Experimental and numerical study on the small scale features of turbulent entrainment

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY
ZURICH

for the degree of
Dr. sc. ETH Zürich

presented by
MARKUS HOLZNER
Dipl. Ing.
born in May 14, 1978
citizen of Italy

accepted on the recommendation of
Prof. Dr. W. Kinzelbach, examiner
Prof. Dr. D. Poulakakos, co-examiner
Prof. Dr. A. Tsinober, co-examiner

2007
Contents

1 Abstract vii

2 Zusammenfassung ix

3 Introduction 1
   3.1 Overview of previous studies on turbulent entrainment 2
   3.2 Similarity theory and entrainment 3
   3.3 Outline of the thesis 5

4 Methods 7
   4.1 Experimental apparatus 7
      4.1.1 Oscillating grid 7
      4.1.2 Rotating disks 8
      4.1.3 PIV and dye visualization 8
      4.1.4 3D Scanning Particle Tracking Velocimetry 10
   4.2 Direct Numerical Simulation 10

5 Scanning PTV 13
   5.1 Introduction 13
   5.2 Experimental apparatus and setup 16
      5.2.1 Hardware 16
      5.2.2 Flow 19
Seite Leer / Blank leaf
Chapter 1

Abstract

The main aim of this study is an analysis of small scale enstrophy and strain dynamics in proximity of a turbulent/non-turbulent interface in a flow with weak mean shear. The main tools used for the investigation are Particle Tracking Velocimetry (3D-PTV) and Direct Numerical Simulation (DNS). In both experiment and simulation, turbulence is generated at the plane $x_2=0$ and propagates along $x_2 > 0$. The Taylor microscale Reynolds number is $Re_\lambda=50$. On the experimental side, before the Lagrangian measurements were conducted, several preliminary steps were carried out, namely (i) further development of the 3D-PTV technique, (ii) a feasibility study of measuring the Laplacian of vorticity, $\nu \nabla^2 \omega$, and (iii) benchmark of detection techniques for the turbulent/non-turbulent interface. First, the experimental method of 3D-PTV was further developed through changes of the illumination, image acquisition and analysis. In particular, a scanning system was developed allowing for larger particle densities and larger observation windows. Checks based on precise kinematical relations were performed to validate the modifications of the new scanning technique for 3D-PTV. Compared to classical PTV, both the observation volume and the spatial resolution could be nearly doubled. As a next step, experiments in quasi-homogeneous turbulence were conducted to assess the feasibility of measuring the Laplacian of vorticity through PTV. The same experiments were also used to analyze some simple aspects of the contribution of this term to vorticity dynamics. The experimental error associated with the measurements of $\nu \nabla^2 \omega$ is considerable and therefore the results are valid for a filtered data-set only. However, comparison with DNS demonstrated that the results based on this restricted data-set are (i) representative of the flow and (ii) in good agreement with the simulation, at least on the qualitative level. In view of preparation for PTV measurements in proximity of a turbulent/non-turbulent interface, a preliminary study on to the identification of the interface and its propagation dynamics was carried out. Namely, Particle Image Velocimetry (PIV) and dye visualization experiments were conducted to bench-
mark detection techniques of the turbulent/non-turbulent interface generated by an oscillating grid. In this study, some important properties of the interface were confirmed, such as the steep gradient of vorticity at the interface and the propagation velocity of the front. The theoretically predicted $t^{1/2}$ law of propagation of the turbulent front in time, $t$, could be verified with both experimental methods by using simple level-based detection techniques. The similarity of the results derived from the dye concentration, velocity and vorticity measurements and the consistency with results in the literature, suggests that it is possible to utilize the same level-based interface detection method also in the three-dimensional Lagrangian measurements. Finally, small scale enstrophy and strain dynamics in proximity of a turbulent/non-turbulent interface were analyzed using PTV and DNS. The comparison of flow properties in the turbulent (A), intermediate (B) and non-turbulent (C) regions in the proximity of the interface allowed for direct observation of the key physical processes underlying the entrainment phenomenon. It was found that the behavior of vorticity-related quantities is very different from their strain-related counterparts. For example, viscosity is important for the increase of enstrophy, $\omega^2$, in regions B and C, while it is not responsible for building up strain. It was also found that both $\omega_j \omega_j s_{ij}$ and $\nu \omega_i \nabla^2 \omega_i$ are responsible for the increase of $\omega^2$ at the interface, where $\omega_i$ and $s_{ij}$ are the the components of the vorticity vector and rate of strain tensor, respectively. The study indicates that the term $\nu \omega_i \nabla^2 \omega_i$ can be interpreted as viscous interaction of vorticity and strain, in analogy to $\omega_j \omega_j s_{ij}$, commonly referred to as the inviscid interaction of vorticity and strain. Furthermore, the results showed that the properties of enstrophy production are substantially different in such regions. In particular, vorticity is more aligned with the vortex stretching vector in the intermediate region B, as compared to regions A and C. In addition, region C is characterized by a strong suppression of the preferential ($\omega, \lambda_2$) alignment and increased probability of ($\omega, \lambda_1$) alignment, where $\lambda_i$ are the eigenvectors of the rate of strain tensor. The analysis also revealed that there is a pointwise relation between $\nu \omega_i \nabla^2 \omega_i$ and $\frac{D \omega^2}{Dt}$ in region C and it was found that $\omega_j \omega_j s_{ij}$ and $\nu \omega_i \nabla^2 \omega_i$ contribute together to the increase of enstrophy in regions B and C, which is quite anomalous if compared to the general behavior of the two terms in fully developed turbulence.
Chapter 2

Zusammenfassung

Das Hauptziel dieser Arbeit ist eine Analyse der Dynamik feinskaliger Größen, wie etwa Wirbelstärke und Spannungsraten, in der Nähe einer turbulenten Grenzschicht in einer Strömung ohne mittlerer Scherung. Für die Arbeit wird hauptsächlich particle tracking velocimetry (3D-PTV) und direct numerical simulation (DNS) benutzt. Sowohl im Experiment als auch in der Simulation wird Turbulenz in der Ebene $x_2 = 0$ erzeugt, die sich in der Zeit entlang $x_2 > 0$ ausbreitet. Die Taylor-microscale Reynoldszahl ist $\text{Re}_\lambda = 50$. Auf experimenteller Ebene wurden einige vorbereitende Schritte eingeleitet, bevor die Lagrange'schen Messungen durchgeführt wurden, und zwar sind dies (i) die Weiterentwicklung der 3D-PTV Messtechnik, (ii) eine Machbarkeitsstudie zur Messung des Laplace-Operators der Wirbelstärke, $\nu \nabla^2 \omega$, und (iii) die Überprüfung von Detektierungsmethoden einer turbulenten Grenzschicht. Die experimentelle Methode, 3D-PTV, wurde weiterentwickelt durch Änderungen an der Beleuchtung, Bildersammlung und Analyse. Insbesondere wurde ein optisches Scanning-system implementiert, welches das Messen mit höheren Partikeldichten und größeren Beobachtungsvolumen ermöglicht. Tests basierend auf präzisen kinematischen Beziehungen wurden durchgeführt, um das neue Messsystem zu validieren. Im Vergleich zur klassischen Methode konnte sowohl die räumliche Auflösung als auch das Beobachtungsvolumen beinahe verdoppelt werden. Als weiterer Schritt wurden Experimente in quasi-homogener Turbulenz durchgeführt, um die Messbarkeit von $\nu \nabla^2 \omega$ durch PTV zu überprüfen. Dieselben Experimente wurden in der Folge auch benutzt, um einige allgemeine Aspekte der Rolle dieser Größe in der Dynamik von Wirbelstärke zu untersuchen. Der Messfehler, der in Verbindung mit $\nu \nabla^2 \omega$ auftritt, ist erheblich und die Resultate sind deshalb nur für eine gefilterte Datenmenge gültig. Dennoch beweist der Vergleich mit DNS, dass die auf dieser Daten-Teilmenge basierenden Resultate (i) repräsentativ für die Strömung sind und (ii) zumindest auf qualitativer Ebene gut mit der Simulation übereinstimmen. Zur Vorbereitung auf die 3D-Messungen in der Nähe einer turbulenten Grenzschicht wurde eine
Studie zur Identifizierung dieser Grenzschicht mittels einfacherer Messmethoden durchgeführt. Particle image velocimetry (PIV) und Farbvisualisierung wurde eingesetzt, um Methoden zur Detektion einer turbulenten Grenzschicht, die mittels eines vibrierenden Gitters erzeugt wird, zu testen. Einige wichtige Eigenschaften der Grenzschicht konnten in dieser Studie bestätigt werden, wie etwa der scharfe Gradient von Wirbelstärke an der Grenzschicht, sowie deren Ausbreitungsgeschwindigkeit. Mittels einfacher Grenzwert-basierender Detektionsmethoden konnte das aus der Theorie bekannte Ausbreitungsgesetz $t^{1/2}$ in der Zeit, $t$, durch beide Messmethoden bestätigt werden. Die Ähnlichkeit der aus Farbkonzentrations- und Geschwindigkeitsmessungen gewonnenen Resultate und die Übereinstimmung dieser Resultate mit der existierenden Literatur deutet an, dass dieselbe Grenzwert-basierende Detektionsmethode auch für die nachfolgenden Lagrange'schen Messungen gut anwendbar sein müsste. Abschließend wurde die Dynamik von feinskaliger Wirbelstärke und Spannungsraten in der Nähe der turbulenten Grenzschicht mit Hilfe von PTV und DNS untersucht. Der Vergleich der Strömungseigenschaften in der turbulenten (A), Übergangs- (B), und nicht-turbulenten Region in der Nähe der turbulenten Grenzschicht erbringt direkte Information über die Schlüsselvorgänge in Einmischprozessen. Es wurde entdeckt, dass das Verhalten von Größen, die an Wirbelstärke geknüpft sind wesentlich anders ist, als die analogen Größen für Streckungsraten. Beispielsweise spielt in Regionen B und C die Zähigkeit der Flüssigkeit eine wichtige Rolle für das Aufbauen von Wirbelstärke, während das für den Fall von Streckungsraten nicht so ist. Genauer sind $\omega_{ij}^2$ sowie $\nu \omega_{ij} \nabla^2 \omega_i$ für die Zunahme von $\omega_i$ an der Grenzschicht verantwortlich, wobei $\omega_i$ die Komponenten des Wirbelstärke Vektors und $\omega_{ij}^2$ die Komponenten des Spannungstensors sind. Die Analyse zeigt, dass die Größe $\nu \omega_{ij} \nabla^2 \omega_i$ als viscous interaction von Wirbelstärke und Streckungsraten interpretiert werden kann, in Analogie zur Größe $\omega_{ij}^2 s_{ij}$, die als inviscid interaction von Wirbelstärke und Streckungsraten geläufig ist. Zudem zeigen die Resultate, dass die Eigenschaften von $\omega_{ij}^2 s_{ij}$ in solchen Regionen unterschiedlich sind. Insbesondere sind Wirbelstärke und ihre Streckung in Region B tendenziell stärker ausgerichtet, als in Regionen A und C. Region C ist zusätzlich von starker Unterdrückung der dominanten Ausrichtung von Wirbelstärke mit dem mittleren Eigenvector des Spannungstensors, $\lambda_2$, und häufiger Ausrichtung mit $\lambda_1$ gekennzeichnet. Die Untersuchung zeigte ausserdem, dass in Region C eine punktuelle Beziehung zwischen $\nu \omega_{ij} \nabla^2 \omega_i$ und $\frac{D \omega_i^2}{Dt}$ besteht. Es zeigte sich, dass $\omega_{ij}^2 s_{ij}$ und $\nu \omega_{ij} \nabla^2 \omega_i$ zusammen für die Zunahme von Wirbelstärke sorgen, welches, verglichen mit dem Verhalten in voll ausgebildeter Turbulenz, ein aussergewöhnliches Merkmal solcher Grenzschichten darstellt.
Chapter 3

Introduction

Turbulent entrainment is a process of continuous transitions from laminar to turbulent flow through the boundary (hereafter referred to as interface, turbulent/non-turbulent interface, turbulent front) between the two coexisting regions of laminar and turbulent state. This process is one of the most ubiquitous phenomena in nature and technology since, in fact, most turbulent flows are partly turbulent: boundary layers, all free shear turbulent flows (jets, plumes, wakes, mixing layers), penetrative convection and mixed layer in the atmosphere and ocean, gravity currents, avalanches and clear-air turbulence. An important aspect of the phenomenon of turbulent entrainment is its Lagrangian nature, that is, fluid portions that are initially located in the non-turbulent region will eventually cross the interface and become so part of the turbulent flow field. The motivation for this thesis is to investigate some basic properties of the mechanism of turbulent entrainment in a homogeneous fluid, using the Lagrangian method of three dimensional Particle Tracking Velocimetry (3D-PTV). Recent further developments of this technique allow for the full gradient of velocity derivatives to be measured along particle trajectories [37]. Among other things, the present work involves further development of 3D-PTV in order to assess the feasibility of measuring the Laplacian of vorticity. Direct numerical simulation (DNS) was employed for comparison with the experimental results. Due to the Lagrangian nature of the problem, the Lagrangian approach is essential for the understanding of the properties of turbulent / non-turbulent interfaces and the physical processes underlying the mechanism of turbulent entrainment.
3.1 Overview of previous studies on turbulent entrainment

The first physically qualitative distinction between turbulent and non-turbulent regions, made by Corrsin [11] and Corrsin and Kistler [12], is that turbulent regions are *rotational*, whereas the non-turbulent ones are (practically) potential, thus employing one of the main differences between turbulent flow and its random *irrotational* counterpart on the 'other' side of the interface separating them. In particular, Corrsin found that (i) the boundary between the two regions is essentially a thin interface which he called the 'viscous superlayer', and (ii) the 'effectiveness' of the entrainment process is strongly enhanced by the large scale undulations of the interface due to large scale motions which result in engulfment of irrotational fluid into the turbulent flow.

The main mechanism by which non-turbulent fluid becomes turbulent as it 'crosses' the interface is believed to involve viscous diffusion of vorticity \( \nu \omega_i \nabla^2 \omega_i \) across the interface [12]. The authors also conjectured that the stretching of vortex lines in the presence of a local gradient in vorticity at the interface leads to a steepening of this gradient since the rate of production of vorticity is proportional to the vorticity present. Moffatt [44] criticized this idea, pointing out that the rate of destruction of vorticity is also proportional to the vorticity present. As these processes are associated with small scales, it is thought to be the reason why the interface appears sharp compared to the large scales of the flow. However, at large Reynolds numbers the entrainment rate and the propagation velocity of the interface relative to the fluid are known to be independent of viscosity (see for example [18, 21, 27, 41, 56, 57, 66, 68, 70, 74] for more information and references). This fact is one of the consequences of similarity theory (see Section 3.2 below) and was verified in a number of experiments (e.g., [66]). Therefore the slow process of diffusion into the ambient fluid must be accelerated by interaction with velocity fields of eddies of all sizes, from viscous eddies to the energy-containing eddies, so that the overall rate of entrainment is set by the large-scale parameters of the flow [66]. This means that although the spreading is brought about by small eddies (viscosity), its rate is governed by larger eddies. The total area of the interface, over which the spreading is occurring at any instant, is determined by these larger eddies [70]. This is analogous to independence of dissipation of viscosity in turbulent flows at large Reynolds numbers. In other words, small scales do the 'work', but the amount of work is fixed by the large scales in such a way that the outcome is independent of viscosity. It should be emphasized that it is not known how the slow process of diffusion into the ambient fluid is accelerated by interaction with velocity fields of eddies of all sizes so that the overall rate of entrainment is set by large-scale parameters of the flow. It is also important that becoming rotational is only a necessary condition of becoming
3.2 Similarity theory and entrainment

Turbulent. Once the irrotational fluid acquired some vorticity via viscous diffusion this vorticity is amplified by the process of predominant vortex stretching due to the random nature of the motion in the proximity of the interface. So far, there is no direct evidence though that this really happens. One of the main goals of this thesis is to address this open issue providing direct evidence.

Until recently it was difficult to implement Corrsin's distinction: it requires information on small scale vorticity and strain which experimentally was not accessible. This is why very little is known about the processes at small scales and in the proximity of the entrainment interface. A few exceptions are recent particle image velocimetry (PIV) and planar laser-induced fluorescence (PLIF) experiments by Westerweel et al. [74,75] of a jet and experiments by Holzner et al. [22] on oscillating grid turbulence. Mathew and Basu [41] used direct numerical simulation (DNS) to investigate entrainment at a cylindrical turbulence boundary. They employed the magnitude of vorticity to discriminate between vortical and irrotational regions and analyzed trajectories of fluid elements crossing the turbulent interface. Westerweel et al. [75] used fluorescent dye to detect the turbulent front and PIV for the analysis of flow properties at the interface. Bisset et al. [4] used DNS of a wake and analyzed vorticity and velocity variances pointing out the significance of large eddies for the observed dynamics.

The progress in experimental techniques and computational methods over the last decades allowed to shed some new light on a few open questions. For example, some effort was dedicated to elucidate the significance of small scale motion for the entrainment process as compared to the contribution of large scales. In particular, it is not clear whether small scale 'nibbling' at the interface is more important for the outward spreading of the turbulent interface, than large-scale 'engulfment' by the inviscid action of the large eddies. In contrast to early studies (e.g. [66]), more recent publications [41,75] support the view that the spreading of small eddies is the prevailing mechanism. Recently, Hunt et al. [27] reviewed different aspects of interfacial layers adjacent to layers with weak and strong mean shear. It was pointed out that local dynamics should be described in relation to moving interfaces, since average Eulerian properties smear out intense gradients.

3.2 Similarity theory and entrainment

Traditionally, the problem of turbulent entrainment has been studied in flows with significant mean shear (e.g., jets, plumes, wakes, mixing layers). Experiments on the growth of plumes of fluid led to some widely used model of entrainment, the so called MTT-model, which is
based on a very simple treatment of the turbulence [45, 72]. One of the important aspects of the entrainment model is the similarity theory. This theory proposes that as the plume grows, its structure remains self-similar: statistically speaking, the scales of the plume and its turbulence all increase in direct proportion. Then it can be assumed that the rate at which the plume entrains fluid from its environment is proportional to the typical velocity scales of the turbulence, which are in turn proportional to the velocity of the plume. As a result, the entrainment hypothesis may be stated as

\[ u_e = \alpha V, \]  

where \( u_e \) is the entrainment velocity, \( \alpha \) is a constant with typical value 0.1, and \( V \) is the mean streamwise velocity in the plume. The plume model then assumes that the turbulent mixing is rapid, and we can assume a chosen profile for the density in the plume, such as a ‘top hat’ or a ‘gaussian’ (which is found experimentally to be a reasonable estimate). The problem is solved by balancing the conservation of mass, momentum and buoyancy in the plume: for instance, for a plume in a uniform (unstratified) environment, it is found that the width of the plume increases linearly with height. For more details see [72]. It is noteworthy that (much earlier) Ludwig Prandtl [52] provided a different model (sometimes called the ‘jet diffusion’ model) that leads to a different closure with a somewhat broader applicability. Some of Prandtl’s ideas were recently verified by Westerweel et al. [75] for a jet flow (see also [27]).

Now we consider a turbulent interface propagating in a flow without significant mean shear. The turbulence is generated by a planar source of energy and propagates mainly along the direction normal to this plane. Experimentally, such a flow is usually realized using an oscillating planar grid. Rouse and Dodu [58] designed an experiment in which an oscillating grid generates turbulence in a box of fluid. Similarly, Turner [71], Wolanski [77], Wolanski and Brush [78], used such a configuration with the purpose to find the speed of propagation, \( u_e = DH/ Dt \), of the front between a turbulent mixed layer of depth \( H \) and the non-turbulent fluid below. The fluid in this flows was stratified, with a density increase with depth. The first experiments in a homogeneous fluid were reported several years later, despite the importance of the problem and its relative simplicity compared to the stratified one. Thompson and Turner [65] and Hopfinger and Toly [24] found in their experiments of steady grid turbulence that the r.m.s velocity, \( u_{rms} \), scales with the coordinate normal to the grid, \( x_2 \), as \( u_{rms} \propto x_2^{-1} \). The integral length scale, \( l \), was found to scale as \( l \propto x_2 \). Based on these results, Long [35, 36] developed a mixing length model for grid-induced turbulence and defined a constant \( k \), termed the grid action, \( k = u_{rms} l \). The Long model assumes that \( l \propto x_2 \) is the relevant length scale for specifying grid-induced turbulent flow and that \( t = l/ u_{rms} \) is the relevant time scale. Since \( u_{rms} \propto x_2^{-1} \), this implies that the turbulent region will advance as \( x_2 \propto t^{1/2} \).
3.3 Outline of the thesis

The main focus in this thesis is on small scale aspects of entrainment in a flow without significant mean shear. The results are based on both experiments and direct numerical simulation. Experimentally, the turbulence is generated using an oscillating grid. The main (but not the only) experimental tool used for the measurements is 3D particle tracking velocimetry. Two other imaging techniques were employed for a preliminary analysis with focus on the larger scales of the flow, namely Particle Image Velocimetry (PIV) and dye visualization. One major milestone of the thesis is the further development of the 3D-PTV technique. This was achieved primarily through changes of the illumination. In particular, a scanning device was implemented allowing to cover larger flow regions than those allowed by the present system and/or providing a better spatial resolution. Before the new 3D Scanning PTV system could be used in the grid experiments, the technique was tested for the case of quasi homogeneous turbulence using a different apparatus. Namely, the flow was forced mechanically from two sides by two sets of four rotating disks. These experiments were also used to assess the feasibility of measuring the Laplacian of vorticity. In this context, some aspects of the contribution of this term to vorticity dynamics were analyzed.

Chapter 4 gives an overview of the experimental setups used in this thesis. Also the technical details for the direct numerical simulation are given therein. The 3D Scanning PTV technique is described in detail in Chapter 5. Chapter 6 is devoted to the feasibility study of measuring the Laplacian of vorticity along with the analysis of some simple aspects of the contribution of viscosity to the evolution of vorticity. Chapter 7 presents experimental results on the propagation of a turbulent front using PIV and dye visualization. The results on the small scale aspects of flows in proximity of the turbulent/non-turbulent interface are presented in Chapter 8. Finally, the conclusions are drawn in Chapter 9.
Chapter 4

Methods

This Chapter presents technical details on the experimental and numerical tools used for the investigation. On the experimental side, two different setups were used for mechanical excitation of turbulence. The first apparatus is an oscillating grid capable of forcing turbulence that is quasi-homogeneous in planes parallel to the grid. The second one consists of sets of rotating disks capable of generating quasi-homogeneous turbulence in the center of the liquid container. For the measurements, in addition to 3D-PTV also PIV and dye visualization were used.

4.1 Experimental apparatus

4.1.1 Oscillating grid

A sketch of the oscillating grid setup is shown in Figure 4.1. The grid is a fine woven screen (circular bars of \( d = 1 \) mm, mesh-size is \( d_0 = 4 \) mm), installed at the top of a water filled glass tank with dimensions of \( 200 \times 200 \times 300 \) mm\(^3\). The distance between the grid and the water surface is about 50 mm and the gap between the grid and the walls is less than 1 mm. The grid is connected to a linear motor able to induce vertical oscillation through the supporting frame of 4 rods of 4 mm in diameter. The motor, operated in a closed loop with feedback from a linear encoder, runs at a frequency of 6 Hz and an amplitude \( \epsilon = \pm 4 \) mm for all the experiments. A time sample of the grid velocity obtained from the encoder signal is shown in the upper right corner of the figure.
4.1.2 Rotating disks

The apparatus is the same as the one used in [34]. The forced flow domain is a rectangular box, 120 × 120 × 140 mm³. The flow is forced mechanically from two sides by two sets of four rotating disks. The disks have artificial roughness elements and are driven by a closed loop controlled servo motor. The motor is installed on top of the forcing unit and drives the counter-rotating disks through a fixed gear chain, where all disks rotate at the same rate according to the scheme shown in figure 4.2.

