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Numerical approaches to 3D magnetic MEMS

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Diploma Thesis

Numerical Approaches
To
3D Magnetic MEMS

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Anyone who has never made a mistake has never tried anything new.

Albert Einstein (1879-1955)
Preface

I would like to express my gratitude to the persons who motivated, encouraged, and supported me throughout all my studies and especially during the work on this diploma thesis. In particular, I am indebted to

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My parents, who supported me during my studies.

Sylwia, for her love and patience, especially during the final stage of this work.

All the other people, whom I forgot to mention.
Abstract

The present work investigates the potential of the finite element method (FEM) in the design process of magnetic Micro-Electro-Mechanical-Systems (MEMS). The magnetic forces and torques acting on a magnetic body are of great importance in wireless actuating principles. Good models are required to allow for precise and predictable motion of the magnetic body. However, analytical results are only available for simple geometries and experiments are often time consuming and may have a certain number of uncertain parameters that may influence the results. Numerical methods, and in particular the finite element method, offer the possibility to study a magnetic body with known material properties in a well defined environment.

Consequently, in this work, a method is proposed to calculate the net body torque on arbitrarily shaped bodies in a homogeneous magnetic field using the commercial finite element software ANSYS. In addition, a procedure to determine the demagnetization factors of these bodies is given. The code is first validated by the known analytical results for an ellipsoid. As an application, the demagnetization factors, as well as the net magnetic torque on brick shaped bodies and the IRIS Microrobot are calculated. A method is proposed to predict the torque acting on the Microrobot analytically. However, experimental results are necessary to confirm this method.

Furthermore, ANSYS is used to model magneto-structural coupling that is, the motion and deformation of a magnetic body due to an external magnetic field. Two devices are presented (as case studies rather than as actual design concepts), the magnetic resonator and the magnetic scratch drive actuator (MSDA). A quasi-analytical model for the static deflection of the magnetic resonator is given and good agreement with the finite element model is obtained. The MSDA is modeled to show the potential of ANSYS in modeling MEMS devices, as additional to the coupling effects, contact elements and spring elements are introduced. Again, experimental results are required.
Zusammenfassung


# Contents

- **Abstract** iii
- **Zusammenfassung** iv
- **List of Tables** vii
- **List of Figures** vii
- **Nomenclature** ix

## 1 Introduction
   - 1.1 Scope of the Thesis 1
   - 1.2 Structure of the Report 2

## 2 Theoretical Considerations
   - 2.1 Magnetostatics 3
   - 2.2 Magnetic Force and Torque 7
   - 2.3 Analytical Model 9

## 3 Finite Element Model
   - 3.1 Introduction to the Finite Element Method 11
   - 3.2 ANSYS and the Maxwell Stress Tensor 11
   - 3.3 The Meshing 12
   - 3.4 Boundary Conditions 13
   - 3.5 Material Properties 15
   - 3.6 Finite Elements 18
   - 3.7 Scripting 21

## 4 Demagnetization Factors and Magnetic Torque
   - 4.1 Demagnetization Factors 22
   - 4.2 Magnetic Torque 25
   - 4.3 Saturation Effects 34
5 Coupled Magneto-Structural Analysis 37
  5.1 Overview ................................................. 37
  5.2 Implementation of Magneto-Structural Coupling ............ 38
  5.3 Magnetic Resonator .................................... 42
  5.4 Magnetic Scratch Drive Actuator .......................... 50

6 Summary and Outlook 56

References 58

A CD Content 61

B Mathematical Expressions 62

C ANSYS Script to determine the Magnetic Torque on the IRIS Microrobot 64

D Modeling BH curves using the Langevin Function 74
List of Tables

1. The KEYOPT(1) setting for the SOLID5 element .................................. 39
2. Schematics of the indirect coupling ...................................................... 40
3. Dimensions and parameters of the magnetic resonator .......................... 42
4. Dimensions and parameters of the modeled MSDA ............................... 53

List of Figures

1. The considered magnetic and air domains ........................................... 6
2. Force and torque calculation using the Maxwell stress tensor ............... 9
3. Analytical model of the torque on a prolate ellipsoid in a constant external magnetic field ......................................................... 10
4. Mesh of the prolate ellipsoid that is used to validate the finite element code. (Only 1/8 of the model is shown) .......................... 13
5. Example of a BH curve defined in ANSYS ............................................ 17
6. The SOLID5 and SOLID96 Finite Elements ....................................... 19
7. The SOLID98 and PLANE13 Finite Elements ..................................... 21
8. Validation of the procedure to determine the demagnetization factors .... 23
9. Demagnetization factor $n_x$ of brick shaped bodies compared to their equivalent ellipsoids ....................................................... 24
10. Validation of the calculation of the magnetic torque ............................. 26
11. Magnetic torque on brick shaped structures ........................................ 27
12. Finite Element Model of the Microrobot ............................................ 28
13. Torque per volume on the Microrobot predicted by FEM and compared to analytical results ......................................................... 29
14. Fit of the relationships between the demagnetization factors and the size of the equivalent ellipsoid and $E$ ........................................... 30
15. Mapping between the Microrobot, its equivalent ellipsoid and the ellipsoid $E$ ................................................................. 31
16. Different torque results for brick with $b/a = 0.8$ and $c/a = 0.3420$ .... 33
17. The BH curve allows to study saturation effects on the torque behavior .................................................................................. 34
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Experimental results for the torque on the microrobot compared to the analytical value for an ellipsoid</td>
<td>35</td>
</tr>
<tr>
<td>19</td>
<td>Model of the magnetic resonator</td>
<td>43</td>
</tr>
<tr>
<td>20</td>
<td>Mesh of the magnetic resonator</td>
<td>43</td>
</tr>
<tr>
<td>21</td>
<td>Static deflection of the magnetic cantilever calculated by FEM and compared to the analytical model</td>
<td>47</td>
</tr>
<tr>
<td>22</td>
<td>The first two mode shapes of the magnetic resonator</td>
<td>49</td>
</tr>
<tr>
<td>23</td>
<td>Bode plot showing the frequency response of the magnetic resonator</td>
<td>49</td>
</tr>
<tr>
<td>24</td>
<td>The electrostatic scratch drive actuator</td>
<td>50</td>
</tr>
<tr>
<td>25</td>
<td>Working principle of the ESDA. See text for explanations.</td>
<td>51</td>
</tr>
<tr>
<td>26</td>
<td>Model of the Magnetic Scratch Drive Actuator</td>
<td>52</td>
</tr>
<tr>
<td>27</td>
<td>Mesh of the Magnetic Scratch Drive Actuator</td>
<td>54</td>
</tr>
<tr>
<td>28</td>
<td>Deflection of the MSDA plate</td>
<td>55</td>
</tr>
<tr>
<td>29</td>
<td>Total reaction force at the bushing–substrate interface</td>
<td>55</td>
</tr>
</tbody>
</table>
Nomenclature

Roman letters

1  Unity tensor  -

$a, b, c$  Semi-axes of ellipsoids  m

$B_{\text{air}}$  Magnetic flux density in the air (or external)  T

$B_m$  Magnetic flux density in a magnetic material (or internal)  T

$B$  Magnetic flux density  T

$D$  Electric flux density (or Electric displacement)  C/m$^2$

$\mathcal{E}$  Ellipsoid with the same torque per volume as the Microrobot

$E$  Electric field vector  V/m

$e_i$  Base vector of the Euclidian space in the $i$-th direction

$F$  Net magnetic body force  N

$H_{\text{air}}$  Magnetic field in the air (or external)  A/m

$H_m$  Magnetic field in a magnetic material (or internal)  A/m

$H$  Magnetic field vector  A/m

$M$  Magnetization  A/m

$N$  Demagnetization tensor  -

$n$  Normal unity vector  -

$n_i$  Demagnetization factor in the $i$-th direction  -

$r$  Position Vector  m

$T$  Maxwell stress tensor  N/m$^2$

$V$  Volume of the magnetic body  m$^3$
Greek letters

$\beta$ Ratio of the semi axes $b$ to $a$ of an ellipsoid

$\chi_a$ Apparent susceptibility tensor

$\chi$ Magnetic susceptibility tensor

$\delta$ Mapping between the demagnetization factors of the equivalent ellipsoid and $\mathcal{E}$

$\varphi$ Magnetic Potential

$\gamma$ Mapping between the aspect ratios of the equivalent ellipsoid and $\mathcal{E}$

$\mu_0$ Free space permeability

$\mu_r$ Magnetic relative permeability tensor

$\mu$ Magnetic permeability tensor

$\Omega$ Closed surface containing the magnetic body

$\rho$ Free electric charge density

$\theta$ Angle between the $x$-axis and $\mathbf{H}_{\text{air}}$

$\tau$ Net magnetic body torque

Abbreviations & Acronyms

APDL ANSYS Parametric Design Language

DOF Degree of freedom

ESDA Electrostatic Scratch Drive Actuator

FEM Finite Element Method

MEMS Micro Electro Mechanical System

MSDA Magnetic Scratch Drive Actuator

RMSE Root Mean Squared Error
1 Introduction

The integration of magnetic materials onto flexible silicon supports has been demonstrated several years ago [1]. Moreover, these structures have been successfully actuated by external magnetic fields. Thus, magnetic MEMS (Micro-Electro-Mechanical-Systems) offer the potential for wireless sensing and actuation applications.

However, due to the planar fabrication technology (lithography), only two dimensional devices with a given thickness can be produced. 3D MEMS with arbitrary shapes in all three directions are difficult to fabricate. One approach to overcome this limitation is to assemble three dimensional bodies from planar structures.

The knowledge of the forces and torques acting on a magnetic body that is placed in a magnetic field is necessary in order to predict its motion and consequently to allow for successful control strategies. For example, the Institute of Robotics and Intelligent Systems (IRIS) at ETH Zürich has proposed a 3D MEMS Microrobot assembled from electroplated magnetic parts that is actuated by external magnetic fields [2]. As one of the applications of the Microrobot is supposed to be eye surgery, a precise and predictable motion is required.

As will be shown later, the calculation of forces and torques is straightforward as long as the magnetization of the body is known. However, the magnetization depends on the magnetic field inside the magnetic body, which in turn is a function of the external field and the shape of the body. Hence, analytical solutions are only available for simple geometries and magnetic field configurations, e.g. the torque on an ellipsoid in a constant external magnetic field can be predicted.

1.1 Scope of the Thesis

Currently, several projects at IRIS aim at deepening the understanding of the magnetic properties of microassembled 3D magnetic bodies. Amongst others, the dependency of the torque on the shape of the mentioned Microrobot is experimentally investigated in a semester project [3].
The present work will contribute to this by proposing a finite element method (FEM) to predict the torques acting on the Microrobot. Since no experimental results are available yet, the code is validated by the analytical results for an ellipsoid. Similarly, a method for the determination of the demagnetization factors of symmetrical bodies is proposed.

Finally, the coupling of the magnetic and the mechanical field is examined to provide a method for the optimization of magnetic MEMS devices. Two devices are investigated, a magnetic resonator and the magnetic scratch drive actuator. These studies are only feasibility studies to show the capabilities of the finite element code and there is no intention to give design propositions.

1.2 Structure of the Report

Section 2 presents the theoretical background of this work. The governing equations, that is, Maxwell’s equations in the magnetostatic case, as well as the magnetic force and torque equations are introduced and applied to the case of a uniformly magnetized ellipsoid.

The basics of the finite element method are discussed in Section 3. The usage of the finite element software Ansys [4] for magnetostatics, as well as specific topics such as the meshing, the boundary conditions, and material parameter settings are explained.

The results of the finite element simulations on the calculation of the demagnetization factors and the global body torque are presented in Section 4. In addition, the torque on the Microrobot is predicted and compared to the analytical torque expression of an ellipsoid.

Coupled Field Analysis is introduced in Section 5. After presenting the basics on the coupling of different physical fields in the finite element code, two devices, the magnetic resonator, and the magnetic scratch drive actuator are studied.
2 Theoretical Considerations

After introducing Maxwell’s equations to describe electric and magnetic fields, the constitutive relationships for magnetic materials are given. Next, boundary and continuity equations are discussed. Furthermore, two equivalent expressions for the net magnetic force and torque acting on a soft magnetic body are presented. Finally, the analytical expression for the net magnetic torque on an ellipsoid is derived.

2.1 Magnetostatics

2.1.1 Maxwell’s Equations

The behavior of electromagnetic fields as well as their interactions with matter are described by Maxwell’s equations, which in the differential form are given by

\[
\begin{align*}
\text{Gauss’s Law} & \quad \nabla \cdot \mathbf{D} = \rho \quad (2.1) \\
\text{Gauss’s Law for Magnetics} & \quad \nabla \cdot \mathbf{B} = 0 \quad (2.2) \\
\text{Faraday’s Law of Induction} & \quad \nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt} \quad (2.3) \\
\text{Ampère’s Law} & \quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{d\mathbf{D}}{dt} \quad (2.4)
\end{align*}
\]

where \( \mathbf{H} \) and \( \mathbf{E} \) are the magnetic and electric field respectively, \( \mathbf{D} \) and \( \mathbf{B} \) are the electric and magnetic flux density, and \( \rho \) and \( \mathbf{J} \) are the free electric charge and free current density.

