Time-resolved single-electron detection in semiconductor nanostructures

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Abstract

In the work presented in this thesis we use time-resolved charge detection techniques to investigate single-electron tunneling in semiconductor quantum dots. The ability to detect individual charges in real-time makes it possible to count electrons one-by-one as they pass through the structure. The setup can thus be used as a high-precision current meter for measuring ultra-low currents, with resolution several orders of magnitude better than that of conventional current meters. A single-electron detector setup is therefore envisioned to be used as a natural definition for a current standard.

In addition to measuring the average current, the counting procedure also makes it possible to investigate correlations between charge carriers. Electron correlations are conventionally probed in noise measurements, which are technically challenging due to the difficulty to exclude the influence of external noise sources in the experimental setup. Using real-time charge detection techniques, we circumvent the problem by studying the electron correlation directly from the counting statistics of the tunneling electrons. In quantum dots, we find that the strong Coulomb interaction makes electrons try to avoid each other. This leads to electron anti-bunching, giving stronger correlations and reduced noise compared to a current carried by statistically independent electrons.

In our setup, the charge detector is implemented by monitoring changes in conductance in a near-by capacitively coupled quantum point contact. In a series of measurements on a double quantum dot structure, we find that the quantum point contact not only serves as a detector but also causes a back-action onto the measured device. Electron scattering in the quantum point contact leads to emission of microwave radiation. The radiation is found to induce an electronic transition between two quantum dots, similar to the absorption of light in real atoms and molecules. Using a charge detector to probe the electron transitions, we can relate a single-electron tunneling event to the absorption of a single photon. Moreover, since the energy levels of the double quantum dot can be tuned by external gate voltages, we use the device as a frequency-selective single-photon detector operating at microwave energies. The ability to put an on-chip microwave detector close to a quantum conductor opens up the possibility to investigate radiation emitted from mesoscopic structures and give a deeper understanding of the role of electron-photon interactions in quantum conductors.
A central concept of quantum mechanics is the wave-particle duality; matter exhibits both wave- and particle-like properties and can not be described by either formalism alone. In the previously mentioned experiments, the electrons were treated as particles tunneling back and forth between the quantum dots. To investigate the wave properties of the electrons, we perform experiments on a structure containing a double quantum dot embedded in the Aharonov-Bohm ring interferometer. Aharonov-Bohm rings are traditionally used to study interference of electron waves traversing different arms of the ring, in a similar way to the double-slit setup used for investigating interference of light waves. In our case, we use the time-resolved charge detection techniques to detect electrons one-by-one as they pass through the interferometer. We find that the individual particles indeed self-interfere and give rise to a strong interference pattern as a function of external magnetic field. The high level of control in the system together with the ability to detect single electrons enables us to make direct observations of non-intuitive fundamental quantum phenomena like single-particle interference or time-energy uncertainty relations.
Zusammenfassung

In dieser Doktorarbeit wird das Tunneln von einzelnen Elektronen in Quantendots mit Hilfe eines hochempfindlichen Ladungsdetektors zeitaufgelöst untersucht. Die Fähigkeit, einzelne Elektronen in Echtzeit zu messen, ermöglicht das Zählen von Elektronen während sie durch die Struktur fließen. Damit kann die experimentelle Anordnung als ein Strommesser betrachtet werden, bei dem die Empfindlichkeit mehrere Größenordnungen höher ist, als bei einem herkömmlichen Amperemeter. Der Einzel-elektronendetektor wird deshalb als primäres Messgerät zur Definition des Stromstandards diskutiert.


für Mikrowellenstrahlung auffassen. Die Fähigkeit, einen Mikrowellen-detektor auf
dem Chip in direkter Nähe eines mesoskopischen Systems zu platzieren, ermöglicht
die Untersuchung ausgesendeter Strahlung mesoskopischer Strukturen und kann
damit hoffentlich zu einem tieferen Verständnis der Rolle der Elektron-Photon-
Wechselwirkungen für den Elektronentransport durch Nanostrukturen beitragen.

Ein zentrales Konzept der Quantenmechanik ist der Welle-Teilchen-Dualismus;
Materie zeigt sowohl Wellen- als Teilcheneigenschaften. In den oben erwähnten Ex-
perimenten wurden die Elektronen als Teilchen behandelt, die zwischen Quantendots
hin- und hertunneln. Um die Wellennatur der Elektronen zu untersuchen, haben wir
Experimente an einem Doppelspaltexperiment durchgeführt, der in ein Aharonov-Bohm
Interferometer eingebettet ist. Aharonov-Bohm-Ringe werden häufig zum Studium
der Interferenz von Elektronenwellen in Nanostrukturen verwendet; sie funktionieren
ähnlich, wie das Doppelspaltexperiment für die Untersuchung der Interferenz von
Lichtwellen. In unserem Experiment schicken wir einzelne Elektronen so durch das
Interferometer, dass sie nach dem Durchgang mit dem Ladungsdetektor in Echtzeit
detektiert werden können. Wir finden dass die einzelnen Teilchen tatsächlich mit
sich selbst interferieren, was zu einem starken Interferenzmuster als Funktion des
äußereren Magnetfeldes führt. Die hohe Kontrolle über unser System zusammen mit
der Möglichkeit, einzelne Elektronen zu messen ermöglicht die direkte Beobachtung
von nicht-intuitiven fundamentalen Quantenphänomenen wie der Interferenz von
einzelnen Teilchen oder der Unschärfebeziehung bezüglich Zeit und Energie.
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<tr>
<td>2DEG</td>
<td>two dimensional electron gas</td>
</tr>
<tr>
<td>ac</td>
<td>alternating current</td>
</tr>
<tr>
<td>AFM</td>
<td>atomic force microscope</td>
</tr>
<tr>
<td>CB</td>
<td>Coulomb blockade</td>
</tr>
<tr>
<td>dc</td>
<td>direct current</td>
</tr>
<tr>
<td>DOS</td>
<td>density of states</td>
</tr>
<tr>
<td>DQD</td>
<td>double quantum dot</td>
</tr>
<tr>
<td>ES</td>
<td>excited state</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width at half maximum</td>
</tr>
<tr>
<td>GS</td>
<td>ground state</td>
</tr>
<tr>
<td>QD</td>
<td>quantum dot</td>
</tr>
<tr>
<td>QPC</td>
<td>quantum point contact</td>
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<tr>
<td>ST</td>
<td>singlet-triplet</td>
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<td>$-e &lt; 0$</td>
<td>electron charge</td>
</tr>
<tr>
<td>$h = 2\pi\hbar$</td>
<td>Planck’s constant</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>$\mu_B$</td>
<td>Bohr magneton</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$B$</td>
<td>magnetic field</td>
</tr>
<tr>
<td>$C_\Sigma$</td>
<td>self-capacitance of a quantum ring/dot</td>
</tr>
<tr>
<td>$E_C$</td>
<td>constant interaction energy</td>
</tr>
<tr>
<td>$\alpha_G$</td>
<td>gate lever arm</td>
</tr>
<tr>
<td>$E_n$</td>
<td>single particle energy of the $n$th level</td>
</tr>
<tr>
<td>$\Delta E_n$</td>
<td>spin degenerate single particle level spacing</td>
</tr>
<tr>
<td>$\mu$</td>
<td>electrochemical potential</td>
</tr>
<tr>
<td>$Q$</td>
<td>charge</td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
</tr>
<tr>
<td>$V$</td>
<td>voltage</td>
</tr>
<tr>
<td>$G$</td>
<td>conductance</td>
</tr>
<tr>
<td>$R$</td>
<td>resistance</td>
</tr>
<tr>
<td>$\Gamma_{S,D}$</td>
<td>tunnel coupling</td>
</tr>
<tr>
<td>$v_F$</td>
<td>Fermi velocity</td>
</tr>
<tr>
<td>$k_F$</td>
<td>Fermi wavenumber</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td>Fermi wavelength</td>
</tr>
<tr>
<td>$m^*$</td>
<td>effective electron mass in GaAs</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>electron mobility</td>
</tr>
<tr>
<td>$g$</td>
<td>electron g-factor</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$p_{x_0}(N)$</td>
<td>probability distribution for registering $N$ counts within time $t_0$</td>
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Chapter 1

Introduction

In this chapter we look at the basic concepts of electron transport in low-dimensional semiconductor nanostructures. In particular, we investigate the properties of single and double quantum dot structures. The chapter is not meant to give a comprehensive overview of the subject but rather to serve as a general introduction for non-specialists. More details as well as experimental realizations of various mesoscopic systems are given in later chapters.

1.1 Two-dimensional electron systems

The structures presented in this work are fabricated using a two-dimensional electron gas (2DEG) formed in a GaAs-Al$_x$Ga$_{1-x}$As semiconductor heterostructure. Figure 1.1 shows a schematic diagram of the different layers in the heterostructure. Because of different energies of the conduction bands, which depend on the fraction of aluminium contents in the layers, and the remote doping in the donor layer, an approximately triangular potential in z-direction is formed 34 nm below the surface. The potential confines the electrons to the x-y plane. The width of the potential is comparable to the Fermi wavelength of the electrons, so that the electron energy is quantized in z-direction. If only the lowest energy state is occupied, the electrons are
only free to move in the x-y plane and we have a two-dimensional electron system
in the quantum limit.

The heterostructures used in this thesis were grown by D.C. Driscoll and A.C.
Gossard at University of California, Santa Barbara, USA, M. Reinwald and W.
Wegscheider at University of Regensburg, Germany, and Silke Schön, ETH Zurich.

1.2 One-dimensional conductors

When introducing an additional confinement in the x- or y-direction of the two-
dimensional electron gas, we get a one-dimensional conductor or a quantum point
contact (QPC). The QPC is the electronic analogue of a photon waveguide. If the
width of the constriction is of the order of the electron wavelength, constructive and
destructive interference only allow electron wavefunctions corresponding to stand-
ing waves in the confinement direction. We postpone the detailed investigation of
quantum point contacts to chapter 3.

1.3 Quantum dots

All experiments described in this thesis involve quantum dots (QD) in one form
or another. A quantum dot is a structure where the electrons are confined in all
three spatial dimensions. In semiconductor systems, quantum dots are defined by
introducing additional lateral confinement in a two-dimensional electron gas. This
can be done by depleting parts of the 2DEG with the use of local gates, etching
techniques or local oxidation. Most samples in this work are fabricated by local
oxidation. The method is described in more detail in section 2.1. For a general
review of properties of QDs in semiconducting systems, see [1].

Quantum dots are conveniently described by the capacitive model shown in Fig.
1.2(a). The QD is tunnel-coupled to source and drain leads, allowing electrons to
tunnel into or out of the QD. The gate is used to shift the electrochemical potential
of the QD and thereby change its electron population. The spectrum of a semi-
conductor quantum dot is dominated by two energy scales. The Coulomb repulsion
between the charge carriers together with the small size of the structure leads to a
finite charging energy $E_C = e^2/C_\Sigma$ for adding an electron to the QD. Here, $C_\Sigma$ is the
total capacitance between the QD and its environment. The effect leads to a sup-
pression of the tunneling current called Coulomb blockade and is of purely classical
nature involving only the discreteness of charge.

The second energy scale is set by the energy separation $\Delta E$ between excitations
of quantum-mechanical states in the confinement potential. The energies of the
many-body states $\varepsilon_n$ depend not only on the confinement but also on interactions
between the electrons in the QD. This is a many-body problem that is difficult to
solve even if the exact shape of the potential is known. The problem is simplified
in the constant-interaction model [2, 3]. Here, we assume (1) that $E_C$ does not
1.3. Quantum dots

Figure 1.2: (a) Capacitive model describing a single quantum dot. The QD is tunnel coupled to source and drain leads. A voltage applied to the gate shifts the electrochemical potential of the QD relative to the Fermi level in the leads. We have not included the capacitive coupling between the leads and the QD. (b) Diagram depicting the energy level configuration of the QD.

Transport measurements are commonly used to probe the properties of the states in the quantum dot. Depending on the temperature and the alignment of the Fermi levels in the leads and the levels of the QD, we can have single-level or multi-level transport as well as strong suppression of the current if no states are available. The effects are conveniently visualized if the QD current is plotted versus bias voltage and an external gate voltage, which gives a charge stability diagram of the QD. In Fig. 1.3(a) we plot a sketch of such a stability diagram, showing the characteristic Coulomb blockade diamonds. The gray areas correspond to current flow, while white regions represent regions of Coulomb blockade. With increased gate voltage, the electrochemical potential of the QD is lowered and electrons are filled into the dot one by one.

Figure 1.3(b) shows the energy level configuration of the QD for three positions of the Coulomb diamonds in Fig. 1.3(a). For all three configurations there is a fixed bias applied across the source and drain contacts. In case I, transport is possible as the electrochemical potential $\mu_n$ is within the transport windows set by the Fermi levels in source and drain leads. When increasing the gate voltage to proceed to case II, $\mu_n$ is lowered and electron tunneling is prohibited due to Coulomb blockade. Continuing to case III, the electrochemical potential of the n+1 state ($\mu_{n+1}$) is...
Chapter 1. Introduction

Figure 1.3: (a) Schematic charge stability diagram of a quantum dot, showing the so called Coulomb diamonds. The QD population increases by one between consecutive diamonds. (b) Energy level configuration of the QD for a few positions within the diamonds of (a).

lowered inside the bias window and transport is again possible. Excited states of the QD will give rise to additional lines in the charge stability diagram; this is discussed in detail in section 4.5.

1.4 Double quantum dots

We now proceed to describe double quantum dots (DQDs). Again, this section only gives a quick overview of the subject; for a more complete review of the physics of double quantum dot systems, see Ref. [4]. The properties of DQDs are also treated in chapter 6.

A serial DQD contains two tunnel-coupled quantum dots connected in series. We describe the system using the capacitive model shown in Fig. 1.4. The two gates are used to shift the electrochemical potentials of the QDs and to control their electron population. Due to capacitive cross-talk, the voltage on gate 1 will also influence the electrochemical potential of QD2 and vice versa. We have not included the capacitive coupling between the leads and the DQD; this leads to a shift of the DQD potentials when changing the DQD bias. The influence can be minimized by applying the DQD bias symmetrically across the source and drain leads. The QDs are tunnel coupled with a coupling energy $t$. In addition, there is a capacitive coupling $C_{QD1-QD2}$ between the two QDs; this gives rise to the mutual charging energy $E_{Cm}$. The mutual charging energy leads to an increase of the electrochemical potential for QD1 by $E_{Cm}$ when an electron is added to QD2, and vice versa.

The energy spectrum of the DQD depends strongly on the strength of the interdot tunnel coupling $t$. If the coupling energy is low compared to the single-level spacing in the individual QDs, we can simplify the situation by only considering the
1.4. Double quantum dots

Figure 1.4: Capacitive model of a double quantum dot. The QDs are tunnel coupled to source and drain, the interdot tunneling is characterized by a coupling energy \( t \). The two gates control the DQD electron population. The gate designed to shift the electrochemical potential of QD1 also has a capacitive coupling to QD2 and vice versa. In addition there is capacitive coupling between the two QDs. For simplicity, we have not included the capacitive coupling between the leads and the DQD.

In this case, the DQD spectrum is obtained by combining the spectra of the individual QDs while taking the mutual charging energy into account. On the other hand, if tunnel coupling is strong, the states in the individual QDs hybridize and form delocalized states extending over both QDs. In the extreme limit where \( t \) becomes comparable to the charging energies, we approach the case of a single QD where the electron wavefunctions distribute over the whole structure.

To visualize how the various DQD parameters influence the DQD energy spectra and the electron population, it is instructive to draw the charge stability diagrams for different DQD parameters. This is done in Fig. 1.5, showing the DQD population plotted versus voltages on the two gates. The bias across the DQD is assumed to be zero. The numbers in brackets denote the DQD population, the blue lines correspond to charge transitions in QD1 while red lines represent transitions in QD2. In Fig. 1.5(a), the two QDs are completely decoupled and there is no capacitive cross-talk between Gate 1 and QD2 or Gate 2 and QD1. Changing the voltage on Gate 1 thus only influences the population of QD1 and vice versa for Gate 2/QD2.

In Fig. 1.5(b), we turn on the capacitive cross-talk between the gates and the QDs. This leads to a slanting of the charge transition lines; the slopes represent the differences in the capacitive lever arms of the two gates, with the ratio given by \( C_{G2-QD2}/C_{G1-QD1} \) for QD1 and \( C_{G2-QD2}/C_{G1-QD1} \) for QD2. In Fig. 1.5(c), we include the capacitive coupling \( C_{QD1-QD2} \) between the two QDs. This gives a shift of the DQD energy at the crossing points of the transition lines; the size of the shift is equal to the mutual charging energy \( E_{Cm} \). The points where charge transitions
Figure 1.5: (a-d) Charge stability diagrams of a DQD, visualizing the influence of various DQD properties. The numbers in brackets denote the DQD charge population; the blue lines correspond to the addition of electrons in QD1, the red lines represent filling of QD2. In (a), the two QDs are decoupled and their is no cross-coupling between Gate 1 and QD2 or Gate 2 and QD1. In (b), the cross-coupling is turned on, which gives the slope of the charge transition lines. In (c), there is also a capacitive coupling between the QDs, leading to shifts of the DQD energy at crossing points of the charge stability lines. The shift is equal to the mutual charging energy $E_{Cm}$. Finally, the sketch in (d) depicts the influence of the tunnel coupling energy $t$. Close to the triple points, the states in QD1 and QD2 hybridize and form bonding and antibonding molecular states extending over both QDs. This leads to an anticrossing of the charge transition lines.

lines meet up are called triple points; in such configurations three DQD populations are degenerate.

Next, we look at the influence of the interdot tunnel coupling $t$. Figure 1.5(d) shows a magnification around two triple points in Fig. 1.5(c), with the tunnel coupling energy set to $t = E_{Cm}/2$. In addition to the shift due to the mutual charging energy, the tunnel coupling induces an anticrossing at the position of the triple points. Here, electrons are delocalized over both QDs. The lower branch repre-
sents the bonding \( (n+m+1) \)-electron state, while the upper branch belongs to the antibonding \( (n+m+2) \)-electron state.

Finally, when applying a finite bias across the DQD, the charge transition lines shift due to the change of Fermi levels in source and drain. This gives rise to triangle-shaped regions of electron transport around the triple points. In the triangle around the lower triple point, the current through the DQD is described by an *electron* cycle. For the upper triple point, the DQD contains an additional electron and the transport is more conveniently expressed in terms of a *hole* cycle. Finite bias measurements may also be used to perform transport spectroscopy of excited states in the DQD. This is studied in greater detail in chapter 6.
Chapter 2
Sample fabrication and measurement setup

In this chapter we describe the sample fabrication and give a brief overview of the electronic setup. The measurement equipment is conveniently controlled from a computer. For this purpose we have developed a computer program for automating measurement sequences and for providing convenient handling of the measured data. The software is designed to be generally applicable and is therefore not limited to a particular measurement setup. At the end of the chapter we take a look at the data acquisition methods and signal processing tools necessary for performing time-resolved charge detection measurements.

2.1 AFM-lithography

Most of the samples presented in this thesis (structures A-C in appendix A) were fabricated by local oxidation [5, 6] of a heterostructure surface using the tip of an atomic force microscope (AFM). The principle of the technique is sketched in Fig. 2.1. We start by increasing the humidity inside the AFM to allow a water film to form on the surface of the semiconductor. The relative humidity is typically held around 40%; higher humidity generally leads to higher but broader oxide lines. When the AFM tip approaches the surface, a water bridge is formed between the tip and the heterostructure. When applying a negative voltage to the tip ($V_{\text{tip}} \sim -20 \, \text{V}$), a current will flow through the semiconductor and oxidize the AlGaAs in a small region below the tip. By moving the tip around we can create oxide lines on the heterostructure surface. The oxide lines are typically around 12 – 16 nm high and extend approximately the same distance into the bulk of the semiconductor. The change in material composition leads to depletion of the 2DEG at the positions of the oxide lines. In this way we cut the 2DEG into separate conducting parts and may create nanostructures like quantum dots or quantum point contacts. The surrounding regions of the 2DEG are used as in-plane gates. The breakdown voltage between two neighboring regions of the 2DEG is typically in the range 300 – 400 mV,
Chapter 2. Sample fabrication and measurement setup

Figure 2.1: (a) Setup used for performing AFM-lithography. By applying a negative voltage to the tip, we can locally oxidize the semiconductor surface. This depletes the 2DEG at the position of the oxide line. (b) Artistic view of the same process.

the exact value depends strongly on the height and the width of the oxide line.

2.2 Measurement electronics

The main part of the thesis considers time-resolved detection of single electrons using a quantum point contact. In order to measure electrons as fast as possible, we need a sensitive detector combined with an electronic setup allowing large measurement bandwidths with low electrical noise. The sensitivity of the charge detector depends strongly on the sample; this is considered in detail in chapter 3. In this section we describe the experimental setup and estimate the noise and the bandwidth limitations set by the electrical circuit.

We model the system using the circuit shown in Fig. 2.2(a). The QPC is described as a variable resistor $R_S$. The capacitance $C_C$ represent the $\sim 2\text{ m}$ of cables connecting the sample to the room-temperature electronics; as we will see later, the cable capacitance is the main limiting factor for operating the system at high bandwidths. The current through the resistor $R_S$ is measured with a current-to-voltage (I-V) converter. The I-V converter is implemented using an operational amplifier (OPA 627) in feedback mode [7], with feedback resistor $R_F$. The output voltage signal is sent through an 8-th order 80 kHz low-pass filter and afterwards digitized with a data acquisition card (National Instruments PCI 6035-E) or a fast oscilloscope (LeCroy Wavepro 7000). The 80-kHz filter serves as an anti-aliasing filter for the data acquisition card, which has a maximum sampling rate of 200 kS/s. The setup is similar to the one described in Ref. [8]. The I-V converter used in the actual experiment is more complex than shown in the sketch in Fig. 2.2(a); the bias voltage
2.2. Measurement electronics

Figure 2.2: (a) Schematic drawing of the circuit model used for estimating the noise. The sample is modeled as a resistor $R_S$ in parallel with the capacitance $C_C$ of the cables in the cryostat. The various noise sources under consideration are marked by the yellow boxes. (b) Measured frequency response of the I-V converter when driving the input with a fixed current. The bandwidth is set by a low-pass filter formed by the feedback resistor $R_F$ and the capacitor $C_F$.

is applied symmetrically to both sides of the sample and the real converter allows the potential of the sample to be shifted relative to ground. However, the principle of operation and the limitations are well described by the model in Fig. 2.2(a).

2.2.1 Bandwidth

We start by looking at the bandwidth of the setup. From a first glance, one would expect the bandwidth to be limited by a low-pass RC-filter formed by the cable capacitance $C_C$ and the feedback resistor $R_F$. For typical values of $R_F$ and $C_C$, this would give a bandwidth of only a few tens of Hz. However, if the gain $A$ of the op-amp is large, we operate the circuit in feedback mode and the low-frequency input impedance of the I-V converter becomes equal to $R_F/A$. This allows the bandwidth to be increased considerably. In our setup, the feedback resistor is shunted by a small capacitance $C_F$. The capacitor is used to compensate for the capacitive noise gain (described later in this section) and allows stable operation of the converter [9]. The shunt capacitance $C_F$ also forms an RC-filter together with $R_F$; this filter sets the bandwidth of the converter. This is true for frequencies much lower than the unity gain bandwidth of the amplifier, which is 16 MHz for the OPA 627.

In Fig. 2.2(b), we measure the bandwidth of the converter for different feedback resistors. The experiment is performed by driving the input with a fixed AC-current and detecting the response with a lock-in amplifier. Whenever the feedback resistor is increased by a factor of ten, the bandwidth is lowered by the same factor. This
Another important property of the setup is the electronic noise. If the noise is too high, the bandwidth used in the actual measurement may have to be reduced compared to the limits set by the electrical circuit. This turns out to be the case for most experiments presented in this thesis. Noise in general is characterized by a power spectral density $S(f)$ (unit W/Hz) describing the power contents of the fluctuations per frequency interval. Since we are mostly interested in measuring currents, we choose to describe the noise using a current spectral density $S_I(f)$ (unit A/√Hz), with $S_I^2(f) \propto S(f)$. The fluctuations may also be expressed as a voltage spectral density $S_V(f)$ (unit V/√Hz). In the following, we list the main noise sources in the system. The magnitudes of the different noise contributions are visualized in Fig. 2.3, calculated using parameters relevant for our setup (see caption of Fig. 2.3).

QPC shot noise. This is the fundamental current noise and arises because the
current is carried by discrete charge carriers. In the low-frequency region of interest here, the power spectrum is flat and scales linearly with the magnitude of the current. We have \( S_I = \sqrt{D(1 - D)2eI} \), where \( D \) is the transmission coefficient of the QPC.

**Thermal noise** or *Johnson-Nyquist* noise is generated by the thermal agitation of the charge carriers in a conductor and appears regardless of applied voltage. The thermal noise is given by \( S_I = \sqrt{4k_B T G} \), where \( G \) is the conductance of the conductor. In the model of Fig. 2.2(a), thermal noise appears over both the sample resistance \( R_S \) and the feedback resistance \( R_F \). Since the sample is held at very low temperatures, its thermal noise becomes negligible. However, the feedback resistor is operated at room temperature and its thermal noise may therefore become appreciable.

**Amplifier noise.** An operational amplifier is characterized by equivalent voltage and current noise sources appearing at its input. For the amplifier used here (OPA 627), the current noise is of the order of \( S_I \sim 1.6 \text{fA/}\sqrt{\text{Hz}} \) and can be neglected compared to other noise contributions. On the other hand, the voltage noise plays a major role. In the frequency range of interest it amounts to \( S_V \sim 5 \text{nV/}\sqrt{\text{Hz}} \). To convert this value to current noise, we need to calculate the voltage noise at the output of the amplifier and scale the result with the feedback resistor. The voltage noise at the output is given by

\[
S_{\text{amp}}^\text{out}(f) = \left(1 + \frac{Z_F}{Z_L}\right) S_{\text{amp}}^V, \tag{2.1}
\]

where \( Z_F = 1/(1/R_F + i2\pi fC_F) \) is the complex impedance of the feedback circuit and \( Z_L = 1/(1/R_S + i2\pi fC_C) \) is the impedance of the load. The result of Eq. (2.1) is scaled with the feedback resistor and plotted in Fig. 2.3. To understand the shape of the curve we start in the low-frequency region \((f < 1 \text{kHz})\). At these frequencies the capacitances in the system do not allow a current to flow and the voltage appearing at the output is set by the purely resistive voltage divider formed by \( R_S \) and \( R_F \). As the frequency is increased above \( f > 1/(2\pi R_S C_C) \) (corresponding to a few kHz in our system), the impedance of the capacitance \( C_C \) becomes lower than \( R_S \) and the voltage noise \( S_{\text{amp}}^V \) will start to drive a noise current through \( C_C \). This current is converted to a voltage noise at the output over the feedback resistor. Since the impedance of the capacitance goes down with frequency, the current and thereby the output voltage noise will increase with frequency. The effect is called *capacitive noise gain* and is clearly visible in Fig. 2.3. To limit the noise gain, the amplifier contains an extra capacitance \( C_C \). This leads to a decrease of the gain at \( f > 1/(2\pi R_F C_F) \) and serves to stabilize the operation of the amplifier [9]. At the same time the bandwidth of the I-V converter starts to limit the output.
Chapter 2. Sample fabrication and measurement setup

An important property of noise is that contributions from independent sources add up incoherently, so that

\[ S_{I}^{\text{tot}}(f) = \sqrt{S_{1}^{2}(f) + S_{2}^{2}(f) + S_{3}^{2}(f) + \ldots} \quad (2.2) \]

This means that if one noise source is slightly stronger than the others, it will dominate the total spectrum. For the case of the noise shown in Fig. 2.3, we expect the voltage noise of the amplifier in combination with cable capacitance to dominate the spectrum. The capacitive noise gain is a major problem for performing high-bandwidth charge detection measurements; to get around the issue one may reduce the capacitance by shortening the cable and mounting a cryogenic amplifier much closer to the sample \[10\]. A different approach is to incorporate the QPC into an rf-circuit using impedance matching techniques \[11–13\].

2.2.3 Measured noise spectrum

Finally, we take a look at the experimentally obtained noise spectrum, measured at the output of the setup. Figure 2.4(a) shows the current noise of the setup with the QPC connected, measured for two different feedback resistors. The capacitive noise gain is clearly visible in both curves. The increase is the same for both feedback resistors, while the bandwidth and the corresponding cut-off in the capacitive gain appears at a lower frequency for \( R_F = 10 \, \text{M}\Omega \). At low frequencies, the noise is higher for \( R_F = 1 \, \text{M}\Omega \) due to larger thermal current noise in the feedback resistor. The curve for \( R_F = 1 \, \text{M}\Omega \) corresponds to the calculations shown in 2.3; the measured and calculated curves are in reasonably good agreement, especially in the low-frequency regime. For higher frequencies, there is an unexpected increase in noise at \( f \approx 20 \, \text{kHz} \) and the bandwidth is slightly lower than expected from Fig. 2.3. The discrepancies could possible come from the limited gain and bandwidth of the amplifier [9]. The spikes appearing at high frequencies are due to external noise sources; these can in principle be avoided by proper shielding and avoiding ground loops. At \( f = 80 \, \text{kHz} \) there is a strong cut-off due to the external anti-aliasing filter.

In Fig. 2.4(b), we plot the noise spectrum for \( R_F = 10 \, \text{M}\Omega \) but measured with and without feed-through filters mounted between the sample and the I-V converter at the top of the cryostat. The purpose of the feed-through filter is to minimize the noise entering the cryostat, but it will also introduce an extra capacitance of around 1 nF parallel to the cable capacitance. The difference is clearly seen in Fig. 2.4(b); without the filters, the capacitive noise gain is much weaker than with the filters mounted. Operating the setup without filters thus provides a possibility to lower the noise and increase the bandwidth. On the other hand, without filters external noise can more easily couple into the cryostat. This effect is also visible in Fig. 2.4(b); the spikes due to external noise are considerably stronger without the filters mounted. Moreover, noise at even higher frequencies (GHz) may enter the cryostat and lead to a higher electron temperature in a setup without filters.
2.3 Measurement software

Almost all measurements described in this thesis involve the investigation of some quantity (current through the sample, microwave intensity, etc) as a function of one or several external parameters (gate voltages, magnetic field, temperature, etc). Such experiments are conveniently performed using a computer; the computer can communicate with the experimental equipment and is thus capable of generating a sequence of output signals and measure the corresponding response of the device under test. These kind of measurement procedures are very general and commonly practiced in most areas of science and engineering. In order to standardize our measurements, we decided to develop a computer program for setting up and organizing measurements, providing a common platform for instrument communication and for handling the measured data. The software was written in the LabVIEW programming environment from National Instruments. To keep the measurement software generally applicable, it was designed to fulfill the following criteria:

Modularity Since the measurement procedure described above is generic, the part of the software used for defining and executing a measurement sequence should not depend on the instruments used in a particular experiment. This makes it possible to split the software into a main part responsible for the measurement configuration and data handling and into several smaller parts responsible for the actual instrument communication.

Expandability The measurement requirements and the experimental equipment
change continuously. Therefore, it is important to (1) allow easy integration of new hardware and (2) keep the software architecture flexible enough to allow the implementation of new measurement procedures.

**Data storage** The handling and especially storage of measured data requires some planning. Log files containing raw data without any reference to properties of the measured signals may lead to confusion and in worst cases to mixing-up of data. Since the computer is capable of communicating with the measurement hardware, it is possible for the software to automatically retrieve relevant instrument settings such as integration time, dynamic range, bandwidth, etc. In addition, by allowing the user to add comments to the log files the software can be used as an electronic logbook.

### 2.3.1 Realization

With the requirements listed in the previous section kept in mind, we developed the software Step&Log. In the following, we give a short description of the program. This section should neither be considered as a user manual nor as a technical description of the architecture of the program; instead we give a general introduction to the central concepts of the software. For information about how to extend the program with new instrument drivers, see Appendix C.

The software is built around the concept of *channels*, with each channel representing a single physical quantity in the measurement. A typical channel could be the voltage applied to a gate, the frequency of an ac-voltage or a measured dc current. The channel consists of two data constructs; one *signal* part describing the physical properties of the channel and one *instrument* part describing the hardware used for measuring or generating the signal. Some typical data fields for the signal- and instrument constructs are listed in Fig. 2.5(a); in addition we have given a concrete example of a channel describing a dc-current measured using a current-to-voltage converter and a digital multimeter. Note that the data fields in the signal construct are the same for all channels, whereas instrument construct only contains two common data fields, namely ”Instr. type” and ”gpib”. The remaining data fields depend on the details of the particular experimental equipment. Again, the reason for splitting the channel into two parts is that we can handle the signal part without making any reference to the hardware used for measuring it. This makes it possible to generalize the software.

We now describe the procedure for setting up and performing a measurement. After hooking up the sample to the experimental setup, the user needs to configure all required channels in the measurement program. This may seem unnecessarily complex and cumbersome at first, but it only needs to be performed once at the very beginning when configuring the experimental setup. The configuration provides several advantages at later stages in the measurement phase. Most notably, for each measurement performed the software keeps track of the values and configurations of
all equipment and saves the information into the measurement log file.

We distinguish between input and output channels, describing signals being either measured or generated by the program. They are described by the same kind of data construct, but handled separately by the measurement program. Figure 2.5(b) shows the user interface used for configuring an input channel, filled out with the same parameters as in table in Fig. 2.5(a). For the example in Fig. 2.5, we have configured a channel called "QPC current", containing signal information about the physical unit (Ampere), measured unit (Voltage), conversion factors and the instrument to use for acquiring the signal (HP 34401 Multimeter). Note that we only specify what kind of instrument to use; the actual instrument configuration is performed in another subprogram specific for that experimental hardware. The fields named "output limits" and "sweep rates" are only applicable to channels describing outputs.

After configuring the channels, we are ready to perform a measurement. Figure 2.6 shows the user interface of the measurement configuration window, in this particular case configured for performing a measurement on a quantum dot. The window consists of three lists; the bottom list contains all input channels, the middle list contains all output channels. In the uppermost list, we define which of the output channels to change during the measurement. In the example of Fig. 2.6, we have defined a two-dimensional sweep involving a gate voltage ("DownGate") and
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Figure 2.6: Main user interface for defining measurements. The channels are separated into inputs and outputs. The step list defines which output channels to sweep during the measurement sequence.

the magnetic field. From a conceptual point of view, the channels are equivalent and independent of the physical quantities they represent. This means that we can easily define multidimensional sweeps using any combination of output channels belonging to any kind of instrument or physical quantity. Moreover, it is possible to define mathematical relations between the output channels to allow two or more output channels to be swept in parallel.

Finally, we are ready to perform the measurement. The program steps through the points in the sweep list, applies the sweep values to the corresponding output channels and records the values of all input channels at each measurement point. Figure 2.7 shows a screen-shot of the user interface during the measurement phase. The acquired data is updated into the log graph (left window of Fig. 2.7), while the measurement sequence is monitored and controlled from the control panel (right window). The resulting data matrix together with all configuration details are saved into a log file. In addition to the measurement program, we have developed a...
2.4. Time-resolved measurements

In the previous section we have described measurements involving stepping some output channels while monitoring the single-valued response of the input channels. The time-resolved measurement techniques needed for detecting single-electron tunneling involves a slightly more complex scheme. The method of counting electrons essentially relies on the ability to detect transitions between two fixed levels in the current flowing through a quantum point contact. Each transition corresponds to an electron entering or leaving the structure under investigation. The details of the technique of charge detection using a quantum point contact is described in detail in chapter 3; here we focus on the requirements for a software package used to automate the counting procedure.

To be able to count electrons, one needs to acquire time traces of the QPC current with a sampling rate higher than the typical tunneling times of the electrons. From such a time trace, we need to perform some basic signal processing, extract quantities relevant for the measurement (for example counting transitions in the QPC current corresponding to tunneling of an electrons) and finally save those quantities to a log file. Moreover, we also need to save the raw time traces to disk, since it in general turns out to be difficult to optimize the algorithm for detecting electrons without beforehand knowing the properties of the measured data. By keeping the raw data,
Chapter 2. Sample fabrication and measurement setup

the signal processing and the electron detection algorithm can always be performed again at a later point.

2.4.1 Realization

Since we want to detect electrons as a function of external parameters like gate voltages or magnetic field, it is desirable to incorporate the electron-detection procedures into the Step&Log program used for performing low-frequency, single-valued measurements. This is accomplished by realizing the high-frequency software as a virtual instrument driver for the main program. When Step&Log calls the high-frequency driver for counting electrons, the driver will acquire a time trace, perform the data analysis and at the end return one or a few single-valued quantities to Step&Log. The time traces are only handled internally by the high-frequency driver and thus remain invisible to Step&Log. Second, it is important to keep the high-frequency routines modular and expandable. At the start of an experiment, it is often difficult to know what kind of data processing or acquisition hardware that will become necessary at a later stage of the measurement. Therefore, all routines involving high-frequency data (acquisition, signal processing, data analysis) have been incorporated into separate modules that are independent of each other. The purposes of the main high-frequency instrument driver is to (1) serve as a platform for setting up and configuring the modules, (2) transfer data between the modules and (3) provide a link to the Step&Log program.

In Fig. 2.8 we show an example of a sequence of modules used for acquiring, processing and analyzing QPC conductance traces. The time traces before and after performing a certain module are plotted in both time- and frequency domain to illustrate the module’s functionality. The particular scheme of Fig. 2.8 consists of the following five modules:

**Acquire data** Data is acquired using the experimental hardware. In our case we used either a data acquisition card or a digital oscilloscope. For the data shown in Fig. 2.8 we acquired data from a Lecroy oscilloscope using a sampling rate of $f = 1 \text{ MHz}$ for a time duration of $T = 0.2 \text{s}$. The sharp decrease in the frequency spectrum at 80 kHz corresponds to a hardware 8th-order 80 kHz-lowpass filter mounted between the output of the amplifier and the input of the oscilloscope.

**Filtering** Depending on the ratio between the signal and the noise (S/N), one may need to perform extra software filtering to reduce the noise and enhance the visibility of the transitions in the QPC current traces. For the data shown in Fig. 2.8, the S/N is rather good and filtering is not strictly necessary. Still, we have incorporated a 6th order 40 kHz low-pass filter to illustrate the procedure.

**Resample** After low-pass filtering, the part of the spectrum above the filter bandwidth does not contain any useful information and can be discarded. This is
2.4. Time-resolved measurements

Figure 2.8: Typical signal processing scheme used for acquiring, processing and evaluating QPC conductance traces. The plots to the left shows the time domain signal before and after performing the operation of an individual module, the plots to the right show the corresponding data in the frequency domain. The frequency domain data is plotted in log-log scale.

done by resampling the data at a new, lower sampling rate. According to the Nyquist sampling theorem, we need to sample the data with at least twice the bandwidth of the filter. The procedure reduces the amount of experimental data which speeds up the analysis processes and reduces the demands on computer memory and hard disk space. For the data in Fig. 2.8, the resampling was performed at $f = 100 \text{kHz}$. As a consequence, the spectrum is cut at $f/2 = 50 \text{kHz}$. However, since the data was already filtered at a lower fre-
frequency, the information contents in the discarded data is small and the time
domain signal remains essentially unaffected by the process.

**Save to disk** In this step, the trace is saved into a binary file on the hard disk. Each trace is given a unique reference number so that data can be identified if it becomes necessary to do more processing at a later point. The files with raw data may quickly become very large; a typical two-dimensional sweep consisting of \(200 \times 200\) time traces of length \(T = 0.5\) s sampled at \(f = 100\) kHz and saved with 8-bit resolution creates a log file with 2 Gigabyte of data.

**Count events** Finally, we perform the actual counting of electrons by analyzing the time traces. For this we need to identify the two QPC current levels; this is done by creating a histogram of the data and looking for peaks (see Fig. 2.8). When the levels have been found, the module extracts the transitions and returns physically interesting quantities like number of counts/s, position of the two levels, tunnel rates, S/N, etc.

The procedure described above is preferably performed in real-time during the measurement. This puts some requirements on the computer hardware, but it turns out that the workload is within the capabilities of an ordinary modern computer. Using a desktop computer equipped with an Intel Core 2 Duo 2.13 GHz processor and acquiring data with a sampling rate of 100 kHz, the time needed for data processing and writing data to disk corresponds to roughly half the acquisition time.