4.1.3 PIV and dye visualization

Both the PIV and dye visualization experiments were conducted using a high-speed camera (Photron Ultima APX, operated at 1024 × 512 pixels resolution) at a frame rate of 50 Hz, with an exposure time of 5 ms. The beam of a continuous 25 Watt Ar-ion laser is expanded through two cylindrical lenses and forms a planar laser sheet less than 1 mm thick, which passes
4.1. Experimental apparatus

Figure 4.2: A servo-motor (not shown), installed on the top of the forcing unit, drives 8 counter-rotating disks, 4 each rotating according to the scheme. The flow volume is located at the center of the forcing unit, mid-way between the disks.

through the mid-plane of the tank, as it is shown schematically in Figure 4.1. The field of view extended over the whole width of the tank and had the dimensions of $200 \times 100$ mm$^2$. For the PIV experiments, the camera recorded the light scattered by neutrally buoyant Polystyrene tracer particles with a diameter of 40 $\mu$m. This particle size is within the range of diameters typically used for such configurations (see for example Table 4 in [42]). The PIV images were processed using the commercial software Insight 3.3 from TSI Inc. [67], with an interrogation window of 16 $\times$ 16 pixels, 50 % overlap, yielding about 8000 two-component velocity vectors per realization, denoted as $u$ and $v$ in $x_1$ and $x_2$ direction (Figure 4.1), respectively. During the post-processing the standard global outlier and local median filters were applied and about 5% of erroneous vectors were found, removed and linearly interpolated by using the values of the nearest neighbors. The $\omega_3$ (out-of-plane) vorticity component was calculated using a least squares differentiation scheme (e.g., [55]). For the flow visualization experiments, the fluid in proximity of the grid was marked with fluorescent dye before oscillation started. We used an array of syringes and did several tests injecting the dye in the whole region above the grid or just into a small region and observed that the details of injection did not influence the final outcome. The fluorescent dye (Uranin) has a Schmidt number of approximately 2000, so that the molecular diffusion of the dye is much smaller than its turbulent mixing.
4.1.4 3D Scanning Particle Tracking Velocimetry

For the three dimensional measurements, 3D scanning particle velocimetry (3D-SPTV) was used. The technique is described in detail in Chapter 5, see also [26]. 3D-SPTV is a flexible flow measurement technique based on the processing of stereoscopic images of flow tracer particles. The technique allows obtaining Lagrangian flow information directly from measured 3D-trajectories of individual particles. For the experiments described in this thesis, two different recording systems were used. The first system consists of the same camera mentioned above. Namely, the Photron Ultima APX camera was used in combination with an image splitter to mimic a multi-camera setup, see Figure 4.3 (left). The camera is capable of achieving a maximum frame rate of 2 kHz at full resolution (1024 x 1024 pixels). With a built in memory of 2 GB, the system is able to record a time series of 2000 images. Recently, this memory could be upgraded to 8 GB. The second system consists of 4 Mikrotron high-speed CMOS cameras with a resolution of 1280 x 1024 pixels and 500 Hz frame rate, see Figure 4.3 (right). The cameras are connected to a long time streaming storage from I/O industries.

4.2 Direct Numerical Simulation

Direct numerical simulation (DNS) was performed in a box (side-length $5L_1$, $5L_2$, $3L_3$) of initially still fluid. Random (in space and time) velocity perturbations are applied at the boundary $x_2=0$. The procedure of generating the boundary conditions is as follows. For a fixed time and in the discrete set of points, $x_1 = k\Delta t$, $x_3 = l\Delta z$ (k, l - integers), each velocity component, $u_i$ ($i = 1, 2, 3$), is calculated as $u_i = V_i \xi$, where $\xi$ is a random number within the

Figure 4.3: Photron Ultima APX with 4 way image splitter (left) and Mikrotron multi-camera system (right).
4.2. Direct Numerical Simulation

interval $[-1, 1]$ and $V_i$ is a given velocity amplitude. For other times and spatial points $(x_1, x_3)$ boundary velocities are obtained by cubic interpolation in time and bilinear interpolation in space. At each time the three boundary velocity components yield zero average value over the boundary plane. The method of boundary velocity assignment determines the velocity scale, $V = \max(V_i)$ and the length scale $\Delta_l$. Together with the viscosity of a fluid, $\nu$, these parameters define the Reynolds number $Re = V\Delta_l/\nu = 1000$ of the simulation. The time scale can be defined as $\Delta_t = \Delta_l/V$. The Navier-Stokes equations

$$\frac{\partial u}{\partial t} = -u\nabla u + \frac{1}{Re} \nabla^2 u - \nabla p \tag{4.1}$$

are solved with the incompressibility constraint

$$\nabla \cdot u = 0. \tag{4.2}$$

Shear-free conditions $\partial u_1/\partial x_2 = \partial u_3/\partial x_2 = u_2 = 0$ are imposed at the boundary $x_2 = 3L_2$. A finite-difference method is used for the spatial discretization and the time advancement is computed by a semi-implicit Runge-Kutta method [46, 47]. The resolution is $256 \times 256 \times 256$ grid points in $x_1, x_2$ and $x_3$ direction. Previously, also a mixed spectral-finite-difference method was tested for the simulation, together with different Reynolds numbers, boundary conditions and box sizes. The respective discussion can be found in Appendix 1. Figure 4.4 shows a vorticity iso-surface for a given snapshot at time $t/\Delta_t = 5$, obtained through the simulation.

The paths of 4000 fluid particles have been calculated using

$$\frac{Dx}{Dt} = u(x, t), \tag{4.3}$$

where $x$ is the position of the fluid particle at time $t$. For the time integration of the particle position a 3rd order explicit Runge-Kutta scheme was used. The scheme is of the form

$$(x(1) - x(0))/dt = u(0), \tag{4.4}$$

$$(x(2) - x(0))/dt = (u(0) + u(1))/4, \tag{4.5}$$

$$(x(3) - x(0))/dt = (u(0) + u(1) + 4u(2))/6, \tag{4.6}$$

where

$$u(0) = u(t(0), x(0)), \tag{4.7}$$

$$u(1) = u(t(0), x(1)), \tag{4.8}$$

and

$$u(2) = u(t(0+dt/2), x(2)). \tag{4.9}$$
The velocity and other quantities of interest were interpolated to the trajectory point using a bilinear interpolation in space. The fluid particles were released at $t/\Delta t=5$ and integrated until $t/\Delta t=10$. Their initial positions are regularly distributed in a subregion of the computational domain ($2.5 < x_1/L_1 < 3.5$, $1.2 < x_2/L_2 < 2.2$ and $2.5 < x_3/L_3 < 3.5$), in proximity of the vorticity surface shown in Figure 4.4.
Chapter 5

Scanning PTV

This Chapter illustrates the experimental setup and data processing schemes for 3D Scanning Particle Tracking Velocimetry (SPTV), which expands on the classical 3D Particle Tracking Velocimetry (PTV) through changes of the illumination, image acquisition and analysis [26]. 3D-PTV is a flexible flow measurement technique based on the processing of stereoscopic images of flow tracer particles. The technique allows obtaining Lagrangian flow information directly from measured 3D-trajectories of individual particles. While for a classical PTV the entire region of interest is simultaneously illuminated and recorded, in SPTV the flow field is recorded by sequential tomographic high-speed imaging of the region of interest. The advantage of the presented method is a considerable increase of maximum feasible seeding density. Results are shown for an experiment in homogenous turbulence and compared with PTV. In this experiment, SPTV yielded an average 3500 tracked particles per time step, which implies a significant enhancement of the spatial resolution for Lagrangian flow measurements.

5.1 Introduction

Particle Tracking Velocimetry (PTV) has been established in an automated form as a valuable tool for Lagrangian flow measurements in the 1980’ies. Chang and Tatterson [9] introduced a stereoscopic 3D-PTV system, further developed by Racca and Dewey [54]. Nishino et al. [48] presented a 3D PTV system that allowed arbitrary viewing directions and yielded on average about 160 velocity vectors in a measurement of an unsteady laminar Couette flow. Later, Kasagi et al. [28] applied the system to a turbulent channel flow where they presented flow measurements and flow statistics from about 440 instantaneous velocity vectors. Papantoniou et al. [50] succeeded in tracking about 700 particles simultaneously using a three camera sys-
tem. Thereafter, Maas et al. [40] were able to increase the number of simultaneous velocity vectors to about 1300 per time step in a 3D turbulent channel flow. The method has been developed, tested and validated during the course of several flow studies (most recently in [38], see also [39], [76], [15]). However, the number of $O(10^3)$ velocity vectors still represents the achievable limit when using a classical PTV system consisting of four standard CCD cameras. This limit is imposed mainly by ambiguities of particle positions arising from particle image overlap on the imager chip of the camera. Ambiguities can be tolerated to a certain extent, become however prohibitive with further increase of the seeding density.

For classical 3D-PTV, a multi camera system is needed to image an observation volume with an aspect ratio of $O(1)$, i.e. the extent of the observation volume in the direction of the optical axes of the cameras has a dimension comparable to the width and the height. The three velocity components are obtained by identifying and tracking the position of the particles as they move through the observation volume. Lüthi et al. [38], measured the full set of velocity derivatives in a Lagrangian way in quasi-homogeneous turbulence. As pointed out by the authors in [38], for a better accuracy of the velocity derivatives, both long particle trajectories and high spatial resolution are desirable. As mentioned above, with increasing seeding density, ambiguities in individual particle recognition occur due to the large third dimension of the observation volume. This results in noisy and incomplete data and thus hinders the tracking of particles over many time steps. Long trajectories could be determined so far only if the probability of ambiguities is reduced by lowering the seeding density, however concurrently reducing the spatial resolution.

In scanning PTV, this problem is overcome by subdividing the observation volume into a series of thin slices. This allows for an increase in seeding density accordingly, since the saturation of particle images will be limited to a single slice only. The concept of a scanning light sheet is not new in whole field velocity measurements. Whole field velocity measurements were reported by [61], [62], [43] using tomographic Laser Induced Florescence (LIF) and also by [7], who used a scanning laser beam which provided overlapping illumination to correlate the particle images not only in the 2D in plane coordinates $x_1$ and $x_2$, but also to obtain a correlation in the out of plane direction $x_3$, to measure velocity components in the third dimension. The necessary considerable overlap of the light sheets for the correlation technique can also be avoided when using stereoscopic scanning PIV [5] or color coded PIV [6]. A combination of 2D correlation with subsequent 2D tracking of individual particles allowed Guezenneec at al. [20] to increase the in-plane spatial resolution of their stereoscopic scanning PIV images considerably compared to the use of correlation only. Using an adaptive Gaussian filter, they interpolated the velocities onto a regular grid for each of their stored images and thus obtained an in-plane high resolution velocity vector map of the three velocity components combining two camera projections. Both tomographic LIF and tomographic PIV require fairly thin illuminated layers. In LIF this is
especially true because of the necessity to have non-blurred images with a resolution similar in
the scanning direction as in the laser plane itself. Therefore with tomographic LIF one obtains
volumetric data of the scalar concentration field with very high spatial resolution. However, the
scalar gradient field has to be mixed and folded sufficiently to have measurable scalar gradients
down to the smallest eddy scales, i.e. the Kolmogorov dissipation scale. For scanning PIV,
both 2-D and stereo, the light sheet thickness is determined by the fairly high seeding density
of tracer particles together with the desired small interrogation volume. Additionally, in PIV,
the uncertainty of the position of the velocity vector is determined by the light sheet thickness
and also for this reason one needs to keep the sheet as thin as possible. These restrictions
require the experimenter to carefully weigh trade offs with the variation of possible parameters.
These are mainly related to light intensity, sheet thickness, particle size, seeding density, over¬
al achievable volume scan rate with a given camera, measurement location and especially in
tomographic imaging the restriction of the depth of field. In the presented SPTV technique,
the accuracy of an individual particle position is mainly determined by viewing angles, camera
resolution and the calibration quality, while the thickness of the light sheet plays no direct role.
Therefore, in SPTV the restriction of the light sheet thickness is nowhere as critical as required
by LIF and PIV. In fact, the sheet thickness can vary together with the seeding density such
that we are close to the established limit of \(O(10^3)\) particles per slice. For a fixed camera frame
rate, one can therefore vary the volume scan rate together with the number and thickness of
the image planes and the particle seeding density to the optimum values for the experiment.
In other words, depending on the experimental conditions, one can scan with a high seeding
density and a larger number of thin slices using a slower volume scan rate or one can scan with
a lower seeding density, a smaller number of thicker slices and a higher volume scan rate.
Tomographic LIF was also performed by [59] in the measurement of a 3D temperature field us¬
ing a mixture of Rhodamine B and Rhodamine 110 fluorescent dies. The temperature field was
determined from the previously calibrated temperature dependent ratio of the two dies' fluores¬
cent intensities, which were separated using a color beam splitter. Although the primary goal
was the measurement of the temperature field, the temperature itself could be used as a flow
marker to derive the velocity field. Recently, [25] published results from a scanning PIV system
measuring a 100 mm\(^3\) observation volume of a round jet of Reynolds number 1000. They
obtained velocity vectors on a grid spaced in \(x_1, x_2\) and \(x_3\) direction of 2.5mm\( \times \)3.5mm\( \times \)2mm
respectively. This is equivalent to measuring on a grid of 40\( \times \)28\( \times \)50 mesh points. The volume
scan rate of 3 Hz achieved in their experiment however restricts the application considerably.
When one compares the different techniques, all have advantages in some respect and compro¬
mise elsewhere, mainly to benefit either the spatial or temporal resolution of the field of view.
PTV / SPTV however is the only method that directly yields Lagrangian flow information.
This is of utmost importance for the study of mixing and transport phenomena and for the investigation of turbulence characteristics in general.

The main processing steps in Particle Tracking are (1) in situ calibration, (2) stereoscopic matching, (3) tracking of individual particles and (4) post-processing of trajectories. With the exception of the in-situ calibration, all processing and post-processing steps require modifications for SPTV. The necessary modifications on the image acquisition side are discussed in the experimental Section 5.2, the processing modifications are discussed in Section 5.3. Results for an experiment in homogenous turbulence are presented in Section 5.4 and concluding remarks are drawn in Section 5.5.

5.2 Experimental apparatus and setup

5.2.1 Hardware

SPTV uses images that are obtained by a high-speed camera system recording scattered light from particles illuminated by a scanning laser light sheet. To obtain a time resolution comparable to the classical PTV technique, the SPTV system has to scan through a whole volume in the same time as the classical PTV system needs to record a single frame. Therefore the camera recording rate has to be faster, proportionally to the number of slices in the observation volume. The camera frame rate \( f_c \) and the volume scan rate \( f_v \) are therefore linked in the following form through the number of slices \( n_s \):

\[
f_c \geq n_s f_v \quad (5.1)
\]

The additional degree of freedom that one has in selecting the number of slices, allows for fine-tuning of the seeding density to the necessary spatial and time scales for resolving the flow. This means that, for a given camera frame rate, one has the choice to either scan with few slices and a fast volume scan rate or with more slices and smaller volume scan rate, thereby increasing the spatial resolution by reducing the time resolution or vice versa.

An important parameter that has to be taken into account for this decision is the so-called "tracking parameter". The tracking procedure must establish correct links of individual particles between successive volume scans. A general criterion for the trackability of particles is given by the tracking parameter \( p \), defined as the ratio between the mean displacement \( d_0 \) to the mean inter-particle distance \( d_p \) (see [15]): for a small ratio, i.e. \( d_0 < d_p \) tracking is trivial, because in most cases the nearest neighbor in the following time step is the right choice. For a high ratio, i.e. \( d_0 > d_p \) tracking becomes difficult and if the temporal resolution can not be
5.2. Experimental apparatus and setup

Improved by hardware, the seeding of particles has to be decreased. Our light source for illumination is a continuous 20 Watt Argon Ion Laser. A beam expander comprising two cylindrical lenses spreads the beam to a sheet. A spherical lens of focal length \( f = 1 \) m is used to adjust the thickness of the light sheet. Following this set of lenses, the vertical light sheet passes through an octagonal Plexiglas cylinder (index of refraction \( n_2 = 1.49 \)). This cylinder can rotate at a set speed around its axis while the laser light sheet passes through the parallel facing planes of the cylinder. Compared to the discrete step scan of mirror setups, with this setup the light sheet is scanned in a continuous way. It is flexible, since the control of the light sheet thickness is achieved through the camera timing. The maximum sweep of the laser sheet however is tied to the geometry and requires a different octagon according to the desired region of interest. The cylinder axis is mounted vertically and carefully aligned with the incoming light sheet. Through its rotation, the angle of incidence of the incoming laser light sheet varies as \( \phi_0 = 2\pi \) rad/sec and is limited by the octagonal shape of the cylinder to \(-\pi/8 < \phi_0 < \pi/8\). Each time the limiting angle is passed, the incident laser light sheet enters through the next face of the octagon starting at a new angle shifted by \(-\pi/4\). For each eighth of a rotation of this cylinder, the exiting light sheet shifts parallel by an amount of approximately \(-10 \text{ mm} < x_3 < 10 \text{ mm}\) (\(x_3\) is the coordinate parallel to the scanning direction and to the symmetry axis of the camera arrangement respectively, see figure 5.1). This parallel shift depends nonlinearly on the rotation angle, and can be calculated using Snell’s law as follows:

\[
x_3 = h \sin \theta \left[ 1 - \frac{\cos \theta}{\sqrt{(n_1)^2 - (\sin \theta)^2}} \right]
\]

(5.2)

where \(h = 85 \text{ mm}\) is the inner circle diameter, \(n_1 = 1.0\) is the refractive index of air, \(n_2 = 1.49\) the refractive index of the cylinder material and \(\theta\) the angle of incidence. Snell’s Law is used in the presented technique for the determination of the slice position during data processing.

The recording system used for the experiment is the one consisting of the Photron high-speed camera with image splitter described in Section 4.1.4. We used a Nikor Micro 60 mm objective with an aperture set to \(f/16\), which ensured good focus over the entire scanned depth of 20 mm. The classical PTV system uses four synchronized cameras [38], [39] to enable robust stereoscopic matching within the established seeding limit, whereas in SPTV one single camera is used in combination with a four-way image splitter to mimic the classical four camera setup. This four-way splitter consists of a set of four fixed primary bevelled mirrors assembled on a regular pyramid and four secondary mirrors, which are mounted on regular pan/tilt fixtures to
Figure 5.1: A schematic of the experimental setup. The laser beam is expanded to a light sheet and scanned through the observation volume using an eight face prism.

individually adjust the field of view for each viewing direction. All are front coated mirrors. Through the projection of four viewing directions on a single camera chip one has the advantage, that exact synchronization of the cameras, which is critical for the stereoscopic matching and estimation of the particle positions, is automatically achieved. Through the four-way splitter a single camera image of 1024×1024 pixels resolution is split into four images, each of 512×512 pixels resolution, which is about the resolution of one CCD camera in the classical PTV system. As mentioned above, the minimum frame rate of the camera is determined by the revolution speed of the prism multiplied by the number of slices per volume scan. For the data presented, each volume scan consists of 10 illuminated light sheets. With the revolution frequency of the octagonal cylinder set to 6.25 Hz, we achieve a volume scan rate of 50 Hz, which results in a minimum camera frame rate of 500 Hz. With this setup we are able to record the observation volume 200 times during four seconds. The rotating cylinder and the camera are synchronized by an external timing signal. This signal is generated by an infrared diode aligned over an 8-hole pattern, which is fixed and aligned with the corners of the rotating cylinder. Every time a cylinder corner crosses the light sheet, the camera records 10 frames at a fixed rate of 500 Hz. The exposure time is set to the inverse of the frame rate, therefore the camera always integrates
over the entire path of the moving light sheet. As mentioned above, the thickness of one single light sheet \(d_s\), depends on the length of the scanning path during the exposure and on the 'static' light sheet thickness, which can be adjusted with a long focal length lens. Depending on these parameters, one achieves an illumination overlap region of variable size in which particles are exposed twice in successive scans. Although we use a continuous wave laser for the illumination and integrate the frame exposure over the inverse of the frame rate, we do not observe streaked particle images in this experiment. The time that a particle is illuminated by the incident laser light sheet during one scan depends on the static laser sheet thickness and the relative normal velocity of the particle with respect to the scan velocity. A particle that moves with the sheet (i.e. along positive \(x_3\) axis) has a smaller relative normal velocity and therefore is illuminated for a longer time than a particle that moves in the opposite direction. Since the flow velocities are considerably smaller than the scan velocity \(v_s\), this effect is very small. With a scan velocity of \(v_s \approx 1\) m/s and the static light sheet thickness of about 1 mm we expect a particle illumination time of roughly 1 ms and the travel path of a particle during this time of about 10\(\mu\)m. The travel path during the exposure therefore is smaller than the diameter of the particles. The highest resolvable frequency of the position signals in the flow is determined by the Nyquist criterion given a sample rate equal to the volume scan rate. With a volume scan rate of 50 Hz the Nyquist cutoff frequency lies at 25 Hz. This cutoff is considerably higher than the highest expected frequency of the position signal of 4 Hz estimated from the Kolmogorov time scale \(\tau_\eta = 0.25s\) in our flow.

5.2.2 Flow

The flow apparatus is the one described in Section 4.1.2. The goal was to reproduce the same turbulent flow field as in [34] and [38], for a comparison to the classical PTV system. The actual observation volume of approximately 20\(\times\)20\(\times\)20 mm\(^3\) was centered with respect to the forced flow domain, mid-way between the disks. The presented data were recorded with disk rotational speed of 40 rpm. During the four seconds of the experiment the mean flow over the entire observation volume showed negligible mean velocity. The fluctuating \(r.m.s.\) velocity within the observation volume, \(\sqrt{u'^2}\), is of order \(O(0.01\) ms\(^{-1}\)). The characteristic properties of the flow are given in table 5.1.

The flow is seeded with neutrally buoyant 40 \(\mu\)m polystyrene particles \((\rho = 1.02\text{mg/m}^3)\) with a number density of about 600 particles per cm\(^3\). This corresponds to an average inter-particle distance of 1.2 mm. It is well established that these particles follow water flows at our
Reynolds number accurately. The observation volume was scanned using 10 slices of about 3 mm thickness. Neighboring slices were overlapping by a ratio of about 25%.

### 5.3 Software Adaptations

#### 5.3.1 Determination of 3D particle coordinates

The 3D-PTV measurement principle is based on the synchronous acquisition of image sequences of the motion of tracer particles from four different viewing angles. After recording, the image sequences are processed to detect the particle images and to extract their image coordinates, as shown in [39]. With the knowledge of the previously calibrated camera position and orientation with respect to a fixed point at the wall of the flow cell, it is possible to establish corresponding particle images between the four different views and successively determine their 3D coordinates. This is achieved by ray-tracing through a multi-media geometry (air, glass, water; see [39] for details). With SPTV a significant increase of the yield of 3D particle coordinates per unit time step is possible, i.e. it provides a denser input of particles per unit time step for the subsequent tracking routine. To find the trajectories from the 3D point clouds, a tracking procedure developed by [76], based both on image and object space information is applied. For the presented scanning technique, some modifications of the classical PTV method both in the determination of the particle positions and in the tracking of detected particles in time are necessary.

