Private Provision of Public Goods: 
Incentives for Donations

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Private Provision of Public Goods: Incentives for Donations

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Abstract

In many countries the government supports individuals’ and companies’ donations dedicated to charity organizations or – more general – to public goods. Yet the effects of governmental support with respect to the provision of public goods has been and still is subject to an extensive debate in the economic literature. Starting from Warr’s (1982, 1983) famous neutrality result an array of conditions has been identified under which this result holds or not.

In this paper we examine the commonly used policy approach to subsidize the private provision of public goods by granting agents deductions with respect to their income or corporate tax burden. We especially take into account that most income tax schemes are progressive and that deductibility is limited. The problems that arise from these specific properties of the considered tax-refund schemes are pointed out first. We then turn towards the effects which such a tax-refund scheme has with respect to the provision of the public good on the one hand and individual as well as aggregate welfare on the other hand. We show that the effects of this commonly practised method of supporting private public good provision depend crucially on the specific properties of the progressive tax scheme and the preference structure of agents. While Pareto-improvements and even Pareto-efficiency can result from the implementation of such a scheme, it is also conceivable that at least some agents perceive a utility reduction. Due to the dependency of welfare effects on the tariff structure, income tax reforms as they are planned in many countries might not only induce a reduction in private public good provision, but might also alter the induced welfare effects.

Keywords: public goods, sponsoring, neutrality

JEL classification: H23, H42

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1 Introduction

In many countries like Germany, Switzerland and the US, the government supports individuals’ and companies’ donations dedicated to charity organizations. Governments support donations by granting reductions in the donators’ income and corporate taxes.

Among the abetted organizations are those espousing science, social welfare and culture. The activities of these organizations can widely be considered to represent public goods (see e.g. Kingma 1989, Jones and Posnett 1994 as well as Khanna, Posnett and Sandler 1995).

However, the effect of governmental support on the public good provision level is questionable, as economic literature suggests. This is due to the neutrality result (re)discovered by Warr (1982, 1983). It states that, as long as interior Nash equilibria prevail, redistribution of income among agents is neutral. Income transfers are called neutral if they do not affect the total public good provision and the individual agent’s consumption of private goods. Prior to this, the neutrality result had already been noticed by Becker (1974), while Barro (1974) had formally demonstrated that neutrality may even hold for intergenerational transfers.

Warr’s analysis has been discussed extensively in the literature on the private provision of public goods. Kemp (1984) extends Warr’s ‘neutrality theorem’ to the case of more than one public good. His analysis has been further developed by Bergstrom, Blume and Varian (1986), Cornes and Schweinberger (1996) as well as recently by Cornes and Itaya (2003). Boadway, Pestieau and Wildasin (1989) point out that transfers may be neutral even when there are distortions in the shape of taxes and subsidies on private goods or factors, strictly local public goods, or on goods that are public to all. Varian (1994) finds that neutrality may also occur for Stackelberg equilibria. Shibata (2003) as well as Shibata and Ihori (2003) show that the Nash equilibrium quantity of a negative public good may be independent not only of income distribution but also of the aggregate income of the set of contributors.

These results suggest that unconditional income transfers would only cause a shifting of public good provision among individuals while leaving the overall provision level, as well as individual welfare, unchanged.

Nevertheless, the literature also identifies several reasons why transfers may be non-neutral with respect to private public good provision. Bergstrom, Blume and Varian (1986) show that income redistribution may affect private provision, when corner solutions are possible, i.e. some agents do not contribute to the provision of the public good.1 Income transfers are also considered to be non-neutral when there are cost differentials in the production of public goods among the providing agents.

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1 Implicitly, Cornes and Sandler (1985: 107) already gave a first hint that the neutrality result may become invalid, if not all agents make strictly positive contributions to the public good.
(see Buchholz and Konrad 1995, Konrad and Lommerud 1995 as well as Ihori 1996). Furthermore, income redistribution has an impact on the the overall provision level when impure public goods are considered (see Cornes and Sandler 1984, 1996 as well as Andreoni 1986, 1989, 1990). When impure public goods are considered, the non-neutral outcome may be additionally perturbed by cost differentials in the independent generation of the private characteristic of the impure public good (see Rübbelke 2002). And finally, transfers of income are non-neutral when conditional income transfers in the shape of subsidies are provided (see Bergstrom 1989 as well as Buchholz 1990).

In our analysis, we investigate common tax refund schemes as applied in Germany or the US. We illustrate how these systems promote private contributions to public goods. The considered schemes feature two important characteristics which induce interesting deviations from the standard strand of literature.

First, the schemes are systems of progressive income taxation. Progressive income taxation implies degressive tax refund or subsidy rates. Consequently, in contrast to former analyses of the influence of subsidies on the private provision of public goods, we allow for subsidy rates that do not only vary among agents but also vary across incomes and public good provision efforts of an agent.

Second, a maximum level of donations is specified which is potentially deductible from the income tax base. This implies that donors who pass this threshold receive a quasi lump-sum subsidy which is independent of the amount they contribute. In this case the effective price of the public good remains unchanged by the subsidy. We allow the threshold to vary among agents in absolute monetary values, while the maximum deductible donations in terms of a percentage of taxable income is equal among agents.

The combination of deductibility from a progressive income tax and introduction of maximal deductible contribution levels bears interesting and complex consequences for the private provision of public goods. We show that even in this case neutrality with respect to the provision level may exist, yet a variety of scenarios may arise in which subsidization is non-neutral. In the latter cases we observe interesting changes of agents’ welfare which are due to subsidy-induced price effects. Furthermore, we demonstrate that the tax refund schemes provide no effective means to achieve an efficient public good provision level.

We proceed as follows: The next section shortly recapitulates the basic results on subsidizing private donations to a public good, given constant subsidy rates. Thereafter, in section 3, we extend the analysis by considering a system where the subsidies are provided in the shape of income tax refunds. Since the income taxation is progressive, the subsidization is degressive. Then, in section 4, we further extend...
the model by introducing a maximum deductible level of donations. In section 5, we draw conclusions on the appropriateness of the analyzed refund schemes. Throughout, we stick to interior solutions and since the analysis already proves to be rather involved, we neglect the implications of cost-differentials in the public good provision.

