Doctoral Thesis

Local manipulation of single electrons in semiconductor nanostructures

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Local manipulation of single electrons in semiconductor nanostructures

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July 2005
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<th>Explanation</th>
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<tbody>
<tr>
<td>e</td>
<td>Elementary charge</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Dielectric permittivity</td>
</tr>
<tr>
<td>( \epsilon_0 )</td>
<td>Vacuum dielectric constant</td>
</tr>
<tr>
<td>( G_0 = \frac{e^2}{h} )</td>
<td>Conductance quantum</td>
</tr>
<tr>
<td>( h = 2\pi\hbar )</td>
<td>Planck’s constant</td>
</tr>
<tr>
<td>( k_B )</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>( m )</td>
<td>Electron rest mass</td>
</tr>
<tr>
<td>( \mu_B )</td>
<td>Bohr magneton</td>
</tr>
<tr>
<td>( R_0 = \frac{h}{e^2} )</td>
<td>Resistance quantum</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>2DEG</td>
<td>two-dimensional electron gas</td>
</tr>
<tr>
<td>AFM</td>
<td>atomic force microscope</td>
</tr>
<tr>
<td>DOS</td>
<td>density of states</td>
</tr>
<tr>
<td>FQHE</td>
<td>fractional quantum Hall effect</td>
</tr>
<tr>
<td>IQHE</td>
<td>integer quantum Hall effect</td>
</tr>
<tr>
<td>I-V converter</td>
<td>current to voltage converter</td>
</tr>
<tr>
<td>IVC</td>
<td>inner vacuum chamber</td>
</tr>
<tr>
<td>LL</td>
<td>Landau level</td>
</tr>
<tr>
<td>LO</td>
<td>local oxidation</td>
</tr>
<tr>
<td>LP</td>
<td>low-pass</td>
</tr>
<tr>
<td>MBE</td>
<td>molecular beam epitaxy</td>
</tr>
<tr>
<td>MFM</td>
<td>magnetic force microscope</td>
</tr>
<tr>
<td>MOSFET</td>
<td>metal oxide semiconductor field effect transistor</td>
</tr>
<tr>
<td>PCB</td>
<td>printed circuit board</td>
</tr>
<tr>
<td>PID controller</td>
<td>proportional integral differential controller</td>
</tr>
<tr>
<td>PLL</td>
<td>phase-locked loop</td>
</tr>
<tr>
<td>QHE</td>
<td>quantum Hall effect</td>
</tr>
<tr>
<td>QPC</td>
<td>quantum point contact</td>
</tr>
<tr>
<td>RPA</td>
<td>random phase approximation</td>
</tr>
<tr>
<td>SCA</td>
<td>subsurface charge accumulation</td>
</tr>
<tr>
<td>SdH</td>
<td>Shubnikov-de Haas (oscillations)</td>
</tr>
<tr>
<td>SET</td>
<td>single electron transistor</td>
</tr>
<tr>
<td>SFM</td>
<td>scanning force microscope</td>
</tr>
<tr>
<td>SNOM</td>
<td>scanning near-field optical microscope</td>
</tr>
<tr>
<td>SP</td>
<td>sorption pump</td>
</tr>
<tr>
<td>SQUID</td>
<td>superconducting quantum interference device</td>
</tr>
<tr>
<td>STM</td>
<td>scanning tunneling microscope</td>
</tr>
<tr>
<td>TF</td>
<td>tuning fork</td>
</tr>
<tr>
<td>VCA</td>
<td>voltage controlled amplifier</td>
</tr>
<tr>
<td>VCO</td>
<td>voltage controlled oscillator</td>
</tr>
<tr>
<td>VTI</td>
<td>variable temperature insert</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$A$</td>
<td>vector potential</td>
</tr>
<tr>
<td>$\alpha_{\text{gate}}$</td>
<td>gate lever arm</td>
</tr>
<tr>
<td>$B$</td>
<td>magnetic field</td>
</tr>
<tr>
<td>$C_{\Sigma}$</td>
<td>self-capacitance of a quantum dot</td>
</tr>
<tr>
<td>$\Delta N$</td>
<td>Single particle level spacing</td>
</tr>
<tr>
<td>$E_c$</td>
<td>charging energy</td>
</tr>
<tr>
<td>$E_F$</td>
<td>Fermi energy</td>
</tr>
<tr>
<td>$\epsilon_N$</td>
<td>single particle energy of the Nth level</td>
</tr>
<tr>
<td>$G$</td>
<td>conductance</td>
</tr>
<tr>
<td>$G_0$</td>
<td>conductance quantum</td>
</tr>
<tr>
<td>$g^*$</td>
<td>effective g-factor</td>
</tr>
<tr>
<td>$g_s$</td>
<td>spin degeneracy</td>
</tr>
<tr>
<td>$g_v$</td>
<td>valley degeneracy</td>
</tr>
<tr>
<td>$\Gamma_{r,l,s,d}$</td>
<td>tunnel coupling</td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
</tr>
<tr>
<td>$k_F$</td>
<td>Fermi wavenumber</td>
</tr>
<tr>
<td>$L,W$</td>
<td>system size (length, width)</td>
</tr>
<tr>
<td>$\ell_B$</td>
<td>magnetic length</td>
</tr>
<tr>
<td>$\ell_e$</td>
<td>elastic mean free path</td>
</tr>
<tr>
<td>$\ell_\phi$</td>
<td>phase coherence length</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td>Fermi wavelength</td>
</tr>
<tr>
<td>$m^*$</td>
<td>effective mass</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>electron mobility</td>
</tr>
<tr>
<td>$\nu$</td>
<td>filling factor</td>
</tr>
<tr>
<td>$n_s$</td>
<td>electron sheet density</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>cyclotron frequency</td>
</tr>
<tr>
<td>$Q$</td>
<td>charge</td>
</tr>
<tr>
<td>$R$</td>
<td>resistance</td>
</tr>
<tr>
<td>$R_0$</td>
<td>resistance quantum</td>
</tr>
<tr>
<td>$\rho$</td>
<td>resistivity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of states</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>transport scattering time</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$v_d$</td>
<td>drift velocity</td>
</tr>
<tr>
<td>$v_F$</td>
<td>Fermi velocity</td>
</tr>
<tr>
<td>$V$</td>
<td>voltage</td>
</tr>
</tbody>
</table>
Abstract

In this thesis we report scanning gate experiments performed at low temperatures on a number of nanostructures defined on two-dimensional electron gases (2DEGs), with the aim to get insight into the local mechanisms governing electron transport in such samples.

Scanning gate measurements require a local scanning apparatus which, in our case, consists of a home-built scanning force microscope (SFM) which can be operated at a temperature of 300 mK and in magnetic fields up to 9T.

In the first part we focus on the quantum Hall effect (QHE) and investigate a Hall-bar in the quantum Hall regime between filling factor $\nu = 3$ and $\nu = 12$. By using the negatively biased SFM tip as a local scatterer, we are able to modify the backscattering rate in the sample, which is recorded as a change in its resistance as a function of tip position. Changes in the local resistance are measured for different filling factors. At high filling factors we observe fluctuations of the longitudinal resistance spread over the entire sample, which reduce themselves to the edge regions for lower filling factors. This effect is related to the formation of extended states leading to edge channels at the borders of the Hall bar. At even lower filling factors we can identify features in the local resistance which reflect the $1/B$ periodicity of the QHE. The robustness of the quantization of the Hall resistance is demonstrated by scanning at integer $\nu$ within the Hall cross, where no change in the Hall resistance is observed. The results are discussed in the framework of the Landauer-Büttiker formalism, which allows us to interpret the local resistance in terms of the backscattering rate induced by the scanning tip.

In the second part we investigate a quantum dot in the Coulomb blockade regime. The scanning gate technique allows us to map out the potential of the quantum dot, and by using the dot as a sensitive electrometer, we are able to quantitatively describe the interactions between the electrostatic potential of the SFM tip and the electron states in the quantum dot. The results allows us on one hand to reconstruct the interaction potential, and on the other hand to simulate the behavior of a quantum dot in the presence of an external local potential. The effects are discussed in terms of electrostatic interactions. By focusing on the dot interior we observe changes in the electrochemical potential at the resolution limit of our experiment which may be a first step towards the probability distribution mapping of electrons in a quantum dot.

In the third part we focus on a quantum point contact (QPC). We observe strong
changes in its conductance depending on the position and bias voltage of the SFM tip as well as smaller changes appearing in the vicinity of the constriction. We relate the first effect to the bare electrostatic interaction between the QPC and the SFM tip, as demonstrated by simple simulations. We relate the smaller changes to the ability of the QPC to detect charging of local impurity sites induced by the scanning SFM tip.
Zusammenfassung

In dieser Doktorarbeit wurden ortsaufgelöste spektroskopische Messungen mittels Rasterkraft-Mikroskopie bei tiefen Temperaturen an Halbleiternanostrukturen durchgeführt. Ziel dieser Arbeit ist es, Elektronentransport in Nanostrukturen ortsaufgelöst auf mikroskopischer Ebene zu studieren und zu verstehen.

Die Sonde, die benötigt wird, um ortsaufgelöste Messungen durchzuführen, besteht bei diesen Experimenten aus der Spitze eines selbstgebauten Tieftemperatur-Rasterkraftmikroskops, das bei einer Temperatur von 300 mK und in magnetischen Feldern bis zu 9T betrieben werden kann.

Im ersten Teil der Arbeit wird der quanten Hall Effekt (QHE) untersucht, und wir messen den Widerstand einer Hall-Geometrie im Bereich zwischen den Füllfaktoren $\nu = 3$ und $\nu = 12$. Die negativ geladene Spitze des Mikroskops wird dabei als lokale Störstelle verwendet, die uns erlaubt, die Rückstreuwahrscheinlichkeit in der Probe zu verändern, was sich in einer Änderung des Probenwiderstandes als Funktion der Position der Mikroskopspitze äussert. Diese Änderungen werden für verschiedene Füllfaktoren gemessen. Bei hohen Füllfaktoren beobachten wir Fluktuationen im lokalen longitudinalen Widerstand, die sich über die gesamte Probenbreite verteilen, während sich bei kleineren Füllfaktoren die Fluktuationen auf die Probenränder beschränken. Dieser Effekt entsteht durch Bildung von Randkanälen bei den Rändern der Probe. Bei noch kleineren Füllfaktoren (um $\nu = 4$) können wir lokale Widerstandsänderungen identifizieren, die die $1/B$-Periodizität des QHE auch in der lokalen Widerstandsverteilung bestätigen. Die Stabilität der Quantisierung des Hall-Widerstandes wurde bewiesen, indem der Hall-Widerstand im Bereich der Probe zwischen den Elektroden gemessen wurde, und keine Änderung bei ganzzahligen Füllfaktoren detektiert wurde. Die Resultate werden im Rahmen der Landauer-Büttiker Transporttheorie diskutiert, was uns ermöglicht, die lokalen Widerstandsänderungen in einer Änderung der Rückstreuwahrscheinlichkeit quantitativ zu verstehen.

Bereich im Inneren des Quantenpunktes, wo wir Änderungen im lokalen electrochemischen Potential detektieren, die einen ersten Schritt zur Spektroskopie von der Aufenthaltswahrscheinlichkeit der Elektronen im Quantenpunkt darstellen.

Im dritten Teil der Arbeit wird ein Quantenpunktkontakt (QPC) gemessen. Wir beobachten starke Änderungen der Leitfähigkeit je nach Position der Mikroskopspitze und angelegter Spannung, sowie schwache Fluktuationen bei bis zu einem Mikrometer Abstand vom Punktkontakt. Der erste Effekt entsteht aus der elektrostatischen Wechselwirkung, während wir die schwächeren Änderungen mit der Fähigkeit eines QPCs verbinden, sehr kleine Spitzen-induzierte Ladungsänderungen auf lokализierte Zustände zu detektieren.
Chapter 1

Introduction

Today's world of electronics is dominated, although in a rather subtle manner, by two-dimensional systems: Everyone is using computers and mobile phones, whose key components are still based on the evolution of a very simple device: The metal-oxide-semiconductor field effect transistor (MOSFET). The first device was built in 1960 [1], based on the simpler junction-FET (JFET) [2], and its realization led to a very unusual evolution with respect to the “normal” course of action in physical research: A whole field of fundamental and applied physics was born, first the device, then the physics!

While applications became one of the most booming industrial domains, fundamental research in the field of two-dimensional systems was triggered by the discovery of the integer quantum Hall effect (IQHE) in 1980 [3], where steps in the transversal resistance in a Hall geometry appear for different magnetic fields. Although the classical Hall effect, basically also a 2D effect, was discovered already in 1879 [4], the observation of quantum mechanical effects like conductance quantization would not have been possible without 2DEGs, where electrons are confined in one direction, but are free to move in the other two directions. The confinement is obtained by growing two semiconductor materials with different band gap, but a similar lattice constant, on top of each other, with the result of getting a narrow potential well at the interface, where a 2DEG can form. The fractional quantum Hall effect (FQHE) discovered two years after the IQHE [5] added new effects, like the formation of quasi-particles of fractional charge, and was also awarded with a Nobel prize.

The increase in purity of the sample materials and the development of more elaborate lithographical techniques, allowing to reduce the size of small semiconductor structures also in the plane of the 2DEG, made mesoscopic conductors become possible. A mesoscopic conductor has typical size parameters which are comparable or smaller than the length scales relevant for electron transport, the elastic mean free path \( \ell_e \) or the phase coherence length \( \ell_\phi \), reaching sizes of the same order of the Fermi wavelength \( \lambda_F \). The probably best known representatives for this kind of conductors are quantum dots, which are basically zero-dimensional mesoscopic
systems. Due to their weak coupling to the leads, charge quantization becomes very important, leading to the so-called Coulomb blockade effect, where single electron charging is observed. The ability to control the number of electrons in a quantum dot one by one is the reason for their definition as artificial atoms \[6, 7\], and is an important step for the detection and control of single spins. Quantum dots have also been proposed as building blocks within quantum information processing schemes \[8\], since the charge or the spin of the confined electrons can be used as an elementary unit (qubit) for quantum computation algorithms.

The realization of such an information processing demands full control over the parameters of the devices involved, requiring a complete knowledge of their response to external (wanted or unwanted) perturbations. Such a complete knowledge is impossible to gain with the standard measurement techniques, where basically the only parameter which can be measured is the current flowing in and out of the sample. The simple question of how the electrons behave inside the device cannot be answered. This question is not only interesting in the context of quantum computation, but also for more fundamental physical effects, such as the geometrical quantization of current channels in the QHE or in constrictions, or the probability density of electrons in a quantum dot.

Local probing techniques have a high potential to give very detailed insight into the local properties of a semiconductor sample, and their technical realization was made possible by the invention of the scanning tunneling microscope (STM) in 1981 \[9\], and the scanning force microscope (SFM) four years later \[10\]. Both techniques permit to resolve and manipulate objects as small as atoms, even if the tips of such microscopes are macroscopic objects with typical diameters of thousands of times the size of an atom. The same is valid for the scanning gate technique, where a metallic SFM tip is used as a moving gate: it is possible to resolve changes in the conductance of mesoscopic samples over a distance which roughly corresponds to the extension of some hundreds of atoms, even though the diameter of the scanning tip is much larger.

Due to the technical complexity of low temperature local probe experiments, and therefore to the limited number of experiments performed up to now, the technique becomes extremely interesting for studying almost every two-dimensional mesoscopic system. In this thesis we focus our attention on the QHE, on a quantum dot, and on a quantum point contact.

The IQHE is known for its $1/B$ periodicity and for the quantization of the Hall resistance at multiples of $h/e^2$, which is known to be independent of sample properties and purity. A local back-scatterer like the potential induced by the scanning SFM tip is an ideal test for such robustness, since it can be placed at any position of the Hall bar sample, and can be tuned in amplitude. Moreover, the scanning gate technique can be used to verify the $1/B$ periodicity from the point of view of the resistance distribution in the sample, delivering at the same time a better understanding of physical phenomena related to the localization of quantum mechanical
states and the formation of extended states at the edges of the sample.

The ultimate goal for scanning gate experiments on quantum dots is the spatial mapping of the probability density of electrons, which would deliver the complete information about the electronic states in the dot. The reason why this hasn’t been achieved yet on tunable quantum dots with an SFM is that it is a formidable task, which requires the knowledge of all interaction parameters in the system. This information can be provided by the quantum dot itself, since it is a very sensitive electrometer. A first impression of how the electrochemical potential in the dot may look like, can be gained since scanning gate measurements provide a very high electrical resolution. In addition the scanning gate setup allows to manipulate single electrons by means of the interaction potential between scanning tip and electronic states in the dot, and by the charge quantization inherent to the Coulomb blockade effect.

Additional knowledge is provided by experiments on one of the most important building blocks of nanostructures, a quantum point contact (QPC). By studying its conductance it is possible to gain additional information about the mechanisms governing sequential tunneling in the QPCs of a quantum dot. Its high sensitivity as a charge detector allows to observe the charge distribution around the constriction forming the QPC.

The thesis begins with a short overview over the physical principles needed for understanding the observed effects (chapter 2). The measurement setup and in particular the low-temperature SFM are discussed in detail in chapter 3, with a short overview of other techniques and experiments in the field of scanning probe. Chapter 4 deals with the physics inherent to the interactions between the potential induced by the scanning tip and the different kinds of samples, described in the following chapters. The experimental part begins in chapter 5, where we determine the local properties of a Hall bar in the quantum Hall regime, using the scanning tip as a local back-scatterer. The backscattering amplitude is found to be the relevant parameter, and it will be discussed in the framework of a suitable theoretical model. The complete mapping of interaction potentials and the manipulation of single electrons are discussed in chapter 6, where we present the first local probe experiments on a fully tunable quantum dot. Although experimental constraints inherent to the measurement setup limit the measurements, we are able to give a first insight into the details of the electrochemical potential inside the quantum dot. The experimental part concludes with measurements on a QPC (chapter 7), where we discuss the general properties and the detection of charging effects observable in the proximity of the constriction. The last chapter is a brief summary of the reported data and an outlook for future experiments.
Chapter 2

Electron transport in semiconductor nanostructures

2.1 Two-dimensional electron gas (2DEG)

2.1.1 Heterostructures

The field of two-dimensional electron gas (2DEG) research started after the invention of the silicon based MOSFET (metal-oxide semiconductor field effect transistor) in the late sixties. It was a unique situation, where a technological discovery opened a new field of fundamental research. First experiments were done already in that decade \[11\], but it was only after the discovery of the quantum Hall effect (QHE) in 1980 \[3\], that a large number of scientists started working on these structures. With todays molecular beam epitaxy (MBE) it is possible to deposit layers of semiconducting material with atomic precision, allowing an exact control of the electronic parameters of a sample. At the present day, experiments are mostly performed on III-V compounds like Gallium Arsenide (GaAs), Aluminum Arsenide (AlAs), or Indium Phosphide (InP), as well as ternary and quaternary compounds of these elements.

The experiments discussed in this thesis are done on GaAs heterostructures, which contain a very high mobility 2DEG due to the high purity of the Ga[Al]As system. These structures are manufactured by growing a GaAs layer on a GaAs substrate, allowing an atomically flat 001 surface. The potential well where the 2DEG forms, is obtained by growing a layer of Al\(_x\)Ga\(_{1-x}\)As on top of it. This material has almost the same lattice constant, thus almost no strain will be present at the boundary between the two layers, but a different bandgap \(E_g\), which can be tuned by the aluminum concentration \(x\) (GaAs: \(E_g = 1.52\) eV, AlAs: \(E_g = 2.23\) eV). Therefore the potential shape of the conduction band can be engineered to any shape (e.g. triangular, parabolic). On top of these layers follows the donor layer (Si \(n\)-type \(\delta\)-doping) and a GaAs cap layer. Figure 2.1 shows the layer sequence, and a scheme of the band diagram of such a sample. The doping layer consists of
Si atoms 17 nm above the 2DEG (modulation doping) which mainly replace the Ga atoms. About 50% of these donor atoms get ionized and their loosely bound electrons diffuse into the GaAs, which has a lower conduction band edge. The conduction band edge is lowered (see Fig. 2.1), forming a confinement potential in z-direction, which keeps the electrons at the interface between GaAs and AlGaAs. Since the electrons can move freely in the $x - y$ plane, they form a two-dimensional electron gas where, at low temperatures, only the ground state is occupied; states with higher quantum number in $z$-direction are called higher subbands. Modulation doping allows samples with very high mobilities, since no scattering at donor atoms can occur, and is therefore used e.g. in MOSFETs for high frequency applications. An extensive review on two-dimensional systems is given in [12]. The 2DEG is electrically connected to the measurement setup by Ohmic contacts obtained by diffusing AuGe or TiN through the layers on top of the 2DEG.

Lateral patterning can be done with different techniques. Among these: Optical lithography, electron-beam lithography, or AFM lithography. In optical lithography the sample is first covered with a photosensitive resist and then exposed with ultraviolet light through a mask, which contains the pattern of the structure. The exposed surface of the sample is then etched down by typically 100 nm, thus removing also the 2DEG in these regions. The Hall-bar used for the measurements discussed in chapter 5 was obtained with this technique. The main limitation of the resolution
is the wavelength of the light (larger than 200 nm) and the mechanical precision of the exposing equipment (mask aligner). Electron beam lithography overcomes this limitation with the shorter wavelength of the electrons and is widely used for the fabrication of semiconductor nanostructures like quantum dots and quantum point contacts (QPCs). The samples discussed in chapters 6 and 7 were patterned using a different technique: AFM lithography.

### 2.1.1 AFM lithography

The high resolution which can be achieved with scanning force microscopes (SFMs) makes them the ideal tool for patterning semiconductor nanostructures (more details about SFMs can be found in the next chapter). First attempts were done by mechanically removing the top layers of the sample by scratching [13], a technique which has since then been refined, allowing a very high resolution [14, 15, 16, 17, 18]. The technique used here is local oxidation (LO): a negative voltage with respect to the sample is applied to the tip of a SFM, using the water film that covers every surface at ambient conditions as the electrolyte for the process. The conductive tip acts therefore as the cathode and the 2DEG (which is connected to ground) as the anode. The OH$^-$ ions, which arise from the dissociation of the water molecules, combine with the GaAs surface and form an oxide layer, under which the 2DEG is depleted (if it is buried less than 70 nm below the surface). The technique was first used in the early nineties [19], and later [20] on GaAs to increase the resistance of the 2DEG below the oxide lines, until it was shown that the 2DEG can be depleted completely [21, 22, 23]. Many research groups use this technique to pattern semiconductor nanostructures [15, 24, 25, 26, 27], but it can also be used to pattern thin metallic films on top of the semiconductor material [28, 29]. Using the SFM tip as a pencil, it is possible to “draw” any pattern on the surface of a semiconductor material, including quantum dots, quantum point contacts, quantum rings, antidot arrays and quantum wires.

### Quantum effects and sample size

The sizes of these nanostructures are still large compared to typical atomic sizes, but it is still possible to observe quantum mechanical effects: This is due to the basic length scales in these systems. In two dimensions the kinetic energy of the conduction electrons is given by $E(k) = \frac{\hbar^2 k^2}{2m^*}$, where $m^*$ is the effective mass of an electron ($m^* = 0.067m_e$ in GaAs) and $k$ the wavevector, and the density of states (DOS) is constant and given by

$$\rho(E) = \frac{m^*}{\pi \hbar^2} = \text{const.},$$  \hspace{1cm} (2.1)

where spin degeneracy has been assumed. Due to the constant DOS, the sheet density of electrons $n_s$ is related to the Fermi energy $E_F$ via $n_s = E_F \rho(E)$, and
therefore the Fermi wavevector $k_F = \sqrt{2m^*E_F/\hbar^2}$ is related to the density by

$$k_F = \sqrt{2\pi n_s}. \quad (2.2)$$

The Fermi wavelength is therefore given by

$$\lambda_F = \sqrt{2\pi n_s}, \quad (2.3)$$

and can reach 50 nm in Ga[Al]As samples, where the density is relatively small (compared to metals). Since the impurities concentration is very low in GaAs, very high mobilities $\mu_e = e\tau_t/m^*$ can be achieved, leading to elastic mean free paths (with $v_F = \hbar k_F/m^*$ the Fermi velocity)

$$\ell_e = \tau_tv_F \quad (2.4)$$

of up to several tens of microns. These dimensions are usually much larger than the size of semiconductor nanostructures, explaining why quantum effects can be measured without requiring nanostructures of atomic size.

### 2.2 Diffusive transport: the Drude model

#### 2.2.1 Basics

The simplest model for describing the conductivity in a 2D system is the Drude model for the DC electrical conductivity in a metal. The basic assumptions of the Drude model are the following:

- In absence of external fields electrons move uniformly on a straight line. They are considered as classical point charges with an effective mass $m^*$ and a velocity $\vec{v}$. Electron-electron interaction and electron-ion interaction are neglected (this is the independent- and free-electron approximation resp.).

- Collisions are instantaneous, with instantaneous change in the velocity of an electron, and are due only to electron-ion collisions.

- The probability for a collision per unit of time is given by $1/\tau_t$, where $\tau_t$ is the transport scattering time, an average over all possible scattering times between two collisions. This means that an electron will travel on average for a time $\tau_t$ between two collisions.

- Local thermodynamic equilibrium is maintained only through collisions. This is a logical consequence from the independent- and free-electron approximation.
In the presence of an external electric field $\vec{E}$, the isotropic velocity $\vec{v}$ averages to the drift velocity, which can be expressed as $\vec{v}_d = -e\vec{E}\tau/m^*$. The drift velocity can be written in terms of the electron mobility $\mu_e = e\tau/m^*$ and yields

$$v_d = -\mu_e \vec{E}. \quad (2.5)$$

Combining the expression for the current density $\vec{j} = -ne\vec{v}_d = \sigma \vec{E}$ with eqs. (2.2) and (2.5), one gets the result for the Drude conductivity [30, 31]:

$$\sigma = en_s\mu_e = \frac{e^2 n_s \tau}{m^*} = \frac{e^2}{h} k_F \ell_e. \quad (2.6)$$

The Drude model fails as soon as the local electric field fluctuates on a length scale comparable to $l_e$, since then the basic assumptions are not valid anymore.

### 2.2.2 Magnetoresistance

Taking the classical equation of motion for an electron in two dimensions, and assuming the relaxation time approximation for the average momentum, the steady state current $\vec{j} = -n_s e\vec{v}_d = \sigma \vec{E}$ contains the conductivity tensor $\sigma$ given by:

$$\sigma = \frac{en_s\mu_e}{1 + \mu_e^2 B^2} \begin{pmatrix} 1 & -\mu_e B \\ -\mu_e B & 1 \end{pmatrix}. \quad (2.7)$$

Equation (2.7) contains only two sample dependent quantities: The electron density $n_s$ and mobility $\mu_e$. By inverting the conductivity tensor, one gets the resistivity tensor

$$\rho = \sigma^{-1} = \frac{1}{en_s\mu_e} \begin{pmatrix} 1 & \mu_e B \\ -\mu_e B & 1 \end{pmatrix} \quad (2.8)$$

where the diagonal and off-diagonal elements

$$\rho_{xx} = \rho_{yy} = \frac{1}{en\mu_e} \quad \text{and} \quad \rho_{xy} = -\rho_{yx} = \frac{B}{en} \quad (2.9)$$

are the longitudinal resistivity and the Hall resistivity (or resistance, in 2D) respectively. In this model $\rho_{xx}$ does not depend on the magnetic field, while $\rho_{xy}$, and thus the Hall voltage $V_H$ increases linearly with $B$. At low magnetic fields, where this model is still valid, one can determine the electron density $n_s$ and the mobility $\mu_e$ by measuring $\rho_{xx}$ and $\rho_{xy}$:

$$n_s = \left| e \frac{\partial \rho_{xx}}{\partial B} \right|^{-1} = \left[ \frac{|e|}{I} \frac{dV_H}{dB} \right]^{-1} \quad (2.10)$$

(for the second expression $I = \text{const.}$ was assumed) and

$$\mu_e = \frac{1}{|e|n_s \rho_{xx}} = \left| e |n_s \frac{L \ V_H}{W \ I} \right|^{-1}. \quad (2.11)$$
Also here, a constant current was assumed, and the aspect ratio of the sample \(L/W\) was inserted. At higher magnetic fields one enters the quantum Hall regime, and the situation becomes very different, (see section 2.4). For more detailed informations about this model and the following theory sections see [32] and [33, 34, 35].

2.3 Ballistic transport: Landauer-Büttiker formalism

2.3.1 Conductance from transmission: Landauer formula

A very effective method to describe electronic transport in mesoscopic systems is to express the current through a conductor in terms of the probability that an electron can be transmitted through it. In these systems, due to the reduced size, electrons behave like photons in a electromagnetic waveguide and occupy single discrete energy levels (subbands or modes). Assuming a channel which is infinitely long in \(y\) direction, the wavefunction of such electrons can be written as:

\[
\psi_{n\vec{k}}(\vec{r}) = \chi_n(x,z) \frac{1}{\sqrt{L}} e^{ik_yy},
\]  

(2.12)

where the mode quantization is contained in the function \(\chi_n(x,z)\), while \(\frac{1}{\sqrt{L}} e^{ik_yy}\) is a normalized plane wave in \(y\) direction. The simplest description is to consider a narrow channel carrying \(N\) modes, connected to two reservoirs (the contacts) at electrochemical potentials \(\mu_L\) and \(\mu_R\), where \(\mu_L - \mu_R = eV\) is kept constant by an external voltage source. If the conductor is not ideal, part of the current injected into the channel from the contact with electrochemical potential \(\mu_L\) will go to the contact with electrochemical potential \(\mu_R\), while a part will be reflected back. At a temperature \(T = 0\) the current starting from the left contact is given by

\[
I_L = g_s(e/h)N (\mu_L - \mu_R),
\]  

(2.13)

while the current reaching the right contact is given by

\[
I_R = I_{tot} = g_s(e/h)NT (\mu_L - \mu_R).
\]  

(2.14)

\(T\) is the transmission probability from the left contact to the right contact, assuming that all \(N\) modes carry the same current. The conductance \(G = \frac{eI}{\mu_L - \mu_R}\) is then given by

\[
G = g_s \frac{e^2}{h} NT,
\]  

(2.15)

which is the so-called Landauer formula [36]. If the transmission probability is different for different modes, eq. (2.15) has to be rewritten as

\[
G = g_s \frac{e^2}{h} \sum_{n=1}^{N} T_n,
\]  

(2.16)
where $T_j$ is the transmission probability of the $j$-th mode, neglecting inter-mode scattering.

### 2.3.2 Quantum point contacts

The Landauer formula can be used to describe the conduction through a narrow constriction separating to electron reservoirs, a quantum point contact (QPC). If $T = 0$ K, the Fermi distribution will be a sharp step, and the current can be written like in eq. (2.14). If $T \neq 0$ K the distribution function has to be taken into account. For the current one obtains

$$I_R = g_s e \sum_n \int_0^\infty dE T_n(E) [f_L(E) - f_R(E)],$$

(2.17)

where $f_L(E)$ and $f_R(E)$ are the distribution functions in the left and right reservoir respectively. In linear response, the difference $f_L(E) - f_R(E)$ can be expanded if the difference in electrochemical potential $\delta \mu$ is a small quantity:

$$f_L(E) - f_R(E) = -\frac{\partial f_L(E)}{\partial \mu} \delta \mu,$$

(2.18)

giving a total current of

$$I_R = g_s e \sum_n \int_0^\infty dE T_n \left( -\frac{\partial f_R(E)}{\partial \mu} \right) \frac{\delta \mu}{e}.$$

(2.19)

Leaving out the term $\delta \mu/e$, one gets the Landauer formula (2.15) for finite temperatures in linear response. For very low temperatures, the conductance will depend only on the transmission probability at $E_F$, and will be quantized at the values

$$G = g_s e^2 h N,$$

(2.20)

multiples of the conductance quantum $G_0 = e^2/h$, with every mode contributing with $g_s e^2/h$ to the total conductivity. The number of modes can be estimated with $N \sim \frac{W}{\lambda_F/2}$, where $W$ is the width of the constriction.

### 2.3.3 Büttiker formula

The Büttiker formula is the generalization of the Landauer formula for multi-terminal geometries. The transmission is now not only from the left contact to the right, but between every pair of contacts: The transmission from contact $j$ to contact $i$ is given by the transmission matrix element $T_{ij} = T_{i-j}$, while the diagonal elements of the transmission matrix given by $T_{ii} = N_i - R_i$, where $N_i$ is the number of modes and $R_i$ the backscattering probability, describe the probability of an electron to enter
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the device at the lead \( i \). For \( T = 0 \) the current flowing through contact \( j \) can be written as

\[
I_j = \frac{2e}{h} \sum_i [T_{ij} \mu_j - T_{ji} \mu_i],
\]

(2.21)
or, with \( V = \mu/e \),

\[
I_j = \sum_i \left[ G_{ij} V_j - G_{ji} V_i \right] \text{ with } G_{ij} = \frac{2e^2}{h} T_{ij}.
\]

(2.22)

If all voltages are the same, the current has to be zero, leading to \( \sum_j G_{ij} = \sum_j G_{ji} \), and therefore eq. (2.22) becomes

\[
I_j = \sum_i G_{ji} \left[ V_j - V_i \right].
\]

(2.23)

Also in this case, for \( T \neq 0 \) K the distribution functions in the contacts have to be taken into account. The current through contact \( j \) is then

\[
I_j = -g_s \frac{e}{h} \int dE \left\{ [N_j(E) - R_j(E)] f_j(E) - \sum_{i,i\neq j} T_{ji}(E) f_i(E) \right\}.
\]

(2.24)

In linear response, the same argument as above holds, and the conductance matrix becomes

\[
G_{ji} = g_s \frac{e^2}{h} \int dET_{ji}(E) \left( \frac{\partial f_j(E)}{\partial \mu} \right),
\]

(2.25)

where the diagonal and the off-diagonal elements of the matrix are related through

\[
N_j - R_j = \sum_{i,i\neq j} T_{ji} = \sum_{i,i\neq j} T_{ij}.
\]

(2.26)

Equation (2.24) allows to calculate the transmission coefficients of any geometry, if the currents and voltages are known. If the resistance is measured, the experimental value \( R_j \) of the resistance in contact \( j \) can be connected to \( T_{ij} \) using the relation \( R_j = V_j/I \), where \( I \) is the total current, which can be reduced to a single parameter with a suitable wiring setup.