The determination of corresponding particles in two or more images is based on the epipolar line intersection technique [39]. Stereoscopic matching is equivalent to finding the image of the same particle in the other camera views. In the strict mathematical formulation the epipolar lines intersect at corresponding particle images. The range of the illuminated region in scanning direction, which depends on the slice number 'n', limits the start and end points on the epipolar line in-between which the corresponding particle must lie. The depth of the instantaneous observation volume, together with the tolerance band and the camera orientation, define the search area for corresponding particles in image space. So the reduced thickness of the

<table>
<thead>
<tr>
<th>$\sqrt{u^2}$ (mm/s)</th>
<th>$\eta$ (mm)</th>
<th>$\tau_\eta$ (s)</th>
<th>$\lambda$ (mm)</th>
<th>$Re_\lambda$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.5</td>
<td>0.25</td>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 5.1: Characteristic properties of the flow.
instantaneous observation volume not only reduces the chance of overlapping particle images, but also reduces the number of possible candidates on the epipolar lines to match. The stereo matching thus becomes easier. For our setup, the scanned depth for one single acquisition is one order of magnitude smaller than the whole depth of the observation volume used in the classical method. The determination of corresponding particles was adopted without any modification from the classical method, apart from changing the depth of the observation volume to the slice limits as the light sheet is scanning through the volume.

The correspondences and 3D-particle positions are established sequentially thereby filling the observation volume. Therefore, not only is the point cloud of one observation volume skewed in time, but also particles can move from one slice to the next between volume scans. Both effects need to be accounted for in the tracking and necessitate interpolation of all particle positions on a single volume time stamp. In our adapted tracking procedure we assume to find the future position of a particle in the next volume scan either in the same slice $n$, or in one of the neighboring slices at $n + 1$ or $n - 1$. Thus, the time interval between two particle position measurements is either $1/f_v$ or $1/f_c$ or $1/f_c$ respectively (see figure 5.2 a). We therefore assume that we scan sufficiently fast so that a particle never moves further than one slice thickness in $x_3$-direction between two volume scans. For this reason an estimate of the highest flow velocities in the flow have to known, as will be discussed in the next subsection.

It was shown above that the effective light sheet thickness results from a dynamic part due to the sweeping motion of the sheet and a static part which results from the Gaussian beam profile across the light sheet. The resulting shape of the light intensity distribution of one slice is a "Gaussian tapered window". The dynamic width of the scan, which has been scanned by the intensity maximum, is $\Delta x_{3,100\%} = v_s/f_c \approx 2.5 \text{ mm}$, where $v_s$ is the local scan velocity (figure 5.3). Additionally the window is tapered on each side by half of the Gaussian shape of the laser sheet intensity profile.

The main parameters governing the visibility of a particle are the particle size, viewing direction with respect to the illumination, scatter properties of the surface and/or from inside the particle, and illuminating light intensity. Thus on the tapered ends of the individual slices some particles remain undetected. In this region, however, the slices overlap, so that each particle is swept over by the maximum light intensity once in either one of the frames. This contributes to the robustness of the method, as the probability for particles to remain undetected in the overlap region of both slices is low. Identifying double-exposed particles is straightforward, as their relative displacement for our experimental conditions is very small and lies within the order of the positioning error. To avoid that double-exposed particles are considered twice, they are
identified using a nearest neighbor test with small threshold. Depending on its $x_3$-position within the scan, the particle that has been scanned by the intensity maximum is chosen from the pair. This intermediate identification of doubles is necessary before the data can be further processed by the tracking routines.

5.3.2 Particle tracking

The goal of the tracking procedure is to repeatedly link positions of the same particles in time from one volume scan to the next. Willneff (2003) [76] implemented a particle tracking method based on combined image and object space information, which led to a higher tracking efficiency for the classical PTV system. In this method, information extracted both from image and object space is used simultaneously. The same principle was implemented for the scanning technique presented. As for the classical PTV system, the tracking procedure is based on kinematic motion modelling using the following criteria for a reliable and effective assignment.
5.3. Software Adaptations

Figure 5.3: Schematic illustration of the light intensity distribution through scanning with velocity $v_s$.

(see [15]): (1) the velocity of a particle is limited in all three components of the motion vector (2) the Lagrangian acceleration of a particle is limited, and (3) in cases of ambiguities the assignment with the smallest Lagrangian acceleration is the most probable one. The first criterion defines a search volume for the positions of possible candidates, therefore an upper bound for the expected maximum velocity needs to be known (usually readily obtainable from the boundary conditions of the flow). The Lagrangian acceleration is not directly used as a threshold, but takes the part of a weighting function together with the change of Lagrangian acceleration. As stated in [15], the penalization of large acceleration magnitudes and its large changes, respectively, is a heuristic approach. However, one can expect that molecular viscosity will damp acceleration and its change at small scales. There is some evidence that this happens, for example in the Direct Numerical Simulation (DNS) data of Yeung and Pope ([79]), who concluded that the magnitude of the Lagrangian acceleration of particles changes slowly on a time scale of approximately $0.6T_L$, where $T_L$ is the integral time scale. It was verified that the chosen parameters led to stable tracking results, i.e. there is no bias towards low intensity events.

In the classical PTV system all particles in the observation volume recorded in one frame correspond to one time step. For the scanning technique, each of the slices building up one volume scan corresponds to a different time step. The volume scan rate is set in a way that particles completely leaping one slice do not have to be considered, as the thickness of one slice is much higher than the displacement of the fastest particles between two volume scans (as said above, an upper bound for the velocity has to be known to ensure this to be true). Both for the kinematic particle motion modelling and for the decision criteria, no modifications
were necessary to correctly track particles moving from one slice to a neighboring one, since their relative time difference is only $0.1 \cdot f_v^{-1}$. After tracking the particles along their path across different slices, the derived trajectories consist of points separated by non-uniform time intervals. However, the velocity post-processing calls for a single time stamp for all particles of a volume scan. In the following we describe how this is accomplished for our scanning technique.

5.3.3 Trajectory processing

Lüthi et al. [38] presented a method that yielded filtered Lagrangian velocities and accelerations from the trajectories using a moving spline interpolation scheme, since the velocities and especially the accelerations obtained from a simple central differences of position data showed significant noise, caused mainly by positioning errors. The interpolated positions were derived from a 3rd order polynomial moving spline function fitted from an over-determined set of 10 points prior and post to the current point along the trajectory. The same method was modified for the presented technique not only to filter out the positioning noise, but also to interpolate particle position, velocity and acceleration in the entire observation volume at the same time stamp $t_*$ (defined as the time when the laser sheet passes the center of the observation volume). This way it is possible to obtain all the quantitative information of one volume scan not only filtered along trajectories, but also synchronized in time. For clarity, the interpolation procedure for a single trajectory is presented in more detail. In SPTV, the particle positions are measured at variable time intervals depending on the position of the particle inside a volume scan. If we consider a particle in a volume scan $j$, we denote the corresponding slice number of this particle as $n_j$ (as the particle can move from one slice to the next, this number can easily change several times during 21 scans). For all three position coordinates $x_i$ ($i = 1, 2, 3$) of each point, the spline function is expanded around the time $t_*$ ($t_*$ is set as the origin of the time axis, see figure 5.2 b)). Relative to $t_*$, the time information of each particle over the 21 trajectory points can be expressed as $t(j, n_j) = j/f_v + (n_j - n_s)/f_c =: t'(j)$, where $n_s = 10$ is the total number of slices. The constants $c_i$ are the coefficients of a 3rd order polynomial spline function of the form

$$x_i(t) = c_{i,0} + c_{i,1}t + c_{i,2}t^2 + c_{i,3}t^3$$

approximating the true function around the point $x_i(t_*)$ and are determined as
5.3. Software Adaptations

\[ c_i = (A^T A)^{-1} (A^T x_i) \] \hspace{1cm} (5.4)

where

\[ A = \begin{bmatrix}
1 & t'(-10) & t'^2(-10) & t'^3(-10) \\
1 & t'(-9) & t'^2(-9) & t'^3(-9) \\
\vdots & \vdots & \vdots & \vdots \\
1 & t'(10) & t'^2(10) & t'^3(10)
\end{bmatrix} \] \hspace{1cm} (5.5)

and

\[ x_i = \begin{bmatrix}
x_i[t'(-10)] \\
x_i[t'(-9)] \\
\vdots \\
x_i[t'(10)]
\end{bmatrix} \] \hspace{1cm} (5.6)

Position, velocity and acceleration, \( \hat{x}_i(t_*) \), \( \hat{v}_i(t_*) \), \( \hat{a}_i(t_*) \) are consequently obtained as

\[ \hat{x}_i(t_*) = c_{i,0} \] \hspace{1cm} (5.7)

\[ \hat{v}_i(t_*) = c_{i,1} \] \hspace{1cm} (5.8)

and

\[ \hat{a}_i(t_*) = c_{i,2} \] \hspace{1cm} (5.9)

As shown in [38], the procedure effectively acts as a low pass filter with a cut off frequency of about 10 Hz, which reduces noise originating from particle position inaccuracies. In analogy, our cut off frequency of about 8 Hz is well above the maximal position signal frequency of 4 Hz and well below the highest resolvable frequency 25 Hz. Now that velocities are obtained at regular time steps and time intervals with the procedure described above, spatial and temporal velocity derivatives are interpolated for every particle trajectory point as shown in [38].
5.4 Checks and results

5.4.1 Checks

The intensity of the velocity field measured by [38] was reaching the limit of the system capabilities with respect to trackability. Thus it was decided not to increase the spatial resolution too much and use the higher seeding also for a larger field of view. The goal was to reproduce the same flow conditions as [38], extend the observation volume and enhance spatial resolution within the limits of trackability. In fact, both the observation volume and spatial resolution were nearly doubled as shown in table 5.2.

<table>
<thead>
<tr>
<th>observed volume (cm$^3$)</th>
<th>inter-particle distance (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTV</td>
<td>4.5</td>
</tr>
<tr>
<td>SPTV</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 5.2: Some characteristics of the PTV and SPTV experiments.

For our experiment on the average 3900 particles per volume scan could be assigned a position in space. The linking efficiency, defined as the ratio between the number of linked and matched particles respectively, was about 85%. About 3000 particles per volume scan could be followed over 13 time steps (equivalent to one Kolmogorov time scale $\tau_\eta$) or longer. Table 5.3 compares the particle numbers through the different processing steps for the classical and the new technique.

<table>
<thead>
<tr>
<th>Nr. of particles (-)</th>
<th>Nr. of correspondences (-)</th>
<th>Nr. of links (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTV</td>
<td>1500</td>
<td>800</td>
</tr>
<tr>
<td>SPTV</td>
<td>5000</td>
<td>3900</td>
</tr>
</tbody>
</table>

Table 5.3: Comparison of particle numbers for the PTV and SPTV experiments along the chain of processing steps.

Due to the limited memory of the camera, the recorded time series is 4 s long. A total number of $6 \times 10^5$ points of the data set belong to trajectories longer than 1 $\tau_\eta$ and $2 \times 10^5$ points to
trajectories longer than $4 \tau_\eta$.

<table>
<thead>
<tr>
<th>recording time (s)</th>
<th>$\tau_\eta$ (s)</th>
<th>Frames/ $\tau_\eta$ ($\frac{1}{s}$)</th>
<th>recording time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTV 100</td>
<td>0.23</td>
<td>14</td>
<td>435</td>
</tr>
<tr>
<td>SPTV 4</td>
<td>0.25</td>
<td>13</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 5.4: Characteristics of the statistical sets of the PTV and SPTV experiments.

Checks based on precise kinematical relations are presented to validate the modifications of the introduced technique and to assess the quality of the data itself. Due to the incompressibility of water ideally the trace of $\frac{\partial u_i}{\partial x_j}$ should be zero, i.e.:

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial u_j}{\partial x_j} + \frac{\partial u_k}{\partial x_k}$$

Figure 5.4 shows joint PDFs of $-\frac{\partial u_i}{\partial x_i}$ versus $\frac{\partial u_j}{\partial x_j} + \frac{\partial u_k}{\partial x_k}$ (no summation over $i,j,k$ is applied). The aspect ratios of the contour surfaces are significantly smaller than the ones shown in Lüthi et al. [38] (the values are compared in table 5.5). This is a first indication that the lower inter-particle distance (and thus higher spatial resolution) leads to a higher quality of velocity derivatives.

A more strict check involves several quantities, each of which is derived in a different manner, the Lagrangian accelerations $a_i = \frac{Du_i}{Dt}$, local accelerations, $a_{i,i} = \frac{\partial u_i}{\partial t}$, and convective accelerations, $a_{c,i} = u_j \frac{\partial u_i}{\partial x_j}$. These quantities are related by the following equation:

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}$$

The left hand side of the equation is derived from the trajectory of a single particle, whereas the right hand terms need common information from neighboring particles. In a spatially under-resolved lowpass-filtered flow field, the divergence check might still be satisfactory with underestimated velocity gradients. The acceleration check provides a better assessment of the accuracy of velocity derivatives, because $a_i$, compared to the other two quantities, is obtained in a more straight-forward manner. Joint PDFs of $a_i$ versus $a_{i,i} + a_{c,i}$ are shown in figure 5.5. Similar to the divergence check, the aspect ratios of the contour surfaces are lower than the ones obtained applying the classical PTV system, suggesting again that the quality of the data has improved (the values are shown in table 5.5). The higher aspect ratio for the third
acceleration component is associated with the \( x_3 \)-component inaccuracy as explained by Lüthi et al. [38]. Due to the arrangement of the 4 view directions with respect to the observation volume, the \( x_3 \) position signal is less accurate than \( x_1 \) and \( x_2 \).

The relative divergence, \( \delta \), defined as

\[
\delta = \frac{\left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right)}{\left( \frac{\partial u_1}{\partial x_1} \right)^2 + \left( \frac{\partial u_2}{\partial x_2} \right)^2 + \left( \frac{\partial u_3}{\partial x_3} \right)^2} \tag{5.12}
\]

is used for a weighted fit along particle trajectories as a measure for the local interpolation quality [38]. For the experiment presented, 67% of the data points are of high quality, i.e. \( \delta \leq 0.1 \). Therefore, the whole data set is more reliable and the effect of the weighted polynomial is reduced. This effect is also visible in the last check presented. The figure 5.6 shows the mean relative divergence, \( \langle \delta \rangle \), as defined in expression 5.12, plotted as a function of the trajectory length \( \ell \). Compared to the plot shown in Lüthi et al. [38], \( \langle \delta \rangle \), after a lower peak, starts to decay earlier towards a common value of 0.1.
Figure 5.5: The expression \( \frac{\partial u_i}{\partial \tau} + u_j \frac{\partial u_i}{\partial x_j} \) is checked for each component \( i \), with joint PDFs of \( a_{i,j} + a_{c,i,j} \) versus \( a_i \).

<table>
<thead>
<tr>
<th></th>
<th>divergence check</th>
<th>acceleration check</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTV</td>
<td>( 1 \div 5 )</td>
<td>( 1 \div 3.5 )</td>
</tr>
<tr>
<td>SPTV</td>
<td>( 1 \div 10 )</td>
<td>( 1 \div 4.5 )</td>
</tr>
</tbody>
</table>

Table 5.5: Aspect ratios of the contour surfaces obtained from the "divergence check" and "acceleration check" compared for the PTV and SPTV experiments.

The above checks show not only that velocity derivatives can be obtained correctly with the presented technique, they show also that their quality is significantly improved compared to the classical PTV method. Following the approach shown in Luthi et al. [38] (section 2.4.1), the accuracy analysis was repeated for SPTV, using the aspect ratios shown in table 5.5. With a measured \( \text{rms}(\frac{\partial u}{\partial \tau}) \) of 1.5 s\(^{-1}\) the accuracy of \( \frac{\partial u}{\partial \tau} \) relative to its \( \text{rms}, \epsilon_{\text{rms}} \), is approximately 7\%. 


Thus, compared to the experiment in [38], the accuracy of velocity derivatives is improved by a factor 2. Further, with a measured \( \text{rms}(a_{ci}) \sim 13 \text{mm/s}^2 \), the accuracy for the convective acceleration (assumed to be of equal order as the local acceleration) relative to its r.m.s. gives 15%, while Lüthi et al. [38] estimated an accuracy of 20% for this quantity.

### 5.4.2 Results

The statistics shown in this section are derived from data points stemming from particle trajectories longer than 13 frames, with a relative divergence, \( \delta \), equal or smaller than 0.1. The number of data points that satisfy this criterion are \( 3.8 \times 10^5 \).

Some selected results on the statistics of vorticity and strain are briefly presented in this section, for the sake of comparison between the two methods. First, the PDFs of enstrophy and strain production, \( \omega_j \omega_j s_{ij} \) and \( -s_{ij} s_{jk} s_{ki} \), normalized with their respective mean values are shown in figure 5.7. We compare the results obtained by SPTV to the results of Liberzon et al. [34], who used the classical PTV of [38] in the same experimental facility and with the same experimental parameters. The agreement between the results in figure 5.7 is satisfactory, where PDFs of the production terms show clearly positively skewed distributions, as shown also in [38] and [62].

Second, we take a look at the PDFs of the eigenvalues \( \Lambda_{1,2,3} \) of the rate of strain tensor, \( s_{ij} \). Figure 5.8 shows a fundamental property of turbulence, i.e. the positive skewness of the PDF.
5.4. Checks and results

Figure 5.7: PDFs of $\omega_i \omega_j s_{ij}$ and $-4/3s_{ij}s_{jk}s_{ki}$ normalized with their mean values $\langle \omega_i \omega_j s_{ij} \rangle$ and $-4/3\langle s_{ij}s_{jk}s_{ki} \rangle$ respectively, as obtained by SPTV and the PTV experiment of [34].

The magnitude of $\langle \lambda_2 \rangle \approx 0.48 \text{ s}^{-1}$ is small compared to $\langle \lambda_1 \rangle \approx 2.4 \text{ s}^{-1}$ and $\langle \lambda_3 \rangle \approx 2.9 \text{ s}^{-1}$. The ratio of $\langle \lambda_1 \rangle : \langle \lambda_2 \rangle : \langle \lambda_3 \rangle$ is consistent with [34], [38] and references therein.

Figure 5.9 shows the PDFs of $|\cos(\omega, \lambda_i)|$ conditioned on weak and strong strain, illustrating the orientation of vorticity $\omega$, relative to the eigenframe of the rate of strain tensor $s_{ij}$. For strong events a predominant alignment of $\omega$ with the intermediate eigenvector $\lambda_2$ of the rate of strain tensor persists (see also [38]). As stated in [68], this result is one of the intrinsic property of turbulent flows.

Last, figure 5.10 shows a joint PDF plot of vortex stretching versus $\frac{D}{Dt}\frac{\omega^2}{2}$. For a flow with a statistically stationary level of enstrophy, it is clear that enstrophy production is balanced by
Figure 5.8: PDFs of the eigenvalues, $\Lambda_{1,2,3}$, of the rate of strain tensor $s_{ij}$. $\nabla$: $\Lambda_1$, $\circ$: $\Lambda_2$, $\square$: $\Lambda_3$.

its viscous destruction [68] as

$$\langle \omega_i \omega_j s_{ij} \rangle = -\langle \nu \omega_i \nabla^2 \omega_i \rangle. \quad (5.13)$$

As already pointed out by [38], the variations of $\frac{D}{Dt} \frac{\omega_i^2}{2}$ are much larger than those for vortex stretching. Apparently there is no simple relation between production and change of magnitude of enstrophy. The action of the viscous term $\nu \omega_i \nabla^2 \omega_i$ in balancing overall enstrophy production must be strongly non-local in space, since (see [68])

$$\frac{D}{Dt} \int_V \omega^2 dV \ll \frac{1}{V} \int_V \omega_i \omega_j s_{ij} dV \sim -\frac{\nu}{V} \int_V \omega_i \nabla^2 \omega_i dV. \quad (5.14)$$
5.5 Concluding remarks

In this chapter, a scanning technique for Particle Tracking Velocimetry (SPTV) was presented. So far, SPTV yielded 3500 velocity vectors per unit time step, using just one single high speed camera. Compared to classical PTV, both the observation volume and the spatial resolution were nearly doubled. Limited memory of the camera allowed a recording time of 4 s only, thus posing limitations on the convergence of some statistics. However, hardware improvements can easily overcome the mentioned limitation. SPTV offers enhanced data-quality compared to classical PTV and it represents an excellent tool for turbulence investigation. In particular, SPTV has the potential to provide a spatial resolution high enough for attempting an investigation on the role played by the viscous term in the enstrophy balance.
Figure 5.10: Joint PDF plot of vorticity production versus change of enstrophy, both axes normalized with $\langle \omega_j \rangle$.
Chapter 6

Observations on the viscous contribution to
vorticity dynamics

The Laplacian of vorticity $\nabla^2 \omega_i$ is obtained experimentally by means of three dimensional particle tracking velocimetry in $Re \lambda = 50$ quasi homogeneous turbulence. This Chapter focuses on the local enstrophy balance equation, namely $\frac{D}{Dt} \omega_i^2 = \omega_i \omega_j s_{ij} + \nu \omega_i \nabla^2 \omega_i$, as well as the analog equations for vorticity $\omega_i$. As it was expected, the PDF of $\nu \omega_i \nabla^2 \omega_i$ is found to be negatively skewed and $\nu \nabla^2 \omega$ is found to be statistically anti-aligned with vorticity. A closer inspection reveals that the alignment between $\nu \nabla^2 \omega$ and $\omega$ depends on both, the local alignment between $\omega$ and the eigenframe $\lambda_i$ of the rate of strain tensor $s_{ij}$, and on the magnitudes of strain $s^2$ and enstrophy $\omega^2$. It is shown that under some conditions, the viscous term $\nu \omega_i \nabla^2 \omega_i$ contributes less to the attenuation of enstrophy, while $\nu \nabla^2 \omega$ contributes to strong tilting of vorticity. We demonstrate how a vorticity self-moderation process governed by $\nu \nabla^2 \omega$ counteracts the production of enstrophy through the vortex stretching mechanism. The experimental results are compared to a Direct Numerical Simulation (DNS).

6.1 Introduction

The motivation of this work stems from recent developments of particle tracking velocimetry (PTV) which allow to access the field of velocity derivatives such as vorticity and strain and a number of associated key quantities and to obtain a number of results concerning these quantities in a Lagrangian setting [26, 38]. It is well known that, for example, the evolution of the field of vorticity is governed by two mechanisms (e.g., [29, 64]): the inviscid stretching/tilting and the corresponding contribution from the viscous term. So far only the first process was
accessible in the above experiments. Apart from looking at some basic aspects, the main purpose of this study is to assess the feasibility of evaluating the viscous contribution from PTV experiments. It is not possible to obtain the viscous term directly as it involves second derivatives of the vorticity field. Therefore, the Laplacian of vorticity was obtained from the local balance equation of vorticity in the form

$$\nabla \times \mathbf{a} = \nu \nabla^2 \omega$$  \hspace{1cm} (6.1)

by evaluating the term $\nabla \times \mathbf{a}$ from the Lagrangian tracking data.

