Single-shot electron bunch-length measurements with a spatial electro-optical-auto-correlation interferometer using coherent transition radiation at the 100MeV SLS pre-injector LINAC

Author(s):
Sütterlin, Daniel Oliver

Publication Date:
2006

Permanent Link:
https://doi.org/10.3929/ethz-a-005243797

Rights / License:
In Copyright - Non-Commercial Use Permitted
Single-Shot Electron Bunch-Length Measurements with a Spatial Electro-Optical-Auto-Correlation Interferometer using Coherent Transition Radiation at the 100 MeV SLS pre-injector LINAC

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH

for the degree of
Doctor of Natural Science

presented by
DANIEL OLIVER SÜTTERLIN
Dipl. Phys. ETH
born 13. 04. 1974
citizen of Zürich

accepted on the recommendation of
Prof. Dr. H. Jäckel, examiner
Dr. V. Schlott, co-examiner
Dr. D. Erni, co-examiner
Dr. H. Sigg, co-examiner

2006
Single-Shot Electron Bunch-Length Measurements with a Spatial Electro-Optical-Auto-Correlation Interferometer using Coherent Transition Radiation at the 100 MeV SLS pre-injector LINAC

ABHANDLUNG
Zur Erlangung des Titels
Doktor der Naturwissenschaften

der
EIDGENÖSSISCHEN TECHNISCHEN HOCHSCHULE
ZÜRICH

vorgelegt von
DANIEL OLIVER SÜTTERLIN
Dipl. Phys. ETH
geboren am 13. 04. 1974
von Zürich

Angenommen auf Antrag von:
Prof. Dr. H. Jäckel, Referent
Dr. V. Schlott, Korreferent
Dr. D. Erni, Korreferent
Dr. H. Sigg, Korreferent

2006
For my parents and for my sister as acknowledgment of my gratitude for their great support and love during the past years
# Contents

1 Transition radiation: generation and diagnostic methods .................................. 1
   1.1 General description of transition radiation ................................................. 3
   1.2 Coherent emission of transition radiation ................................................. 5
      1.2.1 Emission of CTR at the SLS pre-injector LINAC .............................. 9
      1.2.2 Bunch length measurement methods using CTR ............................. 14

2 Emission characteristics of long-wavelength TR ............................................. 21
   2.1 Introduction ......................................................................................... 21
   2.2 Electromagnetic fields of relativistic electrons ........................................ 24
   2.3 Transition radiation emitted from an oblique target screen ....................... 26
      2.3.1 On simplification and error estimation ........................................... 35
   2.4 Discussion ......................................................................................... 36
      2.4.1 Long wavelength limit .................................................................... 39
      2.4.2 Short wavelength limit .................................................................. 40
   2.5 Summary ............................................................................................. 42

3 Novel spatial interferometer with single-shot capability .................................. 43
   3.1 Martin-Puplett interferometer measurements .......................................... 44
   3.2 Design principles for the quasi-optical set-up of the EOA experiment ....... 47
      3.2.1 Optical diagnostic port: out-coupling of CTR ................................ 48
      3.2.2 Transfer optics: off-axis parabolic mirrors .................................... 50
   3.3 Spatial interferometer .......................................................................... 51
3.3.1 Theoretical description of spatial auto-correlation .......................... 52
3.3.2 Optical layout of spatial interferometer ........................................... 57
3.3.3 Proof of principle of the spatial interferometer ................................. 60
3.3.4 Measurement with the spatial interferometer at the SLS pre-injector LINAC ................................................................. 67
3.4 Transfer function of the complete quasi-optical set-up ......................... 71
3.4.1 Wavelength dependent CTR spot size .............................................. 76

4 Electro-optical imaging system and synchronization ............................... 81
4.1 Active-mode-locked laser system ........................................................ 83
4.1.1 Active-active-mode-locking-master oscillator with negative feedback and regenerative amplifier .................................................... 84
4.1.2 Measurement of Nd:YAG time profile .............................................. 87
4.2 Electro-optical imaging system ............................................................ 88
4.2.1 Electro-optical crystal ................................................................. 89
4.2.2 Cross-polarization configuration .................................................... 91
4.2.3 InGaAs linear image sensor .......................................................... 94
4.3 Synchronization scheme ................................................................. 95
4.4 Summary ......................................................................................... 98

5 Single-shot bunch length measurements ................................................. 101
5.1 Measurements .................................................................................. 101
5.1.1 Measurement pre-alignement ...................................................... 101
5.1.2 Data acquisition ........................................................................ 102
5.2 Signal analysis of the auto-correlation profiles .................................... 106
5.2.1 Bunch length fits ....................................................................... 107
5.3 Error analysis: sensitivities on the parameters .................................... 113
5.4 Conclusions .................................................................................... 116

6 System limitations and improvement of the monitor set-up ..................... 117
6.1 Sensitivity and noise sources of the present set-up ........................................ 117
6.1.1 Sensitivity of the set-up ................................................................. 119
6.1.2 Noise sources of the present set-up ..................................................... 123
6.1.3 Signal to noise ratio ................................................................. 130
6.2 System improvements ................................................................. 131
6.2.1 Quasi-optical system ................................................................. 131
6.2.2 Electro-optical imaging system ...................................................... 136
6.2.3 Conclusions ................................................................. 140
6.3 Summary ................................................................. 141

A Electromagnetic fields of relativistic electrons ........................................ 143

B Spatial auto-correlation ................................................................. 145

C Electro-optical effect ................................................................. 147
   C.1 The index ellipsoid ................................................................. 148
   C.2 The Pockels effect ................................................................. 148

D Influence of alignment errors of the spatial interferometer ....................... 153

E Short pulses from fibre lasers ................................................................. 155
Abstract

Structural research on an atomic and subatomic level is of interest in many fields of science. In order to resolve the microscopic structure of a material the wavelength of the light needed for investigation has to be smaller than the structure itself. Wavelengths of 0.1 nm are needed to resolve atomic structures.

This kind of radiation is commonly supplied from synchrotron radiation facilities. The next generation of light sources will offer brilliances which are higher by some ten orders of magnitude as well as unprecedented properties such as coherence and time structure. This next generation of light sources will be based on the principle of self amplifying spontaneous emission (SASE), which has been successfully demonstrated in a number of test facilities such as the TESLA Test Facility (TTF) at DESY in Hamburg. This sources will be of a LINAC\(^1\)-based design with electron beams requiring small transverse emittances (transverse beam size), small energy spreads and extremely short bunch lengths.

Before such a source is operational many technological challenges have to be solved especially in the field of electron beam diagnostics. In particular the measurement of extremely short electron bunches (< 1 ps) is demanding and challenging. Different methods have been proposed and applied in order to understand and characterize the bunching process of the electron beam throughout the LINAC. Different measurement techniques include step-scan interferometers and the correlation of coherently emitted radiation with femto-second lasers and electro-optical (EO) switches.

In this thesis a novel concept of a bunch-length monitor is presented. The monitor set-up offers single-shot capability and sub-picosecond time resolution. The single-shot capability is obtained by a novel interferometer design which produces a spatial interference pattern of the Fourier components of the coherent transition radiation (CTR) as produced by a LINAC. The single-shot read-out is achieved by electro-optical techniques using a comparatively long (500 ps) Nd:YAG probe laser pulse.

The replacement of the femto-second Ti-Sapphire laser system as commonly used for EO-sampling experiments by a simpler active-mode-locked Nd:YAG laser promises a highly

\(^1\)Linear Accelerator
robust as well as a more affordable set-up. Especially the severe requirements regarding synchronization are dramatically relaxed by using a probe laser pulse which is longer than the CTR pulse under investigation.

A thorough investigation of the emission process of long wavelength transition radiation (TR) allows an efficient design of the quasi-optical set-up (comprising the out-coupling of the TR, the transfer optics as well as the spatial interferometer). The principle of the novel interferometer was demonstrated by performing laboratory measurements with cw-sources. The set-up was then implemented at an optical beam port of the SLS pre-injector LINAC where the auto-correlation of the broadband CTR could be successfully demonstrated. Single-shot bunch length measurements were performed at the SLS LINAC using an electro-optical read-out. Bunch lengths in the range between 1.5 and 4 ps have been measured with sub-picosecond time resolution.
Zusammenfassung

Die Erforschung der Materiestruktur auf einer atomaren und sub-atomaren Basis ist in zahlreichen Wissenschafts- und Wissenschaften von zentraler Bedeutung. Um die mikroskopische Struktur von Materie aufzulösen, wird Licht benötigt, dessen Wellenlänge kürzer ist als die Struktur der zu untersuchenden Materie. Um atomare Strukturen auflösen zu können werden Wellenlängen von 0.1 nm gebraucht.


Vor allem der Ersatz des Femtosekunden Ti:Sapphir Lasers, wie er üblicherweise bei der Elektro-Optischen Abtastung zum Einsatz kommt, mit einem wesentlich unkomplizierteren aktiv-moden-gelocktem Nd:YAG System ermöglicht einen robusten sowie auch erschwinglichen Aufbau. Im Speziellen werden die strengen Anforderung an die Synchronisation zwischen Laser und Übergangsstrahlungs Pulsen wesentlich entspannt. Dies ist eine Folge davon, dass der Laser Puls in unserem Experiment wesentlich länger als der zu untersuchende Übergangsstrahlungs Puls ist.

Eine gründliche Untersuchung der Emissionscharakteristik von langwelliger Übergangsstrahlung erlaubt die Auslegung eines effizienten quasi-optischen Aufbaus (Auskopplung und Transfer der Übergangsstrahlung sowie das spatiale Interferometer).

Chapter 1

Transition radiation: generation and diagnostic methods

The bunch length monitor presented in this thesis analyzes the spectral composition of coherent radiation emitted by the relativistic electron bunch. In this respect coherent radiation sources like transition radiation (TR) as well as synchrotron radiation (SR) can be used for these purposes.

Optical transition radiation (OTR) is commonly used in electron beam diagnostics for the determination of the emittance and the energy spread of the relativistic bunches. Therefore, diagnostics port for transition radiation are widely available at accelerator facilities. The measurements presented in this thesis were performed at the pre-injector LINAC of the Swiss-Light-Source (SLS). In this context transition radiation emitted at the SLS LINAC was selected as the source of the measurements. In this chapter we introduce the emission characteristics of transition radiation.

Transition radiation (TR) is widely used in electron beam diagnostics providing information about the transverse and longitudinal beam properties. It is emitted when charged relativistic particles cross the boundary between two media with different dielectric properties. In a typical setup, a thin metallic target screen which is rotated by 45° with respect to the electrons trajectory, is inserted into the electrons beam path allowing the backward emitted TR to be observed through a vacuum window (see figure 1.1).

It has to be distinguished between the TR emission in the optical range and the emission of long wavelength TR. In this chapter it will be shown that the spectrum of long wavelength TR carries information about the bunch charge configuration.

Optical transition radiation (OTR) is commonly used for the investigation of the electron beam transverse phase space. The radiation is broadband, instantaneous, linear to the
Chapter 1. Transition radiation: generation and diagnostic methods

Figure 1.1: Emission of transition radiation.

number of particles in the beam (beam charge) and diagnostic devices using OTR provide high spatial resolution, which is independent of the beam energy [Fio02]. Likewise, the spectral and angular distribution of OTR are also used to measure the electron beam divergence, trajectory angle and emittance [Fio94].

The emission of OTR which is instantaneous, is also used for time-resolved measurements of electron beams [Lum92]. The temporal profile of the emitted OTR can be characterized using a streak camera. Up-to-date streak cameras offer time resolutions down to 200 fs [Ham]. As a streak camera represents a considerable asset, these devices are not placed directly in the radiation bunker of the accelerator. An optical transmission line has to be used to transport the OTR onto the entrance slit of the streak camera. The light transport has to be performed creating minimum time dispersion and minimum light losses [Bae92]. Hence the use of streak cameras is a complex and expensive task and it is still not possible to resolve X-FEL beams down to some 10 fs level.

In the first section of this chapter a brief description of the TR emission characteristics is given according to the well-known model of Ginzburg and Frank [Gin45] for a single electron. The second part of the chapter considers the emission process of a bunch of \( N \) particles. It will be shown that long-wavelength (\( \lambda \geq \sigma_z \), where \( \sigma_z \) is the bunch length) TR carries information of the longitudinal charge distribution in its power density spectrum. For this range of the TR spectrum the ensemble of electrons radiate coherently. The succeeding section of the chapter shows the experimental observation of coherent TR at the SLS pre-injector LINAC. The chapter concludes with a discussion of different bunch length measurement methods in electron beam diagnostics using coherent TR.
1.1 General description of transition radiation

The existence of transition radiation was predicted in 1945 by the two Russian physicists V.L. Ginzburg and I. Frank. The relation of the phase velocity of the electromagnetic field of an uniformly moving relativistic charge to the speed of light is continuously changing in an inhomogeneous dielectric medium resulting in a separation of the electromagnetic field from the charge. For an observer in the laboratory rest frame the separating field is interpreted as electromagnetic radiation originating from the relativistic charge. Actually, there are several concepts of the generation process of transition radiation. One perception is using image charge theory. Transition radiation is then just the radiation emitted in a pair annihilation (or creation) process taking place at the boundary. Another prominent view is that of the virtual photon picture of the electron’s electric field. Due to the introduction of boundary conditions the field is reflected and these photons become real, resulting in the observed TR. A more detailed description of another model for the emission process of long-wavelength TR is found in the second chapter. The well-known result derived by Ginzburg and Frank assumes two medias with different dielectric constants $\epsilon_1$ and $\epsilon_2$ and a boundary of infinite extent in between. The description of Ginzburg and Frank is based on the pair annihilation of the electron with its mirror particle. For short wavelengths (optical transition radiation) the assumption of an infinite target is valid while for large wavelengths the emission characteristics differ strongly from the predictions of Ginzburg and Frank due to the finite aperture size of the target. This results from the fact that the low-frequency components of the electromagnetic source fields of the relativistic electrons can be considerably larger ($\gamma \lambda$) than the dimension of the target screen (c.f. chapter 2). Thus, for optical frequencies the TR source seems to be point-like, while at longer wavelengths this does not hold anymore due to the larger size of the electromagnetic source fields. Considering the transition from vacuum ($\epsilon_1 = 1$) to metal ($\epsilon_2 \rightarrow \infty$) yields the well-known Ginzburg-Frank formula for the spectral energy flux emitted in a solid angle $d\Omega$ by an electron with the speed $v = \beta c$:

$$
\frac{d^2 I}{d\nu d\Omega} = \frac{e^2}{2\pi^2\epsilon_0c} \frac{\beta^2 \sin^2(\theta)}{(1 - \beta^2 \cos^2(\theta))^2}.
$$

(1.1)

A complete derivation of formula (1.1) can be found in [Gin45] or [TM72].

The transverse emission characteristic for the 100 MeV SLS pre-injector LINAC (relativistic Lorentz factor $\gamma = 200$) according to equation (1.1) is shown in figure 1.3. The maximum in emission occurs at $\theta = \pm \frac{\pi}{\gamma}$. The emitted TR spectrum ranges up from the microwave to the x-ray region to the plasma frequency $\omega_p$ of the material used as target.
Chapter 1. Transition radiation: generation and diagnostic methods

Figure 1.2: The solid angle $d\Omega$.

The most obvious difference of TR compared to SR is the very special polarization: The electrical field of the emitted TR is radial polarized resulting from the radial polarization of the electrical field of the relativistic electron. As a consequence, the intensity of the emitted TR vanishes at $\theta = 0$. This is a consequence from the fact that the sum of all electrical field vectors cancels at the symmetry point.

The above relation which is valid for the assumption of the target screen of infinite extent, is frequency independent (c.f. formula (1.1)). This is due to the fact that the extend of the electromagnetic fields of the electrons are much smaller than the size of the target. Therefore, the source can be seen as point-like, independent on the wavelength of the emitted radiation. Hence, this applies typically in the optical range, where the assumption of an infinite target does hold. However for increasing wavelengths the electromagnetic source fields will become longer than the screen itself as will be shown in chapter 2. Therefore, at long wavelengths/low frequencies the emission characteristics can be expected to differ strongly from the predictions of Ginzburg and Frank. At these wavelengths the TR source can not be viewed as point-like anymore and therefore, the target screen size is of great importance.
1.2 Coherent emission of transition radiation

So far the emission of TR from a single electron has been considered. A bunch in an RF accelerator consists of a large number of particles (up to $10^{12}$). The description of Ginzburg and Frank yields a frequency independent emission characteristics of TR. The emission of $N$ particles leads to a spectral dependence of TR due to coherence effects. For wavelengths which are equal or larger than the extent of the electron bunch the electrons will emit in phase. Hence the individual field amplitudes sum up constructively and the resulting radiated energy flux is coherently enhanced. The degree of coherence is given by the so-called form factor $f(\nu)$.

The total radiated electromagnetic field from a bunch of $N$ electrons is the superposition of that of each individual electron with its phase. Hence the total radiation power is given by [Hir91], [Wan97].

**Figure 1.3:** Transverse emission characteristics of transition radiation according to equation (1.1) for an electron energy of 100 MeV as behind the SLS pre-injector LINAC. The maxima in emission occurs at $\theta = \pm \frac{1}{\gamma}$.

**Figure 1.4:** Calculation of the radiation emitted by a bunch of electrons. The position of the individual electron in the bunch is given by $\vec{r}_k$. 
\[ I_{\text{total}}(\nu) = I_1(\nu) \cdot \left| \sum_{n=1}^{N} e^{i \frac{2\pi \nu \vec{n} \cdot \vec{r}_k}{c}} \right|^2 = I_1(\nu) \cdot T(\nu), \]  

(1.2)

where \( I_1 \) is the radiated power of a single electron\(^1\). According to [Nod54] the term \( T(\nu) \) can be expressed as follows:

\[ T(\nu) = \left( N + \sum_{k=1, l=1, k \neq l}^{N} e^{i \frac{2\pi \nu \vec{n} \cdot (\vec{r}_k - \vec{r}_l)}} \right). \]  

(1.3)

The term \( N \) is resulting from the summation with \( k = l \). It denotes the incoherent part of the emitted radiation, where the \( N \) individual electrons radiate independently.

Equation 1.3 can be reformulated as given by [Hir91]:

\[ T(\nu) = \left( N + N(N + 1)f(\nu) \right), \]  

(1.4)

where the so-called form factor \( f(\nu) \) is introduced which is calculated by considering an average distribution over the particle ensemble. Thus, we define the normalized distribution function \( S(\vec{r}) \), where \( S(\vec{r}) \) is a continuous probability function such that \( N \cdot S(\vec{r})dV \) is the probability of finding a particle in \( dV \) at \( \vec{r} \) [Hir91].

\( N \) is usually a large number \( (10^7 - 10^{12}) \). Thus in the limit \( \lim_{N \to \infty} \) only the coherent terms \( \propto N^2 \) have to be considered as the incoherent background given by the \( \propto N \) terms is much less significant. The form factor \( f(\nu) \) is finally computed [Hir91] by:

\[ f(\nu) = \left| \int e^{i \frac{2\pi \nu \vec{n} \cdot \vec{r}}{c}} S(\vec{r})dV \right|^2 \]  

(1.5)

as the squared modulus of the Fourier transform of the average charge distribution \( S(\vec{r}) \).

In the limit \( \nu \to 0 \) the form factor \( f(\nu) \) converges to 1.

If the wavelength of the emitted radiation is larger than the dimension of the electron bunch the individual electrons radiate in phase (sum up over their field amplitudes, \( \propto N^2 \)) while at shorter wavelengths the phase relation is lost and the sum signal of the amplitudes averages out (sum up over individual intensities, \( \propto N \)) and the form factor \( f(\nu) \) equals zero (see figure 1.5). Thus information about the average bunch charge configuration is contained in the coherent power density spectrum of the emitted radiation which is therefore of great interest for electron beam diagnostics. Namely the longitudinal bunch charge distribution is in the focus of the analysis of coherent radiation emitted by a short bunch of electrons. This is one of the most sensible parameters for the operation of any future X-FEL light sources. Distinct longitudinal bunch shapes will lead to specific

\(^1I_1 \) is independent from \( \nu \) for OTR.
1.2. Coherent emission of transition radiation

![Coherent emission process](image)

**Figure 1.5:** Upper figure: coherent emission process for wavelengths larger than the bunch length; lower figure: at short wavelengths the average amplitude signal vanishes.

![Form factors for Gaussian and rectangular distributions](image)

**Figure 1.6:** Form factors for Gaussian and rectangular longitudinal charge distributions. Left side: bunch lengths 2 ps ($2\sigma$ for the Gaussian distribution). Right side: bunch lengths 5 ps.

Form factors $f(\nu)$ which allow to reconstruct the longitudinal bunch charge distribution $S(z)$. A bunch with a Gaussian temporal structure will result in the narrowest coherent spectrum (which is given by: $\Delta \nu \propto 1/\Delta t$) while a rectangular shaped bunch leads to large overtones which are broadening the coherent spectrum. The form factors for Gaussian and rectangular bunch charge distributions are shown in figure 1.6.

In figure 1.6 we have neglected the transverse size of the bunch and we have considered only the longitudinal dimension. For a cylindrical charge distribution of length $l$ and radius $a$ (c.f. figure 1.7) this requires $l \gg a$. The form factor $f(\omega)$ for the above charge density function is given by [Men05], [Lih96] as:

$$f(\omega) = \left| 2 \frac{J_1 \left( \frac{\omega a}{c} \sin(\theta) \right)}{\frac{\omega a}{c} \sin(\theta)} \cdot \frac{\sin \left( \frac{\omega l}{2c} \cos(\theta) \right)}{\frac{\omega l}{2c} \cos(\theta)} \right|^2$$

(1.6)
where $\omega = 2\pi \nu$, $J_1$ denotes the Bessel function of first order and $\theta$ is the angle between the electron beam axis and the detector.

For $\theta = 0$ there is no dependence of the form factor of the radius $a$. For large angles $\theta$ or big transverse beam sizes $a$ the transverse contribution will result in a more narrow coherent spectrum as shown in figure 1.8.

According to [Lih96] the transverse size of the bunch can be neglected when the following condition is fulfilled:

$$\frac{2\pi a}{3.83 \tan(\theta)} \ll l \quad (1.7)$$

The maximum intensity of transition radiation is emitted at an angle $\theta = 1/\gamma$. Therefore, the above condition is valid even when we consider the broadening of the emission pattern for increasing wavelengths as pointed out in the following chapter. For the SLS LINAC (100 MeV , $\gamma = 200$) the angle $\theta$ is $\approx 0.05$ rad. Here we have taken the broadening of the emission pattern with a factor of 10 into account (c.f. section 2.4). This yields $a \ll 12 \cdot l$. The transverse size of the electron bunch at the optical monitor ALIDI-SM-5...
of the SLS LINAC is typically in the order of 0.5 mm [Süt01]. According to PARMELA\textsuperscript{2} simulations (energy versus phase) [Ped], [You] a longitudinal bunch length of 2 ps rms can be expected at the SLS pre-injector LINAC. Thus, for the expected bunch lengths it is justified to neglect the transverse dimension of the bunches at the SLS pre-injector LINAC.

1.2.1 Emission of CTR at the SLS pre-injector LINAC

Even though the SLS pre-injector LINAC is not predestined to produce shortest (sub-ps) electron bunches, it has been chosen as a source of CTR for the experiment described in this thesis. The first part of this section gives a description of the SLS LINAC and its working principle. The section is concluded with CTR measurements showing the quadratic dependence of the signal as a function of the bunch charge as predicted by equation (1.4). Measurements of the CTR emission characteristics show an angular dependence which differs strongly from the predictions by the Ginzburg and Frank formalism. For the optical range, the size of the electromagnetic source fields of the relativistic electron is much smaller than the dimension of the target screen. Therefore, the source can be considered as point-like in this frequency range. However, at longer wavelengths the extend of the electromagnetic fields is larger than the diameter of the target screen. Thus, the assumption of an infinite screen of Ginzburg and Frank becomes void. In the extreme case of long-wavelengths or high beam energies ($\propto \gamma$), the fields drop with $1/r$, where $r$ is the transverse distance from the electron beam. It is convenient to consider this problem in cylindrical coordinates. Due to the Jacobian of the coordinate transformation we must multiply with $r$. Thus, in the extreme case of long wavelengths or high beam energies the fields extend over the entire screen (c.f. section 2.2). Therefore, the source can not be treated as point-like as in the limit of the optical range. As a consequence this will result in a broadening of the angular emission pattern (c.f. figure 1.13).

The SLS pre-injector LINAC

The SLS is a third generation synchrotron light source. The accelerator complex consists of a 100 MeV S-band linear accelerator (LINAC) used as pre-injector, a full energy booster synchrotron, and a 2.4 GeV storage ring. Both booster and storage ring run at a frequency

\textsuperscript{2}PARMELA is an electron-linac particle-dynamics code. The name is from the phrase, "Phase and Radial Motion in Electron Linear Accelerators." It is a versatile multi-particle code that transforms the beam, represented by a collection of particles, through a user-specified linac and/or transport system. It includes a 2D space-charge calculation and an optional 3D point-to-point space-charge calculation.
During normal operation the LINAC is run in top-up mode to refill the storage ring to a constant beam current of 350 mA. The LINAC can be run independently from the rest of the facility. This mode was used for the measurements described in this thesis.

In order to explain the generation of short electron bunches in the SLS pre-injector the working principle of the SLS LINAC and its main components is described in brief [Str], [Ped99], [Ped00]. The layout of the LINAC is presented in figure 1.9 and a more detailed view is found in figure 1.10.

The electron source is a triode gun consisting of the cathode, a grid and the anode. The electrons are emitted by a thermionic Barium dispenser cathode and accelerated by the 90 kV anode voltage. The temporal structure of the electron beam and the bunch charge are controlled by the voltage applied to the grid. The grid pulse length is \(< 1 \text{ ns}\) producing either single bunches (as used in the experiments described in this thesis) or a train of bunches with a 2 ns spacing and an overall length of up to 700 ns, the so-called multi-bunch mode.

The bunching process in the SLS LINAC relies on velocity bunching utilizing the fact

![Figure 1.9: Schematic overview of the SLS pre-injector LINAC.](image)
1.2. Coherent emission of transition radiation

Figure 1.10: The SLS LINAC: electron gun, bunching section and solenoid magnets.

that electrons below energies of 3 to 4 MeV are non-relativistic \((v < c)\). This means that two electrons with slightly different energies will travel at different velocities. Thus, applying a RF voltage with the correct phase setting allows to compress the bunch. A particle with higher energy which is arriving too early can be decelerated by negative RF voltage while a low energy particle which arrives too late is accelerated by positive RF voltage. Hence, the tail particles catch up, the leading particles slow down and the bunch is compressed after some drift space (see figure 1.11). The bunching of the SLS LINAC is performed using three bunching cavities and drift spaces after the cavities. The first cavity, the subharmonic pre-buncher (ALIRF-SPB) is operated at 500 MHz. The standing wave subharmonic pre-buncher compresses the bunches coming from the gun. The RF frequency of the SLS LINAC is 3 GHz. The purpose of the 4 cell traveling wave pre-buncher (ALIRF-PBU) is to make the beam acceptable for the 3 GHz accelerating structures. The 16 cell traveling wave final-buncher (ALIRF-FBU) is also operated at 3 GHz and is also used to further compress the non-relativistic electron bunches. Secondly it accelerates the electrons to quasi-relativistic energies of 4 MeV and thus freezes the temporal structure of the electron bunches.

Focusing along the low energy bunching section of the SLS pre-injector LINAC is provided by 16 solenoid magnets (ALIMA-0/-1 to 16) compensating the space charge effects at low energies. The two 5.2 m long traveling wave structures (ALIRF-AS-1/2) are accelerating
Chapter 1. Transition radiation: generation and diagnostic methods

Figure 1.11: Velocity bunching of a non-relativistic bunch of electrons. The electron bunch is given by the temporal distribution \( \rho(t) \). The high energy particles \( t < 0 \) arrive too early and are decelerated by the RF voltage \( V(t) \) (red line) while low energy particles \( t > 0 \) arriving too late are accelerated. After some drift space the bunch is compressed as the tail catches up and the head of the bunch is slowed down.

Observation of CTR at the SLS pre-injector LINAC

The proof of CTR emission was achieved by recording the square dependence of the emitted radiation power as a function of the bunch charge. According to formula (1.4) the total radiated power is proportional to \( I_{\text{total}} \propto T(\nu) = (N + N(N + 1)f(\nu)) \) and for \( n \to \infty \) \( I_{\text{total}} \propto N^2f(\nu) \). For coherent emission (dominating form factor \( f(\nu) \)) the signal is rising with the square of the number of charges, as \( N \) is very large (0.5 nC \( \Rightarrow N = 3 \cdot 10^9 \)). Figure 1.12 shows a measurement of CTR behind the SLS LINAC at the optical beam port ALIDI-SM-5. The signal of a pyro-electric detector, which is sensitive to long-wavelength FIR radiation, is plotted against the measured bunch charge at the integrating current transformer ALIDI-ICT-2 [Sch99] behind the LINAC. The fitted curve shows the expected quadratic dependence of the signal.

The result derived by Ginzburg and Frank predicts a two-lobed emission of the TR with their maxima separated by an angular distance of \( 2/\gamma \). The forming of the distinct lobes is a direct consequence of the radial polarized characteristics of TR. The lobes were measured at the SLS LINAC. They are separated by more than 50 mrad as can be seen in figure 1.13 below. The broadening of the emission pattern clearly indicates a spectral dependence for the emission pattern of long wavelength CTR. This results from the fact that the electromagnetic source fields of the relativistic electron can be considerably larger at these wavelengths than the size of the target screen. Furthermore, the measurements indicate an
1.2. Coherent emission of transition radiation

asymmetry in emission in horizontal direction (c.f. section 2.4) In the limit of the optical frequency range the electromagnetic source of the TR can be treated as point-like, while at these long wavelengths a considerable portion of the target screen radiates, which means that the size of the screen becomes important. For the design of an efficient quasi-optical system an accurate source description is needed. Clearly the formalism of Ginzburg and Frank is not applicable in this context as it can not explain the broadening of the emission pattern. In this context the description of the emission process of long wavelength CTR is one of the findings of this thesis and is described in detail in the succeeding chapter.

Figure 1.12: Signal of pyro-electric detector against charge. The quadratic dependence indicates the coherent emission.

Figure 1.13: Measurement of CTR emission characteristics at the SLS LINAC. Left side: measurement set-up. Right side: The two maxima of the two CTR lobes are separated by more than 50 mrad indicating that the Ginzburg and Frank model is not applicable for the emission of long wavelength coherent TR.
1.2.2 Bunch length measurement methods using CTR

Several methods are used to measure the longitudinal bunch charge distribution. This section presents the two most prominent approaches. The first one are scanning interferometric measurements of the coherent spectrum and the second approach uses the advantage of THz-gating technologies with electro-optical crystals allowing the direct determination of the bunch shape and width. The section is concluded with the presentation of novel measurement methods offering single-shot capability.

Michelson type interferometers

The coherent spectrum can be measured using a Michelson type interferometer [Mar82], [Gei99a], [Gei99b]. The intensity auto-correlation of the radiation pulse is measured with a Martin-Puplett interferometer and then Fourier transformed to yield the coherent radiation power density spectrum and thus the bunch form factor.

![Figure 1.14: Schematic set-up of the Martin-Puplett interferometer.](image)

The advantage of interferometer measurements is the high frequency resolution. The drawback, however, is that as a step-scan method the interferometric measurements are averaged over a large number of shots. Furthermore, the Fourier transformation of the intensity auto-correlation function yields only the absolute magnitude $|f(\nu)|$ of the form factor. The Kramers-Kronig dispersion relation can be used to reconstruct the minimal phase of the CTR.

The low frequency part of the power density spectrum is suppressed by cut-off effects due to truncations of the quasi-optical beam at the apertures of the optics as well as diffraction effects. Hence, for an exact reconstruction of the longitudinal bunch distribution the transfer function of the set-up must be known which is difficult to determine at low
frequencies. In this respect, an accurate description of the emission characteristics of long wavelength CTR is needed. The error of the analysis is dominated by the systematic low-frequency effects and is estimated to be in the order of 20% - 30% [Gei99b]. It will be shown in chapter 5, that the assumption of a distinct (Gaussian) longitudinal bunch charge distribution $S(z)$ can lead to a smaller sensitivity (in the order of 5 %) of the retrieved bunch lengths on the low frequency attenuation.

**Electro-optical techniques**

Electro-optical techniques are another possibility to determine the longitudinal structure of a short electron bunch by using the fast response capability of electro-optical crystals. The most prominent approach is the so-called electro-optical sampling (EOS) which was successfully demonstrated at a large number of facilities [Win04b], [Oep99], [Fit99]. A short (< 100 fs) titanium-sapphire (Ti:Sa) laser pulse is used to sample the birefringence which is induced in a nonlinear optical crystal by the co-moving electric field of the CTR [Win04b]. The initial linear polarization of the laser pulse is converted into a slightly elliptical polarization. The most sensitive method to measure this ellipticity is to pass the beam through a quarter wave plate, transforming the slight elliptic polarization into a slightly perturbed circular polarization, and then through a Wollaston prism, which serves for a spatial separation of the two orthogonal polarization components. These are then coupled into optical multimode fibres and guided to a balanced diode receiver which act as detector [Win04b]. By shifting the timing of the laser pulse relative to the electron bunch in sub-picoseconds steps the time profile is obtained by sampling over many bunches (see figure 1.16).

![Figure 1.15: Principle of electro-optical sampling.](image-url)
Chapter 1. Transition radiation: generation and diagnostic methods

Figure 1.16: Simplified view of signal detection using a quarter wave plate, a Wollaston prism and a balanced diode detector.

Single-shot techniques

The method of electro-optical sampling is a time domain technique but has the same major drawback as interferometric techniques of not being single-shot measurements. Successful single-shot measurements\(^3\) using electro-optical techniques have been recently conducted at the FELIX Linac in Rijnhuizen (NL) [Ber04].

The EO spectral decoding technique (see figure 1.17) uses a short TiSa laser pulse which is linearly chirped to a duration which exceeds the measurement window. The time profile of the local electric field of the electron bunch is encoded onto the intensity envelope of the chirped optical probe beam and subsequently decoded through a single shot measurement of the probe spectrum [Ber04].

In the novel temporal decoding method (see figure 1.17) the chirped pulse which carries the modulation due to the presence of the electron bunch is cross-correlated with a part of the original optical pulse, which has been split off before the optical stretching. The single-shot cross-correlation is based on the temporal to spatial conversion that occurs through the spatial overlap of non-collinear beams in a second-harmonic crystal [Ber04].

This measurements also use a short Ti:Sa laser pulse. The synchronization of such lasers with short electron bunches is very complex task and the lasers itself represents large asset. However, the active stabilization for sub-picosecond laser pulses using RF-locking techniques has been recently achieved down to timing jitter level of 3 fs [Win05].

---

\(^3\)Both measurement methods (spectral and temporal decoding) commonly use the wakefields (electro-magnetic source fields) of the electron bunches which thus yields much higher signal levels than obtained with CTR.
1.2. Coherent emission of transition radiation

Figure 1.17: The general layout for the techniques of spectral decoding (top), and temporal decoding (bottom). Only the ZnTe crystal is placed within the beam pipe [Ber05] to observe the wake fields (electromagnetic fields) of the electron bunch.

Electro-optical auto-correlation

This thesis presents another novel approach for single-shot bunch length measurements with high resolution combining the advantages of interferometric techniques with those of the electro-optical methods but with less demanding requirements on the laser system and therefore on the synchronization scheme. Hence, the method represents a robust and affordable alternative to the approaches presented above.

