Characterization of EnviroSMART™ Capacitance Sensors* for Measuring Soil Water Content

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Abstract

Volumetric soil water content $\theta$ can be estimated from the bulk soil dielectric constant $\varepsilon$ measured using ring-capacitor sensors inserted into a plastic access tube augured into soil. The present laboratory experiments were designed to characterize the sensor response over a full range of environmental conditions including $\varepsilon$ values from air to water and over a range of temperatures. First, resonant frequency $f_r$ values were recorded for surface-mounted devices of known capacitance $C$ soldered to the sensor electronics, confirming a linear relationship between $f_r^{-2}$ and $C$. Next, water-dioxane mixtures were placed into a solvent-resistant container equipped with custom tools for heating and mixing the fluid, removing air bubbles from sensitive surfaces, measuring permittivity in-situ, and creating an axisymmetric metal disturbance to the electric field. Total $C$ was measured using a Vector Network Analyzer connected to one sensor, while four other sensors provided replicated $f_r$ readings. The measured temperature response of free water permittivity was linear with a negative slope, which is consistent with theory. A precise nonlinear relationship between $\varepsilon$ and normalized sensor reading is derived, where the instrumental error in $\varepsilon$ ($RMSE_\varepsilon = 0.226$ for $3 < \varepsilon < 43$) corresponds to an error (i.e., precision) in soil water content $\theta(\varepsilon)$ derived from Topp’s equation of $RMSE_\theta = 0.0034 \text{ m}^3\text{m}^{-3}$. Different models for $\theta(\varepsilon)$ were compared with the manufacturer’s default calibration equation, which tends to underestimate $\theta$ by approximately 0.066 $\text{m}^3\text{m}^{-3}$. Axisymmetric numerical simulations of the electric field were used to extend the experimental results. The characteristic length scale of an exponential model for the distance measured from the access tube is approximately 12 mm over a range of $\varepsilon$ values typical of soils, meaning that 95% of the signal is sensed within 36 mm of the access tube. The results are crucial for scientific applications of the sensor to environmental media including future investigations of physically based dielectric mixing models.
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1 Introduction

1.1 Applications of Capacitance Sensors

Capacitance sensors have been developed commercially and are being used globally for estimating soil water content. For field applications, basically two electromagnetic techniques for measuring soil moisture are available: time-domain reflectometry (TDR) and capacitive techniques. Several TDR probes with different dimensions and different numbers of rods representing the transition line exist today (Roth et al. 1990). In all TDR probes, the travel time of the electrical signal along the probe is measured, from which the permittivity \( \varepsilon \) of the soil in the vicinity of the TDR probe is calculated (Nussberger 2005). The permittivity acts as a proxy for the volumetric soil water content \( \theta \) to be measured. In this regard, TDR and capacitive techniques coincide; both are indirect techniques using soil permittivity \( \varepsilon \) as a proxy for determining \( \theta \).

The characterized capacitance, or frequency domain, sensor is designed to measure resonant frequency, rather than directly measuring capacitance, and the permittivity of a medium is most directly related to the effective capacitance. However, the relationship between the sensor reading (or resonant frequency) and added capacitance or permittivity of the measured medium has not been well characterized. Instead, water content is estimated by empirical correlation with the sensor reading. This approach does not provide the information needed to determine the fundamental property measured by a capacitance sensor: permittivity or "dielectric constant" of the bulk soil and its temperature sensitivity. Thus, additional experiments in the laboratory were required to accurately determine the relationship between sensor reading and permittivity.

Field data from near-surface sensors (30 to 60 cm depths) display variations at diurnal and other time scales associated with measured temperature fluctuations. The exact causes and quantification (i.e., correction) of these temperature effects on apparent water content measurements are currently unknown, though several theories have been postulated. One theory relates the temperature effects to “bound water” versus “free water” where bound water is defined as water very near a solid surface, such that its dielectric relaxation properties are modified relative to free water.

Of the currently available instruments, capacitance probes are preferable for field installations where the distances between probes and the datalogger(s) are large. TDR probes have a limited cable length (typically less than 30 m) due to attenuation of the analogue signal, whereas the analogue signal of the EnviroSMART\textsuperscript{TM} and EnviroSCAN\textsuperscript{TM} family of capacitance probes is processed within each access tube. Digital data is then transmitted from each probe to the datalogger, which can be placed off of the field hundreds of metres away. In addition, probe installation is more efficient for soil profile installations (to 1.6 m) using an access tube for the capacitance probe instead of trenching or drilling multiple holes for TDR.

To gain insight into dielectric mixing processes in soils from data collected with capacitive techniques, the relation between the sensor reading and the soil permittivity \( \varepsilon \) has to be known. Such knowledge is also required for validating and improving dielectric mixing models used for soil moisture estimation from measured proxy data \( \varepsilon \).
1.2 Design and Deployment of the EnviroSMART™ Probe

The Sentek EnviroSMART™ probe and capacitance sensors have been designed to measure soil water content in the field with or without surface access. Figure 1a shows field installation of the plastic access tube using a hand auger to ensure contact between the soil and outer surface of the tube. Sensors are attached to a probe (Figure 1b “sensor stick”) with integrated circuits for signal processing and analogue to digital conversion. The probe is inserted in the access tube, and a 5-wire cable connects each probe with multiple sensors to a datalogger using digital communication. Thus, there is no signal degradation between the probes and the datalogger. Probes have been operated successfully up to 450 m from a datalogger in Colorado, USA, and longer distances may be possible.

Figure 1c-e provides a close-up view of the ring-capacitor sensor assembly with schematic electric field lines (torus-like, axisymmetric pattern), the electronic circuit board inside the rings, and an equivalent circuit diagram. The external capacitance \( C(\varepsilon) \) is a function of the permittivity \( \varepsilon \) of the medium surrounding the access tube, and \( C(\varepsilon) \) includes the series capacitance of the plastic tube plus any air gap between the rings and inner diameter of the tube. There is also an internal capacitance \( C_{\text{int}} \) acting in parallel to \( C(\varepsilon) \), comprised of the on-board sensor capacitance and the effective \( C \) of the materials in the probe inside the capacitor rings.

1.3 Basic Dielectric Theory

The relative permittivity \( \varepsilon = \varepsilon' - i \varepsilon'' \) of a material is a dimensionless complex number (relative to the absolute permittivity of air \( \varepsilon_a \approx 1 \)). The real part \( \varepsilon' \) describes the ability of the material to interact with an external electric field in terms of energy storage (or wave propagation velocity). The imaginary part \( \varepsilon'' \) is the sum of a conductivity term and a relaxation term describing the ohmic and the relaxation losses of the material. The relative permittivity of air and the solid phase (matrix) of a soil are \( \varepsilon_a = 1 \) and \( \varepsilon_m \approx 3 - 5 \) respectively. The permittivity of pure free water at 25°C and frequencies smaller than 1 GHz is \( \varepsilon_w \approx 78 \). Due to the large contrast between \( \varepsilon_w, \varepsilon_a \) and \( \varepsilon_m \), the overall permittivity \( \varepsilon \) of a wet soil is a strong function of the volumetric water content \( \theta \). Thus, permittivity \( \varepsilon \) is an appropriate proxy quantity for determining soil water content. However, it is known that \( \varepsilon \) can not be calculated as the linear weighting of the permittivities of the constituents according to their volume fraction. Therefore, more sophisticated dielectric mixing approaches considering the morphology of the dielectric constituents have to be used.

The permittivity \( \varepsilon \) of a soil is a function of volumetric water content, salinity, temperature, electromagnetic frequency or wavelength, volume fraction of bound and free water related to the specific soil surface area, soil bulk material, and the shapes of the water inclusions (Dobson et al. 1985). Consequently, various empirical and semi-empirical models are used to relate permittivity \( \varepsilon \) with water content \( \theta \) of different soil types.
Figure 1  a) Installation of the access tube in the field; b) EnviroSMART™ soil water content probe with capacitance sensors; c) sensor with symbolized field lines; d) sensor electronic board; and e) equivalent circuit diagram. The dashed line in (e) represents the sensor electronics board, neglecting capacitors and resistors on the board.
1.4 Inductor-Capacitor (L-C) Circuit Theory

Because a sensor reading $R$ is proportional to the corresponding resonant frequency $f_r$ of the sensor, the normalized sensor reading $N$ defined by equation (7) can also be expressed in terms of resonant frequencies. Furthermore, $f_r$ is understood to be the resonant frequency of the LC-oscillator given by the inductance $L$ of the coil mounted on the electronic board and the total sensor capacitance $C$ connected to the sensor electronics:

$$f_r = \frac{1}{\sqrt{uC}}$$  \hspace{1cm} (1)

With $u$ defined as $u = 4\pi^2 L$. The total sensor capacitance $C$ comprises several capacitances representing contributions of the interior of the ring electrodes $C_{\text{int}}$, the access tube $C_{\text{acc}}$ and the part which is sensitive to the environmental permittivity $C_{\varepsilon}$. As illustrated in Figure 1e these capacitances are in series and in parallel to each other. Therefore, the total $C$ connected to the sensor electronics is:

$$C = \frac{1}{C_{\varepsilon}^{-1} + C_{\text{acc}}^{-1}} + C_{\text{int}}$$  \hspace{1cm} (2)

In the following derivation, it will be shown that a change $\Delta C$ of the capacitance $C$ is proportional to the difference $\Delta(f_r^{-2}) = f_r^{-2A} - f_r^{-2B}$ of inverse squared resonant frequencies $f_{rA}$ and $f_{rB}$.

$$\Delta C = \Delta(f_r^{-2}) \cdot \frac{dC}{d(f_r^{-2})}$$  \hspace{1cm} (3)

The proportionality factor $dC/d(f_r^{-2})$ can be expressed using the chain rule as:

$$\frac{dC}{d(f_r^{-2})} = \left( \frac{d(f_r^{-2})}{dC_{\varepsilon}} \cdot \frac{dC_{\varepsilon}}{dC} \right)^{-1}$$  \hspace{1cm} (4)

The derivatives $d(f_r^{-2})/dC_{\varepsilon}$ and $dC_{\varepsilon}/dC$ are calculated from (1) and (2) leading to:

$$\frac{d(f_r^{-2})}{dC_{\varepsilon}} = \frac{u}{(1 + C_{\varepsilon}/C_{\text{acc}})^2} \quad \text{and} \quad \frac{dC_{\varepsilon}}{dC} = (1 + C_{\varepsilon}/C_{\text{acc}})^2$$  \hspace{1cm} (5)

This simple derivation shows that the proportionality factor $dC/d(f_r^{-2}) = u$ in (3) does not depend on $C$ if the equivalent circuit diagram sketched in Figure 1e is used. This is in excellent agreement with the measurements shown in Figure 9b which confirms the validity of the equivalent circuit diagram. Furthermore, relation (3) is used in the Appendix for validating the correction of normalized readings caused by the FEP shrink fit.
2 Methods and Materials

2.1 Resonant Frequency Measurements using Capacitor Devices

To gain an improved understanding and characterization of the instrument, it is desirable to measure the resonant frequency response to known values of added capacitance. “Added” capacitance is emphasized, because the sensor electronic circuit is likely to have its own internal capacitance, which may act in parallel to the ring capacitance (Kelleners et al. 2004).