Figure 2.9 shows a screen-shot of the user interface used for configuring the high-frequency instrument driver. The list in the upper-left corner of the figure shows the active modules. The list below displays the output parameters of the modules; for this particular case we see that the ”Count events”-module locates the two QPC current levels at \(I_{QPC} = 14.67\) nA and \(I_{QPC} = 21.84\) nA and counts 400 transitions from the lower to the upper level. The graphs to the right are configured to show the time- and frequency domain data before and after the module selected in the upper-left list (in this case the filter module); this provides a convenient tool when developing new modules as well as for manual inspection of the data. The small free-floating window shows the configuration panel for the filter module; the module is configured to apply an 8th-order low-pass Bessel filter.
2.4. Time-resolved measurements

Figure 2.9: User interface for the instrument driver handling high-frequency data in Step&Log. The upper-left list shows the active modules, while the lower list gives the output values of the modules. The small free-floating window is the configuration panel for the filter module. The program is configured so that the two graphs show the signal before and after the filter operation in both time- (upper graph) and frequency domain (lower graph).
Chapter 3

Charge detection with a quantum point contact

A quantum point contact is a narrow constriction connected to well-conducting leads. The width of the constriction is comparable to the electron wavelength, allowing only for a few modes to transmit electrons. The conductance of the quantum point contact depends strongly on its electrostatic surroundings. This may be utilized to detect charge fluctuations close to the constriction with single-electron resolution. In this chapter, we show how to operate the quantum point contact as a charge detector and investigate how to optimize the device to obtain the best charge sensitivity.

3.1 The quantum point contact

The quantum point contact (QPC) is the electron analogue of a photon waveguide. Since the width of the constriction is of the order of the electron wavelength, constructive and destructive interference only allow electron wavefunctions corresponding to standing waves in the directions of the confinement. Due to the Fermionic nature of electrons, each mode within the QPC carries a fixed conductance of $G_0 = e^2/h$. The conductance of a QPC with $N$ available modes is thus equal to [14]

$$G = N G_0,$$

with $N$ integer. The effect is called conductance quantization. If the measurement is performed in the absence of magnetic fields, the electron spin states are degenerate and the conductance quantization appears in units of $2e^2/h$ instead of $e^2/h$.

We are concerned with quantum point contacts (QPC) formed in a two-dimensional electron gas (2DEG). For such structures, the confinement in growth direction is usually much stronger than in the lateral direction. In the following, we assume the part of the electron wavefunction in the growth direction to be in its ground state and consider additional modes only in the lateral direction. Quantum point contacts may be fabricated using a variety of methods, for example by depleting the 2DEG.
Figure 3.1: (a) Quantum point contact, defined by etching trenches (marked in blue in the figure) in a GaAs heterostructure containing a 2DEG 37 nm below the surface. A quantum dot defined in an InAs nanowire (purple) is lying on top of the structure. (b) Conductance of the QPC versus voltage applied to the 2DEG. The measurement was performed in a two-terminal setup, with $V_{\text{QPC-SD}} = 200 \mu V$ applied across the QPC. A series resistance of 4 k$\Omega$ was subtracted because of the ohmic contact resistance. The measurement was performed at a temperature of $T = 1.7$ K.

by applying negative voltages to metallic gates put on the heterostructure surface [14, 15]. Here we investigate structures formed by etching or by local oxidation of the heterostructure surface.

Figure 3.1(a) shows an example of a QPC defined by etching; the blue areas are etched trenches separating the QPC from the rest of the 2DEG. The parts of the 2DEG marked by L and R are used as in-plane gates to control the electrostatic width of the QPC. The purple object in the figure is a nanowire lying on top of the surface, electrically isolated from the QPC. The nanowire contains a quantum dot (QD), sitting directly on top of the QPC. The QD is formed in the same etching step as the QPC, which ensures perfect alignment between the two devices [16].

In Fig. 3.1(b), we plot the conductance of the QPC, measured when shifting the voltage on the part of the 2DEG connected to the QPC ($V_{\text{2DEG}}$) and keeping the other contacts grounded. Making $V_{\text{2DEG}}$ more positive has the same effect as making the surrounding gates more negative, leading to pinch-off of the QPC. As $V_{\text{2DEG}}$ is lowered, the constriction opens up to allow the first electron mode to populate the QPC. Further lowering $V_{\text{2DEG}}$ makes more modes available and the conductance increases stepwise.
3.2 Charge detection

From Fig. 3.1(b) it is clear that the QPC conductance depends strongly on the confinement potential $U_{QPC}(\vec{r})$. When operating the QPC in the region between pinch-off and the first plateau ($\sim 375$ mV in the figure), a small perturbation $\delta U_{QPC}(\vec{r})$ leads to a large change in conductance $\delta G$. If a QD is placed in close vicinity to the QPC, we expect a fluctuation $\delta q$ in the QD charge population to shift the QPC potential $U_{QPC}(\vec{r})$ and thus give rise to a measurable change in QPC conductance. A figure of merit for using the QPC as a charge detector is then

$$\frac{\delta G}{\delta q} = \left(\frac{\delta G}{\delta U_{QPC}(\vec{r})}\right) \times \left(\frac{\delta U_{QPC}(\vec{r})}{\delta q}\right).$$

(3.2)

The first factor describes how the conductance changes with energy, which depends strongly on the operating point of the QPC. The second factor describes the electrostatic coupling between the QD and the QPC. In the following sections, we will investigate how to optimize the two factors in order to reach the best charge sensitivity.

To simplify the problem, we describe the system using the capacitive model shown in Fig. 3.2. The coupling between QD and QPC is then determined by the capacitances in the system. The QPC is modeled as a single barrier with potential $\mu_{QPC}$. Equation (3.2) simplifies to

$$\frac{\Delta G}{\Delta q} = \frac{\Delta G}{\Delta \mu_{QPC}} \frac{\Delta \mu_{QPC}}{\Delta q}.$$

(3.3)
Chapter 3. Charge detection with a quantum point contact

Figure 3.3: Current in the QPC and the QD for the structure shown in Fig. 3.1(a), measured vs voltage on the 2DEG. A second gate is used to keep the potential of the QPC roughly constant during the sweep. As the voltage of the 2DEG is lowered, electrons are unloaded from the QD. At each transition there is a corresponding increase of the QPC conductance. At the same gate voltages, sequential tunneling gives rise to peaks in the QD current.

An additional electron on the QD will induce a shift of the QPC potential

$$\Delta \mu_{\text{QPC}} = e^2 \frac{C_{\text{QPC:QD}}}{C_{\text{QD:QD}}} \frac{1}{C_{\text{QPC:QD}} + C_{\text{QPC:G}} + C_{\text{QPC:S}} + C_{\text{QPC:D}}}.$$  \hspace{1cm} (3.4)

Here, $C_{\text{QD:QD}}$ specifies the total capacitance between QD and its environment. The first factor states how well a charge on the QD couples to the QPC, the second factor describes how the charge is screened by the surrounding gates. From Eq. (3.4), we see that the potential shift $\Delta \mu_{\text{QPC}}$ may be increased by either fabricating the QPC and QD close to each other (increasing $C_{\text{QPC:QD}}$), or by avoiding screening of the QD and the QPC (by making $C_{\text{QD:QD}}$ or $C_{\text{QPC:G}} + C_{\text{QPC:S}} + C_{\text{QPC:D}}$ small). How this can be achieved in practice is discussed in section 3.3.2.

Figure 3.3 displays simultaneous measurements of QPC and QD currents for the structure of Fig. 3.1(a). As the gate voltage is lowered, electrons are unloaded from the QD and the QD current shows clear Coulomb peaks at each charge transition. At the same time, the QPC conductance changes in steps at the positions of the Coulomb peaks. The QPC was voltage biased with $V_{\text{QPC-SD}} = 200 \mu V$ and operated between pinch-off and the first plateau. The QPC conductance is kept roughly constant during the sweep by applying a compensation voltage to the side gate marked by L in Fig. 3.1(a).
3.2. Charge detection

Figure 3.4: (a) Current through a QD, measured with bias voltage $V_{\text{QD--SD}} = 10 \mu \text{V}$. For low gate voltages, the tunneling couplings between the QD and the leads are too small to allow the current to be detected with a conventional current meter. (b) Conductance of the QPC, measured for the same region as in (a). Even for the regions of (a) where the current is too low to be measurable, the QD transitions are clearly visible in the charge detector signal. The measurement was performed with sample A (see appendix A).

The charge detection method has some advantages compared to a standard current measurement. A conventional current meter has a resolution of $\sim 10 \text{ fA}/\sqrt{\text{Hz}}$, meaning that the tunneling rates of a QD must be kept larger than $\Gamma > 10 \text{ fA}/e \sim 60 \text{ kHz}$ for reasonable integration times. Moreover, in order to measure current through the QD it needs to be hooked up to two leads. On the other hand, a charge detector can measure electron tunneling which occurs on much slower timescales as well as detect equilibrium fluctuations between a QD and a single lead. Figure 3.4 shows a comparison of the two measurement methods. The measurement was performed on a QD-QPC structure fabricated by local oxidation (see sample A in appendix A). In Fig. 3.4(a), we plot the current through a QD as a function of two gate voltages tuning both the QD population and the opening of source and drain barriers. Coulomb peaks are visible for high gate voltages, but for lower voltages the current signal is lost as the source and drain barriers are made more opaque. At the same time, the charge detector signal [Fig. 3.4(b)] shows a regular pattern of Coulomb resonances for the whole range of gate voltages in the figure. The additional lines with slope $\Delta V_{\text{G1}}/\Delta V_{\text{G2}} \neq -1$ are due to charge traps in close vicinity to the QPC. These are treated in more detail in section 3.4.
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Figure 3.5: (a) Time trace of the current through the quantum point contact, showing a few transitions due to electrons tunneling into and out of the QD. The two traces are taken with 10 kHz and 50 kHz bandwidth. (b) Blow-up of one switching event from the trace in (a). The rise time is clearly different for the data taken with 10 kHz and 50 kHz bandwidth. (c) Histogram showing the distribution of the current for the data in (a). The two levels are easily distinguished.

3.3 Time-resolved operation

If the tunnel couplings between the QD and the leads are tuned below the QPC measurement bandwidth, charge transitions may be detected in real-time. This allows a wealth of experiments to be performed, like investigating single-electron dynamics or probing interactions between charge carriers in the system. Figure 3.5(a) shows a measurement of the QPC current, measured in a configuration where the coupling between the QD and the source lead is below 1 kHz, and the other lead is completely pinched off. The QPC current shows two levels corresponding to \( n \) and \( (n + 1) \) electrons on the QD, with transitions between the two levels occurring on a millisecond timescale. We postpone the detailed investigation of single-electron tunneling in quantum dots to chapters 4 and 5; here we focus on how to optimize the QPC in order to perform the best possible charge detection measurement.

The time resolution available for detecting charge transitions as seen in Fig. 3.5(a) is set directly by the bandwidth of the QPC measurement circuit. On the other hand, increasing the bandwidth also increases the noise in the measurement, leading to a trade-off between noise and bandwidth. The effect is visualized in Fig. 3.5, where
the two curves show the same set of data but filtered with different bandwidths, 10 kHz (black) and 50 kHz (red). The filtering was performed numerically with a 6th-order Bessel low-pass filter. In Fig. 3.5(b), we zoom in on one of the transitions of Fig. 3.5(a). The data taken with lower bandwidth shows a considerably slower time response than the trace taken with higher bandwidth. The lower-bandwidth filter also introduces a time offset; this is not a major problem since we are interested in determining the time intervals between transitions rather than the absolute transition times. In Fig. 3.5(c), we plot the distribution functions for the two traces shown in Fig. 3.5(a). The distributions contain two peaks associated with the two QPC current levels. The distance between peaks gives directly the change in QPC current ($\Delta I_{\text{QPC}}$) for one electron entering the QD, while the standard deviation of the current distribution $p(I)$ around each peak ($I_{\text{noise}}$) reflects the amount of noise in the measured signal. As a consequence of the increased bandwidth for the red trace, the data contains noise contributions from a broader frequency spectrum and the peaks in the distribution function become significantly broader.

### 3.3.1 Signal-to-noise

The ratio between the change in current $\Delta I_{\text{QPC}}$ and the noise $I_{\text{noise}}$ is conveniently expressed as a signal-to-noise (S/N) ratio. To maximize the useful information that can be extracted from the measurement, we need to maximize the signal and minimize the noise. In this subsection we consider the effects of the noise, in the following sections we describe how to optimize the signal by tuning the operation point of the QPC.

The noise of the QPC signal can be separated into intrinsic and extrinsic contributions. With intrinsic noise we refer to noise generated by the QPC itself, while extrinsic noise is due to amplifiers and other external noise sources. It turns out that the main source of noise in the setup is given by amplifier noise. Since the noise is extrinsic, it is essentially independent of both operating point and biasing conditions of the QPC. The only way to reduce this noise is to use an amplifier with lower noise figures or to reduce cable capacitances. The amplifier noise spectrum is not flat and depends on the details of the setup (see section 2.2); a rough estimate for the noise contribution in the relevant frequency range is $\sim 400 \text{ fA}/\sqrt{\text{Hz}}$.

The fundamental intrinsic noise of the QPC current is the shot noise, which arises due to the fact that the current is carried by discrete charged particles. The shot noise has a flat power spectrum in the region of interest which scales linearly with the magnitude of the current. For typical currents used here ($\sim 10 \text{ nA}$), the white noise power is $\sim 30 \text{ fA}/\sqrt{\text{Hz}}$, which is considerably lower than the amplifier noise. The thermal noise or Johnson-Nyquist noise is generated by the thermal agitation of the charge carriers in a conductor and appears regardless of applied voltage. Another form of intrinsic noise arises because of fluctuations of trapped charges close to the QPC. The charge traps sit at lattice defects or at the heterostructure surface and may be activated by a large current passing the QPC. Such noise is
usually referred to as burst noise or popcorn noise. In GaAs QDs, it is believed that the 1/f-noise is generated by fluctuations in an ensemble of charge traps distributed uniformly in the device [17]. The magnitude of the noise depends on the quality of the heterostructure and on the abundance of traps close to the QPC. As we will see later in this chapter, this kind of noise becomes a major contribution at high QPC current levels.

The noise is described by a power spectral density $S(\omega)$, which depends on the physical process responsible for generating the fluctuations. The amplitude of the current noise in a trace as shown in Fig. 3.5 is given by integrating the spectral density over the measurement bandwidth

$$I_{\text{noise}} \sim \sqrt{P_{\text{noise}}} = \left( \int_{0}^{2\pi f_{\text{BW}}} S(\omega) \, d\omega \right)^{1/2}.$$  (3.5)

Here, $P_{\text{noise}}$ is the noise power and $f_{\text{BW}}$ the measurement bandwidth. If we assume for simplicity the spectrum to be independent of frequency ($S(\omega) = \text{const.}$), then the current noise scales with the square root of the bandwidth,

$$I_{\text{noise}} \sim \sqrt{f_{\text{BW}}}.$$  (3.6)

Increasing the bandwidth thus increases the noise and lowers the S/N, as visualized in Fig. 3.5. A single-electron detector must be able to reliably detect transitions between the two levels in the QPC current. How much can the bandwidth and the noise be increased before the detection mechanism becomes unreliable? A qualitative answer would be when $I_{\text{noise}}$ is comparable to the step height $\Delta I_{\text{QPC}}$. To investigate the issue quantitatively, we need to estimate the probability of detecting false transitions due to the noise. The problem is well understood in the language of information theory [18]; here we make a simplified analysis to get a quick estimate of the risk of detecting false counts.

For this purpose, we assume the distribution of the QPC current $p(I_{\text{QPC}})$ to be Gaussian around each of its two levels and evaluate the part of the distribution deviating by more than $\Delta I_{\text{QPC}}/2$ from the peak value,

$$p_{\text{out}} = \int_{\Delta I_{\text{QPC}}/2}^\infty p(I_{\text{QPC}}) \, dI_{\text{QPC}}.$$  (3.7)

This fraction is beyond the midline between the two peaks of the distribution $p(I_{\text{QPC}})$ and gives rise to false counts. The number of false counts $n_{\text{false}}$ registered during a time interval $\Delta t$ is equal to $p_{\text{out}}$ multiplied with the number of measurements performed in the interval, which according to sampling theorem needs to be $n_{\text{meas}} \sim 2 \Delta t f_{\text{BW}}$. For the false counts, we get

$$n_{\text{false}} = p_{\text{out}} n_{\text{meas}} \sim 2 p_{\text{out}} \Delta t f_{\text{BW}}.$$  (3.8)

In Fig. 3.6(a) we plot $p_{\text{out}}$ as a function of S/N. As a consequence of the Gaussian distribution, the risk of detecting false counts falls off stronger than exponential with
increased S/N. Figure 3.6(b) shows the risk of detecting a false count, calculated using Eq. (3.8) with $\Delta t = 1 \text{s}$ and $f_{BW} = 10 \text{kHz}$. For S/N=7, we find that the detector will register an average of four false counts per second.

### 3.3.2 Tuning the QPC operation point

As mentioned in section 3.2, the performance of the charge detector depends strongly on the operation point of the QPC. The best sensitivity is expected when the QPC is tuned to the steepest part of the QPC conductance curve [see Fig. 3.1(b)]. This corresponds to maximizing the factor $\Delta G/\Delta \mu_{\text{QPC}}$ in Eq. (3.3). In Fig. 3.7(a) we plot the conductance change $\Delta G$ for one electron entering the QD versus QPC conductance, in the range between pinch-off and the first conductance plateau ($0 < G_{\text{QPC}} < 2e^2/h$). The change $\Delta G$ is maximal around $G_{\text{QPC}} \sim 0.4 \times 2e^2/h$ but stays fairly constant over a range from 0.3 to 0.6 $\times 2e^2/h$. The dashed line in Fig. 3.7(a) shows the numerical derivative of $G_{\text{QPC}}$ with respect to gate voltage. The maximal value of $\Delta G$ coincides well with the steepest part of the QPC conductance curve. The inset in the figure shows how the conductance changes as a function of gate voltage.

In Fig. 3.7(b), we plot the relative change in conductance $\Delta G/G_{\text{QPC}}$ for the same set of data. Surprisingly, the relative change increases monotonously with decreased conductance, reaching above 50% at $G_{\text{QPC}} = 0.02 \times 2e^2/h$. To be able to understand these results, we consider the energy dependence of the QPC conductance in the limit $G_{\text{QPC}} \ll 2e^2/h$. Here, transport can be described as electrons tunneling through a potential barrier. The tunneling probability is expected to depend exponentially on the barrier height and therefore also on QPC potential (see section 4.6),

$$G_{\text{tunneling}} = a \exp[b \mu_{\text{QPC}}].$$

(3.9)
Figure 3.7: (a) Change of QPC conductance as one electron enters the QD, measured at different values of average QPC conductance. The dashed line is the numerical derivative of the QPC conductance with respect to gate voltage. The change is maximal at $G_{QPC} = 0.4 \times 2e^2/h$, which coincides with the steepest part of the QPC conductance curve [see inset in (b)]. (b) Relative change of QPC conductance for one electron entering the QD, defined as $(G_{\text{high}} - G_{\text{low}})/G_{\text{high}}$. The relative change increases with decreased $G_{QPC}$, reaching above 50% at $G_{QPC} = 0.02 \times 2e^2/h$. The inset shows the variation of $G_{QPC}$ as a function of gate voltage.

Here, $a$ and $b$ are constants. To estimate the charge sensitivity, we use Eq. (3.3) together with the derivative of Eq. (3.9),

$$\frac{\Delta G_{\text{tunneling}}}{\Delta q} = \frac{\Delta G_{\text{tunneling}}}{\Delta \mu_{QPC}} \frac{\Delta \mu_{QPC}}{\Delta q} = a b \exp[b \mu_{\text{QPC}}] \frac{\Delta \mu_{QPC}}{\Delta q}. \quad (3.10)$$

Both the conductance and the charge sensitivity have the same exponential dependence on QPC potential. If we form the relative conductance change by combining Eqs. (3.9-3.10), the energy dependence cancels out,

$$\frac{\Delta G_{\text{tunneling}}}{G_{\text{tunneling}}} = b \frac{\Delta \mu_{QPC}}{\Delta q}. \quad (3.11)$$

The result thus predicts the relative conductance change to be independent of QPC potential (and independent of $G_{QPC}$) in the limit $G_{QPC} \ll 2e^2/h$. This is in dis-
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The relative change in QPC conductance in this particular device is extraordinarily large compared to top-gate defined structures, where the change $\Delta G$ is typically around one percent $[8, 12]$. We attribute the large sensitivity to the close distance between the QD and QPC and to the absence of metallic gates on the heterostructure surface, which reduces the screening.

3.3.3 Changing the QPC bias

In the previous subsection we investigated how to optimize the QPC operating point in order to maximize the charge sensitivity. The results of Fig. 3.7 show that the optimal operation point is different depending on if we want to maximize the absolute or the relative conductance change. Which one to choose depends on the details of the experimental setup and on the properties of the noise in the system.

For our setup, the conductance is measured by applying a fixed bias voltage
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Figure 3.9: (a) Change of QPC current as one electron tunnels out of the QD, measured vs bias voltage over the QPC. The two curves correspond to different operating points of the QPC. The current step saturates at $V_{\text{QPC-SD}} \sim 4 \text{ mV}$. (b) Noise of the QPC current vs QPC bias, measured with 20 kHz bandwidth. The noise was extracted from histograms of time traces as shown in Fig. 3.5. (c) Noise of the QPC current plotted versus $I_{\text{QPC}}$. The dotted line shows the extrinsic noise level of the setup. (d) Signal-to-noise ratio (S/N) vs QPC bias. The maximal S/N is achieved at $V_{\text{QPC-SD}} \sim 2.4 \text{ mV}$.

$V_{\text{SD}}$ across the QPC and monitoring the current. In the linear response regime, both the average current $I_{\text{QPC}}$ and the step height $\Delta I_{\text{QPC}}$ scale linearly with applied bias. If the noise of the QPC current is dominated by extrinsic noise sources that are independent of the QPC operating point and the applied bias, it is better to optimize $\Delta G$ rather than $\Delta G/G$ and operate the QPC close to $G_{\text{QPC}} = 0.5 \times 2e^2/h$. At this point, the signal $\Delta I_{\text{QPC}} = \Delta G_{\text{QPC}}V_{\text{SD}}$ is maximized relative to the fixed noise level. On the other hand, if the current noise is intrinsic and increases with $I_{\text{QPC}}$, both the noise and the signal will go up with applied bias. If the noise increases faster than the signal, it becomes more important to maximize the relative conductance change. Here it might be preferable to operate the QPC close to pinch-off.

In Fig. 3.9(a), we plot the absolute current step $\Delta I_{\text{QPC}}$ versus bias, measured at two different QPC operating points. Both curves first increase linearly with applied bias, but show deviations from linear behavior at higher bias voltages. The data taken at $G_{\text{QPC}} = 0.05 \times 2e^2/h$ displays considerably lower signal $\Delta I_{\text{QPC}}$, as expected.
from the more pinched-off configuration. Figure 3.9(b) shows the noise extracted from the two sets of data, extracted by evaluating the standard deviation of the current distribution \( p(I_{QPC}) \) around one of its peaks. Surprisingly, the noise depends heavily on applied bias, staying flat only up to \( V_{SD} = 0.4 \text{ mV} \) for \( G_{QPC} = 0.2 \times 2e^2/h \) and \( V_{SD} = 1 \text{ mV} \) for \( G_{QPC} = 0.05 \times 2e^2/h \). We attribute the increase in noise to charge traps in the heterostructure or on the surface close to the QPC. The traps are activated by the current flowing in the QPC and lead to fluctuations of the QPC potential, which transform into current noise. It should be noted that the intrinsic QPC shot noise is expected to be orders of magnitude lower than the noise seen in Fig. 3.9(b).

In Fig. 3.9(c), we plot the noise versus QPC current for data corresponding to the low-bias range of Fig. 3.9(b). The dotted line indicates the noise contribution from the experimental setup. The noise has almost the same current dependence at both operating points, which indicates that the fluctuations are triggered by the current in the QPC and not by the applied bias. The charge traps are investigated in more detail in section 3.4.

Coming back to the question of the optimal operating point of the QPC, we plot the S/N versus applied bias for the two different configurations [Fig. 3.9(d)]. At low QPC bias, the noise level stays relatively constant and the data taken at \( G_{QPC} = 0.2 \times 2e^2/h \) gives a considerably higher S/N than the data taken at \( G_{QPC} = 0.05 \times 2e^2/h \). For higher bias the noise increases with QPC current and the difference in S/N between the two operating points is decreased. The optimal S/N is reached at around \( V_{SD} = 2.4 \text{ mV} \) for both operating points. However, at high bias voltages the current in the QPC may excite electrons in the QD and thus exert a back-action on the measured system [see chapter 7]. For such reason the practically usable bias voltage may be much smaller.

Finally, we note that both the step height \( \Delta I_{QPC} \) and the noise at the two different operating points approach the same values at the largest bias (\( V_{SD} > 3 \text{ mV} \)). Most likely, in this regime the bias voltage exceeds the level separation of the QPC, meaning that several QPC channels are contributing to the transport.

3.3.4 Current biasing the QPC

In the previous section we saw that the best S/N for a voltage-biased QPC is reached when the QPC is operated in a regime where the step height \( \Delta G_{QPC} \) is maximized. The situation is different if we compare the S/N for different operating points while adjusting the QPC bias to keep the average current \( I_{QPC} \) constant. This resembles current biasing the QPC, which is the measurement method to use if one wants to keep a fixed current through the QPC rather than a fixed bias voltage. As seen in Fig. 3.9(c), the noise scales with current in roughly the same way at both QPC operating points. If the average current is put to the same value in the two configurations, the step height \( \Delta I_{QPC} \) is given by the relative change \( \Delta G_{QPC}/G_{QPC} \). Since this quantity is larger at the more pinched-off operation point [see Fig. 3.7(b)],
we expect that configuration to give the larger S/N.

The S/N plotted versus QPC current is plotted in Fig. 3.10, extracted from the same data as in Fig. 3.9. For low $I_{\text{QPC}}$, the pinched-off configuration indeed shows the better S/N. For larger QPC currents, there is a crossover and the more open configuration gives the better S/N. The result demonstrates a difficulty with current biasing: to apply a large current when $G_{\text{QPC}} \ll 2e^2/h$, it is necessary to increase the bias over the QPC. This drives the QPC out of the linear regime at a lower current compared to when $G_{\text{QPC}} \sim 0.5 \times 2e^2/h$, which causes the increase in noise and the decrease in S/N seen in Fig. 3.10. The same argument applies to the detector back-action due emission of photons; as discussed in chapter 7, higher bias voltages over the QPC increase the energy range of the emitted radiation and thus makes a larger number of QD states available for excitation. The measurement presented in the following chapters were performed with voltage biasing and with the QPC conductance in the range $G_{\text{QPC}} \sim (0.2 - 0.6) \times 2e^2/h$.

3.4 Charge traps in the vicinity of the QPC

In the previous sections, we argued that the excess noise in the QPC current is due to charge fluctuations in traps in the vicinity of the QPC [17]. If the trap is close enough and if the fluctuations occur on a timescale slower than the measurement bandwidth, the charge dynamics of the individual traps can be investigated using the time-resolved charge detection methods. By comparing the conductance change $\Delta G_{\text{trap}}$ due to charge fluctuations in a trap with the conductance change $\Delta G_{\text{QD}}$ due to an electron in the QD, we get an idea of the position of the trap relative to the QD. The trap position may be further pinned down by checking the influences of
Weak gate dependence

Figure 3.11: Count rates for a single QD, measured vs voltage on the two gates G1 and G2. Apart from the transitions due to electrons tunneling into and out of the QD, there are several other lines present in the figure. These origin from charge traps sitting in the substrate close to the QPC. Such events can be distinguished from tunneling in QD by investigating the change of the QPC conductance or looking at how the switches depend on gate voltages. The boxes describe some of the transitions, where $\alpha_{G1}/\alpha_{G2}$ is the capacitive lever arms of the gates relative to the trap.

Figure 3.11 shows electron counts registered by the QPC charge detector for a QD-QPC structure defined by local oxidation (sample A in appendix A). The two voltages $V_{G1}$ and $V_{G2}$ are applied to gates to the left and right of the structure that have roughly the same capacitive lever arms ($\alpha_{G1}/\alpha_{G2} \sim 1$) on the QD states. The lines with slope $\Delta V_{G1}/\Delta V_{G2} \sim -1$ in Fig. 3.11 all give the same $\Delta G_{QPC}$ and thus belong to tunneling in the QD. In the lower-left region of the graph the tunneling in the QD disappears due to pinch-off of the QD leads.

Various other lines are seen in the plot; their gate voltage dependences and their influence on the QPC conductance are given in the figure. Traps with $\alpha_{G1}/\alpha_{G2} > 1$ are situated closer to gate G1, traps with $\alpha_{G1}/\alpha_{G2} < 1$ are closer to gate G2. The trap with $\alpha_{G1}/\alpha_{G2} = 4.8$ seen to the left in the graph is probably relatively close to the QD; the lines from the trap and the lines from the QD anticross due to their mutual charging energy, similar to a double quantum dot system. Almost all traps give a smaller $\Delta G_{QPC}$ compared to the QD, showing that the major influence on QPC conductance still originates from the QD. We note that the method only shows traps where the charge fluctuates on timescales slower than the measurement...
Figure 3.12: Same as Fig. 3.11, this time measured for a QPC defined by etching. The switches due to traps are more frequent than in the AFM-defined sample, possibly because of surface states formed in the etched trenches. The data was also taken at a higher electron temperature \( T = 1.7 \text{ K} \) instead of 200 mK.

bandwidth; traps with faster fluctuations will give an overall increase of the noise floor.

It is not clear whether the charge traps are formed inside the heterostructure or if they are sitting on the surface. In Fig. 3.12 we present a measurement similar to the one shown in Fig. 3.11, but this time for a QPC defined by etching (see Fig. 3.1(a) or sample D in appendix A). This sample shows a greater trap density compared to the structure defined by local oxidation used in Fig. 3.11. The difference could be due to the fabrication method; the etching procedure will bring surface states closer to the QPC. For a structure defined by local oxidation, the surface is kept further away. On the other hand, it is dangerous to draw too far-going conclusions from the two sets of data; the structures were fabricated on different (but similar) wafers, and the measurement of Fig. 3.12 was performed at a higher electron temperature \( T = 1.7 \text{ K} \) compared to \( T = 200 \text{ mK} \). Further experiments are necessary to clarify the issue.
Chapter 4

Time-resolved single-electron detection

In this chapter, we show how time-resolved charge-detection is used to investigate properties of electron transport in a single quantum dot. We start with describing the dynamics of electron tunneling between one lead and a single QD state, before moving on to more complex situations involving multiple leads, finite bias, excited states and degenerate states. Finally, we show how the potential landscape forming the tunnel barriers is influenced by changing the gate voltages.

4.1 Sample and experimental setup

The sample investigated in this chapter is shown in Fig. 4.1(a). It consists of a QD [dotted circle in Fig. 4.1(a)] and a nearby QPC. The charging energy of the QD is 2.1 meV and the mean level spacing is 200 – 300 µeV. From the geometry and the characteristic energy scales, we estimate that the QD contains about 30 electrons. The QD is connected to source and drain leads through tunnel barriers. The transparency of the tunnel barriers is controlled by changing the voltage on gates G1 and G2. In the experiment, we tune the tunnel coupling rates between the QD and the leads to below 10 kHz. This allows electron tunneling to be detected in real-time with the low-bandwidth (∼30 kHz) detector. The P gate is used to tune the conductance of the QPC to a regime where the sensitivity to changes in the QD charge is maximal. As described in chapter 3, the voltage on gate P is adjusted to keep the QPC in the region of maximum sensitivity whenever changing the voltage on another gate. The measurements were performed in a dilution refrigerator with a base temperature of 60 mK.
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Figure 4.1: (a) Quantum dot with integrated charge read-out investigated in this chapter. (b) Schematic drawing depicting the electrochemical potentials of the system. By making the barrier between source and the QD very opaque, electron tunneling is only possible between the QD and the drain lead. (c) Current through the QPC as a function of time, showing a few electrons tunneling into and out of the QD. The lower current level corresponds to a situation where the QD holds one excess electron. Transitions between the two levels occur whenever an electron enters or leaves the QD. The quantities $\tau_{\text{in}}$ and $\tau_{\text{out}}$ specify the time it takes for an electron to tunnel into and out of the dot, respectively.

4.2 Electron tunneling with one lead connected to the quantum dot

First, we investigate the case of electron tunneling between a QD and one lead. This is achieved by keeping the drain barrier open but making the source barrier very opaque, allowing electron tunneling only between the QD and the drain lead [Fig. 4.1(b)]. Coulomb blockade prohibits the QD to hold more than one excess electron. When an electron enters the QD, the conductance through the QPC is reduced due to the electrostatic coupling between the QD and the QPC. As the electron leaves, the QPC conductance returns to the original value. This gives rise to a QPC current switching between two levels, as shown in Fig. 4.1(c). The low level corresponds to a situation where the dot holds an excess electron. Transitions between the two levels occur whenever an electron enters or leaves the QD. The duration between transitions gives directly the time it takes for an electron to tunnel...
4.2. Electron tunneling with one lead connected to the quantum dot

into or out of the QD. In Fig. 4.1(c), these times are marked by $\tau_{\text{in}}$ and $\tau_{\text{out}}$.

In the regime of single-level transport, the process of an electron tunneling into or out of the dot is described by the rate equation

$$\dot{p}_{\text{in/out}}(t) = -\Gamma_{\text{in/out}} \times p_{\text{in/out}}(t). \quad (4.1)$$

Here, $p_{\text{in/out}}(t)$ is the probability density for an electron to tunnel into or out of the dot at a time $t$ after a complementary event. Solving the differential equation and normalizing the resulting distribution gives

$$p_{\text{in/out}}(t) \, dt = e^{-\Gamma_{\text{in/out}} t} \times \Gamma_{\text{in/out}} \, dt. \quad (4.2)$$

The tunneling rates $\Gamma_{\text{in/out}}$ in Eqs. (4.1, 4.2) are effective rates involving the dot-lead tunnel coupling $\Gamma$ and the thermal population of the states in the lead, with

$$\Gamma_{\text{in}} = \Gamma \times f(\Delta \mu/k_B T) \quad (4.3)$$

$$\Gamma_{\text{out}} = \Gamma \times (1 - f(\Delta \mu/k_B T)). \quad (4.4)$$

Here, $f(x) = 1/(1 + \exp(x))$ is the Fermi distribution function, $T$ is the electron temperature in the lead and $\Delta \mu$ is the energy difference between the electrochemical potential of the QD and the Fermi level in the lead. Equations (4.3-4.4) are valid in a small range around $\delta \mu = 0$ where the tunnel coupling $\Gamma$ can be assumed to be independent of energy and gate voltages. The gate-voltage influence on the tunnel coupling is investigated in greater detail in section 4.6. Also, Eqs. (4.3-4.4) assume the QD state to be non-degenerate. In the case of degenerate states, the rates should be multiplied with the appropriate degeneracy factor. Here, we assume non-degenerate states and postpone the discussion of degenerate states to section 4.7.

The method of time-resolved charge detection makes it possible to test the validity of the model described in Eqs. (4.1-4.4). The tunneling rates $\Gamma_{\text{in}}, \Gamma_{\text{out}}$ are determined directly from time traces such as the one shown in Fig. 4.1(c). Using Eq. 4.2, we find

$$\Gamma_{\text{in}} = 1/\langle \tau_{\text{in}} \rangle, \quad \Gamma_{\text{out}} = 1/\langle \tau_{\text{out}} \rangle, \quad (4.5)$$

where $\langle \tau_{\text{in}} \rangle$ and $\langle \tau_{\text{out}} \rangle$ are the average tunneling times extracted from the time trace. It should be noted that the expression in Eq. (4.5) is valid only for an infinite-bandwidth detector, and that the finite bandwidth of the detector leads to a systematic under-estimation of the actual rates. However, knowing the bandwidth makes it possible to correct for the deviations [20]. The influence of the detector bandwidth is discussed in greater detail in section 5.4.

Combining Eqs. (4.3-4.5) gives an expression for the Fermi function

$$f(\Delta E/k_B T) = \langle \tau_{\text{out}} \rangle / (\langle \tau_{\text{in}} \rangle + \langle \tau_{\text{out}} \rangle) = \langle n_{\text{excess}} \rangle, \quad (4.6)$$

with $\langle n_{\text{excess}} \rangle$ being the average excess charge on the QD. The average dot population can be determined by monitoring the average conductance of the QPC [21]. By
Chapter 4. Time-resolved single-electron detection

Figure 4.2: (a) Average dot population versus voltage on gate G2. The fit shows the Fermi distribution function with $T = 230$ mK. (b) Counts per second for the same data as in (a). The data was fit to Eq. (4.7), giving $\Gamma = 9.2$ kHz and $T = 230$ mK. (c) Tunneling rates for electrons entering (squares) and leaving (circles) the QD, extracted from the same set of data as in (a, b). The solid lines are the results of Eqs. (4.3-4.4) in the text, with $\Gamma = 9.2$ kHz and $T = 230$ mK. (d) Distribution of tunneling times for electrons entering (squares) and leaving (circles) the QD, extracted at $V_{G2} = -96.3$ mV [marked by arrow in (c)]. The solid lines show the exponential behavior given by Eq. (4.2) in the text, with $\Gamma_{\text{in}} = 1/(\tau_{\text{in}}) = 7.2$ kHz, $\Gamma_{\text{out}} = 1/(\tau_{\text{out}}) = 2.0$ kHz. The length of the time trace for the data shown in the figure is 0.5 s.

Adding time resolution to the detector and counting electrons one by one as they enter the QD, we can extract not only the Fermi function of the lead but also the tunnel coupling $\Gamma$. Assuming sequential tunneling and using Eqs. (4.3-4.4), we find that the rate for electrons entering the dot $r_E$ is given by

$$r_E = 1/(\langle \tau_{\text{in}} \rangle + \langle \tau_{\text{out}} \rangle) = \Gamma \times f(1 - f).$$

(4.7)

Measuring the count rate $r_E$ thus directly determines the tunnel coupling $\Gamma$.

In Fig. 4.2(a, b) we plot the average QD population and the number of counts per second as gate G2 was used to change the electrochemical potential of the QD. The solid lines are the fits to Eq. (4.6) and Eq. (4.7), demonstrating the good agreement between the data and the expected relations. By first determining the lever arm between gate G2 and the dot from standard Coulomb diamond measurements [1],
4.3 Electron tunneling with two leads connected to the quantum dot

it was possible to extract the electronic temperature \( T = 230 \text{ mK} \) from the width of the Fermi function. The same temperature was found by checking the width of standard Coulomb blockade current peaks \cite{1}, measured with the QD in a more strongly coupled regime.

The time-resolved detection method also allows the tunneling rates \( \Gamma_{\text{in}} \) and \( \Gamma_{\text{out}} \) to be determined separately. The rates are plotted in Fig. 4.2(c), extracted from the same set of data as shown in Fig. 4.2(a, b). The solid lines are fits to Eqs. (4.3-4.4), with \( \Gamma = 9.2 \text{ kHz} \) and \( T = 230 \text{ mK} \). The figure clearly demonstrates an exponential falloff of the tunneling rates as the QD electrochemical potential is shifted above or below the Fermi level of the lead. This is a direct consequence of the Fermi distribution for the electrons in the lead. The fact that both \( \Gamma_{\text{in}} \) and \( \Gamma_{\text{out}} \) can be fitted with a single tunneling rate \( \Gamma \) shows that the QD state is non-degenerate. This is not always the case, as will be seen in section 4.7.

The results presented so far rely on the assumption that Eq. (4.2) is correct. The validity of this assumption can be tested by extracting the experimental distribution function \( p_{\text{in/out}}(t) \) of tunneling times \( \tau_{\text{in}}, \tau_{\text{out}} \) from a time trace containing a large number of events. Such distributions are shown in Fig. 4.2(d), taken at the position marked by the arrow in Fig. 4.2(c). The data exhibit the expected exponential behavior of Eq. (4.2), with dashed lines being fits with \( \Gamma_{\text{in}} = 7.2 \text{ kHz} \) and \( \Gamma_{\text{out}} = 2.0 \text{ kHz} \).

The measurements presented so far only involve tunneling between the QD and one lead. These tunneling events are due to equilibrium fluctuations and do not give rise to a net current through the QD. Consequently, it is impossible to investigate such effects with conventional current measurement techniques. This demonstrates the power of time-resolved charge detection methods for probing properties of mesoscopic structures. The overall good agreement between Eqs. (4.2-4.4) and the results of Fig. 4.2 makes us confident that the model of single-electron tunneling is well capable of describing the system. Next, we move on to the case where the QD is connected to two leads.