2.4 Quantum Hall Effect (QHE)

The Drude model of conductivity predicts a linear increase of the Hall resistance with the magnetic field. But at low temperatures and at high magnetic fields, the Hall resistance shows plateaus at resistance values which correspond to integer fractions of the resistance quantum \( R_0 = h^2/e^2 \). This quantization is very stable since it does not depend on the geometry of the sample, on the purity of the sample,
and on the material. For these reasons, and due to its very high relative precision, it is used as the international standard for resistance [38]. In contrast to the Hall resistance $R_{xy}$, the longitudinal resistance $R_{xx}$ strongly depends on the properties of the sample, but also shows striking differences to the Drude model, since it is only constant at very low fields, and starts oscillating with periodicity $1/B$, dropping to 0 when the Hall resistance is on a plateau. Figure 2.2 shows such a resistivity measurement, as well as the Hall bar geometry used for such an experiment.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{(a) Measurement geometry for measuring the longitudinal and Hall resistance (b) Typical measurement of Shubnikov-de Haas oscillations in $R_{xx}$ and Hall resistance $R_{xy}$ (from [39]).}
\end{figure}

2.4.1 Electrons in a magnetic field: Landau levels

In order to understand the $1/B$ periodicity of the longitudinal resistance, it is necessary to solve Schrödinger’s equation for a single electron confined in 2D:

$$
\left[ \frac{(-i\hbar\nabla + eA)^2}{2m^*} + U(z) \right] \psi(x, y) = E \psi(x, y),
$$

where $U(z)$ is the confining potential of the 2DEG. For a magnetic field in $z$-direction, the Landau gauge $\vec{A} = (-Bz, 0, 0)$ can be used, and the Hamiltonian separates in its $z$- and $xy$-components

$$
H_z = -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + V(z) \quad \text{and} \quad H_{xy} = \left( \frac{i\hbar\nabla_x - eBz}{2m^*} \right)^2 + p_y^2.
$$

The $z$-component does not depend on the magnetic field, while the $xy$-component can be solved with an Ansatz $\psi(x, y) = e^{ik_zz} \chi(y)$, leading to the equation of a
harmonic oscillator

\[
\left( \frac{i \hbar \vec{\nabla}_x}{2m^*} + \frac{1}{2} m^* \omega_c (y - y_c)^2 \right) \chi(y) = E \chi(y),
\]

with a center coordinate \( y_c = \frac{\hbar k_e B}{e} \), where \( \omega_c = \frac{eB}{m^*} \) is the cyclotron frequency. The energy levels of an electron in magnetic field are therefore those of an harmonic oscillator:

\[ E_n = \hbar \omega_c \left( n + \frac{1}{2} \right), \]

and are called Landau levels (LLs). Since they do not depend on \( k_x \), they are energetically degenerated. The electrons occupy quantized orbits (see for example [30][40]), and the number of electrons per LL can be found by dividing the area between two orbits \( 2\pi eB/\hbar \) by the area in \( k \)-space occupied by a single orbital \( (2\pi/L)(2\pi/W) \) \((L \text{ and } W \text{ are the length and width of the sample respectively})\), leading to

\[ N_L = \frac{eLW}{2\pi\hbar} B. \]

Dividing the total number of electrons in the sample \( N_{tot} = n_sWL \) by \( N_L \), one gets the number of occupied LLs in a sample with density \( n_s \) at a magnetic field \( B \):

\[ \nu = \frac{N_{tot}}{N_L} = \frac{n_s \hbar}{eB}. \]

This number is called “filling factor”. Since \( E_F \) oscillates with \( \nu \) periodically in \( 1/B \) at a fixed density \( n_s \), it is possible to extract the density from the period of the oscillations, since \( \Delta E = \frac{\nu \hbar}{n_s \hbar} \) (the factor 2 comes from spin degeneracy). The electron spin can be added to the energy term \( (2.30) \), leading to

\[ E_n^\pm = \hbar \omega_c \left( n + \frac{1}{2} \right) \pm \frac{1}{2} g^* \mu_B B_z, \]

where the Zeeman term \( \frac{1}{2} g^* \mu_B B_z \) contains the effective g-factor \( g^* \) and the Bohr magneton \( \mu_B = e\hbar/2m^* \). The DOS looks like sketched in Fig. 2.3. In absence of a magnetic field it is constant, and it shows quantized energy levels if a field is applied. The separation between two spin-degenerated LLs is given by \( \hbar \omega_c \), while the separation between the spin up and down states of the same LL corresponds to the Zeeman term \( g^* \mu_B \).

The energy levels are broadened due to local potential inhomogeneities which cause backscattering (see Fig. 2.3). If the scattering rate is given by \( 1/\tau \), the broadening is given (due to the uncertainty principle) by \( \hbar/\tau \). The transition from the Shubnikov-de Haas (SdH) to the quantum Hall regime is determined by the formation of mobility gaps between different LLs: On the tail of the broadened LL peak, the states are localized, and therefore do not contribute to transport, in contrast to the extended states on top of the peak. As soon as the broadening of the peaks becomes smaller than the separation between LLs, i. e. above a magnetic field \( B = \frac{\omega_c}{e\tau} \), there is a transition to the quantum Hall effect.
2.4.2 Edge channels and the integer quantum Hall effect

Each LL has a nearly constant energy in the bulk of the 2DES, but this energy becomes very large, if an electron tries to leave the system (like in a potential well), see Fig. 2.3. If the Fermi energy aligns with a Landau level (half-integer filling factor), the electrons will occupy states over the bulk of the sample, since the energy there is equal to the Fermi energy. This means that, if a scattering event occurs, the electron in a state $|n,k\rangle$ will find a state $|n, -k\rangle$ to which it can scatter, and the resistivity is not zero. If the Fermi energy is between two Landau levels (integer filling factors), only electrons at the border of the system will contribute to a current through it, since there the Fermi energy “intersects” the Landau levels. But since the Landau levels are quantized, the number of current channels will also be quantized (corresponding to the number of intersections between Fermi energy and Landau levels): these are called edge channels. They can be understood by adding a potential $U(y)$, which represents the sample boundaries in $y$-direction, in the Hamiltonian $H_{2D}$, as displayed in Fig. 2.3. In first order perturbation theory, the energy $E_{2D}$ is then

$$E_{n,k_x} = \hbar \omega_c \left( n + \frac{1}{2} \right) + \langle n, k_x | U(y) | n, k_x \rangle \approx \hbar \omega_c \left( n + \frac{1}{2} \right) + U(y_c)$$  \hspace{1cm} (2.34)

The states have a nonzero group velocity $v_x = \frac{1}{\hbar} \frac{\partial E_{n,k_x}}{\partial k_x} = \frac{1}{eB} \frac{\partial U(y)}{\partial y}$. In this situation

\[ \text{Energy} \]

\[ \text{Density of states} \]

\[ g^*mB \]

\[ \text{Extended states} \]

\[ \text{Localized states} \]

\[ \text{DOS at } B = 0 \]

\[ \text{y} \]

\[ E_F \]

\[ U(y) \]

\[ \text{Figure 2.3: (a) Density of states with spin split Landau levels (b) Effect of the confining potential } U(y): \text{ The Landau levels are bent at the edges of the sample leading to edge channels, while potential fluctuations in the bulk lead to localized states.} \]
the resistivity $\rho_{xx}$ drops to zero since there are no free states to which the electrons
could scatter. With a change in the magnetic field, the Landau levels move with
respect to the Fermi energy, causing the resistivity to oscillate, and drop to zero
every time the Fermi energy is between two levels (this is the equivalent to the de
Haas-van Alphen effect).

2.4.3 QHE in the Landauer-Büttiker picture

The Landauer-Büttiker picture \[41\] is ideal for describing the IQHE in terms of
transmitted channels. The quantization of the Hall resistance is connected to the
behavior of the longitudinal resistance: When $R_{xx}$ goes to zero (when $E_F$ is between
two Landau levels), the edge channels act as an ideal one-dimensional channel.
Thus, all contacts on one side of the Hall-bar (contacts 6, 5, and 4 in Fig. 2.2
resp. contacts 1, 2, and 3) will have the same electrochemical potential $\mu_T$ and $\mu_B$
respectively. The Hall voltage is measured between contacts 2 and 6, and is given
by $V_H = \frac{\mu_T - \mu_B}{e}$, while the current at a given filling factor $\nu$ is given by $I = \nu \frac{e^2}{h} V_H$.
The Hall resistance is then

$$R_{xy} = \frac{V_H}{I} = \frac{1}{\nu} \frac{e^2}{h}.$$  \hspace{1cm} (2.35)

The Hall conductance, which can be obtained by tensor inversion from the lon-
gitudinal and Hall resistances, is also quantized, and is given by $G_{xy} = \sigma_{xy} = \nu \frac{e^2}{h}$.
This is the analogue of the Landauer formula (2.20) for a quantum point contact,
and a Hall bar can be treated in the same way, if the Hall resistance is on a quantum
Hall plateau. In that situation the Fermi energy lies exactly between two Landau
levels in the bulk, which is then dominated by localized states. The extended states
reduce to edge channels, which are spatially well separated and can be considered as
ideal one-dimensional channels. The situation at filling factor $\nu = 4$ (two spin degene-
rate channels) is displayed in Fig. 2.4, where reflection-less contacts are assumed.
Equation (2.23) becomes:

$$\begin{pmatrix}
    I_1 \\
    I_2 \\
    I_3 \\
    I_4 \\
    I_5 \\
    I_6
\end{pmatrix} = \frac{e^2}{h} \begin{pmatrix}
    \nu & 0 & 0 & 0 & -\nu & 0 \\
    -\nu & \nu & 0 & 0 & 0 & 0 \\
    0 & -\nu & \nu & 0 & 0 & 0 \\
    0 & 0 & -\nu & \nu & 0 & 0 \\
    0 & 0 & 0 & -\nu & \nu & 0 \\
    0 & 0 & 0 & 0 & -\nu & \nu
\end{pmatrix} \begin{pmatrix}
    V_1 \\
    V_2 \\
    V_3 \\
    V_4 \\
    V_5 \\
    V_6
\end{pmatrix}$$  \hspace{1cm} (2.36)

Because contact 4 is connected to ground, $I_1 = -I_2 = I$, and in a four-point setup
the voltage measurement is without current: $I_2 = I_3 = I_5 = I_6 = 0$. Since all
contacts on one side of the Hall bar are at the same potential, the current can be
written as

$$I = \frac{e^2}{h} \nu (V_4 - V_1).$$  \hspace{1cm} (2.37)
Figure 2.4: Schematic of how the Landauer-Büttiker formalism can be applied to a quantum Hall measurement, here for $\nu = 4$. The current is flowing from contact 1 to contact 4 (which is connected to ground), while the resistances $R_{xx}$ and $R_{xy}$ are measured in a four-point setup.

The Hall resistance and the longitudinal resistance can then be found to be

$$R_{xy} = R_{26,14} = \frac{V_2 - V_6}{I} = \frac{h}{e^2} \frac{1}{\nu} \quad \text{and} \quad R_{xx} = R_{65,14} = \frac{V_6 - V_5}{I} = 0,$$

(2.38)

where $R_{ij,kl}$ means the resistance between contact $i$ and $j$ when the current flows between contact $k$ and $l$. If the Fermi energy lies on the maxima of the DOS, the bulk will be dominated by extended states, thus backscattering (i.e., scattering from one edge channel to another on the other side of the Hall-bar) is possible, and not all 0-elements of the transmission matrix in eq. (2.36) will still be vanishing. In this case also the quantization of the Hall resistance disappears, leading to the transition to the next quantized plateau.

### 2.5 Quantum dots

The QPCs discussed in section 2.3.2 can be used to separate a small conducting island with a self capacitance $C_\Sigma$ from two contacts, used as drain and source, as sketched in Fig. 2.5. At very low temperatures the energy needed to add an electron to the island may exceed the thermal energy ($e^2/C \gg k_B T$), and the current through the island becomes blocked. This effect is called “Coulomb blockade”. By placing metallic gates close to the island, connected capacitively to it with a coupling capacitance $C_G$, it is possible to control the number of electrons on the island. The voltage on the gate is increased until the Coulomb repulsion is compensated, and
an extra electron can hop on the island. At this point the conductance shows a thermally broadened peak, indicating that current is flowing between source and drain. Since only one electron at a time is allowed to enter the island, this current is determined by sequential tunneling, where the electron has to leave the island before the next one enters. For this reason such an island is also called single electron transistor (SET). This effect was first observed in thin granular metallic films and explained in the early 50’s [42, 43], but more than 30 years later it was also observed in an artificially built metallic SET [44] and 2DEGs [45]. The next sections are a short introduction to the physics of quantum dots, more detailed reviews can be found in [46, 47, 48, 49, 50, 51, 52].

![Figure 2.5:](image)

**Figure 2.5:** (a) Schematic drawing of a quantum dot environment. (b) Current through a quantum dot as a function of the voltage on the gate. Between the conductance peaks the current is blocked because of the Coulomb blockade.

### 2.5.1 Single electron charging

The simplest way to describe Coulomb blockade is to take a metallic island and consider only the classical electrostatic problem. The total charge of a system of \( n \) electrodes is given by

\[
Q_i = \sum_{j=0}^{n} C_{ij} V_j + Q_{i}^{(0)},
\]

where \( C_{ij} \) is the capacitance matrix describing the geometry of the system (with \( \sum_{i=0}^{n} C_{ij} = 0 \) from charge neutrality), and \( V_j \) is the electrostatic potential of electrode \( j \). The index \( i,j = 0 \) is reserved for the metallic island, and its charge is given
by

\[ Q_0 = C_{00}V_0 + \sum_{j=1}^{n} C_{0j}V_j. \]  \hspace{1cm} (2.40)

Since the total charge of the system has to be constant, the self-capacitance of the island can be defined as

\[ C_{00} = - \sum_{j=1}^{n} C_{0j} := C_{\Sigma}. \] \hspace{1cm} (2.41)

Solving with respect to \( V_0 \) gives the electrostatic potential on the island:

\[ V_0 = \frac{1}{C_{\Sigma}} \left( Q_0 - Q_0^{(0)} - \sum_{j=1}^{n} C_{0j}V_j \right), \] \hspace{1cm} (2.42)

where \( Q_0^{(0)} \) is the charge on the island if all the other potentials are put to zero. If the charge on the island is quantized (\( Q_0 = -eN \)), then the energy needed to put \( N \) electrons on the dot is given by:

\[ U(N) = \int_{0}^{-eN} dQ_0 V_0(Q_0) = \frac{e^2N^2}{2C_{\Sigma}} + \frac{eN}{C_{\Sigma}} \left( Q_0^{(0)} + \sum_{j=1}^{n} C_{0j}V_j \right). \] \hspace{1cm} (2.43)

The last term is governed by the voltages applied on the gates, which can continuously change the charge \( \sum_{j=1}^{n} C_{0j}V_j \). The energy needed to add one single electron can then be defined as

\[ E_C = U(N + 1) - U(N) = \frac{e^2}{C_{\Sigma}} \left( N + \frac{1}{2} \right) + \frac{e}{C_{\Sigma}} \left( Q_0^{(0)} + \sum_{j=1}^{n} C_{0j}V_j \right), \] \hspace{1cm} (2.44)

and increases proportionally to the number of electrons on the island. In the Coulomb blockade regime the number of electrons on the island is fixed, while it can change between \( N \) and \( N - 1 \), or \( N \) and \( N + 1 \), if the voltage applied on the gate electrodes allows current to flow (i.e. on conductance peaks, like those shown in Fig. 2.5). An important quantity can be extracted from eq. (2.44): If all gates except gate \( i \) are connected to ground, and gate \( i \) is raised to 1 Volt, then the electrostatic potential is given by

\[ \alpha_i := - \frac{C_{0i}}{C_{\Sigma}}. \] \hspace{1cm} (2.45)

\( \alpha_i \) is called the characteristic function of gate \( i \). Its expectation value for a state \( n \) is given by \( \langle n | \alpha_i(\vec{r}) | n \rangle \) and is called the lever arm of gate \( i \) on the state \( n \).
2.5.2 Coulomb blockade in quantum dots: constant interaction model

By adding the sum $\sum_{i=1}^{N} \epsilon_i$ of all single-particle energies to the charging energy (2.44), and then using $\mu_N = E(N) - E(N-1)$, one gets the electrochemical potential:

$$\mu_N = \epsilon_N^{(0)} + E_c \left( N - \frac{1}{2} \right) + \frac{e}{C_{\Sigma}} \left( Q_0^{(0)} + \sum_{j=1}^{n} C_{0j} V_j \right), \quad (2.46)$$

where $E_c := \frac{e^2}{C_{\Sigma}}$ is the charging energy (A more detailed approach is given in appendix B). Electron transport through a quantum dot can now be described in terms of the electrochemical potentials of a structure consisting of two tunnel barriers, like displayed in Fig. 2.6, in the case of low bias and low temperatures ($e V_{\text{bias}}, k_B T \ll e^2/C_{\Sigma}$). Starting with $N$ electrons on the dot, the electrochemical potential $\mu_N$ inside the dot is raised (by changing the gate voltage) until it aligns with the electrochemical potential $\mu_D$ of the drain contact. At this point the $N$-th electron can leave the dot. If the electrochemical potential $\mu_S$ of the source contact is close to $\mu_D$, and $\mu_D \leq \mu_N \leq \mu_S$ electrons can enter the dot and current will flow. As soon as $\mu_N$ increases further, the dot is again in the Coulomb blockade, and there will be $N-1$ electrons on it. The same procedure repeats then for the $N-1$ electron case, if the chemical potential $\mu_{N-1}$ is raised.

Figure 2.6: Schematic energy diagram of a quantum dot. (a) The dot is in the Coulomb blockade regime with $N$ electrons on it. (b) The gate voltage is changed until the $N$-electron level is aligned with the drain level. At this point the $N$th electron can leave the dot, and an electron from the source can enter (c) leading to a finite current through the dot. (d) The gate voltage is changed again and the dot returns to the Coulomb blockade with $N-1$ electrons on it.
On a Coulomb peak, when $\mu_D \approx \mu_N \approx \mu_S$, eq. (2.46) can be solved with respect to the gate voltage, giving the voltages at which there is a Coulomb peak:

$$V_{\text{gate}}(N) = \frac{1}{e\alpha_{\text{gate}}} \left[ \epsilon_N + E_c \left( N - \frac{1}{2} \right) - e \sum_{j \neq \text{gate}} \alpha_j V_j + eQ^{(0)}_0 - \mu_S \right]. \quad (2.47)$$

The separation between two peaks is then:

$$\Delta V_{\text{gate}}(N) = V_{\text{gate}}(N) - V_{\text{gate}}(N - 1) = \frac{1}{e\alpha_{\text{gate}}} (\epsilon_N - \epsilon_{N-1} + E_c), \quad (2.48)$$

where $\Delta N := (\epsilon_N - \epsilon_{N-1})$ is the single-particle level spacing. The separation between two peaks depends therefore only on the level spacing and on the charging energy. Thus it is possible to determine the single-particle spectrum of the dot by subtracting $E_c$ from the peak spacing. This is valid if one assumes that $E_c$ is constant for every electron number on the dot, which is not perfectly true, but can sometimes be a very good approximation.

### 2.5.3 Charge stability diagrams

Quantitative spectroscopy on a quantum dot can be done using eq. (2.48). However, the value of the gate lever arm $\alpha_{\text{gate}}$ is missing. The simplest way to determine it is to measure the current-voltage characteristic of the dot as a function of the voltage applied on the gate, leading to a charge stability diagram and to the so-called “Coulomb blockade diamonds” explained below. For a positive bias voltage $V_{SD} > 0$, and if $\mu_S$ is kept constant, the conditions for having the dot in a stable $N$-electron configuration are:

$$\begin{align*} 
\mu_N &< \mu_S - eV_{SD} \\
\mu_{N+1} &> \mu_S 
\end{align*}$$

and for a negative bias:

$$\begin{align*} 
\mu_N &< \mu_S \\
\mu_{N+1} &> \mu_S - eV_{SD}. 
\end{align*}$$

If the bias voltage is applied symmetrically to the dot (like in the experiments discussed in this thesis), which means $\mu_S = \mu_0 + eV_{SD}/2$ and $\mu_D = \mu_0 - eV_{SD}/2$, where $\mu_0$ is the source and drain electrochemical potential without bias, the above conditions become for $V_{SD} > 0$:

$$\begin{align*} 
\mu_N &< \mu_0 - eV_{SD}/2 \\
\mu_{N+1} &> \mu_0 + eV_{SD}/2, 
\end{align*}$$

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and for $V_{SD} < 0$:

\[
\begin{align*}
\mu_N &< \mu_0 + eV_{SD}/2 \\
\mu_{N+1} &> \mu_0 - eV_{SD}/2.
\end{align*}
\]

Putting these conditions into eq. (2.47) leads to the lines bordering the Coulomb diamonds in the charge stability diagram. For $V_{SD} > 0$ they are

\[
V_G = \frac{1}{e\alpha_{\text{gate}}} \left[ \epsilon_N + E_c \left( N - \frac{1}{2} \right) - \mu_0 
+ e(1 - \alpha_S + \alpha_D)V_{SD}/2 - e \sum_{j=4}^n \alpha_j V_j + eQ_o(0) \right]
\]

\[
V_G = \frac{1}{e\alpha_{\text{gate}}} \left[ \epsilon_{N+1} + E_c \left( N + \frac{1}{2} \right) - \mu_0
- e(1 + \alpha_S - \alpha_D)V_{SD}/2 - e \sum_{j=4}^n \alpha_j V_j + eQ_o(0) \right],
\]

(2.49)

where $\alpha_S$ and $\alpha_D$ are the lever arms of the source and drain contacts. The slope of the lines is given by the terms $\frac{1}{2\alpha_G}(1 - \alpha_S + \alpha_D)V_{SD}/2$ and $-\frac{1}{2\alpha_G}(1 + \alpha_S - \alpha_D)V_{SD}/2$; if the coupling is the same, i.e. the source and drain tunnel barriers have the same geometry, then $\alpha_S = \alpha_D$ and the lines have the slopes $\frac{1}{2\alpha_G}$ and $-\frac{1}{2\alpha_G}$. The two lines with positive and negative slope cross at the points $eV_{SD} = \Delta_{N+1} + \frac{e^2}{\epsilon_S}$, and the crossing points are separated by $eV_G = \frac{1}{\alpha_G} \left( \Delta_{N+1} + \frac{e^2}{\epsilon_S} \right)$. Also if the bias is not symmetric, as in the case of eq. (2.49), the difference between the two slopes is always $1/\alpha_G$, allowing a direct determination of the gate lever arm. The result is summarized in Fig. 2.7, which includes also the situation of excited states. These states appear, if not only a single level contributes to the transport, which is the case if $eV_{SD} > \Delta_{N+1}$. Then more single-particle levels contribute to the transport, and additional lines appear in the stability diagram, outside the Coulomb blockade diamonds.

### 2.5.4 Lineshape of the conductance peaks

The considerations about quantum dots made above do not take into account that a quantum dot is not a perfectly isolated system and do not consider temperature effects. This would lead to perfectly sharp conductance peaks. Due to the overlap of the wavefunctions in the dot with those in source and drain, the conductance peaks will have a finite width, which can be different for each energy level $\epsilon_N$, given by $h\Gamma^{(N)} = h \left( \Gamma_S^{(N)} + \Gamma_D^{(N)} \right)$. This is independent of temperature and is the dominant effect in the strong coupling regime, where $k_B T \leq h\Gamma$. If one neglects the
2.5. Quantum dots

Figure 2.7: Schematic of Coulomb blockade diamonds. The number of electrons is constant inside the coloured diamond, and a conductance peak appears at the points where the borderlines of the diamond intersect the $V_{\text{gate}}$ axis. The excited states are sketched outside the $N$-electron diamond, with the indication of their relative single-particle level spacing.

Differences in coupling between different levels, the system becomes very similar to a simple double barrier, and the lineshape of a conductance peak is given by the Breit-Wigner formula

$$G = \frac{2e^2}{\hbar} \left( \frac{1}{\Gamma_S} + \frac{1}{\Gamma_D} \right)^{-1} \frac{h^2\Gamma}{\alpha_{\text{gate}}^2 (V_{\text{max}}^{\text{gate}} - V_{\text{gate}})^2 + (h\Gamma/2)^2}.$$  \hspace{1cm} (2.50)

The first term in brackets is the inverse conductance of the two tunnel barriers, and it determines the height of the conductance peaks. In the weak coupling regime, where $h\Gamma \ll k_B T$, and for the quantum-Coulomb blockade, where $k_B T \ll \Delta, e^2/C_S$, the lineshape can be described by a model discussed in [46]. The result for the lineshape is:

$$G = \frac{e^2}{4k_B T} \left( \frac{1}{\Gamma_S} + \frac{1}{\Gamma_D} \right)^{-1} \cosh^{-2} \left( \frac{\Delta_{\text{min}}}{2k_B T} \right),$$  \hspace{1cm} (2.51)

where $\Delta_{\text{min}} = \Delta(N_{\text{min}})$ is the smallest value of $\Delta(N) = \epsilon_N + U(N) - U(N+1) - E_F$, equal to $\mu_N - \max(\mu_S, \mu_D)$. The peak amplitude decreases therefore as a function of
temperature with \( G(T) \approx 1/T \). By comparing with eq. (2.47), the result becomes

\[
G = \frac{e^2}{4k_B T} \left( \frac{1}{\Gamma_S} + \frac{1}{\Gamma_D} \right)^{-1} \cosh^{-2} \left( \frac{e\alpha_G \left( V_G^{(N)} - V_G \right)}{2k_B T} \right),
\]

allowing to estimate the temperature of the electrons in the dot.
Chapter 3

Measurement setup

3.1 Introduction: low-temperature scanning force microscopes

3.1.1 History and motivation

Scanning microscopy started with the invention of the scanning tunneling microscope (STM) by Binnig, Rohrer and Gerber [9] in 1981. Four years later, the scanning force microscope (SFM) was invented also at IBM by Binnig, Quate and Gerber [10], and has since then become a standard instrument for the investigation of surfaces down to the atomic scale. The resolution which can be achieved makes a SFM the ideal tool for studying the physics of mesoscopic semiconductor structures. In conventional magnetotransport experiments one gets only indirect information about microscopic processes taking place within the system, since the only measured quantity is the current flowing through the sample at the leads connecting it to the measurement equipment. A scanned probe can indeed offer the opportunity to locally (on the nanometer scale) modify the properties of a system, thus giving direct insight into processes occurring in the sample. Due to the technical difficulties in realizing such a measurement system, only a few successful experiments have been performed up to now, even though low temperature SFMs are now commercially available. For an overview about cryo-SFMs see [54].

3.1.2 Different scanning microscope principles

Scanning tunneling microscope (STM)

The STM uses the tunnel current between a sharp metallic tip and a conducting sample as a sensor signal. The tunnel resistance $R$, for sufficiently small separation $s$ between tip and sample, is given by

$$R(s) \sim \exp(A\phi^{1/2}s),$$

(3.1)
where the constant $A$ is given by $(4\pi/h)(2m)^{1/2}$ with $m$ the electron mass in the tunnel barrier, and $\phi$ is the effective electrostatic potential between tip and sample $[3]$. The exponential dependence explains the very high spatial resolution which can be achieved by this technique: Since the tunneling electrons leave the STM tip at the last atom, atomic resolution was obtained already in early experiments $[55,56]$, where the $7\times7$ reconstruction of the Si(111) surface and the $2\times1$ structure of chemisorbed oxygen on Ni(110) were resolved. The technology developed rapidly, making the STM a standard tool for surface investigation on conducting surfaces (see for example $[57,58]$ for reviews). STMs are nowadays mainly used for two purposes: surface studies and manipulation of single molecules or atoms. Experiments of the first kind include investigations of the structure of atoms in real space $[59]$, local density of states measurements on 1D wires $[60]$, wave function mapping of strain induced quantum dots on the surface of InAs(110) $[61,62,63]$, wave function mapping in plain 2DEGs $[64]$ in quantum corrals $[65,66,67]$, as well as measurements of the energy needed to flip the spin of single Mn atoms adsorbed on a Al$_2$O$_3$ islands on a NiAl surface $[68]$.

Manipulation of single molecules and atoms on surfaces is another application of the STM technique, which is widely applied. First attempts were made at IBM in 1990 $[69]$. More recent experiments include patterning of quantum corrals, which allow the formation of standing electron waves $[65,66,67]$, the construction of a single atom switch $[70]$, or the construction of one-dimensional gold chains, allowing the direct determination of band structure $[71]$. A similar experiment is presented in $[72]$, where nanostructures composed of copper and phthalocyanine are bonded to gold chains, and then the band structure is determined, or the photon emission from single atom silver chains on a NiAl(110) surface is studied $[73]$. Another interesting application is the construction of classical logic gates, using the principle of “molecular cascades” $[74]$: Here CO molecules are assembled on a Cu(111) surface, and arranged on unstable positions. At sufficiently low temperatures thermally activated hopping is strongly reduced, and the cascade can be started with the STM tip.

**Atomic force microscope (AFM)**

The AFM technique is used for the experiments presented in this thesis. Although, from the technical point of view, an AFM is very similar to a STM, the physical principles of operation are completely different. The main advantage over an STM is the fact that any sufficiently smooth surface can be scanned, without need of a conducting material. The invention of the AFM $[10]$ was a consequence of the observation of several different forces acting on an STM tip. The interactions causing these forces are of very different nature, and include:

- *Pauli principle* (repulsive).
- *Electric interactions* (attractive), divided into Van-der-Waals forces, due to
3.1. Introduction: low-temperature scanning force microscopes

the electromagnetic interaction of fluctuations in the atoms of the tip and the surface, and capacitive forces, due to the potential difference between tip and surface.

- **Chemical bonding** which allows atomic resolution and the manipulation of single atoms.
- **Capillary forces** in the presence of a water film on the surface of the sample.
- **Magnetic forces** (only with magnetic tips).
- **Dissipative mechanisms**.

Extensive modeling of these interactions can be found in [75]. The AFM tip is usually mounted on a cantilever, and the interaction force is measured by determining the deflection of the cantilever. The different readout techniques are described in section 3.5.1. While approaching the tip to the surface, the attractive forces dominate at a relatively large distance. One of those is the Van-der-Waals force, whose potential is of the kind $V(r) = -\frac{\sigma}{r^6}$. If the tip gets very close to the surface, the force due to the Pauli principle becomes more important, and the tip gets into the repulsive regime. Both forces can be described by a Lennard-Jones potential:

$$U(d) = -E_b \left[ 2 \left( \frac{\sigma}{d} \right)^6 - \left( \frac{\sigma}{d} \right)^{12} \right]$$  \hspace{1cm} (3.2)

where $E_b$ is the bond energy, $\sigma$ the equilibrium distance and $d$ the tip-to-sample separation. The potential for the electrostatic force is given by

$$U_E(d) = \frac{1}{2} C(d) \phi^2$$  \hspace{1cm} (3.3)

where $\phi$ is the electrostatic potential difference between tip and sample, and $C(d)$ is the distance-dependent capacitance between sample and tip.

All these forces are extremely weak: The repulsive component is of the order of $10^{-9}$ N, and the attractive one about $10^{-12}$ at small distances $[76, 77]$ ($d < 1$ nm).

Topographic images of a sample can be obtained in two operating modes of an AFM: The **contact mode** or the **dynamic mode**. In the first case, the deflection of the cantilever is kept constant by a $z$-feedback, which controls the voltage on the $z$-electrode of the scanning piezo. This mode requires operation in the repulsive regime and may therefore damage the surface of the sample. In the second case, the cantilever is excited at its resonance frequency, and feedback is provided by measuring the frequency shift of the resonance, since this depends on the gradient of the force acting on the tip and on the amplitude of the oscillation. More details are given in section 3.4.2.

A short overview of AFM applications is now given (for more details see [78, 79]). Atomic resolution was demonstrated in several experiments, for example by
visualizing single adatoms on a silicon (111)-(7x7) surface \[80\], by measuring short-range electrostatic interactions on the same material \[81\], or by visualizing single atoms on a NiO(001) surface at low temperatures \[82\] or KBr(001) surface \[83\]. Another interesting measurement is the quantitative determination of short-range chemical bonding forces \[84\]. AFMs are also widely used in chemistry and molecular biology (see for example \[85\] or a review in \[79\]). More related to the field of this thesis are experiments on carbon nanotubes \[86, 87\], as well as many measurements on structured 2DEGs, discussed later. Another important field is the patterning of semiconductor nanostructures, as the ones discussed in this thesis, with local AFM oxidation \[21, 22, 28\], or even the patterning of ferromagnetic films \[88\].

**Other techniques**

The scanning techniques discussed up to now, basically allow to map the topography of a surface. Since most of the cantilevers supporting the AFM tips are made of Silicon, it is straightforward to integrate other kinds of sensors on the tip. These allow for example to probe the local magnetic or electric field or the local optical properties of the sample. Some experiments are now discussed:

- **local magnetic field:** Three kinds of sensors have been used to probe the local magnetic field: Hall sensors, superconducting quantum interference devices (SQUIDs) and magnetic tips. Hall sensors use a small Hall-bar for the detection of the local magnetic field, and the measured quantity is the Hall voltage. In order to keep the distance to the sample constant, a thin Au film is evaporated onto the edge of the Hall sensor chip, allowing STM operation mode. With this technique, it was possible to image single vortices in high-\(T_c\) superconductors \[89, 90\]. SQUID sensors can be used for the same purpose \[91, 92\], or for microwave \[93\] and current density \[94\] measurements on integrated circuits. Magnetic force microscope (MFM) measurements with magnetic tips include the observation of vortices in high-\(T_c\) superconductors \[95\], measurements on magnetic thin films \[96, 97\], or, more recently, the determination of the spin density in CoO/(CoPt) multilayers \[98\].

- **local electric field:** A single electron transistor (SET) is a very sensitive probe of the local electric field. However, its integration on standard cantilevers is difficult. Measurements on GaAs/AlGaAs SETs have been performed in \[99\], showing that this kind of sensors are suitable for the integration on standard cantilevers. Using an SET fabricated on the tip of a glass fibre it was possible to image localized states in the quantum Hall regime, showing the microscopic nature of localization in this regime \[100, 101, 102\], and to image fractionally charged quasi-particle \[103\].