2 Subsidizing Private Donations to Public Goods

Let us consider an economy in which conditional transfers in the form of subsidies are paid by the government for private donations to the provision of public goods.

2.1 The Basic Model

An individual living in this economy derives utility from the consumption of a bundle of private goods \( y \) and the aggregate level of a public good \( x \). The public good is produced via provision of donations whereby we assume that there are no productivity differentials between agents. The agent receives some gross income \( I^g_i \), \( i = 1, ..., n \), which he spends on private goods \( y_i \) and his contribution \( x_i \) to the public good. The aggregate provision of the public good \( x \) is given by the sum of the agent’s own contribution and the other agents donations \( \bar{x}_i (\bar{x}_i = \sum_{j \neq i} x_j) \).

In the absence of any governmental intervention agent \( i \) faces the following maximization problem:

\[
\max_{y_i, x_i} U(y_i, x_i) = U(y_i, x_i + \bar{x}_i) \tag{1}
\]

s.t. \( y_i + x_i = I^g_i, \quad i = 1, ..., n. \) \tag{2}

For simplicity the prices of the public and private goods are set equal to unity. Consequently, the effective price of the public good \( p^e_i \) - the price of \( x \) in terms of the price of \( y \) - is equal to unity.

Without loss of generality we can rewrite agent \( i \)’s private budget constraint in the following way: \( y_i + x = I^g_i + \bar{x}_i = I^v_i \). Thus the RHS represents the virtual or social income \( I^v_i \), while the LHS denotes total expenditures associated with agent \( i \)’s utility level \( u(y_i, x) \).

Maximizing utility subject to either the private or the social budget constraint yields the following condition for an optimal allocation of private and public goods consumption:

\[
\frac{\partial U_i}{\partial x} = \frac{\partial U_i}{\partial y_i} = 1 \quad \Leftrightarrow \quad MRS_i(y_i, x) = p^e_i. \tag{3}
\]

The result describes a unique Nash equilibrium (see Bergstrom, Blume and Varian 1986, 1992 as well as Cornes, Hartley and Sandler 1999).
Condition (3) determines the income-expansion path (IEP) of the individual agent. It represents the locus of all utility maximizing combinations of consumption of the private good and the public good in the private-good/public-good plane for different income levels. That means, all combinations for which the marginal rate of substitution between the individual’s public and private goods consumption equals the effective price $p_e^i = 1$. Assuming that both types of goods, the public good and the bundle of private goods, represent normal goods, the expansion paths are strictly monotonic increasing.

By comparing (3) with the condition for a Pareto-optimal provision of the public good (the so-called Samuelson condition)

$$\sum_{i=1}^{n} MRS_i(y_i, x) = 1$$

it becomes obvious that private provision without governmental intervention induces a suboptimally low provision of the public good.

2.2 Governmental Intervention

Now consider that the government supports private donations to public goods by granting a uniform subsidy rate $z$ per unit contributed to the public good. These subsidies are financed by an income tax $T^I_i$ and a head tax $T^H$ designed to close the gap between the politically determined income tax revenues and the payment for subsidies. Hence the budget constraint of the government is given by

$$\sum_{i=1}^{n} T^I_i(I^g_i) + nT^H = \sum_{i}^{n} zx_i,$$

assuming that debt financing is not permitted. In case that the subsidies are lower than the overall revenues from taxation, the head tax will become negative, thus representing a lump-sum transfer to the agents. Given the income tax tariff, the revenue from income taxation is determined exogenously by the gross income of individual agents. Consequently, changes in the provision of public goods only affect the level of the head tax.

In the following we distinguish between naive and non-naive agents, showing the consequences of naivety for optimal subsidization schemes. Naive agents are assumed not to realize the impact of their own provision of the public good on the level of the head tax (budget illusion). They perceive no wedge between net and gross subsidy rates and neglect the effect on the head tax in their optimization. As already stressed by Boadway, Pestieau and Wildasin (1989) this seems to be a fairly realistic assumption for economies with a large number of agents. Yet, the literature on the private provision to public goods often assumes agents to be
non-naive. As Falkinger (1996: 416) puts it: “the assumption of budget illusion is rather questionable”. Which of the two scenarios is more appropriate, can hardly be judged in general. To capture both extremes - complete budget illusion and no budget illusion at all - we take a look at two scenarios: Case N, where it is assumed that agents take the head tax to be exogenous to their decisions, and Case NN, where agents have perfect information about the repercussions of their actions on the governmental budget, and therefore, take the head tax to be endogenous.

The naive agent’s budget constraint in the presence of taxation and subsidization is given by:

\[
\text{Case N: } y_i + (1 - z) x_i = I^i_n - T^i_n(I^i_n) - \hat{T}^H
\]

with \(I^i_n\) denoting net income after income taxation.

Non-naive agents on the other hand realize their effect on the head tax. They know that of the received \(z\) per unit of the public good they have to self-finance \(\frac{1}{n} z\), such that the net subsidy rate is reduced to \(\frac{n-1}{n} z\). Their budget constraint therefore reads

\[
\text{Case NN: } y_i + (1 - \frac{n-1}{n} z) x_i = I^i_n - \frac{1}{n} \left( \sum_{j=1}^{n} z x_j - \sum_{j=1 }^{n} T^j(I^j_n) \right)
\]

Maximization of (1) subject to (6) for Case N and (7) for Case NN respectively, leads to the following optimality conditions with respect to the two cases:

\[
\text{Case N: } MRS_i(y_i, x) = p^e_i = 1 - z
\]

\[
\text{Case NN: } MRS_i(y_i, x) = p^e_i = 1 - \frac{n-1}{n} z.
\]

The effective price that an agent expects declines in both cases compared to the non-intervention scenario, whereby the decline is larger in the naivety case. This is due to the fact that the naive agent does not take into account that his own higher provision of the public good increases the head tax, which he has to pay (or in case of a negative head tax: decreases the amount he receives).