We will consider the special case with no electrical charges (\( \rho = 0 \)), no electric fields (\( \mathbf{E} = 0 \)), no currents (\( \mathbf{J} = 0 \)) and static fields (\( \frac{d(\mathbf{D})}{dt} = 0 \)). Then Maxwell’s equations reduce to

\[
\begin{align*}
\nabla \cdot \mathbf{B} & = 0 \quad (2.5) \\
\nabla \times \mathbf{H} & = 0 \quad (2.6)
\end{align*}
\]

\textsuperscript{1}See Appendix B for an overview of the used mathematical symbols

\textsuperscript{2}\( D \) is also referred to as the electric displacement
2.1 Magnetostatics

2.1.2 Magnetic Materials

In a magnetic material (as well as in vacuum) \( \mathbf{B} \) and \( \mathbf{H} \) are related by the constitutive law

\[
\mathbf{B} = \mu \mathbf{H}
\]  

(2.7)

where \( \mu \) is the magnetic permeability tensor. This relationship can be anisotropic and nonlinear (\( \mu = \mu(\mathbf{H}) \)). For air (treated as vacuum) and a linear soft magnetic material (2.7) reduces to [5]

\[
\mathbf{B}_{\text{air}} = \mu_0 \cdot \mathbf{H}_{\text{air}} \\
\mathbf{B}_m = \mu_0 \mu_r \cdot \mathbf{H}_m
\]  

(2.8)  

(2.9)

where \( \mu_0 \) is the free space permeability (scalar) and \( \mu_r \) is the relative permeability tensor that reduces to a scalar in the isotropic case. The subscript \( m \) refers to a soft magnetic material. When dealing with magnetic materials in magnetic fields the quantities \( \mathbf{B}_{\text{air}} \) and \( \mathbf{H}_{\text{air}} \) are often referred to be the external (or applied) fields, that is the field without the magnetic material, whereas \( \mathbf{B}_m \) and \( \mathbf{H}_m \) are considered to be the internal fields.

A magnetic material in an applied magnetic field \( \mathbf{H}_{\text{air}} \) responds to the field by producing a magnetic field: its magnetization \( \mathbf{M} \). Depending on their demagnetization properties, magnetic materials can be classified into two groups:

**Hard Magnetic** materials need high external fields to reduce the magnetization and are thus difficult to demagnetize. This fact is also described by designating hard magnetic materials as having a high coercivity.

**Soft Magnetic** materials on the other hand show a low coercivity and are consequently easily demagnetized.

In the following, we will only concentrate on soft magnetic materials. The magnetization \( \mathbf{M} \) of a soft magnetic material is defined by

\[
\mathbf{M} = \mathbf{B}_m / \mu_0 - \mathbf{H}_m = (\mu_r - 1) \mathbf{H}_m := \chi \mathbf{H}_m
\]  

(2.10)

where \( \mathbf{1} \) is the unity tensor, the second equality is obtained using (2.9), and \( \chi \) is the magnetic susceptibility tensor defined by \( \chi := \mu_r - 1 \) that relates the mag-
netization of a material to its internal field $H_m$.

In addition, the magnetization can be used to relate the internal field $H_m$ to the external field $H_{\text{air}}$ by \[ 2.11 \]

$$H_m = H_{\text{air}} - N \cdot M$$

where $N$ is demagnetization tensor\(^3\) that is diagonal if the coordinate frame is aligned with the axes of symmetry (if existent) of the magnetic body:

\[ 2.12 \]

$$N = \begin{bmatrix} n_x & 0 & 0 \\ 0 & n_y & 0 \\ 0 & 0 & n_z \end{bmatrix}$$

where $n_i$ is the demagnetization factor in the $i$-th direction.

By inserting (2.11) into (2.10) and assuming a constant and isotropic susceptibility ($\chi = \chi \mathbb{1}$, $\chi \in \mathbb{R}$) we find

\[ 2.13 \]

$$M = \chi H_m = \chi (H_{\text{air}} - N \cdot M).$$

Solving for $M$ we can relate the magnetization to external field $H_{\text{air}}$ by

\[ 2.14 \]

$$M = (\mathbb{1} + \chi N)^{-1} \chi H_{\text{air}} := \chi_a H_{\text{air}}$$

where $\chi_a$ is the apparent susceptibility tensor defined by

\[ 2.15 \]

$$\chi_a = \begin{bmatrix} \frac{\chi}{1+n_x \chi} & 0 & 0 \\ 0 & \frac{\chi}{1+n_y \chi} & 0 \\ 0 & 0 & \frac{\chi}{1+n_z \chi} \end{bmatrix}$$

Note, that all tensors are formulated with respect to the coordinate frame of the magnetic body.

### 2.1.3 Boundary and Continuity Conditions

The transition of the magnetic field from one material to another is governed by boundary and continuity conditions. We assume no electrical current flow and

\(^3\)The magnetic field $\mathbb{N} \cdot M$ is also referred to as the demagnetization field.
2.1 Magnetostatics

Figure 1: The considered magnetic and air domains.

the case of an isotropic, soft magnetic material with permeability $\mu_r$ and magnetization $M$ in a magnetic field $H_{\text{air}}$ (see Figure 1).

Mathematically, we consider the magnetic domain $\Gamma_m$ and the air domain $\Gamma_{\text{air}}$. The total domain $\Gamma = \Gamma_{\text{air}} \cup \Gamma_m$ is bounded by the union of their outer boundaries $\partial \Gamma = \partial \Gamma_{\text{air}} \cup \partial \Gamma_m$. Finally, $\partial \Gamma_{\text{air},m}$ is the boundary between $\Gamma_{\text{air}}$ and $\Gamma_m$, that is the boundary of the soft magnetic material that is in contact with air.

On $\partial \Gamma$ the boundary conditions are defined as [7]

$$B \cdot n = 0 \quad (2.16)$$
$$H \times n = 0 \quad (2.17)$$

where $n$ is the normal unit vector. Thus, the normal component of $B$, as well as the tangential component of $H$ vanishes. Note, that in the finite element model, we will prescribe these boundary conditions in order to achieve the desired field on $\Gamma_{\text{air}}$ (see section 3.4).

The continuity conditions on $\partial \Gamma_{\text{air},m}$ require that

$$B_m \cdot n_m + B_{\text{air}} \cdot n_{\text{air}} = 0 \quad (2.18)$$
$$H_m \times n_m + H_{\text{air}} \times n_{\text{air}} = 0 \quad (2.19)$$
where $\mathbf{n}_{\text{air}}$ and $\mathbf{n}_{\text{m}}$ are opposed normal unit vectors, that is

$$\mathbf{n}_{\text{air}} = -\mathbf{n}_{\text{m}}. \quad (2.20)$$

Thus, conservation of the normal component of the magnetic induction $\mathbf{B}$ and the tangential component of the magnetic field $\mathbf{H}$ is demanded.

Writing $\mathbf{B}_i$ and $\mathbf{H}_i$ $(i = \text{air,m})$ in their normal and tangential components

$$\mathbf{B}_i = (\mathbf{B}_{i,n}, \mathbf{B}_{i,t}) \quad (2.21)$$

$$\mathbf{H}_i = (\mathbf{H}_{i,n}, \mathbf{H}_{i,t}) \quad (2.22)$$

we can rewrite (2.18) and (2.19) as

$$\mathbf{B}_{\text{air},n} = \mathbf{B}_{\text{m},n} \quad (2.23)$$

$$\mathbf{H}_{\text{air},t} = \mathbf{H}_{\text{m},t} \quad (2.24)$$

We now can express the normal and tangential component of the magnetization on the boundary $\partial \Gamma_{\text{air,m}}$ as [5]

$$\mathbf{M}_n = \frac{\mathbf{B}_{\text{m},n}}{\mu_0} - \mathbf{H}_{\text{m},n} = \frac{\mathbf{B}_{\text{air},n}}{\mu_0} - \mathbf{H}_{\text{air},n} - \mathbf{H}_{\text{m},n} \quad (2.25)$$

$$\mathbf{M}_t = \frac{\mathbf{B}_{\text{m},t}}{\mu_0} - \mathbf{H}_{\text{m},t} - \frac{\mathbf{B}_{\text{m},t}}{\mu_0} - \mathbf{H}_{\text{air},t} = \frac{1}{\mu_0} (\mathbf{B}_{\text{m},t} - \mathbf{B}_{\text{air},t}) \quad (2.26)$$

where in each line the second equality is obtained by applying the continuity equations (2.23) and (2.24) respectively, and the last equality is given by the constitutive law 2.8.

### 2.2 Magnetic Force and Torque

In general the net body force $\mathbf{F}$ and torque $\mathbf{\tau}$ on a soft magnetic body in a magnetic field $\mathbf{H}_{\text{air}} = \frac{1}{\mu_0} \mathbf{B}_{\text{air}}$ can be found as [8]

$$\mathbf{F} = \int_V (\mathbf{M} \cdot \nabla) \mathbf{B}_{\text{air}} \, dV \quad (2.27)$$

$$\mathbf{\tau} = \int_V \left[ \mathbf{r} \times (\mathbf{M} \cdot \nabla) \mathbf{B}_{\text{air}} + \mathbf{M} \times \mathbf{B}_{\text{air}} \right] \, dV \quad (2.28)$$
where $V$ is the volume of the body and $\mathbf{r}$ is the position vector. From this we see that for a constant field ($\nabla B_i = 0, i \in \{x, y, z\}$), the net force on the body will vanish and only a torque will act on the body. This torque tends to align the magnetization vector with the external field vector.

Since (2.27) and (2.28) are only valid for a known magnetization, expressions that are not based on $\mathbf{M}$ are required. Using Poynting’s theorem combined with Maxwell’s equation it can be shown that [9]

$$
\mathbf{F} = \oint_{\Omega} \mathbf{T} \cdot \mathbf{n} \, d\Omega \quad (2.29) \\
\mathbf{\tau} = \oint_{\Omega} \mathbf{r} \times (\mathbf{T} \cdot \mathbf{n}) \, d\Omega \quad (2.30)
$$

where $\Omega$ is a closed surface containing the magnetic body, $\mathbf{n}$ is the normal vector pointing outwards and $\mathbf{T}$ is the maxwell stress tensor given by

$$
\mathbf{T} = \mathbf{H} \otimes \mathbf{B} - \frac{1}{2} (\mathbf{H} \cdot \mathbf{B}) \mathbf{1}.
$$

(2.31)

Here $\otimes$ is the dyadic product\footnote{The dyadic product $\mathbf{P} = \mathbf{u} \otimes \mathbf{v}$ of a column vector $\mathbf{u}$ and a row vector $\mathbf{v}$ is a tensor of rank two. The entries are given by $P_{ij} = u_i v_j$ (using Einstein’s summation convention).} and $\mathbf{H}$ and $\mathbf{B}$ are related by the constitutive relationship corresponding the material crossed by $\Omega$. (For example, if $\Omega$ crosses only air, we have $\mathbf{H} = \mathbf{H}_{\text{air}}$ and $\mathbf{B} = \mathbf{B}_{\text{air}} = \mu_0 \mathbf{H}_{\text{air}}$. (see Figure 2))

The volume integral in (2.27) and (2.28), has been transformed into a surface integral over a closed surface $\Omega$. Other useful properties of this approach are

1. (2.29) and (2.30) are not approximations of (2.27) and (2.28). In fact, it can be shown that these expressions are equivalent, provided that the exact values for $\mathbf{B}$ and $\mathbf{H}$ are known.

2. The choice of $\Omega$ is arbitrary as long as only empty space (defined by $\mu_r = 1$) is crossed.

Because of its surface integral nature, the Maxwell stress tensor method has been implemented and widely used in ANSYS and other finite element software for over 15 years [10]. A common application for example is the calculation of
2.3 Analytical Model

For the validation of the finite element code, analytical results are required. Therefore we consider a soft magnetic ellipsoid in a constant homogeneous external magnetic field $H_{\text{air}}$. The semi-axis of the ellipsoid are assumed to be $a \geq b \geq c > 0$ along the $x, y$ and $z$ axis respectively. Assuming the magnetic field only in planes parallel to the $(x - y)$-plane, we can write

$$H_{\text{air}} = H_{\text{air}}(x, y) = H_0(\cos \theta e_x + \sin \theta e_y) \quad (2.32)$$

where $H_0$ is the magnitude of the magnetic field, $\theta$ is the angle between the $x$-axis and $H_{\text{air}}$ (See Figure 3) and $e_i$ are the base vectors of $\mathbb{R}$.

Since the magnetization of an ellipsoid is uniform [6], we can plug in (2.32) into (2.14) and using (2.15) we find

$$M = \left( \frac{1}{1 + n_x \chi} \cos \theta e_x + \frac{1}{1 + n_y \chi} \sin \theta e_y \right) \chi H_0 \quad (2.33)$$

**Figure 2:** Force and torque calculation using the Maxwell stress tensor
Inserting (2.33) into (2.28) we can find the $z$-component of the torque to be

$$
\tau_z = \frac{(n_y - n_x)\chi^2}{1 + (n_y + n_x)\chi} \frac{\sin 2\theta}{2} \mu_0 V H_0^2
$$

which in the case of a prolate ellipsoid ($b = c$) reduces to

$$
\tau_z = \frac{\chi^2(1 - 3n_x)}{(1 + n_x\chi)(2 + \chi - n_x\chi)} \frac{\sin 2\theta}{2} \mu_0 V H_0^2
$$

Consequently we are expecting a quadratic behavior of the torque with respect to $H_0$. Furthermore, the maximal torque will be at $\theta = 45^\circ$.

Finally, the demagnetization factor $n_x$ for a prolate ellipsoid can be calculated by

$$
n_x = \frac{1 - e^2}{2e^3} \left[ \ln \left( \frac{1 + e}{1 - e} \right) - 2e \right], \quad e = \sqrt{1 - \frac{b^2}{a^2}}.
$$

However, since tabulated data is available, these values will be taken from literature [6].