- **optical methods:** One method, which overcomes the limits of optical microscopy (the diffraction limit of half the wavelength of the light source), is scanning near-field optical microscopy (SNOM). The principle is the following: The
3.2 Basic requirements of low temperature operated SFMs for studying semiconductor nanostructures

sample is illuminated by a laser beam, and an optical probe (usually an optical fiber) is brought very close to the surface. If this distance is smaller than the wavelength, also waves which are exponentially damped and do not propagate can be detected. A review of the technique can be found in [104]. The technique can be used for the optical spectroscopy of quantum dots [105], and due to the high resolution, also for optical lithography [106, 107]. It has also been suggested to use single molecules as a light source replacing the optical probe [108].

In addition to these three methods, shear force microscopy or friction microscopy can be used to determine the surface properties of a sample [109, 110].

3.2 Basic requirements of low temperature operated SFMs for studying semiconductor nanostructures

Magnetotransport experiments have to be performed at low temperatures and often in high magnetic fields. SFMs are operated best in an environment free of vibrations and gas turbulence. Moreover, semiconductor nanostructures are sensitive to light (persistent photoeffect). These are the “boundary conditions” for the construction of a cryo-SFM, and make its realization very complicated.

Since the energy scales important for semiconductor nanostructures are in the range of a few $\mu$eV to a few meV, low temperatures are needed to perform experiments. If phase coherence effects are the goal of the experiment, the temperature should be well below 1 K. A conventional $^4$He cryostat, which has a high cooling power of many milliwatts, can reach a base temperature of typically 1.7 K, which is too high to study coherence effects. A $^3$He system like the one used here, with a base temperature of 300 mK, has a cooling power of the order of 0.1 mW. A dilution refrigerator would have the same cooling power at its base temperature. A low thermal load is therefore high priority, but exactly this is very difficult due to the cabling required for an SFM. Moreover, temperatures below 300 mK cannot be reached in an optical cryostat with split coil magnets, making it very difficult to know the exact position of all moving parts inside the cryostat. This is required, since the typical size of the area of interest for measurements on semiconductor samples is of the order of $20 \times 20 \mu m^2$.

The high magnetic fields up to 15T required by magnetotransport experiments, limit the range of materials which can be used for the construction of such a microscope, since they all have to be non-magnetic. Usual commercially available (and still affordable) superconducting magnets have a bore with a diameter of 5-7 cm, limiting the maximum diameter of the SFM to 3.5-5.5 cm. The field is homogeneous (within 0.1 %) only over a range of about 1 cm$^3$, which reduces the possibilities of designing a SFM furthermore, also considering that the sample space has a length
of about 15 cm, and the whole SFM has to be contained in this length. An SFM should be isolated as good as possible from all sources of vibrations, since these would make it very difficult to keep the scanning tip at a constant distance (for constant height measurements) or constant force (for feedback mode measurements) on the surface. The microscope assembly has to be put into a vacuum environment, in order to avoid temperature gradients, as well as helium flow artifacts, which would be unavoidable if the microscope would just be put into a variable temperature insert (VTI).

Commercially available cantilevers with piezoresistive actuation usually use a laser beam for position detection. Since semiconductor nanostructures are sensitive to light, this detection scheme cannot be used, at least not without additional shielding (large cantilevers), which is not commercially available. The solution consists in a purely electrical detection mechanism, with the disadvantage that it has to be custom built.

The next sections illustrate how these problems have been solved in the experimental setup used for the experiments described in this thesis.

3.3 A SFM operated at 300 mK: Design

3.3.1 The cryostat

The cryostat is a commercially available model from Janis Research Corporation, with a base temperature of 300 mK. Figure 3.1 (left) shows a drawing of the cryostat setup. The cryostat is placed in a hole drilled in the concrete floor of the lab, which is mechanically decoupled from the rest of the building. This ensures a mechanically very quiet environment. Additional microphony effects are reduced by a rubber mat tightly wrapped around the cryostat. In order to minimize vibrations, which may arise from the large surface of the can, the $^3$He insert is decoupled from the cryostat and suspended from a separate platform, which also allows a precise vertical alignment. The platform itself is placed on four sand filled steel supports, screwed to the floor, and is of heavy construction, lowering the resonance frequency of the system. The only mechanical connection between the insert and the can consists of a rubber sleeve, which has to be $^4$He tight. The pump, needed for pumping the 1 K pot and the sorption cooling (see the next section), is placed on a separate floor, and all pumping lines are flexible plastic tubes, for minimizing vibrations. Cooling is provided by a 50 l reservoir of liquid helium, which is shielded by a LN$_2$ tank placed around it. The cryostat can be kept at 300 mK for about 7 days without recondensing, if the magnet is not driven, and both the nitrogen and the helium tanks are filled. The magnet is a TiNb type with a maximum field of 9T and a persistent current switch. No lambda fridge is available in this cryostat.
3.3. A SFM operated at 300 mK: Design

Figure 3.1: Schematic of the cryostat setup (left), including the superconducting magnet and the $^3$He insert. On the right, a detail of the $^3$He insert as delivered from the factory.

Basic principle of a single shot $^3$He cryostat

The basic principle is to condense $^3$He gas by bringing it in thermal contact with a pumped 1.7 K $^4$He reservoir. Low temperatures are achieved by decreasing the vapor pressure on top of the $^3$He pot with an internal sorption pumping system. The sorption pump (SP) cooling power is controlled by a variation of its temperature between 4K and 40K. A typical cool-down cycle starts by heating the SP, while cooling the 1K pot by pumping liquid $^4$He through the 1K pot capillary (see Fig. 3.1). By heating the sorption pump $^3$He gas is released; the gas condenses in the 1 K-pot and the liquid dripples down into the $^3$He pot cooling it to 300 mK (and with it the microscope). The SP is then cooled by pumping liquid $^4$He through a capillary wound around it. The whole condensing process takes about 4 hours. The sample can be kept at the base temperature of 300 mK for 6 to 8 days, depending on the kind of experiment. Afterwards, the condensation process has to be repeated.

The $^3$He insert

Figure 3.1 (right) shows a schematic of the $^3$He insert, without the modifications required by the 300 mK SFM operation. These include additional anchor points for all sample cables at the 1K pot, as well as anchor points for all cables including high voltage at the $^3$He pot. Care has to be taken with the high voltage cables, since
electrical breakthrough could be a problem. Moreover, all metallic parts have been electrically connected to the central $^3$He tube, which is connected to the electrical ground. Figure 3.2 shows the actual $^3$He insert, as well as the magnet assembly. The anchor points and all the different cable types are visible. Vacuum sealing of the IVC is provided by a brass taper seal, onto which a Cu can is pressed. This seal is crucial, since a small leak at this point can compromise the experiment. The pressure, which can be reached in the IVC can at room temperature by pumping it with a turbo-molecular pump, is usually around $10^{-6}$ mbar.

### 3.3.2 The microscope

The low temperature AFM is a custom designed and manufactured microscope and has been improved during its operation for the measurements presented in this thesis. An early version of a low temperature AFM, operating at 2K is presented in [111, 112]. After this model, two generations of AFMs were built, until the present one was operational. It was constructed by Tobias Vancura, Thomas Ihn and Cecil Barengo [113, 114]. Details are discussed in the following sections.

#### The z-module

The z-module contains the scanning piezo unit with the scanning sensor as well as the slip-stick motor allowing coarse tip-sample approach. The unit is displayed in Fig. 3.3. The frame of the z-module is made of non-magnetic CuBe, which is a good material from the point of view of thermal conductivity, but has the disadvantage of a large thermal expansion factor, which requires a lot of care during the cool down procedure. In order to minimize shifts in position between SFM tip and sample, the microscope is designed to be as symmetric as possible. The z-motor consists of a macor prism, which contains the scanning piezo, and of the shear piezo stacks, which provide the motion in z-direction (see Fig. 3.4). The travel range of the z-motor is about 12 mm. The prism is held in position by a CuBe spring, which presses it against the six commercially available piezo stacks. The design is similar to that used in [54, 115].

The scanning piezo is a lead-zirconate-titanate ceramic tube with copper electrodes, and allows a lateral scan range of more than 60 $\mu$m in x-y direction at room temperature and about 5 $\mu$m in z direction. At 300 mK, the lateral scan range is much smaller, and goes down to 12.5 $\mu$m in x-y direction and 1.95 $\mu$m in z direction. The tuning fork sensor is mounted on a PCB, which is screwed on a support, placed on top of the scanning piezo. Three of the six screws provide the electrical connection to the coaxial cables connecting the tip and the two prongs of the tuning fork. The driving voltage of the tuning fork goes through a 1/1000 voltage divider, directly connected to the coaxial plug at the $^3$He pot. This reduces the noise on the tuning fork, because any noise picked up by the long cables is divided as well. The cabling consists of 4 coaxial cables (2 for the tuning fork, one for the tip and one
3.3. A SFM operated at 300 mK: Design

Figure 3.2: (a) $^3$He insert with complete cabling and microscope attached. (b) Magnet assembly with $^3$He insert as delivered from the factory. (c) Detail of the $^3$He pot with all thermal anchors.
for the z-positioning sensor), as well as of high voltage cables for the scanning piezo and the z-motor. These cables are shielded by putting them into metallic tubes.

Figure 3.3: Schematic drawing (left) and photography (right) of the microscope. The x-y table is hidden in the photography.

Figure 3.4: Microscope seen from the side (left). x-y table in the same position as during an experiment (right).
3.3. A SFM operated at 300 mK: Design

The z-positioning sensor consists of three conducting plates, two of them attached to the outer frame, one on the macor prism. The setup is displayed in Fig. 3.5. An AC voltage $U_{AC}$ with frequency $\omega$ is applied to the two fixed plates, with one of the two signals phase shifted by 180 degrees. The current through the capacitor is measured over a I-U-converter and is given (in a plate capacitor model) by

$$I = U_{AC} \omega \frac{W \varepsilon \varepsilon_0}{d} (L_1 - L_2),$$

where $W$ is the width of the capacitor plates and $d$ the separation of the plates. If $L_1 = L_2$, which means that the moving plate is positioned exactly in the center of the two fixed plates, the current is zero. The positioning sensor is the only way to monitor the motion of the z-motor, once the IVC has been evacuated, and is therefore of crucial importance for the experiment.

![Figure 3.5: Schematic of the capacitive positioning sensor.](image)

The x-y table

The x-y table is shown in Fig. 3.4 and consists of an x-y motor, a thermometry block and the sample holder. The x-y motor uses the same slip-stick technique as the z-motor and allows coarse positioning of the sample in the x-y plane. Instead of shear piezo stacks, piezo tubes, glued with single crystal sapphire balls, are used in this case. Their 4-quadrant contacts allows bending in x and y direction similar to the scan piezo, thus providing movement of the sample. They are glued to a shapal support, which is an electrically insulating ceramic material with good thermal conduction and a thermal expansion coefficient similar to the one of the tubes. The travel range of the table is about 5 mm, which corresponds to the size of a typical semiconductor chip. The thermometry block contains a heater, a Pt100 thermometer, a ruthenium-oxide thermometer and an Allen-Bradley thermometer. It is thermally coupled to the sample and allows therefore precise temperature reading. The heater is used for evaporating the water film on the sample during cool-down, or for keeping the sample at a higher temperature than the $^3$He pot. The sample holder is a commercial 32-pin model for ceramic chip carriers, used as a support for the samples. Thermal coupling between sample and $^3$He pot is mainly provided by the sample and thermometry cables, which are thermally anchored at the $^3$He pot.
Chapter 3. Measurement setup

The sample is placed upside-down in the microscope for two reasons: The probability that particles may fall on the sample is smaller, and it is easier to withdraw the tip with the z-motor (since withdraw means going down) in case the tip-to-sample distance becomes too small during a cool-down. The proper function of the x-y table is very important, since it is the only way to re-position the sample during a cool-down (typical shifts in the x-y direction between room temperature and 300 mK are of the order of 40 \(\mu m\)). This is quite a difficult task, since the scanning range is only 12.5 by 12.5 \(\mu m\), while the whole area, where the tip may have been shifting, can be about 40 times larger, as found during the experiments.

3.4 Scanning sensors: overview and modelling

Many different kinds of scanning microscope tips are used for low temperature measurements, but not all are suitable for experiments on semiconductor nanostructures. The experiments, which can be performed with the different scanning sensors, have been presented in section 3.1.2. This part is devoted to the technical realization and to the advantages and disadvantages, if used for scanning semiconductor nanostructures at low temperatures. The technique of laser beam deflection, which is commonly used on room temperature SFMs, is not suitable for measurements on semiconductor nanostructures, since they are sensitive to light. This kind of sensors is therefore not presented below.

STM tips

STM tips are the most simple scanning sensors, since they are not operated in dynamic mode or need any detection of their deflection. However, they cannot be used for scanning semiconductor nanostructures, since the 2DEG is buried several nanometers (in the samples discussed here 35 nm) below the surface, requiring relatively high voltages in order to get any tunneling current. Such high voltages would cause an irreversible charging of the sample surface and also massive distortion of the confinement potential by capacitive coupling, thus making any experiments meaningless. Successful measurements have been done with the subsurface charge accumulation (SCA) technique [116], where an AC voltage, applied on the 2DEG, is detected via the charge induced on the STM tip.

Piezoresistive cantilevers

Piezoresistive cantilevers are fabricated from silicon wafers and use a Wheatstone bridge for measuring the change of resistance due to the piezoresistive effect. The relation between the deflection of the cantilever \(\Delta z\) and the relative change in resistance \(\Delta R/R\) is given by:

\[
\frac{\Delta R}{R} = \alpha_{pr} \Delta z,
\]

(3.5)
where the constant $\alpha_{pr}$ depends on the geometry of the lever. The cantilevers can be driven to their resonance frequency and can therefore be used for all SFM operating modes. Due to the relatively high voltages, needed for reading the resistance change, high power is dissipated (up to a milliwatt), which could be enough to exceed the cooling power of the cryostat, making low temperature measurements impossible. Nevertheless, the resolution which can be achieved is very high: A vertical resolution of 0.1 Å has been demonstrated [117]. Due to their commercial availability, piezoresistive cantilevers are used in a wide range of applications, including lithography and biosensing. Since they are made out of a wafer, scalability is not a problem, leading to an interesting application for future high density (up to 1 Tbit/in\(^2\)) data storage [118, 119].

**Scanning single electron transistors**

Scanning SET experiments have been already discussed in section 3.1.2, showing their suitability for low temperature measurements. Their working principle (see for example [47]) allows to use them as a very sensitive electrometer for measuring the local electric field. Since the sensor itself cannot be used for detecting the tip-to-sample distance, either the distance dependence of the induced charge on the SET island or the capacitance have to be used. A different approach, which is compatible with the piezoelectric sensors used in this thesis and discussed below, can be used. The SET is defined on the apex of a cantilever, which is glued on the two prongs of a piezoelectric tuning fork [120, 121]. The cantilever allows also force detection and conventional scanning gate experiments, thus increasing the flexibility of the experiment. First measurements did not yet succeed [122], but the principle seems to be very promising.

### 3.4.1 Piezoelectric quartz tuning forks

Piezoelectric quartz tuning forks (TFs) are an alternative to piezoresistive cantilevers, if (like in the present case) a non-optical detection mechanism is needed. They are usually used as a clock reference in watches, and are therefore available at a very low cost. The forks are fabricated from $\alpha$-quartz and are excited to their resonance frequency, using the piezoelectric effect. Due to their very high quality factors (typical $Q$ values go from 150'000 to 1 million for the sensors used in this experiment), the power loss on resonance is much below 1 $\mu$W, which is ideal for using them in low cooling power cryostats like the $^3$He system used here or a dilution refrigerator. The main disadvantage is that the high $Q$ factor requires a more complex electronic setup than in the case of piezoresistive cantilevers. Moreover, it is not possible to buy complete tuning fork based SFM sensors, making it necessary to build them specifically before every experiment.

Tuning fork sensors are used in a wide variety of experiments, including SNOM [123, 124], MFM [125] and shear-force microscopy [126, 127], and their suitability...
for high resolution or high speed measurements has also been shown \cite{128,129,130}.

**Fabrication**

The tuning fork itself does not allow any STM or SFM operation, since it does not include the scanning tip. For STM operation and for scanning gate measurements (where the tip is coupled capacitively to the 2DEG) in the SFM mode, a conductive tip is needed. The tuning forks (with prongs of 8 mm length) are delivered in a sealed casing, which has first to be removed. The fork is then soldered on a round piece of PCB with an angle of about 30°, supported on a small brass support, see Fig. 3.6 The tip itself consists of a 15 µm PtIr wire, which is glued on the top of one of the two prongs of the tuning fork and on a Cu post placed close to it, which provides the electrical connection of the tip. In order to avoid the tip moving inside the glue at low temperatures, it is bent by 90° exactly at the point where it is glued. This operation is very difficult, since great care has to be taken not to make the prong too heavy (resulting in a very low $Q$), and not to touch one of the electrodes, used for the excitation of the tuning fork (resulting in a short circuit between tip and tuning fork driving voltage). The wire is then electrochemically etched, resulting in a tip which has a diameter of less than 50 nm. The length of the tip is typically around 200 µm. In order to connect the casing of the tuning fork to electrical ground, a grounding plane has been added to the PCB plate.

![Tuning fork assembly](image.png)

*Figure 3.6:* Three different views of the sensor plate with the tuning fork soldered on it. The PtIr tip visible on the right image has not been etched yet.
3.4. Scanning sensors: overview and modelling

3.4.2 Modeling

Scanning sensors, used for dynamic mode SFM, consist of an oscillating cantilever, to which the tip is attached, and can therefore be described with a harmonic oscillator model \[131\]. In the case of tuning forks, where the two prongs are oscillating one against the other, the situation is more complex, also because the tip and the glue provide an additional mass on one of the prongs, and the force between sample and tip acts as well only on this prong. However, it has been shown that the harmonic oscillator model can also be applied to tuning fork sensors \[114\].

Since the damping of the tuning fork is compensated by the driving voltage, the equation of motion of the tip reduces to the form

\[ m_{TF} \ddot{z} = -k_{TF} z + F_{ts}(z), \tag{3.6} \]

where \( m_{TF} \) is the effective mass of the tuning fork prong, \( k \) the spring constant, and \( F_{ts} = -\partial V_{ts}(z)/\partial z \) is the interaction force between tip and sample. If \( F_{ts} \) were constant, the solution \( z(t) \) would just be a harmonic oscillation with \( \omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m_z}} \). For small oscillation amplitudes, \( F_{ts} \) can be approximated by the first two terms of a Taylor expansion in \( z \),

\[ F_{ts}(z) \approx F_{ts}(z_0) + \frac{\partial F_{ts}(z_0)}{\partial z} (z - z_0), \]

and the solution of the equation of motion becomes:

\[ z(t) = z_0 + A \cos \left( 2\pi \left( f_0 + \Delta f \right) t \right), \tag{3.7} \]

where the frequency shift \( \Delta f \) at a tip-to-sample distance \( d \) can be written as

\[ \Delta f \approx -\frac{f_0}{4k} \frac{\partial F_{ts}(d)}{\partial z}. \tag{3.8} \]

For conventional cantilevers, the result has to be multiplied by two, since in the case of tuning forks both prongs are oscillating, but only one senses the force \( F_{ts} \) \[132\].

From eq. (3.8) it is possible to see that by measuring the frequency shift one can determine the tip-to-sample distance.

Characterization

The resonance curve of a tuning fork sensor is displayed in Fig. 3.7. The upper plot shows the amplitude vs. frequency for a driving voltage of 2 mV. The asymmetric shape of the resonance arises from the electric capacitance parallel to the piezo oscillator, leading to an anti-resonance. The lower plot shows the relative phase between excitation voltage and output current. If a force acts on the sensor, both curves shift to the left or to the right. Since the phase signal is very steep at the resonance frequency, it can easily be used for phase locking in a phase-locked loop (PLL) detection scheme. The properties of a tuning fork sensor strongly depends on the environment. The following list illustrates, how the sensor parameters react to external changes:
Figure 3.7: Typical tuning fork sensor current amplitude (top) and frequency shift (bottom) vs. frequency. The asymmetry in the amplitude is due to the additional mass of the PtIr wire and the glue attached to one of the two tuning fork prongs.

- **Mechanical coupling:** Since the PtIr wire forming the tip is attached to a Cu post close to the tuning fork, there is a force acting perpendicularly to the oscillation direction of the two prongs. This force can completely destroy the resonance.

- **Pressure:** The $Q$ factor of the tuning fork increases by a factor of about two, if the pressure is lowered from 1 bar to $10^{-6}$ mbar, while the resonance frequency increases.

- **Temperature:** Also in this case, by lowering the temperature, the $Q$ factor increases by a factor of 3-4, and the resonance frequency increases. Combining the two effects, a tuning fork with a $Q$ of 20'000 at room temperature and 1 bar, can have a $Q$ of almost half a million in vacuum and at the base temperature of the cryostat, limiting the maximum speed, at which it is possible to scan the surface of the sample.

- **Magnetic field:** The tuning forks have a ferromagnetic housing, therefore the frequency shift depends on $B$. This dependence can vary from sensor to sensor, some seem to be stable at magnetic fields larger than 1 T, others show changes
in the frequency shift up to 9 T. Due to this behavior it is not possible to sweep
the magnetic field while keeping the tip at a constant distance with respect to
the sample.

3.5 Electronics

The high stiffness and resulting high $Q$ factor of piezoelectric quartz tuning forks has
advantages for experiments in a low temperature environment, but requires more
complex readout electronics than it would be necessary for piezoresistive cantilevers.
This is due to the very low bandwidth available for the detection of the phase shift
of a tuning fork sensor [133]: The bandwidth is given by $f_0/2Q$, where $f_0$ is the
resonance frequency. Typical sensors have a resonance frequency $f_0$ of 32 kHz, and
a $Q$ factor of about 300’000, leading to a bandwidth below 1 Hz. This would be too
slow for any application, therefore the readout electronics have to compensate for
the low speed of the sensor.

3.5.1 Frequency detection with phase-locked loop

The solution consists in using a phase-locked loop (PLL) for the detection of the
resonance frequency shift and the dissipated force. This scheme, suggested by [134],
uses a first feedback loop, based on lock-in technique and a second loop inside the
microscope controller. A schematic diagram of the system is displayed in Fig. 3.8,
and the principle of operation is explained below.

Excitation and readout of a TF sensor are very simple: The excitation voltage
is applied to one of the two electrodes, while the other electrode is connected to
the input of a current to voltage converter (I-U converter). The TF is driven at its
(original) resonance frequency $f_0$ by a voltage controlled oscillator (VCO), with an
amplitude of 0.1 to 2 mV, in order to get an oscillation amplitude of 2-5 nm (the
actual voltage at the VCO is 0.1 to 2 V, but is then reduced with a voltage divider
at low temperatures). The resulting current at the other electrode is converted to a
voltage with a gain of $10^6$. This signal is fed into a lock-in amplifier, which measures
the amplitude and the phase relative to the driving signal provided by the VCO.
The phase can be locked to 0 when no force is acting on the sensor, so, as soon as
a frequency shift appears, the phase will change, and a signal will be present at the
phase output of the lock-in amplifier.

This signal is fed back to the VCO through the PLL, which compensates the
frequency of the driving voltage, in order to minimize the phase-signal. The same
principle works with the amplitude of the signal: The signal at the output of the
VCO is amplified by a voltage controlled amplifier (VCA), whose gain is controlled by
the amplitude output of the lock-in amplifier, after passing the PID controllers. This
loop keeps the oscillating amplitude of the TF sensor constant. The two signals are
filtered by a PID-controller (proportional, integral, differential), which adjusts the
response function of the system, and is crucial for optimum operation of the feedback loop. Since the phase shift contains the main information about the distance between the tip and the sample, the phase signal is fed into the \( z \)-feedback of the microscope controller (another PID-controller and a high voltage amplifier), which compensates the tip-to-sample distance by applying a voltage on the \( z \)-electrode of the scanning piezo.

\[ \text{Figure 3.8: Schematic of the feedback loops, showing in red the PLL, in blue the amplitude feedback and in green the } z \text{-feedback.} \]

\section*{Optimization of the feedback parameters}

The optimization of the parameters of the PID controllers is extremely important for the operation of the SFM in feedback mode. If the P and I parts are set to high (gain) and low (bandwidth, therefore high speed) respectively, the \( z \)-feedback will tend to overreact, withdrawing the tip at the smallest perturbation. Moreover, a high P part leads to a noisy measurement, since it amplifies also the input noise. The opposite situation can be dangerous to the tip, since the feedback may react too slowly at an obstacle, or reduce the resolution of the measurement. The optimal parameters can be computed analytically, if the resonance curve of the sensor is known, like demonstrated in [114], independently of tip-sample interactions, which may depend on the quality of the tip. It is important that all three feedback loops (or at least the phase and \( z \)-feedback) are optimized at the same time. Experience shows that the parameters depend on the quality of the tip in the sense that a
very sharp tip allows a wide range of parameter settings, while a flat tip requires extremely careful optimization.

3.5.2 The TOPS III microscope controller

The microscope controller is a commercial model from Oxford Instruments, controlled by a specialized software running on a standard PC. The controller consists mainly of three parts: The $z$-feedback (discussed above), the data acquisition AD-converters and the high-voltage amplifiers. Eight 16-bit AD-converters are available, three of them are used for the sensor signals ($z$-position, phase, and amplitude), the other five can be used for recording the signals, obtained during a scanning gate measurement. The input range of the converters is $\pm 10$ V, which corresponds to the maximum output voltages of the lock-in amplifier. The high-voltage amplifiers have to provide the voltage for the scanning piezo (maximum voltage: 150 V) and for the slip-stick motors (the $x-y$-table and the $z$-motor). Originally, the controller was able to deliver a voltage output of $\pm 240$ V, which is not enough for driving the $z$-motor at low temperatures. The complete amplifier has been rebuilt and can now deliver up to 700 V. Another modification includes a compensation for the sample tilt, which is necessary if one wants to scan the tip at constant height over the surface. This is achieved by adding an adjustable fraction of the voltages, applied on the $x$ and $y$ electrodes of the scanning piezo, on the $z$ electrode. The result of such a compensation is displayed in Fig. 3.9.

![Figure 3.9: Effect of the sample-tilt compensation circuit. The left picture shows the not-compensated situation, where a strong tilt of the sample surface can be observed (blue line). The right picture shows an almost perfectly horizontal topography.](image-url)
Chapter 4

Scanning gate technique: principle and overview

4.1 Principle

The term “scanning gate” is used as a general definition for every scanned probe technique using electrostatic coupling between the probe and the sample which is being scanned. The principle is the following: While the resistance or conductance is measured like in a conventional transport measurement, the conductive SFM tip is scanned (in constant height or in feedback mode) over the portion of sample of interest, with a voltage applied on it. Due to the electrostatic potential of the tip, the potential of the underlying 2DEG is modified locally, leading to a variety of effects. In the simplest case, if one takes a plain 2DEG, only the local density $n_s(\vec{r}_{\text{tip}})$ and the local scattering rate at the tip-induced potential will be changed. As soon as more complex transport effects, like edge channel transport, sequential tunneling, or charge quantization appear, the effect of the scanning tip will be more pronounced: In the case of the edge channels it acts as a local perturbation, which scatters the electrons of an edge channel either to another edge channel or to the bulk of the sample. In the case of sequential tunneling and charge quantization, important for quantum dots, the tip can manipulate single electrons, making them hop on and off the dot, or can change the tunnel coupling of a QPC. The result of a scanning gate measurement is a 2D image, where for every position of the tip in the X-Y-plane either the resistance or the conductance are measured. The basic concept is illustrated in Fig. 4.1.

Even if the tip has a macroscopic size (typically 100 nm diameter), its function is not limited to that of a gate of reduced size (and small lever arm), but it can also, due to the gradient in the electrostatic potential, resolve features much smaller than its actual size. This is the ideal starting point for studying the interior of a quantum dot, which is not accessible in a conventional transport measurement. Due to the technical difficulties in the realization of a low temperature STM and the time consumption of typical experiments, the number of experimental data in this
Figure 4.1: Typical setup for a scanning gate measurement on a quantum dot. An AC voltage is applied to the source contact, and the current through the drain is measured, for every position of the local potential perturbation (colors) induced by the scanning tip (black). The dot is tuned in the right regime by applying the voltages $V_{\text{Gate1-3}}$ to the in-plane gates.

Field has been small and limited to QHE studies, constrictions, quantum wires and quantum dots in carbon nanotubes.

4.2 Overview of published experiments

The number of experiments performed with the scanning gate technique and published in literature is small. A short overview of published data is given below, focusing on experiments, which are related to those discussed in this thesis.

Local properties of the QHE have been studied either with a scanning gate setup [135, 136, 137, 138] or with the SCA [139] or scanning SET [140] technique. The scanning gate measurements in [135] focus on Hall voltage profiles across a Hall bar. A linear profile is observed in the bulk for the transition region between quantum Hall plateaus, while a nonlinear profile is observed on the plateaus with localized voltage drops at the edges of the sample. Similar measurements are discussed in [136], while interedge-state scattering is observed in [137]. Here the tip is used to locally couple edge channels on opposite sides of the Hall bar, allowing to localize individual scattering centers. Electrical imaging of the quantum Hall state is discussed in
4.2. Overview of published experiments

Here, compressible and incompressible strips as well as the pattern of the current along edge channels are directly imaged, providing a qualitative picture of the current flow.

Figure 4.2: (a) Scanning gate image of the conductance of a QPC scanned close to the constriction. Sets of concentric conductance halos appear (from [141]) (b) Coherent branched flow observed through a constriction. The fringes are spaced by $\lambda_F/2$ (from [142]).

Experiments on constrictions include the observation of local charging in a QPC [141], measurements of electron flow [143, 144, 145] and density in a QPC [146] and the observation of a coherent branched flow on either side of a electron beam defined QPC [142, 147, 148]. The experiments in [141] show that a QPC behaves like a local electrometer, detecting the charge of a number of quantum dots, which are believed to form in the donor layer close to the constriction. The conductance of the QPC shows concentric “halos” forming close to the QPC, whose size depends on the voltage applied on the scanning tip. A set of halos is shown in Fig. 4.2 (a). The other QPC experiments are similar, in the sense that they image the electron flow through or close to the constriction. Highly detailed scanning gate measurements are presented in [142, 147], where branching of the electron flow can be observed. These measurements were performed very close to the surface of the sample and show fringes in the conductance with a spatial separation of $\lambda_F/2$, see Fig. 4.2 (b).

Quantum wires are ideal if one wants to study longitudinal modes in the wire, as shown in [149, 150], or even for erasable low temperature lithography [151]. Experiments on quantum dots include dots formed in single-wall carbon nanotubes, where the typical concentric rings are observed [152]. Recently, results on a small quantum dot have been published [153], showing similar features, but for a tunable number of electrons down to a single electron dot.
Figure 4.3: (a) Measurement configuration (b) Scanning gate and electrostatic force microscopy image of carbon nanotubes. (c) Scanning gate images for different tip bias voltages. Details are discussed in the text (from [152]).

The measurement configuration is displayed in Fig. 4.3 (i), the dots form inside the carbon nanotube. Figure 4.3 (ii) (A) displays the conductance of two dots as a function of tip position, showing concentric rings corresponding to conductance resonances. (B) and (C) are electrostatic force microscopy images, showing
the force between SFM tip and sample and the relative change in $Q$ factor of the cantilever. They both were performed on different nanotubes. Tip bias dependent measurements confirm that the changes in force and $Q$ factor correspond to the conductance oscillations of the dots formed in the nanotube. Figure 4.3 (iii) shows the evolution of the conductance features with tip voltage between -100 mV (A) and +250 mV (H). The conductance features increase in size if electrons are added to the dot(s) (positive tip bias) and decrease in size if electrons are removed (negative bias). In all images these features form complicated noncircular pattern. The main limitation of these measurements is that the location of the quantum dots is not controllable and that their coupling to the leads, as well as the electron number, are not tunable. Gated nanostructures offer a very important advantage with respect to the configuration used in [152].
Chapter 5

Probing the quantum Hall effect

5.1 Introduction

The quantum Hall effect (QHE) is one of the fundamental quantum phenomena occurring in 2DEGs at low temperatures. Since its discovery in the early 80’s [3], a huge amount of experimental and theoretical work has been done in this field. A remarkable property of the QHE, which will be tested in the experimental part of this chapter, is the robustness and precision of the quantization irrespective from the detailed properties of the sample (material and purity), allowing to use the effect as a resistance standard [38]. Another important property is the formation of edge states at the sample boundaries, which arise from the Landau quantization of the DOS, discussed in section 2.4. These edge channels, forming extended states, lead to the formation of compressible and incompressible stripes [154], determined by the screening properties of the 2DEG. The Landauer-Büttiker formalism, discussed in section 2.4.3, is a good framework for the description of transport in the QHE [41], but it gives only a rough insight into the current distribution inside the sample. In order to obtain more, a local probing technique is needed, and many attempts have already been made to gather more information about the internal structure of the 2DEG in the QHE regime.

The first attempt, an almost local approach, was done by placing gate stripes across a Hall bar, and by measuring selective backscattering of edge states [155]. The first really local techniques used the electron-phonon interaction [156], optical techniques [157, 158, 159] with limitations in the resolution (not less than 1 μm), inductive coupling [160, 161] or a SET placed near the edge of a 2DEG [162]. Scanning probe experiments in the QHE regime include measurements with a scanning SET [102, 140], scanning potential microscopy [135], Kelvin probe technique [136, 163, 164], SCA technique [139, 165, 166] and tunneling between edge channels [137, 138].

The results presented in this chapter were obtained with the scanning gate technique, where the tip is used to enhance the backscattering probability of the 2DEG electrons. The experiments are divided into two main parts: First the scanning gate
measurements in the regime of high filling factor (low magnetic fields around 1 T),
and then at low filling factor (with magnetic fields up to 9 T). The aim of these
measurements is to show the periodicity of the QHE effect in its local properties,
along with the robustness of the resistance quantization in the presence of an external
perturbing potential, and to extract the tip-induced backscattering probability for
the different measurements.