The basic idea is to produce a spatial auto-correlation of CTR [Sig00] and to read out the interference pattern using electro-optical techniques in a single-shot (see figure 1.18). The spatial intensity auto-correlation \( I(x, \nu) \) of a monochromatic wave of frequency \( \nu \) is given by (c.f. section 3.2):

\[
I(x, \nu) = I(\nu) \cdot (1 + \cos(2(2\pi\nu/c) \sin(\theta)x)).
\] (1.8)
Chapter 1. Transition radiation: generation and diagnostic methods

The intensity auto-correlation pattern which thus results from a broadband source like coherent transition radiation represents nothing else than the cosine transform of the underlying power density spectrum $I_{\text{spec}}(\nu)$:

$$I(x) = \int_{0}^{\infty} \left( I_{\text{spec}}(\nu)(1 + \cos(2(2\pi \nu/c) \sin(\theta) x)) \right) d\nu$$ (1.9)

Here $2 \cdot \theta$ is the angle between the two CTR rays. The intensity profile $I(x) = E_{\text{CTR}}^2(x)$ of the auto-correlation pattern is then translated into a polarization modulated transverse profile of the probe laser pulse in the electro-optical crystal according to the following relation:

$$I_{\text{mod}} = I_{\text{laser}} \cdot \sin(\pi d \frac{V_{\lambda/2}}{E_{\text{CTR}}(x)})^2,$$ (1.10)

where $V_{\lambda/2}$ is the so called half wave voltage which is needed to rotate the linear polarization by an angle of $\pi/2$ and $d$ is the thickness of the crystal. Thus, we can write for the modulated laser profile:

$$I_{\text{mod}}(x) = I_{\text{laser}} \cdot \sin\left(\frac{\pi d}{V_{\lambda/2}}\right)^2 I(x),$$ (1.11)

Here, it must be noted that in contrast to the EOS experiment, the polarizers used in the presented EO set-up are not orientated at $\pi/4$ [Win04b], [Win04a] but in cross-polarization $\pi/2$ in order to have maximum suppression of the probe laser at zero incidence of the CTR on the crystal. The transmitted laser signal will thus depend on the square of the rotation angle of the polarization (c.f. eq. (1.11)). As this angle is proportional to the electrical field amplitude of the CTR applied to the crystal, the probe laser signal is therefore proportional to the CTR intensity making the readout inherently less sensitive than in the sampling technique\(^4\). Still, if the CTR is sufficiently strong, EO auto-correlation patterns of high quality can be recorded in single-shot as is shown by our work. It must be noted also that similar to the bunch length measurement with step-scan interferometers all phase relations are lost which makes the reconstruction of the longitudinal bunch shapes demanding.

The scheme of the experiment is presented in figure 1.19. The set-up comprises three major elements: The coherent transition radiation which is extracted through the optical view port, the quasi-optical system which consists of the transfer optics and the spatial interferometer and the electro-optical read-out comprising the active-mode-locked probe laser system with a pulse width much longer than the duration of the signal, the electro-optical crystal, the polarizer optics and the linear image sensor. The three major components as

---

\(^4\)This is a consequence of the fact, that the CTR field strengths at the SLS LINAC are expected to be in the order of 2 kV/cm, which thus lead to phase rotations of $10^\circ = 0.17$ rad. The modulated laser power is proportional to the squared value of the phase rotation, whereas the EOS method yields signal levels which are proportional to the phase rotation. Thus, the signal levels are reduced by nearly one order of magnitude compared to the ones obtained in the EOS experiment.
1.2. Coherent emission of transition radiation

Figure 1.18: Principle of the electro-optical auto-correlation (EOA) experiment. The spatial auto-correlation of the CTR pulse is read out using an electro-optical crystal. The interference pattern thus translates to a modulation of the transverse profile of the probe laser pulse.

listed above are described and analyzed in details in the following chapters. The entire experiment except the probe laser system was placed in the radiation protection bunker of the SLS pre-injector LINAC. This necessitates that most of the components are remote controllable.

The method which is referred to in the following as electro-optical auto-correlation (EOA) thus offers the capability of single-shot measurements of electron bunch lengths. The monitor set-up was successfully installed at the SLS pre-injector LINAC and was used for the determination of pulse widths as short as 1.5 ps. As intended, changes of the electron bunch lengths due to changes of the machine setting of the accelerator could be determined with sub-picosecond time resolution. Furthermore, the monitor offers the capability to be used as on line diagnostics, which thus would represent an indispensable tool for any operator in order to optimize the accelerator for shortest bunches.
Figure 1.19: Scheme of the electro-optical auto-correlation monitor. The set-up comprises the CTR source, the quasi-optical system and the electro-optical read-out system.
Chapter 2

Emission characteristics of long-wavelength TR

2.1 Introduction

The well-known relation derived by Ginzburg and Frank for transition radiation (TR) predicts frequency independent emission characteristics [Gin45] [TM72]. This result however is valid only when the source fields impinging onto the radiating screen are of considerably smaller extent ($\gamma \lambda$) than the target dimension, which is typically the case in the optical range of the spectrum.

Several approaches have been pursued to describe the generation of TR. Generally one can distinguish three different perceptions for the emission process. A prominent one is the model of the virtual quanta which is based on the assumption of virtual photons, constituting the electromagnetic source fields of the particle, which are converted into real photons by reflection at the finite metallic interface.

Prerequisite for this model is the fact that the electromagnetic field of a highly relativistic electron in the rest frame of the laboratory is confined in a flat disk perpendicular to the direction of motion. Hence, this yields only tangential fields as required in the approach. Here it is taken full advantage of the boundary condition that the tangential electrical field $E_t$ at the boundary of a perfect electrical conductor (PEC) must vanish. This necessitates a change in sign of the incoming to the outgoing electrical field, which thus corresponds to the reflection of the incident wave at the metallic target interface.

This is expressed in [Ver00], where the tangential radiation field of backward TR is formulated by the above mentioned boundary condition $E_t = E^{(q)} + E^{(r)} = 0$ set for a perfect conducting infinite boundary. The suffix ($q$) denotes the electromagnetic source field, while ($r$) gives that of the radiating field. In [Dob03] the aforementioned boundary condition is
formulated in a more general way in order to describe both the forward and the backward radiation fields. Additionally the outgoing electromagnetic wave can be computed in the sense of a scattering problem by applying the Huygens-Fresnel principle [Sch05].

However, this scheme can also be applied for a tilted screen where the normal projections of the tangential fields mediate the change in sign of the outgoing wave but which is then reflected in another direction as the incident wave.

The second approach for the generation of TR is that of the moving mirror charge [Gin45]. Here the electron and its mirror-(anti) particle which is incident from the opposite side of the target are stopped abruptly at the screen boundary while their charge is annihilated. In this view only their electromagnetic fields are remaining which thus can be identified as radiation from the target. However, it must be noted that this description does only hold in the very final moment when the electron hits target because during the relativistic propagation no mirror-(anti) particle can be anticipated. This is simply due to the fact that the electromagnetic field of the highly relativistic charges is contracted in longitudinal direction and thus is confined in a flat disk perpendicular to the direction of motion.

A more realistic perception of the aforementioned emission process of transition radiation is that of Bremsstrahlung [Gin89]. Similar to the annihilation model the relativistic electron is abruptly stopped at the target. In accordance with the fundamental principle of energy conservation the kinetic energy of the electron is emitted as radiation which can be identified as TR.

In this chapter we introduce a fourth view for the generation of transition radiation in which the metallic target screen itself is considered as source of the radiation using the so-called Physical-Optics (PO) technique. In this perception the magnetic field of the relativistic electron induces surface current density $\vec{j}_s$ in the finite circular metallic target, from which the vector potential representation of the radiated field is acquired. The use of the well-known PO approach yields an analytical solution for the electromagnetic fields of the emitted TR. Thus, it represents a very fast method for the calculation of the emerging fields, without the need to solve integral equations. When calculating the components of the resulting electrical and magnetic fields a further formalism has been introduced that provides accurate approximations for both cases the far-field and the radiating near-field. Similar to the model of the virtual quanta only the backward TR emission is considered. The derivation of TR emission based on virtual quanta model is a scalar theory. Thus, in contrast our model introduces a descriptive presentation of the generation mechanism of TR which is based on a vectorial formalism.

Since standard diagnostic ports use targets rotated by $\pi/4$ into the electron beam path, a circular tilted radiator is considered. The induced surface currents corresponds to the boundary condition for the magnetic field, which may become redundant to the one of the
2.1. Introduction

electrical field in the case where a perfect electrical conducting (PEC) target interface is assumed.

The concept of our model is illustrated in the following figure 2.1:

![Figure 2.1: Model for the emission characteristics of long wavelength TR: The surface current density (\(\vec{j}\)) induced by the magnetic field (\(\vec{H}_\phi\)) of the relativistic electron is used to compute the vector potential (\(\vec{A}\)) representing the emitted TR. At the point of observation \(\vec{r}_{\text{obs}}\).](image)

The far-field condition \(L \gg \gamma^2 \lambda\) for the radiating target as given by comprehensive references [Ver00], [Dob03] yields distances up to several hundreds of meters for TR in the FIR region. This is a consequence of the source fields which transverse extent is given by \(d = \gamma \lambda \ (L \gg d^2/\lambda)\). In our description the transition from the near-field to the far-field zone is characterized by the Rayleigh distance, which is approximated according to \(L_R = 4r_s^2/\lambda\) where we have considered the source dimension to be \(r_s\) denoting the radius of the radiating screen.

In the second section an electromagnetic description for the source field formation induced by the relativistic electron passage through the target is introduced. Based on this electromagnetic source term a closed form expression for the backward CTR from an oblique target screen is then presented in the third section. The introduced formalism yields accurate results for the wave zone (or the far-field, i.e. for the observation points farther than the Rayleigh distance) and provides very good approximations for the radiating near-field (or the Fresnel zone, i.e. for observation points within the Rayleigh distance), both for finite targets with lateral sizes that may become even smaller than the transversal extent \(\gamma \lambda\) of the relativistic electron’s electromagnetic-field Fourier-component. A proper validation of the emission formalism is given in the fourth section along with a typical test example: This confirms the asymmetric emission characteristic occurring for the horizontal polarized CTR lobes. This asymmetry predicted by the formalism was experimentally confirmed at the pre-injector LINAC of the Swiss Light Source (SLS) [Süt03], [Süt05]. Finally the chapter concludes with a short summary.
2.2 Electromagnetic fields of relativistic electrons

For a relativistic electron of which its rest frame \((x', y', z')\) moves in free space along the \(z\)-axis with respect to the laboratory frame \((x, y, z)\) we define the longitudinal Fourier transformation of the time dependent function \(A(t)\) by:

\[
\hat{A}(\nu) = \int_{-\infty}^{+\infty} A(\eta) \cdot e^{-i\eta 2\pi \nu} d\eta = e^{i2\pi \nu z/v_c} \int_{-\infty}^{+\infty} A(t) \cdot e^{-i2\pi \nu t} dt \tag{2.1}
\]

with the longitudinal coordinate \(\eta = t - \frac{z}{v} = t - \frac{z}{\beta c}\), where \(v = \beta \cdot c\). It is convenient to consider the problem in cylindrical coordinates. From symmetry arguments only the radial and longitudinal components of the electric field \((E_r, E_z)\) and the azimuthal components of the magnetic field \((B_\phi)\) have to be considered. \(\rho\) denotes the charge density and the current \(\vec{j}\) has only a longitudinal component originating from the moving charge distribution. It will be shown that the electrical field in the laboratory frame is contracted in longitudinal direction as illustrated in figure 2.3

In vacuum with \(\vec{D} = \epsilon_0 \vec{D}\) and \(\vec{B} = \mu_0 \vec{H}\) the macroscopic Maxwell equations in the laboratory frame in cylindrical coordinates are written as follows:

\[
\frac{1}{r} \partial_r (r E_r) + \partial_z E_z = \frac{\rho}{\varepsilon_0} \tag{2.2}
\]

\[
\frac{1}{r} \partial_\phi B_\phi = 0 \tag{2.3}
\]

\[
\frac{1}{r} \partial_\phi E_z = 0 \tag{2.4}
\]

\[
\partial_z E_r - \partial_r E_z = -\partial_t B_\phi \tag{2.5}
\]

\[
\frac{1}{r} \partial_\phi E_r = 0 \tag{2.6}
\]

\[
-\partial_z B_\phi = \frac{1}{c^2} \partial_t E_r \tag{2.7}
\]

\[
\frac{1}{r} \partial_r (r B_\phi) = \mu_0 j_z + \frac{1}{c^2} \partial_t E_z \tag{2.8}
\]

Applying the above defined Fourier transformation (see equation (A.1)) to Maxwell’s equa-
2.2. Electromagnetic fields of relativistic electrons

\[ E_r E_z B_\phi / c^2 \]
\[ e^{-\nu \Box} = c / c^9 \]
\[ e^{z} e / c^{10} \]

Figure 2.3: Electrical and magnetic field of a charge moving with velocity \( v = \beta \cdot c \) along the z-axis. The field in the laboratory frame is contracted in longitudinal direction. From symmetry considerations only the radial and longitudinal components of the electric field (\( E_r \) and \( E_z \)) and the azimuthal components of the magnetic field (\( B_\phi \)) have to be considered.

\[
\frac{1}{r} \partial_r (r \hat{E}_r) + \frac{i2\pi\nu}{\beta c} \hat{E}_z = \frac{\hat{p}}{\varepsilon_0} \tag{2.9}
\]
\[
\frac{1}{r} \partial_\phi \hat{B}_\phi = 0 \tag{2.10}
\]
\[
\frac{1}{r} \partial_\phi \hat{E}_z = 0 \tag{2.11}
\]
\[
\frac{i2\pi\nu}{\beta c} \hat{E}_r - \partial_r \hat{E}_z = i2\pi\nu \hat{B}_\phi \tag{2.12}
\]
\[
\frac{1}{r} \partial_\phi \hat{E}_r = 0 \tag{2.13}
\]
\[
- \frac{i2\pi\nu}{\beta c} \hat{B}_\phi = - \frac{i2\pi\nu}{c^2} \hat{E}_r \tag{2.14}
\]
\[
\frac{1}{r} \partial_r (r \hat{B}_\phi) = \mu_0 j_z - \frac{i2\pi\nu}{c^2} \hat{E}_z \tag{2.15}
\]

Maxwell’s equations are then solved in Fourier space obtaining the Fourier components of the magnetic field which are given by [Dob03], [Gei99a]

\[
\hat{B}_\phi = \frac{q}{(2\pi)^{1/2} \varepsilon_0 \beta^2 c^{-1} \gamma} \cdot K_1 \left( \frac{2\pi\nu}{\beta c \gamma} r \right), \tag{2.16}
\]

where \( K_1 \) is the modified Bessel function of first order. From equation (2.14) it follows
directly that
\[
\hat{E}_r = \frac{q}{(2\pi)^{1/2} \epsilon_0 \beta^2 c} \frac{2\pi \nu}{\beta \gamma} \cdot K_1(\frac{2\pi \nu}{\beta \gamma} r).
\] (2.17)

A derivation of the above solutions can be found in the appendix. The longitudinal component is obtained by inserting equations (2.16) and (2.17) into (2.12)
\[
\partial_r \hat{E}_z \propto \frac{i(2\pi \nu)^2}{\gamma^2 \beta} \cdot K_1(\frac{2\pi \nu}{\beta \gamma} r)(1 - \beta^2) = \frac{i(2\pi \nu)^2}{\gamma^2 \beta^2} \cdot K_1(\frac{2\pi \nu}{\beta \gamma} r).
\] (2.18)

For simplicity we have neglected here the pre-factor. With \(\partial_r (K_0(ar)) = -a \cdot K_1(ar)\) and by inserting the pre-factor the longitudinal component is readily calculated:
\[
\hat{E}_z = -\frac{i(2\pi \nu)}{\gamma^2 \beta} \frac{q}{(2\pi)^{1/2} \epsilon_0 \beta^2 c} \cdot K_0(\frac{2\pi \nu}{\beta \gamma} r).
\] (2.19)

For typical kinetic energies of some hundred MeV the \(\gamma\)-factor becomes large and the longitudinal component \(\hat{E}_z\) can therefore be neglected. Thus, it indicates that the electromagnetic field is confined in a disk perpendicular to the direction of motion. In the laboratory frame the longitudinal components are contracted by a factor of \(\gamma\) with respect to the transverse/tangential field components: \(\hat{E}_r, \hat{B}_\phi \propto 1/\gamma, \hat{E}_z \propto 1/\gamma^2\).

Figure 2.4 depicts two plots of the radial electrical field \(r \cdot \hat{E}_r(r)\) (due to the Jacobian for the cylindrical coordinate transform) for different values of the relativistic Lorentz factor \(\gamma\). For each \(\gamma\) the fields are computed for five different wavelengths \((\lambda = 0.1 \text{ mm, } 0.5 \text{ mm, } 1.0 \text{ mm, } 2.0 \text{ mm and } 5.0 \text{ mm})\). For large wavelengths the fields are nearly constant over the complete radius of the screen (c.f. figure 2.4). The effect is more pronounced for higher values of \(\gamma\). Since, \(\hat{B}_\phi = (\beta/c) \hat{E}_r\) the same behaviour applies for the magnetic field.

## 2.3 Transition radiation emitted from an oblique target screen

A standard TR diagnostic port consists of a target screen rotated by an angle of \(\pi/4\) to the direction of the electron beam allowing the extraction of the backward TR through a vacuum window (c.f. figure 1.1). In the following a tilted circular thin metallic screen representing the radiating source is considered. Most important the metallic target screen itself is treated as a source of the emitted TR using the Physical-Optics (PO) approach [Bor01]. The PO technique is a well-known and widely used optical technique for the calculation of the electromagnetic field scattered from a PEC surface by an incident electromagnetic
2.3. Transition radiation emitted from an oblique target screen

The radial field \( r \cdot \vec{E}_r(r) \) (due to the Jacobian for the cylindrical coordinate transform) for different wavelengths and two different \( \gamma \) values.

Field [Sef05], [Bor01]. It is a very fast method since it does not require an integral equation to be solved. The idea is to describe the surface currents \( \vec{j}_s \) induced by the magnetic fields \( \vec{H} \) which then represents the source term in the inhomogeneous Helmholtz equation describing the propagation of the resulting vector potential \( \vec{A} \).

Assuming a PEC target physically means that the fields do not penetrate into the conductor. Even for realistic metallic conductors one obtains skin depths in the order of 100nm [Jac75] which compares only to tiny a fraction of the wavelength involved (\( \lambda = 1\text{mm} \)). The conductor can therefore be assumed to be field-free, and, hence, only reflected fields are generated in the framework of the virtual quanta model. The connection to PO is revealed along the boundary conditions, which provides an exact measure of the surface current, i.e. for the source term of the backward TR.

**PEC boundary conditions**

We start with the well known boundary conditions at the interface of a dielectric medium \( i \) adjacent to a PEC

\[
\begin{align*}
\vec{E}_{it} &= \vec{0} \\
\vec{H}_{it} &= \vec{j}_s \land \vec{n} \\
\varepsilon \vec{E}_{in} &= \zeta \\
\mu \vec{H}_{in} &= 0
\end{align*}
\]

(2.20) \hspace{1cm} (2.21) \hspace{1cm} (2.22) \hspace{1cm} (2.23)

where \( \zeta \) is the surface charge density, \( n \) labels the field component normal to the interface, \( \vec{n} \) denotes the normal unit vector directed into region \( i \), index \( t \) labels the tangential field component and \( \vec{j}_s \) is the surface current density, which can be retrieved by inverting
equation (2.21)

\[ \vec{j}_s = \vec{n} \wedge \vec{H}_{it} \]  
(2.24)

It is found that not all of the above boundary conditions are linear independent [Leu05]: It is sufficient to require, that the tangential components $\vec{E}_t$ and $\vec{H}_t$ are continuous at the boundary surface.

Setting a cartesian coordinate system which $x$-axis stands normal on the surface $\partial S$ and using the first Maxwell equation for the normal $x$-component yields:

\[ \partial_y E_z - \partial_z E_y = -\partial_t B_x \]  
(2.25)

Since we demand that the tangential field components $E_y$ and $E_z$ are continuous at the boundary, the same applies for their derivatives. This means that $\partial_t B_x = \partial_t B_n$ must also be continuous. $B_n$ itself is continuous when the continuity is guaranteed at any point in time. This is always the case when the field is switched on as before all field strengths must be zero. Thus, we conclude, that the fourth boundary conditions is a direct consequence of the first boundary condition which states that the tangential electrical field components $E_y$ and $E_z$ are continuous.

Similarly by considering only the normal ($x$-component) in the second Maxwell equation we find for the third boundary condition:

\[ \partial_y H_z - \partial_z H_y = J_n + \epsilon \partial_t E_n \]  
(2.26)

Once again by demanding that the tangential field components $H_y$ and $H_z$ are continuous and $J_n$ is zero, which is true for a perfect electrical conductor the continuity of the normal component of $\vec{D} = \epsilon \vec{E}$ follows in agreement with above considerations. Thus, we find, that it is sufficient to require only the two first boundary conditions. In a typical problem the surface charge $\zeta$ and the surface currents $\vec{j}_s$ are unknown. Therefore, the second and the third boundary conditions must be used to determine $\zeta$ and $\vec{j}_s$ and thus, only the very first boundary condition is required at a surface of a perfect electrical conductor.

**Surface currents induced by the magnetic field**

The unknown quantity is the surface current $\vec{j}_s$ induced by the magnetic field. $\vec{j}_s$ is determined by the boundary condition $\vec{j}_s = \vec{n} \wedge \vec{H}_{(tot)}$ ($\vec{H}_{(tot)} = \vec{H}_{(inc)} + \vec{H}_{(sc)}$, incident and scattered field) for the tangential magnetic field at the PEC surface.
2.3. Transition radiation emitted from an oblique target screen

As the electromagnetic field of the relativistic electron has negligible longitudinal components and hence only transversal (i.e. radial) field components are present the first boundary condition to be satisfied concerns the continuity of the tangential electrical field at the target interface. For the PEC interface this field yields zero which renders the outgoing field to be a reflected one with opposite signs of the tangential electric field components:

\[ \vec{n} \wedge \vec{E}_{\text{tot}} = 0 = \vec{n} \wedge (\vec{E}_{\text{inc}} + \vec{E}_{\text{sc}}) \Rightarrow \vec{E}_{\text{inc}} = -\vec{E}_{\text{sc}}. \] (2.27)

This is tantamount to the picture of the virtual quanta where the electromagnetic field of the relativistic electron is set equivalent to an electromagnetic pulse that, afterwards, may undergo specular reflection at the target interface.

Our description of the emission process of TR is based on the Physical-Optics (PO) approach, which yields a direct expression of the induced surface current \( \vec{j}_s \) [Atl06]. Let us consider the problem where an incoming magnetic field (traveling along \( \vec{m} \)) \( \vec{H}_{\text{inc}} = \vec{h} \cdot \text{e}^{-ik(\vec{m} \cdot \vec{r})} \) illuminates a PEC plane at \( z = 0 \) and produces a scattered field \( \vec{H}_{\text{sc}} \) (similar to the model of the virtual quanta) thus, satisfying the first boundary condition for the PEC target. Here we neglect the contributions from edges and all mutual interactions (multiple reflections), thus, what remains is the main reflection at the PEC surface (incoming field \( \rightarrow \) scattered field).

The polarization is orthogonal to the direction of the magnetic field, which means \( \vec{h} \cdot \vec{n} = 0 \). The direction of the field is reflected in the normal direction, but remains the same in the tangential direction [Atl06]. Since the polarization \( \vec{h} \) is orthogonal to the direction, we also get reflection in the polarization and the scattered field becomes \( \vec{H}_{\text{sc}} = (h_x, h_y, -h_z)e^{-ik(m_x m_y, m_y m_z, -m_z) \cdot \vec{r}} \). It holds that with \( \vec{n} = \vec{e}_z \),

\[ \vec{n} \cdot \vec{H}_{\text{tot}} = 0, \vec{n} \wedge \vec{H}_{\text{sc}} = \vec{n} \wedge \vec{H}_{\text{inc}}. \] (2.28)

Thus, the PO-approximation is written as follows:

\[ \vec{j}_s = \vec{n} \wedge \vec{H}_{\text{tot}} = 2 \cdot \vec{n} \wedge \vec{H}_{\text{inc}}. \] (2.29)

Since everything is linear, this generalizes trivially to planes other than \( z = 0 \). Thus, we can conclude, that the model is also valid for tilted target screens where normal projections of the transversal electrical field on the target interface emerge. The normal components only mediate the sign reversal in the tangential electrical field projections with respect to the oblique incident virtual quanta and its corresponding reflection.

Dealing with a PEC interface has thus revealed a sort of redundancy: Both boundary conditions for the electrical field are surplus relations if the emerging TR is modeled using solely the magnetic field (and its corresponding boundary condition) where the resulting
surface current density is the proper source of TR. But then enforcing the boundary conditions for the electrical field components only has led us to the independent model of e.g. virtual quanta. In the case of a general target material (i.e. non PEC material) the redundancy is removed and always two types of boundary conditions have to be taken into account.

**Edge effects and multiple reflections**

Within the framework of our source model we have assumed an anechoic behaviour for the induced currents at the screen boundaries. Here we must point out that a more detailed investigation of possible fringe effects is beyond the scope of this thesis as it would involve extensive numerical models such as e.g. the methods of moments and would therefore not lead anymore to an analytical solution of the problem as intended. Furthermore, the analytical model presented in this chapter is able to explain the broadening of the emission pattern with respect to the Ginzburg and Frank formalism and the asymmetry in emission (see figure 1.13).

Here we must note that the discrepancies due to possible fringe effects at the screen boundary cannot be larger than the difference of the individual but converging models for the emission of transition radiation. The virtual quanta model considers the incoming field to be reflected at the metallic target screen which thus acts as scattering target. The field outside of the target is diffracted in forward direction around the screen. Possible fringe effects for our source description must be of same order of magnitude as these diffraction effects in the perception of the virtual quanta description. It can be concluded, that the diffraction at the screen boundary must have a similar influence than the fringe effect in our formalism where the target is treated as source of the radiation. However, possible fringe effects could be considered by applying a modified surface current density calculated by perturbation theory, but noteworthy without changing our formalism. Furthermore, it must be noted, that including the fringe effects necessitates a numerical approach which therefore renders the formalism much more complex.

An additional question may arise with respect to temporal perturbations that could be caused by a potential surface (current) wave resonance on the finite target screen. There are several arguments against this conjecture. First, the PEC like target barely supports surface waves. Second, referring to the scattering model (i.e. the model of virtual quanta) a finite-sized target would only be excited through the target edges; hence, the strength of the resulting temporal perturbation is below the order of magnitude of the aforementioned edge effects. In turn a potential distortion of the outgoing wave must be radiated by the
2.3. Transition radiation emitted from an oblique target screen

sides of the screen. Thus, a possible perturbation can only be a second order effect of the already small coupling efficiency. The third rationale is displayed within the proposed model. Here, the surface current density is induced virtually instantaneously in the PEC, while the generation of the current density by the magnetic fields happens on a very short timescale due to the fact that the electromagnetic fields of the relativistic electron are confined into a flat disk perpendicular to the direction of motion. Therefore, we can presume that the stimulation is too short compared to its strength to efficiently excite potential resonant surface waves within the finite target screen.

Multiple reflections at the metallic target screen can be neglected. This is mainly due to the fact, that we are dealing with a plane metallic scatterer which hardly causes multiple reflections (meaning that a scattered field might be reflected at another point of the surface again). Here we only have to consider the main reflection at the target screen \(\vec{H}_{inc} \rightarrow \vec{H}_{sc}\). Hence, this justifies the PO approach used in our model [Bor01]. However, it must be noted, that mutual interactions must be taken into account when considering curved targets, especially surfaces of concave form.

Vector potential representation

The resulting surface currents are then acting as electrical current sources, which generate a vector potential\(^1\) calculated by the following expression:

\[
d\vec{A} = \frac{\mu_0}{4\pi} \cdot \vec{j}_s \cdot \frac{e^{i\frac{2\pi}{c}R}}{R} dS \quad \text{with} \quad dS = r'dr'd\phi'
\]  

(2.30)

\(R\) is the distance from a point \(\vec{r}_s = (x', y', z')\) on the target screen to a point of observation \(\vec{r}_{obs} = \rho \cdot \vec{e}_\rho\) (see figure 2.5). Here we have used the fact, that the vector potential \(\vec{A}\) does satisfy the inhomogeneous Helmholtz equation, which describes the propagation of the vector potential \(\vec{A}\). The solution to the inhomogeneous Helmholtz equation is given by the superposition of the spherical waves \(e^{i(\frac{2\pi}{c}R)/R}\) [Jac75].

Coordinate transformation: \(\pi/4\) rotation of the target screen

In order to determine the TR from a tilted target the screen is parametrized according to Fig. 2.5:

\[
x' = r' \cos(\phi')
\]  

(2.31)

\(^1\)The vector potential \(\vec{A}\) is defined by \(\vec{B} = \vec{\nabla} \wedge \vec{A}\).
Chapter 2. Emission characteristics of long-wavelength TR

\[ y' = r' \sin(\phi') \]  \hspace{1cm} (2.32)
\[ z' = 0 \]  \hspace{1cm} (2.33)

The original coordinates are obtained from the screen coordinates by a \( \pi/4 \) rotation around the \( y' \)-axis:

\[ x = \frac{1}{\sqrt{2}} x' \]  \hspace{1cm} (2.34)
\[ y = y' \]  \hspace{1cm} (2.35)
\[ z = \frac{1}{\sqrt{2}} x' \]  \hspace{1cm} (2.36)

**Figure 2.5:** Schematic for the target geometry. The point of observation is parametrized by spherical coordinates \((\rho, \theta, \varphi)\). It is convenient to use polar coordinates \((r', \phi')\) for describing the position on the tilted screen. Right side: top-view

The transversal plane normal to the electron trajectory is described with the following polar coordinates \((r, \phi)\); \(x = r \cos(\phi), y = r \sin(\phi)\):

\[
r(r', \phi') = \sqrt{x'^2 + y'^2} = \sqrt{\frac{1}{2}(r'^2 \cos(\phi'))^2 + (r'^2 \sin(\phi'))^2} \]  \hspace{1cm} (2.37)

\[
\phi(r', \phi') = \arg(x + i \cdot y) = \arg\left(\frac{1}{\sqrt{2}} r' \cos(\phi') + i \cdot r' \sin(\phi')\right). \]  \hspace{1cm} (2.38)

Due to the strong longitudinal confinement of the relativistic electron’s field, the magnetic field will hit the \( \pi/4 \)-tilted target at continuous subsequent positions \(z_1, z_2, ...\) where surface current densities \( \vec{j}_s \) are then induced at the associated time delays \(t_1, t_2, ...\). The moving impact position as well as the associated impact time are characterized by a phase relation which is interrelated to the target screen’s coordinate transformation. Therefore, we set the phase on the screen at position \(z\) to \( \Psi(z) = e^{i \frac{2\pi \varphi z}{\lambda c}} \).
2.3. Transition radiation emitted from an oblique target screen

The Fourier-components of the magnetic field on the screen are given by equation (2.16) multiplied with the phase term $\Psi(z)$ leading to:

$$\hat{B}_\phi = \frac{q}{(2\pi)^{1/2}\epsilon_0\beta^2 c} \frac{2\pi\nu}{c\gamma} e^{\frac{i2\pi z}{\beta c\gamma}} K_1 \left( \frac{2\pi\nu}{\beta c\gamma} \right) r = \frac{\sqrt{2}q}{(2\pi)^{1/2}\epsilon_0\beta^2 c} \frac{2\pi\nu}{c\gamma} e^{\frac{i2\pi z}{\beta c\gamma}} r' \cos(\phi) K_1 \left( \frac{2\pi\nu}{\beta c\gamma} r' \left( r', \phi' \right) \right).$$

(2.39)

The x- and y-components are then:

$$\hat{B}_x = -\sin(\phi(r', \phi')) \frac{q}{(2\pi)^{1/2}\epsilon_0\beta^2 c} \frac{2\pi\nu}{c\gamma} e^{\frac{i2\pi z}{\beta c\gamma}} r' \cos(\phi) K_1 \left( \frac{2\pi\nu}{\beta c\gamma} r' \left( r', \phi' \right) \right).$$

(2.40)

$$\hat{B}_y = \cos(\phi(r', \phi')) \frac{q}{(2\pi)^{1/2}\epsilon_0\beta^2 c} \frac{2\pi\nu}{c\gamma} e^{\frac{i2\pi z}{\beta c\gamma}} r' \cos(\phi) K_1 \left( \frac{2\pi\nu}{\beta c\gamma} r' \left( r', \phi' \right) \right).$$

(2.41)

With $\hat{j} = 2 \cdot \hat{n} \wedge \hat{H}$ following the PO approach the components of the surface current density induced by the magnetic field are calculated according to:

$$\mu_0\hat{j}_x = -\frac{2}{\sqrt{2}} \hat{B}_y$$

(2.42)

$$\mu_0\hat{j}_y = \frac{2}{\sqrt{2}} \hat{B}_x$$

(2.43)

$$\mu_0\hat{j}_z = -\frac{2}{\sqrt{2}} \hat{B}_y.$$

(2.44)

It is convenient to consider polar coordinates $(e_r, e_\phi, e_z)$. Thus, it follows that $\hat{H}_{(inc)} = (0, \hat{H}_\phi, 0)$. Since the normal vector $\hat{n}$ on the radiator surface only has radial ($r$) and longitudinal ($z$) components we can write: $\hat{n} = (n_r, 0, n_z)$. Thus, it must be that $\hat{j}_s = 2 \cdot \hat{n} \wedge \hat{H}_{(inc)} = 2 \cdot \hat{H}_\phi(n_z, 0, -n_r)$. Therefore, the surface current density only has radial and longitudinal components and points to the origin of the screen. The longitudinal
component is due to the $\pi/4$ rotation of the screen.