2.1.1 Installing Surface Mounted Devices (SMDs)

Surface Mounted Devices (SMDs) of known capacitance $C_{SMD}$ values are soldered to one sensor, with and without the ring capacitor attached. $C_{SMD}$ values are selected such that the measured frequency range with the ring capacitor disconnected falls between sensor measurements in air and water (i.e., $1.03\ pF \leq C_{SMD} \leq 21.25\ pF$).

2.1.2 Air Measurements

All measurements with SMD capacitors are taken in air without the plastic access tube. Thus, sensor readings with no added capacitance ($C_{SMD} = 0$) exceed the air count when the probe is inserted into the access tube (i.e., standard procedure for sensor calibration (2001)). The sensor is held at a sufficient distance from other objects to avoid any disturbance of the electrical field when the ring capacitor is attached for the first set of measurements. Subsequently, the ring capacitor is disconnected from the sensor electronics, such that $C_{SMD}$ is the only variable. Each sensor reading $R$ ($25000 < R < 37000$) is converted to a resonant frequency using the linear relationship (Sentek technical support, personal communication, 2004):

$$f_r = 4.096 \cdot 10^{-3} R$$

where $f_r$ has units of MHz and $R$ is dimensionless.

2.2 Definition of Normalized Sensor Reading

The characteristics of individual EnviroSMART™ sensors are not perfectly uniform. As a consequence, the readings $R_a^k$ of different sensors $k$ are different even for identical materials with permittivity $\varepsilon$ surrounding the sensors. To eliminate the effect of these sensor specific differences we normalized the readings $R_{\varepsilon}^k$ of any sensor $k$ to the normalized sensor reading $N^k$ defined as:

$$N^k = \frac{R_a^k - R_{\varepsilon}^k}{R_a^k - R_w^k}$$

where $R_a^k$ and $R_w^k$ are the air and water readings recorded with the EnviroSMART™ sensor $k$ installed in the access tube, surrounded by air and immersed in pure water at 25°C. The normalized sensor reading $N^k$ is a dimensionless number having the values $N^k = 0$ for an air reading and $N^k = 1$ for a measurement in pure water of 25°C.

As it will be shown in section 3.3.1, the normalized sensor readings $N^k$ yield very similar results for different sensors $k$ under identical environmental conditions. This allows for using normalized sensor readings $N = N^k$ to make sensor independent statements.
2.3 Equipment for Dielectric and Capacitance Measurements

2.3.1 Dielectric measurements

Permittivities of solutions with different dioxane-water mixing ratios and temperatures, plus permittivities of EnvironSMART™ sensor components, are measured separately. The permittivity $\varepsilon$ of a material is deduced from the measured reflection coefficient determined by the permittivity of the material in contact with the coaxial electrodes of the dielectric probe (Figure 2).

The commercially available dielectric measurement system HP 85070M is used for these measurements allowing for measuring the intrinsic electrical properties of materials in the RF and microwave frequency bands. The system includes a HP 85070B high-temperature dielectric probe, Vector Network Analyzer HP 8753E (VNA), software and all necessary accessories to measure the complex permittivity of liquids and semi-solids. More details are available from the manufacturer at (http://www.home.agilent.com/USeng/nav/-536894858.536879746/pd.html).

Before measuring environmental data, the system is calibrated by a three-step procedure: a) probe in air (open ended); b) inner and outer electrode of the probe is shorted with a well fitting metal; c) probe immersed into distilled water at 25 °C. A Sucoflex 104 cable is used to connect the VNA with the dielectric probe representing an open-ended coaxial line (Figure 5). Due to the influence of mechanical tension on the phase- response of the cable, it is important to avoid moving this cable after calibrating the system.

We recorded 101 data in the restricted measurement band 100 MHz – 2 GHz. However, the permittivities of the water-dioxane solutions are calculated as the average over the limited frequency range 300 MHz $\leq f \leq$ 500 MHz. These frequencies are higher than the resonant frequency band of the EnviroSMART™ sensor (100 MHz -200 MHz). However, the measured real parts of the permittivities are also representative for these lower frequencies. The 300 MHz restriction is chosen because of increasing uncertainties of the measurements and the limitation to 500 MHz is chosen to avoid relaxation effects causing a reduction of the water permittivity.

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**Figure 2** Sketch of dielectric probe and preparation of the sample of the access tube material for measuring the $\varepsilon_{acc}$. 
2.3.2 Permittivity of Access Tube

The permittivity $\varepsilon_{\text{acc}}$ of the access tube material is measured with the dielectric sensor described in section 2.3.1. The dielectric probe and the preparation of the access tube sample material is sketched in Figure 2. Two slices are cut from the tube, glued together, and flattened at the outer surface. This procedure is required to obtain a sufficient sample thickness of minimum 5 mm at the sensitive region of the dielectric probe and to ensure a tight contact between the sample and the probe. The dielectric sensor is pressed manually onto the sample material while the contact quality is ensured by observing the temporal stability of the reflection coefficient measured with the Vector Network Analyzer (see section 2.3.1).

The permittivity of the access tube material was measured to be $\varepsilon_{\text{acc}} = 3.35$ for frequencies not exceeding 1 GHz. Because the plastic material comprising the access tube is identical with the electrode holder material, the same permittivity $\varepsilon_{\text{hold}} = \varepsilon_{\text{acc}} = 3.35$ is used in the electromagnetic numerical simulations presented in section 3.3.3.

2.3.3 Capacitance Measurements

The capacitance between the two ring electrodes of one EnviroSMART™ sensor is measured with the Vector Network Analyzer (VNA) HP 8751A. The connection from the VNA to the ring electrodes (load) is comprised of a Sucoflex 104 cable connected to a short (≈ 10 cm flexible) 50 $\Omega$ cable soldered directly to the inside of the ring electrodes. Before the measurement, the reference plane (zero phase) is adjusted to the end of the short flexible 50 $\Omega$ cable. The Sucoflex 104 cable is held in the same position for each measurement.

The frequency response of the complex load reflection coefficient $\Gamma_L$ is measured for 201 frequencies $f$ in the range $0.1 \text{ MHz} \leq f \leq 500 \text{ MHz}$. The spectrum of the total capacitance $C$ between the electrodes is calculated from $\Gamma_L$.

The reflection coefficient $\Gamma_L$ measured for a load with impedance $Z_L$ connected behind the reference plane is given by ($Z_0 = 50 \Omega = $ line impedance):

$$\Gamma_L = \Gamma_r + i \Gamma_i = \frac{Z_L - Z_0}{Z_L + Z_0}$$  \hspace{1cm} (8)

The VNA setup allows us to interpret the measurements $\Gamma_L$ as the result of the reflection caused by the mismatch between $Z_0$ and the impedance $Z_L$ of the ring electrodes of the sensor. Because the load is assumed to be exclusively capacitive,

$$Z_L = \frac{-i}{2\pi f C}$$  \hspace{1cm} (9)

combining relation (8) with (9) and solving for the capacitance $C$ allows one to derive the frequency response of the sensor capacitance $C$. 
2.4 Materials Selection, Container Design and Fabrication

The experiments are designed to measure dielectric properties of different well-mixed liquids representative of a range of environmental soil-air-water permittivities. The ideal liquids are water and another liquid fully miscible in water with \( \varepsilon \) near \( \varepsilon_a = 1 \). We selected Dioxane (or Dioxan) with \( \varepsilon_d \approx 2.2 \) (Maurel and Price 1973). Dioxane has a boiling point of 101 °C, melting point of 12 °C, and flash point of 11 °C. The low flash point and high volatility require special handling and ventilation, as well as the need to avoid direct localized heating. Dioxane is also a strong solvent, used as a co-solvent in the pharmaceutical industry, requiring solvent-resistant materials in contact with the liquid. Furthermore, the melting point and flash point limit the range of temperatures tested with the liquid mixtures. The full range of possible permittivities using various dioxane-water mixtures outweighs the difficulties of working with such a chemical.

2.4.1 Design Criteria

The criteria for designing the measurement container depicted in Figure 3 are:

a. solvent-resistant materials for containing the dioxane;
b. a volume large enough to avoid signal disturbance at the container boundaries, yet as small as possible to reduce the amount of dioxane used;
c. robust containment of toxic volatile liquids;
d. ability to cool and heat the liquids for measurements at various temperatures up to 50 °C;
e. complete chemical and thermal mixing capability;
f. visibility for monitoring and removing any accumulation of air bubbles on sensor surfaces; and
g. near-simultaneous measurement of permittivity and capacitance sensor readings.

2.4.2 Components and Tools

Additional functionality required in 2.4.1 is achieved using the following special tools and components (Figure 3):

a. Teflon valve and glass stem at the bottom to drain liquids;
b. Sealable access port on top for adding fluids;
c. Submersible glass 300 W heater (commonly used in fish tanks). The commercial thermostat was shorted to obtain a higher temperature range (up to 50 °C for the present experiments), and a variable load regulator (i.e., “dimmer switch”) was installed to control the heating power. A hot air blower provides additional heating to the sidewalls as needed.
d. Submersible, solvent-resistant housing for the permittivity sensor;
e. Stainless steel thermistor and digital thermometer;
f. Magnetic mixer, including a machined cavity to hold the mixing stir bar in place;
g. Solvent-resistant, submersed brush for cleaning the permittivity probe (Figure 4); and
h. Collar suspended around the access tube for removing air bubbles and providing additional vertical mixing during heating.
Figure 3  Sketch of the solvent-resistant container with the used tools. The heater, dielectric probe, the thermometer, and the tools for removing air bubbles from the access tube and the dielectric probe are retracted during the capacitive measurements and the sensor readings. Dimensions are not to scale.
Items c, d and e above can be retracted during capacitance measurements to avoid signal disturbance. The container (Figure 5) is mounted on three legs, one of which is the mixer, on a table with rollers for transportation to and from a cold room.

