Sliding Friction of Polyethylene on Snow and Ice

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PRESENTED BY

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Abstract

The low friction in skiing on snow is due to water films generated through frictional heating. There is, however, uncertainty about the thickness and the distribution of these water films. Since direct observation of the water films is difficult, tribometer measurements including temperature measurements are carried out, and the contact area between ski and snow is investigated. The work is divided into four parts: the design of a tribometer, tribometer measurements, investigations on the contact area, and numerical modeling of snow and ice friction. Due to difficulties in conducting experiments with snow, the work focuses on friction between polyethylene - the principal component at the snow-contacting face of skis - and ice.

A large-scale tribometer (diameter 1.80 m, pin-on-disc geometry) for friction measurements on ice has been designed and built. The apparatus is placed in a cold chamber with an accessible temperature range of $T_{env}=-20{\degree}C$ to $+1{\degree}C$. IR thermocouples measure the temperature of the track before and after the slider. In addition, integrated thermocouples are used to measure temperature inside the polyethylene slider. The friction coefficient ($\mu$) can be determined with an accuracy of $\pm 5\%$. The kinetic friction between polyethylene and ice is measured as a function of temperature, velocity, load, apparent contact area, and surface topography. The friction coefficient, as well as the temperature increase in the slider depends on all of these parameters. Interpretations are given on the basis of hydrodynamic friction, taking into account the generation and shearing of thin water films at the contact spots.

For the experimental investigation of the contact area between polyethylene and snow, scanning electron microscopy and X-ray computer tomography have been used. Contact spot size can be estimated, and a dependence of the real contact area on load and snow type can be seen. The investigation of the contact area between polyethylene and ice (tribometer experiment) is carried out on imprints of the polyethylene slider and the ice surface, by means of optical profilometry. The effect of polishing of the ice by the slider during friction experiments is observed. All methods described give a precise surface characterization, and the results are used in the prediction of the contact area and contact spot size evolution in the friction process.

A numerical model for sliding on ice including dry friction and generation of and
lubrication by water films is described. Alternative energy dissipation mechanisms are discussed. The model is verified by comparing it with experimentally determined temperature evolution and friction coefficients.

The main conclusions are: sliding on snow and ice can be explained by hydrodynamic principles. Unevenly distributed thin water films are responsible for the low friction observed. Water film thickness ranges from below 100 nm at low temperatures to about 1 µm close to 0°C. Average static contact area with snow is around 5%, with contact spot diameters of approximately 100 µm. Behavior of the water films and size of the real contact area can explain the friction process, no capillary attachments are needed. The most critical parameter determining friction between skis and snow or ice is the real contact area. Ski friction can be optimized by adjusting the size and the topography of the ski base.
Zusammenfassung


Ein grosser Tribometer (Durchmesser 1.80 m, pin-on-disc Geometrie) für Reibungsmessungen auf Eis wurde entwickelt und gebaut. Die Anordnung steht in einer Kältekammer mit einem Temperaturbereich von -20°C bis +1°C. IR Thermoelemente messen die Temperatur der Eisspur vor und hinter der Probe. Integrierte Thermoelemente messen die Temperatur in der Polyethylen Probe. Der Reibungskoeffizient ($\mu$) kann mit einer Genauigkeit von ±5% bestimmt werden. Die dynamische Reibung zwischen Polyethylen und Eis wird in Funktion von Temperatur, Geschwindigkeit, Last, scheinbarer Kontaktfläche und Oberflächentopographie bestimmt. Der Reibungskoeffizient, sowie die Temperaturerhöhung in der Probe hängt von allen diesen Parametern ab. Interpretationen werden gemacht unter der Annahme hydrodynamischer Reibung, welche die Generierung und Scherung dünner Wasserfilme mit einbezieht.

Für die experimentelle Untersuchung der Kontaktfläche zwischen Polyethylen und Schnee wurden Rasterelektronenmikroskopie und Röntgen-Computertomographie benutzt. Die Kontaktstellengrösse kann abgeschätzt werden, und die wahre Kontaktfläche und ihre Abhängigkeit von der Last und der Schneeart kann bestimmt werden. Für die Untersuchung der Kontaktfläche zwischen Polyethylen und Eis (Tribometer Experiment) wurde optische Profilometrie von Abdrücken der Probe und des Eises durchgeführt. Der Poliereffekt des Eises durch die Probe während Reibungsmessungen kann beobachtet werden. Für alle diese Methoden resultieren präzise Oberflächencharakterisierungen, welche für die Vorhersage der Entwicklung
der wahren Kontaktfläche und der Kontaktstellengröße verwendet werden.

Ein numerisches Modell des Gleitens auf Eis wird beschrieben. Es beinhaltet Trockenreibung und die Generierung der und Schmierung durch Wasserfilme. Weitere Mechanismen der Energieabgabe werden diskutiert. Das Modell wird anhand experimentell bestimmter Temperaturentwicklung und Reibungskoeffizienten verifiziert.

Fazit: Gleiten auf Schnee und Eis kann mit hydrodynamischen Prinzipien beschrieben werden. Unregelmäßig verteilte, dünne Wasserfilme sind für die tiefe Reibung verantwortlich. Die Wasserfilmdicke reicht von unter 100 nm bei tiefen Temperaturen bis ca. 1 µm nahe 0°C. Mittlere statische Kontaktfläche auf Eis ist ca. 5%, Durchmesser einer Kontaktstelle ist ca. 100 µm. Das Verhalten der Wasserfilme und die Grösse der wahren Kontaktfläche kann den Reibungsprozess erklären; keine Kapillarkräfte sind dazu nötig. Der für die Reibung zwischen Ski und Schnee oder Eis kritischste Parameter ist die wahre Kontaktfläche. Die Reibung von Skis kann optimiert werden durch Verändern der Grösse und der Topographie der Lauffläche.
Acknowledgments

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Chapter 1

Introduction

There has been much interest in snow and ice friction but considerable uncertainty about the underlying mechanisms. Nevertheless, a lot of evidence supports the idea of melt-water lubrication (water generated through frictional heating). The outstanding questions require increased knowledge of the contact phenomena between a slider and snow or ice, the role of melt-water lubrication, including thickness and distribution of the water films, possible occurrence of capillary bonding, and the presence of dry frictional processes. Knowledge of the effects of load, speed, temperature, snow type, and ski properties on all of these processes and parameters is critical to understanding the behavior.

1.1 Sliding Friction on Snow and Ice

Although the ultimate goal is to understand snow friction as in a ski sliding on snow, most of the problems regarding kinetic friction on snow can be addressed by looking at the kinetic friction on ice. The most important stages in the investigation of ice friction shall therefore be summarized briefly.

Bowden and Hughes [BH39] found that the low kinetic friction of ice is due to a partial water layer formed by frictional heating. The kinetic friction coefficient ($\mu$) is independent of speed, load, and apparent contact area, but decreases with decreasing temperature. Experiments on snow showed similar trends at higher friction coefficients, attributed to the extra work done in displacing and compressing the snow grains.

Bowden [Bow53] showed that a highly conductive slider has more friction than a well-insulated one and that the difference between them decreases as temperature decreases. Indication that the conduction of heat plays a major role in ice friction.
Evans et al. [ENC76] developed a quantitative model of the frictional heating theory of Bowden and Hughes for the friction of ice. Frictional heat is conducted into both slider and ice, thereby raising the surface to its melting point. The heat used in melting is small, most of is conducted into slider and ice. They calculated the friction coefficient to be proportional to the temperature below the melting point and to the sliding velocity according to $v^{-1/2}$, which was in reasonable agreement with their experiments.

Kuroiwa [Kur77] measured the real contact area between a slider and snow by observing the snow surface after a sliding experiment through a microscope. He found the average contact size to be around 0.04 mm$^2$, which results in a contact diameter of around 200 µm (circular contacts assumed). An estimate showed that only 1% of the frictional energy is consumed to melt snow, therefore most of it is transferred to the snow and the slider.

Ambach and Mayr [AM81] measured the thickness of the water films in sliding on snow using a capacitive probe built into the ski. Their values of several µm are generally assumed to be too high.

Oksanen and Keinonen [OK82] further developed Evans' theory by assuming that the frictional force is caused by viscous shear of the water layers between slider and ice. Assuming a true contact area calculated from normal load divided by indentation hardness of ice, they calculated the friction coefficient and its dependence on temperature, velocity, and normal load. In tribometer experiments, they found the same dependence, validating their theory.

Glenne [Gle87] presented a general macroscopic view of the friction between ski and snow. The resistance to sliding is the sum of dry and wet friction, and resistance caused by displacing and compressing the snow.

Colbeck [Col88, Col92] refined the mathematical description of the process of sliding on snow. The following mechanisms are assumed to be present: dry friction, boundary lubrication through melt water, and capillary attachments due to excess water production (water bridges exerting a drag). In the dry friction regime, not enough frictional heat is generated to allow for melt water to be present. Yet heat accumulates in the snow grains, eventually generating water to lubricate the contacts between ski and snow. The frictional process is thus one of boundary and mixed lubrication. Based on this, analytical solutions describing the dependence of friction
on e.g. temperature, velocity, or snow grain size are developed. Colbeck focuses on snow friction, and points out the difference between snow and ice friction, but also states that a snow surface that has seen repeated passes of a slider will exhibit a contact area of nearly 50% and resemble an ice surface.

Dosch et al. [DLB95] addressed the issue of pre-melting of ice surfaces (also denoted quasi-liquid layer) by X-ray scattering. They concluded that a pre-melted layer starts to be built up at around -12°C, reaching several nm at -5°C. Other studies using different measurement principles [DB00, BOF+02] reported similar effects. Still the question remains as to whether this layer also exists in contact with a solid; e.g. static friction should then be low, which is not the case.

Strausky et al. [SKLA98] used fluorescence spectroscopy combined with a pin-on-rotating-ice-disc experiment to detect possible water films. Water films, if present, must be below about 100 nm (detection limit of their experiment), thus much smaller than predicted earlier. Their experiment was limited to velocities below 0.1 ms\(^{-1}\), however.

Buhl et al. [BFR01] conducted both field measurements with real skis and laboratory experiments on ice using a pin-on-disc tribometer. They found the friction coefficient to be lowest at around -3°C and to increase for low snow and ice temperatures as well as for snow and ice temperatures close to 0°C. The influence of load can only be seen at low temperatures, higher load resulting in lower friction.

Persson [Per00] describes general concepts of heat flow during sliding, and gives a short summary on sliding on ice and snow. His ideas will be the theoretical basis for the explanations of snow and ice friction and the development of a numerical simulation (see Chapter 3 and 5).

### 1.2 Outline

The work is divided into four parts: the design of a tribometer, tribometer measurements, investigations on the contact area, and numerical modeling of snow and ice friction. Due to difficulties in conducting experiments with snow, the work focuses on friction between polyethylene - the principal component at the snow-contacting face of skis - and ice (if not stated otherwise).

**Tribometer Design** A large-scale tribometer (pin-on-disc geometry) is built. Development of friction force and temperature measurements is described. Mea-
surement and evaluation procedures are developed.

**Tribometer Measurements** Friction coefficients as a function of temperature, normal load, velocity, apparent contact area, and slider surface topography are measured in the ranges of interest for real skiing. Temperature measurements using thermocouples built into the slider, and IR measurements of the ice surface are conducted.

**Contact Area** The real contact area between polyethylene and snow is determined using an x-ray computer tomograph. The contact area between polyethylene and ice is estimated from profilometer data of both surfaces. Theoretical considerations on the contact mechanisms conclude the studies.

**Modeling** A model including dry friction and generation of and lubrication by water films is designed. Other possible energy dissipation mechanisms are discussed. The model is developed and verified by comparing it with experimentally determined temperature evolutions and friction coefficients.
Chapter 2

Tribometer Design

Abstract
A large-scale tribometer (diameter 1.80 m, pin-on-disc geometry) for friction measurements on ice has been designed and built. The apparatus is placed in a cold chamber with an accessible temperature range of $T_{env}=-20^\circ C$ to $+1^\circ C$. Velocity can be varied between $v=0.5\,\text{ms}^{-1}$ and $20\,\text{ms}^{-1}$, load can be varied between $F_n=25\,\text{N}$ and $84\,\text{N}$, frictional force being measured using a load cell. Problems associated with the construction of tribometers are discussed. IR thermocouples measure the temperature of the track before and after the slider. In addition, integrated thermocouples are used to measure temperature inside the polyethylene slider. The friction coefficient ($\mu$) can be determined with an accuracy of $\pm5\%$. An error analysis is carried out.

2.1 Introduction
A number of investigators have set up experiments in order to measure friction of different materials on snow and ice (see e.g. [BH39, GG72, Kei78, Leh89, BFR01]). It was recognized that the warming of the ice track presents a problem, thus either small tribometers (different designs: pin-on-disc, rotating drum, linear devices) with low sliding velocities ($v<1\,\text{ms}^{-1}$) were used, or larger-scale devices were built. Most of the latter encountered problems with vibrations, reducing the accuracy of the measurement. Although it was recognized that the low kinetic friction is due to the melting of the snow or ice surface, and thus strongly influenced by temperature, none of the earlier experiments had adequate temperature control. See [God95, Flo83] for a general discussion of the design of tribometers.

In this project, the requirements for the tribometer were as follows:

- Measurements under conditions encountered in real skiing: temperature, pres-
sure, and especially sliding velocity ($v=10\text{ms}^{-1}$ and above).

- Sufficient accuracy to measure differences between current ski-base-preparation techniques, and to evaluate new ideas of surface preparations or ski-base materials.
- Model experiments with monitored temperature evolution at or near the interface should answer questions as to how much heat is generated in the friction process.

The original idea was to measure friction of ski-base materials on snow. Due to practical problems with the preparation of a smooth, reproducible snow surface, ice as a substitute surface for hard-packed snow is used. The limitations of this will be discussed.

### 2.2 Experimental Setup

![Tribometer](image)

Figure 2.1: Tribometer. Loading arm with force sensor (1), ice surface preparation arm (2), slider (3), ice annulus (4), foundation and motor (5), ice temperature measurement (6).

![Tribometer](image)

Figure 2.2: Tribometer. Side and top view. Outside diameter of the table is 1.80 m, diameter of the track at the position of the slider is 1.60 m.
2.2. EXPERIMENTAL SETUP

2.2.1 Tribometer

A 1.80 m-diameter tribometer has been built and placed in a cold chamber. The apparatus consists of the rotating table, carrying the ice annulus, and two arms for holding the slider and the ice surface preparation tools. It is powered by a 4 kW 3-phase AC motor, maximum 1435 rpm, coupled to the axis of the rotating table by a toothed drive belt (transmission ratio 90:22). The motor is controlled by a frequency transformer (Invertek Optidrive). Drawings and photographs of the tribometer are shown in Figures 2.1 and 2.2.

The compressor and the fans of the chamber-cooling system induce vibrations, and thus the device must be decoupled from the structure of the cold chamber. It is therefore directly attached to the concrete floor of the room. Even a very stiff construction can at high rotational velocities induce considerable vibrations if not balanced well. The tribometer is therefore balanced using a Brüel & Kjaer Vibrotest 60 device.

2.2.2 Friction Force Measurement

Figure 2.3 shows the assembly of the friction-force-measurement unit. A strain-gauge load cell (Interface MB-10) measures the shear force between the upper (fixed) aluminium plate and the lower (movable) aluminium plate holding the slider. The normal force is transmitted via two vertical brass plates. These plate springs provide a frictionless force measurement. Ball bearings introduce friction and can therefore not be used. FEM calculations show that in principle, using just the right thickness of the brass plates to avoid buckling (0.3 mm), the stiffness of the two plates is less than 1/100 of the stiffness of the load cell beam. It does therefore hardly influence the shear force measurement. To be on the safe side (vibrations of the device, unevenness of the track), thicker brass plates are used (0.8 mm). The stiffness of the plate springs then amounts to 15% of the stiffness of the load cell beam, and thus, the measured force has to be corrected. Also considered in the calibration is the (small) influence of the load on the friction measurement via deformation of the brass plates. The normal force is applied using dead weights of roughly 10 N, 20 N, and 30 N. The slider size is varied from 40 mm to 200 mm (length) and from 5 mm to 50 mm (width). Construction of the sliders (see Figure 2.4): a block of polyethylene (height 9 mm) is attached to a 10 mm thick foam rubber, which is then attached to a top aluminium plate. Double-faced scotch tape is used for the connections. The aluminum plate is screwed to the lower plate of the friction measurement device or slider holder, respectively. The foam rubber is necessary to assure a flat, parallel contact, and to compensate for small unevenness of the track and vibrations. The signal from the load cell is amplified, read into a PC via a National Instruments
PCI-6034E card, and processed in Labview.

Figure 2.3: Sensor and slider holder. View from front (photograph) and back (drawing). Load cell (1), overload protection (2), copper plates (3). The slider is attached to the lower part of the slider holder.

(a) Slider equipped with thermocouples (slider B). Running surface area: 4 cm x 4 cm. (b) Ski base sample. Running surface area: 6 cm x 10 cm.

Figure 2.4: Construction of the sliders: (a) A block of polyethylene is attached to a layer of foam rubber, which is then attached to a top aluminum plate. The lower aluminum plate is needed for accommodating the thermocouple plugs. (b) An approximately 1 mm thick sample of ski-base material is glued onto a stiff aluminum plate. This is again attached to a layer of foam rubber and a top aluminum plate. Such samples can be ground on a commercial ski-base-grinding machine.

2.2.3 Velocity Measurement

In order to have an exact velocity control, a magnet encoder (MDFK 08G2124/N4 from Baumer Electric, Switzerland) measures the rotational speed of the tribometer. The digital signal is read into the PC and processed in Labview.

2.2.4 Ice Surface Preparation

For the ice preparation, a 2 to 5 cm thick ice annulus is frozen onto the table layer by layer. The thickness should not surpass some millimeters per layer, otherwise
crack formation and air inclusions become severe. To account for the expansion of the ice, foam rubber is used as side wall on the outer side of the track. For the last couple of layers, or to refresh an existing ice surface, only thin layers of water are frozen on using a wet towel. Freezing of ice in this way still leads to a relatively uneven surface. The surface is therefore shaved down to a constant height using the principle of a lathe. A vertical steel bar of width 2mm at the tip can be moved manually in vertical and radial direction using linear stages. For most of the experiments, an elevated track is carved into the ice, to prevent the slider from cutting into the ice (edge effect), see Figure 2.5. A tangential speed of 5 m/s and a feed rate of 0.5-1 mm/s are used for the carving. This procedure often results in the formation of holes and cracks; in that case, new water has to be added, or an industrial dryer is used to melt the cracked ice, after which it is allowed to refreeze. Different water preparation techniques were used (distilled water, water degassed by boiling), with no noticeable change to the quality of the ice or the measured friction coefficients. Consequently, tap water is used for all the measurements.

