. However, its exact location is difficult to determine in a clinical environment. The pupillary axis (PA) is perpendicular to the cornea and passes through the center of the entrance pupil (EPC). It is mainly used as an objective measure to judge the amount of eccentric fixation, where a retinal point other than the center of the fovea is used for fixation. The line of the coaxially sighted corneal reflex (LCCR) joins the fixation target and the center of curvature of the anterior corneal surface (CC) and is thus normal to the cornea.

The angle between the visual axis and the optical axis is referred to as the angle $\alpha$. It is positive if the visual axis is on the nasal side of the optical axis in object space. The mean value in an emmetropic eye is +5 degrees in the horizontal
plane, and 2-3 degrees downwards in the vertical plane.

2.2 Centration of corneal refractive procedures

2.2.1 Corneal reference points in dependance of refractive error

The above axes intersect the cornea at different locations. Current discussions on the ideal corneal centration point focus on either the corneal intersect of the line of sight LOS (called the corneal sighting center by Mandell, 1995), the corneal intersect of the visual axis VA (called the ophthalmometric pole by Le Grand and ElHage, 1980), or the coaxially sighted corneal reflex CR.

*The eye as a centered optical system*

If the eye were a centered optical system with the fovea located on the optical axis, the above axes would coincide and corneal ablation had to be centered on the corneal apex for optimal correction. Fig. 2.2 (left) shows the example of a well-centered Navarro schematic eye with 10 Diopters of axial myopia. Two-dimensional ray tracing was performed using a custom software in Matlab. The Navarro wide angle schematic eye is well suited for ray tracing applications beyond the paraxial region [2]. A spherical retina of -12 mm radius of curvature was added according to Escudero-Sanz and Navarro [3]. Fig. 2.2 right shows the eye after a simulated myopic correction centered on the corneal apex. Subsequent ray-tracing reveals a perfect post-operative focus at the intersect of the optical axis with the retina.

*The eye as a non-centered optical system*

Fig. 2.3 shows a modified version of the Navarro schematic eye with the fovea located off-axis by an angle \( \alpha = 5 \) degrees. Ray tracing through the emmetropic eye for a focus at infinity reveals some amounts of cylinder and coma due to the off-axis position of the fovea (detail below). It has to be assumed, that in a real eye a significant portion of the cylinder and the coma is eliminated by the excentric orientation of the refracting surfaces relative to each other.

In a systematic approach to the topic of centration the terms 'centration axis' and 'reference axis' would have to be distinguished. A reference axis has to be stable
2.2 Centration of corneal refractive procedures

Figure 2.2: Well-centered Navarro schematic eye with 10 diopters of axial myopia (left) and after simulated correction centered on corneal apex (right).

and easy to locate in a clinical environment. In contrast, the actual centration axis needs to have a significance for the visual function of the eye. In order to effectively use such a two-reference approach, the exact relationship between the two axes would have to be known. Unfortunately, the relationships between the ocular axes are not fixed. They depend on the properties of the individual eye and undergo changes during the treatment. Fig. 2.4 shows the corneal intercept locations of three ocular axes in dependence of the refractive error of the eye. Calculations were done on a Navarro schematic eye with off-axis fovea (see Fig. 2.3). Axial myopia and hyperopia were induced by shifting the retinal plane. Thereby it was assumed, that the distance of the foveola from the optical axis remained constant at 1.43 mm (resulting from the assumption \(\alpha = 5\) degrees in the emmetropic eye) in both myopic and hyperopic eyes while the angle \(\alpha\) changed accordingly with the shift of the retinal plane. The original angle of 5 degrees in the emmetropic case decreased to 3.94 degrees for 10 diopters axial myopia and increased to 5.57 degrees for 5 diopters axial hyperopia. Lateral distances on the cornea were measured from the optical axis (OA). Fig. 2.4 indicates that the
2.2 Centration of corneal refractive procedures

Figure 2.3: Emmetropic Navarro schematic eye with the fovea located off-axis by an angle \( \alpha = 5 \) degrees. Ray tracing reveals some amounts of cylinder and coma due to the off-axis position of the fovea (detail below)
2.2 Centration of corneal refractive procedures

corneal intersects of the axes are closer to the optical axis for myopic eyes and further away for hyperopic eyes. The line of sight (LOS) appears to change least with refractive error. It is located 210 microns nasally from the OA in case of 10 diopters myopia, and 300 microns for 5 diopters hyperopia. The coaxially sighted corneal reflex changes from a distance of 530 microns at 10 diopters myopia to 750 microns at 5 diopters hyperopia.

![Line-of-sight (LOS) vs Pre-op refractive error of eye](image)

*Figure 2.4: Corneal intersect locations of three ocular axes in dependence of the refractive error of the Navarro schematic eye with off-axis fovea. Distances are measured from the optical axis (OA).*

2.2.2 Prismatic effect

A small decentration of a spherical correction in an aberration-free optical system introduces tilt [4]. If correction of higher-order wavefront aberrations is aspired, decentration introduces additional types of aberrations besides tilt as discussed under 3.3.
2.2 Centration of corneal refractive procedures

Retinal tilt characterizes a transverse displacement of the retinal point spread function away from the center of the fovea. This shift, also referred to as the prismatic effect, can be quantified in terms of prism diopters. One prism diopter corresponds to the deflection of a straight beam of light of one centimeter when measured one meter away from the prism. Prentice's rule (Eq. 2.1) states that prism in diopters ($\Delta$) is equal to the deccentration distance ($c$) in centimeters multiplied by the lens power ($D$).

$$\Delta = c \cdot D \quad (2.1)$$

Fig. 2.5 depicts the induced prism diopters of tilt in dependence of the chosen centration point on the pre-operative eye after correction of -10 D myopia (left) and +5 D hyperopia (right). The results, which were obtained from chief ray tracing in the modified Navarro eye, are consistent with the values calculated using Prentice's rule (Eq. 2.1). 0.5 prism diopters correspond to a retinal shift of the image of approximately 0.15 mm. Ray tracing through the centered Navarro eye with off-axis fovea has shown that centration on the optical axis (OA) causes no prismatic effect as the balance between the centered optical elements is preserved. It can be presumed that also in real eyes the angle $\alpha$ changes least if ablation on the cornea is done according to the best fit optical axis. As shown in Fig. 2.5, the prismatic effect is highest when centering on the coaxially sighted corneal reflex which is farthest from the optical axis.

Regarding the prescription of spectacle lenses the American National Standard Institute (ANSI) states that the tilt should not exceed 2/3 prism diopters in the horizontal plain to be acceptable [5]. This means that a refractive correction of ±10 diopters could have a deccentration of 0.67 mm and still be within tolerance. A refractive correction of ±5 diopters might tolerate a deccentration as large as 1.33 mm. Thus, centration on either of the discussed reference points would produce tilt that is within the tolerance limit for corrections in the range of -10 to +5 diopters.
2.2 Centration of corneal refractive procedures

Figure 2.5: Induced prism diopeters of tilt in dependance of the chosen centration point on the pre-operative eye (OA at zero) after correction of 10 D myopia (left) and 5 D hyperopia (right). Centration on the corneal intersects of the optical axis (OA), the line of sight (LOS), the visual axis (VA) and the coaxially sighted corneal reflex (CR) was simulated.

2.2.3 Dislocation of corneal reference axes through refractive correction

If the decentration from the optical axis is within the specified tolerance range, the induced prismatic effect is compensated by the eye after the treatment by adjusting the fixation angle $\alpha$. Thus, the retinal image is moved back to the fovea.

Both, the adjustment of the angle $\alpha$ and the shift of the nodal points through the refractive correction move the corneal reference points relative to their pre-
operative locations. Refractive corrections were simulated on the Navarro schmetic eye with off-axis fovea to investigate this effect. Corneal ablation was performed according to Munnerlyn’s formulas [6]. For each centration axis the main corneal intersection points were calculated post-operatively and compared to the corresponding locations before the treatment. Thereby it was assumed that the optical axis (OA) remained constant even after corrections centered on axes different than the OA.

**Figure 2.6**: Change of the corneal intersect locations pre-op to post-op (10 D myopia correction) in dependance of the chosen centration point on the pre-operative eye (OA at zero). Centration on the corneal intersects of the optical axis, the line of sight, the visual axis and the coaxially sighted corneal reflex was simulated.

Fig. 2.6 shows the results for the correction of 10 diopters axial myopia. The x-axis indicates the centration of the procedure on the pre-operative eye with the OA at zero. Centration on the corneal intersects of the optical axis, the line of sight, the visual axis and the coaxially sighted corneal reflex was simulated. The
y-axis shows the change of the corneal intersect locations pre-op to post-op with positive values indicating a shift in the temporal direction. The CR undergoes the largest change after centration on the OA by moving 180 microns in the nasal direction, while the VA is shifted by only 20 microns in the same direction. The LOS remains approximately in the same distance to the OA as before the treatment. Centration on the CR causes an equal shift of all the intercepts of about 15 microns in the temporal direction. The LOS shows the highest stability towards changes in centration of the procedure.

Fig. 2.7 shows the results for the correction of 5 diopters axial hyperopia. The CR and the VA undergo the largest change after centration on the OA by moving 100 microns in the temporal direction, while the LOS is shifted by only 10 microns in the same direction. Centration on the pre-operative corneal reflex CR shifts the post-operative CR in the nasal direction by 40 microns. Again the LOS shows the highest stability.

The changes in the positions of the ocular axes can have a negative effect on the comparison of post- and pre-operative measurement data such as corneal topography if such measurements are aligned with an axis that undergoes considerable shifts during the treatment. The LOS was shown to be the most stable axis in this respect. Centration of measurement devices used to assess the success of refractive treatments should therefore be done on the line of sight.

2.2.4 Ideal centration axis

As mentioned earlier, current discussions on the ideal corneal centration point focus on either the corneal intersect of the line of sight LOS, the corneal intersect of the visual axis VA or the coaxially sighted corneal reflex CR.

In 1986 Uozato et al. proposed to use the line of sight for centration as it marks the center of the ray bundle that enters the eye [7]. They further analyzed the error associated with various methods of corneal centering as a result of non-coaxial or binocular sighting. In the early days of refractive surgery the visual axis was emphasized for centering corneal surgical procedures. As its exact location is difficult to determine in a clinical environment, the corneal light reflex
2.2 Centration of corneal refractive procedures

5D hyperopia correction

![Graph showing change of corneal intersect locations pre-op to post-op (5D hyperopia correction) in dependence of the chosen centration point on the pre-operative eye (OA at zero). Centration on the corneal intersects of the optical axis, the line of sight, the visual axis and the coaxially sighted corneal reflex was simulated.](image)

Figure 2.7: Change of the corneal intersect locations pre-op to post-op (5 D hyperopia correction) in dependence of the chosen centration point on the pre-operative eye (OA at zero). Centration on the corneal intersects of the optical axis, the line of sight, the visual axis and the coaxially sighted corneal reflex was simulated.

was often used as an approximation for the VA. Uozato et al. have shown that the error associated with this approximation can amount to 0.8 mm in optical devices where the fixation light is not coaxially mounted to the observation system. The results presented in Fig. 2.4 were obtained for monocular viewing and coaxial sighting. They revealed a maximum deviation of the corneal reflex from the visual axis of only 0.06 mm which is consistent with Uozato’s results for the same viewing conditions. However, Uozato and coworkers stated, that even if the true visual axis could be located, it is the center of the entrance pupil that should be used for centration as it marks the center of the ray bundle that enters the eye.

In 1992 Pande et al. proposed to use the corneal intercept of the VA for centration as, in contrast to the LOS, this point does not shift with changing pupil size.
They stated that the effect of image degradation due to decentration was minimal if the optical zone is centered on the VA. However, their statement was not supported by any geometrical optical examinations. Instead, they studied the differences in corneal reference points in 50 eyes finding that the coaxially sighted corneal reflex yields the best approximation of the VA and should therefore be used for centration. In a letter to the editor, Mandell criticized Pandc's statement as the VA axis would not represent the light that actually passed through the eye [9]. He strongly emphasized to use the corneal intersect of the LOS for centration as it defines the center of the area of the cornea that is intersected by the bundle of light that finally enters the eye through the pupil, thus supporting the statement of Uozato.

In 2000 the VSIA taskforce on standards for reporting optical aberrations of the eye recommended the use of the LOS as the reference axis for the purpose of calculating and measuring the optical aberrations of the eye [10]. As the LOS defines the chief ray of the light bundle entering the eye, aberrations measured with respect to this axis will have the pupil center as the origin of a Cartesian reference frame.

Decentration of a correcting lens in a centered optical system primarily introduces non-rotationally symmetrical aberrations (see 3.3). Translation of a spherical aberration for instance causes coma, tilt and small amounts of astigmatism.

Ideal centration of a rotationally symmetrical correction on the cornea can thus be defined as the procedure that induces no non-rotationally symmetrical optical errors. Even though in case of a pure myopia the light rays entering the eye are not in perfect retinal focus, they form a symmetrical pattern around the chief ray (LOS) which defines the central ray of the light cone entering the pupil (Fig. 2.8). Ideal correction preserves the symmetry of the marginal rays to the chief ray and moves the focal point backwards along the chief ray to the fovea. This symmetry is best preserved when centering the correction on the axis of symmetry itself i.e. the corneal intersect of the line of sight (LOS).

The LOS has further advantages over the other ocular axes (VA, LCCR). Its corneal intercept location shows the least dependance on the pre-operative re-
2.2 Centration of corneal refractive procedures

Figure 2.8: Bundle of light rays entering a myopic eye. The light cone passing through the pupil is symmetrical to the chief ray that is equal to the LOS. Ideal correction moves the focal point backwards along the chief ray to the fovea.

A drawback for using the corneal intercept of the line-of-sight for centration might be the fact, that the pupil center shifts with changing pupil size [11, 12, 13]. Shifts of up to 0.4 mm [11], 0.6 mm [12] and even 0.7 mm [13] equally distributed in all directions were found by different groups. A result of this phenomenon might be, that a post-operative eye with only some amounts of spherical aberration left at one pupil size might suddenly exhibit some additional amounts of coma if measured at another pupil size. This is due to the fact that coma is a pupil-position-dependant aberration. That means that coma can not only result from intrinsic asymmetry of the refracting elements of the eye, but also from pupil decentration in an otherwise symmetrical system. As the deteriorating effect of
higher-order aberrations on the visual quality is known to increase for growing pupil size, ablation should be performed on the center of the dilated pupil. Thus, the optical performance of the eye is optimized for a large pupil and possible coma measured at smaller pupil sizes due to a pupil shift can be neglected.

2.3 Summary

The exact corneal intersect locations of three ocular reference axes were calculated in dependence of the refractive error of a Navarro schematic eye with off-axis fovea. It was shown that the corneal intersects of the axes are closer to the optical axis for myopic eyes and further away for hyperopic eyes. The line of sight appeared to change least with refractive error and refractive correction.

Decentration of refractive corrections primarily introduces tilt i.e. a transverse displacement of the retinal point spread function away from the center of the fovea. It was shown, that centration on either of the discussed reference points would produce tilt that is within the ANSI tolerance limits for corrections in the range of -10 to +5 diopters.

Refractive corrections have the potential to shift the intercept locations of the ocular reference axes. These shifts depend on the chosen centration axis. The coaxially sighted corneal reflex was shown to undergo the largest change after centration on the optical axis. The line of sight showed the highest stability towards changes in centration of the procedure, which makes it the most suitable reference axis to compare pre- and post-operative measurement data.

Geometrical optical considerations led to the conclusion, that an ideal correction can be defined as the procedure that preserves the symmetry of the marginal rays to the chief ray. This symmetry is best preserved when centering the optical zone on the axis of symmetry itself i.e. the corneal intersect of the line of sight.
References


Chapter 3

Constant misalignment of refractive laser corrections

3.1 Introduction

3.1.1 Constant misalignment

Objective assessment of the eye’s refractive state, be it through autorefraction, ocular topography or wavefront aberrometry, requires exact alignment of a measurement device relative to the eye. For diagnostic purposes alone, exact registration might play a minor role. If correction of a refractive error is aspired, however, each coordinate point in the acquired map needs to be assigned to the corresponding point in the eye. Consequently, exact registration is crucial.

In refractive corneal surgery, a constant misalignment of the ablation zone occurs, if the reference coordinate system of the treatment does not exactly coincide with the eye’s reference axes. Alignment of the procedure relative to the eye is a task with six degrees of freedom. Fig. 3.1 illustrates the situation for an ablation profile that is horizontally translated by the amount $\Delta x$ and rotated around the z-axis by the angle $\Delta \gamma$.

The correct axial z-distance of the laser guarantees the delivery of the desired fluence onto the cornea. The lateral placement of the ablation zone is defined by the coordinates x and y. In-plane rotation around the longitudinal axis z (torsion or cyclorotation) has to be avoided in non-rotationally symmetrical corrections.
3.1 Introduction

such as cylinder or coma. Rotations of the ablation around the lateral axes x and y do not have a definite appearance in refractive surgery. In contrast to spectacle lenses, which can be tilted relative to the eye, the refractive correction applied by the laser can not exhibit a pure tilt on the cornea. Yet, the correction can be laterally shifted due to a relative tilt between the laser beam and the eye. For contact lenses, small tilts and longitudinal displacements as a side effect to lateral translation of the lens on the cornea were shown to be negligible compared to the effect of the lateral translation itself [1, 2].

Lateral alignment errors might arise from an inaccurate centration by the surgeon, or if measurement and treatment are performed at different pupil sizes, as the pupil center was shown to shift with changing pupil diameter [3, 4]. In-plane rotation errors can result from an unnoticed lateral tilt of the patient’s head under the laser, as well as from ocular cyclotorsion [23, 24].

![Ablation profile](image)

Figure 3.1: Misalignment of the ablation zone relative to the eye’s reference coordinate system (origin on the corneal anterior surface). The ablation is horizontally translated by the amount $\Delta x$ and rotated around the z-axis by the angle $\Delta \gamma$

3.1.2 Literature review

The theoretical effects of decentered optical elements on image quality have been well investigated in technical optics [5, 6, 1]. It was recognized a decade ago that decenteration of the ablation zone is among the most serious complications follow-
3.1 Introduction

ing photorefractive keratectomy (PRK) and laser in-situ keratomileusis (LASIK) [7, 8]. It can pose serious, visually disabling side effects such as halos, glare and diplopia [7, 8, 9, 10, 11]. Since then, a number of reports have dealt with the topic of misalignment in refractive surgery, mostly analyzing cases of displaced ablations by means of post-operative corneal topography measurements.