4.3 Electron tunneling with two leads connected to the quantum dot

In order to perform time-resolved measurements of electron transport through the dot, the tunnel barriers have to be symmetrized so that both give similar tunneling rates. The rates must be kept lower than the bandwidth of the setup, but still high enough to give good statistics. Figure 4.3(a) shows the number of events per second as a function of the two gates voltages \( V_{G1} \) and \( V_{G2} \). In the upper left corner of the figure, \( V_{G1} \) is high and \( V_{G2} \) is low, corresponding to the case where the source lead is open and the drain lead is closed. In the bottom right corner, the configuration is inverted. For the region in between, marked by the ellipse in Fig. 4.3(a), the data indicate that both leads are weakly coupled to the dot.
Figure 4.3: (a) Counts per second versus $V_{G1}$ and $V_{G2}$. For low values of $V_{G1}$ and $V_{G2}$, both the source lead and the drain lead are pinched off. For high voltages, the barriers open up so that tunneling occurs on a timescale faster than the measurement bandwidth. (b) Temperature (squares) and tunnel coupling (crosses), extracted from data shown within the ellipse in (a). As $V_{G2}$ is increased, $V_{G1}$ is decreased, in order to keep the dot at a constant potential. For low $V_{G2}$, tunneling occurs between the source lead and the dot, for high $V_{G2}$, the electrons tunnel between the drain and the dot. For intermediate gate values, both leads contribute to the tunneling. The electron temperature was found to be the same for both leads, within the accuracy of the experimental data.

For zero voltage bias across the QD, the measurement method does not enable us to distinguish whether an electron that tunnels into the dot arrives from the left or from the right lead. Therefore, when both leads are connected to the dot, the rates in Eqs. (4.3-4.4) must be adjusted to contain one part for the left lead and one part for the right lead,

$$
\Gamma_{in} = \Gamma_{in}^L + \Gamma_{in}^R = \Gamma_L f_L + \Gamma_R f_R,
\Gamma_{out} = \Gamma_{out}^L + \Gamma_{out}^R = \Gamma_L (1 - f_L) + \Gamma_R (1 - f_R). \tag{4.8}
$$

Here, $f_L$ and $f_R$ are the Fermi distribution functions of the left and the right lead, respectively. Using Eq. (4.8), we calculate the rate of events for the case when both leads are coupled to the dot with rates accessible for the detector,

$$
r_E = \frac{[\Gamma_L f_L + \Gamma_R f_R][\Gamma_L (1 - f_L) + \Gamma_R (1 - f_R)]}{\Gamma_L + \Gamma_R}. \tag{4.9}
$$

With no bias applied across the dot, the two distributions functions $f_L$ and $f_R$ are identical except for a possible difference in electronic temperature in the two leads. However, assuming $T_L = T_R = T$, we have $f_L = f_R = f$, and Eq. (4.9) simplifies to $r_E = (\Gamma_L + \Gamma_R) \times f (1 - f)$. Fitting this expression to curves similar to that shown in Fig. 4.2(b), we extract the temperature and combined tunneling rate $\Gamma_L + \Gamma_R$ from the data within the ellipse of Fig. 4.3(a). The result is presented in Fig. 4.3(b). The
4.4 Finite bias

With the barriers properly symmetrized, we apply a finite bias voltage between source and drain leads and measure electron transport through the QD. Figure 4.4(a) shows Coulomb blockade diamonds measured by counting electrons entering the QD. The bias is applied symmetrically, with

$$\mu_S = |e|V_{SD}/2, \quad \mu_D = -|e|V_{SD}/2.$$  \hspace{1cm} (4.10)

The gate $G_1$ is used as a plunger gate to control the dot electrochemical potential. However, the gate also strongly affects the source tunnel barrier. For low $G_1$ voltages, the source lead is closed, giving strong charge fluctuations only when the drain lead is in resonance with the dot [see case I in Fig. 4.4(a, b)]. At higher gate voltages, the source lead opens up and a current can flow through the dot. In point II of Fig. 4.4(a), the QD electrochemical potential $\mu_n$ lies within the bias window but far away from the thermal broadening of the Fermi distribution in the leads. The condition can be expressed as

$$|\pm eV/2 - \mu_n| \gg k_BT,$$  \hspace{1cm} (4.11)

where the "+" case refers to the source contact and the "-" case refers to the drain. Whenever Eq. (4.11) is fulfilled, electrons can only enter the dot from the source lead and only leave through the drain. This makes it possible to determine the individual tunnel couplings $\Gamma_S/\Gamma_D$, with

$$\Gamma_S = \Gamma_{in} = 1/\langle \tau_{in} \rangle, \quad \Gamma_D = \Gamma_{out} = 1/\langle \tau_{out} \rangle.$$  \hspace{1cm} (4.12)

In this regime, we measure the current through the dot by counting events. This opens up the possibility to use the QD as a very precise current meter for measuring sub-fA currents [22, 23]. Since the electrons are detected one by one, the noise and higher order correlations of the current can also be experimentally investigated. This is explained in more detail in section 5.2.

When the bias exceeds the dot charging energy, $E_C \sim 2.1$ meV, and the electrochemical potentials of the $(n+1)$ and the $(n+2)$ states are within the bias window [see case III of Fig 4.4(a,b)], transport processes are allowed where the dot may contain 0, 1 or 2 excess electrons. A time trace measured at point III of Fig.
Figure 4.4: (a) Coulomb diamonds, measured by counting electrons entering the QD. For low values of $V_{G1}$, the source lead is pinched off and tunneling can only occur between the dot and the drain lead. As $V_{G1}$ increases, the source lead opens up and a current can flow through the dot. (b) Diagrams depicting the energy levels of the dot at points I, II and III. In case III, the bias is higher than the charging energy of the dot, meaning that the dot may hold 0, 1 or 2 excess electrons. (c) Time trace taken at point III. The three possible dot populations ($n$, $n+1$ or $n+2$ electrons) are clearly resolvable.

4.4(a) is shown in Fig. 4.4(c). The sensitivity of the QPC charge detector allows to measure switching between three different levels, corresponding to ($n$), ($n+1$) and ($n+2$) electrons on the dot. It is not possible to make this distinction in a standard current measurement.

4.5 Excited states

If there are excited states inside the bias window, tunneling may occur into any of the available states. In this regime, the rates $\Gamma_S$ and $\Gamma_D$ of Eq. (4.12) will not be the tunneling rates of the single ground state but rather a sum of rates from all states contributing to the tunneling process. A further complication with excited states is that there may be equilibrium charge fluctuations between the lead and the excited state, thereby removing the unidirectionality of the electron motion. However, if the relaxation rate of the excited state into the ground state is orders of magnitude faster than the tunneling out rate, the electron in the excited state will have time
4.5. Excited states

Figure 4.5: (a) and (b): Blow-up of the upper left region of Fig. 4.4(a), showing the rates for electrons tunneling into (a) and out of (b) the QD, respectively. The while solid lines mark the positions where the source lead is lining up with the electrochemical potential of the QD ground state. The dashed lines mark the lower edge of the region where condition of Eq. (4.11) in the text is fulfilled. The color scales are different for the two figures, the rate for tunneling out is roughly 10 times faster than tunneling in. (c) Diagram depicting the energy levels along the dashed lines in (a) and (b). As the source lead is raised [corresponds to going upward along the dashed lines in (a)], excited states become available for tunneling. (d) Energy diagram for the configuration marked by the arrow in (b). Here, the excited states is visible in the rate for electrons tunneling out of the QD. (e) Tunneling rate for electrons entering the dot, measured along the dashed line in (a). Three excited states are clearly resolvable.

to relax to the ground state before equilibrium fluctuations can take place.

The separate rates $\Gamma_{in}$ and $\Gamma_{out}$ for a close-up of the upper-left region of Fig. 4.4(a) are plotted in Fig. 4.5(a, b). It is important to note that the requirement of Eq. (4.11) is met only for the region along and above the dashed lines in the figures. At the lower left end of the dashed lines, the energy levels of the dot are aligned as shown in Fig. 4.5(c). Going diagonally upward along the lines corresponds to raising the Fermi level of the source lead, while keeping the energy difference between the dot and the drain lead fixed.

Starting at low bias and low voltage on the gate $V_{G1}$, the dot is in the Coulomb blockade regime, and no tunneling is possible. Following the dashed line upwards, the QD ground state becomes available for tunneling at $V_{bias} = 0.3$ mV. The transition is marked by the white solid lines in Fig. 4.5(a, b). At these low gate
voltages, the source tunnel barrier is almost completely pinched off, meaning that the rate for electrons entering the QD is still low [Fig. 4.5(a)]. Even so, some electrons do enter the QD, as can be seen from the few points of measurements of $\Gamma_{\text{out}}$ within the corresponding region of Fig. 4.5(b).

We first concentrate on the tunneling-in rate in Fig. 4.5(a). As the source level is further raised, excited states become available for transport. The first excited state (at $V_{\text{bias}} = 0.85$ mV along the dashed line) is more strongly coupled to the lead than the ground state, giving a tunneling rate of $\sim 70$ Hz for electrons entering the dot. The large difference in the tunneling-in rate between the ground and the excited state can be understood if the wavefunctions of the ground and excited state have different spatial distributions. If the overlap with the lead wavefunction is larger for the excited state, the tunneling rate will also be larger. Similar differences in tunneling rates have been found between the singlet and triplet states in a two-electron dot [24, 25].

By further raising the source level, tunneling can also occur through a second excited state. The measured tunneling-in rate will now be the sum of the rates from both excited states; by subtracting the contribution from the first state, the rate for the second state can be determined. Using this method, we can resolve three excited states, with excitations energies $\epsilon_1 = 0.55$ meV, $\epsilon_2 = 1.0$ meV, $\epsilon_3 = 1.3$ meV and with tunneling rates $\Gamma_1 = 70$ Hz, $\Gamma_2 = 190$ Hz, $\Gamma_3 = 190$ Hz. The excited states are clearly seen in Fig. 4.5(d), which is a cut along the dashed diagonal line in Fig. 4.5(a).

Focusing on the rates for electrons tunneling out of the QD [Fig. 4.5(b)], there is a noisy region where the ground state but no excited states are within the bias window ($0.3 < V_{\text{bias}} < 0.85$ mV along the dashed line). In this regime, few electrons will enter the dot, meaning that the statistics needed for measuring the rate of electrons leaving the dot is not sufficient. However, for bias voltages higher than the first excited state, the tunneling-out rate remains constant along the dashed line. This is in contrast to the steps seen in the tunneling-in rates, indicating that the rate for tunneling out of the QD does not depend on the state used for tunneling into the QD. Since the individual excited states are expected to have different rates also for tunneling out of the dot, the data is consistent with the interpretation that an electron entering the dot into an excited state will always have time to relax to the ground state before it tunnels out. The rate for tunneling out is $\sim 6$ kHz, giving an upper bound for the relaxation time of $\sim 170$ $\mu$s.

The main relaxation mechanism in quantum dots is thought to be electron-phonon scattering [26]. Measurements on few-electron vertical quantum dots have shown relaxation times of 10 ns [27]. Recent numerical investigations have shown that the electron-electron interaction in multi-electron dots can lead to reduced relaxation rates [28]. Still, the relaxation rate is expected to be considerably faster than the upper limit we give here.

The rate for tunneling out is actually not constant for the whole region of the Coulomb diamond, but shows a change at the position marked by the arrow in
4.6 Tuning the tunnel couplings

Fig. 4.5(b). This transition occurs along a line perpendicular to the ones seen in \( \Gamma_{\text{in}} \). This is expected assuming the transition seen in Fig. 4.5(b) involve changes in \( \Gamma_{out} \) instead of in \( \Gamma_{in} \). Going perpendicular to the dashed lines in Figs. 4.5(a, b), we keep the QD and source potential constant while lowering the drain lead. At some point, the Fermi level of the drain is low enough so that an electron in the QD \((n+1)\)-electron ground state may tunnel out and leave the QD in an \(n\)-electron excited state. The process is sketched in Fig. 4.5(d). Comparing Figs. 4.5(c-d), we see that the rate \( \Gamma_{in} \) probes the excitation spectrum of the \((n+1)\)-electron QD, while \( \Gamma_{out} \) reflects the spectrum of the \(n\)-electron QD.

4.6 Tuning the tunnel couplings

Changing a gate voltage does not only shift the electrochemical potential of the QD, but also affects the height of the tunnel barrier connecting the QD to the leads. The effect was mentioned already in relation with the results of Figs. 4.3 and 4.4. Here we investigate the behavior more carefully. Figure 4.6(a) shows a sketch of the potential landscape for a QD with a bias voltage applied between the source and drain contacts. Electrons entering the QD from the source lead need to tunnel through a potential barrier of height \((U_{SB} - \mu_{QD})\), while the barrier height for electrons tunneling from the QD to drain is \((U_{DB} - \mu_{QD})\). By changing the voltages on gates \(G_1\) and \(G_2\), we expect to be able to tune the potentials \(U_{SB}\) and \(U_{DB}\) and thereby control the tunneling rates.

The tunneling probability also strongly depends on the width of the barrier as well as on the exact shape of the electrostatic potential forming the QD and the barriers. These details are not known, but for small perturbations to the barrier potential \(\delta U_{SB/DB}\) and QD potential \(\delta \mu_{QD}\), the tunneling rate is expected to depend exponentially on the energy difference \((\delta U_{SB/DB} - \delta \mu_{QD})\) \[29\]

\[
\Gamma \sim \Gamma_0 \exp[-\kappa (\delta U_{SB/DB} - \delta \mu_{QD})].
\]

Here, \(\Gamma_0\) and \(\kappa\) are constants given by the exact shape of the potential. To make quantitative comparisons with the experiments, we use a capacitor model to estimate the influence that gate voltages have on the different potentials in the system \[30\]

\[
\begin{bmatrix}
\delta \mu_{QD} \\
\delta U_{SB} \\
\delta U_{DB}
\end{bmatrix}
= \begin{bmatrix}
\alpha_{S-QD} & \alpha_{D-QD} & \alpha_{G1-QD} & \alpha_{G2-QD} \\
\alpha_{S-SB} & \alpha_{D-SB} & \alpha_{G1-SB} & \alpha_{G2-SB} \\
\alpha_{S-DB} & \alpha_{D-DB} & \alpha_{G1-DB} & \alpha_{G2-DB}
\end{bmatrix}
\begin{bmatrix}
\delta \mu_S \\
\delta \mu_D \\
\delta |e| V_{G1} \\
\delta |e| V_{G2}
\end{bmatrix}.
\]

(4.14)

The coefficients \(\alpha\) are the capacitive lever arms between the gates and the various sample potentials. It should be noted that both the gate voltages \(V_{G1}, V_{G2}\) and the source and drain potentials \(\mu_S, \mu_D\) have gating effects on the QD and on the barriers. In the following, we focus on the influence of the gate voltages \(V_{G1}, V_{G2}\) and assume a fixed bias voltage \(V_{SD}\) applied symmetrically across the QD, with \(\mu_S = |e| V_{SD}/2\),
Figure 4.6: (a) Potential landscape of the QD when a fixed bias voltage is applied between the source and drain contacts. (b) Tunneling rates $\Gamma_S/\Gamma_D$ measured versus $V_{\text{diff}} = V_{G2} - V_{G1}$. The solid lines are fits to Eq. (4.17) in the text. The measurements were performed by sweeping both gate voltages $V_{G1}, V_{G2}$, with $V_{G1} = -0.142 - V_{G2}$.

$\mu_D = -|e| V_{SD}/2$. Also, by operating the QD at fixed bias and ensuring that electron transport is unidirectional [Eq. (4.11)], we can use the relations of Eq. (4.12) to determine the tunnel couplings $\Gamma_S$ and $\Gamma_D$ separately.

As seen in Eq. (4.13), the tunneling strength depends on the difference $\delta U_{SB/DB} - \delta \mu_{QD}$. To simplify matters we want to fix the QD potential $\mu_{QD}$ and investigate only the influence that the gate voltages have on the barrier potentials $U_{SB}$ and $U_{DB}$. This is done by sweeping the two gate voltages $V_{G1}$ and $V_{G2}$ against each other in a way that $\mu_{QD}$ remains constant. Setting $\delta \mu_{QD} = 0$ in Eq. (4.14), assuming a fixed bias voltage ($\delta \mu_S = \delta \mu_D = 0$) and solving for $V_{G1}$ gives the prescription

$$\delta V_{G1} = -\frac{\alpha_{G2-QD}}{\alpha_{G1-QD}} \delta V_{G2}. \tag{4.15}$$

Due to the symmetry of the device, we have $\alpha_{G1-QD} \approx \alpha_{G2-QD}$ so that the above expression reduces to $\delta V_{G1} \approx -\delta V_{G2}$. Introducing $V_{\text{diff}} = V_{G2} - V_{G1}$ we find from Eqs. (4.13-4.14)

$$\Gamma_S \sim \exp[\kappa_S |e| \delta V_{\text{diff}} (\alpha_{G2-SB} - \alpha_{G1-SB})] \equiv \exp[\gamma_S \delta V_{\text{diff}}], \tag{4.16}$$

$$\Gamma_D \sim \exp[\kappa_D |e| \delta V_{\text{diff}} (\alpha_{G2-DB} - \alpha_{G2-DB})] \equiv \exp[\gamma_D \delta V_{\text{diff}}]. \tag{4.17}$$

Thus we expect the tunneling rates to depend exponentially on the voltage difference $V_{\text{diff}}$. The sign of the factors $\gamma_S$ and $\gamma_D$ determine if the rates are increasing or
4.7 Degenerate states

In this section, we discuss how degenerate states may influence the measured statistics. For simplicity, we limit the discussion to the case where the QD is connected only to one lead, with the other lead being completely pinched off. In this configuration, the tunneling is due to equilibrium fluctuations between the QD and the lead. Fig. 4.7(a) shows the average dc current through the QPC when sweeping the two gates $G_1$ and $G_2$. The diagonal lines correspond to electrons being loaded/unloaded from the QD. Along these lines, the electrochemical potential of the QD is aligned with the Fermi level of the right lead. From the slope of the line we see that the voltages on the two gates $G_1$ and $G_2$ have roughly the same influence on the energy levels of the QD, as expected from the device geometry. We now focus on determining the tunneling rates for three electronic states along the dotted line in Fig. 4.7(a).

Starting at low $V_{G_1}$ voltages, the dot gets successively populated as the voltage on $G_1$ is increased. At each charge degeneracy point, we use the time-resolved measurement techniques to determine the rates for electrons entering and leaving the dot. The results are shown in Fig. 4.7(b).

Taking the possibility of degenerate states into account, the results of Eqs. (4.3-4.4) are extended to

$$\Gamma_{\text{in}} = g_{\text{in}} \Gamma_R \times f_R(\Delta \mu/k_B T),$$

$$\Gamma_{\text{out}} = g_{\text{out}} \Gamma_R \times [1 - f_R(\Delta \mu/k_B T)].$$

Here, the factors $g_{\text{in}}$ and $g_{\text{out}}$ account for possible degeneracies. For electrons entering the QD, the factor $g_{\text{in}}$ should include the number of degenerate empty states. For tunneling out, only the degeneracy of occupied states is relevant. The tunnel coupling $\Gamma_R$ is assumed to be independent of energy and of the QD level within the small gate voltage range considered here. The energy level for three different gate voltages are drawn schematically in Fig. 4.7(c). The middle plot of Fig. 4.7(b) indicates the gate voltage ranges corresponding to the drawings shown in Fig. 4.7(c).
Chapter 4. Time-resolved single-electron detection

Figure 4.7: (a) Current through the QPC as a function of voltage on gates $G_1$, $G_2$. The diagonal lines show positions where the population of the QD changes by one electron. The numbers specify the dot occupation in the different regions. (b) Effective tunneling rates for electrons entering and leaving the dot, measured at the three charge degeneracy points marked by circles along the dashed line in (a). The solid lines are fits using Eqs. (4.18, 4.19), with $T = 230$ mK and the other fitting parameters given in Table 4.1 in the text. (c) Alignment of the QD electrochemical potential relative to the Fermi level of the lead for the gate voltage configurations shown in the middle plot in (b).

The effective rates for electrons tunneling into and out of the QD involve the density of states and the occupation probability in the lead. This gives a strong dependence on the alignment between the Fermi level in the right lead and the electrochemical potential of the dot. Starting at low $V_{G1}$ voltages in Fig. 4.7(b) [case I in Fig. 4.7(c)], the QD potential is far above the Fermi level of the lead. At this point, the density of occupied states in the lead is low and the effective rate for tunneling into the QD is low. If an electron eventually manages to tunnel in, the effective rate for tunneling out again will be high, since there are many empty states in the lead to tunnel into. As the gate voltage is increased, the QD potential goes down to the Fermi level of the lead [case II in Fig. 4.7(b, c)]. In this configuration, the effective rates for tunneling into and out of the QD are roughly equal. As the gate voltage is further increased, the potential of the QD is pushed below the Fermi
4.7. Degenerate states

Initially empty

Initially occupied

Tunneling in

Tunneling out

Figure 4.8: Effective tunneling rates for spin degenerate states in different configurations. The empty circles represent empty spin states, filled circles represent occupied ones. The arrows depict the number of possible tunnel processes.

level. Here, the density of occupied states in the lead is large, giving a high effective rate for electrons entering the QD. Conversely, the effective rate for leaving the dot is low [case III in Fig. 4.7(b, c)].

Looking at the shape of the data in Fig. 4.7(b), we see that they indeed follow a Fermi function. The solid lines in the figure are fits using Eqs. (4.18-4.19), with $T = 230$ mK. The parameters used in the fitting procedure are summarized in Table 4.1.

![Table 4.1: Fitting parameters for the solid lines in Fig. 4.7(b), fitted using Eqs. (4.18, 4.19).](image)

Comparing the numbers of Table 4.1, we see that the effective coupling $g_{in/out} \Gamma_R$ differs strongly depending on whether it was extracted from the tunneling in or from the tunneling out data. One possible explanation for the difference is degeneracy due to the electron spin. Assuming a spin-degenerate state with both the spin-up and the spin-down state initially empty, electrons from the lead could tunnel into either of the two states. This makes $g_{in} = 2$. Once the electron has tunneled into the QD, it sits in either the spin-up or the spin-down state. Since only one of the spin-degenerate states is occupied, the degeneracy for tunneling out will be $g_{out} = 1$. The situation is different if we start with a QD with one of the spin-degenerate states already occupied. For the tunneling-in process, there is only one empty state available, giving $g_{in} = 1$. For the tunneling-out process, any of the two
Chapter 4. Time-resolved single-electron detection

electrons sitting on the dot may tunnel out. This leads to \( g_{\text{out}} = 2 \). The different situations are shown schematically in Fig. 4.8. The model discussed here assumes that the spin states are not influenced by Coulomb interactions, which may be an oversimplification considering that we are dealing with a many-electron system. Still, spin pairing has been observed in chaotic QDs containing a large number of electrons [31].

The experimental method described here can only determine the ratio \( g_{\text{in}}/g_{\text{out}} \). In the following we assume the degeneracies to be due to spin to be able to extract tunnel couplings and absolute degeneracies from the data. The results of this model are shown in Table 4.2. For the first resonance at \( V_{G1} = -30.35 \) mV we extract \( g_{\text{in}} = 2 \) and \( g_{\text{out}} = 1 \), indicating a two-fold degeneracy with both states initially empty. At the next resonance, the degeneracy factors are exchanged, with \( g_{\text{in}} = 1 \) and \( g_{\text{out}} = 2 \). For the third resonance, the degeneracy factors are the same as for the second resonance, with \( g_{\text{in}} = 1 \) and \( g_{\text{out}} = 2 \).

<table>
<thead>
<tr>
<th>( V_{G1} )</th>
<th>( \Gamma )</th>
<th>( g_{\text{in}} )</th>
<th>( g_{\text{out}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30.35 mV</td>
<td>110 Hz</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>15.70 mV</td>
<td>220 Hz</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>45.35 mV</td>
<td>307 Hz</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4.2: Possible interpretation of the data shown in Table 4.1, assuming spin-degenerate states.

The first and second resonance could be attributed to consecutive filling of spin states, meaning that the two first electrons would form a so-called spin pair. The third electron does not follow the rules expected from simple spin-filling. The reason could be due to many-body effects between the electrons in the quantum dot or due to a charge rearrangement taking place between the second and third resonance (at \( V_{G1} \sim 30 \) mV). Also, we stress that there are other possible explanations for the measurement results, like energy-dependent tunneling rates or accidental degeneracies of orbital states. To prove the spin degeneracy, one would need to perform measurements at non-zero magnetic fields. This would lift the spin degeneracy and make \( g_{\text{in}} = 1 \) and \( g_{\text{out}} = 1 \).

As seen in section 4.6, changing a gate voltage also affects the tunnel couplings in the system. Since the tunneling rates \( \Gamma_{\text{in}}/\Gamma_{\text{out}} \) are measured at slightly different gate voltages, it could be that the differences seen in Fig. 4.7(b) are due to tuning of the tunneling barrier. To avoid such influences, we used gate G1 to tune the QD electrochemical potential, since it is expected to have a smaller effect on the tunnel barrier between the QD and drain than gate G2. From Eq. (4.17) and the results of section 4.6, we estimate the change of tunneling rates within the gate voltage range shown in Fig. 4.7(b) to be well below 10%. Also, the gating effect of G1 on the tunnel barrier would make it more likely for \( \Gamma_R \) to increase with \( V_{G1} \). Since \( \Gamma_{\text{in}} \) is determined at a slightly higher gate voltage than \( \Gamma_{\text{out}} \), we would expect \( \Gamma_{\text{in}} \) to be larger than \( \Gamma_{\text{out}} \). This is in contradiction with the results of Table 4.1.

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Chapter 5

Statistics of electron transport

In this chapter we investigate the statistical properties of single-electron tunneling through a quantum dot. In the general case, we find that the fluctuation of the current due to shot noise is suppressed because of Coulomb blockade. Electrons tend to avoid each other, giving anti-bunching or sub-poissonian noise. In other regimes we find bunching of electrons, or super-poissonian noise. Finally, we investigate how the finite bandwidth of the detector influences the measured statistics and discuss the possibilities of using a quantum dot combined with a charge-detector as a current meter.

5.1 Electron transport and shot noise

Electrical current is carried by electrons passing through the conductor. The current is given as
\[ I = \frac{e}{\langle t \rangle}, \]
where \( e \) is the electron charge and \( \langle t \rangle \) the average time between electrons. The discreteness of the charge carriers gives rise to temporal fluctuations in the current. These statistical fluctuations are called shot noise. The principle behind the shot noise is illustrated in Fig. 5.1. In Fig. 5.1(a), we show an idealized current flow. Each spike corresponds to one electron passing the conductor. The time interval between two electrons \( \Delta t \) is constant, so that the current is given as \( I = e/\Delta t \). Figure 5.1(b) displays a more realistic current, where the time intervals between electrons show random fluctuations. If the average time between electrons \( \langle t \rangle = \Delta t \), then a measurement of the time-averaged current in case (a) and (b) will give the same value.

Still, the currents in the two cases are obviously different. This becomes clear in Fig. 5.1(c-d), where we plot the distribution function \( P_{t_0}(N) \) of the number of electrons \( N \) that pass through the conductor within a fixed time-interval \( t_0 \). The time \( t_0 \) is chosen so that the average number of transmitted electron \( \langle N \rangle = 5 \) in both cases. The distribution function for the idealized current in (a) is simply a single peak with \( P_{t_0}(5) = 1 \) [Fig. 5.1(c)]. On the other hand, the realistic current...
Figure 5.1: (a) Idealized current flow. Each spike corresponds to an electron passing the conductor, with the time intervals between electrons being equidistant. (b) Same as (a), but for a realistic current. The electron flow shows random variations. (c-d) Distribution function for the currents shown in (a-b). The distribution function is formed by counting the number of electrons passing the conductor within a time $t_0$.

This gives a broad distribution due to the statistical fluctuations in the current.

In this way, the shape of the distribution function is a measure of the statistical fluctuations of the current. To be more quantitative, we calculate the central moments of the distribution

$$
\mu_1 = \langle N \rangle, \quad \mu_i = \langle (N - \langle N \rangle)^i \rangle, \text{ for } i = 2, 3, \ldots \tag{5.2}
$$

Here, $\langle \ldots \rangle$ represents the mean over a large number of periods of length $t_0$. The first moment (mean) gives access to the average current, $I = e\mu_1/t_0$. The second central moment (variance) defines the shot noise power, with

$$
S_I = 2e^2\mu_2/t_0. \tag{5.3}
$$

Equation (5.3) is valid if $t_0$ is much larger than correlation times in the system. In the following section, we will also evaluate the third central moment, $\mu_3$. It describes the asymmetry (skewness) of the distribution function around its maximum.
The noise of a current is often expressed as the Fano factor, which is the width of the distribution divided by its mean,

\[ F = S_I/2eI = \mu_2/\mu. \]  

(5.4)

For processes governed by Poisson statistics, like electron tunneling through a single barrier, the Fano factor is equal to one. If the Fano factor is smaller than one, we speak of sub-Poisson noise. This generally means lower noise power and electron correlation in time. Conversely, if the Fano factor is greater than one, the noise is super-Poissonian and the electron transport is less regular than in the Poissonian case.

If the charge is transferred in units of \( q \) instead of \( e \), the Fano factor will be modified by a factor \( q/e \). By measuring both the shot noise \( S_I \) and current \( I \), one can use this relation to directly determine the fractionality of the charge of the carriers. Such measurements have been performed to demonstrate the charge of quasi-particles in the fractional quantum Hall effect \([32, 33]\) as well as the double charge of Cooper pairs in superconductors \([34]\). These are examples where noise measurements provide additional information about the system that cannot be extracted from a standard current measurement \([35]\).

In electron transport through a semiconductor quantum dot (QD), the noise is typically suppressed compared to the Poisson distribution. This is due to the Coulomb blockade, which enhances the temporal correlation between successive electrons and thereby reduces the noise \([36–40]\). The Pauli exclusion provides an additional noise suppression mechanism \([41, 42]\). However, when several channels with different coupling strengths contribute to electron transport, interactions can lead to more complex processes and to an enhancement of the noise \([43–46]\). Furthermore, there are predictions that entangled electrons may lead to super-Poissonian noise, thus making noise measurements a possible way of detecting entanglement in mesoscopic systems \([47–49]\).

The above examples demonstrate that noise measurements are important tools for characterizing properties of mesoscopic systems. However, due to the very low current levels involved, it is difficult to perform the experiments with conventional measurement techniques. One has to carefully eliminate other noise sources like Johnson-Nyquist thermal noise and the noise of the amplifiers. Recent attempts include using a resonant circuit together with a low-temperature amplifier \([50, 51]\), a superconductor-insulator-superconductor junction \([52]\) or a second QD acting as a high-frequency detector \([53]\).

A different approach is to use time-resolved charge detection methods as described in Chapter 4 to count the electrons one-by-one as they pass through the conductor. From such a measurement, one can directly determine the probability distribution function \( p_0(N) \). The distribution function is then used to calculate both the shot noise as well as higher-order moments. This way of measuring is analogous to the theoretical concept of full counting statistics (FCS), which was introduced as a new way of examining current fluctuations \([54]\). In the following
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5.2 Sequential transport – Sub-Poissonian noise

In order to use a charge detector for measuring current and current noise, one has to avoid that electrons tunnel back and forth between the dot and the source or drain lead due to thermal fluctuations [Fig. 5.2(a)]. This is achieved by applying a finite bias voltage between source and drain, i.e.

\[ k_B T \ll |\pm eV/2 - \mu_n| \ll E_C. \]  \hspace{1cm} (5.5)

Here, \( E_C \) is the charging energy, \( \mu_n \) is the electrochemical potential of the QD and \( V \) is the bias voltage, symmetrically applied to the QD [Fig. 5.2(b)]. With a finite bias applied to the QD, and with the Fermi levels of the leads far away from the electrochemical potential of the QD, the probability for electrons to tunnel in the opposite direction is exponentially suppressed. In this regime, we attribute each transition \( n \to n+1 \) to an electron entering the QD from the source contact, and each transition \( n+1 \to n \) to an electron leaving the QD to the drain contact. The charge fluctuations in the QD then correspond to a non-equilibrium process, and are directly related to the current through the dot. The current is determined by the tunneling rates \( \Gamma_{in} \) and \( \Gamma_{out} \), with

\[ I = e \frac{\Gamma_{in}\Gamma_{out}}{\Gamma_{in} + \Gamma_{out}}. \]  \hspace{1cm} (5.6)

From the tunneling rates, one could calculate all the higher moments of the current distribution as well [55]. However, the results are only valid assuming that Eq. (4.2)
5.2. Sequential transport – Sub-Poissonian noise

Figure 5.3: Statistical distribution of the number \( N \) of electrons entering the QD during a given time \( t_0 \). The two panels correspond to two different values of the tunneling rates, obtained for different values of the gate voltage \( V_{G1} \). The time \( t_0 \) is chosen in order to have the same mean value of number of events, \( \langle N \rangle \approx 3 \), for both graphs. The line shows the theoretical distribution calculated from Eqs. (5.10) and (5.9). The tunneling rates are determined experimentally by the method described in Chapter 4, and no fitting parameters is involved in the curves showing theoretical results.

is correct. In order to measure the current and the current distribution function for any experimental configuration, we instead focus on extracting the current distribution function \( p_{t_0}(N) \) from the experimental data.

The distribution is found by splitting a time trace of length \( T \) into \( m = T/t_0 \) intervals of length \( t_0 \) and counting the number of electrons entering the QD within each interval. Examples of such distributions are shown in Fig. 5.3, taken at two different gate configurations. The noise and the higher moments are then extracted directly from the measured distribution using Eqs. (5.2-5.4), giving for \( I = 792 \, \text{e}/\text{s}, \, F = \mu_1/\mu_2 = 0.52 \) for case (a) and \( I = 626 \, \text{e}/\text{s}, \, F = 0.89 \) for case (b). The noise is relatively close to Poissonian for case (b), but clearly sub-Poissonian in case (a). This difference is easily seen by eye by comparing the width of the two distributions. In order to understand why the shape is different in the two situations in Fig. 5.3(a, b), we need to calculated the noise expected from the QD. This is the subject of the next section.

5.2.1 Theory and model description

The noise properties of a QD in the sequential tunneling regime was investigated in detail by Bagrets and Nazarov [55], using the framework of full counting statistics. Here, we summarize their results, apply conditions appropriate for our experimental configuration and compare the theoretical results with experimental data.

The QD occupancy in the low bias, single-level transport regime is modeled by
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Figure 5.4: State diagram of a two-state model describing electron tunneling in a QD in the single-level regime. Transitions between the states occur with rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$. The counting field $e^{i\chi}$ is introduced for the transition involving an electron entering the QD, as marked by the dashed circle.

A two-state rate equation

$$\frac{d}{dt} \begin{pmatrix} p_n \\ p_{n+1} \end{pmatrix} = \begin{pmatrix} -\Gamma_{\text{in}} & \Gamma_{\text{out}} \\ \Gamma_{\text{in}} & -\Gamma_{\text{out}} \end{pmatrix} \begin{pmatrix} p_n \\ p_{n+1} \end{pmatrix}.$$  (5.7)

Here, $p_n$ and $p_{n+1}$ give the occupation probability for the states with $n$ and $n+1$ electrons, respectively. The two states and the possible transitions are depicted in Fig. 5.4.

To evaluate the counting statistics of the system, we need to introduce a counting field $e^{i\chi}$ into the rate equation. We choose to count electrons tunneling into the QD, which changes the matrix in Eq. (5.7) to:

$$M(\chi) = \begin{pmatrix} -\Gamma_{\text{in}} & \Gamma_{\text{out}} \\ \Gamma_{\text{in}} e^{i\chi} & -\Gamma_{\text{out}} \end{pmatrix}.$$  (5.8)

In the limit $t_0 \gg \Gamma_{\text{in}}^{-1}, \Gamma_{\text{out}}^{-1}$, the normalized distribution $p_{t_0}(N/t_0)$ is independent of $t_0$. In the same limit, the cumulant-generating function $S(\chi)$ is related to the lowest eigenvalue of $M(\chi)$, $\lambda(\chi)$ as [55]

$$S(\chi) = \lambda(\chi) t_0 = t_0 \left[ \Gamma_{\text{in}} + \Gamma_{\text{out}} - \sqrt{\left( \Gamma_{\text{in}} - \Gamma_{\text{out}} \right)^2 + 4\Gamma_{\text{in}}\Gamma_{\text{out}} e^{-i\chi}} \right].$$  (5.9)

The distribution function for the number of electrons tunneling through the quantum dot during a time $t_0$ is generated from the cumulant-generating function $S(\chi)$ [see Appendix B.1]:

$$p_{t_0}(N) = \int_{-\pi}^{\pi} \frac{d\chi}{2\pi} e^{-S(\chi)-iN\chi}.$$  (5.10)

The solid lines in Fig. 5.3 are distributions calculated from Eqs. (5.9-5.10). The tunneling rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ are determined separately, as explained in Chapter 4.
The agreement with the experimental distribution is very good, in particular, given that the curves involve no fitting parameters. As mentioned earlier, the graphs show a clear qualitative difference: Figure 5.3(b) has a broader and more asymmetric distribution than Fig. 5.3(a). We will see later that this difference comes from the different asymmetries of the source and drain tunneling rates.

In order to perform a more quantitative analysis, we evaluate the three first central moments \( \mu_i \) of the current distribution, which coincide with the first three cumulants \( C_i \) [see Appendix B.1 for a discussion about the difference between moments and cumulants]. The cumulants are generated directly from the cumulant-generating function \( S(\chi) \). The mean current is then given by the first cumulant \( C_1 \) of the distribution:

\[
I = \frac{e}{t_0} C_1 = \frac{e}{t_0} \left( -i \frac{dS}{d\chi} \right)_{\chi=0} = e \frac{\Gamma_{\text{in}}\Gamma_{\text{out}}}{\Gamma_{\text{in}} + \Gamma_{\text{out}}} . \tag{5.11}
\]

The symmetrized shot noise is calculated from the variance, or the second cumulant \( C_2 \), of the distribution:

\[
S_I = \frac{2e^2}{t_0} C_2 = \frac{2e^2}{t_0} \left( - \frac{d^2S}{d\chi^2} \right)_{\chi=0} , \tag{5.12}
\]

from which we get the Fano factor:

\[
F_2 = \frac{S_I}{2eI} = \frac{C_2}{C_1} = \frac{\Gamma_{\text{in}}^2 + \Gamma_{\text{out}}^2}{(\Gamma_{\text{in}} + \Gamma_{\text{out}})^2} = \frac{1}{2} \left( 1 + a^2 \right) , \tag{5.13}
\]

where \( a = (\Gamma_{\text{in}} - \Gamma_{\text{out}})/(\Gamma_{\text{in}} + \Gamma_{\text{out}}) \) is the asymmetry of the coupling. This result recovers the earlier calculations for the shot noise in a quantum dot [36], and shows the reduction of the noise by a factor 1/2 for a QD symmetrically coupled to the leads, while the Poissonian limit, \( F_2 = 1 \), is reached for an asymmetrically coupled QD \((a = \pm 1)\). The reduction of the noise is a direct consequence of Coulomb blockade; when one electron occupies the QD, a second electron cannot enter before the first one leaves. This leads to correlations in the current fluctuations, and to a reduction of the noise. The reduction is maximal when the tunnel barriers are symmetric. For an asymmetrically coupled QD, the transport is essentially governed by the weakly transparent barrier and the noise approaches the value for a single tunneling barrier, \( S_I = 2eI \). The results discussed here assume tunneling with transmission coefficients much smaller than one.

Finally, we want to calculate the third cumulant \( C_3 \), of the fluctuations, which characterizes the asymmetry of the distribution (skewness):

\[
C_3 = i \left( \frac{d^3S}{d\chi^3} \right)_{\chi=0} . \tag{5.14}
\]
The asymmetry can also be normalized to the mean of the distribution:

\[
F_3 = \frac{C_3}{C_1} = \frac{\Gamma_{\text{in}}^4 - 2\Gamma_{\text{in}}^3 \Gamma_{\text{out}} + 6\Gamma_{\text{in}}^2 \Gamma_{\text{out}}^2 - 2\Gamma_{\text{in}}^3 \Gamma_{\text{out}}^3 + \Gamma_{\text{out}}^4}{(\Gamma_{\text{in}} + \Gamma_{\text{out}})^4}
\]

\[
= \frac{1}{4} \left( 1 + 3a^4 \right).
\]

The result shows that for a symmetrically coupled QD, the third moment is reduced by a factor \(1/4\) compared to the Poissonian limit. For an asymmetrically coupled dot with \(a \to \pm 1\), we recover \(F_3 \to 1\).