---

$^5$using symmetry ($n_y = n_z$) and the constrain that for an ellipsoid $\sum_i n_i = 1$
3 Finite Element Model

The finite element method is introduced. After showing how in ANSYS the magnetic force and torque are calculated using the maxwell stress tensor, the importance of a good mesh is discussed. Furthermore, the necessary boundary conditions for a desired magnetic field are presented. Finally, the treatment of the material properties in ANSYS as well as the used elements are shown.

3.1 Introduction to the Finite Element Method

It is beyond the scope of the thesis to give a complete overview on theory of the finite element method. A thorough introduction can be found in [11]. While [12] presents a theoretical approach to FEM in electromagnetics, in [13] the focus is on the practical application of FEM to electrical machines. Furthermore, [7] and [14] propose algorithms to study magneto-structural couplings.

3.2 ANSYS and the Maxwell Stress Tensor

The calculation of the total body force and torque acting on a (soft or hard) magnetic body using ANSYS is performed using the following macros

\textbf{fmagbc (2D/3D)} This macro is invoked after selecting all elements of the magnetic body. It applies to the air elements adjacent to the magnetic material the Maxwell surface flag, which is used during the solution process to identify the elements where the Maxwell stress tensor needs to be evaluated.

\textbf{fmagsum (2D/3D)} When the magnetic field distribution is available (after solving the problem) this macro calculates the total magnetic force acting on the body on which the surface flag has been set before.

\textbf{torqsum (2D)} Similar to the previous macro, this calculates the total magnetic torque. This macro is only available in the 2D case, however it is straightforward to calculate this quantity in 3D as will be shown later.

Note, that it is also possible to set the surface flag manually, or even to specify an arbitrary path (and use manual evaluation) instead of using this approach. However, we will use this \textit{macro-based} approach as it is assumingly the least error-prone approach.
3.3 The Meshing

A good mesh is always important to achieve the highest necessary accuracy using a finite element approximation. However, the evaluation of the Maxwell stress tensor with the method described above requires some additional attention.

As mentioned, the \texttt{fmagbc} macro applies the surface flag to the air elements adjacent to the magnetic material. This means that each element on the surface of the magnetic body \textit{must have} an adjacent air element, such that the surface nodes are shared by both elements. This however constrains the shapes of the possible elements to the hexahedral (or brick) elements. Also sharp corners should be avoided (using fillets) if possible, in order to allow for accurate surface integration. Otherwise the calculated force and torque values may have errors up to 30% [15]

Meshing with brick elements is only available as a \textit{mapped} (or regular) mesh (as opposed to a \textit{free} mesh only available for non-brick elements). A mapped mesh however has specific demands on the volumes that the body and the surrounding air is modeled, e.g. they have to be fairly regular\textsuperscript{6}.

Consequently, it is proposed that the air volume is subdivided into several parts, and that a thin air film is modeled around each surface of the magnetic body. The magnetic body, as well as this tiny air film will be meshed regularly with hexahedral elements, while the remaining air is meshed freely with tetrahedral elements. This sets an additional constrain on the used elements, as transition elements (typically pyramid and wedge shaped) that connect the mapped (hexahedral) mesh with the free (tetrahedral) mesh are required. Section 3.6 discusses the used elements.

Figure 4 shows the mesh of the validation model, the prolate ellipsoid. Note that the ellipsoid is placed into a big air block to avoid the constrain of the magnetic field around the body by the boundary conditions: As the biggest semi axis of the ellipsoid is $a$, the side length of the cube shown in fig. 4(d) is $6a$\textsuperscript{7}.

\textsuperscript{6}See ANSYS Documentation [4]: \textit{Modeling and Meshing Guide}, Chapter 7

\textsuperscript{7}This number is arbitrary, however simulations using $20a$ instead show the same results
3.4 Boundary Conditions

Apart from the Maxwell surface flag, we need to specify boundary conditions for the magnetic field such that the magnetic body is studied in the desired environment. We will give these boundary conditions in a more general case, and then reduce them to the specific case we are interested in.

First, because of $\nabla \times \vec{H} = 0$, $\vec{H}$ can be written as the gradient of a scalar potential $\varphi$

$$\vec{H} = -\nabla \varphi \quad (3.1)$$

Figure 4: Mesh of the prolate ellipsoid that is used to validate the finite element code. (Only 1/8 of the model is shown)
This scalar potential $\varphi$ is the degree of freedom MAG that ANSYS uses to solve 3D magnetic field problems.\(^8\)

Second, we restrict the analysis to magnetic fields of the form

$$
\mathbf{H}_{\text{air}} = \begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix} = \begin{bmatrix}
H_x(x) \\
H_y(y) \\
H_z(z)
\end{bmatrix}.
$$  \hfill (3.2)

Thus, the component in the $i$-th direction can only be dependent on $i$. Now, we find the necessary expression for $\varphi(x, y, z)$ for a given $\mathbf{H}_{\text{air}}$ at the boundary of the cube showed in Figure 4(d). Plugging in (3.2) in (3.1) we have

$$
\begin{bmatrix}
H_x(x) \\
H_y(y) \\
H_z(z)
\end{bmatrix} = - \begin{bmatrix}
d\varphi \\
d\varphi \\
d\varphi
\end{bmatrix}. \hfill (3.3)
$$

Integrating each component separately, we get

$$\begin{align*}
\varphi_1(x) &= - \left[ \int H_x(x) dx + k_1(y, z) \right] \hfill (3.4) \\
\varphi_2(y) &= - \left[ \int H_y(y) dy + k_2(x, z) \right] \hfill (3.5) \\
\varphi_3(z) &= - \left[ \int H_z(z) dz + k_3(x, y) \right] \hfill (3.6)
\end{align*}$$

Note that the assumption (3.2) decouples these three equations. Finally, we can satisfy (3.3) by setting the integration constants $k_i$ to zero and summing up (3.4)–(3.6), thus

$$
\varphi(x, y, z) = \sum_{i=1}^{3} \varphi_i = -\left[ \int H_x(x) dx + \int H_y(y) dy + \int H_z(z) dz \right]. \hfill (3.7)
$$

\(^8\)Note, that because of $\nabla \cdot \mathbf{B} = 0$, we have also $\mathbf{B} = \nabla \times \mathbf{A}$ where $\mathbf{A}$ is the magnetic vector potential and is used in 2D analysis (DOF = AZ). The boundary conditions are changing accordingly. The use of the magnetic vector potential is not recommended in 3D problems due to reported errors.

\(^9\)Of course, this is not the most general expression. However, we are only interested in constant fields as will be explained. Thus, the restriction allows us to define gradients of the field along the specific axes.
To be able to compare the simulations to the analytical model described in Section 2.3, $H_{\text{air}}$ is considered as

$$H_{\text{air}} = H_0 \begin{bmatrix} \cos \theta \\ \sin \theta \\ 0 \end{bmatrix}$$

(3.8)

that is, a constant magnetic field with magnitude $H_0$, direction $\theta$ and no $z$-component. Then, (3.7) reduces to

$$\varphi(x, y, z) = -H_0 (x \cos \theta + y \sin \theta)$$

(3.9)

This expression is used to set the magnetic potential $\varphi$ (= MAG in ANSYS) at the boundary node with coordinates $(x, y, z)$. As boundary nodes, we consider the nodes at the external areas of the air cube that contains the magnetic body (see Fig. 4).

### 3.5 Material Properties

We focus only on the magnetic material properties; mechanical, electrical and other properties are defined in an analogous manner. ANSYS has the ability to model soft and hard magnetic materials, as well as nonlinear material behavior and anisotropy.\(^{10}\) In general, a material parameter is set by the command $\texttt{MP}$ as

$$\texttt{MP, MatProp, MatNum, PropValue}$$

where $\texttt{MathProp}$ is the name of the property to be defined, $\texttt{MatNum}$ is the material number and $\texttt{PropValue}$ is the value of $\texttt{MathProp}$. By assigning the material number to an area (or volume), e.g. using the $\texttt{mat}$ command, the area (volume) will behave as expected during solution.

As for the magnetic properties, the permeability matrix

$$\begin{bmatrix} \mu_{rx} & 0 & 0 \\ 0 & \mu_{ry} & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix}$$

(3.10)

\(^{10}\)See ANSYS Documentation [4]: Theory Reference, Chapter 5
where $\mu_{ri}$ is the relative permeability in the $i$-th direction, is specified using the variables $\text{MURX}, \text{MURY}$ and $\text{MURZ}$, corresponding to the relative permeabilities $\mu_{rx}$, $\mu_{ry}$ and $\mu_{rz}$ respectively. For example $\text{MP,MURX,2,1000}$ means that material 2 is defined to have a relative permeability $\mu_{rx} = 1000$. For isotropic material, only $\text{MURX}$ needs to be specified.

If nonlinear material behavior ($\mu = \mu(H)$) is desired, $\text{MURX,MURY}$ and $\text{MURZ}$ must not be defined. Instead, an isotropic BH curve can be specified by first writing

$tb,bh,MatNum,,n$

This means, that the $bh$ values for the material $MatNum$ will be input as a table ($tb$) with $n$ ($H_i, B_i$) pairs. Second (in the next line), these $n$ ($H_i, B_i$) pairs are specified by

$tbpt,,H_1,B_1$
$tbpt,,H_2,B_2$
$\ldots$
$tbpt,,H_n,B_n$

For example, the following code creates the BH curve for the material 2 with 25 entries, as shown in Figure 5

$\text{TB,BH,2,,25}$
$\text{TBPT,,100, .46512}$
$\text{TBPT,,200, .72993}$
$\text{TBPT,,300, .90090}$
$\text{TBPT,,400, 1.0204}$
$\text{TBPT,,500, 1.1086}$
$\text{TBPT,,600, 1.1765}$
$\text{TBPT,,700, 1.2302}$
$\text{TBPT,,800, 1.2739}$
$\text{TBPT,,900, 1.3100}$

11It is only possible to model one BH curve per material. Consequently the material behavior will be isotropic unless any $\text{MURi}$ are specified. Then, the BH curve is used only for directions along which no other parameters are defined.
Figure 5: Example of a BH curve defined in ANSYS

TBPT,,1000,1.3405
TBPT,,1400,1.4257
TBPT,,1800,1.4778
TBPT,,2200,1.5131
TBPT,,2600,1.5385
TBPT,,3000,1.5576
TBPT,,3400,1.5726
TBPT,,3800,1.5847
TBPT,,4200,1.5945
TBPT,,4600,1.6028
TBPT,,5000,1.6098
TBPT,,7000,1.6332
TBPT,,9000,1.6465
TBPT,,11000,1.6551
TBPT,,13000,1.6611
TBPT,,15000,1.6656
3.6 Finite Elements

Note that this method allows us to define a BH relationship only in the first and third quadrant passing through the origin[^11]. In order to additionally model hard magnetic material (e.g. a hysteresis loop), the coercivity needs to be defined by specifying $MGXX$, $MGYY$ and $MGZZ$.

Of course, the different material models can be used simultaneously producing a not-necessarily realistic, yet well-defined material behavior. For example, linear soft magnetic material can be specified in the $x$-direction ($MURX > 1$), a permanent magnet in the $y$-direction ($MGYY \neq 0$) and nonmagnetic behavior in the $z$-direction ($MURZ = 0$).

Finally, $\mu_0$ can be specified using the `emunit` command. This is necessary if a unit system different from the default ($MKS$) is used. For example, in the $\mu MKSV$ system that is preferably used for the simulation of MEMS, since the structural dimensions are in the range of $\mu$m, we find $\mu_0 = 4\pi 10^{-25} T \mu m / \mu A$.

In the simulations, the magnetic body will be considered to have linear and isotropic magnetic properties, if nothing else is mentioned. For this, the material property $MURX$ will be set to a constant value of 1000. This value corresponds to a typical value for soft magnetic materials [8]. In addition, choosing a relatively big value for $\mu_r = \chi + 1$ allows us, to study the effects of the material shape, as for $\chi \to \infty$, the apparent susceptibility tensor defined in (2.15) reduces to

\[
\lim_{\chi \to \infty} \chi_a = \begin{bmatrix}
\frac{1}{n_x} & 0 & 0 \\
0 & \frac{1}{n_y} & 0 \\
0 & 0 & \frac{1}{n_z}
\end{bmatrix}
\]

(3.11)

and hence the biggest possible effect on the susceptibility and consequently on the magnetic force and torque due to the shape of the magnetic body is achieved.

3.6 Finite Elements

Section 3.3 already introduced several requirements and constrains on the meshing and consequently on the elements that may be used:

[^11]: If only the first quadrant ($B_i > 0, H_i > 0$) is specified, ANSYS completes the third quadrant by symmetry. In addition, for $H > H_n$ the BH curve is extrapolated using the last slope.
3.6 Finite Elements

(a) The Solid5 Element  
(b) The Solid96 Element

Figure 6: The finite elements used in the 3D models. Note the possibility for transition elements, due to the degenerated cases of the Solid96 element.

1. The magnetic body, as well as the surrounding thin air film are meshed with a mapped mesh using hexahedral elements.

2. The surrounding bulk air may be meshed freely with tetrahedral elements. Consequently, transition elements are required between the mapped mesh and the free mesh of the surrounding air.

Additional constrains are

3. The option to set the Maxwell surface flag has to be available.

4. Obviously, the element has to have magnetic degrees of freedom (DOFs). Furthermore, for coupled magneto-structural problems (see Section 5), additional structural DOFs may be required.