Maloney has estimated the potential effects of decentration of excimer PRK. He estimated, that decentrations of as little as 1.0 mm could be optically significant with small central optical zones. Kawcsch and Maloney introduced the concept of quantifying centration by using subtraction maps, the data of which were then applied to a center of mass equation [12].

Cavanaugh et al. assessed PRK centration in 49 patients by visual estimation of the center of the post-PRK flat zone on the topographic color map, and found an average decentration of 0.44 mm from the center of the pupil [13]. Mean uncorrected visual acuity in a series of 110 patients treated for low myopia reported by the same authors was 20/20 for decentrations up to 1.0 mm, but fell to 20/30 for decentrations greater than 1.0 mm [10].

Amano and coauthors found a mean decentration of 0.51 ± 0.31 mm in 51 eyes after PRK [14]. Verdon et al. have shown a significant decrease in visual acuity under low-contrast situations with increasing decentration of the ablation zone [15]. The reduction in low-contrast visual acuity was positively correlated with the decentration of the ablation zone, providing evidence of an association between corneal topography and the functional outcome of PRK.

Mulhern et al. compared the post-operative topography of PRK and LASIK patients finding significantly higher mean decentrations in the LASIK group (0.90 mm) than in the PRK group (0.48 mm) [8]. They further suggested, that decentrations of up to 1.00 mm may be tolerated, with minimal or no subjective visual disturbance.

Tsai et al., however, found no significant difference between PRK (0.33 ± 0.32) and LASIK (0.35 ± 0.26) [16]. Table 3.1 which was taken from Tsai et al. gives an overview of the mean decentrations found in different studies.
Table 3.1: Summary of mean decentrations measured in different studies.

<table>
<thead>
<tr>
<th>Study</th>
<th>Procedure</th>
<th>Total mean decentration</th>
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<tbody>
<tr>
<td>Terrell et al. [17] 1995</td>
<td>PRK</td>
<td>0.41</td>
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<tr>
<td>Amano et al. [14] 1994</td>
<td>PRK</td>
<td>0.51</td>
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<tr>
<td>Cavanaugh et al. [13] 1993</td>
<td>PRK</td>
<td>0.40</td>
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<tr>
<td>Schwartz-Goldstein et al. [18] 1995</td>
<td>PRK</td>
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<tr>
<td>Deitz et al. [19] 1996</td>
<td>PRK</td>
<td>0.62</td>
</tr>
<tr>
<td>Lin [20] 1993</td>
<td>PRK</td>
<td>0.34</td>
</tr>
<tr>
<td>Pallikaris et al. [21] 1994</td>
<td>PRK</td>
<td>0.81</td>
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<tr>
<td></td>
<td>LASIK</td>
<td>0.96</td>
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<tr>
<td>Mulhern et al. [8] 1997</td>
<td>PRK</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>LASIK</td>
<td>0.90</td>
</tr>
<tr>
<td>Tsai et al. [16] 2000</td>
<td>PRK</td>
<td>0.33</td>
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<td></td>
<td>LASIK</td>
<td>0.35</td>
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Whereas all the above studies dealt with the issue of lateral decentration of the ablation zone, Tjon-Fo-Sang et al. investigated ocular cyclotorsion as a possible cause of rotational misalignment of the ablation pattern in 15 subjects [22]. Only two subjects (13%) showed statistically significant cyclotorsion of more than 13 degrees when changing from binocular to monocular fixation in a seated position, and three subjects (20%) in a supine position. Changing from the seated to the supine position however had no influence on ocular torsion. Equally, Smith et al. [23, 24] found no statistically significant difference between the two measurements in the seated and in the supine positions (n = 50), even though they observed cyclotorsion of 7 to 16 degrees in 25% of eyes.

Clinical trials presented by SMI (SMI product information, ScensoMotoric Instruments GmbH, Teltow, Germany) have shown a mean value and standard deviation for ocular torsion of 2.8 ± 3.6 degrees, with a maximum of 9.4 degrees. Enright found minimal cyclotorsion of up to 2 degrees due to changes in fixation distance [25].
A number of reports approached the topic of alignment analytically.

Bara et al. [2] studied the positioning tolerances for phase plates used to compensate human eye aberrations. Lateral displacements, in-plane rotations, and axial translations were considered. The general trend of results suggested that lateral positioning, followed by angular positioning, were the key factors affecting compensation performance in practical setups, whereas positioning along the longitudinal axis turned out to be less critical.

Mrochen et al. demonstrated that decentrations as small as 0.2 mm significantly increase coma- and spherical-like wavefront aberrations and, therefore, deteriorate the optical quality of the retinal image [27]. Numerical simulations were supported by clinical results.

Guirao et. al. [28] discussed the effect of cyclorotations and translations of an ideal correction pattern on the ocular modulation transfer function and the RMS of the residual wavefront error based on 6-mm pupil data from 10 eyes. The image quality did not fall below the Rayleigh limit for tolerances (measure for the diffraction limit) for decentrations smaller than 0.1 mm and for cyclotorsions smaller than 3 degrees.

In the following, a method to analyze the optical effect of decentered ablations will be introduced and applied to measured wavefront data. It was the aim of this study to provide information on the maximum permissible alignment errors in wavefront-guided refractive surgery to improve the ocular optics to a sought level. In contrast to earlier investigations our calculations were based on measured aberration data from a large cohort of eyes.

### 3.2 Concept of wavefront correction pattern

Wavefront aberrations of the eye can be expressed by a series of Zernike monomials [29],

\[
W_A\text{eye}(x,y) = \sum_{n,m} a_n^m Z_n^m \quad n = 1, 2, ..., 6
\]
3.2 Concept of wavefront correction pattern

where $Z_n^m$ is the Zernike monomial of radial order $n$ and meridional order $m$. $a_n^m$ is the corresponding coefficient. The relative pupil coordinates $x$ and $y$ take values between -1 and 1 and correspond to the absolute pupil positions $X$ and $Y$ normalized by the pupil radius $R$. Using polar coordinates $\rho$ and $\theta$ instead of cartesian coordinates $x$ and $y$, the Zernike monomials are:

$$Z(\rho, \theta) = R_n^m(\rho) \cdot \begin{bmatrix} \cos(m\theta) \\ \sin(-m\theta) \end{bmatrix}$$

where $R_n^m(\rho)$ is the radial polynomial. This terminology follows the recommendations of the VSIA taskforce on standards for reporting optical aberrations of the eye [37].

Refractive corrections intend to either completely or selectively remove the existing wavefront aberrations. A so called wavefront correction pattern can be introduced in terms of Zernike monomials:

$$WA_{corr}(x, y) = \sum_{n,m} c_n^m Z_n^m \quad n = 1, 2, ..., 6$$

The residual wavefront aberrations after the treatment are obtained from the difference between the eye's aberrations and the wavefront correction pattern:

$$WA_{residual}(x, y) = WA_{eye} - WA_{corr} = \sum_{n,m} a_n^m Z_n^m - \sum_{n,m} c_n^m Z_n^m$$

The correction pattern is deduced from the wavefront aberrations of the eye. In case of a perfectly aligned correction of all the assessed Zernike terms, the coefficients $a_n$ and $c_n$ are equal. A selection of the measured Zernike terms might be used in the correction pattern if only the treatment of some specific aberrations is aspired. $WA_{corr}$ can be rotated and translated relative to $WA_{eye}$ as a result of an inaccurate alignment of the treatment. The correction pattern can then be calculated using coordinate transformations:
3.3 Effect of translation and rotation on Zernike representation

\[
WA_{corr}(x, y) = WA_{eye}(x', y')
\]

(3.4)

Where

\[
x' = (x - \Delta x)\cos(\Delta \gamma) + (y - \Delta y)\sin(\Delta \gamma)
\]

\[
y' = (y - \Delta y)\cos(\Delta \gamma) - (x - \Delta x)\sin(\Delta \gamma)
\]

\(\Delta \gamma\) is the angle of rotation around the eye's longitudinal axis, and \(\Delta x, \Delta y\) are the translations in the x and y direction. These new coordinates can be introduced in each Zernike monomial of the eye's aberrations \(WA_{eye}\) (Eq. 3.1), and by rearranging terms to isolate the original Zernike monomials, the coefficients \(c_i\) can be obtained analytically from functions of the original coefficients \(a_k\) (see A). Those analytical functions can be converted to matrix equations defining a translation matrix \(T\) and a rotation matrix \(R\) [28]. For the investigations presented here, the coordinate transforms were performed numerically on the aberration maps. Zernike monomials were then fitted to these data using a least-square-fit routine to obtain the coefficients \(c_i\).

3.3 Effect of translation and rotation on Zernike representation

3.3.1 Translation

The analytically determined translation matrix \(T\), which can be used to directly convert the coefficients \(a_k\) to \(c_i\) is of very complex nature (see A). However, assuming translations smaller than 0.15 times the pupil radius, the matrix can be simplified by only considering the linear translation terms \(\Delta x/R\) and \(\Delta y/R\) [28]. Using this simplification, it turns out that with translation, an original aberration of order \(n\) mainly introduces additional aberrations of the orders \(n-1, n-3, n-5\) and so on. Thus, defocus or astigmatism mainly produce tilt, coma generates astigmatism, and spherical aberration primarily produces coma and tilt. Additional consideration of the second order translation terms \((\Delta x/R)^2, (\Delta y/R)^2\) and
3.3 Effect of translation and rotation on Zernike representation

\[(\Delta x/R) \cdot (\Delta y/R)\] in the matrix \(T\) shows, that further aberrations of the orders \(n-2, n-4, n-6\) and so on are introduced, especially for translations larger than 0.15 times the pupil radius.

![Figure 3.2: Example of a coma (left), that is translated by the amount \(\Delta x/R = 0.2\) (middle). The difference corresponds to the residual wavefront aberration after translated correction (right).]

Our results obtained with numerical coordinate transformation and subsequent Zernike polynomial fitting support these findings. Tab. 3.2 (left) shows the change in the Zernike coefficients when translating a vertical coma \((C_{31}^{-1} = 0.6\) microns, normalization factor included, 6 mm pupil diameter) by the amount \(\Delta x/R\) as depicted in Fig. 3.2, where \(R\) is the pupil radius. Incremental translation of the coma produces increasing amounts of astigmatism (order \(n-1\)) and some tilt (order \(n-2\)), while the coma term remains constant at the initial value of 0.6 microns. Tab. 3.2 (right) shows the change in the Zernike coefficients when translating a spherical aberration \((C_{40}^{0} = 0.6\) microns, normalization factor included, 6 mm pupil diameter). The translation produces coma (order \(n-1\)) and tilt (order \(n-3\)) and smaller amounts of astigmatism and defocus (order \(n-2\)), while the original term remains constant at the initial value of 0.6 microns. Subtraction of the original terms from the coefficients produced by the translation provides the coefficients of the residual wavefront aberration after misaligned correction.

The Strehl intensity ratio for a 6-mm pupil was calculated from the residual wavefront aberration as a measure for the retinal image quality after misaligned ablation. Eq. 3.5 provides a good estimation of the Strehl intensity ratio \(S\) from the root-mean-squared (RMS) wavefront error \(\sigma\) for \(S > 0.1\) (at the wavelength \(\lambda\)).
3.3 Effect of translation and rotation on Zernike representation

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Table 3.2: Change of Zernike coefficients with increasing translation of a primary coma (left) and a primary spherical aberration (right). Coefficients are given in microns. The translation is normalized by the pupil radius R.

\[ S \approx \exp[-\left(\frac{2\pi}{\lambda} \sigma\right)^2]\]  \hspace{1cm} (3.5)

The Strehl value of the residual aberration was then set into relation with the Strehl value of the original wavefront aberration to be corrected. This ratio, that can be considered a benefit factor, decreases with growing translation. Values higher than 1 indicate an improvement of the image quality compared to the pre-operative state. Values smaller than 1 characterize a decrease due to translation of the correction. The magnitude of the aberrations corresponded to 0.5, 1.0 and 3.0 times the standard deviation of the Zernike coefficients measured in a large population by Porter et al. [30].

Fig. 3.3 (left) depicts the results for a vertical coma. The larger the original aberration to be corrected, the higher is the benefit factor in case of a good centration, and the stronger is the decrease with growing decentration. The image quality can be improved by a factor of 13 when correcting a large coma \( (C_3^{-1} = 1.8 \text{ microns}, 6 \text{ mm pupil diameter, normalization factor included}) \) with
no decentration. The quality is degraded compared to the pre-operative value for translations larger than 0.15 - 0.20 mm. Fig. 3.3 (right) shows the change in the Strehl ratio for a spherical aberration of three different magnitudes. Again, the larger the aberration to be corrected, the stronger the decrease. The image quality is degraded compared to the pre-operative value for translations larger than 0.13 - 0.20 mm.

Figure 3.3: Ratio of Strehl post-op to pre-op values with increasing translation of a primary coma (left) and a primary spherical aberration (right). Values higher than 1 indicate an improvement of the image quality compared to the pre-operative state.

The prismatic effect

The primary effect of a decentered ablation is to produce a transverse displacement of the retinal point spread function. This displacement is expressed by the two tilt terms in the Zernike expansion. The shift of the retinal image away from the fovea, also referred to as the prismatic effect, can further be quantified in terms of prism diopters. One prism diopter corresponds to the deflection of a straight beam of light of one centimeter when measured one meter away from the prism. Fig. 3.4 shows the induced prism as a function of ablation decentration (spherical correction). Ablation of ±5 diopters 2 mm off center introduces 1 prism diopter. A correction of ±10 diopters 2 mm of center causes prisma of about 1.9 diopters. These results were obtained from simulated corneal ablation in a Navarro schematic eye and subsequent two-dimensional ray-tracing. They are consistent with the results obtained from Prentice’s rule (Eq. 2.1).
3.3 Effect of translation and rotation on Zernike representation

Figure 3.4: Induced prism as a function of ablation decentration for different corrections. Myopic and hyperopic corrections produce the same absolute amounts of prism.

3.3.2 Rotation

Figure 3.5: Example of a coma (left), that is rotated by 30 degrees (middle). The difference corresponds to the residual wavefront aberration after rotated correction (right).

The rotation matrix $R$, which can be derived analytically from the Zernike polynomials (see A), converts the original terms $a_k$ to the coefficients $c_4$ after rotation. It turns out that this conversion depends exclusively on the rotation angle $\Delta \gamma$ as well as on the meridional degree $m$ of the Zernike term, but is independent of the radial degree $n$. This is due to the rotational invariance of the Zernike polynomials. Thus, rotation only produces aberrations of the same radial order $n$. Tab. 3.3 (left) lists the induced coefficients when rotating an astigmatism ($C_2^{-2} = 2.0$ microns, normalization factor included, 6 mm pupil diameter) by 30
and by 60 degrees. The rotation only produces complimentary astigmatism $C_2^2$. Tab. 3.3 (right) shows the change in the Zernike coefficients when rotating a vertical coma ($C_2^{-1} = 0.6$ microns, normalization factor included, 6 mm pupil diameter) as depicted in Fig. 3.5. Again, only complimentary coma $C_3$ is produced.

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**Table 3.3:** Change of Zernike coefficients with increasing rotation of a primary astigmatism (left) and a primary coma (right). Coefficients are given in microns.

Fig. 3.6 (left) depicts the ratio between the Strehl post-op and pre-op values due to rotation of a vertical coma. The larger the original aberration to be corrected, the higher the benefit factor in case of a good alignment, but the stronger the decrease with growing rotational misalignment. The ratio becomes smaller than 1 for rotations larger than 20 to 30 degrees. Fig. 3.6 (right) shows the change in the Strehl ratio for a 6-fold aberration of three different magnitudes. Due to the periodic nature of the meridional Zernike terms, the alignment accuracy starts to increase again after a minimum at $\pi/m$. The retinal image quality is degraded compared to the pre-operative value for rotations above 10 degrees.

**Rotation of cylinder axis**

Holladay et al. investigated the tolerable angular error to achieve certain absolute levels of precision for different magnitudes of cylindrical corrections [26]. It was shown, that with an angular error of 30 degrees, the achieved precision is always equal to the magnitude of the measured cylinder.
3.3 Effect of translation and rotation on Zernike representation

Figure 3.6: Ratio of Strehl post-op to pre-op values with increasing rotation of a primary coma (left) and a 6-fold aberration (right). Values higher than 1 indicate an improvement of the image quality compared to the pre-operative state.

The same result is obtained when comparing the post-op and pre-op Strehl intensity ratios of an astigmatic correction. Fig. 3.7 depicts the change of the benefit factor for three cylinders of 0.5, 1.0 and 3.0 diopters. The factor is equal to 1 for an axis misalignment of 30 degrees for all cylinder magnitudes. The minimal ratio is reached at 90 degrees rotation.

Figure 3.7: Ratio of Strehl post-op to pre-op values with increasing rotation of an astigmatic correction. The benefit factor becomes 1 for a rotation of 30 degrees.
3.3.3 Discussion

Misalignment of the correction has the potential to decrease the post-operative image quality. The ratio between the post- and pre-op Strehl values as shown in Fig. 3.3 and 3.6 expresses the relative benefit of a refractive treatment. The above calculations were performed for each type of aberration of the Zernike orders 2 to 6. For translation, the amount of misalignment at which this benefit ratio falls below 1 was found to only slightly depend on the magnitude of the corrected aberration. It is in the order of 0.15 mm for larger aberrations, and 0.20 mm for small aberrations. The relative translation tolerance is almost independent of the Zernike order n i.e. of the type of aberration. The tolerance for rotational misalignment strongly depends on the meridional degree m of the Zernike polynomials. As mentioned above, the post-op Strehl ratio takes a minimum at a rotation angle of $\pi/m$ due to the periodic nature of the meridional Zernike terms. Thus, the larger m, the smaller the tolerance angle. For instance, the benefit ratio falls below 1 after a 5 degree rotation of a 6th order 6-fold aberration and reaches a minimum at 30 degrees. In contrast, it takes a rotation of about 30 degrees of a 3rd order 3-fold aberration to decrease the image quality to the same relative limit. The minimal ratio is reached at 60 degrees. A 5th order coma ($m = 1$) can even tolerate rotations up to 60 degrees until the correction benefit factor becomes one. The minimum ratio occurs at an in-plane rotation of 180 degrees. The rotation tolerance was found to be practically independent of the magnitude of the corrected aberration.