### 5.2.2 Experimental results

From experimental distributions as the ones shown in Fig. 5.3, we can easily obtain moments of any order using the relations in Eq. (5.2). We first focus on the mean \(\mu\) of the distribution. By measuring \(\mu\) as a function of the voltage applied on gate G1 and the bias voltage \(V\), we construct the Coulomb diamonds [see Fig. 5.5(a)]. We observe clear Coulomb blockade regions as well as regions of finite current. Figure 5.5(b) shows a cross section taken at \(V_{G1} = -44\) mV, the position is indicated by the dashed line in Fig. 5.5(a). As the bias voltage is increased, we see steps in the current. As explained in Section 4.5, the first step in Fig. 5.5(b) (see left arrow) corresponds to the alignment of the chemical potential of the source contact with the ground state in the QD, and the following steps with excited states in the QD. From the resolution of the Coulomb diamonds, we see that the sample is stable enough such that background charge fluctuations do not play a significant role on the time scales relevant for this experiment [17].

In addition to the mean, we evaluate the second and third central moments from the measured counting statistics. These two moments are plotted in Fig. 5.5(b) as a function of the bias voltage. The second moment (blue dotted line) reproduces the steps seen in the current. These two moments can be represented by their reduced quantities \(F_2 = \mu_2/\mu\) (Fano factor) and \(F_3 = \mu_3/\mu\), as shown in Fig. 5.5(c). Both normalized moments are almost independent of the bias voltage, and show a reduction compared to the values \(\mu_2/\mu = \mu_3/\mu = 1\) expected for classical fluctuations with Poissonian counting statistics.

As described in section 4.6, the tunnel couplings can be tuned by adjusting the gate voltages \(V_{G1}\) and \(V_{G2}\). In this way, we are able to continuously change the symmetry of the barriers from symmetric to very asymmetric coupling. In Fig. 5.6, we show the normalized second and third central moments as a function of the asymmetry \(a\). The tunneling rates are directly measured as described in section 4, and the inset of Fig. 4.6(b) shows the variation of asymmetry with gate voltage in the region of interest. As expected from the discussion in the previous section, the noise is reduced for symmetric barriers. The experimental data follow the theoretical predictions given by Eqs. (5.13, 5.15) very well. We note in particular that no fitting parameters have been used since the tunneling rates are determined separately.
5.2. Sequential transport – Sub-Poissonian noise

Figure 5.5: (a) Average number $\mu$ of electrons entering the QD, measured as a function of the gate voltage $V_{G1}$ and the bias voltage $V_{SD}$. Far from the edges of the Coulomb blockade region, i.e. for $|\pm eV_{SD}/2 - E_d| \gg k_B T$, the fluctuations of $n$ are directly related to current fluctuations. The dashed line correspond to the cross-section shown in panel (b). (b) Three first moments of the fluctuations of $n$ as a function of the bias voltage $V_{SD}$ and at a given gate voltage $V_{G1} = -44$ mV. The ground state (GS) as well as two excited states (ES) are clearly visible. The moments are scaled so that $\mu$ corresponds to the number of electrons entering the QD per second. In the gray region, the condition $|\pm eV_{SD}/2 - \mu_n| \gg k_B T$ is not valid, and the number of electrons entering the QD cannot be taken as the current flowing through the QD. The width of this region is $9 \times k_B T/e \approx 300 \mu V$, determined from the width for which the Fermi distribution is between 0.01 and 0.99. (c) Normalized second and third moments as a function of the bias voltage $V_{SD}$ and at a given gate voltage $V_{G1} = -44$ mV.

5.2.3 Time statistics

A complementary way of investigating the correlations is to look at the temporal statistics of electron transport. Instead of evaluating the probability distribution for the number of electrons that are transferred within a fixed time $t_0$, we examine the continuous distribution $p_N(t)$ describing the time needed for a fixed number of $N$ electrons to pass through the QD. With the rates for tunneling into and out of the QD given by Eq. (4.2), we find for $N = 1$

$$p_{N=1}(t) = \int_0^t p_{in}(t')p_{out}(t - t')dt' =$$

$$= \frac{\exp(-\Gamma_{in} t) - \exp(-\Gamma_{out} t)}{1/\Gamma_{in} - 1/\Gamma_{out}}.$$  \hspace{1cm} (5.16)
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Figure 5.6: (a) Second and (b) third normalized central moments of the fluctuations of \( n \) as a function of the asymmetry of the tunneling rates, \( a = (\Gamma_{\text{in}} - \Gamma_{\text{out}})/(\Gamma_{\text{in}} + \Gamma_{\text{out}}) \). To increase the resolution, each point at a given asymmetry is obtained by averaging over about 50 points at a given voltage \( V_{G1} \) and in a window of bias voltage \( 1.5 < V_{SD} < 3 \text{ mV} \). Error bars correspond to the standard error of this averaging process, and are of the size of the points if not shown. The dashed lines are the theoretical predictions given by Eqs. (5.13, 5.15). No fitting parameters have been used, since the tunneling rates are fully determined experimentally. Inset of (b): Variation of the asymmetry of the tunneling rates, \( a \), as a function of \( V_{G1} \).

In Fig. 5.7, we show the experimentally determined distribution \( p_{N=1}(t) \) for two different values of the asymmetry together with the results of Eq. (5.16). For the symmetric case \( [a = -0.07 \text{ in Fig. 5.7}] \), there is a clear suppression of transfer probability for short time scales. Again, this is due to the Coulomb blockade. We measure anti-bunching of electrons and sub-Poisson noise levels. For the more asymmetric case \( [a = 0.9 \text{ in Fig. 5.7}] \), the anti-bunching is less prominent and the probability distribution approaches the exponential behavior expected for a single tunnel barrier.

The ability to measure the counting statistics of electron transport relies on the high sensitivity of the QPC as a charge detector. Given the bandwidth of our experimental setup, \( \Delta f = 30 \text{ kHz} \), the method allows to measure currents up to 5 fA, and we can measure currents as low as a few electrons per second, i.e., less than 1 aA. The low-current limitation is mainly given by the length of the time trace and the stability of the QD, and is well below what can be measured with conventional current meters. In addition, as we directly count electrons one by one, this measurement is not sensitive to the noise and drifts of the experimental setup. It is also a very sensitive way of measuring low current noise levels. The precision and limitations of the measurement method are described in more details in sections 5.4 and 5.5.
5.3 Bunching of electrons

So far, we have analyzed data where the tunneling events can be well explained by a rate equation approach with one rate for electrons tunneling into and another rate for electrons leaving the dot. For the trace shown in Fig. 5.8(a), the behavior is distinctly different. The electrons come in bunches; there are intervals where tunneling occurs on a fast time scale (>10 kHz), in-between these intervals there are long periods of time (>1 ms) without any tunneling. The data was taken with a bias applied so that the Fermi level of the source lead lines up with the electrochemical potential of the dot, while the drain lead is far below the electrochemical potential of the dot, thus prohibiting electrons from entering the QD through the drain lead. The voltage on gate $V_{G1}$ was set to 34 mV, which is outside the range of the Coulomb diamonds presented in Fig. 5.5(a). Since the QPC current is at the high level during the intervals without tunneling, the dot contains one electron less when the fast tunneling is blocked.

In order to explain the two different time scales, we assume a mechanism where there are two almost energy-degenerate dot states within the thermal broadening of the distribution in the source lead. Because of Coulomb blockade, the dot may hold one or zero excess electrons. Hence, the model includes three possible dot states as shown in Fig. 5.8(b). State $S_A$ is the $n$-electron ground state, state $S_B$ is an excited $n$-electron state and state $S_{n+1}$ is the ground state when the dot contains $(n + 1)$
Figure 5.8: (a) Time trace of the QPC current showing bunching of electrons. (b) Dot states included in the model used to describe the bunching of electrons. The red circles correspond to electron occupation. State $S_A$ is the $n$-electron ground state, state $S_B$ is an excited $n$-electron state and state $S_{n+1}$ is the ground state when the dot contains $(n+1)$ electrons. (c) Energy diagram for the model. The two dot transitions are both within the thermal broadening of the lead. Electrons enter the dot from the left lead and may leave through either the left or the right lead. (d) Possible transitions between the different states of the model. The rates $\Gamma_A^\text{in}$, $\Gamma_B^\text{in}$ refer to electrons entering the QD from the left lead, thus taking the dot from state $S_{A/B}$ to state $S_{n+1}$. The rates $\Gamma_A^\text{out}$, $\Gamma_B^\text{out}$ describe electrons leaving the dot to the left lead, giving transitions from state $S_{n+1}$ to $S_{A/B}$. $W_{AB}$ and $W_{BA}$ are the direct transition rates between states $S_A$ and $S_B$. Finally, the rates $\Gamma_A^\text{right}$, $\Gamma_B^\text{right}$ refer to electrons leaving the dot through the right lead.

electrons. Transitions between the $S_A/S_B$ states and the $S_{n+1}$ state occur whenever an electron tunnels into or out of the dot.

The tunnel coupling strength between the dot and the lead is given by the overlap of the dot and lead electronic wavefunctions. Since the wavefunctions corresponding to the two states $S_A$ and $S_B$ may have different spatial distributions, the coupling strength $\Gamma_A$ of the transition $S_A \leftrightarrow S_{n+1}$ may differ from the coupling $\Gamma_B$ of the $S_B \leftrightarrow S_{n+1}$ transition. The energy levels of the dot and the leads for the configuration where we measure bunching of electrons are shown in Fig. 5.8(c), while the possible transitions of the model are depicted in Fig. 5.8(d).
5.3. Bunching of electrons

Starting with one excess electron on the dot [state $S_{n+1}$ in Fig. 5.8(d)], at some point an electron will tunnel out, leaving the dot in either state $S_A$ or state $S_B$. Assuming $\Gamma_B \gg \Gamma_A$, it is most likely that the dot will end up in the excited state $S_B$. If the tunneling rate $\Gamma_B$ is faster than the relaxation process $S_B \Rightarrow S_A$, an electron from the lead will have time to tunnel onto the dot again and take the dot back to the initial $S_{n+1}$ state. The whole process can then be repeated, leading to the fast tunneling in Fig. 5.8(a).

However, at some point the dot will end up in state $S_A$, either through an electron leaving the dot via the $\Gamma_A$ transition, or through relaxation of the $S_B$ state. To get out of state $S_A$, there must be either a direct transition back to state $S_B$, or an electron tunneling into the dot through the $S_A \Rightarrow S_{n+1}$ transition. With $\Gamma_B \gg \Gamma_A$ and assuming $\Gamma_B \gg W_{BA}$, both processes are slow compared to the tunneling between the lead and state $S_B$. This mechanism will block the fast tunneling and produce the intervals without switching events seen in Fig. 5.8(a). Similar arguments can be used to show that the blocking mechanism will be possible also if $\Gamma_B \ll \Gamma_A$.

From the above reasoning, we see that the fast time scale is set by the fast tunneling state, while the slow time scale is determined either by the relaxation process $S_B \Rightarrow S_A$ or by the slow tunneling rate, depending on which process is the fastest. Either way, it is crucial that the relaxation rate is slower than the fast tunneling rate (in our case $W_{AB} \ll \Gamma_B \sim 20 \text{ kHz}$). We speculate that the slow relaxation rate may be due to different spin configurations of the two states. For a few-electron QD, spin relaxation times of $T_1 > 1 \text{ ms}$ have been reported [24, 56, 57].

To make quantitative comparisons between the model and the data, we use the methods of full counting statistics to investigate how the dot charge fluctuations change as the source lead is swept over a Coulomb resonance. Theoretical investigations of multi-level quantum dots have lead to predictions of electron bunching and super-Poissonian noise [45]. Following the lines of Refs. [45, 55], we first write the master equation for the system,

$$\frac{d}{dt} \begin{bmatrix} p_A \\ p_B \\ p_{n+1} \end{bmatrix} = M \begin{bmatrix} p_A \\ p_B \\ p_{n+1} \end{bmatrix},$$

with

$$M = \begin{bmatrix} -\Gamma_A - W_{BA} & W_{AB} & (\Gamma_{out}^A + \Gamma_{right}^A) * e^{i\chi} \\ W_{BA} & -\Gamma_B - W_{BA} & (\Gamma_{out}^B + \Gamma_{right}^B) * e^{i\chi} \\ \Gamma_A^{in} & \Gamma_B^{in} & -\Gamma_{out} \end{bmatrix}.$$  

(5.18)

Here $\Gamma_{out} = (\Gamma_{out}^A + \Gamma_{out}^B + \Gamma_{right}^A + \Gamma_{right}^B)$ and $p_A$, $p_B$ and $p_{n+1}$ are occupation probabilities for states $S_A$, $S_B$ and $S_{n+1}$, respectively. The effective tunneling rates are determined by multiplying the tunnel coupling constants for each state with the Fermi distribution of the electrons in the lead,

$$\Gamma_{in/out}^{A/B} = f[\pm(eV - \mu_{A/B})] \Gamma_{A/B}.$$  

(5.19)
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The tunneling rates $\Gamma^A_{\text{right}}$ and $\Gamma^B_{\text{right}}$ are included to account for the possibility for electrons to leave through the right barrier. The Fermi level of the right lead is far below the electrochemical potential of the dot, so that the states in the right lead can be assumed to be unoccupied. Finally, $W_{AB}$ and $W_{BA}$ are the direct transition rates between states $S_A$ and $S_B$. These rates obey detailed balance,

$$W_{AB}/W_{BA} = \exp \left[ (\mu_A - \mu_B)/k_B T \right].$$

The phenomenological relaxation rate between the two states is given as $1/T_1 = W_{AB} + W_{BA}$.

In Eq. (5.18), we introduce charge counting by multiplying all entries of $M$ involving an electron leaving the dot with the counting factor $\exp(i\chi)$ [55]. We do not distinguish whether the electron leaves the dot through the left or the right lead. In this way we obtain the counting statistics $p_{t_0}(N)$, which is the probability for counting $N$ events within the time span $t_0$. The distribution describes fluctuations of charge on the dot, which is exactly what is measured by the QPC detector in the experiment. We stress that this distribution is equal to the distribution of current fluctuations only if it can be safely assumed that the electron motion is unidirectional. This is the case if the condition in Eq. (5.5) is fulfilled, i.e. if the tunneling due to thermal fluctuations is suppressed. Here, we are in a regime where there is a mixture of tunneling due to the applied bias and tunneling due to equilibrium fluctuations. But since the model defined in Eq. (5.18) is valid regardless of the direction of the electron motion, it can still be used for analyzing the experimental data.

Using the method of Ref. [55], we calculate the lowest eigenvalue $\lambda_0(\chi)$ of $M$ and use it to obtain the cumulant generating function (CGF) for $p_{t_0}(N)$,

$$S(\chi) = -\lambda_0(\chi)t_0.$$  

(5.21)

The CGF can then be used to obtain the cumulants of any order using the relation $C_n = -(-i\partial_\chi)^n S(\chi)|_{\chi=0}$. In order to compare the theory with the experiment we extract the first three cumulants of $p_{t_0}(N)$ from the experimental data. The cumulants were found by taking a trace of length $T = 0.5$ s and splitting it into $m = T/t_0$ independent traces. By counting the number of electrons $N$ leaving the dot in each trace and repeating the procedure for all $m$ sub-traces, the distribution function $p_{t_0}(N)$ could be experimentally determined. The experimental cumulants were then calculated directly from the measured distribution function. The time $t_0$ was chosen such that $\langle N \rangle \approx 3$.

Figure 5.9(a) shows the first three cumulants versus voltage applied to the source lead. The points correspond to experimental data, while the solid lines show the cumulants calculated from the CGF of our model, with fitting parameters $\Gamma_A = 1.6$ kHz, $\Gamma_B = 20.5$ kHz, $\Gamma^A_{\text{right}} = 4.6$ kHz, $\Gamma^B_{\text{right}} = 310$ Hz, $T_1 = 8$ ms and $\mu_A - \mu_B = 13 \mu$eV. The electronic temperature in this measurement was 400 mK. The figure shows good agreement between the model and the experimental data.
5.3. Bunching of electrons

Figure 5.9: (a) First, second and third cumulants of the distribution of charge fluctuations. The symbols show values extracted from the experimental data, while the solid lines are calculated from the model given in the text. Fitting parameters are: $\Gamma_A = 1.6$ kHz, $\Gamma_B = 20.5$ kHz, $\Gamma_{\text{right}}^A = 4.6$ kHz, $\Gamma_{\text{right}}^B = 310$ Hz, $T_1 = 8$ ms and $\mu_A - \mu_B = 13$ $\mu$eV. The electronic temperature was 400 mK. (b) Normalized cumulants $C_3/C_1$ and $C_2/C_1$ versus bias voltage. The noise is clearly super-Poissonian in the central region of the graph. (c) Calculated maximal value of $C_3/C_1$ as a function of the relaxation time between the two states. The values are calculated by varying the relaxation time while keeping the other parameters to the values given by the fit shown in (a). The maximum value $C_3/C_1$ extracted from the experimental data is 15.9.

Figure 5.9(b) shows the normalized cumulants $C_2/C_1$ and $C_3/C_1$ for the experimental data; we notice that both the second and the third cumulants vastly exceed the first cumulant when the Fermi level of the source lead is aligned with the electrochemical potential of the dot ($V_{\text{bias}} = 1.3$ mV). For a Poissonian process one expects $C_2/C_1 = C_3/C_1 = 1$; here, the noise is clearly of super-Poissonian nature, as expected from the bunching behavior of the electrons.

When the bias voltage is further increased ($V_{\text{bias}} > 1.5$ mV), the source lead is no longer in resonance with the electrochemical potential of the dot and the equilibrium fluctuations between the source and the dot are suppressed. In this regime, the measured charge fluctuations are due to a current flowing through the dot. Electrons enter the dot from the source lead and leave the dot through the drain lead. The blocking mechanism is no longer effective and the transport process
will predominantly take place through state \( S_A \), since the tunnel coupling to the drain lead is stronger for this state \( (\Gamma_{\text{right}}^A \gg \Gamma_{\text{right}}^B) \). The transport through the dot can essentially be described by a rate equation, with one rate for electrons entering and another rate for electrons leaving the dot. For such systems, it has been shown in chapter 5.2 that the Coulomb blockade will lead to an increase in correlation between the tunneling electrons compared to a single-barrier structure, giving sub-Poissonian noise \([36, 40]\). The effect is seen for \( V_{\text{bias}} > 1.5 \text{ mV} \) in Fig 5.9(b); both the second and third cumulants are reduced compared to the first cumulant.

The value of \( T_1 = 8 \text{ ms} \) obtained from fitting the experimental data is of the same order of magnitude as previously reported values for the spin relaxation time \( T_1 \). We stress that the bunching of electrons and the super-Poissonian noise can only exist if the relaxation time is at least as long as the inverse tunneling time. This is demonstrated in Fig. 5.9(c), which shows the maximum value obtained for the ratio \( C_3/C_1 \) calculated for different \( T_1 \) while keeping the rest of the fitting parameters at the values given in the caption of Fig. 5.9.

5.4 Higher order moments and limitations of the detector

So far, we have presented measurements of the second and third cumulants or central moments. As mentioned in section 5.1, the shot noise is a direct consequence of the discreteness of the charge carriers in the system. A measurement of the second moment (Fano factor) thus provides a way to determine the charge of those discrete carriers. The third moment of a tunneling current has been shown to be independent of the thermal noise \([23, 58]\), thus making it a potential tool for investigating electron-electron interactions even at elevated temperatures.

What about the higher order moments? In strongly interacting systems, they are predicted to depend strongly on both the conductance \([59]\) and on the internal level structure \([45]\) of the system. Determining higher order moments may therefore give a more complete characterization of the electron transport process. This can be of importance for realizing measurements of electron correlation and entanglement effects in quantum dots \([47, 48]\). In quantum optics, higher order moments are routinely measured in order to study entanglement and coherence effects of the electromagnetic field \([60]\).

In this section, we present measurements of the fourth and fifth cumulant of the distribution function for charge transport through a QD. As demonstrated in section 5.2, we determine the cumulants by first generating the experimental probability density function \( p_{t_0}(N) \). This is done by splitting a time trace of length \( T \) into \( m = T/t_0 \) intervals and counting the number of electrons entering the dot within each interval. The higher cumulants describe more subtle features of the distribution function. To extend the methods of section 5.2 to higher cumulants, it is therefore necessary to increase the measurement time to collect more statistics. This requires
5.4. Higher order moments and limitations of the detector

A stable sample without any fluctuating charge traps close to the QD.

In the experiment, we use a single QD (sample B in Appendix A) with the same design as the one described in section 4 and section 5.2 (sample A). The coupling between the QD and the QPC was weaker in sample (B) compared to sample A, meaning that the bandwidth had to be reduced below 10 kHz. On the other hand, the stability of the structure allowed the measurement of time traces of length $T = 10$ minutes. In the experiment, the QPC was voltage biased with $V_{QPC} = 250 \mu V$. The current signal was sampled at 100 kHz, software filtered at 4 kHz using an 8th order Butterworth filter and finally resampled at 20 kHz in real-time to keep the amount of data manageable.

![Figure 5.10](image)

Figure 5.10: (a-d) Normalized cumulants $C_n/C_1$ versus dot asymmetry, $a = (\Gamma_{in} - \Gamma_{out})/(\Gamma_{in} + \Gamma_{out})$. The solid lines are theoretical predictions assuming a perfect detector, $C_2/C_1 = (1 + a^2)/2$, $C_3/C_1 = (1 + 3a^4)/4$, $C_4/C_1 = (1 + a^2 - 9a^4 + 15a^6)/8$ and $C_5/C_1 = (1 + 30a^4 - 120a^6 + 105a^8)/16$. The dashed lines show the cumulants calculated from the model defined by Eq. (5.24) in the text. The inset in (c) shows the variation of the total tunneling rate $\Gamma_{tot} = \Gamma_{in} + \Gamma_{out}$ for the different measurement points.

The results are shown in Fig. 5.10, where we plot the normalized cumulants for different values of the asymmetry of the tunneling rates, $a = (\Gamma_{in} - \Gamma_{out})/(\Gamma_{in} + \Gamma_{out})$. The asymmetry is tuned by shifting the voltage on gate $G_1$ by an amount $\Delta V$ and at the same time applying a compensating voltage $-\Delta V$ on gate $G_2$. With the two
gates having a similar lever arm on the dot, the electrochemical potential of the QD remains at the same level, but the height of the tunneling barriers between the dot and the source and drain leads will change. Doing so, we could tune the asymmetry from $a = -0.94$ to $a = +0.25$ while still keeping both tunneling rates within the measurement bandwidth and avoiding charge rearrangements. To get data for the full range of asymmetry, we did a second measurement at a different gate voltage configuration. For the second set of data, the asymmetry was tuned from $a = 0.07$ to $a = 0.93$. The stars and the circles in Fig. 5.10 represent data from the two different sets of measurements. The measurements were performed with a QD bias of $V_{\text{bias}} = 2.5 \, \text{mV}$, with the electrochemical potential of the dot far away from the Fermi levels of the source and drain leads. This is to ensure that tunneling due to thermal fluctuations is sufficiently suppressed.

The solid lines in Fig. 5.10 depict the theoretical predictions calculated from a two-state model [55]. The analytical expressions are given in the figure caption. The higher cumulants show a complex behavior as a function of the asymmetry, with local minima at $a = \pm 0.6$ for $C_4/C_1$ and at $a = \pm 0.8$ for $C_5/C_1$. The fifth cumulant even becomes negative for some configurations. The experimental data qualitatively agrees with the theory, but for small values of the asymmetry there are deviations from the expected behavior. The deviations are stronger for the first set of data (stars). Since the tunneling rates in the first measurement was about a factor of three higher than in the second measurement [see inset of Fig. 5.10(c)], we suspect the finite bandwidth of the detector to be a possible reason for the discrepancies.

In general, experimental measurements of FCS for electrons are difficult to achieve due to the need of a sensitive, high-bandwidth detector capable of resolving individual electrons [22, 61, 62]. However, a more fundamental complication with the measurements is that most forms of the FCS theory assume the existence of (1) a detector with infinite bandwidth and (2) infinitely long data traces. Since no physical detector or experiment can fulfill these requirements, every experimental realization of the FCS will measure a distribution which is influenced by the properties of the detector. In the following, we investigate how the violation of the two assumptions modifies the measured statistics.

Naaman et al. [20] pointed out that measurements of the transition rates of a Poisson two-state system using a finite bandwidth detector always leads to an underestimate of the rates. As a result, the measured probability distribution for the times needed for an electron to tunnel into or out of the QD no longer follow the expected exponential $p_{\text{in/out}}(t) = \Gamma_{\text{in/out}} \exp (-\Gamma_{\text{in/out}} t)$. Due to the finite detection time, very fast tunneling events are less likely to be detected, giving a cut-off for short time scales in the measured distribution. Moreover, since the fast events are not detected, the measurement will over-estimate the occurrence of slow events. The long-time tail of the measured distribution will still decay exponentially, but the tunneling rate extracted from the distribution will be under-estimated. To determine the rates correctly, the detection rate $\Gamma_{\text{det}}$ of the detector must be taken into account [20].
5.4. Higher order moments and limitations of the detector

Figure 5.11: (a) Probability density of time needed for an electron to tunnel into the dot. Note the sharp decrease in counts for \( t < 100 \mu s \) due to the finite bandwidth of the detector. The black curve is a long-time exponential fit with \( \Gamma = 1.39 \text{ kHz} \). (b) Model for the dot-detector system. A state \((n, m)\) corresponds to \( n \) electrons on the dot while the detector at the same time is measuring \( m \) electrons.

An example of a probability distribution taken from measured data is shown in Fig. 5.11(a). The long-time behavior is exponential, but for times \( t < 100 \mu s \) there is a sharp decrease in the number of counts registered by the detector. From the figure, we can estimate \( \tau_{\text{det}} = 1/\Gamma_{\text{det}} \), which is the average time it takes for the detector to register an event. We find \( \tau_{\text{det}} = 70 \mu s \), giving a detection rate of \( \Gamma_{\text{det}} = 1/\tau_{\text{det}} = 14 \text{ kHz} \). Note that the detection rate \( \Gamma_{\text{det}} \) does not only depend on the measurement bandwidth but also on the signal-to-noise ratio of the detector signal as well as the redundancy needed to minimize the risk of detecting false events [63]. The compensations for the tunneling rates are given as [20]

\[
\Gamma_{\text{in}} = \Gamma_{\text{in}}^* \frac{\Gamma_{\text{det}}}{\Gamma_{\text{det}} - \Gamma_{\text{in}}^* - \Gamma_{\text{out}}^*}, \tag{5.22}
\]

\[
\Gamma_{\text{out}} = \Gamma_{\text{out}}^* \frac{\Gamma_{\text{det}}}{\Gamma_{\text{det}} - \Gamma_{\text{in}}^* - \Gamma_{\text{out}}^*}. \tag{5.23}
\]

Here, \( \Gamma_{\text{in/out}}^* \) are the true tunneling couplings and \( \Gamma_{\text{in/out}}^* = 1/\langle \tau_{\text{in/out}} \rangle \) are rates extracted from the measurement. All tunneling rates presented in the following have been extracted using Eqs. (5.22-5.23) with \( \Gamma_{\text{det}} = 14 \text{ kHz} \).

The finite bandwidth will also influence the FCS measured by the detector. Following the ideas of Ref. [20], we account for the finite bandwidth by including the states of the detector into the two-state model of section 5.2.1. Figure 5.11(b) shows the four possible states of the combined dot-detector model. The state \((n+1, n)\) refers to a situation where there are \( n+1 \) electrons on the dot, while the detector at the same time reads \( n \) electrons. The transition from the state \((n+1, n)\) to the state \((n+1, n+1)\) occurs when the detector registers the electron. This process
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occurs with the rate of the detector, $\Gamma_{\text{det}}$.

To calculate the FCS for the QD-detector system, we write the master equation $\dot{P} = M P$, with $P = [(n, n), (n + 1, n), (n, n + 1), (n + 1, n + 1)]$ and

$$M_x = \begin{bmatrix}
-\Gamma_{\text{in}} & \Gamma_{\text{out}} & \Gamma_{\text{det}} & 0 \\
\Gamma_{\text{in}} & -(\Gamma_{\text{out}} + \Gamma_{\text{det}}) & 0 & 0 \\
0 & 0 & -(\Gamma_{\text{in}} + \Gamma_{\text{det}}) & \Gamma_{\text{out}} \\
0 & \Gamma_{\text{det}} e^{i\chi} & \Gamma_{\text{in}} & -\Gamma_{\text{out}}
\end{bmatrix}. \quad (5.24)$$

In the above matrix, we have included the counting factor $e^{i\chi}$ at the element where the detector registers an electron tunneling into the dot [see dashed circle in Fig. 5.11(b)]. The statistics obtained in this way relates directly to what is measured in the experiment. Using the methods of Ref. [55], we calculate the first few cumulants for the above expression as a function of relative bandwidth $k = \Gamma_{\text{det}}/(\Gamma_{\text{in}} + \Gamma_{\text{out}})$ and asymmetry $a = (\Gamma_{\text{in}} - \Gamma_{\text{out}})/(\Gamma_{\text{in}} + \Gamma_{\text{out}})$. The normalized second and third cumulants take the form

$$C_2/C_1 = \frac{1 + a^2}{2} - \frac{k(1 - a^2)}{2(1 + k)^2}, \quad (5.25)$$

$$C_3/C_1 = \frac{1 + 3a^4}{4} - \frac{3k(1 + k + k^2)}{4(1 + k)^4} - \frac{6a^2k^2}{4(1 + k)^4} + \frac{3a^4k(1 + 3k + k^2)}{4(1 + k)^4}. \quad (5.26)$$

In Fig. 5.12(a) we plot the second and third cumulants from Eq. (5.25) and Eq. (5.26) for different values of asymmetry $a$ and relative bandwidth $k$. The cumulants have been normalized to the values for the infinite bandwidth detector.

Fig. 5.12(b) shows the corresponding results for the forth and fifth cumulants. With $\Gamma_{\text{det}} \gg \Gamma_{\text{in}} + \Gamma_{\text{out}}$, the cumulants approach the infinite bandwidth result, as expected. However, even with $\Gamma_{\text{det}} = 10(\Gamma_{\text{in}} + \Gamma_{\text{out}})$ and perfect symmetry ($a = 0$), the second cumulant deviates by almost 10% and the third cumulant by more than 20% from the perfect detector values. As the bandwidth is further decreased, the deviations grow stronger and reach a maximum as $\Gamma_{\text{det}} = \Gamma_{\text{in}} + \Gamma_{\text{out}}$. With $\Gamma_{\text{det}} \ll \Gamma_{\text{in}} + \Gamma_{\text{out}}$, the cumulants once again approach the perfect detector values. When the detector is much slower than the underlying tunneling process, it will only sample the average population of the two states. In this limit, the dynamics of the system does not interfere with the dynamics of the detector and we recover the correct relative noise levels. It should be noted that this is true only for the noise relative to the detected mean current. Since the detector will miss most of the tunneling events, the absolute values of both the current and the noise will be underestimated.

Over the full range of bandwidth and asymmetry, we find that the noise detected with the finite bandwidth system is always lower than for the ideal detector case. The reduction can be qualitatively understood by considering the probability distribution $p_{\text{no}}(N)$. The finite bandwidth makes it less probable to detect fast events, meaning
5.4. Higher order moments and limitations of the detector

Figure 5.12: Higher cumulants versus relative detection bandwidth $\Gamma_{\text{det}}/(\Gamma_{\text{in}} + \Gamma_{\text{out}})$, calculated from the model in Fig. 5.11(b). The cumulants are normalized to the results from the infinite-bandwidth case. The influence of the finite bandwidth is maximal when the asymmetry $a = (\Gamma_{\text{in}} - \Gamma_{\text{out}})/(\Gamma_{\text{in}} + \Gamma_{\text{out}})$ is zero.

that the probability of detecting a large number of electrons within the interval $t_0$ will decrease more than the probability of detecting few electrons. This will cut the high-count tail of the distribution and thereby reduce its width ($C_2$) and its skewness ($C_3$). An interesting feature is that the cumulants calculated for a less symmetric configurations [$a = 0.9$ in Fig. 5.11(c)] show less influence of the finite bandwidth.

A second limitation of a general FCS measurement is the finite length of each time trace. In order to generate the experimental probability density function $p_{t_0}(N)$, the total trace of length $T$ must be split into $m = T/t_0$ intervals, each of length $t_0$. Most FCS theories only predict results for the case $t_0 \gg 1/\Gamma$, where $\Gamma$ is a typical transition rate of the system. In the experiment, it is favorable to make $t_0$ as short as possible in order to increase the number of samples $m = T/t_0$. This will improve the quality of the distribution and help to minimize statistical errors. However, if $t_0$ is made too short, this will influence the extracted distribution. This is visualized in Fig. 5.13, where distribution functions for different $t_0$ are extracted from the same set of experimental data. The distributions give the same current $I = e\langle N \rangle/t_0$, but their properties are clearly different. In the extreme case of $t_0 \ll 1/\Gamma (\langle N \rangle \ll 1)$, the
distribution approaches the Bernoulli distribution, for which only $p_{t_0}(0)$ and $p_{t_0}(1)$ are non-zero.

The condition $t_0 \gg 1/\Gamma$ is imposed by the approximation that the cumulant generating function (CGF) $S(\chi)$ for $p_{t_0}(N)$ only depends on the lowest eigenvalue $\Lambda_{\min}$ of the master equation matrix $M_\chi$, with $S(\chi) = -t_0 \Lambda_{\min}$. A FCS valid for finite $t_0$ must include all eigenvalues and eigenvectors of $M_\chi$ \[55\]. The corresponding expression is

$$ \exp[S(\chi)] = \langle q_0| p^{(n)} \rangle \exp(-t_0 \Lambda_n)\langle q^{(n)}| p_0 \rangle, \tag{5.27} $$

where $\langle q^{(n)}| \rangle$ and $| p^{(n)} \rangle$ are the left and right eigenvectors of the matrix $M_\chi$, $\Lambda_n$ are the eigenvalues of $M_\chi$ and $\langle q_0|$, $| p_0 \rangle$ are the eigenvectors corresponding to the lowest eigenvalue $\Lambda_{\min}$. The cumulants generated from the CGF in Eq. (5.27) will in general be a function of $t_0$.

To investigate how small $t_0$ can be before systematic errors become relevant, we calculate the cumulants from the CGF of Eq. (5.27) with the master equation matrix $M_\chi$ of Eq. \[54\]. The results are shown in Fig. 5.14, where we plot the normalized cumulants as a function of the mean number of counts per interval, $\langle N \rangle = t_0/(1/\Gamma_{\text{in}} + 1/\Gamma_{\text{out}})$. The symbols show cumulants extracted from measured data ($T = 10$ minutes, $a = 0.053$, $\Gamma_{\text{in}} + \Gamma_{\text{out}} = 3062$ Hz and $\Gamma_{\text{det}} = 14$ kHz), while the solid lines are results from the CGF for the same set of parameters. The dashed lines are the asymptotes for the limiting case $t_0 \to \infty$.

In general, the data and the theory are in good agreement. There are some deviations in the fourth and fifth cumulants for large $t_0$ ($\langle N \rangle > 6$ in Fig. 5.14), but these are statistical errors in the experiment due to the shortness of the total time trace. For short $t_0$, all cumulants converge to $C_n/C_1 \to 1$. This is because as $\langle N \rangle \ll 1$, the probability distribution $p_{t_0}(N)$ will be non-zero only for $N = 0$ and $N = 1$, with $p_{t_0}(0) = 1-q$, $p_{t_0}(1) = q$ and $q = \langle N \rangle$. This is the definition of a Bernoulli distribution, for which the normalized cumulants $C_n/C_1 \to 1$ as $q \to 0$ \[64\].

Focusing on the other regime, $\langle N \rangle > 1$, we see that cumulants of different orders...
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Figure 5.14: Normalized cumulants evaluated for different lengths of the time interval $t_0$. The symbols show the experimental data, extracted from a time trace of length $T = 10$ minutes, containing 350595 events, with $a = 0.053$, and $\Gamma_{\text{tot}} = 3062$ Hz. The solid lines are calculations from the FCS given by Eq. (5.27) in the text, while the dashed lines are the asymptotes for $t_0 \to \infty$. The inset shows a magnification of the vertical axis (horizontal axis unchanged) for $C_4/C_1$ and $C_5/C_1$ for $\langle N \rangle > 0.6$.

converge to their asymptotic limits for different values of $t_0$. The second cumulant needs a longer interval $t_0$ to reach a specified tolerance compared to the higher cumulants. This is of interest for the experimental determination of higher cumulants. By choosing a shorter value of $t_0$ when calculating higher cumulants, the amount of samples $m = T/t_0$ can be increased. For the data in Fig. 5.10, the cumulants were calculated with intervals $t_0$ giving $\langle N \rangle = 15$ for $C_2$, $\langle N \rangle = 6$ for $C_3$, $\langle N \rangle = 3$ for $C_4$ and $\langle N \rangle = 2$ for $C_5$. The maximal deviations between the correct cumulants and the ones determined with a finite length $t_0$ can be estimated by checking the convergence for all values of the asymmetry. For the data shown in Fig. 5.10, we find $\Delta C_2/C_1 = 0.007$, $\Delta C_3/C_1 = 0.009$, $\Delta C_4/C_1 = 0.01$ and $\Delta C_5/C_1 = 0.03$.

Coming back to the results of Fig. 5.10, we are now able explain why the measured cumulants show lower values compared to the perfect-detector theory. The dashed lines in Fig. 5.10 are the cumulants calculated from the combined QD-detector model of Eq. (5.24), with $\Gamma_{\text{det}} = 14$ kHz. The overall agreement is good, especially since no fitting parameters are involved. Higher cumulants end up to be slightly lower than theory predicts. We speculate that the deviations could be due to low-frequency fluctuations of the tunneling rates over the time of measurement.
5.5 Measurement precision

In this section we investigate the precision possible to achieve with a current meter based on single-electron counting. For this purpose, we assume a QD in the high-bias regime with a single state available for transport, i.e., the model defined by Eq. (5.7) in section 5.2.1. As derived in section 5.2.1, the current $I$ and the shot noise are

$$I = e \frac{\Gamma_{\text{in}} \Gamma_{\text{out}}}{\Gamma_{\text{in}} + \Gamma_{\text{out}}},$$

$$S_I = 2e^2 \frac{\Gamma_{\text{in}} \Gamma_{\text{out}} (\Gamma_{\text{in}}^2 + \Gamma_{\text{out}}^2)}{(\Gamma_{\text{in}} + \Gamma_{\text{out}})^3}.$$  

When counting electrons passing through the QD, we use the tunneling electrons to probe the tunnel couplings $\Gamma_{\text{in}}/\Gamma_{\text{out}}$. Since tunneling is a statistical process, it involves a certain degree of randomness and we need to detect an ensemble of electrons in order to be able to form the average $\Gamma_{\text{in}/\text{out}} = 1/\langle \tau_{\text{in}/\text{out}} \rangle$. The statistical variations of the tunneling times imply that there is a relation between the duration and the precision of the measurement. More precisely, assuming that the tunneling rates $\Gamma_{\text{in}}/\Gamma_{\text{out}}$ in Eqs. (5.28-5.29) are constant, for how long is it necessary to measure in order to reach a certain precision in the current or the noise level? This is investigated in the following section. The theoretical findings are then compared with experimental results.