Equation (4) shows that for the provision of the public good to be Pareto-efficient the sum of the marginal rates of substitution over all agents has to be equal to unity. Yet, summing up the conditions for an individually optimal provision (8), resp. (9), over all \(i, i = 1, ..., n\), yields:

\[
\text{Case N: } \sum_{i=1}^{n} MRS_i = (1 - z)n
\]

\[
\text{Case NN: } \sum_{i=1}^{n} MRS_i = (1 - z)n + z.
\]
Consequently, if agents are identical, the Pareto-optimal solution can be implemented by using a uniform subsidy rate. Then, $z$ would have to be set at $\frac{n-1}{n}$ in the naive case and unity in the non-naive case.

Yet it was shown by Falkinger (1996) that for $z = 1$ in the non-naive case only the aggregate provision $x$ but not the individual’s provision $x_i$ is determined. With $z = 1$ the budget constraint of the agent is given by $y_i = I_i - (1/n)(x - \sum_j n T_j)$. Replacing $y_i$ by this relation in (8) shows that the optimal allocation between consumption of the private good and the public good only depends on the aggregate provision of $x$. This result is quite straightforward as with $z = 1$ it does not make a difference with respect to the individual’s budget constraint, whether an additional unit of the public good is provided by the agent himself or by one of the other agents. Of a unit provided by himself or by somebody else he equally has to bear $\frac{1}{n}$th of the price of the public good, such that he is indifferent with respect to whom is contributing to the public good.

Applying the same line of reasoning in the case of naive agents, we get $y_i = (1/n)(I_i^p - x_i) - TH$, which depends on $x_i$, such that the aggregate as well as the individual contributions are determined in equilibrium.

However, if we allow for agents with differing preferences, it is not possible to induce a Pareto-efficient outcome by means of the suggested scheme, neither, if we allow the agents to coordinate their activities. Consider that the individual agents face the same effective prices within our scheme. If some agents have lower preferences for the public good than others their individual marginal rates of substitution in (4) differ. Since the effective prices of all agents are identical, some agents’ Nash conditions (8) or (9) are not met. They will become non-contributors of the public good and total public-good provision remains on a suboptimal low level. In order to achieve a Pareto-optimum, the government has to implement a subsidization scheme which allows effective prices to vary across agents, such that each individual agent’s optimality condition can be met in the Pareto-optimum. Subsidization rates have in this case to be tied to some specific characteristic of the agent as it is considered in the next section.

3 The Case of Income Tax-refunds

3.1 The General Idea

In many countries donors can credit their donations to specific public goods with respect to their income tax. More precisely, the donations are deducted from the income tax base - in our case gross income - and therefore reduce income tax liabilities. The refund is conditional on the contribution to the provision of the public good and can therefore be considered as a conditional transfer/subsidy of the government
to the donors. For simplicity we first assume the preference structure of agents to be identical while incomes can differ. Furthermore, we now focus our analysis on non-naive agents, in order to make the analysis more stringent. Hence, we do not have to recurrently distinguish between naive and non-naive case and the reader’s attention is not detracted from the analysis’ main line of reasoning.

In the general set-up of the modified optimization problem we take account of the fact that in many countries the income tax rate is not constant, but rather depends on some personal characteristic of the agent, most commonly his after donation income \( I^g_i - x_i \). With tax rates depending on after donation income, income tax payments of the individual agent can be rewritten as

\[
T^I_i(I^g_i, x_i) = T^I_i(I^g_i) - Z_i(I^g_i, x_i) = \int_0^{I^g_i} t(I_i) dI_i - \int_{I^g_i-x_i}^{I^g_i} t(I_i) dI_i = \int_{I^g_i-x_i}^{I^g_i} t(I_i) dI_i, \quad i = 1, \ldots, n, \tag{12}
\]

where \( t(I_i) \) is the marginal tax rate and \( Z_i(I^g_i, x_i) \) denotes the tax refund or subsidy as a function of gross income and the donation to the public good. Utility maximization which is now subject to

\[
y_i + x_i = I^g_i - T^I_i(I^g_i) + Z_i(I^g_i, x_i) - \frac{1}{n} \left( \sum_j^n (T^I_j(I^g_j) - Z_j(I^g_j, x_j)) \right) \tag{13}
\]

yields the following optimality conditions:

\[
MRS_i = 1 - \frac{n-1}{n} t(I^g_i, x_i). \tag{14}
\]

Hence, the marginal rate of substitution has to be equal to the effective price of the public good from an agent’s point of view. Summing up over all agents gives:

\[
\sum_j^n MRS_j = n - \frac{n-1}{n} \sum_j^n t(I^g_j, x_j), \tag{15}
\]

which implies that for the provision of the public good to equal the Pareto-optimal amount the sum over all marginal tax rates has to equal \( n \).

This result can only be obtained by imposing a linear tariff with \( t = 1 \). Otherwise, the considered agents, with identical preferences, would face different effective prices in the Nash equilibrium, such that this equilibrium would not be compatible with the Pareto-optimal outcome.

It can easily be shown that in the case of a linear income tax the effects of introducing tax deductibility or applying the subsidy scheme discussed in the previous
section are equivalent if the subsidy rate is set equal to the income tax rate. Yet the distribution of income after taxation and subsidization may differ under the two schemes. In the economy we considered in the previous section, a subsidy rate of unity does not imply anything per se about taxation, such that subsidy rates and income tax rates may differ. When we deal with tax deductibility, tax and subsidy rates are equal.

Let us now drop the assumption of identical preferences among agents. Rewriting the condition for an implementation of the Pareto-optimal outcome \( \sum_j^n t(I_j^g-x_j) = n \) as \( \frac{\sum_j^n t(I_j^g-x_j)}{n} = 1 \) shows that on average the marginal tax rate in the optimum has to be unity. This implies that in the equilibrium marginal income tax rates have to be above unity for some agents and below unity for others.