Figure 6 shows the elements used for 3D problems. For the magnetic body the 3D Coupled Field Solid element SOLID5 is used, whereas for the air film, as well as for the bulk air, the 3D Magnetic Scalar Solid element SOLID96 has been chosen.\footnote{See ANSYS Documentation [4]: Element Reference, Part I, Element Library}

These two elements fulfill all the stated requirements: Both have a main hexahedral shape to mesh the magnetic body and the surrounding thin air film and
offer the option to set the *Maxwell surface flag*. The SOLID96 element has degenerated forms (tetrahedrons, wedges and pyramids) to mesh freely the surrounding bulk air and connect it to the thin air film. Finally, the SOLID5 element has the option to activate structural DOFs additionally to the magnetic DOFs.

However, they show the drawback of being linear elements; that is, they have no mid-nodes on the edges. This means, that the degree of freedom \( (\varphi = \text{MAG}) \) is allowed to vary only linearly throughout the element. Recalling (3.1) \[ H = -\nabla \varphi \] we observe that consequently the magnetic field in one element will be constant.\(^{14}\) This means that if the elements are too big unrealistic gradients in the magnetic field can occur. Thus, in regions where changes of the magnetic field are expected, namely around the magnetic body, a denser mesh is required than in the outer part of the bulk air. Since mapped mesh is used for the magnetic body and the thin air film, the mesh size on the edges (=lines) can be set with the command `lesize`.

There are other element types available in ANSYS. However, they are not equally well suited to model the problems we consider. We discuss briefly the advantages and drawbacks of two elements, the SOLID98 and SOLID62.

The SOLID98 element (see Figure 7(a)) is a second order *Coupled-Field Solid* element. However it is a tetrahedral element only, without the option for either hexahedral or transition shapes. Hence, it could be used only for the free bulk air mesh. Yet, connecting two meshes with elements of different order (if SOLID5 was used for the magnetic body and the thin air film) is a non trivial and fault-prone problem.

Another element that cannot be used is the *3-D Magneto-Structural Solid* SOLID62. It uses the magnetic vector potential \((A_X, A_Y, A_Z)\) as DOF (as opposed to the magnetic scalar potential \(\text{MAG}\) used by SOLID5, SOLID96 and SOLID98).

\(^{14}\) The mechanical analogon is that since the stress in an element is the derivative of the displacement, for linear elements, the stress is constant throughout an element.
Figure 7: The Solid98 element cannot be used as it has no transition element options. The Plane13 element is used in 2D Simulations and the solution (in the 3D case) has been found to be incorrect when the normal component of the vector potential is significant at the interface between elements of different permeability.\footnote{See ANSYS Documentation \([4]\): Element Reference, Part I, Element Library}

Finally, for two dimensional problems, the Plane13 element (the 2D version of the Solid5 element) fulfills every requirement and can be used for meshing the magnetic body as well as the air (see Figure 7(b)). In 2D everything is meshed using a mapped mesh, hence avoiding the thin air film and the need for transition elements. Note that Plane13 uses the \(z\)-component of the magnetic vector potential as DOF (\(AZ\)), yet the errors mentioned for Solid62 only apply for the 3D case.

3.7 Scripting

ANSYS uses its own scripting language APDL (ANSYS Parametric Design Language)\footnote{See ANSYS Documentation \([4]\): APDL Programmer’s Guide, Chapter 1} to allow for advanced modeling capabilities, such as conditional loops, parametric model creation, macros etc. Throughout this project, all models have been created using such scripts. They can be found on the CD that comes with this work, together with the MATLAB scripts used for the evaluation of the results.\footnote{See Appendix A for the contents of the CD}. In addition, the script for the calculation of the magnetic torque on the Microrobot, as well as its demagnetization factors, is printed in Appendix C.
4 Demagnetization Factors and Magnetic Torque

In this section, the finite element method is used to calculate the (shape dependent) demagnetization factors as well as the magnetic torque on soft magnetic bodies. Before results for arbitrarily shaped bodies are given, the method is validated by the known analytical results of the prolate ellipsoid from Section 2.3. Finally, a method to calculate the torque acting on the IRIS Microrobot analytically is proposed.

4.1 Demagnetization Factors

We recall that (2.10) and (2.14) give two different expressions for the magnetization $M$ of a magnetic body:

$$M = \chi H_{m}$$

$$M = \chi_a(n_i)H_{air}$$

where it is indicated that the apparent susceptibility $\chi_a$ is dependent on the demagnetization factors $n_i, i \in \{x, y, z\}$. In each of these two equations, there is one unknown quantity in general: in (4.1) the (internal) magnetic field is unknown, whereas in (4.2) the demagnetization factors are unknown.

However, since in the finite element model the magnetic field is calculated in each point, $H_{m}$ can be easily found after the solution of the problem. Thus, (4.1) and (4.2) can be equated and solved for each $n_i$:

$$n_i = \frac{H_{air,i}}{H_{m,i}} \frac{1}{\chi}, \quad i \in \{x, y, z\}.$$  \hspace{1cm} (4.3)

Consequently, the following procedure can be used to calculate the demagnetization factors:

1. Apply an external field $H_{air}$ in a given direction $i \in \{x, y, z\}$ as boundary condition.

2. After the solution, extract the component $H_{m,i}$ of the magnetic field in the $i$-th direction inside the magnetic body.
3. Calculate the mean value of $H_{m,i}$

4. Use (4.3) to determine $n_i$.

The script $3d_{\text{demagfacs.txt}}$ creates the geometry of the ellipsoid, applies the desired magnetic field and determines the demagnetization factors (see Appendix A).

### 4.1.1 Validation

For different $b/a$ and $c/a$ ratios of an ellipsoid, the demagnetization factors have been calculated and compared to values from literature [6]. Figures 8(a)–8(c) show the results of the validation and Figure 8(d) shows the relative error. Very good agreement with the values from literature can be achieved, the maximum error being around 2%.

![Figure 8: Validation of the procedure to determine the demagnetization factors](image-url)
4.1 Demagnetization Factors

4.1.2 Arbitrary Shapes

As an application, the script has been extended to be able to determine the demagnetization factors of brick shaped bodies. To be able to compare the results for bricks to the results of ellipsoids, we define the equivalent ellipsoid to a brick, as the ellipsoid whose semi axes are half of the side lengths of the brick. Thus, an ellipsoid with semi axes $a$, $b$ and $c$ along $x$, $y$ and $z$ direction, is equivalent to a brick with side lengths $2a$, $2b$ and $2c$ along the respective directions.

Figure 9 shows the results for $n_x$ for different bricks compared to their equivalent ellipsoids. It seems that the values for ellipsoids are not well suited to describe the demagnetization factors of bricks for all aspect ratios. For thin MEMS structures however ($c/a \leq 0.1$), this approximation seems valid, as it is also confirmed in literature [1]. Note that simulations for $c/a \leq 0.1$ cannot be performed for ellipsoids due to the high aspect ratio that prohibits the creation of the ellipsoidal geometry.

In addition, the results show also that the ellipsoid with $b/a = 0.4$ seem to be
comparable to the brick with $b/a = 0.8$. Consequently, it is possible that the magnetic torque $T_{m}^{(2.28)}$ in a homogeneous magnetic field on both bodies is the same. We will continue investigation on this result in Section 4.2.3.

### 4.2 Magnetic Torque

Similar to the demagnetization factors, the torque calculation is validated by the analytical model given in Section 2.3. Recalling the analytical torque expression in (2.35)

$$
\tau_{z} = \frac{\chi^{2}(1 - 3n_{x})}{(1 + n_{x}\chi)(2 + \chi - n_{x}\chi)} \frac{\sin 2\theta}{2} \frac{\mu_{0}VH_{0}^{2}}{\mu_{0}VH_{0}}
$$

(4.4)

the parameters will be examined as follows:

1. A constant value of $\chi = 1000$ is used.

2. The torque per volume $\tau_{z}/V$ will be compared to account for the volume effect.

3. Two different ellipsoids (and hence two different $n_{x}$ values) will be studied at a constant angle $\theta = 45^{\circ}$ and for a varying magnetic field $H_{0}$.

4. The dependence on $\theta$ is shown for two different values of $H_{0}$ for one ellipsoid.

Again, a script has been created that creates the geometry, applies a magnetic field with the desired magnitude and at the desired angle, sets the Maxwell surface flag, and finally calculates the net torque on the body.

#### 4.2.1 Validation

Figure [10] shows the validation results. Again, very good agreement with the theoretical results can be found. The error for the prolate ellipsoid ($b/a = c/a = 0.5$) is below 1% and the discrepancy (error about 4%) for the other configuration is due to the fact that the analytical formula holds only for prolate ellipsoids ($b/a = c/a$).

\[\text{assuming uniform magnetization of the brick}\]
4.2 Magnetic Torque

(a) Torque per volume as a function of the external field $H_{\text{air}}$ ($\theta = 45^\circ$)

(b) Torque per volume as a function of the angle $\theta$ ($c/a = b/a = 0.5$)

**Figure 10:** Validation of the calculation of the magnetic torque

### 4.2.2 Torque on Bricks

Brick shaped elements, such as cantilevers are often used in MEMS devices as sensing or actuating elements. Consequently, for magnetically actuated cantilevers the magnetic torque has to be known accurately (preferably as an analytical expression) to allow for an optimal design. Since analytical results are only available for ellipsoids, we will calculate the torque on bricks by finite element simulation and compare it to the analytical result of its equivalent ellipsoid. In addition, the torque value is calculated analytically, using demagnetization factors for bricks determined by FEM.

For the simulations $a = 430\mu m$ and $c = 130\mu m$ (from [1]). Thus $c/a = 0.302$ is used and $b$ is varied such that $b/a \in (0.02, 0.302)$. Next, $\theta = 45^\circ$ and the torque expression is normalized by $V, \mu_0$ and $|H_{\text{air}}|^2$.

Figure 11 shows the results. As can be seen in Figure 11(a) the assumption to normalize can be made, as all three curves show a comparable behavior with respect to $b/a$. However, as shown in Figure 11(b), the analytical approximations have a different error behavior. Approximating the torque on the brick by its equivalent ellipsoid with less than 10% error seems valid for $b/a \leq 0.035$. This result is also confirmed in literature [1]. For increasing $b/a$-ratios, the error increases and reaches $\approx 25\%$ for $b/a = 0.302$. Thus, for the equivalent ellipsoid,
4.2 Magnetic Torque

![Graphs showing magnetic torque and ratio of finite element to analytical results.]

(a) Normalized Torque \((c/a = 0.302)\)

(b) Ratio of the finite element result to both analytical results

**Figure 11:** Magnetic torque on brick shaped structures

...the error is a function of the \(b/a\)-ratio.

On the other hand, note that using the demagnetization factors from FEM, an almost constant error of \(\approx 28\%\) is obtained. This means, that the demagnetization factors cannot be determined by FEM and used directly in the analytical torque expression. Instead, it is probable that either a modification of the demagnetization factors or of the analytical torque expression will yield the expected result. Note, that we already concluded at the end of the Section 4.1.2, that the ellipsoid with \(b/a = 0.4\) shows the same demagnetization factor as the brick with \(b/a = 0.8\). We will recall this discussion again after determining the torque on the IRIS Microrobot in the next Section.

### 4.2.3 Torque on Assembled 3D MEMS

Since the proposed method has been successfully validated by the analytical results for an ellipsoid, it is now applied to arbitrarily shaped bodies. We are particularly interested in MEMS devices that are assembled from planar structures.

Therefore, the IRIS Microrobot [2] is modeled in Ansys as two ellipses with the same semi-axes \(a_{mr}\) and \(b_{mr}\) and the same thickness \(t_{mr} = 55 \mu m\), rotated by 90° along their long axis with respect to each other. Again, we define the
4.2 Magnetic Torque

Figure 12: Finite Element Model of the Microrobot

**equivalent ellipsoid** to the Microrobot as the ellipsoid with the same geometrical dimensions, that is with the semi-axes \((a_{mr}, b_{mr}, b_{mr})\).

Figure 12 shows the mesh of the IRIS Microrobot as well as the mesh of the thin air film around the microrobot.

Choosing \(a_{mr} = 1000\,\mu m\) and \(b_{mr} = \beta a_{mr}\) with the aspect ratio \(\beta = b_{mr}/a_{mr} = 0.5\) and calculating the torque on the Microrobot using the proposed method yields the results shown in the Figures 13(a) and 13(c). The curves show a quadratic behavior with respect to \(H_0\), as well as a \(\sin 2\theta\) behavior as it is predicted by the analytical formula. Hence, it is probable that the same formula can be used to describe the torque on the microrobot. Since, the only variable that now can be varied is the demagnetization factor \(n_x\), at least three different values are possible:

1. \(n_x = n_{x,FEM}\) calculated by the finite element method as proposed in the previous section,
2. \(n_x = n_{x,e}\) of the equivalent (prolate) ellipsoid, or
3. an arbitrary value for \(n_x\)
4.2 Magnetic Torque

(a) Torque predicted by FEM

(b) Torque calculated using the analytical formula with different $n_x$

Figure 13: Torque per volume on the Microrobot as a function of the external field $H_{\text{air}}$ and the angle $\theta$. It can be seen that the torque on the Microrobot predicted by FEM can be described using the analytical formula for the ellipsoid $E$ ($n_x = n_{x,r}$)
4.2 Magnetic Torque

Figure 14: Fit of the relationships between the demagnetization factors and the size of the equivalent ellipsoid and $E$

As can be seen in the Figures 13(b) and 13(d), the first two approaches do not lead to satisfactory results. The third however, shows that there exists one particular value $n_{x,r}$ that allows to describe the torque. Ideally, $n_{x,r}$ should be related to the dimensions of the microrobot. Since this is not possible without considerable computational effort, $n_{x,r}$ will be related to the dimensions of the equivalent ellipsoid of the microrobot. For this, the torque on Microrobots with different aspect ratios $\beta \in \{0.17, 0.34, 0.50, 0.64, 0.86\}$ is determined by FEM and the corresponding $n_{x,e}$ value is found manually. Then the $n_{x,e}$ values of the equivalent ellipsoids are plotted against the $n_{x,r}$ values. Note, that the thickness $t_{mr}$ of the ellipses of the microrobot is kept constant.