5.5.1 Theoretical precision

In the single-level regime, the process of an electron tunneling into or out of the dot is described by the rate equation

$$\dot{p}_{\text{in}/\text{out}}(t) = -\Gamma_{\text{in}/\text{out}} \times p_{\text{in}/\text{out}}(t).$$  

Here, $p_{\text{in}/\text{out}}(t)$ is the probability density for an electron to tunnel into or out of the dot at a time $t$ after a complementary event. Since the expressions for electrons entering and leaving the dot are the same, we drop the subscripts (in/out) and use the notations $p(t)$ and $\Gamma$ to describe either one of the two processes. Solving the differential equation and normalizing the resulting distribution gives

$$p(t)dt = \Gamma e^{-\Gamma t}dt.$$  

In the experiment, we measure a time trace containing a sequence of tunneling times $\tau_k, k = 1, 2, 3, \ldots$. To estimate $\Gamma$ and its relative accuracy from such a sequence, we need to calculate the probability distribution for extracting a certain value $\Gamma$, given a fixed sequence of tunneling times. We start by dividing the time axis into bins of width $\Delta \tau$ and number them with $i = 0, 1, 2, \ldots$ A tunneling event $\tau_k$ will be counted in bin $i$ if $i\Delta \tau \leq \tau_k < (i+1)\Delta \tau$. Using Eq. (5.31) and assuming $\Delta \tau \ll 1/\Gamma$, we find that the probability to get a count in bin $i$ for a given value of $\Gamma$ is equal to

$$p(i|\Gamma) = \Gamma \Delta \tau e^{-\Gamma \Delta \tau i}.$$  

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A certain sequence \( \{i_n\} \) is realized with probability

\[
p(\{i_n\}|\Gamma) = \prod_{n=1}^{N} \Gamma \Delta \tau e^{-\Gamma \Delta \tau i_n} = (\Gamma \Delta \tau)^N e^{-\Gamma \Delta \tau \sum_{n=1}^{N} i_n}
\]

\[
= (\Gamma \Delta \tau)^N e^{-\Gamma \Delta \tau \sum_{i=0}^{\infty} n_i}
\]

\[
= (\Gamma \Delta \tau)^N e^{-\Gamma \Delta \tau N \langle i \rangle}.
\]

(5.33)

Here, \( n_i \) is the number of times an event falls into bin \( i \), \( \sum_{i=0}^{\infty} n_i = N \) is the total number of events in the trace and \( \langle i \rangle = \frac{1}{N} \sum_{i=0}^{\infty} n_i i \) is the average of \( i \). A certain set of bin occupations \( \{n_i\} \) can be achieved with many different \( \{i_n\} \)-series, namely \( \frac{N!}{\prod_{i=0}^{\infty} n_i!} \). Assuming that they all occur with the same probability \( p(\{i_n\}|\Gamma) \), we find

\[
p(\{n_i\}|\Gamma) = \frac{N!}{\prod_{i=0}^{\infty} n_i!} (\Gamma \Delta \tau)^N e^{-\Gamma \Delta \tau N \langle i \rangle}.
\]

(5.34)

This is our sampling distribution. For an estimate of \( \Gamma \) we use Bayes theorem

\[
p(\Gamma|\{n_i\}) = \frac{p(\Gamma)}{p(\{n_i\})}.
\]

(5.35)

Because we have no information on the prior probabilities \( p(\Gamma) \) and \( p(\{n_i\}) \), the principle of indifference requires them to be constants, giving

\[
p(\Gamma|\{n_i\}) = C (\Gamma \Delta \tau)^N e^{-\Gamma \Delta \tau N \langle i \rangle},
\]

(5.36)

where \( C \) is constant. Normalization \( \int_{0}^{\infty} p(\Gamma|\{n_i\}) d\Gamma = 1 \) leads to

\[
p(\Gamma|\{n_i\}) = \frac{N^{N \langle i \rangle} (N+1) \Delta \tau^N (\Gamma \Delta \tau)^N e^{-\Gamma \Delta \tau N \langle i \rangle}}{N!}
\]

\[
= \frac{N^N}{N!} \langle \tau \rangle (\langle \tau \rangle)^N e^{-N \langle \tau \rangle}.
\]

(5.37)

The most likely value of \( \Gamma \) is therefore \( \Gamma^* = 1/\langle \tau \rangle \). The relative accuracy of this estimate is given by the width of the distribution. Setting \( x = \Gamma \langle \tau \rangle \) and evaluating the width at half maximum gives

\[
x^N e^{-xN} = \frac{1}{2} e^{-N}
\]

\[
\Rightarrow \ln(x) = x - 1 - \frac{1}{N} \ln(2).
\]

(5.38)

For large \( N \) we can expand \( \ln(x) \) in a Taylor series around \( x = 1 \). Keeping only the first two terms, it follows

\[
\frac{1}{2} (x - 1)^2 = \frac{1}{N} \ln(2)
\]

\[
\Rightarrow x = 1 \pm \sqrt{\frac{2 \ln(2)}{N}}.
\]

(5.39)

Thus the relative accuracy is

\[
\Delta \Gamma/\Gamma = \sqrt{2 \ln(2)/N}.
\]

(5.40)
5.5.2 Experimental precision

In order to compare the results of Eq. (5.40) with the measurement, we take a data set \( \{t_{\text{in}}^i, t_{\text{out}}^i\} \) containing 120000 events, extracted from a trace such as the one shown in Fig. 4.1(c). The tunneling rates are \( \Gamma_{\text{in}} = 1/\langle \tau_{\text{in}} \rangle = 594 \text{ Hz} \) and \( \Gamma_{\text{out}} = 1/\langle \tau_{\text{out}} \rangle = 494 \text{ Hz} \). In the following, we choose to investigate the precision of \( \Gamma_{\text{in}} \) and drop the subscript. We have also performed the analysis for \( \Gamma_{\text{out}} \), with similar results.

To proceed, we use Eq. (5.31) to calculate the probability that a certain set of tunneling times \( \{\tau_i\} \) belongs to a physical process characterized by the tunnel coupling \( \Gamma \)

\[
p(\Gamma|\{\tau_i\}) = \prod_{i=1}^{N} \Gamma \tau_i e^{-\Gamma \tau_i}. \tag{5.41}
\]

In Fig. 5.15 we plot the probability distributions of Eq. (5.41) for subsets of \( \{\tau_i\} \) with different lengths \( N \). As the size of the subset is increased, the probability distribution gets focused around \( \Gamma = \Gamma_{\text{in}} = 594 \text{ Hz} \). This simply reflects the fact that the larger the amount of experimental evidence available, the less likely it becomes that the data is generated by a tunneling process with \( \Gamma \neq \Gamma_{\text{in}} \).

The experimental uncertainty in \( \Gamma \) is given by the width of the distributions in Fig. 5.15. Figure 5.16 shows the normalized uncertainty \( \Delta \Gamma/\Gamma \) versus subset size \( N \). The solid line is the result of Eq. (5.40), showing very good agreement with the experimental data. The results validate Eq. (5.31) and demonstrate the stability.
of the sample; a sudden change in the tunnel coupling $\Gamma$ during the relatively long measurement time of 10 minutes would introduce deviations between Eq. (5.40) and the measured precision. For the full data set $N = 120000$, we find $\Gamma_{\text{in}} = 593.8 \pm 1.7$ Hz and $\Gamma_{\text{out}} = 494.2 \pm 1.4$ Hz.

For simplicity, we have assumed a perfect detector with infinite bandwidth. We have also performed the analysis for a model incorporating the detector bandwidth as explained in section 5.4, and we obtain very similar results. The analysis is slightly more involved since a tunnel coupling $\Gamma_{\text{in}}$ will depend not only on the set $\{\tau_{\text{in}}^i\}$, but also on $\{\tau_{\text{out}}^i\}$.

### 5.5.3 Current meter precision

Knowing the precision of the tunneling rates $\Gamma_{\text{in}}/\Gamma_{\text{out}}$, we use the relations in Eqs. (5.28-5.29) to determine the precision of the current and the noise. For the data set with $N = 120000$ discussed in the previous section, we find

$$I = (292.87 \pm 0.64) \text{ e/s} = (46.917 \pm 0.10) \text{ aA.} \quad (5.42)$$

The shot noise of the current is equal to

$$S_I = (7.5772 \pm 0.017) \times 10^{-36} \text{ A}^2/\text{Hz.} \quad (5.43)$$
Conventional measurement techniques are usually limited by the current noise of the amplifiers (typically $10^{-29}$ $A^2$/Hz) [32, 33, 37, 39]: here we demonstrate a measurement of the noise power with a sensitivity better than $10^{-37}$ $A^2$/Hz. The limits in precision investigated here are not due to a measurement apparatus but appears because of the discreteness of charge; the precision of the shot noise measurement is limited by the shot noise itself. In the experiment more uncertainty occurs if (1) the correction for the finite bandwidth in Eq. (5.22) is incorrect or (2) because of the detection of false events due to an insufficient signal-to-noise ratio in the measurement of the QPC conductance (see section 3.3.1).

5.6 Conditional statistics

In this section we investigate the noise of the combined QD-QPC system of Fig. 4.1(a). The ideas presented here follow from theoretical work by Eugene Sukhorukov from University of Geneva, Switzerland and Andrew Jordan from University of Rochester, New York. The work was published in Ref. [65]. In the following, we present the main results of the paper together with experimental data. For a detailed discussion of the model and a derivation of the theoretical results, the reader is kindly referred to Ref. [65]. Similar analysis can also be found in Ref. [66].

5.6.1 Model

The QPC current as shown in Fig. 4.1(c) is a typical example of telegraph noise, where there are random switchings between two stable states [67]. Telegraph noise is a common phenomenon in nature; it can originate from such diverse origins as thermal activation of an unstable impurity [68–70], nonequilibrium activation of a bistable system [71–74], switching of magnetic domain orientation [75–77], or a reversible chemical reaction in a biological ion channel [78]. In the case of the QD-QPC structure, the switching of the QPC current occurs because of the electron tunneling through the QD. Therefore, the shot noise of the QD current $J$ (randomness in the number of switches in a given time interval) is intimately linked with telegraph noise in the QPC current $I$ (randomness of duration time in each current value) [65, 66].

To model the combined QD-QPC system, the detector signal is assumed to show switching between two noiseless values $I_{1,2}$. The two levels are identified with the two current levels experienced by the QPC. When an electron enters the QD, the current switches from $I_1$ to $I_2$, and when the electron leaves the QD, the reverse switching happens. The number of “down” or “up” switching events $M$ in a given time trace of duration $t$, is identified with the number of different transport electrons that occupy the QD in that time interval, naturally defining a QD current variable $J = M/t$ (we set $e = 1$ to count in single electron charge units). The analogous number of electrons $N$ passed by the QPC in this same time interval defines the QPC current variable $I = N/t$. The assumption of noiseless current levels implies that $I_1 \leq I(t) \leq I_2$. 

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while the unidirectional nature of the QD transport implies that $0 \leq J(t) < \infty$. Stochastic, statistically independent quantum tunneling into and out of the QD is described with rates $\Gamma_{1,2}$. For convenience, we define the average and difference variables $I_0 = (I_1 + I_2)/2$, $\Gamma_0 = (\Gamma_1 + \Gamma_2)/2$, $\Delta I = (I_2 - I_1)/2$, $\Delta \Gamma = (\Gamma_2 - \Gamma_1)/2$. Here, $\Gamma_1$ and $\Gamma_2$ are the tunneling rates of the left and right lead.

The current in the QPC and the QD can be characterized by their own probability distributions $P(I,t), P(J,t)$ of finding a given number of electrons transmitted in a given time, or equivalently, all current cumulants $\langle I^n \rangle, \langle J^m \rangle$. For example, the first two cumulants are the average current $\langle I \rangle$, and the shot noise power $\langle I^2 \rangle = \int dt \langle \delta I(t) \delta I(0) \rangle$, where $\delta I = I(t) - \langle I \rangle$. For the QPC and QD, the average current $\langle I \rangle, \langle J \rangle$ and shot noise power $\langle I^2 \rangle, \langle J^2 \rangle$, are given respectively by

$$\langle I \rangle = (I_1 \Gamma_2 + I_2 \Gamma_1)/(2\Gamma_0), \quad \langle I^2 \rangle = (\Delta I)^2 \Gamma_1 \Gamma_2 / \Gamma_0^3,$$
$$\langle J \rangle = \Gamma_1 \Gamma_2 / (2\Gamma_0), \quad \langle J^2 \rangle = \langle J \rangle (\Gamma_1^2 + \Gamma_2^2) / (4\Gamma_0^2).$$

(5.44)

By describing the QD and the QPC using two separate distributions as in Eq. (5.44), we miss the important fact that the two conductors are strongly correlated by the Coulomb interaction between them. In order to specify the statistical correlation between the conductors, one needs to introduce the joint counting statistics of both conductors. More specifically, the correlations may be quantified by the joint probability distribution $P(I,J,t)$ of finding current $I$ and current $J$ in a time $t$ (equivalently, all cross cumulants $\langle I^n J^m \rangle$), or may also be specified by the conditional distribution functions $P(I|J)$ or $P(J|I)$, the probability of observing one current, given an observation of the other. These distributions are all related to one another by $P(I,J) = P(I|J)P(J) = P(J|I)P(I)$, where the last equality is an expression of Bayes’ theorem. The joint generating function is given by

$$H(\lambda, \chi) = \lambda I_0 - \Gamma_0 + \sqrt{(\lambda \Delta I - \Delta \Gamma)^2 + \Gamma_1 \Gamma_2 \exp \chi}.$$  

(5.45)

The results in Eq. (5.44) follow from Eq. (5.45). The function $H(0, \chi)$ generates the current cumulants of the QD [55], while $H(\lambda, 0)$ generates the current cumulants of the QPC [79].

### 5.6.2 Statistical distribution of the detector current

To confirm the validity of the model in Eq. (5.45), we compare it against experimental data from two separate configurations. The data in configuration A is characterized by $\Gamma_1 = 160 \text{ Hz}$, $\Gamma_2 = 586 \text{ Hz}$, and configuration B is characterized by $\Gamma_1 = 512 \text{ Hz}$, $\Gamma_2 = 345 \text{ Hz}$. For each configuration, we collected traces of length $T = 700 \text{ s}$, containing around $10^8$ tunneling events. The conductance of the QPC was tuned close to $0.5 \times 2e^2/h$. We apply a dc bias voltage between the source and the drain of the QPC, $V_{QPC} = 250 \mu V$, and measure the current through the QPC. The current signal was digitized with a sampling frequency of 100 kHz and software filtered at 4 kHz using a 8th order Butterworth filter.
The statistics of the QD current was investigated in section 5.2, here we start by looking at the statistics of only the QPC current. Assuming the stochastic switching between the two levels to be characterized by the rates $\Gamma_1, \Gamma_2$, the detector current distribution $P(I)$ follows from Eq. (5.45) and is predicted to follow an elliptical shape [79]:

$$\log P(I)/t = - (\mathcal{G}_1 - \mathcal{G}_2)^2/(2\Delta I),$$

(5.46)

where $\mathcal{G}_{1,2} = \sqrt{\Gamma_{1,2}|I - I_{2,1}|}$. This prediction is experimentally confirmed in Fig. 5.17 for data sets A (blue asterisks) and B (red boxes). The overall agreement is very good, especially considering that the theoretical curves do not contain any fitting parameters.

### 5.6.3 Joint distribution function

The joint generating function (5.45) directly gives the joint probability distribution $P(I, J)$ of measuring current $I$ and current $J$. The logarithm of this distribution has been measured and is given in Fig. 5.18(a, c) for configurations A and B respectively. The theoretical prediction for this quantity is given in Fig. 5.18(b, d) with striking agreement. The experimental probability distributions were generated by splitting the data into a large number of subtraces, each containing on average seven tunneling events.
5.6. Conditional statistics

Having described the joint statistical properties of both currents, we now return to the detection question. It is important to distinguish between physical backaction and statistical/informational backaction. The quasi-noninvasive QPC detector changes its physical current state depending on whether or not the QD is occupied by an extra electron, while the physical dynamics of the QD is unaffected by the state of QPC. However, by observing a particular outcome of the detector variable \( I \), this leads to conditional (Bayesian) backaction of the detector on the system variable \( J \). This informational backaction is introduced in the concept of conditional counting statistics: The statistical current fluctuations of one system, given the observation of a given current in the other. These statistics may be calculated.

Figure 5.18: The logarithm of the joint probability distribution of detecting QPC current (x-axis) and QD current (y-axis) is given as a color density plot, where red indicates high probability, and blue indicates low probability. (a, c) Experimental construction for data set A and B respectively. The experimental probability distributions were generated by splitting the data into a large number of subtraces, each containing on average seven tunneling events. (b, d) Theoretical prediction for configurations A and B respectively.
Chapter 5. Statistics of electron transport

Figure 5.19: (a) Conditional QD current $\langle J \rangle_c$ plotted as a function of $I$. (b) Conditional QD noise $\langle \langle J^2 \rangle \rangle_c$ plotted as a function of $I$. Solid line is theory from Eq. (5.47) describing universal semi-circles. Solid blue dots and open red squares denote data set A and B respectively. (c) Conditional QPC current $\langle I \rangle_c$ plotted as a function of $J$. (d) Conditional QPC noise $\langle \langle I^2 \rangle \rangle_c$ plotted as a function of $J$. Solid line in (c) and (d) is theory, given by Eq. (5.48). In (d) the horizontal dashed lines indicate the unconditional noise level, illustrating the effect of "conditional noise enhancement" for data set B described in the text. The experimental statistics were generated by splitting the data into a large number of subtraces, each containing on average four tunneling events. The error bars show the standard error, evaluated by dividing the total amount of data into 10 subsets and calculating the moments for each subset individually.

from the joint generating function (5.45), giving the (normalized) conditional statistics of the QD, given the observation of a current $I$,

$$\langle J^n \rangle_c = \Omega / 2^n, \text{ with } \Omega = \frac{\sqrt{\Gamma_1 \Gamma_2}}{\Delta I} \sqrt{(I - I_1)(I_2 - I_1)}.$$  \hfill (5.47)

In Fig. 5.19(a, b) we compare the experimental values of the first two conditional cumulants to the theoretical results of Eq. (5.47), with excellent agreement.

Turning the perspective around, we can pose similar questions about the conditional detector statistics, given an observation of the system current $J$. The condi-
5.6. Conditional statistics

tional current $\langle I \rangle_c$, and the conditional noise $\langle I^2 \rangle_c$ are given by

$$
\langle I \rangle_c = I_0 - \frac{\Delta I \Delta \Gamma}{J + S}, \quad \langle I^2 \rangle_c = \frac{(\Delta I)^2 J}{S(J + S)},
$$

where $S = \sqrt{J^2 + (\Delta \Gamma)^2}$. These conditional cumulants are evaluated from experiment and compared with Eq. (5.48) in Fig. 5.19(c, d). As a function of the QD current $J$, the conditional current tends to either $I_1$ or $I_2$ as $J \to 0$, depending on the sign of $\Delta \Gamma$. This corresponds to the most likely detector current configuration in the case of no switches observed: the QPC current stays on one value, also implying that the system becomes noiseless in this limit. This is easily seen in (5.48) because $\langle I^2 \rangle_c$ is proportional to $J$. The exception to this rule is the perfectly symmetric situation $\Gamma_1 = \Gamma_2$, where the QPC conditional average current is $I_0$. This situation corresponds to rare symmetric switching between the states, whose effective rate is the conditional QD current $J$. The corresponding QPC conditional noise actually diverges in this limit, because the effective switching rate is vanishing. We refer this effect, where the noise in one system (monitored by another) can be dramatically larger than the unmonitored noise, as conditional noise enhancement. The same effect persists in the asymmetric situation, and the maximum of the conditional noise occurs at $J^2 = (\Delta \Gamma)^2(\sqrt{5} - 1)/2$. For the conditional noise peak to exceed the unconditional noise, the ratio $R = |\Delta \Gamma| \Gamma_1 \Gamma_2 / \Gamma_3^3$ must be less than $[(\sqrt{5} - 1)/2]^{5/2} \approx 0.3$. For data set A and B, $R_A \approx 0.38$ and $R_B \approx 0.19$, so only data set B exhibits conditional noise enhancement. In the opposite limit, $J \to \infty$, the conditional current tends to $I_0$, and the noise tends to zero. This situation corresponds to rapid symmetric switching between the current states, whose effective rate is again controlled by $J$. In both limits, the typical dynamics of the telegraph process gets completely taken over by the transport condition.
Chapter 6

Double quantum dot systems

The double quantum dot is the mesoscopic analogue of a diatomic molecule. In weakly coupled dots, the electrons are well localized within the individual dots, their wavefunctions are spatially separated and electron transport is described by sequential tunneling between discrete single-dot states. With increased interdot coupling, the single-dot wavefunctions hybridize and form molecular states extending over both dots. The ability to tune both the interdot coupling and the energy levels of the individual QDs make the double quantum dot an interesting model system for studying interactions in coupled quantum systems. In this chapter we show how to use time-resolved charge detection techniques to probe various properties of double quantum dots.

6.1 The double quantum dot

The measurements presented in this chapter were performed on the sample shown in Fig. 6.1(a). The structure is fabricated with local oxidation techniques and consists of two QDs (marked by 1 and 2 in the figure) connected by two separate tunnel barriers. For the results presented here only the upper tunnel barrier was kept open; the lower was pinched-off by applying appropriate voltages to the surrounding gates. For the purpose of this chapter, the system may be described as a standard serial double quantum dot (DQD); the ring-shape properties of the sample are investigated and utilized in chapter 8.

The DQD is coupled to source and drain leads via tunnel barriers. Several in-plane gates [marked by T, B, L and R in Fig. 6.1(a)] are used to tune the various tunnel couplings. Two quantum point contacts are located in the upper-left and lower-right parts of Fig. 6.1(a). In the measurement, it was only possible to operate the upper-left QPC as a charge detector; the one in the lower-right corner was always pinched off. The conductance of the upper-left QPC was measured by applying a bias voltage of 200 – 400 µV and monitoring the current (I_{QPC} in the figure). The QPCs were also used as in-plane gates to control the electron population in the DQD. This was achieved by applying fixed voltages \( V_{G1} \), \( V_{G2} \) to both sides of the QPCs in addition to the bias voltage.
In Fig. 6.1(b) we sketch the energy levels in the system. The QD states are coupled to source and drain leads with tunneling rates $\Gamma_S$ and $\Gamma_D$, while the interdot coupling is described by a coupling energy $t_C$. The electrochemical potentials of the two QDs are denoted by $\mu_1$ and $\mu_2$, measured relative to the Fermi levels of the source and drain leads. The next unoccupied QD states are separated by the charging energies $E_{C1}$ and $E_{C2}$.

In a first set of measurements, we operate the sample in an open regime where the tunneling rates $\Gamma_S$ and $\Gamma_D$ are large enough to allow the current through the DQD to be measured. Figure 6.2 shows the double quantum dot current $I_{DQD}$ measured versus voltages on the two gates $G1$ and $G2$. Several Coulomb resonances are visible, but the lines are relatively parallel and in more resemblance with the behavior of a single QD than with the hexagon pattern expected for a double QD [4]. Some slight bending of the lines is visible, but the coupling $t_C$ between the QDs is strong enough to delocalize the electrons over the whole structure resulting in data approaching the single QD case.

To reach a better-defined DQD configuration we apply more negative gate voltages in order to close the constrictions between QD1 and QD2. The gate voltages also influence the tunneling coupling to source and drain; as a consequence the DQD current $I_{DQD}$ drops below the measurable limit and we need to operate the charge detector to measure charge transitions in the QDs. Figure 6.3 shows the numerical derivative of the QPC current with respect to the gate voltage $V_{G2}$. A compensation voltage was applied to the QPC gate [upper-leftmost part of Fig. 6.1(a)] to keep the QPC conductance relatively constant within the gate voltage of interest. This gives the uniform light-bluish background of Fig. 6.3. On top of that there is a clear
6.1. The double quantum dot

Figure 6.2: Current through the DQD versus voltage on the two gates G1 and G2, measured with $V_{DQD-SD} = 10 \mu V$ applied between the source and drain contacts. The two QDs are strongly coupled, giving only a slight bending of the conductance resonances versus gate voltage compared to the single QD case.

hexagon pattern emerging, with all features expected from a DQD as described in section 1.4.

The numbers in brackets denote the electron population of the two QDs. The charge transitions occurring at the borders between different regions of fixed charge give rise to different changes of $dI_{QPC}/dV_{G2}$. To understand these features we first note that the QPC is asymmetrically positioned with respect to the DQD, with QD1 being much closer than QD2. Charge fluctuations in QD1 are therefore expected to give a stronger influence on the QPC conductance than fluctuations in QD2. Now, starting within the hexagon marked by $(n,m)$ and increasing $V_{G2}$ will lower the DQD potentials $\mu_1$ and $\mu_2$ and eventually allow an additional electron to enter the DQD. As the transition takes place, the QPC conductance decreases, giving a sharp peak with negative $dI_{QPC}/dV_{G2}$ in Fig. 6.3. Depending on the energy level configuration of the two QDs, the electron may enter into either QD1 or QD2. The dip in $dI_{QPC}/dV_{G2}$ is stronger for the transition $(n,m) \rightarrow (n+1,m)$ than for $(n,m) \rightarrow (n,m+1)$, reflecting the stronger coupling between the QPC and QD1.

Since the gate G2 is located closer to QD2 the gate voltage $V_{G2}$ has a larger influence on $\mu_2$ than on $\mu_1$. Increasing $V_{G2}$ may thus lead to a situation where $\mu_2 + E_{C2}$ is shifted below $\mu_1$. At the transition an electron will tunnel from QD1 over to QD2. The process takes an electron further away from the QPC, leading to an increase in QPC conductance and a positive peak in $dI_{QPC}/dV_{G2}$. The effect is clearly seen at the transition $(n+1,m) \rightarrow (n,m+1)$ in Fig. 6.3.
Figure 6.3: Numerical derivative of the QPC current with respect to the voltage on gate G2. A positive derivative reflects an increase in QPC conductance, which means that an electron is moving away from the QPC. For a negative derivative, an electron is coming closer to the QPC. The horizontal white line most likely originates from electron fluctuations of a charge trap. The numbers in the figure refer to the number of electrons in the two QDs. The data was taken with QPC bias $V_{\text{QPC}-\text{SD}} = 400 \mu V$ and zero bias across the DQD.

An interesting feature of Fig. 6.3 is that the blue lines corresponding to interdot transitions grow broader and fainter at higher gate voltages. This is a consequence of increased interdot coupling $t_C$; if the coupling is strong enough the interdot transition is smeared out over the gate voltage region where the electron is delocalized over both QDs. Measuring the width of these transitions thus provides a convenient way to determine the tunnel coupling between the two QDs that works even if the electron tunneling is occurring on timescales much faster than the detector bandwidth. The method is investigated in more detail in section 6.2.2.

### 6.2 Time-resolved detection

Next, we add time resolution to the detector and investigate the electron transitions in real-time. The tunneling rates $\Gamma_S$ and $\Gamma_D$ to the leads are tuned to values below 10 kHz, while the coupling $t_C$ is kept so large that the interdot transitions are occurring at GHz frequencies. The charge detector will therefore only resolve transitions where electrons are entering or leaving the DQD; tunneling between the QDs is much faster than the detector bandwidth.

Figure 6.4(a) shows a charge stability diagram for the DQD, measured by counting electrons tunneling into and out of the DQD. The data was taken with a bias voltage of 600 $\mu V$ applied across the DQD, giving rise to finite-bias triangles of se-
6.2. Time-resolved detection

![Figure 6.4: (a) Charge stability diagram of the DQD, measured by counting electrons entering and leaving the DQD. The data was taken with a voltage bias of $V_{DQD-SD} = 600 \mu V$ applied over the DQD. The diagrams on the right depict the DQD level configuration at different positions of the charge stability diagram. (b) Time traces of the QPC current measured at the position of the red and black points in (a). The QPC conductance was measured with $V_{QPC-SD} = 300 \mu V$. The count rates in (a) were extracted from traces of length $T = 0.5 s$ with a bandwidth of 5 kHz.](image)

Sequential transport in addition to the features present in Fig. 6.3. The diagrams to the right of the plot show schematics of the DQD energy levels for three different positions in the charge stability diagram. Depending on the level alignment, different kinds of electron tunneling are possible.

At the transition between the $(n, m)$ and $(n + 1, m)$ configuration [red point in Fig. 6.4(a)], the potential of QD1 is aligned with the Fermi level of the source lead. The tunneling at this point is due to equilibrium fluctuations between source and QD1. Using the methods of section 4.2, measuring the count rate as a function of the potential $\mu_1$ provides a way to determine both the tunneling rate $\Gamma_S$ and the electron temperature in the source lead. The situation is reversed at the transition...
between the \((n, m)\) and \((n, m + 1)\) configuration [black point in Fig. 6.4(a)]. Here, electron tunneling occurs between QD2 and the drain, thus making it possible to perform an independent measurement of \(\Gamma_D\) and the electron temperature of the drain lead.

Typical time traces of the QPC current for the two DQD configurations are shown in Fig. 6.4(b). As described in chapter 3, the transitions between the two current levels correspond to electrons tunneling into and out of the QD. The red trace corresponding to electron fluctuations in QD1 shows a larger change \(\Delta I_{QPC}\) in QPC current compared to fluctuations in QD2; again, this reflects the stronger coupling between QPC and QD1 due to the geometry of the device.

At the grey point within the triangle of Fig. 6.4(a), the levels of both QD1 and QD2 are within the bias window and the tunneling is due to sequential transport of electrons from the source lead into QD1, over to QD2 and finally out to the drain. The electron flow is unidirectional and the count rate relates directly to the current flowing through the system. As mentioned previously, the interdot transitions occur on timescales much faster than the measurement bandwidth. Therefore, a trace of the QPC conductance taken within the triangle of Fig. 6.4(a) only shows two levels, even though the electron transport cycle involves the three charge configurations \((n, m)\), \((n + 1, m)\) and \((n, m + 1)\).

### 6.2.1 Electron transport cycles

In Fig. 6.5 we investigate the charge localization more carefully by plotting the absolute change in QPC current \(\Delta I_{QPC}\) as one electron enters the DQD for the same set of data as in Fig. 6.4(a). The step height \(\Delta I_{QPC}\) was extracted from time traces similar to the ones shown in Fig. 6.4(b). As seen in the figure, the detector essentially only measures two different values of \(\Delta I_{QPC}\); either \(\Delta I_{QPC} \sim 0.3\, \text{nA}\) (blue regions of Fig. 6.5) or \(\Delta I_{QPC} \sim 0.6\, \text{nA}\) (red regions). Comparing the results of Fig. 6.5 with the sketches in Fig. 6.4(a), we see that regions with high \(\Delta I_{QPC}\) match with the regions where we expect the counts to be due to electron tunneling in QD1, while the regions with low \(\Delta I_{QPC}\) come from electron tunneling in QD2. Again, this is in agreement with the fact that the QPC is located closer to QD1 than QD2.

The regions inside the bias triangles are described in detail in the energy level diagrams of Fig. 6.5(a), together with idealized QPC conductance traces shown in Fig. 6.5(b). To simplify the discussion we neglect the bound electrons inside the DQD and consider only the excess charge population compared to the \((n,m)\) configuration. In the lower triangle, the current is carried by a sequential electron cycle. Starting with an empty DQD, an electron will tunnel in from the source lead at a rate \(\Gamma_S\) making the transition \((0, 0) \rightarrow (1, 0)\). The electron then passes on to QD2 at a rate \(\Gamma_C\) \(([1, 0] \rightarrow (0, 1))\) before leaving to drain at the rate \(\Gamma_D\) \(([0, 1] \rightarrow (0, 0)]\). Since the rate \(\Gamma_C\) is much faster than the detector bandwidth (and \(\Gamma_C \gg \Gamma_S, \Gamma_C \gg \Gamma_D\)), the detector will only register transitions between the
6.2. Time-resolved detection

Figure 6.5: (a) Change of QPC current $\Delta I_{QPC}$ as one electron enters the DQD, extracted from the same set of data as shown in Fig. 6.4. The two levels correspond to the QPC detector registering electron tunneling in QD1 and QD2, respectively. The energy level diagrams describe the hole and the electron cycle of sequential transport within the finite bias triangles. (b) Idealized QPC current traces for the hole and electron cycle shown in (a). The solid lines show the expected response of an ideal detector. For the low-bandwidth setup, the detector is not be able to resolve the time spent in $(1,0)$, giving the response shown as dashed lines in the figures.

Therefore, we expect the step height $\Delta I_{QPC}$ within the lower triangle to be equal to $\Delta I_{QPC}$ measured for electron fluctuations in QD2, in agreement with the results of Fig. 6.5(a).

For the upper triangle, the DQD holds an additional electron and the current is carried by a hole cycle. Starting with both QDs occupied $[(1,1)]$, an electron in QD2 may leave to the drain $[(1,1) \rightarrow (1,0)]$, followed by a fast interdot transition.
6.2.2 Determining the coupling energy

In the previous section we have shown that interdot transitions occur much faster than the detector bandwidth, but so far we did not try to quantify the tunnel coupling. As already mentioned, the coupling can be determined by looking at the delocalization of charge as a function of energy separation of the QD states [21]. To simplify the problem, we consider the DQD as a tunnel-coupled two-level system containing one electron, isolated from the environment [see Fig. 6.6(a)].

We introduce the basis states \( \{ \Psi_1, \Psi_2 \} \) describing the electron sitting on the left or the right QD, respectively. The two states are tunnel coupled with coupling \( t \) and separated in energy by the detuning \( \delta = \mu_1 - \mu_2 \). The Hamiltonian of the system is

\[
H = \begin{bmatrix}
-\delta/2 & t \\
t & \delta/2
\end{bmatrix}.
\]  

(6.1)

The eigenvectors of the Hamiltonian in Eq. (6.1) form the bonding \( \Psi_B \) and antibonding states \( \Psi_A \) of the system. The eigenvalues give the energies \( E_B, E_A \) of the
two states, with
\[ E_B = -\frac{1}{2} \sqrt{4t^2 + \delta^2}, \quad E_A = \frac{1}{2} \sqrt{4t^2 + \delta^2}. \] (6.2)

The energies are plotted in Fig. 6.6(b); at zero detuning, the states anticross due to the coupling energy. For a finite temperature \( T \), the system will be in a statistical mixture of the bonding and antibonding states. The occupation probabilities \( p_B \) and \( p_A \) of the two states are determined by detailed balance,
\[ p_B = 1 - \frac{1}{1 + e^\frac{E_A - E_B}{k_B T}} = 1 - \frac{1}{1 + e^\frac{\sqrt{4t^2 + \delta^2}}{k_B T}}, \]
\[ p_A = \frac{1}{1 + e^\frac{E_A - E_B}{k_B T}} = \frac{1}{1 + e^\frac{\sqrt{4t^2 + \delta^2}}{k_B T}}. \] (6.3)

In the measurement, we use the change of the QPC current (\( \Delta I_{\text{QPC}} \)) when one electron enters the DQD to determine the amount of charge localized in the individual QDs. To evaluate this quantity from Eqs. (6.1-6.3), we take the thermal population of the bonding and antibonding states and project them onto the states \( \Psi_1 \) and \( \Psi_2 \)
\[ p_1 = (p_B \Psi_B + p_A \Psi_A) \cdot \Psi_1 = \frac{1}{2} \left( 1 - \frac{\delta \tanh \left( \frac{\sqrt{4t^2 + \delta^2}}{2k_B T} \right)}{\sqrt{4t^2 + \delta^2}} \right), \]
\[ p_2 = (p_B \Psi_B + p_A \Psi_A) \cdot \Psi_2 = \frac{1}{2} \left( 1 + \frac{\delta \tanh \left( \frac{\sqrt{4t^2 + \delta^2}}{2k_B T} \right)}{\sqrt{4t^2 + \delta^2}} \right). \] (6.4)

Next, we compare the results of Eq. (6.4) with experimental data. Figure 6.7 shows the measured electron population on QD2 versus detuning, extracted from the change in QPC current \( \Delta I_{\text{QPC}} \). The signal has been normalized to the levels measured for complete localization in QD1 and QD2. The different data sets are taken for different gate voltages, demonstrating the possibility to tune the tunnel coupling. The dashed lines are fits to Eq. (6.4), showing good agreement with the data. It should be noted that this method for determining the tunneling coupling can only be used as long as the coupling is larger than the thermal broadening. For a temperature of \( T = 100 \) mK, the limit corresponds to \( t \gtrsim 10 \) \( \mu \)eV.

### 6.2.3 Excited states

The excitation spectrum of the DQD can be investigated with finite bias spectroscopy, similar to the case of a single QD discussed in section 4.5. If the coupling between the QDs is weak \( (t_C \ll \Delta E^{(1)}, \Delta E^{(2)}, \text{with} \Delta E^{(1,2)} \text{being the mean level spacing in each QD}) \), the DQD spectrum essentially consists of the combined excitation spectrum of the individual QDs. For a more strongly coupled DQD the QD states residing in different dots will hybridize and delocalize over both QDs. In
this section we consider a relatively weakly coupled configuration ($t \sim 25 \mu eV$) and assume the excited states to be predominantly located within the individual QDs.

Figure 6.8 shows a blow-up of the central triangles of Fig. 6.4(a), measured with both negative and positive bias applied across the DQD. The band-shaped regions of low-intensity counts outside the triangles are attributed to cotunneling processes and are discussed in greater detail in section 6.3. The length of each side of the triangles is related to the applied bias, meaning that the finite bias measurement provides a way to determine the characteristic energies of the sample. From the data of Fig. 6.4 and Fig. 6.8 we extract the individual QD charging energies $E_{C1} \sim 1.2 \text{meV}$, $E_{C2} \sim 1.3 \text{meV}$ and the mutual charging energy $E_{Cm} \sim 800 \mu \text{eV}$.

Excited states are visible within the triangles, especially for the case of negative bias [marked with arrows in Fig. 6.8(a)]. Transitions between different excited states occur at parallel lines along which the potential of QD2 is held constant; this indicates that the excited states are located in QD2. To investigate the states more carefully, we measure the separate tunneling rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ along the dashed lines in Fig. 6.8. The results are presented in Fig. 6.9, together with a few sketches depicting the energy level configuration of the system.

We begin with the results for the negative bias case, which are plotted in Fig. 6.9(a). Going along the dashed line in Fig. 6.8(a) corresponds to keeping the detuning $\delta$ between the QDs fixed and shifting the total DQD energy. The measurements were performed with a small detuning ($\delta \approx -100 \mu \text{eV}$) to ensure that the electron transport is unidirectional. Because of this, the outermost parts of the
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Figure 6.8: Finite-bias spectroscopy of the DQD, taken with negative (a) and positive (b) bias. The figures are constructed by counting electrons entering and leaving the DQD. Excited states are visible, especially for the negative bias data [marked with arrows in (a)]. The data was taken with $V_{\text{DQD-SD}} = \pm 500 \mu V$, $V_{\text{QPC-SD}} = 250 \mu V$ and $f_{\text{BW}} = 5 \text{kHz}$.

Traces in Fig. 6.9(a) correspond to regions where transport is due to cotunneling [compare the dashed line with the position of the triangle in Fig. 6.8(a)]; the regions where transport is sequential are shaded gray in Fig. 6.9(a).

Starting in the regime marked by I in Fig. 6.9(a,c), electrons may tunnel from drain into the ground state of QD2, relax down to QD1 and tunnel out to the source lead. Assuming the relaxation process to be much faster than the other processes, the measured rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ are related to the tunnel couplings of the source and drain $\Gamma_{\text{in}} \approx \Gamma_{\text{D}}$ and $\Gamma_{\text{out}} = \Gamma_{\text{S}}$. Now, going to higher gate voltages we lower the overall energy of both QDs. At the position marked by an arrow in Fig. 6.9(a), there is a sharp increase in the rate for tunneling into the DQD. We attribute this to the existence of an excited state in QD2; as shown in case II in Fig. 6.9(c), the electron tunneling from drain into QD2 may enter either into the ground $(n,m+1)$ or the excited state $(n,m+1^*)$, giving an increase in $\Gamma_{\text{in}}$. When further lowering the DQD energy another excited state comes into the bias window and $\Gamma_{\text{in}}$ increases even more [second arrow in Fig. 6.9(a)]. The rate for tunneling out of the DQD shows only minor variations within the region of interested. This supports the assumption that the excited states relax quickly and that the electron tunnels out of the DQD from the ground state of QD1, similar to what was measured for the single QD (see section 4.5).

Finally, continuing to the edge of the shaded region ($V_{G2} \sim -9.55 \text{ mV}$), the potential of QD1 goes below the Fermi level of the source. Here, electrons get
Figure 6.9: (a,b) Tunneling rates for electrons entering and leaving the DQD, measured along the dashed lines in Fig. 6.8(a,b). In (a), we show the results for negative bias across the DQD, in (b) the results for positive bias. The shaded areas mark the regions where electron transport is sequential, either in the electron or the hole transport cycle. The arrows indicate the positions of excited states. The data was extracted from QPC conductance traces of length $T = 5\, \text{s}$, taken with $V_{\text{QPC–SD}} = 250\, \text{mV}$. (c) Schematics of the DQD energy configuration at three different positions in (a,b).

trapped in QD1 and the tunneling-out rate drops drastically. At the same time, $\Gamma_{\text{in}}$ increases; when the electron in QD1 eventually tunnels out, the DQD may be refilled from either the source or the drain lead. The picture described above is repeated in the triangle with hole transport ($-9.25\, \text{mV} < V_{G2} < -8.9\, \text{mV}$). This is expected, since the hole transport cycle involves the same QD states as in the electron case. An interesting feature is that $\Gamma_{\text{in}}$ shows essentially the same values in both the electron and the hole cycle, while $\Gamma_{\text{out}}$ increases by a factor of three. The presence of the additional electron in QD2 apparently affects the tunnel barrier between source and QD1 more than an additional electron in QD1 affects the barrier between QD2 and drain.

Next, we move over to the case of positive bias [Fig. 6.9(b)]. Here, the roles of QD1 and QD2 are inverted, meaning that electrons enter the DQD in QD1 and leave
6.3 Cotunneling

from QD2. Following the data and the arguments presented for the case of negative bias, one would expect this configuration to be suitable for detecting excited states in QD1. However, looking at the tunneling rates within the sequential region of Fig. 6.9(b), the rate for entering QD1 ($\Gamma_{in}$) stays fairly constant, while the rate for tunneling out decreases at the point marked by the arrow. Again, we attribute the behavior to the existence of an excited state in QD2.