As a next step we express the radiated field at an observation point $\mathbf{r}_{\text{obs}} = (\rho, \theta, \varphi)$ by the vector potential. For large distances $\rho$ from the screen compared to the screen radius ($\rho \gg r$) the vector potential in spherical coordinates is thus expressed using equation (2.30) in conjunction with the transformation from cartesian to spherical coordinates:

\[
\begin{align*}
\mathbf{e}_\rho &= \sin(\theta) \cos(\varphi) \mathbf{e}_x + \sin(\theta) \sin(\varphi) \mathbf{e}_y + \cos(\theta) \mathbf{e}_z \\
\mathbf{e}_\theta &= \cos(\theta) \cos(\varphi) \mathbf{e}_x + \cos(\theta) \sin(\varphi) \mathbf{e}_y - \sin(\theta) \mathbf{e}_z \\
\mathbf{e}_\varphi &= -\sin(\varphi) \mathbf{e}_x + \cos(\varphi) \mathbf{e}_y
\end{align*}
\] (2.45)

\[
\begin{align*}
\mathbf{e}_\rho &= \sin(\theta) \cos(\varphi) \mathbf{e}_x + \sin(\theta) \sin(\varphi) \mathbf{e}_y + \cos(\theta) \mathbf{e}_z \\
\mathbf{e}_\theta &= \cos(\theta) \cos(\varphi) \mathbf{e}_x + \cos(\theta) \sin(\varphi) \mathbf{e}_y - \sin(\theta) \mathbf{e}_z \\
\mathbf{e}_\varphi &= -\sin(\varphi) \mathbf{e}_x + \cos(\varphi) \mathbf{e}_y
\end{align*}
\] (2.46)

\[
\begin{align*}
\mathbf{e}_\rho &= \sin(\theta) \cos(\varphi) \mathbf{e}_x + \sin(\theta) \sin(\varphi) \mathbf{e}_y + \cos(\theta) \mathbf{e}_z \\
\mathbf{e}_\theta &= \cos(\theta) \cos(\varphi) \mathbf{e}_x + \cos(\theta) \sin(\varphi) \mathbf{e}_y - \sin(\theta) \mathbf{e}_z \\
\mathbf{e}_\varphi &= -\sin(\varphi) \mathbf{e}_x + \cos(\varphi) \mathbf{e}_y
\end{align*}
\] (2.47)

\[
d \hat{A} = \frac{\mu_0}{4\pi} \frac{e^{ik\rho}}{\rho} \left( \begin{array}{c}
\cos(\varphi) \sin(\theta) \hat{j}_x + \sin(\theta) \sin(\varphi) \hat{j}_y + \cos(\theta) \hat{j}_z, \\
\cos(\theta) \cos(\varphi) \hat{j}_x + \cos(\theta) \sin(\varphi) \hat{j}_y - \sin(\theta) \hat{j}_z, \\
-\sin(\varphi) \hat{j}_x + \cos(\varphi) \hat{j}_y
\end{array} \right) dS. \\
\end{equation}
\] (2.48)

The magnetic induction $\hat{B} = (\hat{B}_\rho, \hat{B}_\theta, \hat{B}_\varphi)$ is found according to $\hat{B} = \text{rot}(\hat{A})$.

\[
d \hat{B} = \frac{\mu_0}{4\pi} \left( 0, \frac{e^{ik\rho}}{\sin(\theta) \rho} (-ik \sin(\theta)(\cos(\varphi) \hat{j}_y - \sin(\varphi) \hat{j}_x)) + \frac{\sin(\theta)(\cos(\varphi) \hat{j}_y - \sin(\varphi) \hat{j}_x)}{\rho}, \\
\frac{e^{ik\rho}}{\rho} (ik(\cos(\theta) \cos(\varphi) \hat{j}_x - \sin(\theta) \hat{j}_z + \cos(\theta) \sin(\varphi) \hat{j}_y) - \frac{\cos(\theta) \cos(\varphi) \hat{j}_x - \sin(\theta) \hat{j}_z + \cos(\theta) \sin(\varphi) \hat{j}_y}{\rho}) \right) dS. \\
\end{equation}
\] (2.49)

Taking only the first order terms $\frac{1}{\rho}$ into account, the above expression simplifies to:

\[
d \hat{B} \approx \frac{\mu_0}{4\pi} \left( 0, \frac{e^{ik\rho}}{\sin(\theta) \rho} (-ik \sin(\theta)(\cos(\varphi) \hat{j}_y - \sin(\varphi) \hat{j}_x)) + \frac{e^{ik\rho}}{\rho} (ik(\cos(\theta) \cos(\varphi) \hat{j}_x - \sin(\theta) \hat{j}_z + \cos(\theta) \sin(\varphi) \hat{j}_y)) \right) dS. \\
\end{equation}
\] (2.50)

The influence of this approximation is estimated in the next section.

The electric field $\hat{E} = (\hat{E}_\rho, \hat{E}_\theta, \hat{E}_\varphi)$ is computed by:

\[
\hat{E} = \frac{1}{i \kappa \epsilon_0} \text{rot}(\hat{H}) = \frac{1}{i \kappa \epsilon_0 \mu_0} \text{rot}(\hat{B}). \\
\end{equation}
\] (2.51)
2.3. Transition radiation emitted from an oblique target screen

Referring to the aforementioned first order approximation, the field $\hat{\mathbf{E}}$ is thus written as:

$$d\hat{E}_\rho = 0$$  \hspace{1cm} (2.52)

$$d\hat{E}_\theta \approx \frac{k}{4\pi i\epsilon_0} \frac{1}{\rho} e^{ik\rho}(\cos(\theta) \cos(\varphi) \hat{j}_x - \sin(\theta) \hat{j}_z + \cos(\theta) \sin(\varphi) \hat{j}_y) dS$$  \hspace{1cm} (2.53)

$$d\hat{E}_\varphi \approx \frac{k}{4\pi i\epsilon_0 \mu_0} \frac{1}{\rho} e^{ik\rho}(\cos(\varphi) \hat{j}_y - \sin(\varphi) \hat{j}_x) dS$$  \hspace{1cm} (2.54)

Equations (2.52) to (2.54) and (2.50) give an analytic electromagnetic field description of TR as emitted from an oblique, thin metallic target screen. The field components are calculated by integrating the above expressions over the thin circular target screen. The spectral energy flux of the emitted radiation is calculated from the Poynting vector. Expressing the magnetic field $\mathbf{H}$ through the electrical field $\mathbf{E}$ [Dob03] yields that the spectral energy flux at the point of observation must be proportional to the module square of the transverse electrical field $\hat{E}$. Using Parseval’s theorem [Jac75] it is found that

$$\frac{d^2I}{dv d\Omega} = 2\epsilon_0 c |\hat{E}|^2 = 2\epsilon_0 c \int_0^{\pi} \int_0^{2\pi} (d\hat{E}_\theta + d\hat{E}_\varphi) r' dr' d\phi'^2.$$  \hspace{1cm} (2.55)

2.3.1 On simplification and error estimation

In order to define a range of validity of the developed formalism, a detailed error estimation and discussion including the applied assumptions, will be performed.

Phase error

In equation (2.48) we have used the simplification $R \approx \rho$ by inferring that $\rho \gg r$. Considering the function $f(x_1, ..., x_n) = h(x_1, ..., x_n) \cdot e^{g(x_1, ..., x_n)}$, where $h$ and $g$ are non-exponential functions, it follows that $\partial_{x_i} f = e^g(\partial_{x_i} h + h \partial_{x_i} g)$. Hence it must be that $\text{rot}(\hat{\mathbf{A}}) \propto e^{ikR(\rho, \theta, \varphi)}$ with $e^{ikR(\rho, \theta, \varphi)}$ being the sole exponential term. Therefore the inferred constraint is dramatically relaxed by allowing to substitute $e^{ik\rho}$ with $e^{ikR}$ in the final expression for the Fourier components of the electrical field, where the exact distance $R$ is given by the following expression:

$$R = \sqrt{(\rho \sin(\theta) \cos(\varphi) - x)^2 + (\rho \sin(\theta) \sin(\varphi) - y)^2 + (\rho \cos(\theta) - z)^2}.$$  \hspace{1cm} (2.56)

Here $\mathbf{r}_{\text{obs}} = (\rho, \theta, \varphi)$ represents the position of the observer and $\mathbf{r}_s = (x, y, z)$ yields a point on the circular target screen.
Comparison of first order and second order terms

When using back substitution of the exact phase term the remaining error is estimated by comparing second order against the first order terms. Expressing $R'$ by $R' \approx \rho - (\vec{r}_{\text{obs}} \cdot \vec{r}_{s})/\rho$, is valid for $\rho \gg r$, independent of the value $k\rho$, providing an adequate approximation even in the radiating near-field zone [Jac75]. By using $e^{ikR'}/\rho$ instead of $e^{ik\rho}/\rho$ one computes the first order and the second order terms in the Fourier-components of the electrical field:

$\vec{E}_1(1) \propto k/\rho$ whereas $\vec{E}_2(2) \propto 1/\rho^2$ and $\vec{E}_2(2) \propto kr/\rho^2$, with $(i), i = 1, 2$ denoting the respective order. Therefore the second order terms are negligible when $\rho \gg r$ and $k\rho \gg 1$ is valid. In our experimental set-up the typical distance from the radiating target screen (i.e. to the observation plane respectively the first optical element) is 250 mm. For such a distance and frequencies higher than 30 GHz $k\rho \gg 1$ is easily fulfilled. The corresponding electrical field errors for a target screen of radius 28 mm are then estimated to be in the order of 10 percent. The maximum error is expected to occur normal to the target screen, since at this angle the largest projection of the tilted screen is found. Due to the radially polarized character of TR the intensity must vanish in this symmetrical center. Hence the absolute value of the differences is negligible.

As already mentioned a conservative definition for the Rayleigh distance with $d_R = \frac{d_{R}^2}{\lambda}$ is found in classical antenna theory. Furthermore the IEEE definition describes the near-to-far-field boundary by the criteria that the phase of the emitted radiation differs by $\lambda/8$ to the phase of a spherical wave. Both definitions yield Rayleigh distances larger than one meter for a frequency of 120 GHz, proving that the formalism presented here provides a very good approximation for the radiating near-field.

2.4 Discussion

The introduced analytical model yields a frequency dependent description for the emission process of the long-wavelength TR. In the following the intensity distribution is calculated using our analytical model for a single electron of 100 MeV, which corresponds to the beam energy of the SLS pre-injector LINAC. In Fig. 2.7 two-dimensional intensity distributions according to equation (2.55) are plotted for three wavelengths: $\lambda = 10.0$ mm (30 GHz), $\lambda = 5.0$ mm (60 GHz) and $\lambda = 2.5$ mm (120 GHz). The plots are computed on a plane at a distance $x = 250$ mm. The figures clearly reproduce the expected angular broadening of the emission profile for increasing wavelengths. Furthermore an asymmetry in the emission pattern is observed with respect to the horizontal dimension. Similar to the increased radiation divergence the emission asymmetry is pronounced for longer wave-
lengths. Fig. 2.8 shows one-dimensional angular representations of the aforementioned intensity profiles, i.e. their corresponding projections on a sphere of 250 mm radius. The horizontal profile is parametrized with $\theta$ whereas the vertical one is expressed in $\phi$ (see Fig. 2.5). The asymmetry is exclusively apparent in the calculated horizontal intensity profile. For both profiles the maximum in emission occurs at angles much larger than $\frac{1}{7}$ as predicted by the formalism of Ginzburg and Frank for the infinite target size.

![Figure 2.7: Computed CTR intensity plot (single electron of 100 MeV) for (a) $\lambda = 10.0$ mm, (b) $\lambda = 5.0$ mm and (c) $\lambda = 2.5$ mm on a plane of $x = 250$ mm distance to the circular target screen of radius $r_s = 28$ mm.](image)

![Figure 2.8: The calculated CTR radiation pattern (single electron of 100 MeV) for a target screen of radius $r_s = 28$ mm at a spherical distance of $R = 250$ mm. Left side: at $\phi = 0$ (horizontal polarization); Right side: at $\theta = \frac{\pi}{2}$ (vertical polarization). The normalized distribution for three wavelengths is computed: blue line $\lambda = 10.0$ mm, red line $\lambda = 5.0$ mm and black line $\lambda = 2.5$ mm. Left side: the model predicts an emission at 90 degrees to the electron beam path with an asymmetry in emission.](image)
These characteristics of the transverse CTR emission patterns as resulting from the introduced model have been experimentally confirmed at the SLS pre-injector LINAC [Süt03], [Süt05] for the first time in the coherent long wavelength range. The measurements were conducted using a Golay cell far infrared detector mounted on two motorized linear stages thus allowing two-dimensional scans at a distance $x = 250$ mm to the target screen. Since the monitor presented in this thesis uses polarized CTR (c.f. section 4.2) the measurements were done for both horizontal and vertical polarization using a wire-grid polarizer in front of the vacuum window of the diagnostic port. In Fig. 2.9 the intensity profiles are computed for a discrete set of frequencies and weighted according to the electron bunch power density spectrum as measured with a Martin-Puplett interferometer (c.f. Fig. 3.4). The curves (see figure 2.8) are finally convoluted with the 6 mm aperture of the detector window. The measurements in Fig. 2.9 show reasonable agreement with the theoretical expectations. The asymmetry due to the rotation of the target screen was confirmed for the horizontal profile of the two-lobed emission pattern. It was found that the maximum in emission occurred at an angle of 50 mrad which is a factor 10 larger than the one predicted by Ginzburg and Frank, showing the influence of the finite target size as predicted by the formalism. The underlying power density spectrum used in the simulation has been measured on another occasion with different machine setting of the LINAC. Furthermore only an estimate of the transmission function of the Martin-Puplett interferometer (MPI) which was used to measure the spectrum was included in the simulations. Hence the discrepancies between measurements and simulations can be attributed mainly to deviations in the assumed power density spectrum.

![Figure 2.9](image)

**Figure 2.9:** Measurement of CTR intensity at the SLS LINAC. Left side: horizontal polarization. Right side: vertical polarization. The scans are compared against computed intensity profiles according to the formalism presented in this chapter (solid line). The underlying CTR power density spectrum has been measured with an MPI and is shown in figure (3.4).
The emitted radiation is also characterized by its phase front. Fig. 2.10 gives a comparison between the phase front of a spherical wave and the CTR. It is found that the phase difference of CTR with respect to a perfect spherical wave originating from $\vec{r}_s = (0,0,0)$ is oscillating with a swing of about $\frac{\lambda}{4}$ for the shortest wavelength indicated (2.5 mm). This compares well with the other definition of the Rayleigh distance (such as the aforementioned IEEE definition in antenna theory) at which the mentioned phase difference should amount $\frac{\lambda}{8}$ indicating that the observation point lies within the near-field. The oscillating behavior is attributed to diffraction at the screen boundary. Therefore the emitted long wavelength CTR at this distance can approximated as a spherical wave. The change in sign at $\theta = \frac{\pi}{2}$ respectively at $\varphi = 0$ reproduces the radial polarized character of the emitted TR.

![Figure 2.10](image.png)

**Figure 2.10:** Computed difference in phase between electric field (left side: $\theta$-component (horizontal polarization) at $\varphi = 0$; right side $\varphi$-component (vertical polarization) at $\theta = \frac{\pi}{2}$) of CTR (single electron of 100 MeV) and a spherical wave at a distance of $R = 250$ mm for a target screen of radius $r_s = 28$ mm. The phase is shown in fractions of the wavelength, for three different wavelengths: blue line $\lambda = 10.0$ mm, red line $\lambda = 5.0$ mm and black line $\lambda = 2.5$ mm. Note the change in sign at $\theta = \frac{\pi}{2}$ respectively in $\varphi = 0$, indicating the radially polarized character of CTR.

### 2.4.1 Long wavelength limit

It has been shown in the previous chapter that the emitted spectral energy flux becomes frequency dependent [Schb], [Gei99a] over the complete range of the coherent spectrum. This results from the fact that the electromagnetic source field propagating further away from the electron trajectory miss the target screen of radius $r_s$ and do not contribute to the radiation process (c.f. figure 2.4). Therefore the radiated energy decreases toward smaller frequencies, because of an increasing part [Gei99a] of the electron electromagnetic
source fields of the electron does not interact with the screen.

The fraction of intensity that hits the screen can be calculated with the following expression:

\[
I_{\text{fraction}}(\nu) = \frac{\left| \int_0^{r_s} \frac{2\pi \nu}{c\gamma} r K_1\left(\frac{2\pi \nu}{c\gamma} r\right) dr \right|^2}{\left| \int_0^{r_s} \frac{2\pi \nu}{c\gamma} r K_1\left(\frac{2\pi \nu}{c\gamma} r\right) dr \right|^2}
\] (2.57)

For large frequencies (respectively short wavelengths) the function \(I_{\text{fraction}}\) will become constant with \(I_{\text{fraction}}(\nu) \to 1\) for sufficient large frequencies \(\nu\). In this spectral range the emitted radiation flux is frequency independent.

Figure 2.11 shows the fractional intensity \(I_{\text{fraction}}\) for three different electron energies. The dots present a polynomial fit (of the form \(P(\nu) = a_1 \nu + a_2 \nu^2 + a_3 \nu^3\)) to the curve with \(\gamma = 200\), which corresponds to the beam energy of the SLS LINAC. The radiated spectral energy flux must be scaled with the function \(P(\nu)\). Hence the coherent spectrum thus not only depend on the form factor \(f(\nu)\), but also on the function \(P(\nu)\). The effect becomes more dominant for larger values of \(\gamma\).

**Figure 2.11:** The calculated electromagnetic source field intensity that hits the target screen \((r_s = 28 \text{ mm})\) computed for three different values of \(\gamma\). For \(\gamma = 200\) (SLS LINAC beam energy) a polynomial fit is given \(P(\nu) = a_1 \nu + a_2 \nu^2 + a_3 \nu^3\) up to frequencies where coherent emission can be expected.

### 2.4.2 Short wavelength limit

In the short wavelength limit (towards the FIR range) the presented CTR emission model correctly merges into the well known radiation pattern as predicted by the Ginzburg and Frank formalism (see formula 1.1). The plot in figure 2.12 shows the one-dimensional horizontal intensity profile simulated on a sphere of 1 m radius. The asymmetry due to
The \( \pi/4 \) rotation of the target screen in respect to the electrons trajectory is not apparent at these wavelengths and the maximum in emission occurs near \( \gamma \). At these short wavelengths the source fields impinging on the screen are much smaller than the target size corresponding to the assumption of the infinite target screen.

In the case of normal incidence the prediction of Ginzburg and Frank can be reproduced analytically too. Applying the formalism on the special case of normal incidence the spectral energy in the far-field (\( r_s \ll \rho \Rightarrow R = \rho \)) is found to be:

\[
\frac{d^2I}{d\nu d\Omega} \propto \left( \frac{2\pi\nu}{\gamma} \right)^2 \left| \int_0^{r_s} J_1(\sin(\theta)\frac{2\pi\nu}{c \gamma r})K_1(\frac{2\pi\nu}{\beta \gamma \rho}r)rdr \right|^2
\]

where \( J_1(r) \) is the first order Bessel function. The integral can be solved analytically [Sch05]:

\[
\frac{d^2I}{d\nu d\Omega} \propto \frac{\beta^2 \sin(\theta)^2}{(1 - \beta^2 \cos(\theta)^2)^2} [1 - T(\nu, \theta)]
\]

using

\[
T(\nu, \theta) = \frac{2\pi\nu/c r_s}{\beta \gamma} J_0(2\pi\nu/c r_s \sin(\theta)) K_1(\frac{k r_s}{\beta \gamma}) + \frac{k r_s}{\beta^2 \gamma^2 \sin(\theta)} J_1(k r_s \sin(\theta)) K_0(\frac{2\pi\nu/c r_s}{\beta \gamma}).
\]

For \( r_s \to \infty \) (infinite target screen) the term \( T(\nu, \theta) \) vanishes and the formula of Ginzburg and Frank is revealed.

![Figure 2.12: Calculated intensity distribution (single electron of 100 MeV) for an oblique target screen of radius \( r_s = 28 \) mm at a spherical distance of \( R = 1000 \) mm and at \( \varphi = 0 \). The normalized distribution is computed for the wavelengths: blue line \( \lambda = 30.0 \) microns, red line \( \lambda = 100.0 \) microns. The black line presents the prediction of Ginzburg and Frank.](image-url)
2.5 Summary

At the SLS pre-injector LINAC bunches as short as 2 ps can be expected yielding a CTR spectrum up to frequencies of about 500 GHz (c.f. figure 1.6). In this frequency range the transversal extent of the electromagnetic source fields (\(\gamma \lambda\)) is considerably larger than the target dimension (2.8 cm). Hence the model of Ginzburg and Frank is clearly not directly applicable and the formalism presented in this chapter has to be considered.

The predictions of our model, such as the asymmetrical emission pattern and the angular broadening of the emission, were experimentally confirmed at the SLS LINAC. Therefore, the computed emission patterns are used as input for simulations of the quasi-optical system of the EOA experiment in order to determine its transfer function. It must be further noted that the model predicts an emission which is frequency dependent not only in its transverse pattern, but also in the spectral energy flux. This results from the fact that at large wavelengths the electromagnetic source fields are considerably larger than the target itself and therefore do not contribute completely to the radiation process. Hence the emitted CTR spectrum is not only depending from the form factor \(f(\nu)\) but also from the ratio of source field dimensions and target size (c.f. equation 2.57).

For small wavelengths in the FIR range the model is in agreement with the predictions of Ginzburg and Frank. For the emission from an oblique target screen the characteristic asymmetry in emission vanishes at high frequencies and the two maxima in the emitted energy flux are separated by the well-known angular distance of \(2/\gamma\) as given by the Ginzburg and Frank formalism. The model introduced in this chapter has considered the generation of long wavelength TR from a single electron. For the bunch of electrons we will apply the superposition principle.
Chapter 3

Novel spatial interferometer with single-shot capability

The bunch length measurement methods which have been mentioned in the first chapter offer several advantages but also have some major drawbacks (c.f. section 1.2.2):
Interferometric analysis of coherent transition radiation (CTR) as performed with Michelson or Martin-Puplett interferometers (MPI) offer excellent frequency resolution, but measurements are averaged over many electron pulses and the reconstruction of the bunch shape is difficult. In contrast the EOS method is a time domain technique with sub-picoseconds time resolution but does not offer single-shot capability either.
Single-shot measurements have been successfully demonstrated using the spectral- respectively the temporal decoding methods. Both techniques offer sub-picoseconds time resolution but similar to the EOS method use expensive short pulse TiSa laser systems which necessitate a complex synchronization scheme.
The spatial interferometer described in this chapter is combining the advantages of interferometric measurements with single-shot capability [Sig00]. This is achieved by splitting the two vertically polarized CTR lobes and refocus them under an oblique angle onto the focal plane of the interferometer. This optical arrangement produces a phase difference between the two rays which intersects in the focal plane at off axis. The thereby obtained spatial interference pattern is closely related to the auto-correlation of the CTR pulse, and thus can be used to extract the pulse spectral distribution and the underlying pulse width. The single-shot readout is achieved by electro-optical techniques using an active mode-locked Nd:YAG probe laser with a pulse width of some hundred picoseconds. Hence the synchronization scheme is much more relaxed and the experiment offers a unique robustness. The major drawback of the technique is the fact, that the experiment is not a direct measurement in the time space. Similar to the conventional step-scan interferomet-
ric methods the phase information is lost. This makes reconstruction of the longitudinal profile demanding.

The first section of this chapter is devoted to measurements of the averaged CTR power density spectrum at the SLS pre-injector LINAC using a conventional Martin-Puplett interferometer (MPI). The measurements were performed to determine the CTR power density spectrum at the optical diagnostic port ALIDI-SM-5 downstream the SLS LINAC (see figure 1.9).

Based on the thorough investigations of the emission process of long wavelength CTR the fundamental principles for the quasi-optical set-up of the EOA experiment are formulated in section 3.2. The quasi-optical\(^1\) set-up comprises the out-coupling and transfer optics of the CTR pulse to the experiment and the spatial interferometer.

The detailed description of the spatial interferometer is given in section 3.3. First a theoretical characterization of the interferometer assuming plane wave propagation in the focal plane is given. This is followed by a characterization of the optical layout of the spatial interferometer and the description of its alignment procedures. The working principle of the interferometer was demonstrated in measurements using monochromatic cw-sources in the millimeter wavelength range\(^2\). The results of the measurements were confirmed by simulations using the commercial software package GRASP\(^3\) [TIC].

In the last part of section 3.3 measurements with the spatial interferometer set-up at the SLS pre-injector LINAC are presented. The measurements show the interference of the two broadband signal paths which was reproducibly observed both in phase scans and in transverse horizontal scans in the focal plane of the interferometer. The chapter concludes with GRASP simulations of the complete quasi-optical set-up for the experiments which yields its overall intensity transmission.

### 3.1 Martin-Puplett interferometer measurements

In order to design an efficient quasi-optical set-up for the experiment, the CTR power density spectrum was determined at the ALIDI-SM-5 optical diagnostic port behind the SLS LINAC by measurements with a Martin-Puplett interferometer. As will be shown by the Martin-Puplett interferometer (MPI) measurements the emitted coherent radiation spectrum is pronounced in a range between 200 GHz and 500 GHz. Below 200 GHz the

\(^1\)THz beams are only a few wavelengths in diameter as compared to conventional optics, and a different approach, termed quasi-optics, is required to control and analyse them.

\(^2\)Measurements were performed at the Institute of Applied Physics (Microwave Department) of the University of Bern.

\(^3\)GRASP uses Physical Optics (PO) and Physical Theory of Diffraction (GTD).
spectrum is strongly suppressed mainly due to diffraction effects at the optics apertures. Hence, a transmission of 50 % at a frequency $\nu$ of 100 GHz for the quasi-optical set-up of the EOA experiment is set as a reasonable target.

A Martin-Puplett interferometer is a Michelson type interferometer used in the millimeter and sub-millimeter range. A schematic view of the interferometer as installed at the SLS LINAC is depicted in figure 3.1. The typical MPI consists of two parabolic mirrors, three wire grid polarizers, two roof mirrors and two detectors. The paraboloid mirrors are used to collimate and focus the CTR onto the detector. Wire grids (wires with 12 $\mu$m diameter thickness with a spacing of 100 $\mu$m) act as polarizer, beam splitter and analyzer. The incident radiation pulse is polarized horizontally by the first wire grid and is then splitted by the beam divider into components of different polarization entering the two interferometer arms. The polarization is flipped by $\pi/2$ by the roof mirrors, hence the polarization component first transmitted at the beam splitter is now reflected and vice versa. The recombined radiation is in general elliptically polarized, depending on the path difference in the two arms. The analyzing grid transmits one polarization component into detector 1 and reflects the orthogonal component into detector 2. The measurement presented in this section was conducted with a single Golay cell detector (detector 2). A detailed description of the working principle of the Martin-Puplett interferometer can be found in [Gei99a], [Gei99b].

Figure 3.3 shows three consecutive scans and their averaged profile. The later is multiplied by a triangle apodisation function as shown in figure 3.3, which is used to bring the interference pattern smoothly down to zero at the edges of the sample region. This suppresses side lobes which would otherwise be produced, but at the expense of widening
the lines and therefore decreasing the resolution [Wol]. Figure 3.4 shows the power density spectrum as obtained through a Fourier transformation of the profile in figure 3.3. The transmission function of the Martin-Puplett interferometer was estimated by simulations using the GRASP [TIC] software package\(^4\). The low frequency attenuation which is mainly due to truncation losses and diffraction at the limited apertures of the optics in the interferometer, is described by the transmission curve as fitted to a discrete set of frequencies is shown in figure 3.2. The transmission had been calculated with the GRASP software package. Only the two parabolic mirrors separated by 1000 mm are included in the simulation. Truncation losses and edge diffraction from the other optical components between them (wire grids, plane mirrors, rooftop mirrors) are neglected. The water absorption in air for the 1.40 meter long path of quasi-optical system is included according to [Bur68]. The spectrum was derived by applying a Fourier transformation on the smoothed interference pattern. The resulting coherent power spectrum was found to be in a range of up to 600 GHz and is pronounced between 200 GHz and 500 GHz. Therefore, the MPI measurements are in reasonable agreement with the expectation of a 2 ps long bunch of Gaussian shape (c.f. figure 1.6) as predicted by PARMELA simulations of the SLS pre-injector LINAC. The low-frequency attenuation at 100 GHz was simulated to be in the order of 15%. Furthermore, it must be noted that the real transmission can be expected to be even smaller as only the two focusing mirrors are included in the simulation.

\[^4\]The simulations were performed at the Institute of Applied Physics (Microwave Department) of the University of Bern.
3.2 Design principles for the quasi-optical set-up of the EOA experiment

As a result of the detailed studies of the CTR emission characteristics (c.f. chapter 2), two basic issues have been considered in the design of the out-coupling and the quasi-optical
system for the EOA experiment at the SLS LINAC.

The diameters of the optical components (wire grid polarizer, paraboloid, plane and toroidal mirrors) are chosen large enough to account for the angular broadening of the CTR emission with increasing wavelengths. This means, that the apertures should be big enough to collect most of the fields of the CTR beam.

As shown in chapter 2 the tilt of the TR screen by 45° results in an asymmetric emission pattern for the horizontal CTR polarization components. Thus, only the symmetric, vertically polarized CTR components were selected using a wire grid polarizer.

The coherent power density spectrum, which was obtained from interference patterns measured with a Martin-Puplett interferometer (MPI) (c.f. section 3.1), is more pronounced in a wavelength range above 0.5 mm (below 600 GHz) where mainly the aforementioned effects occur.

![Diagram](image)

**Figure 3.5:** Out-coupling and transfer optics for the CTR of the EOA experiment. The CTR is produced at the optical diagnostic port ALIDI-SM-5 and is coupled out through a crystalline quartz window. Two off-axis parabolic mirrors transport the CTR pulse to the experiment.

### 3.2.1 Optical diagnostic port: out-coupling of CTR

All experiments described in this work have been carried out at the optical beam port ALIDI-SM-5 downstream the SLS LINAC. There are several such optical stations along the LINAC to measure the transverse beam parameters with high resolution [Sch99], [Süt01]. A typical station consists of a three stage pneumatics allowing to insert either a highly sensitive YAG:Ce detector to visualize the transverse beam distribution or a thin Al screen.
producing TR.
The screen monitor at the optical port ALIDI-SM-5 is commonly used to determine the emittance of the beam, whereas the ALIDI-SM-E monitor, which is situated behind the 45° ALIMA-BY bending magnet in front of the beam dump is utilized to measure the energy and the energy spread of the 100 MeV beam (see figure 1.9).
The CTR is coupled out of the UHV system through a crystalline quartz window. According to [Gri90] the absorption of the crystalline quartz window is negligible below frequencies of 1 THz. The optical port at ALIDI-SM-5 was modified to enhance the efficiency of extraction of the CTR (c.f. chapter 2). The angular broadening of the CTR emission pattern towards long wavelengths necessitates a larger solid angle of the beam port which thus improves the out-coupling of the long wavelength CTR. The original TR target screen (Al-foil of 38 mm diameter) was replaced with a larger size (56 mm diameter, 380 micron thickness) Al-plated Silicon waver to increase the spectral flux of the emitted CTR emission at long wavelengths according to the considerations in chapter 2 (c.f. equation (2.57)). The quantitative improvement is presented in figure 3.7. A schematic view of the old and new diagnostic port is shown in figure 3.6.

Figure 3.6: Old and new optical port installed at ALIDI-SM-5. The geometry of the new port offers a larger usable solid angle for the extraction of the emitted TR.
Chapter 3. Novel spatial interferometer with single-shot capability

Figure 3.7: Improvement in spectral flux at low frequencies due to the larger radius of the target screen. This results from the fact, that a larger fraction of the electromagnetic fields of the relativistic electron hit the screen.

3.2.2 Transfer optics: off-axis parabolic mirrors

The vertical polarized CTR is transported to the entrance focus of the interferometer using two off-axis parabolic mirrors of 200 mm focal length and an aperture size of 90 mm\(^5\). The first paraboloid mirror contains a hole of 12 mm diameter for extracting the OTR used for synchronization in the EOA experiment. A wire grid polarizer of 80 mm diameter is placed in front of the vacuum window (crystalline quartz of 63 mm diameter, 4.65 mm thickness) in order to obtain vertical polarized CTR. First experiments were performed in order to prove the ability to focus the CTR using the two off-axis parabolic mirrors. A horizontal scan in the focal plane of the second mirror is depicted in figure 3.8 below. The finite window size of the Golay cell (6 mm) and the high bandwidth (50 GHz-15 THz) of the cell cause an average over a broad frequency range and the central minimum resulting from the radial polarization of the CTR is not resolved in the experiment. The peak field strengths of the CTR as measured with the Golay cell are calculated to be in excess of 2 kV/cm. This number was computed by using the responsivity of the detector. A field strength of 2 kV/cm rotates the polarization of the probe laser beam by 10\(^\circ\) (see equation (4.1)), which is sufficient for the experiment.

As a result, it has been shown, that the use of the off-axis parabolic mirrors retains the radial polarization of the CTR. Figure 3.9 shows a scan of horizontal polarized respectively vertical polarized CTR in a distance of \(d = 70\) mm behind the focal plane of the second mirror. The asymmetry in the horizontal polarization is retained but flipped as expected. Since, the beam splitting in the spatial interferometer is achieved by the separation of

\(^5\)The combination of the 200 mm focal length and the 90 mm aperture diameter offers the largest usable solid angle which could be delivered by the manufacturer of the mirrors.
3.3. Spatial interferometer

In this section the novel interferometer producing a spatial auto-correlation of the CTR pulse is introduced. Since the spatial interference pattern in the focal plane of the interferometer is generated by each single radiation pulse this set-up allows in principle single-shot measurements of the power density spectrum of the emitted radiation. A theoretical description of the auto-correlation is followed by an experimental and simulated proof-of-principle and measurements of the spatial auto-correlation of the broadband CTR source from the SLS pre-injector LINAC.

Figure 3.8: Horizontal scan in the focal plane of an off-axis paraboloid mirror.

the two vertically polarized CTR lobes, the conservation of the radial polarization by the transfer optics is one of the prerequisites for the EOA experiment.

Figure 3.9: Scan after two off-axis parabolic mirrors. Left side: horizontal polarization the asymmetry is retained but flipped as expected. Right side: vertical polarization.
3.3.1 Theoretical description of spatial auto-correlation

To illustrate the principle of spatial auto-correlation two plane waves intersecting at the angle $\theta$ are assumed. The two beams are spatially coinciding at $r_3 = 0$ in the focal plane as depicted in figure 3.10. The two complex amplitudes of frequency $\nu$ are given by:

$$E_1(\vec{r}, t) = E_1 \cdot e^{2\pi \nu i (t - \frac{\vec{k}_1 \cdot \vec{r}}{2\nu} - \Delta t_1)}, \quad (3.1)$$
$$E_2(\vec{r}, t) = E_2 \cdot e^{2\pi \nu i (t - \frac{\vec{k}_2 \cdot \vec{r}}{2\nu} - \Delta t_2)}, \quad (3.2)$$

where $\Delta t_2 - \Delta t_1$ is the phase delay of wave 2 with respect to wave 1. Here we also have assumed that the two waves have the same polarization $\vec{E}_{1,2}$ parallel to $\vec{r}_2$. This is justified as we will use wire grid polarizers in the experiment.

![Figure 3.10: Two plane waves incident under the angle $\theta$.](image)

The intensity at position $\vec{r}$ is computed by:

$$I(\vec{r}, t) = |E(\vec{r}, t)|^2 = |E_1(\vec{r}, t) + E_2(\vec{r}, t)|^2 =$$

$$(E_1(\vec{r}, t) + E_2(\vec{r}, t))(E_1^*(\vec{r}, t) + E_2^*(\vec{r}, t)), \quad (3.3)$$

where $E_i^*$ denotes the complex conjugate of $E_i$.

After some algebra it follows that (see appendix):
3.3. Spatial interferometer

\[ I(\vec{r}) = E_1^2 + E_2^2 + 2E_1E_2 \cos((\vec{k}_2 \vec{r} - \vec{k}_1 \vec{r}) + (\phi_2 - \phi_1)), \quad (3.4) \]

where \( \phi_l = 2\pi \nu \Delta t_l \) \((l = 1, 2)\).

In the case depicted in figure 3.10 it is that:

\[ \vec{k}_1 = k \cdot (- \sin(\theta), 0, - \cos(\theta)) \quad (3.5) \]
\[ \vec{k}_2 = k \cdot (+ \sin(\theta), 0, - \cos(\theta)) \quad (3.6) \]

with \( k = \frac{\omega}{c} = \frac{2\pi \nu}{c} \).

Therefore, it is found that the intensity distribution of the interfering plane waves in the focal plane can be calculated to:

\[ I(\vec{r}, \nu) = \left( E_1^2 + E_2^2 + 2E_1E_2 \cos(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot r_1 + (\phi_2 - \phi_1)) \right) = \left( I_1 + I_2 + 2\sqrt{I_1I_2} \cos(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot r_1 + (\phi_2 - \phi_1)) \right). \quad (3.7) \]

The modulation depth which is dependent on the intensity ratio between the two signal paths is given by:

\[ \alpha_m = \frac{2\sqrt{I_1 \cdot I_2}}{I_1 + I_2}. \quad (3.8) \]

Thus, the above equation is transformed to:

\[ I(\vec{r}, \nu) = (I_1 + I_2) \left( 1 + \alpha_m \cdot \cos(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot r_1 + (\phi_2 - \phi_1)) \right). \quad (3.9) \]

In figure 3.11 the modulation depth \( \alpha_m \) is plotted against the intensity \( I_2 \) in the second signal arm of the interferometer with \( I_1 = 1 \). For typical intensity ratios \( I_1/I_2 > 0.5 \) the modulation depth \( \alpha_m \) is near to unity.