From the technical point of view, two important aspects of this work are that (i) the results reported below were obtained from the same experiment but by two different systems, namely 3D-PTV [38], and 3D-SPTV [26] and (ii) all the results are compared to a Direct Numerical Simulation (DNS).

We address a number of relatively simple issues relating to the contribution of viscosity to the evolution of vorticity. These include aspects of geometrical statistics and tilting of vorticity. The latter is characterized by the equation for the unit vector $\hat{\omega} = \omega / \omega$

$$\frac{D\hat{\omega}_k}{Dt} = \eta^i_{\omega_k} + \eta^v_{\omega_k},$$  \hspace{1cm} (6.2)

where $\eta^i_{\omega_k} = \frac{\omega_j s_{ij}}{\omega} - \frac{\omega_k s_{ij}}{\omega} \omega_k$ and $\eta^v_{\omega_k} = \nu \nabla^2 \omega_k - \frac{\omega_k \nabla^2 \omega_k}{\omega} \omega_k$ represent the inviscid and viscous tilting respectively. Additionally, of particular interest in this study are terms appearing in the local balance equations of vorticity and enstrophy, written as

$$\frac{D\omega_i}{Dt} = \omega_j s_{ij} + \nu \nabla^2 \omega_i,$$  \hspace{1cm} (6.3)

$$\frac{D\omega^2}{Dt} = \omega_j s_{ij} + \nu \omega_i \nabla^2 \omega_i.$$  \hspace{1cm} (6.4)

After a description of the experiment and post processing, we present the results and a discussion concerning the mentioned quantities, followed by the concluding remarks.

6.2 Experimental method and post processing

The experimental results presented here are obtained through three dimensional particle tracking velocimetry. A set of experiments were performed by using the 3D-PTV system of [38]
6.2. Experimental method and post processing

and the 3D-SPTV system described in the previous Chapter, see also [26]. The experiment was carried out in a glass tank, in which the flow is forced mechanically from two sides by two sets of four rotating disks carrying artificial roughness elements (the details are given in Section 4.1.2). The observation volume of approximately $15 \times 15 \times 20 \text{mm}^3$ was centered with respect to the forced flow domain, mid-way between the disks. The characteristic properties of the flow are: r.m.s. velocity $10 \text{mm/s}$, $Re_\lambda = 50$ (based on the Taylor microscale) and Kolmogorov length and time scales are estimated as $0.5 \text{mm}$ and $0.25 \text{s}$, respectively.

As mentioned in the introduction, the viscous term of the vorticity equation (Eq. 6.3) was accessed through the Lagrangian acceleration. In order to obtain the spatial derivatives of the Lagrangian acceleration, $\partial a_i/\partial x_j$, which are necessary to estimate the term $\nabla \times a$, the postprocessing of [38] was extended. The spatial acceleration derivatives are obtained for every point $x_0$ along particle trajectories from information on accelerations $a$ of particles in the proximity of $x_0$. The procedure involves a local quadratic interpolation of the acceleration field and a weighted polynomial fitting procedure along particle trajectories, making use of the autocorrelations of the Lagrangian derivatives. For $\hat{a}_i(x_0)$, $i = 1, 2, 3$, the quadratic ansatz

$$\hat{a}_i(x_0) = c_{i,0} + c_{i,1}x_1 + c_{i,2}x_2 + c_{i,3}x_3 + c_{i,4}x_1^2 + c_{i,5}x_2^2 + c_{i,6}x_3^2 + c_{i,7}x_1x_2 + c_{i,8}x_1x_3 + c_{i,9}x_2x_3,$$

(6.5)

can be made with $\partial a_i/\partial x_1 = c_{i,1}, \partial a_i/\partial x_2 = c_{i,2}, \partial a_i/\partial x_3 = c_{i,3}, etc.$

Expression (6.5) for $\hat{a}_i(x_0)$ with its ten unknowns $c_{i,0-9}$ can be solved for $c_{i,j}$ with the information of at least nine points in the proximity of $x_0$, and the point $x_0$ itself, as described in [38]. The quadratic interpolation is followed by a weighted fitting procedure (see [38]) in which, in addition to relative divergence, $\delta_u$, we use relative acceleration, $\delta_a$, defined as:

$$\delta_u = \left( \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} \right) \delta_a = \frac{1}{3} \sum_{i=1}^{3} \frac{|a_i - a_{i,i} - a_{c,i}|}{|a_i| + |a_{i,i}| + |a_{c,i}|},$$

(6.6)

where $a_i = Du_i/Dt, a_{i,i} = \partial u_i/\partial t$ and $a_{c,i} = u_j \partial u_i/\partial x_j$, ($i = 1, 2, 3$) denote Lagrangian, local and convective acceleration, respectively. At the end of the processing, about 45% of the processed trajectory points satisfy both $\delta_u < 0.1$ and $\delta_a < 0.2$ for the two recordings.

The need of an independent experimental verification and validation of the results was addressed in the following ways: i) the experiment was accomplished in the same facility with the same experimental parameters by two different systems, namely 3D-PTV [38] and 3D-SPTV [26]; ii) the measurement error and its propagation through the post-processing was estimated (see Appendix 2) and iii) the results are compared to the DNS described in Section 4.2, see also [46, 47]. The properties of turbulence in the simulation are not exactly the same as
in the experiments. First, in the DNS turbulence is quasi-homogeneous in plane, while in
the experiments it is quasi-homogeneous in all 3 directions. Second, the turbulence in the
experiment is fully developed, while in the computational domain turbulent flow coexists with
regions in irrotational state. The irrotational regions in the simulated flow field were excluded
from the analysis simply by setting a threshold on enstrophy. Since the focus in the study is on
small scale properties, these differences have only a minor relevance, as our results will prove.

The error analysis is carried out for the two recordings by comparison of the terms of equations
6.3, 6.4 and also using
\[ \nabla \cdot \mathbf{a} + 2Q = 0, \tag{6.7} \]
where \( Q = \frac{1}{4}(\omega^2 - 2\omega_i^2) \) is the second invariant of the velocity gradient tensor. It is important
to note that the single terms in these equations are derived independently of each other. For
instance, acceleration \( \mathbf{a} \) is measured by following the particles along their trajectories, while \( Q \)
is derived by solving the ansatz for velocity derivatives [38]. The distribution of the different
quantities is presented by joint probability density functions (JPDF). The shape of the JPDFs
of \( \nabla \cdot \mathbf{a} \) versus \( 2Q \) is shown in Figure 6.1a, and the JPDF of the left-hand-side and right-
hand-side terms of Eq. 6.4 is given in Figure 6.1b. The JPDFs show that the whole data-set
includes a non negligible number of points with a significant error. This error is estimated from
both the JPDF and from the error propagation analysis (following the method used in [38]) to
be below 23\% for acceleration derivatives, and below 41\% for the viscous term, \( \nu \omega_i \nabla^2 \omega_i \) (see
Appendix 2 for details).

The large error that affects the enstrophy balance equation (and that is reflected in the JPDF
in Figure 6.1b) is mostly due to the experimental error which propagates along the higher order
derivatives, i.e. the viscous term. To limit the influence of the experimental error, we select a
subset of the data which fulfills the following condition based on the balance of the enstrophy
equation:
\[ \delta_r = \frac{| \frac{\partial}{\partial t} \omega_i^2 - \omega_i \omega_j s_{ij} - \nu \omega_i \nabla^2 \omega_i |}{| \frac{\partial}{\partial t} \omega_i^2 | + | \omega_i \omega_j s_{ij} | + | \nu \omega_i \nabla^2 \omega_i |} \leq 0.2. \tag{6.8} \]
In Figure 6.1c we present the JPDF of the selected subset, which includes \( 2 \times 10^5 \) (SPTV)
and \( 4 \times 10^5 \) (PTV) points satisfying all the conditions and represents roughly 17\% of the raw
datasets. The results in the next Section are based on this subsets of data and compared to a
Direct Numerical Simulation. The balance check of Eq. 6.4 for the DNS is shown in Figure 6.1d.
For the experimental part we show that the selected subset of data is still representative for the
entire flow in the sense that the statistics of the quantities of interest, excluding the viscous
term itself, are not affected by the imposed condition. This implies that the selection according
6.2. Experimental method and post processing

Figure 6.1: Joint PDFs of the terms in Eq. 6.4 (a), unconditioned (b) and conditioned (c) terms of Eq. 6.4 from PTV. Joint PDF of the terms of Eq. 6.4 from DNS (d)

to Eq. 6.8 indeed mainly filters out the more accurate estimates for \( \nabla^2 \omega \) without systematically biasing e.g. \( \omega_j \omega_j \) or \( \frac{D \omega^2}{Dt} \). The unconditioned and conditioned statistics are presented in a comparative way in Figure 6.2a-c. In Figure 6.2 we depict univariate probability density functions (PDF) of the modulus of velocity \( |v| \), and acceleration \( |a| \) (6.2a), enstrophy \( (\omega^2) \) and strain \( (s^2) \) (6.2b), normalized over their respective means, and the three terms of Eq. 6.4 in (6.2c). It is visible that the PDF of the velocity is almost unaffected, while the selection seems to slightly favor events with lower acceleration and higher enstrophy. Also in the last plot (Figure 6.2c) we observe that the conditioned PDFs of \( \frac{D \omega^2}{Dt} \) and \( \omega_j \omega_j s_{ij} \) change slightly as compared to the unconditioned data. We note that, consistently with our previous results ([26, 34, 38]), the PDF of \( \omega_j \omega_j s_{ij} \) is positively skewed and the PDF of \( \frac{D \omega^2}{Dt} \) is symmetric. In summary, we found that the sorting leads to some (small) bias of the data. However, this is
one of the reasons why all the results are compared to DNS.

Figure 6.2: PDFs of the normalized quantities of the unconditioned (solid lines) and conditioned (dashed lines) datasets from the experiments: a) velocity (triangles), acceleration (circles), b) enstrophy (triangles) and strain (circles), c) terms in Eq. 6.4
The analysis in this section is based on the experimental results obtained from the selected subsets of data. In addition, the results are compared to a Direct Numerical Simulation. It is noteworthy that all the results from the two mentioned experiments were found to be practically identical. The PDF's of $\omega_i\omega_j s_{ij}$ and $\nu \omega_i \nabla^2 \omega_i$ as obtained from PTV and DNS shown in Figure 6.3 are the first significant result and, to the best of our knowledge, represent the first attempt to estimate the viscous contribution to enstrophy dynamics experimentally. As expected, the PDF of $\nu \omega_i \nabla^2 \omega_i$ is strongly negatively skewed. On the qualitative level the experimental result agrees well with the simulation. Generally, geometrical properties (e.g. alignments between two vectors) allow for a better comparison than results based on magnitudes of quantities. Therefore, in this Section we will estimate the statistical properties of the viscous contribution by focusing not only on the scalar quantity $\nu \omega_i \nabla^2 \omega_i$ but mostly on invariant quantities related to $\nu \nabla^2 \omega$.

First, we represent enstrophy production as a scalar product of the vorticity vector and the vortex stretching vector, $W_i = \omega_j s_{ij}$, as $\omega_i \omega_j s_{ij} = \omega \cdot W$. In analogy we can write the viscous
term as $\nu \omega_i \nabla^2 \omega_i = -\nu \nabla^2 \omega$. In Figure 6.4 we show the alignment between $\omega$ and $\nu \nabla^2 \omega$ and we compare it with the alignment between $\omega$ and $W$. On the left panel we present the PDFs of these alignments for the whole dataset and conditioned on high enstrophy-low strain and high strain - low enstrophy events. The comparison between unconditioned and conditioned PDFs suggests that in high enstrophy regions vorticity is less aligned with the vortex stretching vector (i.e. weaker enstrophy production, Figure 6.4a) and more anti-aligned with $\nu \nabla^2 \omega$ (Figure 6.4c). This indicates that in highly rotational regions the viscous contribution is mostly damping the growth of enstrophy. In high strain regions we observe higher enstrophy production (in agreement with [38, 68] among others) and, as it was inferred in the results of [19], the effect of the viscous contribution is less 'damping', but more 'tilting' [29].

Similar to [19], where it was found that tilting of vorticity exhibits a substantially different behavior depending on the alignment of $\omega$ with the different eigenvectors $\lambda_i$, here we also condition the data on situations with different alignment of vorticity with the principal strain frame. In the right panel of Figure 6.4 we present the PDFs of the aforementioned alignments divided into the three subsets depending on the local alignment between $\omega$ and $\lambda_i$. The subsets are divided according to the condition of $\cos(\omega, \lambda_i)^2 \leq 0.7$, corresponding to a cone of roughly 33°, as in [19]. If one writes

$$W_i = \omega_j s_{ij} = \omega \Lambda_i \cos(\omega, \lambda_i), \quad \cos(\omega, W) = \frac{\Lambda_i \cos^2(\omega, \lambda_i)}{(\Lambda_i^2 \cos^2(\omega, \lambda_i))^{1/2}},$$

it is clear that alignment between $\omega$ and $W$ (i.e. positive cosine of the angle between the two vectors) occurs when $\omega$ is aligned with $\lambda_1$, since $\Lambda_1 > 0$. As pointed out by [68], due to the positive skewness of $\Lambda_2$, $\omega$ is also predominantly aligned with $W$ when $\omega$ is aligned with $\lambda_2$ (see Figure 6.4b). In Figure 6.4d we show the PDF of $\cos(\nu \nabla^2 \omega, \omega)$. Consistently with the negative skewness of $\nu \omega_i \nabla^2 \omega_i$ it appears that $\omega$ and $\nu \nabla^2 \omega$ are significantly anti-aligned when $\omega$ is aligned with $\lambda_2$ and $\lambda_1$. This reflects the effect of viscous destruction of enstrophy, $\nu \omega_i \nabla^2 \omega_i < 0$. In addition we note that, when $\omega$ is aligned with $\lambda_3$, $\nu \nabla^2 \omega$ retains a significant component perpendicular to $\omega$. This indicates that when vorticity is compressed, the viscous destruction of enstrophy is strongly reduced (i.e. $D\omega^2 / Dt \approx \omega_i \omega_j s_{ij} < 0$), while $\nu \nabla^2 \omega$ is mostly contributing to vorticity tilting (see Eq. 6.3). In Figure 6.4e+f we show the alignment between $W$ and $\nu \nabla^2 \omega$. We see that $\nu \nabla^2 \omega$ counteracts vortex stretching (i.e. $\nu \nabla^2 \omega$ is predominantly anti-aligned with $W$), particularly when $\omega$ is aligned with $\lambda_2$ and even more so when $\omega$ is aligned with $\lambda_1$. The behavior changes significantly when $\omega$ is on $\lambda_3$, which is associated with a compressive $W$.

To summarize these observations, we identify two distinct behaviors of the viscous term: (i) In the situation in which $\omega$ is aligned with $\lambda_3$ and $W$ is anti-aligned with vorticity (compressing
Figure 6.4: PDFs of $\cos(\omega, W)$, $\cos(\nu \nabla^2 \omega)$, and $\cos(\nu \nabla^2 \omega, W)$ conditioned on enstrophy and strain (left panel) and $(\omega, \lambda_i)$ alignments (right panel)
vorticity, Figure 6.4b) the viscous contribution is preferably perpendicular to $\omega$ (Figure 6.4d). Hence, in this case the viscous term is mostly tilting vorticity, rather than changing its magnitude. (ii) In the (most common) situation in which $\omega$ is aligned with $\lambda_2$, $\nu \nabla^2 \omega$ is anti-aligned with both $\omega$ and $\mathbf{W}$ (Figure 6.4e-f). In this case, the viscous term is contributing more to balance the stretching of vorticity rather than to change its direction. This is also true for the case of $\omega$ aligned with $\lambda_1$.

When we compare the numerical and experimental results in Figure 6.4, we note that the sharp match of the curves related to inertial terms (Figure 6.4a and especially Figure 6.4b) is not achieved for the results related to the viscous term. In particular, the very high probabilities at pronounced anti-alignment between $\omega$ and $\nu \nabla^2 \omega$ obtained from DNS (Figure 6.4c+d) are not reached by the experimental data. Nevertheless, the qualitative trends are found to be in good agreement.

In Figure 6.5 we present the orientation of the inviscid contribution to tilting, $\eta_{\omega}^i$, and the viscous one, $\eta_{\omega}^v$, (equation 6.2) with respect to $\lambda_i$. It is natural to expect that the tilting vectors, appear to be statistically perpendicular to the plane formed by $\lambda_1$ and $\lambda_2$, due to the fact that vorticity mostly "lives" in this plane and that both tilting vectors, by definition, are orthogonal to vorticity. In Figure 6.5 we see that $\eta_{\omega}^i$ is indeed predominantly aligned with $\lambda_3$. It is a bit surprising to see that $\eta_{\omega}^v$ is aligned with $\lambda_1$ with a higher probability as compared to $\lambda_3$. This

![Figure 6.5: PDFs of the cosines between a) $\eta_{\omega}^i$, b) $\eta_{\omega}^v$ and the three eigenvectors of the rate of strain tensor.](image_url)
6.3. Results and discussion

Effect is more pronounced in the DNS as compared to the experiments. As in the previous figure, the agreement between experimental and numerical results is striking as far as inviscid quantities are concerned and remains qualitatively reasonable for the viscid ones.

As it was expected from the conditional statistics given in [19], the total tilting $\frac{D\omega}{Dt} = D\omega_{\lambda}/Dt$ (see Eq. 6.2) significantly depends on the mutual orientation of vorticity and strain eigenframe. In Figure 6.6 (top panel) the PDFs of the total tilting are shown for the whole dataset and conditioned on the $\omega, \lambda_1$ alignment. High $\Omega_\omega^2$ is observed in the case of $\omega$ aligned with $\lambda_3$, which is in agreement with the results of [19]. We also note from conditional averages in Table 6.3 that the viscous tilting is significantly larger than the inviscid one. It is noteworthy that, when $\omega$ lies in the plane of $\lambda_1$ and $\lambda_2$ (frequent), both viscous and total tilting are weaker, compared to the situation of $\omega$ aligned with $\lambda_3$ (less frequent). The inviscid, viscous and mixed contributions to tilting, conditionally averaged on $\lambda_2$, are shown in Figure 6.6 (bottom panel). Consistent with the previous observation and as it is inferred in [19], we see that high $\Omega_\omega^2$ is indeed mostly associated with viscous effects. For this last experimental result (Figure 6.6c) we used a different (more restrictive) constraint for the filtering of the data. The new constraint is analogous to Eq. 6.8 for the scalar quantity $\omega^2$, but based on the balance equation for the vorticity vector. Qualitatively we obtain the same result by using the usual filter, but the tilting balance is better maintained with this alternative selection.

Finally, we emphasize the different relationships between the enstrophy production and the viscous destruction of enstrophy by using Joint PDFs. Figure 6.3 shows JPDF's of the terms in Eq. 6.4 and Figure 6.8 (a+c) the same terms with condition on $(\omega, \lambda_1)$ alignments. From the shapes of the JPDFs we observe that:

$\omega \parallel \lambda_1$ (Figure 6.8a): strong enstrophy production co-occurs with slightly weaker viscous destruction.

$\omega \parallel \lambda_2$ (Figure 6.8b): weak enstrophy production is likely to appear with slightly stronger viscous destruction.

$\omega \parallel \lambda_3$ (Figure 6.8c): strong enstrophy destruction by negative $\omega_j \omega_j s_{ij}$ co-occurs with a low

<table>
<thead>
<tr>
<th></th>
<th>all</th>
<th>$\omega \parallel \lambda_1$</th>
<th>$\omega \parallel \lambda_2$</th>
<th>$\omega \parallel \lambda_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\langle \eta_\omega^2 \rangle_{\omega}^2)$ $[s^{-2}]$</td>
<td>2.0</td>
<td>1.9</td>
<td>1.5</td>
<td>2.9</td>
</tr>
<tr>
<td>$(\langle \eta_\omega^2 \rangle_{\omega}^2)$ $[s^{-2}]$</td>
<td>6.2</td>
<td>6.0</td>
<td>5.9</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Table 6.1: Average values of the viscous and inviscid contributions (see Eq. 6.2) to $\Omega_\omega^2$ conditioned on $(\omega, \lambda_1)$ alignments obtained from PTV.
viscous contribution, which in this situation was shown to strongly contribute to the tilting of vorticity.
6.4 Concluding remarks

The viscous terms $\nu \nabla^2 \omega_i$ and $\nu \omega_i \nabla^2 \omega_i$ are obtained experimentally by means of Particle Tracking Velocimetry, through 3D-PTV ([38]), 3D-SPTV ([26]) and numerically by DNS in $Re_\lambda = 50$ quasi homogeneous turbulence in statistically steady-state conditions. The effect of $\nu \nabla^2 \omega_i$ on the vorticity vector was investigated under different strain and enstrophy magnitudes and for three situations of preferential alignment between $\omega$ and the eigenframe $\lambda_i$ of the rate of strain tensor $\dot{s}_{ij}$. We found that in regions of high enstrophy and low strain, the viscous term contributes more to the reduction of the magnitude of $\omega$, than to changing its direction. In regions with high strain and low enstrophy the tilting contribution on average increases. In addition, when $\omega$ is aligned with $\lambda_2$, $\nu \nabla^2 \omega_i$ appears to act predominantly against the direction of $\omega$ (attenuating enstrophy), while when $\omega$ is aligned with $\lambda_3$, there is a significant contribution of $\nu \nabla^2 \omega_i$ in the direction perpendicular to $\omega$ which induces its tilting. The viscous contribution to the total tilting of vorticity was found to be significantly larger than the inviscid contribution.

In analogy with the common interpretation of the process of enstrophy production $\nu \dot{s}_{ij} \omega_i \omega_j$ as a self-amplification of velocity derivatives, the processes of viscous destruction and tilting of vorticity might be interpreted as a "self-moderation" of velocity derivatives.

We remind that the present experimental results are strictly valid for only a limited, though representative portion of our data-set. In addition, the analysis was limited to one Reynolds number case $Re_\lambda = 50$. However, since it was shown e.g. in [30] that almost every genuine
Figure 6.8: Joint PDFs of the terms in Eq. 6.4, conditioned on $(\omega, \lambda_i)$ alignments from PTV (left) and DNS (right)
feature of turbulence can be found both at low $Re_\lambda$ of $O(10^2)$ as well as in highly turbulent flow with $Re_\lambda \sim 10^4$, we hope that our conclusions related to the Laplacian of vorticity are valid also for higher Reynolds numbers. From the technical point of view, we note that results obtained by both experimental systems were practically the same, which - along with the physical results reported - is considered as a clear indication of the reliability of both techniques. The agreement between experimental and numerical results is striking as far as inviscid quantities are concerned and remains satisfactory, at least on the qualitative level, for the viscous quantities.

This study is focused on the typical situation of positive $\omega_i \omega_j s_{ij}$ and negative $\nu \omega_i \nabla^2 \omega_i$. It is also of great interest to understand the physics underlying the viscous production of enstrophy (i.e. $\nu \omega_i \nabla^2 \omega_i > 0$), which, as can be seen from Figure 6.8a, is statistically relevant.
Chapter 7

Generalized detection of a turbulent front generated by an oscillating grid

This Chapter reports experimental results on the propagation of a turbulent front induced by an oscillating grid starting from rest. The purpose of this preliminary investigation is to implement and validate detection methods of the turbulent / non-turbulent interface, which are based on flow measurements (velocity and vorticity) and scalar intensity, for oscillating grid turbulence. This is done using particle image velocimetry (PIV) and fluorescent dye visualization, separately. The results of both techniques describe the spreading of the turbulent front, confirming the known dependency of the front location, \( H \), on time, \( t \). It is demonstrated, that the level-based detection of a turbulent front can be applied to an unsteady flow, such as grid turbulence advancing into a fluid at rest.