### 2.4.3 Simultaneous Measurements of Permittivity and Capacitance

The functional relation $\varepsilon(N)$ between the permittivity $\varepsilon$ of a material outside the EnviroSMART™ soil moisture sensor and the normalized reading $N$ is deduced by measuring the permittivity of the encompassing dioxane-water mixture simultaneously with taking sensor readings. Furthermore, the capacitance $C$ between the two sensor ring electrodes disconnected from the sensor electronics is measured under the same conditions.

Figure 5 shows the experimental setup used for these measurements. The Vector Network Analyzer (VNA) to the right, connected to the dielectric probe as described in section 2.3.1, is used for in-situ measuring the dioxane-water permittivity $\varepsilon$. The dielectric probe is mounted in a sealed tube-housing allowing for immersion of the probe into the solvent without bringing the coaxial cable into contact with the chemically aggressive solvent.

The dielectric sensor is taken out of the sensitive region of the soil moisture sensor while readings are taken. The entire VNA is placed on a lifting table and mechanically connected with the holder of the dielectric probe. Alternating mechanical tension acting on the Sucoflex 104 cable after repositioning the probe is minimized with this setup allowing for in-situ measurements of $\varepsilon$ between capacitance measurements without recalibrating the system.

The uppermost of the five capacitance sensors mounted on the sensor stick is used for direct capacitance measurements. The electronics of this sensor are disconnected from the electrodes and a short 50 $\Omega$ cable is soldered to the inside of the ring electrodes.

The VNA depicted in the left side of Figure 5 is thus connected for measuring the capacitance $C$ independently of the Sentek proprietary electronics and signal processing. Altering mechanical tension acting on the Sucoflex 104 cable is minimized by disconnecting the short coaxial cable to the capacitor from the VNA, such that the Sucoflex cable is always in the same position. Furthermore, the Sucoflex cable is mechanically stabilized at two locations (not shown entirely in Figure 5).
2.5 Cylindrical Metal Interference

To estimate the sampling volume of the sensor, we introduce a cylindrical disturbance at distance $D$ from the access tube. For $0 \leq D \leq 96$ mm sensor readings are taken and compared with the undisturbed reading at the same environmental $\epsilon$. The distance $\Lambda$ representing the sampling volume of the sensor is estimated and compared with numerical calculations (section 3.5).

2.5.1 Mechanical Design and Geometry

The coaxial metal disturbance surrounding the access tube is realized with a brass foil of thickness 0.3 mm. Variable diameters are achieved by rolling up the foil and holding it in place with removable sticks. The soil moisture sensor mounted in the experimental container filled with water and supplemented with metal disturbance is shown in Figure 6a-c, where the coaxial metal disturbance is photographed at mean distances $D = 13.8$ mm, 35.5 mm and 96.3 mm respectively. Nine discrete distances $D$ and four environmental permittivities ($\epsilon = 1, 16.4, 20.3, 78.38$) are realized in the experiment presented in section 3.5.1. The rolled metal foil is held in each position by four vertical rods. The experiment is started with the metal disturbance installed in the tightest position $D_{\text{min}} = 4.55$ mm around the access tube. The larger distances are realized by subsequently removing the four holding rods from the top of the container. All of the rods are removed at the largest distance $D_{\text{max}} = 96.3$ mm where the foil is in contact with the container wall.

Figure 5  Picture of the container constructed for characterizing the EnviroSMART™ Soil Water Content sensor.
The resulting shapes of the brass roll are not perfectly cylindrical (Figure 6d). From the main axes $d_1$ and $d_2$ the corresponding distances $D_1$, $D_2$ from the access tube with diameter 57.4 mm are calculated from:

$$D_k = \frac{d_k - 57.4\,\text{mm}}{2} \quad k = 1, 2$$

(10)

The mean distance $D = (D_1 + D_2)/2$ used in section 3.5.1, the absolute differences $\Delta D = D_1 - D_2$ and the relative difference $\delta D = \Delta D / D$ for each of the nine sizes of the cylinder are shown in Table 1.

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<th>Position $i$</th>
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<th>$\Delta D_i$ [mm]</th>
<th>$\delta D_i$ [%]</th>
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2.6 Axisymmetric Electromagnetic Simulations

The electric field distributions that arise from the two electrodes of the EnviroSMART™ sensor are simulated using the commercial available finite element software Maxwell®2D, which is distributed freely and can be downloaded from http://www.ansoft.com/maxwellsv/. The software allows for calculating the two dimensional field solution. The full three dimensional solution can be calculated from the axially symmetric problem.

Figure 7 shows the rotation symmetrical configuration used for representing the EnviroSMART™ soil moisture sensor. This model setup makes it possible to calculate the total capacitance \( C \) between the two ring electrodes. At the margin of the simulation area, the boundary condition is set to “balloon” which models the case in which the structure is infinitely far away from all other electromagnetic sources. The electric potential of the positive (black) and the negative (white) ring electrodes are set to +1 V and -1 V respectively. This setting does not affect computed \( C \), but the absolute values of the calculated field strength |\( E \)| depicted in Figure 8 are scaled proportionally to the potential difference \( U \) between the electrodes.

The permittivities of the access tube and the electrode holder used in the calculations are set to \( \varepsilon_{\text{acc}} = \varepsilon_{\text{hold}} = 3.35 \). This value was determined experimentally using the procedure described in section 2.3.2. All of the dielectric components are assumed to be lossless, and the metallic components (ring electrodes and brass sheet) are represented by ideal conductors.

The model does not consider air gaps possibly occurring between the electrodes and the access tube nor between the access tube and the environmental material. Furthermore, the actual sensor includes an electronic board inside the electrode holder which is not considered due to its highly asymmetric geometry and unknown dielectric properties. The medium inside the electrode holder is assumed to be the same as air with permittivity \( \varepsilon_a = 1 \). These two model restrictions may lead to an error in the computed \( C \) values.

In section 2.6.1, two versions of the electromagnetic model used for parametric investigations are described: First, the setup for computing the capacitance \( C \) between the two ring electrodes of the sensor embedded in homogeneous environmental material with permittivity \( \varepsilon \) is described (Figure 8a). Second, the setup for computing \( C \) under consideration of an additional coaxial disturbance at distance \( D \) from the access tube of the sensor is presented (Figure 8b).

2.6.1 Calculation of Sensor Capacitances

The surfaces of two electrodes represented by ideal conductors are equipotential with potential difference \( U \), which varies linearly with the total charge \( \pm Q \) accumulated onto the electrode surfaces. Given that

\[
Q \equiv CU
\]

defines the capacitance \( C \) between two conductors. The capacitance \( C \) between the electrodes of the EnviroSMART™ sensor is calculated from the computed electric field \( \mathbf{E} \) and the corresponding dielectric displacement \( \mathbf{D} = \varepsilon \mathbf{E} \) which allows for calculating the electric field energy \( \phi \) storage within a volume (vol):

\[
\phi = \frac{1}{2} \int_{\text{vol}} \mathbf{E} \cdot \mathbf{D} \, dv
\]
Figure 7  Sketch of the rotation symmetrical model setup implemented for simulating the electric field caused by a potential difference $U$ applied to the ring electrodes of the sensor.

This field energy $\phi$ has to be consistent with the energy spent for transferring the charge $Q$ from one electrode to the other. This gives an alternative definition of capacitance $C$ which is used for computing $C$ from the field $E$. The energy needed for transferring the infinitesimal charge $dq$ over the potential difference $U$ is $d\phi = U dq = q dq/C$. Consequently, the energy $\phi$ used to transfer the total charge $Q$ from one conductor to the other is:

$$\phi = \int_0^Q d\phi = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C} = \frac{CU^2}{2} \quad (13)$$
Combining (12) with the last expression of (13) allows for calculating $C$ from the electric field $\mathbf{E}$.

The finite element software calculates capacitances $C_{i,j}$ between electrodes $i, j$ as described above. Thereby, the capacitances $C_{i,j}$ are given in terms of a capacitance matrix $[C_{i,j}]$. The solver calculates $C_{i,j}$ from the fields $\mathbf{E}$ resulting from $U = 1$ V applied to the electrode $i$ and grounding all other electrodes $i \neq j$.

Figure 8 shows cross sections of the field strength $|\mathbf{E}|$ in the sensor region computed for the two described model layouts. In both cases, the permittivity of the environmental material is $\varepsilon = 20$ and the potential difference between the two ring electrodes is $U = 2$ V. The potential of the coaxial metal disturbance placed at the distance $D = 27$ mm from the access tube is defined to be floating (electrically not connected with a fixed potential, e.g., ground).

The fields are concentrated within a narrow range around the sensor electrodes. The highest field strength $|\mathbf{E}|$ actually occurs within the sensor structure (electrode holder and access tube). Just outside the access tube the field $|\mathbf{E}|$ is already reduced by a factor of 10 compared with $|\mathbf{E}|$ in the electrode holder between the positive and the negative ring electrode.

The coaxial metal disturbance leads to a general increase of the field strength $|\mathbf{E}|$ between the access tube and the metal sheet compared to $|\mathbf{E}|$ at the corresponding location of the undisturbed situation. The field outside the metal sheet is shielded ($|\mathbf{E}| = 0$) as the result of the constant potential on the conducting disturbance.

The field energy $\phi$ (12) needed for calculating a capacitance in (13) is computed for the total simulation volume. Consequently, a capacitance comprises contributions of partial capacitances originating from different regions. The contribution of the region inside the electrode holder is the most uncertain due to the electronic board which is not considered in the simulation. An adequate representation of the electronic board would have required full 3d capability of the finite element software. Neglecting the unsymmetrical sensor internal material might be the main reason for deviations between measured and simulated sensor capacitances. Furthermore, internal and external fields are interdependent, such that the inner capacitance cannot be treated as a fixed, parallel capacitor. A more rigorous investigation of these problems goes beyond the goal of this work and is not discussed here.

Capacitances between all the conducting elements $i, j$ ($i \neq j$) of the model are calculated as described above and displayed as a capacitance matrix $[C_{i,j}]$.