**Auto-correlation**

The intensity auto-correlation of the complex amplitude \( E(t) = E \cdot e^{i2\pi \nu t} \) with itself shifted by the delay \( \tau \) is given by\(^6\):

\[ I(\tau) \propto \text{Re}\left( \frac{1}{T} \int_{0}^{T} E(t)E^*(t + \tau)dt \right). \quad (3.10) \]

\(^6\)It must be noted that when recording the intensity auto-correlation all phase relation is lost.
Thus it becomes that:

\[ I(\tau) \propto c_0 + \cos(\tau), \quad (3.11) \]

where \( c_0 \) denotes the integration constant. Here the delay \( \tau \) which depends on the spatial position \( r_1 \) is calculated by:

\[ \tau = 2\pi \nu \left( \frac{2 \sin(\theta) r_1}{c} \right) + (\phi_2 - \phi_1). \quad (3.12) \]

Thus it becomes that

\[ I(\tau) \propto c_0 + \cos(2\pi \nu \left( \frac{2 \sin(\theta) r_1}{c} \right) + (\phi_2 - \phi_1)). \quad (3.13) \]

The expression (3.13) is similar to equation (3.9), which thus is representing the intensity auto-correlation of the complex amplitude \( E(\vec{r}, t) \).

The maximum and minimum position of the resulting fringes are determined by considering the derivative of the above calculated intensity distribution. Assuming that \( I_1 = I_2 = I_0 \) and setting \( x = r_1 \) it is found that:

\[ I(x, \nu) = 2I_0 \left( 1 + \cos(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot x + (\phi_2 - \phi_1)) \right) \]

\[ I \propto (1 + \cos(bx + a)) \quad (3.14) \]

with

\[ a = \phi_2 - \phi_1 \]
\[ b = 4\pi \sin(\theta) \cdot \frac{\nu}{c} \]

When solving \( \partial_x (1 + \cos(bx + a)) = -b \sin(bx + a) = 0 \), the maximum and minimum position of the resulting fringes are located at:
3.3. Spatial interferometer

\[ x_n = \frac{\pi n - a}{b}. \]  

(3.15)

Inserting the above substitutions for the parameters \(a\) and \(b\) yields:

\[ x_n = \frac{\pi n - (\phi_2 - \phi_1)}{4\pi \sin(\theta) \cdot \frac{c}{
u}} \]

\[ x_n = \frac{\pi n - 2\pi \nu \Delta t}{4\pi \sin(\theta) \cdot \frac{c}{
u}}, \]  

(3.16)

where \(\Delta t = \Delta t_2 - \Delta t_1\).

The position of the zeroth order\(^7\) fringe \((n = 0)\) is frequency independent and is always given by \(\frac{\Delta t_c}{2\sin(\theta)}\) as shown in figure 3.12.

![Interference pattern](image)

**Figure 3.12:** Interference pattern as calculated with equation (3.9) for 3 frequencies (60 GHz, 90 GHz and 120 GHz). The position of the maximum at \(x = r_1 = 0\) is frequency independent and thus is identified as the zeroth order fringe (for \(\Delta t = 0\)). The pattern of a broadband signal will thus show a characteristic spike at position \(x = 0\).

**broadband signal**

So far we only have considered the interference of two monochromatic waves. The CTR source is broadband as shown in section 3.1. The interference pattern of the broadband CTR pulse is calculated by the following expression:

\[ I(\vec{r}) = \int_0^\infty I(\vec{r}, \nu) I_{\text{spec}}(\nu)d\nu \]  

(3.17)

\(^7\)For notation: the number \(n\) of the fringes will be called the \(n\)th order fringe in the reminder of this thesis.
where $I(\vec{r}, \nu)$ is the interference pattern as generated by the monochromatic wave of frequency $\nu$ (c.f. equation (3.9)) and $I_{\text{spec}}(\nu)$ denotes the power density spectrum in the focal plane of the interferometer. The angle of incidence is chosen to be $\theta = 30^\circ$, which is justified in the following section.

Formula (3.17) with its superposition of intensities relies on the fact that the different frequencies $\nu$ do not interfere when averaged over time. The fields at different frequencies $\nu$ do interfere. However, constructive interference will occur as often as destructive interference. Hence, when averaged over time (as done with the intensity auto-correlation (see equation (3.13))) the individual frequency components transmit independently [Pea06].

Figure 3.14 shows the simulated interference pattern of the broadband CTR source according to equation (3.17). The underlying power density spectrum $I_{\text{spec}}(\nu)$ is the power density spectrum as measured with the Martin-Puplett interferometer (MPI) depicted in figure 3.4. The simulation takes the radial polarization of the CTR source in to account. Hence the interference of the two vertical polarized lobes results in an anti-correlated signal as illustrated in figure 3.14. This is explained in figure 3.13: The sign of the electrical field of the two vertically polarized CTR lobes changes at the symmetry axis between the two lobed emission pattern. Thus, we must rewrite equation (3.3):

$$I(\vec{r}, t) = |E(\vec{r}, t)|^2 = |E_1(\vec{r}, t) - E_2(\vec{r}, t)|^2. \quad (3.18)$$

Hence, after some algebra (see appendix), equation (3.9) slightly changes as follows:

$$I(\vec{r}, \nu) = (I_1 + I_2)\left(1 - \alpha_m \cdot \cos(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot r_1 + (\phi_2 - \phi_1))\right). \quad (3.19)$$

Figure 3.13: The vertical polarization component of the radial polarized electrical field of the CTR is transmitted through the wire grid polarizer. Thus, the sign of the electrical field changes at the symmetry axis between the two distinct lobes.
This produces the characteristic central dip which corresponds to the zeroth order fringe and which position is independent on the frequency as shown in equation (3.16). The width of the zeroth order fringe contains the information about the broadness of the coherent spectrum which is to be determined. The smaller fringes correspond to over tones of the coherent spectrum resulting from edges and modulations of the longitudinal bunch shape. The $30^\circ$ angle of incidence of the signal arms yields a phase shift of 3 ps over 1 mm. Thus, we can roughly estimate the width of the pulse.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.14.png}
\caption{Simulated interference pattern of a broadband CTR source. The underlying power density spectrum is measured with a Martin Puplett interferometer (MPI) shown in figure 3.4.}
\end{figure}

### 3.3.2 Optical layout of spatial interferometer

The spatial interferometer is comprising two plane mirrors which act as beam splitters of the two vertical lobes of the emitted CTR and two toroid mirrors to refocus the two lopes onto the focal plane under an oblique angle $\theta$. The layout is shown in figure 3.15.

The optics is based on the transformation between the two foci of an ellipsoid. The ellipsoid is hereby approximated by toroidal mirror characterized by its two radii of curvature which are determined in the following. The general set-up is further specified by the angle of incidence of the two rays onto the focal point. This angle is chosen to be $30^\circ$ in order to obtain a sufficient large phase shift over the crystal in the focal plane (3 ps over 1 mm) needed to resolve the auto-correlation pattern with a pixel array detector with 50 micron pitch (150 fs per pixel). Hence, this configuration yields a frequency resolution of $\delta\nu = 2$ THz, which is more than sufficient for the CTR pulses at the SLS LINAC. Additionally
Chapter 3. Novel spatial interferometer with single-shot capability

Figure 3.15: Geometrical layout of the spatial interferometer

This angle allows to use optics with diameters of 10 centimeter or even larger in order to avoid truncation and diffraction losses without obstructing the two beam paths. Figure 3.16 depicts the optical path of one arm of the interferometer $\text{ABCD}$ together with the defolded path $\text{A}'\text{C'D}$. $A'$ and $D$ are the focal points of the ellipsoid with eccentricity and main axes defined by $\epsilon$ and $e$, respectively.

Figure 3.16: Schematic set-up of the spatial interferometer
From figure 3.16 it is evident that the conditions

\[ a = b + c \]  
\[ \cos(30^\circ) = \frac{a/2}{b} \]  

must be fulfilled. The parts b and c are then computed to be:

\[ b = \frac{a}{\sqrt{3}} \]  
\[ c = b(\sqrt{3} - 1). \]  

In order to avoid obstructions of the two beam paths the parameter \( a \) is chosen large enough. Setting \( a = 300 \text{ mm} \), yields the following dimensions of \( a \) and \( b \):

\[ b = 173 \text{ mm} \]  
\[ c = 127 \text{ mm}. \]

**Spherical approximation**

The ellipsoid is spherically approximated and a toroidal mirror is used as focusing element. Considering the two-dimensional case the equations for the ellipse [Bro93] and the circle are written as follows:

\[ 1 = \frac{x^2}{e^2} + \frac{z^2}{d^2} \]  
\[ R^2 = x^2 + z'^2, \]

where \( z' = z + (R - d) \).

The above equations can be transformed to:

\[ z(x) = \pm \frac{d\sqrt{e^2 - x^2}}{e} \]

for the ellipse and

\[ z(x) = d - R \pm \sqrt{R^2 - x^2} \]

for the circle.

The spherical approximation necessitates that the radii of curvatures at \( x = 0 \) are the same for both ellipse and circle:

\[ \left. \frac{\partial^2 z_{\text{ellipse}}}{\partial x^2} \right|_{x=0} = \left. \frac{\partial^2 z_{\text{circle}}}{\partial x^2} \right|_{x=0} \]
Hence, the following equation for the large radius of the toroid mirror $R$ is obtained:

$$R = \frac{e^2}{d}. \quad (3.29)$$

Using the optical property of the ellipse $2e = r_1 + r_2$ as found in [Bro93] it must be that $e = a$. With $d = a \cdot \cos(\alpha)$, where $\alpha$ is the angle of incidence onto the focusing mirror it can be found that:

$$R = \frac{a}{\cos(\alpha)} \quad (3.30)$$

$$d = a \cdot \cos(\alpha). \quad (3.31)$$

Therefore, the following radii of curvature have been specified for the toroidal mirrors:

$$R_1 = R = 310.6 \text{mm}$$

$$R_2 = d = 289.8 \text{mm}. \quad (3.32)$$

The interferometer set-up was constructed according to the above guidelines by PSI’s mechanical engineering department. The mirrors were fabricated by Kugler Precision Optics. They are made of anticorodal alloy (AlMgSi1) and coated with 400 nm of gold. The surface roughness was specified to be below than 10 nm and the surface accuracy better than 2 $\mu$m (guaranteed by the manufacturer), which represents excellent optical quality for the CTR spectral range of interest ($< \lambda/100$).

The interferometer can be aligned using two diaphragms which can be set to mark the two focal points. For pre-alignment a small electric bulb is used to emulate the diverging source. The light is focused onto the first diaphragm using a lens of 20 mm focal length. The beam splitting mirrors are mounted on mirror holders adjustable in two axes enabling to align the two signal paths exactly onto the second diaphragm. The set-up is shown in figure 3.17 below.

### 3.3.3 Proof of principle of the spatial interferometer

The proof of principle of the spatial interferometer was achieved by measurements using continuous wave sources in the millimeter wavelength range. The simulations and the measurement showed the spatial interference of the two signal arms of the interferometer. These experiments as well as the microwave optics simulations of the interferometer were

---

$^8$r$_l$, $l = 1, 2$ denotes the two focal lengths of the ellipse.
3.3. Spatial interferometer

Figure 3.17: Alignment set-up for the spatial interferometer

carried out in collaboration with the microwave department of the University of Bern [Mur04].
The measurements were performed at frequencies between 70 and 120 GHz using an ABmm vector network analyzer [ABm] and a planar scanner. The source feed and the detector probe consisted of two slightly flared rectangular WR-10 waveguides with an aperture of $3\text{mm} \times 3\text{mm}$ each. The simulations are carried out with Physical Optics (PO) and Physical Theory of Diffraction (GTD) using the commercial software package GRASP [TIC]. All measurements and simulations were performed with vertical polarization, which is perpendicular to the symmetry plane of the interferometer.

GRASP simulations

The beam pattern has been simulated at the position of the two beam splitting plane mirrors. Both mirrors have a rectangular hole (see Figure 3.18) at the beam center which was included in all following simulations. (The hole was originally intended to insert the probe laser beam from the front side of the interferometer.) Figure 3.18 presents the simulated radiation pattern at the position of the beam splitter at two different frequencies together with the projected outline of one of the flat mirrors. The simulated interference pattern for the diverging beam of the waveguide feed at 70 and 100 GHz are depicted in figure 3.19). The tilt of the interference fringes is a result of the asymmetric layout of the interferometer, as the beams of the two signal paths intersect the
focal plane from opposite vertical directions. This tilt could be avoided in an arrangement where the deflection plane of the two beams coincides with the interferometer plane. This could be obtained by either mounting the interferometer vertically, which is not very practical or by transfer optics such that the CTR is coupled out downwards. in this case a configuration is possible which would be symmetrical with respect to the vertical plane resulting in a perfectly symmetric fringe pattern.

![Figure 3.18](image1.png)

**Figure 3.18:** -1dB and -3dB field contours at the position of the beam splitter at 70 and 100 GHz and projected rim of the upper planar mirror.

![Figure 3.19](image2.png)

**Figure 3.19:** Simulated interference pattern of both signal paths in the focal plane of the interferometer. Left side: at 70 and right side: at 100 GHz.
3.3. Spatial interferometer

Measurements with the ABmm vector network analyzer

The measurements are performed with an ABmm network analyzer, which measures the complex, or vector, impedance (a real and an imaginary part of the impedance or an amplitude and a phase of the microwaves) in the millimeter and sub-millimeter frequency domain [ABm]. The waveguide feed of the source was placed in the nominal focus position in front of the beam splitting planar mirrors. An identical waveguide probe with the detector was mounted on a planar xy-scanner in the focal plane with an additional translation stage for the z axis (see figure 3.17). The axial distance $z$ was adjusted until the beam patterns of the individual signal paths were approximately centered on each other. To measure a pattern of an individual interferometer arm the other signal path was blocked with microwave absorber.

Measurements of both signal paths at the same time result in the interference patterns which are shown in figure 3.20. The modulation depth was found to exceed 10 dB.

In figure 3.21 two-dimensional and horizontal scans for 70, 100 respectively 120 GHz are depicted. The position of the zeroth order fringe is frequency independent, as follows from theory (c.f. equation (3.16)). The zeroth order fringe is found to be shifted by 2 mm out of the center. This results from a path length difference between the two interferometer arms. Since the CTR pulse is expected to be as short as 2 ps ($\approx 0.7\,\text{mm}$) the phase between the two signal arms must be adjustable. Otherwise, the interference can easily be shifted out of the finite CTR distribution in the focal plane of the interferometer (c.f. section 3.4). This is achieved by mounting one of the focusing toroid mirrors on a motor-

Figure 3.20: Measured interference pattern of both signal paths in the focal plane of the interferometer at left: 70 GHz; right 100 GHz.
ized linear translation stage which is moving the mirror along the bisecting line along the angle between incidence and reflectance (see figure 3.17).

Figure 3.21: Interference pattern measured in consecutive 1D scans at 70 and 100 GHz (left) and -10dB contours of measurements at 70, 100 and 120 GHz (right). In both plots the zero order fringe is found at about +2mm off the nominal focus position.

Figures 3.22 to 3.23 show the unwrapped phase of the single path measurements. Solid black lines are phase contours of multiple 360°. The dashed lines present field contours (-3 dB, -10 dB and -20 dB). Where there is significant intensity the field contours are nearly linear. Thus, the effect of deformation of the phase contours at the edges is negligible. By fitting the tilt of the equiphase surface it is possible to determine the angle of incidence on the measurement plane of the scanner. For that purpose the beam center in k-space was calculated after a 2D Fourier transformation of the complex data. The fit results for the angles of incidence $\delta x$ and $\delta y$ are summarized in table 3.1. The table also gives the lateral offsets $\Delta x$ and $\Delta y$ of the center of the amplitude contours. The lateral offset between the two beams indicates that the distance between the EOA optics and the scanner plane (focal plane) was not adjusted perfectly during the measurements.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Angle of Incidence</th>
<th>Lateral Offsets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta x$ [deg]</td>
<td>$\delta y$ [deg]</td>
</tr>
<tr>
<td>left 70 GHz</td>
<td>+26.43</td>
<td>-7.01</td>
</tr>
<tr>
<td>right 70 GHz</td>
<td>-26.13</td>
<td>+6.64</td>
</tr>
<tr>
<td>left 100 GHz</td>
<td>+25.86</td>
<td>-5.73</td>
</tr>
<tr>
<td>right 100 GHz</td>
<td>-26.02</td>
<td>+4.99</td>
</tr>
</tbody>
</table>

Table 3.1: The fit results for the angles of incidence.
The measurements and simulations show a distinct tilt of the interference fringes. This results from the asymmetric optical layout in which the beams of the two signal paths intersect the xy-plane from opposite directions. The effect depends on the illumination of the beam splitting planar mirrors and will be smaller for the rays which are closer to the optical axis. This effect was verified with a series of measurements where the central
or the outer half of the flat mirrors was blocked with microwave absorber. Figure (3.24) shows that the tilt is smaller when only the inner half of the mirrors is used, and larger for the outer half.

![Image](image.png)

**Figure 3.24:** Measurements at 70 GHz with different areas of the beam splitter mirrors covered with absorber. These results demonstrate that the tilt of the fringes is affected by the off-axis angle. a) without absorber b) outer area blocked ⇒ smaller tilt than a) c) inner area blocked ⇒ larger tilt than a)

**Summary**

The measurements proved that the interferometer optics works as designed and produces the expected interference pattern. The GRASP simulations showed perfect agreement with the measurements even for the fine details of the side-lobes. The significant diffraction effects resulting from the complex shape of the beam splitting mirrors and the high edge tapers on all mirrors are predicted by the simulations. Thus, it is justified to use the GRASP software package to simulate the quasi-optical set-up of the experiment. The measurements indicated a path length difference in the two signal paths in spite of the precise machining of the set-up’s assembly board and careful pre-alignment procedure applied. To avoid any loss of the interference of the CTR due to misalignments a motorized linear stage has been implemented to remotely adjust the phase between the two beam paths.
3.3.4 Measurement with the spatial interferometer at the SLS pre-injector LINAC

In order to produce the spatial auto-correlation of the broadband CTR pulse the interferometer was placed at the optical beam port ALIDI-SM-5 at the SLS pre-injector LINAC. The interferometer entrance focus was set in the focal point of two parabolic mirror as depicted in figure 3.25 below. After the vacuum window a wire grid polarizer was placed to obtain the vertical polarized CTR. Behind the interferometer the Golay Cell detector, which is sensitive to long wavelength FIR radiation, was installed on three motorized linear translation stages thus allowing to move the device in horizontal, vertical and axial directions in order to scan the focal plane of the interferometer.

![Interferometer set-up at the SLS LINAC.](image)

The entrance focus of the spatial interferometer must be set in the focal point of the second off-axis parabolic mirror of the transfer optics, as the designed optics of the interferometer is based on the transformation between the two foci of an ellipsoid. Hence the position of the focal point of the second off-axis paraboloid must be determined. The use of a HeNe alignment laser allows marking the focal length of the second parabolic mirror. The laser beam is thereby inserted from the opposite side of the optical beam port and is focused onto the target screen in the middle of the vacuum beam pipe. Thus, the laser beam is set to emulate the diverging CTR source on the target screen (see figure 3.26). The exact position of the CTR focal spot can then be found by performing a transverse scan with the Golay cell detector in the focal plane of the transfer optics.
Chapter 3. Novel spatial interferometer with single-shot capability

Figure 3.26: The HeNe alignment laser beam is inserted from the opposite site of the optical diagnostic station ALIDI-SM-5. The laser beam is focused onto the target screen to emulate the diverging CTR source.

The entrance focus of the interferometer must now coincide with the focus of the second parabolic mirror of the transfer optics. If the interferometer is not properly placed at that point the two signal arms will not overlap in the focal plane of the interferometer. The proper positioning of the interferometer thus can be verified by measuring the position of the center of the two individual signal arms in the focal plane of the interferometer. Hence the other beam path is blocked using radar absorbent material (RAM).

The modulation depth of the spatial auto-correlation depends on the intensity ratio between the two signal arms of the interferometer. The largest modulations can be expected when the same intensity is transported in both of the two beam paths. This necessitates a clean splitting of the two vertically polarized CTR lobes. The divergence of the transverse CTR emission pattern is found to be smaller than the divergence of the cw-sources used in the laboratory measurements. Hence the CTR intensity distribution is centered only on the very inner part of the two beam splitting mirrors. Therefore, the two beam splitting plane mirrors must be properly aligned in respect to the impinging CTR pattern. Thus the interferometer board was vertically adjustable using three fine thread screws. When the height of the interferometer is properly set the intensity ratio between the two signal arms must be near to unity. This can be verified by performing a horizontal scan behind the focal plane of the interferometer \((z < 0)\). Such a scan is shown in figure 3.27. The two diverging signal arms are clearly distinguishable. Their maxima are comparable \((I_1/I_2 > 90\%)\) which indicates that the same intensity is transported in both signal paths, and which thus leads to optimum modulation depths (c.f. figure 3.11).
3.3. Spatial interferometer

As a consequence of the laboratory measurements, which showed a difference in length of the two signal paths the right toroid mirror was mounted on a linear translation stage in order to adjust the phase between the two interferometer arms (see figure 3.17). The position of the CTR spot in the focal plane does not change when adjusting the path length of the right interferometer arm. This was verified with measurements in the focal plane of the interferometer (see figure 3.28). When moving the mirror, even by a large distance of 12 mm, from its nominal position, the center of the corresponding beam spot of the right signal path in the focal plane changed only by less than 0.5 mm, which is small compared to the size of the focal spot of about 4 mm FWHM. This behaviour was consistently observed during the alignment procedure of the interferometer using visible light.

Figure 3.27: Horizontal scan 10 mm behind the nominal focal plane of the interferometer.

Figure 3.28: Two separate horizontal scans in the focal plane of the interferometer. The solid line corresponds to a scan with the toroid mirror in nominal position while the dotted line presents a scan for the toroid mirror moved away by 12 mm from the nominal position. The change in position of the spot is less than 0.5 mm.
The CTR auto-correlation between the two signal arms was reproducibly observed. In order to enhance the spatial resolution of the Golay cell detector (diamond window with 6 mm diameter), a circular wave guide feed was mounted in front of the baffle of the detector. Other arrangements, such as a vertical slit, introduce an additional phase shift, which varies with the angle of incidence onto the detector. Thus the interference signal of the broadband CTR pulse is smeared out by the integrating detector.

The detector was placed at the maximum of the CTR spot in the focal plane of the interferometer. The position of the right toroid mirror which corresponds to equal path length between the two signal arms is determined with a longitudinal scan with the toroid mirror. Such a scan is shown in figure 3.29. The length of the two beam paths is equal when the toroid mirror is placed at about 0.6 mm away from its nominal position. Due to the finite size of the circular wave guide feed (1.9 mm) frequencies above 200 GHz are smeared out. Hence the power density spectrum can only be retrieved with the EO read-out which offers much higher spatial resolution (50 micron corresponding to the pitch of the pixels of the InGaAs linear array detector).

![Figure 3.29: Longitudinal phase scan in the center of the beam spot.](image)

At the fixed phase, the horizontally distributed intensity is measured by scanning the detector along the horizontal plane. Figure 3.30 shows a horizontal scan in the focal plane of the interferometer. The characteristic minimum which corresponds to the zeroth order fringe is clearly visible. The modulation depth is found to be 50 percent. According to equation (3.16) the position of the zeroth order fringe does shift with the phase between the interferometer arms. Therefore, a phase shift results in a shift of position of the zeroth order fringe which is shown in figure 3.30.
3.4 Transfer function of the complete quasi-optical set-up

As indicated in section 3.1 the interference pattern of the broadband CTR pulse depends on the power density spectrum of the CTR pulse \( I_{\text{spec}}(\nu) \) in the focal plane of the interferometer.

\[
I(\vec{r}) = \int_0^\infty I(\vec{r}, \nu) I_{\text{spec}}(\nu) d\nu. \tag{3.33}
\]

Here, \( I(\vec{r}, \nu) \) is the interference pattern as generated by two monochromatic plane waves described by their complex amplitudes \( E_1(\vec{r}, t) \) and \( E_2(\vec{r}, t) \) (c.f. equation (3.9)). The spectrum \( I_{\text{spec}}(\nu) \) depends on the overall transmission function of the quasi-optical system.
which is derived in this section using the commercially available GRASP\textsuperscript{9} software package. The source is emulated with transverse (electrical) field patterns at different frequencies which have been calculated using the formalism described in the second chapter. The computed field patterns are then propagated through the quasi-optical set-up as shown in figure 3.31.

According to [Gri90] the absorption of the crystalline quartz window of 4.65 mm thickness is negligible below frequencies of 1 THz, where the CTR emission is expected at the SLS pre-injector LINAC. Since the quasi-optical components offer excellent optical quality for the CTR spectral range of interest, reflection losses are neglected, whereas the absorption in humid air must be included.

The section is concluded with a description of the CTR spot size in the focal plane of the interferometer which depends on the wavelength.

\textbf{Figure 3.31:} Quasi-optical set-up of the EOA experiment used for the GRASP simulations.

\textsuperscript{9}GRASP uses Physical Optics (PO) and Physical Theory of Diffraction (GTD).
3.4. Transfer function of the complete quasi-optical set-up

The relative intensity propagated to each intermediate point (see figure 3.31) is calculated for a set of different frequencies. An intensity transfer function is then fitted\(^{10}\) to the resulting relative intensities that are transported through the entire quasi-optical system consisting of the aperture of the vacuum window, the two paraboloid mirrors and the interferometer. The GRASP simulations are fitted by an intensity transfer function which is the product of a high-pass filter followed by a low-pass filter. The low-pass filter is introduced to account for the hole in the first paraboloid mirror. The following definition of the high-pass filter function proved useful to account for the low-frequency attenuation:

\[
T_{\text{highpass}}(\nu) = \frac{1}{\sqrt{1 + \left(\frac{\nu}{\nu_{HP}}\right)^2}}. \tag{3.34}
\]

while the low-pass function is defined as follows:

\[
T_{\text{lowpass}}(\nu) = \frac{1}{\sqrt{1 + \left(\frac{\nu}{\nu_{LP}}\right)^2}} \tag{3.35}
\]

The resulting overall transmission of the set-up is shown in figure 3.32. The low frequency attenuation of the quasi-optical system is described by the filter function \(T_{\text{highpass}}\) with \(\nu_{HP} = 130\) GHz and \(T_{\text{lowpass}}\) with \(\nu_{LP} = 1340\) GHz accounts for the hole in the first parabolic mirror. The transmission at 100 GHz is in the order of 60 % (above 50 % as we have set as target in section 3.2). Thus, this represents a significant improvement over the transmission of the conventional Martin-Puplett interferometer, which is calculated to be below 15 %.

The GRASP simulations in figure 3.34 show the electrical field patterns of the both interferometer arms at the measurement plane. The vertical mismatch of the two signal arms in the focal plane of the interferometer is pronounced for long wavelengths where as for high frequencies the signal arms will overlap. The resulting low frequency attenuation due to the vertical mismatch (see figure 3.32) is again described by the intensity filter function \(T_{\text{highpass}}(\nu)\) with \(\nu_{HP} = 270\) GHz and must be taken into account in the signal analysis presented in chapter 5. This behaviour is mainly a consequence of the transfer optics which at long wavelengths introduces a frequency depending focus. At these large wavelengths the focal point of the second parabolic mirror does not coincide with the entrance focus of the interferometer [Mur].

Furthermore the water absorption in humid air is considered. The water absorption in air for the 1.25 meter long path of the quasi-optical system is calculated according to [Bur68].

---

\(^{10}\)The fits have been done using the build in function "NonlinearFit" in Mathematica, which allows to perform a least-squares fit to a list of data. The estimates of the model parameters are chosen to minimize the \(\chi^2\) merit function given by the sum of squared residuals \(\sum_i e_i^2\).
Chapter 3. Novel spatial interferometer with single-shot capability

Figure 3.32: Left side: Transmission as fitted to the the GRASP simulations. Right side: Overlap of the the two signal paths of the interferometer as fitted to the GRASP simulations.

Since, the expected spectral range is more pronounced below 500 GHz (c.f. section 3.1) the absorption in humid air must not be considered to pose a real problem.

The total power transmission of the quasi-optical system is the product of all individual transmission functions and is shown in figure 3.33:

\[ T_{\text{total}}(\nu) = T_{\text{quasioptical}}(\nu) \cdot T_{\text{overlap}}(\nu) \cdot T_{\text{H}_2\text{O}}(\nu). \]  

\hspace{1cm} (3.36)

Figure 3.33: Overall transmission according to equation (3.36)
Figure 3.34: Simulated electrical field pattern for the two individual signal arms of the interferometer (upper signal path: solid red; lower signal path: dashed blue line) in the measurement plane of the interferometer for four frequencies (a): 60 GHz; (b): 120 GHz; (c): 240 GHz and (d): 360 GHz. Each plot shows the -3 dB, -10 dB, -20 dB and -30 dB contours. For longer wavelengths (60 GHz) the two signal paths are vertically displaced, while for higher frequencies (360 GHz) the two coincide.
3.4.1 Wavelength dependent CTR spot size

Simulations and measurements with the spatial interferometer yielded a finite CTR spot in the focal plane of the interferometer. Thus, the interference pattern (equation (3.9)) is convoluted with the spatial CTR distribution in the focal plane of the spatial interferometer. In the following description a perfect horizontal overlap (at $r_1 = 0$) of the two signal arms is assumed. In both simulations and measurements the spot size was found to scale with the wavelength of the source (e.g. see figure 3.34). Hence, a frequency dependence of the spot size is presumed similar as for a Gaussian beam, where the beam diameter also scales with the wavelength:

$$I(\nu, r_1) \propto e^{-\frac{d^2 r_1^2}{(c/\nu)^2}}. \quad (3.37)$$

The dimensionless parameter $d$ is determined using a horizontal scan with a pyro-electric detector in the focal plane of the interferometer. Thus, the phase between the two signal arm is adjusted in a way that no interference is observed within the CTR spot. The underlying CTR power density spectrum at the measurement plane is calculated by:

$$I(\nu) = f(\nu) \cdot P(\nu) \cdot T_{total}(\nu) \quad (3.38)$$

where $f(\nu)$ is the form-factor (chapter 1) and $P(\nu)$ accounts for the CTR spectral energy flux as introduced in chapter 2. $T_{total}(\nu)$ is given by equation (3.36).

$$I_{CTR}(x) = (I_1 + I_2) \cdot \int_{0}^{\infty} I(\nu)e^{-\frac{d^2 x^2}{(c/\nu)^2}} d\nu \quad (3.39)$$

In order to account for the form factor $f(\nu)$ a Gaussian longitudinal charge distribution of $2\sigma = 2$ ps is assumed (c.f. section 5.2). The parameter $d$ is then obtained by a nonlinear fit (using equation (3.39)) to the measurement data of the horizontal scan.

The spatial auto-correlation pattern is convoluted with the transverse distribution of the CTR spot in the focal plane of the spatial interferometer. This results in a degrading of the higher order interference fringes (equation (3.16)) as shown in figure 3.36. For a parameter $d = 0.51$ the first order fringes are attenuated to $\alpha_i \approx 0.7$ $(10 \log(\alpha_i) = -2 \text{ dB})$ while the second order fringes are reduced to $\alpha_i \approx 0.3$ $(10 \log(\alpha_i) = -5 \text{ dB})$ and the third order ones to $\alpha_i \approx 0.1$ $(10 \log(\alpha_i) = -10 \text{ dB})$ with respect to the CTR spot of infinite size. This must be taken into account in the analysis of the measured interference patterns.

Furthermore, the convolution leads to a reduction of the width of the interference pattern as shown in figure 3.37. The plot shows the simulated pattern at 60 GHz respectively at 240 GHz. The width (FWHM) of the center dip, which corresponds to the zeroth order
3.4. Transfer function of the complete quasi-optical set-up

Figure 3.35: Horizontal scan of the CTR spot in the focal plane of the interferometer. The black line represents the fit for the parameter $d$ assuming a Gaussian longitudinal bunch charge distribution of $2\sigma = 2$ ps. The parameter $d$ is calculated to be 0.51.

Figure 3.36: The convolution of the spatial auto-correlation pattern with the frequency dependent CTR spot results in a degrading of the higher order fringes. The figure shows two plots for 100 GHz and 300 GHz respectively.

The influence of the finite CTR spot size on the auto-correlation pattern is shown in figure 3.38. The simulation includes the frequency dependent tilt of the fringes. The underlying power density spectrum $I_{\text{spec}}(\nu)$ is the power density spectrum as measured with the Martin-Puplett interferometer (MPI) depicted in figure 3.4. The zeroth order fringe is
Figure 3.37: Influence of the finite CTR spot on the width (FWHM) of the zeroth order fringe. Left side: at 60 GHz. Right side: at 240 GHz. The decrease is frequency independent. The solid line gives the simulation for the finite spot size, while the dashed line shows the one for the spot of infinite extend.

clearly visible in both plots. The smaller fringes are attenuated due to the convolution of the interference pattern with the finite focal point of the interferometer. The frequency dependent tilt of the fringes is included in the simulation. The electro-optical read out of the EOA experiment uses a cylindrical lens which is implemented to focus the probe laser pulse onto the linear image sensor (see chapter 4). The use of the cylindrical lens thus corresponds to an integration of the interference pattern in vertical direction. Hence, the tilt of the fringes results in an attenuation of the recorded modulation depth of the recorded pattern to -1 dB as shown in figure 3.38.

Summary

The auto-correlation pattern as produced by the spatial interferometer depends on the overall power transmission function of the complete quasi-optical system. This was computed with the GRASP software package.

The GRASP simulations showed a vertical mismatch of the two signal arms of the interferometer pronounced at low frequencies due to the fact that the focal point of the transfer optics does not coincide with the entrance focus of the interferometer at these large wavelengths. This yields a low-frequency attenuation of the spatial auto-correlation pattern in the focal plane of the interferometer.

The auto-correlation pattern is convoluted with the spatial distribution of the CTR spot in the focal plane of the interferometer. A frequency dependent description of the diameter is found, assuming a Gaussian distribution. The finite CTR spot size results in a degrading
Figure 3.38: The influence of the finite CTR spot size on the auto-correlation. Left side: convolution of the spatial auto-correlation with the finite frequency dependent CTR spot size. Right side: the pattern is integrated in vertical direction similar to the cylindrical lens used in the electro-optical read out. This procedure results in a degrading of the modulation depth in the order of -1 dB.

of the higher order interference fringes as well as a change of the width (FWHM) of the interference pattern. In a robust data treatment (presented in section 5.1) this convolution is taken into account by subtracting the non-interfering CTR background which is identified as the finite frequency dependent CTR spot. The influence of this procedure on the measurement of the bunch lengths is in the order of 2 % as estimated by the reduction in width (FWHM) of the interference pattern.
Chapter 4

Electro-optical imaging system and synchronization

The novel interferometer presented in the previous chapter is designed to produce a spatial auto-correlation of short CTR pulses. The corresponding interference pattern is produced by every single CTR pulse thus, offering single-shot capability. The single-shot read-out of the spatial interference pattern is achieved by applying electro-optical techniques. The basic idea is that the birefringence which is induced in a nonlinear optical crystal by the electric field of the CTR modulates the initial linear polarization of the laser probe pulse. Thus, the spatial auto-correlation pattern of the CTR in the focal plane of the interferometer is translated into a polarization modulation of the transverse profile of the transmitted probe laser pulse. The experimental set-up is presented in figure 4.1.