Scanning gate experiments were performed both in constant height mode and in
feedback mode; since the latter allowed a higher resolution, only the results obtained
from measurements in feedback mode will be discussed, unless stated otherwise. One
of the crucial parameters is the bias applied on the tip. Test images were taken at tip
biases ranging from -6 V to +2 V, and the voltage selected for the measurements is
-2 V, since it allows a high contrast without the risk of creating irreversible changes
in the sample, observed for a lower voltage. The maximum scanning range for these
measurements is only 9 µm, due to a different scanning piezo than the one used
later. Measurements performed with the tip at a fixed position (which will be called
“nonlocal”), in order to determine the screening properties of the sample, can be
found in appendix C.

5.2 Sample and measurement setup

The sample consists of a 10 µm long and 4 µm wide Hall bar fabricated from a
shallow Ga[Al]As heterostructure with a 2DEG 34 nm below the surface. The Hall
bar is defined by photolithography as can be seen in Fig. 5.1 Etched markers
ease the procedure of positioning the sample relative to the SFM tip. A back gate
electrode, made of highly doped GaAs, is buried 1.3 µm below the 2DEG and is
separated from it by a layer of ErAs islands [167]. The electron density at 300 mK
and zero back gate voltage is $n_s = 5.5 \times 10^{15}$ m$^{-2}$, while the mobility is $\mu = 8.5$
m$^2$/Vs, both determined from Hall and Shubnikov-de Haas measurements. From
these values one can extract a mean free path (eq. 2.4) of 1 µm, ensuring that
electron motion is diffusive on the scale of the sample size.

Scanning gate measurements were done while applying a current of 100 nA at
a frequency of 623.6 Hz through the Hall bar and by measuring the longitudinal
resistances $R_{xx1}$ and $R_{xx2}$, as well as the Hall resistance $R_{xy}$ with lock-in amplifier
technique. The first characterization of the Hall bar was done at a frequency below
20 Hz, which was then increased, in order to achieve a higher measurement band-
width, up to the point where the parasitic capacitances in the system started to
affect the measurements, leading to small dips at the upper field edge of the Hall
plateaus. Figure 5.2 shows a transport measurement as a function of magnetic field,
where these dips can clearly be observed. The density of the sample allows to reach
filling factor $\nu = 3$ at a magnetic field of 8T. By applying a negative voltage to the
back gate it is possible to reach filling factor 2.
5.3 Scanning tip as a source of local backscattering and density changes

Scattering in semiconductor nanostructures is usually induced by local potential fluctuations due to impurities or other lattice imperfections, thus leading to Coulomb repulsion between the impurity and the electrons of the 2DEG. Since the location of the impurity perpendicular to the plane of the 2DEG is not determining the scattering event, but only the shape and strength of the potential, the external potential, induced by the scanning tip, can act as a source of scattering. Its strength depends on the voltage on the tip and on the tip-to-sample separation, and its lateral position can be varied by moving the tip. Significant backscattering can be induced, if the electrons in the 2DEG have to move on well defined channels, like it is the case in the quantum Hall regime or in constrictions like QPCs. In the first case backscattering means that the electrons in an edge channel on one side of the sample are scattered to an edge channel on the other side of the sample, as shown schematically in Fig. 5.3 thereby increasing the resistance. In the second case, the electrons are just scattered back into the constriction, reducing its conductance.

Depending on the voltage on the tip, the density \( n_s \) will be locally modified by the presence of the tip. If the voltage is negative enough, the 2DEG can be depleted.

Figure 5.1: (a) Photography of the sample showing the markers used for positioning the Hall bar. (b) Schematic measurement setup, showing a topography picture taken at 300 mK with the microscope used for the measurements.
Figure 5.2: Hall resistance and Shubnikov-de Haas oscillations as a function of magnetic field. The two traces $R_{xx1}$ and $R_{xx2}$ correspond to the longitudinal resistances measured at both sides of the sample.

Figure 5.3: Effect of the scanning tip in a Hall-bar geometry. The potential of the tip induces backscattering between edge channels at opposite sides of the bar.
tip is to the surface, the stronger the induced density changes will be. In a more elaborate model with an axially symmetric conical tip, the resulting density change is given by

$$\Delta n(r) = \frac{n_s}{1 + (r/d)^2} \left( \frac{V}{V_d} \right),$$

where $d$ is the 2DEG depth, $r$ is the radial distance between the tip and the plane of the 2DEG, and $V_d$ is the voltage applied on the tip at which the 2DEG is depleted.

5.4 Effect of high tip voltage on the sample

The appropriate tip bias voltage for scanning gate experiments has been determined by scanning the tip at different voltages (from -6 V to +2 V), and by measuring Kelvin parabolas. The voltages required are relatively high, and later experiments confirmed that they can lead to permanent changes in the sample, which disappear only at higher temperatures. These changes are most pronounced if the voltage applied to the tip is negative. The effect is amplified by the fact that the contact potential difference is compensated for a voltage of about 0.5 V (see [168] for Kelvin probe experiments on a similar sample). A tip voltage of -2 V corresponds then to an effective voltage difference between tip and 2DEG of about -2.5 V.

While the consequences of such voltages during scanning gate experiments are difficult to measure, the effect of Kelvin probe measurements, where the tip is always at the same position in the x-y plane of the sample, can be measured. This is achieved by scanning the tip in constant height mode over the sample while measuring the frequency shift of the tuning fork sensor. The image obtained (Fig. 5.4) reveals two features: The contour of the Hall bar and a black spot, corresponding to a negative frequency shift on one edge of the Hall bar.

The first can be explained with the height of the mesa edge in the Hall bar region, which increases the attractive force between tip and sample. The second does not have any equivalent feature in the topography images, indicating that the interaction has to be of electrostatic nature. Moreover, the spot can only be observed when a negative voltage is applied on the tip. The position of the spot corresponds exactly to the position of the tip during Kelvin parabola measurements, which are the obvious reason for the spot. Its shape allows to estimate the electrical resolution of the tip, which is below 150 nm. The consequences for the scanning gate experiments will be discussed later.

5.5 Quantum Hall regime: High filling factor

The measurements discussed here were made in the Shubnikov-de Haas regime, between the classical Hall regime and the quantum Hall regime. Here the number of edge channels is large, and the current should be spread all over the sample. High
filling factor means here a value of $\nu$ between 11 and 25, corresponding to magnetic fields of 2.03 T and 1.015 T.

The scanning gate images (Fig. 5.5) show a very complex resistance landscape in the bulk of the Hall bar, while at the same time the relative Hall resistance changes are restricted to the Hall cross on the left, and are very small in amplitude. This is mainly due to the low level of the absolute resistance signal. Also the $\Delta R/R$ ratio is small for the Hall resistance, and its increase for increasing magnetic field is proportional to the increase in absolute signal. The same ratio for the longitudinal resistance, however, increases much more with magnetic field: from 5% to more than 10%, although the absolute value of the resistance does not increase that strongly. At the same time a reduction of the fluctuations combined with a concentration on certain regions (the upper right region, close to the Hall cross, and the upper right edge of the Hall bar) can be observed at higher fields. The fact that the changes in resistance are spread over the entire Hall bar suggests that the local potential is extremely inhomogeneous between the two edges, and the tip is able to create complex backscattering patterns in the sample.

5.6 Quantum Hall regime: Low filling factor

In contrast to the previous section, where due to the large number of channels the resistance changes were observed over the entire Hall bar, at lower filling factors one would expect larger resistance changes concentrated in smaller regions (see appendix C). At the same time, due to the larger separation between the different filling
5.6. Quantum Hall regime: Low filling factor

Factors in magnetic field, the quantum Hall regime offers the opportunity to study local properties of the QHE in terms of the magnetic field dependence of the effect. The measurements discussed below were performed in feedback mode at magnetic fields between 2.5 and 7 T, corresponding to the filling factors 10 and 3.5, with a scan range of 8.8 \( \mu \)m.

5.6.1 Longitudinal resistance at different filling factors

Figure 5.5 shows scanning gate images for \( R_{xx2} \) for the whole magnetic field range, with their position relative to the normal 4-terminal resistance measurement indicated in the plot. Looking carefully at the images, one can recognize features in the resistance distribution, which occur in images at different magnetic field at the same position and with similar amplitudes. The images correspond to measurements at a filling factor \( \nu \) and \( \sim \nu + 2 \), indicating that the periodic oscillations in the 4-terminal resistance have a correspondence with the scanning gate measurements.

The images, which can be compared, are discussed in detail below. Figure 5.7 shows the measurement for the filling factors \( \nu = 6.5 \) (\( B = 3.74 \) T) and \( \nu = 4.6 \) (\( B = 5.3 \) T). The bulk of the sample is much more homogeneous than at higher filling factors, while the resistance decrease at the edges is much stronger (indicated by “A”) and more pronounced at the upper edge, the one opposite to the voltage...
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Figure 5.6: Scanning gate measurements of the resistance $R_{xx2}$ at the magnetic fields indicated in the SdH-oscillations plot.

Figure 5.7: Scanning gate measurements of $R_{xx2}$ at $\nu = 6.5$ and $\nu = 4.6$. The details are discussed in the text.

probes used for measuring $R_{xx2}$. The opposite effect appears at the lower voltage probe (indicated by “C”), where the resistance increases by 4% at 3.74 T and almost 10% at 5.3T. A slight increase in the bulk at the point “B” can also be observed in both images. There is a dip in the resistance (dotted circle), which also appears in both cases and in all other measurements, except those at odd filling factor.
The measurements in Fig. 5.8, although the difference in filling factor is very small (here $\nu = 6.25$ and 4.17), completely lack the feature “C” and the increase in resistance “B”, but maintain the resistance reduction at the edges (“A”) and the dip at the lower edge (dotted circle). A new increase in resistance appears in the middle of the Hall cross (“D”) and is much stronger at the lower filling factor (31% change at 6.05 T) than at the higher one (12% change at 3.9 T).

**Figure 5.8:** Scanning gate measurements of $R_{xx}$ at $\nu = 6.25$ and $\nu = 4.17$. The details are discussed in the text.

The last two measurements, which can be compared because of the features in the scanning gate image, are for filling factors $\nu = 5.08$ ($B = 4.8$ T) and $\nu = 3.59$ ($B = 6.8$ T) (Fig. 5.9). The resistance distribution in the bulk is completely flat, while the feature “D”, observed before in the Hall cross, appears again, along with a decrease in resistance (“E”) also in the Hall cross. The amplitude of the effect is of the order of 7% at 4.8 T and 12.5% at 6.8 T. From the images discussed above it is possible to conclude that the scanning resistance maps in the quantum Hall...
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regime are similar, if the number of spin-degenerate channels differs by an integer number, although detailed features may differ. The $1/B$ periodicity is present in the resistance maps, taken at different $B$. However, in order to make a quantitative analysis, a much larger number of scanning gate images would be necessary. A more detailed analysis of the magnetic field dependence of the longitudinal resistance is presented in [122].

It is also possible to extract a general behavior of local features in magnetic field: The tendency is that these features become more localized at the edges with increasing $B$, while the structure in the bulk disappears (compare the measurements at high filling factor in section 5.5). This may be related to the transition from the SdH regime to the QHE, where localization of states in the bulk and the formation of extended edge states is very important.

**Longitudinal resistance for the opposite voltage probes pair**

In all measurements discussed above, the longitudinal resistance was measured between the pair of voltage probes, located at the bottom of the images. Most of the changes in the resistance could be observed in the central region of the Hall bar or in the Hall cross at the top, while the only feature, which appears at a different position, is the one labeled with “C” in Figs. 5.7 and 5.9, located on the voltage probe at the lower left of the image. Figure 5.10 shows the same set of measurements of

![Figure 5.10](image)

Figure 5.10: Scanning gate measurements at the same magnetic fields as in Fig. 5.6 but for the resistance $R_{xx}$ measured at the upper voltage probe pair.

Fig. 5.6, but for the voltage probes located at the top of the images (i.e. the ones measuring $R_{xx1}$). Most of the observed features do not change, with the exception of the average resistance and the complete absence of the feature “C”. The first
effect can be derived from the difference in four-terminal resistance (visible in Fig. 5.2) between $R_{xx1}$ and $R_{xx2}$, while the second effect indicates that backscattering in the voltage probes can be detected only if the tip is scanned on the voltage probe where the resistance is measured. This effect is also observed in the constant height measurements discussed in section 5.6.5.

### 5.6.2 Hall resistance at different filling factors

The measurements of the Hall resistance were performed at the same time as those discussed in the previous section, the filling factors match therefore exactly. The result is displayed in Fig. 5.11. It is not difficult to see that the situation is completely different. On one hand most of the images have very little contrast, indicating that the Hall resistance is independent of tip position, on the other hand changes in the resistance appear only within the region of the Hall cross. The relative change in resistance is also much smaller than in the longitudinal resistance: The ratio $\Delta R/R$ reaches only 2 to 3%. The images which show changes in the Hall resistance are those, which correspond to measurements at magnetic field where the transition between Hall plateaus occurs.

The lack of changes in resistance in the other images is in perfect agreement with the observation that the quantized values of the plateaus are independent of

---

**Figure 5.11:** Scanning gate measurements of the Hall resistance $R_{xy}$ between 2.5 and 7 T.
the microscopic details of the sample interior. If this would not be like this, the tip, which induces a local potential, would be able to change the Hall resistance. The changes in resistance are limited to the region of the Hall cross, in agreement with the concept of having only localized states in the bulk, and also in agreement with the previous measurements, where also the longitudinal resistance was flat in the bulk of the sample at these filling factors ($\nu = 5.08$ and $\nu = 3.59$).

5.6.3 Measurements in terms of relative backscattering

The Landauer-Büttiker formalism, presented in section 2.4.3, could be a good framework for describing the effects observed in the scanning gate measurements. Starting with the longitudinal resistance, one can, in the simplest approximation, compare the tip to a gate placed on top of the Hallbar, which is able to induce a certain amount $\Gamma$ of backscattering from one sample edge to the other [155]. The situation is displayed in Fig. 5.3, where one edge channel is partially scattered from contact 2 to contact 6 and from contact 5 to contact 3. The scattering amplitude can be summarized in the transmission coefficients $T_{23}$ and $T_{53}$, while the amplitude transmitted from contact 2 to 3 and 5 to 6 with $T_{26}$ and $T_{56}$ respectively. Equation 2.36 has now to be rewritten and becomes

$$
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0
\end{pmatrix} = \frac{e^2}{h}
\begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & -T_{23} & \nu & 0 & -T_{53} & 0 \\
0 & 0 & -\nu & \nu & 0 & 0 \\
0 & -T_{26} & 0 & 0 & -T_{56} & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}.
$$

(5.2)

It was assumed that the current flows from contact 1 to contact 4, and that contact 4 is connected to ground. Since the sum over all rows and columns of the transmission matrix has to be zero, one finds $T_{26} = \nu - T_{23} = \nu - T_{56}$ and $T_{26} = T_{53}$ as well as $T_{23} = T_{56}$. Putting $T_{26} = T_S$ and $T_{23} = T_T$, by solving eq. (5.2), one gets the main equations for the problem:

$$
\begin{align*}
I &= \nu(V_1 - V_6) \\
V_5 &= 0 \\
V_3 &= V_1 - V_6 \\
0 &= (T_S - \nu)V_2 + \nu V_3 \\
0 &= \nu V_6 - T_S V_2.
\end{align*}
$$

(5.3)

The longitudinal resistance $R_{56,14} = \frac{V_5 - V_5}{I}$ then becomes [155] [169]

$$
R_{56,14} = \frac{h}{e^2 \nu - T_S},
$$

(5.4)
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which can be rewritten by putting \( \gamma = T_S/\nu \), as \( R_{56,14} = \frac{h}{e^2 \nu} \frac{\gamma}{1 - \gamma} \). \hspace{1cm} (5.5)

The parameter \( \gamma \) represents the relative backscattering from one edge of the Hall bar to the other. It is one, if all channels are backscattered completely, and therefore the longitudinal resistance diverges, and zero, if no backscattering occurs. The prefactor \( h/(e^2 \nu) \) is the Hall resistance in the absence of backscattering.

It turns out that the Hall resistance is not affected by scattering as considered here: If the eqs. \( (5.3) \) are solved for the Hall resistance \( R_{62,14} = \frac{V_6 - V_3}{I} \), the result is \( R_{62,14} = \frac{h}{e^2 \nu} \), \hspace{1cm} (5.6)

independent of the backscattering probability \( T_S \). This is consistent with the experiment, since the scanning gate measurements, where the Hall resistance was monitored, did not show any change in the resistance, if the tip was scanned across the Hall bar between longitudinal voltage probes. On the other hand a change of the Hall resistance is possible, if the tip is scanned on the Hall cross and the resistance is not quantized. In the framework used above one out of the many possible backscattering configurations can be like that displayed in Fig. 5.12. The solution

**Figure 5.12:** (a) Possible tip-induced scattering in the region of the Hall cross. (b) Scanning gate image showing a change of the Hall resistance in the Hall cross.

of eq. \( (2.36) \) for this scheme and for the Hall resistance reads

\[
R_{35,14} = \frac{h \cdot \nu - 2T_S}{e^2 \nu \cdot \nu - T_S}, \hspace{1cm} (5.7)
\]

where \( T_S \) is the scattered amplitude from contact 3 to contact 4. In this case a change of the Hall resistance can be induced by the scanning tip, but it requires a
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scattering amplitude between all of the four contacts of the Hall cross. For more possibilities see appendix E and the detailed measurements in [122].

The relative backscattering parameter $\gamma$ can directly be related to the measurements by solving eq. (5.5) with respect to $\gamma$ and using eq. (5.6). The result is

$$\gamma = \frac{R_L}{R_L + R_H} = \frac{R_L/R_H}{1 + R_L/R_H},$$

where $R_L = R_{56,14}$ and $R_H = R_{62,14}$. Transport through the sample can be characterized with $\gamma$, by plotting its amplitude with respect to the magnetic field. Figure 5.13 displays the result for the nonlocal measurement as well as the results for the scanning experiments. The parameter $\gamma$ oscillates as a function of magnetic field

**Figure 5.13:** The relative backscattering parameter $\gamma$ obtained from a 4-terminal resistance measurement (upper curve) and from the scanning gate images. The crosses in the upper plot indicate the value of $\gamma$ of the scanning gate measurements taken outside the Hall bar region, while the violet bar indicates the range of $\gamma$ for the relative scan. The lower curve displays $\Delta \gamma$ the change in $\gamma$ for the scanning gate measurements.

and shows minima corresponding to the minima of the SdH oscillations at even integer filling factor. $\gamma$ is maximum at odd integer filling factors, where a new spin degenerate LL becomes occupied and can reach values over 0.27, meaning that 27% of the electrons are backscattered from one sample edge to the other. The same analysis can be done for every tip position in the scanning gate images, and the results are also displayed at the top and the bottom of Fig. 5.13.
5.6. Quantum Hall regime: Low filling factor

The scanning gate images look very similar to those of the longitudinal resistance, since the value of $\gamma$ is basically the longitudinal resistance normalized to the Hall resistance (eq. (5.8)), which is constant over the whole scan area, except for the magnetic fields $B = 4.8$ T and $B = 6.8$ T. The maximum values of $\gamma$ are obtained close to the lower voltage probes, where the longitudinal resistance is measured, indicating that at that point the sample is very sensitive to the presence of the scanning tip. Like observed in capacitance measurements (see appendix C), the influence of the leads in the 4-terminal measurements manifests itself in a difference between the value of $\gamma$ taken from scanning gate images, but with the tip outside the Hall bar (crosses in the upper curve), and the nonlocal measurement. The values taken from the scanning gate measurements are slightly lower than the 4-terminal ones, indicating that additional backscattering may be present in the leads.

The maximum change of $\gamma$ is very small (lower curve): The tip can change it by less than 0.05, meaning that the tip can induce a change in the amount of backscattered electrons of less than 5%. It also shows a minimum at filling factor $\nu = 4$, reflecting the fact that the change in longitudinal resistance at this filling factor is very small. This is because the resistance is dominated by edge currents, and the coupling between edge channels is very difficult. If one moves away from $\nu = 4$, the states at the Fermi energy become more extended, and the tip can more easily couple states between the edges of the sample.

5.6.4 Results in the framework of a percolation model

The relative backscattering parameter $\gamma$ is a good empirical measure for the coupling between edge channels on opposite edges of the Hall bar, but it cannot explain how this coupling comes about. The Landauer-Büttiker approach is also a good framework for describing the QHE effect, as long as the resistance is quantized. The transition between different quantum Hall plateaus cannot be explained in this framework in its local details and is usually discussed in a model of a localization-delocalization transition in a percolating network of states, localized at potential fluctuations in the bulk. In this model the localization length is studied in the transition region, showing the importance of quantum tunneling between localized states at saddle points of the local potential [171, 172]. The measurements will be discussed in the framework of this model, which is briefly presented below.

The model is based on the Chalker-Cossington [171] network model, which describes the quantum mechanics of a system of non-interacting electrons in 2D with the presence of a strong perpendicular magnetic field, adding the components needed to understand the origin of localized states. The model assumes a long-range correlated, random spatial distribution of the potential energy (as it is the case for the potential originated by doping atoms and impurities), with a correlation length $l_c$ much larger than the magnetic length $l_B = \sqrt{\hbar/eB}$, which is the case at high fields. Schrödinger’s equation is then solved for this potential distribution, leading to eigenfunctions which are superpositions of the Landau states related to a certain
equipotential line with energy $E$. These wavefunctions are of the form

$$\psi(x, y) \propto f(v)e^{i\kappa(u,v)u},$$  

(5.9)

where $u$ and $v$ are local coordinates in a coordinate system, having one axis parallel and one perpendicular to an equipotential line $V(x, y) = E$. $\kappa(u, v)$ is the local wavenumber and $f(v)$ a function which is non-vanishing only within the distance $l_B$ from the equipotential. A net current is carried only along an equipotential line, and the transport problem has to be solved like a classical percolation problem.

It becomes clear that a net current through the percolating network is carried only if the equipotential lines are connected, i.e. if a certain critical energy is reached, where the wavefunction associated to it becomes extended. If one considers tunneling events, this condition has not to be fulfilled exactly, since electrons can tunnel from one localized state to the other, according to a transmission probability $T(\epsilon) = \frac{1}{1 + e^{-\pi \epsilon}}$, where $\epsilon$ is the energy separation between the energy of the tunneling electron and the saddle point energy. This allows to describe two effects observed in the experiments: The insensitivity of the Hall resistance to the scanning tip in the magnetic field region of a quantized plateau is due to the absence of compressible strips in the bulk of the sample. Since there are no free states in the bulk where electrons could scatter, the Hall resistance remains quantized. At the transition between two quantized plateaus the situation is different: The bulk is dominated by a percolating network of extended states at the Fermi energy. The effect of the scanning tip is to change locally the transmission probability between these states leading to a tunneling current. This is possible even if the change in backscattering rate is very small, since the transmission probability depends exponentially on the energy separation between the tunneling electron and the empty state.

For the longitudinal resistance the situation is exactly the opposite: Whereas for the Hall resistance a coupling between different voltage probes is needed in order to change the resistance, for the longitudinal resistance it is enough to create backscattering from one sample edge to the other, in order to record a change in the signal. If the energy of the localized states is close to the critical energy leading to transport, the effect of the tip will be small, since the changes in local potential can be avoided by “using” many different available states. The electrons are therefore able to go around the potential induced by the scanning tip. This is the case at the maximum of the SdH oscillations, where the changes in longitudinal resistance are very small. If there is no percolating network in the bulk of the sample, the only states to which electron can scatter are the states in the edge channel at the opposite edge of the Hall bar, leading to the sensitivity to the tip position at the edges observed in the scanning gate images of the longitudinal resistance. The small changes in the backscattering rate suggest that the potential configuration intrinsic to the sample is not changed by the presence of the tip, and that only the transmission probabilities are locally modified.
5.6.5 Local charging

The sensitivity of the edge regions of the sample to the potential of the scanning tip leads to another interesting effect: Concentric resistance oscillations appear close to the edges of the Hall bar. Figure 5.14 shows measurements where these oscillations can be observed for both longitudinal resistances $R_{xx1}$ and $R_{xx2}$. They form concentric resistance “halos” around a dip located either on the edges of the Hall bar or in the voltage probes. This last point is crucial for the measurement at $\nu = 3.6$, since there the halos appear at opposite edges, depending if one looks at $R_{xx1}$ or $R_{xx2}$, since only the voltage probes, where the resistance is actually measured, can be sensitive to the presence of the tip. The measurements were performed by scanning the tip at a constant height of 50 nm over the surface of the sample and with a bias of -4 V. Therefore they cannot be directly compared to previously presented measurements.

A more detailed analysis can be done by looking at a measurement at $\nu = 2.7$ ($B = 9$ T), where a large set of concentric rings appears at the upper edge of the Hall bar. The concentric rings can be better recognized and seem to extend much further away from their center. The black line in Fig. 5.15 (a) is a profile taken horizontally through the resistance dip and allows to recognize several different resistance peaks. If the separation between the peaks $\Delta \rho$ is plotted as a function of their distance from the center $\rho$, one gets a linear increase for small $\rho$’s, whose slope increases for larger distances. This evolution can be understood in a simple capacitive model,
Figure 5.15: Another resistance halo appears at a magnetic field of 9 T. A profile taken through the resistance dip (black line) shows a series of resistance peaks. The right plot shows the spacing of the peaks as a function of the distance from the center of the dip (red crosses) fitted with the curve given by eq. 5.10 for $\epsilon_{eff} = 6 \times 10^{-13}$.

where the electric field of the tip is assumed to be that of a point charge, and the halos are the result of single electron charging of a quantum dot in the donor layer. The capacitance between the tip and a dot in the donor layer (also considered as a point charge) is then given by

$$C(\rho, d) = 4\pi\epsilon_{eff}\left(\rho^2 + d^2\right)^{-1/2},$$

(5.10)

where $d$ is the distance in $z$-direction between the tip and the donor layer, and $\epsilon_{eff}$ is an effective dielectric constant including screening by surface charges. The charge induced on the dot is given by (see eq. (2.39)) $Q = V_{tip}C(\rho, d)$. The single resistance peaks correspond to the regime where the dot is in the Coulomb blockade, with a fixed number of electrons $N$ on it. The difference in charge on the dot between two peaks is therefore $\Delta Q = e = V_{tip}\partial C(\rho, d)/\partial \rho \Delta \rho$, leading to a spacing between the peaks in terms of the in-plane distance $\rho$ of

$$\Delta \rho = \frac{e}{4\pi\epsilon_{eff}} \frac{1}{\rho V_{tip}} \left(\rho^2 + d^2\right)^{3/2}.$$  

(5.11)

For large distances $d \ll \rho$ the spacing goes with $\rho^2$, reflecting the behavior observed in Fig. 5.15 (b), where the ring spacing is plotted against the ring radius. The parameters are a tip voltage of -4V, a tip-to donor layer distance of 67 nm and an effective dielectric constant of $\epsilon_{eff} = 6 \times 10^{-13}$. Since $\epsilon_{eff}$ contains also the permittivity constant $\epsilon_0 = 8.85 \times 10^{-12}$, the dielectric constant is $\epsilon = 6.7 \times 10^{-3}$. This is a very small number compared to the dielectric constant of GaAs ($\epsilon = 12.7$), indicating that the model may be too simplified, or that oscillations between those observed could not be detected.
Thus the sensitivity of the edge region can be used to detect charging effects in the proximity of the edges of the sample. If there is a potential well close to an edge channel, it can offer additional quantized states to which electrons can tunnel, acting like a quantum dot. If the tunnel coupling to these states is lower than to localized states in the bulk, electrons will tend to fill the levels in the well, and a change in resistance will be detected. By moving the tip, the energy of the well is changed, and the current oscillates like in the Coulomb blockade.

Effect of a local deposited charge

In section 5.4 a dip in the frequency shift of the tuning fork was explained with the assumption that a charge deposited locally on the surface could modify the force between the sample and the scanning tip. This assumption can be proven further by looking at the position of the charge relative to the features observed in the resistance. Figure 5.16 shows a scanning gate image performed in constant height mode, with the position of the local charge marked by a black spot. The resistance dip (observed also in other measurements at the same position) is very close to the position of the charge, and a set of halos originates from that position.

![Figure 5.16: Position of the local charge deposited by the tip obtained from Fig. 5.4 with respect to the resistance in a scanning gate image. A reduced resistance region is observed close to the charge.](image)

5.7 Conclusions

In summary, the experiments presented in this chapter allow to characterize the QHE with respect to the local resistance distribution. One main feature of the QHE
is the $1/B$ periodicity, which has been shown to be visible in the local properties of both the longitudinal and the Hall resistances. The formation of edge channels could be observed in the lack of changes of the longitudinal resistance in the bulk of the sample in the quantum Hall regime, compared to the complex resistance patterns observed at high filling factor in the bulk of the sample. The changes in local resistance of the longitudinal resistance could be explained in the Landauer-Büttiker picture by correlating them to an effective backscattering parameter which varies with the position of the scanning tip. The same picture allows to understand the lack of changes in the Hall resistance on quantized plateaus, providing a further proof of the robustness of the quantization and its suitability as a resistance standard. The changes in the Hall resistance, observed in the transition region between two quantized plateaus, could be related to a percolation of localized states in the bulk of the sample, which also explains the complexity of the resistance patterns in the longitudinal resistance, and allows a qualitative understanding of the local properties of the sample. Additional effects could be related to local potential fluctuations, leading to charging effects in the region where the sample is most sensitive to change in the backscattering rate, i.e. at the edges of the Hall bar.
Chapter 6

Single electron manipulation in a quantum dot

6.1 Introduction

Quantum dots are a research subject which allows to study many fundamental physical effects, and they may be used in the future as a component of next generation information processing devices. The physical effects which govern the electrical transport through a quantum dot, single-electron charging and tunneling, are not only important in artificial nanodevices, but also in molecules and atoms. These effects can be studied in a controllable way in quantum dots [47, 48, 176], and control over single electron charges [47], spins [177, 178, 179, 180], and orbital quantum states [181] can be gained by experimental means. State-of-the-art quantum dot design allows the fabrication of single or two electron dots, which show the physical properties similar to those of hydrogen and helium atoms [182, 183, 184, 185]. Moreover, quantum dots have been suggested as building blocks within quantum information processing schemes [8].

The possibility to confine electrons in quantum mechanical states inside a small area, is the ideal premise for using a scanning probe technique to study local details of the quantum mechanical effects involved. However, only one experiment of local spectroscopy on quantum dots, formed in carbon nanotubes [152], has been reported to date, where the electrical tunability of a lithographically defined dot was not available. The AFM lithography technique allows the construction of complex nanostructures with full tunability of their electrical transport properties. For quantum dots these are the number of electrons and the tunnel coupling to the leads as well as the symmetry of the confining potential.

The measurements discussed in this chapter were performed on two different samples, all defined by AFM lithography, one of these measured during two different cool-downs. The first sample is a four-terminal quantum ring, and the original goal of the experiment was to measure the local properties of electron interference. Unfortunately one of the segments of the ring could not be opened by the lateral
gates, so that the experiments focused on the single quantum dots formed in the open segments of the ring. Since the quality of the data obtained by later experiments is much higher, only a few measurements are presented in the first section of this chapter. The second sample is a quantum dot with an adjacent QPC, which is, however, not suitable for simultaneous charge detection \[186 \text{ 187}\]. Two sets of measurements were performed on this sample, leading to sets of data which partially differ due to the change in experimental parameters (different scanning tip, and differences inherent to the cool-down procedure). Common to all the measurements presented from now on is a new scanning piezo, which allows to increase the scanning range from less than 9 $\mu$m to 12.5 $\mu$m.

6.1.1 Motivation: Manipulation of single electrons and wave-function mapping

The first and the third term in eq. 6.1 contain the direct influence of the tip on the electrochemical potential of the dot. By choosing an appropriate tip voltage, or an appropriate tip position, it is possible to control the number of electrons in the dot, since this quantity depends directly on the electrochemical potential (see eq. 2.46). Figure 6.1 shows schematically how the conductance of the dot depends on the tip position for a constant tip voltage. If the tip is scanned in a 2D-plane, the conductance peaks will form concentric rings around the center of the quantum dot, and the shape of the rings will depend on the shape of the tip-induced potential. Since every conductance resonance corresponds to a fixed electron number on the dot, scanning the tip along one of these resonance rings means scanning it on a line of constant energy in the dot. On the other hand, by moving the tip between the positions corresponding to resonance peaks, one can control the energy and therefore the number of electrons on the dot.

Wave-function mapping

Scanning probe experiments can provide a very direct insight into the properties of semiconductor nanostructures, and their high spacial resolution can allow the visualization of effects which arise from space or energy quantization. The quantum Hall effect and the conductance through a QPC are examples for the first case, while a quantum dot is a good example for the second case. Here energy quantization leads to a quantized number of electrons and therefore to different probability densities inside the dot. One of the main experimental goals of the scanning probe technique is to visualize the probability density inside a quantum dot, which is the squared wavefunction of the electrons in the dot. The requirements for achieving this are briefly discussed below, following a numerical model presented in 188. While the visualization of wavefunctions in self-assembled QDs using an STM has been achieved 63, no comparable results have been obtained with a SFM on electrically tunable quantum dots.
6.1. Introduction

The model considers the transmission probabilities through a resonant double barrier system, which corresponds to an open quantum dot. This is the first constraint to the technique, since the scanning potential has to be weak enough not to close the barriers. An interesting point is that the extent of the scanning potential is not crucial for the determination of the wavefunctions, even if it reaches half the size of the dot. Most important is the condition of a low tip-induced potential height below the single-particle level spacing. The measurements presented in this thesis were performed on quantum dots much larger than the ones used in the model, but the relative size of the tip-induced potential with respect to the size of the dot are comparable. The low potential height requirement can be met by minimizing the contact potential difference with the technique discussed in section 6.1.2 and the signal can be maximized by scanning the dot in a regime, where the dot is most sensitive to changes in the potential, i.e. on the slope of a sharp conductance peak. More recent calculations [189] confirm that this method leads to a mapping of the probability density, since the probability density is reflected by the energy shift of a conductance resonance. By measuring the conductance changes on the slope of a sharp peak, where it changes almost linearly with a shift in energy, this requirement can be fulfilled.