### 3.2 Progressive Tax Schemes

Income tax schemes are often characterized by a progressive tariff, so let us take a closer look on progressive taxation. Progressive tax schemes give rise to two problematic properties with respect to the considered tax-refund systems: Firstly progressive income taxation implies degressive subsidy rates when donations are deductible from taxable income, implying that those agents with the highest income level receive the highest subsidy rates. Secondly, it holds for each agent, regardless of his gross income level, that the first unit of private provision of the public good is funded at the highest rate, with rising provision the subsidy rate then declines. In the equilibrium marginal benefit and effective price are equalized (Figure 1: \( x_1 \)).

It is obvious that the same amount of \( x \) would have been provided by the agent if instead of degressive funding, an, e.g., progressive scheme would have been applied, which is associated with a lower amount of subsidies to be paid by the government to the agent.

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\( ^3 \)The ‘modified’ budget constraint of agent \( i \) under the tax refund system and linear taxation reads

\[
\begin{align*}
y_i + x_i &= I_i^g - t \cdot (I_i^g - x_i) - T^H \\
\Leftrightarrow y_i + (1-t)x_i &= I_i^g - T_i^t(I_i^g) - T^H
\end{align*}
\]

which is equivalent to (6) for \( t = z \).

\( ^4 \)Equality of tax and subsidy rates implies for Case NN and \( t = 1 \) that the government appropriates the complete gross income and redistributes it back to the agents in a lump-sum fashion which leaves all agents with the same after tax income. Consequently, given that all agents have identical preferences, all will contribute the same amount to the provision of the public good.

\( ^5 \)The result of marginal tax rates that exceed unity may seem counterintuitive at first, yet, non-naive agents realize that although the marginal tax rate is above unity, part of their tax payment is redistributed to them via the head tax. Through this redistribution the net tax rate falls below unity and part of the marginal income is left with the agent.

\( ^6 \)As a simple example for a progressive tax scheme, let us assume that marginal tax rates are given by \( t(I_i^g-x_i) = a \cdot (I_i^g-x_i) \) where \( a > 0 \) is exogenous to the agent. For this tax scheme to
price, marginal benefit

excess funding with degressive subsidy rates

effective price under degressive funding

effective price with progressive subsidy rate

Figure 1: Progressive vs degressive funding

induce a Pareto-optimal provision of the public good

\[
\sum_{i} a \cdot (I_i^g - x_i) = n \iff a = \frac{n}{\sum_{i} I_i^g - x}
\]  

(16)

has to hold. It can be seen that although agents perceive \( a \) to be exogenous, it does depend on the aggregate provision of the public good which is endogenous to the model. For this tax scheme to be implementable two further conditions have to be satisfied:

For effective prices – as perceived by the agents – to remain positive in the presence of subsidization, an upper limit for the marginal tax rate in the post-subsidization optimum has to be specified. From (14) it can be shown that the marginal tax rate has to stay below \( t(I_i^g, x_i) < n/(n-1) \). Higher tax – and consequently subsidy – rates would be associated with a negative effective price of the public good.

It can further be shown that for the tax scheme to be implementable the spread of incomes in the society must not be too high. Making use of (16) and rearranging gives \( I_i^g - x_i < (\sum_j I_j^g - x)/(n-1) \) which shows that the suggested tax scheme is only admissible if incomes do not vary too much across agents. It should be noted though that it is the relative and not the absolute spread of incomes that matters. Assume e.g. that all tax bases change by the same factor. This would change the absolute spread of the income distribution, while the relative spread remained unchanged. As the marginal tax rate is homogeneous of degree zero in \( I_i - x_i, i = 1, ..., n \), this exerts no effect on marginal taxes.
3.3 Preliminary Results

The analysis showed that an income-tax refund scheme can – in general – induce a Pareto-efficient outcome. However, then marginal income-tax rates equal or above unity have to be raised. Yet with respect to real-world income tax schemes this would hardly be politically realizable. On the contrary, income tax rates have even declining during recent years in many countries. In the US, e.g., the top marginal income-tax rate was lowered from 38.6% to 35% in 2003. And in Germany the ongoing tax reform will lower the top marginal rate from 48.5% in 2003 to 42% in 2005.

Furthermore, the progression causes the effective price of the public good to increase with rising provision level (subsidy rate declines). This seems to be counterintuitive, given the fact of diminishing marginal utility of the public good consumption. We can conclude that the income-tax refund scheme is no effective means to induce a Pareto-efficient private provision of public goods. This holds even more, because tax schemes often limit the level of income-tax deduction. To illustrate the problems arising from this limitation, we will subsequently have a closer look at such income-tax schemes with limited deductibility.

4 Limited Tax Deductibility

To limit the amount of charitable contributions that are subsidized, tax laws often specify an upper bound to deductibility in the form of a maximal deductible percentage of the income tax base: In the US up to 50% of a taxpayer’s contribution base (i.e. adjusted gross income) can be deducted for the tax year,\footnote{Furthermore individuals in the USA may carry forward charitable contributions that exceed the deductible ceiling for the contribution year for five years. The corporate limit in the US is much lower at 10%.
} while in Germany the maximum deductible amount is even limited to only 10% of an individual’s gross income. For donors who donate more than maximal deductible the tax rebate turns into a quasi lump-sum subsidy, which is independent of the amount they further contribute. In this case the effective price of the public good remains – at the margin – unchanged by the subsidy.

4.1 Budget Constraint and Effective Price

Following the above examples, we assume that the maximal deductible amount is defined in terms of the agent’s gross income level \(I_i^{\text{max}}(I^g_i)\). More specifically we take the maximal deductible amount to be a linear function of gross income \(I_i^{\text{max}}(I^g_i) = b \cdot I^g_i, 0 \leq b \leq 1\). The subsidy schemes we discussed in the previous
section would simply imply \( b = 1 \) and \( I_{i}^{\text{max}} = I_{i}^{g} \). If the private provision of public goods is not abetted by the government, \( b \) would be equal to zero.