Not shown in figure 4.1 is the active-mode-locked Nd:YAG laser (of 500 ps pulse width) which is placed outside of the LINAC area in the SLS technical gallery. The laser beam is guided into the radiation bunker of the SLS LINAC by a 15 m long optical transfer line. It consists of an ISO-100 vacuum pipe (100 mm in diameter), 5 plane mirrors for deflecting the beam and two achromatic lenses ($f_1 = 3750$ mm, $f_2 = 5250$ mm) to collimate the radiation. The shot-to-shot position stability of the laser beam behind the 15 meter long transfer line was measured to be in the order of ± 80 micrometer as measured by the InGaAs linear image sensor. This has a consequence on the data processing as introduced in the following chapter. These transverse fluctuations cause modulations in the auto-correlation patterns which are analyzed in chapter 6. Figure 4.2 shows a histogram of the shot-to-shot variations of 1000 successive pulses. The intensity transmission of the transfer line was measured to be in the order of 40 %. The Zinc Telluride (ZnTe) crystal is placed in the focal plane of the interferometer. The laser pulse passes the first polarizer before it is deflected by a small mirror placed between the two signal arms of the interferometer.
onto the ZnTe crystal in the focal plane. Behind the second polarizer the laser beam is focused in vertical direction by a cylindrical lens onto the InGaAs linear image sensor. A detailed description of the working principle of the laser is given together with temporal measurements of the laser pulse in section 4.1. The succeeding section presents the electro-optical imaging system which consists of the two polarizers, the ZnTe crystal and the InGaAs linear image sensor. The comparatively (with respect to the CTR pulse) long
laser pulse necessitates almost perfect extinction in cross-polarization (better than $10^{-5}$). Extinction levels of $10^{-6}$ were achieved using commercially available Glan laser polarizers. However, the crystal degrades the total extinction by nearly two orders of magnitude due to strain induced birefringence as will be shown in section 4.2.2. The best achieved extinction levels are $4 \cdot 10^{-5}$. The InGaAs array detector was selected for the experiment due to the high responsivity at the fundamental wavelength of the Nd:YAG probe laser, $\lambda = 1064$ nm (c.f. section 4.2.3). The chapter is concluded with a characterization of the synchronization scheme of the experiment.

4.1 Active-mode-locked laser system

The probe laser system is one of the key components of the EO read-out system. The phase velocity of the sub-THz CTR wave has to be matched to the group velocity of the probe laser pulse in order to ensure a minimum phase walk between the CTR and the laser radiation. We intend to use a Zinc-Telluride (ZnTe) crystal in the experiment. Thus, the use of an infrared laser system is favourable. Furthermore, InGaAs detectors offers high sensitivity in the infrared wavelength range. The measurement set-up is intended to be used as a robust, easy-to-handle online bunch length monitor. Since Nd:YAG lasers ($\lambda = 1064$ nm) offer good beam quality as well as high efficiency, and are also maintenance-free, these lasers are the best choice in the EOA experiment.

The great advantage of the monitor presented in this thesis is the robust timing of the set-up. This is achieved to a large extent with a synchronization scheme which is much more relaxed than the synchronization needed in other bunch length methods based on sub-picoseconds optical pulses (c.f. chapter 1). Therefore, the measurement window, as given by the laser pulse width, is selected to be considerably larger than the electron bunch length. For the 2 ps long electron bunch at the SLS pre-injector the probe laser pulse width was chosen to be in the order of 500 ps. The long laser pulse eases the overlap with the short CTR pulse.

The following requirements are specified for the probe laser system: Since the SLS LINAC is phase-locked to the 500 MHz master oscillator of the SLS an active mode-locked laser system is preferred. The possibility of active mode-locking by an acousto-optical modulator allows easy synchronization to the aforementioned SLS master oscillator. Furthermore, a Nd:YAG active mode-locked laser system produces pulses with widths in the order of some hundred picoseconds and smooth temporal profiles. The timing jitter of the laser system is specified to be $1/10$ of the laser pulse width, which is sufficient to produce a stable overlap between laser pulse and CTR. The laser beam must be linearly polarized.
Using the Jones Vector representation in order to describe the polarization state of the laser beam, it is found, that every polarization can be described using two Jones vectors only [Yar84]. Thus, let us assume, that the polarization of our laser beam shall be described by both linear and a circular polarized components. The use of a polarizer can only extinct 50% of the circular polarized component. We intend to achieve an extinction of $10^{-4} \cdot I_{\text{laser}}$. Therefore, using two Glan optical polarizers, the linear polarization grade of the probe laser beam must be better than 1000:1 (in intensity). Since, the optical cavity of the laser system used in the experiment is based on mirrors and polarizers, a polarization grade better than 1000:1 must be achievable performing multiple roundtrips (mode-locked laser). The laser beam is required to be of single transverse mode ($TEM_{00}$) in order to obtain a smooth transverse profile. A flash lamp pumped, external triggerable (repetition rate 1 - 10 Hz) active mode-locked laser system from Quanta System (SYL-A1 NF) was selected for the EOA experiment. In the following a description of the working principle of the laser is given.

### 4.1.1 Active-active-mode-locking-master oscillator with negative feedback and regenerative amplifier

The laser oscillator is based on a single 7 mm diameter 115 mm long Nd:YAG rod with end faces cut parallel at an angle of 3.5 degrees. It is pumped with 30 J of energy by a single flash lamp [SPA]. The laser consists of a standard stable resonator optimized for single transverse mode ($TEM_{00}$) oscillation using an intra cavity aperture. The master oscillator (MO) is based on the concave high reflective mirror M1 and the partial reflective plane mirror M2 and is folded by a prism. Mode-locking is achieved by an acousto-optic standing-wave modulator which is fed with about 5 W of RF power at 50 MHz RF obtained by division of the SLS master clock signal by a factor of 10. The acousto-optic modulator provides optical modulation at two times the 50 MHz RF, matching the 10 ns optical round trip time of the cavity.

Since Nd:YAG is a high gain medium Nd:YAG lasers are typically used in high power applications. The mode-locking process however depends on the number of round trips in the cavity [Sie86]. Hence, this necessitates a mechanism allowing to stabilize the intra-cavity pulse energy to a low level in order to maximize the number of round trips. This is achieved by the so-called negative feedback (NF), which modulates dynamic losses in the MO cavity (c.f. figure 4.3). The mirror M2 transmits 40% of the light pulse and thus introduces steady losses in order to minimize the intra-cavity pulse energies. The feedback depth is proportional to the light transmitted by the mirror M2 and reflected
by mirror M4 on to the photo conductive element NF. The voltage proportional to the detected pulse of light is applied to the Pockels’ cell PC1. Applying high voltage to the Pockels’ cell PC1 starts the evolution of the laser process in the MO (Q-switch) as is shown in figure 4.3. The additional steady losses introduced by the quarter-wave plate QWP1 and the polarizer P1 are compensated by the rotation of polarization due to the Pockels’ cell PC1. Correct setting of the steady losses and fine alignment of the light signal onto the negative feedback cell allows the NFC to stabilize the intra cavity pulse energy and to obtain a stable oscillation with slightly modulated pulse energy over a duration of some microseconds. Therefore, around 200 round trips are achieved resulting in a final pulse duration of 300-500 ps.

![Graph](image)

**Figure 4.3:** Q-switch (black line) and signal of photo diode PD1 (red line) monitoring the MO cavity. The evolution of the laser process in the MO starts when high voltage is applied to the Pockels’ cell PC1 (Q-switch). The NF stabilizes the intra cavity pulse energy over a duration of some microseconds.

The stabilized light pulse produced in the MO is injected into the regenerative amplifier (RA) (consisting of mirror M1 - polarizers P1 and P2 and mirror M3) for amplification. The MO cavity is dumped when the quarter-wave voltage is applied to the double Pockels’ cell optical switch (PC2 and PC3) rotating the polarization by $\frac{\pi}{2}$ and reflecting the pulse by polarizer P2. The RA features the same active medium as the MO with low optical losses for efficient extraction of the energy stored in the mode volume of the laser rod. The optical pulse is finally extracted after a fixed delay by the optical switch based on Pockels’ cell PC4 and polarizer P1 (see figure 4.4). Single pulse energy of 0.5-1 mJ is available at the output of the RA.
Figure 4.4: Left side: Photo diode PD2 monitoring the regenerative amplifier. The RA cavity dump is disabled. Right side: Photo diode PD2 monitoring the regenerative amplifier (black line). The RA cavity dump is engaged. Photo diode PD3 detects the extracted pulse (orange line).

Figure 4.5: Schematic overview of active-active-mode-locked Nd:YAG laser SYL-A1 NF (Negative-Feedback) by Quanta System S.P.A.

List of optical components

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Mirror HR at 1064 nm, $\theta = 0^\circ$, RoC -5000 mm, diameter 25 x 6 mm</td>
</tr>
<tr>
<td>M2</td>
<td>Mirror R = 60 % at 1064 nm, $\theta = 0^\circ$, PL/PL, diameter 25 x 6 mm</td>
</tr>
<tr>
<td>M3,4</td>
<td>Mirror HR at 1064 nm $\theta = 0^\circ$, PL/PL, diameter 25 x 6 mm</td>
</tr>
<tr>
<td>M5</td>
<td>Mirror HR at 1064 nm $\theta = 45^\circ$, diameter 25 x 6 mm</td>
</tr>
<tr>
<td>M6</td>
<td>Mirror HT at 1064 nm $\theta = 45^\circ$, diameter 25 x 6 mm</td>
</tr>
</tbody>
</table>
4.1. Active-mode-locked laser system

FP Folding prism
WP BK7, 1° wedged optical plate, AR/AR at 1064 nm, diameter 25 x 5 mm
PC1,2,3,4 $KD^*P$ Pockels’cell, AR/AR at 1064 nm, 8.3 x 8.3 x 20 mm
HWP Half-wave plate AR/AR at 1064 nm
QWP1,2 Quarter-wave plate AR/AR at 1064 nm
P1,2,3,4 Thin film polarizer, $T_p$ 98 %, $T_s$ 0.2 %, diameter 30 x 1.8 mm
Nd:YAG 7 x 115 Nd:YAG rod, end faces 3.5° parallel, AR/AR at 1064 nm
A 1.7 mm diameter limiting aperture for TEM00 operation
SHG $KD^*P$ type II, 14 x 14 x 25 mm, AR/AR at 532 and 1064 nm
AOM Acousto Optic Modulator, end faces tilted parallel, 45 mm, quartz
PD1,2,3 Photo diodes
NF Negative feedback cell

Abbreviation Explanation
HR High reflectivity
HT High transmission
R Reflectivity
RoC Radius of Curvature
PL Planar
AR Antireflective

4.1.2 Measurement of Nd:YAG time profile

The temporal profile of the active-active-mode-locked Nd:YAG laser SYL-A1 NF by Quanta System S.P.A. was measured using a streak camera with 2 ps time resolution\(^1\), the results

\(^1\)Optoscope dual sweep, synchroscan streak camera system, which is equipped with one fast (horizontal) and two selectable slow (vertical) time bases. The 250 MHz, fast deflecting time base of the streak camera
are presented in figure 4.6 below. Alternatively an ultrafast photo diode (Alphalas UPD series InGaAs with a bandwidth of 5 GHz) together with a 8 GHz digital oscilloscope was used to monitor the laser pulse. This configuration was selected as on line diagnostics for the laser system during the EOA experiment.

The average (over 20 shots) pulse width measured is 300 ps with a standard deviation of 30 ps. The acousto-optic modulator provides optical modulation at two times the RF of 50 MHz. The master oscillator was optimized for highest number of cavity round trips. The measured pulses were extracted at the end of the Q-switch after 4 $\mu$s, yielding 400 round trips in the master oscillator (MO). According to [Sie86] the smallest pulse width is given by $\tau_0 \approx 0.5/\sqrt{Nf_m}$ where $N$ denotes the number of round trips in the cavity and $f_m$ is the RF frequency of the acousto-optical standing-wave modulator. Therefore the shortest pulse width expected for 400 round trips is calculated by $\tau_0 \approx 250$ ps. For these measurements the laser was optimized for maximum number of round trips in the MO. However, due to thermal stability problems an extraction after 200 round trips leads to a more stable and reproducible situation which results in a typical pulse width of 500 ps $\pm$ 50 ps. Hence, the normal pulse width during the experiment is in the order of 500 ps.

The jitter in respect to the 500 MHz SLS master clock was determined with the streak camera, as it is synchronized to 250 MHz obtained by division of the SLS master clock. Hence, the timing jitter in respect to the 500 MHz is given by the standard deviation of the center of mass $\mu$ of the fitted Gaussian temporal distribution. The jitter which can be attributed to jitter of the electronics of the laser is found to be in the order of 60 ps. Hence, the jitter could be lowered by an improved electronic board of the laser. However, the current timing jitter is sufficient for the synchronization of the laser with the accelerator as the width of the laser pulse is still one order of magnitude larger than the jitter.

### 4.2 Electro-optical imaging system

The electro-optical imaging system consists of a 10 mm $\times$ 10 mm $\times$ 1 mm Zinc-Telluride (ZnTe) crystal, two Glan optical polarizers and an InGaAs linear image sensor (LIS) with 256 pixels of 50 micron pitch. In the first part of this section the linear electro-optical effect in a crystal of cubic symmetry as for Zinc Telluride (ZnTe) is described yielding the preferred orientation of the crystal and the respective polarization of the CTR pulse. The following part presents measurements of the extinction in cross-polarization. The section concludes with a description of the InGaAs linear image detector.
4.2. Electro-optical imaging system

![Graph showing a laser pulse with a time profile and counts in arbitrary units.](image)

**Figure 4.6:** Streak camera measurement of laser pulse optimized for smooth temporal profile. The bold black line gives a gaussian fit.

### 4.2.1 Electro-optical crystal

For the EOA experiment an electro-optical crystal is used which, under an externally applied electrical field, becomes optically active, i.e. the polarization of a linear polarized wave is rotated as it passes through the medium. In the absence of an external field, the crystal should be optically inactive, which is the case for crystals of cubic symmetry [Yar84] such as the "Zincblende" structure with symmetry $43m$. Another requirement is that the phase velocity in the low THz regime must be equal to the group velocity of the laser pulse. This is important in order to avoid phase walk-off between the sub-THz CTR field and the infrared probe laser pulse. For a Nd:YAG laser at its fundamental wavelength (1064 nm) two electro-optical crystals Zinc Telluride (ZnTe) and Gallium Phosphide (GaP) can be considered. ZnTe is favoured as the electro-optical effect is considerably stronger than in GaP [Yar84]. For ZnTe the linear Pockel’s effect is dominating and the quadratic Kerr effect can be neglected.

The crystal used in the experiment is cut in the [110]-plane as shown in figure 4.7. The probe laser pulse impinges along the $\vec{e}_3$ direction. The largest modulation can be observed when the CTR pulse is parallel to the X-axis (see figure 4.7). In this configuration the polarization of the incoming laser pulse is rotated by the angle $\Gamma(E_{CTR})$ which is calculated to be:

$$\Gamma(E_{CTR}) = \frac{2\pi d}{\lambda} n_0^3 r_{41} E_{CTR} = \frac{\pi d}{V_{\lambda/2}} E_{CTR}. \quad (4.1)$$

$V_{\lambda/2}$ is the so called half wave voltage which is needed to rotate the linear polarization by

---

The nomenclature refers to the contracted indices used to describe the Pockel’s tensor of the crystal (c.f. appendix B).
Chapter 4. Electro-optical imaging system and synchronization

Figure 4.7: Left side: base system of the crystal lattice. The crystal is cut in the [110]-plane. Right side: The incident probe laser pulse is parallel to the vector $\vec{e}_3$. The electrical field vector of both the CTR and the laser pulse lie in the [110] plane.

an angle of $\pi/2$

$$V_{\lambda/2} = \frac{2n_0^3r_{41}}{\lambda}, \quad (4.2)$$

where $\lambda$ is the wavelength of the probe laser, $n_0$ the index of refraction and $r_{41}$ denotes the Pockel’s coefficient. For ZnTe the half wave voltage $V_{\lambda/2}$ is 3.8 kV at 1064 nm. A detailed derivation of the angular dependence of the electro-optical effect of the ZnTe crystal is given in the appendix. At the SLS pre-injector LINAC we expect peak field strengths in the order of 2 kV/cm leading to a $10^\circ$ rotation of the polarization, which should be sufficient for the experiment.

According to equation 4.1 the intensity of the laser beam which is modulated is given by:

$$I_{mod} = \sin(\Gamma(E_{CTR}))^2. \quad (4.3)$$

Thus for small modulations $\Gamma(E_{CTR})$ the intensity of the modulated signal is proportional to the CTR intensity. Equation 4.3 highlights the major difference between the electro-optical auto-correlation (EOA) and the electro-optical sampling (EOS) experiment. The intensity of the laser beam which is modulated by the electrical field of the CTR is proportional to the square of the electrical field.

$^3$For a crystal of thickness 1 mm this corresponds to a field strength of 19 kV/cm.
4.2. Electro-optical imaging system

4.2.2 Cross-polarization configuration

One of the challenges of the EOA experiment is to achieve almost perfect extinction \(< 10^{-5}\) in cross-polarization configuration in order to achieve a sufficient signal to background ratio for resolving the spatial CTR auto-correlation pattern in the focal plane of the interferometer. This is challenging since the length of the Nd:YAG probe laser pulse is 500 ps compared to the signal itself which is expected to be as short as 2 ps.

Background considerations

The most dominant background sources, which contribute during the entire duration of the laser pulse (500 ps), are: the stray light scattered into the detector due to the limited extinction of the polarizer and the strain induced birefringence of the electro-optical crystal. The contributions are given by \(\epsilon_{\text{polarizer}}\) and \(\epsilon_{\text{strain}}\). Although polarizers with \(\epsilon_{\text{polarizer}} < 10^{-6} \cdot I_{\text{laser}}\) are used (which indicates, that the initial polarization grade of the laser is better than \(< 10^{-5}\)), the experimentally achievable total extinction \(\epsilon_{\text{total}} = \epsilon_{\text{polarizer}} + \epsilon_{\text{strain}}\) may not be better than \(10^{-4} \cdot I_{\text{laser}}\) due to the strain induced birefringence in the ZnTe crystal.

During the electro-optical sampling measurements at the SLS pre-injector LINAC maximum phase shifts in the order of \(10^6\) have been measured. According to equation (4.1) this yields an electrical field strength of \(2 \, \text{kV/cm}\) \((V_{\lambda/2} = 3.8 \, \text{kV/cm} \, \text{for ZnTe and } d = 1 \, \text{mm})\). This results in a peak signal of \(C_{\text{peak}} = 2 \cdot 10^{-2} \cdot I_{\text{laser}}\) for the EOA experiment (see equation (4.3)), where \(I_{\text{laser}}\) denotes the intensity of the probe laser pulse. The non-interfering background \(B_{\text{background}}\) (see equation (3.9)) is in the same order of magnitude as the signal \(C_{\text{peak}}\) itself. Both the signal levels and the background must be weighted as follows:

\[
\overline{C}_{\text{peak}} = \frac{\sigma_{\text{CTR}}}{\sigma_{\text{laser}}} \cdot C_{\text{peak}}
\]

\[
\overline{B}_{\text{background}} = \frac{\sigma_{\text{CTR}}}{\sigma_{\text{laser}}} \cdot B_{\text{background}}
\]

where \(\sigma_{\text{CTR}}\) corresponds to the electron bunch length and \(\sigma_{\text{laser}}\) is the width of the laser probe pulse. This yields \(\overline{C}_{\text{peak}} \approx 10^{-4} \cdot I_{\text{laser}}\) which is thus comparable to the limited extinction in cross-polarization \(\epsilon_{\text{total}}\). This should be sufficient to clearly determine the coincidence between CTR and probe laser pulse. Since \(\sigma_{\text{laser}} \gg \sigma_{\text{CTR}}\) the contribution \(\epsilon_{\text{total}}\) can be subtracted from the measurement profile as systematic background. The same applies to the non-interfering background \(\overline{B}_{\text{background}}\).

Since, the laser pulse is significantly longer than the CTR signal, the background due to the limited extinction (strain induced birefringence) can be considered to be uncoupled
from the interference signal. Therefore, the background \( (\epsilon_{\text{total}}) \) which corresponds to the stray light of the laser pulse without CTR applied on the ZnTe crystal) can be subtracted ideally from the measurement profiles. Since, it is not possible to record simultaneously a reference pattern (without CTR) with the measurement profile itself, fluctuations of the probe laser pulse might produce an additional noise background resulting from the subtraction. This problem is described in details in chapter 6 in which the limitations of the present monitor set-up are analyzed.

Similar to the above mentioned subtraction the non-interfering background \( (B_{\text{background}}) \) can be taken into account, however, this poses the same problem as described before. The intensity fluctuations of both the probe laser pulse and the CTR can be considered by averaging the reference measurements and by normalizing the signal profile as well as the reference patterns with respect to the intensity. However, the jitter of the transverse profile of the probe laser beam is difficult to compensate. The influence of the stability of the laser with respect to the signal-to-noise ratio is analyzed in details in chapter 6.

**Measurements in cross-polarization configuration**

Two identical Glan laser polarizers (Newport, Glan laser calcite, uncoated) were selected. The measurement set-up shown in figure 4.8 is used to determine the grade of extinction of the electro-optical imaging system.

\[
\mathbf{J} = \begin{pmatrix} \sin(\delta) \\ \cos(\delta) \end{pmatrix}
\]

\[
\mathbf{J} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

\[ \mathbf{J} = (1, 0). \] (4.4)

\( \mathbf{J} \) denotes the Jones vector [Yar84] which describes the polarization state of the laser beam. The second polarizer (analyzer) which is mounted in a motorized rotation stage (Newport SR50PP compact high-resolution rotation stage with stepper motor) is set at an angle \( \delta \) in

![Figure 4.8: Experimental set-up of the electro-optical imaging system.](image-url)
4.2. Electro-optical imaging system

respect to the vertical polarization $\vec{J} = (0, 1)$. The intensity which is transmitted through the set of two polarizers is given by:

$$T = I_0 \cdot \sin(\delta)^2$$  \hspace{1cm} (4.5)

where $I_0$ is the intensity of the incident laser pulse (the laser is horizontally polarized).

Figure 4.9 depicts a measurement of the intensity as transmitted through the set of the two polarizers in dependence of the angle $\delta$ of the second polarizer. The measurement is in good agreement with the theoretical expectations $(\sin(\delta)^2)$ as given by the solid red line. The best achieved extinction level is $8 \cdot 10^{-7}$ which corresponds to an angle $\delta = 0.05^\circ$ as depicted in figure 4.9.

The laser pulse is focused onto the linear image sensor by a cylindrical lens. It was found that multiple reflections occurred within the electro-optical imaging system. These reflections are vertically displaced. By adjusting the vertical position of the LIS detector, these spurious signals were suppressed.

When the crystal is placed between the two polarizers the extinction is degraded by nearly two orders of magnitude. This is mainly a result of strain induced birefringence. The smallest level achieved in cross-polarization was measured to be $4 \cdot 10^{-5}$ as found in figure 4.10. The extinction in cross-polarization with the ZnTe crystal of 1 mm thickness is determined from the crossing of maximum extinction and the sinus-square dependence of the transmission versus angle.

Several crystals were selected and measured. The results are presented in table 4.1. The crystal with the smallest strain induced birefringence (Nikko-Materials) was thus selected for the experiment.

![Figure 4.9](image-url)
<table>
<thead>
<tr>
<th>crystal</th>
<th>size [mm]</th>
<th>$\delta_0$ [deg]</th>
<th>extinction</th>
</tr>
</thead>
<tbody>
<tr>
<td>eV-Products</td>
<td>$10 \times 10 \times 1.0$</td>
<td>0.5</td>
<td>$8 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>eV-Products</td>
<td>$10 \times 10 \times 2.0$</td>
<td>1.0</td>
<td>$3 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>MaTeck</td>
<td>$10 \times 10 \times 0.5$</td>
<td>0.5</td>
<td>$8 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Nikko-Materials</td>
<td>$10 \times 10 \times 1.0$</td>
<td>0.4</td>
<td>$4 \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 4.1: Measured extinction levels achieved with different ZnTe crystals.

**Figure 4.10:** The extinction with the ZnTe placed between the polarizers is determined from the crossing of maximum extinction and the sinus-square dependence of the transmission versus angle as given by the solid line. The best extinction ($4 \cdot 10^{-5}$) was found with a crystal from Nikko-Materials [Asa04].

Thus, the achievable grade of extinction $\epsilon_{total}$ of the electro-optical read out system is in the order of $\epsilon_{total} = 10^{-4}$ which is sufficient to clearly see if coincidence between laser pulse and CTR is achieved.

### 4.2.3 InGaAs linear image sensor

The detector used to record the Nd:YAG probe laser pulse is a Hamamatsu G9203-256D InGaAs linear diode array of 256 pixels of 50 micron pitch and 250 micron height. InGaAs was selected due to the high sensitivity (about 0.75 A/W) at the fundamental wavelength (1064 nm) of the Nd:YAG laser. Figure 4.11 shows the wavelength dependent sensitivity [Pho]. The signal dynamics of the detector was measured to be 35 dB.

The readout electronics were designed by Dipl.-Ing. Kramert GmbH. The analog signal of the linear image sensor is digitized by a 16-bit ADC and then serialized in a CPLD\(^4\). The

\(^4\)Complex Programmable Logic Device
4.3. Synchronization scheme

signal is transferred to the VME\textsuperscript{5} dual ported RAM of the line image sensor interface [Dip]. The detector is controlled via the SLS controls network using an IDL [IRS] application.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{sensitivity.png}
\caption{Sensitivity of the Hamamatsu G9203-256D InGaAs linear diode array [Pho].}
\end{figure}

4.3 Synchronization scheme

Remote control of the EOA experiment is achieved by the SLS controls network. Since the main components of the experiment are located in the radiation area of the SLS pre-injector LINAC, remote control of the experimental parameters such as the phase between the interferometer arms or the rotation angle of the analyzer is of great importance. All relevant parameters are controllable over EPICS\textsuperscript{6} channels using a VME crate which contains specialist modules (VME cards). Thus, all SLS timing signals can be accessed through the so-called event receiver (ER) card. These timing signals can be further distributed by

\textsuperscript{5}In 1979 Motorola were developing their new 68000 CPU, and one of their engineers decided to set about creating a standardized bus system for 68000-based systems. Engineers of Motorola-Europe added a mechanical specification to the system, basing it on the Eurocard standard that was then late in the standardization process. The result is known as VMEbus, for VERSAmodule Eurocard bus (although some refer to it as Versa Module Europa VME) [WIK].

\textsuperscript{6}Experimental Physics and Industrial Control System. EPICS is a set of software tools, libraries and applications developed collaboratively and used worldwide to create distributed soft real-time control systems for scientific instruments such as a particle accelerators, telescopes and other large scientific experiments.
means of the event generator (EG) card. The EG card has both TTL and NIM outputs. The width of the pulses and their delays can be adjusted in steps of 20 ns.

The SLS pre-injector LINAC is operated at a frequency of 3.125 Hz and has to be synchronized to the 50 Hz AC line frequency, since the heating of the two klystrons is fed by the 50 Hz line current. Thus, the operation frequency of 3.125 Hz is generated by dividing the 50 Hz sine wave with a digital divider module. The gun trigger signal of the LINAC is phase-locked to the 500 MHz master oscillator of the SLS and the gun trigger can be delayed in steps of 2 ns in order to target specific RF buckets in the booster synchrotron. The SLS timing system distributes the LINAC pre-trigger event which is fixed in respect ($\approx$ 51 microseconds advanced) to the LINAC gun trigger.

The synchronization between the pre-injector LINAC and the active mode-locked laser pulse and the LIS detector is assured by the following means:

- Both the LINAC and the active mode-locked laser are phase-locked to the 500 MHz master oscillator of the SLS. The laser is synchronized to the 500 MHz clock since the acousto-optic modulator is fed with 50 MHz obtained by division of the SLS master clock signal by 10. The electronics of the laser itself (c.f. figure 4.12) is clocked by $1/4$ of the 50 MHz (12.5 MHz).

- The laser is triggered by the laser pre-trigger event which is generated by the SLS timing system and which is therefore phase-locked to the 500 MHz clock. The laser pre-trigger event fires the flash lamp. All succeeding events in the laser, such as the Q-switch, the MO cavity dump and RA cavity dump (see chapter 4) are fixed by the electronics of the laser and can be set by dip-switches on the electronic board of the laser (see figure 4.12).

  The laser pre-trigger can be delayed by steps of 880 ns in respect to the LINAC pre-trigger event. The EG card allows to adjust the pulse width and the delay of the laser pre-trigger in steps of 20 ns.

- The bunch of the LINAC can be shifted in steps of 2 ns by selecting the booster RF bucket in which the electron bunch is to be injected.

- The laser pulse can be fine-tuned within a time window of 2 ns by using a mechanical phase shifter in order to adjust the phase of the 50 MHz RF input which feeds both the acousto-optical modulator and the electronics of the laser.

\footnote{The SLS booster synchrotron with its circumference of 270 m has 450 RF buckets (500 MHz) which can be filled.}
4.3. Synchronization scheme

- The LIS detector is triggered by the LINAC pre-trigger event which can be delayed in steps of 20 ns by using the EG card. The integration time of the sensor is given by the width of the trigger pulse (minimum is 6000 ns) (see figure 4.13).

The timing jitter between the laser and the CTR pulse is given by the jitter between the laser and the 500 MHz master clock. This jitter was measured to be in the order of 50 ps (c.f. section 4.1). The relative jitter between the electron bunches and the linac RF was determined during the electro-optical sampling (EOS) experiments at the SLS LINAC [Win04b] to be in the order of 250 fs. Hence, the timing jitter is dominated by the relative jitter between the laser pulse and the SLS master clock, which corresponds to only 1/10 of the laser pulse width as specified and thus yields sufficient stability in order to produce a stable overlap between laser pulse and CTR. Furthermore, we can point out, that this figures underline the relaxed synchronization scheme of the experiment, which is the basis of the unique robustness of the presented monitor set-up.

![Diagram of synchronization scheme](image)

**Figure 4.12:** Timing of the EOA experiment.
An active-mode-locked Nd:YAG probe laser system was evaluated and set-up in the technical gallery of the SLS. The laser beam was inserted into the experimental area within the radiation bunker of the SLS pre-injector LINAC through a 15 m long optical transfer line.

The active-mode-locked Nd:YAG probe laser system was successfully synchronized to the electron bunch down to the level of 50 ps. This corresponds to 1/10 of the typical laser

**Figure 4.13:** Synchronization scheme of the EOA experiment.

### 4.4 Summary

An active-mode-locked Nd:YAG probe laser system was evaluated and set-up in the technical gallery of the SLS. The laser beam was inserted into the experimental area within the radiation bunker of the SLS pre-injector LINAC through a 15 m long optical transfer line.

The active-mode-locked Nd:YAG probe laser system was successfully synchronized to the electron bunch down to the level of 50 ps. This corresponds to 1/10 of the typical laser
pulse width as specified, and is sufficient to produce a stable overlap between laser and CTR pulse. The background as determined by the stray light scattered into the detector due to the limited extinction in cross-polarization configuration and the strain induced birefringence in the ZnTe electro-optical crystal is determined to be $\varepsilon_{\text{total}} \leq 10^{-4} \cdot I_{\text{laser}}$. The peak signal levels of the electro-optical sampling (EOS) experiment indicates that highest polarization rotations up to $10^\circ$ can be expected at the SLS pre-injector LINAC. This yields signals of $2 \cdot 10^{-2} \cdot I_{\text{laser}}$ (for a ZnTe crystal of 1 mm thickness). However, the comparably long probe laser pulse yields a much lower signal level. Since, the CTR pulse is much shorter than the laser pulse only a fraction of the probe laser pulse is modulated.

The signal to noise ratio can be improved by subtracting the two background profiles $\varepsilon_{\text{total}}$ (without CTR) and the non-interfering CTR background ($B_{\text{background}}$) from the actual measurement patterns. Here we must consider fluctuations of the transverse profile of the probe laser beam thus resulting in noise modulations of the difference profiles. The influence of the stability of the probe laser are analyzed in details in chapter 6, which presents the limitations of the present set-up.
Chapter 5

Single-shot bunch length measurements

In this chapter single-shot bunch length measurements performed at the SLS pre-injector LINAC are presented. In section 5.1 a description of the measurement procedures and the data acquisition is given. In the following section the signal analysis is introduced. The curve resulting from the analysis is fitted to the single-shot measured profiles for different machine settings of the SLS LINAC. The analysis includes several parameters which were determined by measurements or simulations. These parameters include the wavelength dependent spot size of the CTR on the ZnTe crystal (c.f. chapter 3), the spot size of the Nd:YAG probe laser pulse and the transfer function of the quasi-optical system as well as the overlap of the two signal paths in the focal plane of the spatial interferometer (see chapter 3). An error analysis yielding the sensitivity of the resulting bunch lengths on the aforementioned parameters is given in the third section.

5.1 Measurements

5.1.1 Measurement pre-alignement

The Nd:YAG laser has to be properly aligned onto the CTR spot in the focal plane of the spatial interferometer. Therefore the maximum of the CTR distribution is determined in a horizontal scan using a pyro-electric detector respectively a Golay cell detector. The background level of the measurement is minimized by achieving the optimum extinction in cross-polarization. Therefore, the signal in dependence of the rotation angle of the analyzer (see figure 4.8) is observed on the linear image sensor (LIS) and minimized.
Further optimization is possible by displacing the LIS detector in vertical direction. The detector is vertically adjusted in a way that multiple reflections within the optical imaging system, which could not be completely eliminated, are avoided (c.f. chapter 4). The optimum position (best extinction $\leq 10^{-4}$) is found where the signal level of the LIS is sensitive to small changes ($\pm 0.1^\circ$) of the rotation angle $\delta$ of the analyzer. The zeroth order fringe must coincide with the maximum of the CTR distribution in order to maximize the signal level. Thus, the phase of the interferometer is adjusted for equal path lengths. The detector is placed at the CTR maximum and the phase between the signal arms is scanned while moving the right toroid mirror (see figure 3.29).

The coincidence between CTR and the Nd:YAG probe laser pulse is established. Therefore the signal of the optical transition radiation (emitted through the hole in the first paraboloid) and the laser pulse is observed on the photomultiplier. In order to achieve coarse synchronization the OTR pulse is then set about 1 ns in advance of the laser. This is necessary since the CTR pulse is following a path within the spatial interferometer which is 30 cm longer than the one of the probe laser pulse (see figure 4.1). Fine-tuning is then achieved by adjusting the phase of the 50 MHz RF input of the active mode-locked laser. For monitoring, the LIS signal is integrated over all 256 pixels and is displayed on a strip-chart tool. Figure 5.1 depicts a strip-chart plot showing the coincidence between the CTR and the Nd:YAG probe laser pulse on the ZnTe crystal in the focal plane of the spatial interferometer. The integrated signal rises by a factor of 2 if the coincidence is achieved. The noise is due to intensity fluctuations of both the probe laser and the CTR source. The noise level is doubled when the the screen is inserted and CTR is applied on the crystal.

### 5.1.2 Data acquisition

Due to the fact that the interference pattern is convoluted with the spatial CTR distribution (see equation (3.37)) a background subtraction of the CTR spot is performed. Recalling equation (3.17) the auto-correlation pattern convoluted with the finite CTR spot is given by the following expression:

$$S(x) = \int_0^\infty I(x, \nu) I_{\text{spec}}(\nu) e^{-\frac{x^2}{\omega^2}} d\nu$$

(5.1)

where $I(x, \nu)$ is the interference pattern as generated by the monochromatic wave of frequency $\nu$ (c.f. equation (3.9)), and $I_{\text{spec}}(\nu)$ denotes the power density spectrum in the focal plane of the interferometer. The frequency dependent CTR spot is described by the Gaussian distribution (c.f. equation 3.37)).
5.1. Measurements

Figure 5.1: Coincidence between CTR and Nd:YAG laser pulse. The plot shows a strip-chart of the integrated signal of the LIS detector. The bold red line is the sliding average over 30 shots. The background is given at the start of the strip-chart when the laser is not inserted into the LINAC bunker. The target screen was then pulled out again several times indicating a rise by a factor of 2 due to the CTR induced birefringence.