6.1.2 Quantum dot as a sensitive electrometer

The effect of gate electrodes on the chemical potential of a quantum dot can be summarized with eqs. (2.46) and (8.3). The scanning tip enters there as a simple additional gate with a voltage $V_{\text{tip}}$ applied on it. However, the movement of the tip
can induce various additional effects, which are described in detail in appendix C. In a more intuitive way, the movement of the tip will change the effective lever arm of the in-plane gates, due to its own electric field (superposition principle), which modifies the local electrochemical potential in the gate region. In addition to that, local potential changes, due for example to fixed charges, will also be influenced by the presence of the tip.

The consequences of eq. (8.12) can now be interpreted in terms of eq. (8.3) for a quantum dot. The change in electrochemical potential, due to the external perturbation potential, can be summarized as follows:

\[
\Delta \mu = -e \langle \alpha_{\text{tip}}(\mathbf{r}, \mathbf{r}_{\text{tip}}) \rangle \Delta V_{\text{tip}} + \sum_i \langle \alpha_{\text{gate}}^i(\mathbf{r}, \mathbf{r}_{\text{tip}}) \rangle \Delta \mu_i
\]

\[
- e V_{\text{tip}} \langle \nabla_{\mathbf{r}_{\text{tip}}} \alpha_{\text{tip}}(\mathbf{r}, \mathbf{r}_{\text{tip}}) \rangle \Delta \mathbf{r}_{\text{tip}} + \langle \nabla_{\mathbf{r}_{\text{tip}}} \delta U(\mathbf{r}, \mathbf{r}_{\text{tip}}) \rangle \Delta \mathbf{r}_{\text{tip}}. 
\]

In this model the four terms can be determined experimentally, using a quantum dot as a sensitive electrometer, as described below:

- The first term in eq. (6.1) depends on two quantities: the tip lever arm \( \alpha_{\text{tip}} \) and the tip voltage \( V_{\text{tip}} \). The voltage can be set externally, while the lever arm and its position dependence have to be determined in the experiment. In order to do that, the tip is scanned along a line close to the dot, and for every point the DC bias voltage of the dot is changed. This corresponds to a measurement for determining the charge stability diagram, but instead of varying the energy of the dot by changing the voltage on a lateral gate, the tip position is changed. Since the lever arm depends on the position \( \mathbf{r}_{\text{tip}} \) of the tip, its absolute value and its position dependence can be determined with this procedure. The whole term can be zero, if the voltage difference \( \Delta V_{\text{tip}} \) is zero, i.e. if the contact potential difference between tip and 2DEG is compensated. This potential arises from the different materials (PtIr for the tip and GaAs for the sample) and also from all the charges in the system. These include the charges on the gates, leading to the potential

\[
U_{\text{gate}}(\mathbf{r}, \mathbf{r}_{\text{tip}}) = -e \sum_j V_j \alpha_{j, \mathbf{r}=\mathbf{r}_{\text{tip}}}(\mathbf{r}, \mathbf{r}_{\text{tip}}),
\]

the image charges induced in the 2DEG by the presence of the tip

\[
U_{\text{img}}(\mathbf{r}, \mathbf{r}_{\text{tip}}) = \frac{e^2}{2} G(\mathbf{r} - \mathbf{r}_{\text{tip}}, \mathbf{r} - \mathbf{r}_{\text{tip}}),
\]

and the fixed charges in the sample

\[
U_{\text{fc}}(\mathbf{r}, \mathbf{r}_{\text{tip}}) = -e \int_\Omega d\mathbf{r}' \rho_{\text{fc}}(\mathbf{r}') G(\mathbf{r} - \mathbf{r}_{\text{tip}}, \mathbf{r}' - \mathbf{r}_{\text{tip}}).
\]

The density \( \rho_{\text{fc}} \) can be separated in a component \( \rho_{\text{bg}} \), given by the average charge of the doping atoms and the average surface charge, and a component
\( \delta \rho(\vec{r}) \), which expresses position-dependent fluctuations in charge density. The potential can thus be written as

\[
U(\vec{r}, \vec{r}_{\text{tip}}) = U_{\text{gate}}(\vec{r}) + U_{\text{img}}(\vec{r}) + U_{\text{fc}}(\vec{r}) + \delta U(\vec{r}, \vec{r}_{\text{tip}}) \tag{6.5}
\]

The first term does not depend on the position of the tip, and can be compensated by choosing an appropriate tip voltage, the second term cannot be compensated, and depends only on the sample. The determination of the tip voltage, where the contact potential is compensated, can be performed experimentally with the Kelvin probe method, or again by using a quantum dot as a detector. In the Kelvin probe method the frequency shift of the SFM cantilever is measured for different tip-to-sample distance as a function of the voltage on the tip. The apex of the obtained parabolas corresponds to the voltage, where the contact potential is compensated \[168\].

This method has to be employed with care, since it requires relatively high voltages on the tip. This problem can be avoided by using again a quantum dot as an electrometer, and in analogy to the determination of the lever arm, the tip is scanned along a line close to the quantum dot. In this case, however the tip bias is the parameter which is varied, the contact potential is compensated when the voltage where conductance peaks of the quantum dot appear does not depend on the position of the tip.

- The second term in eq. (6.1) depends on the lever arm of the gates (which also depends on the position of the tip), and the electrochemical potential on the gates. Since the electrochemical potential is constant, by keeping all voltages except one constant, the lever arm can be determined by a standard stability diagram measurement, where the conductance of the dot is measured as a function of the number of electrons and the DC current through it. The lever arm depends also on the position of the tip, but due to the very small lever arm of the tip this dependence is small.

- The third term contains the direct dependence of the electrochemical potential of the quantum dot on the tip position. This term can also be made disappear, if the tip lever arm is zero, i.e. if the contact potential difference is compensated. If not, this term can be determined by scanning the tip along a line close to the dot, and by measuring the conductance as a function of tip position and dot energy (electron number). The position of the conductance peaks as a function of the position of the tip reflects the interaction potential between quantum dot and tip, and, in contrast to the determination of the contact potential difference, it cannot be set to zero by changing the energy of the dot.

- The fourth term does not depend on neither the tip position and voltage nor the voltage on the other gates, but depends only on potential inhomogeneities.
in the sample. It cannot be determined directly in the measurement, but it can be extracted from the above measurements, if they show tip-position dependent features, which should not appear in the absence of local potential changes.

In reality the contact potential difference has also a spatial dependence [122], complicating the details of the above analysis.

6.2 Experiment I: Dot forming in a quantum ring

6.2.1 Sample and measurement setup

The quantum ring sample, originally intended for interference experiments [181], is displayed in Fig. 6.2 (a).

A constant AC voltage of 30 $\mu$V is applied to the sample, and the current is measured through an I-V converter (Fig. 6.2 (c)). The I-V converter consists of a two stage preamplifier with an OPA627 in the current-to-voltage conversion stage and an INA108 as a voltage amplifier, which provide a better SNR than the converters incorporated in the lock-in amplifier.

The ring has a diameter of 1 $\mu$m, and its four segments have a width of less than 200 nm. By applying a positive voltage to the gate PG3 placed close to one of the segments, it is possible to form a quantum dot in that segment. Electrons enter and exit the dot through two constrictions (QPCs) at the opposite sides of the segment (see Fig. 6.2 (b)) as soon as the voltage on the gate PG3 is above 90 mV. A typical trace of the conductance vs. gate voltage is shown in Fig. 6.3 (a).

The influence of the tip is summarized in Fig. 6.3 (b): A change in tip bias can not only change the energy of the dot by a considerable amount, but can also change the tunneling coupling of the QPCs, leading to a change in the amplitude of the Coulomb peaks. Concerns about a possible influence of the oscillating tuning fork on the conductance turned out to be unnecessary, since no change in the amplitude of the peak could be observed, even with the high sensitivity of a quantum dot (as displayed on the central peak in Fig. 6.3 (b)).

6.2.2 Energy dependence

Scanning gate images of this sample show a quite complicated conductance distribution, with a lot of fine-structure down to a length scale of 20 nm. Nevertheless the change in dot energy (and therefore number of electrons) can be mapped out. Figure 6.4 (a)-(e) displays a series of scanning gate images taken at different voltages of PG3, all with the same conductance scale, ranging from 0 to 0.4 $e^2/h$. The images can be divided into three main regions: A region of increased conductance on the lower right, a region of zero conductance on the upper right and a region with several “concentric” conductance peaks. The region of increased conductance
6.2. Experiment I: Dot forming in a quantum ring

**Figure 6.2:** Sample and measurement setup for the ring structure. The dot is formed in one of the segments of the ring, by applying a positive voltage on the gate called PG3. The topography images were taken at room temperature after lithography.

will be discussed later; its origin is difficult to understand, since the tip is negatively biased (-0.2 V). The region of zero conductance, on the other hand, arises because of the negatively biased tip: The tip-induced potential pinches off the source-drain coupling, leading to a completely blockaded quantum dot. As soon as the tip is further away from the dot, electrons can hop on and off the dot, leading to concentric conductance peaks, following the principle discussed in section 6.1.1. The circles do not close since the scanning range was not large enough to include the entire region of interest. (This could have been changed only by moving the x-y table, which is a risky procedure once a sample is positioned and working properly.)

Similar experiments on other dots will be described in more detail later on, showing more convincing results. The resistance drop, which appears in the region of increased conductance on the lower right (indicated by the dotted black circle in Fig. 6.4 (d)), is not due to the sample, but is an artifact of external noise sources. The lock-in amplifier lost the signal locking, due to interference with transformer noise on the signal line, and due to the very rapid resistance changes during the measurement. The relative position of the sample could be found by taking a topography image at the end of the experiment and by fitting the conductance rings with an ellipse, as shown in Fig. 6.4 (f). The dot (red circle) is outside the scan range, and the tip-
Figure 6.3: (a) Conductance peaks as a function of the voltage on the gate PG3. (b) Combined plot showing the effect of a change in tip bias and of the excitation voltage of the tuning fork. A change in tip voltages shifts the position of the peaks in energy and changes their amplitude, while a change in excitation voltage has no effect.

Figure 6.4: Scanning gate images for different voltages of PG3, and for a tip voltage of -0.2 V. The number of electron in the dot increases with increasing voltage and is reflected by the conductance peaks getting closer to the center. Image (f) shows the position relative to the sample, and a “fit” of the conductance peaks. The actual shape of the tip induced potential in the x-y plane could correspond to the feature indicated by the white arrow. The regions of zero conductance (black circle) are discussed in the text.
induced potential seems to have a very elongated shape, as shown also by a small conductance ring, appearing at the lower right (white arrow). A small region of increased conductance just on the left of the quantum dot is placed exactly over the particle, which appears on the topography images at room temperature (see inset of Fig. 6.2).

### 6.2.3 Determination of the local interaction potential

The interaction potential between tip and quantum dot can be determined by scanning the tip along a line close to the quantum dot, and measuring the conductance for different voltages on PG3 (see Fig. 6.5), as described in section 6.1.2. The lines of resonant conductance, which appear in the measurement plot, correspond to single conductance peaks observed already in the scanning gate images. Since they correspond to lines of constant interaction energy, their change in energy with respect to the position of the tip allows to map out the interaction potential quantitatively. Figure 6.5 shows the result of the measurement; the energy scale has been calculated from the lever arm of PG3, using eq. (2.49) for the crossing points of the Coulomb blockade diamonds boundaries in the stability diagram and their relative separation:

$$\Delta E = eV_{SD} = \Delta N + 1 + \frac{e^2}{C_{\Sigma}}$$

and

$$\Delta V = eV_G = \frac{1}{\alpha_G} \left( \Delta N + 1 + \frac{e^2}{C_{\Sigma}} \right).$$

(6.6)

Thus one can calibrate the gate voltage axis to energy, by multiplying the gate voltage with the respective lever arm (obtained from the charge stability diagram measurement). The change in energy, due to the movement of the tip, reaches about 22 meV, a value close to the typical Fermi energy in such a sample which is 19 meV. This indicates that the tip is able to pinch off the source-drain coupling completely.

The right part of Fig. 6.5 (a) is related to the abrupt increase of conductance observed in the scanning gate images. It seems to be caused by some kind of screening effect, which lowers the electrochemical potential in the dot and leads to high conductance. A different set of Coulomb peaks with a different slope as a function of tip position appears, leading to energy level anti-crossing (Fig. 6.5 (b) and (c)). This indicates the presence of another quantum dot coupled to the first.

The change in lever arm of the tip can be determined by measuring tip-position-dependent Coulomb diamonds, (as explained in section 6.1.2 for the determination of the first term in eq. (6.1)) but the strong increase in conductance close to the dot did not allow to measure enough conductance peaks for getting a reasonable result. The measurement shown in Fig. 6.6 demonstrates the feasibility of such a measurement. More convincing data obtained on another sample will be presented later.

The three different experiments (scanning gate images, tip-induced potential and tip-position-dependent diamonds) can be related through a set of common parameters: The scanning gate measurement was done at a voltage of PG3 of 140 mV,
**Figure 6.5:** (a) Local potential measurement by scanning the tip on a line close to the dot and changing the voltage of PG3. A fitted potential curve is drawn in the left plot. (b) shows the left region of the measurement, where anti-crossing due to a second dot appears. (c) was obtained by a graphical fit of the measurement.

**Figure 6.6:** (a) Coulomb blockade induced by scanning the tip along a line close to the dot. This trace correspond to the line of zero bias in (b) (without the leftmost part), where Coulomb diamonds were measured as a function of the position of the tip.

the same as for the diamond measurement, whereas both the local potential and the diamond measurements were done by scanning the tip on the same line. If one draws the line (in x direction) in the scanning gate image, the points, where it crosses conductance peaks, should match the points where the potential contour
lines cross the 140 mV level in the local potential plot and the crossing points in the diamonds. This is shown in Fig. 6.7, where the color scales are adapted for clarity. The figure shows the excellent stability of the sample and the reproducibility of the measurements.

![Diagram](image)

**Figure 6.7:** The three experiments discussed above related with respect to their common parameters.
6.2.4 Resolution of the experiment and quality of the sample surface

The measurements in the QHE regime on the Hall bar (chapter 5) showed that with the scanning gate technique it is possible to achieve a very high resolution, of the order of 100 nm, in the resistance images. On this sample it is possible to reduce the size of the conductance “rings” until they get very close to the central region of enhanced conductance, by increasing the voltage on the gate PG3 and by using a positive tip voltage of the order of 1 V. The features observed in this region can be resolved down to a size of about 20 nm, as it can be seen in Fig. 6.8. In order to get close enough to the region of interest without moving the x-y-table of the microscope, an additional offset voltage was applied to the scanning piezo, adding almost 2 $\mu$m of scan range, and allowing to scan on top of the quantum dot. The region above the dot cannot be resolved, since the conductance enhancement covers all the features which may exist close to the dot, leaving only the filaments around it. The reason for this conductance enhancement is unclear, but it may be related to screening due to charges between the tip ad the 2DEG.

Figure 6.8 (b) shows a similar scan positioned with respect to a topographic image of the sample. The points of highest conductance correspond to the positions of the sample, where a particle is lying on the surface, indicating that the tip potential may be screened by surface charges, which accumulate close to the impurity. However, it is still puzzling, why screening should be able to increase the conductance.

Figure 6.8: (a) Scanning gate image with the tip scanned over the center of the dot at a bias of 1 V. Fine filament structure is resolved down to a resolution of 20 nm (inset) (b) Two particles on the surface of the sample seem to be a possible origin of the conductance enhancement.
6.2.5 Conclusions and motivation for the next experiment

The experiments on a dot formed in a segment of a quantum ring demonstrated that it is possible to visualize Coulomb blockade in space, and to map interaction potentials between the SFM tip and a quantum dot with a scanning gate setup. The quality of the data is not yet satisfying and can be improved by considering a couple of issues:

- The surface of the sample has to be extremely clean. Any kind of particles lying on the surface can potentially be a source of unwanted screening effects, or may be charged by the tip. A clean surface will also minimize the risk of tip-crashes, and allows to approach the tip even closer to the surface.

- Quantum dots need more degrees of freedom in their tunability. In the previously discussed experiment, by changing the number of electrons on the dot, one changes automatically also the coupling to the leads. This is an effect which has to be avoided, since it is not possible to measure many conductance resonances without increasing at the same time the background conductance.

- Scanning gate experiments on semiconductor nanostructures are easier to understand, if the sample has a simple geometry. The large number of oxide lines around the dot of interest may be a source of additional features in the images, making their interpretation more difficult.

These requirements are fulfilled for the sample discussed in the next section, and special care has been taken not to expose the sample surface to the environment.

6.3 Experiment II: Quantum dot

6.3.1 Sample and measurement setup

The sample has been fabricated on a AlGaAs-GaAs heterostructure with a 2DEG 34 nm below the surface, with a density of $5 \times 10^{11}$ cm$^{-2}$ and a mobility of 450'000 cm$^2$/V s at a temperature of 4.2 K. The quantum dot and three QPCs have been defined by room temperature local oxidation with a SFM. Figure 6.9 shows two topography images of the sample surface, one taken at room temperature with the SFM used for the lithography, the other one taken at 300 mK before beginning the measurements.

The number of electrons in the quantum dot can be controlled by the lateral plunger gate pg, while the two gate regions qpc1 and qpc2 tune the tunnel coupling to source and drain. The QPCs of the regions labeled with qpc1 and pg could not be tuned in a conductance quantization regime, and were therefore not used in the experiment. The QPC in region qpc2 will be discussed in chapter 7. For all the measurements presented below, the dot is operated in the Coulomb blockade regime, requiring negative voltages applied on the gates. The measurements were performed.
with the optimized electronic setup, discussed in Appendix A, which allows to reduce the current noise floor below 20 fA at a bandwidth of 200 Hz. A bias voltage of 20 µV is applied to the quantum dot, at frequencies between 80 to 120 Hz, measured with lock-in technique.

The charging energy of the dot, obtained from Coulomb blockade diamonds (see Fig. 6.10), is about 1.4 meV. The single-particle level spacing can be estimated from the geometry of the dot to be about 30 µeV, assuming a depletion length of 100 nm. The conductance resonances are thermally broadened with an electron temperature of 550 mK. The average single-particle level spacing is smaller than the thermal broadening of the conductance peaks. Therefore the dot is operated in the regime of few-level transport.

6.3.2 Effect of the scanning tip on the oxide lines

Nanostructures, defined by local oxidation, have many differences with respect to those written by electron beam or optical lithography. One of these is that the voltages, which can be applied on the gates, are lower. This is due to leakage currents between different gates, which have to be kept small in order to avoid unwanted electrical signals superimposed on the dot signal. The typical voltages, which can be applied on the gates if one wants to have less than 5 pA leakage current, are of the order of ± 500 mV. For this sample, the maximum voltages were +650/-520 mV for the gate qpc1, +520/-500 mV for the gate qpc2, and +510/-490 mV for the gate pg. These voltages have been measured in the absence of the tip. The applied tip voltages are of the same order of magnitude as those which can be applied on the lateral gates. Therefore one could expect a change in the insulating properties of the oxide lines, when the tip is present, and therefore changes in the

Figure 6.9: (a) Topography of the sample measured at room temperature (b) Topography measured at 300 mK. The quantum dot and the QPC labeled with qpc2 can be measured simultaneously.
6.3. Experiment II: Quantum dot

![Graph](image.png)

**Figure 6.10:** (a) Charge stability diagram from which the charging energy could be extracted (logarithmic plot) (b) Conductance resonances as a function of the plunger gate voltage $V_{pg}$.

Since the scanning gate images did not show any clear evidence of this effect, a set of leakage current measurements was made at the end of the cool-down. The measurement is done by applying a voltage to one gate up to the point where the leakage current is of the order of 5-10 pA, while all other areas of the sample are connected to ground. The tip is then scanned with a relatively high negative or positive voltage over the sample, while monitoring the current over the oxide lines. Figure 6.11 (a) shows the result of the measurements for the gates $pg$ and qpc2. For gate qpc1 the images showed no contrast, even if the leakage current was already of the order of 5 pA. This indicates that the point, where the oxide line is leaking, is placed outside the scanning range. By combining the measurements it is possible to locate exactly the oxide lines in the scanned images (Fig 6.11 (b)). At certain points the leakage currents increase strongly, indicating that their insulating behavior changes locally. This effect may be due to inhomogeneous oxide thicknesses. Even though the tip is able to increase the magnitude of the leakage currents, this is of no relevance for the experiments shown in the remainder of this thesis, since the voltages, which have to be applied on the gates, exceed by far those used in the measurements. The measurements would be safe even if the measurement voltages would be separated by only 10 to 20 mV from the critical setting, since the exponential decrease of the leakage current eliminates any effect of the scanning tip. Moreover, the points at which an increase of the current is observed, are outside the line sections delimiting the quantum dot.
6.3.3 Local properties of the quantum dot

Tip-position-dependent lever arms of the gates

The gates of semiconductor nanostructures defined by local oxidation are basically regions of 2DEG, separated from the measured region by one or more oxide lines. It is therefore not clear if the density changes, induced by the scanning tip in the gate regions (see section 5.3), may influence also the lever arm of the gates with respect to the dot. This would lead to a shift in its chemical potential (see eq. (6.1)). These changes can be studied by placing the tip at different positions above the surface of the sample and then measuring the lever arms of all the other gates. This procedure requires that at least one of the lever arms is known with respect to the dot (from the charge stability diagram), the others can then be determined by varying the relative voltages and tracking the energy shift of the conductance peaks. The measurement was performed by placing the tip 200 nm above the surface of the sample as shown in Fig. 6.12 at seven different positions on top of the quantum dot and of the gates.

Table (6.1) shows that the lever arm of the tip is changing significantly for the different positions, reaching changes of more than 600%, while the change in the lever arms of the three gates qpc1, qpc2 and pg is smaller (less than 40%) but still well measurable. The fact that the changes seem to be unsystematic in the tip positions can be attributed to the shape of the scanning tip: During this cool-down
6.3. Experiment II: Quantum dot

Figure 6.12: Positions of the tip for the measurements of the tip-position dependence of the lever arms.

Figure 6.13 (a) shows a scanning gate image where the dot is tuned in a regime of relatively high conductance close to the Coulomb blockade regime [190]. The tip bias used for this scanning gate image is 0.2 V, which corresponds to a negative tip bias, considering the contact potential difference. Four different regions labeled I to IV can be distinguished: (I) A central peak of increased conductance, when the tip is exactly above the quantum dot, (II) the region of Coulomb blockade, where no current flows through the dot (due to the negative bias of the tip), (III) a region of ring-shaped conductance oscillations around the blockaded region and (IV) a region of weakly varying conductance close to the value in the absence of the tip. In the region where the tip crosses the dot, a series of horizontal stripes of modified conductance appear, leading to an outwards shift of the conductance.

Local interaction potential

(mentioned one on this sample), a quite asymmetric double tip was observed, leading to a highly non-symmetric electrostatic potential. Also the charging energies, see the last column of table [6.1] show a similar unsystematic behavior. The numbers in brackets are the relative error. From eq. (6.2) one would expect the sum of all lever arms to be 1. However, this would be the case only if all electric field lines starting from the 2DEG would also end on a gate surface. This is impossible for this sample since no backgate is present, and many metallic surfaces are positioned close to the sample. Nevertheless, the sum is higher if the tip is very close to the dot.
peaks. This stripes do not rotate with the scan direction (see Fig. 6.13 (b)) and can be attributed to tip-induced charge rearrangements when the tip is close to the dot.

While region I cannot be interpreted directly, and is a feature similar to the conductance increase observed in the measurements of the quantum ring, regions II to IV can be interpreted straightforwardly: In region IV the conductance is almost constant since the tip is far away from the quantum dot and its influence is very small. When the tip approaches the dot in region III, the conduction starts to oscillate, since the energy in the dot is increased and electrons start to tunnel off the dot one by one. For the oscillations closest to the center, the tip is able to pinch off the source and drain tunnel barriers completely, so that clear Coulomb blockade can be observed. In region II this effect is enhanced further, so that the barriers are almost completely depleted, and no current can be detected at all. The ring-like conductance peaks observed in region III indicate that a single electron is entering or leaving the quantum dot. These closed lines correspond to a specific conductance peak and therefore to a specific energy of the quantum dot. Therefore these lines map out the interaction potential between the tip and the quantum dot, and by scanning the tip along one of those lines the energy of the dot (and its electron number) are not changed. The noncircular shape of the contour lines is an indication for an asymmetry of the tip around the z axis, originating from the electrochemical etching procedure. Figure 6.13 (c) shows the same measurement in a 3D representation, where the actual height of the different oscillations is better visible.

The interaction potential can be mapped out by scanning the tip on a line (dashed line in Fig. 6.13 (a)) and varying plunger gate voltage $V_{pg}$. The line is about 1.3 $\mu$m away from the dot, and the tip bias for the measurement is 0 V. The results are summarized in Fig. 6.14. The light conductance peaks, changing in energy with tip position, correspond to the change in interaction potential. By following a single peak when the tip position is varied, one can map out the potential, as indicated by

<table>
<thead>
<tr>
<th>Pos.</th>
<th>PG</th>
<th>Tip</th>
<th>qpc1</th>
<th>qpc2</th>
<th>$E_C(\mu eV)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.129(0.01)</td>
<td>0.013(7x10^{-4})</td>
<td>0.107(0.008)</td>
<td>0.065(0.005)</td>
<td>733(49)</td>
</tr>
<tr>
<td>2</td>
<td>0.114(0.017)</td>
<td>0.0014(6x10^{-4})</td>
<td>0.093(0.008)</td>
<td>0.054(0.008)</td>
<td>772(10)</td>
</tr>
<tr>
<td>3</td>
<td>0.138(0.011)</td>
<td>0.0017(4x10^{-4})</td>
<td>0.111(0.009)</td>
<td>0.069(0.007)</td>
<td>808(8)</td>
</tr>
<tr>
<td>4</td>
<td>0.119(0.008)</td>
<td>0.0018(1x10^{-4})</td>
<td>0.101(0.006)</td>
<td>0.058(0.004)</td>
<td>786(12)</td>
</tr>
<tr>
<td>5</td>
<td>0.132</td>
<td>0.004</td>
<td>0.111</td>
<td>0.0654</td>
<td>732(34)</td>
</tr>
<tr>
<td>6</td>
<td>0.133</td>
<td>0.005</td>
<td>0.111</td>
<td>0.0653</td>
<td>810(10)</td>
</tr>
<tr>
<td>7</td>
<td>0.096(0.016)</td>
<td>0.009(0.001)</td>
<td>0.082(0.014)</td>
<td>0.044(0.008)</td>
<td>766(42)</td>
</tr>
</tbody>
</table>

Table 6.1: Changes in the lever arms of the gates and the tip and of the charging energy of the dot due to changes in tip position. The numbers in brackets are the relative error (0.01 means ±0.01).
6.3. Experiment II: Quantum dot

Figure 6.13: (a) Scanning gate image of the dot at a plunger gate voltage $V_{pg}$ of -420 mV (dotted line in Fig. 6.14) scanned in feedback mode a few nm above the surface. The oxide lines have been drawn in the figure. Regions I to IV are discussed in the text. (b) By turning the scanning direction by 90 degrees the horizontal stripes crossing the dot do not turn, indicating that they are due to charge rearrangements. (c) 3D image of the scanning gate measurement of (a).

The solid lines in Fig. 6.14. The measurement was performed with an ADC at fixed resolution, leading to the horizontal lines in the figure every time the input range of the lock-in amplifier was changed. The tip-induced potential maximum at this distance from the dot is more than 14 meV, just slightly less than the Fermi energy in the unpatterned 2DEG. The 2DEG can thus considered to be depleted in the dot region under the tip for a tip voltage $V_{tip} = 0$ V.

The insert shows a detail of the top region, where the change in potential reaches its maximum: A set of enhanced conductance features at lower energy appears, indicating that the interaction potential is lowered at that point. These features are

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related to the enhanced conductance region I in Fig. 6.13 (a), and are similar to the potential lowering of Fig. 6.5. The conductance increase, appearing in region I, depends on the cool-down (it did not appear during the second cool-down of the same sample) and is unexpected, since the tip should be repulsive at that voltage.

**Local contact potential**

Using the method briefly discussed in section 6.1.2 it is possible to minimize the tip-induced potential. Line scans along the line displayed in Fig. 6.13 (a) have been performed for different tip voltages \( V_{tip} \), ranging from 0 V to 1 V. The result is shown in Fig. 6.15 (a). A symmetry line can be placed at about \( V_{tip} = 0.56 \) V. Below this voltage the curvature is concave, while above it is convex. The reason is that the contact potential difference between sample surface and tip is compensated at a tip voltage of 0.56 V. The tip is therefore repulsive for \( V_{tip} < 0.56 \) V and attractive for \( V_{tip} > 0.56 \) V. This voltage can be understood by taking the contributions of the work function difference between PtIr (electron work function: 5.57 eV for 80% Pt and 20 % Ir) and GaAs (electron work function: 3.98 eV) and the Fermi-level pinning at the GaAs surface (around 0.9 eV [191]) together, leading to a contact potential voltage between 0.4 V and 0.8V.

A zig-zag motion of the conductance peaks, appearing in the \( x \)-position range between 5.5 \( \mu m \) and 7 \( \mu m \), does not disappear even for the voltage at which the tip is...
expected to have no influence on the sample. This additional features do not depend on the tip bias, indicating that the tip-induced potential contribution, leading to the
effect, is voltage independent. The position of the features close to the quantum dot relates them to the conductance increase observed on top of it. This can be observed also in Fig. 6.15 (b), where a series of scanning gate measurements for different tip voltages between 0 V and 0.5 V is displayed. The region of increased conductance in the center of the conductance oscillation rings is changing only slightly with tip voltage. The voltage-dependent contribution manifests itself in a change in size of the conductance oscillations: When the tip bias increases, the rings shrink until they merge with the central region of enhanced conductance. By further increasing the voltage, the rings would expand again as soon as the voltage on the tip is larger than 0.56 V.

Local electric field

The electric field can be expressed as the change of the electrostatic potential with position. The electric field caused by the tip can be determined by measuring Coulomb diamonds as a function of tip position. The tip is scanned along the same line already used for the local potential and for the contact potential difference (Fig. 6.13 (a)), while the source-drain voltage $V_{SD}$ is varied between -2 and +2 mV. The potential energy of the dot is changed by the movement of the electrostatic potential of the tip, while the change in energy of a single-electron level can be varied with the source-drain voltage $V_{SD}$. The slope of the position dependent diamonds boundaries (Fig. 6.16) corresponds to $\frac{\Delta V_{SD}}{\Delta x} = E_x$, the electric field in $x$-direction. The electric field varies between about 50 (far away from the dot) and 12000 V/m close to the dot. When the tip approaches the dot, the diamonds become more extended in the $V_{SD}$ direction, while at the smallest tip-dot spacing (in the center of the image) the diamonds become very small. This effect is related to the conductance increase in region I of Fig. 6.13 (a). The measurement illustrates in a very intuitive way, how the lever arm of the tip changes with its position, and how the dot reacts to that change. The elongation of the diamonds in $x$ direction depends on the lever arm of the tip, since it is given by $\frac{1}{\alpha_{tip}} \left( \Delta N + 1 + \frac{e^2}{C_S} \right)$. It therefore decreases if the lever arm of the tip increases, and vice versa. The elongation in $V_{SD}$ direction does not depend on the lever arm, but only on single-particle level spacing and charging energy (see Fig. 2.7). Thus, if the diamonds become more extended in $V_{SD}$ direction, the charging energy and the single particle level spacing have to become larger. Both parameters are correlated: If the size of the dot is reduced due to the repulsive potential of the tip, the single particle level spacing will increase. At the same time the self capacitance of the dot decreases, leading to a larger charging energy. The latter effect will dominate, since the charging energy for this dot is 1.4 meV, while the single-particle level spacing can be estimated to be around 30 $\mu$eV, almost 50 times smaller.

In the central region of Fig. 6.16 the diamonds become smaller in size, both in the $x$- and $V_{SD}$ direction. This means that the lever arm of the tip is larger than in the outer regions, while the single-particle level spacing and the charging energy
6.4 Experiment III: Same dot, different cool-down

6.4.1 Local contact potential

During the second cool-down of the same sample, the measurement of the local contact potential led to a surprising result: There is no single symmetry line (like in Fig. 6.15 (a)), and the conductance resonances seem to follow a more complicated pattern in space. Figure 6.17 shows the result of two different measurements, one performed by scanning the tip on a line crossing a region with many conductance resonances, the other one on a line crossing the region where the tip seems to be attractive for most of the voltages applied. It seems that there are two different voltages where the contact potential difference is zero, indicating that a double tip, possibly made of different materials, may be present. Figure 6.17 (a) shows two different sets of curves, each corresponding to what seems to be a different tip. The indicated equilibrium point corresponds to the equilibrium voltage obtained in the previous cool-down and is related to the PtIr tip. Figure 6.17 (b) on the other hand shows a completely different equilibrium voltage, and its relatively small value suggests that it may be related to a particle of the same material as the sample, which has been picked up by the tip. The next section shows, how this difference in

Figure 6.16: Coulomb diamonds as a function of tip position in $x$-direction and source-drain bias voltage $V_{SD}$. The tip is scanned 150 nm above the surface, with a bias of +0.2 V.

become smaller.
Figure 6.17: (a) Determination of the contact potential for the second cool-down. The influence of two tips with different contact potential differences with respect to the sample, manifests itself in the two different curvatures of the conductance peaks. The compensation voltage for one of the tips is marked by the white line, and is in agreement with the one determined during the first cool-down. (b) Same measurements, but focusing on the set of conductance peaks originated from the other tip. Here the contact potential seems to be compensated by a slightly negative voltage.

contact potentials influences the scanning gate images.

6.4.2 Tip bias dependence

The larger number of conductance peaks per area, observed in the second cool-down, allows to obtain a higher resolution in tip voltage, since more conductance resonances can be tracked. Moreover, the increased conductance region does not appear again, allowing to observe the origin of conductance peak rings from their center. Two different sets of measurements are shown in Figs. 6.18 and 6.19. Figure 6.18 shows the measurements for a voltage difference between the gates qpc1 and qpc2 of 245 mV on a scan area of $2.5 \times 2 \mu m^2$, with a positive voltage of 130 mV on the gate pg. Figure 6.19 is for a highly asymmetric voltage (400 mV difference) between the gates qpc1 and qpc2, and for $V_{pg} = -40$ mV on a scan area of only 700 by 700 nm$^2$. Both measurements show two sets of conductance peak rings, which react depending on changes of the tip bias. Since the conductance resonances can be attributed to a constant energy and delimit regions of constant electron number in the dot, their shape reflects the interaction potential between the tip and the quantum dot. However, one scanning gate image is not enough to determine the sign of this potential. This requires changing the tip bias as displayed in the figures.