According to the two conducting elements (two ring electrodes) comprised in the undisturbed model (Figure 8a) the computed $[C_{i,j}]$ is a $2 \times 2$ matrix. For the model parameters used in Figure 8a the capacitance matrix $[C_{i,j}]$ is (in units of pF):

$$[C_{i,j}] = \begin{bmatrix} 0 & 18 \\ 18 & 0 \end{bmatrix}$$ (14)

The matrix element $C_{1,2} = 18$ pF is the capacitance between the two ring electrodes and thus interpreted as the sensor capacitance $C$. In section 3.3.3 (Figure 14a) computed $C$ for environmental permittivities $\varepsilon$ between air and water are presented.

According to the three conducting components in the model configuration of Figure 8b, $[C_{i,j}]$ is a $3 \times 3$ matrix. For the model parameters used in the field calculation depicted in Figure 8b, $[C_{i,j}]$ is (in units of pF):
Figure 8  Electric field strength $|E|$ resulting from the potential difference $U = 2V$ between the ring electrodes calculated using the model configuration depicted in Figure 7a) without and b) with the metal sheet disturbance at distance $D = 27$ mm from the access tube. Permittivities of access tube, electrode holder and environmental material are $\varepsilon_{acc} = \varepsilon_{hold} = 3.35$ and $\varepsilon = 20$ respectively.
\[
\begin{bmatrix}
0 & 2.85 & 32.25 \\
2.85 & 0 & 32.25 \\
32.25 & 32.25 & 0
\end{bmatrix}
\] (15)

The sensor capacitance is calculated according to the equivalent circuit drawn in Figure 8b:

\[
C = C_{1,2} + \frac{C_{1,3}}{2} = 18.98 \text{ pF} \tag{16}
\]

As expected, this is slightly more than the EnviroSMART\textsuperscript{TM} sensor capacitance calculated for the model without the metal disturbance. The dependence of \( C \) on the distance \( D \) of the metal is evaluated for various permittivities \( \varepsilon = 1, 5, 10, 20, 30, 40, 78.38 \) of the environmental material. The results are presented in section 3.5.2 (Figure 17) in terms of normalized sensor readings and compared with measurements.
3 Results and Discussion

3.1 Surface Mounted Devices

Before taking measurements with liquids of known permittivities outside of the access tube, one sensor was used for measurements with known capacitance $C_{SMD}$ values as described in section 2.1. The results are shown in Figure 9, where the almost perfectly linear responses in Figure 9b are expected for L-C circuits (1), but the non-zero intercept even without the ring capacitor indicates additional capacitance in the electronics. The difference between the linear regression lines in terms of capacitance is approximately 6.3 pF. Because the SMD and ring capacitors were soldered in parallel, this difference is the total ring capacitance in air, including the effects of the sensor electronics and plastic sensor stick (Figure 3) inside the ring, but without the access tube.

Inverting the linear regression equation with the ring capacitor attached (Figure 9b), leads to a useful expression (see Appendix) for the change $\Delta C$ in capacitance with change $\Delta f_r = f_{r,A} - f_{r,B}$ in measured resonant frequency under two different conditions A and B:

$$\Delta C = 2.982 \times 10^3 \cdot \Delta \left( f_r^{-2} \right)$$

(17)

where $\Delta C$ has units of pF and $f_r$ has units of MHz. This empirical result is consistent with the theory presented in Section 1.4.

![Figure 9](image-url) Sensor responses to SMD added capacitance $C_{SMD}$ with and without the ring capacitor attached to the instrument: 

a) Resonant frequency $f_r$, derived from measured $R$ using (6) versus $C_{SMD}$; and

b) $f_r^{-2}$ versus $C_{SMD}$ (linear response).
3.2 Permittivities of Dioxane-Water Mixtures

Permittivities of samples with volumes of 100 ml and volumetric mixing ratios $\phi_d$ from pure water ($\phi_d = 0$) to pure dioxane ($\phi_d = 1$) are measured at 25°C. Each $\varepsilon$ value shown in Figure 10 (hollow circles) represents the average permittivity in the frequency range $300 \text{ MHz} \leq f \leq 500 \text{ MHz}$ as measured with the reflection method described in section 2.3.1. This is slightly higher than the relevant frequency range of the EnviroSMART™ sensor which is between 100 MHz and 150 MHz. However, permittivities can be expected to remain essentially unchanged at lower frequencies (the permittivity of pure water at 25°C and 500 MHz is 78.35 and at 100 MHz it is 78.40 (Meissner and Wentz 2004).

The measured $\varepsilon$ of the mixtures deviate significantly from the linear weighting of the permittivities $\varepsilon_w$ and $\varepsilon_d$ of the water and dioxane constituents. The main nonlinearity is quantified using the power-law fitting approach (e.g., (Sihvola 1999)):

\[
N_{\varepsilon} = 78.4
\]

\[
\varepsilon_{\text{d}} = \frac{2.2}{N_{\varepsilon}}
\]

\[\Delta \varepsilon \left[\text{-}\right] \]

\[\phi_d \left[\text{-}\right] \]

**Figure 10**  Permittivities $\varepsilon$ of solutions with volumetric ratios $\phi_d$ of dioxane before mixing from pure water ($\phi_d = 0$) to pure dioxane ($\phi_d = 1$) measured (hollow circles) with the reflection method (section 2.3.1). The solid line represents the power law fit $\varepsilon(\phi_d)$ (equation (18)). The residuals $\Delta \varepsilon$ are represented by the solid dots and the dashed line is the corresponding polynomial fit (20).
\[
\varepsilon(\phi_d) = \left[ \phi_d \varepsilon_d^{\beta} + (1-\phi_d) \varepsilon_w^{\beta} \right]^{1/\beta}
\] (18)

The optimized parameter \( \beta = 0.813 \) describes the main deviation from the linear dielectric mixing (fine dashed line); \( \varepsilon_w = 78.38 \) and \( \varepsilon_d = 2.2 \) are the literature values of the permittivities of pure water and dioxane respectively (which are in agreement with our measurements within the given uncertainty); \( \phi_d \) is the volumetric dioxane ratio before mixing (might be slightly different from the volumetric dioxane concentration in the dioxane-water solution after mixing):

\[
\phi_d = \frac{V_d}{V_d + V_w}
\] (19)

where \( V_d \) and \( V_w \) are dioxane and water volumes, respectively.

The residuals \( \Delta \varepsilon = \varepsilon - \varepsilon(\phi_d) \) depicted in the lower graph of Figure 10 (solid dots) show a clear additional dependency on \( \phi_d \). From this systematic deviation, one can postulate a physical process that is not considered in the dielectric mixing approach (18) adapted to the measurements. If the residuals are approximated with a polynomial expression \( \Delta \varepsilon(\phi_d) \) (dashed line) and added to the semi-empirical mixing approach (18), the data approximation using \( \varepsilon^*(\phi_d) = \varepsilon(\phi_d) + \Delta \varepsilon(\phi_d) \) can be improved:

\[
\Delta \varepsilon(\phi_d) = k_0 + k_1 \phi_d + k_2 \phi_d^2 + k_3 \phi_d^3 + k_4 \phi_d^4
\] (20)

The computed least square fitting parameters are: \( k_0 = 0.258959, k_1 = 4.8317, k_2 = 7.17616, k_3 = -51.1773, k_4 = 39.2371 \) and the remaining RMSE between the approximation \( \Delta \varepsilon(\phi_d) \) and the data is 0.221. For \( \phi_d < 0.57 \) the residuals \( \Delta \varepsilon(\phi_d) \) are negative and for \( \phi_d > 0.57 \) they are positive. This coincides with the experimental observation that the chemical reactions for \( \phi_d < 0.6 \) are exothermic and endothermic for \( \phi_d > 0 \).

### 3.3 Normalized Sensor Readings and Capacitances

The permittivity \( \varepsilon \) of the material encompassing the EnviroSMART\textsuperscript{TM} sensor affects the capacitance \( C \) and thus the normalized reading \( N \). In the practical application as a soil moisture sensor, the environmental material is represented by soil and its permittivity \( \varepsilon \) acts as a proxy for determining the volumetric water content \( \theta \) of the soil. Therefore, knowing the relation between \( \varepsilon \) and the normalized sensor reading \( N \) is of high practical importance for the quantitative soil moisture detection using adequate models describing the relation between soil water content \( \theta \) and \( \varepsilon \).

Normalized sensor readings \( N \) measured in environmental material with well controlled permittivity \( \varepsilon \) are presented in section 3.3.1. The reference permittivities \( \varepsilon \) are measured with the reflection method described in section 2.3.1. Sensor capacitances measured with a Vector Network Analyzer (VNA) are presented in section 3.3.2. Simulated \( C \) and \( N \) are presented in section 3.3.3 and compared with measurements.

#### 3.3.1 Measured Normalized Sensor Readings

Sensor raw counts \( R_i^k \) were recorded with the four sensors \( k = (1, 2, 3, 4) \) inside the access tube, while permittivity \( \varepsilon_i \) of the liquid encompassing was varied. Sensor readings for \( \varepsilon_i (i = 1 - 22) \) between the dioxane permittivity \( \varepsilon_d \approx 2.2 \) and the water permittivity \( \varepsilon_w \approx 78.38 \) at 25°C were investigated by realizing volumetric dioxane-
water mixing ratios $\phi_d$ from 0 (pure dioxane) to 1 (pure water). The normalized readings $N_i^k$ ($i = 1 - 22$) of the sensors ($k = 1 - 4$) were derived from the four sensor raw counts $R_i^k$ (equation (7)) recorded for the permittivities $\varepsilon_i$. The measurements of $\varepsilon_i$ were taken in-situ with the reflection equipment (see section 2.3.1). For quality control, samples are taken after completing the measurements at each dioxane-water mixing ratio $\phi_d$, to measure the corresponding $\varepsilon_i$ with the reflection method using a static probe setup.

Figure 11 shows the measured relation between the permittivity $\varepsilon_i$ of the dioxane-water mixture encompassing the access tube and the normalized reading $N_i = (N_i^1 + N_i^2 + N_i^3 + N_i^4)/4$ averaged over the four sensors. The data represented by the black circles are the readings $N_i$ corrected for the calculated effect $\Delta N(\varepsilon_i)$ of the FEP coating (see Appendix). The solid lines are the approximations (23) and $\sigma$ is the standard deviation between $N_i^k$ measured with the sensors $k = 1$ to 4.

Figure 11 Measured relation between permittivities of the dioxane-water mixture outside the access tube and normalized sensor readings defined by equation (7). The light grey data $N_i$ are the laboratory measurements and the data $N_i$ represented by the black circles are corrected for the calculated effect $\Delta N(\varepsilon_i)$ of the FEP coating (see Appendix). The solid lines are the approximations (23) and $\sigma$ is the standard deviation between $N_i^k$ measured with the sensors $k = 1$ to 4.