7.1 Introduction

Many fluid flows in nature are 'partly' turbulent [60], which implies that fluid regions in laminar and turbulent state often co-exist close to each other. One prominent feature of the non-turbulent flow region is that it is irrotational [12]. Indeed, as confirmed in the recent studies of Bisset et al. [4] and Westerweel et al. [74], vorticity shows a very sharp variation at the interface. Similar to vorticity (but less steep), passive scalars also show a finite gradient across the interface, e.g. [4]. For this reason, both vorticity and passive scalars were used for the identification of the outer bounds of the turbulent regions in a number of studies. For example, Kovasznay et al. [31] used the level of one component of velocity derivatives (which is related to the level of spanwise vorticity in this case) for the separation of the vortical and
non-vortical zones in the outer part of a boundary layer. Chen and Blackwelder [10] used a heated wall to slightly increase the temperature of the turbulent flow, which could be distinguished from (cooler) non-turbulent fluid using an array of cold wires. The same approach was followed by LaRue and Libby [33] to study the wake of a heated cylinder. Although the method of using temperature as a passive marker makes it possible to detect whether the probe is in a turbulent or non-turbulent region, it is not able to determine accurately the interface position. Westerweel et al. [74] (see also [75]) used particle image velocimetry (PIV) and fluorescent dye simultaneously for the investigation of the turbulent / non-turbulent interface of a turbulent jet. The fluorescent dye was used in the study to detect the interface with a high spatial resolution. Recently, Aguirre and Catrakis [1] (see also [8]) used laser-induced fluorescence for their investigation of the properties of the outer fluid interfaces of a turbulent jet. Bisset et al. [4] used velocity fields from direct numerical simulations (DNS) of a turbulent wake behind a flat plate for the study of the turbulent / non-turbulent interface. The vorticity magnitude was employed to detect the boundaries of the turbulent regions. The authors in [4] substantiated that the advancement of the vortical interface into the irrotational flow is driven by large-scale eddy motion and stated that the zone between turbulent and non-turbulent motion is likely to be less energetic and more diffuse in the case of flows with lower mean shear across the interface.

In the present experimental investigation, a turbulent / non-turbulent interface is induced by the motion of an oscillating grid, as in [13], [14], [73], among others. When a horizontal grid starts oscillating vertically in a water tank, it produces a layer of fluid in turbulent motion. This turbulent layer propagates vertically in the tank via entrainment of the irrotational fluid. Long [35] predicted by theoretical arguments, that the depth $H$ of the turbulent layer increases with time $t$ as $H \propto (kt)^{1/2}$, where $k$ is defined as the 'action' of the grid, a quantity with dimensions and characteristics of an eddy viscosity. Dickinson et al. [13] confirmed this theory presenting experiments using an oscillating grid in a cylindrical tank. Later, the same authors [14] presented more elaborate results on experiments in a square tank with and without rotation. Voropayev and Fernando [73] provided additional experiments in a grid-stirred cylindrical tank over a wider range of parameters and presented a semi-empirical model based on external parameters (grid frequency, oscillation amplitude, grid diameter, grid mesh size, fluid viscosity) to explain this behavior. In this and in the aforementioned studies, the flow was visualized using neutrally buoyant tracers and was recorded photographically. The position of the turbulent / non-turbulent interface was determined visually from the recorded images.

For the investigation of the entrainment mechanism, three-dimensional time-resolved measurements of turbulent quantities, such as velocity and velocity derivatives at the interface, are of utmost importance. So far, vorticity was not used directly for the detection of a turbulent
front in experimental studies or in transient flows, although the azimuthal vorticity component was accessed in the studies of Westerweel et al., [74], [75]. In view of preparation for three-dimensional measurements (Particle Tracking Velocimetry), the main point of the present work is to validate the vorticity-based method for the detection of the turbulent / non-turbulent interface. Since this interface is propagating in time, the applicability of level based techniques previously employed for stationary flows, is discussed. We resolve the larger scales of the flow and concentrate on the implementation and comparison of the different techniques used for detection. This is done by using PIV to determine the two-dimensional velocity and out-of-plane vorticity components, in addition to the more commonly used flow visualization with fluorescent dye. The experimental facility used for the experiments is the oscillating grid described in Section 4.1.1 and the PIV and dye visualization techniques are described in Section 4.1.3. The recording time for the experiments was set to 30 seconds, well above the average travel time of the turbulent front through the field of view (~ 20 s). In total, 4 experiments were carried out for the case of dye visualization and 7 for PIV measurements, respectively. We did two sets of experiments to investigate the influence of the free surface on the front propagation (see also [73] for a theoretical estimate of this influence). In the first set, the distance between the grid and the water surface is 5 cm, in the second set the distance is 10 cm (experiments marked by an asterisk in Table 7.1). The front detection techniques are described in the following Section 7.2. The results are shown in Section 7.3, followed by the concluding remarks in Section 7.4.

7.2 Detection of the turbulent / non-turbulent interface

Detection through scalar

The instantaneous concentration field of the fluorescent dye is commonly used for the detection of the interface (e.g., [74]). We used the procedure described in [74] to detect the interface from the images of the concentration field (a typical image is shown in Figure 7.1a).

The four main steps of the procedure are illustrated in Figure 7.1 and summarized as follows:

(i) A median filter is applied to the image in order to remove noise (single-pixel objects) in the background.

(ii) The gray level intensity images are transformed into binary images by means of a threshold, i.e. all pixels with a gray value above the threshold are labelled with '1', the remaining pixels are labelled with '0' (Figure 7.1b). A fixed gray-value threshold (normalized by
the maximum value of 255), $c_f$, is chosen using the technique proposed by the authors in [53]: in each image, the mean gray level intensity of all the pixels above the varying threshold is calculated. This conditionally sampled mean gray level intensity is shown of Figure 7.2a, where the different curves correspond to different times (in the figure, the axes are normalized by the maximum gray-value). As the initially highly concentrated dye diffuses, the mean gray level is decreasing in time. We also computed the time averaged curve and used one fixed threshold for all images. In the time averaged curve shown in Figure 7.2b, we detect two nearly linear regions indicated by two dashed lines. According to Prasad and Sreenivasan [53], the intersection point of the two dashed lines determines the value of the threshold (in our case $c_f$ ranged from 0.23 to 0.26 for the different experiments).

(iii) In the binary images we observed patches of entrained ambient fluid in the turbulent region (which appear as 'holes') and 'islands' of marked fluid outside of the continuous region. The detached patches of dye (see arrow in Figure 7.1b) were removed (Figure 7.1c).
(iv) The boundary of the continuous object is detected as the lowest point \( \max(x_2) \) at each \( x_1 \) (see Figure 7.1d). The resulting discontinuous line is only a rough representation of the presumably continuous and smooth interface, reflecting mainly the large scales. However, for our purpose of analyzing the propagation of the turbulent front, this level of representation is regarded as sufficiently accurate.

Given the large Schmidt number of the fluorescent dye and the limited resolution of the camera, the smallest scales of the dye interface are not resolved by our measurements, as discussed in the following. Usually the scales of the flow quantities and the concentration field are defined on the basis of dimensionless parameters, such as the Reynolds number of the flow. It is difficult to define a Reynolds number associated with our experiments, since the flow is transient and the scales of the flow are expected to change in space and time. Voropayev and Fernando [73] provided a Reynolds number based on the geometry and forcing parameters of the grid for their experiments. According to their definition (Eq. 9 in [73]), the global Reynolds number of the flow is \( Re = 1800 \) for our experiments. The smallest scales of the dye concentration field are of the order of the Batchelor scale, defined as

\[
\eta_B = \frac{\eta}{\sqrt{Sc}},
\]

where \( \eta \) is the Kolmogorov scale and \( Sc \) is the Schmidt number. The Kolmogorov scale is expected to be smallest in the vicinity of the grid, where turbulence intensities are high. The Kolmogorov scale can be calculated using

\[
\eta = \frac{\nu^{3/4}}{\langle \epsilon^{1/4} \rangle},
\]

where \( \nu \) is the viscosity of the fluid and \( \langle \epsilon \rangle \) is the mean dissipation rate. Here, by the term 'mean' we refer to the average of a quantity over a period that is longer than the turbulence time scales in a given location of the flow domain. As mentioned previously, the flow properties change in space and time. However, once the turbulent front is far from the grid, the turbulent quantities close to the grid are expected to be statistically stationary. This was verified by our measurements. We selected a thin horizontal window 1 cm away from the grid and averaged the velocity gradients from the PIV measurement over the last few seconds of the experiment. With the 2D approximation, \( \langle \epsilon \rangle \) was estimated to be of the order of \( \mathcal{O}(10^{-4}) \) m²/s⁴, which leads to an estimate of \( \eta \) of about 0.3 mm in that region. Hence, the Batchelor scale can be estimated to be of the order of \( \mathcal{O}(10^{-2}) \) mm. The large scales of the dye concentration field will be comparable to the large scales of the flow, which grows from a size that is comparable to the grid mesh size in the vicinity of the grid to several cm's further away. A good estimate for
the upper limit of the large scale of the concentration field in the considered flow region could be the half width of the tank, i.e. 10 cm. Given that the resolution for the dye visualization is about 0.2 mm/pixel for our experiments, the smallest scalar scales are expected to be one order of magnitude smaller than the smallest resolved scales.

Detection through velocity and vorticity

It is known (e.g., [12]), that a sharp interface separates irrotational, non-turbulent fluid from vortical turbulent fluid. That is why vorticity is an adequate quantity for the detection of the turbulent / non-turbulent interface (e.g., [4]). Since also velocity fluctuations, among other turbulent quantities, decrease sharply at the interface (although less steeply than vorticity, e.g. [4]), the velocity magnitude was also tested for the detection. Differently from vorticity, velocity fluctuations in the non-turbulent region are often non-zero, because of irrotational fluctuations induced by large-scale turbulent motions (previously observed in [4] and references therein). Nevertheless, these fluctuations are typically smaller than the ones inside the turbulent region and they decay very rapidly with distance from the interface. The detection was based on the instantaneous two-dimensional velocity or vorticity component realizations and implemented as a level-based method. A typical vector plot of velocity, overlayed by the contours of the instantaneous vorticity component $\omega_3$ is shown in Figure 7.3.

As in all level-based methods, the threshold has to be set appropriately, but there is no unique, 'best' choice. For instance, the 'zero-threshold' is not applicable, because noise associated

![Figure 7.2: Average intensity of all pixels above the threshold as a function of the threshold (see [53]) for a given experiment. a) variation in time $(-20s)$ b) time averaged curve.](image)
7.2. Detection of the turbulent / non-turbulent interface

Figure 7.3: An example of an instantaneous PIV realization. Vectors show the direction and the magnitude of the velocity field and contours are of $\omega_3$. Solid lines denote the positive values and the dashed lines are for the negative values (the lowest level is 0.3 s$^{-1}$).

with PIV measurements and analysis leads to non-zero vorticity and velocity in the irrotational region. Nevertheless, due to the steep spatial gradient of turbulent quantities across the interface (see e.g., [4, 74]), genuine vorticity / velocity could be distinguished from the noise. We estimated the noise level of vorticity from the root-mean-square (r.m.s.) values of $\omega_3$ in the lower (non-turbulent) part of the field of view, for the first few seconds of the experiment, when the flow was still undisturbed. It was found that the noise level is of the order of $O(0.1)$ s$^{-1}$. Similarly, the noise level of velocity was estimated to be of the order of $O(0.5)$ mm/s.

The boundary of the turbulent region is detected similarly to step (d) in the previous section. For each time instance $t$, and for each $x_1$ the turbulent boundary is the lowest point, $x_1^t(x_1,t)$, in which the magnitude of the signal exceeds a fixed (for all times and $x_1$ locations) threshold ($c_w$ and $c_u$ for vorticity and velocity, respectively). The procedure is depicted in Figure 7.4, in which three typical vertical profiles of vorticity are shown, along with the threshold and the detected points $x_1^t(x_1,t)$.

Contours of $\omega_3$ were plotted for many time steps (as for example shown in Figure 7.3) and it was found that a level of the threshold of four times the noise level (for both vorticity and velocity) is the best to delineate the vortical regions for all experiments. For thresholds above this level, some low vorticity regions were left out, while below this level the profile shows excursions into non-rotational regions.
Moreover, we applied the test proposed by Bisset et al., [4] in which the threshold \( c_\omega \) was evaluated by analyzing the conditionally averaged (over time and horizontal coordinate \( x_1 \)) vertical profiles of \( |\omega_3(x_2)| \), where \( x_2 = x_2 - x_2^* \). The conditionally averaged vertical profiles of \( |\omega_3(x_2)| \) and \( |u(x_2)| = \sqrt{u(x_2)^2 + v(x_2)^2} \) (\( |\cdot| \) denotes the absolute value) are shown in Figure 7.5. Since the vertical velocity component is of interest because of its relation to the entrainment rate (e.g. [75]), also \( |u(x_2)| \) is plotted in the figure. Note that the steepness of the gradient of the profile at the interface (the location where vorticity/velocity magnitude changes from the low to the high level) is the important parameter of this test, commonly related to the sharpness of the identified interface (e.g., [4,12]). As expected, the steepness of the gradient of the velocity profiles is less pronounced than the one of vorticity.

Figure 7.6 shows the spatial distributions of the magnitude of vorticity and velocity, respectively (high gray level corresponds to high intensity). The detected interface is visualized on both panels by assigning white markers to the position of \( x_2^*(x_1, t) \) for three different time instances. We observe that the propagation of the turbulent front, as detected by vorticity and velocity, is very similar in both right and left panels. The slight differences can be associated with the fact that the PIV measurements yield two velocity components and only one vorticity component, i.e. the out-of-plane component. The measured component of vorticity can be zero in regions

![Figure 7.4: Vertical profiles of vorticity magnitude for 3 different time instances at x = 50mm.](image)
where the other two components are not. This leads to holes in the turbulent regions visible in the vorticity magnitude plots of Figure 7.6. This effect is less pronounced in the case of velocity. It is also visible in the figure, that the contour obtained by using the velocity is more continuous than the one for vorticity. As we shall show below, the lack of 'continuity' is not problematic for the present analysis as we concentrate on the overall front propagation in terms of a horizontal average.

7.3 Results

The present study focuses on the propagation (in time) of the turbulent / non-turbulent interface, detected using the techniques described above. When the grid starts to oscillate each grid bar generates, alternatively, small regions of positive and negative vorticity, because of the no-slip condition at the surface of the grid elements [73]. The induced velocity field advects the vorticity away from the boundary. It was inferred by [73] (among others) that in the initial stage of the grid movement, when the typical width of the small jets / vortices is less than the grid mesh size $d_0$, there is little interaction between them. However, as these structures grow with time (and their size becomes comparable to $d_0$) these jets / vortices start

![Figure 7.5: Conditionally sampled vertical profiles of the magnitudes of vorticity (left axis, solid line) and of velocity vector and vertical velocity component (right axis, dashed and dashed dotted lines).](image-url)
Chapter 7. Generalized detection of a turbulent front

Figure 7.6: Vorticity (left panel) and velocity (right panel) magnitude maps for 3 time instances, a) $t = 2\, \text{s}$, b) $t = 6\, \text{s}$, c) $t = 18\, \text{s}$. The white spots represent the detected front, i.e. $x_2^*(x_1, t)$.

to interact and the vortical flow becomes turbulent. In the semi-empirical model of Voropayev and Fernando [73], the expression for the initial stage, $t_0$, is given as

$$ t_0 = \frac{d_0^2}{2\nu c_1^2 c_2 R^{3/5} \nu^2}. \quad (7.3) $$

where $\nu$ is the kinematic viscosity of water, $c_1 \simeq 2.5$ and $c_2 \simeq 0.14$ are two constants associated with the shape of the grid bars, see [73]. For our setup, equation 7.3 yields $t_0 \simeq 0.1\, \text{s}$. For the later times it was predicted by Long [35] and verified in many subsequent studies (e.g. [13,73]) that the interface propagates as $H \propto (kt)^{1/2}$, where $H$ is the distance from the grid to the interface (typically $H$ and $t$ are given in dimensional units of centimeters and seconds, respectively).

The propagation of the turbulent front, as it is detected by the aforementioned methods, is
7.3. Results

depicted in Figure 7.7a-c. Here each point of $H(t)$ is the average of $x_2^*(x_1, t)$ over $x_1$ for the different experiments. In addition to the values for each experiment marked by symbols, the trend line, which is an average of $H(t)$ for the different experiments, is plotted in the figure as a continuous line. It can be noticed that the single experiments show significant scatter due to the direct influence of the grid oscillation. However, this scatter is not larger than the range of variation between the different experiments. Some experiments show a somewhat smaller slope within the first $1\div2$ seconds of the experiment, visible also in the trend lines. We understand that the theoretical prediction of 0.1s for the initial stage, where turbulence is not yet developed, may underestimate the real time span needed. The slopes of the plotted curves were estimated by regression analysis (similar to [13]), using:

$$H = k^{1/2} t^n$$  \hspace{2cm} (7.4)

or

$$\ln(H) = n \cdot \ln(t) + \frac{1}{2} \ln(k).$$  \hspace{2cm} (7.5)

The results of the regression analysis are summarized in Table 7.1 for the dye visualization and PIV experiments, respectively. The table also includes the calculated values for the trend lines. Their best fits are plotted as dashed lines in Figure 7.7a-c. We observe that the theoretical value for the slope of 0.5 is reasonably met by the experiments with the two extreme values of 0.42 and 0.66 in experiments 4 and 7, respectively. Also, we observe that the slopes of the averaged curves are slightly higher than the theoretical value. This is probably due to the limited number of experiments. The slopes found by the detection through velocity and vorticity are not equal, due to the different characteristics of the detected interface mentioned above.

As mentioned in Section 7.2, we carried out two sets of experiments to test the influence of the distance between the water surface and the grid on the front propagation. In the first set the distance between the grid and the water surface was 10 cm (experiments labelled by an asterisk in Table 7.1) and in the second set it was 5 cm, respectively. It is visible in Table 7.1 that there is no systematic dependence of the values on the distance between the grid and the water surface. It is also noteworthy that the theory of Long [35] is valid for a semi-infinite space, in which the effect of the bottom or the side walls was not taken into account. In our experiments, strong vorticity was observed shedding from the grid border and propagating along the walls. This is also visible in the vorticity contours of Figure 7.3. Similar observations can be found in [73]. We limited the influence of this effect excluding 2cm of horizontal distance on either side of the observation window from the computation of $H(t)$. By using this exclusion, generally the estimated slopes were closer to the $H \propto (kt)^{1/2}$ law, than the ones
obtained over the whole width of the container. The mean value of the slopes obtained by all three detection methods is \( \langle n \rangle = 0.52 \) with a standard deviation of 0.07 (or 14%). In the above discussion, the regression analysis was done using \( k \) as a free parameter. However, \( k \) can be characterized in terms of grid geometry and forcing parameters [13], [14] and should not vary between the experiments. If we fix \( k \) to the mean value over the coefficients obtained from the vorticity based detection, i.e. \( \langle k \rangle = 0.84 \text{cm}^2/\text{s} \), the mean value of the slope is almost the same, \( \langle n \rangle = 0.53 \), but the standard deviation is reduced to 0.04 (or 8%).

The important observations are: 

1. the results for the three different quantities are consistent with each other and with the existing literature; 
2. applying the threshold evaluation technique described above, two-dimensional velocity data and the derived one-component vorticity fields are valid for a reliable and robust detection of the propagating turbulent front.

<table>
<thead>
<tr>
<th>Exp. nr.</th>
<th>Symbol</th>
<th>( k(\text{cm}^2/\text{s}) )</th>
<th>( n )</th>
<th>( R^2 )</th>
<th>( \langle k(\text{cm}^2/\text{s}) \rangle )</th>
<th>( \langle n \rangle )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>dye</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1*</td>
<td>o</td>
<td>1.2</td>
<td>.43</td>
<td>.87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2*</td>
<td>x</td>
<td>.73</td>
<td>.56</td>
<td>.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \nabla )</td>
<td>.80</td>
<td>.63</td>
<td>.92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( \cdot )</td>
<td>1.4</td>
<td>.42</td>
<td>.88</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \langle \cdot \rangle )</td>
<td>-</td>
<td>.96</td>
<td>.53</td>
<td>.99</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>velocity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>o</td>
<td>1.0</td>
<td>.51</td>
<td>.91</td>
<td>.82</td>
<td>.53</td>
<td>.92</td>
</tr>
<tr>
<td>6</td>
<td>x</td>
<td>.73</td>
<td>.52</td>
<td>.89</td>
<td>1.0</td>
<td>.52</td>
<td>.93</td>
</tr>
<tr>
<td>7</td>
<td>( \nabla )</td>
<td>.52</td>
<td>.65</td>
<td>.94</td>
<td>.58</td>
<td>.66</td>
<td>.95</td>
</tr>
<tr>
<td>8</td>
<td>( \cdot )</td>
<td>.97</td>
<td>.51</td>
<td>.96</td>
<td>.87</td>
<td>.55</td>
<td>.95</td>
</tr>
<tr>
<td>9*</td>
<td>+</td>
<td>.82</td>
<td>.49</td>
<td>.92</td>
<td>.99</td>
<td>.45</td>
<td>.89</td>
</tr>
<tr>
<td>10*</td>
<td>*</td>
<td>.83</td>
<td>.49</td>
<td>.95</td>
<td>.63</td>
<td>.56</td>
<td>.91</td>
</tr>
<tr>
<td>11*</td>
<td>( \Diamond )</td>
<td>1.0</td>
<td>.46</td>
<td>.93</td>
<td>1.0</td>
<td>.46</td>
<td>.90</td>
</tr>
<tr>
<td>( \langle \cdot \rangle )</td>
<td>-</td>
<td>.76</td>
<td>.54</td>
<td>.98</td>
<td>.78</td>
<td>.54</td>
<td>.98</td>
</tr>
</tbody>
</table>

Table 7.1: Values of \( k \) and exponent \( n \) obtained from the regression analysis for the three different detection methods. \( R^2 \) is the correlation coefficient. The values in the rows of the table marked with \( \langle \cdot \rangle \) refer to the regression of the averaged curve over experiments 1\( \sim \)4 and 5\( \sim \)11, respectively. For experiments marked by an asterisk, the distance between the free surface and the grid was 10 cm, for all other experiments it was 5 cm.
Figure 7.7: Vertical position of the interface, $H$, versus time, detected by using (a) dye concentration, (b) velocity and (c) vorticity. The different experiments are listed in Table 7.1. The continuous and dashed lines represent the average curve over the experiments and its best fit estimated by regression analysis (Eq. 7.4), respectively. For the sake of clarity the number of plotted data points is reduced.
7.4 Concluding remarks

The propagation of the turbulent / non-turbulent interface, generated by an oscillating grid starting from rest, was analyzed using flow visualization and particle image velocimetry (PIV). Both techniques have already been applied for the detection of the turbulent front for flows with significant mean shear. The present study shows that they are also valuable tools for oscillating grid experiments. Some important properties of the front were confirmed, such as the steep gradient of vorticity at the interface and the propagation velocity of the front. The theoretically predicted \((kt)^{1/2}\) law of propagation of the turbulent front in time could be verified with both experimental techniques.