For the lowest tip bias (60 mV) in Fig. 6.18 a set of fringes is centered around the point indicated by the yellow cross, and the size of the fringes decreases with
6.4. Experiment III: Same dot, different cool-down

**Figure 6.18:** Scanning gate images for different tip voltages (60 mV to 480 mV). At 120 mV a new set of rings starts to appear (red cross), while the number of rings related to the other center (yellow cross) decreases.

**Figure 6.19:** Scanning gate images for different tip voltages (0 V to 0.7 V), showing the action of the double tip on the quantum dot. One of the tips is always in the attractive regime, and the number of electrons increases, while the other one is repulsive, and the number of electrons is reduced by the presence of the tip. The dotted line indicates the energy where the dot has N electrons.

Increasing tip bias. At the same time, a second set of fringes centered around the red cross start to appear above a tip bias of 120 mV, and their size increases with increasing tip bias. From this evolution it is possible to conclude that the interaction potential has to be repulsive (negative) for the first set of rings, while it has to be positive for the second set of rings, since in the latter case the number of electrons increases. This difference can be attributed to the presence of a double tip made
Chapter 6. Single electron manipulation in a quantum dot

of different materials, where one tip is responsible for an attractive potential in the voltage range used in the experiment, while the other one is repulsive.

Comparing these observations with the results obtained in section 6.3.3 leads to the following explanation: The tip responsible for the lower set of rings (yellow cross) is the PtIr tip, for which the contact potential difference is zero at 560 mV, and which is always repulsive for the voltage range of the measurement. The other set of rings is due to the second tip, which shows the same effect at about -100 mV. Therefore, the second tip is always in the attractive regime for the voltage range of the experiment. The same observations can be made in Fig. 6.19, where the voltage range is shifted to higher positive voltages. The size of the fringes of the lower set decreases also here for an increasing tip voltage, but for the last two voltages (0.6 and 0.7 V) it seems to stay almost constant, indicating that the tip may be in the attractive regime. The same effect can be observed if one places a boundary of N electrons on top of one of the conductance peak lines: This boundary moves away from the center with increasing number of rings, but its position becomes stable for the highest voltages.

Another proof of the existence of a double tip is the shift in position of the center of the conductance peaks rings. The image in Fig. 6.18 for a tip voltage of 480 mV shows the shift in the position of the center, which is much stronger for the upper set of rings. This is due to the starting voltage (60 mV) close to the equilibrium voltage for that tip, which has to be close to a saddle point in the potential between the two tips.

The influence of the tip on the height of the conductance peaks can be observed in both figures: The conductance is very high on the top right of the lower set of rings and on the lower left of the upper set of rings. This region of enhanced conductance has on its left a region of low conductance (visible in Fig. 6.18 but not 6.19). It appears also on the left of the lower set of rings, which corresponds to the tip which has a repulsive potential. The centers of these regions are about 300 nm away from the centers of the conductance peak rings, a distance just slightly larger than the separation between the center of the dot and the left qpc. Thus, when the tip which has an attractive potential is scanned over the qpc, the height of the conductance peaks increases, while it decreases when the other tip is at the same position.

This behavior can be understood by looking at eq. (2.51), which gives the lineshape of the conductance peaks. The first term giving the maximum amplitude contains the influence of the qpcs through their tunnel coupling to the dot, and it can be rewritten putting $\Gamma_S = \Gamma_L$ and $\Gamma_D = \Gamma_R$ for the left and right qpcs of the sample as

$$G_{\text{max}}(\vec{r}_{\text{tip}}) = \frac{e^2}{4k_BT} \frac{\Gamma_L(\vec{r}_{\text{tip}})\Gamma_R(\vec{r}_{\text{tip}})}{\Gamma_L(\vec{r}_{\text{tip}}) + \Gamma_R(\vec{r}_{\text{tip}})}. \quad (6.7)$$

The change in amplitude can thus be justified with a tip position dependent tunnel coupling of the qpcs, and the fact that the changes appear only on the left of the centers of the two sets of rings indicates that the left qpc dominates the value of
6.4. Experiment III: Same dot, different cool-down

$G_{\text{max}}(r_{\text{tip}})$, since its tunnel coupling $\Gamma_L(r_{\text{tip}})$ is smaller and therefore more sensitive to the position of the tip. A more detailed analysis of this effect can be found in section 7.6.

The potential itself can be reconstructed using scanning gate images, assuming that the charging energy term in eq. (6.6) is dominating the energy separation between two neighboring conductance peaks, meaning that $\Delta \mu_n = E_c$. The lines, corresponding to a constant energy in the dot, can thus be used as equipotential lines for the tip-induced potential, allowing to reconstruct the three-dimensional shape of the potential. The first step is to create a “top view” of the system by inverting the spatial coordinates of the measurement, as shown in Fig. 6.20 (a) and (b) for a tip bias of 0 mV and 510 mV of the same measurement series of Fig. 6.18. The contour plot can then be converted to the actual shape of the interaction potential, as shown in (c) and (d). The top of the potential is truncated, since the dot is pinched off in the center of the rings, and does not allow to reconstruct the shape of the apex of the potential. Figure 6.20 (c) is for the PtIr tip in the repulsive regime, while in (d) both contributions are visible, the smallest one is the remaining part of the PtIr
tip contribution close to the equilibrium voltage. The potential close to the positive contribution is flat, therefore the center of a set of fringes is only weakly bound to a certain $x - y$ position for small changes in tip voltage, while the potential is very steep in the region between the two tips close to the negative contribution.

**An intuitive model for the width and separation of the peaks**

The separation and the width of the conductance peaks depends on the position of the tip relative to the dot. In Fig. 6.18, for example, the outer ring-shaped conductance resonances are broader and wider spaced than the inner ones, especially if the inner ones are in the region between the two sets of rings. On the other hand, the ring-shaped conductance peaks closest to the center of the set become again broader.

These effects are related to the shape of the interaction potential as shown in Fig. 6.21 (a), where a Lorenzial-shaped tip has been used as a model. The separation between two peaks, if the energy separation is constant, depends on the gradient of the interaction potential. In a linear approximation one gets

$$|\Delta \vec{r}| = \frac{\Delta E}{|\nabla \mu_n(\vec{r})|}. \quad (6.8)$$

If the gradient of the interaction potential is small, the peaks will be broad and far apart (like in Fig. 6.20 (a) on the lower left), if it is large, the peaks will be very sharp and close together (like in the region between the two tips in Fig. 6.20 (b)). The maximum values of $\Delta \mu_n$ can be extracted from the scanning gate images of Fig.

![Figure 6.21](image.png)

**Figure 6.21:** (a) Schematic drawing of the tip induced potential. $\Delta E$ is the spacing or the width of the Coulomb peaks, while $\Delta x$ is the distance necessary to go from one peak to the next one or through one peak. The regions at the tail and on top of the potential (small $\Delta \mu_n$) as well as on the steep part (high gradient) are indicated in the scanning gate image (b).
In the region between the two sets of ring-shaped conductance the gradient reaches 20 \( \mu \text{eV/nm} \).

### 6.4.3 Simulations of the tip bias dependence

The relation between the chemical potential inside a quantum dot and the voltages on the gates is given by eq. (6.1). In this model, the third and fourth term of eq. (6.1) will be neglected, leading to a chemical potential, which depends only on the voltage applied on the gates, the voltage applied on the tip and the position dependent lever arm of the tip. The changes in lever arm of the other gates due to the movement of the tip are also neglected. Thus the expression becomes

\[ \Delta E = \langle \alpha_{\text{tip}}(\vec{r}, \vec{r}_{\text{tip}}) \rangle \Delta V_{\text{tip}} + \langle \alpha_{\text{gate}}(\vec{r}, \vec{r}_{\text{tip}}) \rangle \Delta V_{\text{gate}}, \]  

which, for the case of the double tip observed in the experiment, and for a single gate, can be written as

\[ \Delta E = \alpha_{\text{pg}} V_{\text{pg}} + \alpha_{\text{tip},1}(\vec{r}_{\text{tip}}) (V_{\text{tip}} - V_{\text{contact},1}) + \alpha_{\text{tip},2}(\vec{r}_{\text{tip}}) (V_{\text{tip}} - V_{\text{contact},2}). \]  

The term \( (V_{\text{tip}} - V_{\text{contact},i}) \) is the effective voltage \( V_{\text{eff}}^{\text{tip},i} \) on the tip with respect to the sample, when the contact potential difference is subtracted. The lever arm of the tip \( \alpha_{\text{tip},i}(\vec{r}_{\text{tip}}) \) is approximated by a Lorentzian curve \(^{202}\) of the form

\[ \alpha_{\text{tip},i} = \frac{w_i^2}{w_i^2 + (r - r_i)^2}, \]  

where \( w_i \) is the full width at half maximum. The conductance through the quantum dot can now be written as the sum of these terms:

\[ G(r_{\text{tip}}) = G \left( V_{\text{pg}} + V_{\text{tip},1}^{\text{eff}} \alpha_{\text{tip},1}(\vec{r}_{\text{tip}}) + V_{\text{tip},2}^{\text{eff}} \alpha_{\text{tip},2}(\vec{r}_{\text{tip}}) \right). \]  

Using eq. (6.12), it is now possible to simulate the scanning gate experiment, if the conductance of the quantum dot as a function of \( V_{\text{pg}} \) is known. Figure 6.22 shows the result of such a simulation, where the measured contact potential voltages of -100 mV and 560 mV have been inserted. The tip separation is 750 nm, while the widths of the Lorentzian functions, used for the simulation, are \( w_1 = 2 \mu \text{m} \) and \( w_2 = 1.5 \mu \text{m} \). The image is obtained by shifting the voltage \( V_{\text{pg}} \) by the local potential of the tip, and by inserting the conductance from the gate sweep, displayed in Fig. 6.23, at the voltage obtained after the shift. The potentials, displayed in Fig. 6.22 (a), correspond to the potentials displayed in Fig. 6.20 here for different voltages and with inverted sign. The potential surfaces look very similar, and mainly differ in the symmetry and in some details. The simulation leads to a symmetric potential, since the Lorentzian is symmetric around the axis perpendicular to the plane of the 2DEG, while the actual tip is rather non-symmetric, due to the etching procedure. This difference is visible in the simulated scanning gate images in Fig 6.22 (b), where
Figure 6.22: (a) Simulated tip potential for an applied voltage of 100 and 300 mV. (b) Simulated scanning gate measurements for the voltage range of Fig. 6.19.

the detailed shape of the ring-shaped conductance resonances is different compared to the measurement shown in Fig. 6.19. In order to understand the differences in terms of the conductance through the dot, eq. (2.51) can be rewritten using eq. (6.7) as

\[
G(\vec{r}_{\text{tip}}) = G_{\text{max}}(\vec{r}_{\text{tip}}) \cosh^{-2} \left( \frac{\mu_n(\vec{r}_{\text{tip}})}{2k_BT} \right).
\] (6.13)

In the simulation \(\mu_n(\vec{r}_{\text{tip}})\) is determined by the Lorentzian approximation of the tip-induced potential. By using the potentials reconstructed in Fig. 6.20 the simulation would better match the measurements, since those potentials are determined not only by the exact shape of the tip, but also by local potential changes in the sample (local charges, oxide lines, responsible for the last term in eq. (6.1)), and thus include all contributions, which change the electrochemical potential in the quantum dot. In the simulation, the amplitude of a specific conductance resonance is constant for all positions, while the experiment shows a varying amplitude. This is due to the approximation made for \(G_{\text{max}}(\vec{r}_{\text{tip}})\), which it is taken as a tip-position independent constant, determined for one tip position during a sweep of the voltage \(V_{pg}\). Moreover, the contact potential difference may vary with tip position, an effect, which has not been accounted for in the simulation, leading to different states of the dot. At 0.7 V for example, the simulation shows the influence of two attractive tips (the ring-shaped resonances deform towards the center of the lower tip), while in the measurement the lower tip seems to be still repulsive. Despite these simplifications, the simulated scanning gate images show, how well the experiment can be simulated even in the presence of a complicated tip-induced potential.
6.4.4 Looking inside the quantum dot

The absence of a region of increased conductance in the center of the dot allowed to take a closer look at the interior of the dot. The aim of such measurements would be to directly image the probability density related to the wavefunction of the electrons in the dot. This could not be performed with the actual setup, since particles on the surface of the sample did not allow to approach it closer than 130 nm to the surface, which is a large number compared to the Fermi wavelength. In addition the tip was not sharp enough.

By reducing the scan range to 300 nm by 300 nm (thus considering only the interaction with one of the two tips), allowing an electrical resolution of 3 nm, it is still possible to recognize substructure in the scanning gate images. One of the requirements is to keep the operating regime of the dot stable, without destroying Coulomb blockade due to the presence of the tip, while the tip is kept at a constant position over the dot. This was achieved in the experiment as shown in Fig. 6.23 (a), where conductance resonances are broadened only by temperature. The conductance trace was recorded with the tip placed on top of the dot at a distance of 130 nm from the surface and with a bias voltage of 205 mV.

\[\text{Figure 6.23: (a) Conductance resonances measured with the SFM tip on top of the quantum dot at a distance of 200 nm from the surface, with a bias voltage of 205 mV. (b) The scanning range used for the experiments covers almost the whole area of the dot interior.}\]

Figure 6.23 (b) shows the area of the dot which has been scanned in the experiments, covering most of the interior of the quantum dot. The actual electronic size of the dot is smaller than its geometric size, due to the depletion length of the oxide lines. The resolution of the measurement is 3.37 nm per pixel, a number much smaller than the Fermi wavelength (about 30 nm). Two kinds of measurements were performed: a first series, varying the tip voltages from 85 mV to 290 mV in steps of 5 mV, and a second series, in which \(V_{pg}\) was changed between -138 mV and
-121 mV in steps of 0.2 mV. The complete results for the second series are shown in Appendix B. Part of the measurements of the first series are shown in Fig. 6.24 for tip bias voltages between 90 and 200 mV.

**Figure 6.24:** Scanning gate images (300×300nm) of the central dot region for different tip bias voltages. The ring-shaped conductance resonances expand continuously when the tip bias increases.

The measurement covers an energy range of about one and a half conductance peaks, as can be seen by comparing the images: The measurement at 90 mV can be compared to the one at 160 mV, the same for higher voltages. Therefore, it is necessary to change the tip voltage by 70 mV, in order to add or remove one electron from the dot. Three different states of the quantum dot can be observed: at 90 mV and 160 mV a small spot of increased conductance appears in the center, with a size (100 by 50 nm) much smaller than the estimated electronic size of the dot. By increasing the tip voltage, the spot expands to a plateaus region of about the size of the dot (100 and 170 mV), before a conductance decrease starts to appear in the center, and the plateau transforms itself into a ring-shaped structure (120 and 190 mV). The final state is that of a larger ring-shaped conductance resonance and no current in the center (130-150 mV and 200 mV). Similar results have been published in [153].

All measurements were performed by scanning the area influenced by the tip in the attractive regime, which means that at lower tip bias voltage the tip is closer to the least invasive voltage. This is reflected by the fact that the size of the conductance increase in the dot center is smaller for higher than for lower voltages. It would be expected that the conductance resonance in the dot center becomes as large as the electronic size of the dot at the least invasive voltage. This could
not be verified, since the influence of the repulsive tip became too strong at negative voltages. Measurements at higher voltages (not shown) show an even smaller conductance increase region in the center.

The increasing influence of the repulsive potential of the other tip is the reason for the shift in position (to the lower right) of the conductance increase. Apart from a decrease of conductance at the lower right of the conductance increase plateau for $V_{\text{tip}} = 90$ and 100 mV (dashed ellipse), no additional substructure can be resolved within this region. The reason for the conductance decrease can be attributed to a parametric charge rearrangement in the sample close to the quantum dot.

The main reason for the lack of substructure on the conductance plateau is the tip to sample separation of 130 nm, which is close to the electronic size of the dot. At such a distance, even with a sharper tip no finer features could be resolved.

The interesting quantity is the change in local electrochemical potential inside the quantum dot region when the tip is scanned on top of it. In contrast to the measurement of section 6.4.2, where the gradient $\nabla \mu_n(\vec{r})$ is determining the peak spacing and width, and the influence of the source and drain QPCs cannot be neglected, here the gradient is always relatively small, and one is interested in small fluctuations of $\mu_n(\vec{r})$. By inverting eq. (6.13), one can get the local electrochemical potential $\mu_n(\vec{r})$ as a function of $G$ and $G_{\text{max}}$, which is

$$\mu_n(\vec{r}) = \pm 2k_B T \arccosh \left( \sqrt{\frac{G_{\text{max}}(\vec{r})}{G(\vec{r})}} \right), \quad (6.14)$$

where the two signs correspond to the two branches of arccosh. $G_{\text{max}}(\vec{r})$ has been obtained from the experiments by fitting a plane, defined by three points placed on a ring shaped conductance resonance. For the tip bias voltage dependent measurements, the image for a tip voltage of 110 mV was chosen (see Fig. 6.24), since there a ring-shaped resonance is fully developed. This method is a first order approximation, but allows to effectively reduce the influence of the $G_{\text{max}}(\vec{r})$ term. Using eq. (6.14), it is possible to plot $\mu_n(\vec{r})$ as a function of tip position for the measurement of Fig. 6.24 as displayed in Fig. 6.25.

The electrochemical potential is low when the conductance through the quantum dot is high, forming a low potential flat plateau at tip bias voltages of 100 and 170 mV. The flatness of this region indicates that the tip is sharp enough to probe the interior of the dot, otherwise the dot would be probing the tip. Due to the nonlinearity of $\cosh^{-1}$, small changes in the conductance can lead to a large change in electrochemical potential. This can be seen at 150 mV, where the potential decreases to about half its value, while the conductance increases only by a small amount, compared to the maximum conductance.

The resolution, which can be achieved by such a measurement, depends on the size of the tip, but this does not mean that a large tip will not lead to conductance features smaller than the size of the dot. From eq. (2.51), one can see that, as long as the interaction potential $\mu_n(\vec{r})$ is changing (which means that $\Delta \mu$ can become
zero), a conductance peak can appear. As long as the tip induced potential has any curvature, changes of the conductance appear in the scanning gate images, even if their size is smaller than the electronic size of the dot. However, the disadvantage of a flat tip-induced potential is that small fluctuations may wash out due to the insufficient sharpness.

More details can be extracted by comparing profiles of the electrochemical potential for different tip bias voltages, as displayed in Fig. 6.26. The four curves correspond to tip voltages between 85 and 100 mV and show very similar behavior. The electrochemical potential first decreases stays at an almost constant value and

![Figure 6.25: Local electrochemical potential obtained from the data of Fig. 6.23](image)

![Figure 6.26: Local electrochemical potential profiles taken from the scanning gate images of Fig. 6.25 for the indicated tip voltages. (a) shows the conductance used for extracting the term $G_{\text{max}}(\vec{r})$.](image)
then increases again when the tip moves away from the dot. The flat region in the center is evidence that the tip is probing the dot, and that the tip-induced potential in the plane of the dot is smaller than the extension of the plateau, which is approximately 80 nm for the curves in Fig. 6.26 (c). In the perpendicular direction (i) the plateau is washed out and is broader (about 150 to 200 nm). This reflects that the dot is more elongated in this direction, and that the approximation for $G_{\text{max}}(\vec{r})$ is not as accurate as for the perpendicular direction. The charge rearrangement, which is marked by the dashed line A in Fig. 6.26 (a), is visible as a kink in the electrochemical potential in both profiles. The feature B is related to the ring-shaped resonance, which encloses the central high conductance region. The value of the electrochemical potential for that feature is completely independent of the tip bias voltage. This is not the case for the other curves, which are equally spaced in electrochemical potential. By increasing the tip voltage above 100 mV, the central plateau region splits into two valleys, while the electrochemical potential raises in the center of the former plateau. As soon as this is the case, the two minima in electrochemical potential do not depend on the bias voltage of the tip, as it is the case for feature B.

The second series of measurements where the plunger gate voltage $V_{\text{pg}}$ was varied gives similar results, but the higher energy resolution allows to look at the plateau region in more detail. Two $V_{\text{pg}}$ ranges were selected, one from -139 to -138 mV (before the onset of the first Coulomb peak, see appendix B), the other from -128 to -127 mV, where the maximum conductance is 2.5 times larger. The voltage ranges were selected in order to consider the evolution from a single conductance increase spot to the onset of a ring-shaped conductance increase. The tip was scanned 200 nm over the surface of the sample with a bias voltage of +100 mV. The results are displayed in Fig. 6.27 along with a sample scanning electrochemical potential image taken at $V_{\text{pg}} = -138$ mV. The upper row of Fig. 6.27 is for the profile along the line (i), which has a smaller angle with respect to the direction of the current through the dot. The profiles show a similar behavior to those dependent on tip bias, but don’t seem to be exactly equally spaced. The curves are equally spaced in $V_{\text{pg}}$ for the left and right regions, but tend to flatten out in the central region. This is visible in Fig. 6.27 (b) for profile (i), where the curves merge at the bottom of the plateau. The general behavior can be explained considering that the central conductance increase develops to a ring-shaped resonance, leading to two valleys with constant electrochemical potential. The slope of the central region can be attributed to a gradient in $G_{\text{max}}$ between source and drain inside the dot, and it is less steep in (c), where a higher number of electrons is in the dot. This indicates that, if a larger number of electrons is on the dot, it becomes less sensitive to the tunnel couplings to source and drain, contained in the term $G_{\text{max}}$ of eq. (6.14). The step, indicated by A in Fig. 6.27 (c), can be attributed to a parametric charge rearrangement, as for the tip bias dependent measurements.

The lower row (ii) shows first a dip (B) in the electrochemical potential, then a steep decrease, a plateau-like region and a almost linear increase on the right.
Chapter 6. Single electron manipulation in a quantum dot

Figure 6.27: (a) Local electrochemical potential obtained from the $V_{pg}$ dependent measurements ($V_{pg} = -138$ mV). The lines indicate where the contour plots of (b) and (c) have been taken. (b) Contours as indicated in (a) for $V_{pg}$ between -139 mV and -138 mV. (c) Same plot, but for $V_{pg}$ from -128 to -127 mV. A parametric charge rearrangement can be observed as a step at the point A, while the reduction in $\mu_n$ at the point B is less tip bias voltage dependent.

The dip marked by B changes only little with $V_{pg}$ and belongs to the ring-shaped conductance increase around the center of the dot. The small change indicates that the gate is mostly effective only on the electron with the highest energy, which is the one related to the central feature. The central plateau-like region shows almost no slope (which is logical since the profile is taken almost perpendicular to the source-drain direction) and is not as well defined, as for the tip bias dependent measurements. The charge rearrangement observed in the upper row, indicated by A, is visible also in this profile.

The lack of substructure in the central region, which would be of interest for wavefunction mapping, is an indication for two main limitations of the experiment: The tip to sample distance is too large, and the number of electrons on the dot is also too large. The last point can be understood with the simple model of a particle in a box: The higher the energy, the more the wavefunction becomes extended over the whole area, and the probability density is high also at the borders of the box.
6.4.5 QPC conductance and wavefunction mapping

If the consequences of the source and drain QPCs on the dot conductance would be exactly known, it would be easier to extract the local electrochemical potential much more exactly than presented in section 6.4.4. The changes in the conductance of a QPC caused by the scanning tip are due to a change in tunnel coupling of the barrier. The function $G_{\text{max}}$, which describes the amplitude of the conductance oscillations (see eq. 6.13), depends only on the tunnel coupling of the source and drain barriers:

$$G_{\text{max}}(\vec{r}_{\text{tip}}) = \left( \frac{\Gamma_S(\vec{r}_{\text{tip}}) + \Gamma_D(\vec{r}_{\text{tip}})}{\Gamma_S(\vec{r}_{\text{tip}})\Gamma_D(\vec{r}_{\text{tip}})} \right) \frac{e^2}{4k_BT\hbar}. \quad (6.15)$$

So, if the tip-position dependent couplings are known, it could be possible to determine the position dependent function $G_{\text{max}}(\vec{r}_{\text{tip}})$. By dividing the measured conductance of the quantum dot by $G_{\text{max}}(\vec{r}_{\text{tip}})$, one can, at least partially, exclude the influence of the source and drain QPCs, and get a step closer to a detailed insight into the probability distribution of a quantum dot. The problem is that $\Gamma_S(\vec{r}_{\text{tip}})$ and $\Gamma_D(\vec{r}_{\text{tip}})$ have to be determined independently in a regime, where the dot will be measured later on. However, by doing so, their relation to $G_{\text{max}}$ will be changed, since the energy level formation in a quantum dot is relevant for $G_{\text{max}}$. An approximated form for $G_{\text{max}}(\vec{r}_{\text{tip}})$ could be obtained with a three-terminal quantum dot, where the complete conductance matrix $G$ can be measured. The three diagonal components of $G_{\text{max}}$ are given by [192]

$$G_{ii} = -\frac{e^2}{4k_BT} \frac{\Gamma_i(\Gamma_j + \Gamma_k)}{\Gamma_1 + \Gamma_2 + \Gamma_3}, \quad \text{for } i \neq j, i \neq k, j \neq k, \quad (6.16)$$

while the off-diagonal elements are

$$G_{ij} = \frac{e^2}{4k_BT} \frac{\Gamma_i\Gamma_j}{\Gamma_1 + \Gamma_2 + \Gamma_3}, \quad \text{for } i \neq j. \quad (6.17)$$

Thus, by measuring all the elements of the conductance matrix, the individual tunneling rates can be determined. However, this is a time consuming experiment, since the matrix elements have to be determined for every position of the tip in scanning gate images.

6.4.6 Magnetic field

The behavior of quantum dot states in parallel and perpendicular magnetic fields has been studied in many experiments, but no information about local effects is available. The main reason is purely technical: There are not many low temperature local probe measurement setups which allow to apply a magnetic field, and it is almost impossible, without a complete redesign, to apply a parallel field to the sample. But exactly a parallel field would be needed to observe changes in the conductance peak spacing and amplitude, as well as a diamagnetic shift. Moreover, in order to tune
the sample in a suitable regime, back- and top-gated structures would be the best choice, but a top gate does not allow to perform local probe measurements. Since fluctuations in the peak spacing can be observed in a perpendicular magnetic field (although not in the weak coupling regime, where the sample is operated for these measurements), it is of interest to perform measurements in magnetic field. However, magnetic field measurements can be critical, since the ferromagnetic properties of the piezoelectric tuning fork can lead to an unpredicted behavior.

The tip to sample separation dependence of the sensor used for this series of measurements on the magnetic field, was found to be very strong, allowing to reach only a field of 0.167 T, without using the z-motor. This magnetic field range is insufficient for observing fluctuations, at least if they are related to the Zeeman term $g \mu_B B$. The changes in tip-to sample separation due to magnetic field are the main limitation for the measurements, as shown in Fig. 6.28. Figure 6.28

\[ \text{Figure 6.28: (a) Change of the position of the Coulomb blockade peaks for different tip to sample separations. (b) Same measurement for a constant tip to sample separation and different magnetic fields. The sudden change on the left is due to an overnight break in the measurement, but no other changes in the position of the conductance peaks can be observed. (c) Same measurement as in (b), without keeping the tip to sample distance constant. The changes in position of the conductance peaks are due to the change in tip to sample separation.} \]

(a) shows a measurement at zero field, where the tip is scanned along a line close to the dot for different tip-to sample distances, over a range of 240 nm. Just by changing the distance by this amount, four electrons can be added to the quantum dot, as visible from the number of peaks appearing in Fig. 6.28(a). The z-position dependence becomes extreme when the magnetic field is changed, as shown in (b), where the tip is kept at a fixed position. By increasing the magnetic field, the tip gains more distance to the sample (less conductance peaks) up to a field of 70 mT and then approaches rapidly. This behavior is confirmed by tip to sample separation measurements for different magnetic fields. If the separation is kept constant (which means readjusting it for every measurement), the number of electrons on the dot, or the position of the conductance peaks, doesn’t change at all, as shown in (c). The abrupt change in the conductance is due to a break in the measurement, and is not
related to the magnetic field.

Scanning gate images for different magnetic fields are displayed in Fig. 6.29. Between the zero field image and the magnetic field measurements, the tip got into contact with the surface of the sample, modifying its electrical properties. Therefore, it can be used only as a reference for the position of the center of the dot, and not be compared to the magnetic field measurements. The tip to sample separation is in all images 350 nm, and all voltages are the same for the four images. Changing the magnetic field seems to have no effect on the Coulomb blockade (small differences can be attributed to an uncertainty in tip to sample separation, caused by the instability of the feedback), although the coupling to source and drain, which manifests itself in regions of high and low amplitude conductance peaks, is affected by the magnetic field.

6.5 Conclusions

The experiments presented in this chapter demonstrate the ability to manipulate single electrons in tunable semiconductor quantum dots, with full control of the electrostatic interactions between the quantum dot and the scanning tip. The interaction potentials can be mapped out quantitatively, both in their spatial extent and in their coupling amplitude, allowing to tune the system in a well defines state.
Moreover, the electrostatic interactions can be simulated, if the conductance through the quantum dot as a function of one of the gates is known, allowing to explain the purely electrostatic interactions. The ability to map out the local electrostatic potential inside the quantum dot is a first step towards imaging of the quantum mechanical probability distribution, although some difficulties encountered in the experiment have to be solved first. The experiment has been pushed to the limits of the measurement setup, demonstrating that the stability of the sample and the electrical resolution would fulfill the requirements for sensitive measurements in the quantum dot interior. However, in order to achieve this, three basic requirements have still to be met:

- The tip to sample separation has to be reduced to less than 30 nm. This requires extremely clean samples, and probably additional treatment of the samples in order to etch away the oxide lines.

- The number of electrons has to be lowered in order to allow imaging of the first excited states, where the probability distribution is less flat in space. This requires quantum dots of smaller size, and would at the same time increase the single particle level spacing.

- The electron temperature has to be lowered in order to reach the single-level transport regime, requiring a SFM operated in a dilution refrigerator

The experiments discussed are a step towards an imaging technique, which can allow to image and design quantum states in mesoscopic systems, a technique, which cannot be found already well described in the literature, and which still needs to be developed.
Chapter 7

Scattering at a quantum point contact

7.1 Introduction

A quantum point contact (QPC) is the simplest nanostructure which can be engineered, since it consists only of a constriction which shows conductance quantization. The opening of this constriction can be tuned either by lateral topgates, as it is the case for electron beam defined QPCs, or by lateral in-plane gates defined in the 2DEG, like it is the case here. QPCs are interesting samples for local probe techniques for two main reasons: On one hand they allow to study the electrons flowing through the constrictions \cite{142, 148} or the effect of local charges in its proximity \cite{143}, on the other hand they are a key component of a quantum dot. The experiments discussed in this chapter can be divided into two main parts: The first one is devoted to the electrostatic interactions between the scanning tip and the QPC. This becomes very important if one wants to understand the changes in maximum conductance of the conductance oscillations in a quantum dot, which depend on the tunnel coupling of the source and drain QPCs. The second part relates the global transport behavior of the QPC to the local charge distribution around the constriction, using the QPC as a sensitive charge detector.

7.2 Sample and measurement setup

The sample used for these experiments has already been described in chapter \cite{6}. The QPC used is the one placed about 1 \textmu m away from the quantum dot on the upper right in Fig. \cite{6.9}. Its large separation from the quantum dot does not allow to perform simultaneous Coulomb blockade and charge detection measurements. However, the analysis of the QPC data allows to understand transport through the dot better. The measurement setup is a standard AC setup with lock-in technique, where a voltage of 100 \textmu V is applied between the source and drain contacts of the
QPC, and the current is measured in a two-terminal setup. The disadvantage of the two point measurement is that the contact resistances are added to the resistance of the sample. This effect can be accounted for in the data analysis, since the conductance on a quantized plateau is known to be a multiple of $2e^2/h$. The contact resistance can be estimated from the conductance of the first plateau by using $G_1 = \frac{1}{R_{\text{contact}} + h/2e^2}$. In the measurements the first plateau appears at $1.55 \frac{e^2}{h}$, see Fig. 7.1, leading to a contact resistance of 3.6 kΩ. Figure 7.1 also shows a typical scanning gate image, which can be used as a reference for positioning later images with respect to the oxide lines defining the QPC.

The oscillations in the conductance, appearing on what should be a quantized plateau, can be attributed to transmission resonances (see e.g. [193, 194, 195] for examples and reviews). They appear as soon as impurities or the confinement potential of the constriction induce a partial multiple reflection of the transmitted amplitude. The result is that the transmission coefficient for a mode can be smaller than 1, even for an energy higher than the quantization energy of that mode. Already the simplest model for the problem, a quadratic 1D potential step, leads to an oscillatory behavior similar to that observed in the measurements (see for example [196]). The conductance through the QPC was tuned by applying a constant voltage on the gate pg and the region of the quantum dot. In order to reduce the asymmetry in the sample, the region labeled qpc1 (see Fig. 6.9) is at the same electrostatic potential of the QPC being measured.
7.3 Gate and tip bias dependence of the QPC conductance

A QPC is a much simpler nanostructure compared to a quantum dot, and the only parameters which can be tuned in our measurement setup are the gate and tip bias voltage. Figure 7.2 (a) shows the conductance of the QPC as a function of gate voltage for tip bias voltages of 300 and 1100 mV, when the tip is placed over the constriction at a z distance of 200 nm. An increase in tip voltage mainly shifts the whole curve to the left, with some little changes on the conductance oscillations on the plateaus. The scanning gate images (b) show much richer features in the conductance, which can be related to different effects.

\[
\text{Conductance } [e^2/h] \\
\text{Log conductance} \\
\text{Current } [nA]
\]

\[
V_{\text{tip}} = 300 \text{ mV} \\
V_{\text{tip}} = 1100 \text{ mV}
\]

\[
V_{\text{pg}} = 140 \text{ mV} \\
V_{\text{pg}} = 255 \text{ mV} \\
V_{\text{pg}} = 370 \text{ mV} \\
V_{\text{pg}} = 475 \text{ mV}
\]

Figure 7.2: (a) Conductance through the QPC as a function of $V_{\text{pg}}$ for two different tip bias voltages. (b) Scanning gate images taken at $V_{\text{pg}}$ indicated by the red lines in (a), with a tip bias voltage of 300 mV. The gradient of the measured data is plotted on a logarithmic scale in (c). Details are discussed in the text.