The result is shown in Figure 14(a) together with the fitting line. An affine relationship $\delta$, such that $n_{x,r} = \delta(n_{x,e})$ can be found

$$\delta : \mathbb{R}_+ \rightarrow \mathbb{R}_+$$

$$n_{x,e} \mapsto 0.4725 \cdot n_{x,e} + 0.0121.$$  \hspace{1cm} (4.5)

This means that to a given microrobot (described by the ellipsoid $n_{x,e}$), a unique ellipsoid $E$ (corresponding to $n_{x,r}$) can be related, such that the magnetic torque on both shapes is given by the analytical formula (2.35). In other words, the microrobot is mapped to its equivalent ellipsoid through its dimension and to $E$ by the torque per volume. In addition, the demagnetization factors of both
4.2 Magnetic Torque

\[ n_{x,r} = \delta(n_{x,e}) \]
\[ \beta_E = \gamma(\beta_e) \]

\[ n_{x,r}, \beta_E \]

Ellipsoid \( E \)

\[ n_{x,F.E.M}, \beta = \frac{b_{mr}}{a_{mr}} \]

Microrobot

**Figure 15:** The Microrobot is related to its equivalent ellipsoid by its size (aspect ratio \( b/a \)) and to the ellipsoid \( E \) by the same torque per volume \((\tau/V)\). Both ellipsoids can be related by the affine functions \( \delta \) and \( \gamma \).

Ellipsoids are mapped by \( \delta \). This is illustrated in Figure [15].

The facts that this relationship is found using ‘guessed’ \( n_{x,r} \) values and that the goodness of the fit (as measured by the Root Mean Squared Error (RMSE)) is 0.0036 are remarkable.\(^{19}\)

Figure [14(b)] shows the mentioned relationship between the size of the microrobot (aspect ratio \( b/a = b_{mr}/a_{mr} = \beta_e \) of the equivalent ellipsoid) and the aspect ratio \( \beta_E \) of the ellipsoid \( E \) (assuming that \( E \) is prolate). Again, an affine function \( \gamma \), such that \( \beta_E = \gamma(\beta_e) \), can be fitted to the data with an RMSE value of 0.0079

\[
\gamma : (0, 1) \to (0, 1) \\
\beta_e \mapsto 0.4229 \cdot \beta_e + 0.0785. \quad (4.6)
\]

\(^{19}\)An RMSE value close to 0 indicates a good fit, see *MatLab Documentation* [16].
4.2 Magnetic Torque

To summarize, the torque per volume on the microrobot can be determined analytically by

1. Find the equivalent ellipsoid \( \beta_e = \frac{b_m}{a_m} \) and its corresponding demagnetization factor \( n_{x,e} \)

2. Use \( \delta \) to determine the demagnetization factor \( n_{x,r} \) of the ellipsoid \( E \)

3. Calculate the torque using (2.35) and \( n_x = n_{x,r} \)

(4.5) and (4.6) are promising results\(^{20}\) for the ongoing projects on the Micro-robot at IRIS. Note however, that they are only numerical results without any experimental confirmations. But parallel to this work, a semester project aims at the experimental examination of the torque on the Microrobot and similar microassembled structures \(^3\). If the simulations are validated by the experiments, the proposed method can be used for the design and optimization of arbitrarily shaped 3D magnetic MEMS devices.

Now we recall the results for brick shaped elements. We found, that

1. the demagnetization factors for the ellipsoid \( b/a = 0.4 \) is comparable to the brick with \( b/a = 0.8 \), and that

2. the torque predicted by the analytical formula using the demagnetization factors of both, the equivalent ellipsoid and the finite element model showed errors.

Next, consider Figure \(^16\) where different torque per volume values are shown for the brick with \( b/a = 0.8 \) and \( c/a = 0.3420 \)

1. The result of the FEM torque calculation

2. Analytical result of the equivalent ellipsoid \( n_x = n_{x,e} \)

3. Analytical result using \( n_x = n_{x,FEM} \)

4. Analytical result using a guessed value \( n_x = n_{x,r} \)

\(^{20}\)Of course, finding the size of \( E \) using \( \gamma \) and then determining \( n_{x,r} \) is an equivalent approach.

\(^{21}\)Note, that it would also have been possible to use another approach by multiplying the torque equation (2.35) by a microrobot shape factor \( K_m \) which then again could be related to the equivalent ellipsoid.
4.2 Magnetic Torque

Figure 16: Different torque results for brick with $b/a = 0.8$ and $c/a = 0.3420$.

5. Analytical result of the ellipsoid with $b/a = 0.4$. This ellipsoid was assumed to show the same torque behavior, because it has a similar demagnetization factor (see Section 4.1.2).

Comparing Figure 16 to Figure 13(b), the same conclusions as for the micro-robot can be drawn: Neither the demagnetization factor from the finite element model nor from the equivalent ellipsoid is suitable to predict the torque accurately enough. As a consequence, the torque on the ellipsoid that has the same demagnetization factor as calculated by FEM is neither suitable. However, again a demagnetization factor can be 'guessed' to model the torque adequately. Considering the results for the microrobot, it is probable that mappings similar to $\delta$ and $\gamma$ may exist.

However, before investigating this in detail by FEM, experimental results are required to validate this approach. Once validated, FEM can be used to find mappings for arbitrary shapes and consequently, predict the torque on any magnetic body.
4.3 Saturation Effects

We consider again the torque on ellipsoids. So far, we assumed linear material behavior described by the relative permeability \( \mu_r = 1000 \). Now, a more realistic material model is studied by specifying the BH curve shown in Figure 17(a).

The nonlinear BH curve is generated using a modified Langevin function (see Appendix D for the derivation) such that its linear part still corresponds to \( \mu_r = 1000 \) (in fact \( \chi = 1000 \) is specified, but the error is negligible since \( \chi/\mu_r \approx 1 \)). The saturation magnetization is \( M_s = 8 \times 10^5 \text{A/m} \) at the saturation internal field \( H_s = 6000 \text{A/m} \). This model allows us to study the influence of saturation on the torque by applying a field \( H_{\text{air}} \) such that the internal field \( |H_m| > H_s \).

Figure 17(b) shows the torque on an ellipsoid as a function of \( \theta \) for different \( |H_{\text{air}}| \) values for the linear model and using the nonlinear BH curve. For low values of \( |H_{\text{air}}| \), no difference between the linear and the nonlinear model can be seen. However, if \( |H_{\text{air}}| \) is increased to sufficiently high values, two effects can be observed:

1. The maximum torque value shifts from \( \theta = 45^\circ \) to a higher value (still below \( 90^\circ \) however).
4.3 Saturation Effects

Figure 18: Experimental results for the torque on the microrobot compared to the analytical value for an ellipsoid

2. The curve is not symmetric anymore about $\theta = 45^\circ$ as it is predicted by the linear model. Instead, the slope of the curve decreases for small angles.

This result predicts that the analytical formula for the torque may be used only for linear material behavior. Hence, the magnetic properties of the used material need to be known to allow for accurate actuation and consequently precise motion of the magnetic body.

In addition, consider the experimental results\textsuperscript{22} shown in Figure 18. The torque on the microrobot found experimentally, as well as the analytical torque curve for an ellipsoid are presented. Note the different $\theta$-dependence of both curves. We now give a possible explanation for this discrepancy by noting that

1. It has been demonstrated that the microrobot can be related to an ellipsoid $\mathcal{E}$, such that the same torque is acting on both bodies.

2. The torque curve for an ellipsoid in high magnetic fields (see figure 17(b)) shows a similar behavior to the experimental torque curve for the microrobot.

\textsuperscript{22}The experiments were performed by Dr. Jake Abbott and Olgaç Ergeneman before the present project has been started.
4.3 Saturation Effects

Hence it is proposed, that the differences in the curves in Figure [18] are due to the saturation of the magnetic material of the microrobot. This hypothesis may also be strengthened by the fact that the magnetic field used in the experiments had a magnitude of $|B_{\text{air}}| \approx 1.5\text{T}$. It seems realistic in micromagnetics that such a magnetic field is able to saturate the material.
5 Coupled Magneto-Structural Analysis

This section presents the results for coupled magneto-structural simulations. After giving an overview on coupled-field phenomena, the implementation of the sequential coupling of the magnetic and mechanical field in ANSYS is presented. As an application, two magnetic devices are studied, the magnetic resonator and the magnetic scratch drive actuator.

5.1 Overview

In general, the discretization of a static problem using FEM results in the linear equation system

\[ K \cdot u = f \]  \hspace{1cm} (5.1)

where \( K \) is the stiffness matrix, \( u \) is the vector of the degrees of freedom (DOF) that is solved for, and \( f \) is the source vector. For example, in a structural field problem \( K \) represents the stiffness of the structure, \( u \) is the nodal displacement vector and \( f \) is the nodal force vector.

If more than one physical phenomena are considered simultaneously, (5.1) is extended to the system (in the case of two fields)

\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix}
\cdot
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
=
\begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
\]  \hspace{1cm} (5.2)

where \( K_{ii} \) are the stiffness matrices for the uncoupled problem and \( K_{ij}, (i \neq j) \) represent the coupling terms. Next, \( u_i \) and \( f_i \) are the DOF and the source vector respectively for the physical field \( i \). Consequently, if

\[ K_{ij} \neq 0, \quad (i \neq j) \]  \hspace{1cm} (5.3)

the problem is considered to be a coupled-field problem, since the result of one field \( (u_i) \) influences the determination of the other \( (u_j) \).

Typical examples of coupled-field problems are fluid-structural, electro-mechanical, fluid-electromagnetical, magneto-mechanical, and other combinations, typically
5.2 Implementation of Magneto-Structural Coupling

also involving the temperature field. For a more detailed overview of coupled-field problems, particularly in MEMS, see [7, 14, 17].

5.1.1 Coupled-Field Analysis in Ansys

There are two different possibilities in ANSYS to solve a coupled-field problem as given by the system (5.2):

**Direct coupling** using elements with multiple DOFs. In 2D, as well as in 3D problems, several elements are available that have the option of different active DOFs (activated by the `KEYOPT(1)` setting). The SOLID5 element in 3D (and its 2D equivalent PLANE13) for example, can be used among others for structural, magnetic, piezoelectric, or coupled magneto-structural problems (see Table 1).

**Indirect (or sequential) coupling** solves the coupling problem iteratively if no direct coupling is possible or desired. Here, the two fields are set up independently using the physics environment. Then the first field is solved, the coupling informations, such as forces, displacements, etc. are extracted and applied as a boundary conditions to the second field. The results of the second field are then again used as boundary conditions for the first field and the solving continues iteratively until a convergence criteria is met. If necessary, mesh morphing can be performed, that is the changing of the mesh according to structural displacements (after solving the structural field) in order to perform the next calculation in the deformed state. In addition, predefined load transfers between different fields are also available.

5.2 Implementation of Magneto-Structural Coupling

Because convergence problems have been experienced with the direct coupling method, indirect coupling is applied as shown in Table 2. As mentioned, first the geometry and the mesh is set up for the different fields and then the uncoupled boundary conditions are set. In the following, we discuss specific parts of the implementation, such as load transfer, mesh morphing, convergence check and the used element type.

---

In particular, unrealistic high values for material parameters, such as the Young’s modulus have to be used to achieve convergence.
5.2 Implementation of Magneto-Structural Coupling

<table>
<thead>
<tr>
<th>KEYOPT(1)</th>
<th>Element DOFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>UX, UY, UZ, TEMP, VOLT, MAG</td>
</tr>
<tr>
<td>1</td>
<td>TEMP, VOLT, MAG</td>
</tr>
<tr>
<td>2</td>
<td>UX, UY, UZ</td>
</tr>
<tr>
<td>3</td>
<td>UX, UY, UZ, VOLT</td>
</tr>
<tr>
<td>8</td>
<td>TEMP</td>
</tr>
<tr>
<td>9</td>
<td>VOLT</td>
</tr>
<tr>
<td>10</td>
<td>MAG</td>
</tr>
</tbody>
</table>

Table 1: The KEYOPT(1) setting for the SOLID5 element activates different DOFs and allows for a direct coupled field analysis.

5.2.1 Load Transfer and Mesh Morphing

These crucial parts of the coupling procedure are implemented as follows

Load Transfer from the magnetical to the mechanical field  First, from the solution of the magnetic field distribution, the magnetic torque $\tau_{\text{MAG}}$ is determined by the macro `torqsum` (see Section 3.2). Then the magnetic body force couple $(f_m, -f_m)$ is applied to the body as two nodal forces, where $f_m$ is determined by

$$f_m = \frac{\tau_{\text{MAG}}}{d} \quad (5.4)$$

and $d$ is the distance between the two nodes. The direction of the forces is such that they are perpendicular to the line joining the two nodes.