The similarity of the results derived from concentration, velocity and vorticity measurements and the consistency with available data, suggests that it is possible to utilize the level-based interface detection method in the future three-dimensional Lagrangian measurements of the interface. The robust determination of the threshold will allow for detection of the Lagrangian trajectories that cross the sharp interface between turbulent and non-turbulent regions.
Chapter 8

Small scale aspects of flows in proximity of the turbulent/non-turbulent interface

This Chapter is devoted to a first of its kind study of the properties of turbulent flow without strong mean shear in a Newtonian fluid in proximity of the turbulent/non-turbulent interface, with emphasis on the small scale aspects. Some of the presented results are also published in Holzner et al. [23]. The main tools used are a three-dimensional particle tracking system (3D-SPTV) allowing to measure and follow in a Lagrangian manner the field of velocity derivatives and direct numerical simulations (DNS). The comparison of flow properties in the turbulent (A), intermediate (B) and non-turbulent (C) regions in the proximity of the interface allows for direct observation of the key physical processes underlying the entrainment phenomenon. The differences between small scale strain and enstrophy are striking and point to the definite scenario of turbulent entrainment via the viscous forces originating in strain.

8.1 Introduction

As pointed out in Chapter 3 a Lagrangian investigation of the small scale properties of turbulent/non-turbulent interfaces is of fundamental importance for the understanding of the phenomenon of turbulent entrainment. Studies of this kind are lacking in literature, also because technically up to now they were very difficult to conduct. The main objective of the presented study is a systematic analysis of the small scale dynamics associated with turbulent entrainment. The special emphasis is on the processes involving the field of vorticity, \( \omega_i \), and its production/destruction by inertial \( \omega_i \omega_j s_{ij} \) and viscous \( \nu \omega_j \nabla^2 \omega_i \) processes in the proximity of the interface (\( s_{ij} \) are the components of the fluctuating rate-of-strain tensor, \( \nu \) is the kine-
matic viscosity). Studying the production of vorticity requires access to the field of strain as well (and thereby also to the dissipation, $2\nu s_{ij}s_{ij}$). The focus is not only on the local balance equation for enstrophy, Eq. 6.4, but also on the analogous equation for the rates, i.e.

$$\frac{D}{Dt} \frac{\omega^2}{\omega^2} = \omega_i \omega_j s_{ij}/\omega^2 + \nu \omega_i \nabla^2 \omega_i/\omega^2. \quad (8.1)$$

In addition, some attention is dedicated to the terms of the equation for strain, namely

$$\frac{D}{Dt} s_{ij}^2 = -s_{ij}s_{jk}s_{ki} - 1/4\omega_i \omega_j s_{ij} - s_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} + \nu s_{ij} \nabla^2 s_{ij} \quad (8.2)$$

and to the analogous equation for the rates,

$$\frac{D}{Dt} \frac{s_{ij}^2}{s^2} = -s_{ij}s_{jk}s_{ki}/s^2 - 1/4\omega_i \omega_j s_{ij}/s^2 - s_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} /s^2 + \nu s_{ij} \nabla^2 s_{ij}/s^2. \quad (8.3)$$

Unfortunately, the pressure and the viscous term in the strain equations can presently not be obtained through PTV, but they are available in the DNS.

On the one hand, the study is based on the statistical analysis of flow tracers crossing the turbulent/non-turbulent interface. On the other hand, particular focus is on the relation of small scale quantities with the distance to the interface. This requires the availability of information both in the Lagrangian and Eulerian frames of reference (for both simulation and experiment). The technical operations and checks that were necessary to achieve this for the experimental data are described in the next Section 8.2. The results obtained from the analysis of Lagrangian trajectories are summarized in Section 8.3. Section 8.4 describes small scale properties of the flow in proximity of the interface. The Chapter closes with the concluding remarks in Section 8.5.

### 8.2 Method

The experimental apparatus used for the experiment is the oscillating grid described in detail in Section 4.1.1, see also [22]. The details of the direct numerical simulation are given in Section 4.2. For the three dimensional measurements, 3D scanning particle velocimetry (3D-SPTV) was used. The technique is described in detail in Chapter 5, see also [26]. The Mikrotron multi-camera system shown in Section 4.1.4 was operated at a frame rate of 500 Hz in order to obtain a volume scan rate of 50 Hz. An observation volume of $20 \times 20 \times 16 \text{ mm}^3$ (whose center is located 15 mm below the grid mid-position, see Figure 4.1) was tomographically scanned by a laser sheet yielding about 7000 3D particle positions per volume scan. The
obtained trajectories were post-processed to obtain the fields of velocity and velocity derivatives along particle trajectories, as described in Chapter 5. Out of the 6000 linked particles per frame, about 4000 belong to trajectories that are long enough so that velocity, Lagrangian acceleration and velocity derivatives could be successfully obtained [38]. The computation of $\frac{\partial u_i}{\partial x_j}$ is validated by checking both, the local divergence, $\frac{\partial u_i}{\partial x_j}$, which should vanish due to incompressibility, and the kinematic relation, $Du_i/Dt = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}$, between Lagrangian acceleration $Du_i/Dt$, directly obtained from particle position, and Eulerian acceleration, $\frac{\partial u_i}{\partial t}$, and convective acceleration, $u_j \frac{\partial u_i}{\partial x_j}$, which both involve the computation of spatial and temporal velocity derivatives. All the checks are close to the ones shown in Section 5.4.1. The Laplacian of vorticity, $\nabla^2 \omega$, was obtained from derivatives of Lagrangian acceleration, as described in Section 6.2. Inaccurate measurements of $\nabla^2 \omega$ were filtered using the constraint given in Eq. 6.8. The checks based on Eq. 6.7 and Eq. 6.4 look similar to the ones illustrated in Figure 6.1(a-c) and are not shown here again.

Some results in this Chapter are based on terms in the balance equation for the rate of change of enstrophy, Eq. 8.1. In analogy to Figure 6.1(c,d), Joint PDF’s of the RHS versus LHS of this equation are plotted in Figure 8.1 for the experiment (left) and the simulation (right). It is visible that most of the points lie on the diagonal and hence the check gives some additional confidence on the reliability of the experimental and numerical methods.

The Lagrangian measurements were interpolated on an Eulerian grid using the same local interpolation procedure described for the case of acceleration in Chapter 6 (see Eq. 6.5). The quantities interpolated in this way are velocity, acceleration and their derivatives. After that, instead of the weighted fit along trajectories [38], a weighted polynomial fit in time is applied to each quantity and for each point on the Eulerian grid. The grid spacing was taken equal to

![Figure 8.1: Joint PDFs of the terms in Eq. 8.1 for PTV (a) and DNS (b).](image-url)
the inter-particle distance in the three directions, i.e. $\Delta x_1 = \Delta x_2 = \Delta x_3 = 1$ mm and the number of grid points is $20 \times 20 \times 16 = 6400$. There are a number of checks that can be performed to test the quality of the interpolated data. One of them is to compare the material derivative of a quantity directly obtained through differentiation along a trajectory path with the sum of local and convective contributions obtained from Eulerian data, i.e.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla.$$ (8.4)

The quantities that will be checked here in this way are the 3 components of the vorticity vector and enstrophy. It is known, e.g. for the case of acceleration [69], that there is a strong mutual cancelation effect between the local and convective terms, such that the sum of the two terms is generally much smaller in magnitude than the two single terms themselves.

Joint PDF’s of $\frac{\partial}{\partial t} \frac{\omega^2}{2}$ versus $\mathbf{u} \cdot \nabla \frac{\omega^2}{2}$ are shown in Figure 8.2 for the case of PTV (left) and DNS (right). Apparently, also for the case of enstrophy the local and convective terms are strongly anti-correlated. As a consequence, small inaccuracies associated with the two terms will reflect into large errors in the estimation of their sum. In other words, the absolute error of the sum of local and convective terms is of the same order as the error of the single terms themselves, but its relative error is much larger (since generally $\frac{D}{Dt} \ll \frac{\partial}{\partial t} \mathbf{u} \cdot \nabla$). For the experiment, an inaccurate estimation of the interpolated $\frac{\partial}{\partial t} \frac{\omega^2}{2}$ would also lead to problems for the filtering of $\nu \omega \nabla^2 \omega$, since the term enters in Eq. 6.8.

Figure 8.3 shows Joint PDF’s of LHS versus RHS of Eq. 8.4 for the components of the

![Figure 8.2: Joint PDF of $\frac{\partial}{\partial t} \frac{\omega^2}{2}$ versus $\mathbf{u} \cdot \nabla \frac{\omega^2}{2}$ for PTV (left) and DNS (right).](image-url)
vorticity vector, $\omega_1$, obtained from PTV. It can be seen that the shapes of the iso-contours are not perfectly sharp, also for the reasons mentioned above. However, the aspect ratio of the iso-contours is comparable to the checks shown e.g. in Figure 5.5 and they are closely aligned with the diagonal. Figure 8.4 shows the same check for the case of $\omega^2$ as obtained from the experiment (left) and the simulation (right). Compared to the previous figure, the reliability of the measurements appears to be somewhat reduced with the higher order, nevertheless most of the points lie on the diagonal. The analogous result obtained from DNS (Figure 8.4 right) shows very good agreement between $\frac{\partial \omega_2}{\partial t}$ derived from the Lagrangian and Eulerian data sets, separately. Whenever possible, for the analysis below the Eulerian data-set was used for the case of DNS and the Lagrangian data-set for the experiment, consistently with the 'nature' of the two techniques.

8.3 Lagrangian analysis

This Section focuses on the statistical analysis of Lagrangian flow information. The characteristic properties of the flow are given in Table 8.3 for the experiment and the simulation. The Kolmogorov length scale, $\eta$, and time scale, $\tau_\eta$, are estimated from the dissipation, $c = 2\nu \omega^2$, by taking $\eta = (\nu^3/\epsilon)^{1/4}$ and $\tau_\eta = (\nu/\epsilon)^{1/2}$. In Figure 8.5 we show a typical trajectory measured through PTV with initial position in the non-turbulent region.

To obtain a first impression of the entrainment process in a Lagrangian frame, time series of several quantities along this trajectory are plotted in Figure 8.6. The time axis is centered at the point $t^*$, when a fixed threshold on $\omega^2$ is exceeded, i.e. $\dot{t} = t - t^*$, and normalized by $\tau_\eta$. 

![Figure 8.3: Joint PDF's of LHS versus RHS of Eq. 8.4 for the components of the vorticity vector, $\omega_1$, from PTV.](image)
Figure 8.4: Joint PDF's of LHS versus RHS of Eq. 8.4 applied to \( \omega \), from PTV (left) and DNS (right).

\[
\frac{\partial}{\partial t} \frac{\partial u_{ij}}{\partial x_j} + u \cdot \nabla \frac{\partial u_{ij}}{\partial x_j} = -\frac{\partial}{\partial x_j} \left( \frac{\partial u_{ij}}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \frac{2}{\beta} \frac{\partial u_{ij}}{\partial x_j} \right) + \nu \frac{\partial^2 u_{ij}}{\partial x_k \partial x_k} + 2 \nu \frac{\partial u_{ij}}{\partial x_k} \frac{\partial u_{ik}}{\partial x_k} + \nu \frac{\partial^2 u_{ij}}{\partial x_k \partial x_k} + 2 \nu \frac{\partial u_{ij}}{\partial x_k} \frac{\partial u_{ik}}{\partial x_k}.
\]

Table 8.1: Characteristic properties of the flow for the experiment and the simulation.

<table>
<thead>
<tr>
<th></th>
<th>( \Delta x_k )</th>
<th>( \Delta t )</th>
<th>( \tau_{\eta}/\Delta t )</th>
<th>( \eta/\Delta x_k )</th>
<th>( \lambda/\Delta x_k )</th>
<th>Re_\lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D-SPTV</td>
<td>0.02s</td>
<td>1mm</td>
<td>0.6</td>
<td>15</td>
<td>0.6</td>
<td>15</td>
</tr>
<tr>
<td>DNS</td>
<td>8 \cdot 10^{-3} \Delta t</td>
<td>2 \cdot 10^{-3} \Delta t/V</td>
<td>2.0</td>
<td>300</td>
<td>28</td>
<td>50</td>
</tr>
</tbody>
</table>

Figure 8.5: Single trajectory plotted in real space. The symbol ◆ indicates the initial position.
Figure 8.6a shows the evolution of $\omega^2$, $2s^2$ and the second invariant of the velocity gradient tensor, $Q = \frac{1}{4}(\omega^2 - 2s^2)$. We observe that $\omega^2$ is initially very low and increases attaining values close to the intensity of strain. In contrast to enstrophy, strain is already significantly high in the non-turbulent region and increases more gradually. This difference in magnitudes between $\omega^2$ and $s^2$ is an important feature of the process, since in fully developed turbulence enstrophy and strain are 'equal partners' (e.g., [68]). Notably, the invariant $Q$ reaches a local minimum in correspondence of $\dot{t}/\tau_\eta = 0$.

Figure 8.6b illustrates the evolution production terms of enstrophy and strain together with the third invariant of the velocity gradient tensor, $R = -\frac{1}{3}(s_{ij}s_{jk}s_{ki} + 3/4\omega_i\omega_j\omega_k)$. Similar to $\omega^2$ and $s^2$, $\omega_i\omega_j\omega_k$ is low in the non-turbulent region, while $-s_{ij}s_{jk}s_{ki}$ is not as small. With time, they both grow in magnitude and become of comparable intensity. Similar to $Q$ before, the third invariant $R$ shows a local maximum close to the origin. Next, the evolution of the terms of Eq. 6.4 are shown in Figure 8.6c. It turns out that both, $\omega_i\omega_j\omega_k$ and $\nu\omega_i\nabla^2\omega_i$ are positive and contribute to the growth of $\omega^2$. Compared to $\nu\omega_i\nabla^2\omega_i$, the production term is initially smaller in magnitude, but grows quickly and becomes the major responsible term for the growth of $\omega^2$, while the viscous term reaches a local maximum before it becomes negative. Figure 8.6d illustrates the evolution of the rates of the same set of quantities. The term $\nu\omega_i\nabla^2\omega_i/\omega^2$ initially attains very high positive values and drops to negative values after a few $\tau_\eta$. The rate of the production term, $\omega_i\omega_j\omega_k/\omega^2$, instead is relatively small in magnitude and remains relatively constant during the evolution.

For the statistical analysis, the measured and simulated particle trajectories are averaged defining an ensemble of events, similar to the procedure described in Section 7.2. All trajectories with initial position in the non-turbulent region are centered at the point $t^*$ introduced above and subsequently they are ensemble averaged. For the experiment, about $3 \cdot 10^3$ trajectories with an average length of $4\tau_\eta$ were processed in this way. The number of points considered for the statistics is $2 \cdot 10^5$. In the simulation, out of the $4 \cdot 10^3$ trajectories about 300 could be processed with a total number of $5 \cdot 10^5$ data points. The value of the threshold was set to 5% of the mean value of $\omega^2$ in the turbulent region. This parameter was varied between 1-25% (numerically, this is equivalent to $0.1 - 2.5s^{-2}$) and it was verified that, at least on the qualitative level, all the results remained valid.

The top panel of Figure 8.7 shows the conditionally averaged Lagrangian evolution of $\omega^2$ and $2s^2$ for PTV (Figure 8.7a) and DNS (Figure 8.7b). Both figures illustrate similar trends as discussed for the individual trajectory above. That is, $\omega^2$ is initially very small in magnitude while $s^2$ is already significant. Enstrophy grows steeply and reaches values comparable to strain after a few $\tau_\eta$. The top panel of Figure 8.7 illustrates the conditionally averaged evolution of
the invariants $Q$ and $R$ obtained from the experiment (left) and the simulation (right). An additional invariant based on the two viscous terms, namely $S = \nu \omega_i \nabla^2 \omega_i - 2\nu \delta_{ij} \frac{\partial}{\partial t} \delta_{ij}$ is displayed in Figure 8.7d. Apparently, the invariants reach an extremum near $h_{\text{att}}/\tau_\eta = 0$, before they decrease again to lower values. Since the mean values of $Q$, $R$ and $S$ vanish identically for homogeneous turbulence, their nonzero values indicate that the fluid paths cross regions
with some degree of inhomogeneity.

It is also interesting to investigate the role of production and viscous terms in Eq. 6.4 and 8.2 during this evolution. The conditionally averaged production and viscous terms are plotted in the top panel of Figure 8.8 from PTV (left) and DNS (right), the bottom panel shows the corresponding rates of quantities. Remarkably, the term $\nu \omega_i \nabla^2 \omega_i$ shows a positive peak at $\hat{t} = 0$. It is reminded that in fully developed turbulence, $\nu \omega_i \nabla^2 \omega_i$ mainly contributes to the destruction of $\omega^2$, i.e. it is negative in the mean. Similar to $\omega^2$, also its production term, $\omega_i \omega_j s_{ij}$, is small in the non-turbulent region, while the term $-s_{ij} s_{jk} s_{ki}$ is already significant and
Figure 8.8: Conditionally averaged Lagrangian evolution of inertial and viscous terms (top) and their rates (bottom) obtained from PTV (left) and DNS (right).

grows anticipating the growth of $\omega_i \omega_j s_{ij}$. Interestingly, the behavior of the two viscous terms is very different, as $\nu s_{ij} \nabla^2 s_{ij}$ is mainly negative (Figure 8.8b). This is the case also for the rates of the two quantities (Figure 8.8d). The term $\nu \omega_i \nabla^2 \omega_i / \omega^2$ is initially high and positive and decreases to smaller negative values during the evolution, while $\nu s_{ij} \nabla^2 s_{ij} / s^2$ is always small and mainly negative. The agreement between numerical and experimental results in Figure 8.8 is not that striking as in the previous one, but the important trends are the same.

8.4 Small scale properties of the interfacial region

This Section is devoted to the analysis of small scale quantities with the distance to the interface. Firstly, the interface is identified at $x_i^2(t)$, using a fixed threshold of enstrophy, (as
8.4. Small scale properties of the interfacial region

Figure 8.9: Average profiles of $\omega^2(\hat{x}_2)$ and $2s^2(\hat{x}_2)$ from PTV (symbols), and DNS (lines) relative to the interface, $\hat{x}_2 = x_2 - x_2^*$, on linear scale (left) and on log-scale (right). The values are normalized by the respective maxima ($\max(\omega^2) \approx \max(2s^2) \approx 3.6 \, s^{-2}$ for PTV and 3.0 for DNS). The error bars display the sensitivity of the experimental result on the enstrophy threshold, 0.5-2.5 $s^{-2}$. The symbol ($\cdot$) denotes the ensemble average of the respective quantity.

in Section 7.2, see also [22] and references therein) and the analysis is done with respect to the interface location, as in Figure 8.9, in which profiles of enstrophy, $\omega^2$ and strain rate, $s^2$, averaged over homogeneous $x_1, x_3$ directions for the case of the experiment (left) and the simulation (right), are shown. The distance to the interface, $\hat{x}_2$ is normalized by the Kolmogorov length scale, $\eta$. The terminology 'proximity' or 'region of the interface' refers to the interval $-5 < \hat{x}_2/\eta < 5$. We observe that the rate of strain on the non-turbulent side of the interface remains high in contrast to enstrophy which drops much more steeply. Different from the DNS, experimentally it is not possible to obtain enstrophy lower than a small (but finite) level of noise. Figure 8.10 shows profiles of production and viscous terms of strain and enstrophy (left) and their rates (right). Consistently with the previous observation in the Lagrangian setting, we note that the viscous term, $\nu \omega_i \nabla^2 \omega_i$, exhibits a remarkable behavior showing a distinct maximum in the region of the interface. The individual Lagrangian trajectories (examples are shown in the insert in Figure 2, see also Figure 8.6 and 8.7a,b) possess such an extremum. Therefore, we use the maximum of the viscous term as the exact location of the interface, defined in a physically more appealing way than the threshold-dependent crossing of $\hat{x}_2/\eta=0$. For the further analysis we define three physically distinct regions of the interface with respect to the maximum of $\nu \omega_i \nabla^2 \omega_i$ (marked in Figure 8.10): (A) the turbulent region, in which the behavior
Figure 8.10: Average profiles of production and viscous destruction terms of strain and enstrophy (left) and their rates (right). The insert shows the individual Lagrangian trajectories of \( \nu \omega_i \nabla^2 \omega_i \) obtained from PTV. Lines are from DNS, symbols are from PTV.

of the viscous term is 'normal', i.e. it is negative in the mean, (B) the interval between the peak and the point where \( \langle \nu \omega_i \nabla^2 \omega_i \rangle = 0 \) is termed intermediate region (with the 'abnormal' viscous production) and, (C) the non-turbulent region from the peak to \( x_2/\eta = 5 \). It is possible to define these 3 regions both in the Euierian and Lagrangian setting. The positiveness of both \( \omega_i \omega_j s_{ij} \) and \( \nu \omega_i \nabla^2 \omega_i \) is a peculiar feature of the regions B and C, in contrast to region A, where, in the mean, \( \nu \omega_i \nabla^2 \omega_i \) contributes to the destruction and \( \omega_i \omega_j s_{ij} \) to the production of \( \omega^2 \). It is noteworthy that strain behaves rather differently from vorticity. In particular, the viscous term, \( \nu s_{ij} \nabla^2 s_{ij} \), is negative in the mean in all three regions, i.e. it is not building up \( s^2 \). In Figure 8.10 we see also that strain production, \( -s_{ij} s_{jk} s_{ki} \), is significant and it is (in the mean) not balanced by \( \nu s_{ij} \nabla^2 s_{ij} \) in region C. When the rates of quantities are considered it appears (Figure 8.10, right) that the role of viscous production is even more important: the negative average \( \nu \omega_i \nabla^2 \omega_i / \omega^2 \) in region A (balancing the average \( \omega_i \omega_j s_{ij} / \omega^2 \)) becomes positive in region B and grows throughout region C. In contrast, the term \( \omega_i \omega_j s_{ij} / \omega^2 \) and the analogous rates of the strain viscous and production terms do not change as drastically and remain of the same sign. All the observations are consistent with the Lagrangian results presented in the previous Section.

Figure 8.11 presents the estimations of probability density functions (PDFs) of the relevant terms from the different regions (A, B, and C, from left to right; PTV top, DNS bottom). Consistently with the other results, the PDFs of both \( \omega_i \omega_j s_{ij} \) and \( \nu \omega_i \nabla^2 \omega_i \) are positively skewed in regions B and C. In region B we note that the probability of negative events of
8.4. Small scale properties of the interfacial region

Figure 8.11: PDF of various quantities from experiment (top) and simulation (bottom) according to the division in 3 regions: turbulent (left), intermediate (center) and non-turbulent (right). $\omega_i \omega_j s_{ij} (-)$, $\nu \omega_i \nabla^2 \omega_i (- -)$, $-s_{ij} s_{jk} s_{ki} (-)$, $\nu s_{ij} \nabla^2 s_{ij} (\cdots)$, only bottom)

$\nu \omega_i \nabla^2 \omega_i$ and positive events of $\omega_i \omega_j s_{ij}$ increases as compared to region C. Finally, as expected, in region A the PDF of $\nu \omega_i \nabla^2 \omega_i$ is negatively skewed. The changes of the strain production and viscous terms between the regions A-C are less drastic. Essentially, $-s_{ij} s_{jk} s_{ki}$ is positively and $\nu s_{ij} \nabla^2 s_{ij}$ is negatively skewed in all the three regions.