The measurement at 140 mV tip bias voltage shows two regions, one of reduced and one of enhanced conductance, which can be related to the two tips. The details of these regions can be understood by looking at the conductance for different gate voltages in (a): If the voltage is lowered, the conductance will drop continuously to zero. While if the voltage is enhanced, it will first decrease, then increase to the
plateau value and at the end oscillate. The low conductance region in the scanning gate image corresponds to the drop in conductance for lower voltages than 140 mV on the $V_{pg}$ scale. The decrease for voltages slightly higher than 140 mV is visible as a faint halo on the left of the region between high and low conductance. The region of high conductance reflects the conductance oscillations on the first plateau, visible as concentric red and yellow stripes. The shape of this region is closely related to the shape of the tip-induced potential, and roughly corresponds to the shape of the conductance resonances observed on the quantum dot.

The measurement at 370 mV is very similar, but since the operating voltage is at the beginning of the conductance increase, the regions of reduced conductance are more important here. The scanning gate measurements on the first (255 mV) and second (475 mV) plateau show smaller changes in the conductance, since by shifting the conductance curve to the left or to the right does not lead to substantial changes in the conductance (other than those related to the conductance oscillations). Here also the features in the scanning gate image can be related to single conductance oscillations in the $V_{pg}$-dependent trace, but the more interesting point are ring-shaped steps in the conductance indicated by the arrows. These closed lines have the same shape as the regions of enhanced conductance (related to the tip in the attractive regime) and basically do not change at different gate voltages. By plotting the gradient of the measured data on a logarithmic scale (Fig. 7.2 (c)), these features become better recognizable. One can see that they appear in all the images at fixed positions, and all have the same shape.

The main changes in the conductance can be understood by making the same kind of simulations already done for the quantum dot (see section 6.4.3), with the difference that the conductance trace used here is the one of Fig. 7.2 (a). The optimal contact potential differences are also slightly different: it turns out that 620 mV and -400 mV are ideal, the width $w_i$ of the tip potentials has been determined to be 2 $\mu$m and 1.5 $\mu$m respectively. The results of this analysis for the gate voltages $V_{pg}$ of the scanning gate images of Fig. 7.2 are shown in Fig. 7.3.

The simulated measurements show the basic features observed in the real measurement, although most of the details do not match. This is partially due to the choice of the parameters of the simulation, which may not correspond exactly to the situation during the experiment. On the other hand, the simulation takes into account only the effects arising from a shift in energy of the QPC due to the change in potential caused by the scanning tip, without considering the geometry of the QPC or the effect of local potential fluctuations. These effects will be discussed in the next sections. Nevertheless, the simulation allows to exclude other effects: In the measurement at $V_{pg} = 140$ mV a region of low conductance, which may look like a backscattering effect due to an enhanced electron flow in that region, can be related to a low conductance region in the simulation. This effect is due to the transmission resonance on the QPC conductance curve, which causes a lower conductance, if the gate voltage is raised slightly over 140 mV, before the conductance increases to the value corresponding to the first quantized plateau. With this argu-
7.3. Gate and tip bias dependence of the QPC conductance

Figure 7.3: Simulated scanning gate images for the voltages \( V_{pg} \) of Fig. 7.2.

ment, most of the features, which are centered around the maxima and minima of the conductance, corresponding to the repulsive and attractive tip positioned over the QPC constriction, can be understood as a “gating” effect of the tip. Since transmission resonances are suppressed in magnetic field \[193\], scanning gate images in a magnetic field should prove this assumption.

7.3.1 Suppression of resonances in a weak magnetic field

The scanning gate images shown in Fig. 7.4 were taken at magnetic fields between -200 and +200 mT, with a gate voltage \( V_{pg} \) of 140 mV and a tip bias of 1.1 V. They are centered over the region of reduced conductance on the left of the QPC. The dashed circle in the logarithmic plot has a radius corresponding to the cyclotron radius \( r_c = \frac{\hbar k_F}{eB} \), while the white circle has a diameter corresponding to the magnetic length \( \ell_B = \sqrt{\frac{\hbar}{eB}} \), which is the extent of the lowest Landau level state. At 50 mT the \( r_c \) is 2.3 \( \mu \)m, exceeding the scanning range, while at 200 mT it is 580 nm. The dashed ellipses fit to the conductance steps, which appear close to the QPC and have a shape similar to that of the attractive tip. Almost all these features are centered inside the radius of \( r_c \), and are suppressed, if the magnetic field is increased. This can be observed at \( \pm 200 \) mT, where most of them disappear completely outside the region marked by the dashed circle. Their suppression means that less electrons are backscattered into the QPC, and it is a purely classical effect, since the field
Figure 7.4: Scanning gate images on the left of the QPC for magnetic fields between -200 and 200 mT. The lower row contains logarithmic plots of the gradient of the conductance. The different features are discussed in the text.

\( \phi_0 \), which would suppress coherent backscattering (which leads to weak localization [197, 198, 199, 200]), is much smaller than the fields applied, for \( l_e = 5.25 \, \mu m \) as in our sample.

Also the features marked with A and B become very weak at 200 mT, indicating that they are related to conductance resonances. The remarkable observation is that all the ring-shaped conductance steps observed up to now appear at the same position. This may indicate that they are related to fixed charges in the sample and not directly to the QPC. In order to verify this, linescans for different tip bias voltages close to the QPC were employed. They give more precise information about tip-position dependent small conductance changes, as will be shown in the next section.

7.4 Conductance changes due to change in backscattering rate

The electrostatic interaction between the scanning tip and the QPC has been mapped out in the scanning gate images, and simulations have confirmed the experimental observations. However, like in the Hall-bar measurements, the relation between conventional transport measurements and local properties of the sample has not yet been found. A first indication that the conductance of the QPC may be influenced by local charges placed in proximity of the constriction, was given in the previous section. On the other hand, simulations of a QPC in the presence of a scanning probe tip [201] indicate that the presence of the tip alone is already able to change significantly electrical transport. The measurements discussed here allow to correlate resonances observed on the conductance curve of the QPC as a function of the gate voltage with local backscattering.

The tip is scanned along several lines indicated in Fig. 7.5 (a) for different gate
7.4. Conductance changes due to change in backscattering rate

Figure 7.5: Linescan performed on the lines indicated in the scanning gate image on top. The lower rows of images allow to better recognize the typical QPC conductance curve.

voltages $V_{pg}$. The measurement can be better understood, if one thinks about placing the tip at different positions on the lines indicated and changing the gate voltage for every position. It means measuring the conductance of the QPC with a perturbation potential placed at different positions close to the constriction. If changes in the conductance are observed, which do not depend on the purely electrostatic gating effect, they have to be related to the local perturbation.

The images 7.5 (b) to (g) show the result of the measurements. The curve marked
by the solid line i) in Fig. 7.5 (b) corresponds to the gating curve of the tip. This is
the case, since the onset of the first conductance increase before the first and second
plateaus follows this curve, which can be related to a lever arm of the tip normalized
to 1. If the curve is shifted to different $V_{pg}$’s, it still follows the contour lines of the
measured data.

But also other resonances appear, such as curve ii) in Fig. 7.5 (b), and in that
case the effect of the tip is different: Although the shape of these resonances in the
tip-position-$V_{pg}$ plane is still the same, the lever arm of the tip is much larger. If for
the gating curves the tip is able to induce a change of 50 mV in gate voltage, for the
other ones (dotted lines) the shift reaches 200 mV. This means that by measuring
the conductance of the QPC at different tip positions, certain resonances will change
their position in $V_{pg}$ faster than others. For line (d) the change becomes very steep,
as marked by the dotted lines iii). In image (e) the energy of the QPC is raised,
if the tip is placed at certain positions, as indicated by the dotted lines iv). Since
the tip interacts directly with the QPC only over its gating effect, the additional
resonances have to be attributed to something not directly related to the QPC.

The number of these features increases if the tip is close to the constriction: In
all images their number is maximized in the region where the energy increase is
maximized, and the absolute number is also the largest for images (d), (f), and (g).
This effect has its correspondence in the ring-shaped conductance steps observed in
the scanning gate images, which appear in large number close to the constriction,
and can be attributed to local tip-induced charge rearrangements in the vicinity
of the QPC. This will be verified in the next section, where the behavior of such
ring-shaped structures is studied in more detail.

### 7.5 Local charging

Local charges have been involved as an explanation of ring-shaped concentric con-
ductance oscillations already in the experiments on the QHE (chapter 5). In these
measurements several of such features appear in the scanning gate images, with the
important difference that the conductance does not oscillate, but shows single steps,
leading to disks of enhanced conductance. The difference to the QHE observations
of section 5.6.5 can be understood in terms of the percolation model of section 5.6.4.
For a Hall bar there is a high probability to directly measure sequential tunneling
events like the charging of a quantum dot, when the sample is in the regime of the
percolating network. This is the case, since the percolating current is the measure-
ment current, and it may oscillate, if quantum dots form on its path. In the case of
the QPC, the area where a current can flow is not limited to a certain region (like
the edges of a Hall bar), therefore such events would not influence the conductance
of the QPC. On the other hand, a QPC is a sensitive charge detector, and charge
detection measurements show a step in the conductance every time the number of
electrons in an adjacent quantum dot is changed by one. If the scanning tip is able
to put electrons in empty states related to local impurities, the QPC should be able to detect them, as long as they are close enough to the constriction.

The fact that the shape of such conductance step disks corresponds to that of the attractive tip, indicates that the energy of these states is slightly above the Fermi energy in the 2DEG. Another condition is that, if the steps are related to local quantum dots, the size of the disk should change with tip voltage in a similar way like in the quantum dot measurements. This is verified by performing several scanning gate measurements close to the constriction for different tip bias voltages. Since most of the details are visible only by taking the gradient of the measured data, the scanning gate images were taken with a transconductance measurement setup, where a small AC amplitude is added to the DC bias voltage of the tip, and the current of the QPC is measured at the frequency of the AC amplitude. The measured quantity is therefore the derivative of the QPC conductance with respect to the applied AC tip voltage.

The result is shown in Fig. 7.6. In order to obtain the response of the conductance changes with respect to the tip voltage, single lines from the scanning gate image have been put together, forming linescans for different tip voltages. The scanning gate image shows a large number of ring-shaped peaks and dips in the transconductance signal. A peak corresponds to an electron, hopping onto a local impurity state, while a dip corresponds to an electron, hopping off such a state. The rings are never closed, indicating that an electron may get off the local state (in the case of the light rings, corresponding to a conductance increase) without being detected. Scanning gate measurements have been performed for tip bias voltages between 140 and 1090 mV, with a gap between 500 and 670 mV, visible as a gray stripe in the lower images.

By taking a line for a specific \( y \)-position in every scanning gate image and putting them together, it is possible to obtain the images of the lower part of Fig. 7.6, which correspond to linescans for different tip bias voltages. With these images it is possible to track the position of the ring-shaped features and study their evolution for different voltages. A positive slope indicates that the line is moving to the right with increasing tip bias, a negative slope that is moving to the left. Lines belonging to the same ring are marked with the same color, and therefore it is possible to see that all rings enlarge, if the voltage is increased (black, blue and green marked lines). In the case of the peak marked by the black crosses, it is possible to observe that the ring shrinks in size until it disappears completely. The shape of the fitted line corresponds to an equipotential for that specific impurity state, and the minima corresponds to the voltage where the contact potential difference of the tip is compensated. The measurement is therefore equivalent to the one displayed in Fig. 6.15.

The observations lead to the conclusion that the ring-shaped features behave like conductance resonances in a quantum dot, proving that they have to be related to potential fluctuations in the vicinity of the QPC, which is able to detect tip-mediated charging of states related to these impurities. The energy of these states lies probably 100 - 300 meV above the Fermi energy. This can be extracted from
Figure 7.6: Transconductance measurements close to the QPC constriction. The image on top is a scanning gate image taken at a tip bias of 670 mV (DC). The lower figures have been obtained by putting together the lines A to F indicated of all scanning gate images in the tip bias range from 140 to 1090 mV. The tip bias voltage dependence of the single peaks (light) and dips (dark) can be tracked, and some examples are marked by the colored crosses. The gray region corresponds to a tip bias voltage range not measured.
7.5. Local charging

Fig. 7.6 by looking at the tip voltages where the slope of the lines becomes almost flat. An example is the black line for the case A, where the lines with positive and negative slope join at a tip voltage of about 150 mV. For several lines, the slope decreases at the bottom of the figures (the lower tip voltage region), indicating that a similar situation may occur at tip voltages slightly below 150 mV. Certain scanning gate images also show concentric ring-shaped features, indicating that a second electron has been charged to the impurity state. The range, where charging events are detected by the QPC, is about 800 nm, confirming the observations of section 7.3.1.

It is very difficult to find out the exact position of the impurity states as well as the origin of the electrons which are charged, since it is not possible to determine their electrostatic properties. Possible situations are discussed below [202]:

- **Surface states** could be the origin of the observed features. At the surface of the sample the electric field of the tip is much higher than below the surface, since there it is screened by the high dielectric constant of GaAs ($\epsilon = 12.9$, see appendix L). Due to the excellent reproducibility of the scanning gate images it is rather improbable that the charge states are dragged along by the tip, so they have to be bound to particles lying on the surface. These could be charged by electrons in surface states moved by the tip (which is always positively biased, thus attractive for electrons), and behave similar to self-assembled quantum dots. Since the charging energy of the impurity states could not be determined, it is impossible to estimate the size of the particles, and thus explain, why they have not been detected in a topography scan performed after the measurements. And since the number of charging events (i.e. the number of ring-shaped features) is about 50/µm², they should be detected in a topography image.

- **Surface defects**, leading to a change in the conduction band edge at the 2DEG could be another possibility. This effect is possible, since the Fermi level is pinned at the surface of GaAs, thus any change in distance between surface and 2DEG will lead to a shift in the conduction band edge with respect to the Fermi energy. However, no surface defects have been detected in the topography image, and it would be impossible that a defect not detected by the tip would be able to change the conduction band in a way that single electron charging becomes possible.

- **Donors in or close to the doping plane** are another candidate. However, the ionization energy of the Si donors in the doping plane is 120 meV, an energy scale far beyond the typical energy changes induced by the tip (typically well below 20 meV, see appendix L). If the donor atoms build DX centers, this energy can reach even higher values [203]. But, if single doping atoms diffuse into the spacer layer between 2DEG and doping plane during epitaxial growth, it may be possible that their ground state energy levels gets very close to the
electrochemical potential of the electrons in the 2DEG. This would allow tip-induced charging events, even if it is unclear, how the QPC could be able to detect them at distances up to 800 nm. This possibility is supported by the low number of charging events (about 50/µm²), since the number of donors, which diffuse into the spacer layer, is very small compared to the total number of donors per area.

Even if none of the discussed possibilities is completely convincing, the experiments do not allow to exclude completely any of these options. Considering that the surface of the sample was free of particles and defects, it seems that the most probable location for the charging centers is inside the sample, close to the 2DEG.

7.6 Conductance of the QPC and measurements on the quantum dot

The measurements on the quantum dot did not allow to determine exactly the tip-position dependent coupling to the source and drain QPCs. This term, given by eq. \[2.51\], can be understood better with the scanning gate measurements presented in this chapter, although the different geometry of the constriction leads to results which cannot be directly applied to the constrictions of the dot. A comparison between the local conductance of the QPC and the conductance through the dot is displayed in Fig. 7.7. The bordering of the conductance increase and reduction regions have been marked in (a) and superimposed to (b) and (c), where the dot conductance is displayed. The bordering does not match exactly the shape of the conductance oscillations of the dot, but the shape is very similar. The differences are due to the different geometry of the source and drain QPCs and to different parameters of the experiment. Another reason is that in a quantum dot both source and drain QPCs are influenced by the scanning tip at the same time, leading to a different interaction potential. The fit in Fig 7.7(c) suggests that the shape of the

\[\text{Figure 7.7: (a) QPC conductance for } V_{pg}=140 \text{ mV and } V_{tip}=300 \text{ mV. (b), (c) Scanning gate image on the dot. The shape of the conductance changes of the QPC marked in (a) has been drawn in the figures. In (b) (dashed line) it has been fitted to the conductance resonances, while in (c) it reflects the changes in conductance peaks height of the quantum dot measurement.} \]
increase in amplitude of the conductance resonances is related to the coupling of the left QPC, since it is similar to the feature observed on the QPC alone. Nevertheless, a good result can be obtained with the simulation method of section 7.3 as displayed in Fig. 7.8. The idea is to multiply the simulated scanning gate image of the dot with the one on the QPC, normalized to one for its maximum conductance. The image is for a tip voltage of 250 mV and gate voltage $V_{pg} = -100$ mV, and reflects very well the observed behavior. The height of the dot conductance peaks is modulated by the conductance through the QPC, leading to an oscillating behavior in the range of conductance fluctuations on the QPC conductance. This effect explains, why some scanning gate images (e.g. Fig. 6.20 in the region between the centers of the ring-shaped conductance oscillations) show sets of concentric rings, which seem to be related to additional dots: If the number of measurement points is not high enough, the superposition of two (dot and QPC) centers of ring-shaped conductance oscillations leads to a Moiré effect, which may lead to think that an additional dot is being charged.

### 7.7 Conclusions

The measurements on the QPC presented in this chapter are of interest for two main reasons: They allow to get insight into the local potential around a QPC and its
consequences for the conductance, and they can be used for a better understanding of the scanning gate experiments on a quantum dot. The conductance changes induced by the scanning tip can be understood by the gating effect of the tip, which is a convolution between the tip-induced potential and the conductance of the QPC, depending on the voltage applied to a lateral gate controlling its tunnel coupling. This can be modeled, if the conductance through the QPC is known, as shown above. Part of the transmission resonances can be related to the presence of localized potential fluctuations in the 2DEG, since they strongly depend on the position of the scanning tip. The origin of such potential fluctuations can also be observed, using the QPC as a charge detector for local charges in its close proximity. This is confirmed by the interaction between the tip and these local charges, which is similar to the one observed in the quantum dot. The information about the tip-position dependent conductance of a QPC allows to determine the form of the transmission $G_{\text{max}}(\vec{r}, \vec{r}_{\text{tip}})$ more exactly, and permits to eliminate the effect of this term in the analysis of the local electrochemical potential inside a quantum dot. This would be a step further towards true wavefunction mapping. However, these measurements would require a three-terminal quantum dot, which was not available for these experiments.
Chapter 8
Summary and outlook

In this thesis we have described scanning gate measurements on semiconductor nanostructures performed with a low-temperature scanning force microscope.

The microscope construction and operation were described, focusing on the requirements of a low temperature local probe setup realized in the actual instrument. The home built tuning fork sensors with metallic PtIr tip give a high resolution paired with the advantages of full electronic readout and low power dissipation.

Scanning gate experiments on a Hall bar in the quantum Hall regime allowed to study the quantum Hall effect in the presence of a local potential perturbation. We observe the $1/B$ periodicity of the effect in both the longitudinal and the Hall resistance, and an increase in sensitivity to the tip induced potential in the edge region of the sample for certain higher magnetic fields. The Hall resistance is unaffected by the presence of the SFM tip on quantum Hall plateaus, and shows changes only in the region between the Hall voltage probes, if the hall-resistance is between two Hall plateaus. The formation of rich structure in the bulk of the sample and the changes at the edges can be described by the interactions between a percolating network of localized states and backscattering of edge channels. The last effect can be summarized in a local backscattering parameter in the framework of the Landauer-Büttiker formalism. More measurements would be needed to investigate the observed effects further. While a detailed analysis of the magnetic field dependence has been presented in [122], there is no data available about the transition region between two Hall plateaus. High resolution measurements in that regime may show the real appearance of the proposed percolating network of localized states discussed in this thesis.

With the scanning gate measurements on a quantum dot we have demonstrated the ability to manipulate single electrons, using the electrostatic interaction between the scanning SFM tip and the quantum dot. The information about the interaction potential between dot and tip was obtained by using the quantum dot as a sensitive electrometer, with the result that a quantitative mapping of the potential becomes possible in all spatial and electrical parameters of the system. Simulations of the scanning gate measurements model the experimental data, and allow to separate
purely electrostatic interactions from effects arising from the change in coupling to the leads and from local potential fluctuations. Focusing on the interior of the dot we have observed a flat response in the electrochemical potential, with small oscillations on top of it. Although, for the reasons discussed in the text, it is not possible to achieve a higher resolution, these measurements give a first impression of the details in the electrochemical potential distribution and a further proof that the measurement technique allows the necessary resolution. These measurements can be considered as a first step towards the mapping of the quantum mechanical probability distribution inside a quantum dot, which is one of the main goals of scanning probe measurements on semiconductor nanostructures.

Measurements on a QPC reveal that the main conductance changes are related to electrostatic interaction. In addition we can visualize, how the transport properties of a QPC are influenced by the local impurities in its vicinity by means of local charging induced by the scanning tip. The QPC is used as a sensitive detector for charge induced on impurity sites by the presence of the tip.

The number of experiments, which could be performed with a setup like the one presented in this thesis, is very large: Modes in 1D quantum wires could be studied, magnetic tips could be used to detect the spatial origin of interference in the Aharonov-Bohm effect in quantum rings, or the tuning between rings and dots, or between two dots in a double dot device could be tuned by a scanning tip, just to cite a few possibilities. Why aren’t all these nice experiments presented in this thesis? One limitation of the scanning gate measurements cannot be avoided: A scanning gate image is a 2D-image, and therefore needs a measurement time which is larger by a factor corresponding to the number of lines in the image, and thus the resolution of the measurement. And since most of the experiments weren’t performed before, the mechanisms, governing the system, have first to be understood. Further work is also required for the investigation of electron wavefunctions in a quantum dot. While technical parameters, as the tip to sample separation, or the number of electrons in the dot can be modified at the sample processing stadium, the problem of an insufficiently low temperature cannot be solved in our setup. The construction of a low-temperature SFM, operated in a dilution refrigerator, will probably give some answers to the open questions in this field.
Appendix A

Measurement electronics

The detection of the very low currents of single electrons tunneling into and out of a quantum dot require a very careful electronic setup. The first conductance peaks of a quantum dot in the Coulomb blockade regime can have typical currents of less than 100 fA, therefore, the noise floor has to be lower than this level. Two kind of noise sources have to be distinguished: Internal and external noise sources. Internal noise sources are:

- Thermal noise (Johnson-Nyquist) of the sample and of all resistors in the measurement circuit. Every resistor produces a thermal noise with a voltage amplitude of $V_{\text{noise}} = \sqrt{4k_BTR_B}$, where $B$ is the bandwidth of the noise signal, leading to a noise current $I_{\text{noise}} = V_{\text{noise}}/R$. The thermal noise of the sample itself is extremely small, but it could make sense to cool down the feedback resistor of the operational amplifiers located outside the cryostat, with the disadvantage that the capacitive noise, arising from the cables to the feedback resistor, will again increase the noise level. However, this is not the case for low-frequency measurements.

- Amplifier noise from all active components. This noise source cannot be avoided, and the noise can only be reduced by choosing low-noise components designed specifically for a certain input signal (amplitude and frequency), and by operating them in their optimum regime.

External noise sources are:

- Low frequency acoustic noise, coming from the cryostat and the cabling. The first can be reduced with the measures discussed in chapter 3. Noise from the cabling is mainly produced by triboelectric effects (charge generation due to friction), as well as capacitance and inductive changes due to the movement. Drilling cables together is an effective solution.

- 50 Hz (and harmonics) noise from power transformers and mains. This is the main noise source and can be avoided with the measures discussed below.
• Noise from light tubes and various screens (<100 kHz). Can be reduced by using careful shielding of all cables.

• Noise from radio and television stations, computers and other digital instruments (clock), WLAN, GSM, GPS, weather radar and microwaves, in the range from 150 kHz to more than 50 GHz. Signal up to 10 MHz can enter the setup through all metallic connections to the lab (power, water, He-recovery). High frequency is very difficult to shield and can provide considerable electron heating.

In order to minimize the 50 Hz noise, the whole setup was modified eliminating all ground loops. The change is illustrated schematically in Fig. 8.1. The main improvement lies in the central point (the mains ground) of electrical ground, where all other instruments and metallic parts are connected, carefully avoiding additional connections between them. The whole measurement electronic was designed in order to take advantage of a central electrical ground and of differential amplification, and is displayed in Fig. 8.2. The mains ground is connected to the case of the metallic box containing the different components, but it is decoupled through a buffering

\[\text{Figure 8.1: (a) Electrical connections between the different components of the measurement setup before the optimization. Ground loops are indicated in color (b) Situation after the optimization, where all instruments are connected to a reference ground marked by the black dot. The dotted lines indicate that the signal passes DA and subsequent AD conversion.}\]
amplifier (OPA27) from the electronic components themselves. These include a symmetric current source for quantum dot measurements, a current source for a two point measurement, a current source for a four point measurement (current measurement at constant voltage drop) and four gate connections. The circuits are placed on four different boards, whose reference ground can either be directly connected to the buffered ground, or at the output of a gate drive. The last solution allows to apply a defined voltage on the sample structure connected to the board, allowing to use it as a gate. The reference ground is also connected to the amplifier providing the tip bias voltage, in order to have the AFM tip and the sample at the same reference potential, and to the cryostat insert, which is electrically decoupled from the dewar. In order to avoid ground loops caused by the external sources, all DC and AC inputs are buffered over a differential amplifier (the AC outputs do not need this decoupling, since the lock-in amplifiers have floating inputs), and special cables are used for the electrical connection. The lock-in amplifiers measure the difference between the output signal of the IU-converters and the reference ground, in order to avoid offsets between the different electrical grounds. The gain of every board can be adjusted externally from unity to $10^{10}$, the usual amplification is $10^7$ to $10^9$. In a typical amplifier setup, the AC and (DC) signals are buffered with a INA118 (AD624, due to the lower drift) operational amplifier, and then the amplitude of the signal is divided by a factor of 1000 (100) in order to reduce the input noise on the sample. The sample signal is amplified by a OPA627AP IU-converter and then amplified by a INA118 voltage amplifier. For the tuning fork, positioning sensor and tip-current detector, an additional guard amplifier is used to drive the guard of the triaxial cables, in order to reduce the capacitive component of the signal.

The DC sources are the 7651 model from **Yokogawa**, while the AC source is the internal clock generator of the **Stanford SR830** DSP lock-in amplifiers. Only for the tuning fork sensor the signal is generated by a **Yokogawa FG200** synthesized function generator.
Figure 8.2: Schematics of the measurement electronic boxes, including the TF-sensor, tip and positioning sensor electronics (on top), and the complete sample gate supplies and IU-converters.
Appendix B

Model for interactions in a scanning gate setup

Constant interaction model

The charging energy (2.44) was obtained by only considering the electrostatics of a metallic island. In a more detailed treatment, a quantum dot can be considered as a system of interacting \( N \) particles, described by a Hamiltonian of the form:

\[
H_N = \sum_{n=1}^{N} \left\{ -\frac{\hbar^2}{2m^*} \Delta - e \int_{\Omega} d\Omega' \rho(\vec{r}) G(\vec{r}, \vec{r}') + \frac{e^2}{2} G(\vec{r}, \vec{r}) - e \sum_i V_i \alpha_i(\vec{r}) \right. \\
+ \left. e^2 \sum_{m=1}^{n-1} G(\vec{r}_m, \vec{r}_n) \right\},
\]

where \( G(\vec{r}, \vec{r}') \) is the Green function of the system, solving Poisson’s equation for a point charge with Dirichlet boundary conditions, and \( \Omega \) is the volume over which the integration is performed. \( \alpha_i \) is the characteristic function of gate \( i \), defined as \( \alpha_i : = -\frac{C_0}{C_\Sigma} \). The Hamiltonian (8.1) can for example be solved in the Hartree or in the Hartree-Fock approximation. If one neglects the exchange term, the dot reduces to a system of particles interacting through the Coulomb interaction and with a quantized single-particle energy \( \epsilon_n \), arising from the reduced size of the dot. The single-particle energy can then be obtained by looking at \( \langle n|H_N|n \rangle \) for a given potential \( V_i^{(0)} \) on the gate \( i \). The single-particle energy can then be written as

\[
\epsilon_n = \epsilon_n^{(0)} - e \sum_i \left( V_i - V_i^{(0)} \right) \langle n|\alpha_i(\vec{r})|n \rangle,
\]

where \( \langle n|\alpha_i(\vec{r})|n \rangle \) is the lever arm of the gate \( i \), which determines how much the energy of the dot changes with a change of the voltage on gate \( i \). In the constant interaction model the electrochemical potential can thus be written as

\[
\mu_N = \epsilon_N^{(0)} - e \sum_i \alpha_i \left( V_i - V_i^{(0)} \right) + (N - 1) \frac{e^2}{C_\Sigma},
\]

where \( E_c : = \frac{e^2}{C_\Sigma} \) is the charging energy.
Model for interactions

Starting from eq. (8.1) it is possible to describe the system in the presence of the scanning tip. In this case the position of the tip is the most important parameter, since it is the main addition to the other (in-plane) gates. Neglecting the two particle operator, eq. (8.1) for a single electron becomes

$$h(r, r_{\text{tip}}) = -\frac{\hbar^2}{2m^*} \Delta - e \int_{\Omega} d\Omega' \rho(r') G(r - r_{\text{tip}}, r' - r_{\text{tip}}) + \frac{e^2}{2} G(r - r_{\text{tip}}, r - r_{\text{tip}})$$

$$- e \sum_i V_i \alpha_i(r, r_{\text{tip}}).$$

(8.4)

The aim of the next calculations [204] is to determine the change of the chemical potential in a quantum dot as a function of the tip position $r_0$, the chemical potential $\mu_i$ on the gate $i$, and on the voltage $V_{\text{tip}}$ applied on the tip. The assumptions are that the many-electron problem has been solved for a given set of the above parameters, and that $E_n$ and $\psi_n$ are known. The resulting self-consistent screening problem of finding the lever arms of the various electrodes as a function of the parameters can be solved in the random-phase approximation (RPA).

Lever arm of the gates

The total change $\delta U_{\text{tot}}(r)$ in the external potential can be written as the sum of the change of the induced potential $\delta U_{\text{ind}}(r)$ on the gate and the change of the external perturbation potential $\delta U_{\text{ext}}(r)$. The change in total potential is then

$$\delta U_{\text{tot}}(r) = \int d\Omega' \epsilon^{-1}(r, r') \delta U_{\text{ext}}(r'),$$

(8.5)

where $\epsilon(r, r')$ is the Linhard dielectric function. Since the origin of $\delta U_{\text{ext}}(r)$ is a change of charge density, it can be written as $\delta U_{\text{ext}}(r) = \int d\Omega' G(r, r') \delta \rho_{\text{ext}}(r')$, where the charge density is proportional to the change in chemical potential on the gate in consideration and its local density of states: $\delta \rho_{\text{ext}}(r) = -e \rho_i(r) \Delta \mu_i$. The change in total potential becomes

$$\delta U_{\text{tot}}(r) = -e \Delta \mu_i \int d\Omega' \epsilon^{-1}(r, r') \int d\Omega' G(r, r') \rho_i(r'),$$

(8.6)

and the lever arm can be defined as

$$\alpha_{\text{gate}} = \frac{\delta U_{\text{tot}}(r)}{\Delta \mu_i}.$$

(8.7)

Lever arm of the tip

The same calculation for the tip, using the relation $\delta U_{\text{ext}}(r) = -e \alpha_{\text{tip}}(r) \Delta V_{\text{tip}}$, leads to a change in total potential

$$\delta U_{\text{tot}}(r) = -e \Delta V_{\text{tip}}(r) \int d\Omega' \epsilon^{-1}(r, r') \alpha_{\text{tip}}(r'),$$

(8.8)
and also in this case the lever arm can be written as

$$\alpha_{\text{tip}} = \frac{\delta U_{\text{tot}}(\vec{r})}{\Delta V_{\text{tip}}(\vec{r})}. \quad (8.9)$$

**Movement of the tip**

The determination of the total potential as a function of the position of the tip is more difficult, since in the other cases the Green’s function $G(\vec{r}, \vec{r}')$ did not depend on the tip position. Therefore, in the Hamiltonian $[8.4]$, the coordinates have to be changed in $\vec{r} \rightarrow \vec{r} - \vec{r}_{\text{tip}}$ and $\vec{r}' \rightarrow \vec{r}' - \vec{r}'_{\text{tip}}$. The term $h$ will depend only on $\vec{r}$ and $\vec{r}_{\text{tip}}$ and is given by

$$h(\vec{r}, \vec{r}_{\text{tip}}) = -\frac{\hbar^2}{2m^*} \Delta + U_{bg}(\vec{r} - \vec{r}_{\text{tip}}) + \delta U(\vec{r}, \vec{r}_{\text{tip}}) - e \sum_j V_j \alpha_j,\text{tip}(\vec{r} - \vec{r}_{\text{tip}}), \quad (8.10)$$

where $U_{bg}$ is a constant background potential, which includes the potential due to image charges, and the constant component of the potential due to fixed charges in the system (see section 6.1.2), while $\delta U$ contains the non-constant component of the fixed charges potential, and the last term the tip-induced potential. If the change in tip position $\Delta \vec{r}_{\text{tip}}$ is small, $\vec{r}_{\text{tip}}$ can be replaced by $\vec{r}_{\text{tip}} + \delta \vec{r}_{\text{tip}}$, and the Hamiltonian can be expanded in $\delta \vec{r}_{\text{tip}}$. The external perturbation potential is then given by

$$\delta U^i_{\text{ext}} = \nabla_{\vec{r}_{\text{tip}}} \left( U_{bg}(\vec{r} - \vec{r}_{\text{tip}}) - e \sum_j V_j \alpha_j,\text{tip}(\vec{r} - \vec{r}_{\text{tip}}) + \delta U(\vec{r}, \vec{r}_{\text{tip}}) \right) \quad (8.11)$$

$$-e \sum_k \int_{\Omega_k} d\Omega' \rho^k(\vec{r}') G_{\text{tip}}(\vec{r} - \vec{r}_{\text{tip}}, \vec{r}' - \vec{r}_{\text{tip}}) \Delta \vec{r}_{\text{tip}}. \quad \delta U^i_{\text{ext}}$$

The density $\rho^k$ can also be separated in a constant component $\rho_{bg}$ and a position dependent component $\delta \rho(\vec{r})$. Since all the constant contributions do not depend on $\vec{r}_{\text{tip}}$, the external perturbation potential can be simplified, and the total potential becomes

$$\delta U^i_{\text{tot}} = \nabla_{\vec{r}_{\text{tip}}} \left( -e \sum_j V_j \alpha_j,\text{tip}(\vec{r} - \vec{r}_{\text{tip}}) + \int d\Omega' \epsilon^{-1}(\vec{r}, \vec{r}') \delta U(\vec{r}, \vec{r}_{\text{tip}}) \right) \quad (8.12)$$

$$-e \int d\Omega'' \int d\Omega' \epsilon^{-1}(\vec{r}, \vec{r}') G_{\text{tip}}(\vec{r}' - \vec{r}_{\text{tip}}, \vec{r}' - \vec{r}_{\text{tip}}) \delta \rho(\vec{r}') \Delta \vec{r}_{\text{tip}}. \quad \delta U^i_{\text{tot}}$$

The first term can be controlled externally by changing the voltage applied on the tip, the other two terms are determined by the properties of the sample, and cannot be changed with experimental parameters.
Appendix C

Nonlocal measurements in the QHE

Capacitance, transparency and edge channels

With “nonlocal” measurements, we intend measurements performed with the scanning tip placed at a fixed position over the center of the Hall bar, at a tip to sample separation of 100 nm.