Mesh morphing  The structural displacement due to $(f_m, -f_m)$ is calculated and the command `damorph` is used to move the nonstructural part (air) of the mesh according to this displacement. The new position is saved and the distribution of the magnetic field in this new position is calculated. Then, the structural analysis is started from the saved position.

Note the assumption that is made here: we consider the magnetic body as rigid and apply the net body torque as a body force couple, instead of distributed forces (as obtained by the direct method). Consequently, the position of the nodes where the forces are applied is not relevant, as long as the perpendicular condition mentioned above is fulfilled.

An advantage of using the indirect method is that the results (magnetic torque and structural displacements) of the first iteration can be compared to analyt-
Table 2: Schematics of the indirect coupling as applied for the simulations of the magnetic resonator and the magnetic scratch drive actuator
5.2 Implementation of Magneto-Structural Coupling

ical results if available. In addition, specific solvers and solver settings for the solution of each field problem can be used. For example, for structural problems undergoing large displacements (geometric nonlinear problem, activated by setting nlgeom, on) ANSYS offers the possibility of automated time stepping, that is, ANSYS automatically determines the size of the next increment in the nonlinear (and hence iterative) analysis. This option is not available for the direct coupling method.

5.2.2 Convergence Check

For the convergence criterion of the coupled field problem, the maximal structural displacement $u_{i}^{\text{max}} = \max (u_{x,i} + u_{y,i})$ obtained in the $i$-th iteration is monitored during solution and convergence is reached if

$$\left| \frac{u_{i}^{\text{max}} - u_{i-1}^{\text{max}}}{u_{i-1}^{\text{max}}} \right| \leq r_{\text{tol}} \iff \left| u_{i}^{\text{max}} - u_{i-1}^{\text{max}} \right| - r_{\text{tol}} \cdot u_{i-1}^{\text{max}} \leq 0.$$  (5.5)

The right equation is the implemented version and $r_{\text{tol}}$ is the relative tolerance criterion that is set to $5 \times 10^{-3}$. In addition, (5.5) requires that at least two iterations are executed. Finally, the maximum number of iterations is set to 25 to allow for a robust break if no convergence is reached.

5.2.3 Finite Elements

The 2D Coupled Field Solid element PLANE13 (see Section 3.6) is used to mesh the magnetic as well as the nonmagnetic (in particular air) parts of the geometry. This element has no option for specifying its thickness in the third dimension. Instead, it uses unity thickness (in the used unit system). Hence to compare with analytical values, unity thickness of the 2D structures will be assumed.

For indirect coupling, only the elements specific to the active field have to be activated\(^{25}\), that is

- all elements for the magnetic field, and
- only structural elements for the mechanical field.

\(^{24}\)Note that a structural nonlinear problem requires an iterative solution by itself

\(^{25}\)Using the `ekill` and `ealive` commands
5.3 Magnetic Resonator

5.3.1 Overview

The magnetic resonator consists of a flexible silicon (nonmagnetic) beam that is clamped at one end. At the other end, a less flexible magnetic beam is rigidly attached. In a magnetic field, the torque acting on the magnetic material bends the whole structure. The studied structure is shown in Figure 19 and its mesh in Figure 20. The dimensions and parameters are given in Table 3 (The subscript \(m\) stands for the magnetic material, whereas the subscript \(b\) refers to the nonmagnetic (silicon) beam).

We will first give an analytical model to describe the bending of the structure in a static constant magnetic field to be able to validate the coupled field analysis by comparing it to the first iteration of the finite element simulation. Then, we study the harmonic response of the magnetic resonator.

5.3.2 Static Analysis

Consider the structure shown in Figure 19. Assuming, that the magnetic torque \(\tau_{\text{MAG}}\) acts in the middle of the magnetic part, we can subdivide the structure in three parts, on each of which the deflection is described by a function \(v_i(y)\), \(i = 1, 2, 3\)

<table>
<thead>
<tr>
<th>Name</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Magnetic Beam</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>(l_m)</td>
<td>400µm</td>
</tr>
<tr>
<td>Width</td>
<td>(w_m)</td>
<td>44µm</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>(Y_m)</td>
<td>200GPa</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>(\mu_r^m)</td>
<td>1000</td>
</tr>
<tr>
<td><strong>Nonmagnetic (Si) Beam</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>(l_b)</td>
<td>400µm</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>(Y_b)</td>
<td>170GPa</td>
</tr>
<tr>
<td>Width</td>
<td>(w_b)</td>
<td>2µm</td>
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<tr>
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<td>1</td>
</tr>
<tr>
<td><strong>Common Parameters</strong></td>
<td>Poisson’s ratio</td>
<td>(\nu)</td>
</tr>
</tbody>
</table>

Table 3: Dimensions and parameters of the magnetic resonator
5.3 Magnetic Resonator

Figure 19: Model of the magnetic resonator

Figure 20: Mesh of the magnetic resonator
where \( L := l_b = l_m \) is the length of both the magnetic and the nonmagnetic part. Since the bending torque is constant \( M_b(y) = \tau_{\text{MAG}} \) for \( 0 \leq y \leq (3/2)L \) and vanishes for \( y \geq (3/2)L \), we find from elementary beam mechanics [18] for \( i = 1, 2 \)

\[
v_i(y) = -\frac{\tau_{\text{MAG}}}{2E_iI_i}y^2 + c_{1,i}y + c_{2,i}
\]

where \( E_i \) and \( I_i \) are the Young’s moduli and the moment of inertia respectively and the \( c_{j,i} \) are integration constants. Next, for \( i = 3 \) we find

\[
v_3(y) = c_{1,3}y + c_{2,3}.
\]

We require that the nonmagnetic beam is clamped at its end and the transition between the three parts is continuous. Hence, the six integration constants are constrained by the following boundary and transition conditions

\[
\begin{align*}
v_1(y = 0) &= 0 \quad (5.11) \\
v_1'(y = 0) &= 0 \quad (5.12) \\
v_2(y = L) &= v_1(y = L) \quad (5.13) \\
v_2'(y = L) &= v_1'(y = L) \quad (5.14) \\
v_3(y = \frac{3}{2}L) &= v_2(y = \frac{3}{2}L) \quad (5.15) \\
v_3'(y = \frac{3}{2}L) &= v_2'(y = \frac{3}{2}L) \quad (5.16)
\end{align*}
\]

Where (\( \bullet \))’ designates the derivation with respect to \( y \).
Using *Mathematica* [19], we can solve the equation system (5.11)–(5.16) to find

\begin{align}
  c_{1,1} & = 0 \quad \text{(5.17)} \\
  c_{2,1} & = 0 \quad \text{(5.18)} \\
  c_{1,2} & = \tau_{\text{MAG}} L \left( \frac{1}{E_2 I_2} - \frac{1}{E_1 I_1} \right) \quad \text{(5.19)} \\
  c_{2,2} & = \tau_{\text{MAG}} L^2 \frac{E_2 I_2 - E_1 I_1}{2E_1 I_1 E_2 I_2} \quad \text{(5.20)} \\
  c_{1,3} & = -\tau_{\text{MAG}} L \left( \frac{1}{E_1 I_1} + \frac{1}{2E_2 I_2} \right) \quad \text{(5.21)} \\
  c_{2,3} & = \tau_{\text{MAG}} L^2 \frac{5E_1 I_1 + 4E_2 I_2}{8E_1 I_1 E_2 I_2} \quad \text{(5.22)}
\end{align}

Inserting these values into (5.9) and (5.10) and solving for \( v_{\text{max}} = v_3(2L) \) we can determine the maximal displacement of the structure to be

\[ v_{\text{max}} = -\frac{3}{8} \tau_{\text{MAG}} L^2 \frac{(E_1 I_1 + 4E_2 I_2)}{E_1 I_1 E_2 I_2} \quad \text{(5.23)} \]

This expression simplifies for \( E_2 I_2 \gg E_1 I_1 \) to

\[ v_{\text{max}} = -\frac{3}{2} \tau_{\text{MAG}} L^2 \frac{E_1 I_1 + 4E_2 I_2}{E_1 I_1 E_2 I_2} \quad \text{(5.24)} \]

which corresponds to considering the magnetic part as a rigid body and hence moving the magnetic torque at the end of the nonmagnetic beam.

Next, we need to find an analytical expression for the magnetic torque \( \tau_{\text{MAG}} \). We will use (2.28), that in a constant field \( H_{\text{air}} \) reduces to

\[ \tau = \mu_0 \int_V M \times H_{\text{air}} \, dV. \quad \text{(5.25)} \]

We will assume a constant magnetization in the \( y \) direction and with a magnitude still to determine, and hence the integration reduces to a multiplication by the volume of the magnetic part \( V = l_m \cdot w_m \cdot 1\mu m^{26} \). Finally the magnitude of the

\footnotesize
\[ 1\mu m \text{ is the unity thickness in 2D expressed in the } (\mu MKSV) \text{ unit system, see Section 3.5} \]
torque can be found to be

\[
\tau_{\text{MAG}} = \mu_0 V |M| |H_{\text{air}}| \sin \theta \tag{5.26}
\]

where \(\theta\) is the angle between the magnetization (assumed to be along the \(y\) axis) and \(H_{\text{air}}\). A constant value of \(\theta = 45^\circ\) will be used in all simulations. The magnitude of the external field \(|H_{\text{air}}|\) is known, as it is a boundary condition of the simulation. To determine the magnitude of the magnetization, we first note that it is given by

\[
|M| = \chi |H_{\text{m}}| \tag{5.27}
\]

where \(\chi = 1000\) will be used. Since we assume a constant magnetization, we may use the finite element model to extract the mean magnitude of the internal magnetic field \(|H_{\text{m}}|\) given by

\[
|H_{\text{m}}| = \frac{\sum_{k=1}^{N} |H_{\text{m,k}}|}{N} \tag{5.28}
\]

where \(N\) is the total number of nodes in the magnetic part and \(|H_{\text{m,k}}|\) is the magnitude of the magnetic field at the node \(k\).

Note that this approach reduces the analytical model to a quasi-analytical model, since the result of the finite element simulations for \(M\) is required to calculate the analytical value of the magnetic torque. However, the made assumptions (constant direction and magnitude of \(M\)) are often used in micromagnetics [1] together with the assumption that the magnetization has reached a saturated value and reduces to a scalar. In addition, in the finite element model, the torque will be calculated using the maxwell stress tensor, which is a different approach involving surface integration as introduced in Section 2.2.

The simulations are done with the parameter values shown in Table 3. The only parameter that is varied is the applied field \(H_{\text{air}}\) = from \(0.39 \times 10^4\)A/m to \(2.39 \times 10^4\)A/m, corresponding to \(B_{\text{air}} = 5\)mT and 30mT respectively. For the comparison, we extract the magnetization values after the first iteration and calculate the torque given by (5.26).
Figure 21: Static deflection of the magnetic cantilever calculated by FEM and compared to the analytical model

Figure 21(a) shows the deflection of the beam and Figure 21(b) shows the mean ratio of the analytical and the finite element result for the different $|B_{air}|$ values. In addition, the number of iterations required for each solution is shown. Note that the error is constant ($\approx 5\%$) for $|B_{air}| \leq 15\text{mT}$ but it rapidly increases for higher values. This correlates with the shift of the number of iterations from two to three. The execution of only two iterations means that the results of the second are comparable to the first. Consequently, the analytical values are expected to be close to the simulated results, since they have been determined by the results of the first iteration.

In addition, Figure 21(b) also shows that the ratio values can be fitted to a quadratic curve with an RMSE value of 0.0043. Of course, this fit is dominated by the values for three iterations, and hence we may conclude that for $|B_{air}| \in \{B_{3,\text{min}}, B_{3,\text{max}}\}$ the error will increase quadratically with $|B_{air}|$. By $B_{3,\text{min}}$ and $B_{3,\text{max}}$ we mean the minimum and maximum value for $|B_{air}|$, for which three iterations have to be executed.

Finally, the error of 5% seems rather big. However, this is due to the fact how the analytical torque is calculated. Here, the biggest error source is the determination of the magnetization. As in reality (and in the finite element model), neither the magnitude, nor the direction of $\vec{M}$ is constant, other approaches than
calculating the mean value are possible. Since it effects linearly the calculation of the torque, even slight changes may decrease the error values. In fact, it can be shown, that using the FEM torque values, the error can be reduced to below 0.1%.

### 5.3.3 Harmonic Analysis

To specify that a harmonic analysis is conducted, the command `antype,harmic` needs to be issued. Next, with `hafreq,\omega_{\text{min}},\omega_{\text{max}}` the studied frequency range is specified and `nsubst,n` sets the number of substeps (that is the number of equidistant frequencies at which results will be available).

The harmonic analysis consists of applying a sinusoidal magnetic field. Since we are interested in the deflection of the resonator for a given frequency, we do not need a fully coupled analysis. Instead, neglecting transient effects, we assume that the magnetic torque acts instantaneously on the structure. Hence, we need to extract the torque value for a given (non-harmonic) external field and apply the force couple (5.4) at the frequencies that are to be studied.

The modal analysis of the structure (using ANSYS) has found that the first two resonant frequencies are $\omega_0 = 495\text{Hz}$ and $\omega_1 = 6550\text{Hz}$ (see Figure 22 for the mode shapes). Consequently, the applied frequencies will be varied from $\omega = 0\text{Hz}$ to 7000Hz and 150 substeps are calculated\(^{27}\). The applied magnetic field is $|B_{\text{air}}| = 10\text{mT}$. In addition, the density of both materials is set to $9 \times 10^{-15}\text{kg}/\mu\text{m}^3$. A constant (independent of the excitation frequency) damping ratio $\xi \in \{1\%, 10\%, 99\%\}$ is added to the system\(^{28}\).