Different methods for snow preparation were developed and evaluated. The two main problems encountered are the icing of the track through repeated passes of the slider and the shaving off of snow grains, eventually leading to the track becoming too uneven and excessive vibrations. Only very few measurements with large sliders (resulting in a badly defined contact, due to waviness and curvature of the sliders) and at low velocity and load were carried out on snow, due to these difficulties and the consequently poor reproducibility. Later, a cylinder with machined-in grooves was used to structure the ice and render a surface exhibiting a snow-like roughness (Figure 2.6).

![Figure 2.5: Schematic cross section of the ice annulus, perpendicular to the sliding direction (v). Width (w) of the elevated track can be varied.](image)

### 2.2.5 Temperature Measurements

A number of techniques can be used to measure transient interface temperature rise at sliding interfaces, e.g. thermocouples, thermistors, radiation detection techniques
CHAPTER 2. TRIBOMETER DESIGN

(a) Cylinder. (b) Structured ice surface.

Figure 2.6: The ice surface is structured using a cylinder with machined-in grooves, mounted on the preparation arm. A brush cleans the cylinder of the ice dust accumulating.

(for a complete summary, see [Bhu02], pp. 314-326). In this work, an infrared (IR) camera was used for a first estimate of the heat generated in the frictional process. Afterwards, thermocouples were built into a polyethylene slider, and IR thermocouples measured the temperature of the ice track.

Infrared Camera

A Varioscan 3021-ST IR camera is used to take images of the track behind the slider (Figure 2.7). A considerable increase of the temperature can be seen. However, the obtained values cannot be interpreted quantitatively, since they represent an average of the temperatures encountered at the surface; i.e. spots that have been in contact with the slider (real contact area), and where melting took place, as well as regions that have not experienced a major temperature increase. Moreover, the radiation measured by the camera is emitted not only from the very surface of the ice, but also from regions slightly below the surface. This effect, too, makes it hard to obtain quantitative temperature values using IR measurements. Commercially available IR cameras cannot resolve single ice contact spots and are too slow to capture dynamic processes. In the future, faster IR cameras combined with suitable optics may possibly allow for the visualization of such frictional processes.

Infrared Thermocouples

IR thermocouples (Omega OS36-K-50F) are used to measure the temperature of the track in front and behind the slider. IR thermocouples convert radiation emitted from the ice surface into a voltage, which can be interpreted as a thermoelectric potential, and thus can be treated like standard thermocouples. The IR, as well as the standard thermocouple voltages are read in via an USB TC08 data recorder from Pico Technology, and processed in Labview.
2.2. EXPERIMENTAL SETUP

Figure 2.7: Heating of the track behind the slider. Infrared image. Temperature of the surrounding ice is -6°C, maximum temperature measured is -3°C. $F_n=84\,\text{N}$, $A_{\text{app}}=10\,\text{cm}^2$, $v=5\,\text{ms}^{-1}$.

Embedded Thermocouples

Since temperature measurement at the very interface is not easily accomplished, temperature is measured just above the interface within the slider. Holes of diameter 1.5 mm are drilled into the slider from the top to within 0.2±0.1 mm of the lower surface. Thin thermocouples (Omega CHAL-002, wire diameter of 50 $\mu$m) are glued into the holes using a two component resin (Araldit AW 136H and Aradur 5049-1). Heat conductivity and capacity of polyethylene and the resin differ only slightly and should therefore not affect the heat conduction into the slider. Monitoring of the location of the thermocouples as well as detection of possible air inclusions are conducted using an X-ray computer tomograph. From X-ray scans, it is possible to determine the position of the thermocouple to ±50 $\mu$m (Figures 2.8 and 2.9). 3D scans show negligible air inclusions. The slider with the built-in thermocouples is calibrated in an ice-water mixture. Since the exact position of the thermocouples relative to the surface, as well as the thermal properties of the polyethylene used are known, it is possible to calculate the temperature at the surface.

2.2.6 Humidity Measurements

The air humidity in the cold chamber was measured using a Hydrolog NT3-D data logger. It was found to be around 70%, varying only little.
2.3 Measurements

2.3.1 Parameters

Temperature in the cold chamber is varied from $T_{\text{env}}=1^\circ\text{C}$ to $-15^\circ\text{C}$, resulting in ice temperatures of roughly $T_{\text{ice}}=0^\circ\text{C}$ to $-17^\circ\text{C}$ (see discussion below). Ice is very brittle at $-15^\circ\text{C}$, it becomes increasingly difficult to prepare an even ice surface without cracks.

Load is varied from $F_n=25\ \text{N}$ (weight of the loading arm) to $84\ \text{N}$ (using additional dead weights), corresponding to mean pressures of $p=25\ \text{kPa}$ to $84\ \text{kPa}$ for the most frequently used slider size and track width. This is about one order of magnitude higher than the mean static pressure exerted by a skier on the snow surface. The pressure distribution under a ski is, however, quite inhomogeneous, and can be considerably higher e.g. in a carved turn. Pressures in the range of up to $100\ \text{kPa}$ can thus be expected in real skiing. For larger sliders (lower pressure), the reproducibility of the measurement suffers.

Velocity is varied from $v=0.55\ \text{ms}^{-1}$ to $10\ \text{ms}^{-1}$. At even lower velocities, the torque of the motor is not sufficient to surmount friction. At $v=10\ \text{ms}^{-1}$ and higher, vibrations due to the not perfectly balanced table (with every ice preparation, the weight distribution changes slightly) cause a deterioration of the track through the slider. This could be improved by balancing after each preparation (time and cost intensive). Yet even with a perfectly balanced table, vibrations induced by the not perfectly flat ice surface will limit the achievable velocity.
2.3. MEASUREMENTS

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<tr>
<td>TC3</td>
<td>30 mm</td>
<td>0.40 mm</td>
</tr>
</tbody>
</table>

Figure 2.9: Positions of the thermocouples in the polyethylene sliders. Thermocouples are in the center of the slider if not denoted _l (10 mm to the left) or _r (10 mm to the right), seen from top in the sliding direction v. Accuracy: 0.05 mm. Other sliders used are not equipped with thermocouples (e.g. slider C).
Measurements at high velocities can momentarily only be carried out for a short time (1-2 minutes).

**Apparent contact area** size is changed by varying the width of the track (see Figure 2.5) from \( w=25 \text{ mm} \) to 5 mm, resulting in apparent contact areas of \( A_{\text{app}}=10 \text{ cm}^2 \) to 2 cm\(^2\) (the length of the slider is usually kept constant at \( l=40 \text{ mm} \)).

Considerable scatter (about a factor of 2) is observed for some sets of parameters, especially at very low and very high velocities, and at low normal forces. This is due to a not well defined contact, and to stronger vibrations at high velocities. In the course of the experiments, it could be seen that a clearly defined contact which allows for well reproducible measurements can be achieved for apparent contact areas of up to \( A_{\text{app}}=4 \text{ cm}^2 \).

### 2.3.2 Measurement Procedure

Measurements are conducted according to the following procedure:

1. Preparation of the ice surface. Set track width.
2. Run-in period: the slider to be measured is slid for about 5 minutes at high load \( (F_n=84 \text{ N}) \) and velocity \( (v=5-10 \text{ m s}^{-1}) \).
3. Sample and ice is left to cool for 5 minutes. With the tribometer still running, this can be sped up (forced convection).
4. Tribometer is set to desired velocity. Recording of the data is started. Sample is lowered on the running tribometer.
5. Stop recording. Lift up slider. Proceed with step 3.

Along with the measurements, repeated optical checking of the ice surface and detection of possible vibration is carried out. A new ice surface is prepared if necessary.

### 2.3.3 Interpretation of the Measurement

Tribology experiments are generally not straightforward. Many parameters (e.g. surface contaminants, humidity), sometimes hard to control, can influence the result. A relatively large scatter of the data is common. For tribometer experiments on ice, this is no different. This section explains how a measurement is analyzed, and illustrates some peculiarities of ice friction measurements. All measurements shown relate to polyethylene sliding on ice.
Temperature-Controlled Friction and Polishing

Figure 2.10 shows a typical measurement. The sharp increase of the friction coefficient (black circles) after about 10 seconds represents the lowering of the slider onto the ice. Explanation for the initial increase in friction is the wearing off of sharp asperities (refrozen water, possibly surface hoar), resulting in a fast increase in real contact area. The following slow increase of the friction curve is attributed to a further increase in real contact area due to the polishing of the ice track. In order to get a single friction coefficient value, the slow increase is fitted by an exponential function (dashed black line) approaching a constant value, and the latter is taken as the steady state friction coefficient. Time resolution of the friction force measurement is usually set to 10 Hz but can be increased if higher resolution is desired, e.g. for the measurement of static friction. Time resolution of the temperature measurement is 1 Hz. The temperature recorded by the thermocouples (red curves) in the slider show a delayed increase but also reach a steady state after about 2 minutes. Specifications of the built-in thermocouples report an accuracy of about ±0.2°C. The temperatures recorded by the IR-thermocouples in front of (magenta curve) and behind (blue curve) the slider can only be interpreted qualitatively, they show an average ice surface temperature of former contact spots (having been at 0°C) and regions that have not made contact (always been below 0°C).

The influence of slider and ice temperature on the friction force and vice versa can be seen in Figure 2.11. After 5 minutes of sliding (corresponds to time t=0 s in the figure), a steady state is reached. The slider is then lifted up for different periods and
Figure 2.11: Slider lifted up for about 3 s, 10 s, and 40 s during the measurement. $T_{env}=-5^\circ C$, $v=5\text{ ms}^{-1}$, and $A_{app}=4\text{ cm}^2$ (slider B)

lowered again. After the lowering, temperature of the ice behind the slider (IRback) and friction ($\mu$) increase in parallel. Interpretation: friction increases due to the polishing effect (increased contact area), and coupled therewith, overall temperature in the slider and the ice increases. To what degree the slider temperature itself influences friction is not evident but could be clarified using a heatable slider. Note the ”bumps” in the IRfront-curve some 10 seconds after each lowering of the slider. See also Figure 2.12, which shows the evolution of the friction force and the slider temperature for two different loads. Slider temperatures and surface temperature behind the slider (IRback) instantly increase at the beginning of the measurement (lowering of the slider on the rotating tribometer), surface temperature in front of the slider shows a delay (at least one turn, corresponding to 1 second at $v=5\text{ ms}^{-1}$). The ”bump” in the IRfront-curve in the first 30 seconds of the measurement is typical for most of the experiments in all temperature ranges. It describes a kind of overshooting, however, no satisfactory explanation for this could be found.

Figure 2.13 shows three measurements carried out successively, with a varied waiting time in-between. The regular routine is to measure for 5 minutes, and then allow for cooling for 5 minutes (10 minutes interval). Friction values indicated correspond to fits of exponential functions, approaching a constant value ($\mu_{0\text{ min}}$ to $\mu_{40\text{ min}}$). The second curve (10 min, squares) is shifted to slightly higher values, but otherwise increases similar to the first curve (0 min, circles). After 25 minutes waiting between measurements, the curve starts at lower values, but then increases faster (40 min, triangles). Later measurements tend to result in higher friction, this is likely to be due to a polishing of the track and therefore increased real contact area. The initially
2.3. MEASUREMENTS

Figure 2.12: Friction force and temperature increase of ice surface and slider for loads of $F_n=52$ N and 84 N, $T_{env}=-5$°C, $v=5$ ms$^{-1}$, and $A_{app}=2$ cm$^2$ (slider B).

Figure 2.13: Consecutive measurements with different waiting times in-between: 5 mins, 25 mins, $T_{env}=-5$°C, $F_n=84$ N, $v=5$ ms$^{-1}$, and $A_{app}=4$ cm$^2$ (slider B).
lower friction after longer waiting times between measurements can be a result of a change in the track due to evaporation. Other researchers [GG72] who measured friction of polymers on ice using a tribometer claim that surface hoar builds up on the ice during the time it is exposed to air. These sharp ice crystals would then provide low real contact area, or even act as a "ball bearing". It is questionable whether this effect is relevant, compared to the relatively fast evaporation. Both of these effects would lead to initially less conforming sliding partners and therefore lower real contact area.

Temperatures

Figure 2.14 shows the cooling of the slider and the ice after a measurement at \( T_{env}=-1 \)°C. After about 5 minutes, the initial temperature (before the measurement) is reached again. Note the difference between slider and ice temperature. The oscillations in the temperature curves reflect the cooling cycles of the cold chamber: about every 5 minutes, the compressor starts up. The initial temperature of the ice can deviate considerably from the environmental temperature, due to evaporation and accompanying heat drain. However, small changes in the initial temperature of the ice and the slider (attributed to the limited precision of the temperature control in the cold chamber) showed no noticeable influence on the friction coefficient.

![Figure 2.14: Cooling of the slider (TC_mean) and the ice (IRfront and IRback) after a measurement at \( T_{env}=-1 \)°C. Lift up of the slider at \( t=0 \) min, tribometer is kept running at \( v=5 \) ms\(^{-1}\) (forced convective cooling).](image)
2.3. MEASUREMENTS

Contact Problems

Figure 2.15 illustrates the ongoing polishing during measurements. Interpretations: the track conforms to the slider, leading to increased contact area, and therefore higher friction. The effect disappears if the waiting time between measurements is increased. Evaporation leads to a more even ice surface, which initially provides less real contact with the slider. Longer waiting times or fresh preparation between measurements can lessen the effect of polishing. Such ice polishing effect must be kept in mind when carrying out comparative studies.

![Figure 2.15: Effect of polishing. 5 minutes waiting time between measurements, except between measurement 4 and 5: 30 minutes waited.](image)

Figure 2.15: Effect of polishing. 5 minutes waiting time between measurements, except between measurement 4 and 5: 30 minutes waited. $T_{env}=-5^\circ C$, $F_n=84 N$, $v=5 \text{ ms}^{-1}$, and $A_{app}=4 \text{ cm}^2$ (slider B).

Figure 2.16 illustrates the problems arising when measuring larger sliders (here: $A_{app}=10 \text{ cm}^2$). During the measurements, the distribution of the contact spots and/or thickness of the water films at the contacts changes. E.g. TC2 (middle-center) and TC2_r (middle-right) oscillate counter-cyclically, before seemingly approaching a steady state value of about $T=-6^\circ C$ (see Figure 2.9 for the positions of the thermocouples). This leads to quite high "noise" in the friction coefficient measurement. Figure 2.17 shows an experiment at lower apparent contact area ($A_{app}=2 \text{ cm}^2$). The contact is better defined, no oscillations can be seen, and a steady state for both friction and temperature measurements is reached quickly. Conclusion: pressure distribution under the slider has a large impact on the measurements, and is, the larger the slider (the apparent contact area), the harder to control. When preparing sliders, attention has to be paid in order to produce a
Figure 2.16: Friction coefficient and temperatures of ice surface and slider at $T_{env}=-10^\circ$C, $F_n=84$ N, $v=3.3$ ms$^{-1}$, and $A_{app}=10$ cm$^2$ (slider A).

Figure 2.17: Friction coefficient and temperatures of ice surface and slider at $T_{env}=-10^\circ$C, $F_n=84$ N, $v=3.3$ ms$^{-1}$, and $A_{app}=2$ cm$^2$ (slider A). Note that only the center of the slider is in contact with the ice, and therefore only TC1, TC2, and TC3 (red curves) show a marked temperature increase.
2.4. ERROR ANALYSIS

macropscopically flat surface.

**Tribometer vs. Real Skiing**

Both comparative studies of ski bases at laboratory scale and experiments to explain the process of snow and ice friction have been conducted on the tribometer. The differences between skiing in the field and laboratory tests shall be addressed briefly. On the tribometer, the slider is heated constantly (as in real skiing [CP04]), while the ice is heated periodically (laboratory "artefact" because of the pin-on-disc geometry). As a consequence, $T_{\text{ice}}$, initially below $T_{\text{env}}$, is increased (usually not exceeding $T_{\text{env}} + 1^\circ \text{C}$). The experiment thus runs at a slightly higher (ice) temperature than is set in the cold chamber.

As mentioned above, measurements are carried out on ice, due to difficulties in preparing an even and durable snow surface. The structural difference between the two materials manifests in different thermal and mechanical properties, and surface topography. The impact of these properties on the friction process depends on many factors (e.g. temperature, hardness of the snow) and needs to be discussed in detail for each set of parameters. In general, it can be said that ice is similar to the "old snow" situation of large polished grains exhibiting low friction. What we can expect is the same tendencies for temperature, load, and velocity dependence.

Results from laboratory measurements have to be interpreted carefully when wanting to draw conclusions for real situations. Scale up can be problematic, especially in the ski friction case, where water films are expected to build up at the front of the ski and increase in thickness and area covered towards the end.

**2.4 Error Analysis**

**2.4.1 Input Parameters**

**Temperature** can be controlled to within $\pm 0.2^\circ \text{C}$, but can vary considerably depending on the location in the cold chamber. Average initial temperature of the slider at otherwise same conditions (same $T_{\text{env}}$, cooling time allowed) can vary by about $\pm 0.4^\circ \text{C}$.

**Load** The mass of the loading arm and the dead weights is determined to $\pm 0.01 \text{ kg}$. Accuracy of the load determination is therefore $\pm 0.1 \text{ N}$. Should slider and ice not be aligned perfectly parallel, then the foam rubber is expected to compensate for that and provide a constant pressure under the slider.
**Velocity** is set by controlling the torque of the AC-motor. However, sliding friction of the slider can slow the tribometer down. Independently measured velocity shows that a velocity set to (e.g.) $v_{\text{set}}=5\text{ ms}^{-1}$ varies about $v_{\text{meas}}=4.8\pm0.02\text{ ms}^{-1}$.

**Apparent contact area** is measured to $\pm0.5\text{ mm}$ using a ruler. For small apparent contact areas (up to $A_{\text{app}}\approx4\text{ cm}^2$), the slider is expected to be perfectly flat.