Capacitance and transparency measurements allow to determine two fundamental quantities of a 2DEG in the quantum Hall regime: the actual “metallic” surface in the 2DEG, which is related to the current distribution in the sample, and the local DOS at the Fermi level. The original idea of the experiments presented here is to probe these quantities locally, in a more detailed way than possible with single fixed top gates [205]. Due to an insufficiently sharp tip and due to the strong influence of external noise sources, which could be corrected only after the experiment, no local dependence was observed. However, these measurements allow to determine the influence of the SFM tip as a top gate and are therefore presented below.

The concept of edge channels, already presented in section 2.4.2 will now be discussed in more detail. Close to the edge of the sample the Landau levels start to follow the confining potential (see Fig. 2.3), and the electron density drops to zero. In this model, the electron density would change abruptly from one LL to the next one. This is a rather unrealistic situation, which doesn’t account for screening effects. In a self-consistent approach which considers the Coulomb interaction [154], this unphysical situation can be avoided leading to the picture of compressible and incompressible strips, see Fig. 8.3, where the electron density changes gradually. In one region the Landau levels are flat and the topmost level, which is partially filled, is pinned at the Fermi level. This means that electrons can easily scatter to free states, leading to good screening (metallic behavior); the 2DEG is thus compressible in these regions, due to its finite compressibility [207]:

\[ \kappa = \left( n_s^2 \partial \mu / \partial n \right)^{-1}. \] (8.13)

The electron density increases steadily towards the center of the sample, while the diamagnetic equilibrium currents are proportional to \( dn/dx \). In the regions between
Figure 8.3: (a) Landau levels in the single particle picture without screening, like in Fig. 2.3, leading to a stepwise increase of the electron density (b). Taking self-consistent screening into account, the Landau levels show a stepwise behavior (c), forming stripes of compressible and incompressible strips. The electron density raises continuously with plateaus at the incompressible strips (d). The equilibrium currents are shown in (d) [206].

the compressible strips the Fermi energy is in the gap between two LLs, and no free states are available. The 2DEG has therefore very poor screening properties, and is locally incompressible. The LLs shift by an energy corresponding to $\hbar \omega_c$, while the electron density is constant, and the paramagnetic equilibrium currents are proportional to $dU/dx$. If no bias is applied to the sample, the equilibrium currents cancel each other, and the net current is zero. A bias voltage will create a difference in electrochemical potential, and a Hall voltage can be measured across the sample, meaning that the 2DEG is not in equilibrium anymore. The situation for different filling factors can be described as follows [164, 206, 208]:

- At a filling factor slightly larger than integer, the Fermi level aligns with a LL, which is constant over the bulk of the sample. The potential drop over this region is almost zero, and a dissipation-free net current flows over the incompressible edge regions.
- At a filling factor slightly smaller than integer, the coupling between the edge and bulk regions increases, and the probability that an electron scatters between these regions is higher. This leads to a dissipative current, which is carried over the entire width of the Hallbar, mostly over the compressible bulk region. The potential drops also over the whole width of the sample.

- At integer filling factor the Bulk is incompressible, and the dissipation free net current is carried only in this region. The potential drop depends on the local potential in the bulk.

The different situations are illustrated in Fig. 8.4.

**Figure 8.4**: Schematic representation of the nonequilibrium current distribution over a Hall bar of width $w$, at filling factors between 1.2 and 2.5. At integer filling factor (lower left, $\nu = 2$) dissipation free bulk currents dominate, and the sample becomes transparent.

The screening properties of a 2DEG are related to the ability to move without dissipation enough charge from an external reservoir. This is possible only for compressible regions, which have a metallic behavior. Therefore, one would expect that the transparency of the 2DEG (i.e. the ability to screen an external field) is
proportional to the surface of the incompressible regions, which are badly screening [209, 210, 211]. This surface is largest at integer filling factors, where the bulk region of the sample is incompressible, while it is small for the situation, where the incompressible regions appear only at the sample edges. The capacitance signal will behave the opposite way: Here a large (metallic) compressible surface will lead to a large capacitance signal, and this is the case at half integer filling factor, where the bulk is (mostly) compressible.

The relation between capacitance and the DOS in a 2D system has been discussed extensively in [212, 213, 214, 215, 216, 217, 218]. A brief summary is given below. The measured capacitance of a 2DEG can be expressed as follows:

\[
\frac{A}{C_{\text{meas}}} = \frac{A}{C_I} + \frac{\gamma z_0}{\epsilon_S} + \frac{1}{e^2 \frac{dn}{d\mu}},
\]

where \( A \) is the area of the capacitor, \( C_I \) is the capacitance of the doping plane between 2DEG and electrode (in our case the AFM tip), \( \gamma \) is a numerical constant between 0.5 and 0.7, \( z_0 \) is the average position of the electrons in the channel, \( \epsilon_S \) is the dielectric constant of the semiconductor, and \( dn/d\mu \) is the thermodynamic DOS at the Fermi energy. The first two terms in (8.14) do not depend on the magnetic field, and thus changes in the capacitance are directly related to the DOS. It is also known [217, 218] that at very low temperatures \((dn/d\mu)^{-1}\) can become negative, indicating that the energy of sublevels decreases with increasing \( n \).

The inverse thermodynamic density of states can be obtained from eq. (8.14) and yields:

\[
\left( \frac{dn_s}{d\mu} \right)^{-1} = \frac{\pi \hbar^2}{m^*} + e^2 A \left( \frac{1}{C(B)} - \frac{1}{C(B = 0)} \right) \\
\simeq D_0^{-1} + e^2 A \frac{\Delta C}{C^2} \quad \text{for} \quad \Delta C \ll C.
\]

\( C \) is the backgate-2DEG capacitance, \( D_0 \) is the density of states at \( B = 0 \) and \( T = 0 \), and \( \Delta C = C(B = 0) - C(B) \). The compressibility (8.13) is also related to the DOS, and it is very difficult to measure, since the geometric capacitance term in (8.14) dominates. For this reason, a setup like the one displayed in Fig. 8.7 is used. The tip voltage is tuned such that the contact potential difference between tip and 2DEG is compensated. In a perfect plate capacitor the electric field of the backgate would be perfectly screened by the 2DEG. But, since the DOS of the 2DEG is finite, some field penetration occurs [215]. This is due to the fact that the capacitance of the 2DEG (or of any 2D system) consists of a geometric capacitance plus a “quantum capacitance” \( C_Q \) per area which is given by the last term of eq. (8.14)

\[
C_Q = g_\nu D_0 e^2 = g_\nu \frac{m^*}{m_0} \times 6.0 \times 10^7 \text{cm}^{-1},
\]

where \( g_\nu \) is the valley degeneracy factor, and \( m^* \) is the perpendicular effective mass.
Capacitance measurements

The setup used for the capacitance measurements is displayed in Fig. 8.5 and allows to set the tip and the backgate at different electrostatic potentials. The measurements were performed with an AC voltage of 150 mV applied to the back gate and to the 2DEG, at a frequency of 5 kHz. Typical oscillations as a function of magnetic field are shown in Fig. 8.6. The measured current ranges from 0.7 to about 1 nA, leading to typical capacitances of the order of 170 fF, obtained with the simple relation $C = \frac{I}{\omega V}$. The change in density due to a DC backgate voltage manifests itself in a shift of the maxima: With -3.5 V on the back gate (low density), filling factor 4 can be reached already at about 4 T, while a magnetic field of 6.7 T is necessary to reach the same filling factor with an applied voltage of +1.5 V. A comparison to magnetoresistance measurements shows that the measured density is slightly higher (by about 12%), if measured with the described capacitance measurement method. This may be due to the fact that the capacitance between tip (placed at the center of the Hall bar) and the 2DEG is less affected by the density in the leads. The fact that the capacitance is not disappearing at integer filling factors may be attributed to stray fields between 2DEG, tip and other conductors in their vicinity, and to mobile charges. Using eqs. (8.15) and (8.13), it is possible to calculate directly the DOS and the compressibility of the 2DEG. The result is displayed in Fig. 8.7.

Transparency measurements

Transparency measurements require a different measurement setup, since the current measured on the tip has to “go through” the 2DEG to the back gate, where the AC
Figure 8.6: (a) Magnetocapacitance measurement for different back gate voltages. (b) Comparison between the magnetocapacitance (not in scale) measurement at a backgate voltage of 1.5 V and the longitudinal resistance measurements for the same voltage. The slower oscillations of the magnetocapacitance are due to a slightly higher density. The inset shows a detail of the low field region.

Figure 8.7: Density of states (DOS) and compressibility extracted from the capacitance measurement at a back gate voltage of 1.5 V.

voltage is applied. The detected current will therefore be much smaller than in the case of a capacitance measurement, and higher measurement frequencies are needed in order to obtain a higher capacitive current. The experiments were performed
with the tip close to the surface in the middle of the Hall bar, by applying an AC voltage of 50 mV on the back gate at a frequency of 13.848 kHz. Higher frequency could not be applied, since parasitic capacitances and resistive effects \[219\] started to affect the measurement. The measurement scheme and the result as a function of the magnetic field are displayed in Fig. 8.8. The current level increases significantly at integer filling factors, where the screening is not efficient.

**Remarks on non-locality of the measurements**

Scanning capacitance and scanning transparency measurements showed no evidence for position dependent effects, although the tip was scanned over the entire Hall bar. The probable reasons are on one hand the insufficient sharpness of the tip, on the other hand the unsuitability of some components of the measurement setup for this kind of measurements. The wiring in the cryostat is not designed for measurements at high frequencies, thus, the maximum frequency, which can be applied in the measurement is limited, and, as a consequence, the strength of the detected signal is very low. Stray capacitances can also be avoided by building a bridge low-temperature amplifier setup mounted on the sensor plate, which would also increase the sensitivity. Finally, a more careful shielding of the tip (for example a coaxial tip) and of the tuning fork-sensor plate unit, with respect to the sample, could also reduce parasitic effects and considerably increase the measurement sensitivity. However, part of these measures would decrease the flexibility of the measurement setup, which is built with the aim of performing scanning gate measurements.
Appendix D

Gradient of $R_{xx}$ in the quantum Hall regime

Scanning gate images in the quantum Hall regime show complicated resistance changes in various regions of the Hall bar, but it remains difficult to see, if a general behavior of the resistance appears over the whole surface of the sample. In order to verify this, profiles of the scanning gate images have been taken for several positions, always perpendicular to the direction of the current, as shown in Fig. 8.9. The general features discussed in chapter 5 can be recognized also in the profiles: Most of the resistance changes appear at the edge of the Hall bar, while the bulk shows a flat response to the SFM tip. A gradient in the resistance is observed only for the measurements labeled by 1 and 5. They correspond to the transition regions between the plateau for $\nu = 4$ and $\nu = 5$ (measurement 1), and between $\nu = 4$ and $\nu = 3$ (measurement 5), where the formation of a percolating network of localized states is expected. Note the opposite sign of the resistance change for $R_{xx1}$ and $R_{xx2}$ in the case of measurement 5.
Figure 8.9: (a) Lines where the profiles through the scanning gate images have been taken. (b) Resistance plot showing the magnetic fields at which the measurements have been performed. (c) Scanning gate images and corresponding profiles. The lowest curve corresponds to the profile 1, all curves are offset for clarity.
Appendix E

Landauer-Büttiker approach for the Hall cross

The Hall resistance can depend on the position of the SFM tip on the Hall cross. A total of 17 scattering configurations (for two channels) is now analyzed. The different situations are illustrated in Fig. 8.10. The following matrices and the following Resistances are obtained:

- **Case 1**

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0
\end{pmatrix}
= \frac{e^2}{\hbar} \begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & -T_{23} & \nu & 0 & -T_{53} & 0 \\
0 & 0 & -\nu & \nu & 0 & 0 \\
0 & 0 & 0 & -\nu & \nu & 0 \\
0 & -T_{26} & 0 & 0 & -T_{56} & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]

This is the same situation as in the previous calculation. The resistances are therefore the same:

\[
R_{65,14} = R_{23,14} = \frac{\hbar}{e^2} \left( \frac{1}{T_{23}} - \frac{1}{\nu} \right)
\]  

(8.17)

and

\[
R_{62,14} = R_{53,14} = \frac{\hbar}{e^2} \left( -\frac{1}{\nu} \right)
\]  

(8.18)

- **Case 2**

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0
\end{pmatrix}
= \frac{e^2}{\hbar} \begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & -\nu & \nu & 0 & 0 & 0 \\
0 & 0 & -\nu & \nu & 0 & 0 \\
0 & 0 & 0 & -T_{45} & \nu - T_{55} & 0 \\
0 & 0 & 0 & -T_{46} & -T_{56} & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]
Figure 8.10: Possible situations where the SFM tip can induce scattering between one or two edge channels.

One parameter cannot be determined, so one voltage remains free.

\[ R_{65,14} = \frac{\hbar}{e^2} \left( \frac{V_5}{I} \right) \]  \hspace{1cm} (8.19)

and

\[ R_{53,14} = \frac{\hbar}{e^2} \left( \frac{1}{\nu} - \frac{V_5}{I} \right) \]  \hspace{1cm} (8.20)

- Case 3
\[ \begin{pmatrix} I \\ 0 \\ -I \\ 0 \end{pmatrix} = \frac{e^2}{h} \begin{pmatrix} \nu & 0 & 0 & 0 & -\nu \\ -\nu & \nu & 0 & 0 & 0 \\ 0 & -\nu & \nu & 0 & 0 \\ 0 & 0 & -T_{34} & \nu & -T_{44} \\ 0 & 0 & -T_{35} & -T_{45} & \nu \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} \]

This situation causes no change to the resistance:

\[ R_{56,14} = 0 \] (8.21)

and

\[ R_{35,14} = \frac{h}{e^2} \left( \frac{1}{\nu} \right) \] (8.22)

• Case 4

\[ \begin{pmatrix} I \\ 0 \\ -I \\ 0 \end{pmatrix} = \frac{e^2}{\pi} \begin{pmatrix} \nu & 0 & 0 & 0 & 0 & -\nu \\ -\nu & \nu & 0 & 0 & 0 & 0 \\ 0 & -T_{23} & \nu & -T_{33} & 0 & 0 \\ 0 & -T_{24} & -T_{34} & \nu & 0 & 0 \\ 0 & 0 & 0 & -\nu & \nu & 0 \\ 0 & 0 & 0 & 0 & 0 & -\nu & \nu \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} \]

Also in this case there is no change:

\[ R_{56,14} = 0 \] (8.23)

and

\[ R_{35,14} = \frac{h}{e^2} \left( \frac{1}{\nu} \right) \] (8.24)

• Case 5

\[ \begin{pmatrix} I \\ 0 \\ -I \\ 0 \end{pmatrix} = \frac{e^2}{h} \begin{pmatrix} \nu & 0 & 0 & 0 & 0 & -\nu \\ -\nu & \nu & 0 & 0 & 0 & 0 \\ 0 & -T_{23} & \nu & -T_{43} & 0 & 0 \\ 0 & 0 & -\nu & \nu & 0 & 0 \\ 0 & 0 & 0 & -T_{45} & \nu & -T_{55} \\ 0 & 0 & -T_{26} & 0 & 0 & -T_{56} & \nu \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} \]

\[ R_{56,14} = \frac{h}{e^2} \left( \frac{1}{T_{23}} - \frac{1}{\nu} \right) \] (8.25)

and

\[ R_{35,14} = \frac{h}{e^2} \left( \frac{1}{\nu} \right) \] (8.26)
• Case 6

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0 \\
0
\end{pmatrix}
= \frac{e^2}{\pi}
\begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & -\nu & \nu & 0 & 0 & 0 \\
0 & 0 & -T_{34} & \nu - T_{44} & 0 & 0 \\
0 & 0 & 0 & -T_{45} & \nu - T_{55} & 0 \\
0 & 0 & -T_{36} & 0 & -T_{56} & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]

This situation gives 0 resistance everywhere:

\[R_{56,14} = 0\] (8.27)

and

\[R_{35,14} = 0\] (8.28)

• Case 7

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0 \\
0
\end{pmatrix}
= \frac{e^2}{\pi}
\begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & -T_{23} & \nu - T_{33} & 0 & 0 & 0 \\
0 & 0 & -T_{34} & \nu - T_{44} & 0 & 0 \\
0 & 0 & 0 & -T_{45} & \nu & 0 \\
0 & 0 & 0 & 0 & -\nu & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]

This situation gives the same result as the unperturbed situation:

\[R_{56,14} = 0\] (8.29)

and

\[R_{35,14} = \frac{\hbar}{e^2} \left( \frac{1}{\nu} \right)\] (8.30)

• Case 8

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0 \\
0
\end{pmatrix}
= \frac{e^2}{\pi}
\begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & -T_{23} & \nu - T_{33} & 0 & 0 & 0 \\
0 & 0 & -T_{34} & \nu - T_{44} & 0 & 0 \\
0 & 0 & 0 & -\nu & \nu & 0 \\
0 & 0 & -T_{26} & 0 & -T_{56} & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]

\[R_{56,14} = \frac{\hbar}{e^2} \left( \frac{1}{T_{56}} - \frac{1}{\nu} \right)\] (8.31)

and

\[R_{35,14} = \frac{\hbar}{e^2} \left( \frac{1}{T_{56}} \right)\] (8.32)
• Case 9

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0
\end{pmatrix}
= \frac{e^2}{h} \begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & 0 & \nu & 0 & -\nu & 0 \\
0 & 0 & -\nu & \nu & 0 & 0 \\
0 & 0 & 0 & -\nu & \nu & 0 \\
0 & 0 & 0 & 0 & -\nu & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]

In this case the Hall resistance is zero, while the longitudinal resistance depends on one voltage:

\[R_{56,14} = \frac{h}{e^2} \left( \frac{V_2}{I} \right)\] (8.33)

and

\[R_{35,14} = 0\] (8.34)

• Case 10

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0
\end{pmatrix}
= \frac{e^2}{h} \begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & 0 & \nu & 0 & -\nu & 0 \\
0 & 0 & -\nu & \nu & 0 & 0 \\
0 & 0 & 0 & -\nu & \nu & 0 \\
0 & 0 & 0 & 0 & -\nu & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]

Also in this case one parameter remains:

\[R_{56,14} = \frac{h}{e^2} \left( \frac{V_5}{I} \right)\] (8.35)

and

\[R_{35,14} = \frac{h}{e^2} \left( \frac{1}{\nu} - \frac{V_5}{I} \right)\] (8.36)

• Case 11

\[
\begin{pmatrix}
I \\
0 \\
0 \\
-I \\
0
\end{pmatrix}
= \frac{e^2}{h} \begin{pmatrix}
\nu & 0 & 0 & 0 & 0 & -\nu \\
-\nu & \nu & 0 & 0 & 0 & 0 \\
0 & 0 & \nu & 0 & -\nu & 0 \\
0 & 0 & -\nu & \nu & 0 & 0 \\
0 & 0 & 0 & -\nu & \nu & 0 \\
0 & 0 & 0 & 0 & -\nu & \nu
\end{pmatrix}
\begin{pmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
V_5 \\
V_6
\end{pmatrix}
\]
Since the grounded contact is pinched off, the resistance drops to zero:

\[ R_{56,14} = 0 \]  \hspace{1cm} (8.37)

and

\[ R_{35,14} = 0 \]  \hspace{1cm} (8.38)

- **Case 12**

\[
\begin{pmatrix}
  I \\
  0 \\
  0 \\
  -I \\
  0
\end{pmatrix}
= \frac{e^2}{\hbar}
\begin{pmatrix}
  \nu & 0 & 0 & 0 & 0 & -\nu \\
  -\nu & \nu & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & -\nu & 0 & \nu & 0 \\
  0 & 0 & 0 & -\nu & \nu & 0 \\
  0 & 0 & 0 & 0 & -\nu & \nu \\
\end{pmatrix}
\begin{pmatrix}
  V_1 \\
  V_2 \\
  V_3 \\
  V_4 \\
  V_5 \\
  V_6
\end{pmatrix}
\]

\[ R_{56,14} = 0 \]  \hspace{1cm} (8.39)

and

\[ R_{35,14} = \frac{h}{e^2} \left( \frac{1}{\nu} \right) \]  \hspace{1cm} (8.40)

- **Case 17**

\[
\begin{pmatrix}
  I \\
  0 \\
  0 \\
  -I \\
  0
\end{pmatrix}
= \frac{e^2}{\hbar}
\begin{pmatrix}
  \nu & 0 & 0 & 0 & 0 & -\nu \\
  -\nu & \nu & 0 & 0 & 0 & 0 \\
  0 & -T_{23} & \nu & -T_{43} & 0 & 0 \\
  0 & 0 & -\nu & \nu & 0 & 0 \\
  0 & -T_{25} & 0 & -T_{45} & \nu & 0 \\
  0 & 0 & 0 & 0 & -\nu & \nu \\
\end{pmatrix}
\begin{pmatrix}
  V_1 \\
  V_2 \\
  V_3 \\
  V_4 \\
  V_5 \\
  V_6
\end{pmatrix}
\]

\[ R_{56,14} = 0 \]  \hspace{1cm} (8.41)

and

\[ R_{35,14} = \frac{h}{e^2} \left( \frac{2}{\nu} - \frac{1}{T_{45}} \right) = \frac{h}{e^2} \frac{1}{\nu} \left( \nu - 2T_{34} \right) \]  \hspace{1cm} (8.42)

The other cases (13 - 16) are of less interest. Three main situations appear:

- Situations with no change with respect to the unperturbed system, like 3, 4, 7, and 12.
- Situations where the resistance drops to zero, like 6 and 11.
- Situations where backscattering influences the resistance. These can be exactly be calculated, like 1, 5, 8, and 17, or depend on a voltage, like 2, 9, and 10.
Appendix F

Parallel and series measurements on two quantum dots

The measurements were performed on the quantum ring, creating quantum dots in two different arms by applying suitable gate voltages. In a first measurement the current was applied to the lead between PG3 and PG4, see Fig. 8.11, while the conductance was measured at the leads between PG1 and PG3 and between PG2 and PG4. The dots are too far apart in order to interact with each other, but the experiment is a good test for simultaneous scanning gate measurements. The figures show slightly different features, indicating that crosstalk is very small. The same measurement has been repeated by measuring through the two dots in series, see Fig. 8.12. Here the signal is very weak, and no significant information can be gained.

**Figure 8.11:** (a) Measurement setup for measuring the two quantum dots in parallel. (b) Conductance through the dot at the gate PG3, (c) conductance through the dot at PG4.
Figure 8.12: (a) Measurement setup for measuring the two quantum dots in series. (b) Conductance through the dot at the gate PG3, (c) conductance through the dot at PG4. The measured signal is extremely weak.
Appendix G

Quantum dot tip bias dependence: More measurements

These measurements were the first performed on the quantum dot during the second cool-down. The scanning gate images were taken for tip bias voltages between 0.1 and 0.7 V in constant height mode. They are similar to the measurements already shown in section 6.4.2 and do not provide additional information. On the other hand they confirm the measurements already shown, as well as the simulations explained in section 6.4.3. In the right column of Fig. 8.13 the measurements are plotted in a logarithmic scale.
Figure 8.13: Scanning gate images of the quantum dot, taken at the beginning of the second cool-down of that sample. The images in the right column are plotted on a logarithmic scale.
Appendix H

Energy dependence inside a quantum dot

The idea behind these measurements is to determine, if small changes in the energy of the dot can lead to changes in the local conductance inside the dot. The energy was changed by tuning the voltage $V_{pg}$ in steps of 0.2 mV, and the tip was scanned over an area of 300 by 300 nm. The position of the scanning gate images relative to the conductance trace was determined with the tip at the scanning height in the center of the scanning range of the shown images. The result is that it is not possible to discern more than a periodicity of the general features, comparable to the position relative to a conductance peak. The reasons are discussed in section 6.4.4.
Figure 8.14: Scanning gate images of the interior of the quantum dot for a fixed tip bias voltage and for different energies. The conductance scale varies from 0 to 0.4 $e^2/h$ for the lowest energies to 0 to 1.8 $e^2/h$ for the highest energies.
Figure 8.15: Scanning gate images of the interior of the quantum dot for a fixed tip bias voltage and for different energies (continuation of the previous figure). The conductance scale varies from 0 to $1.8 \frac{e^2}{h}$ for the lowest energies to 0 to $4.5 \frac{e^2}{h}$ for the highest energies.
Appendix J

Gate and tip bias dependence of the QPC conductance

These measurements can be considered as a supplement to those already discussed in section 7.3. Although the physics behind these images is based on the electrostatic interactions between the scanning SFM tip and the QPC, they allow to observe the evolution of the QPC conductance, depending on tip bias voltage or gate voltage.

Gate voltage dependence

The tip bias voltage for these measurements is always 300 mV, while the gate voltage is varied between 60 and 240 mV. The conductance trace for different gate voltage is displayed in Fig. 7.1. In order to discern more details in the scanning gate images, also the gradient $\nabla = \sqrt{(d/dx)^2 + (d/dy)^2}$ is plotted. The gradient of the data is displayed in Fig. 8.17, while the bare measurement data are shown in Fig. 8.16.

![Figure 8.16: Gate voltage dependence of the QPC conductance for a tip bias voltage of 300 mV.](image)

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**Figure 8.17:** Gradient of the data displayed in Fig. 8.14. Small changes in the conductance become visible.

**Tip bias dependence of the upper region of the QPC**

The idea behind these measurements is to check, whether it could be possible to observe the electron flow through the QPC. The images are therefore centered on the upper-left part of the constriction, including the oxide line on the left, which delimits the region qpc1 and qpc2 (see Fig. 6.9). However, no such flow could be observed, and features of small size can be related to charging of local potential fluctuations, see section 7.5. The images allow to observe several centers of charging events. The tip bias voltages were chosen in order to have the highest sensitivity to small changes in the conductance.

**Figure 8.18:** Tip bias dependence of the QPC conductance in the upper-left region of the constriction. The two lower rows are the gradient of the data.
Appendix K

QPC measurements at 2T

The measurements shown here are basically the measurements discussed in section 7.5 and in addition measurements not done as transconductance. Figures 8.19 and 8.20 show scanning gate images taken at 100 nm from the surface for tip a tip bias voltage of 1.1 V and gate voltages $V_{pg}$ between 395 and 485 mV. Figure 8.20 is a differential picture. Figures 8.21 and 8.22 show the transconductance measurements for a gate voltage $V_{pg}$ of 410 mV and a tip bias voltage range between 170 mV and 1100 mV. It is clearly visible how the size of the ring-shaped conductance oscillations reduces with decreasing tip bias voltage. All measurements were done at a magnetic field of 2 T.

![Scanning gate images](image)

**Figure 8.19:** Scanning gate images on the left of the QPC for the indicated gate voltages.
Figure 8.20: Gradient of the data displayed in Fig. 8.16.

Figure 8.21: Transconductance measurements for different tip bias voltages.
Figure 8.22: Transconductance measurements for different tip bias voltages.
Appendix L

Tip over 2DEG

A general difficulty in scanning gate experiments is that one doesn’t know, how the tip induced potential looks like for different positions above and below the surface of the sample. In this section the problem is solved by assuming a spherical tip with radius \( r \), with its center at a distance \( Z_0 \) from the surface of the sample. The 2DEG is considered as a perfect metallic surface at a distance \( D \) below the surface. The space between the 2DEG and the surface is filled with a dielectric material of dielectric constant \( \epsilon_2 = 12.9 \) (like GaAs), while the tip is scanned in vacuum, with \( \epsilon_1 = 1 \). Since all surfaces are assumed as perfect conductors or perfect dielectric materials, the electric field can be calculated by using image charges placed outside the region of interest [220].

The first step is to place a charge \( Q_0 \) in the center of the sphere of radius \( r \), representing the tip, at a distance \( z = Z_0 \) from the surface, which has the coordinate \( z = 0 \). The electrostatic potential in the region \( z > 0 \) can be calculated by placing an image charge \( q'_0 = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} Q_0 \) at a position \( z'_0 = -Z_0 \), and yields

\[
\phi_0(\rho,z) = \frac{Q_0}{4\pi \epsilon_0 \epsilon_1} \left( \frac{1}{\sqrt{\rho^2 + (z - Z_0)^2}} + \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \frac{1}{\sqrt{\rho^2 + (z + Z_0)^2}} \right), \quad \text{for} \quad z > 0.
\]

This potential doesn’t fulfill the boundary conditions for the electric field at the surface of the sphere. In order to solve this problem, a charge \( Q_1 = \frac{r^2}{2Z_0} \) is placed inside the sphere at a coordinate \( Z_1 = Z_0 - \frac{r^2}{2Z_0} \). In the region below the surface of the sample, the original charge \( Q_0 \) is seen as a charge \( q''_0 = \frac{2\epsilon_2}{\epsilon_1 + \epsilon_2} Q_0 \) at the same coordinate \( Z_0 \) of \( Q_0 \). The potential of this charge in the region \( -D < z < 0 \),

\[
\phi_0(\rho,z) = \frac{Q_0}{2\pi \epsilon_0 (\epsilon_1 + \epsilon_2)} \frac{1}{\sqrt{\rho^2 + (z - Z_0)^2}}, \quad \text{for} \quad -D < z < 0,
\]

does not fulfill the boundary condition for the electric field at the surface of the 2DEG at \( z = -D \). For this reason, an image charge \( q_1 = \frac{-2\epsilon_2}{\epsilon_1 + \epsilon_2} Q_0 \) has to be placed at the position \( z_1 = -2D - Z_0 \). Two new charges have been introduced in the
system, thus the electrostatic potential of those charges has also to be taken into account. For the charge $q_1$ in the region below the surface the potential yields

$$\phi_{q_1}^1(\rho, z) = \frac{q_1}{4\pi \epsilon_0 \epsilon_2} \left( \frac{1}{\sqrt{\rho^2 + (z - z_1)^2}} + \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1} \frac{1}{\rho^2 + (z + z_1)^2} \right), \text{ for } -D < z < 0,$$

while it is given by

$$\phi_{q_1}^1(\rho, z) = \frac{q_1}{2\pi \epsilon_0 (\epsilon_1 + \epsilon_2)} \frac{1}{\rho^2 + (z - z_1)^2}, \text{ for } z > 0.$$

The potential $\phi_1(\rho, z)$ for $-D < z < 0$ doesn’t fulfill the boundary condition at the 2DEG, requiring a charge $q_2 = \frac{Q_0 - q_1}{\epsilon_2 + \epsilon_1}$ placed at a position $z_2 = -2D + z_1$. In the case $z > 0$ the boundary conditions at the sphere are not fulfilled, therefore an image charge $Q_2 = \frac{r}{z_0 - z_1} \frac{2q_1}{\epsilon_2 + \epsilon_1}$ has to be placed inside the sphere at a position $Z_2 = Z_0 - \frac{r^2}{z_0 - z_1}$. The potential due to $Q_1$ can be calculated in a similar way as for $Q_0$, leading to

$$\phi_{Q_1}^1(\rho, z) = \frac{Q_1}{4\pi \epsilon_0 \epsilon_1} \left( \frac{1}{\sqrt{\rho^2 + (z - Z_1)^2}} + \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \frac{1}{\rho^2 + (z + Z_1)^2} \right), \text{ for } z > 0,$$

and

$$\phi_{Q_1}^1(\rho, z) = \frac{q_1}{2\pi \epsilon_0 (\epsilon_1 + \epsilon_2)} \frac{1}{\rho^2 + (z - Z_1)^2}, \text{ for } -D < z < 0.$$

Again, two new charges have been introduced in the system, leading to additional components of the total electrostatic potential. The calculation can be carried on by iteration, leading to $2^{n+1} - 1$ charges (real and virtual) in the system, if the charge pair $Q_n, q_n$ is considered. The series is a very good approximation already for $n = 3$, but it requires more steps, if the sphere is very close to the surface of the sample, or the radius of the sphere is small. Figure 8.23 shows the result of such a calculation with 10 iterations for the tip-to sample separation and tip voltages used for the experiments described in section 7.5, assuming a tip radius of 60 nm. The calculation shows that even if a bias voltage of 1 V is applied to the tip, the effective tip-induced electrostatic potential at the donor layer below the tip is only 15 mV.
Figure 8.23: (a) and (b): Electrostatic potential as a function of $z$ position, assuming that the tip is a sphere placed 60 nm over a dielectric. The tip bias is 140 mV in (a) and 1 V in (b), a contact potential difference of -100 mV was assumed. (c) and (d): Electrostatic potential in $x$ direction at the surface of the sample (blue) and at the donor layer (red).
Publications

Quantum Hall effect
S. Kičin, A. Pioda, T. Ihn, K. Ensslin, D. C. Driscoll and A. C. Gossard
S. Kičin, A. Pioda, T. Ihn, K. Ensslin, D. C. Driscoll and A. C. Gossard

Quantum dots
Physica E, to be published
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Curriculum Vitae

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