The situation is described in sketch III of Fig. 6.9(c). The electrochemical potential of QD2 is high enough to allow the electron in the $(n, m + 1)$-state to tunnel out to the drain and leave the DQD in an excited state $(n, m^*)$. Since the energy difference $E[(n, m^*)] - E[(n, m + 1)]$ is smaller than $E[(n, m)] - E[(n, m + 1)]$, the transition involving the excited state appears below the ground state transition. As the overall DQD potential is lowered, the transition energy involving the excited state goes below the Fermi level of the drain, resulting in a drop of $\Gamma_{out}$ as only the ground state transition is left available. Similar to the single QD case (section 4.5), we see that the tunneling-in rate samples the excitation spectrum for the $(n, m + 1)$-configuration, while the tunneling-out rate reflects the excitation spectrum of the $(n, m)$-DQD.

To conclude the results of Fig. 6.9, we find two excited states in QD2 in the $(n, m + 1)$ configuration with $\Delta E_1 = 180 \mu eV$ and $\Delta E_2 = 340 \mu eV$, and one excited state QD2 in the $(n, m)$ configuration, with $\Delta E_1 = 220 \mu eV$. No clear excited state is visible in QD1. This does not necessarily mean that such states do not exist; if they are weakly coupled to the lead they will only have a minor influence on the measured tunneling rates. Excited states in both QDs have been measured in other configurations; there, we find similar spectra of excited states for both QDs.

6.3 Cotunneling

In the previous sections, we mentioned that weak tunneling occurs in regions outside the boundaries expected from sequential transport. The effect is clearly visible in the high-resolution charge stability diagram shown in Fig. 6.10(a). The data was taken with a bias $V_{DQD-SD} = -500 \mu V$ across the DQD, giving well-defined triangles of sequential transport. Between the triangles, there are broad, band-shaped regions with low but non-zero count rates where sequential transport is expected to be suppressed due to Coulomb blockade [two examples are marked with I and II in Fig. 6.10(a)]. The finite count rate in this region is attributed to electron tunneling involving virtual processes.

6.3.1 Virtual processes

The DQD energy level configuration for two regions marked in Fig. 6.10(a) are shown in Fig. 6.10(b). In case I, the electrochemical potential of QD2 is within the bias window, but the potential of QD1 is shifted below the Fermi level of the
source and not available for transport. We attribute the non-zero count rate for this configuration to be due to electrons cotunneling from QD2 to the source lead. The time-energy uncertainty principle still allows electrons to tunnel from QD2 to source by means of a higher order process. In case II, the situation is analogue but the roles of the two QDs are reversed; electrons cotunnel from the drain into QD1 and leave sequentially to the source lead.

To investigate the phenomenon more carefully, we measure the rates for electrons tunneling into and out of the DQD in a configuration similar to the configuration along the dashed line in Fig. 6.10(a). The line corresponds to keeping the electrochemical potential of QD1 fixed within the bias window and sweeping the electrochemical potential of QD2. The data is presented in Fig. 6.11. In the region marked by I in Fig. 6.11, electrons tunnel sequentially from drain into QD2, relax from QD2 down to QD1 and finally tunnel out from QD1 to the source lead. Proceeding from point I to point II, the electrochemical potential of QD2 is lowered so that an electron eventually gets trapped in QD2. At point II, the electron lacks an
energy $\delta_a$ to leave to QD1. Still, electron tunneling is possible by means of a virtual process [80]. Due to the energy-time uncertainty principle, there is a time-window of length $\sim \hbar/\delta_a$ within which tunneling from QD2 to QD1 followed by tunneling from the source into QD1 is possible without violating energy conservation. An analogous process is possible involving the next unoccupied state of QD2, occurring on timescales $\sim \hbar/\delta_b$, where $\delta_b = E_{C2} - \delta_a$ and $E_{C2}$ is the charging energy of QD2. The two processes correspond to electron cotunneling from the drain lead to QD1.

Continuing from point II to point III, the unoccupied state of QD2 is shifted into the bias window and electron transport is again sequential.

In the sequential regime (regions I and III), we fit the rate for electrons entering the DQD to a model involving only sequential tunneling [solid lines in Fig. 6.11(a)] [1]. The fit allows to determine the tunnel couplings between drain and the occupied ($\Gamma_{Da}$)/unoccupied ($\Gamma_{Db}$) states of QD2, giving $\Gamma_{Da} = 7.5\,\text{kHz}$ and $\Gamma_{Db} = 3.3\,\text{kHz}$. Going towards region II, the rates due to sequential tunneling are expected to drop exponentially as the energy difference between the levels in QD1 and QD2 is increased. In the measurement, the rate $\Gamma_{in}$ initially decreases with detuning, but the decrease is slower than exponential and flattens out as the detuning gets larger. This is in strong disagreement with the behavior expected for sequential tunneling. Instead, in a region around point II we attribute the measured rate $\Gamma_{in}$ to be due to electrons cotunneling from drain to QD1.

The rate for cotunneling from drain to QD1 is given as [80]:

$$\Gamma_{cot} = \Gamma_{Da} \frac{t_a^2}{\delta_a^2} + \Gamma_{Db} \frac{t_b^2}{\delta_b^2} + \cos \phi \sqrt{\Gamma_{Da} \Gamma_{Db}} \frac{t_a t_b}{\delta_a \delta_b}. \quad (6.5)$$

Here, $t_a, t_b$ are the tunnel couplings between the occupied/unoccupied states in QD2 and the state in QD1. The first term describes cotunneling involving the occupied state of QD2, the second term describes the cotunneling over the unoccupied state and the third term accounts for possible interference between the two. The phase $\phi$ defines the phase difference between the two processes. To determine $\phi$ one needs to be able to tune the phases experimentally, which is not possible from the measurement shown in Fig. 6.11(a). In the following we assume the two processes to be independent ($\phi = \pi/2$) and postpone the question of interference to chapter 8.

The dashed line in Fig. 6.11(b) shows the results of Eq. (6.5), with fitting parameters $t_a = 15\,\mu\text{eV}$ and $t_b = 33\,\mu\text{eV}$. These values are in good agreement with values obtained from charge localization measurements as described in section 6.2.2. The values for $\Gamma_{Da}$ and $\Gamma_{Db}$ are taken from measurements in the sequential regimes. We emphasize that Eq. (6.5) is valid only if $\delta_a, \delta_b \gg t_a, t_b$ and if sequential transport is sufficiently suppressed. The data points used in the fitting procedure are marked by filled squares in the figure. It should be noted that the sequential tunneling in region III prevents investigation of the cotunneling rate at small $\delta_b$. This can easily be overcome by inverting the DQD bias.

The rate for electrons tunneling out of the DQD [$\Gamma_{out}$, blue trace in Fig. 6.11(a)] shows only slight variations over the region of interest. This is expected since the
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Figure 6.11: Tunneling rates for electrons entering and leaving the DQD, measured while keeping the potential of QD1 fixed and sweeping the electrochemical potential of QD2. The data is measured in a configuration similar to going along the dashed line in Fig. 6.10(a). The solid lines are tunneling rates expected from sequential tunneling, while the dashed line is a fit to the cotunneling model of Eq. (6.5). Parameters are given in the text. (b) Schematic drawings of the DQD energy levels for three different configurations in (a). At point I, electrons tunnel sequentially through the structure. Moving to point II, the energy levels of QD2 are shifted and the electron in QD2 is trapped due to Coulomb blockade. Electron transport from drain to QD1 is still possible through virtual processes, but rate for electrons entering the DQD drops substantially due to the low probability of the virtual processes. At point III, the next level of QD2 is brought inside the bias window and sequential transport is again possible.

potential of QD1 stays constant over the sweep. The slight decay of $\Gamma_{\text{out}}$ with increased detuning comes from tuning of the tunnel barrier between source and QD1 (see section 4.6).

6.3.2 Cotunneling vs. molecular states

The overall good agreement between Eq. (6.5) and the measured data demonstrates that time-resolved charge detection techniques provide a direct way of quantitatively
Figure 6.12: (a) Cotunneling described as a second-order tunneling process. (b) Cotunneling described using molecular states. Due to the large detuning the empty antibonding state is mainly localized on QD1, but a small part of the wavefunction is still present in QD2 which allows an electron to enter from the drain.

probing the time-energy uncertainty principle. However, a difficulty arises as $\delta \to 0$; the cotunneling rate in Eq. (6.5) diverges, as visualized for the dashed line in Fig. 6.11(a). The problem with Eq. (6.5) is that it calculates the cotunneling rate as a second-order tunneling process in a model which defines the energy levels in the QDs and leads to be fixed. For small detunings $\delta$ the model does not hold; we know from section 6.2.2 that the states in QD1 and QD2 will hybridize and form bonding and antibonding molecular states.

A different approach is to assume the coupling between the QDs to be fully coherent and describe the DQD in terms of the bonding and antibonding states introduced in section 6.2.2 [81]. Both the sequential tunneling and the cotunneling can then be treated as first-order tunneling processes into the molecular states; what we previously referred to as sequential tunneling would be tunneling into the bonding state, whereas the cotunneling is described as tunneling into the antibonding state. The two models are sketched in Fig. 6.12. In Fig. 6.12(a), the cotunneling is a second order process involving the simultaneous transfer of an electron from drain to QD2 and from QD2 to QD1. In Fig. 6.12(b), the DQD is described in terms of molecular states. Here, the bonding state is occupied and in Coulomb blockade. Still, an electron may tunnel from drain into the antibonding state. Due to the large detuning, the antibonding state is mainly located on QD1, the overlap with the electrons in the drain lead is small and the tunneling is weak. Changing the detuning will have the effect of changing the shape of the molecular states and shift their weight between the two QDs.

To calculate the rate for electrons tunneling from drain into the molecular state of the DQD as visualized in Fig. 6.12(b), we use the formalism from section 6.2.2 and project the thermal population $p_B$, $p_A$ of the molecular states $\Psi_B$ and $\Psi_A$ onto the unperturbed state of QD2, $\Psi_2$. This gives the probability $p_2$ of finding an electron in QD2 if making a projective measurement in the $\Psi_2$-basis. This is exactly what electrons in the drain are performing as they try to tunnel into QD2. The measured rate $\Gamma_{in}$ is thus equal to the probability of finding QD2 being empty $(1-p_2)$ multiplied with $\Gamma_D$, the tunneling rate between the drain and the unperturbed state.

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Figure 6.13: (a) Rate for electrons tunneling into the DQD, measured vs DQD detuning. The data is the same as in Fig. 6.11, but this time plotted together with the molecular-states model of Eq. (6.6). The same fitting parameters are used as for the cotunneling model in Fig. 6.11. The model fits the data well over the whole measurement range. (b) Same as (a), but plotted on a log-log scale to enhance the features at small detuning. The dashed line is the results of the cotunneling model in Eq. (6.5), the solid line shows the result of the molecular-state model [Eq. (6.6)].

\[
\Gamma_{in} = \Gamma_D (1 - p_2) = \Gamma_D (1 - (p_B \Psi_B + p_A \Psi_A) \cdot \Psi_2)
\]

(6.6)

For large detuning, the bonding and antibonding states are well localized in QD2 and QD1, respectively. Here, we should recover the results for the cotunneling rate obtained for the second-order process [Eq. (6.5)]. First, we assume low temperature \( T \ll \delta \), so that the electron only populates the bonding ground state \( p_B = 1 \) and \( p_A = 0 \):

\[
\Gamma_{in} = \Gamma_D \frac{1}{2} \left( 1 + \frac{\delta}{\sqrt{4t^2 + \delta^2}} \right).
\]

(6.7)

In the limit of \( \delta \gg t \) the expression reduces to

\[
\Gamma_{in} = \Gamma_D \frac{1}{2} \left( 1 + \frac{\delta}{\sqrt{4t^2 + \delta^2}} \right) \approx \Gamma_D \frac{t^2}{\delta^2}.
\]

(6.8)

As expected, the rate approaches the result of the second-order cotunneling processes in Eq. (6.5). The advantage of the molecular-state model is that it is valid for any detuning, both in the sequential and in the cotunneling regime. Figure 6.13(a) shows \( \Gamma_{in} \) extracted from the same measurement as in Fig. 6.11, but this time plotted
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together with the results of Eq. (6.6). The equation has been evaluated twice, once for the occupied \((n,m)\) and once for the unoccupied state in QD2 \((n,m+1)\); the curve in Fig. 6.13(a) is the sum of the two rates. The same parameters were used as for the cotunneling fit in Fig. 6.11. The model shows very good agreement with data over the full range of the measurement.

To compare the results of the molecular-state and the cotunneling model in the regime of small detuning, we plot the data in Fig. 6.13(a) on a log-log scale [Fig. 6.13(b)]. For large detuning, the tunneling rate follows the \(1/\delta^2\) predicted by both the molecular-state and the cotunneling model. For small detuning, the deviations become apparent as the cotunneling model diverges while the molecular-state model still well reproduces the data.

6.3.3 Inelastic cotunneling

So far, we have only considered cotunneling involving the ground states of the two QDs. The situation is more complex if we include excited states in the model; the measured rate may come from a combination of cotunneling processes involving different QD states. To investigate the influence of excited states experimentally, we go back to the regime of Fig. 6.8 where the finite-bias measurement reveals clear excited states in QD2. Looking carefully at the lower-right regions of the positive-bias triangles in Fig. 6.8(b), we see that the count rates in the cotunneling regions outside the triangles are not constant along lines of fixed detuning (corresponds to going in a direction parallel to the dashed line). Instead, the cotunneling regions seem to split into three parallel bands.

In Fig. 6.14(a), we plot the tunneling rates \(\Gamma_{\text{in}}\) and \(\Gamma_{\text{out}}\) for electrons entering and leaving the DQD, extracted from the same set of data as used in Fig. 6.8(b). The solid lines mark the edges of the finite-bias triangles. Again, the cotunneling rates outside the triangles are not uniform; parallel bands appear in \(\Gamma_{\text{in}}\) for the position marked by I and in \(\Gamma_{\text{out}}\) for the position marked by II in the figures.

To understand the data we draw energy level diagrams for the two configurations [see Fig. 6.14(b)]. Focusing first on case I, we see that the electrochemical potential of QD2 is within the bias window, whereas QD1 is detuned and in Coulomb blockade. The cotunneling occurs via QD1 states; electrons cotunnel from source into QD2, afterwards they tunnel out from QD2 to drain sequentially. The picture is in agreement to what is measured in Fig. 6.14(a); the cotunneling rate (\(\Gamma_{\text{in}}\)) is low and strongly dependent on detuning \(\delta\), while the sequential rate \(\Gamma_{\text{out}}\) is high and essentially independent of detuning. The three bands seen in \(\Gamma_{\text{in}}\) occur because of the excited states in QD2; depending on the average DQD energy, electrons may cotunnel from source into one of the excited states, relax to the ground state and then leave to the drain lead. The state of QD1 remains unaffected by the cotunneling process. For this configuration, we speak of elastic cotunneling.

The situation is different in case II. Here, cotunneling occurs in QD2 as electrons tunnel out from QD1 into the drain lead. This means that \(\Gamma_{\text{in}}\) is sequential while
Figure 6.14: (a) Tunneling rates for electrons entering and leaving the DQD, extracted from the same set of data as used in Fig. 6.8(b). The data was measured with $V_{\text{DQD-SD}} = 500 \mu V$. The solid lines mark the position of the finite-bias triangles. (b) Energy-level diagrams for the two positions marked in (a). In case I, the cotunneling itself is elastic, with energy relaxation occurring after the cotunneling has taken place. In case II, inelastic cotunneling processes are possible.

$\Gamma_{\text{out}}$ describe the cotunneling process. As in case I, the cotunneling rate $\Gamma_{\text{out}}$ splits up into three bands; we attribute this to cotunneling where the state of QD2 is changed during the process. QD2 ends up in one of its excited states. The energy of the electron arriving in the drain lead is correspondingly decreased compared to the electrochemical potential of QD1. Here, the cotunneling is inelastic.

The inelastic cotunneling is described in greater detail in Fig. 6.15. In Fig. 6.15(a) we plot the count rate measured along the dashed line at the right edge of Fig. 6.14(a). Figure 6.15(b) shows energy level diagrams at two positions along the line. Starting in the configuration marked by I, cotunneling is only possible involving the QD2 ground state. The cotunneling is weak, with count rates being well below 1 count/s. Continuing to case II, we rise the average DQD energy while keeping the detuning $\delta$ constant. When the electrochemical potential of QD1 is sufficiently increased compared to the Fermi level of the drain, inelastic cotunneling becomes possible causing a sharp increase in count rate. The process is sketched in Fig. 6.15(b); it involves the simultaneous tunneling of an electron from QD1 to the first excited state of QD2 with an electron in the QD2 ground state leaving to the drain. The process is only possible if

$$\delta - \Delta E_1 = \mu_1 - \mu_2 - \Delta E_1 > \mu_D - \mu_2.$$  \hspace{1cm} (6.9)

Here, $\Delta E_1$ is the energy of the first excited state in QD2. In Fig. 6.15(a) we define the average DQD energy to be zero when $\mu_1$ aligns with $\mu_D$. The position of the step in Fig. 6.15(a) thus directly gives the energy of the first excited state, and we
Figure 6.15: (a) Electron count rate along the dashed line in Fig. 6.14(a), measured for both positive and negative DQD bias. In the trace, the detuning $\delta$ stays constant and we sweep the average DQD energy. The DQD energy is defined from the position where the electrochemical potential of QD1 is aligned with the Fermi level of the drain lead. The steps in the count rate are due to the onset of inelastic cotunneling processes in QD2. The data was extracted from traces of length $T = 10\,\text{s}$, measured with $V_{\text{QPC-SD}} = 200\,\mu\text{V}$. (b) Energy level diagrams for the two configurations marked in (a).

Find $\Delta E_1 = 180\,\mu\text{eV}$.

Further increasing the average DQD energy makes an inelastic process involving the second excited state in QD2 possible, giving $\Delta E_2 = 340\,\mu\text{eV}$. Finally, as the DQD energy is raised to become equal to the applied bias, the electrochemical potential of QD1 aligns with Fermi level of the source lead. Here electron tunneling mainly occurs due to equilibrium fluctuations between source and QD1, giving a sharp peak in the count rate. The excited states energies extracted from the inelastic cotunneling give the same values as obtained from doing finite-bias spectroscopy within the triangles, as described in section 6.2.3. The good agreement between the two measurements demonstrates the consistency of the model.

The dashed line in Fig. 6.15(a) shows data taken with reversed DQD bias; for this configuration the Fermi levels of the source and drain leads are inverted, the electrons cotunnel from drain to QD2 and the peak due to equilibrium tunneling occurs at $\mu_1 = 0$. 

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6.3.4 Noise in the cotunneling regime

Using the methods of chapter 5, we can extract the noise of electron transport in the cotunneling regime. In the single dot sequential-tunneling case, it was found that transport in most configurations is well-described by independent tunneling events for electrons entering and leaving the QD. The Fano factor then becomes a function of the tunneling rates [Eq. (5.13)], which we repeat below for convenience:

\[ F_2 = \frac{S_I}{2eI} = \frac{\mu_2}{\mu} = \frac{\Gamma_{in}^2 + \Gamma_{out}^2}{(\Gamma_{in} + \Gamma_{out})^2} = \frac{1}{2} \left( 1 + a^2 \right), \tag{6.10} \]

with \( a = (\Gamma_{in} - \Gamma_{out})/(\Gamma_{in} + \Gamma_{out}) \). In the case of cotunneling, the situation is more complex. As described in the previous section, cotunneling may involve processes leaving either QD in an excited state. The excited state has a finite lifetime \( \tau_{rel} \); during this time, different states are available for the tunneling and the tunneling rates may be different compared to the ground-state configuration. We therefore expect that on time scales on the order of \( \tau_{rel} \), the existence of an electron in an excited state may induce temporal correlations between subsequent cotunneling events. In this way, the noise of the cotunneling current has been proposed as a tool to probe excited states and relaxation processes in QDs [82, 83].

In Fig. 6.16, we plot the Fano factor measured from the same region as that of Fig. 6.11. The dashed line shows the result of Eq. (6.10), with tunneling rates...
6.4 Spin effects in many-electron dots

So far, we have neglected the spin properties of the electrons by considering them to be spin-less particles. In few-electron double quantum dots, spin effects have been shown to lead to Pauli spin blockade \[84, 85\]; the current is strongly suppressed in configurations where a spin flip is required for electrons to traverse the DQD. The Pauli blockade configuration has been utilized for performing electron spin resonance experiments \[86, 87\] as well as for studying interactions between the electron and nuclei spin systems \[85, 88–90\].

In the system investigated here, the DQD contains a relatively large number of electrons; from the energy scales and from the geometry of the device we estimate each QD to hold \(\sim 30\) electrons. This makes the observation of spin blockade more difficult, since neither the excitation spectrum nor the exact QD spin configuration is well known. For few-electron QDs, the first two electrons fill up spin-degenerate
single-particle states and form a spin-pair \[91\]. Spin pairing has also been reported in many-electron chaotic dots \[31\] and quantum rings \[92\]. If spin-pairing occurs, it is possible to get a spin-zero many-electron ground state and we may neglect the spin-less core of electrons and only consider the spin of the outermost electrons \[89\].

To investigate the occurrence of spin pairing and spin blockade in our device, we use the methods of section 4.7 to determine the degeneracy of the QD ground states. Depending on the occupancy of a spin-degenerate state, the rates for electrons tunneling into and out of a QD should differ by a factor of two. By performing such measurements for consecutive electron filling in the DQDs, we can extract a possible spin configuration of the DQD. The method is visualized in Fig. 6.17, together with a possible spin configuration for the hexagons from Fig. 6.4(a). The numbers in the figure do not refer to the absolute DQD electron population but to the number of excess electrons relative to the configuration indicated by \((0,0)\).

Starting in the Coulomb-blockaded region marked by \((0,0)\), we increase the gate voltage \(V_{G2}\) to add an electron into QD2. At the transition to \((0,d)\) (case I in Fig. 4.7), we find that the tunneling rate for electrons entering QD2 is larger than the rate for electrons tunneling out. Increasing the gate voltage further to add another electron to QD2 (case II), the relation between \(\Gamma_{in}\) and \(\Gamma_{out}\) is reversed. This is in agreement with successive filling of electrons into a degenerate state; if both degenerate states are initially unoccupied (case I), an incoming electron may tunnel into either state with an effective tunneling rate \(\Gamma_{in} = g \times \Gamma\). Here, \(g\) is the degeneracy factor and \(\Gamma\) is the tunnel coupling to the lead. On the other hand, the rate for electrons leaving the QD is determined by the number of occupied degenerate states. With only one electron in the QD we get \(\Gamma_{out} = \Gamma\) and thus expect \(\Gamma_{in}/\Gamma_{out} = g\). The situation is reversed if the QD is initially occupied (case II); here we expect \(\Gamma_{in}/\Gamma_{out} = 1/g\).

If we assume the degeneracy to be due to spin states, the data indicates that QD2 is successively filled up with one spin-down and one spin-up electron. The pattern is repeated if we perform similar measurements for QD1 (cases III-IV). From these measurements we extract the spin configurations shown in Fig. 6.17. It should be noted that the degeneracy factors in cases I and III are lower than the factor \(g = 2\) expected from spin-degenerate states. This might be due to changes in the tunneling coupling \(\Gamma\) within the gate voltage region of interest, although the coupling normally only changes slightly within the small voltage range used here (see section 4.6). Therefore, the spin configurations marked in Fig. 6.17 should not be considered as definite; we can not rule out other explanations for the data.

Keeping this reservation in mind but still assuming the spin configuration of Fig. 6.17 to be correct, we expect spin blockade to occur in the transport triangle involving the configurations \((0,d)\), \((d,d)\) and \((0,ud)\). The principle of the blockade is explained in Fig. 6.18(a). We start in the configuration \((0,d)\), where QD1 is empty and QD2 contains one excess electron. An electron may tunnel from source into QD1 and since the QD is initially empty, the incoming electron may be either spin-up or spin-down. If the spin is opposite to the spin of the electron in QD2, the electron
in QD1 can continue to QD2 to form the spin singlet ground state (0, ud). Finally, an electron may leave to the drain which takes the system back to the state (0, d) and the cycle can be repeated.

However, if the electron tunneling from source into QD1 has the same spin orientation as the electron in QD2, it cannot continue from QD1 to QD2. This is because of the singlet-triplet splitting in QD2; due to the exchange energy the system favors the formation of a spin singlet and the energy of the spin triplet is raised by the single-triplet splitting $E_{ST}$. The electron in QD1 is thus blocked until a spin-flip occurs in either QD1 and QD2. Since spin relaxation is slow, the effect leads to a sharp decrease in the current through the DQD [84]. In our case, we do
Figure 6.18: (a) Sketch of a DQD in the regime of Pauli spin blockade. If the electron entering QD1 has the same spin orientation as the electron sitting in QD2, transport is blocked until either electron flips its spin to allow a singlet state to form in QD2. (b) Charge stability diagram measured by counting electrons entering the DQD. The numbers refer to the assumed excess charge population relative to a state where both QDs have zero spin. The data was extracted from QPC conductance traces of length $T = 0.5\,\text{s}$, taken with $V_{\text{DQD-SD}} = 600\,\mu\text{V}$ and $V_{\text{QPC-SD}} = 400\,\mu\text{V}$. (c) QPC conductance traces, taken at the points marked in (b). For both positions, a third level appears which we attribute to the transition $(1, 1) \rightarrow (0, 2)$. (d) Regions of the charge stability diagram of (b) where the charge detector finds more than two levels in the QPC conductance traces. In the spin blockade model of (a), the width of regions with three levels corresponds to the singlet-triplet spacing in QD2.

not measure the average current but rather count the electrons as they pass through the structure. As mentioned in section 6.2.2, the tunnel coupling between the QDs is too strong to allow interdot charge transitions to be resolved in time. However, in the spin-blockade regime interdot charge transition from QD1 to QD2 should be limited by the spin relaxation rate, which for GaAs QDs has been reported to be several milliseconds or even seconds for magnetic fields of 1 T [56, 57]. This is within
the bandwidth of the charge detector and we thus expect spin blockade to help make
the interdot charge transitions resolvable.

Figure 6.18(b) shows the finite-bias charge stability diagram measured by count-
ing electrons in the regime located between the (d,d) and (0,ud)-region of Fig. 6.17.
The data shows two triangle-shaped regions of electron and hole transport expected
from the applied voltage bias. Figure 6.18(c) shows examples of QPC current traces
taken at the two positions marked in Fig. 6.18(b). Taking a closer look at the data
from position I, we see that the time trace actually contains three levels; starting
at the QPC current level labeled (0,1), the QPC current drops to level (1,1) as an
electron tunnels into QD1. The electron relatively quickly continues to QD2 [level
(0,2) in Fig. 6.18(c)], before tunneling out to the drain and taking the QPC con-
ductance back to level (0,1). The ability of the QPC to determine if the electron is
sitting in QD1 or QD2 comes directly from the geometry of the device; the QPC is
located closer to QD1.

For case II of Fig. 6.18(b,c), the transport is governed by the hole process (0,2) →
(1,2) → (1,1) → (0,2). As for the electron process in case I, the transition that
possibly involves spin relaxation is the one where the electron hops from QD1 to
QD2 [(1,1) → (0,2)]. The timescale of the interdot transitions is marked by \( t_{(1,1)} \)
in the traces of Fig. 6.18(c). From the above discussion, we expect \( t_{(1,1)} \) to be long
enough to be measurable as long as the DQD detuning is smaller than the singlet-
triplet spacing, \( \delta < E_{ST} \). If \( \delta > E_{ST} \), the electron in QD1 may tunnel to QD2
regardless of the spin direction and we expect to resolve only two levels in the QPC
conductance traces. This is visualized in Fig. 6.18(d), where we plot the number of
current levels detected by an automatic level detection algorithm (see section 2.4).
Focusing first on the electron transport cycle, there is a region of three-level traces
situated at the base of the triangle. In the model of spin-blockade, the width of the
region in direction of detuning is equal to the singlet-triplet splitting in QD2, giving
\( E_{ST} \sim 200 \mu eV \). For the hole cycle, the region showing three levels is less regular.
However, this is actually an artifact due to imperfections of the level detection
algorithm; for the electron cycle (case I), the third, fast level occurs below the other
levels, which makes it relatively easy to detect. It is much harder to reliably detect a
fast third level if it occurs in-between the two main levels as in the hole region (case
II). Manual inspection of the traces confirms that the region of three-level traces
indeed has the same extension for the hole as for the electron cycle.

To get more quantitative concerning the time scale of the (1,1) → (0,2) trans-
transition and to investigate if it is really related to spin relaxation, we measure the
average of \( t_{(1,1)} \) as a function of detuning [dashed line in Fig. 6.18(d)] and mag-
netic field. The result is presented in Fig. 6.19(a). As mentioned in the previous
paragraph, the third level is only visible in the region of \( 0 < \delta < E_{ST} \sim 200 \mu eV \).
Figure 6.19(b) shows a cross section taken at \( \delta = 170 \mu eV \). The time spent in the
(1,1)-state is around 400 \( \mu s \) at zero magnetic field, but decays rapidly with increased
B-field and disappears below the time resolution of the detector as \( |B| > 30 \text{ mT} \).

The spin blockade can be conveniently expressed using a model involving two-
electron spin singlet (S) and triplet (T) states distributed over both QDs. In this language, the electron tunneling into the DQD from the source lead can enter either the singlet S(1,1) or the triplet T(1,1) state. The singlet S(1,1) quickly relaxes to S(0,2) followed by an electron leaving the DQD to the drain. On the other hand, if the electron enters into the triplet T(1,1), it can not proceed to T(0,2) since this state is raised by the singlet-triplet splitting in QD2. The triplet T(1,1) first needs to relax to S(1,1) before proceeding to S(0,2), leading to spin blockade.

For GaAs quantum dots, the spin blockade has been observed to be lifted at zero magnetic field because of mixing of the T(1,1) and S(1,1) states due to hyperfine interactions with the nuclear spin bath. The mixing energy is given by the magnitude of the random magnetic field $\vec{B}_N$ generated by the fluctuating nuclear spins, with $E_N = g\mu_B|\vec{B}_N| \sim 0.1 \mu eV$ for a typical quantum dot containing $n \sim 10^6$ nuclei. The mixing can be removed by applying an external magnetic field so that the electron Zeeman splitting becomes larger than the mixing energy $E_N$. This typically occurs on a magnetic field scale of a few mT [85]. In our case, we observe the opposite behavior; the relaxation rate $\Gamma_{\text{rel}} = 1/\langle t(1,1) \rangle$ is minimal at zero magnetic field and increases with external magnetic field. In contrast to the setup of Ref. [85], we are in the strong coupling regime, with $t \sim 30 \mu eV \gg E_N$. As discussed in section 6.2.2, the tunnel coupling will hybridize the S(1,1) and S(0,2) singlet configurations and thereby keep the energy separation to the T(1,1) triplet larger than $E_N$ over the full range of detuning in Fig. 6.19(a). This suppresses the relaxation due to hyperfine mixing, even at zero external magnetic field [93].

A strong increase of the relaxation rate for small magnetic field has been seen in InAs DQD [89]. The behavior was attributed to the strong spin-orbit interactions

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**Figure 6.19:** (a) Average time spent in the (1,1) state, measured vs DQD detuning and magnetic field. The third level in the QPC conductance traces corresponding to the (1,1) state is only visible in the region $0 < \delta < E_{ST} \sim 200 \mu eV$. The data was taken along the dashed line in Fig. 6.18(d). (b) Cross-section of the graph in (a), taken at $\delta = 170 \mu eV$. The time $t(1,1)$ falls of quickly with increased B-field and drops below the time resolution of the detector as $|B| > 30 \text{ mT}$. 
6.5 Weak interdot coupling

of that material system. The main spin relaxation mechanism in few-electron single QDs in GaAs is also due to spin-orbit coupling, with relaxation rates increasing with external magnetic field [57, 94]. However, the relaxation times seen in Fig. 6.19(b) are much shorter and the B-field dependence much stronger than reported for few-electron single quantum dots. It is unclear how the existence of additional electrons in our DQD influence the relaxation process and it is uncertain if it is reasonable to assume electron-electron interactions to be weak enough to allow the QDs to be modeled using independent single-particle states. From the measurements presented here, one can not make a clear statement whether the observed features are due to spin relaxation or not. It would certainly be interesting to repeat the time-resolved measurements on a DQD containing only two electrons.

6.5 Weak interdot coupling

In the last section of this chapter, we treat the case where the three barriers of the DQD are tuned so that all tunneling processes occur on timescales slower than the bandwidth of the charge detector. In this regime it is possible to detect electrons tunneling back and forth between the QDs and thus determine the direction of the tunneling electrons [23].

It turned out to be difficult to reach this regime for the ring-shaped DQD of Fig. 6.1(a). The constrictions between the QDs were generally much more open than the constrictions to source and drain leads, which made it hard to pinch off the middle constriction while at the same time keeping source and drain open and forming well-defined dots. A measurement from one of the few cases where we were partly successful is shown in Fig. 6.20(a). The plot shows the charge stability diagram measured with \(-700\,\mu\text{V}\) bias applied across the DQD. The transport triangles due to electron and hole transport are well visible. There are a few striking things in this measurement compared to charge stability diagrams shown previously in this chapter. First, the size of the triangle due to hole transport is considerably smaller than the electron triangle. Although this is not quantitatively understood, we speculate that it is due to a weakly coupled state in QD2 that blocks transport in parts of the triangles. Second, there are bands of weak tunneling occurring outside the triangles. We attribute this to photon absorption processes driven by the current flowing in the QPC; this is the subject of chapter 7. Finally, there are stripes occurring parallel to the base line of the triangles; these are excited states in the QDs probed by interdot transitions.

Figure 6.20(b) shows the number of QPC current levels found with the automatic level detection algorithm discussed in section 2.4. Three levels are found in most of the hole transport triangle as well as in large parts of the electron transport triangle, showing that tunneling between the QDs is indeed slow enough to be detected by the detector. Figure 6.20(c) shows three QPC conductance traces taken at the positions marked in Fig. 6.20(a). Energy level diagrams for the corresponding configurations
Figure 6.20: (a) Charge stability diagram of the DQD measured by counting electrons entering the DQD. The data was taken with $V_{DQD-SD} = -700 \, \mu V$ and $V_{QPC-SD} = 300 \, \mu V$. (b) Number of levels in the QPC conductance traces, extracted from the same data as in (a). The dashed lines show the extension of the triangle expected from the applied bias and the capacitive lever arms of the gates. (c) QPC conductance traces, taken at three positions marked in (a). In case I, the tunneling is due to equilibrium fluctuations between QD1 and QD2. In cases II-III, a current is flowing through the DQD. (d) Energy level diagrams depicting the DQD configuration for the three position in (a,c).
are shown in Fig. 6.20(d).

Starting at the position marked by I, the two QD levels are aligned but shifted outside the bias window. Here, equilibrium fluctuations occur between the QDs. The QPC conductance trace shows transitions between two levels corresponding to an electron sitting on QD1 and QD2, respectively. The transitions occur on a relatively slow timescale of $\sim 10 \text{ ms}$.

Continuing to case II, we keep the alignment of the levels in the two QDs but shift them inside the bias window of the hole transport cycle. Looking at the trace in Fig. 6.20(c), we see that the transition of electrons from QD2 to QD1 $[(0,1) \rightarrow (1,0)]$ still occurs on a timescale comparable to case I. However, before the electron in QD1 has time to tunnel back to QD2, an electron is quickly refilled into QD2 from the drain lead and takes the QPC conductance to the $(1,1)$ level. Afterwards, an electron may leave from QD1 to source and the system is back in the $(0,1)$ state. Each cycle corresponds to one electron being transferred through the DQD.

The timescale for interdot transition is clearly slower than the tunneling involving the source or drain lead. The DQD current is thus limited by the central barrier. This is clearly visualized if we continue to case III, which corresponds to a slightly lowered electrochemical potential of QD1 relative to QD2. Here, the interdot transition can not occur resonantly; the tunneling electron needs to lose parts of its energy to the environment. This makes the tunneling process less probable and reduces the count rate in the region of case III in Fig. 6.20(a). The QPC conductance trace Fig. 6.20(c) shows that the electron indeed spends most of the time in QD2; once a transition to QD1 occurs it is immediately followed by tunneling from drain to QD2 and from QD1 to source, as discussed for case II. Finally, by further lowering the electrochemical potential of QD1 an excited state of QD1 lines up with QD2 and tunneling may again occur resonantly. This is the reason for the stripes parallel to the triangle baseline occurring inside the triangles.

The above discussion raises a few interesting questions concerning the interdot tunneling. Firstly, what sets the width of the regime with resonant interdot tunneling? In case I, the electrons in the DQD are isolated from the leads and it seems unlikely that the DQD transitions should be influenced by the thermal distribution of the electrons in the leads. Secondly, what are the relaxation processes leading to the slow but non-zero tunneling rates in the non-resonant regime? To answer the first question, we take the data from Fig. 6.20(a) and use the known capacitive lever arms to convert the gate voltages into energy of the DQD. The results is presented in Fig. 6.21(a), where we plot the count rate for the hole transport triangle vs average DQD energy and detuning energy $\delta$. The two axes have the same scaling, which makes it easier to compare energy scales of different processes.

We first focus on the tilted line with slope $1/2$ marked by I in Fig. 6.21(a). The line is due to equilibrium fluctuations between QD1 and the source lead; the broadening of the line is a direct measure of the electron temperature in the source lead (see section 4.2). By converting the energy to temperature we find that the electron temperature in the lead is around $T = 100 \text{ mK}$. However, the width of
Figure 6.21: (a) Same as Fig. 6.20(a), but with the gate voltages converted to average energy and detuning of the double quantum dot. The line coming from tunneling between QD1 and the lead (case I) is considerably broader than the lines due to interdot transitions (case II). (b) Rate for the interdot transition $(0, 1) \rightarrow (1, 0)$, measured along the dashed line in (a). The peaks come from resonant tunneling through excited states in either QD. The inelastic tunneling between resonant peaks increases strongly with increased detuning. The dashed line is a guide to the eye depicting exponential increase.

The thermally-broadened line stands in sharp contrast to the narrow vertical lines coming from interdot transitions (case II). The width of these lines is only around a quarter of the thermal-broadened line, which would correspond to a temperature of 25 mK. This energy scale matches relatively well to the base temperature of the cryostat. A possible explanation could therefore be that the broadening occurs because of scattering with thermally excited acoustic phonons. A straightforward experimental check of this hypothesis would be to investigate how the broadening changes when raising the base temperature of the cryostat. Unfortunately, shortly after measuring the data in Fig. 6.20 we had to warm up the cryostat, and we were not able to reach the same regime in subsequent cool-downs.

A different energy scale is given by the tunnel coupling between the two QDs. If we assume the transport between the QDs to be coherent and convert the measured tunneling rate of $\Gamma \sim 100$ Hz to a coupling energy, we find

$$t \sim hf \sim 0.4 \text{ peV}. \quad (6.11)$$

This is obviously several orders of magnitudes smaller than the width measured in the experiment. Still, the discussion raises some interesting questions concerning
coherence and projective measurements. For a fully coherent system, the electron wavefunctions in the two QDs hybridize and form bonding and antibonding states that delocalize over both dots. At zero detuning both the bonding and antibonding wavefunctions have the same spatial extent, which means that a charge detector would not be able to resolve transitions between the two states independently of how slowly the transitions occur. The very fact that we detect electrons tunneling back and forth between the QDs even at zero detuning is an obvious indication that the system is not coherent. The decoherence rate is faster than the tunnel coupling, meaning that the coherent evolution of an electron between the two QDs is interrupted by a projective measurement taking the electron back into the states of the individual QDs. The rate at which we observe transitions between the two QDs thus depends not only on the tunnel coupling but also on the decoherence in the system. It would certainly be interesting to perform measurements in a regime where the tunnel coupling and the decoherence rate are comparable, and to investigate how the measured transition rates are affected by the presence of the QPC and its ability to perform projective measurements. One would expect an increased QPC bias to introduce additional effects compared to the intrinsic decoherence.

Finally, we come back to the question of the relaxation mechanism leading to the finite count rate between the lines of resonant tunneling in Fig. 6.20(a) and Fig. 6.21(a). Figure 6.21(b) shows the interdot transition rate $\Gamma_{(0,1)-(1,0)} = 1/\langle t_{(0,1)} \rangle$ measured along the dashed line in Fig. 6.21(b), extracted from traces similar to the ones shown in Fig. 6.20(c). The ground state transition as well as transitions due to three exited states give rise to clear peaks in the figure. In between the peaks, the rate of the non-resonant transition increases strongly as the detuning gets larger.