### 2.4.2 Measured Parameters

<table>
<thead>
<tr>
<th>$T$ ($^\circ\text{C}$)</th>
<th>$A_{\text{apparent}}$</th>
<th>$n$</th>
<th>Mean</th>
<th>Scatter</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10°C</td>
<td>0.8 cm$^2$</td>
<td>4</td>
<td>0.048</td>
<td>$\pm0.001\ (\pm2%)$</td>
<td>$0.001\ (\pm2%)$</td>
</tr>
<tr>
<td>-10°C</td>
<td>4 cm$^2$</td>
<td>4</td>
<td>0.082</td>
<td>$\pm0.003\ (\pm4%)$</td>
<td>$0.003\ (\pm4%)$</td>
</tr>
<tr>
<td>-10°C</td>
<td>10 cm$^2$</td>
<td>5</td>
<td>0.108</td>
<td>$\pm0.013\ (\pm12%)$</td>
<td>$0.011\ (\pm10%)$</td>
</tr>
<tr>
<td>-5°C</td>
<td>4 cm$^2$</td>
<td>13</td>
<td>0.073</td>
<td>$\pm0.005\ (\pm7%)$</td>
<td>$0.003\ (\pm4%)$</td>
</tr>
</tbody>
</table>

Table 2.1: Scatter of friction coefficient and temperature (last column) at different temperatures and apparent contact areas for $F_n=84\text{ N}$ and $v=5\text{ ms}^{-1}$ (steady state values, exponential fit). Values at -10°C are for slider A, values at -5°C are for slider B. $n$ is the number of measurements per set of parameters, SD is the standard deviation.

The achievable accuracy of the measurements depends on the conditions. For the same slider, scatter can vary considerably depending on temperature, load, velocity, and apparent contact area. At lower temperatures, it becomes more difficult to prepare an even ice track. Increased load seems to provide better defined contact. For the same reason, lower apparent contact area (i.e. track width) reduces scatter. Higher velocity speeds up the process of conditioning, again leading to better defined contact. In general, it can be said that scatter increases towards lower values of the parameters $T_{\text{env}}$, $F_n$, and $v$, and decreases for decreasing $A_{\text{app}}$.

Temperature scatter usually goes along with friction-force scatter. Friction force is an integral value, whereas thermocouples measure temperatures relatively locally. Temperatures can oscillate considerably, making it hard to find a steady-state value at otherwise ”converging” friction force (see Figure 2.16).

A look at the effect of velocity shall point out the influence of small changes of an experimental parameter on the measured parameters:
A sharp drop in friction is expected for a velocity (and thus heat generation) that is high enough to raise the temperature at a contact to the melting point, and provide water lubrication. This can explain large scatter at a certain velocity, where slightly different velocities decide whether friction is "dry" or "wet".

For low velocities, the ice on the tribometer is allowed for longer cooling between two contacts with the slider. The temperature of the ice does not rise as high. The conformation process of slider and ice is slower, and in addition, the ice can be leveled out by evaporation between two contacts. This can lead to both increased (lower temperatures) and decreased (worse conforming surfaces) friction, an estimation a priori is difficult.

The effects described can be responsible for the scatter observed e.g. at low velocities in Figure 3.5, Chapter 3. Similar considerations can be made for the influence of load, however, velocity seems to be a more critical parameter.

### 2.5 Conclusion

On the tribometer, it is possible to reproducibly measure friction of ski-base materials on ice, as well as the temperature evolution in the slider and in the ice. Temperature ($T_{env}$) and sliding velocity ($v$) can be chosen according to conditions encountered in real skiing, while load ($F_n$) and sample size ($A_{app}$) have to be adjusted to the laboratory scale. Depending on the conditions (above all: $T_{env}$ and $A_{app}$), the friction coefficient can be determined ±2%. This should be sufficient to measure differences between current ski-base-preparation techniques. Some peculiarities of the tribometer measurement, e.g. the heating of the ice, have to be kept in mind, especially when comparing laboratory and field measurements.
Chapter 3

Tribometer Measurements

Abstract

The kinetic friction between polyethylene and ice is measured as a function of temperature, velocity, load, apparent contact area, and surface topography. The friction coefficient, as well as the temperature increase in the slider depends on all of these parameters. Interpretations are given on the basis of hydrodynamic friction, taking into account the generation and shearing of thin water films at the contact spots.

3.1 Introduction

Ice shows very special tribological behavior. This is not surprising, since at most temperatures relevant to e.g. skiing, ice is at a very high homologous temperature ($T/T_m$, where $T_m$ is the melting point temperature). Classical friction laws for plastically deforming materials (which most materials are, at a not-too-low roughness) predict a friction coefficient independent of load ($F_n$), velocity ($v$), and apparent contact area ($A_{app}$). For materials sliding on ice, this does not hold. Rather, friction can be very much dependent on the above parameters. Discussion of the results presented is based on the assumption that the shearing of water films generated through frictional melting processes is responsible for friction. The amount of frictional heat generated is

$$P = \mu \cdot F_n \cdot v = F_f \cdot v,$$

where $F_f$ is the friction force. Note that $\mu$ represents the kinetic friction coefficient (also denoted $\mu_k$), as opposed to the static friction coefficient ($\mu_s$). Alternatively, the total heat flux $q' = P/A$ through the area $A$ into the upper and the lower solid can be written as
\[ q'' = v \cdot \sigma_k, \]  

where \( \sigma_k \) is the shear stress necessary to slide two surfaces with the relative velocity \( v \). For an estimate, the shear stress is written as \( \sigma_k = \mu \sigma_0 \), where the perpendicular stress \( \sigma_0 \) in a contact spot is assumed to be equal to the penetration hardness of the softer material.

Figure 3.1: A solid sliding against ice along a planar surface separated by a thin lubricating layer of water. Assume that the heat flux \( q'' \) flows primarily into the lower solid.

If all heat were to be conducted into one of the solids in contact (Figure 3.1), the temperature increase would amount to

\[ \Delta T = 2 \cdot q'' \cdot \left( \frac{t}{\pi \cdot \lambda \cdot c_p \cdot \rho} \right)^{\frac{1}{2}} \cdot f(z, t), \]  

where \( f \) is between 0 and 1 and accounts for the heat diffusion into the solid (see [Per00], pp. 251-254). \( \lambda \), \( c_p \), and \( \rho \) are the thermal conductivity, the specific heat capacity, and the density of the material, respectively. Under the assumption that heat is primarily conducted into the ice, Equation 3.3 gives an estimate of the temperature increase in the ice. Contact spots (junctions) between two solids usually have a diameter of \( D \sim 1 - 100 \mu m \). For a soft material like ice, \( D=100 \mu m \) is assumed. Using \( v=1 \text{ ms}^{-1} \) results in a time of contact of approximately \( 10^{-4} \) s and, according to Equation 3.3, to a temperature increase at the interface (\( f=1 \)) of 30 K for \( \mu=0.3 \) (dry friction) or 2 K for a \( \mu=0.02 \) (water lubricated friction). One can define a heat penetration depth as the distance into the solid at which the temperature increase equals \( \Delta T/2 \). For the above parameters, this renders a depth of 7 \( \mu m \). Heat diffusion is therefore primarily 1-dimensional, perpendicular to the interface.

The observed velocity dependence of the friction force [OK82] results from the generation of a layer of melt water due to the frictional heating. The exact thickness of the water films is not easy to calculate, since when a water film starts to form, the
sliding friction drops rapidly, resulting in a drop in the frictional heat production. Latent heat must be taken into account for melting to occur. Using

\[ \mu = \frac{\eta \cdot v}{h_{wf} \cdot \sigma_0} \]  

(3.4)

where \( \eta \) is the kinematic viscosity of water at 0°C, even a \( h_{wf} = 10 \text{ nm} \) thick water layer between two (smooth) surfaces would result in a friction coefficient of about 0.01 (penetration hardness of ice: \( \sigma_0 \approx 2 \times 10^7 \text{ Nm}^{-2} \), \( v = 1 \text{ ms}^{-1} \)). It is assumed that the pressure at the contacts does not reach such high values (due to creep), and therefore thicker water layers are required for low friction.

Alternatively, using \( \sigma_0 = F_n / A_{real} \) and \( relRCA = A_{real}/A_{app} \), Equation 3.4 can be rewritten to yield the friction force given by

\[ F_f = \frac{\eta \cdot v \cdot A_{real}}{h_{wf}} = \eta \cdot v \cdot A_{app} \cdot \frac{relRCA}{h_{wf}}, \]  

(3.5)

A higher heat flux, due to e.g. higher velocity, can lead to both increased film thickness (in turn leading to lower friction, see Equation 3.5) or increased real contact area (leading to higher friction). How these two processes are linked depends on the roughness (see Chapter 4) of the sliding partners. For simplicity, it is assumed that a perfectly flat slider slides on ”rough” ice (roughness in the order of \( R_a = 0.1 \) to 1 \( \mu \text{m} \), gaussian-like height distribution), and ”slices off” ice asperities according to the water-film thickness. Figure 3.2 illustrates this mechanism, and shows the relation between water-film thickness and relative real contact area (i.e. bearing ratio). A plot of contact area vs. water-film thickness is shown in Figure 3.3 (dashed line, left axis). The curve can be combined with Equation 3.5 to result in a friction-coefficient dependence on water-film thickness (solid line, right axis), and will help understanding the experimental results. Three regimes can be distinguished: I. Increase in film thickness leads to a sub-proportional growth of the real contact area; overall friction decreases. II. Increase in film thickness leads to an over-proportional growth of the real contact area, overall friction increases. III. Real area of contact reaches 100%, other processes limit water-film thickness (pressing out, self-balanced film thickness: even thicker films lead to less heat available from shearing, in turn leading to thinner films and so on). Thicker films leading to again lower friction are therefore not expected.

Water-film thickness is expected to increase with temperature. Figure 3.3 can therefore be used to explain the temperature dependence of friction (see Figure 3.4).

In brief, the sliding process is an interplay of frictional heat conducted away and available for the melting of ice (very much temperature dependent), water-film thickness, and real contact area.
CHAPTER 3. TRIBOMETER MEASUREMENTS

Figure 3.2: Relation between the water-film thickness and the real contact area (bearing ratio). Melting of ice corresponds to a slicing off, and leads to the growth of existing contacts, and the formation of new contacts.

Figure 3.3: Contact area (dashed line, left axis) and friction (solid line, right axis) vs. water-film thickness. Qualitative curve, calculated according to Equation 3.5.
3.2. RESULTS AND DISCUSSION

The analysis is divided into three different temperature regimes which show different dependencies of the friction force on the parameters velocity and load.

**Low temperatures:** $T_{\text{env}} \leq -10^\circ \text{C}$. High ("dry", meaning badly lubricated) friction. Too little water. Increased velocity and temperature leads to thicker water films and correspondingly lower friction.

**Intermediate temperatures:** $-10^\circ \text{C} < T_{\text{env}} \leq -1^\circ \text{C}$. Regime of lowest friction. Only little load and velocity dependence. Minimum friction is expected at around $-3^\circ \text{C}$ (see Figure 3.4 and [BFR01]).

**Temperatures close to the melting point:** $T_{\text{env}} > -1^\circ \text{C}$. Wet friction regime. Fully water lubricated friction is expected. High friction, due to largely increased contact area.

The concept of a friction coefficient as the ratio between friction force and load can become somewhat ill-defined, especially at low loads (the non-vanishing friction force at zero load leads to infinitely high coefficients).

Table 3.1 lists some roughness parameters of the sliders used in the experiments. Sliders A and B are equipped with thermocouples close to the interface, equally spaced along the sliding direction. For the exact position of the thermocouples in the respective sliders, see Figure 2.9, Chapter 2.

Figure 3.4: Friction coefficient vs. temperature in a velocity range of $v=3 \text{ ms}^{-1}$ to $5 \text{ ms}^{-1}$ and a load range of $F_n=52 \text{ N}$ to $84 \text{ N}$, summarized and plotted for 3 different apparent contact areas (slider A). Dashed line is shown as a guide to the eye.
### Table 3.1: Roughness of the sliders used in the experiments. The surfaces of the sliders are ground with fine emery paper. Slider A was polished in addition. Sliders B and C exhibit similar roughnesses, and still a "hairy" surface originating from the grinding.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Slider A</th>
<th>Sliders B &amp; C</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa</td>
<td>2 µm</td>
<td>3.6 µm</td>
<td>Average roughness</td>
</tr>
<tr>
<td>Pz</td>
<td>22 µm</td>
<td>40 µm</td>
<td>Average maximum height of profile</td>
</tr>
</tbody>
</table>

#### 3.2.1 Low Temperatures

At temperatures of $T_{env}=-10^\circ C$ and below, a major part of the frictional heat generated is expected to be conducted into slider and (mainly) ice, due to high temperature gradients. Only little heat is therefore available for melting water, and only thin water films form. This is very pronounced at low velocities, where the heat generation $P = \mu F_nv$ is small (see Figure 3.5 at $T_{env}=-15^\circ C$). The decreasing friction with increasing velocity can be explained by more heat being available to melt water, and more water increasing the average water-film thickness (while real contact area negligibly increases), which leads to lower friction (corresponds to regime I, see Figure 3.3). Earlier measurements [ENC76, OK82] showed the friction coefficient to follow $v^{-0.5}$. At even higher velocities, the competing processes will be the growth of the water films (decreasing friction) and the shearing of the latter (increasing friction with velocity), leading to a finite friction coefficient, or possibly to an overall increase.

Otherwise the behavior is somewhat classical: friction force increases linearly with load (Figure 3.6), yielding a friction coefficient $\mu \approx 0.1$ at $v=5\,\text{ms}^{-1}$. The extrapolated slider temperature at the interface can be interpreted as an average of contact spots at $T=0^\circ C$ and regions of the slider that have not made contact and are still at $T=-15^\circ C$ (assumption for not too large relative real contact areas, where contact spots are few and fairly well separated). A temperature of $T=-12^\circ C$ therefore corresponds to a relative real contact area of approximately 20%. This however represents an upper limit, since non-contacting regions at the interface have also been warmed by adjacent contacting areas.

If all frictional heat is assumed to be conducted into the slider, a calculation for the slope of the temperature rise vs. load curve (Equations 3.1 and 3.3, after about $t=120\,\text{s}$, a steady state is reached) results in a value that is 100 times higher than the experimentally obtained $0.06\,\text{KN}^{-1}$. This implies that only a small fraction of the frictional heat enters the slider; most if it is conducted into the ice, or used in the phase transfer from ice to water. Apart form the seven times higher thermal conductivity of ice, as compared to polyethylene, the temperature gradients in the ice are higher. The ice is at a lower temperature, since it is constantly "refreshed",
3.2. RESULTS AND DISCUSSION

Figure 3.5: Dependence of friction coefficient on velocity at $T_{env}=-15\,^\circ C$ and $F_n=52\,N$ and $84\,N$ (slider A). Measurements at low velocity ($v < 1\,m/s$) are omitted in the fit, they do not seem to obey the predicted law. See Chapter 2 for a discussion.

while the slider is being heated permanently.

At $T_{env}=-10\,^\circ C$, friction decreases with increasing velocity, similar to the trend observed at lower temperatures (Figure 3.7). Note the lower apparent contact area compared to the measurements at $T=-15\,^\circ C$ for the series of measurements presented. Lower contact area leads to lower friction, however not proportionally. Friction force still increases linearly with load (Figure 3.8), rendering friction coefficients of $\mu=0.046$ (for smaller contact area) and $\mu=0.062$ (for larger contact area). The temperature increase in the slider does not depend on the contact area. This implies that the same friction process integrated over a large area leads to proportionally higher friction.

Contact Area

Friction clearly increases with increasing apparent contact area (Figure 3.9, only small velocity dependence in this range). The plot is divided in three regions:

1. A high pressure region, where the mean pressure is roughly $p=1\,MPa$. The actual pressure at the contacts can be even higher. Note that the load is kept constant for all data shown. At such high pressures, other processes such as squeeze out of the water films, or even considerable elastic and/or plastic deformation of either surface are expected to play a role. This can explain the non-vanishing friction force towards zero contact area.

2. For the linear region, the slope of a fit to the data can be compared to Equation 3.5 to find the quotient of relative real contact area and
Figure 3.6: Dependence of friction force and temperature in the slider close to the interface on load at $T_{env}=-15$°C and $v=5$ ms$^{-1}$ (slider A).

Figure 3.7: Dependence of friction coefficient on velocity at $T_{env}=-10$°C and $F_n=84$ N for 2 different apparent contact areas (slider A). $v^{-0.5}$ fit according to [ENC76, OK82].
3.2. RESULTS AND DISCUSSION

Figure 3.8: Dependence of friction force and temperature in the slider on load at $T_{env}=-10^\circ C$, $v=5\ \text{ms}^{-1}$ and contact areas $A_{app}=0.8\ \text{cm}^2$ and $2\ \text{cm}^2$ (slider A).

water-film thickness. Taking the inverse of that ($h_{wf}$ divided by $relRCA$) yields a value of about $1\ \mu\text{m}$ (this would correspond to $h_{wf}$ for 100% $relRCA$). If the water-film thickness were constant (say 100 nm), this would result in a relative real contact area of about 10%. In other words, the thickness of the water films and the relative real contact area stays constant, but only up to a certain apparent contact area. 3. At some point (in this case around $A_{app}=6\ \text{cm}^2$) the water film starts to be dispersed, and friction force levels off at a constant value (constant region).

3.2.2 Intermediate Temperatures

At intermediate temperatures around $T_{env}=-5^\circ C$, the friction coefficient does not depend on velocity in the range observed (Figure 3.10). This implies that either the water-film thickness stays fairly constant, or that two competing processes balance each other, namely the increase in water-film thickness (for itself leading to lower friction) and the increase in real contact area (for itself leading to higher friction). This should manifest in a relatively flat minimum in the friction coefficient vs. water-film thickness curve (see Figure 3.3). Only for small contact areas, friction increases slightly with velocity, implying that increased velocity does rather lead to larger $relRCA$ than to larger $h_{wf}$.

Load and Pressure

Pressure is a poorly defined concept in friction, since the actual pressure at the contacts is seldom known. Figure 3.11 shows the dependence of the friction force on
CHAPTER 3. TRIBOMETER MEASUREMENTS

Figure 3.9: Dependence of friction force on contact area at $T_{\text{env}}=-10^\circ\text{C}$, $F_n=84$ N and velocities $v=3.3$ ms$^{-1}$ and 5 ms$^{-1}$ (slider A).

Figure 3.10: Dependence of friction coefficient on velocity at $T_{\text{env}}=-5^\circ\text{C}$, $F_n=52$ N, and different apparent contact areas ($A_{\text{app}}=0.8$ cm$^2$ to 10 cm$^2$, slider C). Note: different slider exhibiting higher friction. Lines are included as a guide to the eye.
3.2. RESULTS AND DISCUSSION

the mean pressure in the apparent contact area considered. Friction force at zero pressure (or load) can be calculated according to Equation 3.5. Using $v=5 \text{ms}^{-1}$, $A_{\text{app}}=2 \text{cm}^2$, a film thickness of $h_{wf}=1 \mu\text{m}$, and $\text{relRCA}=1$ (100%), a friction force of 3.6 N results. A relative real contact area of 25% results in a value of about 1 N, coinciding with the measurement (assuming a water-film thickness of 200 nm would correspond to 5% relative real contact area). If friction force were only pressure dependent, all data points shown should be on one straight line. This is clearly not the case, proof that friction depends on both area and load. Explanation for the increasing friction with load: higher load leads to increased contact area and/or thinner water films (squeeze). Note: Friction force at zero load should be area dependent, however, extrapolation from a linear fit does not show this.