3.3.4 Alternative simulation with optical ray tracing

The concept of the wavefront correction pattern was introduced to estimate the residual wavefront error after misaligned refractive correction. It presumes, that the optical system to be corrected consists of only one refracting surface. The human eye, however, is a system with multiple surfaces, and thus, the application of a refractive correction onto the first surface alters the incidence positions and angles of the light rays on the following surfaces of the system. Although the intraocular structures remain unchanged after surgery, their contribution to the total aberrations of the whole system is changed according to [31]. In reality, decentration of the ablation might therefore cause slightly different residual wavefront aberrations than predicted by the wavefront correction method. These
3.3 Effect of translation and rotation on Zernike representation

effects can be considered by performing optical ray tracing through a schematic model eye after correction of the first surface. To evaluate the differences in the results between the two calculation models, a decentred myopic treatment of 3 diopters was simulated with each method.

First, the wavefront aberration pattern of a 3 diopter myopia ($C_2^0 = 6.75$ microns, normalization factor included, 6 mm pupil diameter) was translated by 0.5 and 1.0 mm using coordinate transformation. Zernike polynomials were fitted to the transformed maps to obtain the corresponding coefficients.

Secondly, the corneal ablation profile of a 3 diopter myopic correction (6.5 mm optical zone) was calculated using Munnerlyn’s formula [32]. The ablation profile was subtracted from the anterior corneal surface of an emmetropic Koijmann model eye, three-dimensional optical ray tracing was performed through the entire eye in Matlab (Software version 6.5, The MathWorks, Inc.). Fig. 3.8 depicts the incidence positions of the traced rays on the retina for decentrations of the ablation of 0 mm (left), 0.5 mm (middle) and 1.0 mm (right). The ideal focal point of all the rays in the emmetropic state would be at the position (0,0). The dot pattern is shifted horizontally and becomes more and more irregular with increasing decentration of the ablation. The deviations of the rays from their ideal retinal positions are called transverse aberrations. The wavefront aberrations can be reconstructed from the transverse aberrations using a least-square fitting method (see appendix). Calculations were done for a pupil diameter of 6 mm.
3.3 Effect of translation and rotation on Zernike representation

Table 3.4: Comparison of two methods to predict wavefront aberrations after translated correction. Coordinate transform of wavefront aberrations (columns 2-4) and optical ray tracing (columns 5-7). Zernike coefficients are given in microns.

<table>
<thead>
<tr>
<th>Zernike Index</th>
<th>Translation [mm]</th>
<th>Münnerlyn correction and ray tracing in eye model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>8.7</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>6.8</td>
<td>6.8</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Tab. 3.4 compares the calculated Zernike coefficients of the two simulation methods. The wavefront correction method using coordinate transformation only predicts the introduction of wavefront tilt (Zernike index 2) through translation, whereas the ray tracing method additionally predicts small amounts of astigmatism (index 5) and coma (index 8) beside wavefront tilt. These additional aberration terms can be traced back to the consideration of the intraocular structures such as the corneal back surface and the lens in the schematic eye model. As their magnitude is comparably small for the range of translations investigated in this study, the wavefront correction method provides a good estimate of the wavefront aberrations introduced by misalignment of the correction.
3.4 Investigation with measured wavefront data

The effect of lateral and rotational misalignment of the ablation zone on the post-operative optical outcome was simulated based on measured wavefront aberration data from 130 eyes undergoing refractive surgery. The above concept of the wavefront correction pattern was used to simulate misaligned refractive corrections. Simulations included lateral decentrations and rotations around the longitudinal axis of the eye. The post-operative optical quality was rated by means of the root-mean-squared residual wavefront error. The aim was to provide information on the maximum permissible misalignments in wavefront-guided refractive surgery to improve the ocular optics to a sought level. The results were published in [Bueeler M, Mrochen M, Seiler T. Maximum permissible lateral decentration in aberration-sensing and wavefront-guided corneal ablation. J Cataract Refract Surg. 2003; 29(2): 257-63] and [Bueeler M, Mrochen M, Seiler T. Maximum permissible torsional misalignment in aberration-sensing and wavefront-guided corneal ablation. J Cataract Refract Surg. 2004; 30(1): 17-25].

3.4.1 Subjects and Methods

Subjects:
Data from one hundred and thirty eyes were taken from an earlier study on normally aberrated eyes [33]. In brief, the mean age of the 90 individuals enrolled in the study was 32.9 years (range: 18–63). Subjects (eyes) were eligible to be included in the study if (1) they were at least 18 years of age, (2) were free of ocular diseases, (3) had a best-spectacle-corrected visual acuity of 20/20 or better, (4) the spherical equivalent of the refraction was between 8.0 D and +2.0 D, (5) the manifest refractive cylinder was less than 3.0 D, and (6) wavefront measurements were done on pupils of 7 mm in diameter using the technique described below. The median of the 2\textsuperscript{nd} to the 6\textsuperscript{th} order root-mean-squared wavefront aberration error was 0.830 microns with a 25\textsuperscript{th} percentile of 0.498 microns and a 75\textsuperscript{th} percentile of 2.370 microns. Neglecting the sphero-cylindrical errors, the median of the higher-order (3\textsuperscript{rd} to the 6\textsuperscript{th} order) root-mean-squared wavefront error for the 7-mm pupils was 0.230 microns with a 25\textsuperscript{th} percentile of 0.167 microns and a 75\textsuperscript{th} percentile of 0.297 microns. The demographic and refractive data of the investigated subjects are summarized in Tab. 3.5.
### 3.4 Investigation with measured wavefront data

| Table 3.5: Demographic and refractive data of the investigated eyes. |
|---|---|---|
| Age | Mean ± SD | Range |
|     | 32.9 ± 11.3 years | 18 to 63 years |
| Sex | Female | 82 (63.1 %) |
|     | Male | 48 (36.9 %) |
| Eye | OD | 64 (49.2 %) |
|     | OS | 66 (50.8 %) |
| Manifest refraction | Sphere | -1.70 ± 2.16 D | -7.50 to 2.25 D |
|     | Cylinder | -0.44 ± 0.49 D | 0 to -2.75 D |

### Wavefront sensing:

A wavefront sensor of the Tscherning-type was used in this study, that has been described in earlier publications [34, 35, 36]. In brief, a collimated laser beam (wavelength 532 nm) illuminates a mask with regular matrix pin holes which forms a bundle of thin parallel rays. These rays form a spot pattern on the retina that is more or less distorted according to the optical errors of the eye. This retinal spot pattern is imaged onto the sensor of a low-light video camera using the principle of indirect ophthalmoscopy. The beam pattern attenuated by the shutter is centered onto the line of sight by means of a second video camera depicting the iris while the examined eye was fixating a fixation target aligned coaxially to the optical axis of the measuring device.

From the deviations between the observed retinal spot positions and their ideal positions, the actual wavefront aberrations were calculated in terms of Zernike polynomials up to the 6th order. The wavefront aberrations were measured at pupils larger than 7 mm in diameter. These data were further processed to obtain the wavefront aberrations for virtual pupils of 3, 5 and 7 mm in diameter. The nomenclature used for the Zernike representation is according to the standards of the Optical Society of America (OSA) [37].

The optical quality was rated by using the root-mean-squared (RMS) wavefront error as a single-value representation of the calculated wavefront aberrations. The RMS-wavefront error is the square root of the variance of the deviations between
the real wavefront and the ideal reference wavefront at the entrance pupil plane. It is a conventional statistic to describe the overall performance of an optical system and can be calculated directly from the Zernike coefficients.

Previous to the subject measurements, the calibration and the reproducibility was tested by means of an artificial eye including several phase plates (WaveLight Laser Technologic AG, Erlangen, Germany) with defined wavefront aberrations. The reproducibility for the total RMS-wavefront error in case of a defined aberration of RMS = 1.14 mm was ±0.08 mm at a pupil size of 6 mm in diameter.

Numerical simulations:
The wavefront correction patterns were calculated from the measured aberration maps using coordinate transformations (Eq. 3.4). Lateral shifts up to $\Delta x_{\text{max}} = 2.0$ mm (1.5 mm for the 3-mm pupil data) in increments of 0.05 mm, and torsions up to $\Delta \gamma_{\text{max}} = 45$ deg (increments of 0.5 deg) were calculated. Calculations were done for the 3-, 5- and 7-mm pupil data considering 2nd to 6th Zernike orders. Piston and tilt (Zernike orders 0 and 1) terms were neglected.

Each wavefront correction pattern was numerically subtracted from the original aberration surface to simulate a systematically displaced ablation (Fig. 3.9). Subtraction of the two patterns results in a residual wavefront aberration. Zernike polynomials up to the 6th order were numerically fitted to the residual wavefront aberrations using a three dimensional least-square-fit routine in Matlab (Software version 6, The MathWorks, Inc.). Complete correction of all the error orders 2nd to 6th (sphero-cylindrical and higher-orders) as recorded by the wavefront measurement was assumed. In case of an ideal i.e. perfectly aligned treatment, the residual wavefront error would be zero over the entire entrance pupil zone.

The amount of the RMS residual wavefront error (2nd to 6th Zernike orders) was studied as a function of the alignment error parameters $\Delta x$ and $\Delta \gamma$. 
3.4 Investigation with measured wavefront data

Coordinate transformation

\[
\begin{align*}
\text{WA}_{\text{pre}}(x,y) & \quad \text{WA}_{\text{post}}(x,y) \\
= & \quad \text{WA}_{\text{rot}}(x',z') \\
\end{align*}
\]

Figure 3.9: Ratio of Strehl post-op to pre-op values with increasing rotation of a primary coma.

Rating criteria:
The RMS wavefront error resulting from a misaligned wavefront correction was rated by three different image-quality criteria as summarized in Tab. 3.6.

The so-called Maréchal criterion (abbreviated MA in figures) states, that a well corrected, diffraction limited optical system has an RMS-wavefront error not exceeding 1/14 of the wavelength. In our case, a RMS-value of 0.038 microns serves for the Maréchal criterion corresponding to the wavelength of 0.532 microns used for the wavefront measurements. The MA criterion is the strictest rating criterion and therefore very difficult to achieve, especially in pupils as large as 7 mm in diameter.

The lower 10\textsuperscript{th} percentile of the preoperative higher-order (orders 3-6) RMS-wavefront errors at a certain pupil size served as another rating criterion (abbreviated \(P_{10}\) in figures). This is the 10\% of eyes that have the lowest amount of higher-order aberrations (i.e. best optical quality) in the group. The value for the \(P_{10}\) criterion was found to be 0.075 microns for the 3-mm pupils, 0.078
Table 3.6: Values of the RMS error criteria used to rate the optical quality.

<table>
<thead>
<tr>
<th></th>
<th>Value [microns]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 mm</td>
</tr>
<tr>
<td>MA</td>
<td>0.038</td>
</tr>
<tr>
<td>$P_{10}$</td>
<td>0.075</td>
</tr>
<tr>
<td>OT</td>
<td>0.054 – 3.130</td>
</tr>
</tbody>
</table>

microns for the 5-mm pupils and 0.131 microns for the 7-mm pupil data. The 1st and 2nd order wavefront aberrations were not taken into account for the $P_{10}$ criterion, as their comparably high values would render the criterion too mild.

Beside the two mentioned criteria, each eye's own total preoperative RMS-wavefront aberration (orders 2-6) at the respective pupil size was used as a third rating criterion (abbreviated OT in figures) to provide information about the required alignment accuracy in order not to degrade the optical quality compared to the preoperative state (i.e. not to increase the RMS value through the treatment). Thus, for most of the examined eyes, the OT criterion can be expected to be the mildest criterion to rate the effect of misalignment.

The Maréchal criterion is extremely strict for the evaluation of refractive corrections, especially within treatment zones as large as 7 mm in diameter. The two statistical values derived from the measurement data (criteria $P_{10}$ and OT), however, are more realistic thresholds to rate the optical outcome after corneal ablations. Degrading vision below the original error (as represented by the criterion OT) is not acceptable in any refractive treatment. However, the criterion OT was chosen in order to set up a boundary value for a worst case scenario. The Maréchal criterion (MA) on the other hand, which represents the best optical quality achievable in theory, provided the data for the other side of the quality spectrum.

Data evaluation:
Intersecting the functions $RMS_{\text{residual}} = f(\Delta x)$ and $RMS_{\text{residual}} = f(\Delta \gamma)$ for each eye with the rating criteria results in the alignment error parameters $\Delta x_{\text{crit}}$ and $\Delta \gamma_{\text{crit}}$ at which the corresponding criterion is violated (Fig. 3.10). Small values
of the critical parameters indicate a high sensitivity of the particular eye for the respective type of alignment error. A set of 130 $\Delta x_{\text{crit}}$- and another set of 130 $\Delta \gamma_{\text{crit}}$-values were obtained (corresponding to the 130 examined eyes) for each of the three criteria MA, P8 and OT. Each set was then split up into percentile values (increments of 5%) and the corresponding statistical distribution of the $\Delta x_{\text{crit}}$- and $\Delta \gamma_{\text{crit}}$-values was summarized.

![Figure 3.10](image.png)

**Figure 3.10**: Schematic illustration of the relationship between the root-mean-squared residual wavefront error and the lateral misalignment for one single eye. Intersecting the curve with the RMS-rating criteria results in the critical misalignment at which the corresponding criterion is violated.

Furthermore, the torsional alignment accuracies required to meet the criteria MA and $P_{10}$ for wavefront-guided treatments, were correlated with the original cylinder of each examined eye. The simulations reported above were repeated for refractive treatments designed to correct only sphere and cylinder using the 7-mm pupil data. The higher-order aberrations, which were left uncorrected in these simulations, where neglected when calculating the RMS residual wavefront aberration, as their correction was not planned. Again, the torsional alignment accuracies needed to meet the criteria MA and $P_{10}$ were correlated with the original cylinder. Comparison of the two cases was intended to illustrate the effect of treating higher-order aberrations on the required torsional alignment accuracy.
3.4 Investigation with measured wavefront data

3.4.2 Results

Translation:
Fig. 3.11 shows a choice of four statistically relevant functions $RMS_{residual} = f(\Delta x)$ for 7-mm pupils. 99% of the treated eyes can be expected to end up with RMS residual aberrations (orders 2-6) below the uppermost curve after refractive treatments with decentrations up to 2 mm in magnitude. 50% of the examined eyes are expected to have RMS residual wavefront aberrations that are located below the median curve. Intersecting these curves with the rating criteria (MA, OT, $P_{10}$) provides the data that were statistically evaluated for Fig. 4.

![Graph showing RMS residual wavefront error as a function of lateral translation for 7-mm pupil data.](image)

**Figure 3.11: Rms residual wavefront error (Zernike orders 2 to 6) as a function of the lateral decentration (7-mm pupil data).** 99% of the treated eyes can be expected to end up with RMS residual aberrations (orders 2-6) below the uppermost curve after refractive treatments with decentrations up to 1.5 mm in magnitude. 50% of the examined eyes are expected to have RMS residual wavefront aberrations that are located below the median curve.

Fig. 3.12, 3.13 and 3.14 show the expected percentage of normal eyes to meet the three image quality criteria MA, $P_{10}$ and OT at a certain amount of decentration for the three pupil sizes. In general, alignment becomes less critical with smaller pupil sizes, as the influence of the higher-order aberrations decreases. Assuming
a misalignment of 0.25 mm, the diffraction limit could only be achieved in 10% of the measured eyes with a 7-mm pupil, but in 90% with a 3-mm pupil (Fig. 3.12). Increasing the centration error to 0.5 mm, the Maréchal criterion wouldn't have been fulfilled in any eye with a 7-mm pupil. However, in 35% of the eyes with a 3-mm pupil, this goal would have been reached. In Fig. 3.13, the criterion \( P_{10} \) would still be met in 45% of the eyes with 7-mm pupils, and even in 85% with 3-mm pupils. If the decentration could always be kept below 0.45 mm, none of the examined eyes with a 7-mm pupil would suffer from a loss of optical quality (i.e., an increase in the RMS value) after surgery, as the criterion OT is met by all of them (Fig. 3.14).

Figure 3.12: Expected percentage of normal eyes to meet the MA criterion at a certain amount of lateral decentration for the three pupil sizes.

The alignment accuracies required to fulfill the three criteria in 95% of the examined eyes are summarized in Tab. 3.7.

Rotation:
The correlation of the original cylinder of each eye and the corresponding torsional alignment accuracy for 7-mm pupils is shown in Fig. 3.15 for treatments designed to correct only sphero-cylindrical errors and for treatments aspiring correction of both, sphero-cylindrical and higher-order aberrations. In Fig. 4a the
optical quality after the treatment was expected to fulfill the criterion MA. The criterion $P_{10}$ was met for 7-mm pupils in all the cases depicted in Fig. 4b. An exponential fit with $R^2 > 0.999$ was found for the data of the pure sphero-cylindrical
3.4 Investigation with measured wavefront data

<table>
<thead>
<tr>
<th></th>
<th>Pupil diameter</th>
<th>95% fulfill criterion MA</th>
<th>95% fulfill criterion $P_{10}$</th>
<th>95% fulfill criterion OT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral centration</td>
<td>3 mm</td>
<td>0.21 mm</td>
<td>0.41 mm</td>
<td>0.85 mm</td>
</tr>
<tr>
<td></td>
<td>7 mm</td>
<td>0.07 mm</td>
<td>0.22 mm</td>
<td>0.62 mm</td>
</tr>
</tbody>
</table>

Table 3.7: Required alignment accuracies to fulfill the three criteria in 95% of the examined eyes.

treatments (solid lines in Fig. 4). Since correction of the higher-order aberrations was not planned in these cases, the RMS value after the treatment was only rated relative to the preoperative RMS value. An original cylinder of 1.0 D was found to tolerate torsional misalignments of up to 8 degrees until the $P_{10}$ criterion, and 2 degrees until the MA criterion is violated for a 7-mm pupil.

![Figure 3.15: Correlation of the original (i.e., pre-operative) cylinder of each eye and the required torsional alignment accuracy (7-mm pupil data) for treatments designed to correct only sphere and cylinder and for treatments aspiring correction of both sphero-cylindrical and higher-order aberrations. Optical quality after the treatment was expected to fulfill the criterion MA (left) or the criterion $P_{10}$ (right).](image)

Data points of the treatments trying to correct both sphero-cylindrical and higher-order aberrations are scattered and mostly located below the curve for the pure cylinder, as shown in Fig. 3.15. The non-rotationally symmetric higher-order aberrations render the corrections more sensitive to torsional misalignments compared to pure cylindrical corrections. In some cases with small amounts of original
3.4 Investigation with measured wavefront data

cylinder (< 1.0 D), the alignment tolerance range can decrease from 30 degrees down to 5 degrees (data points formerly located on the solid line moved downwards in Fig. 3.15, right) when trying to achieve the same improvement in image quality through the treatment ($P_{10}$ criterion in this case).