The solid lines in Figure 11 represent the least mean square approximations to the data $[N_i, \varepsilon_i]$ (light grey circles) and the corrected data $[N_i - \Delta N(\varepsilon_i), \varepsilon_i]$ (black circles) ($i = 1 - 22$). The fitting approach considers a quadratic and an exponential factor:
\[ \varepsilon(N) = \left( a_0 + a_1 N + a_2 N^2 \right) \cdot e^{kN} \]  \hspace{1cm} (21)

As will be shown, this approach provides a macroscopically accurate representation of the data within the entire range \(0 \leq N \leq 1\). As indicated by the dashed portion of the fitted lines and discussed below, the fit near \(N = 1\) may be less accurate than at steeper portions of the curve. In accordance with the definition (7) of \(N\), equation (21) must fulfill two constraints:

\[ \varepsilon(N = 0) = \varepsilon_a = 1 \quad \text{and} \quad \varepsilon(N = 1) = \varepsilon_w = 78.35 \]  \hspace{1cm} (22)

This allows for eliminating two of the four fitting parameters \(a_0, a_1, a_2, k\) in (21):

\[ \varepsilon(N) = \left( \varepsilon_a + \left( \varepsilon_w e^{-k} - 1 - a_2 \right) N + a_2 N^2 \right) \cdot e^{kN} \]  \hspace{1cm} (23)

The computed values of the remaining fitting parameters \(a_2\) and \(k\) representing the approximation of the uncorrected data \([N_i, \varepsilon_i]\) are \(a_2 = 1.15008\) and \(k = 6.66056\). For the data \([N_i - \Delta N(\varepsilon_i), \varepsilon_i]\) corrected for the effect of the FEP coating one finds: \(a_2 = 1.12819\) and \(k = 6.64846\). The latter values should be used for calculating permittivities \(\varepsilon(N)\) from field-measured data \(N\).

From the sensor reading (raw count) \(R_\varepsilon\) of a sensor placed in an environmental material with unknown permittivity \(\varepsilon\), one can calculate the normalized reading \(N\) using the definition (7) with the air and water counts \(R_a\) and \(R_w\) of the sensor. From this, the permittivity \(\varepsilon\) of the environmental material can be estimated using relation (23). This is of practical importance for the quantitative estimation of volumetric soil water content \(\theta\) from the sensor readings. The applicability of relation (23) is limited to low permittivities \((\varepsilon < 40)\) due to increasing uncertainty of calculated \(\varepsilon\) from measured sensor readings \(N\) as the result of decreasing sensitivity of \(N\) with respect to \(\varepsilon\) at higher permittivities (dashed line in Figure 11). However, permittivities of soils with realistic water contents are typically smaller than 40 allowing for estimating \(\varepsilon\) from \(N\). An exception is the estimation of \(\varepsilon_w\) at different temperatures (section 3.4).

The resulting Root Mean Squared Error \((RMSE)\) between the corrected data \(N_i - \Delta N(\varepsilon_i)\) and the approximation \(\varepsilon(N)\) evaluated at \(N = N_i - \Delta N(\varepsilon_i)\) and the measured permittivities \(\varepsilon\) is \(RMSE_\varepsilon = 0.859\) for \(1 < \varepsilon < 80\) and only \(RMSE_\varepsilon = 0.226\) for the measured range of \(3 < \varepsilon < 43\) pertaining to soil water. The overall \(RMSE_\varepsilon\) of 0.859 is affected primarily by one large deviation at \(\varepsilon(N = 0.9925) = 77.12\). If this is excluded the \(RMSE_\varepsilon\) is 0.314 for \(1 < \varepsilon < 64\) which is more indicative of the expected deviations.

### 3.3.2 Measured Sensor Capacitances

The capacitance \(C\) between the electrodes of one EnviroSMART\textsuperscript{TM}\ sensor is computed from VNA measurements of the complex reflection coefficient \(\Gamma_L\) as it is described in section 3.3.3. Figure 12a shows the imaginary part \(\Gamma_i\) plotted versus the real part \(\Gamma_r\) (polar diagram, Smith Chart) of the reflection coefficient \(\Gamma_L = \Gamma_r + i \Gamma_i\) measured for \(\varepsilon = 20\).

Due to resonance phenomena in the ring electrode circuit occurring at various frequencies (see loops in the polar diagram of Figure 12a) we assumed the average of the measurements in the frequency range \(80\) MHz \(\leq f \leq 100\) MHz (bold dots in Figure 12b) to be representative for the low-frequency sensor capacity \(C\). Figure 13 shows the resulting low-frequency sensor capacities \(C\) measured at the same
**Figure 12**  
(a) Polar diagram (Smith Chart) for \(0.1 \text{ MHz} \leq f \leq 500 \text{ MHz}\) measured for the ring electrodes. The reflection coefficient data \(\Gamma_L = \Gamma_r + i \Gamma_i\) between 80 MHz – 100 MHz (bold dots) are used for estimating the capacitance \(C\) at low frequencies. The dashed line is the unit circle for \(1 = \Gamma_r^2 + \Gamma_i^2\).

(b) Sensor Capacitance \(C\) versus frequency \(f\) deduced from \(\Gamma_L\) using relations (8) and (9). \(C\) is calculated as the average of the capacitances between 80MHz – 100MHz (bold dots).

**Figure 13**  
Measured relation between sensor capacitance \(C\) and permittivity \(\varepsilon\) of the dioxane-water mixture outside the access tube.
environmental permittivities $1 \leq \varepsilon \leq 78.4$ as the normalized sensor readings presented in 3.3.1.

The relative values of $C$ and shape of the $C(\varepsilon)$ curve in Figure 13 give the following insights: 1) $C(\varepsilon = 1)$ is approximately half of the value of $C(\varepsilon = 25)$, which may represent a relatively wet soil. Thus, more than half of the total capacitance measured in soils is instrumental (affected by the inner ring capacitance and the access tube); and 2) $C(\varepsilon)$ is very nonlinear in this range, so the geometric factor relating $C$ to $\varepsilon$ is not a constant, which indicates large changes in the electromagnetic field pattern with changes in the environmental $\varepsilon$ values. These factors are related to the instrumental design and affect the measurement sensitivity and accuracy.

### 3.3.3 Calculated Capacitances and Normalized Sensor Readings

The experimental results presented in the previous sections 3.3.1 and 0 are modeled using the procedure described in section 2.6. The stars in Figure 14 show sensor capacitances $C_{\text{model}}$ computed for a set of permittivities $1 \leq \varepsilon \leq 78.38$ between air and water.

![Figure 14](image)

**Figure 14**

a) Modeled and measured capacitances $C_{\text{model}}$ and $C_{\text{exp}}$ between the ring electrodes as a function of the permittivity $\varepsilon$ of the environmental material outside the access tube. The model setup described in section 2.6 is used with $\varepsilon_{\text{acc}} = \varepsilon_{\text{hold}} = 3.35$.

b) Modeled normalized sensor readings $N_{\text{model}}$ and normalized readings $N_{\text{exp}}$ measured in the laboratory (with the FEP shrink tube on the access tube) together with the interpolation $\varepsilon(N_{\text{exp}})$. 
Using the relationship between resonant frequency and capacitance, the normalized sensor reading \( N \) can be expressed by the sensor capacitances \( C_a, C_w \) and \( C_\varepsilon \) if it is embedded in air, water and an arbitrary material with permittivity \( \varepsilon \):

\[
N = \frac{1}{\sqrt{C_a}} \frac{\frac{1}{1}}{\sqrt{C_\varepsilon}} \frac{1}{\sqrt{C_w}}
\]

(24)

The asterisks in Figure 14b show normalized sensor readings \( N_{\text{model}} \) calculated from relation (24). Thereby, the modeled capacitance \( C_{\text{model}} \) for \( 1 \leq \varepsilon \leq 78.38 \) and \( C_a = 4.62 \text{ pF} \) for \( \varepsilon = \varepsilon_a = 1 \) and \( C_w = 24.65 \text{ pF} \) for \( \varepsilon = \varepsilon_w = 78.38 \) (at 25°C) are used.

The corresponding experimental data \( C_{\text{exp}} \) (circles), \( N_{\text{exp}} \) (circles) and the data fit \( \varepsilon(N_{\text{exp}}) \) (solid line) already presented in Figure 11 and Figure 13 are also shown in Figure 14 for comparison.

The disparity between measured and simulated results is likely due to the two factors mentioned in section 2.6: 1) air gaps between the sensor (capacitor rings) and the access tube; and 2) neglecting the unknown internal capacitance due to the electronic board. Indeed, Figure 14a shows a large \( C \) value for \( \varepsilon = 1 \) relative to the total \( C \) at higher \( \varepsilon \) values. In the following using relation (24) for comparing experimental with calculated normalized sensor readings is avoided and explaining the discrepancy is left for future investigations.

3.4 Temperature Dependence of Environmental Permittivity

Below the relaxation frequency (which is below 10 GHz), the permittivity of pure free water \( \varepsilon_w \) is known to decrease with increasing temperature \( T \). Qualitatively this is explained by the increasing thermal distortion of the dipoles with increasing temperature which hinders the alignment of the water molecules with the applied electric field. Meissner and Wentz (Meissner and Wentz 2004) provide a semi empirical model for measured values of \( \varepsilon_w \) in terms of an approach based on two Debye relaxation frequencies. Water permittivity measured in the temperature range between -20°C and +40°C including supercooled water are reproduced for frequencies up to 500 GHz. The temperature gradient of approximately \( d\varepsilon_w/dT \approx -0.35 \text{ K}^{-1} \) for frequencies smaller than 500 MHz can be estimated from the model.

