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# **Pricing Credit Derivatives**

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# Abstract

The size of global credit derivatives markets rose from USD 586 billion at the beginning of this millennium to more than USD 20 trillion of outstanding notional by the end of 2006 as recently reported by the *British Bankers' Association*. This outstanding growth and the fact that the mathematics of credit derivatives are in some respects quite different from classical equity and fixed income derivatives made the valuation of credit derivatives products an active research area of mathematical finance. In this thesis we address the challenge of pricing credit derivatives in presence of different sorts of default dependencies: Dependence across the underlying obligors of a credit portfolio and dependence between these underlying credits and other related financial factors such as interest-rates and foreign exchange rates.

In part I we present a modelling framework for portfolio credit risk which incorporates the dependence between risk-free interest-rates and the aggregated portfolio loss process. The innovation in this approach is that –besides the traditional diffusion-based covariation between the portfolio loss intensity and interest-rates– a direct dependence between interest-rates and the portfolio loss process is allowed. The model is set up using a set of loss-contingent forward interest-rates  $f_n(t, T)$  and loss-contingent forward credit protection rates  $F_n(t, T)$  to parameterize the market of default-free bonds and credit portfolio-sensitive assets such as single-tranche collateralized debt obligations. We show that existence of such a parameterization is equivalent to absence of static arbitrage opportunities in the underlying assets and we give necessary and sufficient conditions on the stochastic evolution of the parameterization to ensure absence of dynamic arbitrage strategies. Moreover we identify the particular form of the model-implied portfolio spot loss intensity under an equivalent martingale measure.

We then turn away from the issue of loss-dependent interest rates and address the question of what type of generic spot model for the portfolio loss intensity produces flexible term structures of loss-contingent forward credit

protection rates  $F_n(t, T)$ . In single-obligor default risk modelling, using a background filtration in conjunction with a suitable embedding ( $\mathbb{H}$ ) hypothesis has proven a very successful tool to separate the actual default event from the default intensity model. We analyze the conditions under which this approach can be extended to the credit portfolio case with focus on the so-called top-down view. We introduce the natural  $\mathbb{H}$ -hypothesis of this setup –the *successive*  $\mathbb{H}$ -hypothesis– and we show that it is equivalent to a seemingly weaker *one-step*  $\mathbb{H}$ -hypothesis. Furthermore, we provide a canonical construction of loss processes in this setup and we show that under little regularity every loss process satisfying the successive  $\mathbb{H}$ -property actually stems from a canonical construction. In a special case, we provide closed-form solutions for some pricing problems.

As will be made obvious in the analysis of part I on the example of loss-dependent interest-rates, in credit risk modelling one has to take into account that related financial factors may show dependence with the underlying credits not only through diffusion-type covariation, but also via joint jumps at the default times of the underlying obligors.

In part II this observation is taken up in the case of a related foreign exchange rate (FX). We analyze the connections between the credit spreads that the same single-name credit risk commands in different currencies. We show that the empirically observed differences in these credit spreads are mostly driven by the dependence between the default risk of the underlying obligor and the exchange rate. In our model there are two different channels to capture this dependence: First, the diffusions driving FX rate and default intensities may be correlated, and second, an additional jump in the exchange rate may occur at the time of default. The differences between the default intensities under the domestic and foreign pricing measures are analyzed and closed-form prices for a variety of securities affected by default risk and FX risk –including credit default swaps (CDS) are given. In the empirical analysis we find that a purely diffusion-based correlation between exchange rate and default intensity is not able to explain the observed differences between JPY and USD CDS rates for a set of large Japanese corporate obligors. The data implies a significant additional jump in the FX rate at default.

The contents of this thesis are summarized in Ehlers and Schönbucher [2004, 2006b, 2006a].

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