Fig. 3.16 shows a choice of four statistically relevant functions $RMS_{\text{residual}} = f(\Delta \gamma)$ for 7-mm pupils, when correction of the orders 2-6 is planned. 99% of the treated eyes can be expected to end up with RMS residual aberrations (orders 2-6) below the uppermost curve after refractive treatments with torsional misalignments up to 23 degrees in magnitude. 50% of the examined eyes are expected to have RMS residual wavefront aberrations that lie below the median curve. The functions $f(\Delta \gamma)$ of 80% of the sampled normal eyes are located in between the 10th and the 90th percentile curves. Intersecting these functions with the rating criteria provides the data that was statistically evaluated for Fig. 6.

![Figure 3.16: RMS residual wavefront error (Zernike orders 2 to 6) as a function of the torsional alignment error (7-mm pupil data). 99% of the treated eyes can be expected to end up with RMS residual aberrations (orders 2-6) below the uppermost curve after refractive treatments with torsional misalignments up to 23 degrees in magnitude. 50% of the examined eyes are expected to have RMS residual wavefront aberrations that lie below the median curve, and so on.](image)
Fig. 3.17, Fig. 3.18 and Fig. 3.19 show the expected percentage of normal eyes to meet the different image quality criteria MA, \( P_{10} \) and OT at a certain amount of torsional misalignment for the three pupil sizes. These results were obtained for treatments designed to correct both, sphero-cylindrical errors as well as higher-order aberrations. In general, alignment becomes less critical with smaller pupil sizes, as the influence of the higher-order aberrations decreases. Assuming a misalignment of 5 degrees, the diffraction limit could only be achieved in 15% of the measured eyes with a 7-mm pupil, but in 85% with a 3-mm pupil (Fig. 3.17). Increasing the torsional alignment error to 10 degrees, the Maréchal criterion would not have been fulfilled in any eye with a 7-mm pupil. However, in 45% of the eyes with a 3-mm pupil, this goal would have been reached. In Fig. 3.18, with a 10 degree alignment error, the criterion \( P_{10} \) would still be met in 45% of the eyes with 7-mm pupils, and even in 85% with 3-mm pupils. If the torsional alignment error could always be kept below 15 degrees, none of the examined eyes with a 7-mm pupil would suffer from a loss of optical quality after surgery, as the criterion OT is met by all of them (Fig. 3.19).

![Graph showing expected percentage of normal eyes to meet the MA criterion.](image)

**Figure 3.17**: Expected percentage of normal eyes to meet the MA criterion at a certain amount of torsional misalignment for the three pupil sizes.

The alignment accuracies required to fulfill the three criteria in 95% of the examined eyes are summarized in Tab. 3.8.
3.4 Investigation with measured wavefront data

Figure 3.18: Expected percentage of normal eyes to meet the P_{10} criterion at a certain amount of torsional misalignment for the three pupil sizes.

Figure 3.19: Expected percentage of normal eyes to meet the OT criterion at a certain amount of torsional misalignment for the three pupil sizes.
3.4 Investigation with measured wavefront data

<table>
<thead>
<tr>
<th>Pupil diameter</th>
<th>95% fulfill criterion MA</th>
<th>95% fulfill criterion $P_{10}$</th>
<th>95% fulfill criterion OT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 mm</td>
<td>3 deg</td>
<td>6 deg</td>
<td>29 deg</td>
</tr>
<tr>
<td>7 mm</td>
<td>1 deg</td>
<td>4 deg</td>
<td>21 deg</td>
</tr>
</tbody>
</table>

*Table 3.8: Required alignment accuracies to fulfill the three criteria in 95% of the examined eyes.*

### 3.4.3 Discussion

This study investigated the effect of constant rotational and lateral misalignment of the ablation zone on the post-operative optical outcome. The aim was to provide information on the lateral and rotational alignment accuracy needed in wavefront-guided refractive surgery to improve the ocular optics to a sought level in a certain percentage of normally aberrated eyes.

It was shown, that in order to achieve the diffraction limit in 95% of the measured normal eyes for a 7-mm pupil, alignment of wavefront-guided treatments would have to be performed with a lateral precision of approximately 70 microns or better (Fig. 3.12). Centration with a precision of 200 microns or better is enough to improve the optics in a majority of eyes with 7-mm pupils to the level of the best 10% in an uncorrected population (Fig. 3.13).

Rotational precision of approximately 1 degree or better (Fig. 3.17) was shown to be necessary to achieve the diffraction limit in 95% of the measured normal eyes for a 7-mm pupil. As alignment becomes less critical in smaller pupil sizes, the same optical quality would result in 95% of the 3-mm pupils providing a lateral accuracy of 0.2 mm or better, and a rotational accuracy of 3 degrees or better. Rotational misalignment smaller than 5 degrees is necessary to improve the optics in a majority of eyes with 7-mm pupils to the level of the best 10% in an uncorrected population (Fig. 3.18).

The comparison of wavefront-guided treatments with classical sphero-cylindrical procedures for a 7-mm pupil demonstrated stricter requirements for torsional alignment when trying to correct higher-order aberrations, especially in eyes with small original cylinder. The data points in Fig. 3.15, which are all lo-
Investigation with measured wavefront data

3.4 Investigation with measured wavefront data

cated on the solid line for pure sphero-cylindrical corrections, move downwards to smaller alignment tolerances when wavefront-guided surgery is simulated. In some cases (Fig. 3.15, right), the torsional alignment tolerance decreased by 25 degrees (points moved downwards in diagram) if correction of not only the sphero-cylindrical errors, but also of the higher-order aberrations is planned.

Guirao et. al. [28] discussed the effect of cyclorotations and lateral shifts of an ideal correction pattern on the ocular modulation transfer function and the RMS of the residual wavefront error based on 6-mm pupil data from 10 eyes. The mean value and standard deviation of the RMS wavefront aberrations including defocus and astigmatism was 0.97 ± 0.47 mm. The image quality did not fall below the Rayleigh limit for tolerances until the rotation angle was 3 deg and until the translation was at least 0.1 mm, which is consistent with our results obtained for a cohort of eyes with RMS wavefront aberrations including defocus and astigmatism of 1.22 ± 1.05 mm for 6-mm pupils. Furthermore, they found, that the benefit of additionally correcting higher-order aberrations decreases when the misalignment increases. These results, however, were based on a very small number of eyes.

Alignment of the ablation zone, whether lateral or torsional, appears to be one of the most critical factors in corneal laser surgery. Rough centration based on the operators judgment might not be accurate enough to achieve a significantly improved optical quality in a high percentage of the treated eyes. Our results demonstrated, that precise torsional alignment is of much higher importance in aberrometry-guided treatments than in conventional sphero-cylindrical treatments.
3.5 Summary

The theoretical calculations on single types of aberrations have shown that the amount of translation at which the post-op to pre-op Strehl ratio falls below 1 is in the order of 200 microns for 6-mm pupils. This ratio, that can be considered a benefit factor, decreases with growing translation. Values higher than 1 indicate an improvement of the image quality compared to the pre-operative state. Values smaller than 1 characterize a decrease due to translation of the correction. The above criterion OT, which is based on wavefront RMS, is similar to the Strehl benefit factor of the preliminary study. It suggests a maximal translation of about 400 microns in order not to decrease the optical performance compared to the pre-operative state. The study performed with patient data further suggests tolerable translations of about 200 microns to achieve the level of the best 10% of the investigated 130 eyes in an uncorrected population for 7-mm pupils, and translations of 70 microns or better to achieve a nearly diffraction limited optics for that pupil size.

The tolerance for rotational misalignment strongly depends on the meridional degree m of the Zernike polynomials. The post-op Strehl ratio takes a minimum at a rotation angle of $\pi/m$ due to the periodic nature of the meridional Zernike terms. Thus, the larger m, the smaller the tolerance angle to achieve the same relative benefit from the treatment. It should be noted that rotation of a 5th order aberration might induce smaller absolute errors than rotation of cylinder, the relative benefit of the correction however might be significantly smaller. The study performed with patient data has shown that rotational misalignment smaller than 5 degrees is necessary to improve the optics in a majority of eyes with 7-mm pupils to the level of the best 10% in an uncorrected population. Rotational precision of approximately 1 degree or better is necessary to achieve the diffraction limit in 95% of the measured normal eyes for a 7-mm pupil.

The comparison of wavefront-guided treatments with classical sphero-cylindrical procedures for a 7-mm pupil demonstrated stricter requirements towards torsional alignment when additionally trying to correct higher-order aberrations, especially in eyes with small original cylinder.
Translation and rotation of the optical zone was shown to be tolerable up to a certain degree, that depends on each eye’s individual amount of original aberrations. Procedures have to be elaborated assuring that the translatory and rotational alignment of the ablation is done within a certain tolerance range. Rough centration based on the surgeons judgement is not accurate enough to achieve a significantly improved optical quality in a high percentage of the treated eyes. Exact registration requires information on the spatial orientation of the measured wavefront aberration pattern relative to the eye. This could either be done by means of artificial marks on the limbus or by registering natural iris structures.
References


Chapter 4

Single spot misalignment

4.1 Introduction

4.1.1 Scanning spot laser surgery

In scanning spot laser surgery, the ideal corneal surface is approximated by directing and overlapping a finite number of laser pulses. Displacements of single laser shots from their ideal overlap position might have the potential to significantly increase the post-operative surface roughness. Furthermore, it might even introduce significant contour deviations from the ideal post-operative surface, which can be described through modal aberrations.

Natural eye movements and improper fixation can not be avoided during refractive surgery. Even though fixating a target significantly suppresses the power spectral density of the eye motion, involuntary movements are not eliminated. Whereas early treatments with broad beam lasers were done without movement compensation, the introduction of scanning beam lasers was believed to necessitate the development of active eye-tracking systems as the exact placement of each laser pulse had become more crucial than ever before.

Fig. 4.1 (top) illustrates a displacement of a single spot ablation paraboloid from its ideal position at $x = 0$. Only the cross volume between the two paraboloids contributes to the intended ablation at the position $x = 0$. Fig. 4.1 (bottom) shows the ablated percentage of the intended ablation volume at position $x = 0$ as a function of the spot radius (normalized with the spot displacement) for a constant spot displacement.
4.1 Introduction

Figure 4.1: Above: Displacement of a single spot ablation paraboloid from its ideal position at $x = 0$. Only the cross volume between the two paraboloids contributes to the intended ablation at the position $x = 0$. Below: Percentage of the intended ablation volume at position $x = 0$ as a function of the spot radius for a constant spot displacement. The larger the spot radius for a constant displacement, the smaller the portion of misguided ablation energy, and thus, the higher the accuracy of the ablation.

The larger the spot radius for a constant displacement, the smaller the portion of misguided ablation energy, and the higher the accuracy of the ablation. For spot radii smaller than or equal to half the displacement, none of the incoming pulse
energy is absorbed within the paraboloid volume at the intended position. For spot radii larger than three times the displacement, the gradients of the curve significantly decrease and the relative effect of the displacement becomes less critical. Consequently, as laser systems move to smaller spot sizes the sensitivity towards transverse placement errors increases.

### 4.1.2 Reasons for spot displacement

If no active eye tracking is performed during the procedure, spot displacement can result from ocular movement and from inaccurate shot placement by the laser delivery system. The later would also include systematic displacements due to an initial offset error.

If the procedure is performed with active eye tracking, possible spot displacements would result from incomplete compensation of the ocular movements due to technical deficiencies of the eye tracking system. The following list provides an overview of some critical points:

- In video eye-tracking, the accuracy of the movement registration depends on the contrast between the pupil and the iris provided by the illumination. Taylor et al. [1] examined an eye-tracking system used in refractive surgery and found an accuracy of 0.06 mm for an intact cornea and 0.1 mm for a cornea with a thin flap removed.

- The measurement of the eye position and the subsequent adjustment of the laser beam requires some time. As the eye can move again during this period, it must be kept as small as possible to reduce the latency error.

- Additional inaccuracies can emerge from an effect referred to as the parallax error of video eye-tracking. In case of eye movements the entrance pupil is observed excentrically by the camera and the overlying target point on the cornea is shifted. This effect will be investigated in 4.3.

Only limited data on the theoretical impact of intra-operative eye movements on the optical outcome of refractive treatments with a scanning spot laser are published. Shimmick et al. analyzed the effects of axial and transverse displacement of the ablation on the optical quality after excimer laser photoablation with a
broad beam laser (spot diameters as large as 7 mm) [2]. They concluded, that transverse displacement during laser treatment has the potential to induce refractive aberrations. However, definite tolerances for transverse displacement could not be determined until the smoothing properties of the corneal healing process were better understood. Ludwig et al. have simulated the retinal image contrast after photorefractive keratectomy with a diaphragm mask [3]. The simulations indicated that PRK can reduce retinal contrast markedly. The most critical factor for such a reduction was shown to be the step height of the ablation pattern. Micromovements during PRK were also found to lead to partial loss of retinal image contrast. Taylor et al. investigated the effect of different eye movement scenarios on PRK results. They found that though eye movements reduce the contrast sensitivity of the image, they have minimal effect on the final visual outcome [4].

There is a lack of information on the demands on eye-tracker specifications for scanning spot refractive surgery. Unanswered questions on the ideal combination of treatment parameters such as the ablation depth per pulse and the spot diameter for a given eye tracker performance are of importance in that context.

4.1.3 Eye movements

*Types of movements:*
There are various types of eye movements with different characteristics and serving different purposes [5]. The voluntary movement of the eyes to a new location of fixation is performed by executing a saccade. The average velocity is in the order of 200 degs/s with peaks up to 500 degs/s. Amplitudes up to 15 degs were measured. Drifts, microsaccades and tremors occur during fixations and are physiologically determined. Slow drifts away from the point of fixation with amplitudes up to 6.0 mins are followed by fast microsaccades (amplitudes up to 50 mins) that appear to have a drift-correcting function. Tremors are high-frequency oscillations of the eye with amplitudes up to 20 mins that serve to continuously shift the image on the retina, thus calling fresh retinal receptors into operation.

There are a number of reports on eye movements during refractive surgery to be found. Shimmick et al. [2] presented a video analysis of transverse eye movements
acquired during seven refractive surgeries. However, only data from one patient was discussed in their report. They found lateral shifts up to ±0.4 mm from the offset axis. Gobbi et al. [6] reported the development of a video-based eye tracking system for refractive surgery. In their report of the system, they didn’t report the mean and variance of the eye motion unfortunately. Schwicgerling et al. [7] studied movements in 5 eyes, finding a standard deviation of approximately 0.10 mm for the lateral shift. Peak movements up to 0.35 mm from the centration offset were observed. Our group has performed eye movement measurements in 6 eyes undergoing corneal laser surgery (unpublished data, Mrochen et al., 1997). A standard deviation of 0.135 mm was determined for the lateral movements.

Methods of maintaining alignment:
There are several methods used to maintain alignment between the patient’s eye and the laser beam. They range from completely subjective to fully automated measurement and control systems [8]:

- **Passive fixation**: This method requires the patient to fixate on an alignment light. It is completely subjective and is not capable of eliminating involuntary eye movements. A major risk with this method is that the patient may change the leading eye for fixation during the process.

- **Suction rings**: Alignment may be improved by a suction ring fixed onto the cornea. A significant portion of the high frequency eye movements might be suppressed, however, suction rings may be decentered, lose suction during the procedure, distort the cornea, and interfere with airflow around the cornea.

- **Eye tracking**: This is the only method that is completely independent of subjective influences by the patient and doctor. A sensing device acquires an image of the patient’s eye. A processing subsystem calculates the position of the eye from that image, and a control system moves the laser beam to compensate for any change in eye position.

Movement compensation through active eye tracking:
A number of different eye tracking techniques are used in a variety of applications. The main techniques applied in refractive surgery are photoelectric and video-based eye tracking. Electro-oculography (EOG) is not useful for this application:
4.2 Effect of eye-tracker latency

- **Electro-Oculography**: EOG is one of the oldest and most common techniques for measuring eye movements. Electrodes are mounted around the eye on the skin and impedance changes are measured. This method is not applied in refractive surgery, as it does not allow an accurate absolute measurement of the eye position.

- **Photoelectric techniques**: These techniques detect eye movements from changes in reflected light. Either focused spots, slits, or rings of light are projected onto the cornea and the response from multiple light detectors is analyzed using analog signal processing.

- **Video eye tracking**: In video eye tracking, infrared light illuminates the eye and IR-sensitive cameras acquire the image of the eye. Image processing algorithms are used to determine the pupil center and to assess its shift from the offset position.

*Latency in active eye tracking:*

The temporal delay between the actual eye movement and the subsequent compensation through the laser delivery system is one of the most important parameters in active eye tracking. In detail, this latency is determined by the following components: The time required to measure the eye position (in turn determined by the processing speed and the image integration time). The time necessary to adjust the lasers firing position by moving the deflection mirrors. Finally, the latency caused by a potential timing difference between firing and tracking. As the eye can move again during this time, latency must be kept as small as possible. In current laser systems the latency is specified between 2 and 90 ms [9]. Eye tracker latency leads to incomplete compensation of the eye movements, leaving so called movement artefacts.

4.2 Effect of eye-tracker latency

The aim of this study was to investigate the stability of scanning spot laser corrections of higher-order aberrations in the case of incomplete eye movement compensation due to eye-tracker latency.

All simulation procedures were programmed in our lab using Matlab (Software version 6, The MathWorks, Inc.).
4.2 Effect of eye-tracker latency

<table>
<thead>
<tr>
<th>Radial order n</th>
<th>Type of WA</th>
<th>Zernike polynomial</th>
<th>[microns]</th>
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<tr>
<td>3</td>
<td>Primary coma</td>
<td>$Z_{3}^{-1}$</td>
<td>0.600</td>
</tr>
<tr>
<td>4</td>
<td>Primary spherical</td>
<td>$Z_{4}^{0}$</td>
<td>0.750</td>
</tr>
<tr>
<td>5</td>
<td>Secondary coma</td>
<td>$Z_{5}^{-1}$</td>
<td>0.075</td>
</tr>
<tr>
<td>6</td>
<td>Secondary spherical</td>
<td>$Z_{6}^{0}$</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Table 4.1: Zernike coefficients used for the simulated corrections. The values are reported according to the standards of the VSIA task force for a 5.7-mm pupil.