With regard to field experiments where this soil moisture sensor is applied, investigating the temperature dependency of the permittivity \( \varepsilon(T) \) of environmental material is of particular importance (Or et al. 1999, Baumhardt et al. 2000, Logsdon and Laird 2004). The temperature dependencies of pure water \( \varepsilon_w(T) \), pure dioxane \( \varepsilon_d(T) \) and a mixture of 98% dioxane and 2% water \( \varepsilon_{dw98}(T) \) are estimated from measured sensor readings, which are converted to permittivities using the fit (23). Permittivity measurements are performed simultaneously using the reflection method described in section 2.3.1. Therefore, the permittivity sensor is installed in the solution as depicted in Figure 5. Figure 15a shows \( \varepsilon_w(T) \) measured in-situ with the reflection method (solid dots) and \( \varepsilon_w(T) \) deduced from the sensor readings (hollow circles) for 5°C ≤ \( T \) ≤ 50°C. Both water permittivities \( \varepsilon_w(T) \) show a negative gradient \( d\varepsilon_w/dT \) as predicted from model. The linear regression of \( \varepsilon_w(T) \) measured with the VNA electromagnetic reflection method yields a gradient of \( d\varepsilon_w/dT \approx -0.36 \text{ K}^{-1} \) which quantitatively agrees with the model.
Figure 15  

a) Temperature dependency $\varepsilon_w(T)$ of the permittivity of pure water ($\phi_d = 0$) measured with the VNA dielectric sensor (solid dots) and derived from the capacitance sensor readings (hollow circles).

b) Temperature dependencies $\varepsilon_d(T)$ and $\varepsilon_{d_{98}}(T)$ of pure dioxane ($\phi_d = 1$, hollow squares) and a solution with $\phi_d = 0.98$ (hollow diamonds) deduced from the capacitance sensor readings.
One could correct the slope $d\varepsilon_w/dT$ based on the fit of $N(\varepsilon)$ near $\varepsilon_w = 80$ by adjusting the parameter values in (23). However, this would not be useful for all $\varepsilon$ values. Another option is to find a different mathematical form for the calibration equation, which is beyond the scope of this report. It is worth noting here that $d\varepsilon_w/dT$ is negative even without localized fitting, which is not the case for measurements of soil water in the field. Thus, the laboratory experiment confirms that the dominant field results are not due to instrumental error.

The temperature dependencies of pure dioxane $\varepsilon_d(T)$ and of the 98% dioxane-water mixture $\varepsilon_{dw98}(T)$ for $12^\circ C \leq T \leq 25^\circ C$ are depicted in Figure 15. Dioxane permittivities $\varepsilon_d(T)$ display no temperature dependence within the accuracy of our measurements. The literature value for the permittivity of dioxane is $2.20 \pm 0.11$ (Maurel and Price 1973). This is consistent with the average $\varepsilon_d = 2.089 \pm 0.008$ deduced from the 9 sensor readings at the temperature between $12.5^\circ C$ and $25.4^\circ C$. The average permittivity $\varepsilon_{dw98}$ and the temperature gradient $d\varepsilon_{dw98}/dT$ determined from the readings of the sensors immersed in the 98 % dioxane-water mixture at 8 temperatures between $13.7^\circ C$ and $22.4^\circ C$ is $\varepsilon_{dw98} = 2.679 \pm 0.026$ and $d\varepsilon_{dw98}/dT = -0.009 K^{-1}$ respectively. The permittivity of the solution estimated from the volumetric mixing ratio $\phi_d = 0.98$ (for $V_d = 8820 ml$, $V_w = 180 ml$ with relation (19)) using the dielectric power law (18) is $\varepsilon_{dw98} = 3.074$ which is within the uncertainty $\Delta \varepsilon \approx 0.15$ at $\phi_d = 0.98$ of the model (Maurel and Price 1973). If one assumes a linear effect of water on the temperature dependence, the estimated slope would be -0.012 based on using (23) to estimate $\varepsilon(N)$. This value is close to -0.009 from the direct measurements.

### 3.5 Characteristic Distance of Influence

The field caused by the voltage between the two ring electrodes is concentrated in a narrow region around the electrodes (see Figure 8). Consequently, the capacitance determining the sensor reading is influenced predominantly by the permittivity $\varepsilon$ of the environmental material that is in the vicinity of the access tube. Measuring the effect of a dielectric disturbance located at distance $D$ within the outside material allows for indicating a distance $\Lambda$ representing the characteristic distance of influence of the sensor. Such sensitivity information is relevant for practical application of the sensor.

#### 3.5.1 Measured Distance of Influence

A concentric metal sheet with changeable diameter is installed around the access tube as described in section 2.5. This setup enables measuring the change in sensor readings as a result of an extreme coaxial disturbance in the environmental material. Figure 16 shows the normalized sensor readings $N = N_{exp}(D)$ measured in the lab plotted versus the distance $D$ between the access tube and the coaxial metal sheet. We performed measurements at environmental permittivities $\varepsilon = 1, 16.4, 20.3, 78.38$ each with 10 distances $0 \leq D \leq 96 mm$, and without any metal disturbance (i.e., $D = \infty$).

When the metal sheet is in contact with the access tube, the normalized sensor readings $N_{exp}(D = 0) = N_0 \approx 1.02$ are the same for all $\varepsilon$. This is due to the fact that the electric field outside the conducting sheet is totally screened and thus the external permittivity $\varepsilon$ has no effect on the total capacitance.
The measurements \( N_{\text{exp}}(D = 96.3 \text{ mm}) \) are taken at the largest possible distance limited by the diameter of the container and \( N_{\infty} = N_{\text{∞}} \) are the undisturbed normalized readings. As can be seen, the diameter of the container is large enough to ensure that the outer container wall does not affect the sensor readings. The permittivities \( \varepsilon = (\varepsilon_a = 1, 16.4, 20.3, \varepsilon_w = 78.38) \) of the dioxane-water mixtures are determined from the corresponding \( N_{\infty} \) using the fit (23).

To estimate \( \Lambda \) representing the characteristic distance of influence of the sensor at the investigated permittivities, the experimental data \([D, N_{\text{exp}}]\) are fitted using the following approach (with \( N_0 = 1.02 \) and \( N_{\infty} = 0, 0.752, 0.804, 1 \)):

![Figure 16](image)

**Figure 16** Normalized sensor reading \( N \) measured for 10 distances \( D = (0, 4.55, 9.3, 13.8, 20.55, 35.05, 47.3, 66.55, 81.05, 96.3) \text{ mm} \) between the access tube and the metal sheet and for the permittivities \( \varepsilon = (\varepsilon_a = 1, 16.4, 20.3, \varepsilon_w = 78.38) \). The solid lines are the interpolations using the approach (25).

The measurements \( N_{\text{exp}}(D = 96.3 \text{ mm}) \) are taken at the largest possible distance limited by the diameter of the container and \( N_{\text{exp}}(D = \infty) = N_{\infty} \) are the undisturbed normalized readings. As can be seen, the diameter of the container is large enough to ensure that the outer container wall does not affect the sensor readings. The permittivities \( \varepsilon = 16.4 \) and \( \varepsilon = 20.3 \) of the dioxane-water mixtures are determined from the corresponding \( N_{\infty} \) using the fit (23).

To estimate \( \Lambda \) representing the characteristic distance of influence of the sensor at the investigated permittivities, the experimental data \([D, N_{\text{exp}}]\) are fitted using the following approach (with \( N_0 = 1.02 \) and \( N_{\infty} = 0, 0.752, 0.804, 1 \)):

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>( \Lambda ) [mm]</th>
<th>( \sigma ) [10^{-3}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_a = 1 ) (Air)</td>
<td>4.321</td>
<td>22.9</td>
</tr>
<tr>
<td>16.4</td>
<td>12.259</td>
<td>5.51</td>
</tr>
<tr>
<td>20.3</td>
<td>13.176</td>
<td>5.65</td>
</tr>
<tr>
<td>78.38 (water @ 25°C)</td>
<td>55.957</td>
<td>3.53</td>
</tr>
</tbody>
</table>

**Table 2** Fitting parameters \( \Lambda \) and standard deviations \( \sigma \) between the approximations and the measured data plotted in Figure 16 according to approximation (25).
\[ N(D) = N_{\infty} + (N_0 - N_{\infty}) \cdot \exp \left( -\frac{D}{\Lambda} \right) \]  

(25)

The solid lines in Figure 16 show the approximations corresponding to the above approach. The calculated least square fit parameter values for \( \Lambda \) and the corresponding RMSE values between the approximations and the measured values are listed in Table 2 for the data measured at \( \varepsilon = 1, 16.4, 20.3, 78.38 \).

The data measured at the permittivities \( \varepsilon = 16.4 \) and 20.3 are approximated well by the simple exponential approach (25) whereas the exponential approximations of the data measured at \( \varepsilon = 1 \) and 78.38 are not adequate. For the air environment this accounts for an extremely low value of \( \Lambda \approx 4.321 \text{ mm} \) and the relatively large fitting error. For the pure water environment, the exponential fit is not adequate due to the almost constant \( N \)-values for different distances \( D \).

However, realistic permittivities \( \varepsilon \) of natural soils are typically between 4 and 25 for very dry and water saturated soils respectively. The characteristic distance of influence \( \Lambda \) calculated from the data measured within this range (\( \varepsilon = 16.4 \) and 20.3, bold solid lines) is approximately 12 - 13 mm.

Where the exponential model (25) is appropriate, it may be helpful to think about the percent difference induced by the metal border and different distances. The relative errors at \( D = \Lambda, 2\Lambda, \) and \( 3\Lambda \) are 37\%, 14\%, and 5\%, respectively. For the fitted exponential influence with distance of the medium around the access tube, we expect 95\% of the signal to be based on material within 36 mm of the access tube. If there are heterogeneities in a soil system, the soil nearest the access tube has the greatest influence. Likewise, any soil disturbances (e.g., air gaps, stones) immediately outside the access tube will be weighted heavily, and the exponential model could be used to predict such effects, particularly when combined with (23) to predict effects on the apparent permittivity.

### 3.5.2 Calculated Distance of Influence

The characteristic distance \( \Lambda \) of a sensor embedded in the environmental material with permittivity \( \varepsilon \) is computed and compared with the measurements presented in 0. For this purpose, sensor capacitances are computed for different distances and a set of permittivities between air and water using the model described in section 2.6.

The resonant frequency \( f_r \) of an LC-oscillator is given by the basic expression (1) The capacitance \( C \) is derived from the electrical field computed with the finite element software considering the metal sheet disturbance (see section 2.6, Figure 8b).

Twelve finite distances \( 0 \leq D \leq 162 \text{ mm} \) are simulated, each with permittivities \( \varepsilon = 1, 5, 10, 20, 30, 40, 78.38 \).

The undisturbed \( C_{\infty} \) with \( D = \infty \) are calculated without the metal sheet present. Modeled normalized sensor readings \( N \) are plotted in Figure 17 and interpolated using the fitting approach (25) likewise the measured \( N \) depicted in Figure 16.