![Figure 3.11: Dependence of friction force on mean pressure at $T_{\text{env}}=-5 \text{C}$ and $v=5 \text{mss}^{-1}$ for $A_{\text{app}}=2 \text{cm}^2$ and $4 \text{cm}^2$ (slider B).](image)

**Temperature**

Although the friction force does not depend on velocity, and only slightly on load, the measured temperatures in the slider are very much dependent on these parameters (Figure 3.12). Temperature increases proportionally to heat generation (Equation 3.1) up to $v=3 \text{ms}^{-1}$. Assuming that all frictional heat is conducted into the slider, a calculation for the slope of the temperature rise vs. velocity curve (Equations 3.1 and 3.3) results in a value that is 150 times higher than the experimentally obtained 1.6 $\text{Km}^{-1}\text{s}$. This confirms that only a small fraction of the frictional heat enters the slider; most if it is conducted into the ice, or used in the phase transfer from ice to water. Temperature in the slider at a depth of 0.2 mm...
reaches approximately $T = -0.5^\circ\text{C}$ for $v \geq 5 \text{ ms}^{-1}$. The phase transition at $0^\circ\text{C}$ constitutes an upper temperature limit, the increase flattens for higher velocities. Heat is consumed in melting ice, the expected lower friction due to thicker films is balanced by increased real contact area and the shearing of the films being proportional to velocity. Assuming a steady state, and thus a linear temperature distribution in the slider (room temperature at the upper side of the slider), the average temperature at the interface can be calculated to $T \approx -0.4^\circ\text{C}$ (uncertainty of the thermocouple temperature measurement: $\pm 0.2^\circ\text{C}$). This temperature is an average of the area in contact at $T = 0^\circ\text{C}$ and the non-contacting region at a temperature below $T = 0^\circ\text{C}$. An upper limit for the contact area can be calculated to $92\% \pm 4\%$ by assuming a temperature of the non-contacting region of $T = -5^\circ\text{C}$. For velocities $v < 5 \text{ ms}^{-1}$, the temperature in the slider does not rise as high, meaning that less contacting area at $T = 0^\circ\text{C}$ contributes. E.g. the value of $T = -2.7^\circ\text{C}$ at $v = 1.7 \text{ ms}^{-1}$ corresponds to an average interface temperature of $T = -2.6^\circ\text{C}$. An upper limit for the contact area can again be calculated to $47\% \pm 4\%$.

Figure 3.12: Dependence of friction force and slider temperature close to the interface (after 5 minutes of measurement) on velocity at $T_{\text{env}} = -5^\circ\text{C}$, $F_n = 84 \text{ N}$, $A_{\text{app}} = 4 \text{ cm}^2$ (slider B).

Figure 3.13 shows the slider temperature to increase linearly with friction force. Although the thermocouples measure temperature at slightly different distances from the interface (TC1 closest, TC2 and TC3 about twice the distance from the interface), it can be concluded that the slider heats up more at the front. A cross-check with the slider running backwards resulted in the same trend. This implies that the water films are thinnest at the front, and become thicker along the slider. The temperature in a different slider (A) increases from front to back (see Figure 3.14).
Again, a cross-check with the slider running backwards resulted in the same trend. The apparent contact area has no influence on this trend. The two explanations for the low slider temperature at the front are: 1. At the front, the slider comes into contact with cold ice, while the back of the slider already sees heated ice. This is, however, true for all sliders. 2. The deviation from the trend seen for slider B is due to a generally lower friction. I.e. for slider A, relative real contact area is small and increases towards the end, while for slider B, relative real contact area is generally high, and the water-film thickness increases towards the end. These findings show that surface topography can have a large impact on friction. The higher friction for slider B is attributed to the "hairy" surface.

Figure 3.13: Temperature (after 5 minutes measurement) at different positions in the slider depending on friction force ($T_{env}=-5\,\text{deg C}$, $v=5\,\text{ms}^{-1}$, $A_{app}=2\,\text{cm}^2$, slider B). For zero friction force, the initial temperature of the slider is used.

**Contact Area**

The same as for low temperatures, friction increases with increasing $A_{app}$ (Figure 3.15). The slope of a fit to the data can be compared to Equation 3.5 to find the quotient of relative real contact area and water-film thickness. Again, as for low temperatures, a value of $h_{wf}/relRCA\approx1\,\mu m$ is found. Temperature evolution in the slider on the other hand does not depend on the size of the apparent contact area (Figure 3.16). Thermocouples always "feel" the same temperature. This is not evident, since load stays constant, leading to higher average pressure. The ice surface temperature behind the slider measured by the IR thermocouple also increases, due to the aperture angle of the sensor, which results in a measurement spot diam-
Figure 3.14: Temperatures (after 5 minutes measurement) at different positions in the sliders ($T_{env} = -5^\circ C$, $F_n = 84 N$, $v = 5 \text{ ms}^{-1}$, $A_{app} = 4 \text{ cm}^2$). Dashed lines are included as a guide to the eye.

At an environmental temperature of $T_{env} = -1^\circ C$, the friction coefficient is again constant in the velocity range observed (Figure 3.17). For a high enough load, temperature close to the interface reaches $0^\circ C$ (Figure 3.18, uncertainty of the temperature measurement: $\pm 0.2^\circ C$). When temperature reaches $T = 0^\circ C$, the friction curve flattens, or vice versa.

To sum things up for intermediate temperatures, it can be said that the friction coefficient depends on the apparent contact area but not on velocity and only slightly on load. Thickness of the films and relative real contact area remain fairly constant over a large range of these parameters. The relative real contact area is difficult to estimate but should be around 50% for intermediate values, with average water-film thicknesses of several hundred nanometers.

### 3.2.3 Temperatures Close to the Melting Point

At temperatures between roughly $T_{env} = -0.5^\circ C$ to $+0.5^\circ C$, heat conduction plays a minor role, and behavior of relatively thick water films can explain the dependence of the friction coefficient on velocity and load.

From [OK82], water-film thickness is proportional to $\sqrt{v}$, thus from Equation 3.5 an overall dependence of $F_f \sim \sqrt{v}$ results. At zero velocity, the friction coefficient is
3.2. RESULTS AND DISCUSSION

Figure 3.15: Dependence of friction coefficient on apparent contact area for different loads (slider C) at $T_{env}=-5^\circ$C. Velocities of $v=3.3\text{ ms}^{-1}$ and $5\text{ ms}^{-1}$ are combined since there is hardly any velocity dependence of the friction coefficient.

Figure 3.16: Temperature increase of ice surface and slider for $T_{env}=-5^\circ$C, $F_n=84\ N$, $v=5\text{ ms}^{-1}$ and track widths of $w=5\text{ mm}$ and $10\text{ mm}$ (slider B, length 40 mm).
CHAPTER 3. TRIBOMETER MEASUREMENTS

Figure 3.17: Dependence of friction coefficient on velocity at $T_{env}=-1^\circ C$ for different loads and apparent contact areas (slider A). Lines are included as a guide to the eye.

Figure 3.18: Dependence of friction force and slider temperature on load at $T_{env}=-1^\circ C$, $v=3.3\,\text{ms}^{-1}$ and $5\,\text{ms}^{-1}$ (combined) and $A_{app}=2\,\text{cm}^2$ (slider A).
3.2. RESULTS AND DISCUSSION

zero. A linear fit ("valid" in velocity range \(v=2\text{ ms}^{-1}\) to \(8\text{ ms}^{-1}\)) to the experimental data in Figure 3.19 yields a slope of 0.0082. Assuming that the relative real contact area is close to 100% at 0°C, an average water-film thickness of about 4 \(\mu\text{m}\) results (Equation 3.5). This assumption is further confirmed by the fact that the slider reaches temperatures above 0°C in some measurements. Based on the present understanding, this is only possible if the relative real contact area has reached 100%, otherwise additional heat would be used to melt more water and increase the real contact area.

![Figure 3.19](image_url)

**Figure 3.19:** Dependence of friction coefficient on velocity at temperatures close to the melting point (wet conditions) for \(F_n=52\text{ N}\) and \(A_{\text{app}}=10\text{ cm}^2\) (slider A).

The extrapolated friction force (see Figure 3.20) at zero load of \(F_f=2.9\text{ N}\) corresponds well with Equation 3.5 for a relative real contact area close to 100% and an average water-film thickness of about 3 \(\mu\text{m}\). Lower contact area would correspond to proportionally thinner films. Note that the friction coefficient becomes ill-defined for a non-vanishing friction force at zero load. A common construction is to plot the friction coefficient against the quotient of velocity (times viscosity) divided by load, the so-called Stribeck curve. For hydrodynamic friction, a linear relationship results (Figure 3.21).

### 3.2.4 Influence of Surface Topography

Since a strong dependence of the friction coefficient on the apparent contact area is observed in the measurements, samples exhibiting defined surface structures were prepared (see Figure 3.22). The grooves were cut parallel to the sliding direction, in a 60° angle to the surface, and spaced 0.9 mm (‘fine’) and 1.8 mm (‘coarse’) from
Figure 3.20: Dependence of friction force on load at temperatures close to the melting point for \( v = 5 \text{ ms}^{-1} \) and \( A_{\text{app}} = 10 \text{ cm}^2 \) (slider A).

Figure 3.21: Dependence of friction coefficient on velocity divided by load at temperatures close to the melting point (\( A_{\text{app}} = 10 \text{ cm}^2 \), slider A).
3.2. RESULTS AND DISCUSSION

Figure 3.22: Polyethylene samples with longitudinal (parallel to the sliding direction) structures. Ice dust accumulation in the grooves. Width of the elevated track is $w = 2$ cm.

each other. Two samples exhibiting a smooth (machined) surface were measured for comparison. Striking is the rather large difference between the samples 'smooth 1' and 'smooth 2' (Table 3.2, above). It was observed before that samples machined the same way can exhibit quite different friction coefficients. This is likely to be due to the orientation of the pattern (circles) resulting from the machining. In order to produce smooth surfaces that exhibit similar friction coefficients, the samples must be ground using fine emery paper. Yet it can be seen that sample 'fine' shows a reduced friction coefficient, while the friction coefficient of sample 'coarse' is similar to the smooth samples. The explanation goes as follows: there are two competing effects, namely the reduction of the real contact area due to the linear grooves, and the carving of the sliders into the ice. The latter leads to increased real contact area, and therefore friction. In addition, there seems to be more ice dust accumulating in the grooves of sample 'coarse'. This can again lead to increased contact area and friction, namely between ice dust and ice. Both these effects that lead to increased contact area (carving and ice dust accumulation) are time dependent (Table 3.2, below). Four consecutive measurements of sample 'fine' show a constantly low friction coefficient for the first three measurements, before friction starts to increase. Since the surfaces are cleaned (dusted) between measurements, this is more likely to be due to the carving effect.

The friction of sample 'smooth 1' was measured on structured ice (see Figure 2.6, Chapter 2). Again, friction is reduced considerably. Such effects were also observed by other researchers [DZM+05].

These few measurements demonstrate the possibility of influencing friction using defined surface structures. A reduction of friction based on the principle of reduced real contact area is possible. A wider study of different surface structures could help clarify the mechanisms mentioned. It must be kept in mind that these observation hold for sliding on ice. On snow, both carving effect and snow and ice dust
CHAPTER 3. TRIBOMETER MEASUREMENTS

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>smooth 1</td>
<td>0.043</td>
</tr>
<tr>
<td>smooth 2</td>
<td>0.033</td>
</tr>
<tr>
<td>fine</td>
<td>0.025</td>
</tr>
<tr>
<td>coarse</td>
<td>0.039</td>
</tr>
<tr>
<td>smooth 1 on structured ice</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 3.2: Friction coefficients measured for 4 samples 'smooth 1', 'smooth 2', 'fine', and 'coarse'. Mean value of 3 measurements (upper table). Evolution of friction coefficient of sample 'fine' in order of measurements (lower table). $T_{env}=-10^\circ\text{C}$, $F_n=52\text{ N}$, $v=5\text{ m s}^{-1}$, and $A_{app}=8\text{ cm}^2$.

accumulation in the grooves are likely to be more dominant, especially for coarse structures.

3.3 Conclusion

A hydrodynamic approach is successfully used to explain the friction forces and temperature evolutions measured. The main factors determining friction are the thickness of the water films and the relative real contact area. Both these factors can vary over more than an order of magnitude, depending mostly on temperature and velocity (see Figure 3.23).

The friction coefficient decreases with increasing load, especially at higher temperatures. This implies that a heavy skier is advantaged.

The main implication towards an improved sliding behavior of ski bases in real skiing is that surface topography and apparent contact area matters. In world cup skiing, ski size and shape is regularized to within narrow margins. However, ski base structure is not. Relatively broad linear grooves along the ski are likely to reduce friction.
at 0 degC: $F_n = 52$ N, $A_{app} = 10$ cm$^2$

at −1 and −10 degC: $F_n = 84$ N, $A_{app} = 2$ cm$^2$

Figure 3.23: Dependence of friction coefficient on velocity at different temperatures. Note: different loads and apparent contact areas (slider A).
Investigations of the Contact Area between Polyethylene and a Snow or Ice Surface

Abstract

A good estimate of the relative real contact area and the contact spot size is essential to deduce a model for the sliding friction of polyethylene on snow and ice. Real contact area is in general hard to determine experimentally, especially for dynamic processes (see [Bhu02], pp. 154-161). For the experimental investigation of the contact area between polyethylene and snow (“real” skiing situation), scanning electron microscopy and X-ray computer tomography have been used. Contact spot size can be estimated, and a dependence of the real contact area on load and snow type can be seen. The investigation of the contact area between polyethylene and ice (tribometer experiment) is carried out on imprints of the polyethylene slider and the ice surface, by means of optical profilometry. The effect of polishing of the ice by the slider during friction experiments is observed. All methods described give a very precise surface characterization, and the results will be used in the prediction of the contact area and contact spot size evolution in the friction process. Note, however, that the measurements are merely static, and can therefore only yield rough estimates.

4.1 Introduction

The real contact area between polymers (here: polyethylene) and snow or ice is of great interest with regard to snow and ice friction. In most tribological systems, the real contact area \(A_{real}\) between two surfaces is generally less than the apparent
contact area \((A_{\text{app}})\) because the surfaces are in contact only at a number of small asperities. There is considerable debate regarding the type of deformation associated with a pair of conforming rough surfaces in contact. Persson (see [Per00], pp. 49-51) gives an analytical solution, linking roughness and mechanical properties, to determine whether elastic or plastic deformation occurs:

\[
f_p = e^{20 \cdot \frac{r}{l} \left( \frac{\sigma_c}{E} \right)^2},
\]

where \(f_p\) is the fraction of junctions that have undergone plastic deformation, \(r\) is the radius of curvature of an asperity, \(l\) is the rms width of the amplitude distribution function (see Section 4.3.3), \(\sigma_c\) is the indentation hardness and \(E\) the Young's modulus of the softer material. For ground and polished surfaces (such as a flat ice surface), \(r/l\) is generally about \(10^4\), yielding a fraction of \(f_p=0.45\). Thus both elastic and plastic deformation can play a role in ice friction. For rougher surfaces (such as a snow surface), \(r/l\) is about 10, yielding a fraction of \(f_p=0.98\). Thus deformation is mainly plastic. Note that these estimates only hold for purely elasto-plastic materials. The mechanical behavior of ice is very much time dependent, however. It creeps under the slightest load [Hob74].

**Elastic Deformation**

Hertzian contact theory (see [Joh85], pp.90-93) is applied between (hexagonal) closely packed spherical ice grains of diameter \(400\ \mu\text{m}\) and a flat polyethylene surface; see Figure 4.1, representing a simple geometry of a ski sliding on snow. Corresponding to a skier, a (static) load of \(F_n=800\ \text{N}\) on \(A_{\text{app}}=0.3\ \text{m}^2\) is assumed. Radius of the contact spots and mean pressure at the contacts are given by

![Figure 4.1: A simple geometrical model for a slider on snow. Elastic deformation on Closely packed spherical ice grains of radius \(r\). Load \(F_n\), mean pressure at the contact \(p_m\), contact spot radius \(a\).](image-url)
4.1. INTRODUCTION

\[ a = \left( \frac{3 \cdot F^*_{n} \cdot r}{4 \cdot E^*} \right)^{\frac{1}{2}} \]  

(4.2)

and

\[ p_m = \frac{F^*_{n}}{a^2 \cdot \pi}, \]  

(4.3)

where

\[ \frac{1}{E^*} = \frac{1 - \nu_{PE}}{E_{PE}} - \frac{1 - \nu_{ice}}{E_{ice}}. \]  

(4.4)

\( E^* \) is the reduced modulus and \( E \) and \( \nu \) are Young’s moduli and Poisson’s ratios of the contacting surfaces, respectively. The load is being supported by \( n \) contacts, resulting in \( F^*_n = F_n/n \) per contact. \( F^*_n = 0.37 \text{ mN} \) (corresponding to a skier) leads to a contact spot radius of \( a = 4 \mu m \) and a mean pressure at the contact of \( p_m = 7.1 \text{ MPa} \) and a maximum pressure at the contact of \( p_0 = 3/2 p_m = 10.7 \text{ MPa} \). Note that \( p_0 \) is of the same magnitude as the indentation hardness of ice \( (\sigma_c = 40 \text{ MPa}) \). Plastic yield is therefore likely to occur. The relative real contact area \( (\text{relRCA}) \) can then be calculated as 0.04%. Keep in mind that poly-crystalline ice creeps under the slightest load, and that the calculation therefore underestimates \( A_{\text{real}} \). The value of \( \text{relRCA} \) therefore presents a lower limit. The study could be refined using a Gaussian height distribution, but this has not been undertaken.