Simulations of PRK treatments were performed for various combinations of laser spot diameter, laser ablation depth per pulse and eye tracker latency. The laser parameter combinations that provide the best optical outcomes in the presence of eye movements were determined. In addition, this study was intended to provide an indication on the tolerable delay times of eye tracking systems for the correction of certain types of higher order aberrations. The results were published in [Bueeler M, Mrochen M. Interaction of eye-tracker latency, spot size, and ablation pulse depth on the correction of higher-order wavefront aberrations with scanning spot laser systems. Accepted for Publication in J Refract Surg. 2004].

4.2.1 Subjects and Methods

Correction of higher-order wavefront aberrations:
The corneal tissue to be ablated corresponds to the difference between an imperfect pre-operative corneal surface and the ideal contour of the cornea. The profile of corneal tissue to be ablated can be deduced from the measured wavefront aberrations (see appendix). For these simulations, one representative type of wavefront error was chosen from each of the higher Zernike orders three to six. The magnitude of each wavefront error to be corrected was set to a physiologically high value that corresponds to three times the mean plus standard deviation ($C_{ref}$) of the Zernike coefficients measured in a large population by Porter et al. [10]. The coefficients in Tab. 4.1 are reported in VSIA notation [11] for a 5.7-mm pupil. The magnitude of the sixth order aberration was extrapolated from the data presented by Porter and coworkers.
4.2 Effect of eye-tracker latency

*Laser shot list calculation:*  
Ablation profiles for a 6-mm optical zone were calculated for the four different types of wavefront aberrations. The corresponding laser shot positions were determined for a Gaussian beam profile with a software program currently used in our research lab (Wavelight Laser Technologie AG, Erlangen, Germany). For the calculation of the laser shot list, the central depth \( d_c \) and the base diameter \( D \) of the ablation paraboloid had to be provided. Scanning spot ablation was simulated for four different laser spot diameters \( D \) (250, 500, 750, 1000 microns) and four different central ablation depths per pulse \( d_c \) (0.25, 0.50, 0.75, 1.00 microns). Assuming an ablation efficiency of \( m = 0.34 \) microns and a threshold fluence of \( f_{th} = 50 \) \( mJ/cm^2 \) [13, 14, 15], the following peak fluences can be deduced from the central ablation depths \( d_c \) using Eq. 4.1: 105 \( mJ/cm^2 \) for \( d_c = 0.25 \) microns, 218 \( mJ/cm^2 \) for \( d_c = 0.50 \) microns, 454 \( mJ/cm^2 \) for \( d_c = 0.75 \) microns and 947 \( mJ/cm^2 \) for a depth \( d_c = 1.0 \) microns. Eq. 4.1 is a Beer’s law approximation of the ablation process [12].

\[
f_{peak} = f_{th} \cdot e^{\frac{d_c}{m}}
\]

(4.1)

*Eye movement data:*  
Eye movement data were taken from an earlier study performed in 6 eyes undergoing corneal laser surgery (unpublished data, Mrochen et al., 1997). An infrared video camera with an acquisition rate of 250 Hz (temporal resolution of 4 ms) was used to track the movements of the eyes during surgery. A resolution of 0.005° was provided for the determination of the eye position within a range of ±30° horizontally and ±20° vertically.

The readings from one particular eye were further processed to serve as a model data set for further investigations (Fig. 4.2, top left). Ocular drift was eliminated by fitting and subtracting a linear function to the measurement points. Data offset was eliminated by shifting the mean value to zero. The original standard deviations of 0.092 mm for the horizontal, and 0.075 mm for the vertical movement components, were adjusted to be 0.10 mm in both directions to match
the reported data of Schwiegerling et al. [7]. The power spectral density of
the motion data is comparable to the values reported by Huppertz et al. [8],
with the main frequency contents being below 60 Hz (Fig. 4.2, top right). The
lower graph on the right side shows amplitude distribution of the movement data.

In order to directly link the laser shot list with the measured eye movement data
obtained at 250 Hz, it was assumed, that ablation was taking place at the same
repetition rate i.e. at 4 ms intervals. For treatments performed without eye
tracking, the movement data and the ideal shot position data were merged to
obtain the actual impact locations on the cornea.

Due to the latency of eye tracking systems, intra-operative eye movements are
only partially compensated. The movement artifacts resulting from a certain eye
tracker latency $\Delta t$ can be deduced from the measured eye movement data array
by subtracting measurement points with the time interval $\Delta t$ in between. At the
moment $t$ the eye is at the horizontal position $x(t)$. The tracking system however
still tries to compensate the movement registered at the moment $t-\Delta t$, resulting
in a movement artifact of $e(t) = x(t)-x(t-\Delta t)$ (Fig. 4.3). As the eye movements
were acquired at 250 Hz, the simulated latencies $\Delta t$ were multiples of the basic
measurement interval (4 ms). The difference of neighboring measurement points
for instance would thus correspond to a latency of 4 ms. The movement artifacts
are depicted in Fig. 4.2 (middle and bottom, left) for two different latencies, 96
ms and 16 ms. The standard deviation of the untracked horizontal eye movements
can be reduced from 0.100 mm (top) to 0.061 mm (middle) if eye tracking with a
latency of 96 ms is performed. Further improvement of the delay characteristics
to 16 ms reduces the standard deviation of the movement artifacts to 0.020 mm
(bottom).

For the simulation of treatments performed with latency-afflicted movement com-
pensation, the ideal shot position data and the movement artefact data were
merged to obtain the actual impact locations of the laser shots on the cornea.

The following cases of eye tracker performance were simulated in this study: 1)
no latency i.e. perfect compensation of all eye movements, 2) latencies of 4, 32
and 96 milliseconds, 3) no eye tracking i.e. full impact of all eye movements.
4.2 Effect of eye-tracker latency

Eye movements measured at 250 Hz
No compensation
Std = 100 microns

Compensation with 96 ms latency
Std = 61 microns

Compensation with 16 ms latency
Std = 20 microns

Figure 4.2: Eye movement model data of one subject measured over 30 sec at a rate of 250 Hz (top left) and corresponding movement artifacts after incomplete compensation by an eye-tracker working with a latency of 96 ms (middle left) and 16 ms (bottom left). The power spectral density and the amplitude distribution of the original motion data are shown in the right column.
4.2 Effect of eye-tracker latency

Figure 4.3: Eye movement artifacts resulting from a certain eye tracker latency $\Delta t$. At the moment $t$ the eye is at the position $x(t)$. The tracking system however still tries to compensate the movement registered at the moment $t-\Delta t$, resulting in a movement artifact of $e(t) = x(t)-x(t-\Delta t)$.

Scanning spot ablation and data analysis:
Scanning spot ablation was numerically simulated on a plane surface matrix of the size 10x10 mm (801 x 801 elements). The ablation paraboloid corresponding to a certain spot size and ablation depth was discretized using the same discretisation interval. Consecutive subtraction of the these paraboloids from the plane surface at the various shot positions yielded the approximation of the overall ablation volume. Fig. 4.4 depicts the ablated profiles of a primary spherical aberration. The ideal ablation profile and a cross section are shown in the first column. Columns two and three depict the profiles in the cases of 0 and 96 ms eye-tracker latency. In the course of the data analysis process, these approximated profiles were subtracted from the ideal profile, and the variance of this profile deviation was calculated to obtain a first measure for the surface quality after the correction. Fig. 4.5 depicts the central sector (2 x 2 mm) of the cornea after a simulated myopic correction of -4D. Here, ablation was simulated on a spherical surface with a radius of 7.5 mm. The cornea on the left resulted from a treatment performed with ideal movement compensation (0 ms latency) and features a very smooth surface. The rough surface on the right is the result of the same correction done with a tracker latency of 96 ms.

The variance of the post-operative profile deviation was calculated over a pupil diameter of 6 mm, and set into relation with the variance of the pre-operative surface deviation, which corresponds to the inverse of the ablation profile. A ratio of these two values that is smaller than 1.0 represents an increase in surface
4.2 Effect of eye-tracker latency

quality. In contrast, the surface quality is decreased for values above 1.0.

Figure 4.4: Example of ablated profiles of a primary spherical aberration: ideal ablation profile (first column), profile ablated with ideal movement compensation i.e. 0 ms latency (middle column), profile ablated with tracker latency of 96 ms (right column).

Figure 4.5: Central section of the cornea of 2x2 mm after a simulated myopic correction of -4D. The surface on the left results from a treatment with ideal movement compensation. The surface on the right results from a correction performed with an eye-tracker latency of 96 ms.
The differentiation of contour and roughness is important for the characterization of the optical quality after scanning spot ablation. While the contour contributes to the modal aberrations (e.g. Zernike modes), the surface roughness causes scattering that will be more or less suppressed by the epithelial smoothing process after PRK or the repositioning of the flap after LASIK.

Following the mathematical model of corneal surface smoothing after laser refractive surgery by Huang et al. [16], a first-order Butterworth low-pass filter (Eq. 4.3) was applied to the ablated profiles (Eq. 4.2).

\[ \Delta H' = F' \cdot A' \]  

(4.2)

with

\[ F'(\omega_x', \omega_y') = \frac{1}{1 + (\omega_x'/\omega_c')^2 + (\omega_y'/\omega_c')^2} \]  

(4.3)

\( A'(\omega_x', \omega_y') \) is the two-dimensional Fourier transform of the ablated profile and \( \Delta H' \) is the smoothed profile in the frequency domain. \( \omega_x' \) and \( \omega_y' \) are the spatial radian frequencies of the spatial variables \( x' \) and \( y' \). \( F' \) is the filter transfer function in the frequency domain, and \( \omega_c' = 1/s \) is the cutoff frequency calculated from the smoothing constant \( s \).

A smoothing constant of \( s = 0.25 \) mm was assumed for the low-pass filtering. The smoothed profile was then subtracted from ideal ablation profile to determine the contour deviation.

The optical path difference (OPD) can be calculated in a first order approximation from a refracting interface using the difference of the refractive indices in front of and behind the surface. In the calculation model presented here, it was assumed, that the superficial tear film follows the contour deviation of the epithelium after re-epithelisation. Hence the anterior contour of the tear film separates the two media air and water. The posterior surface of the tear film follows the epithelial layer that smoothed the rough stromal surface after the ablation. Hence
4.2 Effect of eye-tracker latency

the surface separates the two media water and epithelium. The epithelial back surface follows the complete post-operative stromal surface deviation including both contour and roughness (Fig. 4.6).

Figure 4.6: The optical path difference resulting from the anterior corneal surface is calculated using a model with three refractive interfaces embedded in the following media: air, tear film, epithelium and stroma.

For each of the three interfaces, the optical path difference was calculated separately:

\[
\begin{align*}
\text{OPD1} &= S_{\text{contour}} \cdot (n_{\text{water}} - n_{\text{air}}) \\
\text{OPD2} &= S_{\text{contour}} \cdot (n_{\text{epithel}} - n_{\text{water}}) \\
\text{OPD3} &= S_{\text{contour+roughness}} \cdot (n_{\text{stroma}} - n_{\text{epithel}})
\end{align*}
\]

The refractive indices used in the above calculations are listed in Tab. 4.2. According to the principle of superposition of aberrations [17], the overall OPD for this model was obtained from the sum of the three path differences:

\[
\text{OPD} = \text{OPD1} + \text{OPD2} + \text{OPD3}
\]

To quantify the post-operative retinal image quality, the point spread function (PSF) and the Strehl intensity ratio were calculated from the OPD for a pupil.
4.2 Effect of eye-tracker latency

The OPD is first used in the calculation of a complex-valued pupil function \( P(x, y) \) (Eq. 4.4).

\[
P(x, y) = D(x, y) \cdot \exp(i \frac{2\pi}{\lambda} OPD)
\]  

where \( D(x,y) \) is a pupil attenuating function. It is normally 1 over the entire pupil, but offers the possibility to consider the angular dependence of retinal sensitivity (Stiles-Crawford effect) in the calculation of the retinal point spread function.

The amplitude point spread function \( A \) with the spatial variables \( x' \) and \( y' \) in the image plane is the Fourier transform of the pupil function (Eq. 4.5).

\[
P(x, y) \longrightarrow A(x', y')
\]

Finally, the intensity point spread function \( PSF(x', y') \) is the squared modulus of the complex amplitude point spread function (Eq. 4.6).

\[
PSF(x', y') = |A(x', y')|^2
\]

The post-operative Strehl intensity ratio was further put into relation with the original Strehl ratio that resulted from the pre-operative wavefront error. A value above 1.0 represents an increase in the post-operative Strehl ratio, that is equal to an improvement in image quality. A value below 1.0 is indicating a loss in image quality.
4.2 Effect of eye-tracker latency

<table>
<thead>
<tr>
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</thead>
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<tr>
<td>Air ( (n_1) )</td>
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</tr>
<tr>
<td>Tear film (salty water) ( (n_2) )</td>
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</tr>
<tr>
<td>Epithelium ( (n_3) )</td>
<td>1.401</td>
</tr>
<tr>
<td>Stroma ( (n_4) )</td>
<td>1.3771</td>
</tr>
</tbody>
</table>

Table 4.2: Refractive indices \( (\lambda = 589\text{nm}) \) used to calculate the OPD according to \cite{18, 19}.

4.2.2 Results

Surface variance:

Fig. 4.7 shows the ablated profiles after scanning spot correction of a vertical coma \( Z_3^{-1} \). The clear contour lines of the ideal profile (first column) become increasingly distorted with increasing latency. Even in the case of a perfect ablation without eye tracker latency, the contour lines are slightly distorted as a result of the scanning spot approximation with a finite number of laser pulses. It appears that in case of zero latency a combination of a small spot diameter and a small ablation depth per pulse provides the best approximation of the ideal profile. For increasing latencies the original structure of the coma appears to remain most recognizable for the combination of a large spot diameter and a small ablation depth per pulse.

The surface variance of the simulated post-operative profile was calculated over a pupil diameter of 6 mm, and set into relation with the variance of the ideal ablation profile. Fig. 4.8 and Fig. 4.9 show the change of this variance ratio with increasing eye-tracker latency for the primary spherical aberration \( Z_1^0 \) and the secondary coma \( Z_5^{-1} \) respectively. The two hatched areas in each graph belong to the extreme spot diameters of 250 and 1000 microns and indicate the investigated range of the central ablation depth per pulse between the two boundary values of 0.25 and 1.00 microns. For all four types of aberrations it can be observed, that the larger the eye tracker latency, the higher the surface variance. Thus, increasing the eye tracker latency increases the deviations from the attempted profile.
4.2 Effect of eye-tracker latency

Figure 4.7: Ablated profiles after simulated scanning spot correction of a vertical coma $Z_{3}^{-1}$. Each row corresponds to a different combination of spot size $D$ and central ablation depth per pulse $d_c$. The first column shows the ideal ablation profile. The following columns depict the approximations of the same profile resulting from scanning spot laser ablation performed with increasing eye tracker latency.

In case of perfect eye movement compensation (no latency), the variance is lowest for corrections performed with a small spot of 250 microns and a small central ablation depth per pulse of 0.25 microns. Thereby, the benefit of a smaller ablation depth per pulse appears to be significantly higher than the one of a smaller spot diameter.

A different situation can be observed, when eye-tracker latency is considered in the simulations. The largest increase in the variance is found for a small spot diameter (250 microns) and a large ablation depth per pulse (1.0 microns). Corrections done with larger spot diameters appear to be more stable towards eye movement artifacts, as the increase of the surface variance is generally smaller. The most stable surface variance is obtained with combinations of a large spot diameter (1000 microns) and a small ablation depth per pulse (0.25 microns).
4.2 Effect of eye-tracker latency

Figure 4.8: Surface variance ratio post-op to pre-op as a function of the eye-tracker latency for the primary spherical aberration.

Figure 4.9: Surface variance ratio post-op to pre-op as a function of the eye-tracker latency for the secondary coma.

case of a tracker latency of 96 ms, this spot parameter combination enables a reduction in surface variance (variance ratio < 1.0) only for the primary coma and the primary spherical aberration. In contrast, the surface variance of the sec-
4.2 Effect of eye-tracker latency

Secondary coma is increased by a factor of almost 7 compared to the pre-operative value, and the one of the secondary spherical aberration by a factor of 10. Only for the combination of a large spot diameter (1000 microns) and an ablation depth per pulse of less than approximately 0.4 microns, the variance ratio remains below one over the entire range of eye-tracker latencies up to 100 ms.

Point spread function and strehl ratio:
To assess the impact of the post-operative corneal surface deviations on image quality, the retinal point spread function (PSF) was calculated from the OPD using fourier transform. Guirao et al. [20] have shown that image quality metrics such as the point spread function provide significantly better results in predicting subjective refraction than pupil plane metrics like the wavefront aberration.

Fig. 4.10 shows PSF cross sections after correction of a primary spherical aberration ($C_4^u = 0.6$ microns, normalization factor included, 6 mm pupil diameter) with four different combinations of spot diameter and central ablation depth per pulse: 1000/1.0 microns (top), 250/1.0 microns (second from top), 1000/0.25 microns (third from top), 250/0.25 microns (bottom). The PSF intensity was normalized by the maximum intensity of the theoretical diffraction-limited PSF. The increase in post-operative surface variance with increasing eye-tracker latency is reflected in the decrease of the retinal peak intensity. For the laser parameter combination 1000/1.0 microns, the normalized maximum intensity of 0.78 in case of 0 ms latency is reduced to 0.51 if no eye-tracking is performed during the treatment (top). Reduction of the spot diameter to 250 microns at the same central depth of 1.0 microns slightly increases the maximum intensity in the case of 0 ms latency ($PSF_{max} = 0.81$). Yet, the intensity decrease for growing latency values is stronger. The peak intensity reaches a minimum of 0.13 in the case where no eye-tracking was performed (second from top). Reducing the central ablation depth to 0.25 microns while the spot diameter remains at 1000 microns as in the case of Fig. 4.10 (top) produces significantly higher peak intensities in the case of 0 ms latency ($PSF_{max} = 0.96$) as well as in the presence of eye movement artefacts. The peak intensity reaches a minimum of 0.76 in the case where no eye-tracking was performed (third from top). For the combination of a small spot diameter (250 microns) and a small ablation depth (0.25 microns), the peak intensity in the case of 0 ms latency is even slightly higher ($PSF_{max} = 0.97$) than in the
case before. Yet, the peak intensities resulting from latency-afflicted ablations are smaller and reach a minimum of about 0.5 if no eye-tracking is performed (bottom).