Following this specification, the individual’s virtual budget constraint has to be amended to

\[
y_{i} + x = I_{i}^{g} - \int_{0}^{I_{i}^{g}} t(I_{i})dI_{i} + \int_{I_{i}^{g} - x_{i}^{*}}^{I_{i}^{g}} t(I_{i})dI_{i} - T^{H} + \tilde{x}_{i},
\]

where

\[
x_{i}^{*} = \begin{cases} x_{i} & \text{if } 0 \leq x_{i} \leq I_{i}^{\text{max}} \\ I_{i}^{\text{max}} & \text{if } x_{i} > I_{i}^{\text{max}}. \end{cases}
\]

The new governmental budget constraint reads:

\[
\sum_{j}^{n} \left( \int_{0}^{I_{j}^{g}} t(I_{j})dI_{j} \right) + nT^{H} = \sum_{j}^{n} \left( \int_{I_{j}^{g} - x_{j}^{*}}^{I_{j}^{g}} t(I_{j})dI_{j} \right).
\]

The effective price of the public good after subsidization now depends on the level of private donations. For \( 0 \leq x_{i} < I_{i}^{\text{max}} \), the effective price is equal to \( 1 - \frac{n-1}{n} t(I_{i}^{g} - x_{i}) \) and for \( I_{i}^{\text{max}} < x_{i} \) it is equal to 1.

Starting from the first unit of \( x_{i} \) provided, the effective price of the public good \( p_{e_{i}} \) rises, assuming that the income tax tariff is progressive. When the maximal deductible amount is reached, the price jumps back to unity as further provided units of \( x_{i} \) are not subsidized and the marginal subsidy rate drops to zero. Hence, individuals for whom \( I_{i}^{\text{max}} < x_{i} \) holds, receive a quasi lump-sum subsidy. Lump-sum in the sense that the total subsidy is constant in \( x_{i} \), and only quasi lump-sum, since the total amount of the subsidy depends on an individual’s gross income.

### 4.2 Effects of Limited Deductibility on the Isolation Demand

To visualize the effects that the combination of degressive subsidies and limited deductibility have on the individual’s decision problem, let us take a look at agent \( i \)’s isolation demand, i.e. his public-good demand when he is providing the public good while the other agents, \( j = 1, .., n \) and \( j \neq i \), do not provide (\( \tilde{x}_{i} = 0 \)) and also not start to provide despite of the subsidization.\(^8\) In this sense we neglect the public good characteristics of \( x \) in this section. (Yet the exercise seems useful, as it will become clear that even in the case that only two agents are providing (see next section) matters become quite involved which makes it hard to keep track of individual effects.)

In absence of governmental activity the budget line of agent \( i \) is simply a straight line with slope -1 (Figure 2: locus A). By introducing a progressive income tax with

\(^8\)In this case agent \( i \)’s virtual and private income coincide.
The deduction of donations up to a percentage $b$ of gross income, we can observe three effects on the budget curve:

1. Taxation of gross income shifts the budget curve inward, leaving the agent with a lower net income (locus B).

2. Funding of $x_i$ induces the absolute slope of the curve to rise for $0 \leq x_i < bI_i^g$, as the effective price is decreasing (locus C). The higher the gross income level the steeper the respective budget line is in this section as the subsidy rate increases with income. Rising provision of $x_i$ on the other hand leads to a decrease in the slope of each budget line as funding is degressive. For $x_i > bI_i^g$ no more funding per unit is granted for extra units of $x_i$ and we are back to an effective price of one. At $x_i = bI_i^g$ the budget line exhibits a kink with the left-hand limit of the slope being $-1$ and the right-hand limit equalling $-1 + \frac{n-1}{n} t(I_i^g - x_i)$.

3. Redistribution of collected net income tax revenues induces an outward shift of the budget line. As the individual net income tax payment can never be negative, the government runs a budget surplus after income taxation and subsidization. The head tax becomes negative and households receive a lump-sum transfer. As income taxation is progressive this implies a levelling effect on the distribution of net income.

Whether or not governmental intervention leaves the agent better or worse off with respect to his personal income, depends on whether or not his after donation income is above or below the average. Ceteris paribus the agent will
be better off if the following condition holds:

\[
\Leftrightarrow \int_{I_i^{n}}^{I_i^{n}-x_i^*} t(I_i)dI_i - T^H > \int_{0}^{I_i^{n}} t(I_i)dI_i
\]

\[
\Leftrightarrow -T^H > \int_{0}^{I_i^{n}-x_i^*} t(I_i)dI_i.
\]

Considering that

\[
-T^H = \frac{1}{n} \sum_{j} \left( \int_{0}^{I_j^{g}-x_j^*} t(I_j)dI_j \right)
\]

this implies

\[
\frac{1}{n} \sum_{j} \left( \int_{0}^{I_j^{g}-x_j^*} t(I_j)dI_j \right) > \int_{0}^{I_i^{n}-x_i^*} t(I_i)dI_i.
\] (19)

Equation (19) shows that an agent is better off if his net income tax burden is below the average which implies, since we are dealing with progressive taxation, that his after donation income is also below the average. In Figure 2 locus D depicts a situation, in which redistribution leads to a shift of the budget line beyond the initial one (locus A), i.e. the transfer scheme raises the considered agent’s income.

If we integrated other agents’ contributions to the public good \(\tilde{x}_i\), the new budget line would be locus E (Figure 3) where we took account of the fact that agent \(i\) can never spend more than his private income \(I_i^{n} - T^H\) on private good consumption.

![Figure 3: Integration of other agents’ contributions.](image)

However, for simplicity, let us temporarily consider the situation where agent \(i\) is the only contributor and the post-subsidizing and -taxation is given by locus D. We can now check for the effects of the limited-deductibility subsidizing scheme on the optimal allocation between private and public goods. The set of all optima for
varying incomes is given by the income expansion path (IEP) whose slope is (inter alia) determined by the effective price of the public good. Consequently the position of the IEP after subsidization depends crucially on the effect subsidization exerts on the effective price, i.e. on the question on which the segment of D the point of tangency with the indifference curve lies. Three basic types of cases have to be distinguished, yet almost any combination of the three is also possible. Let’s first take a look at the three basic cases:

A) $x_{i}^{\text{new}} > bI_{i}^{g}$.