Taking into account the aforementioned non-interfering CTR background a generic ”out-of-phase” profile is taken at an interferometer setting with a large arm length difference (5 mm). This corresponds to $\alpha_m \rightarrow 0$ (recall eq. (3.8)), which yields

$$S(x)_{\alpha_m \rightarrow 0} = \int_0^\infty I_{\text{spec}}(\nu) e^{-\frac{x^2}{(c/\nu)^2}} d\nu$$

(5.2)

Thus, the signal after the subtraction of the non-interfering background is given by:

$$S(x) - S(x)_{\alpha_m \rightarrow 0}.$$  

(5.3)

A second pattern is used to account for the LIS detector background as determined by a typical laser beam profile without CTR. The last profile is dominated by the the stray light scattered into the detector due to the limited extinction of the polarizer and the strain induced birefringence of the electro-optical crystal. Since it is not possible to take the two background profiles together with the actual measurement profile, the two former patterns are averaged in order to account for the intensity fluctuations of the probe laser and the LINAC. The latter profile is subtracted from both the CTR reference pattern as well as from the actual single-shot profile.

Both differences are normalized with respect to the signal intensity before the reference shot is subtracted. Since both the maximum value and the integrated signal over all pixels $\Sigma$ are strongly correlated as shown in figure 5.2 either of the two can be used to normalize the signal with respect to the signal intensity. The plot in figure 5.2 shows the typical ratio between the integrated signal over all pixels and the maximum value. The standard deviation over 100 shots is in the order of only 2 %. In the following the maximum is used...
for the normalization.

The difference then reveals the expected anti-correlated signal. The described procedure

![Figure 5.2: Ratio between the integrated signal over all pixels \( \Sigma \) and the maximum signal. The standard deviation over 100 shot is only 2 % indicating the strong correlation of the two values.](image)

is automized within an IDL routine [IRS]. The robust data treatment is explained in figure 5.3.

Furthermore, the data treatment reduces the noise background as shown in figure 5.4 which depicts both the normalized single-shot and the non-interfering background pro-

![Figure 5.3: Left side: Raw data. the single-shot LIS signal (bold black line) and the averaged CTR reference (dashed green line) taken with the interferometer set to an out-of-phase position. The averaged LIS signal when CTR is off, is given by the dashed red line. Right side: the detector background is then subtracted from both the single shot (bold black line) and the CTR reference shot (dashed green line), both differences are then normalized. The single-shot interference pattern (bold red line) is given by the difference of the single-shot profile and the averaged reference profile. This data acquisition is automized within an IDL routine.](image)
files. The subtraction of the latter reduces the noise background which is mainly due to imperfections of the electro-optical imaging system (c.f. chapter 6) by a factor of 5.

**Figure 5.4:** Intensity normalized single-shot and non-interfering background profiles. The subtraction reduces the noise background resulting from imperfections of the electro-optical imaging system.

Figure 5.5 shows single-shot profiles for different positions of the toroid mirror (different phases between the signal arms of the interferometer). Moving the toroid mirror alters the phase between the two signal paths of the interferometer. According to equation (3.16) the position of the zeroth order fringe is independent of the frequency but depends on the phase between the two signal arms of the interferometer. The characteristic minimum corresponding to the anti-correlated interference signal carries the information of the longitudinal bunch charge distribution and is moving with the phase between the two signal arms of the interferometer as expected.

**Figure 5.5:** Single-shot profiles for different position of the toroid mirror. The minimum corresponding to the zeroth order fringe moves with the phase as expected.
5.2 Signal analysis of the auto-correlation profiles

In the following the signal analysis is introduced for fitting bunch lengths to the interference profiles. In a first step we consider the signal of a monochromatic CTR source. Therefore, we assume two monochromatic plane waves in the focal plane of the interferometer each under the angle $\theta$ the spatial interference pattern of the CTR$^1$ is given by (c.f. equation (3.9)):

$$I_{CTR}(\nu, x) = (I_1 + I_2) \cdot (1 - \alpha_m \cdot \cos(4\pi\nu \sin(\theta)x/c))$$ (5.4)

where $\alpha_m$ denotes the modulation depth. In the above equation we also have to consider the Gaussian spatial distribution of the CTR in the focal plane of the interferometer:

$$I_i(\nu, x) \propto e^{-\frac{2x^2}{W^2}}, i = 1, 2.$$ (5.5)

Here we have assumed a perfect overlap in horizontal direction at $x = 0$ of the both signal arms of the interferometer and furthermore we presume that the spot size is frequency depending as introduced with the Gaussian distribution (see eq. 3.37). The out-of-phase CTR intensity in the focal plane which is needed for the background subtraction is thus given by

$$I_{background} = (I_1 + I_2) \cdot e^{-\frac{2x^2}{W^2}} = I \cdot e^{-\frac{2x^2}{W^2}},$$ (5.6)

assuming that the CTR intensity $I = (I_1 + I_2)$ is the same for the reference profile.

The size of the detection window is given by the transverse extent of the probe laser pulse. For simplicity we presume here a Gaussian laser spot described by the term $e^{-(2x^2)/(W^2)}$. Hence, the intensity profile of the read-out signal becomes

$$S_{CTR}(\nu, x) = e^{\frac{2x^2}{W^2}} \cdot I_{CTR}(\nu, x)$$ (5.7)

and the CTR background signal is computed by

$$S_{background}(\nu, x) = e^{\frac{2x^2}{W^2}} \cdot I_{background}.$$ (5.8)

So far we only have considered the signal of a monochromatic source of frequency $\nu$. The broadband signal in the focal plane of the interferometer depends on the power transmission of the quasi-optical system of the EOA experiment. The vertical mismatch of the two signal paths of the interferometer and the absorption in humid air must also be included (c.f. chapter 3). Thus, the overall power transmission of the set-up is given by:

$$T_{total}(\nu) = T_{quasioptical}(\nu) \cdot T_{overlap}(\nu) \cdot T_{H_2O}(\nu).$$ (5.9)

$^1$Due to the radial polarization of the CTR an anti-correlated interference signal is expected. Hence the interference must be proportional to $(1 - \alpha \cdot \cos(4\pi\nu \sin(\theta)x/c))$. 
5.2. Signal analysis of the auto-correlation profiles

The power density spectrum of the CTR source is described by:

$$I_{\text{source}}(\nu) = f(\nu) \cdot P(\nu),$$

where $f(\nu)$ denotes the form factor and $P(\nu)$ accounts for the CTR spectral energy flux as introduced in chapter 2. Hence, the broadband interference signal measured by the detector is given by:

$$S(x) = \int_0^\infty (S_{\text{CTR}}(\nu,x) - S_{\text{background}}(\nu,x)) \cdot I_{\text{source}}(\nu) \cdot T_{\text{total}}(\nu) d\nu.$$  (5.11)

This encompasses the overall transfer relation of the set-up.

The thus resulting power density spectrum in the focal plane of the interferometer is given by $I_{\text{spectrum}}(\nu) = I_{\text{source}}(\nu) \cdot T_{\text{total}}(\nu)$ and is shown in figure 5.6 together with the resulting interference pattern assuming a Gaussian longitudinal bunch charge distribution of $2\sigma$ width.

![Figure 5.6: Left: Gaussian longitudinal bunch charge distribution for $2\sigma = 1.5$ ps (red line), 2.0 ps (dashed black line) and 3 ps (dashed-dotted green line). middle: resulting power density spectrum in the focal plane of the interferometer. Right: interference pattern.](image)

5.2.1 Bunch length fits

The optimization of the integrated CTR signal on a FIR detector (Golay cell, resp. pyroelectric detector) generates the shortest possible bunch lengths behind the SLS pre-injector LINAC. However, the energy distribution of the bunches needs to be taken into account at the same time, since longitudinal density modulations within the bunch may lead to overtones in the CTR spectrum resulting in similar or even stronger increases of the integrated CTR intensity. Thus, maximum CTR intensity was generated while maintaining a smooth energy distribution on the ALIDI-SM-E monitor. Although this LINAC optimization procedure may not always lead to the maximum CTR intensity, it justifies the
assumption of a Gaussian longitudinal profile for the EOA data analysis [Scha].

The fixed parameters are summarized in table 5.1. The fit parameters are the bunch length \((2\sigma)\) and the overall sensitivity \(\alpha\) with respect to the intensity modulation depth of the recorded auto-correlation pattern which is defined by substituting \(\alpha_m \to \alpha\) in equation (5.4) and by writing.

\[
\alpha = \alpha_{eo} \cdot \alpha_i \cdot \alpha_m, \tag{5.12}
\]

In this approach the overall sensitivity is a product of the different sensitivity factors \(\alpha_k \leq 1, k = m, i, eo\) regarding the main components of the monitor set-up. \(\alpha_m\) denotes the modulation depth of the interferometer (contrast) according to equation (3.8) which depends on the intensity ratio in the two signal arms. For typical intensity ratios \(I_1/I_2 > 0.5\) the modulation depth \(\alpha_m\) is near to unity.

Setting \(\alpha = \alpha_m\) would assume both a perfect interferometer set-up \((\alpha_i = 1)\) and a perfect electro-optical imaging system \((\alpha_{eo} = 1)\). Hence, \(\alpha_i\) describes the sensitivity of the interferometer which includes the degrading of the auto-correlation pattern due to the tilt of the interference fringes. Where as, the parameter \(\alpha_{eo}\) denotes the sensitivity of the electro-optical imaging system namely the ability to translate the interference pattern into the polarization modulated transverse profile of the probe laser pulse and to sample this profile onto the detector (linear image sensor). Thus, the overall sensitivity is the product of all individual factors.

<table>
<thead>
<tr>
<th>parameter</th>
<th>symbol</th>
<th>value</th>
<th>unit</th>
<th>remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle of incidence</td>
<td>(\theta)</td>
<td>30</td>
<td>(^{\circ})</td>
<td>-</td>
</tr>
<tr>
<td>laser spot width</td>
<td>(W)</td>
<td>4.45</td>
<td>mm</td>
<td>-</td>
</tr>
<tr>
<td>quasi-optical low freq.</td>
<td>(\nu_{LF-QO})</td>
<td>130</td>
<td>GHz</td>
<td>see figure 3.32</td>
</tr>
<tr>
<td>quasi-optical high freq.</td>
<td>(\nu_{HF-QO})</td>
<td>1340</td>
<td>GHz</td>
<td>see figure 3.32</td>
</tr>
<tr>
<td>overlap low freq. cut-off</td>
<td>(\nu_{\text{overlap}})</td>
<td>270</td>
<td>GHz</td>
<td>see figure 3.32</td>
</tr>
<tr>
<td>CTR spot size</td>
<td>(\lambda/d)</td>
<td>(\lambda/0.51)</td>
<td>mm</td>
<td>see figure 3.35</td>
</tr>
</tbody>
</table>

**Table 5.1:** Fixed parameters of the data analysis according to formula 5.11.

The bunching process in the SLS pre-injector relies on velocity bunching as pointed out in chapter 1. This scheme is applicable for non-relativistic electrons \((E < 4 \text{ MeV})\) by using RF bunching cavities. A particle with higher energy arrives too early and is decelerated by negative RF voltage while a low energy particle arrives too late and is accelerated by positive RF voltage. Thus the tail particles catch up, the leading particles slow down and the bunch is compressed after some drift space. Hence, the detuning of the cavity phase allows to lengthen respectively to shorten the bunch. This behaviour was measured in
5.2. Signal analysis of the auto-correlation profiles

single-shot operation while adjusting the phase setting of the first 3 GHz bunching cavity (ALIRF-PBU).
The bunch lengths fitted are based on the analysis summarized in equation (5.11). The fits have been achieved by using the built-in function "NonlinearFit" in Mathematica, which allows to perform a least-squares fit to a list of data. The estimates of the model parameters are chosen to minimize the \( \chi^2 \) merit function given by the sum of squared residuals \( \sum_i e_i^2 \). Figure 5.7 shows single-shot profiles for different phase settings of the PBU bunching cavity. The fitted bunch widths (2\( \sigma \)) and modulation depths (\( \alpha \)) for the profiles in figure 5.7 are summarized in tables 5.2 and 5.3 together with the asymptotic confidence intervals for both the parameters and the asymptotic statistical error (stat.). Additionally for the bunch widths the systematical error (syst.) according to the analysis in section 5.3 is given.

<table>
<thead>
<tr>
<th>PBU phase [°]</th>
<th>2( \sigma ) [ps]</th>
<th>( \pm ) stat. [ps]</th>
<th>( \pm ) syst. [ps]</th>
<th>conf. int. 95 % [ps]</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>4.28</td>
<td>0.14</td>
<td>0.64</td>
<td>[4.00, 4.56]</td>
</tr>
<tr>
<td>64</td>
<td>3.09</td>
<td>0.09</td>
<td>0.46</td>
<td>[2.91, 3.27]</td>
</tr>
<tr>
<td>66</td>
<td>2.07</td>
<td>0.05</td>
<td>0.31</td>
<td>[1.94, 2.19]</td>
</tr>
<tr>
<td>68</td>
<td>1.61</td>
<td>0.05</td>
<td>0.24</td>
<td>[1.50, 1.72]</td>
</tr>
<tr>
<td>70</td>
<td>1.56</td>
<td>0.05</td>
<td>0.23</td>
<td>[1.46, 1.66]</td>
</tr>
<tr>
<td>72</td>
<td>1.50</td>
<td>0.04</td>
<td>0.23</td>
<td>[1.42, 1.58]</td>
</tr>
<tr>
<td>74</td>
<td>1.56</td>
<td>0.05</td>
<td>0.23</td>
<td>[1.46, 1.64]</td>
</tr>
<tr>
<td>76</td>
<td>2.58</td>
<td>0.11</td>
<td>0.39</td>
<td>[2.35, 2.81]</td>
</tr>
</tbody>
</table>

Table 5.2: Fitted bunch widths (2\( \sigma \)) for the profiles in fig. 5.7

<table>
<thead>
<tr>
<th>PBU phase [°]</th>
<th>modulation depth ( \alpha )</th>
<th>( \pm ) stat.</th>
<th>conf. int. 95 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>0.25</td>
<td>0.03</td>
<td>[0.19, 0.31]</td>
</tr>
<tr>
<td>64</td>
<td>0.21</td>
<td>0.02</td>
<td>[0.17, 0.25]</td>
</tr>
<tr>
<td>66</td>
<td>0.35</td>
<td>0.05</td>
<td>[0.28, 0.42]</td>
</tr>
<tr>
<td>68</td>
<td>0.42</td>
<td>0.05</td>
<td>[0.33, 0.51]</td>
</tr>
<tr>
<td>70</td>
<td>0.38</td>
<td>0.04</td>
<td>[0.30, 0.45]</td>
</tr>
<tr>
<td>72</td>
<td>0.43</td>
<td>0.05</td>
<td>[0.36, 0.51]</td>
</tr>
<tr>
<td>74</td>
<td>0.41</td>
<td>0.04</td>
<td>[0.28, 0.40]</td>
</tr>
<tr>
<td>76</td>
<td>0.34</td>
<td>0.05</td>
<td>[0.23, 0.45]</td>
</tr>
</tbody>
</table>

Table 5.3: Fitted modulation depths (\( \alpha \)) for the profiles in fig. 5.7
Discussion

The bunch lengths \(2\sigma\) obtained by the fits on the single-shot data shown in figure 5.7 vary in a range between 1.5 ps and 4.3 ps. The zeroth order fringe is clearly visible in all of the recorded profiles. Changes of the phase setting of the bunching cavity which lead to an increase in the bunch length result in a widening of the characteristic dip. Thus, the effect of this changes could easily be traced on-line with the monitor.

As shown in table 5.3 the peak overall sensitivity \(\alpha\) with respect to the intensity modulation depth (contrast) is found to be in the order of 0.4 (in dB units: \(10 \cdot \log(\alpha) = -4\) dB). The vertical integration by the cylindrical lens onto the linear image sensor results in a degrading \(\alpha_i\) of the auto-correlation pattern caused by the tilt of the interference fringes (see chapter 3) in the order of -1 dB. Hence, by recalling equation (5.12) the efficiency of the electro-optical read out \(\alpha_{eo}\) is computed to be -3 dB.

The zeroth order fringe is clearly visible in all of the recorded profiles. However the signal to background ratio of the EO read out is curtailed by noise sources which had not been considered so far. These additional noise sources are analyzed in more details in chapter 6 which describes the limitations of the present EOA set-up. The results depend mainly on the CTR signal intensity respectively the signal to background ratio of the EO read-out and the Nd:YAG laser stability.
5.2. Signal analysis of the auto-correlation profiles

Figure 5.7: Single-shot measurements for different machine settings (phase of pre-buncher cavity [ALIRF-PBU]). The red bold lines presents a fit according to equation (5.11) for a Gaussian longitudinal charge distribution. The first error indicated accounts for the statistical error resulting from the fit, while the second gives the systematical error according to the error analysis.
The average bunch width of ten single-shot measurements together with their standard deviations for different PBU phase settings are plotted in figure 5.8. Bunch widths \(2\sigma\) between 1.5 and 3.8 ps have been measured for different PBU phases. This is in good agreement with EOS measurements performed at the SLS LINAC [Win04a] and with PARMELA simulations [Ped]. The shortest bunches were measured for a PBU phase between 70° and 72°.

**Figure 5.8:** The bunch length \((2\sigma)\) versus pre-buncher phase. For each PBU phase the average of 10 shots and its standard deviation is given. The solid line represents a quadratic fit.

Bunch length variations due to different PBU phase settings have been measured with a resolution down to 200 fs. The smallest deviation which is clearly distinguishable is in the order of 200 fs observed between a PBU phase of 68° and 70° as is shown in figure 5.9. This is in reasonable agreement with the theoretical optimum resolution of the present set-up of 150 fs, which is determined by the 50 \(\mu\)m pitch of the pixel detector and the angle of incidence \(\theta\) of the two signal arms of the interferometer (c.f. chapter 3).

**Figure 5.9:** The bunch length \((2\sigma)\) versus pre-buncher phase. The smallest measured variations due to different phase settings of the pre-buncher cavity (between 68° and 70°) are in the order of 200 fs.
5.3 Error analysis: sensitivities on the parameters

In this section the sensitivity of the obtained bunch lengths on the fixed parameters is investigated. The bunch lengths are retrieved from the interference profiles using formula (5.11). A Gaussian longitudinal bunch charge distribution is assumed to fit the bunch widths \( (2\sigma) \) to the auto-correlation patterns. The fixed parameters used in the fit are summarized in table 5.1. The bunch lengths obtained by the fit depend on the choice of these parameter values.

The choice of the parameters has more influence on longer electron bunches as the coherent power density spectrum is more narrow band. Thus, the low-frequency attenuation of the quasi-optical set-up and the vertical mismatch of the two signal arms of the interferometer is more pronounced for longer wavelengths. Furthermore, the effect of the CTR spot size and the probe laser diameter is larger as the auto-correlation pattern of a longer bunch is wider. Hence an interference profile which results from a long bunch (pre-buncher (PBU) phase setting of \( 64^\circ \), bunch width \( 2\sigma = 2.97 \) ps) is used to investigate the sensitivity of bunch length fit on the parameters.

The retrieved bunch widths are normalized in respect to the original bunch width which results from the parameters given in table 5.1. In the following a worst case assumption is performed for all of the six parameters used in the signal analysis. The sensitivity \( \Delta \) is obtained by using a linear fit on the normalized bunch lengths as shown in figure 5.10:

- Misalignments of the quasi-optical system can lead to a deviation of the angle between the two signal arms of the interferometer. The laboratory measurements with cw-sources (c.f. chapter 3) of the spatial interferometer yielded an angle \( \theta \approx 26^\circ \) for each of the two interferometer arms. Hence, a maximum variation of \( \delta \theta = \pm 4^\circ \) is expected which results in a sensitivity of \( \Delta \theta = \pm 14.5 \% \) of the normalized bunch lengths. The analysis shows that the angle \( \theta \) is the most sensitive parameter for the bunch widths retrieved from the auto-correlation patterns.

- The target low-frequency attenuation of the quasi-optical system at 100 GHz is 50 \%. The simulation given in section 3.4 yields a transmission in the order of 60 \% at 100 GHz. Presuming that the real transmission varies by \( \pm 10 \% \) leads to a deviation \( \delta \nu_{LF-QO} = \pm 35 \) GHz and results in a sensitivity \( \Delta \nu_{LF-QO} = \pm 2.5 \% \) of the normalized bunch lengths.

- Similar to the above assumptions the real low frequency attenuation due to the vertical mismatch of the two signal arms of the interferometer is presumed to vary by \( \pm 10 \% \) at 100 GHz, which thus results in \( \delta \nu_{overlap} = \pm 90 \) GHz. This leads to a sensitivity of \( \Delta \nu_{overlap} = \pm 2 \% \) of the normalized bunch lengths.
The low-pass filter due to the hole in the first parabolic mirror yields a transmission of 80% at 1 THz. Similar to the above considerations we assume a variation of 10% of the real transmission leading to \( \delta_{\nu_{HF-QO}} = 500 \text{ GHz} \) which results in a negligible sensitivity of \( \Delta_{\nu_{HF-QO}} = \pm 0.2\% \) of the normalized bunch lengths.

The dimensionless parameter \( d \) which describes the frequency dependent CTR spot size (\( \lambda/d \)) in the focal plane of the interferometer was obtained by a fit to the data of a horizontal scan using equation (3.39). For the underlying coherent power spectrum a Gaussian shaped bunch of length \( 2\sigma = 2 \text{ ps} \) was assumed. As a worst case assumption the real width of the underlying bunch is presumed to vary by \( \pm 0.5 \text{ ps} \), which would thus lead to a different coherent power spectrum. This yields \( \delta_d = \pm 0.1 \) and thus results in a sensitivity of \( \Delta_{\lambda/d} = \pm 0.1\% \) of the normalized bunch lengths.

The typical laser spot diameter is in the order of \( W = 4.5 \text{ mm} \). Deviations of \( \delta_W = \pm 0.5 \text{ mm} \) lead to a sensitivity of \( \Delta_W = \pm 0.1\% \) of the normalized bunch widths.

The bunch width retrieved from the auto-correlation pattern are insensitive to the CTR spot size \( \lambda/d \), the laser spot size \( W \) and the high-frequency attenuation due to the hole in the first parabolic mirror. The analysis is summarized in table 5.4 below.

<table>
<thead>
<tr>
<th>parameter</th>
<th>symbol</th>
<th>deviation ( \delta )</th>
<th>unit</th>
<th>norm. sensitivity ( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle of incidence</td>
<td>( \theta )</td>
<td>\pm 4 ( \degree )</td>
<td>( \degree )</td>
<td>14.5 %</td>
</tr>
<tr>
<td>laser spot width</td>
<td>( W )</td>
<td>\pm 0.5 \text{ mm}</td>
<td>mm</td>
<td>0.1 %</td>
</tr>
<tr>
<td>quasi-optical low freq. atten.</td>
<td>( \nu_{LF-QO} )</td>
<td>\pm 35 \text{ GHz}</td>
<td>GHz</td>
<td>2.5 %</td>
</tr>
<tr>
<td>quasi-optical high freq. atten</td>
<td>( \nu_{HF-QO} )</td>
<td>\pm 500 \text{ GHz}</td>
<td>GHz</td>
<td>0.2 %</td>
</tr>
<tr>
<td>overlap low freq. cut-off</td>
<td>( \nu_{overlap} )</td>
<td>\pm 90 \text{ GHz}</td>
<td>GHz</td>
<td>2.0 %</td>
</tr>
<tr>
<td>CTR spot size (( \lambda/d ))</td>
<td>( d )</td>
<td>\pm 0.1 -</td>
<td>-</td>
<td>0.1 %</td>
</tr>
</tbody>
</table>

**Table 5.4:** Sensitivities of the bunch width on variations of the fixed parameters.

Therefore, we can compute an overall systematical error for the bunch lengths due to the above presumed variations of the parameters.

The overall systematical error is then computed by:

\[
\Delta_{\text{systematical}} = \sqrt{\sum_i \Delta_i^2} = 15\%,
\]  

(5.13)

where \( \Delta_i \) denotes the sensitivity of the bunch length on the individual parameter as listed in table 5.4. Thus, according to equation (5.13) for the bunch with length \( 2\sigma_0 = 2.97 \text{ ps} \)
5.3. Error analysis: sensitivities on the parameters

Figure 5.10: Variations of the six parameters summarized in table 5.1. The parameters include the angle of incidence ($\theta$) of the two signal arms of the interferometer; the diameter of the laser spot ($W$); the low-frequency attenuation of the quasi-optical set-up ($v_{LF-QO}$); the influence of the hole in the first paraboloid mirror ($v_{HF-QO}$); the overlap of the two signal arms ($v_{overlap}$) and the CTR spot size $\lambda/d$. For each plot a linear fit is given which yields the sensitivity of the normalized bunch length on the individual parameter. The error bars give the statistical error of the fit.

which was used to derive the above sensitivities a systematical error of $2\sigma_0 \cdot \Delta_{systematical}$ 0.45 ps is calculated.
5.4 Conclusions

The single-shot bunch length monitor has been implemented at the optical diagnostic port ALIDI-SM-5 of the SLS pre-injector LINAC. Electro-optical auto-correlation patterns of good quality have been successfully recorded in single-shot operation. The sensitivity of the set-up is found to be in the order of - 4dB. This yields an efficiency $\alpha_{eo}$ (see equation 5.12) of the electro-optical read-out system in excess of 50%. The bunch lengths (2$\sigma$ of a Gaussian bunch charge distribution) retrieved from these profiles using the signal analysis introduced in this chapter are in the range between 1.5 and 4 picoseconds. This is in good agreement with the EOS measurements which have been performed at the SLS LINAC [Win04a] and the expectations from PARMELA simulations of the accelerator [Ped]. Variations in the bunch lengths due to different phase settings of the bunching cavities have been reproducibly observed with a resolution down to 200 femtoseconds.

The current set-up has been successfully applied as an on line monitor for the optimization process of the SLS pre-injector LINAC for short bunches. Any change of the phase setting of the bunching cavities which leads to an increased bunch lengths result in a widening of the interference pattern which thus can be easily traced. Therefore, the monitor described in this thesis represents an indispensable tool for an operator to optimize the machine for shortest bunches.

Furthermore, the monitor set-up has demonstrated an unique robustness mainly due to the relaxed synchronization scheme of the experiment compared to the requirements of other bunch length methods. The synchronization of the CTR and the probe laser pulse is achieved typically in a very brief time due to simplicity of the method. Once the measurement pre-alignment of the the set-up was done the monitor was used during an entire shift without the need for further adjustments. Thus the bunch length monitor represents an easy-to-handle and viable tool for the optimization of the accelerator for short bunches.
Chapter 6

System limitations and improvement of the monitor set-up

The monitor described in this thesis was successfully used to measure bunch length at the SLS pre-injector LINAC in single-shot operation. Bunch lengths retrieved from the measured auto-correlation patterns have been determined to be between 1.5 ps and 4 ps. This chapter describes the limitations of the present EOA monitor set-up and points out possible improvements which should be included in a future set-up. The chapter is concluded with short summary.

6.1 Sensitivity and noise sources of the present set-up

In the first part of this section the sensitivity of the present measurement set-up is investigated. The sensitivity is defined with respect to the intensity modulation depth of the recorded interference patterns (c.f. section 5.2). The inherent degrading of the higher order interference fringes due to the convolution of the auto-correlation pattern with the finite CTR spot size must be considered and we must distinguish between the different orders of the interference fringes (c.f. equation (3.16)).

The second part of the section gives an analysis of the noise level of the present measurement set-up. In this context we must take into account the subtraction which is performed to get rid of the non-interfering CTR background. Due to fluctuations of the transverse profile of the probe laser beam the subtraction leads to the aforementioned noise level. Here we also have to include the distortions from the ideal (Gaussian) transverse laser beam profile which are caused by the strain induced birefringence in the electro-optical crystal as well as diffraction speckles.
Chapter 6. System limitations and improvement of the monitor set-up

Figure 6.1: Signal flow diagram of the EOA monitor.
6.1. Sensitivity and noise sources of the present set-up

All the noise sources are shown in figure 6.1 which presents a signal flow diagram of the present set-up. The figure gives the different sensitivities (c.f. eq. (5.12)) (in the context of contrast degrading of the recorded profile)\(^1\) of the transfer optics, spatial interferometer \((\alpha_m \approx 1, \alpha_i)\) and the electro-optical imaging system \((\alpha_{eo})\) which add up to the total sensitivity of the monitor with respect to the intensity modulation depth (contrast) as well as the different noise sources. The latter are intensity fluctuations of both the CTR and the laser pulses and transverse fluctuations of the probe laser beam. In this context also the disturbed pattern due to the strain induced birefringence and the diffraction speckles caused by the finite optics are considered. The data processing presented in the previous chapter reduces most of the aforementioned noise sources by averaging. The normalization with respect to the intensity of both the actual measurement profile and the non-interfering reference pattern lowers the influence of intensity fluctuations. The background patterns due to strain induced birefringence and diffraction speckles \((\delta_b)\) is reduced by the subtraction of the non-interfering background profile by a factor of 5 (c.f. figure 5.4). However, the subtraction causes modulations \((\delta_f)\) in the resulting difference profile due to transverse fluctuations of the probe laser beam.

The section is concluded with an analysis of the resulting signal to noise ratio of the present set-up.

6.1.1 Sensitivity of the set-up

The present EOA monitor set-up produces auto-correlation patterns of good quality with the zeroth order fringe clearly visible (S/N-ratio in the order of 7 dB). Since the spatial auto-correlation, which is produced in the measurement plane of the interferometer, is convoluted with the focal CTR spot, the signal patterns are obtained after a background subtraction taking a generic ”out-of-phase” profile at an interferometer setting with a large arm length difference into account (c.f. section 5.1).

The principle of this procedure is shown in figure 6.2, which presents a simulation of the auto-correlation pattern with the underlying coherent power density spectrum as measured with the Martin-Puplett interferometer (MPI) (see figure 3.4). The simulation includes the tilt of the interference fringes due to the asymmetrical layout of the interferometer, the frequency dependent CTR spot size (c.f. section 3.4) and the probe laser beam of extent \(W = 4.5\) mm. The two-dimensional plot gives the simulated ideal polarization modulated transverse profile of the probe laser beam behind the ZnTe crystal placed in the focal plane.

\(^1\)The sensitivity is defined with respect to the intensity modulation depth of the recorded auto-correlation patterns. In this context, the contrast degrading corresponds to a loss in the intensity modulation depth of the measured profile.
Figure 6.2: Simulation of the perfect polarization modulated transverse profile of the probe laser behind the ZnTe crystal. The underlying coherent power density spectrum is assumed as measured with the Martin-Puplett interferometer at the ALIDI-SM-5 monitor behind the SLS LINAC. The two (one-dimensional) plots show the resulting profiles which are recorded after the focusing onto the linear image sensor (LIS). Left side: single-shot profile (solid red line) and non-interfering background reference (dashed line). Right side: the difference shot which reveals the auto-correlation pattern.

of the interferometer. The vertical focusing of the laser pulse onto the linear image sensor array using a cylindrical lens corresponds to an integration in vertical direction. This is shown in the two succeeding plots. The left plot gives both the simulated single-shot (solid red line) respectively the non-interfering reference profile (dashed line). The right figure shows the difference of the two aforementioned profiles which reveals the auto-correlation pattern. The detector background resulting mainly from the limited extinction of the probe laser pulse (which is considerably longer than the signal itself) due to the strain induced birefringence in the crystal, is not included in the simulation. Since the laser pulse
is significantly longer than the CTR signal, the background due to the limited extinction (strain induced birefringence) can be considered to be uncoupled from the interference signal. Therefore, it would only represent a sort of background, which can simply be subtracted.

Definition of the sensitivity

In the following analysis we will compare relative intensities. Hence, it is useful to use dB units which are hereby introduced by:

\[
I_1/I_2[dB] = 10 \cdot \log(I_1/I_2)
\]  

(6.1)

What we want to measure are intensity modulations in the transverse laser profile. Therefore, the quantity of interest is the modulation depth of the recorded auto-correlation pattern. Thus, it is useful to define the overall sensitivity \( \alpha \) with respect to the intensity modulation depth (contrast) of the auto-correlation pattern by applying equation (5.4):

\[
I_{CTR}(\nu, x) \propto (1 - \alpha \cdot \cos(4\pi\nu \sin(\theta)x/c))
\]  

(6.2)

with the overall sensitivity respectively modulation depth \( \alpha \) (see equation (5.12)) which is assumed to be the same for all frequencies \( \nu \). This is justified in the following.

\[
\alpha = \alpha_{eo} \cdot \alpha_i \cdot \alpha_m.
\]  

(6.3)

\( \alpha_m \) denotes the modulation depth of the interferometer according to equation (3.8). For typical intensity ratios of \( > 0.5 \) between the two signal arms of the interferometer \( \alpha \rightarrow 1 \) for all frequencies. The term \( \alpha_i \) describes the efficiency of the interferometer which is degraded due to the tilt of the interference fringes as described in the third chapter, where \( \alpha_i \) is estimated to be in the order of \( 10 \log(\alpha_i) = -1 \) dB (see figure 3.38). As shown in section 3.4 the frequency dependent CTR spot size results in an attenuation of the modulation depth of the higher order interference fringes (see figure 3.36). According to the simulation shown in figure 3.36 the modulation depth of the first order fringes are attenuated by \( 10 \log(\alpha_i) = -2 \) dB, the second order by \( 10 \log(\alpha_i) = -5 \) dB and the third order by \( 10 \log(\alpha_i) = -10 \) dB with respect to a spot of infinite extent. Therefore we have to distinguish the different orders \( n \) of the interference fringes (c.f. equation (3.16)) in the analysis. This inherent attenuation of the visibility of the higher order fringes is the fundamental limitation of the monitor set-up. Taking into account the attenuation of the modulation depth due to the tilt of the interference fringes (\( 10 \log(\alpha_i) = -1 \) dB) and the degrading due to the frequency dependent CTR spot size (see figure 3.36) we compute:
Chapter 6. System limitations and improvement of the monitor set-up

<table>
<thead>
<tr>
<th>fringe order n</th>
<th>$10 \log(\alpha_n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1 dB</td>
</tr>
<tr>
<td>1</td>
<td>-3 dB</td>
</tr>
<tr>
<td>2</td>
<td>-6 dB</td>
</tr>
<tr>
<td>3</td>
<td>-11 dB</td>
</tr>
</tbody>
</table>

Since, both the extent of the monochromatic pattern and the finite spot size are correlated with the wavelength, the degrading of the modulation depth of the higher order fringes is expected to be the same for all frequencies (c.f. section 3.4.1). The term $\alpha_i$ is dominated by the degrading of the modulation depth due to the finite CTR spot size. Thus, it is justified to assume that $\alpha_i$ is independent of the frequency.

The term $\alpha_{eo}$ describes the efficiency of the electro-optical imaging system namely the ability to translate the interference pattern into the polarization modulated transverse profile of the probe laser pulse and to image this profile onto the detector (linear image sensor). The measured patterns showed peak modulations $\alpha$ in the order of $10 \log(\alpha) = -4$ dB (c.f. table 5.2) with respect to the zeroth order fringe. This yields an efficiency of the imaging system of $10 \log(\alpha_{eo}) = -3$ dB.

Thus, the overall sensitivity $\alpha$ with respect to the modulation depth adds up to:

<table>
<thead>
<tr>
<th>fringe order n</th>
<th>$10 \log(\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4 dB</td>
</tr>
<tr>
<td>1</td>
<td>-6 dB</td>
</tr>
<tr>
<td>2</td>
<td>-9 dB</td>
</tr>
<tr>
<td>3</td>
<td>-14 dB</td>
</tr>
</tbody>
</table>

The term $\alpha_{eo}$ is independent of the frequency of the CTR pulse. Since, $\alpha_i$ is independent of the frequency according to above considerations and $\alpha_m \to 1, \forall \nu$, the overall sensitivity too must be independent of the frequency. This justifies the above assumption.