The normalized peak intensity of the point spread function is called the Strehl intensity ratio. Fig. 4.11, 4.12, 4.13 and 4.14 show the latency dependent changes in the Strehl intensity ratio compared to the pre-operative Strehl ratio. Each graph illustrates the results for one of the four investigated types of aberrations. The two hatched areas in each graph belong to the extreme spot diameters of 250 and 1000 microns and indicate the investigated range of the central ablation depth per pulse between the two boundary values of 0.25 and 1.00 microns.

The most stable Strehl values in the presence of eye movement artefacts are obtained for combinations of a large spot diameter (1000 microns) and a small ablation depth per pulse (0.25 microns). Even in case of a tracking delay of 96 ms during the treatment of the coma (Fig. 4.11), the Strehl ratio increases by a factor of 12 compared to the pre-op value. The poorest results are obtained for a small spot diameter (250 microns) and a large ablation depth per pulse (1.0 microns).

In general, the smaller the axial dimensions of an optical error to be treated, the smaller the relative benefit of the correction. Even in case of ideal movement compensation (0 ms latency), the Strehl ratio is improved by a factor of only 1.25 at best if an aberration with a small axial depth such as the secondary coma (Fig. 4.13) is corrected. The same would apply for the correction of a small primary coma. Accordingly, spot displacement due to tracker latency was found to have the potential to severely deteriorate image quality when correcting small aberrations with a small spot and a large ablation depth per pulse. If the latency is in the order of 100 ms, the post-operative Strehl ratio can drop to only 60% of the pre-operative Strehl ratio (Fig. 4.13, 4.14). It should be noted that, in general, fifth and sixth order aberrations have smaller axial dimensions than third or fourth order aberrations.
4.2 Effect of eye-tracker latency

Figure 4.10: PSF cross sections after correction of a primary spherical aberration with different combinations of spot diameter and ablation depth per pulse: 1000/1.0 microns (top), 250/1.0 microns (second from top), 1000/0.25 microns (third from top), 250/0.25 microns (bottom). Simulations were performed for different eye-tracker latencies.
4.2 Effect of eye-tracker latency

Figure 4.11: Strehl value ratio post-op to pre-op as a function of the eye-tracker latency for the primary coma.

Figure 4.12: Strehl value ratio post-op to pre-op as a function of the eye-tracker latency for the primary spherical aberration.
4.2 Effect of eye-tracker latency

Figure 4.13: Strehl value ratio post-op to pre-op as a function of the eye-tracker latency for the secondary spherical aberration.

Figure 4.14: Strehl value ratio post-op to pre-op as a function of the eye-tracker latency for the secondary coma.
4.2.3 Discussion

This study investigated the effect of incomplete eye movement compensation during surgery on the optical outcome after attempted correction of various higher order wavefront aberrations. The quality of the ablation was rated by the variance of the post-operative surface deviation and the post-operative Strehl intensity ratio. It was shown, that reduction of the spot diameter reduces the stability of the correction towards spot displacements. Combinations of a large spot diameter (1000 microns) and a small ablation depth per pulse (0.25 microns) yield the least decrease of the Strehl intensity ratio with increasing eye tracker latency. The strongest decrease of the retinal image quality occurs after corrections done with a small spot diameter (250 microns) and a large ablation depth per pulse (1.0 microns).

The results presented here are not only valid for spot displacements originating from eye movements or eye movement artifacts. Positioning errors of single laser shots might as well result from other inaccuracies of the laser system.

As expected, the Strehl values were found to correlate inversely with the post-operative surface variance i.e. the higher the variance of the surface deviation after ablation, the smaller the corresponding Strehl ratio. Furthermore, both measures predict the same behavior for different combinations of laser spot parameters relative to each other. Surface variance and Strehl ratio, however, do not always provide the exact same conclusions regarding improvement or degradation compared to the corresponding pre-operative values. Whereas for instance for a 250 micron spot and an ablation depth of 1.0 microns at a latency of 96 ms the post-op surface variance became significantly smaller than the pre-op variance for the primary spherical aberration (Fig. 4.8), the corresponding Strehl ratio decreased compared to the pre-op value (Fig. 4.12).

Huang et al. have shown, that the theoretical accuracy of the ablation profile approximation increases for smaller spot diameters [21] and our results obtained for the cases of 0 ms latency support these findings. However, Huang and associates did not investigate the influence of the ablation depth per pulse. Fig. 4.15 and Fig. 4.16 show the surface variance ratios plotted over the laser spot diameter.
for the two extreme cases of 0 ms latency and no eye-tracking after treatment of the primary coma. Fig. 4.17 and Fig. 4.18 depict the corresponding Strehl value ratios plotted over the laser spot diameter again for the two extreme cases of 0 ms latency and no eye-tracking. The lines in each graph represent different ablation depths per pulse. As observed earlier, the best approximation of the profile in the case of 0 ms latency is obtained with a small spot (250 microns) and a small ablation depth per pulse (0.25 microns). The post-operative surface variance amounts to only about 0.5% of the pre-operative value and the Strehl ratio increases by a factor of 18. For a spot diameter of 1000 microns and an ablation depth per pulse of 1.0 micron, the post-op value amounts to 5% of the original variance, and the Strehl ratio is 12 times higher than the original value.

![Figure 4.15: Surface variance ratio for the case of 0 ms latency after treatment of the primary coma.](image)

Nevertheless, the reduction of the ablation depth per pulse (i.e. the peak fluence) would provide a significantly higher benefit than reducing the spot diameter. The simulated corrections of the secondary spherical aberration $Z_6^0$ using a deep spot of 1.0 microns have shown, that even in the case of 0 ms latency and a small spot of 250 microns, the post-operative Strehl ratio is lower than the pre-operative value, whereas an improvement can be achieved using the large spot of 1000 mi-
4.2 Effect of eye-tracker latency

Figure 4.16: Surface variance ratio for the case of no eye-tracking after treatment of the primary coma.

Figure 4.17: Strehl value ratio for the case of 0 ms latency after treatment of the primary coma.

crons and a depth of 0.25 microns (Fig. 4.14). Unfortunately, limitations are encountered when minimizing the ablation depth. The central ablation depth
4.2 Effect of eye-tracker latency

Figure 4.18: Strehl value ratio for the case of no eye-tracking after treatment of the primary coma.

Per pulse is associated with the peak fluence of a Gaussian pulse profile. In order to maintain stability of the ablation, the peak fluence should not fall below a certain stability threshold of approximately 250 mJ/cm² (peak fluence). Additionally, both, reducing the beam diameter and minimizing the ablation depth result in a larger number of pulses necessary to ablate a certain volume of tissue. If the laser repetition rate is not appropriately enhanced to match these changes, treatment time is drastically increased. Thus any change of a parameter necessitates appropriate changes of other parameters to achieve an optimum performance.

Even in the case of a treatment performed with perfect eye movement compensation, a small spot size and a small ablation depth per pulse, the general benefit of a wavefront guided treatment appears to be negligible for optical aberrations of small axial magnitude. The examples of the secondary coma and the secondary spherical aberration predict a maximum improvement of the retinal image quality by a factor of only 1.25 and 1.1 respectively.

Both, the quest for higher ablation accuracies in the ideal, latency-free case, and
4.2 Effect of eye-tracker latency

<table>
<thead>
<tr>
<th></th>
<th>Constant ablation depth [microns]</th>
<th>Latency threshold [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary coma</td>
<td>0.25</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>6</td>
</tr>
<tr>
<td>Primary spherical</td>
<td>0.25</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>5</td>
</tr>
<tr>
<td>Secondary coma</td>
<td>0.25</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>5</td>
</tr>
<tr>
<td>Secondary spherical</td>
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<td>0.5</td>
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<td></td>
<td>1.00</td>
<td>0.5</td>
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Table 4.3: Latency thresholds derived from the Strehl intensity data when intending to reduce the spot diameter from 1000 to 250 microns at a constant ablation depth of 0.25 or 1.0 microns. Only for eye tracking systems with latencies below the threshold, changing to the smaller spot size is beneficial.

the quest for higher ablation stability in the presence of eye movement artefacts, ask for a reduction of the peak fluence. The two efforts, however, have oppositional requirements towards the magnitude of the laser spot diameter. According to Fig. 4.11 to 4.14, a latency threshold can be defined for each ablation depth per pulse at the point where the curves of two different spot diameters intersect. For eye tracking systems with latencies below this threshold, changing to the smaller spot size is beneficial. If the known latency characteristics are settled above the threshold value, better results are obtained using the larger spot diameter. It has to be assumed, that this latency threshold not only depends on the type of optical error to be treated, but also on its magnitude. Tab. 4.3 lists the latency thresholds derived from the Strehl intensity data when intending to reduce the spot diameter from 1000 to 250 microns at a constant ablation depth of 0.25 or 1.0 microns. To benefit from the spot size reduction, latency should be smaller than 4 - 6 ms when correcting the primary coma, the primary spherical aberration or the secondary coma, and smaller than 1 ms for the secondary spherical aberration.

Fig. 4.11 to 4.14 show, that even in the presence of considerable amounts of eye movement artefacts there is hardly any decrease in the Strehl ratio for a secondary coma and a secondary spherical aberration, if the treatment is performed
with a large spot diameter of 1000 microns. The reason for this can be found in both, the small axial dimensions of the higher-order ablation profiles, and the post-operative corneal smoothing process. The axial dimensions of these fifth and sixth order ablation profiles are very small (0.89 microns maximum height for the secondary spherical). Depending on the ablation depth per pulse, only 30 to 200 laser pulses are necessary to perform the entire correction. Even if single laser pulses are considerably displaced from their intended position, the low-pass-filter applied to simulate the corneal re-epithelisation is able to almost completely smooth the resulting surface due to the small dimensions of the ablation cavities. Especially when using a larger spot diameter, the number of pulses that are overlapped is reduced to a minimum. Furthermore, potential remains of the original profile after inaccurate ablation are smoothed as well in the simulation model.

The choice of the smoothing constant for the low-pass-filtering influences the magnitude of the absolute Strehl intensity values. We have repeated our calculations for a smoothing constant of 0.50 mm instead of 0.25 mm. Thus, smoothing takes place over larger surface elements and the post-operative Strehl intensity ratios take higher values. The qualitative findings of this study, however, remained unchanged. The most stable retinal image quality in the presence of eye movement artifacts is provided by a combination of a large spot diameter and a small ablation depth per pulse.

An asymptotic approach of the variance and the Strehl ratio towards the corresponding no-eye-tracking values can be observed in Fig. 4.11 to 4.14. Corrections performed with tracker latencies in the order of 100 ms yield similar results as treatments done completely without eye-tracking. The variance ratio values of the primary coma at 96 ms latency are comparable to the values calculated for the no eye-tracking case depicted in Fig. 4.16. In the same way the Strehl ratio values of the primary coma at 96 ms latency found in Fig. 4.11, are similar to the values calculated for the no eye-tracking case depicted in Fig. 4.18.

The combination of a larger spot size with a small peak fluence reflects the historical situation, when treatments were done with broad-beam lasers and no eye-tracking. Results were good due to the excellent stability of this parameter combination towards eye movements. The development of scanning spot laser
systems with smaller beam diameters and larger peak fluences necessitated the application of eye-tracking systems, due to the low stability of this parameter combination as shown in this study.

In conclusion, the smaller the spot diameter and the smaller the ablation depth per pulse, the better the theoretical approximation of the ablation profile. Reduction of the spot diameter, however, drastically decreases the stability of the correction towards spot displacements. A large spot diameter and a small ablation depth per pulse results in a higher stability of the ablation during treatments performed with an eye tracker having certain latency. For any change from a larger to a smaller spot size, a latency threshold can be defined. Only for eye tracking systems with latencies below this certain threshold, changing to the smaller spot size is beneficial.

4.3 Parallax error of video eye tracking

4.3.1 Introduction

Most of the video-based eye trackers determine the position of the eye by locating the entrance pupil center. The entrance pupil is the image of the aperture stop as seen from object space. Its appearance and position changes depending on the eccentricity angle of observation. This is the case for eye movements as they occur during refractive surgery. In refractive surgery, it is actually not the position of the pupil that is of interest, but the exact location of the overlying corneal front surface. Thus, it is assumed, that by always centering the laser on the entrance pupil center, the treatment is constantly centered around the corresponding point on the cornea (we will assume, that this point is the corneal apex, even though the human eye is not a centered optical system). This means, that the center of the camera aperture, the apex and the center of the actual pupil are located on one line. However, in case of eye movements there are two sources of error being connected with this assumption. First, there will be a geometrical parallax, as the three mentioned points are not aligned anymore. Secondly, there will be an optical error due to refraction, as the center of the entrance pupil shifts relative to the center of the actual pupil. The problem can be compared to the effort to track a specific point on the water surface of a randomly moving water filled bin.
by observing an object on the bottom (Fig. 4.19).

![Diagram](image)

*Figure 4.19: Illustration of the investigated problem. An observer is trying to track a point A on the water surface of a randomly moving bin by following a subjacent object on the bottom. Due to the geometrical parallax and the refraction of light, point A appears to be at the position A'.*

Fig. 4.20 shows different alignment scenarios of the eye relative to the eye tracker. Basically, there are three different axes that can be introduced in this context. If we assume, that the eye is a centered optical system, there exists an optical axis, which characterizes the direction of gaze. Thus, this axis always passes through the point the eye is fixating. Furthermore, we assume that this axis passes through the ocular center of rotation CR. Secondly, there is the optical axis of the eye tracker. Thirdly, there is an optical path to be defined along which the entrance pupil center is observed by the camera. This chief ray will be called the "line of observation".

In the ideal case (Fig. 4.20 A), the eye is fixating the point FP while the optical axis of the tracker is aligned to the eyes entrance pupil center. Here, the three mentioned axes coincide. No eye tracking would be necessary, if alignment could always be maintained like this.

In Fig. 4.20 B, the eye is fixating the point FP while the system axis is not aligned to the eye's entrance pupil center i.e. the eye is fixating FP eccentrically.
4.3 Parallax error of video eye tracking

Figure 4.20: Different alignment scenarios of the eye relative to the optical axis of the eye tracker. (A) The eye tracker axis is perfectly aligned to the entrance pupil center while the eye is fixating the fixation target. (B) The eye tracker is not aligned with the entrance pupil center while the eye is fixating the target eccentrically. (C) The eye is laterally shifted from the position illustrated in A and is not fixating the target anymore. (D) The eye is rotated from the position depicted in A and is not fixating the target anymore.

However, the eye’s optical axis coincides with the line of observation and no geometrical parallax occurs.

In case of a pure lateral shift of the eye relative to the tracker (Fig. 4.20 C), the eye is not fixating FP anymore and the tracker axis is not pointing to the entrance pupil center. Such a misalignment might emerge from type B, if the eye is not fixating FP anymore. The line of observation does not coincide with the eye’s optical axis anymore and an error in the determination of the apex emerges.

In case of random eye movements such as microsaccades, the eye rotates around its center of rotation and is not fixating FP anymore (Fig. 4.20 D). The optical
axis of the tracker camera is assumed to still pass through the eye's center of rotation, which is located approximately 14 mm behind the apex. Again, the line of observation does not coincide with the eye's optical axis anymore and a parallax emerges.

The simulations reported here were performed for misalignments of the types depicted in Fig. 4.20 C and D. Alignment situations as shown in Fig. 4.20 A and B enable perfect movement compensation by the eye tracker due to the absence of geometrical parallax.

One has to be aware of the fact, that the parallax error as investigated in this study even emerges in a perfect eye tracking system, independent of its accuracy and latency characteristics. It is beyond our knowledge, if the laser manufacturers are currently considering this effect. Exact compensation of the error would require some custom data of the patient's eye. As far as we know, none of the available laser systems asks for input of such data. Compensation might be based on an estimation for mean sized eyes. However, no laser manufacturer appears to promote such a feature. The fact, that the registered eye movements can not be clearly attributed to either lateral shifts of the head, or rotations of the bulbus, renders compensation a delicate task. The results of this investigation were published in [Bueeler M, Mrochen M. Limitations of pupil tracking in refractive surgery - systematic error in the determination of corneal locations. J Refract Surg. 2004; 20(4):371-378].

4.3.2 Methods

Misalignment in case of a lateral shift of the eye (see Fig. 4.20C):

After a lateral shift of the eye relative to the optical axis of the tracker, both the corneal apex as well as the real pupil center are shifted by the same amount. Thus, the distances $d_A$ and $d_P$ in Fig. 4.21 arc equal. The eye tracker is now observing the pupil center eccentrically along the line of observation. The intersection of the line of observation with the anterior corneal surface at position $d_C$ (Fig. 4.21) was determined by optical ray-tracing. This intersection point is now mistaken for the corneal apex, overlying the entrance pupil. The real apex, however, is located at position $d_A$. The error in the determination of the apex
position is calculated by \( d_A - d_C \). This error is increasing for increasing lateral shifts. Values \( d_A \) ranging from 0 to 1.5 mm were simulated.

![Diagram of eye tracking](image)

**Figure 4.21:** Detailed geometrical situation for the alignment scenario depicted in Fig. 4.20C. The eye is laterally shifted away from the axis of the eye tracker. The real pupil center is shifted by \( d_P \) and the corneal apex by \( d_A \). The pupil center is observed along the line of observation.

When calculating the distance \( d_C \) by optical ray tracing, assumptions have to be made regarding the distance \( D \) between the eye tracker camera and the eye (see Fig. 4.20). With regard to the spatial situation in a refractive laser system, calculations were made for \( D = 100 \) mm, \( D = 200 \) mm and \( D = 500 \) mm.