For the calculations with \( \varepsilon > 5 \), the exponential interpolation (25) is better adapted than for \( \varepsilon \leq 5 \). This is consistent with the analysis performed for the measured \( N \). The values of the fitting parameter \( \Lambda \) representing the characteristic distances of influence and the corresponding standard deviations \( \sigma \) between the interpolation and the model results \( N \) are listed in Table 3. The characteristic distances \((10 < \Lambda < 13 \text{ mm})\) deduced from the electromagnetic calculations are of the same order of magnitude as \( \Lambda \) deduced from the experiment with \( \varepsilon = 16.4 \) and 20.3 \((12 < \Lambda < 13 \text{ mm})\).
Table 3  

Fitting parameters $\Lambda$ and standard deviations $\sigma$ between the interpolations (25) and the calculated data plotted in Figure 17.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\Lambda$</th>
<th>$\sigma [10^{-3}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Air)</td>
<td>6.678</td>
<td>33.68</td>
</tr>
<tr>
<td>5</td>
<td>10.09</td>
<td>11.05</td>
</tr>
<tr>
<td>10</td>
<td>11.27</td>
<td>6.007</td>
</tr>
<tr>
<td>20</td>
<td>12.07</td>
<td>2.914</td>
</tr>
<tr>
<td>30</td>
<td>12.44</td>
<td>1.992</td>
</tr>
<tr>
<td>40</td>
<td>12.61</td>
<td>1.455</td>
</tr>
<tr>
<td>78.34 (water @ 25°C)</td>
<td>13.12</td>
<td>1.009</td>
</tr>
</tbody>
</table>

Figure 17  

Normalized sensor output $N(D)$ calculated from the electromagnetic model. The distance $D$ between the access tube and the metallic cylinder is varied between 0 mm and 162 mm and the permittivity $\varepsilon$ was between the air and the water value.
3.6 Comparison with Default Calibration for Water Content

A power law calibration function is proposed by the vendor of the capacitive soil moisture sensor (2001), for relating normalized sensor reading \( N \) to volumetric soil water content \( \theta \). A default calibration function \( N(\theta) \) with the three parameters \( a = 0.1957, \ b = 0.404 \) and \( c = 0.02852 \) allows for estimating water content \( \theta \) of sands, loams and clay loams from measured normalized sensor readings \( N \):

\[
N = a \left(100 \cdot \theta \right)^b + c
\]

As stated by the vendor, the default calibration (26) may not be adequate for all soil types. Separate calibration has to be performed for soils with very high specific surface area or different textural layers within the sensitivity volume of the sensor. Figure 18 shows volumetric soil water content \( \theta \) calculated from \( N \). The default calibration function (26) is represented by the black solid line.

For comparison we calculated soil moisture content \( \theta(\varepsilon(N)) \) using three different dielectric mixing models for relating soil permittivity \( \varepsilon \) with water content \( \theta \) from (23) which gives the relationship between the environmental permittivity \( \varepsilon \) and the normalized reading \( N \). The thin solid line in Figure 18 shows the data \( \theta \) calculated using the empirical dielectric mixing model of (Topp et al. 1980):

\[
\varepsilon = 3.03 + 9.3 \theta + 146.0 \theta^2 - 76.7 \theta^3
\]

Figure 18  Comparison between Sentek default calibration (26) proposed by the vendor and the water content \( \theta(\varepsilon(N)) \) calculated from three different dielectric mixing models \( \theta(\varepsilon) \) ((Topp et al. 1980), (Roth et al. 1990), (Wang and Schmugge 1980)) using relation (23) for \( \varepsilon(N) \).
This model is purely empirical, but it performs sufficiently well below the relaxation frequency of water (below 10 GHz) for coarse-textured soils with maximum specific matrix surface of 100 m²g⁻¹ and bulk density in the range 1.35 – 1.5 g·cm⁻¹. The main advantage of this relation is that it does not require any additional soil parameters.

The moisture data represented by the dashed line are computed using the semi-empirical mixing model of (Roth et al. 1990). This model is based on an empirical power-law dielectric mixing approach considering the aqueous, solid and gaseous soil phases with corresponding permittivities \( \varepsilon_w, \varepsilon_s, \) and \( \varepsilon_a \) and volumetric fractions \( \theta, (1 - \eta) \) and \( (\theta - \eta) \):

\[
\varepsilon = \left[ \theta \cdot \varepsilon_w^a + (1 - \eta) \cdot \varepsilon_s^a + (\eta - \theta) \cdot \varepsilon_a^a \right]^{1/\alpha}
\]  

(28)

The soil porosity \( \eta \) has to be determined experimentally (here \( \eta = 0.46 \)) and the exponential \( \alpha = 0.46 \pm 0.007 \) was determined from a nonlinear regression applied to measured data.

The gray solid line in Figure 18 shows the calculated water content when the model of (Wang and Schmugge 1980) is applied. This semi-empirical model considers textural effects (clay and sand content) in terms of an adjustable transition point \( \theta_t \) dividing the water content range into two domains. This accommodates the experimental evidence that for \( \theta < \theta_t \) the soil permittivity \( \varepsilon \) increases slowly and for \( \theta > \theta_t \) it increases more rapidly with \( \theta \). The transition point \( \theta_t \) is higher for soils with high clay content than for soils with high sand content. A linear three phase mixture (aqueous, solid and gaseous soil phases) is applied for \( \theta < \theta_t \) whereas four phases

![Figure 19](image-url)

**Figure 19**  Comparison between water content- permittivity (\( \varepsilon-\theta \)) relationships derived from the Sentek default calibration (26) and dielectric mixing models.
are considered for $\theta > \theta_t$. The fourth phase represents the bound water in the vicinity of the solid phase, where water is assumed to be paracrystalline resulting in reduced permittivity.

The water content $\theta$ values derived from the default calibration (26) are biased relative to $\theta$ values calculated from the three dielectric mixing models in combination with relation (23). Where Topp's equation is accurate, the Sentek default calibration equation underestimates $\theta$ for the full range of typical field water contents ($0.05 < \theta < 0.50 \, \text{m}^3\text{m}^{-3}$).

The deviation between the water content derived from the Sentek default calibration and the water content calculated using the Topp model is defined as:

$$\Delta \theta_{\text{Topp}}(N) \equiv \theta_{\text{Sentek}}(N) - \theta_{\text{Topp}}(\varepsilon(N))$$

where $\varepsilon(N)$ is the data fit given by equation (23); $\theta_{\text{Sentek}}(N) = 0.01((N-c)/a)^{1/b}$ is the inverted default calibration (26); and $\theta_{\text{Topp}}(\varepsilon) = 4.3 \cdot 10^{-6} \cdot \varepsilon^3 - 5.5 \cdot 10^{-4} \cdot \varepsilon^2 + 2.92 \cdot 10^{-2} \cdot \varepsilon - 5.3 \cdot 10^{-2}$ is the Topp model inverse of (27).

The $\theta$-interval boundaries $\theta_{\text{min}}$ and $\theta_{\text{max}}$ are the minimum and maximum water contents for which the RMS deviation $\Delta \theta_{\text{RMS}}$ is calculated. $N$-values $N_{\text{min}}$ and $N_{\text{max}}$ corresponding to $\theta_{\text{min}}$ and $\theta_{\text{max}}$ are computed using the Sentek default calibration function (26).

The RMS deviation $\Delta \theta_{\text{RMS}}$ for a $\theta$-interval $[\theta_{\text{min}}, \theta_{\text{max}}]$ is calculated as the following integral which is solved numerically:

$$\Delta \theta_{\text{RMS}} = \sqrt{\frac{\int_{N_{\text{min}}}^{N_{\text{max}}} \{\Delta \theta_{\text{Topp}}(N)\}^2 \cdot dN}{N_{\text{max}} - N_{\text{min}}}}$$

For $0.05 \leq \theta \leq 0.5$ $\Delta \theta_{\text{RMS}}$ equals $0.066 \, \text{m}^3\text{m}^{-3}$, which reflects the negative bias in Figure 18.

Another comparison between the default calibration and soil dielectric mixing models is presented in Figure 19 in terms of the $\varepsilon-\theta$-relation. The bold solid line is computed using the Sentek default calibration (26) for calculating $N$ from $\theta$ first, and then applying relation (23) for calculating $\varepsilon$. For $\theta > 0.3 \, \text{m}^3\text{m}^{-3}$ the discrepancy between the dielectric mixing model-based permittivities and $\varepsilon$ deduced from the Sentek default calibration increases rapidly. The latter reaches unrealistically high values of $\varepsilon$ at very moist soil conditions.

This confirms Sentek’s assertion that power-law form used for the default calibration equation is not appropriate for soils with very high water contents (e.g., clays). In our experience with sandy to silty loams, however, the default calibration yields feasible values of $\theta$ even near saturation ($\sim 0.45 \, \text{m}^3\text{m}^{-3}$). The problem illustrated here involves inversion of (26) to obtain $\varepsilon$. Given our present experimental results, one can now compute $\varepsilon$ directly from $N$ using (23). Therefore, the real problem is an underestimation of $\theta$ at high $\varepsilon$ values. For $\varepsilon > 25$, the default calibration yields values more than $0.05 \, \text{m}^3\text{m}^{-3}$ below those expected from Topp’s model. Users should be aware of these potentially substantial errors and avoid using (25) with the default parameters under such conditions.
4 Summary and Conclusions

The experiments described above led to the following main results:

- An ideal relationship between sensor resonant frequency $f_r$ and SMD capacitance $C_{SMD}$ was established. It confirmed a linear relationship ($R^2 = 1.000$) between $f_r^2$ and $C_{SMD}$, but with a non-zero intercept, implying a parallel capacitance in the circuit, even without having the ring capacitor connected.

- Capacitance sensors displayed appropriate temperature $T$ dependence of the permittivity of free water $\varepsilon_w$, where $\varepsilon_w(T)$ was very linear with a negative slope. Thus, positive slopes in soils under field conditions are not instrumental artifacts. There was no apparent temperature sensitivity in pure Dioxane.

- Dioxane-water mixtures provided a range of $\varepsilon$ values representative of soils from dry to wet ($-5 < \varepsilon < 40$).

- Sensor readings can be normalized using air and water readings to give a new relationship with permittivity: $\varepsilon(N) = \left(\varepsilon_a + (\varepsilon_w e^{-k} - 1 - a_2)N + a_2 N^2\right) e^{kN}$ (see equation (23) in section 3.3.1 for variable definitions), where fitting parameters were found to be: $a_2 = 1.1282$ and $k = 6.6485$. The $RMSE_\varepsilon$ between fitted and measured permittivities was 0.859 for the full range of measurements ($1 < \varepsilon < 80$), and only 0.226 for the range expected for soil-water measurements ($3 < \varepsilon < 43$).