If $x_{i}^{\text{new}} > bI_{i}^{g}$ holds in the new equilibrium, agents receive a quasi lump-sum subsidy. The effective price of the public good remains pegged at unity and the optimality condition for the considered individual is the same as in the case without governmental intervention:

$$MRS_{i} = 1.$$ 

Subsidization results solely in an income effect, which induces a movement outwards along the unchanged IEP (Figure 4 in which IEP\textsuperscript{old} (pre-subsidization IEP) = IEP\textsuperscript{new} (post-subsidization IEP)). No substitution effect arises.

B) $0 < x_{i}^{\text{new}} < bI_{i}^{g}$.

In this case the points of tangency after governmental intervention lie on the new steeper and non-linear section of the budget line. The relevant optimality condition in this case reads:

$$MRS_{i} = 1 - \frac{n-1}{n} t(I_{i}^{g} - x_{i}).$$

Here subsidization not only results in an income effect, but also in a substitution effect. The latter arises as after governmental intervention the effective price of
the public good in the optimum has changed: $p_i^e$ has decreased which leads for a given income level and under the assumption that both goods are normal to an increase in the provision of $x$ by agent $i$ such that the IEP rotates upwards (Figure 5).

In Figure 5 the slope of IEP\textsuperscript{old} is depicted for an effective pre-subsidization price of unity. In case that the effective price of the public good in the post-subsidization optimum is still equal to unity – as in Case A – the slope of the IEP remains, as we have seen, unchanged. Yet for a range of intramarginal units subsidization reduces effective prices. If the post-subsidization optimum lies in this range – as in Case B – the new IEP will be located above the initial one (in the shaded area).

C) $\tilde{x}_{i}^{new} = bI_i^g$.

This case constitutes the switching point between non-zero and zero marginal subsidy rates. As already explained the post-subsidy budget line exhibits a kink at $x_i = bI_i^g$ with the left-hand limit of the slope being $-1$ and the right-hand limit converging to $-1 + \frac{n-1}{n} t(I_i^g - x_i)$. So for $\tilde{x}_{i}^{new} = bI_i^g$ the following condition has to hold in the new equilibrium:

$$\lim_{x \to bI_i^g^-} \frac{du}{dx_i} = 1 - \frac{n-1}{n} t(I_i^g - x_i) < MRS_i(bI_i^g) = \lim_{x \to bI_i^g^+} \frac{du}{dx_i} = 1.$$ 

This case can be considered as a type of "lock-in" situation: Although an agent’s preferences for the public good might rise with rising income, the agent’s contribution to the public good will only increase in proportion to his income as long as the above condition holds. $\tilde{x}_{i}^{new}$ is therefore given by $bI_i^g$ and the IEP is a straight line (Figure 6).

Besides these three basic scenarios almost every combination of the three is conceivable. Suppose, e.g., that at low income levels the agent’s provision of $x_i$ lies below
the threshold level \( I_{i}^{max} \) (Figure 7, Case B). Marginal subsidy rates are positive and the new IEP lies above the old. Then with rising income the agent’s preference for the public good increases, inducing a more than proportional increase in his optimal provision of \( x_i \). At some income level (Figure 7: \( I_{12}^{g} \)) his marginal rate of substitution finally exceeds \( 1 - \frac{n-1}{n}t(I_{i}^{g} - x_i) \), but is still below unity. So for some range of income \( 1 - \frac{n-1}{n}t(I_{i}^{g} - x_i) < MRS_i < 1 \) holds (Case C) and the agent’s contribution rises in proportion \( b \) with his income. Yet at some income level \( (I_{12}^{g}) \) his preference for \( x \) might be high enough for \( MRS_i = 1 \) to hold, such that he is back on the pre-intervention expansion path (Case A).

Of course this scenario is only one of many possible. Depending on the utility
function of the agent many combinations of the three segments are conceivable.

4.3 Effects of Limited Deductibility for $i = 2$

So far we have only analyzed the reaction of one agent to the implementation of a progressive income tax in combination with limited tax deductibility of private donations. This helps of course to understand the effects with respect to the optimization process of a single agent, yet it neglects that the other individuals will also react to the implementation of governmental policy.

To keep the analysis again as tractable as possible we only consider interior solutions in a 2-agent world, i.e. both agents $i, i = 1, 2$, contribute to the public good. The individual agent’s maximization problem is described by (1) subject to (17). From the first-order conditions and the individual and governmental budget constraints we again get the IEPs of the two agents.

Let us reconsider the different cases we distinguished in the previous section. Even in the context of a relative simple two-agent–two-stage game six different combinations of Case A, B and C can potentially characterize the new equilibrium (see Table 1). In the following analysis, we will take a closer look at the combinations of Cases A and B in the post-subsidization equilibrium. Thereby we again focus on interior solutions in the sense that we abstract from the knife-edge Case C.

Table 1: Reaction to implementation of limited deductibility

<table>
<thead>
<tr>
<th>agent 1</th>
<th>agent 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td>Case B</td>
</tr>
<tr>
<td>Case A</td>
<td>AA</td>
</tr>
<tr>
<td>Case B</td>
<td>AB</td>
</tr>
<tr>
<td>Case C</td>
<td>AC</td>
</tr>
</tbody>
</table>

4.3.1 Scenario AA: $x_i > I_i^{max}, i = 1, 2$

In this scenario we assume that both agents provide funds to the public good in excess of their respective deductible ceiling, such that both agents receive a quasi-lump sum subsidy that leaves the effective price of the public good in the new equilibrium unaffected. Consequently, by means of taxation and tax-refunding only an income redistribution takes place which, as will be shown below, exerts no effect on the aggregate level of public good provision, as it is only the sum of incomes, but not their allocation between agents that matters with respect to the optimal provision of $x$. We are back at the familiar neutrality result.
To obtain this result the budget constraint (17) is solved for $y_i$ and the resulting expression inserted into (1). We now face the modified optimization problem

$$\max_x U(y_i, x) = U(I_i^g + T_i^N + x_i - x, x),$$

where $T_i^N$ denotes the net income transfer to agent $i$

$$T_i^N = \int_{I_i^g - x_i}^{I_i^g} t(I_i)dI_i - \int_0^{I_i^g} t(I_i)dI_i - \left( -\frac{1}{2} \sum_j \int_{0}^{I_j^g - x_j} t(I_i)dI_i \right).$$

As under Scenario AA agents are assumed to donate in excess of their respective maximal deductible amount, net transfers in (21) are independent of the actual $x_i$'s and agents receive a fixed subsidy $x_i^* = b I_i^g$. The effective price in the post-subsidization remains pegged at unity. Consequently the IEPs do not shift due to subsidization.