**Misalignment in case of an ocular rotation around CR (see Fig. 4.20D):**
The error in the determination of the corneal apex when tracking the entrance pupil center was further calculated for ocular rotation angles \( \alpha \) ranging from 0 to 6 degrees (increments of 1 deg). Again, the apparent position \( d_C \) of the pupil center i.e. the intersection of the line of observation with the cornea as perceived by the eye tracker was determined by optical ray-tracing. For this, it was assumed, that the distance \( D \) between the video camera and the eye is 500 mm.
Fig. 4.22 illustrates the situation for an eye rotated around the center of rotation (CR) by the angle \( \alpha \) relative to the optical axis of the eye tracker. As a result of the rotation, the center of the real pupil is laterally shifted by the amount \( d_p \). The intersection of the line of observation with the anterior corneal surface is presumed to be the corneal apex overlying the entrance pupil, whereas the real apex is located at position \( d_A \). Again, the parallax error is calculated as \( d_A - d_C \). This error is increasing for an increasing rotation angle \( \alpha \).

**Figure 4.22: Detailed geometrical situation for the alignment scenario depicted in Fig. 4.20D.** The eye is rotated around its center of rotation CR. The real pupil center is shifted by \( d_p \) and the corneal apex by \( d_A \). The pupil center is observed along the line of observation.

**Ray-Tracing:**
A number of different assumptions concerning the geometrical and optical properties of the human eye had to be taken for these simulations. The influence of these parameters on the results with regard to a potential compensation of the parallax effect in clinical practice was further studied. Simulations were done
for two different theoretical eye models. The Le Grand schematic eye featuring spherical corneal front and back surfaces, and the Koijmann eye with two aspherical corneal surfaces. However, results were found to be independent of the chosen cornea model.

Using the corneal shapes and the refractive indices of the Koijman eye model, we further varied the following parameters for the investigation on the ocular rotations as illustrated in Fig. 4.22: the distance R between the corneal apex and the center of rotation (CR) of the eye. According to Le Grand and El Hage [22], R generally varies between 13 and 15 mm. Stenstroem determined the axial length in a large number of eyes finding a range from 20 to 29.5 mm with a mean value of 24 mm [23]. Assuming that R is changing proportionally to the axial length of the eye, values from 11.5 to 17.5 mm with a mean of 14 mm can be estimated for R. The distance L between the anterior corneal surface and the iris (pupil) is reported to vary between 2.8 and 4.6 mm [24]. In schematic eyes, L is normally assumed to be 3.6 mm. The corneal thickness varies over the population. We assumed values from 0.4 to 0.6 mm. The default values for each simulation with the Koijmann schematic eye were: R = 14 mm, L = 3.6 mm, corneal thickness = 0.5 mm.

Hosny et al. studied the relationship between the anterior chamber depth, the refractive state, the corneal diameter and the axial length in 211 eyes [25]. They found, that the anterior chamber depth correlated significantly with the axial length of the globe i.e. the longer the bulbus the deeper the anterior chamber. Assuming that the rotation length R is changing proportionally to the axial length of the eye, it can be further concluded that the larger R the deeper the anterior chamber. Based on this finding, we further calculated the tracking error for physiological combinations of the parameters R and L, relating to a short (L=2.8/R=11.5 mm), a medium (L=3.7/R=14.5 mm), and a long bulbus (L=4.6/R=17.5 mm).
4.3 Parallax error of video eye tracking

4.3.3 Results

Misalignment in case of a lateral shift of the eye:

Fig. 4.23 shows the error $d_A - d_C$ in the determination of the corneal apex as a function of the lateral shift $d_C$ of the entrance pupil center for three different values of the eye tracker distance $D$. These calculations were made for the type of misalignment depicted in Figure 3A, i.e., lateral shifts only. The closer the eye tracker camera is positioned, the flatter the angle of observation is, and the larger is the error in the localization of the corneal apex given the same lateral shift. If the tracker detects a shift of 0.5 mm in the position of the entrance pupil center, the error $d_A - d_C$ would be 0.0025 mm (0.5%) for the camera at 500 mm distance, 0.0075 mm (1.5%) for $D = 200$ mm, and 0.015 mm (3%) for the camera at 100 mm distance.

![Figure 4.23](image)

Figure 4.23: Error in the determination of the corneal apex position as a function of the detected lateral shift $d_C$ of the entrance pupil center. Results were obtained for three different distances $D$ between the eye tracker and the eye.

Misalignment in case of an ocular rotation around CR:

Fig. 4.24 and Fig. 4.25 show the error $d_A - d_C$ in the determination of the corneal apex as a function of the lateral shift $d_C$ of the entrance pupil center. These
4.3 Parallax error of video eye tracking

results relate to the type of misalignment illustrated in Fig. 4.22 i.e. ocular rotations. In Fig. 4.24, results are presented for different ocular rotation lengths $R$.

![Figure 4.24: Error in the determination of the apex position as a function of the detected lateral shift $d_C$. Results were obtained for three different distances $R$ between the corneal apex and the ocular center of rotation.](image)

It appears, that the larger $R$, the smaller the error $d_A - d_C$. In the mildest case of $R = 17.5$ mm, a lateral shift of the entrance pupil of 0.3 mm, which equals an ocular rotation of 1 degree, causes an error of 0.06 mm (20%). For a lateral shift of 1.0 mm ($\approx 3.25$ degrees), which is a common tracking range in video-based eye trackers, the error increases to 0.21 mm. For a very short rotation length of $R = 11.5$ mm, the committed tracking error amounts to 35% of the detected movement. The curves in Fig. 4.25 were obtained for different distances $L$ of the pupil from the corneal apex (anterior chamber depth including corneal thickness). The smaller $L$, the smaller the parallax error. For the mildest case of $L = 2.8$ mm, a lateral shift of 0.2 mm (0.8 degrees ocular rotation) causes an error of 0.04 mm (20%). For a lateral shift of 1.0 mm ($\approx 4$ degrees), the error increases to 0.20 mm (20%). In patients with a deep anterior chamber of $L = 4.6$ mm (including corneal thickness), the committed error increases to 40% of the detected shift.
4.3 Parallax error of video eye tracking

Figure 4.25: Error in the determination of the apex position as a function of the detected lateral shift $d_C$. Calculations were done for three different values of the anterior chamber depth (including corneal thickness) $L$.

Fig. 4.26 depicts the results for combinations of the parameters $R$ and $L$ based on the finding of Hosny et al., that increasing depth of the anterior chamber is associated with increasing bulbus length and thus with increasing rotation length $R$ [25]. The committed error is slightly larger in eyes with a high axial length (30%) than in short eyes (25%). Due to the counteracting effect of the parameters $R$ and $L$, this difference is not significant. All the above results were found to be independent of the corneal thickness (varied between 0.4 and 0.6 mm).

4.3.4 Discussion

The goal of this investigation was to show the theoretical limitations of pupil tracking in refractive surgery. Corneal displacements in case of eye movements were found to be generally underrated when tracking the entrance pupil center. Pure lateral shifts of the eye under the tracking device can cause localization errors of up to 3% of the detected entrance pupil shift. For eye movements originating from ocular rotations, the tracking error can amount to 30% of the
4.3 Parallax error of video eye tracking

![Graph showing the error in the determination of the apex position as a function of the detected lateral shift of the entrance pupil center. Results were obtained for three different physiological combinations of the parameters R and L, relating to three different axial lengths of the bulbus.](image)

Figure 4.26: Error in the determination of the apex position as a function of the detected lateral shift of the entrance pupil center. Results were obtained for three different physiological combinations of the parameters R and L, relating to three different axial lengths of the bulbus.

Detected lateral shift in eyes with a high axial length. That means if the eye tracker assessed a lateral shift of the entrance pupil of 0.1 mm away from the tracking reference axis, the point of interest located on the cornea would essentially be 0.13 mm away from the reference axis. It has to be considered that these results were obtained for a relatively high eye tracker distance of 500 mm. If the tracker camera was mounted closer to the eye, even higher tracking errors have to be expected.

Most of the commercial eye tracking systems are video-based. A video camera, which is originally aligned along the line-of-sight of the eye, is capturing images at a certain frequency. These images are processed and the lateral shift of the entrance pupil center relative to the reference position is determined.

Two different types of misalignments were investigated in this report (Fig. 4.20C and D). However, misalignments of the type Fig. 4.20D, where the eye rotates...
4.3 Parallax error of video eye tracking

around the transverse axes through the ocular center of rotation appear to be more appropriate to simulate eye movements such as saccades, drifts, microsaccades and tremors, where the head of the patient remains fixed.

Assuming a general parallax error of 30%, all the reported eye movement data [2, 6, 7] that are based on pupil tracking would have to be multiplied by a factor of 1.3 to obtain the corresponding displacements of the corneal apex. In case of a peak movement of 0.35 mm of the entrance pupil center as reported by Schwiegerling et al. [7], the corneal apex has moved as far as 0.455 mm away from the reference axis. Thus, a laser pulse fired at that very moment would be displaced by 0.105 mm.

The results were shown to depend on some geometrical parameters of the theoretical eye models. Anterior chamber depth and the distance between the ocular center of rotation and the corneal apex have a counteracting effect on the results. The two parameters, however, are linked over the axial length of the eye. Hosny et al. have shown, that longer eyes also feature a longer anterior chamber [25]. Custom compensation of the systematic parallax errors might be possible, if the above parameters are determined preoperatively in the eye to be treated. Partial compensation could be achieved for a mildest case scenario of a short eye. Assuming only rotational movements of the eye, assuming R=11.5 mm and L=2.8 mm, and considering the eye tracker distance D of the particular laser system would enable a partial compensation of the parallax effect. This could be done by adding an additional percentage (e.g. 25%) to the modulus of the detected lateral shift of the entrance pupil. The study of Hosny has further shown an inverse correlation between bulbus length and spherical equivalent refraction. Highly myopic eyes in the order of 19 D were found the feature bulbus lengths of over 30.0 mm. The committed tracking error can thus be expected to be higher in myopic than in hyperopic eyes.

In our simulations the human eye was assumed to be a centered optical system with a defined optical axis. In reality, however, the optical components of the eye can be decentered and tilted with respect to each other. The optical axis in a real eye can be defined as the axis of revolution of the aspheric refractive surfaces. The Purkinje images of a test object appear in approximate alignment.
along this axis. The fovea is located slightly temporal of the optical axis. Thus, the fixation axis, which is the path of the chief ray from the distant fixation point to the fovea, is not exactly passing through the corneal apex. However, this fact does not change the general conclusion of our study. The parallax effect won't be rotationally symmetric anymore, and thus, the committed error can be expected to become slightly larger than calculated for ocular rotations in one direction, and slightly smaller than predicted for rotations in the other direction. The same can be expected, if the eye tracker camera is not coaxially mounted with respect to the optical axis of the laser system. This is the case in several commercially available refractive lasers. According to Bennett and Rabbetts [24], the pupil is typically thought to be decentered nasally from the optical axis by about 0.25 mm. Both the entrance and the exit pupil would then be located on the nasal side of the optical axis. Reports have further shown, that the pupil does not contract symmetrically [26, 27]. All these characteristics render the investigated problem non-rotationally symmetric.

We provided an estimate of the eye tracking error resulting from the geometrical parallax between the pupil center, the corneal apex and the eye tracker position in case of eye movements. We have shown that this error can amount to 30% (or more for eye trackers mounted closer than 500 mm to the eye) of the detected later shift of the entrance pupil. The considerable magnitude of this number indicates, that this effect should not be considered negligible right away.

### 4.4 Summary

It was shown, that displacements of single laser shots from their ideal overlap position can significantly increase the post-operative surface roughness. Depending on the smoothing characteristics of the epithelial wound healing process as well as on the laser spot parameters, spot displacements can considerably decrease post-operative image quality.

The smaller the spot diameter and the smaller the ablation depth per pulse, the better the theoretical approximation of the intended ablation profile. Reduction of the spot diameter, however, drastically decreases the stability of the correction
towards spot displacements. A large spot diameter and a small ablation depth per pulse result in a higher stability of the ablation during treatments performed with a laser system having certain latency. For any change from a larger to a smaller spot size, a latency threshold can be defined. Changing the laser spot diameter from 1000 to 250 microns was found to be only beneficial when eye movement is neutralized with a tracking system with a latency below 4 ms.

Treatments performed with tracker latencies in the order of 100 ms yield similar results as treatments done completely without eye-movement compensation.

Furthermore, the parallax error associated with localizing corneal positions by tracking the subjacent entrance pupil center was quantified.

The tracking error can amount to 30% (or more for eye trackers mounted closer than 500 mm to the eye) of the detected lateral shift. Thus, if the eye tracker registers a lateral shift of the entrance pupil of 0.2 mm away from the tracking reference axis, the point of interest located on the cornea would essentially be 0.26 mm away from this reference axis. A laser pulse fired at that moment would be systematically displaced by 60 microns.

The fact that the corneal displacement was found to be generally underrated by up to 30% of the measured entrance pupil shift demonstrates the severity of the parallax effect. The results were shown to depend on some geometrical parameters of the eye and the tracking device. Based on some conservative assumptions regarding these geometrical parameters, partial compensation could be done by adding a certain percentage to the modulus of each eye tracker reading.
References


REFERENCES


REFERENCES


Chapter 5

Outlook

The outcome of refractive procedures is determined by the interaction of defined system parameters such as the spot size or the type of aberration to be treated on the one hand, and undesired influence factors such as alignment errors on the other hand.

The exact knowledge of this interaction is important for both the optimization of laser parameters as well as the definition of tolerance limits for the undesired influence factors.

In this thesis a methodology for future parameter optimization under consideration of undesired alignment errors was presented. It was shown, that reducing the spot diameter is only beneficial if the temporal lag of the movement compensation is kept below a certain limit. For a given eye tracker performance, appropriate spot parameters can be determined with respect to the corneal irregularity to be treated. Inversely, tolerance limits regarding eye tracker performance can be determined for defined laser parameters. The study additionally evaluated tolerance limits for static translation and rotation of the optical zone.

Reduction of laser spot diameter was shown to make the correction more susceptible to eye movement induced error. A smaller spot size was shown to be only beneficial when translatory eye movement is neutralized with a tracking system featuring a latency below a certain threshold value. These findings might be of special importance in the future, as current developments towards solid state and femtosecond lasers in refractive surgery consider spot sizes significantly below 0.5
mm. Appropriate enhancement of the tracker response time is indispensable in these systems to maintain stability towards eye movements as pointed out in this thesis. As these new laser systems are going to be designed specifically with a view toward how to optimize custom ablation, the requirements on the accuracy of spot placement might even further increase.

The simulations on single spot displacements presented in Chapter 4 did only focus on translatory eye movements. In a next step, simulations of the ablation process should be performed based on torsional eye movement data in order to evaluate the necessary system performance specifications for cyclotorsion tracking. Unfortunately, these calculations could not be performed in the course of this study due to the lack of torsional eye movement data.

The potential of custom corneal ablation for the topography-guided treatment of corneal irregularities such as steep central islands or decentered optical zones is currently under investigation. The correction of these structures with scanning spot lasers is expected to require utmost precision in the placement of the single laser spots, as their spatial dimensions are significantly smaller than the ones of currently corrected structures. Simulations similar to the ones presented in this thesis should be used to evaluate the necessary performance parameters.

Reducing spot size was shown to be only beneficial if the eye tracker response time is appropriately enhanced. The increased number of pulses necessary for the same correction has to be compensated in turn by increasing the laser repetition rate in order to keep the treatment time at a minimum. Thus, any change of a parameter necessitates appropriate changes of other parameters to achieve an optimum performance.

The decision of up to what order and amplitude wavefront aberrations could be reasonably treated should be based individually on the known performance characteristics of a laser system. Laser spot size, laser fluence or the response time of the eye tracker should be considered in this decision.

Furthermore, this study quantified the parallax error associated with localizing corneal positions by tracking the subjacent entrance pupil center. Compensation
of the parallax error has to be considered mandatory in the future if correction of fine corneal irregularities is aspired. A method for conservative compensation of this effect was presented. More accurate compensation could be achieved by using custom data of the patient’s eye such as ocular length and anterior chamber depth. It can be assumed, that the use of custom data instead of averaged values for the planning of refractive treatments will gain further importance in the future. The comparison of wavefront data with ray tracing results presented under 3.3.4 indicated the importance of the consideration of intraocular structures, especially in the optimization of ablation profiles. The combination of topography, wavefront and biometry data in customized eye models can be expected to provide ablation profiles superior to the ones currently used.

This study investigated the effect of optical zone and single spot misalignment as one type of undesired influence factor in refractive surgery. However, numerous other factors such as the biomechanical response to corneal ablation or epithelial wound healing affect the outcome in an undesired and unpredictable way. Future work will focus on these effects in order to render them more predictable.
Appendix A

Matrices for rotation and translation

The wavefront correction pattern $W_{A_{corr}}$ can be rotated and translated relative to the measured wavefront map of the eye $W_{A_{eye}}$ as a result of an inaccurate alignment of the treatment. The Zernike coefficients of the correction pattern can be calculated using coordinate transformations:

$$W_{A_{corr}}(x, y) = W_{A_{eye}}(x', y')$$

(A.1)

Where

$$x' = (x - \Delta x) \cos(\Delta \gamma) + (y - \Delta y) \sin(\Delta \gamma)$$

$$y' = (y - \Delta y) \cos(\Delta \gamma) - (x - \Delta x) \sin(\Delta \gamma)$$

$\Delta \gamma$ is the angle of rotation around the eye’s longitudinal axis, and $\Delta x, \Delta y$ are the translations in the x and y direction. These new coordinates are introduced in each Zernike monomial of the eye’s aberrations $W_{A_{eye}}$ (Eq. 3.1), and by rearranging terms to isolate the original Zernike monomials, the coefficients $c_i$ can be obtained analytically from functions of the original coefficients $a_k$. The analytical functions can be converted to matrix equations defining a translation matrix $T$ and a rotation matrix $R$ (Eq. A.2):
\[ c_i = \sum_{j,k} T_{ij} R_{jk} a_k \quad (A.2) \]

Tab. A.1 shows a simplified translation matrix \( T \) according to Guirao et al. \( A = \Delta x/r_0 \) and \( B = \Delta y/r_0 \) are the horizontal and vertical translation components normalized by the pupil radius \( r_0 \). For simplicity, only the linear matrix elements depending on \( A \) and \( B \) are shown. \( n' \) means the order corrected and \( n \) is the order affected.

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**Table A.1**: Conversion matrix \( T \) for Zernike coefficients after lateral translation (taken from Guirao et al.).