**Plastic Deformation**

For most (not too smooth) surfaces of elasto-plastic materials, the real contact area can be estimated by assuming that each junction is in a state of incipient plastic flow (see [Per00], p. 49), so that

\[ A_{\text{real}} = \frac{F_n}{\sigma_c}, \]  

(4.5)

However, snow and ice are not simple elasto-plastic materials. The concept of a single indentation hardness value can be misleading. Snow and ice creep as soon as loaded. Hardness is therefore very much deformation rate dependent. In addition, the hardnesses of snow, ice, and polyethylene all depend on temperature in the range of interest [Bow53]. Using Equation 4.5, a lower limit of \( \text{relRCA} \) of polyethylene on snow or ice can be found. For a skier \((F_n = 800 \text{ N}, A_{\text{app}} = 0.3 \text{ m}^2)\) on snow of indentation hardness \( \sigma_c = 50 \text{ kPa} \) [SJSB97], \( \text{relRCA} = 5\% \) results. For the tribometer experiment \((F_n = 84 \text{ N}, A_{\text{app}} = 4 \cdot 10^{-4} \text{ m}^2)\) on ice of indentation hardness \( \sigma_c = 40 \text{ MPa} \), \( \text{relRCA} = 0.5\% \) is calculated.
CHAPTER 4. CONTACT AREA

Frictional Melting

Both estimates so far consider a static contact, and are therefore likely to under-estimate the real contact area developed during the dynamic process of skiing. An estimate of dynamic effects, i.e. melting, shall sum up these theoretical considerations. Starting from the situation depicted in Figure 4.1, the heat $Q$ flowing into one ice grain during the passage of a ski is

$$Q = \mu \cdot F_n \cdot l_{ski},$$  \hspace{1cm} (4.6)

where $\mu$ is the friction coefficient and $l_{ski}$ is the length of the ski. 95% of the heat is assumed to be conducted away into ice and slider [Kur77], and the remaining 5% is used for the melting of small ice caps, corresponding to a slicing off of a spherical segment. The radius of the resulting contact spot then calculates to $a=25 \mu m$ ($\mu=0.02$ and $l_{ski}=2 m$ is used), $\text{relRCA}$ of 1.5% results. The height of the arising cylindrical water film is calculated to $0.8 \mu m$ (squeeze out and shearing out are neglected).

This last estimate seems to be of the right order of magnitude for the values of contact spot size, relative real contact area, and water film thickness. Furthermore, it can be stated that all of the mechanisms discussed can play a role in snow and ice friction. A number of people have tried to approach the problem experimentally. E.g. a snow surface that had seen repeated passing with a slider was examined [Kur77], or the snow conductivity was measured [PS86]. In this work, several methods will be evaluated and, if appropriate, applied.

4.2 Experiments

Several methods were evaluated, see Table 4.1 for a summary. The three experimental methods used in this work are described in more detail hereafter.

4.2.1 Scanning Electron Microscopy

A Philips SEM 515 scanning electron microscope equipped with a liquid nitrogen (l-N$_2$) cooling stage allowing for temperatures down to -160°C was used. Special care has to be taken for the sample preparation, in order to avoid sublimation and/or re-sublimation of air humidity onto the sample. For the preparation of the surface, a flat, piste-like snow surface is passed repeatedly with a small ski. A snow cube of 8 mm x 8 mm x 8 mm is cut out of the ”surface treated” snow, using a warmed blade. Care has to be taken no to introduce heat into the sample. The cube is frozen onto the sample holder, which has been cooled in l-N$_2$, using water and a pipette. For
4.2. **EXPERIMENTS**

<table>
<thead>
<tr>
<th>Method</th>
<th>Resolution</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scanning electron microscopy (SEM)</td>
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<td>For low temperature SEM, the snow sample preparation is difficult. Stero-SEM of imprints at room temperature can render higher resolution than profilometry.</td>
</tr>
<tr>
<td>X-ray computer tomograph ($\mu$CT)</td>
<td>10 $\mu$m</td>
<td>Straightforward measurement. Involves extensive image analysis.</td>
</tr>
<tr>
<td>Optical profilometry</td>
<td>1 $\mu$m</td>
<td>Imprints of ice surfaces used.</td>
</tr>
<tr>
<td>Atomic force microscopy (AFM)</td>
<td>10 nm</td>
<td>Major modifications of standard AFM experiment are necessary [DBKB98, DB00].</td>
</tr>
<tr>
<td>Light microscopy (LM)</td>
<td>1 $\mu$m</td>
<td>Problematic because of transparency of ice.</td>
</tr>
</tbody>
</table>

Table 4.1: Measurement methods for determining surface topography or real contact area, respectively.

longer storage, the sample can be stored at -40°C in a fridge, but has to be covered (lid or polymer foil) to avoid sublimation. For transportation, sample and sample holder are immersed in l-$\text{N}_2$ until placed into the microscope. When taken out of the l-$\text{N}_2$ and put into the pre-vacuum chamber of the microscope, exposure to air should be minimized. In the pre-vacuum chamber, the sample can be left to remove re-sublimated water (surface hoar), before a 6 nm platinum layer is sputtered onto the surface. Other groups have developed more elaborate methods for examining snow in the SEM, however, such instrumentation was not available to us [RWE96].

4.2.2 **X-Ray Computer Tomography**

An X-ray computer tomograph is used for the experimental investigation of the contact area between polyethylene and snow. A Scanco $\mu$CT 80 in a fan beam configuration with a microfocus X-ray source (7 mm diameter) and a line photodiode with 2048 pixels is used for the images. Images are taken with 10-20 $\mu$m nominal resolution, and 10-20 $\mu$m vertical distance between slices. From the original reconstructed volume, a cuboid of size 1000 x 1000 x 100 voxels is extracted, cor-
responding to physical side lengths of 10 mm x 10 mm x 1 mm. Image processing and stereological characterization: the raw sinograms of the computed tomography scanner are transformed to 16-bit images. The stacked images are median filtered, Gaussian filtered (2 x 2 x 2) and segmented by threshold, resulting in a binary set of data. The threshold is determined from the histogram.

The snow is prepared by sieving, using sieves of 500 µm, 700 µm, 1000 µm, and 1400 µm mesh size. For the most often used snow of grain size 500-1000 µm, an average density of about 300 kgm$^{-3}$ results. Only the density at the very surface is relevant for the measurement, however. There, the snow can be very much compressed, depending on the sample preparation. The snow is sieved directly into the sample holder, a cylindrical beaker of diameter 20.5 mm or 32 mm. Special care has to be taken to achieve a horizontal snow surface before pressing on it. Two different ways of applying pressure onto the snow surface are used. One is to press a perfectly fitting polyethylene piston into the cylinder and onto the surface, and applying the desired load using dead weights. Vertical grooves cut into the piston avoid air entrapment. The piston is then kept in place by freezing it to the side wall of the beaker using water. The macroscopic strain is therefore kept constant. Another method is to use a spring with a known spring constant with which a defined initial pressure can be applied. Then, the stress is kept constant (if creep is neglected).

For experimental determination of the influence of frictional heat on the snow surface, a 15 mm x 15 mm peltier element (Quick-Cool 4.6 W) is used to melt the surface of a cylindrical snow sample. For these surface melting experiments, samples are first prepared in a cylinder, pressed, and left to sinter overnight. The snow sample is then taken out of the cylinder, and loaded with the peltier element plus additional load. A thin sheet of polyethylene (non-sticking) is placed between the snow and the heated plate. Heat conductive paste is used to connect polyethylene and peltier element. A fan and cooling fins provide efficient cooling at the upper side of the peltier element. The cooling-heating-cooling cycle is monitored using a relay. A voltage of $U=2$ V, resulting in a current of $I=2$ A is applied and reversed for 4 seconds (procedure: cool - heat for 4 s - cool). The heat flow into the snow surface is $q''=18$ kWm$^2$, or somewhat less, due to the thermal resistance of the polyethylene sheet. The fan is left running during the whole experiment. The snow cylinder is placed in the sample holder. Great care has to be taken not to deposit snow debris on the melted snow surface.
4.2.3 Optical Profilometry

It is not easily possible to carry out topographic measurements on snow and ice surfaces with a high resolution (higher than the 10 µm provided by the X-ray computer tomograph). The transparency of ice makes it impossible to use optical profilometry, and standard contact profilometry cannot be used due to the creep behavior of ice. Imprints of ice surfaces are thus used. Preliminary tests with different imprint materials were carried out. Criteria were the curing behavior at temperatures below 0°C (time of curing, heat of curing) and the resolution. The dimethyl siloxane polymer PROVIL novo Light C.D.2 ‘fast set’ gave very good results: curing time at sub-zero temperatures is about one hour, resolution is in the sub-micrometer range [SKd+06]. For an exact quantitative analysis, the thermal expansion of the imprint material would have to be considered, since the profilometer measurement is carried out at room temperature. This, however, was neglected.

For the topographic measurements, a Fries Research and Technology (FRT) Micro Prof is used. The sample is illuminated by a focused white light. An internal passive optics, using chromatic aberration, splits the white light into different colors (corresponding to different wavelengths). A miniaturized spectrometer detects the color of the light reflected by the sample and determines the position of the focus point. By means of an internal calibration table, the vertical position on the sample surface is determined. Sub-micron resolution is achieved.

4.3 Results and Discussion

4.3.1 Scanning Electron Microscopy

Figure 4.2 shows a snow surface that has seen repeated passes of a miniature ski (smooth polyethylene base) at magnitudes of roughly 10:1 (left) and 100:1 (right). Clearly visible are the flattened regions that have seen contact and are evidence of melting processes. They are very unevenly distributed. The detail image on the right shows one single flat contact of a diameter of about 400 µm. The flakes are an artefact resulting from the delamination of the platinum layer.

It remains to be investigated whether these images are representative of a real ski slope that has been skied on. Yet it can be concluded that electron microscopy is a suitable instrument for determining contact spot size an relative real contact area, if a careful sample collection and preparation is conducted.
4.3.2 X-Ray Computer Tomography

Since the X-ray absorption of ice and polyethylene is very similar, the two materials cannot be distinguished from X-ray data. In order to extract the real contact area from the 3D image, the (usually slightly inclined) interface plane has to be determined first. This is done by scanning through each vertical \( z \) column from the upper ("ski") side and finding the \( z \)-value where polyethylene or ice changes to air (the data set is binary, 1 for ice or polyethylene, 0 for air). For low contact areas, the interface plane can be easily determined, since most of the \( z \)-values lie in that plane. The \( z \)-values that do not lie in the interface plane correspond to contacting ice grains. For larger contact areas, the interface plane is determined by examining the histogram of the \( z \)-value distribution in four corners of the data set.

Constant-Strain Measurements

Figure 4.3 shows a CT image of a polyethylene sample pressed on snow with a grain size of 500-1000 \( \mu \text{m} \). Figure 4.4 shows the real contact area extracted from the 3D image, the black areas represent contact spots (junctions). Contact spot diameter ranges from some 10 \( \mu \text{m} \) to maximum 400 \( \mu \text{m} \), with an average of about 100-200 \( \mu \text{m} \). The relative real contact area amounts to a mean value of \( \text{relRCA}=7\% \).

Constant-Stress Measurements

Figures 4.5 and 4.6 show the results of the constant-stress experiments. Striking is the highly increased contact area compared to the constant-strain measurement. This can have several reasons: 1. The snow creeps under the spring, at the same time densifying. 2. In the constant-strain case, the snow creeps away from the interface, leading to lower real contact area. 3. Friction of the polymer piston at
4.3. RESULTS AND DISCUSSION

Figure 4.3: Computer tomography image of a flat polyethylene sample pressed on snow. Extract is 5 mm x 5 mm x 0.9 mm. Snow grain size is 500-1000 µm.

Figure 4.4: Interface plane determined from CT data. $T_{\text{env}}=-10^\circ$C, apparent pressure $p=40 \text{ kPa}$, constant strain, fine grained snow diameter $D=0.5 \text{ mm}$ to 1.0 mm. Extract is 10 mm x 10 mm, black=snow, white=air. Relative real contact area $\text{relRCA}=6.4\%$. 
CHAPTER 4. CONTACT AREA

the cylinder wall lowers the actual pressure in the constant-strain case. In the fine grained snow case, the snow can pack more easily, again leading to increased density and therefore increased real contact area.

In brief, it is likely that the constant-strain experiment underestimates the real contact area expected in skiing, while the constant-stress experiment overestimates it.

The experiment was carried out with two different snows and three different pressures. However, for the low pressure (about 0.6 kPa), no well defined, flat contact was achieved, and the experiment could not be evaluated. The snow used had been stored for several weeks at -20°C, then sieved to 500-700 µm and 1000-1400 µm to receive fine and coarse grained snow, respectively.

![Figure 4.5: Interface plane determined from CT data. $T_{env}=-10^\circ$C, apparent pressure $p=6$ kPa (left) and 60 kPa (right), constant stress, fine grained snow diameter $D=0.5$ mm to 0.7 mm. Black=snow, white=air, side length 18 mm. Relative real contact area is $relRCA=31\%$ and 87\%, respectively.](image)

**Melting Experiments**

This series of experiments only have preliminary character. It was not possible to extract the interface and determine the real contact area from the tomograph images. Figure 4.7 shows a surface with melted snow grains. They are very unevenly distributed. This indicates that snow is being compacted unevenly, and therefore, contacts, or agglomerates of contacts, respectively, are distributed unevenly. The estimated heat flow at the contacts is $q''=18$ kWm$^{-2}$, which is about one order of magnitude higher than for a skier sliding at $v=10$ ms$^{-1}$. Yet it can be expected that the contact spot diameter increases to well above 1 mm. The experimental design can be improved by including a heat flux measurement and adequate temperature control.
Figure 4.6: Interface plane determined from CT data. $T_{\text{env}}=-10°C$, apparent pressure $p=6\text{kPa}$ (left) and $60\text{kPa}$ (right), constant stress, coarse grained snow diameter $D=1\text{ mm}$ to $1.4\text{ mm}$. Black=snow, white=air, side length $18\text{ mm}$. Relative real contact area is $\text{relRCA}=19\%$ and $45\%$, respectively. Note the hexagonally shaped snow crystals characteristic for coarse, old snow.

Figure 4.7: Surface melted snow sample. Extract is $10\text{ mm} \times 10\text{ mm} \times 0.5\text{ mm}$. Some melted areas are visible. They are very unevenly distributed.
Neck Forming

Figure 4.8 shows the necks formed between ice grains and the polyethylene sample in a constant-strain experiment. For the constant-strain case, negligible neck forming due to creep is expected, and the necks are assumed to be an artefact of the image processing procedure. For the constant-stress case, neck forming due to creep is expected. However, the image processing procedure again leads to a slightly increased contact area.

![Image of neck forming between polyethylene and ice, constant-strain experiment. The stripes in the polyethylene result from the machining of the sample. Extract is 2 mm x 2 mm x 0.9 mm.]

Summary of CT Measurements

<table>
<thead>
<tr>
<th>Snow, µm</th>
<th>Pressure, kPa</th>
<th>relRCA, %</th>
<th>Method</th>
<th>No. measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>500-1000</td>
<td>40</td>
<td>7.0 ± 1.1</td>
<td>Const. strain</td>
<td>6</td>
</tr>
<tr>
<td>500-700</td>
<td>6</td>
<td>31</td>
<td>Const. stress</td>
<td>1</td>
</tr>
<tr>
<td>500-700</td>
<td>60</td>
<td>87</td>
<td>Const. stress</td>
<td>1</td>
</tr>
<tr>
<td>1000-1400</td>
<td>6</td>
<td>19</td>
<td>Const. stress</td>
<td>1</td>
</tr>
<tr>
<td>1000-1400</td>
<td>60</td>
<td>45</td>
<td>Const. stress</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2: Summary of the measurements. All measurements were carried out at a temperature of around $T_{env} = -10^\circ$C.

The reproducibility of the measurement is about ±15% (standard deviation). The relative real contact area values obtained must therefore be interpreted carefully. For an overview, see Table 4.2. The constant-stress experiments clearly lead to increased contact area, since creep progresses even during the measurements (a CT measurement lasts about 30 minutes). The experiment can be considered static, as opposed to the highly dynamic process of sliding friction on snow. In reality,
the formation of lubricating water has an influence, although not necessarily at all temperatures. Although quantitative analysis is critical, rough estimates are possible, and the influence of pressure and snow type on the real contact area is clearly visible.

4.3.3 Optical profilometry

The different ice and polyethylene surfaces examined are listed in Table 4.3.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Material</th>
<th>Preparation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undisturbed</td>
<td>Ice</td>
<td>Tap water left to freeze at -20°C overnight. Smooth, even surface.</td>
</tr>
<tr>
<td>Track</td>
<td>Ice</td>
<td>Surface that has seen repeated passes of a slider (track after performing tribometer measurements).</td>
</tr>
<tr>
<td>Slider A</td>
<td>Polyethylene</td>
<td>Machined, ground, and polished.</td>
</tr>
<tr>
<td>Slider B</td>
<td>Polyethylene</td>
<td>Machined and ground.</td>
</tr>
</tbody>
</table>

Table 4.3: Surfaces examined.

Undisturbed ice surface

Figure 4.9 shows the surface of sample 'Undisturbed'. The dark lines are ice grain boundaries. Figure 4.10 shows a roughness profile along with some roughness parameters. Note that the z-scale (height) is inflated 5000 times, and that the ice surface is in fact very flat; it exhibits an average roughness of about Ra=60 nm, similar to a highly polished surface. For a sinusoidal profile, the root-mean-square roughness Rq should be 1.11 times Ra, here, Rq/Ra=1.27 is calculated. Rz is the maximum height of the profile, it is about one order of magnitude larger than Ra, probably due to the grain boundaries (valleys). Skewness represents the degree of symmetry of the histogram or amplitude distribution function (ADF), see Figure 4.11(a). Surfaces with a positive skewness have relatively high spikes that protrude above a flatter average. Surfaces with negative skewness have relatively deep valleys in a smoother plateau, as is the case here. An Sk value greater than about 1.5 (positive or negative) indicates that the surface does not have a simple shape and a simple parameter such as Ra is not adequate to characterize the surface. Kurtosis (K) relates to the peakedness of the profile and is a measure of the degree of pointedness (K > 3) or bluntness (K < 3) of the ADF. For most surfaces, the ADF follows a Gauss function:
CHAPTER 4. CONTACT AREA

Figure 4.9: Topography measurement of an ice surface at $T_{env}=-20^\circ$C (sample 'Undisturbed'). Grain boundaries are visible (dark lines), grain diameter is about 2 mm.