This procedure allows the extraction of the frequency response\(^{29}\) of the system as shown in the bode plot in Figure 23. Clearly, the resonant peaks and the phase shifts at $\omega_0$ and $\omega_1$, as well as the damping effect can be observed.

\(^{27}\)Corresponding to steps of $\approx 50\text{Hz}$

\(^{28}\)as specified by the command `DMPRAT,\xi`.

\(^{29}\)We still monitor the displacement of the free end of the resonator.
Figure 22: The first two mode shapes of the magnetic resonator. Note that intuitively $\omega_1$ corresponds to the first resonant frequency. However, ANSYS determines another mode as $\omega_0$.

Figure 23: Bode plot showing the frequency response of the magnetic resonator
5.4 Magnetic Scratch Drive Actuator

5.4.1 Overview

Electrostatic scratch drives actuators (ESDA) have been first proposed in [20]. An optimization of the actuation voltage has been introduced in [21]. Figure 24(a) shows the terminology used and Figure 24(b) presents an actual design. The plate is supported by torsional beams that are allowed to slide on the rails.

To achieve motion, a potential difference is applied between the plate and the substrate (see fig. 25). Thus, the plate is deflected towards the substrate due to electrostatic forces. For sufficiently high voltages, the plate touches the substrate and begins to flatten. In the same time, the bushing starts scratching forward and strain energy is stored in the support arms, the plate, and the bushing. Reducing
the voltage results in completing one step, as the stored strain energy is released and pulls the actuator forward. Thus, applying an alternate voltage results in a continuous motion [21].

The IRIS Microrobot may be equipped with actuators to perform specific tasks as a surgery robot. Scratch drive actuators are a possible type of these actuators. Unfortunately, the ESDA has the disadvantage that electrical contact has to be available in order to achieve motion. For the Microrobot however, wireless actuation is required to allow its usage inside the eye for example.

Since magnetic microactuation has been successfully presented in [1], magnetically actuated scratch drive actuators (MSDA) seem theoretically possible and research on them will be continued at IRIS. In this work, an ANSYS model is shown to demonstrate the working principle of the magnetic scratch drive.

### 5.4.2 Used Element Types

Three different element types are necessary to model the motion of the MSDA:
5.4 Magnetic Scratch Drive Actuator

**Coupled Magneto-Structural Elements** For the 2D Problem, again the PLANE13 element is used with the iterative procedure described in Section 5.1.1 and shown in Figure 2.

**Contact Elements** The elements TARGE169 and CONTA171 form a contact couple to model contact and sliding between the plate and the substrate. Since a large number of parameters is required to model the contact (type of contact, behavior of contact surfaces, used contact algorithms, ...), experimental results are necessary to allow for an efficient finite element model and consequently for optimization possibilities.

**Spring Elements** The line element COMBIN14 represents a linear spring-damper element with different possible DOFs. It is used to model torsional support along the z-axis, as well as longitudinal support in both x and y directions (without damping). This approach allows to prescribe exactly the stiffness of the modeled springs.

5.4.3 Modeling

No analytical model will be developed due to the large number of uncertain parameters (contact modeling, magnetic torque, support stiffness). Analytical models for the electrostatic scratch drive actuator are generally based on beam theory and relate the speed and stepsize of the scratch drive to the input voltage. They can be found for example in [20–24].

---

**Figure 26:** Model of the Magnetic Scratch Drive Actuator
Table 4: Dimensions and parameters of the modeled MSDA

<table>
<thead>
<tr>
<th>Name</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of plate</td>
<td>$l_p$</td>
<td>430 $\mu$m</td>
</tr>
<tr>
<td>Thickness of plate</td>
<td>$t_p$</td>
<td>15 $\mu$m</td>
</tr>
<tr>
<td>Height of bushing</td>
<td>$h_b$</td>
<td>10 $\mu$m</td>
</tr>
<tr>
<td>Thickness of bushing</td>
<td>$t_b$</td>
<td>10 $\mu$m</td>
</tr>
<tr>
<td>Bushing-Substrate distance</td>
<td>$d_{bs}$</td>
<td>0 $\mu$m</td>
</tr>
<tr>
<td>Spring Constant in $x$-dir.</td>
<td>$k_x$</td>
<td>0.2296 $\mu$N/$\mu$m</td>
</tr>
<tr>
<td>Spring Constant in $y$-dir.</td>
<td>$k_y$</td>
<td>0.2296 $\mu$N/$\mu$m</td>
</tr>
<tr>
<td>Torsional Spring Constant</td>
<td>$k_\phi$</td>
<td>0.02 $\mu$N$\mu$m/rad</td>
</tr>
<tr>
<td>Young’s Modulus (both)</td>
<td>$Y$</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio (both)</td>
<td>$\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>$\mu_r$</td>
<td>1000</td>
</tr>
<tr>
<td>Angle between $x$-dir. and applied field</td>
<td>$\theta$</td>
<td>45$^\circ$</td>
</tr>
</tbody>
</table>

For the finite element model, consider Figure 26 where the structure of the actuator is shown. Again, the magnetic torque $\tau_{\text{MAC}}$ calculated from the magnetic field distribution is applied as a body force couple $(f_m, -f_m)$ given by (5.4). The support is modeled as two linear and one torsional spring connected to the same point at the right end of the plate. Table 4 shows the values of the used parameters. Figure 27 shows the meshing of the MSDA. Note that the spring elements cannot be seen as their nodes are coincident.

Since the mechanical structure is constrained only at the fixations of the springs, convergence problems occurred. Consequently, the distance between the bushing and the substrate $d_{bs}$ is set to zero and the motion of the nodes touching the substrate is constrained by allowing neither $x$ nor $y$ displacements. Thus, no steps can be observed. Therefore we limit the model to a static analysis and vary the external field $|H_{\text{air}}|$ from 0 to $\approx$800000 A/m (corresponding to $|B_{\text{air}}|$ from 0 to $\approx$1 T).

### 5.4.4 Results

The displacement of the MSDA plate is shown in Figure 28. After a rigid body motion until $|H_{\text{air}}| \approx 300000$ A/m the plate touches the substrate and starts bending.

---

*Again, the width of the model is limited to 1 $\mu$m due to its 2D nature.*
As mentioned, the stepsize cannot be monitored because the bushing is constrained to the substrate. However, this same fact allows to extract the reaction forces at this interface. Of course, the total reaction force is the sum of the nodal reaction forces. Considering the $x$-component of the total reaction force as a measure for the acceleration in the opposite direction, it can be related to the applied field as shown in Figure 29.

As expected, the force is zero during the rigid body motion, until the plate has touched the substrate. Then, it increases in magnitude to attain a maximum negative value at $\approx 30000\text{A/m}$. Finally, the magnitude decreases to attain a relatively constant value after $\approx 30000\text{A/m}$. Consequently, it is expected that the scratch drive has to be actuated in a specific magnetic field range, in order to attain the maximum possible acceleration and hence maximum speed and stepsize.
5.4 Magnetic Scratch Drive Actuator

Figure 28: Deflection of the MSDA plate.

Figure 29: Total reaction force at the bushing–substrate interface. The magnetic field is applied at $\theta = 45^\circ$ to the $x$-direction.
6 Summary and Outlook

The presented results are summarized and an outlook to possible future work is given.

The theory of the behavior of electromagnetic fields, as well as their interactions with matter has been briefly discussed with emphasis on magnetic forces and torques. Two different, yet equivalent approaches for their calculation have been introduced, based on volume integration of the magnetization, and based on surface integration the Maxwell stress tensor. In addition, the analytical model of an ellipsoid in a homogeneous field has been derived to be able to compare with the finite element results.

Next, the finite element package ANSYS has been presented. It has been shown that ANSYS uses the Maxwell stress tensor to calculate magnetic forces and torques on magnetic material. Finally, specific FEM topics, such as the influence of the mesh, the used element types as well as the necessary boundary conditions are discussed.

Then, the finite element method is validated by calculating the demagnetization factors and the magnetic torque on ellipsoid and comparing them to the analytical model. Very good agreement has been found and the same quantities have been calculated for bricks, as well as for the IRIS Microrobot.

A method to predict the torque on the microrobot using the analytical torque formula for an ellipsoid has been derived. It consists of a mapping between the equivalent ellipsoid of the microrobot and the ellipsoid $E$, such that the torque per volume on the microrobot and on $E$ is the same. In addition, it is possible that a similar mapping exists for brick shaped bodies. However, this topic needs further investigation.

In addition, the effects of material saturation has been studied. The finite element model shows that the torque for a saturated ellipsoid differs considerably from the preliminary analytical model. In addition, it is proposed, that
available experimental torque data of the microrobot is explained by saturation effects rather than shape effects.

Finally, magneto-structural coupling has been studied on two devices, a magnetic resonator and the magnetic scratch drive actuator. For the resonator, an quasi-analytical model is given using the magnetization from the finite element model. It is shown, that the analytical and the finite element model are in good agreement as long as the deflection of the resonator is small. Furthermore, the resonance frequencies are found by modal analysis and the frequency response of the resonator is determined by a harmonic analysis.

For the scratch drive actuator, it has been found that the applied field has to be in a specific range in order to achieve the highest possible acceleration and consequently the highest possible force. However, experimental results are required to model the contact behavior of the plate to the substrate more accurately. Only then, a finite element model of the scratch drive actuator can be made that allows for optimization of the device.

As already mentioned, experimental results are indispensable to validate the proposed finite element code. It is proposed, that the presented numerical experiments are first reproduced. Then, the presented approach can be used for further investigation without needing new experiments.

Especially the proposed method of mapping the microrobot to two different ellipsoids needs to be verified experimentally as it is a promising method for the analytical calculation of the magnetic torque on arbitrary shapes.

Finally, test structures are required to validate the magneto-structural coupling. For this, simple cantilever structures can be studied, as they are easy to fabricate and analytical beam theory can be used for the description of their behavior. Adding complexity to the test structures, for example supports, allows to validate the simple spring elements used in the simulations.
References


A CD Content

This work is accompanied by a CD. The following table summarizes the content of the CD.

<table>
<thead>
<tr>
<th>Filename</th>
<th>Folder</th>
<th>Comment</th>
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<tbody>
<tr>
<td>ANSYS Scripts</td>
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<tr>
<td>3d_demagfac.txt</td>
<td>ansys_scripts</td>
<td>Demagnetization factors of ellipsoids and bricks</td>
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<tr>
<td>3d_torque.txt</td>
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<td>Torque on ellipsoids and bricks</td>
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<td>Demagnetization factors and torque on the Microrobot</td>
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<td>ansys_scripts</td>
<td>Static response of magnetic resonator</td>
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<tr>
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<td>ansys_scripts</td>
<td>Harmonic response of magnetic resonator</td>
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<tr>
<td>device_cpl.txt</td>
<td>ansys_scripts</td>
<td>Deformation of magnetic scratch drive</td>
</tr>
<tr>
<td><strong>MATLAB Scripts</strong></td>
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<td>matlab_scripts</td>
<td>Validates torque results</td>
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<tr>
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<td>Validates demagnetization factor results</td>
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<td>matlab_scripts</td>
<td>Microrobot torque results (varying $H_{\text{air}}$)</td>
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<tr>
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<td>matlab_scripts</td>
<td>Microrobot torque results (varying $\theta$)</td>
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<td>matlab_scripts</td>
<td>Determine a BH curve using a Langevin function</td>
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<td>matlab_scripts</td>
<td>Saturation effects of an Ellipsoid</td>
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<td>matlab_scripts</td>
<td>Brick torque results</td>
</tr>
<tr>
<td>cantilever_anal.m</td>
<td>matlab_scripts</td>
<td>Analytical model of the static magnetic resonator</td>
</tr>
<tr>
<td>v.m</td>
<td>matlab_scripts</td>
<td>Function to calculate displacement of the resonator</td>
</tr>
<tr>
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<td>Magnetic scratch drive results</td>
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<td>Mathematica notebook to solve the equation system of the magnetic resonator</td>
</tr>
<tr>
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<td>backup</td>
<td>all files created during this work</td>
</tr>
</tbody>
</table>
B Mathematical Expressions

We give an overview on the mathematical expressions used in this work. First, to fix the notation, we designate a scalar value by

$$\lambda \in \mathbb{R},$$

an $n$ dimensional vector by

$$\mathbf{u} \in \mathbb{R}^n,$$

and a tensor of second order by

$$\mathbf{T} \in \mathbb{R}^{n \times m}.$$

Vectors and tensors are represented with respect to the standard cartesian space, thus for $n = m = 3$

$$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} t_{xx} & t_{xy} & t_{xz} \\ t_{yx} & t_{yy} & t_{yz} \\ t_{zx} & t_{zy} & t_{zz} \end{bmatrix} \tag{B.1} \tag{B.2}$$