For the coherent power density spectrum (see figure 3.4) at the SLS pre-injector LINAC measured with the Martin-Puplett interferometer (MPI) the next modulations beside the characteristic zeroth order fringe are simulated (c.f. figures 3.38 and 6.2) to be positioned at $\pm 1.5$ mm. This coincides with the second order and the third order fringes as the measured spectrum appears in a range between 200 GHz and 400 GHz (c.f. equation (3.16)). Thus, according to the above considerations the modulation depth ($\alpha$) of these adjacent fringes is expected to be in the range between $10 \log(\alpha) = -14$ dB and $10 \log(\alpha) = -9$ dB.
6.1.2 Noise sources of the present set-up

Since the auto-correlation pattern is convoluted with the finite CTR spot a background subtraction of the non-interfering CTR distribution recorded at an "out-of-phase" position of the interferometer is performed within the automated data acquisition procedure. The subtraction reveals the interference pattern (see figure 5.3). Intensity fluctuations of both the CTR and the probe laser are considered by normalizing both the single-shot measurement and the averaged background profiles (see section 5.2). However, shot-to-shot variations in position and size of both the probe laser pulse and the CTR spot result in modulations of the difference profile. In the following we estimate the fluctuations of the transverse profile of both the probe laser pulse and the CTR spot based on the assuming Gaussian profiles for both signals.

Fluctuations of the probe laser

Fluctuations of the transverse laser profile have been observed with the InGaAs linear image sensor (LIS). This jitter leads to modulations when the non-interfering reference pattern is subtracted from the measurement profiles. This is illustrated in figure 6.3, which shows the subtraction of two ideal Gaussian shaped profiles \( S(x) = e^{-2(x-\mu)^2/W^2} \) with \( \Delta \mu = \mu_1 - \mu_2 = 0.1 \text{ mm} \) and \( \Delta W = W_1 - W_2 = 0.1 \text{ mm} \).

![Figure 6.3](image)

**Figure 6.3:** Influence of stability of the transverse profile on the reference subtraction. The solid line gives the measurement profile, the dashed line the reference profile which is subtracted. Both simulations are assumed to be of Gaussian shape and are normalized with respect to the intensity. The variations of \( \Delta \mu = 0.1 \text{ mm} \) and \( \Delta W = 0.1 \text{ mm} \) lead to peak modulations of \( \delta_f = 0.03 \) \( (10 \log(\delta_f) = -15 \text{ dB}) \) in the difference pattern.
Figure 6.4 shows the influence of the pointing stability on the background subtraction (c.f. figure 6.3). The difference $\delta_f$ of two succeeding laser shots measured with the linear image sensor (LIS) are presented. The plots shows intensity normalized difference profiles in cross-polarization configuration with ZnTe crystal respectively without polarizers and crystal. The polarizers and the crystal introduce additional noise due to diffraction and strain induced birefringence in the crystal which will be treated later in this section. The

[Graph showing intensity normalized difference profiles with and without polarizers and crystal.]

**Figure 6.4:** The influence of the typical pointing stability on the background subtraction. Difference of two succeeding intensity normalized laser shots recorded with the linear image sensor (LIS)

IDL software has been used to fit a Gaussian distribution to the resulting profiles. The position stability (center of mass, $\Delta \mu$) was found to be in the order of $\pm 0.1$ mm and the largest standard deviation of the width $\Delta W$ is measured to be $\pm 0.3$ mm. Such variations of the laser beam lead to typical intensity modulations $\delta_f$ in the order of 5% ($10 \log(\delta_f) = -13$ dB) in the difference profile.

**Fluctuations of the CTR spot**

Similar to the transverse fluctuations of the probe laser a spatial jitter of the CTR spot will lead to modulations of the difference profile. As the CTR spot is of similar size as the transverse extent of the probe laser the depth of these modulations would be comparable for the same variations in position and size as measured for the laser. However, due to the fact that dispersion must vanish before the first bending magnet (ALIMA-BY) (c.f. figure 1.9) the beam stability is very high at the LINAC and therefore position variations of the CTR spot are negligible [Scha].
Diffraction due to optics apertures and strain induced birefringence in the EO-crystal

Additionally to the modulations of the difference profiles due to fluctuations of the probe laser beam we have to consider diffraction at the apertures of the polarization optics. The Nd:YAG laser is collimated for optimum injection into the LINAC bunker along the 15 m long transfer line. The transverse extent of the laser beam was typically about $W = 4.5$ mm, where $W$ denotes the Gaussian beam radius [Yar84]. The spot size was chosen large enough to illuminate a large portion of the $10 \text{mm} \times 10 \text{mm} \times 1 \text{mm}$ ZnTe crystal. The apertures of the polarization optics are of 10 mm diameter. Hence the laser beam is diffracted at the optics apertures. This results in a non-Gaussian profile on the LIS detector as depicted in figure 6.5. To avoid diffraction effects optics with larger apertures should be used. As a rule of thumb the radius of the optics should be double the size of the Gaussian beam $W$ [Joh].

Similar to the diffraction speckles we have to consider the background pattern which is produced by the strain induced birefringence resulting from inhomogeneities in the crystal. Due to the fluctuations of the transverse profile of the probe laser beam as described in the preceding section the spikes originating from diffraction and the pattern caused by the strain induced birefringence in the crystal cannot be fully eliminated by the background subtraction (c.f. figure 5.4). The subtraction results in a noise background $\delta_b$ which is assumed to be of similar order of magnitude than the intensity modulations $\delta_f$ due to fluctuations of the probe laser pulse.

Figure 6.5 shows the laser profile recorded with the InGaAs linear image sensor in cross-polarization configuration with the ZnTe crystal placed in the focal plane of the spatial interferometer but without CTR. The figure clearly illustrates the aforementioned effects (strain induced birefringence and diffraction). Since the laser pulse is significantly longer than the CTR signal, the background due to the limited extinction (strain induced birefringence) can be considered to be uncoupled from the interference signal. Therefore, it would only represent a sort of background, which can simply be subtracted.

Measurement of the noise level

In the following the noise level which is recorded during measurements is presented. Figure 6.7 shows 6 raw data profiles recorded with the InGaAs linear image sensor (LIS) before the subtraction of the non-interfering CTR background$^2$. The raw data profiles

$^2$The detector produces maximum signal levels of $V_{\text{max}} = 3.2$ V. The relative detector response gives the signal as normalized to $V_{\text{max}}$. 
show a distinct resemblance in their shape and the variations are mainly due to amplitude fluctuations. The similarity in shape of the raw date profiles is underlined by the strong correlation between the integrated signal $\Sigma$ (integrated over all pixels) and the maximum values of the profiles. The standard deviation (of 100 shots) of the ratio between integrated signal $\Sigma$ and maximum value is found to be smaller than 3 %.

The last two plots in figure 6.7 show the average of 20 single-shot raw data patterns and the averaged reference shot recorded at an "out-of-phase" position of the interferometer. In both profiles the detector background (cross-polarization configuration with ZnTe, but no CTR (c.f. figure 6.5)) is subtracted. Comparing the two last plots in figure 6.7 the characteristic dip of the zeroth order fringe can be anticipated. The subtraction of the non-interfering background (recorded at an "out-of-phase" position) reveals the interference pattern.

The left plot in figure 6.6 presents the standard deviation of 20 single-shot recorded raw data profiles. The pronounced similarity with the laser pattern with no coincidence of CTR and probe laser pulse (see figure 6.5) indicate that the noise is dominated by the laser fluctuations respectively the disturbed beam profile due to strain induced birefringence in the crystal and the diffraction speckles. The right plot in figure 6.6 shows the aforementioned standard deviation which is normalized with the average of the 20 single-shot raw data profiles (c.f. figure 6.7). The normalization reveals a noise background in the order of -8 dB caused by shot-to-shot intensity fluctuations of both the probe laser beam and the CTR pulse. The rise at 7.8 mm corresponds to the position of the zeroth order fringe and thus, can be attributed mainly to intensity fluctuations of the CTR. A background subtraction has to be performed in order to account for the non-interfering CTR background. This is performed by subtracting a reference profile recorded at an

---

**Figure 6.5:** the laser profile recorded with the InGaAs linear image sensor in cross-polarization configuration with the ZnTe crystal. Distortions due to diffraction and strain induced birefringence are clearly visible.
"out-of-phase" position of the interferometer. The detector background, which is recorded while there is no coincidence between probe laser beam and CTR, is subtracted in advance from both the aforementioned non-interfering CTR background and the single-shot recorded profiles. The intensity fluctuations of the probe laser beam and the CTR can be taken into account by normalizing both the resulting single-shot and the reference profiles. Due to the subtraction of the non-interfering reference profile, additional noise sources $\delta_f$ ($10 \log(\delta_b) \approx -13 dB$) resulting from spatial fluctuations of the transverse profile of the probe laser pulse have to be considered. Secondly we must take into account the disturbed profile due to diffraction at the polarizer optics and due to the strain induced birefringence $\delta_b$ ($10 \log(\delta_b) \approx -13 dB$), which as a consequence of the spatial fluctuations of the transverse profile of the probe laser beam do not fully cancel after the subtraction. Thus, according to the considerations which have been made in the previous section a total noise level $\delta_{total}$ (sum of the two aforementioned contributions ($\delta_f + \delta_b$)) in the order of $10 \log(\delta_f + \delta_b) = -10 dB$ is expected after the subtraction of the non-interfering background.

Figure 6.6: Left side: standard deviation of 20 single-shot recorded raw data profiles. Right side: standard deviation normalized with the average of the 20 single-shot raw data profiles.
Figure 6.7: Single-shot recorded raw data profiles. The non-interfering CTR background has to be subtracted in order to reveal the interference pattern. The $\Sigma$ gives the sum over all pixel which is found to be strongly correlated with the maximum value. The last two plots show the average of 20 single-shot raw data patterns and the averaged reference shot recorded at an "out-of-phase" position of the interferometer. The detector background (cross-polarization configuration with ZnTe, but no CTR (c.f. figure 6.5)) is subtracted.
6.1. Sensitivity and noise sources of the present set-up

Figure 6.8: Measurement of background noise intensity. The plot shows the average of 20 single-shot recorded interference profiles. The standard deviation confers to the background noise level and is measured to be in the order of -11 dB.

Figure 6.8 shows the average of 20 single-shot recorded auto-correlation patterns in which the normalization and the background subtraction had been performed. The error bars indicate the standard deviation of the 20 shots. The standard deviation corresponds to the background noise level $\delta_{total}$ which results from the spatial fluctuations and the distortions of the transverse profile of the probe laser beam. $\delta_{total}$ of the profile after the data treating is measured to be in the order of -11 dB which is in agreement with the above considerations. The signal itself (modulation depth of the zeroth order fringe) is in the order of -4 dB. Thus this yields a signal-to-noise ratio of 7 dB.
Influence of noise on bunch fits

In this section we estimate the influence of the above described noise level on the bunch lengths retrieved by the non-linear fit on the data (c.f. section 5.3). Therefore, we presume modulations resulting from spatial fluctuations of the probe laser beam, $\Delta \mu = \pm 0.1$ mm and $\Delta W = \pm 0.3$ mm. Additionally, we include an error $\Delta I_{\text{laser}} = \pm 5\%$ in the normalization with respect to the intensity of both the measurement and the reference profiles. The resulting simulated (equation (5.11)) auto-correlation patterns are shown in figure 6.9 for Gaussian electron bunch shapes of $2\sigma = 1.5$ ps, 3 ps and 4.5 ps. The solid line gives the ideal profile $\Delta \mu = 0$, $\Delta W = 0$ and $\Delta I_{\text{laser}} = 0$. The variations are presented by the dashed lines. The bunch length retrieved by the non-linear fit on all profiles is depicted. It is found, that these modulations result in variations ($\Delta(2\sigma)$) of the bunch lengths obtained from the profiles which are well below 5\%.

Figure 6.9: Simulated auto-correlation patterns for modulations of $\Delta \mu = \pm 0.1$ mm, $\Delta W = \pm 0.3$ mm and $\Delta I_{\text{laser}} = \pm 5\%$ of the probe laser pulse. The influence on the bunch lengths retrieved by the non-linear fit (c.f. equation (5.11)) on the profiles is found to be smaller than 5\%.

### 6.1.3 Signal to noise ratio

It is useful to define the following signal to background ratio $S/N$ for the EOA monitor:

$$\frac{S}{N} = \frac{\alpha}{\delta_{\text{total}}},$$

(6.4)

where $\alpha$ denotes the sensitivity with respect to the intensity modulation depth of the recorded auto-correlation pattern (associated with the contrast) and $\delta_{\text{total}}$ gives the total noise intensity level according to the above made considerations. Hence, this results in a signal to noise ratio $S/N = 6$ dB with respect to the zeroth order fringe. For the adjacent modulations we expect a signal to noise ratio in the range between -4 dB and 1 dB (c.f. section 6.1.1).
The background noise which is caused by the disturbed profile due to diffraction at the polarizer optics and due to the strain induced birefringence is typically on a pixel to pixel basis respectively introduces wiggles with a width of some tenths of a millimeter (c.f. figure 6.4). Thus, in order to clearly distinguish the interference modulations, a signal to noise ratio of 3 dB is needed. Furthermore, it must be noted, that we are talking about an \textit{rms} background noise. Therefore, by assuming a Gaussian noise distribution, a signal to noise ratio \(S/N\) in the order of 0 dB yields that for 32\% of the measurements the background noise can be larger than the signal itself. For a signal to noise ratio \(S/N\) of 3 dB the signal is larger than the noise background in 95\% of the cases and in 68\% the signal is larger by a factor of 2 than the noise. Hence, specifying a signal to noise ratio \(S/N\) of 3 dB should clearly reveal the interference modulations in more than 2 of 3 measurements, which is viable for a single-shot monitor. Therefore, it must be concluded that the smaller adjacent modulations are difficult to be observed with the present monitor set-up.

6.2 System improvements

The present monitor set-up has been successfully used to determine the bunch length of the SLS pre-injector LINAC in single-shot operation. Bunch lengths between 1.5 ps and 4 ps have been measured. Variations due to different machine settings have been observed with a resolution down to 200 fs. The preceding section has highlighted the limitations and drawbacks of the present monitor set-up with regard to the signal to noise ratio. This section points out possible improvements of the monitor in order to enhance the signal to noise ratio. The first part of the section describes possible refinements of the quasi-optical set-up of the experiment. The section is concluded with a summary of improvements of the electro-optical read out system.

6.2.1 Quasi-optical system

The two signal paths of the interferometer do not properly overlap at longer wavelengths. This results in an (additional) low frequency attenuation of the present set-up (c.f. section 3.4). This behaviour is mainly a consequence of the transfer optics of the experiment. The tilt of the interference fringes due to the asymmetrical layout of the interferometer leads to a degrading of the interference fringes. This could be either avoided by a vertical layout of the interferometer and the use of a wedged mirror as beam splitter or with a
vertical out-coupling of the CTR.
A revised optics of the spatial interferometer could produce a larger CTR spot which would thus result in lower attenuation of the higher order interference fringes.

Transfer optics

Two off-axis paraboloid mirrors transport the CTR pulse to the entrance focus of the spatial interferometer as described in chapter 3.
If the focal length of the parabolic mirror is set equal to the actual radius of curvature of the incoming phase front, the out-put waist will occur at the mirror and the outgoing beam will diffract away from the mirror [Wit95]. Due to the definition of the Rayleigh length $L_R = d^2/\lambda$ which characterizes the transition from the near-field to the far-field zone this will occur predominantly at long wavelengths. Hence at long wavelengths the transfer optics consisting of the two paraboloid mirrors will have a frequency depending focal length. This effect is observed while performing the GRASP simulations shown in

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{output_waist_mirror.png}
\caption{The out-put waist at the mirror. The outgoing beam will diffract away from the mirror. Due to the definition of the Rayleigh length this will occur predominantly at long wavelengths. For small enough wavelengths the beam between the parabolic mirrors is parallel.}
\end{figure}

figure 3.34. For longer wavelengths the two signal paths of the interferometer are vertically displaced due to the fact that the focal point of the transfer optics is not coinciding with the entrance focus of the interferometer. Since, the overlap is more pronounced for shorter wavelengths this effect results in a low frequency attenuation as described in chapter 3.

The influence of the transfer optics is shown in figure 6.11. The figure depicts two simulations with the GRASP³ software package. Simulations of the field patterns at 120 GHz are shown for the two interferometer arms in the focal plane of the interferometer. The left plot shows the simulation of the complete quasi-optical system including the transfer

³The simulations were performed at the Institute of Applied Physics (Microwave Department) of the University of Bern.
optics (see figure 3.34) while the right plot shows a simulation containing only the interferometer itself. In the first case the two signal paths are separated by 9 mm while without the transfer optics the interferometer arms are splitted by only 3.5 mm.

Hence, it might be advisable to dispense with the transfer optics and to set the entrance focus of the interferometer directly at the source of the CTR on the target screen. According to the above shown simulation this could improve on the low-frequency attenuation. However, the vertical shift of the two signal paths of the interferometer is not a consequence of the transfer optics alone (c.f. figure 6.11). Since the two CTR lobes are not propagating independently (the characteristic minimum in the emission results from the vectorial cancellation of the electrical fields) the splitting of the two lobes must affect the further propagation of the CTR and results in a truncation of the quasi-optical beam [Mur].

It must be noted once again, that the SLS LINAC is not predestined for the production of ultra-short bunches and that the resulting coherent power spectrum is thus dominated in the millimeter wavelength range, where the low frequency attenuation is particularly more problematic.
Tilt of the interference fringes

The tilt of the interference fringes is a result of the asymmetric layout of the spatial interferometer as shown in section 3.3. The tilt effects the modulation depth of the interference pattern (section 3.4) This could be avoided with a vertically symmetric layout of the interferometer. Such a set-up was simulated with the GRASP software package. Figure 6.12 shows the set-up of the simulation. The resulting vertically symmetric interference pattern is depicted in figure 6.13 for 60 GHz and 120 GHz.

![Quasi-optical set-up with a vertical symmetric layout of the interferometer. The use of a wedged mirror as beam splitter produces a symmetrical interference pattern.](image)

**Figure 6.12:** Quasi-optical set-up with a vertical symmetric layout of the interferometer. The use of a wedged mirror as beam splitter produces a symmetrical interference pattern.

![GRASP simulation of the symmetrical set-up (c.f. figure 6.12). By using a wedged mirror as beam splitter the two rays intercept in the yz-plane. This results in a symmetrical interference pattern.](image)

**Figure 6.13:** GRASP simulation of the symmetrical set-up (c.f. figure 6.12). By using a wedged mirror as beam splitter the two rays intercept in the yz-plane. This results in a symmetrical interference pattern.
Finite CTR spot size

Since the auto-correlation pattern is convoluted with the finite CTR spot size the modulation depth of the higher order fringes is degraded. This leads to an inherent attenuation of the adjacent modulations of the interference profile.

However, a revised interferometer optics with longer focal lengths yields a larger CTR spot in the measurement plane. This is depicted in figure 6.14 below which shows both the original interferometer and the revised one of double the size. The entrance focus is the same for both set-up’s. Figure 6.15 shows a simulation of the interferometer optics assuming Gaussian beams (for a frequency of 120 GHz). The focusing mirror is treated as a thin lens. Increasing the focal length and thus the overall path length of the interferometer results in a larger spot in the measurement plane. The simulation gives both the original optics (dashed line) and an optics with double the focal length, resulting in a spot of double the size (solid line). The CTR source is described by the formalism in chapter 2. On the other hand it must be noted that a larger CTR spot size leads to a smaller intensity, because the intensity is inversely proportional to the illuminated area. Thus, a CTR spot of double the size will result in only $1/4$ of the original intensity.

Figure 6.16 shows the simulation of the spatial auto-correlation pattern with the frequency dependent CTR spot which is assumed to be twice as large as with the present set-up (c.f. equation (3.37)). For the larger CTR spot the first order fringes are attenuated to $10 \log(\alpha_i) = -0.5$ dB while the second order fringes are reduced to $10 \log(\alpha_i) = -1.5$ dB.

![Figure 6.14: Both the original interferometer and the revised one of double the size.](image-url)
and the third order ones to $10 \log(\alpha_i) = -3 \text{ dB}$. Thus, this would represent an improvement of the fringe visibility by a factor of 2 compared to the present set-up.

**Figure 6.15:** Simulation of the interferometer optics as assumed with a Gaussian beam (for a frequency of 120 GHz). The original optics are given by the dashed line, while the filed line shows a simulation for an interferometer with double the focal length and the overall size.

**Figure 6.16:** The convolution of the spatial auto-correlation pattern with the frequency dependent CTR spot results in a degrading of the higher order fringes. The figure shows two plots for 100 GHz and 300 GHz respectively. The frequency dependent CTR spot is assumed to be twice as large as with the present set-up. The right plot shows the simulation corresponding to the present optics.

### 6.2.2 Electro-optical imaging system

As described in section 6.1 the performance of the present monitor set-up is curtailed by the fluctuations of the transverse profile of the Nd:YAG probe laser pulse. Namely the pointing
6.2. System improvements

stability of the laser system and the pattern due to the strain induced birefringence in the electro-optical crystal have to be addressed.

Electro-optical crystal

The achieved total extinction in cross-polarization configuration with the ZnTe crystal is measured to be $< 10^{-4}$. Both the transverse fluctuations of the probe laser beam and the noise pattern resulting from the strain induced birefringence in the crystal are limiting the performance of the present monitor set-up. However, we believe that the crystals from Nikko-Materials, which were used in the experiment, represent the best option available as this crystals are grown with an alternative method which results in lower strain in the crystal as with conventional growing methods [Asa04]. A lower background level due to strain induced birefringence might be obtained by using a thinner crystal than the one of 1 mm thickness. It has to be noted, that according to equation (4.3), a reduction in thickness leads also to a smaller signal level. Thus, a thinner crystal correspondingly necessitates a shorter laser probe pulse.

However, it could be advisable to investigate and specify different electro-optical crystals under laboratory conditions by using a well described THz source. Hence, the most adequate crystals could be selected for the experiment and effects regarding the strain induced birefringence could be investigated systematically.

Alternative laser system

The signal to noise considerations made in the first part of this chapter pointed out that a more stable laser system must lead to a lower background noise level. A viable alternative to a standard solid state pulsed laser system could be realized by a fibre laser. The advantages of fibre lasers are given in [Tue05]: Compared to bulk solid-state lasers, the main advantage of fibre lasers is their outstanding heat-dissipation capability, which is due to the large ratio of surface to volume of a long, thin gain medium like optical fibres. As a result, thermal distortion of the beam is negligible, and the beam quality depends primarily on the physical design of the fibre. Fibre lasers and amplifiers have a very high single-pass gain resulting in low laser thresholds and can be efficiently pumped with diode lasers. Moreover, the broad gain bandwidth, the compactness, robustness and simplicity of operation make fibre lasers attractive for several applications.

Affordable, non-commercial Er-doped (at 1550 nm) fibre laser have been set-up and successfully tested [Win]. Short ps pulses with pulse energies in the order of some nJ can be realized with passively mode-locked systems. Due to the unique configuration of a fibre...
laser excellent beam quality can be expected from such a system. Hence, the position stability is supposed to be greatly improved compared to bulk solid-state lasers [Win]. Improvements in the order of one magnitude or even better can be expected. Figure 6.17 shows a non commercial Er-doped (1550 nm) fibre laser set-up which was successfully commissioned by A. Winter of DESY, Hamburg. Stable synchronization of the system to a given master clock can be realized down to 50 fs levels [Win]. Hence, for a probe laser pulse with a duration in the order of 10 ps width the overlap with the CTR could be easily established and the synchronization is not critical as intended in the EOA monitor.

![Er-doped fibre laser, courtesy of A. Winter, DESY.](image)

Another advantage of a fibre laser is the fact, that such a laser could be easily guided to the experimental through an optical fibre. Thus, an optical transfer line becomes unnecessary. This would present another viable enhancement of the transverse beam quality and the pointing stability. Such a fibre must retain the linear polarization of the laser pulse, while the influence of the chirp due to the dispersion in the fibre can be neglected for short ps long pulses [Win].

**Estimation of probe laser pulse energy**

In this section we estimate the pulse energy of the probe laser which is needed in an improved set-up. We consider a more stable probe laser system which is optimized for shorter pulses. As a basis for the estimation we consider the Hamamatsu G9203-256D InGaAs linear image sensor which was used in the present experiments.
We define $n_{\text{pulse}}$ as the required number of photons of the probe laser pulse which is calculated by:

$$n_{\text{pulse}} = \frac{n_0 \cdot n_{\text{pixel}}}{\eta_{\text{signal}}}, \quad (6.5)$$

where $n_0$ is the required number of photons per pixel on the detector. $n_{\text{noise}}$ is the equivalent number of photons corresponding to the read out noise of the detector which is used, while $d_N$ is the target dynamic range.

$$n_0 = n_{\text{noise}} \cdot d_N \quad (6.6)$$

$\eta_{\text{signal}}$ is the fraction of photons in the probe laser pulse which polarization is rotated in the electro-optical crystal by the electrical field of the CTR. Therefore, we also must consider the duration of both the CTR ($\sigma_{\text{CTR}}$) and that of the laser pulse ($\sigma_{\text{laser}}$).

$$\eta_{\text{signal}} = \frac{\sigma_{\text{CTR}}}{\sigma_{\text{laser}}} \cdot \eta_{\text{modulated}} \quad (6.7)$$

$\eta_{\text{modulated}}$ gives the fraction of the probe laser beam which is modulated in a crystal of thickness $d$ by the electrical field of the CTR with intensity $I_{\text{CTR}}$ (c.f. equation (4.1)). $V_{\lambda/2}$ denotes the half wave voltage of the electro-optical crystal.

$$\eta_{\text{modulated}} = \left(\frac{\pi d}{V_{\lambda/2}}\right)^2 \cdot I_{\text{CTR}} \quad (6.8)$$

Thus, the required number of photons in the probe laser pulse is calculated by:

$$n_{\text{pulse}} = \frac{n_{\text{noise}} \cdot d_N \cdot n_{\text{pixel}}}{\frac{\sigma_{\text{CTR}}}{\sigma_{\text{laser}}} \cdot \left(\frac{\pi d}{V_{\lambda/2}}\right)^2 \cdot I_{\text{CTR}}} \quad (6.9)$$

The $rms$ noise voltage (read out noise) of the Hamamatsu G9203-256D InGaAs linear image sensor is specified to be 0.3 mV. The InGaAs detector shows a peak sensitivity at 1550 nm (see figure 4.11). Here we calculate a response of one electron per incident photon. For the highest resolution the selectable feedback capacitance [Pho] was chosen to be 0.5 pF\(^4\). Hence, using the well-known formula $Q = V \cdot C$ the $rms$ noise in terms of number of photons is calculated to be $n_{\text{noise}} = 10^3$ photons at 1550 nm. We target a signal dynamics $d_N$ of 20 dB, Thus, $n_0 = 10^5$ photons are required per pixel of the detector. Typically, the number of illuminated pixels is $n_{\text{pixel}} = 100$ for an one dimensional array.

\(^4\)The Hamamatsu G9203-256D InGaAs linear image sensor has a selectable capacitance of 0.5 pF respectively of 10 pF. Thus, for highest resolution the smaller capacitance was selected.
respectively $n_{\text{pixels}} = 10^4$ for a 2D detector.

As a conservative estimation we assume peak electrical field strengths of the CTR of 2 kV/cm as measured at the SLS pre-injector LINAC (see section 4.2). According to considerations in the previous section a thinner crystal could lead to smaller background noise levels. A larger CTR spot results in less attenuation of the higher order interference fringes, but simultaneously would lead to lower intensity in the measurement plane. Doubling the size yields $1/4$ of the original intensity. For a crystal of 0.1 mm thickness a signal in the order of $5 \cdot 10^{-4}$. $J_{\text{laser}}$ is calculated for a CTR spot which is assumed to be a factor of two larger than in the present set-up.

We consider a shorter probe laser pulse of $\sigma_{\text{laser}} = 10$ ps width and a CTR pulse as short as $\sigma_{\text{CTR}} = 1$ ps. Hence, $1/10$ of the incident laser beam is modulated. Therefore, the polarization of only $\eta_{\text{signal}} = 5 \cdot 10^{-5}$ of the incident laser beam is rotated.

Thus we need $n_{\text{pulse}} = 2 \cdot 10^{11}$ photons (30 nJ at 1550 nm) for the linear array respectively $n_{\text{pulse}} = 2 \cdot 10^{13}$ (3 $\cdot$ 10$^3$ nJ at 1550 nm) for the 2D sensor. This analysis is based on the assumption of a detector with a similar rms noise than the InGaAs linear image sensor. However, a cooled detector such as the 2D-OMA V:320 (320 x 256 InGaAs array, 30 $\times$ 30 microns) from Princeton Instruments/Acton offers much lower system noise. The system noise of the aforementioned sensor is specified to be 50 electrons [Ins]. this translates to $n_{\text{noise}} = 50$ photons at 1550 nm assuming similar detector sensitivity. Following the above considerations, a pulse energy of 150 nJ would be needed for this 2D detector. Thus, a cooled linear array detector should require pulse energies in the order of 2 nJ which can be obtained with a passive mode-locked Er-doped fibre laser system [Win].

### 6.2.3 Conclusions

By dispensing the transfer optics in a future quasi-optical system the low frequency attenuation could be improved. The tilt of the interference fringes can be corrected by a symmetrical interferometer set-up. A larger CTR spot size would lower the inherent attenuation of the modulation depth of the higher order interference fringes. Thus, the sensitivity $\alpha$ (see equation (6.3)) of the adjacent interference modulations could be enhanced to -10 dB or better (c.f. section 6.1).

The noise level which results from the transverse fluctuations of the probe laser beam could be dramatically decreased by substituting the bulk solid-state laser with a fibre laser. Such a system can be expected to show improvements in pointing stability and stability of the transverse shape of the laser beam of more than one order of magnitude. This results in a similar improvement of the background noise level $\delta_f$ due to the aforementioned fluctuations.
6.3 Summary

A thinner electro-optical crystal might introduce a smaller pattern due to strain induced birefringence. The lower signal level due to the thinner crystal can be compensated with a shorter laser pulse as obtained with a passive mode-locked fibre laser. The synchronization of such a short ps pulse with the CTR can still be established easily and stable. The use of optics of larger diameter leads to smaller diffraction speckles. This could lead to a reduction of the contribution $\delta_b$ in the total background noise level $\delta_{\text{total}}$.

Thus, as a result from better spatial stability of the probe laser beam and a smoother transverse profile, the total background noise level $\delta_{\text{total}}$ can be expected to be improved by one order of magnitude. Therefore, the signal to noise ratio $S/N$ as defined in section 6.1 could be expected to improve up to 20 dB for the zeroth order fringe, while the one for the adjacent modulations could be enhanced to 10 dB.

6.3 Summary

The novel electro-optical-auto-correlation (EOA) monitor was installed at the SLS pre-injector LINAC and was successfully used to measure bunch-lengths in single-shot operation. Bunch widths between 1.5 and 4 picoseconds have been measured. Variations in the bunch lengths due to different phase settings of the bunching cavities have been reproducibly observed with a resolution down to 200 femtoseconds. The current set-up has been successfully applied as an on line monitor for the optimization process of the SLS pre-injector LINAC for short bunches. Any change of the phase setting of the bunching cavities leading to increased bunch lengths result in a widening of the interference pattern which thus can be easily traced. Therefore, the monitor described in this work represents an indispensable tool for an operator of a future X-FEL source to optimize the machine for shortest bunches.

The monitor produces auto-correlation patterns of good quality. The zeroth order interference fringe is clearly visible in all of the recorded profiles. However, the signal-to-noise-ratio is curtailed by the background of the EO read-out system and the Nd:YAG laser stability. Since, the most stable pulses at the SLS LINAC are Gaussian shaped, without distinct longitudinal density modulations within the bunch, it is nevertheless sufficient to clearly reproduce the zeroth order interference fringe. The important quantity is the intensity modulation depth of the interference pattern. The present monitor set-up has a signal-to-noise ratio in the order of 7 dB with respect to the zeroth order fringe. Thus, this is adequate to describe the bunches at the SLS LINAC in single-shot operation.

However, for other machines, which produce bunches with longitudinal density modulations on a sub-picosecond basis the visualization of the zeroth order fringe alone may not
be sufficient. In this context, the side modulations of the interference pattern are important. Thus, in a future set-up the signal-to-noise ratio has to be addressing. The largest improvements can be expected from a revised electro-optical imaging system. Namely, a new set-up could benefit from a more stable probe laser system. A major contribution in the noise background is due to fluctuations of the transverse profile of the laser. A fibre laser would offer much improved spatial stability due to its unique design. The improvements highlighted in this chapter promise an enhancement of the signal-to-noise ratio in the order of 10 dB. This would be sufficient to use the monitor at another machine which produces bunches with distinct modulations in their longitudinal shape.

In this context it must also be noted that the SLS pre-injector LINAC is not predestined to generate ultra-short bunches. Thus, much higher signal levels (due to a broader coherent spectrum and also higher bunch charges) can be expected at a radiation source which is designed to produce shortest bunches.
Appendix A

Electromagnetic fields of relativistic electrons

In this section we will derive the solution for the electromagnetic fields of a relativistic electron. For a relativistic electron of which its rest frame \((x', y', z')\) moves in free space along the z-axis with respect to the laboratory frame \((x, y, z)\) we define the longitudinal Fourier transformation by:

\[
\hat{A}(\nu) = \int_{-\infty}^{+\infty} A(\eta) \cdot e^{-i\eta \frac{2\pi \nu}{\beta c}} \cdot \int_{-\infty}^{+\infty} A(t) \cdot e^{-i2\pi \nu t} dt
\]  

(A.1)

with the longitudinal coordinate \(\eta = t - \frac{z}{v} = t - \frac{z}{\beta c}\), where \(v = \beta \cdot c\).