Figure 4.10: Roughness profile of sample 'Undisturbed'. The deep valleys represent grain boundaries. An Ra-value of below 100 nm indicates a very smooth surface.
4.3. RESULTS AND DISCUSSION

Figure 4.11: Statistical analysis of the topography of sample 'Undisturbed'.

\[ h = h_{max} \cdot e^{-\frac{(h-h_c)^2}{2w^2}}, \]  

(4.7)

where \( h \) is the profile height, \( h_c \) is the center line or height, and \( w \) is the width of the peak at half the maximum. The symmetric Gaussian distribution has a skewness of \( Sk=0 \) and a kurtosis of \( K=3 \). The discrete integration of the ADF curve leads to the bearing-ratio curve (or Abbott curve, see Figure 4.11(b)). A common construction is to find the minimum slope of the curve (shoulder) and extract the parameters \( R_k \) (core roughness depth), \( R_{pk} \) (reduced peak height) and \( R_{vk} \) (reduced valley depth). For engineering surfaces in sliding contact, \( R_{pk} \) relates to the height of the asperities worn off in early use, \( R_k \) is the working part of the surface, and \( R_{vk} \) is an estimate of the depth of valleys which will retain lubricant. Usually, the relationship of bearing ratio to the relative real contact area (\( \text{relRCA} \)) is highly approximate as material is sliced off in the construction of the bearing-ratio curve and the material deformation is not taken into account. In ice friction, the major mechanism of changing the surface is melting. It is therefore considered appropriate here to relate water film thickness to relative real contact area using the bearing-ratio curve. This relation is used in the explanation of the tribometer measurements (see Chapter 3) and it will be implemented in the numerical modeling (see Chapter 5).

Ice and Slider Surfaces from the Tribometer

Figure 4.12 shows the topographies of slider B and ice of the tribometer after a measurement. It could be seen by eye during the friction measurements that the slider carves into the ice, making a "longitudinal" imprint of its perpendicular structure. Also, the increasing friction force implies that the ice surface conforms to the slider, rendering a larger real contact area. Roughness data enables an estimate as to how far this conforming process goes. The real contact area can then in principle be determined between the positive structure of the slider and the corresponding
negative structure in the ice. The data obtained, however, did not allow for an exact quantification of the real contact area. For both slider and ice, roughness is different in the sliding direction and perpendicular to the sliding direction (see Figures 4.13(a) and 4.13(b)). It is assumed that perpendicular to the sliding direction, the two surfaces conform according to the scheme shown in Figure 4.14. If the water film thickness is known (e.g. from numerical modeling), a rough estimate of the relative real contact area is possible. In the case shown, a water film of $h_{wf} \approx 200$ nm would lead to a relative real contact area of some 10%.

Figure 4.12: Topography measurement of slider B (top) and ice (bottom) surface, at equal $z$-scales. The slider surface is turned upside down.

Figure 4.13: Profiles of slider B (top) and ice (bottom) parallel (left) and perpendicular (right) to to sliding direction.

In the evolving grooves along the sliding direction, the ice flattens, and eventually reaches 100% "parallel" contact area. The processes responsible are the water film
4.3. RESULTS AND DISCUSSION

Figure 4.14: Schematic picture of the topography of slider and ice. The slider is about one order of magnitude rougher than the ice, the Ra-value of the slider is about 1 \( \mu m \) (perpendicular to the sliding direction).

formation (corresponding to a slicing off of asperities) and the flattening by elastic and plastic deformation.

**Ground Ski Base**

Figures 4.15 and 4.16 show the structure of coarse and fine ground polyethylene ski bases. For the coarse sample, the roughness perpendicular to the sliding direction (grooves) is \( \text{Ra}_\perp = 14 \mu m \) or \( \text{Rz}_\perp = 50 \mu m \). The grooves are spaced 600 \( \mu m \) from each other. For the fine sample, the roughness perpendicular to the sliding direction is \( \text{Ra}_\perp = 2 \mu m \) or \( \text{Rz}_\perp = 10 \mu m \). The grooves are spaced 200 \( \mu m \) from each other. Figure 4.17 shows a light microscopy image of single snow grains of diameter 500 \( \mu m \). It is clear that the real contact area between ski and snow depends on the snow grain size, as relatively small ice grains can become stuck in the structure of the ski base, leading to more real contact.

Figure 4.15: Coarse ground ski base. Extract is 15 mm x 15 mm. 3D topography (left) with grooves along the sliding direction, and profiles perpendicular and parallel to the sliding direction (right).
Summary

A summary of the roughness parameters for all surfaces examined is given in Tables 4.4 (for the ice and the sliders used in the tribometer experiment) and 4.5 (for snow and real skis). Area parameters are generally not meaningful, since the profile is rather anisotropic. Slider B is about twice as rough as slider A, and 10 times rougher than the ice.

The roughness of skis is some 10 microns (Ra values), with the longitudinal grooves spaced about 1 mm from each other. Diameter of an ice grain is some 100 \( \mu m \), it is therefore well possible that small grains become stuck in the grooves.

4.4 Conclusion

Although some of the studies presented here have only preliminary character, the following can be concluded: Computer tomography is a useful method for estimating snow-ski contact phenomena. The size of the real contact area and the size of the contact spots are important parameters in explaining snow and ice friction. They will be used as input parameters in the numerical modeling of the process. Moreover, the process of the ice surface conforming during the tribometer experiment...
### Table 4.4: Ice and sliders: summary of roughness parameters determined from profiles parallel (∥) and perpendicular (∥) to the sliding direction. The area parameters are deduced from 2D scalar field data, and generally correspond to the values deduced from the perpendicular profiles (not listed here except for Pa and Pz).

- Can be verified. The outstanding questions require e.g. an estimate of the influence of creep of snow and ice. Furthermore, the influence of slider surface chemistry on the formation of the water films (and thus the real contact area) must be examined.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Snow fine</th>
<th>Snow coarse</th>
<th>Ski fine</th>
<th>Ski rough</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa, ( \mu m )</td>
<td>100</td>
<td>400</td>
<td>2</td>
<td>10</td>
<td>Average roughness</td>
</tr>
<tr>
<td>Pz, ( \mu m )</td>
<td>400</td>
<td>1600</td>
<td>20</td>
<td>100</td>
<td>Average maximum height of profile</td>
</tr>
<tr>
<td>( R_{a//} / R_{a\perp} ), ( \mu m )</td>
<td>-</td>
<td>-</td>
<td>0.7 / 2</td>
<td>2 / 14</td>
<td>Average roughness</td>
</tr>
<tr>
<td>( R_{z//} / R_{a\perp} ), ( \mu m )</td>
<td>-</td>
<td>-</td>
<td>4 / 10</td>
<td>8 / 50</td>
<td>Average maximum height of profile</td>
</tr>
<tr>
<td>( Sk// / Sk\perp )</td>
<td>-</td>
<td>-</td>
<td>0 / 0</td>
<td>0 / 0</td>
<td>Skewness</td>
</tr>
<tr>
<td>( K// / K\perp )</td>
<td>-</td>
<td>-</td>
<td>* / 3</td>
<td>* / 2</td>
<td>Kurtosis</td>
</tr>
<tr>
<td>( R_{k//} / R_{k\perp} ), ( \mu m )</td>
<td>-</td>
<td>-</td>
<td>3 / 6</td>
<td>4 / 30</td>
<td>Core roughness depth</td>
</tr>
<tr>
<td>( R_{pk//} / R_{pk\perp} ), ( \mu m )</td>
<td>-</td>
<td>-</td>
<td>0.8 / 3</td>
<td>7 / 6</td>
<td>Reduced peak height</td>
</tr>
<tr>
<td>( R_{vk//} / R_{vk\perp} ), ( \mu m )</td>
<td>-</td>
<td>-</td>
<td>0.6 / 2</td>
<td>3 / 21</td>
<td>Reduced valley depth</td>
</tr>
</tbody>
</table>

Table 4.5: Snow and real ski structures: summary of roughness parameters determined from profiles parallel (\( || \)) and perpendicular (\( \perp \)) to the sliding direction. The area parameters are deduced from 2D scalar field data, and generally correspond to the values deduced from the perpendicular profiles (not listed here except for Pa and Pz). Values for snow are taken from compressed surfaces (\( p=10 \) kPa). *: no meaningful parameter, -: not determined.
Chapter 5

Modeling

Abstract

A numerical model for sliding on ice including dry friction and generation of and lubrication by water films is described. Alternative energy dissipation mechanisms are discussed. The model is verified by comparing it with experimentally determined temperature evolution and friction coefficients.

5.1 Introduction

From estimations for real skiing and from laboratory friction measurements, the friction coefficient of polyethylene sliding on snow and ice is fairly well known. The heat generation involved in the process for a skier sliding at $v=20 \text{ms}^{-1}$ ($\mu=0.03$, $F_n=800 \text{N}$) is roughly 500 W. What is not known \textit{a priori} are the energy dissipation mechanisms involved. Possible mechanisms are: dry friction, water generation, hydrodynamic friction, squeeze out of water films, capillary attachments, electrostatic charges, plowing and compaction of the snow. The mechanisms considered relevant will be discussed and, based thereon, the numerical model is introduced. Validation of the model is done by comparison with the tribometer experiment (see Chapter 3).

5.2 Energy Dissipation Mechanisms

Possible energy dissipation mechanisms are described and estimations of their relevance for the process are made.
CHAPTER 5. MODELING

Dry Friction

Dry friction is friction between two bodies in absence of lubricating films. This is hard to achieve for ice; one has to go to low temperatures and low velocities not to have lubricating water films at the interface of a material sliding on ice (see e.g. [DLB95]). Nevertheless, there are regions of the interface between a material sliding on snow or ice at velocities relevant for skiing where there is badly lubricated sliding [Col92]. Friction arises from elastic and plastic deformation of either surface and from very thin (not exceeding some nm) lubricating water films exhibiting high friction. A detailed discussion of these ("dry") friction processes would go beyond the scope of this chapter, but they can be approximated by a dry friction law and adoption of a generally agreed on value of $\mu_{\text{dry}}=0.3$ [Bow53].

Water Generation

Neglecting wear processes, friction arises from the transfer of translational kinetic energy into heat (lattice vibrations). This frictional heat is generated only at the contacts, is therefore very unevenly distributed, and can cause very high temperature increases (so-called flash temperatures). In ice friction, this provokes the contacting regions of the ice to reach the melting point quite fast, and a phase change is to be expected. A large fraction of the frictional heat is conducted into (primarily) the ice and into the slider, depending on their individual thermal conductivities [Kur77]. The remaining heat is consumed in the phase change process and this is expected to result in water lubrication. Assuming that all heat is conducted into the ice, the temperature increase at the interface can be calculated analytically using

$$\Delta T = 2 \cdot q'' \cdot \left(\frac{t}{\pi \cdot \lambda \cdot c_p \cdot \rho}\right)^{\frac{1}{2}},$$

(5.1)

where $\lambda$, $c_p$, and $\rho$ are the thermal conductivity, the specific heat capacity, and the density of ice, respectively. $q''$ is the heat flux given by

$$q'' = \sigma_k \cdot v = \mu \cdot \sigma_0 \cdot v,$$

(5.2)

where $\sigma_k$ is the shear stress, $\mu$ the kinetic friction coefficient (here: dry friction), and $\sigma_0$ the perpendicular stress (here: indentation hardness). Assuming that the total heat flux passes into the ice, it then takes $3 \mu s$ for the temperature to rise from $T=-10^\circ C$ to $T=0^\circ C$ ($v=1 \text{ ms}^{-1}$). Note that this only holds for a perfectly flat slider in contact with the ice, a slider exhibiting roughness or waviness at any scale will cause intermittent heating of an ice contact, temperature increase will be correspondingly slower. For a rough slider sliding on rough ice, an average contact-spot diameter
5.2. ENERGY DISSIPATION MECHANISMS

Figure 5.1: Transition from dry to wet (lubricated) friction. Friction coefficient vs. water-film thickness (schematically).

of $D=100\,\mu m$ is assumed. The passage of one contact then lasts $100\,\mu s$, lubricated friction for more than 95% of the contact is expected.

Hydrodynamic Friction

A water-film thickness can be calculated where friction from shearing of the water films equals dry friction (see Equation 3.4, Chapter 3). Assuming a relative real contact area of 1% and using the parameters from the tribometer experiment ($v=5\,\text{ms}^{-1}, F_n=84\,\text{N}, A_{\text{app}}=4\cdot10^{-4}\,\text{m}^2$) results in a water-film thickness $h_{\text{trans}}$ of roughly 2 nm. Compared with the ice lattice constant of 4.5 Å to 7.8 Å or the size of a water molecule of 2 Å, this corresponds to a layer 5 to 10 molecules thick. The question arises whether a film this thin can still be described using bulk properties of water (e.g. viscosity). There is still some debate on this [Rav02, Isr90], yet it can be concluded that water films thicker than $h_{\text{trans}}$ can be described by classical hydrodynamic principles (see [Per00], pp. 102-109). Figure 5.1 illustrates how friction is expected to depend on the water-film thickness. The Reynolds number $Re = \frac{vh_{wf}}{\eta}$ for the tribometer experiment is around 1 and thus laminar flow can be assumed (the critical Reynolds number $Re_c$ for turbulent flow is around 1000 for cases relevant to hydrodynamic lubrication).

Squeeze Out

The change in thickness of a lubricating film can be described by

$$\frac{1}{h_{wf}(t)} - \frac{1}{h_{wf}(0)} = \frac{16t\sigma_0}{3\eta D^2} \quad (5.3)$$

where $\sigma_0 = \frac{F_n}{A_{\text{app}} \cdot \text{relRCA}}$ is the perpendicular stress (see [Per00], pp. 120-126). It ranges from 200 kPa for 100% to approximately 20 MPa for 1% relative real contact area for parameters from the tribometer experiment. The upper limit
is in the same range as values reported for the compressive strength of ice [Pet03]. Assuming high contact area (and therefore high average contact-spot diameters) and a water-film thickness not exceeding some µm, completely negligible pressing out results. Assume now a rough slider, where 100 nm thick water films exist at circular contacts of diameter \( D = 100 \mu m \) (aspect ratio of the films \( h_{wf}/D = 0.001 \)) and that these make up for 1% (10%) of the apparent contact area (\( relRCA = 0.01 \) and 0.1 respectively). Figure 5.2 shows the pressing out of such films. The maximum time of 0.1 ms corresponds to the passing of one contact at \( v = 1 \text{ms}^{-1} \). Note that the curves presented only describe the static case of exerting pressure on a water film. It is to be expected, however, that this mechanism limits the growth of a water film in the frictional melting process. It can be concluded that squeeze out must be accounted for for water films thicker than about 10 nm.

![Figure 5.2: Pressing out of a water film for a contact-spot diameter \( D = 100 \mu m \) \((v = 1 \text{ms}^{-1}, F_n = 84 \text{N}, A_{app} = 4 \text{cm}^{-2})\). Smaller \( relRCA \) leads to higher pressure, enhancing the effect.](image)

**Shearing**

A balance of melt and displacement rate suggests that the dominating mechanism of water removal is shearing [Col88]. The removal of water is taken to be half of the volume of the film for a displacement equal to the rms value of the contact diameter, or a change in thickness of the film in the order of

\[
\frac{\partial h_{wf}}{\partial t} = \frac{h_{wf} \cdot v}{2 \cdot D}
\]  

(5.4)
An estimation shows that this mechanism, although responsible for the water removal, does not significantly influence the water-film thickness.

### Capillary Attachments

An explanation used to account for the high friction at temperatures close to the melting point is that of water bridges forming between ice grains (asperities) and a slider. These bridges are thought to first tilt and then elongate, exerting a force opposite to the sliding direction (see [Per00], pp. 141-143 and [Col96]). For all but very wet cases (slush), it is not likely that water films formed will be thicker than some \( \mu \text{m} \), at a diameter of \( 100 \mu \text{m} \) and more. Therefore a tilting and elongating of these films is not likely. Increased friction towards \( 0^\circ \text{C} \) can rather be explained by considerably increased contact area (see Chapter 3).

### Electrostatic Charges

Electrostatic charges, as well as the combined influence of these and contaminations of the surfaces have been studied in detail [PC95] but are not taken into account in this work.

### Plowing and Compaction of the Snow

These can dominate when skiing in soft snow (off-piste). They are not relevant in skiing on groomed, hard packed slopes, and on ice.

### Summary

Dry friction, heat conduction, phase changes, the shearing of water films, and squeeze out will be the energy dissipation mechanisms included in the model. Moreover, the contact area will depend on the water-film thickness, which can explain the temperature dependence of ice friction. The dependence of friction on load (see Chapter 3) will be accounted for by examining the influence of load on the water-film thickness (pressing out) and the real contact area.

### 5.3 Estimation for Snow Using FEM

In order to get an impression of the heat conduction into snow, a 3D structure measured using an X-ray computer tomograph (see Chapter 4) is cut to get a cross-sectional "surface". A finite element mesh is created, then a constant heat
flux applied on the contacts, which make up for about 5% of the cross-sectional area. Figure 5.3 shows the temperature distribution in the snow structure originally at $T = -10^\circ$C after 6 ms of heating with an average heat flux of $q''_z = 1700 \text{ Wm}^{-2}$ ($A_{app} = 100 \text{ mm}^2$). Heat flux is concentrated at the real contact area ($\text{relRCA} \approx 0.05$) and thus multiplied by $1/\text{relRCA}$, resulting in $34 \text{ kWm}^{-2}$. Most of the contacts have reached the melting temperature at this time. Again note that this only holds for a flat slider; for a rough slider, intermittent heating results in a slower temperature increase. In the presented structure, roughly 40 contact spots make up 5% of the apparent contact area. This corresponds to contacts with an average diameter of $D = 0.4 \text{ mm}$, spaced $1.7 \text{ mm}$ from each other. However, average contact-spot diameter is surely overestimated by taking a cross-section as compared to pressing on a snow surface. For the numerical modeling, these findings imply that time steps shall not exceed some milliseconds, and that the topography of the slider and the ice has to be accounted for.

Figure 5.3: Application of a constant heat flux on a real snow structure originally at $-10^\circ$C. Temperature distribution after $t = 6 \text{ ms}$. The gray areas depict the regions of the surface that have reached $0^\circ$C. Cross-sectional surface is $10 \text{ mm} \times 10 \text{ mm}$.

5.4 The Physical Problem

In a first step, the general problem will be developed. It includes dry friction, heat conduction, phase changes, and the shearing of water films. A slider is in contact with an ice grain during the time of passage $t$. Frictional heat is generated at the interface. The conduction of heat is governed by
5.4. THE PHYSICAL PROBLEM

\[ c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (\lambda \frac{\partial T}{\partial z}) + P(z), \]  
(5.5)

where \( c_p \) is the specific heat capacity, \( \lambda \) is the thermal conductivity, and \( P(z) \) is a source term, concentrated at the interface between slider and ice, representing frictional heating. In the case of dry friction, heat generated at the interface is given by

\[ P_{\text{dry}} = \mu_{\text{dry}} \cdot F_{\text{normal}} \cdot v. \]  
(5.6)

Once a water film is present, heat generation is determined by the shearing of the water according to

\[ P_{\text{wet}} = \eta \cdot (\frac{\partial v}{\partial z})^2 \cdot h_{wf} = \frac{\eta \cdot v^2}{h_{wf}}. \]  
(5.7)

Equation 5.7 corresponds to supposing a linear velocity profile in the water film, a situation which is referred to as Couette flow, see Figure 5.4. For a temperature of 0°C at both interfaces, the temperature distribution in the water film is parabolic. For the case of two parallel plates at 0°C in relative motion \((v=10 \text{ m/s})\), separated by a 100 nm water film, a maximum temperature of 0.04°C is reached (see [ID02], pp. 339-343). This temperature increase is negligible, and therefore, a constant temperature of 0°C in the water film is assumed.