The solution to the above optimization problem is given by

$$x = f_i(I_i^g + T_i^N + \tilde{x}_i) = f_i(I_i^v).$$

Thus $x$ is a function of the agent’s full or virtual income $I_i^v$, i.e. the sum of the net income transfer $T_i^N$, the monetary value of the enjoyed externality $\tilde{x}_i$ and gross income $I_i^g$. To the agent all components of $I_i^v$ are exogenous and constitute perfect substitutes. Due to this perfect-substitute property the agent is indifferent between a situation in which his net income transfer is raised by one unit and a situation in which the other agent’s public good provision level is increased by one unit. A net income transfer from one agent to the other therefore only causes a rise in the receiving agent’s public good provision by the same amount, while the other
agent will simultaneously reduce his own provision by this amount. In the new equilibrium public good provision $x$, virtual incomes $I_i^v$, $i = 1, 2$, and consequently also the consumption of the private good $y_i$ remain unchanged. Yet the shares of $x$ that each agent provides rise or fall depending on whether an agent’s private income is increased or decreased by the redistribution. The new and old equilibrium are sketched in Figure 8 where also the relevant sections of the indifference curves and virtual budget lines are depicted.

4.3.2 Scenario AB (or BA equivalently): $0 < x_i < I_i^{max}, x_j > I_j^{max}$

With respect to Scenario AB we have to distinguish two types of effects that arise due to the introduction of governmental policy:

- price effect: the effective price for the agent who stays below $I_i^{max}$ in the new equilibrium decreases while the effective price for the other who provides $x_i > I_i^{max}$ remains pegged at unity (Figure 9: $\Delta p_1^e = 0$ and $\Delta p_2^e \neq 0$).

- redistribution effect: incomes (and public good externalities) are redistributed among agents leading to shifts in the regular budget lines, but itself exerting no effect on aggregate $x$ or the consumption pattern of the $y_i$’s (see reasoning for Scenario AA).

![Figure 9: Scenario AB](image)

So, essentially the reactions of $x$ and the $y_i$’s can be attributed to the change in the effective price of agent 2 only, i.e. to the substitution effect. (Remember that with respect to Scenario AB we assume that the subsidization-induced changes in agent 2’s donations do not cause agent 1 to reduce his contributions to a level where he donates less than deductible from his income tax.)
The decrease in agent 2’s effective price implies that he now faces a steeper budget constraint. As both goods are normal the change in the effective price implies that - taken the contribution of the other agent as fixed for the moment - agent 2 wants to raise \( x \) in relation to \( y_2 \), therefore raising his contribution to \( x \). Since the effective price for agent 1 remains unaltered the resulting combination of \( x \) and \( y_1 \) is suboptimal for him, as it implies that \( MRS_1 \) is below unity. Consequently he reallocates his consumption from \( x \) to \( y_1 \) by decreasing his contribution to \( x \) and raising \( y_1 \). An adaptation process to the new equilibrium starts in which the net effect of the decrease of \( p_2^e \) on \( x \) and consequently also on \( y_1 \) is positive. Yet recalling that the sum of incomes in the economy remains constant as governmental intervention only implies a reallocation of income and also remembering that the transformation rate between \( x \) and \( y \) is unity, the aggregated amount of public and private goods that can be purchased by the agents remains the same.\(^9\) So if \( x \) and \( y_1 \) increase \( y_2 \) has to decrease by more than the increase of \( x \) (\( \Delta y_1 + \Delta y_2 + \Delta x = 0 \Leftrightarrow \Delta y_1 + \Delta x = -\Delta y_2 \Rightarrow \Delta x < -\Delta y_2 \)). Since at the outset the slope of the budget line was equal to -1 this implies that the new equilibrium allocation of agent 2 will lie below this line (Figure 9). To the right of the old equilibrium allocation of agent 2 the slope of the indifference curve which represented the optimal utility level before effective-price reduction is above unity, such that the new equilibrium will always lie below the old indifference curve and utility of agent 2 after subsidization has to be lower which is due to the price-reducing impact of subsidization.

### 4.3.3 Scenario BB: \( 0 < x_i < I_i^{max}, \ i = 1,2 \)

Scenario BB can again be decomposed into a redistribution effect and a price effect. As before the redistribution effect alters the equilibrium composition of \( x \), but neither the aggregate provision of \( x \) nor the equilibrium demands for the private good \( y_i \). Now both agents experience a decrease in the effective price of the public good. As the effective price decreases for both agents, their joint provision of the public good undoubtedly increases and the aggregate consumption of the private good has to decrease. Yet with respect to individual demands and individual utility the development is less clear. Although the tax-subsidization scheme might result in a Pareto-improvement, this is not necessarily the case. The new equilibrium might even be characterized by a reduction in welfare for both agents.

We know that as the transformation rate is equal to one, the decrease in the

\[^9\]Recall that in the utility optimum before as well as after governmental intervention the joint budget constraint of the two agents and the government

\[
I_g^1 + I_g^2 = x^{old} + y_1^{old} + y_2^{old} = x^{new} + y_1^{new} + y_2^{new} \tag{23}
\]

has to bind. This directly implies \( -\Delta x = \Delta y_1 + \Delta y_2 \).
aggregate $y_1 + y_2$ has to be of the same magnitude as the increase in $x$. How much the individual consumption of $y_i$ decreases depends on the shape of the utility functions which determine the slope of the indifference curves and the tax regime that specifies the slope of the budget constraint.