The multiplication of a vector or a tensor with a scalar results in a vector or tensor respectively:

$$\mathbf{v} = \lambda \mathbf{u}, \quad v_j = \lambda v_i \tag{B.3}$$

$$\mathbf{P} = \lambda \mathbf{T}, \quad p_{ij} = \lambda t_{ij} \tag{B.4}$$

Multiplication of a vector by another vector results in three different possibilities. First, the scalar product is obtained by

$$\lambda = \mathbf{u} \cdot \mathbf{v} = \sum_{i=1}^{n} u_i v_i \in \mathbb{R} \tag{B.5}$$
Next, the vector product (or cross product) of two \(n\) dimensional vectors results in an \(n\) dimensional vector

\[
\mathbf{w} = \mathbf{u} \times \mathbf{v} = \begin{bmatrix}
    u_yv_z - u_zv_y \\
    u_zv_x - u_xv_z \\
    u_xv_y - u_yv_x
\end{bmatrix} \in \mathbb{R}^n \tag{B.6}
\]

\[
|\mathbf{w}| = |\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin \theta \tag{B.7}
\]

where the last equation determines the magnitude of \(\mathbf{w}\) and \(\theta\) is the angle between \(\mathbf{u}\) and \(\mathbf{v}\). Furthermore, the dyadic product of two vectors results in a tensor given by

\[
\mathbf{T} = \mathbf{u} \otimes \mathbf{v}, \quad t_{ij} = u_i v_j \tag{B.8}
\]

using Einstein’s summing convention. Finally, multiplication of a tensor by a vector results in a vector defined as

\[
\mathbf{v} = \mathbf{T} \cdot \mathbf{u}, \quad v_i = \sum_{j=1}^{n} t_{ij} u_j. \tag{B.9}
\]

Next, we introduce the nabla operator \(\nabla\). It is defined as

\[
\nabla = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right] \tag{B.10}
\]

and is used to determine the gradient of a scalar field \(\lambda(x, y, z)\) by

\[
\nabla \lambda = \left[ \frac{\partial \lambda}{\partial x}, \frac{\partial \lambda}{\partial y}, \frac{\partial \lambda}{\partial z} \right]^T. \tag{B.11}
\]

In addition, the divergence of a vector field \(\mathbf{u}(x, y, z)\) is given by

\[
\nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}, \tag{B.12}
\]

and finally, the rotation (or curl) of a vector field is defined by

\[
\nabla \times \mathbf{u} = \begin{bmatrix}
    \frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \\
    \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \\
    \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}
\end{bmatrix}. \tag{B.13}
\]
C ANSYS Script to determine the Magnetic Torque on the IRIS Microrobot

! Author: Zoltan Nagy
! Date : 07/2006
!
! Full 3D-Model of Microrobot
!
! Script to calculate
! 1. The Shape Anisotropy Coefficients of
! 2. The Torque acting on
! The IRIS Microrobot

!Usage: 1. Choose Analysis (anal)
! 2. Set Dimensions
! 3a. If anal=0 : Choose Angle (alpha) and Hext
! 3b. If anal=1 : Choose Direction (axis)
!------------------------------------------------------
finish
/clear
/pnum,mat,1 $/num,1 !plotting settings
/filname,microrobot

! Choose Analysis
! 0: torque calculation
! 1: shape anisotropy calculations
anal = 0

! Geometry Parameters
! ---------------------
! uMKSV unit system is used

l = 2000 !length of device: 2mm
w = (0.5)*1 !width of device: 1mm
h = w  !height of device symmetrical
t = 55  !thickness of openings: 55um

elsiz = l/100  !needed for thin air film

/prep7  !enter preprocessor

! ----------------
! CREATE GEOMETRY
! ----------------
/prep7  !enter preprocessor
PI = 3.14159265358979
emunit,muzro,4*PI*(1E-7)*(1E-18)  ! use uMKSV system

! Define Elliptical Coordinate systems
exy = w/l
exz = h/l
eyz = w/h

local,11,1,,,,,,exy
local,12,1,,,,90,,exz
local,13,1,,,,,-90,eyz
local,14,1,t/2,t/2,t/2,,exy
local,15,1,t/2,t/2,t/2,,90,,exz
local,16,1,t/2,t/2,t/2,,,-90,eyz
local,17,1,0,t/2,0,,exy
local,18,1,0,0,t/2,,90,,exz

! Define Keypoints
! ----------------
csys,0
k,1,0,0,0  $k,2,0,0,w/2  $k,3,1/2,2,0,0  $k,4,0,t/2,0
k,5,0,t/2,w/2  $k,6,1/2,t/2,0  $k,7,0,w/2,0  $k,8,t/2,w/2,0
k,9,t/2,t/2,w/2  $k,10,1/2,t/2,t/2  $k,11,0,w/2,t/2
k, 12, 0, 0, t/2 $k, 13, t/2, 0, 0 $k, 14, l/2, 0, t/2 $k, 15, t/2, 0, w/2
k, 16, t/2, t/2, t/2 $k, 17, t/2, w/2, t/2 $k, 18, t/2, t/2, 0
k, 19, w/2+elsiz, 0 $k, 20, 0, w/2+elsiz, t/2 $k, 21, 1/2+elsiz, t/2, 0
k, 22, 1/2+elsiz, t/2, t/2 $k, 23, 1/2+elsiz, 0, t/2 $k, 24, 1/2+elsiz, 0, 0
k, 25, 0, w/2+elsiz $k, 26, 0, 0, t/2+elsiz

! create areas
csys, 17 $a, 4, 6, 7 $a, 4, 21, 19 $csys, 0 $a, 1, 3, 6, 4 $a, 3, 24, 21, 6
avolap, all $numcmp, all

! create volumes
l, 1, 1, 12 $*get, maxline, line, 0, num, max $vdrag, all,,,$, maxline
csys, 18 $a, 12, 14, 2 $a, 12, 25, 23 $avolap, 20, 21 $vdrag, 20, 22,,,$, 23 $csys, 0
nummrg, all
vrotat, 8,,,$, 27, 10, 90
nummrg, all

! create air
block, 0, 6*l, 0, 6*l, 0, 6*l
volap, all

! create whole volume by symmetry
alls
vsymmm, x, all
vsymmm, y, all
vsymmm, z, all

! merge coincident areas, volumes, keypoints and lines
nummrg, all

! -------------------
! MATERIAL PARAMETERS
! -------------------
muair = 1 !relative permeability - air
mumat = 1000 !relative permeability - magnetic material

mp,murx,1,muair
mp,murx,2,mumat

! SET ELEMENT TYPES
et,1,solid5    ! hex element needed to ensure correct torque calculation
keyopt,1,1,10  ! set 'mag'netic potential as only DOF
et,2,solid96   ! allows transition from brick to tetrahedron shaped elements

! MESHING

! microrobot
vsel,s,volu,,2,58,8
vsel,a,volu,,1,11,5
vsel,a,volu,,17,27,5
vsel,a,volu,,23,43,10
vsel,a,volu,,39,59,10
vsel,a,volu,,54,55,1
vsel,a,volu,,7,38,31
vsum    !get volume of robot
*get,volrob,volu,all,volu
type,1
mat,2
lsel,s,length,,1/2,1
lesize,all,,.,10
cm,'robot',volu
vmesh,all

! air film
vsel,s,volu,,4,60,8
vsel,a,volu,,5,61,8
vsel,a,volu,,8,56,16
vsel,a,volu,,3,15,6
vsel,a,volu,,19,31,6
vsel,a,volu,,35,47,6
vsel,a,volu,,51,63,6
cm,'airfilm',volu
type,1
mat,1
vmesh,all

! air
alls
cmsel,u,'robot'
cmsel,u,'airfilm'
type,2
mat,1
lsel,s,length,,6*l
lesize,all,,,10
mshape,1,3d ! needed by solid96
vmesh,all

! merge coincident areas, volumes, keypoints and lines
nummrg,all

! -------------------
! BOUNDARY CONDITIONS
! -------------------
alls
! Maxwell Surface Flag
esel,s,mat,,2
cm,device,elem
fmagbc,'device'
! Create Homogenous Magnetic Field in Arbitrary Direction
*afun,deg ! use degrees for angular units

*if,anal,eq,0,then
alpha = 60 ! angle between B-Field and horizontal
*elseif,anal,eq,1,then
axis = 'x' ! axis to which the magnetic field H is parallel
*endif

PI = 3.14159265358979

! Define values for constant H-field
Hext = (6e8)*7.96E4*1E6 !([pA/um] = [A/m]*1E6)
B_air = 4*PI*(1E-7)*(1E-18)*Hext ! Bfield in air [T] / [TH/um] = [H/m]*1E-18

! Set Boundary Condition
alls
! Select Boundary nodes
nsel,s,loc,x,-6*l $nsel,a,loc,x,6*l
nsel,a,loc,y,-6*l $nsel,a,loc,y,6*l
nsel,a,loc,z,-6*l $nsel,a,loc,z,6*l

*get,nrnodes,node,0,count !get number of nodes
nprev = 0 !previous node number

*do,i,1,nrnodes
  *get,nact,node,nprev,nxth ! find actual node number
  *get,nodex,node,nact,loc,x ! get x component of position
  *get,nodey,node,nact,loc,y ! get y component of position
  *get,nodez,node,nact,loc,z ! get z component of position
  *if,anal,eq,0,then
    ! torque calculation
d,nact,mag,-Hext*(cos(alpha)*nodex+sin(alpha)*nodey)
  *elseif,anal,eq,1,then
! shape anisotropy
*if,axis,eq,'x',then
d,nact,mag,-Hext*nodex
*elseif,axis,eq,'y',then
d,nact,mag,-Hext*nodey
*elseif,axis,eq,'z',then
d,nact,mag,-Hext*nodez
*endif
*endif

nprev = nact ! overwrite parameter

*enddo
/solu
alls
*if,anal,eq,0,then
  nsubst,5 ! calculate in 5 substeps -> get more h values
  outres,all,all
*endif
solv
/post1
alls

*if,anal,eq,0,then

  ! TORQUE CALCULATION
  set,last
  *dim,tmag,array,5
  *do,j,1,5
    set,next
  alls
torque_x = 0
torque_y = 0
torque_z = 0
fxtot = 0
fytot = 0
fztot = 0

! Select Nodes where force values are available
cmsel,,'device' $nsle,,ext $esln
cmsel,u,'device' $nsle,s,1

! Perform Calculation
*get,nrnodes,node,0,count  ! get number of nodes
nprev = 0  ! previous node number

*do,i,1,nrnodes
    *get,nact,node,nprev,nxth  ! find actual node number
    *get,nodex,node,nact,loc,x  ! x comp. of position
    *get,nodey,node,nact,loc,y  ! y comp. of position
    *get,nodez,node,nact,loc,z  ! z comp. of position

    *get,fx,node,nact,fmag,x  ! x comp. of mag. force
    *get,fy,node,nact,fmag,y  ! y comp. of mag. force
    *get,fz,node,nact,fmag,z  ! z comp. of mag. force

    torque_x = torque_x + (nodey*fz-nodez*fy)  ! x comp. of torque
    torque_y = torque_y + (nodez*fx-nodex*fz)  ! y comp. of torque
    torque_z = torque_z + (nodex*fy-nodey*fx)  ! z comp. of torque

    fxtot = fxtot + fx
    fytot = fytot + fy
    fztot = fztot + fz

    nprev = nact  ! overwrite parameter
*endo
! Calculate Volume of body
V = volrob

! Comparison to Analytical Results

! torque per volume
torque_x_v = torque_x/V
torque_y_v = torque_y/V
torque_z_v = torque_z/V

tmag(j) = torque_z_v
! units are in uMKS
! h(MKS) = 1e-6*h(uMKS)
! t/v(MKS) = 1e6*t/v(uMKS)
*enddo

*elseif, anal, eq, 1, then

! SHAPE ANISOTROPY FACTORS

! calculate mean internal magnetic field 'Hi,mean'
esel, s, mat,, 2 $nsle ! select nodes

*get, nrnodes, node, 0, count ! get number of nodes
nprev = 0

Him = 0 ! mean internal field
*do, i, 1, nrnodes
    *get, nact, node, nprev, nxth ! find actual node number
    *get, htemp, node, nact, h, axis ! get required H-value
    Him = Him + htemp
    nprev = nact ! overwrite parameter
*enddo

Him = Him/nrnodes ! Find mean field

! Calculate shape anisotropy factor
*if,axis,eq,'x',then
    nx = ((Hext/Him)-1)/(mumat+1)
*elseif,axis,eq,'y',then
    ny = ((Hext/Him)-1)/(mumat+1)
*elseif,axis,eq,'z',then
    nz = ((Hext/Him)-1)/(mumat+1)
*endif

*endif
alls
D Modeling BH curves using the Langevin Function

For soft magnetic materials a modified Langevin function can be used to model the BH curve [25]. The Langevin function is given by

\[ \mathcal{L}(x) = \coth x - \frac{1}{x}. \] (D.1)

This allows to calculate the magnetization \( |M| \) of the magnetic material by

\[ |M| = M_s \mathcal{L} \left( \frac{|H_m|}{H_0} \right) \] (D.2)

where \( M_s \) is the saturation magnetization of the material, \( H_m \) is its internal field and \( H_0 \) can be approximated using the materials maximum susceptibility \( \chi \) to be

\[ H_0 = \frac{M_s}{3\chi} \] (D.3)

Finally, we note that the magnitude of the magnetic induction \( B_m \) is given by

\[ |B_m| = \mu_0 (|H_m| + |M|) \] (D.4)

Thus, for a given internal field \( H_m \) the BH curve is generated by first calculating the corresponding magnetization values and then determining the internal magnetic induction \( B_m \) as shown in the following MATLAB script:

```matlab
ms = 8e5; % saturation magnetization (A/m)
xi = 1000; % susceptibility
h0 = ms/(3*xi); % approximation
hi = [0.0001:3e5/50:3e5]; % internal field (A/m)
x = hi/h0; % parameter
m = ms*(coth(x)-(1./x)); % magnetization (A/m)
mu0 = 4*pi*1e-7; % free space permitivity
b = mu0*(hi+m); % internal induction (T)
plot(hi,b,'o-') % plot BH curve
```