For a given contact point and the corresponding 1D energy conservation model, the source term in Equation 5.5 is thus given by

\[ P(\text{interface}) = \begin{cases} 
P_{\text{dry}} & \text{if } T < 273 \text{ K} \\
P_{\text{wet}} & \text{if } T \geq 273 \text{ K}.
\end{cases} \]  
(5.8)

Once a water film exists, its behavior is governed by

\[ \frac{\partial h_{wf}}{\partial t} = \frac{1}{L} \left( \frac{\eta \cdot v^2}{h_{wf}} - \lambda \cdot \partial_z T|_{z=0} \right), \]  
(5.9)

where \( L \) is the volumetric latent heat of fusion. \( \partial_z T|_{z=0} \) represents the averaged temperature gradient at the interface. Heat generation during time \( \Delta t \) is then

\[ Q = \int_{t}^{t+\Delta t} P_{\text{wet}} \, dt. \]  
(5.10)

A total friction force can be calculated by summing up over the area covered by water films plus possible dry friction:

\[ F_f = \mu_{\text{dry}} \cdot F_n \cdot \frac{A_{\text{dry}}}{A_{\text{app}}} + \frac{\eta \cdot v \cdot A_{\text{real}}}{h_{wf}} \cdot \left(1 - \frac{A_{\text{dry}}}{A_{\text{app}}}\right), \]  
(5.11)
where $A_{\text{dry}} = 1 - A_{\text{wet}}$ is the part of the apparent area where dry friction is predominant.

The primary goal is to simulate the situation found in the tribometer experiment. Several tribometer-specific problems have to be accounted for. The cooling of the track of ice between two passages of the slider is computed, including convective cooling. Also, the polishing of the track in the course of a measurement will lead to increased real contact area.

### 5.5 Numerical Approximation

The basis is a 1D model of heat generation at the interface and heat flow into the slider and the ice. To account for the changing conditions in the direction of sliding along the slider (change in film thickness and real contact area), several (not coupled) 1D calculations along the slider are conducted (quasi 2D model). The discretization of the slider and the underlying snow or ice, respectively, is depicted in Figure 5.5. A first approach is to move the slider a distance corresponding to the discretization along the direction of movement of the ski ($dx$) and calculate the new temperature distribution. This, however, leads to problems in the water film generation and the corresponding energy from the shearing of the water. Namely, if the system stays in the dry friction regime for too long, the temperature at the interface reaches a value much greater than 0°C. This would then correspond to a very high water-film thickness (see Equation 5.14), which in turn results in too little energy from shearing.
to sustain the water film. The slider is therefore only moved a portion of \( dx \), in order to keep the energy generation of the same order of magnitude (e.g. the water film may not exceed a thickness of 15 nm when first generated). The number of nodes along the slider is kept at a low value to achieve reasonable computational time. In the model, as in the experiment, the slider is kept at a constant position, and the ice is being moved with a constant velocity \( v \). Therefore, the starting temperature of the slider at iteration \( it \) is the one calculated at iteration \( it - 1 \). For the case when the mesh of the slider and the mesh of the ice do not coincide, the starting temperature in the ice must be interpolated from the two nearest set of nodes at a particular position on the slider (Figure 5.6). The interface node belongs to both the slider and the ice, thus its temperature is interpolated from the two temperatures computed:

\[
T_{\text{new}}(if) = \frac{dz(if - 1) \cdot T_{\text{old}}(if) + dz(if) \left( (1 - dx_{\text{tribo}}) \cdot T_{\text{old}}(if) + dx_{\text{tribo}} \cdot T_{\text{old}}(if) \right)}{dz(if - 1) + dz(if)},
\]

(5.12)

where \( T(if) \) is the temperature at the interface and \( dx_{\text{tribo}} \) is the advancement of the slider during one time step. If a water film has existed, it belongs to the ice and is also interpolated. A water film is created if the temperature at the interface node is greater than 273 K (melting point of ice at normal pressure). The temperature in the water film is set to 273 K and the thickness of the film calculated to conform with the energy balance (Figure 5.7). The energy available for melting is given by

\[
E = \frac{dz(if - 1) \cdot c_S \cdot \Delta T}{2} + \frac{dz(if) \cdot c_I \cdot \Delta T}{2},
\]

(5.13)

where \( c_S \) and \( c_I \) are the heat capacities of the slider and the ice, \( \Delta T \) is the temperature above \( T=0^\circ C \), and \( E \) is the energy available per unit area (m\(^2\)) to melt ice.
Figure 5.6: Interpolation of temperatures and water films due to the relative movement between slider and ice. In iteration $i+1$, the ice has moved to the left. The temperatures and water-film thickness at the original position must be interpolated from the new ice positions.

The thickness of the water film created is then

$$h_{wf} = \frac{E}{L} = \frac{(dz(i_f - 1) \cdot c_s + dz(i_f) \cdot c_l) \Delta T}{2 \cdot L},$$

(5.14)

where $L$ is the latent heat of melting ice. A water film can only exist at an interface node. In the heat diffusion equation, the water-film thickness is set to 0 (negligible thickness). The water film is thus only present as a heat source at the interface (Equation 5.7).

**Numerical Parameters and Convergence**

Main focus in figuring out suitable discretization in space and time is on the behavior of the water film. When created, it may not exceed a certain thickness which corresponds to the same order of magnitude for the hydrodynamic friction as for dry friction. Once created, it may not disappear again, and not become as thick as to cause diminishing temperatures in slider and ice. See Table 5.1 for numerical parameters and explanations.
5.5. NUMERICAL APPROXIMATION

Figure 5.7: Generation of a water film at the interface. The shaded area represents a temperature increase in slider and ice that has actually not happened, since the energy has been used for the phase transformation.

Figure 5.8: Discretization in z-direction (perpendicular to interface). The interface is at $z=0\,\text{mm}$, negative values of $z$ refer to the slider, positive values to the ice. Discretization is finer towards the interface.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{z_{ski}}$</td>
<td>Number of nodes in slider perpendicular to interface</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>$N_{z_{ice}}$</td>
<td>Number of nodes in snow/ice perpendicular to interface</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>$D_{ski}$</td>
<td>Thickness of slider</td>
<td>10 mm</td>
<td></td>
</tr>
<tr>
<td>$D_{ice}$</td>
<td>Thickness of ice</td>
<td>20 mm</td>
<td></td>
</tr>
<tr>
<td>$L_{ski}$</td>
<td>Length of the slider</td>
<td>40 mm</td>
<td></td>
</tr>
<tr>
<td>$d_x$</td>
<td>Discretization in x-direction</td>
<td>10 mm</td>
<td>corresponds to spacing of thermocouples</td>
</tr>
<tr>
<td>$d_z$</td>
<td>Discretization in z-direction</td>
<td>2-400 µm</td>
<td>see Figure 5.8</td>
</tr>
<tr>
<td>$N_{x_{ski}}$</td>
<td>Number of elements along slider</td>
<td>4</td>
<td>$d_{x_{ski}} = \frac{L_{ski}}{N_{x_{ski}}}$</td>
</tr>
<tr>
<td>$d_{t_{ski}}$</td>
<td>Discretization in time for advancing slider</td>
<td>$0.1 \cdot \frac{d_{x_{ski}}}{v_{ski}}$</td>
<td></td>
</tr>
<tr>
<td>$d_{t_{1D}}$</td>
<td>Discretization in time for the 1D-models</td>
<td>$1 \cdot d_{t_{ski}}$</td>
<td>will be reduced if problems with water film appear</td>
</tr>
<tr>
<td>$d_{x_{tribo}}$</td>
<td>Advancement of slider during one time step</td>
<td>$0.1 \cdot d_x$</td>
<td></td>
</tr>
<tr>
<td>$T_{env}$</td>
<td>Environmental temperature</td>
<td></td>
<td>fixed at $z = D_{ski}$ and $z = D_{ice}$ (Dirichlet boundary conditions)</td>
</tr>
</tbody>
</table>

Table 5.1: Numerical parameters for the simulation.
5.6 Refinement of the Model

The model presented above will now be compared to experimental data. The measurement shown in Figure 2.10, Chapter 2, will be used (slider B at $T_{\text{env}}=-5^\circ\text{C}$, $F_n=84\text{ N}$, $v=5\text{ m}\cdot\text{s}^{-1}$, and $A_{\text{app}}=4\text{ cm}^2$). Refinements of the model will have to be introduced in order to better describe the sliding process. Figure 5.9 (left) illustrates the assumption of a flat slider on a rough snow or ice surface. This holds for a ski sliding on snow, where the snow is usually rougher than the base of the ski. A spot of the snow or ice surface is in contact with the slider during the entire time of passage of the slider. Its temperature increases continuously, as indicated in Figure 5.9 (right). Figure 5.10 shows the calculated initial temperature increase at and near the interface, and the calculated and measured temperature evolution at the thermocouple (TC) positions. The thermocouples are located close to the interface in the center of the slider, along the slider after (from tip to tail) 1 cm (TC1), 2 cm (TC2), and 3 cm (TC3). Total slider length is 4 cm (see Chapter 2 for the exact positions of the thermocouples). Also shown is the water-film thickness at different positions along the slider, and the measured friction coefficient, along with the friction coefficient calculated according to Equation 5.11. The plot of the initial temperature increase (Figure 5.10 (a)) shows that the temperature at the interface reaches the melting point within a few milliseconds. Temperature increases faster in the ice than in the slider, due to their differing thermal properties. At a later stage, however, the slider reaches higher temperatures. It is being heated constantly, while the ice that participates in the sliding process is always "refreshed". The temperatures calculated for the positions of the thermocouples are too high (Figure 5.10 (b)). The reason for this is that the heat conduction equation is not coupled to neighboring nodes along the sliding direction. Therefore, heat is only conducted in columns perpendicular to the interface. In other words, it is only valid very close to the contact spot, where sideways conduction is negligible. Water-film thickness usually reaches a steady state quite fast, no change is seen after some seconds. This implies that the film thickness can be kept constant for longer simulation times without introducing a large error, rendering faster computation.
Figure 5.9: Schematic representation of a flat slider on a rough snow or ice surface. TC indicates the position of a thermocouple. The plot on the right shows the temperature increase at a contact spot during the passage of the slider.
5.6. REFINEMENT OF THE MODEL

Figure 5.10: Results of a calculation using the 'unrefined' model (lines), and comparison to experimental data (open and closed symbols). \textit{relRCA}=1\%.
CHAPTER 5. MODELING

Contact Temperatures vs. Average Temperatures

As a first improvement, the heat flux calculated at the contacts (which, in a first approach, make up for 1% of the apparent contact area) is distributed over the whole apparent contact area. This should then yield an averaged temperature evolution, which can be compared to the measured temperatures in the sliders. There are now two sets of temperatures to consider: temperatures at and near the contacts ($T_{\text{contact}}$), and average temperatures further away from the contacts that represent the time averaged influence of many contacts ($T_{\text{average}}$). The heat conduction equation is solved twice, i.e. for both sets of temperatures. The formation of the water film is controlled by the contact temperature, since water films only exist at the contacts. Figure 5.11 illustrates the idea.

![Diagram](image)

Figure 5.11: Two sets of temperatures are calculated: temperatures at the contacts (left) and average temperatures in the slider.

Figure 5.12 shows the temperature calculated for the position of the thermocouples, as well as water-film thickness and friction coefficient calculated for two different relative real contact areas: $relRCA=1\%$ (top) and $relRCA=25\%$ (bottom). It can be seen that the real contact area has a large influence on the temperature evolution and the friction coefficient. A relatively good fit to the experimental data can be found by adjusting the relative real contact area. The initial temperature increase in the slider is modeled well by the simulation, and so is the trend in decreasing temperature from tip (TC1) to tail (TC3) of the slider. The experiment shows an increasing friction coefficient with time. Friction increases strongly the first few seconds after the slider is lowered onto the ice, followed by a slow increase. This effect cannot be explained without implementing another effect, namely the increase of the real contact area (polishing effect). This will be considered later. Note that the real contact area influences the temperature evolution in the slider and the friction coefficient, but not the calculated water-film thickness.
5.6. REFINEMENT OF THE MODEL

(a) Temperature at the TC positions - \( \text{relRCA} = 1\% \).

(b) Water-film thickness and friction coefficient - \( \text{relRCA} = 1\% \).

(c) Temperature at the TC positions - \( \text{relRCA} = 25\% \).

(d) Water-film thickness and friction coefficient - \( \text{relRCA} = 25\% \).

Figure 5.12: Comparison of calculation (lines) and experiment (symbols) for two different relative real contact areas.
Temperature Gradients

In the quasi 2D-model, heat conduction is still purely 1-dimensional. In reality, the heat flow at a contact spot is 3-dimensional, providing a much more efficient heat sink than if just a column of ice below the contact is heated (see Figure 5.13, left). Finite element calculations were used to compare the temperature evolution for a constant heat flux at 100% contact area with a heat flux limited to the real contact area. The generated temperature gradients are then compared and from this, correction factors for the thermal conductivity are calculated (see Figure 5.14). The same applies to the slider, in principle. However, heat conduction is proportional to $\lambda \cdot \partial_z T$, and both factors are smaller for the slider (S) as compared to the ice (I): $\lambda_S$ is 7 times lower than $\lambda_I$, and due to the constant heating of the slider, temperature gradients are not as high as for ice, which is always "refreshed". For the thermal conductivity of the slider, the bulk value is therefore taken. Figure 5.15 shows the effect of this modification: more heat drain leads to less heat available for the melting of ice, and correspondingly thinner water films. Thinner films, in turn, lead to more heating at the interface, and therefore a larger temperature increase in the slider (compare to Figures 5.12 (a) and (b)).

Figure 5.13: In order to account for the more efficient heat conduction into the snow or ice, modified thermal conductivities are introduced (left). Moreover, the temperature of the (moving) slider at the beginning of each time step corresponds to the average temperature of the slider at the interface (right).

Temperature gradients at the very interface determine how much of the frictional heat is available for melting or sustaining a water film. So far, the temperature of the slider at the beginning of each time step was assumed to be the temperature valid for the zone near the contact. However, since the slider is moved over a contact spot, the temperature of the slider at the beginning of each time step corresponds to the average temperature of the slider at the interface (see Figure 5.13, right). This leads to higher temperature gradients on the slider side, and correspondingly more
5.6. REFINEMENT OF THE MODEL

Figure 5.14: Correction factors for the heat conductivity of the ice near the interface for different relative real contact areas.

heat drain. Figure 5.16 shows the resulting temperature evolution and water-film thickness for two different contact areas. Again, by adjusting the real contact area, a good fit to the experimental data can be found.
Figure 5.15: Influence of the modified thermal conductivity in the snow or ice on the temperature and water film evolution. The water film thickness is decreased, however, heating of the slider is increased as compared to the calculation with unmodified thermal conductivity. \( relRCA=1\%\).

Figure 5.16: Influence of the assumption of a slider being at average temperature on the temperature and water film evolution. Comparison of calculation (lines) and experiment (symbols) for two different relative real contact areas.
Real Contact Area

The model so far can explain the effect of decreasing friction with increasing temperature. Higher temperatures lead to lower temperature gradients at the interface, more heat is available to melt ice. The water films become thicker and friction decreases. There is, however, a minimum in the friction vs. temperature curve at around $T_{env}=-3°C$ (see Chapter 3 and [BFR01]). The approach here to explain the increasing friction with temperature towards the melting point is with an increasing real contact area. Figure 5.17 illustrates the idea. At the beginning of the measurement, or in the dry friction regime, it is assumed that the relative real contact area is comparable to that measured in static contact area measurements. For snow, this is around 5%, depending on the environmental temperature (see Chapter 4). For ice, an estimation based on plastic deformation yields a relative real contact area of approximately 1%. If a water film is generated, the film thickness is related to the relative real contact area using the bearing-ratio curve of the snow or ice surface (see also introduction to Chapter 3). A bearing-ratio curve which is calculated from the histogram of an ice surface exposed to air is shown in Figure 5.18. The shape of the bearing-ratio curve can vary considerably from smooth ice to rough snow and constitutes a critical parameter to the calculation. Implementing the bearing-ratio curve shown in Figure 5.18 into the model, slider temperature, water-film thickness, and friction coefficient evolution shown in Figure 5.19 results. Some details have to be pointed out. The calculated temperatures at the thermocouple positions are of the right order of magnitude, but do not show the right trend of decreasing temperature along the slider. This is clearly a result of the increase in real contact area along the slider (Figure 5.19 (c)). Also does the increased contact area result in largely increased water-film thicknesses. Note that the calculated friction coefficient now increases at the beginning. The increasing real contact area can therefore explain the friction coefficient evolution seen in the experiments. The transient states at the beginning of a friction measurement are, however, not explained well. For a modified bearing-ratio curve, where the increase in real contact area is shifted to a higher water-film thickness (in this case, from 400 nm to 800 nm), a correct trend in changing friction coefficient with temperature can be calculated (Figure 5.20): friction is lowest at intermediate temperatures, and increases towards lower and higher temperatures. However, the model does not predict properly the magnitude of the friction coefficient. Note the increasing values of water-film thicknesses calculated over a temperature range from -15°C to -0.5°C, ranging from below 100 nm to more than 1 µm.
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Figure 5.17: Relation between the water-film thickness and the real contact area. Melting of ice corresponds to a slicing off, and leads to the growth of existing contacts, and the formation of new contacts.

Figure 5.18: Bearing ratio or relative real contact area vs. water-film thickness for a smooth ice surface exposed to air at -20°C. For a water-film thickness of some 100 nm, the contact area (bearing ratio) starts to increase considerably.
(a) Temperature at the TC positions.

(b) Water-film thickness and friction coefficient.

(c) Relative real contact area.