![Figure 10: Scenario BB – win-win situation](image)

If preferences and tax regime are such that due to the tax-subsidy scheme both agents’ virtual budget lines shift outward at the relevant margin, each agent’s virtual income rises due to increased public good provision levels of the other agent, but the equilibrium consumption of the private good decreases for both agents. The new equilibrium can, but does not have to be associated with a Pareto-improvement.

In Figure 10 the case of a win-win situation is depicted in which both agents’ utility increases due to governmental intervention. The shaded areas in Figure 10 mark all combinations of $x$ and $y_i$ which are associated with higher than pre-subsidization utility. In Figure 10 both agents’ post-subsidization equilibrium allocations are located in these shaded regions. It is however also conceivable that even if both budget lines shift out the new equilibrium is not characterized by a Pareto-improvement. Post-subsidization equilibria in which one agent’s utility is reduced while the other one’s is increased (win-lose) or even a situation in which both agents lose (lose-lose) are also imaginable. Both agents’ utility can e.g. decline if the marginal subsidy rate exceeds the Pareto-optimal rate and the sum of effective prices falls below unity. In this case public good provision rises beyond the Pareto-optimal level while private good consumption falls to a suboptimal low level. If the increase in public good provision does not compensate agents for the associated decrease in $y_i$, utility will decline. In this case the new equilibrium allocation is characterized by $x_i^{new} > x_i^{old}$ and a $y_i^{new}$ which is located between pre-subsidization budget line and pre-subsidization indifference curve.

If one agent’s IEP rotation due to the subsidization is stronger than the rotation...
of the other agent’s path it is also conceivable that he increases his contribution to
the public good by so much that this increase goes along with a fall in his private
consumption that is larger than the increase in $x$ (see Figure 11). In this case his
virtual budget line shifts inward at the relevant margin and the other agent enjoys
an increase not only in the consumption of the public, but also of the private good.
Hence again one agent loses while the other agent gains utility.

![Figure 11: Scenario BB – win-lose situation](image)

Finally, we have to consider the possibility of an inward shift of both agents’
virtual budget lines. As easily conceivable, this cannot be compatible with agents’
welfare maximizing behavior, as in this case the increase in $x$ would be matched by
a larger decrease in the aggregate consumption of the private good. Part of of the
resources in the economy would be left unemployed and the utility of at least one
agent could be raised without making the other agent worse off.

## 5 Conclusions

Last year, the top income tax rate in the USA was reduced and the same happened in
Germany this year. This will, of course, reduce incentives for taxpayers to donate for
public goods, since the tax-refund rate declines, i.e. the effective price of donations
rises. Therefore, the tax-refund (or tax-subsidy) scheme will become less effective
in shifting the public good provision level to a Pareto-efficient outcome.

However, as our analysis demonstrated, the tax-refund scheme is not only inef-
fective due to low refund rates, it is also associated with decreasing subsidy rates
because of the progression in the tax-tariffs. While effective prices of donations
increase, marginal benefits decline. This contradicts economic reasoning.
Another obstacle to generate Pareto-efficiency stems from the limited deductibility. While in the USA the maximum deductible amount is limited to 50 percent of the tax base, in Germany it is even limited to 10 percent.

Keeping the just described problems in mind we examined whether and under which circumstances the described governmental policy might nevertheless lead to a Pareto-efficient outcome or induce at least a Pareto-improvement. Implications of introducing the tax-refund scheme were analyzed with respect to individual as well as aggregate welfare. We demonstrated that even with respect to a still relative simple two-agent-world the effects which arise due to the described governmental intervention can be quite complex. Depending on the progressiveness of the income tax schemes as well as the preferences of agents, Pareto-improvements might, but do not have to be attainable.

Tax progressiveness and preferences determine whether agents donate more or less than maximal deductible which is decisive for the type of reallocation and welfare effects that arise. Different scenarios are conceivable even in a simple two-agent world: both agents might contribute beyond their deductible ceiling, both agents might stay below this threshold or one agent might exceed it while the other donates less than allowable.

Given that both agents donate more than maximal deductible, both agents receive a quasi lump-sum subsidy and subsidization has no impact on the effective price of the public good. Then, of course, the Pareto-optimal solution cannot be implemented and the tax-subsidy scheme only has an income-redistributing but no national-welfare-improving impact. Even the individuals’ welfare levels remain unchanged.

Assuming that only one agent contributes beyond his deductible ceiling while the other donates less, the marginal subsidy rate in the optimum becomes positive for the latter agent. He faces a reduction of the effective price of the public good which induces a reallocation between public and private good. As a consequence the public good provision shifts closer to a Pareto-efficient outcome, yet this will be at the expense of the agent facing the reduced effective price of the public good.

Last but not least both agents might donate less than their maximal deductible amount. In this case, the tax-subsidy scheme may raise both agents’ utility, i.e. a Pareto-improvement may be induced. Even the Pareto-optimum could be implemented. However – again depending on the shape of the tax scheme and agents’ preferences – outcomes in which one or even both agents’ welfare is reduced might arise in the post subsidization equilibrium.

Interesting with respect to these welfare-implications is that the price effect of the subsidization may be advantageous to rich as well as poor people. Provided that rich people contribute in excess of their deductible ceiling, they face no price effect while a poor person contributing less than its ceiling, will face an unfavorable
price effect. On the other hand, given a scenario where both, poor and rich people contribute less than their maximum deductible amount, the unfavorable price effect of the tax-refund scheme will mainly hit the rich people, since they have a higher marginal income tax rate.

Finally, let us summarize that the considered tax-refund schemes generally applied constitute no effective means to reduce suboptimal public good provision levels. Implications with respect to public good provision as well as with respect to welfare are not clear-cut, but depend crucially on e.g. deductibility ceilings and progressiveness of tax rates. The ineffectiveness of the considered schemes is furthermore reinforced by the reduction of income-tax rates, which is due to income tax reforms in many Western countries. These reforms will most likely not only exert an effect on the level of donations but might possibly also jumble welfare effects.
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