It is convenient to consider the problem in cylindrical coordinates. From symmetry only the radial and longitudinal components of the electric field \((E_r \text{ and } E_z)\) and the azimuthal components of the magnetic field \((B_\phi)\) have to be considered. \(\rho\) denotes the charge density and the current \(\vec{j}\) has only a longitudinal component originating from the moving charge distribution. Applying the above defined Fourier transformation (see equation (A.1)) to Maxwell’s equations yields and setting \(k = \frac{2\pi \nu}{\beta c}\):

\[
\frac{1}{r} \partial_r (r \hat{E}_r) + ik \hat{E}_z = \frac{\hat{\rho}}{\varepsilon_0} \quad \text{(A.2)}
\]

\[
\frac{1}{r} \partial_\phi \hat{B}_\phi = 0 \quad \text{(A.3)}
\]

\[
\frac{1}{r} \partial_\phi \hat{E}_z = 0 \quad \text{(A.4)}
\]

\[
-ik \hat{E}_r - \partial_\phi \hat{E}_z = ik\beta c \hat{B}_\phi \quad \text{(A.5)}
\]

\[
\frac{1}{r} \partial_\phi \hat{E}_r = 0 \quad \text{(A.6)}
\]

\[
-ik \hat{B}_\phi = -\frac{ik\beta}{c} \hat{E}_r \quad \text{(A.7)}
\]
\[ \frac{1}{r} \partial_r (r \hat{B}_\phi) = \mu_0 j_z - \frac{ik\beta}{c} \hat{E}_z \tag{A.8} \]

Combining equations (A.5) and (A.7) yields:
\[ \partial_r \hat{E}_z = \frac{ik}{\gamma^2} \hat{E}_r. \tag{A.9} \]

By inserting into equation (A.2) we get:
\[ \partial_r^2 \hat{E}_z + \frac{1}{r} \partial_r \hat{E}_z - \frac{k^2}{(\gamma^2)} \hat{E}_z = (ik)(\frac{\hat{\rho}}{\varepsilon_0 \gamma^2}) \tag{A.10} \]

The two independent solution of the homogenous part of the differential equation (A.10) are the modified Bessel functions \( I_0((k/\gamma)r) \) and \( K_0((k/\gamma)r) \) \cite{Sch05}. Due to equation (A.9) the radial field is then given by the Bessel functions \( I_1((k/\gamma)r) \) and \( K_1((k/\gamma)r) \). However, the field of the relativistic charge \( q \) must decrease with increasing radius \( r \). Therefore, the electrical field is described by the functions \( K_0((k/\gamma)r) \) respectively \( K_1((k/\gamma)r) \) only. Solving the inhomogenous equation we obtain:
\[ \hat{E}_z(r, k) = \frac{-iqk}{(2\pi)^{3/2} \varepsilon_0 \beta \gamma c} K_0((k/\gamma)r) \tag{A.11} \]
\[ \hat{E}_r(r, k) = \frac{qk}{(2\pi)^{3/2} \varepsilon_0 \beta \gamma c} K_1((k/\gamma)r) \tag{A.12} \]

By inserting \( k = 2\pi \nu / (\beta c) \) the above equations transforms to:
\[ \hat{E}_z(r, \nu) = \frac{-iq(2\pi \nu)}{(2\pi)^{1/2} \varepsilon_0 \beta^2 \gamma^2 c^2} K_0(((2\pi \nu) / \beta \gamma c)r) \tag{A.13} \]
\[ \hat{E}_r(r, \nu) = \frac{q(2\pi \nu)}{(2\pi)^{1/2} \varepsilon_0 \beta^2 \gamma^2 c^2} K_1(((2\pi \nu) / \beta \gamma c)r) \tag{A.14} \]
Appendix B

Spatial auto-correlation

To illustrate the principle of spatial auto-correlation two plane waves intersecting at the angle $\theta$ are assumed. The two beams are spatially coinciding at $r_3 = 0$ in the focal plane. The two complex amplitudes of frequency $\nu$ are given by:

$$E_1(\vec{r}, t) = E_1 \cdot e^{2\pi i \nu (t - \frac{\vec{k}_1 \cdot \vec{r}}{2\nu} - \Delta t_1)}$$
$$E_2(\vec{r}, t) = E_2 \cdot e^{2\pi i \nu (t - \frac{\vec{k}_2 \cdot \vec{r}}{2\nu} - \Delta t_2)},$$

where $\Delta t_2 - \Delta t_1$ is the phase delay of wave 2 with respect to wave 1. Here we also have assumed that the two waves have the same polarization $\vec{E}_1, \vec{E}_2$ parallel to $\vec{r}_2$.

The intensity at position $\vec{r}$ is computed by:

$$I(\vec{r}, t) = |E(\vec{r}, t)|^2 = |E_1(\vec{r}, t) + E_2(\vec{r}, t)|^2 =$$
$$= (E_1(\vec{r}, t) + E_2(\vec{r}, t))(E_1^*(\vec{r}, t) + E_2^*(\vec{r}, t)) =$$
$$= E_1(\vec{r}, t)E_1^*(\vec{r}, t) + E_1(\vec{r}, t)E_2^*(\vec{r}, t) +$$
$$+ E_2(\vec{r}, t)E_1^*(\vec{r}, t) + E_2(\vec{r}, t)E_2^*(\vec{r}, t),$$

where $E_i^*$ denotes the complex conjugate of $E_i$.

By inserting the complex amplitudes $E_1(\vec{r}, t)$ and $E_2(\vec{r}, t)$ into above equations we find:

$$I(\vec{r}, t) = E_1^2 + E_2E_1e^{2\pi i \nu (t - \frac{\vec{k}_1 \cdot \vec{r}}{2\nu} - \Delta t_1)}e^{-2\pi i \nu (t - \frac{\vec{k}_2 \cdot \vec{r}}{2\nu} - \Delta t_2)} +$$
$$+ E_2E_1e^{2\pi i \nu (t - \frac{\vec{k}_2 \cdot \vec{r}}{2\nu} - \Delta t_2)}e^{-2\pi i \nu (t - \frac{\vec{k}_1 \cdot \vec{r}}{2\nu} - \Delta t_1)} + E_2^2 =$$
$$= E_1^2 + E_2^2 + E_1E_2\left(e^{i((\vec{k}_2 - \vec{k}_1) \vec{r} - 2\pi \nu (\Delta t_2 - \Delta t_1))} + e^{-i((\vec{k}_2 - \vec{k}_1) \vec{r} - 2\pi \nu (\Delta t_2 - \Delta t_1))}\right)$$

(B.4)
This last expression can be simplified using the identity for the cosine function: \( \cos(x) = \frac{1}{2} \cdot (e^{ix} + e^{-ix}) \):

\[
I(\vec{r}) = E_1^2 + E_2^2 + 2E_1E_2 \cdot \cos((k_2\vec{r} - k_1\vec{r}) + (\phi_2 - \phi_1)),
\]

(B.5)

where \( \phi_l = 2\pi \nu \Delta t_l \) \((l = 1, 2)\).

In the case depicted in figure 3.10 it is that:

\[
\begin{align*}
\vec{k}_1 &= k \cdot (-\sin(\theta), 0, -\cos(\theta)) \\
\vec{k}_2 &= k \cdot (+\sin(\theta), 0, -\cos(\theta))
\end{align*}
\]

(B.6)

(B.7)

with \( k = \frac{\omega}{c} = \frac{2\pi \nu}{c} \).

Therefore, it is found that the intensity distribution of the interfering plane waves in the focal plane can be calculated to:

\[
I(\vec{r}, \nu) = \left( E_1^2 + E_2^2 + 2E_1E_2 \cdot \cos\left(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot r_1 + (\phi_2 - \phi_1)\right) \right) = \\
\left( I_1 + I_2 + 2\sqrt{I_1I_2} \cdot \cos\left(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot r_1 + (\phi_2 - \phi_1)\right) \right),
\]

(B.8)

In the case of our spatial interferometer it is that

\[
I(\vec{r}, \nu) = |E(\vec{r}, \nu)|^2 = |E_1(\vec{r}, \nu) - E_2(\vec{r}, \nu)|^2.
\]

(B.9)

This is due to sign change of the electrical field of the two vertically polarized CTR lobes changes at the symmetry axis between the two lobed emission pattern (c.f. figure 3.13). Then the intensity distribution of the interfering plane waves in the focal plane is

\[
I(\vec{r}, \nu) = \left( I_1 + I_2 - 2\sqrt{I_1I_2} \cdot \cos\left(4\pi \sin(\theta) \cdot \frac{\nu}{c} \cdot r_1 + (\phi_2 - \phi_1)\right) \right).
\]

(B.10)
Appendix C

Electro-optical effect

The polarization $\vec{P}$ of a medium induced by an electromagnetic wave with the electrical field vector $\vec{E}$ is written as

$$\vec{P} = \epsilon_0 \chi \vec{E}, \quad (C.1)$$

where $\chi$ is the so called susceptibility. In case of an isotropic medium the susceptibility is a scalar and therefore the polarization $\vec{P}$ must be parallel to the electrical field vector $\vec{E}$. In anisotropic medias however the susceptibility is not of scalar but of tensor form [Jac75]:

$$\vec{P} = \epsilon_0 \chi_{ij} \vec{E} \quad (C.2)$$

thus the polarization is not necessarily parallel to the electrical field vector. The eigenvectors of the susceptibility tensor are the so called principal axes of the crystal. In this basis the susceptibility tensor is of diagonal form.

The dielectric response of a crystal [Yar84] can be described by the permittivity tensor $\epsilon_{ij}$

$$\epsilon_{ij} = \epsilon_0 (1 + \chi_{ij}) \quad (C.3)$$

which yields

$$D_i = \epsilon_{ij} E_j. \quad (C.4)$$

The phase velocity $v_{phase}$ of a plane electromagnetic wave is thus depending on the direction of propagation and its polarization

$$v_{phase} = \frac{c}{\sqrt{\epsilon}} = \frac{c}{n}. \quad (C.5)$$

For a given direction $\vec{q}$ of propagation there are two linearly independent eigenvectors $\vec{D}_s$ and $\vec{D}_f$ with different phase velocities (the slow ”s” and the fast ”f” axis).
C.1 The index ellipsoid

The energy density of the stored electrical field in the anisotropic medium is given by [Yar84]:

\[ U_e = \frac{1}{2} \vec{E} \cdot \vec{D} = \frac{1}{2} E_i \epsilon_{ij} E_j. \]  

(C.6)

In the basis of the eigenvectors the tensor \( \epsilon_{ij} \) is of diagonal form. In \( \vec{D} \) space the above equation is then written as follows:

\[ 2U_e = \frac{D_x^2}{\epsilon_x} + \frac{D_y^2}{\epsilon_y} + \frac{D_z^2}{\epsilon_z}. \]  

(C.7)

Substituting \( \vec{r} = \vec{D}/\sqrt{2U_e} \) and defining the principal index of refraction by \( n_i^2 = \epsilon_i/\epsilon_0 \) for \( i = x, y, z \), the last equation transforms to [Yar84]:

\[ \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1. \]  

(C.8)

Equation (C.8) describes a general ellipsoid with the major axes parallel to the \( x, y \) and \( z \) directions whose respective lengths are \( 2n_i \), \( i = x, y, z \). The ellipsoid is the so called index ellipsoid (see figure C.1) used to find the two indices of refraction and the two corresponding directions of \( \vec{D} \) associated with the two independent plane waves that can propagate along an arbitrary direction \( \vec{q} \) in a crystal [Yar84].

Define the impermeability tensor \( \eta_{ij} \) as

\[ \eta_{ij} = \epsilon_0 (\epsilon^{-1})_{ij}. \]  

(C.9)

Thus the equation of the index ellipsoid can be rewritten using the impermeability tensor as follows:

\[ \vec{r} \cdot \eta \cdot \vec{r} = 1. \]  

(C.10)

C.2 The Pockels effect

Linear and quadratic electro-optical effects in crystals are described by the electro-optical tensors \( r_{ijk} \) and \( R_{ijkl} \) [Yar84]:
C.2. The Pockels effect

The tensor $r_{ijk}$ describes a linear effect named Pockels effect, whereas $R_{ijkl}$ denotes the quadratic Kerr effect. For ZnTe the linear Pockels effect is dominating and the Kerr effect is negligible.

Due to permutation symmetries of the linear electro-optical tensor [Yar84] it is convenient to use the so-called contracted indices $r_{ijk} \rightarrow r_{mk}$ summarized in table (C.1).

A crystal with cubic symmetry such as ZnTe has only three non-zero entries [Yar84]:

$$r_{41} = r_{52} = r_{63} \neq 0$$  \hspace{1cm} (C.12)

with all other entries being zero.
Appendix C. Electro-optical effect

<table>
<thead>
<tr>
<th>(ij) → m</th>
<th>contracted indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1) → 1</td>
<td>( r_{11k} \rightarrow r_{1k} )</td>
</tr>
<tr>
<td>(2,2) → 2</td>
<td>( r_{22k} \rightarrow r_{2k} )</td>
</tr>
<tr>
<td>(3,3) → 3</td>
<td>( r_{33k} \rightarrow r_{3k} )</td>
</tr>
<tr>
<td>(2,3) → 4</td>
<td>( r_{23k} = r_{32k} \rightarrow r_{4k} )</td>
</tr>
<tr>
<td>(1,3) → 5</td>
<td>( r_{13k} = r_{31k} \rightarrow r_{5k} )</td>
</tr>
<tr>
<td>(1,2) → 6</td>
<td>( r_{12k} = r_{21k} \rightarrow r_{6k} )</td>
</tr>
</tbody>
</table>

Table C.1: Contracted indices for the linear electro-optical effect.

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
r_{41} & 0 & 0 \\
0 & r_{41} & 0 \\
0 & 0 & r_{41}
\end{bmatrix}.
\] (C.13)

The electro-optical effect implies that the equation of the index ellipsoid becomes dependent of the applied electrical field \( \vec{E} \). Using equations (C.8) and (C.11) yields the index ellipsoid to be written as follows:

\[
\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} + \sum_r r_{mk} E_k = 1.
\] (C.14)

ZnTe is of cubic symmetry, thus it becomes that \( n_i = n_0 \) for \( i = x, y, z \). Therefore the above equation transforms to:

\[
\frac{1}{n_0^2}(x^2 + y^2 + z^2) + 2r_{41}(E_x y z + E_y z x + E_z x y) = 1.
\] (C.15)

The EOA experiment uses a crystal cut in the [110]-plane (see figure 4.7). Both the probe and the CTR pulse are perpendicular on this plane. Their electrical field vectors lie in the [110]-plane [Win04a]. The new coordinate system is defined by

\[
X = \frac{1}{\sqrt{2}} \begin{pmatrix}
-1 \\
1 \\
0
\end{pmatrix}, Y = \begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix}.
\] (C.16)

Assuming normal incidence on to the XY-plane and the electrical field of the CTR at an angle \( \psi \) with respect to the X-axis, the electrical field vector of the CTR in the base
C.2. The Pockels effect

The system of the crystal lattice is [Win04a]

$$\vec{E}_{CTR} = E_{CTR} \begin{pmatrix} -\frac{1}{\sqrt{2}} \cos(\psi) \\ \frac{1}{\sqrt{2}} \cos(\psi) \\ \sin(\psi) \end{pmatrix}.$$  \hspace{1cm} (C.17)

Using equation (C.15) the impermeability tensor which is depending from the electrical field of the CTR applied is then given by

$$\eta_{ij} = \begin{pmatrix} \frac{1}{n_0^2} & E_{CTR}r_{41} \sin(\psi) & \frac{E_{CTR}r_{41}}{\sqrt{2}} \cos(\psi) \\ E_{CTR}r_{41} \sin(\psi) & \frac{1}{n_0^2} & -\frac{E_{CTR}r_{41}}{\sqrt{2}} \cos(\psi) \\ \frac{E_{CTR}r_{41}}{\sqrt{2}} \cos(\psi) & -\frac{E_{CTR}r_{41}}{\sqrt{2}} \cos(\psi) & \frac{1}{n_0^2} \end{pmatrix}.$$  \hspace{1cm} (C.18)

To obtain the main refractive indices the eigenvalues of the impermeability tensor \(\eta_{ij}\) have to be computed. These are found to be:

$$\lambda_1 = \frac{1}{n_0^2} - \frac{E_{CTR}r_{41}}{2} (\sin(\psi) + \sqrt{1 + 3\cos(\psi)^2}) \hspace{1cm} (C.19)$$

$$\lambda_2 = \frac{1}{n_0^2} - \frac{E_{CTR}r_{41}}{2} (\sin(\psi) - \sqrt{1 + 3\cos(\psi)^2}) \hspace{1cm} (C.20)$$

$$\lambda_3 = \frac{1}{n_0^2} - \frac{E_{CTR}r_{41}}{2} \sin(\psi). \hspace{1cm} (C.21)$$

The main refractive indices are calculated by

$$n_i = \frac{1}{\sqrt{\lambda_i}} \hspace{1cm} (C.22)$$

Taking into account that \(1/n_0^2 \gg E_{CTR}r_{41}\) [Yar84] yields

$$n_1 = n_0 + \frac{n_0^3 r_{41} E_{CTR}}{4} (\sin(\psi) + \sqrt{1 + 3\cos(\psi)^2}) \hspace{1cm} (C.23)$$

$$n_2 = n_0 + \frac{n_0^3 r_{41} E_{CTR}}{4} (\sin(\psi) - \sqrt{1 + 3\cos(\psi)^2}) \hspace{1cm} (C.24)$$

$$n_3 = n_0 - \frac{n_0^3 r_{41} E_{CTR}}{4} \sin(\psi). \hspace{1cm} (C.25)$$

The third eigenvector is (1,1,0) which is normal to the [110] crystal plane. Therefore the other eigenvectors must lie in the [110] plane itself.

The laser beam is impinging on the ZnTe crystal along the direction \(\vec{e}_3\). The electrical field
vector of both the CTR and the laser pulse of wavelength $\lambda$ must lie in the [110] plane. In a crystal of thickness $d$ the two principle axes will get a relative phase shift $\Gamma(E_{CTR}, \psi)$ due to the applied electrical field of the CTR pulse.

$$\Gamma(E_{CTR}, \psi) = \frac{2\pi d}{\lambda} (n_1 - n_2) = \frac{2\pi d n_0^3 r_{41} E_{CTR}}{\lambda} \sqrt{1 + 3\cos(\psi)^2}. \quad (C.26)$$

Hence the phase shift is only depending from the field strength of the CTR pulse and its angle in respect to the X-axis. The largest modulation can be expected for $\psi = 0$. Thus the above equation transforms as follows:

$$\Gamma(E_{CTR}) = \frac{2\pi d}{\lambda} n_0^3 r_{41} E_{CTR} = \frac{\pi d}{V_{\lambda/2}} E_{CTR} \quad (C.27)$$

where $V_{\lambda/2}$ is the so called halfwave voltage which is needed to rotate the linear polarization by an angle of $\pi/2$.

$$V_{\lambda/2} = \frac{2n_0^3 r_{41}}{\lambda} \quad (C.28)$$
Appendix D

Influence of alignment errors of the spatial interferometer

The interferometer described in chapter 3 of this thesis produces a spatial auto-correlation pattern which thus can be read out in a single shot. In this section we estimate the influence of alignment errors of the interferometer on the interference pattern. This is performed by assuming different angles of incidence $\theta_l$, $l = 1, 2$ of the two individual signal paths of the interferometer.

According to equation (3.4) the interference of the two complex amplitudes $E_l(\vec{r}, t) = E_l \cdot e^{2\pi i (t - (\vec{k}_l \cdot \vec{r})/c)/(2\pi \nu)} + \phi_l$, $l = 1, 2$ can be written as follows:

$$I(\vec{r}) = I_1 + I_2 - 2\sqrt{I_1 I_2} \cdot \cos((\vec{k}_2 \cdot \vec{r} - \vec{k}_1 \cdot \vec{r}) + (\phi_2 - \phi_1)), \quad (D.1)$$

where $I_l = E_l^2$ and $\phi_l$ denotes the phase delay. The two plane waves are coinciding at $\vec{r}_0 = (x, y, z) = (0, y, 0)$ As described by the following expressions:

$$\vec{k}_1 = k \cdot (-\sin(\theta_1), 0, -\cos(\theta_1)) \quad (D.2)$$

$$\vec{k}_2 = k \cdot (+\sin(\theta_2), 0, -\cos(\theta_2)) \quad (D.3)$$

with $k = \frac{\omega}{c} = \frac{2\pi \nu}{c}$. If the two angles of incidence are not equal the interference pattern will depend on the position $z$:

$$I(\vec{r}) \propto 1 - \cos\left(\frac{2\pi \nu}{c}((\sin(\theta_2) + \sin(\theta_1))x + ((\cos(\theta_1) - \cos(\theta_2))z) + (\phi_2 - \phi_1)\right). \quad (D.4)$$

By solving $\partial_z (1 - \cos(ax + b)) = 0$ the position of the zeroth order fringe is calculated to be:
Appendix D. Influence of alignment errors of the spatial interferometer

\[ x_0(z) = \frac{z \cdot \cos(\theta_1) - \cos(\theta_2)}{\sin(\theta_1) + \sin(\theta_2)}. \]  
(D.5)

Here we must include the ZnTe crystal. According to Snellius law the two rays will propagate each under an angle \( \theta'_l, \ l = 1, 2 \) in the ZnTe crystal which is computed by (c.f. figure D.1):

\[ \frac{\sin(\theta_l)}{n} = \frac{\sin(\theta'_l)}{n'}. \]  
(D.6)

![Figure D.1: Snellius law. The two signal paths of the interferometer are coinciding each under an angle of \( \theta_l \) onto the ZnTe electro-optical crystal. In the crystal of thickness \( d \) the two rays will propagate under an angle \( \theta'_l \) which can be calculated using Snellius law.](image)

We assume as a worst case a misalignment of \( \theta_l = \theta \pm 5^\circ, \ l = 1, 2 \) as shown in figure D.1. According to equations (D.5) and (D.6) the shift of the zeroth order fringe over a ZnTe crystal of thickness \( d = 1 \) mm is calculated to be \( \Delta x_0 = x_0(d) - x_0(0) = 25 \) micron. The interference pattern generated by a monochromatic source of frequency \( \nu = 1 \) THz has a spacing of 300 micron (see equation (D.1)). Hence the transverse shift of the interference pattern over the thickness of the crystal is negligible.
Appendix E

Short pulses from fibre lasers

An optical fibre consists of a core surrounded by a cladding layer whose refractive index is slightly lower than that of the core. Thus, the core can guide light by total internal reflection. The principle layout of a fibre laser is shown in figure E.1.

Figure E.1: Principle of a fibre laser.

Short pulses are commonly achieved with either active or passive mode-locking [Win]. Passive mode-locking is performed using nonlinear polarization rotation. The working principle is presented in figure E.2 [Hau97]. After the polarizer the pulse is linear polarized and the quarter-wave plate converts the linear to elliptical polarization which is rotated nonlinearly in the fibre (Kerr-effect, \( n(I) = n_0 + n_2(I) \)). Thus, the peak of the pulse is rotated more than the tails. The \( \lambda/2 \) plate is oriented such that only the center of the pulse passes the second polarizer. Hence, this configuration closely resembles a saturable absorber. This mode-locking scheme yields sub 100 fs to ps pulse durations [Hau97].
Figure E.2: Principle of nonlinear polarization rotation.
Index

A
accelerating structures
   ALIRF-AS-1/2 ..................... 11
active mode-locked Nd:YAG laser 43, 81, 83 f, 86, 96, 102, 125
temporal profile .................... 87
angular broadening in emission 12, 36, 42
asymmetry in emission ............. 37, 42
attenuation of higher order interference fringes .............. 76 f, 79, 135 f
auto-correlation .................... 70

B
background noise .................... 123, 130
Bessel functions ..................... 41
modified ............................ 25
birefringence ......................... 81
booster synchrotron ................ 9, 96
bunch length measurements ....... 101, 116

C
chirped laser pulse .................. 16
coherent transition radiation
   CTR .5, 7, 9, 12 f, 36, 38, 42, 56, 67, 70 f, 81
coincidence between CTR and laser .103
cross-polarization .................... 18, 91 f
   extinction ........................ 83, 92 ff, 101
crystalline quartz window .......... 48 ff, 72
CTR
finite spot size .................... 72, 76 f
wavelength dependent spot size .... 76, 106
cylindrical lens .................... 78, 82, 93, 120

D
diffraction due to optics apertures . 125 f

e
electrical current sources ............ 31
electro-optical auto-correlation
   EOA .......................... 19, 47 f, 89
timing .............................. 97
electro-optical crystal ............... 15, 81, 89, 137
Electro-optical imaging system .... 88
electro-optical sampling
   EOS .......................... 15 f, 18, 43
electromagnetic source field3, 12, 39 f, 42
electron gun ......................... 10
EO read-out ........................ 70, 81, 83, 116
EO spectral decoding technique .... 16 f, 43
EO temporal decoding method ...... 16 f, 43
event generator
   EG ............................ 96 f
event receiver
   ER ............................ 95

F
far-field
   wave zone ........................ 23, 41
fibre laser ........................ 137 f
Er-doped ........................................ 137
final-buncher
ALIRF-FBU .................................. 11
form factor ................................. 5, 7, 14, 42, 107
Fourier transform ......................... 6, 14, 46
fringe order ................................. 55

G

Gaussian longitudinal bunch charge distribution ................. 107
Ginzburg-Frank ... 2 f, 5, 9, 12, 21, 37, 41 f
form factor .................................. 3, 41
Glan laser polarizer ................. 83, 88, 92 f
Golay cell detector .......... 45, 50, 67 f, 101, 107
GRASP ...................................... 44, 61, 66, 72 f, 132

H

half wave voltage ......................... 90
Helmholtz equation ......................... 27
inhomogeneous ......................... 31
high-pass filter function ................. 73

I

improvements of the electro-optical imaging system .......... 136
improvements of the present set-up ........................ 117, 131
improvements of the quasi-optical system ..................... 131
InGaAs
cooled detector .......................... 140
linear array detector ...................... 70
linear image sensor ........................ 81 f, 88, 93 f, 97,
101, 108, 120, 123, 138
integrating current transformer
ALIDI-ICT-1/2 ......................... 12
intensity auto-correlation .................. 53 f

K

Kerr effect .................................. 89
Kramers-Kronig dispersion relation .... 14

L

laser
position stability ....................... 81
laser pre-trigger ....................... 96
limitations of the present set-up ... 117
LINAC pre-trigger ....................... 96 f
linear electron accelerator
LINAC ................................. 5, 9 ff, 36, 42, 47, 67, 72, 82,
96, 101
long wavelength limit ..................... 39
low frequency attenuation ................. 14
low-pass filter function .................... 73

M

Martin-Puplett interferometer
MPI ........................................ 14, 38, 43 – 46, 56, 71, 120
master oscillator
MO ........................................ 84
Maxwell equations .......................... 24
mechanical phase shifter .................. 24
Michelson interferometer ............ 14, 45, 71

N

near-field
Fresnel zone ......................... 23, 36

O

optical beam port
ALIDI-SM-5 ..... 12, 44, 48 f, 71, 82, 116
ALIDI-SM-E ......................... 49, 107
optical transfer line ....................... 81, 125
optical transition radiation
<table>
<thead>
<tr>
<th>Term</th>
<th>Page(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OTR</td>
<td>1, 102</td>
</tr>
<tr>
<td>overall sensitivity $\alpha$</td>
<td>108, 110</td>
</tr>
<tr>
<td>P</td>
<td></td>
</tr>
<tr>
<td>parabolic mirror</td>
<td>45, 48, 50f, 67f, 73, 132</td>
</tr>
<tr>
<td>PARMELA</td>
<td>9, 116</td>
</tr>
<tr>
<td>Pockel's coefficient</td>
<td>90</td>
</tr>
<tr>
<td>Pockel's effect</td>
<td>89</td>
</tr>
<tr>
<td>polarizer</td>
<td>81</td>
</tr>
<tr>
<td>Poynting vector</td>
<td>35</td>
</tr>
<tr>
<td>pre-buncher</td>
<td></td>
</tr>
<tr>
<td>ALIRF-PBU</td>
<td>11, 109</td>
</tr>
<tr>
<td>ALIRF-SPB</td>
<td>11</td>
</tr>
<tr>
<td>pyro-electrical detector</td>
<td>76, 101, 107</td>
</tr>
<tr>
<td>Q</td>
<td></td>
</tr>
<tr>
<td>quarter wave plate</td>
<td>15</td>
</tr>
<tr>
<td>quasi-optical set-up</td>
<td>44, 47, 71</td>
</tr>
<tr>
<td>quasi-optics</td>
<td>44</td>
</tr>
<tr>
<td>R</td>
<td></td>
</tr>
<tr>
<td>radial polarization of transition radiation</td>
<td>12</td>
</tr>
<tr>
<td>Rayleigh distance</td>
<td>23, 36, 39, 132</td>
</tr>
<tr>
<td>regenerative amplifier</td>
<td></td>
</tr>
<tr>
<td>RA</td>
<td>84f</td>
</tr>
<tr>
<td>relativistic Lorentz factor</td>
<td>26</td>
</tr>
<tr>
<td>roof mirror</td>
<td>45</td>
</tr>
<tr>
<td>S</td>
<td></td>
</tr>
<tr>
<td>sensitivity</td>
<td>119</td>
</tr>
<tr>
<td>overall sensitivity</td>
<td>121f, 130</td>
</tr>
<tr>
<td>short wavelength limit</td>
<td>40</td>
</tr>
<tr>
<td>signal to noise ratio</td>
<td>130</td>
</tr>
<tr>
<td>single-shot capability</td>
<td>17, 43, 81, 101, 109, 116</td>
</tr>
<tr>
<td>SLS master oscillator</td>
<td>83, 88, 96</td>
</tr>
<tr>
<td>solenoid magnets</td>
<td>11</td>
</tr>
<tr>
<td>spatial auto-correlation</td>
<td>17, 43, 51f, 67, 81, 91, 145</td>
</tr>
<tr>
<td>convoluted with finite spot size</td>
<td>76, 78, 102, 135</td>
</tr>
<tr>
<td>modulation depth</td>
<td>68, 108, 121</td>
</tr>
<tr>
<td>spatial fluctuations of the CTR</td>
<td>124</td>
</tr>
<tr>
<td>spatial fluctuations of the probe laser</td>
<td>123</td>
</tr>
<tr>
<td>spatial interferometer</td>
<td>43f, 51, 57, 60, 67, 81, 101</td>
</tr>
<tr>
<td>alignment</td>
<td>60</td>
</tr>
<tr>
<td>asymmetric layout</td>
<td>61, 65, 134</td>
</tr>
<tr>
<td>entrance focus</td>
<td>67, 73</td>
</tr>
<tr>
<td>vertical mismatch of the two signal paths</td>
<td>73, 75, 78, 106, 132</td>
</tr>
<tr>
<td>spherical approximation</td>
<td>59</td>
</tr>
<tr>
<td>storage ring</td>
<td>9</td>
</tr>
<tr>
<td>strain induced birefringence</td>
<td>83, 93, 103, 125f, 137</td>
</tr>
<tr>
<td>streak camera</td>
<td>2, 87</td>
</tr>
<tr>
<td>surface current density</td>
<td>33</td>
</tr>
<tr>
<td>Swiss Light Source</td>
<td></td>
</tr>
<tr>
<td>SLS</td>
<td>5, 9ff, 36, 42, 47, 67, 72, 82, 96, 101</td>
</tr>
<tr>
<td>synchronization scheme</td>
<td>43, 83, 95, 98, 102, 116</td>
</tr>
<tr>
<td>T</td>
<td></td>
</tr>
<tr>
<td>target screen</td>
<td></td>
</tr>
<tr>
<td>finite size</td>
<td>3, 12, 26, 35, 39f, 42</td>
</tr>
<tr>
<td>oblique</td>
<td>26, 32, 35</td>
</tr>
<tr>
<td>technical gallery</td>
<td>81</td>
</tr>
<tr>
<td>tilt of the interference fringes</td>
<td>65f, 108, 110, 134</td>
</tr>
<tr>
<td>timing jitter</td>
<td>88</td>
</tr>
<tr>
<td>titanium-sapphire laser</td>
<td></td>
</tr>
</tbody>
</table>
Ti:Sa ......................... 15f
toroid mirror ........ 57, 60, 70, 102, 105
transfer function ..... 14, 46, 71, 73
overall transfer function . 73f, 106
transition radiation
TR .......................... 1, 35

U
ultrafast photo diode ............. 88

V
vector potential .................. 31
velocity bunching .............. 10, 12, 108
VME ............................ 95

W
wire grid polarizer ............ 38, 45, 50
Wollaston prism ................. 15

Z
zeroth order fringe ... 55, 57, 63, 102, 105
Zinc Telluride
ZnTe ........................... 81, 88f, 125
Bibliography

[ABm] ABmm. AB Millimetre. France.


161


[Ins] Princeton Instruments/Acton. Data sheet 2D-OMA V:320, scientific grade In-GaAs camera for low-light NIR imaging applications.
[IRS] IRS. IDL, Interactive Data Language. USA.


[Pho] Hamamatsu Photonics. Data sheet InGaAs linear image sensor G9203-256D, G9204-512D.

[Scha] V. Schlott. private communication.


[Str] A. Streun. private communication.


Curriculum Vitae

Name: Daniel Oliver Sütterlin
Born: 13. April 1974
Citizen of: Zurich, Switzerland
Parents: Heinz Sütterlin
Barbara Sütterlin-Wiedenmeier

Education
1981 - 1989: Primary and secondary school in Glattfelden
1989 - 1994: Kantonsschule Zürcher Unterland in Bülach
1994: Federal Certificate of Maturity, type C (mathematics and science)
1994 - 2001: ETH Zurich, study of physics
2001: Master thesis at the SLS (Paul Scherrer Institute, Villigen)
2001: Physics diploma of ETH Zurich (Dipl. Phys. ETH)
2001 - 2006: Ph.D study at the ETH Zurich and Paul Scherrer Institute
List of publications

D. Sütterlin, et al. Development of a bunch-length monitor with sub-picosecond time resolution and single-shot capability, in *proceedings of DIPAC-03, Mainz, Germany*

D. Sütterlin, et al. Spatial auto-correlation interferometer with single shot capability using coherent transition radiation, in *proceedings of DIPAC-05, Lyon, France*

D. Sütterlin, et al. Single shot bunch length measurements using a spatial auto-correlation interferometer, in *proceedings of FEL-05, Stanford, USA*, paper selected as highlight of the conference


Acknowledgements

I would like to thank the following people for the great support which I enjoyed during the past four years and which enabled me to present this thesis:

Firstly, Prof. Dr. Heinz Jäckel, my doctoral thesis supervisor, for giving me the opportunity to write this thesis and for the discussions and constructive ideas, which helped me to achieve the final results.

Secondly, my supervisor Dr. Volker Schlott for the interesting shifts we spent together measuring at the SLS pre-injector LINAC and for responding to my various questions and ideas. My other supervisor Dr. Hans C. Sigg for the idea of this very interesting project and for our various fruitful discussions and his constructive ideas, which encouraged me to achieve my goals.

My third supervisor Dr. Daniel Erni for his valuable support and for his much appreciated help and advice with the many theoretical aspects of the presented thesis.

Further, the RF group for ensuring operation of the SLS pre-injector LINAC; Dr. Marco Pedrozzi, Dr. Jean-Yves Raguin, Dr. Wolfgang Tron, Dr. Hans-Rudolf Fitze and Mr. Christian Geiselhart for their invaluable support with the accelerator.

Dr. Thomas Schilcher and Dr. Miroslav Dach for their much appreciated support with the programming of the VME crate which ensured the control of the experiment and also for the various valuable discussions with Dr. Thomas Schilcher which gave me renewed motivation for my interesting work.

Dr. Axel Murk of the University of Bern for his much appreciated support with the various issues of the quasi-optical set-up of the experiment.
Dr. Michael Dehler for his kind advice with the description of the emission process of long wavelength CTR.

Mr. Albert Kammerer and Mr. Andreas Jaggi for their invaluable support and their kind help with several last minute changes of the experimental set-up.

Mr. Reinhold Krammert for the support on the readout electronics of the LIS detector.

Dr. Timo Korhonen and Mr. Babak Kalantari for their help with the SLS timing system.

Mr. Martin Heiniger, Mr. Rene Kapeller, Mr. Collin Higgs, Dr. Andreas Luedecke, Dr. Detlev Vermeulen and Dr. Werner Portmann for their support with the control system of the accelerator.

Mr. Peter Hottinger, Mr. Robin Betemps, Mr. Nejat Emek for their appreciated design work of many important components of the experiment and to all technical support personal of the Paul Scherrer Institute which helped to produce many parts of the experiment.

Mr. Patrick Pollet for his help with various electronics problems.

The SLS operators for their support during the shifts.

I am very thankful to Dr. Sara Casalbuoni and Mr. Bernd Steffen for responding to my call of support and also to Mr. Axel Winter for the enjoyable and interesting weeks we all spent measuring at the SLS pre-injector LINAC. I would also like to thank Prof. Dr. Peter Schmueser and Prof. Dr. Manfred Tonutti for the various interesting and helpful discussions both at DESY and at PSI.

Dot. Marco Tagliaferri and the people of Quanta System for their support with the laser.

Dr. Trivan Pal for our interesting discussions and his much appreciated encouragement.

A special thanks to my family for their immense support during the past years. The love and care which I enjoyed have enabled me to finish this thesis. I am grateful to my father, my mother and my sister for their invaluable understanding and encouragement.
Furthermore, the author wishes to acknowledge support from the Swiss National Science Foundation.
Aerial view of the Swiss Light Source (SLS) at the Paul Scherrer Institute (PSI) in Villigen Switzerland. Photograph taken by the author.