- The results for $\varepsilon$ correspond to an $RMSE_\theta$ for soil water content $\theta$ if Topp’s equation is used for $\varepsilon(\theta)$. For the soil relevant range ($3 < \varepsilon < 43$), we found $RMSE_\theta = 0.0034$ m$^3$m$^{-3}$.

- Metal border experiments were simulated using an axisymmetric numerical model. Using an exponential equation (25) to represent the signal disturbance, the characteristic annular distance is estimated to be 12 mm, and 95% of the signal is sensed within 36 mm of the access tube.

- Different models for $\theta(\varepsilon)$ are compared with the manufacturer’s default calibration, showing that the default calibration tends to underestimate $\theta$ by approximately 0.06 m$^3$m$^{-3}$.

The quantitative results from these laboratory experiments provide the information needed to analyze field measurements using EnviroSMART™ probes in terms of soil permittivity and its temperature dependence.
5 Appendix

Fluorinated Ethylene-Propylene (FEP) heat-shrink tubing was placed around the plastic EnviroSMART™ access tube to protect it from the chemically aggressive Dioxane solvent (see Figure 3). The influence of the FEP coating (permittivity $\varepsilon_{FEP} \approx 2$, thickness $\approx 0.5$ mm) on the measured permittivity $\varepsilon$ is estimated using the electromagnetic field simulation software described in section 2.6.

Figure 20a shows capacitances $C_{\text{model}}(\varepsilon)$ computed for environmental permittivities $1 \leq \varepsilon \leq 78.38$ with (hollow circles) and without (solid squares) the FEP shrink fit coating on the access tube. As can be seen in Figure 21a the absolute value of the difference $\Delta C(\varepsilon) = C_{\text{model with}}(\varepsilon) - C_{\text{model without}}(\varepsilon)$ is small when the contrast between $\varepsilon$ and $\varepsilon_{FEP}$ is small but increases for larger $\varepsilon$. Furthermore, the difference $\Delta C(\varepsilon)$ is positive for $\varepsilon \leq \varepsilon_{FEP} = 2$ and negative for $\varepsilon \geq \varepsilon_{FEP}$. This is in accordance with expectations that capacitance is increased by the presence of the FEP shrink fit for $\varepsilon \leq \varepsilon_{FEP}$ and vice versa (see also Table 4).

---

**Figure 20**

a) Calculated sensor capacitances $C_{\text{model, with FEP}}(\varepsilon)$ and $C_{\text{model, without FEP}}(\varepsilon)$ for the case where the access tube is coated (hollow circles) and not coated (solid squares) with the 0.5 mm thick the FEP shrink fit with permittivity $\varepsilon_{FEP} = 2$.

b) Normalized sensor readings $N_{\text{model, with FEP}}(\varepsilon)$ and $N_{\text{model, without FEP}}(\varepsilon)$ derived from $C_{\text{model, with FEP}}(\varepsilon)$ and $C_{\text{model, without FEP}}(\varepsilon)$ using (24) for the case with (hollow circles) and without (solid squares) the shrink fit on the access tube.
The effect of the FEP coating on the measured permittivity \( \varepsilon \) is expressed as the RMSE \( \Delta \varepsilon_{\text{RMS}} \) calculated from the difference \( \Delta \varepsilon(N) = \varepsilon(N_{\text{model, with FEP}}) - \varepsilon(N_{\text{model, without FEP}}) \) computed from the data shown in Figure 20b:

\[
\Delta \varepsilon_{\text{RMS}} = \sqrt{\frac{\int_{N_{\text{min}}}^{N_{\text{max}}} (\Delta \varepsilon(N))^2 dN}{N_{\text{max}} - N_{\text{min}}}}
\]  

(31)

The integration interval \([N_{\text{min}}, N_{\text{max}}]\) is related to corresponding permittivity and soil moisture regimes \([\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]\) and \([\theta_{\text{min}}, \theta_{\text{max}}]\). The interval \([\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]\) is calculated from the boundaries \(\theta_{\text{min}}\) and \(\theta_{\text{max}}\) using the Topp model (27) and \(N_{\text{min}}, N_{\text{max}}\) are calculated from evaluating the interpolation \(N_{\text{model, without FEP}}\) from Figure 20b at the permittivities \(\varepsilon_{\text{min}}\) and \(\varepsilon_{\text{max}}\).

Figure 20b shows normalized sensor readings \(N_{\text{model, with FEP}}\) and \(N_{\text{model, without FEP}}\) derived from \(C_{\text{model, with FEP}}\) and \(C_{\text{model, without FEP}}\) for the case with (hollow circles) and without (solid squares) the shrink fit on the access tube. These \(N_{\text{model}}\)-values are computed using (24) with the calculated air-and water- capacitances \(C_{\text{model, with FEP}}\), \(a = 4.6909\) pF, \(C_{\text{model, with FEP}}\) and \(C_{\text{model, without FEP}}\), \(a = 4.6207\) pF, \(C_{\text{model, without FEP}}\) labelled in Figure 20a.

**Figure 21**

a) Difference \(\Delta C(\varepsilon)\) between simulated capacitances \(C_{\text{model, with FEP}}\) and \(C_{\text{model, without FEP}}\) plotted in Figure 20a.

b) Difference \(\Delta N(\varepsilon)\) between sensor readings \(N_{\text{model, with FEP}}\) and \(N_{\text{model, without FEP}}\) plotted in Figure 20b.
For a realistic soil moisture regime \([\theta_{\text{min}}, \theta_{\text{max}}] = [0.05, 0.5]\) these intervals are 
\([\varepsilon_{\text{min}}, \varepsilon_{\text{max}}] = [3.85, 34.59]\) and \([N_{\text{min}}, N_{\text{max}}] = [0.475, 0.943]\) respectively. The RMSE calculated for the above permittivity range using (31) is \(\Delta_{\text{RMS}} = 0.596\).

The corresponding correction of the normalized readings \(N_i\) used in section 3.3.1, is represented by the modelled deviation \(\Delta N(\varepsilon) = N_{\text{model, with FEP}}(\varepsilon) - N_{\text{model, without FEP}}(\varepsilon)\) plotted in Figure 21b which reaches it maximum of approximately \(11.5 \times 10^{-3}\) at \(\varepsilon \approx 6.2\). This exceeds the maximum standard deviation \(\sigma\) between the normalized readings \(N_i^k\) measured with the four sensors \(k = 1 - 4\) (right axes of Figure 11) and is a systematic bias rather than a random error.

Consequently, the measurements \(N_i\) (section 3.3.1, light grey circles in Figure 11) are corrected for the calculated effect of the FEP coating (i.e., \(\Delta N(\varepsilon)\)) before deriving the approximation of the form (23). Corrected normalized readings \(N_i - \Delta N(\varepsilon_i)\) representing measurements under field conditions (where no FEP shrink fit is present) are calculated from the laboratory measurements \(N_i\) \((i = 1 - 22)\) using the replacement:

\[
N_i \mapsto N_i - \Delta N(\varepsilon_i)
\]

where \(\Delta N(\varepsilon) = N_{\text{model, with FEP}}(\varepsilon) - N_{\text{model, without FEP}}(\varepsilon)\) is represented by a numerical expression plotted in Figure 21b.

Correcting for the effect of the FEP coating present in the laboratory experiments affects the values of the parameters \(a_2\) and \(k\) involved in the approximation (23) relating \(\varepsilon\) with \(N\). For the directly measured data (uncorrected) \([N_i, \varepsilon_i]\) the fit-parameters are: \(a_2 = 1.15008\) and \(k = 6.66056\) and for the data \([N_i - \Delta N(\varepsilon_i), \varepsilon_i]\) corrected for the effect of the FEP one finds: \(a_2 = 1.12819\) and \(k = 6.64846\). The latter parameters together with (23) should be used for converting field-measured data to permittivities.

Sensor readings are taken with and without the FEP coating in water and air. Corresponding readings averaged over the four sensors \(\langle R \rangle\) and the resonant frequencies \(f_r\) computed with (6) are listed in Table 4.

Modeled capacitance \(C_{\text{model}}\) values in Table 4 are the marked data (arrows) from Figure 20a for \(\varepsilon = \varepsilon_a\) and \(\varepsilon = \varepsilon_w\). These values are used to determine the changes in modeled capacitance with and without the FEP coating \(\Delta C\) under “Model” in Table 4. Because the experimental \(\Delta C\) determines \(\Delta R\) and consequently \(\Delta N\), only the relative changes must be simulated properly. Here, we must assume that \(\Delta C(\varepsilon)\) values fall between the end members for air and water. Thus, if the model simulates \(\Delta C\) accurately for air and water, it can be assumed to represent the change correctly for all permittivity values.

The experimental change in capacitance can be calculated directly from equation (13) using the change in resonant frequency with and without FEP. The resulting values in Table 4 are very close to the simulated changes for air and water, where the errors \((\Delta C_{\text{model}} - \Delta C_{\text{exp}})\) are approximately 0.04 pF for air and 0.07 pF for water. These are negligible relative to experimental uncertainties, and this result provides confidence in the correction given above.
Table 4  
Averaged experimental sensor readings $<R>$ and resonant frequencies $f_r$ with and without the FEP coating on the access tube showing corresponding capacitance changes $\Delta C$ compared with model derived $\Delta C$ quantities for air- and water-environments.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Experiment</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$&lt;R&gt;$ [-]</td>
<td>$f_r$ [MHz]</td>
</tr>
<tr>
<td>air</td>
<td>no FEP</td>
<td>36473.5</td>
</tr>
<tr>
<td>air</td>
<td>with FEP</td>
<td>36436.3</td>
</tr>
<tr>
<td>water</td>
<td>with FEP</td>
<td>25919.5</td>
</tr>
<tr>
<td>water</td>
<td>no FEP</td>
<td>24172.5</td>
</tr>
</tbody>
</table>

6 Acknowledgments

The following ETHZ staff members provided important technical assistance: Hanspeter Läser (container design and construction); Hans Wunderli (expansive metal design, construction and digital photography); Hannes Wydler (frabrication of custom tools); Kurt Barmettler (chemical advice); René Saladin (Dioxane handling and safety); Jörg Leuenberger (infrastructure); and Peter Brühwiler (specialty machining).
7 References