Spontaneous energy relaxation in a DQD has been investigated previously using conventional current measurement techniques [95]. In that work, the authors find that the emission rate decreases with increased detuning and attribute the mechanism behind the relaxation to phonon emission. This is in disagreement with the results of Fig. 6.21(b), where the emission rate clearly increases with detuning. It would therefore be interesting to perform further experiments in this regime and investigate the inelastic tunneling of Fig. 6.21(b) in more detail. In addition to checking the obvious influence of the temperature of the phonon bath there could be other explanations for the relaxation such as photon emission to the nearby quantum point contact [96] or to anywhere else in the environment.
Chapter 7

Single-photon detection

In the previous chapters, we used quantum point contacts to measure charge transitions in various mesoscopic structures. While doing so we assumed the point contact to be an idealized detector that does not exert any back-action on the measured object. In reality, this is not true. The scattering of electrons in the quantum point contact leads to emission of microwave radiation. In this chapter, we show that the radiation may drive transitions in a double quantum dot. Turning the perspectives around, the double quantum dot can be seen as a frequency-selective microwave detector. The frequency of the absorbed radiation is set by the energy separation between the levels in the dots, which is easily tuned with gate voltages. By combining this with time-resolved charge detection techniques, we can directly relate the detection of a tunneling electron to the absorption of a single photon.

7.1 Using the double quantum dot as a frequency-selective detector

The interplay between quantum optics and mesoscopic physics opens up new horizons for investigating radiation produced in nanoscale conductors [97, 98]. Microwave photons emitted from quantum conductors are predicted to show non-classical behavior such as anti-bunching [99] and entanglement [100]. Experimental investigations of such systems require sensitive, high-bandwidth detectors operating at microwave-frequency [101]. On-chip detection schemes, with the device and detector being strongly capacitively coupled, offer advantages in terms of sensitivity and large bandwidths. In previous work, the detection mechanism was implemented utilizing photon-assisted tunneling in a superconductor-insulator-superconductor junction [52, 102] or in a single quantum dot (QD) [53].

Aguado and Kouwenhoven proposed to use a double quantum dot (DQD) as a frequency-tunable quantum noise detector [96]. The idea is sketched in Fig. 7.1(a), showing the energy levels of the DQD together with a quantum point contact acting as a noise source. The DQD is operated with a fixed detuning $\delta$ between the elec-
trochemical potentials of the left and right QDs. For an isolated system, the DQD is in the Coulomb blockade regime and there will be no current flowing. However, if the system absorbs an energy $E = \delta$ from the environment, the electron in QD1 is excited to QD2. This electron may leave to the drain lead, a new electron enters from the source contact and the cycle can be repeated. The process induces a current flow through the system. Since the detuning $\delta$ may be varied continuously by applying appropriate gate voltages, the absorption energy is tunable.

The scheme is experimentally challenging, due to low current levels and fast relaxation processes between the QDs [103]. Here, we show that these problems can be overcome by using time-resolved charge-detection techniques to detect single electrons tunneling into and out of the DQD. Apart from giving higher sensitivity than conventional current measurement techniques, the method also allows us to directly relate a single-electron tunneling event to the absorption of a single photon. The system can thus be viewed as a frequency-selective single-photon detector for microwave energies. This, together with the fact that the charge-detection methods allow precise determination of the device parameters, provide major advantages compared to other setups [52, 53, 98, 101, 102].

The measurements were performed on the structure shown in Fig. 7.1(b), which consists of two quantum dots embedded in a ring, together with a nearby QPC (sample C in appendix A). As described in chapter 6, we tune the surrounding gates so that only the upper tunnel barrier connecting the two QDs is kept open. The tunnel coupling between the QDs was set to $t = 32 \, \mu$eV, as determined using charge localization measurements explained in section 6.2.2. The tunneling barriers between the DQD and the source and drain contacts were tuned to a few kHz to enable electron counting in real-time. In the following, we present measurements taken with zero bias across the DQD. Fig. 7.2(a) shows count rates close to the triple point where the $(n+1, m)$, $(n, m+1)$ and $(n+1, m+1)$ states are degenerate [see inset of Fig. 7.2(a)]. The arguments presented below are applicable also for the
Figure 7.2: (a) Count rate for electrons leaving the DQD, measured for a small region close to a triple point (marked by a white point). The inset shows a sketch of the surrounding hexagon pattern. The dashed line denotes the detuning axis, with zero detuning occurring at the triple point. The data was taken with $V_{\text{QPC}} = -300 \, \mu\text{V}$. (b) Blow-up of the lower-right region of (a), measured for different QPC bias voltages. (c) Rates for electron tunneling into and out of the DQD, measured along the dashed line in (a). $\Gamma_{\text{in}}$ falls of rapidly with detuning, while $\Gamma_{\text{out}}$ shows only minor variations.

triple point between the $(n, m)$, $(n + 1, m)$, $(n, m + 1)$ states, but for simplicity we consider only the first case. At the triple point [marked by a white dot in Fig. 7.2(a)], the detuning $\delta$ is zero and both dots are aligned with the Fermi level of the leads. The two strong, bright lines emerging from this point come from resonant tunneling between QD1 and the source lead (lower-right line) or between QD2 and the drain lead (upper-left line). The amplitude of the count rate at the lines gives directly the strength of the tunnel couplings to source and drain leads [20, 104], and we find the rates to be $\Gamma_S = 1.1 \, \text{kHz}$ and $\Gamma_D = 1.2 \, \text{kHz}$.

Along the white dashed line in Fig. 7.2(a), there are triangle-shaped regions with low but non-zero count rates where tunneling is expected to be strongly suppressed
due to Coulomb blockade. The DQD level arrangements inside the triangles are shown in the insets. Comparing with the sketch in Fig. 7.1(a), we see that both regions have DQD configurations favorable for noise detection. The dashed line connecting the triangles is the detuning axis, with zero detuning occuring at the triple point. We define the detuning as $\delta = \mu_1 - \mu_2$, so that the detuning is negative in the upper-left part of the figure.

In Fig. 7.2(b), the lower-right part of Fig. 7.2(a) was measured for four different QPC bias voltages. The resonant line stays the same in all four measurements, but the triangle becomes both larger and more prominent as the QPC bias is increased. This is a strong indication that the tunneling is due to absorption of energy from the QPC. The counts observed above the resonance line for $V_{QPC} = -400 \, \mu V$ are due to electrons being excited from the ground state to the first excited state of the DQD.

The time-resolved measurement technique allows the rates for electron tunneling into and out of the DQD to be determined separately [40]. Figure 7.2(c) shows the rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ measured along the dashed line of Fig. 7.2(a). The rate for tunneling out stays almost constant along the line, but $\Gamma_{\text{in}}$ is maximum close to the triple point and falls of rapidly with increased detuning. This suggests that only the rate for electrons tunneling into the DQD is related to the absorption process. To explain the experimental findings we model the system using a rate-equation approach. For a configuration around the triple point, the DQD may hold $(n+1,m)$, $(n,m+1)$ or $(n+1,m+1)$ electrons. We label the states $L$, $R$ and 2 and draw the energy diagrams together with possible transitions in Fig. 7.3(a). The figure shows the case for negative detuning, with $\delta \gg k_B T$. Note that when the DQD holds two excess electrons, the energy levels are raised by the mutual charging energy, $E_{\text{Cm}} = 800 \, \mu eV$.

In Fig 7.3(b) we sketch the time evolution of the system. The red curve shows the
expected charge detector signal assuming a detector bandwidth much larger than the transitions rates. Starting in state \( L \), the electron is trapped until it absorbs a photon and is excited to state \( R \) (with rate \( \Gamma_{\text{abs}} \)). From here, the electron may either relax back to state \( L \) (rate \( \Gamma_{\text{rel}} \)) or a new electron may enter QD1 from the source lead and put the system into state 2 (rate \( \Gamma_S \)). Finally, if the DQD ends up in state 2, the only possible transition is for the electron in the right dot to leave to the drain lead.

The relaxation rate for a similar DQD system has been measured to be \( 1/\Gamma_{\text{rel}} = 16 \text{ ns} \ [105] \), which is much faster than the available measurement bandwidth. Therefore, the detector will not be able to register the transitions where the electron is repeatedly excited and relaxed between the dots. Only when a second electron enters from the source lead [transition marked by \( \Gamma_S \) in Fig. 7.3(a, b)], the DQD will be trapped in state 2 for a sufficiently long time (\( \sim 1/\Gamma_D, \sim 1 \text{ ms} \)) to allow detection. The measured time trace will only show two levels, as indicated by the dashed line in Fig. 7.3(b). Such a trace still allows extraction of the effective rates for electrons entering and leaving the DQD, \( \Gamma_{\text{in}} = 1/\langle \tau_{\text{in}} \rangle \) and \( \Gamma_{\text{out}} = 1/\langle \tau_{\text{out}} \rangle \). To relate \( \Gamma_{\text{in}}, \Gamma_{\text{out}} \) to the internal DQD transitions, we write down the master equation for the occupation probabilities of the states:

\[
\frac{d}{dt} \begin{pmatrix} p_L \\ p_R \\ p_2 \end{pmatrix} = \begin{pmatrix} -\Gamma_{\text{abs}} & \Gamma_{\text{rel}} & \Gamma_D \\ \Gamma_{\text{abs}} & -(\Gamma_S + \Gamma_{\text{rel}}) & 0 \\ 0 & \Gamma_S & -\Gamma_D \end{pmatrix} \begin{pmatrix} p_L \\ p_R \\ p_2 \end{pmatrix}.
\] (7.1)

Again, we assume negative detuning, with \( |\delta| \gg k_B T \). The measured rates \( \Gamma_{\text{in}}, \Gamma_{\text{out}} \) are calculated from the steady-state solution of Eq. (7.1):

\[
\Gamma_{\text{in}} = \frac{\Gamma_S p_R}{p_L + p_R} = \frac{\Gamma_S \Gamma_{\text{abs}}}{\Gamma_S + \Gamma_{\text{abs}} + \Gamma_{\text{rel}}},
\] (7.2)

\[
\Gamma_{\text{out}} = \frac{\Gamma_D}{\Gamma_D}.
\] (7.3)

In the limit \( \Gamma_{\text{rel}} \gg \Gamma_S, \Gamma_{\text{abs}} \), the first expression simplifies to

\[
\Gamma_{\text{in}} = \Gamma_S \frac{\Gamma_{\text{abs}}}{\Gamma_{\text{rel}}}. \quad (7.4)
\]

The corresponding expressions for positive detuning are found by interchanging \( \Gamma_S \) and \( \Gamma_D \) in Eqs. (7.1-7.4). Coming back to the experimental findings of Fig. 7.2(c), we note that \( \Gamma_{\text{out}} \) only shows small variations within the region of interest. This together with the result of Eq. (7.3) suggest that we can take \( \Gamma_S, \Gamma_D \) to be independent of detuning. The rate \( \Gamma_{\text{in}} \) in Eq. (7.4) thus reflects the dependence of \( \Gamma_{\text{abs}}/\Gamma_{\text{rel}} \) on detuning. Assuming also \( \Gamma_{\text{rel}} \) to be constant, a measurement of \( \Gamma_{\text{in}} \) gives directly the absorption spectrum of the DQD. The measurements cannot exclude that \( \Gamma_{\text{rel}} \) also varies with \( \delta \), but as we show below the model assuming \( \Gamma_{\text{rel}} \) independent of detuning fits the data well.

Equation (7.4) shows that the low-bandwidth detector can be used to measure the absorption spectrum, even in the presence of fast relaxation. Moreover, the
detection of an electron entering the DQD implies that a quantum of energy was absorbed immediately before the electron was detected. The charge detector signal thus relates directly to the detection of a single photon. The efficiency of the detector is currently limited by the bandwidth of the charge detector. However, it should be possible to increase the bandwidth significantly by operating the QPC in a mode analogous to the radio-frequency single-electron transistor [11–13, 106].

To justify the assumption $\Gamma_{rel} \gg \Gamma_{abs}$, we note that even when the detector is too slow to detect individual transitions between the states $L$ and $R$, its dc-response still gives the average population of the two states. In Fig. 7.4, we plot the relative population of state $L$, $p_L/(p_L + p_R)$, for the same gate voltage configuration as in Fig. 7.2(a). The data was extracted by analyzing the absolute change in the QPC conductance for one electron tunneling into DQD (see section 6.2.2). Looking at the region of negative detuning (upper-left part of Fig. 7.4), the average DQD population within the regions of photon-assisted tunneling is very close to the pure $L$-state. The electron spends most of the time in QD1, which validates the assumption $\Gamma_{rel} \gg \Gamma_{abs}$. Similar arguments can be applied for the region of positive detuning.

For fixed DQD detuning, the processes described above only pump electrons in one direction. The system may therefore thought of as a ratchet, giving unidirectional electron flow even at zero bias [103].

### 7.2 Measuring the QPC emission spectrum

In the following, we use the DQD to quantitatively investigate the microwave radiation emitted from the nearby QPC. Figure 7.5(a) shows the measured count rate for electrons leaving the DQD versus detuning and QPC bias. The data was
7.2. Measuring the QPC emission spectrum

Figure 7.5: Count rate measured versus detuning and QPC bias voltage. The dashed line shows the level separation for a two-level system, with $\Delta_{12} = \sqrt{4t^2 + \delta^2}$. There are only counts in the region where $|eV_{\text{QPC}}| > \Delta_{12}$. (b) Count rate versus QPC bias for different values of detuning. The solid lines are guides to the eye. (c) DQD absorption spectrum, measured for different QPC bias. The dashed lines are the results of Eq. (7.6), with parameters given in the text.

taken along the dashed line of Fig. 7.2(a), with gate voltages converted into energy using lever arms extracted from finite bias measurements. Due to the tunnel coupling $t$ between the QDs, the energy level separation $\Delta_{12}$ of the DQD is given by $\Delta_{12} = \sqrt{4t^2 + \delta^2}$. The dashed lines in 7.5(a) show $\Delta_{12}$, with $t = 32 \, \mu\text{eV}$. A striking feature is that there are no counts in regions with $|eV_{\text{QPC}}| < \Delta_{12}$. This originates from the fact that the voltage-biased QPC can only emit photons with energy $\hbar\omega \leq eV_{\text{QPC}} [53, 96, 101]$. The result presents another strong evidence that the absorbed photons originate from the QPC.

To describe the results quantitatively, we consider the emission spectrum of a voltage biased QPC with one conducting channel. In the low-temperature limit $k_B T \ll \hbar\omega$, the spectral noise density $S_f(\omega)$ for the emission side ($\omega > 0$) takes the
form (see [96] for the full expression)

\[ S_I(\omega) = \frac{4e^2}{h} D(1 - D) \frac{eV_{QPC} - h\omega}{1 - e^{-\left(eV_{QPC} - h\omega\right)/k_B T}}, \]  

(7.5)

where \( D \) is the transmission coefficient of the channel. Using the model of Ref. [96], we find the absorption rate of the DQD in the presence of the QPC:

\[ \Gamma_{\text{abs}} = \frac{4\pi e^2 k^2 t^2 Z_l^2 S_I(\Delta_{12}/h)}{h^2} \frac{S_I(\Delta_{12}/h)}{\Delta_{12}^2}. \]  

(7.6)

The constant \( k \) is the capacitive lever arm of the QPC on the DQD and \( Z_l \) is the zero-frequency impedance of the leads connecting the QPC to the voltage source. Equation (7.6) states how well fluctuations in the QPC couple to the DQD system.

Figure 7.5(b) shows the measured absorption rates versus \( V_{QPC} \), taken for three different values of \( \delta \). As expected from Eqs. (7.5, 7.6), the absorption rates increase linearly with bias voltage as soon as \(|eV_{QPC}| > \delta\). The different slopes for the three data sets are due to the \( 1/\Delta_{12}^2 \)-dependence in the relation between the emission spectrum and the absorption rate of Eq. (7.6). In Fig. 7.5(c), we present measurements of the absorption spectrum for fixed \( V_{QPC} \). The rates decrease with increased detuning, with sharp cut-offs as \(|\delta| > eV_{QPC}\). In the region of small detuning, the absorption rates saturate as the DQD level separation \( \Delta_{12} \) approaches the limit set by the tunnel coupling. The dashed lines show the combined results of Eqs. (7.4-7.6), with parameters \( T = 0.1 \) K, \( Z_l = 0.7 \) kΩ, \( D = 0.5 \), \( t = 32 \) \( \mu \)eV, \( k = 0.15 \), \( \Gamma_S = 1.1 \) kHz and \( \Gamma_D = 1.2 \) kHz. Using \( \Gamma_{\text{rel}} \) as a fitting parameter, we find \( 1/\Gamma_{\text{rel}} = 5 \) ns. This should be seen as a rough estimate of \( \Gamma_{\text{rel}} \) due to uncertainties in \( Z_l \), but it shows reasonable agreement with previously reported measurements [105]. The overall good agreement between the data and the electrostatic model of Eq. (7.6) supports the assumption that the interchange of energy between the QPC and the DQD is predominantly mediated by photons instead of phonons or plasmons.

The data for \( V_{QPC} = 400 \) \( \mu \)V shows some irregularities compared to theory, especially at large positive detuning. We speculate that the deviations are due to excited states of the individual QDs, with excitation energies smaller than the detuning. The subject of single-dot excitations is treated in chapter 9. In Fig. 7.6, we convert the detuning \( \delta \) to level separation \( \Delta_{12} \) and use Eq. (7.6) to extract the noise spectrum \( S_I \) of the QPC. The linear dependence of the noise with respect to frequency corresponds well to the behavior expected from Eq. (7.5). Again, the deviations at \( \Delta_{12} = 190 \) \( \mu \)eV are probably due to an excited state in one of the QDs. The excited states are also visible in finite-bias spectroscopy, giving a single-level spacing of \( \Delta E \approx 200 \) \( \mu \)eV. This sets an upper bound on frequencies that can be detected with the detector. The frequency-range can be extended by using DQD in carbon nanotubes [107] or InAs nanowires [108, 109], where the single-level spacing is significantly larger. Photon-absorption processes in InAs nanowires are investigated in chapter 10.
7.3 Finite DQD bias regime

Finally, we apply a voltage bias over the DQD in order to compare the tunneling originating from sequential transport with the tunneling due to photon absorption processes. Figure 7.7(a) shows a charge stability diagram measured with DQD bias $V_{\text{DQD-SD}} = 300 \mu\text{V}$. The two triangles associated with electron and hole transport cycles are clearly visible. Besides that, we have regions of cotunneling (see section 6.3) as well as sharp lines with tunneling due to equilibrium fluctuations whenever the electrochemical potential of QD1 or QD2 lines up with the Fermi levels in the source or drain, respectively. In addition, there are faint triangles appearing in the detuning direction opposite to the transport triangles; we attribute these features to photon-assisted tunneling (PAT).

The DQD energy level configuration in the upper region with faint tunneling (next to the hole transport triangle) is depicted in Fig. 7.7(a). In this regime the DQD may hold one or two excess electrons. For this energy level alignment neither sequential tunneling nor cotunneling is possible. The DQD can only change its state if an electron in QD1 absorbs a photon and is excited to QD2. From this configuration, an electron may enter QD1 from the source lead followed by the electron in QD2 leaving to the drain. In Fig. 7.7(c), we present blow-ups of the region marked by the dashed rectangle in Fig. 7.7(a), measured for different QPC bias voltages. The dashed lines in the leftmost panel in Fig. 7.7(c) show the regions where we expect photon-assisted tunneling. As the QPC bias is increased, we see that the count rate inside these regions indeed goes up significantly. For the highest QPC bias voltage, there are extra features appearing outside the anticipated PAT-region. Again, we attribute this to an excited state in QD2 but postpone the detailed discussion to chapter 9.
Figure 7.7: (a) Charge stability diagram for the DQD, measured with a bias voltage $V_{\text{DQD-SD}} = 300 \, \mu\text{V}$ applied over the DQD. Tunneling due thermal fluctuations, sequential transport, cotunneling and photon-assisted processes (PAT) are visible. The data was taken with $V_{\text{DQD-SD}} = 300 \, \mu\text{V}$. (b) DQD energy level diagram for the upper region of photon-assisted tunneling in (a). The detuning is opposite to the bias direction. (c) Magnifications of the region marked by the dashed rectangle in (a), measured for three different QPC bias voltages. The dashed lines in the leftmost figure show the regions where we expect photon-assisted tunneling. As the QPC bias voltage is increased, the count rate goes up inside the PAT regions.
Chapter 8

Single-electron interference

A central concept of quantum mechanics is the wave-particle duality; matter exhibits both wave- and particle-like properties and can not be described by either formalism alone. Up to this point, we have treated the electrons as particles tunneling back and forth between quantum dots. In this chapter, we investigate their wave properties by studying interference of individual electrons taking two different paths in an Aharonov-Bohm interferometer. The time-resolved charge detection technique enables us to count electrons one-by-one as they pass the interferometer. In this way we make a direct measurement of the self-interference of a single electron. With increased bias voltage across the quantum point contact a back-action is exerted on the interferometer leading to dephasing. We attribute this to emission of radiation from the quantum point contact, which drives non-coherent electronic transitions in the quantum dots.

8.1 The Aharonov-Bohm effect

One of the cornerstone concepts of quantum mechanics is the superposition principle as demonstrated in the double-slit experiment [110]. The partial waves of individual particles passing a double slit interfere with each other. The ensemble average of many particles detected on a screen agrees with the interference pattern calculated using propagating waves [Fig. 8.1(a)]. This has been demonstrated for photons, electrons in vacuum [111, 112] as well as for more massive objects like C_{60}-molecules [113]. The Aharonov-Bohm (AB) geometry provides an analogous experiment in solid-state systems [114]. Partial waves passing the arms of a ring acquire a phase difference due to a magnetic flux, enclosed by the two paths [Fig. 8.1(b)]. Here, we set out to perform the interference experiment by using a quantum point contact to detect single-electron tunneling in real-time.

We first discuss the experimental conditions necessary for observing single-electron AB interference. We make use of a geometry containing two quantum dots within the AB-ring. Figure 8.1(c) shows the structure, with the two QDs (marked by 1 and 2) tunnel-coupled by two separate barriers. It is the same structure as investigated in chapters 6 and 7, but this time tuned to a regime where both barriers connecting
Chapter 8. Single-electron interference

Figure 8.1: (a) Setup of a traditional double-slit experiment. Electrons passing through the two slits give rise to an interference pattern on the observation screen. (b) Schematic drawing of the setup used for measuring single-electron Aharonov-Bohm interference. Electrons are injected from the source lead, tunnel through QD1 and end up in QD2, where they are detected. The interference pattern is due to the applied B-field, which introduce a phase difference between the left and right arm connecting the two quantum dots. (c) Double quantum dot used in the experiment. The yellow parts are lines written with a scanning force microscope on top of a semiconductor heterostructure and represent the potential landscape for the electrons. The QDs (marked by 1 and 2) are connected by two separate arms, allowing partial waves taking different paths to interfere. The current in the nearby QPC ($I_{QPC}$) is used to monitor the electron population in the system.

The QDs are kept open. Following the sketch in Fig. 8.1(b), electrons are provided from the source lead, tunnel into QD1 and pass on to QD2 through either one of the two arms. Upon arriving in QD2, the electrons are detected in real-time by monitoring the conductance of the nearby QPC [8, 62, 104, 115]. Coulomb blockade prohibits more than one excess electron to populate the structure, implying that the first electron must leave to the drain before a new one can enter. This enables time-resolved operation of the charge detector and ensures that we measure interference due to individual electrons.

To avoid dephasing, the electrons should spend a time as short as possible on their way from source to QD2. This is achieved by raising the electrochemical potential of QD1 so that electrons in the source lead lack an energy $\delta$ required for entering QD1 [see Fig. 8.3(b)]. The time-energy uncertainty principle still allows electrons to tunnel from source to QD2 by means of second order processes. The tunneling process is then limited to a short time scale set by the uncertainty relation, with $t = \hbar/\delta$. 

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8.2 Experimental realization

In the experiment, we apply appropriate gate voltages to tune the tunneling rates between the double quantum dot (DQD) and the source and drain leads to values below 15 kHz. The tunneling coupling between the QDs is set to a few GHz, as determined from charge localization measurements (see section 6.2.2). Figure 8.2 shows the charge stability diagram of the DQD systems, measured by counting electrons entering and leaving the DQD within a fixed period of time. The data was taken with 600 $\mu$V bias applied between source and drain. The hexagon pattern together with the triangles of electron transport appearing due to the applied bias are well-known characteristics of DQD systems (see chapter 6). Between the triangles, there are broad, band-shaped regions with low but non-zero count rates where sequential transport is suppressed due to Coulomb blockade. The finite count rate in this region is attributed to electron tunneling involving virtual processes, as described in section 6.3. In the following paragraph we quickly repeat the main results from that section.

Figure 8.3(a) shows the rates for electrons tunneling into and out of the DQD measured along the dashed line in Fig. 8.2. Going along the dashed line corresponds to lowering the electrochemical potential of QD1 while keeping the potential of QD2 constant. In the region marked by I, electrons tunnel sequentially from the source into QD1, continue from QD1 to QD2 and finally leave QD2 to the drain lead. Proceeding to point II in Fig. 8.3(a), the electrochemical potential of QD1 is lowered and an electron eventually gets trapped in QD1 [see sketch in Fig. 8.2(b)]. At position II, the electron lacks an energy $\delta_a$ to leave to QD2. Due to the energy-time uncertainty principle, there is a time-window of length $\sim \hbar/\delta_a$ within which tunneling from QD1 to QD2 followed by tunneling from the source into QD1 is possible without violating energy conservation. An analogous process is possible involving the next unoccupied state of QD1, occurring on timescales $\sim \hbar/\delta_b$. This
Figure 8.3: (a) Tunneling rates for electrons entering (red) and leaving (blue) the DQD, measured along the dashed white line of cotunneling in Fig. 8.2(a). The upper x-axis shows \( \delta_a \), the potential difference between the state in QD2 and the occupied state of QD1. The solid lines are tunneling rates expected from sequential tunneling, while the dashed line is a fit to the cotunneling model of Eq. (6.5) with parameters \( \Gamma_{Sa} = 6.4 \text{ kHz} \), \( \Gamma_{Sb} = 14 \text{ kHz} \), \( t_a = 8.3 \text{ µeV} \), and \( t_b = 13 \text{ µeV} \). The data was taken with \( B = 340 \text{ mT} \). (b) Schematic drawings of the energy levels of the DQD at position II in (a). The energy levels of QD1 are shifted so that the electron in QD1 is trapped due to Coulomb blockade. Electron transport from source to QD2 is still possible through virtual processes. (c) The tunneling processes depicted in the double quantum dot structure. When both barriers between the QDs are kept open, the cotunneling electron may take any arm when going from source to QD2. (d) Number of electrons arriving at QD2 within the fixed period of time indicated in the upper-right corner, measured as a function of magnetic field. The data was taken at point II in (a). The count rate shows an oscillatory pattern with a visibility higher than 90%.

corresponds to electron cotunneling from the source lead to QD2. By continuing to point III, the unoccupied state of QD1 is shifted into the bias window and electron transport is again sequential. The rate for electrons tunneling out of the DQD [\( \Gamma_{out} \), blue trace in Fig. 8.3(a)] shows only slight variations over the region of interest. This is expected, since the potential of QD2 stays constant along the dashed line in Fig. 8.2(a).

Coming back to the sketch of Fig. 8.1(b), we note that the cotunneling configuration of case II in Fig. 8.3(a,b) is ideal for investigating the Aharonov-Bohm effect for single electrons. Due to the low probability of the cotunneling process, the source lead provides low-frequency injection of single electrons into the DQD. The injected electrons cotunnel through QD1 into QD2 on a timescale \( t \sim \hbar/\delta \sim 1 \text{ ps} \)
8.3 Noise in the Aharonov-Bohm regime

much shorter than the decoherence time of the system, which is on the order of a few nanoseconds [116, 117]. This ensures that phase coherence is preserved. Finally, the electron stays in QD2 for a time long enough to be registered by the finite-bandwidth charge detector. The tunneling processes are sketched in Fig. 8.3(c).

Next, we tune the system to case II of Fig. 8.3(a) and count electrons as a function of magnetic field. Figure 8.3(d) shows snapshots taken at three different times. The electrons arriving in QD2 build up a well-pronounced interference pattern with period 130 mT. This corresponds well to one flux quantum $\Phi = h/e$ penetrating the area enclosed by the two paths. The visibility of the AB-oscillations is higher than 90%, which is a remarkably large number demonstrating the high degree of phase coherence in the system. We attribute the high visibility to the short time available for the cotunneling process [118] and to strong suppression of electrons being backscattered in the reverse direction, which is otherwise present in AB-experiments. Another requirement for the high visibility is that the two tunnel barriers connecting the QDs are carefully symmetrized. The overall decay of the maxima of the AB-oscillation with increasing $B$ is probably due to magnetic field effects on the orbital wavefunctions in QD1 and QD2.

In Fig. 8.4(a), we investigate the separate rates for electrons tunneling into and out of the DQD as a function of magnetic field. The y-axis corresponds to the dashed line in Fig. 8.2, i.e., to the energy of the states in QD1. The measurement shows a general shift of the DQD energy with the applied B-field, which we attribute to changes of the orbital wavefunctions in the individual QDs. Within the cotunneling region, $\Gamma_{\text{in}}$ shows well-defined B-periodic oscillations. At the same time, $\Gamma_{\text{out}}$ is essentially independent of the applied field. This is expected since $\Gamma_{\text{out}}$ measures the rate at which electrons leave QD2 to the drain, which occurs independently of the magnetic flux passing through the AB-ring [see Fig. 8.3(a,c)]. In Fig. 8.4(b), the bias over the DQD is reversed. This inverts the roles of $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ so that $\Gamma_{\text{out}}$ corresponds to the cotunneling process. Here $\Gamma_{\text{out}}$ shows B-periodic oscillations while $\Gamma_{\text{in}}$ remains unaffected. In the black regions seen in Fig. 8.4(b), no counts were registered within the measurement time of three seconds due to strong destructive interference for the tunneling-out process. As a consequence, it was not possible to determine $\Gamma_{\text{in}}$ in these regions.

In the sequential regime (upper and lower parts of the color maps in Fig. 8.4), one would also expect AB-oscillations to occur. However, the effect would show up as a modulation of the coupling between the QDs ($\Gamma_C$), which involves timescales of the order $\sim 1/\Gamma_C \sim 1$ ns. The detection of single electron motion on such timescales is presently out of reach due to limited bandwidth of the detector.

8.3 Noise in the Aharonov-Bohm regime

In this section we investigate the noise of the current in the Aharonov-Bohm regime. Using the methods of chapter 5, we can extract the noise and the higher moments
Figure 8.4: (a) Tunneling rates for electrons entering ($\Gamma_{in}$) and leaving ($\Gamma_{out}$) the DQD, measured versus electrochemical potential of QD1 and magnetic field. The y-axis corresponds to sweeps along the dashed line in Fig. 8.2. Within the cotunneling region, $\Gamma_{in}$ shows clear B-field periodicity, while $\Gamma_{out}$ remains constant. This is in agreement with the picture where only the electrons tunneling from source to QD2 encircle the Aharonov-Bohm ring, while electrons leaving to drain remain unaffected by the applied B-field. (b) Same as (a), but with reverse bias over the DQD. Here, the roles of $\Gamma_{in}$ and $\Gamma_{out}$ are inverted.

of the current distribution directly from the QPC conductance traces. Figure 8.5(a) shows a measurement of the current flowing through the DQD, measured in a regime close to the upper region of sequential tunneling in Fig. 8.4(a) [$V_{G1} = 49 \text{ mV}$ at $B = 0 \text{ mT}$, dashed line in Fig. 8.4(a)]. When sweeping the magnetic field, we tune the voltages on gates G1 and G2 to compensate for the shift of the cotunneling region occurring due to orbital effects in the QDs. We chose to measure the AB-oscillation at relatively low DQD detuning; this enhances the cotunneling rates and allows us to collect more statistics within reasonable measurement times. On the other hand, it also increases the contribution of sequential tunneling and photon-assisted tunneling processes, giving the slightly lower visibility compared to Fig. 8.3(d). The small spikes seen at $B = \pm 120 \text{ mT}$ in Fig. 8.5(a) (marked by arrows) are attributed
8.3. Noise in the Aharonov-Bohm regime

Figure 8.5: (a) DQD current in the Aharonov-Bohm regime. (b) Noise \( \mu_2 \) of the DQD current. The curve strongly resembles the average current shown in (a), with the AB-oscillations clearly visible. (c-d) Fano factor \( \mu_2/\mu \) and generalized Fano factor for the third moment \( \mu_3/\mu \), measured within the same region as the traces shown in (a, b). All quantities were extracted from a QPC conductance traces of length \( T = 40 \text{s} \), measured with \( V_{\text{DQD-SD}} = 600 \mu \text{V} \) and \( V_{\text{QPC-SD}} = 300 \mu \text{V} \).

to single-QD excitations; this is the subject of chapter 9.

In Fig. 8.5(b), we plot the shot noise (second moment \( \mu_2 \)) of the current distribution, extracted from the same set of data as used in Fig. 8.5(a). The noise curve shows strong similarities to the current trace in (a), with the AB-oscillations clearly visible. This is reasonable, since we expect the noise to scale with the magnitude of the current. In Fig. 8.5(c), we plot the Fano factor \( \mu_2/\mu \), extracted from the traces in Fig. 8.5(a,b). Also the Fano factor displays AB-oscillations, with a minimum occurring at \( B = 0 \text{mT} \) (with \( \mu_2/\mu = 0.55 \)). We can understand this by considering the noise calculated for a single QD [see Eq. (5.13) in chapter 5]. There, we saw a reduction of the Fano factor due to Coulomb blockade, with the lowest noise given in a configuration where the tunneling rates for entering and leaving the QD were equal.

In the AB-regime, we also measure a current due to two tunneling rates; one is the cotunneling rate showing strong AB-oscillations (in this case \( \Gamma_{\text{in}} \)), while the other (\( \Gamma_{\text{out}} \)) is a sequential rate being independent of external magnetic field [compare the
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rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ in Fig. 8.4(a)]. At zero magnetic field, the cotunneling rate $\Gamma_{\text{in}}$ has a maximum and at this point it becomes comparable to the sequential rate $\Gamma_{\text{out}}$. The two tunneling rates are relatively symmetric, giving a reduction of the Fano factor. For higher magnetic fields, the cotunneling rate $\Gamma_{\text{in}}$ drops drastically while $\Gamma_{\text{out}}$ stays constant. This results in a more asymmetric configuration and a Fano factor close to one.

In the region of higher magnetic fields the experimental precision of the measurement decreases. This is because of the low average count rate, giving less statistical data for extracting the moments compared to the region around $B = 0$ mT. Finally, in Fig. 8.5(d) we plot $\mu_3/\mu$, the generalized Fano factor for the third moment. This quantity also shows indications of AB-oscillations, but the experimental uncertainty in the high B-field range becomes even larger than for the conventional Fano factor.

8.4 Temperature effects

In Fig. 8.3(a), we investigate how the AB-oscillations are influenced by elevated temperatures. The dephasing of open QD systems is thought to be due to electron-electron interaction [119], giving dephasing rates that depend strongly on temperature [120]. Figure 8.6(a) shows the temperature dependence of the AB oscillations in our system. The amplitude of the oscillations remains almost unaffected up to $\sim 400$ mK, indicating that the coherence is not affected by temperature until the thermal energy becomes comparable to the single-level spacing of the QDs.

Figure 8.6(b) shows measurements of the electron count rate vs magnetic field and the average potential of the DQD, taken at $T = 100$ mK and $T = 300$ mK. Contrary to the measurements presented in Fig. 8.3 and Fig. 8.4, the potential difference between QD1 and QD2 is kept constant at $\delta = 350$ $\mu$eV while the overall DQD energy $\varepsilon$ is shifted relative to the leads. The energy $\varepsilon$ is taken to be zero when the level in QD2 is aligned with the Fermi level of the drain [case III in Fig. 8.6(b,c)]. Here, thermal population fluctuations tunneling between QD2 and the drain lead gives rise to a high count rate [strong red line in the lower part of Fig. 8.6(b)]. The width of the resonant line is set by the temperature of the electrons in the lead. Indeed, this line is clearly broader for the $T = 300$ mK data.

Going to point I in Fig. 8.6(b,c), the energy of the DQD is raised compared to the leads and thermal fluctuation are no longer relevant. Here, electrons can only enter QD2 by cotunneling from the source lead. The data shows clear Aharonov-Bohm oscillations at both $T = 100$ mK and $T = 300$ mK, with comparable visibility. At the same time, the effect of the increased temperature is visible in the regime around $\varepsilon = 0$. As the temperature is further increased, the line of thermal fluctuations becomes broader and eventually reaches the dashed line where the AB-oscillations of Fig. 8.6(a) were measured. This leads to the sharp decrease of the AB-visibility demonstrated in Fig. 8.6(a). We conclude that the decreased visibility at higher temperatures is due to an increase in thermal fluctuations of the DQD population.
Figure 8.6: (a) Aharonov-Bohm (AB) oscillations measured at different temperatures. At \( \sim 400 \) mK, the visibility of the oscillations drops drastically. The data was taken along the dashed line in (b). (b) Rate of electrons entering QD2, measured versus B-field and total energy of the DQD, \( \varepsilon \). The two images show data taken at two different temperatures, \( T = 100 \) mK and \( T = 300 \) mK. The DQD energy \( \varepsilon \) is taken to be zero when QD2 is aligned with Fermi level of the drain. Here, tunneling due to thermal fluctuations between QD2 and the lead gives rise to a high count rate (point III). This feature is visibly broadened when the temperature is increased. In the cotunneling region (point I), the count rate shows clear AB oscillations. The elevated temperature only has a slight impact on the AB-visibility. In case II, the cotunneling rate goes up compared to case I. We attribute the increase to tunneling into an excited state in QD2. (c) Diagrams depicting DQD energy levels for the three configurations marked in (b).

8.5 Phase shifts for tunneling involving excited states

In the following, we investigate the phase of the AB-oscillations for different states in QD2. Previous experiments have shown phase shifts of \( \pi \) occurring between consecutive Coulomb resonances in many-electron quantum dots [121, 122]. To measure AB-oscillations for consecutive electron fillings requires a relatively large
shift of the gate voltages. Such measurements are difficult to perform in our setup, since large changes of gate voltages also affect the symmetry of the left and right arm connecting QD1 and QD2, which may strongly reduce the visibility of the AB-oscillations. Instead, we look at excited states of QD2 at fixed electron population [123].

In addition to highlighting temperature effects, the color map in Fig. 8.6(b) also shows the existence of excited states in the QDs. At point II in Fig. 8.6(b,c), the count rate is increased compared to case I. We attribute the increase to cotunneling into an excited state in QD2 (see section 6.3.3). Measuring the AB-oscillations at various DQD energy thus provides a way to investigate relative phases of the excited states in the QDs. From the data in Fig. 8.6(b), we see that the AB-oscillations persist in regions involving several excited states and that the phase of the oscillations seems to remain the same in all regions.

Depending on the direction of the applied bias, we can probe different excited states (see section 6.2.3). For positive bias, electrons cotunnel from source into QD2 and may thereby put QD2 into either the \((m,n+1)\)-electron ground state or an \((m,n+1^*)\)-electron excited state [see case II in Fig. 8.6(c)]. For negative DQD bias, the cotunneling involves an electron leaving from QD2 to the source contact. This involves transitions taking the QD2 into either its \((m,n)\)-electron ground state or into an \((m,n^*)\)-electron excited state. Since the energy difference \(E[(n,m^*)] - E[(n,m+1)]\) is smaller than \(E[(n,m)] - E[(n,m+1)]\), the transition involving the excited state \([(m,n^*)]\) occurs at an energy \(\Delta E\) below the ground state transition [see case II in Fig. 8.7(c)].

Figure 8.7(a) shows a measurement of the electron count rate versus magnetic field and DQD energy \(\varepsilon\) for negative DQD bias. Again, we define \(\varepsilon = 0\) when the electrochemical potential of QD2 is aligned with the Fermi level of the drain lead [see case I in Fig. 8.7(c)]. Here, the tunneling is mainly due to equilibrium fluctuations between QD2 and the drain. As \(\varepsilon\) is reduced, the equilibrium fluctuations between QD2 and drain are no longer possible and electrons can only leave QD2 by cotunneling to the source. The cotunneling region shows AB-oscillations, but the oscillations are less uniform compared to the results for positive bias [Fig. 8.6(b)].

Between the position marked by II and III in Fig. 8.7(a), both the intensity and the behavior of the count rate changes drastically. In Fig. 8.7(b), we plot two cross sections from Fig. 8.7(a), taken at the positions of the dashed lines. Both traces show AB-oscillations, and both curves are symmetric around \(B = 0\) T as expected from the Onsager relations. However, by comparing the positions of the maxima for \(B > 0\) T we see that the phase is shifted by \(0.7\pi\) between the two curves. The reason for the apparent lack of phase rigidity is not understood, further measurements are needed for a more complete understanding of the phenomena.