Figure 5.19: Influence of the coupling of water-film thickness and contact area evolution on the calculation.
Figure 5.20: Water film and friction coefficient evolution at different temperatures. For $T_{env}=258$ K and 268 K, the real contact area does not increase (not shown), for $T_{env}=272.5$ K, the evolution of the real contact area is shown.
5.6. REFINEMENT OF THE MODEL

Squeeze Out

It is desirable that the model can explain the load dependence of the friction coefficient observed in the tribometer experiments. So far, load \( F_n \) does not appear in the model at all. As the estimation in the discussion of the energy dissipation mechanisms showed, squeezing out of the water films should be accounted for. This calls for a modification of the differential equation describing the water film evolution. A third term describing squeeze is added to Equation 5.9:

\[
\frac{\partial h_{wf}}{\partial t} = \frac{1}{L} \left( \eta \cdot v^2 - \lambda \cdot \partial_z T \big|_{z=0} \right) - \frac{8}{3} \cdot h_{wf}^3 \cdot \sigma_0 \cdot \eta \cdot D^2, \tag{5.15}
\]

where \( \sigma_0 \) is the perpendicular pressure, and \( D \) is the contact-spot diameter whose evolution is estimated by

\[
D = D_0 \cdot \sqrt{\frac{\text{relRCA}}{\text{relRCA}_0}}, \tag{5.16}
\]

where \( D_0 \) is the initial (static) contact-spot diameter and \( \text{relRCA}_0 \) is the initial relative real contact area. The water film calculated for each time step can be interpreted as the difference between the water film generated and the water film pressed out:

\[
h_{wf} = h_{total} - h_{squeeze}. \tag{5.17}
\]

\( h_{total} \) determines \( \text{relRCA} \) according to Figure 5.18, while \( h_{wf} \) is the actual film thickness relevant for the shearing. In other words: the ice surface becomes more and more blunt (resulting in higher real contact area), whereas the water film can remain constant. This is illustrated in Figure 5.21. In the model, the "blunting" of the snow or ice surface is implemented by shifting the bearing-ratio curve to the left, i.e. lower water-film thickness due to squeeze does not lead to lower real contact area. Figure 5.22 shows the influence of implementing squeeze. For the parameters chosen, real contact area and contact-spot diameter quickly increase to 100% or maximum contact-spot diameter, respectively, for all but the node at the tip of the slider. This is probably not the case, and the reason for the overestimated temperatures and friction coefficient.

The friction force in the above simulation amounts to \( F_f = \mu F_n = 7.0 \text{ N} \). A simulation for a load of \( F_n = 25 \text{ N} \) results in a friction force of \( F_f = 6.6 \text{ N} \). The effect of the squeeze mechanism is observed: a maximum water-film thickness of \( h_{wf} = 710 \text{ nm} \) is calculated for the lower load, as compared to \( h_{wf} = 625 \text{ nm} \) for the higher load. If the friction coefficient is calculated, however, a lower value results for the higher load, as is expected from the experimental results.
Figure 5.21: Squeeze out process.

(a) Temperature at the TC positions.  
(b) Water-film thickness and friction coefficient.  
(c) Relative real contact area. 
(d) Contact-spot diameter.  

Figure 5.22: Influence of implementing squeeze on the calculated slider temperatures and water-film thickness evolution.
Rough Slider

So far, the model assumes that a portion of the slider comes into contact with the ice at the front of the slider and stays in contact over the length of the slider. This is true for a perfectly flat slider on a rough ice surface. The temperature of the ice at the interface increases according to the square root of time if conduction is neglected. Yet if both the slider and the ice are relatively rough (see Chapter 4), this results in intermittent contact of a point on the slider and the ice. The section of the slider, as well as the ice, is allowed to cool before the next contact takes place, see Figure 5.23.

From profilometer measurements of the contacting surfaces in the tribometer experiment, it could be seen that the slider is actually rougher than the ice. It would therefore be adequate to implement only an intermittent contact between the two. Between two contacts, cooling of both slider and ice would have to be computed. This would demand a major modification of the program code, and is therefore abstained from. A simplified approach is to heat only for a fraction of $relRCA$ of the time step $\Delta t$, and neglect the cooling in-between two contacts. Figure 5.24 shows the results of a calculation with such assumptions. For $relRCA=1\%$, the water-film thickness does not exceed $h_{wf}=50$ nm. If $relRCA$ is increased to 10\%, a quite good match of simulation and experiment is obtained. Water films along the slider then range from 35 nm to 110 nm.

Due to numerical problems, it has not been possible to implement the effect of increasing contact area and squeeze into a calculation for intermittent contact. Note that if the relative real contact area reaches high values, assuming contact throughout the whole time of passage of the slider is not likely to introduce large errors.

![Figure 5.23: Rough slider on rough snow or ice. During the passage of a slider, a contact spot on the snow or ice surface is repeatedly hit and heated by the slider, but allowed to cool in-between.](image)
Figure 5.24: Influence of the assumption of a rough slider on the temperature and water film evolution. Comparison of calculation (lines) and experiment (symbols) for two different relative real contact areas.

Resolution Algorithm and Summary

Figure 5.25 illustrates the general resolution algorithm. In every time step, the 1-dimensional models along the slider are computed independently. As mentioned before, it is not possible to implement all mechanisms introduced for any set of parameters. It has to be decided for each set of parameters, what refinements to include.

Table 5.2 lists the results of all calculations presented. The two calculations that most closely describe the experimental findings are 'Slider' and 'Rough'. A range of water-film thicknesses from 30 nm to 250 nm and relative real contact areas of 10% to 25% results.
5.6. REFINEMENT OF THE MODEL

Setup
Physical parameters
Numerical parameters
Matrices

Start time step
Determine $T_{old}$
(Convection)
Interpolation
Solve 1D models
Checks
(wf-T-incompatibility, disappearance of wf)

Dry friction
$P_{contact} = \mu \cdot F_n \cdot v \cdot w \cdot relRCA$
$P_{average} = \frac{\mu \cdot F_n \cdot v \cdot w}{l_w}$
Solve heat equation for $T_{contact}$ and $T_{average}$
Possible phase change

Wet friction
Determine $relRCA$
Calculate water film
(Equation 5.9)
$P_{contact} = \frac{q \cdot v^2}{h_{wf}}$
$P_{average} = \frac{q \cdot v^2}{h_{wf}} \cdot relRCA$
Solve heat equation for $T_{contact}$ and $T_{average}$

Error handling
(T diminishing, solidification)
reduce time step

End time step
Save Solution

Figure 5.25: Resolution algorithm. In every time step, the 1D models along the slider are computed independently.
5.7 Application of the Model

Of interest for the design of sliders (e.g. skis) is the dependence of friction on slider properties such as thermal conductivity or surface roughness.

Thermal Conductivity

It is to be expected that increased thermal conductivity of the slider leads to higher friction at low temperatures. The simulation (assumption of a rough ski, relRCA=10%, increase of contact area and squeeze neglected) shows that a thermal conductivity increased by a factor of 3 (1 Wm$^{-1}$K$^{-1}$ as compared to 0.34 Wm$^{-1}$K$^{-1}$) in fact leads to a generally decreased water-film thickness (maximum $h_{wf}=85$ nm as compared to $h_{wf}=115$ nm ) and an increased friction coefficient (from $\mu=0.058$ to $\mu=0.075$) for the parameters used above: $T_{env}=-5^\circ\text{C}$, $F_n=84$ N, $v=5$ ms$^{-1}$, and $A_{app}=4$ cm$^2$. At temperatures close to $T_{env}=0^\circ\text{C}$, however, the simulation showed no change in water-film thicknesses and friction coefficient for a thermal conductivity increased by a factor of 3. The conclusion is that thermal conductivity has quite a large influence on friction at lower temperatures, but none close to the melting point.

Colbeck et al. [CP04] observed that skis with black bases run at a higher temperature than equally constructed skis with transparent ski bases. This is explained by enhanced absorption of solar radiation by the black base. The absorbed heat contributes to the production of melt water, facilitating sliding. It is difficult to estimate the magnitude of the heat from solar radiation reaching the ski base. Yet it is again to be expected that this effect leads to thicker water films and therefore lower friction at low temperatures, whereas it does not influence friction at higher temperatures (above about $T_{env}=-2^\circ\text{C}$).

Concluding from the above discussion, the current practice of adding carbon black (soot) to the ski base is beneficial for friction at lower temperatures, due to the absorption of solar heat, but disadvantageous because of increased thermal conductivity. There are, however, other reasons for the addition of carbon black, namely increased abrasion resistance and increased electrical conductivity. An increased electrical conductivity is thought to reduce static charging of the ski base, therefore reducing dirt adsorption, and consequently reducing friction.

Slider Topography

Figure 5.24 showed the results of the simulations for two different real contact areas. Remember that the approach was to heat only for a fraction of relRCA of the time step $\Delta t$, and neglect the cooling in-between two contacts. For a contact that lasts
only $0.01 \cdot \Delta t$, the frictional heating at the interface is only able to generate a water film in the order of 15 nm to 40 nm (front to back of slider). For a contact that lasts for $0.1 \cdot \Delta t$, thicker films are generated (30 nm to 110 nm). However, the real contact area also increases, leading to higher friction. From this point of view, it would be beneficial to have few contact spots, but oriented along the sliding direction. The time of contact must be long enough for the building-up of a thick enough water film to offer low friction.

5.8 Conclusion

The model explains the evolution of slider temperature and friction coefficient observed in the measurements. Approaches for explaining the temperature and load dependence of temperature and friction evolution are presented. The main factors determining friction are the thickness of the water films and the relative real contact area. Thickness of these films is calculated to 30 nm to 250 nm along a slider at intermediate temperatures ($T_{\text{env}}=-5^\circ C$). Relative real contact areas are calculated to 10% to 25%. For lower temperatures ($T_{\text{env}}=-15^\circ C$), water-film thicknesses below 100 nm, and for higher temperatures (close to the melting temperature) water-film thicknesses of around 1 $\mu$m result. The relative real contact area can increase to almost 100% in warm conditions (close to 0$^\circ$C). The model can be used to discover general trends in the dependence of friction on slider properties. However, caution has to be exercised in providing exact values of water-film thickness and real contact area.

The range of physical parameters ($T_{\text{env}}, F_n, v, A_{\text{app}}$) that do not result in numerical problems is currently very limited. Also is the calculation very sensitive to input parameters as bearing-ratio curve, or contact-spot diameter. More work has to be done in order to come up with a comprehensive model that can describe the friction process for a larger range of parameters.
### Table 5.2: Summary of the calculations

<table>
<thead>
<tr>
<th>Refinement</th>
<th>TC1</th>
<th>$\mu$</th>
<th>$h_{wf}$ (min/max)</th>
<th>$relRCA$ (min/max)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>-2°C</td>
<td>0.065</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>None</td>
<td>-0.3°C</td>
<td>0.002</td>
<td>50 nm / 560 nm</td>
<td>1% (fix)</td>
<td>-</td>
</tr>
<tr>
<td>Average</td>
<td>-4.8°C</td>
<td>0.002</td>
<td>50 nm / 560 nm</td>
<td>1% (fix)</td>
<td>-</td>
</tr>
<tr>
<td>Average</td>
<td>-2.3°C</td>
<td>0.045</td>
<td>50 nm / 560 nm</td>
<td>25% (fix)</td>
<td>-</td>
</tr>
<tr>
<td>Lambda</td>
<td>-4.2°C</td>
<td>0.01</td>
<td>10 nm / 200 nm</td>
<td>1% (fix)</td>
<td>-</td>
</tr>
<tr>
<td>Slider</td>
<td>-4.4°C</td>
<td>0.006</td>
<td>40 nm / 100 nm</td>
<td>1% (fix)</td>
<td>-</td>
</tr>
<tr>
<td>Slider</td>
<td>-2.1°C</td>
<td>0.06</td>
<td>100 nm / 250 nm</td>
<td>25% (fix)</td>
<td>-</td>
</tr>
<tr>
<td>RCA</td>
<td>-3.8°C</td>
<td>0.052</td>
<td>300 nm / 800 nm</td>
<td>1% / 100%</td>
<td>Wrong trend in $T$ along slider.</td>
</tr>
<tr>
<td>RCA</td>
<td>-4.6°C</td>
<td>0.003</td>
<td>50 nm / 200 nm</td>
<td>1% / 1%</td>
<td>Modified bearing-ratio curve.</td>
</tr>
<tr>
<td>Squeeze</td>
<td>-1.9°C</td>
<td>0.083</td>
<td>0 nm / 620 nm</td>
<td>1% / 100%</td>
<td>-</td>
</tr>
<tr>
<td>Rough</td>
<td>-4.2°C</td>
<td>0.015</td>
<td>15 nm / 40 nm</td>
<td>1% (fix)</td>
<td>No $relRCA$-increase and squeeze.</td>
</tr>
<tr>
<td>Rough</td>
<td>-2.3°C</td>
<td>0.06</td>
<td>30 nm / 110 nm</td>
<td>10% (fix)</td>
<td>No $relRCA$-increase and squeeze.</td>
</tr>
</tbody>
</table>

Average: the concept of average temperatures is introduced. Lambda: modified thermal conductivity in the ice. Slider: slider is at average temperature. RCA: increase of the real contact area is implemented. Squeeze: squeeze out is implemented. Rough: intermittent heating is introduced. In this last case (‘rough’), increase of the real contact area and squeeze is not implemented. Otherwise, the concepts, once introduced, remain implemented.
Chapter 6

Conclusions and Outlook

A hydrodynamic approach is successfully used to explain the friction forces and temperature evolution measured. The main factors determining friction are the thickness of the water films and the relative real contact area. Unevenly distributed thin water films are responsible for the low friction observed. Thickness of these films is calculated to 30-250 nm along the slider at intermediate temperatures ($T_{env}=-5^\circ C$). Average static contact area with snow is around 5%, with contact spot diameters of approximately 100 $\mu$m (flat PE sample on fine grained snow at $T_{env}=-10^\circ C$). Both these values depend considerably on temperature and snow type. Behavior of the water films and size of the real contact area can explain the friction process, no capillary attachments are needed. The most critical parameter determining friction between skis and snow or ice is the real contact area.

The tribometer at its present state allows for the reproducible measurement of friction on ice. It has been seen that trends seen in the laboratory measurements are not always obeyed in real skiing. This is thought to be due to the difference between sliding on snow as compared to ice. Laboratory measurements on snow are therefore desired. An approach to producing a snow or snow-like surface on the tribometer is to sieve snow onto the track during a friction measurement, with the instrument running. First attempts using an industrial dryer to have the snow grains stick to the underlying ice yielded promising results, but more work is needed. The studies could then be extended to friction on snow.

As far as friction on ice is concerned, the studies with sliders exhibiting defined surface topographies can be extended first to more relatively simple patterns, and later even to complex structures. Apart from that, a systematic examination of the influence of the chemical properties of a surface can be carried out. Attention has to be paid, however, to differentiate between chemical and topographical effects, e.g. the materials compared must exhibit identical surface topographies.

For an improved determination of the contact area between a slider and snow or ice, the influence of creep of snow and ice must be considered. More extensively and
carefully conducted SEM measurements of snow surfaces that have seen passes with a slider seem to be a promising approach. With the present model, the influence of the chemical properties of the slider surface on friction cannot be investigated. There is considerable debate about the use of waxes with regard to rendering the ski base more hydrophobic. In the model, it is assumed that the relative velocity at the ice-water interface is 0, while the relative velocity at the water-slider interface equals the velocity of the slider. For the rather hydrophobic polyethylene, it can be expected that the surface is not wetted by the lubricating water, which may result in a weaker shear mechanism. There are, however, limits to the possibilities of a quasi-2D simulation. A real 2D-simulation is likely to overcome some of the numerical problems.


### Appendix A

#### Notation

Table A.1: List of symbols - physical properties.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>radius of a circular contact</td>
<td>[m]</td>
</tr>
<tr>
<td>$A_{app}$</td>
<td>apparent contact area</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$A_{real}$</td>
<td>real contact area</td>
<td>[m$^2$]</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat capacity</td>
<td>[J kg$^{-1}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter of a circular contact</td>
<td>[m]</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$E^*$</td>
<td>Reduced modulus</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$f_p$</td>
<td>fraction of plastic deformation</td>
<td>[-]</td>
</tr>
<tr>
<td>$F_f$</td>
<td>friction force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_n$</td>
<td>load, normal force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_n^*$</td>
<td>load at a contact</td>
<td>[N]</td>
</tr>
<tr>
<td>$h_{wf}$</td>
<td>water-film thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>$l$</td>
<td>rms width of the ADF</td>
<td>[m]</td>
</tr>
<tr>
<td>$L$</td>
<td>volumetric latent heat of fusion</td>
<td>[J m$^{-3}$]</td>
</tr>
<tr>
<td>$n$</td>
<td>number of contact spots</td>
<td>[-]</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$p_0$</td>
<td>maximum pressure at a contact</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$p_m$</td>
<td>mean pressure at a contact</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$P$</td>
<td>heat generation</td>
<td>[W]</td>
</tr>
<tr>
<td>$q'_z$</td>
<td>heat flux, heat current</td>
<td>[W m$^{-2}$]</td>
</tr>
<tr>
<td>$Q$</td>
<td>heat</td>
<td>[J]</td>
</tr>
<tr>
<td>$r$</td>
<td>radius of curvature of an asperity</td>
<td>[m]</td>
</tr>
<tr>
<td>$relRCA$</td>
<td>relative real contact area</td>
<td>[-]</td>
</tr>
</tbody>
</table>

*continued on next page*
### APPENDIX A. NOTATION

*continued from previous page*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>temperature</td>
<td>[K]</td>
</tr>
<tr>
<td>$T_{\text{env}}$</td>
<td>environmental temperature</td>
<td>[K]</td>
</tr>
<tr>
<td>$T_m$</td>
<td>melting point temperature of ice</td>
<td>[K]</td>
</tr>
<tr>
<td>$v$</td>
<td>sliding velocity</td>
<td>[m s$^{-1}$]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>heat transfer coefficient</td>
<td>[W m$^{-2}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>kinematic viscosity</td>
<td>[m$^2$ s$^{-1}$]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>thermal diffusion coefficient</td>
<td>[m$^2$ s$^{-1}$]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>thermal conductivity</td>
<td>[W m$^{-1}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>kinetic friction coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
<td>[kg m$^{-3}$]</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>normal stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>indentation hardness</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>shear stress</td>
<td>[Pa]</td>
</tr>
</tbody>
</table>

Table A.2: Abbreviations.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF</td>
<td>Amplitude distribution function</td>
</tr>
<tr>
<td>CT</td>
<td>Computer tomography</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element modeling</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning electron microscopy</td>
</tr>
</tbody>
</table>
Bibliography


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SLF Davos. Cooperation in a CTI project "Sliding friction on snow".

2001 - 2002
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2000 (3 months)
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1998 - 2000
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