The rotation matrix \( R \) takes the following form:

\[
R = \begin{bmatrix}
\cos(m\alpha) & -\sin(m\alpha) \\
\sin(m\alpha) & \cos(m\alpha)
\end{bmatrix}
\]

It turns out that this conversion depends exclusively on the rotation angle \( \Delta \gamma \) as well as on the meridional degree \( m \) of the Zernike term, but is independent of the radial degree \( n \).
## Glossary

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<tbody>
<tr>
<td>Ablation</td>
<td>Removal of corneal tissue with a laser.</td>
</tr>
<tr>
<td>Ablation depth per pulse</td>
<td>Depth of tissue removal by one laser pulse. Mostly specified in terms of the “peak ablation depth” indicating the maximum depth in the center of the ablation paraboloid.</td>
</tr>
<tr>
<td>Ablation paraboloid</td>
<td>Shape of corneal tissue removed by one laser pulse with Gaussian energy distribution.</td>
</tr>
<tr>
<td>Anterior chamber depth</td>
<td>Distance between the corneal front surface and the iris.</td>
</tr>
<tr>
<td>Apex</td>
<td>Point of greatest curvature on an aspherical cornea. Also intercept of the optical axis with the cornea in schematic model eyes.</td>
</tr>
<tr>
<td>Astigmatism</td>
<td>An optical error that occurs when the tangential and sagittal foci do not coincide and the system appears to have two different focal lengths. Thus a point object does not produce a point image but instead produces two short lines at right angles to each other. See also cylinder.</td>
</tr>
<tr>
<td>Beam profile</td>
<td>Energy distribution profile of a laser beam. Mostly Gaussian in scanning laser systems.</td>
</tr>
<tr>
<td>Broad beam laser</td>
<td>Laser system that works with a static laser beam of large diameter (typically 6.0 mm). Step-like shapes can be removed from the cornea by placing expanding concentric circular or elliptical appertures into the optical path of the beam.</td>
</tr>
<tr>
<td>Chief ray</td>
<td>The chief ray is the ray that passes through the center of the aperture stop.</td>
</tr>
<tr>
<td>Classical correction</td>
<td>Correction of only sphere and cylinder.</td>
</tr>
<tr>
<td>Coma</td>
<td>Type of wavefront aberration of 3rd (primary), 5th (secondary) or even higher radial order.</td>
</tr>
<tr>
<td>Contour deviation</td>
<td>Deviation from the ideal ablation profile that can be expressed by polynomial expansions. To be distinguished from roughness deviations.</td>
</tr>
<tr>
<td>Cornea</td>
<td>First refractive element of the eye in front of the iris and the pupil.</td>
</tr>
<tr>
<td>Corneal topography</td>
<td>Representation of the corneal anterior surface either by means of diopter or height maps. Measurements are based on ring projection.</td>
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<tr>
<td>Term</td>
<td>Definition</td>
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<tr>
<td>Customized correction</td>
<td>Correction of higher-order optical irregularities. Ablation is either wavefront- or topography-guided.</td>
</tr>
<tr>
<td>Cyclotorsion</td>
<td>Rotational movement of the eye around its longitudinal axis. Often observed when moving the body from an upright to a horizontal position.</td>
</tr>
<tr>
<td>Cylinder</td>
<td>See astigmatism</td>
</tr>
<tr>
<td>Diopters</td>
<td>A unit of measurement of the refractive power of lenses equal to the reciprocal of the focal length measured in meters.</td>
</tr>
<tr>
<td>Drift</td>
<td>Type of eye movement. Slow and constant movement away from the point of fixation.</td>
</tr>
<tr>
<td>Dynamic misalignment</td>
<td>Displacement of single laser pulses from their ideal incidence position on the cornea. To be distinguished from static misalignment that refers to a constant displacement of the optical zone.</td>
</tr>
<tr>
<td>Emmetropia</td>
<td>The condition of the ideal eye when parallel rays are focused exactly on the retina and vision is perfect.</td>
</tr>
<tr>
<td>Entrance pupil</td>
<td>Image of the eye's aperture stop as seen from object space.</td>
</tr>
<tr>
<td>Epithelial smoothing</td>
<td>Regrowth of the epithelium after laser ablation on the cornea. This regrowth has a smoothing effect on the rough postoperative corneal surface acting as a low-pass filter.</td>
</tr>
<tr>
<td>Eye-tracking</td>
<td>Registration of eye movements by means of a video camera or other techniques such as electro-oculography.</td>
</tr>
<tr>
<td>Fluence</td>
<td>Energy per area. Used in specifying laser beams. The peak fluence refers to the maximum fluence in the energy profile of the laser beam.</td>
</tr>
<tr>
<td>Fovea</td>
<td>Area consisting of a small depression in the retina containing cones and where vision is most acute.</td>
</tr>
<tr>
<td>Hyperopia</td>
<td>Farsightedness. It results from the eyeball being too short from front to back, causing images to be focused behind the retina.</td>
</tr>
<tr>
<td>Irregularity</td>
<td>Structure on the cornea or in the eye that requires mathematical description with polynomials higher than second order.</td>
</tr>
<tr>
<td>Kojijmann schematic eye</td>
<td>Wide angle schematic eye model considering aspherical surfaces.</td>
</tr>
<tr>
<td>Laser pulse</td>
<td>Laser that delivers energy in single or multiple pulses. In contrast to continuous-wave lasers that operate in continuous-emission mode.</td>
</tr>
<tr>
<td>Laser shot list</td>
<td>List of coordinates for each single laser pulse to ablate the desired profile.</td>
</tr>
<tr>
<td>LASIK</td>
<td>Laser in-situ Keratomileusis. Type of corneal laser surgery. Prior to laser irradiation a thin corneal flap is cut with a microkeratome and lifted to expose the stromal bed. Reshaping of the cornea is done in the stromal bed and finally the flap is replaced to promote healing.</td>
</tr>
<tr>
<td>Latency</td>
<td>The temporal delay between the actual eye movement and the subsequent compensation through the laser delivery system is one of the most important parameters in active eye tracking.</td>
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<td>Term</td>
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<tr>
<td>Line of sight</td>
<td>The line of sight (LOS) is the chief ray joining the fixation point and the center of the entrance pupil.</td>
</tr>
<tr>
<td>Maréchal criterion</td>
<td>Criterion for optical quality. It states, that a well corrected, diffraction limited optical system has an root-mean-square wavefront error not exceeding 1/14 of the wavelength.</td>
</tr>
<tr>
<td>Microsaccade</td>
<td>Type of eye movement. Microsaccades appear to have a drift-correcting function. See drift.</td>
</tr>
<tr>
<td>Movement artefacts</td>
<td>Incomplete eye-movement compensation due to latency results in movement artefacts.</td>
</tr>
<tr>
<td>Myopia</td>
<td>Nearsightedness. It results from the eyeball being too long from front to back, causing images to be focused in front of the retina.</td>
</tr>
<tr>
<td>Navarro schematic eye</td>
<td>Wide angle schematic eye model considering aspherical surfaces (except for the corneal posterior surface).</td>
</tr>
<tr>
<td>Nodal points</td>
<td>The nodal points are the two points at which two rays with identical directions in the image and the object space appear to intersect the optical axis.</td>
</tr>
<tr>
<td>Optical axis</td>
<td>The optical axis is the line passing through the centers of curvature of a lens. For a multi-lens system it is the optical centerline for all the centers of the optical components. In eye the optical axis is defined as the line of &quot;best fit&quot; through the centers of curvature of the &quot;best fit&quot; spheres to each surface.</td>
</tr>
<tr>
<td>Optical path difference OPD</td>
<td>For a perfect optical system, the optical path or distance from an object point to a corresponding image point will be equal for all rays. In near-perfect systems, slight differences will exist between rays resulting in an optical path difference, usually expressed in fractions of the wavelength being analyzed.</td>
</tr>
<tr>
<td>Optical zone</td>
<td>Area on the cornea where a refractive correction is optically effective. It is followed outwards by the transition zone that provides a smooth transition to the uncorrected part of the cornea.</td>
</tr>
<tr>
<td>Parallax error</td>
<td>An apparent shifting of an object that was originally aligned with another object at a larger distance when viewed eccentrically. The phenomenon can be found in video-based eye trackers that are meant to determine exact corneal positions by locating the underlying pupil center.</td>
</tr>
<tr>
<td>Photoablation</td>
<td>Photoablative decomposition. Breaking of molecular bonds by non-thermal (cool) ablation.</td>
</tr>
<tr>
<td>Piston</td>
<td>Constant term in the polynomial expansion of wavefront aberrations. Does not have any optical meaning and is mostly neglected when reporting aberrations.</td>
</tr>
<tr>
<td>Power spectral density PSD</td>
<td>The power of a measured signal over a certain bandwidth divided by the bandwidth. In case of measured eye movement data, the PSD provides the variance of the movements in a certain frequency range. Practically, the PSD is calculated using the Fourier transform.</td>
</tr>
<tr>
<td>Term</td>
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<tr>
<td>Prismatic effect</td>
<td>This effect characterizes a tranverse displacement of the image away from the ideal focus point as a result of a decentered refractive correction or spectacle lens. It can be quantified using Prentice's rule.</td>
</tr>
<tr>
<td>PRK</td>
<td>Photorefractive Keratectomy. Type of corneal laser surgery. It is characterized by the fact that laser radiation is directly applied to the corneal surface after removal of the epithelial layer. It was recognized that due to the irreversible destruction of the Bowman's layer higher corrections were associated with increased epithelial regrowth reducing the predictability of the treatment.</td>
</tr>
<tr>
<td>Rayleigh criterion</td>
<td>The Rayleigh criterion for the resolution of an optical system states that two points will be resolvable if the maximum of the intensity of the Airy disk from one of them coincides with the first minimum intensity of the Airy disk of the other.</td>
</tr>
<tr>
<td>Re-epithelisation</td>
<td>Regrowth of the epithelium after laser ablation on the cornea. This regrowth has a smoothing effect on the rough post-operative corneal surface acting as a low-pass filter. See epithelial smoothing.</td>
</tr>
<tr>
<td>Refractive surgery</td>
<td>Procedures to surgically improve the optical quality of the eye by applying changes to the refracting elements (corneal laser surgery, radial keratectomy, artificial lens implantation).</td>
</tr>
<tr>
<td>Residual wavefront aberration</td>
<td>Wavefront aberration left after incomplete correction. Incomplete correction can either be planned or result from errors during the treatment such as decentration.</td>
</tr>
<tr>
<td>RMS wavefront aberration</td>
<td>Root-mean-square wavefront aberration. Square root of the wavefront aberration variance.</td>
</tr>
<tr>
<td>Saccade</td>
<td>Type of eye movement. Voluntary movement of the eyes to a new location of fixation.</td>
</tr>
<tr>
<td>Scanning spot/beam laser</td>
<td>Type of laser system, that works with a small diameter beam that moves around on the cornea. A large number of single laser pulses is overlapped in different positions to ablate the desired structure.</td>
</tr>
<tr>
<td>Spherical aberration</td>
<td>Type of wavefront aberration of 4th (primary), 6th (secondary) or even higher radial order.</td>
</tr>
<tr>
<td>Sphero-cylindrical error</td>
<td>Optical error composed of the classical 2nd order aberrations sphere and cylinder.</td>
</tr>
<tr>
<td>Spot size</td>
<td>Base diameter of the paraboloid-shaped ablation crater in the cornea resulting from a single laser pulse. Same as spot diameter.</td>
</tr>
<tr>
<td>Static misalignment</td>
<td>Decentration of the entire optical zone relative to an ideal axis. The decentration is constant during the entire treatment.</td>
</tr>
<tr>
<td>Stiles-Crawford</td>
<td>The Stiles-Crawford effect (discovered in 1933) describes angular dependence of retinal sensitivity. Rays which enter the pupil near its center, which are parallel to retinal receptors, are more effective than oblique rays which enter the pupil near its margins. So, the light passing through the periphery of the pupil is less efficient at stimulating vision than the light passing near the center of the pupil.</td>
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<tr>
<td>Steep central island</td>
<td>Refractive surgery complication in which the laser leaves an &quot;island&quot; of corneal tissue in the concave ablation zone. Symptoms include double vision and distortion.</td>
</tr>
<tr>
<td>Strehl intensity ratio</td>
<td>Peak intensity of the aberrated point spread function normalized by the maximum of the diffraction-limited point spread function.</td>
</tr>
<tr>
<td>Stromal bed</td>
<td>Laid open bed in the cornea after cutting of a small flap where laser ablation takes place in LASIK. The stroma is the connective tissue located under the epithelial top layer.</td>
</tr>
<tr>
<td>Taylor polynomials</td>
<td>Type of polynomial expansion. The polynomial is obtained by truncation of a Taylor series after a finite number of terms.</td>
</tr>
<tr>
<td>Tear film</td>
<td>The liquid layer covering the cornea and conjunctiva. It creates a perfectly smooth liquid outer layer that polishes the corneal surface.</td>
</tr>
<tr>
<td>Tilt</td>
<td>Type of wavefront aberration of 1st order. Tilt is equal to the prismatic effect characterizing a transverse displacement of the image away from the ideal focus point as a result of a decentered refractive correction or spectacle lens. See prismatic effect.</td>
</tr>
<tr>
<td>Topography-guided surgery</td>
<td>Type of customized surgery, where the irregularity to be corrected is measured by means of corneal topography. The corresponding ablation profile is based on a corneal height map. The method of choice for the treatment of high irregularities.</td>
</tr>
<tr>
<td>Torsion</td>
<td>Rotational movement of the eye around its longitudinal axis. Often observed when moving the body from an upright to a horizontal position. See cyclotorsion.</td>
</tr>
<tr>
<td>Transverse aberrations</td>
<td>The lateral deviations of light rays in an aberrated optical system from their ideal positions on the retina.</td>
</tr>
<tr>
<td>Tremor</td>
<td>Type of eye movement. High-frequency oscillations of the eye with amplitudes up to 20 mins that serve to continuously shift the image on the retina, thus calling fresh retinal receptors into operation.</td>
</tr>
<tr>
<td>Visual axis</td>
<td>The visual axis (VA) is the line joining the fixation target and the foveal image through the nodal points (N,N'). It is considered a useful construct for calculating the relation between object and images sizes using Gaussian optics.</td>
</tr>
<tr>
<td>Wavefront</td>
<td>Imaginary surface that joins points of equal phase in a wave traveling through a medium.</td>
</tr>
<tr>
<td>Wavefront aberration</td>
<td>Deviations from an ideal spherical or plane wavefront caused by phase differences. The imaginary surface results from the difference between an ideal and a real wavefront.</td>
</tr>
<tr>
<td>Wavefront sensor</td>
<td>Device for measuring wavefront aberrations. Different techniques are used: Tscherning type sensor, Shack-Hartmann sensor.</td>
</tr>
<tr>
<td>Wavefront-guided surgery</td>
<td>Type of customized surgery, where the irregularity to be corrected is measured by means of wavefront aberrometry. The corresponding ablation profile is based on a wavefront aberration map. The method of choice for the treatment of &quot;normal&quot; irregularities.</td>
</tr>
<tr>
<td>Zernike polynomials</td>
<td>Type of polynomial expansion used to describe wavefront aberrations. It offers the advantage that different modes represent different types of optical aberrations.</td>
</tr>
</tbody>
</table>
Curriculum Vitae

Michael Büeler
born on November 3, 1974 in Lachen SZ, Switzerland
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1981 - 1987 Primary school in Lachen, Switzerland
1987 - 1989 Progymnasium in Lachen, Switzerland
1989 - 1994 Gymnasium Nuolen, Switzerland. Matura Type B (Latin)
1994 - 2000 Studies at the Departement of Mechanical Engineering of the Swiss
Federal Institute of Technology (ETH) in Zürich, Switzerland.
Graduation with degree Dipl. Masch.-Ing. ETH
1999 - 2000 Diploma thesis at the Optical Spectroscopy Lab of the
University of Texas at Austin, USA
2000 Visiting Researcher at the Optical Spectroscopy Lab of the
University of Texas at Austin, USA
2000 - present Doctoral Student and Research Assistant at the
Institute of Biomedical Engineering (Prof. P. Niederer),
University and ETH Zürich, Switzerland.
Publications

First Author

Peer reviewed


Bueeler M, Mrochen M. Interaction of eye-tracker latency, spot size, and ablation pulse depth on the correction of higher-order wavefront aberrations with scanning spot laser systems. Accepted for Publication in J Refract Surg. 2004

Proceedings


Co-Author

Peer reviewed


Proceedings

Presentations

Invited

1st International Wavefront Seminar of Wavelight Laser Technologie, Eye clinic, University Hospital Zürich. "Required Accuracy of Centration in Aberration Sensing and Wavefront Guided Treatment". July 2001

International Wavefront Seminar of Schwind Eye-Tech Solutions, Eye clinic, University Hospital Zürich. "Required Accuracy of Centration in Aberration Sensing and Wavefront Guided Treatment". September 2001

ETH Zürich, Presentation in the lecture "Aktuelle Forschung in der Biomedizinischen Technik". "Zentriergenaugkeit in der refraktiven Chirurgie". April 2002

5th International Congress on Wavefront Sensing and Aberration-Free Refractive Correction, Whistler, Canada. "Limits of higher order correction based on spot size, ablation depth, tracker responsiveness". February 2004

Free papers

International Society of Refractive Surgery 2001 Fall Refractive Symposium, New Orleans, USA. "Required Accuracy of Centration in Aberration Sensing and Wavefront Guided Treatments". November 2001
The International Society for Optical Engineering (SPIE) Photonics West 2002, San Jose, USA. "Required Accuracy of Centration in Aberration Sensing and Wavefront Guided Treatments". January 2002

The International Society for Optical Engineering (SPIE) Photonics West 2002, San Jose, USA. "Correlations between Corneal and Total Wavefront Aberrations". January 2002

3rd International Congress of Wavefront Sensing and Aberration-Free Refractive Correction, Interlaken, Switzerland. "Alignment Accuracy in Wavefront-Guided Refractive Surgery". February 2002

The International Society for Optical Engineering (SPIE) Photonics West 2003, San Jose, USA. "Effect of eye movements during refractive surgery with a scanning spot laser". January 2003

4th International Congress of Wavefront Sensing and Aberration-Free Refractive Correction, San Francisco, USA. "Limitations of Pupil Tracking in Refractive Surgery". February 2003

Posters


Awards

Best Paper Award, International Society of Refractive Surgery, 2001 Fall Refractive Symposium, New Orleans, USA. November 2001

Founder’s Award for best paper, 5th International Congress on Wavefront Sensing and Aberration-Free Refractive Correction, Whistler, Canada. February 2004

Reviews

Reviewer for Journal of Refractive Surgery, International Society of Refractive Surgery, 2004