The probability density functions (PDFs) were also computed for the rates of production and viscous terms for the 3 regions (A,B, and C, from left to right; PTV top, DNS bottom) and are shown in Figure 8.12. As expected, the PDF’s of the rates of the production terms, $\omega_i \omega_j s_{ij}/\omega^2$ and $-s_{ij} s_{jk} s_{ki}/s^2$, are strongly positively skewed and the PDF’s of the analogous viscous terms, $\nu \omega_i \nabla^2 \omega_i/\omega^2$ and $\nu s_{ij} \nabla^2 s_{ij}/s^2$, are negatively skewed in region A. We note that in region B and more so in region C the skewness of the PDF of the term $\nu \omega_i \nabla^2 \omega_i/\omega^2$ becomes strongly positive, while the PDF’s of the other terms roughly maintain the shapes of the PDF’s in region A. The result underlines the inherently different roles of enstrophy and strain related quantities for the entrainment process.

For the understanding of the interaction of strain and enstrophy in the proximity of the interface it is very instructive to look at the invariants of the gradient tensor, $Q$ and $R$, as well
Figure 8.12: PDF of various quantities from experiment (top) and simulation (bottom) according to the division in 3 regions: turbulent (left), intermediate (center) and non-turbulent (right). $\omega_i\omega_j s_{ij}/\omega^2$ (−), $\nu\omega_i \nabla^2\omega_i/\omega^2$ (− ·), $s_{ij}s_{jk}s_{ki}/s^2$ (− ·), $\nu s_{ij} \nabla^2s_{ij}/s^2$ (⋅ ·, only bottom) as $S$ (Figure 8.13, right). As mentioned above, since the mean values of $Q, R$ and $S$ vanish identically for homogeneous turbulence, their nonzero values indicate the degree of inhomogeneity in the proximity of the interface. Apparently, inhomogeneity is the property which is maximal where also $\nu\omega_i \nabla^2\omega_i$ is maximal. In the same context it is also interesting to look at the cosine of the angle between vorticity and its Laplacian, $\nabla^2\omega_i$, shown in Figure 8.13 (left), which exhibits significant changes across the regions A, B, and C. The observed transition from positive (alignment) to negative (anti-alignment) values is in agreement with the qualitatively different behavior of $\nu\omega_i \nabla^2\omega_i$ in these regions. In contrast to that, the alignment between vorticity and the vortex stretching vector, $W_i = \omega_j s_{ij}$, changes only weakly throughout regions A-C and is consistent with the positiveness of $\langle \omega_i\omega_j s_{ij} \rangle$ mentioned above. The results indicate that an interpretation of the viscous term $\nu\omega_i \nabla^2\omega_i$ as interaction between strain and vorticity due to viscosity (i.e. due to the curl of the viscous force originating from the divergence of the strain tensor) is physically more appealing than 'simple' diffusion of vorticity due to viscosity. We emphasize that $\nu\omega_i \nabla^2\omega_i$ is the interaction of vorticity and strain since (e.g., [3]) $\nu\nabla^2\omega = 1/\rho \nabla \times F^s$, where $F^s = 2\nu \partial/\partial x_k\{s_{ik}\}$ and $\rho$ is the fluid density.
For a closer inspection of the nature of enstrophy production in the region of the interface we plot additional PDF's related to this quantity in Figure 8.14 from PTV (left) and DNS (right). The top panel shows PDF's of the cosine between vorticity and the vortex stretching vector. We see that the PDF is clearly positively skewed in region A. The positive skewness of this PDF is a well known genuine property of turbulence and one of the main reasons for the positiveness of $\langle \omega_i \omega_j \rangle$ (see [38, 68] and references therein). Surprisingly, the positive skewness increases in region B, while in region C it becomes again comparable to the initial configuration. The effect is a bit stronger in the simulation as compared to the experiment (Figure 8.14a,b). The effectiveness of $\omega_i \omega_j s_{ij}$ is also related to the orientation of $\omega$ relative to the eigenframe of the rate of strain tensor $s_{ij}$, $cos(\omega, \lambda_i)$ (see Eq. 6.9) shown in the second panel of Figure 8.14. The preferential alignment of $\omega$ with the intermediate eigenvector, $\lambda_2$, visible in region A is another well known characteristic property of turbulence [68]. It looks that this alignment is somewhat decreased in region B in favor of more frequent $(\omega, \lambda_1)$ alignments. Also, the probability of events with $\omega$ oriented normally to $\lambda_3$ is increased in region B. Region C is characterized by a strong suppression of the preferential $(\omega, \lambda_2)$ alignment. In the DNS result (Figure 8.14d) $\omega$ is even preferentially aligned with $\lambda_1$, while the PDF's of $(\omega, \lambda_1)$ and $(\omega, \lambda_2)$ are closer to each other in the experimental result. The probability of $(\omega, \lambda_3)$ alignments appears to be increased in region C. The bottom panel of Figure 8.14 shows PDF's of the eigenvalues of the rate of strain tensor. The positively skewed PDF of the intermediate eigenvalue, $\lambda_2$, comprises another genuine property of turbulence [68]. We see that this property is preserved throughout
Figure 8.14: PDF's of various quantities for the 3 regions from PTV (left) and simulation (right). (a-b) PDF of the cosine between vorticity, $\omega$, and the vortex stretching vector, $W$. (c-d) PDF of the cosine between $\omega$ and the eigenvectors of the rate of strain tensor, $\lambda_i$. (e-f) PDF of the eigenvalues of the rate of strain tensor, $\Lambda_i$. 
8.4. Small scale properties of the interfacial region

the 3 regions. Also the ratio $\Lambda_1:\Lambda_2:\Lambda_3=5:1:6$ remains roughly constant and is consistent with the ratios previously reported by others (e.g. [38,68] and references therein).

Before we proceed with a deeper analysis of the role of inertial and viscous terms, we turn our attention back to the balance equations for $\omega^2$ and $s^2$ (Eq. 6.4 and 8.2). Evidently, the two equations are directly coupled since the term $\omega_j\omega_j s_{ij}$ is present in both equations. Moreover, the equations are also ‘indirectly’ coupled, as $\omega_j\omega_j s_{ij}$ depends on both enstrophy and strain (as mentioned above the term is, in fact, the inviscid interaction of strain and vorticity). In addition to that, pressure enters in the strain equation through the pressure-strain term. Pressure is a non-local quantity since it can be written as an integral of the velocity over the whole flow domain (see e.g., [68]). Velocity, in turn, is directly related to both, vorticity and strain, since it can be expressed as an integral of either strain or vorticity [68]. The peculiar situation in region C is that, also if $\omega_j\omega_j s_{ij}$ is quite small compared to other quantities in Eq. 8.2, still the two equations might be coupled also through pressure. That is, the evolution of strain might in principle be substantially influenced by vorticity through the pressure term. From the other perspective, as already mentioned, the evolution of vorticity is strongly influenced by strain by the viscous forces that act through $\nu \nabla^2 \omega_i$, in addition to the inviscid interaction through $\omega_j\omega_j s_{ij}$. In this context, to further elucidate the mentioned processes, we analyze the relation of quantities by means of Joint PDF’s.

Figure 8.15: Joint PDFs of the enstrophy and strain production terms, $\omega_j\omega_j s_{ij}$ and $-s_{ij} s_{jk} s_{kl}$ from experiment (top) and simulation (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).
An interesting property observed in turbulent flows is that strong production of enstrophy usually does not happen in the same time or location as strong production of strain (e.g., [38, 68]). Joint PDFs of the enstrophy and strain production terms, \( \omega_i \omega_j s_{ij} \) and \(-s_{ij} s_{jk} s_{ki}\) from experiment (top) and simulation (bottom) according to the division in 3 regions (A-C form left to right) are plotted in Figure 8.15. The mean values of each quantity, \( \langle x \rangle \) and \( \langle y \rangle \) respectively, and their correlation value, \( c_{xy} \), are also plotted on the figure (and also on the following figures). We see that in region A strong production of strain inhibits strong production of enstrophy and vice versa and the two quantities are only weakly correlated (see also e.g., [38, 68]). As visible also in the PDF's (Figure 8.11) throughout regions B-C the magnitudes of both terms decrease with respect to A. The magnitudes of \( \omega_i \omega_j s_{ij} \) are even about an order of magnitude lower than \(-s_{ij} s_{jk} s_{ki}\) in region C. The shapes of the Joint PDF's in regions B-C indicate that \( \omega_i \omega_j s_{ij} \) and \(-s_{ij} s_{jk} s_{ki}\) tend to produce \( \omega^2 \) and \( s^2 \) together, as most values are concentrated in the first quadrant. Also, the correlation values are somewhat higher in regions B-C as compared to A.

Analogously, in Figure 8.16 we analyze in the same way the two viscous terms, namely \( \nu \omega_i \nabla^2 \omega_i \) and \( \nu s_{ij} \nabla^2 s_{ij} \) for the 3 regions. Since in a stationary and homogeneous turbulent flow \( \langle \nu \omega_i \nabla^2 \omega_i \rangle \approx -2 \langle \nu s_{ij} \nabla^2 s_{ij} \rangle \) the question raises whether this equilibrium might be reflected in a pointwise relation between the two terms in region A. We see in Figure 8.16 that there is no local balance between the two viscous terms. Even though, most points lie in the 3rd quadrant and hence it appears that enstrophy destruction events are mostly associated with strain destruction events. Interestingly, there is some (small) probability that both terms contribute together to a viscous production of enstrophy and strain. These events become more frequent especially in region B, but also in region C, since there the term \( \nu \omega_i \nabla^2 \omega_i \) is mostly positive. However, it is reminded that, different from \( \nu \omega_i \nabla^2 \omega_i \), in the mean \( \langle \nu s_{ij} \nabla^2 s_{ij} \rangle \leq 0 \) in all the 3 regions, comprising one of the important characteristic features of such interfaces.

Figure 8.16: Joint PDFs of the viscous terms in Eq. 6.4 and 8.2, \( \nu \omega_i \nabla^2 \omega_i \) and \( \nu s_{ij} \nabla^2 s_{ij} \) from DNS according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).
Figure 8.17: Joint PDFs of the terms in Eq. 6.4, $\omega_j \omega_j s_{ij}$ and $\frac{D}{Dt} \omega^2$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).

We now look at the joint PDF of $\omega_j \omega_j s_{ij}$ versus $\frac{D}{Dt} \omega^2$ shown in Figure 8.17. In the turbulent region positive enstrophy changes are mostly associated with positive enstrophy production events, see also [26, 38, 68] among others. In regions B and C we note that, as expected, the mostly positive enstrophy changes are associated with $\omega_j \omega_j s_{ij} > 0$, the correlation between the two terms is higher than in A. However, it seems that the major contribution in terms of magnitude to the positive $\frac{D}{Dt} \omega^2$ in region B (and more so in region C) is not due to the term $\omega_j \omega_j s_{ij}$.

Figure 8.18 shows the Joint PDF of $\nu \omega_i \nabla^2 \omega_i$ versus $\frac{D}{Dt} \omega^2$. The figure indicates that there is a point-wise relation between production of enstrophy and actual change of enstrophy in region C and to a minor degree it is observable also in region B. This is also consistent with the Lagrangian evolution of these quantities shown previously (Figure 8.7). It turns out that the major contribution in terms of magnitude to the positive $\frac{D}{Dt} \omega^2$ is coming from viscous enstrophy production. As expected, on the turbulent side $\nu \omega_i \nabla^2 \omega_i$ is mostly associated with the destruction of $\omega^2$, i.e. $\frac{D}{Dt} \omega^2 < 0$.

For statistically stationary turbulence it is known that, in the mean, $\langle \frac{D}{Dt} \omega^2 \rangle = 0$ and thus $\langle \omega_j \omega_j s_{ij} \rangle = - \langle \nu \omega_i \nabla^2 \omega_i \rangle$ [63]. In addition, there is evidence that this is true at any moment if integrated over the spatial domain [68]. Despite this observation, $\omega_j \omega_j s_{ij}$ appears to be nowhere close to being point-wise balanced by $\nu \omega_i \nabla^2 \omega_i$ (see e.g., [38, 68]). The Joint PDF
Figure 8.18: Joint PDFs of the terms in Eq. 6.4, \( \nu \omega_i \nabla^2 \omega_i \) and \( \frac{\partial \omega_i^2}{\partial x_j} \) from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).

Figure 8.19: Joint PDFs of the terms in Eq. 6.4, \( \omega_i \omega_j \nabla^{2} \omega_i \) and \( \nu \omega_i \nabla^2 \omega_i \) from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).
of $\nu \omega_i \nabla^2 \omega_i$ versus $\omega_i \omega_j \delta_{ij}$ is shown in Figure 8.19. From the mean values we see that there is an approximate balance between production and destruction of enstrophy in region A. We note that, in the turbulent flow region, $\omega_i \omega_j \delta_{ij}$ and $\nu \omega_i \nabla^2 \omega_i$ are preferably of opposite sign, but we confirm that there is no point-wise balance between the two. The behavior of the two terms at the interface is remarkably different. It can be observed that $\omega_i \omega_j \delta_{ij}$ and $\nu \omega_i \nabla^2 \omega_i$ contribute together to the increase of enstrophy, which is quite anomalous if compared to the general behavior in the turbulent bulk.

As a further step we investigate the relation of different quantities with the magnitudes of $\omega^2$ or $s^2$. We start with the Joint PDF of $\omega_i \omega_j \delta_{ij}$ versus $\omega^2$ shown in Figure 8.20 for the 3 regions from PTV (top) and DNS (bottom). It can be noted that in region A higher levels of $\omega^2$ are associated with stronger positive $\omega_i \omega_j \delta_{ij}$. This is consistent with the known observation that the positive skewness of the PDF of $\omega_i \omega_j \delta_{ij}$ increases with higher $\omega^2$ events, see e.g. [38]. The picture in regions B and C remains qualitatively similar, that is, at low values of $\omega^2$ also the production term remains small and grows quickly with higher $\omega^2$.

Figure 8.21 shows the analogous Joint PDF’s for the viscous term, $\nu \omega_i \nabla^2 \omega_i$ versus $\omega^2$. In the turbulent region, as $\omega^2$ grows, $\nu \omega_i \nabla^2 \omega_i$ is more and more contributing to the destruction of $\omega^2$. Contrarily, in region B the viscous term is mostly contributing to the increase of $\omega^2$, but different from $\omega_i \omega_j \delta_{ij}$ before, this action is mainly restricted to lower values of $\omega^2$. Also

![Figure 8.20: Joint PDFs of $\omega_i \omega_j \delta_{ij}$ versus $\omega^2$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).](image-url)
the correlation value is lower than the analogous value for $\omega_i\omega_j s_{ij}$ and $s^2$. In region C the situation looks similar to the one illustrated in the previous figure, that is, $\nu \omega_i \nabla^2 \omega_i$ becomes more positive with higher $\omega^2$.

It is also very instructive to investigate in this way the relation of the terms in the strain balance equation, Eq. 8.2, with the magnitude of $s^2$. The Joint PDF of $-s_{ij}s_{jk}s_{ki}$ versus $s^2$ are shown in Figure 8.22 for the 3 regions from PTV (top) and DNS (bottom). In region A we see that, as strain increases, the strain production increases, consistent with the fact that the positive skewness of the PDF of $-s_{ij}s_{jk}s_{ki}$ increases with higher strain events, see eg., [38]. The shapes of the Joint PDF's in regions B and C look quite similar to the one in region A, with the exception that events with $-s_{ij}s_{jk}s_{ki} < 0$ are strongly suppressed. This difference is especially pronounced at increasing intensities of $s^2$. The correlation values between $-s_{ij}s_{jk}s_{ki}$ and $s^2$ are high and comparable to those between $\omega_i\omega_j s_{ij}$ and $\omega^2$.

Figure 8.23 shows the analogous plot for the pressure term, $-s_{ij}p_{ij}$, (top) and the viscous term, $\nu s_{ij} \nabla^2 s_{ij}$, (bottom) obtained from DNS. We observe that the shape of the Joint PDF of $-s_{ij}p_{ij}$ is symmetric, it is reminded that in homogeneous turbulence $\langle s_{ij}p_{ij} \rangle = 0$. Strong events of $s_{ij}p_{ij}$ appear to be mostly associated with intermediate levels of $s^2$, while at high $s^2$ events the pressure term tends to be of lower magnitude. In region B $s_{ij}p_{ij}$ is mostly contributing to the destruction of strain and this is more and more so with increasing $s^2$. 

Figure 8.21: Joint PDFs of $\nu \omega_i \nabla^2 \omega_i$ versus $\omega^2$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).
8.4. Small scale properties of the interfacial region

Figure 8.22: Joint PDF's of $-s_{ij} s_{jk} s_{ki}$ versus $2s^2$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).

Figure 8.23: Joint PDF's of $-s_{ij} \rho_{ij}$ (top) and $\nu s_{ij} \nabla^2 s_{ij}$ (bottom) versus $2s^2$ from DNS according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).
Differently, in region C the term \(-s_{ij}p_{ij}\) contributes both to the increase and decrease of \(s^2\). We observe that with a considerable probability the magnitudes of \(-s_{ij}p_{ij}\) can be remarkably high (i.e. high compared to the production term, \(-s_{ij}s_{jkl}k_{il}\), and the viscous term, \(\nu s_{ij}\nabla^2 s_{ij}\) shown below) at very low levels of \(s^2\). As expected, from the Joint PDF's of \(\nu s_{ij}\nabla^2 s_{ij}\) versus \(s^2\) shown in the bottom panel of Figure 8.23 we note that, as \(s^2\) grows, \(\nu s_{ij}\nabla^2 s_{ij}\) is more and more contributing to the destruction of \(s^2\). The correlation value is somewhat higher than the analogous correlation between \(\nu \omega_i \nabla^2 \omega_i\) and \(\omega^2\). In regions B and C it can be seen that \(\nu s_{ij}\nabla^2 s_{ij}\) attains both positive and negative values. However, the iso-shapes of the most probable events are mainly concentrated at relatively low magnitudes of \(\nu s_{ij}\nabla^2 s_{ij}\). These results indicate that the contribution of both \(-s_{ij}s_{jkl}k_{il}\) and \(-s_{ij}p_{ij}\) might be particularly important for the initial increase of \(s^2\) in the interfacial regions B and C and underline once again that the dynamics of strain are inherently different from the dynamics of enstrophy in such regions.

8.5 Concluding remarks

In summary, we analyzed small scale enstrophy and strain dynamics in proximity of a turbulent/non-turbulent interface without strong mean shear. The experimental results are in good agreement with the simulation, at least on a qualitative level, which is considered as a clear indication for the reliability of both methods. The behavior of vorticity-related quantities is very different from their strain-related counterparts. For example, the viscous term is not responsible for building up strain as strain is destroyed by \(\nu s_{ij}\nabla^2 s_{ij}\) in all three regions. In addition, the analysis of these quantities with respect to the distance from the interface reveals the range of influence of \(\nu \omega_i \nabla^2 \omega_i\) and \(\nu s_{ij}\nabla^2 s_{ij}\) into the non-turbulent region. We also found that both \(\omega_i \omega_j s_{ij}\) and \(\nu \omega_i \nabla^2 \omega_i\) are responsible for the increase of \(\omega^2\) at the interface and substantiate the physical interpretation of the term \(\nu \omega_i \nabla^2 \omega_i\) as viscous interaction, in analogy to \(\omega_i \omega_j s_{ij}\), commonly referred to as the inviscid interaction of vorticity and strain. Furthermore, we found that the properties of enstrophy production are substantially different in such regions. In particular, vorticity is more aligned with the vortex stretching vector in the intermediate region B, as compared to regions A and C. In addition, region C is characterized by a strong suppression of the preferential \((\omega, A_2)\) alignment and increased probability of \((\omega, A_1)\) alignment. The analysis in terms of Joint PDF's revealed that there is a pointwise relation between \(\nu \omega_i \nabla^2 \omega_i\) and \(\frac{D \omega^2}{Dt}\) in region C. We also found that \(\omega_i \omega_j s_{ij}\) and \(\nu \omega_i \nabla^2 \omega_i\) contribute together to the increase of enstrophy, which is quite anomalous if compared to the general behavior of the two terms in fully developed turbulence. The dependence of the analyzed quantities on the magnitudes of
$\omega^2$ and $s^2$ revealed that viscous production of enstrophy can be particularly significant at low levels of $\omega^2$ (region C), while $\omega_i \omega_j s_{ij}$ becomes dominant at higher $\omega^2$. There is indication that the pressure term, $-s_{ij} p_{ij}$ and the production term $-s_{ij} s_{jk} s_{ki}$ might be mostly responsible for the increase of $s^2$ in such regions.
Chapter 9

Conclusions

The small scale properties of the interface between turbulent and non-turbulent regions of a flow with weak mean shear were analyzed by means of Particle Tracking Velocimetry (3D-PTV) and Direct Numerical Simulation (DNS). In the course of the work, the 3D-PTV technique was further developed through changes of hardware and data processing. In particular, a scanning device was implemented allowing for larger seeding densities and a larger field of view. Compared to the classical 3D-PTV method, both the observation volume and the spatial resolution were nearly doubled.

Experiments in quasi-homogeneous turbulence were conducted to assess the feasibility of measuring the Laplacian of vorticity, $\nabla^2 \omega$, through 3D-PTV. The Laplacian of vorticity was obtained from the local balance equation of vorticity $\nabla \times a = \nu \nabla^2 \omega$ by evaluating the term the curl of the Lagrangian acceleration, $\nabla \times a$, from the tracking data. The analysis revealed important insights in the effects of viscosity on enstrophy dynamics. For example, in regions of high enstrophy and low strain, the viscous term contributes more to the reduction of the magnitude of vorticity, than to changing its direction. In addition, when $\omega$ is aligned with the intermediate eigenvector of the rate of strain tensor, $\lambda_2$, $\nu \nabla^2 \omega$ appears to act predominantly against the direction of $\omega$ (attenuating enstrophy), while when $\omega$ is aligned with $\lambda_3$, there is a significant contribution of $\nu \nabla^2 \omega$ in the direction perpendicular to $\omega$ which induces its tilting. Also, the viscous contribution to the total tilting of vorticity was found to be significantly larger than the inviscid contribution. In analogy with the common interpretation of the process of enstrophy production ($\omega_i \omega_j \delta_{ij}$) as a self-amplification of velocity derivatives, the processes of viscous destruction and tilting of vorticity might be interpreted as a 'self-moderation' of velocity derivatives.

In view of preparation for 3D measurements, the applicability of level-based techniques for the detection of the turbulent/non-turbulent interface were tested using two techniques. The
propagation of a turbulent front generated by an oscillating grid starting from rest was analyzed using flow visualization and Particle Image Velocimetry (PIV). Among other things, known properties of the interface were confirmed, such as the steep gradient of vorticity at the interface and the propagation velocity of the front. The theoretically predicted \((kt)^{1/2}\) law of propagation of the turbulent front in time could be verified with both experimental techniques. The results indicate that it is possible to utilize the same level-based detection methods also in the three-dimensional Lagrangian measurements.

