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On the Algebraic Foundation of Bounded Cohomology

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Abstract

Bounded cohomology for topological spaces was introduced by Gromov in the late seventies, mainly to describe the simplicial volume invariant. It is an exotic cohomology theory for spaces in that it fails excision and thus cannot be represented by spectra. Gromov's basic vanishing result of bounded cohomology for simply connected spaces implies that bounded cohomology for spaces is an invariant of the fundamental group. To prove this, one is led to introduce a cohomology theory for groups and the present work is concerned with the latter, which has been studied by Gromov, Brooks, Ivanov and Noskov to name but the most important initial contributors. Generalizing these ideas from discrete groups to topological groups, Burger and Monod have developed continuous bounded cohomology in the late nineties.

It is a widespread opinion among experts that (continuous) bounded cohomology cannot be interpreted as a derived functor and that triangulated methods break down. We prove that this is wrong.

We use the formalism of exact categories and their derived categories in order to construct a classical derived functor on the category of Banach G -modules with values in Waelbroeck's abelian category. This gives us an axiomatic characterization of this theory for free and it is a simple matter to reconstruct the classical semi-normed cohomology spaces out of Waelbroeck's category.

We prove that the derived categories of right bounded and of left bounded complexes of Banach G -modules are equivalent to the derived category of two abelian categories (one for each boundedness condition), a consequence of the theory of abstract truncation and hearts of t -structures. Moreover, we prove that the derived categories of Banach G -modules can be constructed as the homotopy categories of model structures on the categories of chain complexes of Banach G -modules thus proving that the theory fits into yet another standard framework of homological and homotopical algebra.

Zusammenfassung

Unter Benutzung des Formalismus der exakten Kategorien und der derivierten Kategorien zeigen wir, dass die weitverbreitete Meinung, beschränkte Cohomologie lasse sich nicht via derivierte Funktoren interpretieren, ganz einfach falsch ist. Mittels der Theorie der abstrakten Trunkierung rekonstruieren wir die Waelbroecksche Kategorie von Banachquotienten und einige Verallgemeinerungen. Diese Kategorien treten als Wertebereich unserer Definition der Banachcohomologie für Gruppen in Erscheinung, woraus wir die Theorie von Gromov-Brooks-Ivanov sowie die Burger-Monodsche Theorie rekonstruieren können. Weiter zeigen wir, dass sich diese Theorien auch mittels Modellkategorien behandeln lassen.