Finally, small scale enstrophy and strain dynamics in proximity of a turbulent/non-turbulent interface were analyzed using 3D-PTV and DNS. The comparison of flow properties in the turbulent (A), intermediate (B) and non-turbulent (C) regions in the proximity of the interface allowed for direct observation of the key physical processes underlying the entrainment phenomenon. It was found that the behavior of vorticity-related quantities is very different from their strain-related counterparts. For example, enstrophy and strain are ‘equal partners’ in region A, while regions B and C are characterized by a dominance in terms of magnitudes of strain over enstrophy. Also the behavior of the viscous terms of strain and enstrophy is remarkably different: \(\nu \omega_i \nabla^2 \omega_i\) is important for the increase of \(\omega^2\) in regions B and C, while \(\nu s_{ij} \nabla^2 s_{ij}\) is not responsible for building up strain. The results indicate that an interpretation of the viscous term \(\nu \omega_i \nabla^2 \omega_i\) as interaction between strain and vorticity due to viscosity is physically more appealing than 'simple' diffusion of vorticity due to viscosity. In addition, we found that the properties of enstrophy production are substantially different in such regions. For example, vorticity is more aligned with the vortex stretching vector in the intermediate region B, as compared to regions A and C. Region C is characterized by a strong suppression of the preferential \((\omega,\lambda_2)\) alignment and increased probability of \((\omega,\lambda_1)\) alignment. The analysis also revealed that there is a pointwise relation between \(\nu \omega_i \nabla^2 \omega_i\) and \(\frac{D \omega^2}{Dt}\) in region C. It was also found that \(\omega_i \omega_j s_{ij}\) and \(\nu \omega_i \nabla^2 \omega_i\) contribute together to the increase of enstrophy, which is quite anomalous if compared to the general behavior of the two terms in fully developed turbulence. There is indication that the pressure term, \(-s_{ij} p_{ij}\) and the production term \(-s_{ij} s_{jk} s_{ki}\) might be mostly responsible for the increase of \(s^2\) in such regions.
Chapter 10

Appendix 1

This paragraph summarizes the discussion on the effect of different settings used for the direct numerical simulation, as mentioned in Section 4.2. The Navier Stokes equations are solved by using a finite-difference method for spatial discretization. Also a mixed spectral-finite differences method was previously tested yielding essentially the same results. It was preferred to use the simple finite-difference method, because the computation of Lagrangian particle paths was easier to implement.

In addition to standard grid convergence tests, the effect of different settings on the simulation was tested in the same way, as shown for the experimental results reported in Chapter 7. That is, the effect of different parameters on the propagation of the turbulent/non-turbulent interface was analyzed with focus on the larger scales. The Reynolds number was varied in the range $Re=10^2$ to $Re=10^4$. The flow periods in $x_1$ and $x_3$ directions were changed in the range from 5 to 10 and the size of computational box in $x_2$ direction was varied in the range $3 < X_2 < 40$. The finest spatial resolution for the tests was $127 \times 127$ Fourier modes in $x_1, x_3$ directions and 192 uniformly distributed grid nodes in $x_2$ direction. The qualitative behavior of the spreading of turbulence was investigated by looking at the properties of the flow in the whole computational domain and after that, the details were analyzed using smaller boxes with finer spatial resolution. The amplitudes of boundary velocities were taken equal for all components. Several values of boundary velocity time scale $\Delta_t$ were considered in the range of $0.2 < \Delta_t < 5$. As an illustrative example, the results of three runs with $Re = 10^3$ are shown in Figure 10. In the first run the resolution was set to $63 \times 192 \times 63$, the size of the box was $X_2 = 8$ and the boundary-velocity time-scale was $\Delta_t = 0.5$. The results for 4 time moments: $t = 1, t = 2, t = 5$ and $t = 10$ are well described by the curve $x_2 = a + b\sqrt{t}$ with $a = 0.53, b = 0.46$. In the second run, the resolution was twice as fine in all three directions. The number of grid nodes in $x_2$ direction was the same, but the size of the computational box was taken
two times smaller. In this run, the law of propagation was approximately the same, as in the first run in the initial time moments, but at later time, \( t = 10 \), the interface was found further away. In the third run, all parameters were taken equal the ones in the first run, except the time-scale \( \Delta t \), which is now 0.2, i.e. two and half times less than in the first run. For this run, the propagation of the turbulent/non-turbulent interface in the initial time moments is somewhat faster as compared to run 1 and 2. The best fit for a \( x_2(t) = a + b\sqrt{t} \) for run 3 is \( a = 0.87, b = 0.44 \).

Figure 10.1: Position of the turbulent/non-turbulent interface in time
Chapter 11

Appendix 2

This paragraph is devoted to the error estimation mentioned in Section 6.2. It follows the method used by [38], in which the particle position accuracy $e_x$ was estimated to be $10 \div 40 \mu m$ and the velocity error $(e_u)$ was of the order of $0.5 \text{mm/s}$ or 5% of r.m.s. velocity $(u')$, respectively (an apostrophe ' denotes r.m.s.).

We are not able to estimate the errors of higher order derivatives by using a simple error propagation analysis, because they are calculated by solving a system of linear equations at each time step along the Lagrangian trajectories, rather than by a simple difference scheme. The method we use is based entirely on the estimation of the error as obtained from JPDFs of the balance equations involving the quantities of interest [38]. This method was also applied in [26] to estimate the accuracy of velocity derivatives, $e_{\partial u/\partial x}$, to be approximately 7% of $(\partial u/\partial x)'$. In a similar manner, the error which propagated to the accelerations (defined in Section 6.2) was estimated by [26] to be of the order of $O(15\%)$.

We estimate the error of the acceleration derivatives (Section 6.2), to be of $O(25\%)$, as explained in the following. Figure 6.1a shows the JPDF of the second invariant of the velocity gradient tensor, \( Q = 1/4(\omega^2 - 2s^2) \), versus \( \nabla \cdot a \). Since \( Q \) and \( \nabla \cdot a \) are measured independently of each other (the former involving velocity derivatives, the latter derivatives of Lagrangian acceleration), we can estimate $e_{\nabla \cdot a}$ from the aspect ratio of the iso-contour shapes of the JPDF shown in the figure. By estimating this aspect ratio $n$, to be roughly 3 and by measuring $(\nabla \cdot a)'$ to be $12 \text{s}^{-2}$, we get $e_{\nabla \cdot a} = \frac{(\nabla \cdot a)'}{\sqrt{2n}} \approx 3\text{s}^{-2}$. By using the standard error propagation formula $e_{\nabla \cdot a} = 3/\sqrt{3} \cdot e_{\partial a/\partial x}$, we get $e_{\partial a/\partial x} = 1.5\text{s}^{-2}$. Thus, with a measured $(\partial a/\partial x)'$ of roughly $6.2\text{s}^{-2}$, the accuracy for acceleration derivatives is of order $O(25\%)$.

We can thus estimate the error of the quantity under investigation, $\nu \nabla^2 \omega$, which is calculated by using the curl of acceleration, $\nabla \times a$. The curl operator propagates the error as $e_{\nabla \times a} = 2/\sqrt{2}e_{\partial a/\partial x}$ and this results in approximately $2 \text{s}^{-2}$. The variation of the quantity itself is
measured to be $(\nu \nabla^2 \omega)' = 6.0 \, s^{-2}$, and the relative error is consecutively estimated to be of $O(35\%)$. This can be confirmed from the measurements looking at the JPDF's of the vorticity balance equation (not shown). Taking the aspect ratio of the iso-contours we can estimate $\epsilon_{\nu \nabla^2 \omega} = \frac{\langle \nu \nabla^2 \omega \rangle'}{\sqrt{2}n}$. Since the aspect ratio is 2, we find $\epsilon_{\nu \nabla^2 \omega}$ to be $2.1 \, s^{-2}$, which is equivalent to the relative error of $O(35\%)$. Finally, the error of $\nu \omega \nabla^2 \omega$ is estimated by using the error propagation: $\epsilon_{\nu \omega \nabla^2 \omega} = (\omega + \epsilon_{\omega}) \cdot (\nu \nabla^2 \omega + \epsilon_{\nu \nabla^2 \omega}) - \nu \omega \nabla^2 \omega \approx O(\omega) \cdot \epsilon_{\nu \nabla^2 \omega} + O(\nu \nabla^2 \omega) \cdot \epsilon_{\omega} \approx \frac{12}{s^3}$. This can be confirmed by measurements looking at Figure 6.1b. With an aspect ratio of $n \approx 1.8$ and a measured $(\nu \omega \nabla^2 \omega)' = 29.1 \, s^{-3}$ we get $\epsilon_{\nu \omega \nabla^2 \omega} = 12.0 \, s^{-3}$, or $O(40\%)$. We summarize the error estimation results of all the quantities under investigation in table 11, estimated with the same methodology. Note that the errors of $D\omega^2/Dt$ and $\omega_i \omega_j s_{ij}$ are significantly smaller than the error of $\nu \omega \nabla^2 \omega$.

<table>
<thead>
<tr>
<th>$u$(mm·s$^{-1}$)</th>
<th>$\partial u/\partial x$ (s$^{-1}$)</th>
<th>$\omega_i$(s$^{-1}$)</th>
<th>$\omega^2$ (s$^{-2}$)</th>
<th>$a$ (mm·s$^{-2}$)</th>
<th>$\partial a/\partial x$ (s$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0 ± 0.5</td>
<td>1.5 ± 0.1</td>
<td>3.0 ± 0.15</td>
<td>16.3 ± 0.8</td>
<td>20.0 ± 3.0</td>
<td>6.2 ± 1.5</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>5</td>
<td>5</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>$D\omega_i/Dt$ (s$^{-3}$)</td>
<td>$\omega_i \omega_j s_{ij}$ (s$^{-2}$)</td>
<td>1/2$D\omega^2/Dt$ (s$^{-3}$)</td>
<td>$\omega_i \omega_j s_{ij}$ (s$^{-3}$)</td>
<td>$\nu \nabla^2 \omega_i$ (s$^{-2}$)</td>
<td>$\nu \omega_i \nabla^2 \omega_i$ (s$^{-3}$)</td>
</tr>
<tr>
<td>5.3 ± 0.9</td>
<td>4.6 ± 0.5</td>
<td>26.3 ± 3.8</td>
<td>21.9 ± 4.0</td>
<td>6.0 ± 2.1</td>
<td>29.1 ± 12.0</td>
</tr>
<tr>
<td>17</td>
<td>11</td>
<td>15</td>
<td>14</td>
<td>35</td>
<td>39</td>
</tr>
</tbody>
</table>

Table 11.1: Uncertainty analysis. The first line is r.m.s ± $\epsilon$ and the second line is $\epsilon[\%]$. 
List of Tables

5.1 Characteristic properties of the flow. ................................................. 20
5.2 Some characteristics of the PTV and SPTV experiments. ....................... 26
5.3 Comparison of particle numbers for the PTV and SPTV experiments along the
chain of processing steps. ................................................................. 26
5.4 Characteristics of the statistical sets of the PTV and SPTV experiments. .... 27
5.5 Aspect ratios of the contour surfaces obtained from the "divergence check" and
"acceleration check" compared for the PTV and SPTV experiments. .............. 29

6.1 Average values of the viscid and inviscid contributions (see Eq. 6.2) to \( \Omega_\omega \)
conditioned on \((\omega, \lambda_1)\) alignments obtained from PTV. .................. 45

7.1 Values of \(k\) and exponent \(n\) obtained from the regression analysis for the three
different detection methods. \(R^2\) is the correlation coefficient. The values in the
rows of the table marked with \((\cdot)\) refer to the regression of the averaged curve
over experiments 1=4 and 5=11, respectively. For experiments marked by an
asterisk, the distance between the free surface and the grid was 10 cm, for all
other experiments it was 5 cm. ....................................................... 62

8.1 Characteristic properties of the flow for the experiment and the simulation. ... 70

11.1 Uncertainty analysis. The first line is r.m.s \(\pm\varepsilon\) and the second line is \(\epsilon\)% 96
List of Figures

4.1 Schematic of the oscillating grid setup. A time sample of the grid velocity obtained from the encoder signal is shown in the upper right corner. .................. 8

4.2 A servo-motor (not shown), installed on the top of the forcing unit, drives 8 counter-rotating disks, 4 each rotating according to the scheme. The flow volume is located at the center of the forcing unit, mid-way between the disks. 9

4.3 Photron Ultima APX with 4 way image splitter (left) and Mikrotron multi-camera system (right). .......................................................... 10

4.4 Vorticity iso-surface for a given snapshot in time obtained from DNS. .... 12

5.1 A schematic of the experimental setup. The laser beam is expanded to a light sheet and scanned through the observation volume using an eight face prism. 18

5.2 Schematic illustrations of a) the time evolution versus light sheet position during four volume scans and b) the spline interpolation scheme of a point in a trajectory. 22

5.3 Schematic illustration of the light intensity distribution through scanning with velocity $v_1$. ................................................................. 23

5.4 Joint PDFs of $-\frac{\partial u_x}{\partial x_i}$ versus $\frac{\partial u_j}{\partial x_j} + \frac{\partial u_k}{\partial x_k}$ (here no summation over $i, j, k$ is applied). 28

5.5 The expression $\frac{DA_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}$ is checked for each component $i$, with joint PDFs of $a_{t,i} + a_{c,i}$ versus $a_i$. ........................ 29

5.6 Relative divergence $\langle \delta \rangle$ plotted over trajectory length $l$. ............ 30

5.7 PDFs of $\omega_i \omega_j s_{ij}$ and $-4/3 s_{ij} s_{jk} s_{ki}$ normalized with their mean values $\langle \omega_i \omega_j s_{ij} \rangle$ and $-4/3 \langle s_{ij} s_{jk} s_{ki} \rangle$ respectively, as obtained by SPTV and the PTV experiment of [34]. ........................................... 31

5.8 PDFs of the eigenvalues, $\Lambda_{1,2,3}$, of the rate of strain tensor $s_{ij}$. $\nabla$: $\Lambda_1$, $\circ$: $\Lambda_2$, $\Box$: $\Lambda_3$. .................................................. 32
5.9 PDFs of $\cos(\omega, \lambda_1)$ conditioned on weak and strong $s^2$. -: $\cos(\omega, \lambda_1), s^2 < (s^2)$, $\nabla$: $\cos(\omega, \lambda_2), s^2 > (s^2)$, $\nabla$: $\cos(\omega, \lambda_3), s^2 < (s^2)$, $\nabla$: $\cos(\omega, \lambda_4), s^2 > (s^2)$.

5.10 Joint PDF plot of vorticity production versus change of enstrophy, both axes normalized with $(\omega_j \omega_j s_{ij})$.

6.1 Joint PDFs of the terms in Eq. 6.4 (a), unconditioned (b) and conditioned (c) terms of Eq. 6.4 from PTV. Joint PDF of the terms of Eq. 6.4 from DNS (d).

6.2 PDFs of the normalized quantities of the unconditioned (solid lines) and conditioned (dashed lines) datasets from the experiments: a) velocity (triangles), acceleration (circles), b) enstrophy (triangles) and strain (circles), c) terms in Eq. 6.4.

6.3 PDF's of enstrophy production, $\omega_j \omega_j s_{ij}$, and viscous term, $\nu \omega_j \nabla^2 \omega_i$, normalized over their respective means.

6.4 PDFs of $\cos(\omega, W)$, $\cos(\nu \nabla^2 \omega, \omega)$, and $\cos(\nu \nabla^2 \omega, W)$ conditioned on enstrophy and strain (left panel) and $(\omega, \lambda_1)$ alignments (right panel).

6.5 PDFs of the cosines between a) $\eta_{\omega}$, b) $\eta_{\omega} \omega$ and the three eigenvectors of the rate of strain tensor.

6.6 PDFs of total tilting $\Omega^2$ conditioned on $(\omega, \lambda_1)$ alignments (top panel) and viscid and inviscid contributions to tilting conditionally averaged on $\Omega^2$ (bottom panel). The left panel is from PTV, the right panel from DNS.

6.7 Joint PDFs of the terms in Eq. 6.4 from PTV (left) and DNS (right).

6.8 Joint PDFs of the terms in Eq. 6.4, conditioned on $(\omega, \lambda_1)$ alignments from PTV (left) and DNS (right).

7.1 A typical example of raw and processed images of fluorescent dye. a) raw, gray level image b) binary image c) single object d) object boundary.

7.2 Average intensity of all pixels above the threshold as a function of the threshold (see [53]) for a given experiment. a) variation in time ($0-20s$) b) time averaged curve.
7.3 An example of an instantaneous PIV realization. Vectors show the direction and the magnitude of the velocity field and contours are of $\omega_3$. Solid lines denote the positive values and the dashed lines are for the negative values (the lowest level is $0.3 \text{ s}^{-1}$).

7.4 Vertical profiles of vorticity magnitude for 3 different time instances at $x = 50\text{mm}$.

7.5 Conditionally sampled vertical profiles of the magnitudes of vorticity (left axis, solid line) and of velocity vector and vertical velocity component (right axis, dashed and dashed dotted lines).

7.6 Vorticity (left panel) and velocity (right panel) magnitude maps for 3 time instances, a) $t = 2 \text{ s}$, b) $t = 6 \text{ s}$, c) $t = 18 \text{ s}$. The white spots represent the detected front, i.e. $x^2(x_1, t)$.

7.7 Vertical position of the interface, $H$, versus time, detected by using (a) dye concentration, (b) velocity and (c) vorticity. The different experiments are listed in Table 7.1. The continuous and dashed lines represent the average curve over the experiments and its best fit estimated by regression analysis (Eq. 7.4), respectively. For the sake of clarity the number of plotted data points is reduced.

8.1 Joint PDFs of the terms in Eq. 8.1 for PTV (a) and DNS (b).

8.2 Joint PDF of $\frac{\partial}{\partial t} x_i^2$ versus $u \cdot \nabla \frac{x_i^2}{2}$ for PTV (left) and DNS (right).

8.3 Joint PDF's of LHS versus RHS of Eq. 8.4 for the components of the vorticity vector, $\omega_i$, from PTV.

8.4 Joint PDF's of LHS versus RHS of Eq. 8.4 applied to $\omega_i$ from PTV (left) and DNS (right).

8.5 Single trajectory plotted in real space. The symbol '◎' indicates the initial position.

8.6 Lagrangian evolution of quantities along the trajectory plotted in Figure 8.5. The time axis is normalized by the Kolmogorov time scale, $\tau_\eta$.

8.7 Conditionally averaged Lagrangian evolution of $\omega^2$ and $2s^2$ (top) and the invariants $Q, R, S$ (bottom) obtained from PTV (left) and DNS (right).

8.8 Conditionally averaged Lagrangian evolution of inertial and viscous terms (top) and their rates (bottom) obtained from PTV (left) and DNS (right).
8.9 Average profiles of $\omega^2(x_2)$ and $2s^2(x_2)$ from PTV (symbols), and DNS (lines) relative to the interface, $x_2 = x_2 - x_2^*$, on linear scale (left) and on log-scale (right). The values are normalized by the respective maxima ($\max(\omega^2) \approx \max(2s^2) \approx 3.6$ s$^{-2}$ for PTV and 3.0 for DNS). The error bars display the sensitivity of the experimental result on the enstrophy threshold, 0.5-2.5 s$^{-2}$. The symbol $\langle \cdot \rangle$ denotes the ensemble average of the respective quantity.

8.10 Average profiles of production and viscous destruction terms of strain and enstrophy (left) and their rates (right). The insert shows the individual Lagrangian trajectories of $\nu \omega_i \nabla^2 \omega_i$ obtained from PTV. Lines are from DNS, symbols are from PTV.

8.11 PDF of various quantities from experiment (top) and simulation (bottom) according to the division in 3 regions: turbulent (left), intermediate (center) and non-turbulent (right). $\omega_i \omega_j s_{ij}$ $(-)$, $\nu \omega_i \nabla^2 \omega_i$ $(-)$, $-s_{ij} s_{jk} s_{ki}$ $(- \cdot)$, $\nu s_{ij} \nabla^2 s_{ij}$ $(- \cdots$, only bottom).

8.12 PDF of various quantities from experiment (top) and simulation (bottom) according to the division in 3 regions: turbulent (left), intermediate (center) and non-turbulent (right). $\omega_i \omega_j s_{ij}/\omega^2$ $(-)$, $\nu \omega_i \nabla^2 \omega_i / \omega^2$ $(-)$, $-s_{ij} s_{jk} s_{ki}/s^2$ $(- \cdots)$, $\nu s_{ij} \nabla^2 s_{ij}/s^2$ $(- \cdots$, only bottom).

8.13 (left) Cosines of the angle between vorticity and $\omega (-,\cdot)$ and between vorticity and $\nu \nabla^2 \omega (-,\cdot)$. (right) Invariants, $Q (-,\cdot), R (-,\cdot), S (-,\cdot)$, only DNS. Lines are from DNS, symbols are from PTV.

8.14 PDF's of various quantities for the 3 regions from PTV (left) and simulation (right). (a-b) PDF of the cosine between vorticity, $\omega$, and the vortex stretching vector, $\mathbf{W}$. (c-d) PDF of the cosine between $\omega$ and the eigenvectors of the rate of strain tensor, $\Lambda_i$. (e-f) PDF of the eigenvalues of the rate of strain tensor, $\Lambda_i$.

8.15 Joint PDFs of the enstrophy and strain production terms, $\omega_i \omega_j s_{ij}$ and $-s_{ij} s_{jk} s_{ki}$ from experiment (top) and simulation (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).

8.16 Joint PDFs of the viscous terms in Eq. 6.4 and 8.2, $\nu \omega_i \nabla^2 \omega_i$ and $\nu s_{ij} \nabla^2 s_{ij}$ from DNS according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right).
8.17 Joint PDFs of the terms in Eq. 6.4, $\omega_i\omega_j\delta_{ij}$ and $\frac{D}{Dt} \frac{\omega^2}{2}$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right). .................. 83

8.18 Joint PDFs of the terms in Eq. 6.4, $\nu\omega_i\nabla^2\omega_i$ and $\frac{D}{Dt} \frac{\omega^2}{2}$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right). .................. 84

8.19 Joint PDFs of the terms in Eq. 6.4, $\omega_i\omega_j\delta_{ij}$ and $\nu\omega_i\nabla^2\omega_i$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right). .................. 84

8.20 Joint PDFs of $\omega_i\omega_j\delta_{ij}$ versus $\omega^2$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right). .................. 85

8.21 Joint PDFs of $\nu\omega_i\nabla^2\omega_i$ versus $\omega^2$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right). .................. 86

8.22 Joint PDF’s of $-s_{ij}s_{jk}s_{kl}$ versus $2s^2$ from PTV (top) and DNS (bottom) according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right). .................. 87

8.23 Joint PDF’s of $-s_{ij}s_{kl}$ (top) and $\nu s_{ij}\nabla^2 s_{ij}$ (bottom) versus $2s^2$ from DNS according to the division in 3 regions: turbulent - region A (left), intermediate - region B (center) and non-turbulent - region C (right). .................. 87

10.1 Position of the turbulent/non-turbulent interface in time .................. 94
Bibliography


[37] Lüthi B. (2003) Some aspects of strain, vorticity and material element dynamics as measured with 3D particle tracking velocimetry in a turbulent flow. Dissertation, Diss. ETH Nr. 14893, ETH Zürich


[67] TSI Inc. INSIGHT<sup>TM</sup> 3.3 PIV Evaluation Software (2002) TSI Inc., Shoreview (MN), USA