Starting at point III in Fig. 8.7(a,c), the transition involving the \([(m,n^*)]\)-electron excited state is below the Fermi level of the source so that only cotunneling through the ground state is possible. The trace in Fig. 8.7(b) belonging to point III is qualitatively similar to the data shown in Fig. 8.6(a), with both curves having a maximum

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8.5. Phase shifts for tunneling involving excited states

Figure 8.7: (a) Electron count rate, measured versus magnetic field and total DQD energy relative to the leads, $\varepsilon$. The data was measured with $V_{DQD-SD} = -600 \mu V$ applied to the DQD. (b) Count rates measured at the positions marked by the dashed lines in (a). There is a phase shift of $\sim 0.7\pi$ between the two curves. The trace for $\varepsilon = -580 \mu eV$ has been magnified by a factor of ten for better visibility. (c) Energy diagrams of the DQD for the positions marked by I, II and III in (a). At point I, the potential of QD2 is lined up with the Fermi level in the right lead and the tunneling is mainly due to equilibrium fluctuations between QD2 and the lead. At point II, the DQD potential is shifted downwards, so that electrons in QD2 may only leave by cotunneling to the source lead. The energy level arrangement allows a process involving an excited states of QD2 to contribute to the cotunneling. Finally, at point III only cotunneling involving the ground state of QD2 is possible.

appearing at $B = 0$ T. The similarity is expected, since both measurements involve cotunneling through the ground state of QD2. Moving to point II, the energy of the DQD is raised and the transition involving the excited state may also contribute to transport. The cotunneling rate measured in this regime is a sum of the processes involving the ground state and the excited state. However, since the rates at point II are almost an order of magnitude larger compared to point III, the behavior is to a large extent dominated by cotunneling from the excited state.

From this, we conclude that there is a phase shift occurring in the Aharonov-Bohm signal between tunneling involving the $(m,n)$-electron ground state and a
Chapter 8. Single-electron interference

Figure 8.8: (a) Visibility of the AB-oscillations measured at different QPC bias. The visibility stays roughly constant up to $V_{QPC} = 300 \mu V$ and then decreases drastically with increasing bias voltage. We attribute the reduction in visibility to an increase in photon-assisted tunneling. (b) Energy level diagram of the DQD in the cotunneling configuration. At high QPC bias, both intradot and interdot photon absorption processes become possible.

(m, n*)-electron excited state of QD2. Our findings are in agreement with previously reported results [121–123], but more measurements are needed to map out the complete phase behavior of the QD spectrum.

8.6 Decoherence due to the quantum point contact

In the experiment, we use the current in the QPC to detect the charge distribution in the DQD. In principle, the QPC could also determine whether an electron passed through the left or the right arm of the ring, thus acting as a which-path detector [124, 125]. If the QPC were to detect the electron passing in one of the arms, the interference pattern should disappear. In Fig. 8.8(a), we show the visibility of the AB-oscillations as a function of bias on the QPC. The visibility remains unaffected up to $V_{QPC} \sim 250 \mu eV$, but drops for higher bias voltages.

We argue that the reduced visibility is not due to which-path detection. At $V_{QPC} = 400 \mu V$, the current through the QPC is $\sim 10$ nA. This gives an average time delay between two electrons passing the QPC of $e/I_{QPC} \sim 16$ ps. Since this time is larger than the typical cotunneling time, it is unlikely that the electrons in the QPC are capable of performing an effective which-path measurement. Instead, we attribute the decrease of the AB-visibility to processes where the DQD absorbs photons emitted from the QPC. As described in chapter 7, such processes may indeed excite an electron from one QD to the other, as long as the energy of the excited state is lower than the energy provided by the QPC bias [126]. The radiation of the
8.6. Decoherence due to the quantum point contact

QPC may also drive transitions inside the individual QDs, thus putting one of the dots into an excited state [53]. A few possible absorption processes are sketched in Fig. 8.3(b).

As long as the QPC bias is lower than both the DQD detuning ($\delta = 400 \mu eV$) and the single-level spacing of the individual QDs ($\Delta E \sim 200 \mu eV$), the AB visibility in Fig. 8.3(b) is close to unity. When raising the QPC bias above $\Delta E$, we start exciting the individual QDs. With increased QPC bias, more states become available and the absorption process becomes more efficient. This introduces new virtual paths for the cotunneling process. Since the different paths may interfere destructively, the interference pattern is eventually washed out. In this way, the QPC has a physical back-action on the measurement which is different from informational back-action [65] and which-path detection previously investigated [124, 125]. The properties of the single-QD excitations are investigated in greater detail in chapter 9.
Chapter 9
Quantum dot spectroscopy

Spectroscopy is the study of the interaction between light and matter as a function of radiation wavelength. The method is commonly used to identify substances from their absorption or emission spectra. In this chapter, we use the radiation emitted from a quantum point contact to study the excitation spectrum of a quantum dot. Surprisingly, we find the absorption probability of the quantum dot to be strongly dependent on external magnetic field. We attribute this to shape deformation of the electron wavefunction, leading to varying overlap between ground and excited states.

9.1 Single quantum dot excitations

In chapter 7, we investigated electron transitions between the two QDs driven by absorption of radiation from a quantum point contact. The radiation of the QPC may also drive transitions inside the individual QDs and thereby put one of the dots into an excited state [53]. Since this process does not involve any major spatial displacement of the electron charge distribution, it is normally not possible to detect the effect with a charge detector. However, the phenomenon can be probed by tuning the structure into the configuration sketched in Fig. 9.1(a), with zero bias over the DQD. The electrochemical potential $\mu_1$ of QD1 is put close to the Fermi energy of the source lead and the electrochemical potential $\mu_2$ of QD2 is tuned to be far away from both $\mu_1$ and the Fermi level of the drain. The position in the charge stability diagram is indicated in the inset of Fig. 9.1(a).

In Fig. 9.1(b), we plot the count rate as a function of gate voltage $V_{G1}$ for different QPC bias voltages. The upper x-axis shows the electrochemical potential of QD1 relative to the source lead, calculated using the capacitive lever arms of the gates known from finite-bias measurements. The peak at $V_{G1} = 47$ mV ($\mu_1 = 0 \mu eV$) is due to electron tunneling back and forth between QD1 and the source lead, with the peak width set by the temperature in the lead [46]. For $\mu_1 \gg k_B T$, equilibrium fluctuations are suppressed and we expect no tunneling. However, as the QPC bias is increased, a shoulder appears for negative $\mu_1$. The shoulder corresponds to the process sketched in Fig. 9.1(a), where an electron in QD1 is put into the first excited
state by absorption of a photon. The excited state is above the Fermi level of the lead, so that the electron can tunnel out to the source lead. Afterwards, QD1 is refilled with an electron from the source lead and the cycle is repeated. Direct relaxation from the excited to the ground state in QD1 may limit the efficiency of the process; we will come back to this in section 9.2. The width of the shoulder corresponds to the level spacing $\Delta E$ and is independent of QPC bias. Neither the width nor the height of the main peak is influenced when changing QPC bias, as expected from the model of Fig. 9.1(a).

The processes sketched in Fig. 9.1(a) are independent of the electrochemical potential of QD2. We therefore expect the features described above to show up as shoulders of weak tunneling next to the lines of thermal fluctuations in the charge stability diagram. Such features are indeed present in the charge stability diagrams shown in Fig. 9.2, measured with $V_{\text{DQD-SD}} = -600 \mu\text{V}$ bias applied to the DQD.

The data in Fig. 9.2(a) was extracted from QPC conductance traces measured with QPC bias $V_{\text{QPC-SD}} = 250 \mu\text{V}$. Figure 9.2(b) shows a measurement of the same region with $V_{\text{QPC-SD}} = 400 \mu\text{V}$. A charge rearrangement at $V_{G2} = -36.5 \text{mV}$ causes a minor shift of the DQD energy level configuration, but otherwise both pictures show the finite-bias triangles of sequential transport as well as the regions

Figure 9.1: (a) Energy level diagram depicting the absorption process. The electron in QD1 is excited due to photon absorption, which allows it to tunnel out to the lead. (b) Single quantum dot excitations, measured for different QPC bias voltage. The main peak is due equilibrium fluctuations between the source lead and QD1. As the gate voltage is increased, the electrochemical potential of QD1 drops below the Fermi level of the source lead and only tunneling processes involving excitations of QD1 become possible. The excitation process becomes more likely with increased QPC bias voltage, while the width of the region is given by the energy of the excited state in QD1 and is therefore independent of the QPC voltage. The measurement was performed in a configuration close to case A in Fig. 9.2(b) but without bias applied to the DQD. The position in the charge stability diagram is indicated in the inset of (a).
9.1. Single quantum dot excitations

![Figure 9.2](image)

Figure 9.2: (a) Charge stability diagram of the double QD, measured by counting electrons entering the DQD, measured with $V_{\text{DQD}-\text{SD}} = -600\,\text{µV}$ bias applied to the DQD. The data was extracted from QPC conductance traces of length $T = 1\,\text{s}$, measured with $V_{\text{QPC}-\text{SD}} = 250\,\text{µV}$. A charge rearrangement at $V_{G2} = -36.5\,\text{mV}$ causes a minor shift of the DQD energy level configuration. (b) Same as (a), but measured with $V_{\text{QPC}-\text{SD}} = 400\,\text{µV}$. The solid lines show the position of the finite-bias triangles, while the dashed lines mark regions of cotunneling. The arrows mark regions of tunneling occurring outside the regimes of sequential tunneling and cotunneling. The tunneling is attributed to excitations in the individual quantum dots. (c) Energy level diagrams of the DQD for the three positions marked in (a). In case I, the tunneling is due to cotunneling from drain to QD1. In case II, the electrochemical potential QD1 is lowered and tunneling is mainly due to equilibrium fluctuations between the source lead and QD1. In case III, neither cotunneling nor sequential tunneling is possible. Instead, we attribute the tunneling to an electron in QD1 being excited into the first excited state followed by tunneling to the source contact.

of cotunneling expected from the applied DQD bias (see chapter 6). In Fig. 9.2(b), the finite-bias triangles and the regimes of cotunneling are marked by solid and dashed lines, respectively. The tunneling rates inside these regions are very similar in Fig. 9.2(a) and (b). Outside those regions, the data taken with $V_{\text{QPC}-\text{SD}} = 400\,\text{µV}$ shows additional features occurring in thin stripes next to lines of resonant tunneling [marked by A, B, C and D in Fig. 9.2(b)]. These features correspond to the single-QD excitations described in Fig. 9.1, with cases A, B corresponding to excitations in QD1 and cases C, D to excitations in QD2. The data in Fig. 9.1 was taken in
a configuration close to case A in Fig. 9.2(b), but without any bias applied to the DQD.

Figure 9.2(c) shows energy level diagrams for the three positions marked in Fig. 9.2(a). Starting in position I, the measured count rate is due to cotunneling from the drain lead to QD1. Going to position II, the electrochemical potential of QD1 is resonant with the Fermi level in the source lead, giving a sharp line of tunneling due to equilibrium fluctuations. Proceeding further to position III, both QDs are in the Coulomb blockade and the only possible mechanism for electron tunneling involves excitations in QD1 due to photon absorption. It should be noted that the features due to single QD excitations are more pronounced in Fig. 9.2 than in charge stability diagrams presented in chapter 6 (see, for example Fig. 6.10). This can partly be attributed to the use of more moderate QPC bias voltages in previous chapters. Another difference is that whereas the previous measurements were performed with only one tunnel barrier connecting the two QDs, the charge stability diagrams of Fig. 9.2 are taken with both arms open and tuned to have roughly equal coupling strength (the same configuration was used for measuring the AB-oscillations in chapter 8). The importance of the ring structure for the single-QD excitations is discussed in more detail at the end of this chapter.

9.2 Rate-equation model

To acquire a more quantitative understanding of the single-QD excitations, we go back to the regime shown in Fig. 9.1(b) and look at the separate rates for electrons tunneling into and out of the DQD as a function of QPC bias. Figure 9.3(a) shows the rates \( \Gamma_{\text{in}} \) and \( \Gamma_{\text{out}} \) plotted versus electrochemical potential of QD1, measured for different \( V_{\text{QPC-SD}} \). The rates were extracted from the same measurement as presented in Fig. 9.1(b). The rate for tunneling into the DQD does not show any major QPC bias dependence.

Starting at the position marked by I in Fig. 9.3(a, c), the electrochemical potential of QD1 lines up with the Fermi level of the source lead \( (\mu_1 = 0) \). Here, the tunneling is due to equilibrium fluctuations and the rates for tunneling into and out of the DQD are equal. By lowering the electrochemical potential of QD1 we come into the regime of single-QD excitations [case II in Fig. 9.3(a, c)]. In this region the rate related to photon absorption \( (\Gamma_{\text{out}}) \) varies with bias voltage over the QPC. The bias dependence is remarkably strong; the rate due to photon absorption increases by three orders of magnitude over a bias range of 300 \( \mu \text{V} \). Continuing to case III, the excited state goes below the Fermi level of the source lead and the photon absorption rate drops quickly. At the same time, \( \Gamma_{\text{in}} \) increases as the refilling of an electron into QD1 may occur through either the ground state or the excited state [see sketch III in Fig. 9.3(c)]. This provides a way to determine the tunnel coupling between the source contact and the excited state in QD1 \( (\Gamma_{\text{ES}}^S) \). From the data in Fig. 9.3(a), we find \( \Gamma_{\text{ES}}^S \approx 25 \text{kHz} - 5 \text{kHz} = 20 \text{kHz} \). This value should be considered only as a
9.2. Rate-equation model

Figure 9.3: (a) Rates for electrons tunneling into and out of the DQD, extracted from the same measurement as shown in Fig. 9.1(a). The tunneling-out rate ($\Gamma_{\text{out}}$) was measured for different QPC bias voltages, ranging from $V_{\text{QPC-SD}} = 200, 250, 300, \ldots, 500 \mu V$. The different traces are colored as in Fig. 9.1(b). (b) Tunneling rates vs QPC bias voltage, extracted from the region marked by II in (a). As the QPC bias is raised, the tunneling-out rate increases rapidly while $\Gamma_{\text{in}}$ stays essentially constant. The apparent fluctuations of $\Gamma_{\text{in}}$ at low QPC bias arise due to insufficient statistics because of the low $\Gamma_{\text{out}}$. (c) Energy level diagrams for the three positions marked in (a). For case I, tunneling is due to equilibrium fluctuations between source and QD1. In case II, the tunneling-out rate is due to absorption of radiation which takes QD1 into an excited state. In this region, $\Gamma_{\text{out}}$ is strongly dependent on QPC bias. For case III, an incoming electron may tunnel into either the ground or excited state of QD1. This leads to the increase in $\Gamma_{\text{in}}$ seen in (a).

A rough estimate for $\Gamma_{\text{ES}}$, since the rates are on the limit of what is measurable with the finite-bandwidth detector ($\Gamma_{\text{det}} = 25 \text{kHz}$).

In Fig. 9.3(b) we plot the rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ versus QPC bias, measured with the QD1 potential in the regime of single-QD excitations ($\mu_1 = -80 \mu V$). Below $V_{\text{QPC-SD}} = 180 \mu V$, the bias voltage over the QPC is lower than the excitation energy of QD1 and we observe no counts. As $V_{\text{QPC-SD}}$ is increased above 180 $\mu V$, the rate due to absorption ($\Gamma_{\text{out}}$) goes up almost exponentially until $V_{\text{QPC-SD}} \sim 400 \mu V$. For higher QPC bias voltages $\Gamma_{\text{out}}$ continue to increase, but less dramatic than in the low bias regime. The rate $\Gamma_{\text{in}}$ shows only weak QPC bias dependence, which is expected since it corresponds to refilling QD1 with an electron from the source lead or through cotunneling from the drain lead. The results of Fig. 9.3(b) are in qualitative agreement with the single QD absorption processes sketched in Fig. 9.1(a). However, the very strong QPC bias dependence is problematic. From the results of Eq. (7.5) in chapter 7, we expect the spectral density of the radiation emitted from a QPC to increase linearly with QPC bias voltage.
In order to better understand the results we model the tunneling-out process using a rate-equation approach. Based on the results shown in Fig. 9.2 and Fig. 9.3, we assume that the process can be described by a model involving only three QD states (ground state, first excited state, empty QD). The three configuration together with possible transitions between the states are depicted in Fig. 9.4. The charge detector only provides a measurement of the rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ for electrons entering or leaving the DQD. This corresponds to transitions crossing the dashed line in Fig. 9.4. Even for the relatively simple three-state model, there are a large number of internal rates that we wish to extract from the experimentally obtained rates $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$. To simplify the situation somewhat, we use the results of Fig. 9.2 and Fig. 9.3 to exclude transitions that only give a minor contribution to the measured rates:

**Cotunneling.** The model includes transitions involving cotunneling from QD1 to the drain lead. For cotunneling, one would expect a strong suppression of the measured rates with increased DQD detuning. This behavior is not observed in Fig. 9.2. Moreover, if the electrochemical potential of QD2 is far from $\mu_1$ and the Fermi levels in the leads, we know from section 6.3 that the cotunneling rates are expected to be orders of magnitude weaker than direct sequential rates. For the moment we will neglect transitions involving cotunneling, but we come back to subject in section 9.3 for situations where $\mu_2$ is close to $\mu_1$. 

Figure 9.4: Energy level diagrams of the three states in our model, depicting possible transitions between the states. The charge detector can only detect transitions between states with different QD charge population (marked by the dashed line in the figure).
9.2. Rate-equation model

Direct transitions from the ground state to the lead. One can imagine direct transitions from the ground state to the source lead without involving the excited state. Such transitions can not explain the well-defined shoulder in Fig. 9.3, but they do give rise to features visible in the region of thermal fluctuations around $\mu_1 = 0$ in Fig. 9.3(a). The exponential decrease in $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ when going away from $\mu_1 = 0$ becomes slightly less steep with increased QPC bias (most clearly visible for $\Gamma_{\text{out}}$ at $\mu_1 > 0$). The transitions may be driven by an increase in electronic temperature or by fluctuations in the QPC current. Still, from Fig. 9.3(a) we see that the effect is minor compared to transitions involving the excited state and can be neglected if $\mu_1$ is put sufficiently far away from the Fermi level of the leads.

Transitions from the lead into the excited state. The QD may be refilled not only into the ground state but also into the excited state (rate $\Gamma_{\text{S-ES}}^\text{GS}$ in Fig. 9.4). However, the measured rate $\Gamma_{\text{in}}$ shows no major dependence on neither QPC bias nor electrochemical potential $\mu_1$ within the region of the shoulder in Fig. 9.3(a). Therefore, we only consider refilling into the ground state.

With these simplifications in mind we write down the master equation for the occupation probabilities $p = [p_{\text{GS}}, p_{\text{ES}}, p_0]$ of the three states

\[
\frac{d}{dt} \begin{pmatrix} p_{\text{GS}} \\ p_{\text{ES}} \\ p_0 \end{pmatrix} = \begin{pmatrix} -\Gamma_{\text{abs}} & \Gamma_{\text{rel}} + \Gamma_{\text{em}} & \Gamma_{\text{S-ES}}^\text{GS} \\ \Gamma_{\text{abs}} & -(\Gamma_{\text{S-ES}}^\text{ES} + \Gamma_{\text{rel}} + \Gamma_{\text{em}}) & 0 \\ 0 & \Gamma_{\text{S-ES}}^\text{ES} & -\Gamma_{\text{abs}} \end{pmatrix} \begin{pmatrix} p_{\text{GS}} \\ p_{\text{ES}} \\ p_0 \end{pmatrix}.
\] (9.1)

The rates of the simplified model are visualized in Fig. 9.5. Note that we distinguish between an intrinsic spontaneous relaxation rate $\Gamma_{\text{rel}}$ and a rate $\Gamma_{\text{em}}$ for stimulated emission driven by the QPC. The measured rate $\Gamma_{\text{out}}$ is calculated from the steady-state solution of Eq. (9.1):

\[
\Gamma_{\text{in}} = \Gamma_{\text{S-ES}}^\text{GS}, \quad \Gamma_{\text{out}} = \frac{\Gamma_{\text{S-ES}}^\text{ES} p_{\text{ES}}}{p_{\text{ES}} + p_{\text{GS}}} = \frac{\Gamma_{\text{S-ES}}^\text{ES}}{\Gamma_{\text{S-ES}}^\text{GS} + \Gamma_{\text{abs}} + \Gamma_{\text{rel}} + \Gamma_{\text{em}}}. \tag{9.2}
\]

In the experimentally relevant limit $\Gamma_{\text{S-ES}}^\text{ES} \ll \Gamma_{\text{rel}}$ the expression simplifies to

\[
\Gamma_{\text{out}} = \Gamma_{\text{S-ES}}^\text{ES} \frac{\Gamma_{\text{abs}}}{\Gamma_{\text{abs}} + \Gamma_{\text{rel}} + \Gamma_{\text{em}}}. \tag{9.3}
\]

We estimate the behavior of $\Gamma_{\text{out}}$ in the limit of weak absorption ($\Gamma_{\text{abs}} \ll \Gamma_{\text{rel}}$). Here, Eq. (9.3) simplifies to

\[
\Gamma_{\text{out}} = \Gamma_{\text{S-ES}}^\text{ES} \Gamma_{\text{abs}}/\Gamma_{\text{rel}}. \tag{9.4}
\]

This is the same expression as obtained in Eq. (7.4) in chapter 7 for the case of photon-assisted tunneling in the double quantum dot. In analogy to the DQD case,
we assume that the single QD excitations are driven by the fluctuations in the QPC current. Following the ideas of chapter 7, we combine Eq. (9.4) with the results of Eqs. (7.5-7.6) from chapter 7:

\[
\Gamma_{\text{out}} \propto \Gamma_S^{ES} S_I(\Delta E/\hbar) = \Gamma_S^{ES} \frac{4e^2}{\hbar} D(1 - D) \frac{eV_{\text{QPC}} - \Delta E}{1 - e^{-(eV_{\text{QPC}} - \Delta E)/k_B T}}. \tag{9.5}
\]

Here, \( S_I(\omega) \) is the spectral noise density of the voltage biased QPC and \( D \) is the QPC transmission coefficient. Similar to the double QD case, we thus expect the absorption rate to increase linearly with QPC bias voltage for \( eV_{\text{QPC}} - \Delta E \geq 0 \) and fall off exponentially as \( eV_{\text{QPC}} - \Delta E < 0 \). While this model was well able to explain the QPC bias dependence of the absorption in the double quantum dot case [see Fig. 7.5(b)], it clearly fails to predict the behavior for the single-QD excitation in Fig. 9.3(b). The measured rate \( \Gamma_{\text{out}} \) indeed shows a sharp cut-off for \( eV_{\text{QPC}} - \Delta E < 0 \), but the increase in \( \Gamma_{\text{out}} \) for \( eV_{\text{QPC}} - \Delta E > 0 \) is much stronger than linear.

One could possibly argue that we do not measure transitions to an excited state at \( \Delta E \sim 200 \mu\text{eV} \) but rather to an excited state with \( \Delta E \sim 400 \mu\text{eV} \). In this case we could explain the exponential increase in the region \( V_{\text{QPC-SD}} < 400 \mu\text{eV} \) to be due to temperature. However, two facts speak against such a hypothesis. First, the width of the shoulder of photon-assisted tunneling in Fig. 9.3(a) provides a direct measure of \( \Delta E \), giving \( \Delta E \sim 200 \mu\text{eV} \). Second, the slope extracted from the exponential increase in the region \( V_{\text{QPC-SD}} < 400 \mu\text{eV} \) of Fig. 9.3(b) would correspond to a temperature of \( T \sim 400 \text{mK} \). This does not match the temperature measured for electrons in the QD (\( T = 100 \text{mK} \)). It could well be that the temperature is slightly higher in the QPC than in the DQD due to the current flow in the QPC. Still, in the range \( V_{\text{QPC-SD}} < 400 \mu\text{eV} \) the QPC current is relatively moderate (\( I_{\text{QPC}} < 15 \text{nA} \)) and we find it unlikely that the QPC current could lead to such a strong increase in temperature. The almost exponential increase in Fig. 9.3(b) thus remains unexplained; we will come back to the issue later in this chapter when discussing the magnetic field dependence.
9.3 Magnetic field dependence

Next, we fix $\mu_1 = -100 \mu\text{eV}$ and $V_{\text{QPC-SD}} = 400 \mu\text{V}$ and measure absorption as a function of magnetic field [Fig. 9.6(a)]. Unexpectedly, the absorption rate shows very sharp (width of $\sim 1 \text{mT}$) features at certain magnetic fields. Both the peak width and the separation of peaks involve magnetic field scales much smaller than the period of the AB-oscillations of chapter 8 [see Fig. 8.3(d)]. In Fig. 9.6(b), we repeat the measurement for different values of the potential of QD2. A complex pattern evolves, with the peaks forming parabolae with both positive and negative curvature in the $B - \mu_2$ plane.

In Fig. 9.7(a,b) we plot the separate rates for tunneling into and out of the DQD for the same set of data as shown in Fig. 9.6(b). Only the rate $\Gamma_{\text{out}}$ corresponding to the absorption process shows a strong magnetic field dependence. These sharp

Figure 9.6: (a, b) Photon absorption measured as a function of magnetic field and potential of QD2. The data is measured in the configuration used in Fig. 9.3(a), with $\mu_1 = -100 \mu\text{eV}$ and $V_{\text{QPC-SD}} = 400 \mu\text{V}$. At $\mu_2 = 1.5 \text{meV}$, $\mu_2$ is equal to the charging energy of QD2 and an electron will leave QD2 to the drain. The trace in (a) shows a cross-section taken at $\mu_2 = 1.3 \text{meV}$. 

9.3 Magnetic field dependence

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In Fig. 9.7(a,b) we plot the separate rates for tunneling into and out of the DQD for the same set of data as shown in Fig. 9.6(b). Only the rate $\Gamma_{\text{out}}$ corresponding to the absorption process shows a strong magnetic field dependence. These sharp
features in B-field and the large shift with QD2 potential are completely unexpected from the model in Fig. 9.1(a). The efficiency of an absorption process is expected to depend strongly on the spatial overlap between the ground and excited state in the QD. One may argue that changes in B-field and gate voltage introduce spatial fluctuations in the electron wavefunctions, which could lead to an increased overlap and thus increased absorption rates for certain configurations. However, such fluctuations are expected to appear on magnetic field scales corresponding to one flux quantum penetrating the area of the QD, which is several hundred mT for our sample [127].

Figure 9.7(c) shows the change in QPC current $\Delta I_{QPC}$ for one electron leaving the DQD, also extracted from the same measurement as in Figs. 9.7(a, b). Since the QPC is located closer to QD1, the signal $\Delta I_{QPC}$ reflects the position of the tunneling
electrons. Regions with high $\Delta I_{QPC}$ ($0 \text{ meV} < \mu_2 < 1.4 \text{ meV}$) corresponds to charge fluctuations in QD1, outside this region, the charge fluctuations occur in QD2. At the transitions between regions of low and high $\Delta I_{QPC}$, the tunneling coupling $t$ is comparable to the DQD detuning and the electrons form molecular states extending over both QDs. The small width of the transition regions around $\mu_2 = 0$ and $\mu_2 = 1.4 \text{ meV}$ shows that the tunnel coupling is relatively weak ($t \sim 20 \mu\text{eV}$). For most regions of Fig. 9.7, the DQD detuning is much larger than $t$ which seems to justify the assumption that the excitations are mainly located in QD1.

We now focus at the transition region around $\mu_2 = 1.4 \text{ meV}$ [marked with I in Fig. 9.7(b, c)]. Here, the electrochemical potential of the occupied state in QD2 ($\mu_2 - E_{C2}$) is close to $\mu_1$. The electron wavefunctions will hybridize and form bonding- and antibonding states, as depicted in case I in Fig. 9.7(d). For this situation, the absorption process involves an excited state distributed over both QDs and the excited electron may leave to either source or drain lead. When increasing $\mu_2$ slightly more we come to a configuration where excitations occur mainly in QD2 [case II in Fig. 9.7(d)]. If we raise $\mu_2$ even further, the electrochemical potential of QD2 goes above the Fermi level of the drain. Here, electrons may leave QD2 directly from ground state and the measured rate $\Gamma_{\text{out}}$ goes up drastically. Interestingly, the sharp magnetic-field features seen in Fig. 9.7(b) do not show any radical changes or discontinuities in the region around case I where the weight of the electron wavefunction is shifted from QD1 to QD2. This may indicate that the unexpected peaks in magnetic field do not come from excited states in individual QDs alone but rather arise due to properties of the coupled DQD system.

To get a better understanding of the phenomenon, we have repeated the measurement with different electron numbers in the DQD. Specifically, we add an electron to QD2 and perform the experiments in the regime marked by B in Fig. 9.2(b). We do not change the population of QD1; in a simple picture of independent QDs this means that we expect the same ground and excited state to be responsible for the photon absorption as in case A. In reality this is probably not entirely correct; since the QDs are relatively strongly coupled, the states in QD1 are most likely influenced by the presence of an electron in QD2 by interaction.

We start by measuring the count rate versus magnetic field across the region of absorption marked by B in Fig. 9.2(b). The results are shown in Fig. 9.8(a). The strong red region at the bottom of the graph ($\mu_1 = 0$) comes from equilibrium fluctuations between QD1 and the source contact. Lowering the potential of QD1, we move into the region of photon-assisted tunneling. At $B = 0 \text{ mT}$, the absorption rate is low and we get the shoulder of weak tunneling seen in Fig. 9.2(b). As we change the magnetic field, sharp peaks of strong tunneling appear in the region of absorption. The peaks are symmetric in magnetic field. We want to particularly stress two features of Fig. 9.8(a); first, the magnetic field only changes the intensity of the absorption rate, the width of the shoulder stays roughly constant over the whole magnetic field range. Second, the peak at $\mu_1 = 0$ displays only weak B-field dependence. This shows that the resonant features occurring at certain magnetic
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Figure 9.8: (a) Count rates measured versus potential of QD1 and magnetic field. The strong red line in the lower part of the figure is due to equilibrium fluctuations as the electrochemical potential of QD1 lines up with the source lead ($\mu_1 = 0$). Features occurring at negative $\mu_1$ are due to photon absorption processes. The data was measured across the region of single-QD excitations marked by B in Fig. 9.2(b). (b) Excitation of QD1 due to photon absorption. The measurement is similar to the results shown in Fig. 9.3(a), but this time measured with one additional electron in QD2. The sweep corresponds to a cross section at the position of the arrow in (a), with $B = -51 \text{mT}$ and $V_{\text{QPC–SD}} = 350 \mu\text{V}$.

fields are not due to tuning of the tunnel coupling $\Gamma_S$.

In Fig. 9.8(b), we plot the rates for electrons tunneling into and out of the DQD measured along a cross-section at $B = -51 \text{mT}$ [position marked by an arrow in Fig. 9.8(a)]. The two rates are slightly lower than in Fig. 9.3(a), but the qualitative behavior is the same; at $\mu_1 = 0$ the rates are equal because of equilibrium fluctuations. Going into the region of absorption, the tunneling-out rate $\Gamma_{\text{out}}$ reflects the absorption process, while $\Gamma_{\text{in}}$ is due to refilling of an electron from the source lead. As $\mu_1 < -\Delta E \sim -150 \mu\text{eV}$, both the ground and the excited state are below the Fermi level of the source and the absorption rate drops to zero. At the same time, $\Gamma_{\text{in}}$ increases as the excited state becomes available for refilling QD1. Similar to the case in Fig. 9.3(a), we can use this fact to estimate the tunneling rate between the source contact and the exited state in QD1, giving $\Gamma_S^{\text{ES}} = 2.8 \text{kHz}$. These features are consistent with the model of single-QD excitations shown in Fig. 9.1.

Next, we have fixed $\mu_1 = -90 \mu\text{eV}$ and measured the absorption as a function of $\mu_2$ and magnetic field. The results are shown in Fig. 9.9(a), with the dashed line marking the position of $\mu_2$ used for the data shown in Fig. 9.8. Similar to the case in Fig. 9.6, the peaks in count rate shifts with potential of $\mu_2$, but the pattern is completely different compared to Fig. 9.6. Moreover, the peak at $B = \pm 115 \text{mT}$ seen in Fig. 9.8(a) actually corresponds to a crossing of two peaks. Figure 9.9(b) shows a magnification of the region around the crossing, plotted using a linear color
9.3. Magnetic field dependence

Figure 9.9: (a) Photon absorption measured as a function of magnetic field and potential of QD2. The data is measured in the same regime as shown in Fig. 9.8(a), with $V_\text{QPC-SD} = 350 \, \mu\text{V}$ and $\mu_1 = -90 \, \mu\text{eV}$. (b) Magnification of the crossing region marked by the black rectangle in (a). The vertical line is actually split into two, with the finer structure having a width below 1 mT. The image uses the same colors as in (a), but with a linear scale ranging from 0 to 500 counts/s. (c) Rate for electrons tunneling out of the DQD, measured for different QPC bias voltage. The rate is directly related to photon absorption in QD1. The measurement was performed in two different configuration, corresponding to the configurations marked by \( I \) and \( II \) in (a).

Two things are striking about the measurement; first, the peak showing less shift with $\mu_2$ is actually split into two peaks, with the smaller sub-peak having a full-width half maximum well below 1 mT. Second, there is no anti-crossing visible in the regime where the two main peaks meet. The two processes giving rise to the two peaks seem to be uncoupled.

We now come back to the issue of the efficiency of the absorption processes as a
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function of QPC bias. The data previously presented (see Fig. 9.3) was measured in a regime of a relatively weak absorption; it would certainly be interesting to see how the absorption efficiency behaves at the position of one of the peaks seen in Fig. 9.9(a). In Fig. 9.9(c), we plot the rate connected with absorption ($\Gamma_{\text{out}}$) versus QPC bias voltage, measured in regimes of both strong and weak absorption [positions marked by I and II in Figure 9.9(a)]. The data taken at weak absorption ($B = 0 \, \text{mT}$) shows relatively low rates, but the rate increases stronger than linear with applied bias. This is similar to the behavior seen in Fig. 9.3(b). On the other hand, the data taken on the peak ($B = -51 \, \text{mT}$) shows a remarkably different behavior; after an initial region of strong increase, the absorption rate saturates at $\Gamma_{\text{out}} \approx 1.4 \, \text{kHz}$ for $V_{\text{QPC-SD}} > 500 \, \mu\text{V}$. Coming back to the rate-equation model of Fig. 9.5, we may attribute the saturation to a situation where the rates driven by the QPC current ($\Gamma_{\text{abs}}$ and $\Gamma_{\text{em}}$) completely dominate the behavior of the system. Using the results of Eq. (9.3) and assuming $\Gamma_{\text{abs}} \gg \Gamma_{\text{rel}}$, we get

$$\Gamma_{\text{out}} = \Gamma_{\text{S}}^{\text{ES}} \frac{\Gamma_{\text{abs}}}{\Gamma_{\text{abs}} + \Gamma_{\text{em}}} = \Gamma_{\text{S}}^{\text{ES}} \frac{1}{1 + k},$$

(9.6)

where $k = \Gamma_{\text{em}}/\Gamma_{\text{abs}}$. A measurement of $\Gamma_{\text{out}}$ versus QPC bias thus allows us to compare how the absorption and stimulated emission rates depend on $V_{\text{QPC-SD}}$. The fact that we measure saturation in $\Gamma_{\text{out}}$ implies that the ratio $k = \Gamma_{\text{em}}/\Gamma_{\text{abs}}$ is independent of $V_{\text{QPC-SD}}$ in the region of saturation. Moreover, since the tunnel coupling $\Gamma_{\text{S}}^{\text{ES}}$ is known from the measurement in Fig. 9.8(b), we can insert numbers into Eq. (9.6):

$$k = \frac{\Gamma_{\text{S}}^{\text{ES}}}{\Gamma_{\text{out}}} - 1 = \frac{2.7 \, \text{kHz}}{1.4 \, \text{kHz}} - 1 \approx 1.$$

(9.7)

The results of Eq. (9.7) suggest that the rates due to absorption and stimulated emission are roughly equal ($\Gamma_{\text{abs}} \approx \Gamma_{\text{em}}$) and that they scale in the same way with applied bias. When evaluating the experimental data, we have simplified the situation by assuming that only one excited state is responsible for the measured absorption rate; at high QPC bias ($V_{\text{QPC-SD}} > 400 \, \mu\text{V}$), there are certainly several states available that could contribute to the process. Also, the exponential increase seen at low QPC bias is still not understood. Further investigations will be necessary to understand this phenomenon.

9.4 Excitations in QD2

In the previous sections we treated the case of electron excitations occurring in QD1. Here, we move on to investigate photon absorption processes involving QD2. Looking at regions C and D of Fig. 9.2(b), we see that such processes also occur in QD2. For regions C and D in Fig. 9.2(b), the features due to photon absorption appear at lower gate voltages, meaning that the processes happen in configurations where the electrochemical potential of QD2 is above the Fermi level of the drain.
9.4. Excitations in QD2

Figure 9.10: (a) Energy level diagram depicting the absorption process involving an excited state below the ground state transition. (b) Photon absorption processes occurring in QD2. The main peak is due to equilibrium fluctuations between the QD2 and the drain lead. As the gate voltage is decreased, the electrochemical potential of QD2 is raised above the Fermi level of the drain lead and QD2 needs to be put into an excited state before an electron can enter from the drain. The width of the region of photon-assisted tunneling is given by the energy of the excited state in QD2. The measurement was performed in a configuration close to case C in Fig. 9.2(b) but without bias applied to the DQD. The data was taken with $V_{\text{QPC-SD}} = 350 \mu V$ and $B = 38 \text{ mT}$. 

lead. The absorption thus involves an excited state with transition energy below the ground state transition.

The DQD energy level for such a configuration is sketched in Fig. 9.10(a). The electrochemical potential $\mu_2$ of QD2 is higher than the Fermi level of the lead and we assume the DQD to contain $(n, m)$ electrons. Next, the radiation from the QPC may induce a transition in QD2, thus taking it from the $(n, m)$ electron ground state to an excited state $(n, m^*)$. Since the energy difference $E[(n, m + 1)] - E[(n, m^*)]$ is smaller than $E[(n, m + 1)] - E[(n, m)]$, the transition involving the excited state appears below the ground state transition. We have encountered similar situations before in sections 4.5 and 6.2.3. Now, as long as the DQD stays in the $(n, m^*)$ excited state, an electron may enter QD2 from the source. The experimentally accessible quantity associated with the photon absorption process is therefore the rate for electrons tunneling into the DQD. The rate $\Gamma_{\text{out}}$ describes electrons leaving QD2 to the drain, which occurs independently of the absorption process.

Figure 9.10(b) shows the count rate measured across the region of photon-assisted tunneling marked by C in Fig. 9.2(b). The data was taken with zero bias over the DQD and with $V_{\text{QPC-SD}} = 350 \mu V$. We see a well-defined shoulder appearing next to the peak of resonant tunneling; as for the case of QD1 [Fig. 9.1(b)], the width of the shoulder is equal to the energy of the involved excited state ($\Delta E$). We should
Figure 9.11: (a) Count rates measured versus potential of QD2 and magnetic field. A vertical cross section would correspond to the measurement shown in Fig. 9.10(b). The strong red line in the upper part of the figure is due to equilibrium fluctuations as the electrochemical potential of QD2 lines up with the drain lead ($\mu_2 = 0$). Features occurring at higher $\mu_2$ are due to photon absorption processes. The data was measured across the region of single-QD excitations marked by C in Fig. 9.2(b). The measurement was performed with $V_{\text{QPC-SD}} = 350\,\mu\text{V}$. (b) Horizontal cross section of (a), taken at the position of the dashed line. The different traces corresponds to different QPC bias voltages. Note the sharp increase in counts at the position of the peaks when going from $V_{\text{QPC-SD}} = 250\,\mu\text{V}$ to $V_{\text{QPC-SD}} = 350\,\mu\text{V}$.

perhaps point out that the fact that we observe positive-energy excited state transitions more clearly in QD1 and negative-energy transitions more clearly in QD2 is coincidental; the visibility of the excited states depends strongly on how well they are coupled to the leads. Also, in the charge stability diagram of Fig. 9.2(b), the absorption regions marked by A,B,C and D are better visible due to the direction of the DQD bias. Features appearing on the opposite sides of the lines of resonant tunneling would be concealed by count rates due to stronger processes such as cotunneling and sequential tunneling.

Next, we measure the photon-absorption features in QD2 for different magnetic fields. Figure 9.11(a) shows the count rate versus $\mu_2$ and B-field, measured for the same configuration as in Fig. 9.10(b). The trace of Fig. 9.10(b) corresponds to a vertical cross-section in Fig. 9.11(a). The strong, red line in the upper part of the
9.4. Excitations in QD2

Figure 9.12: Photon absorption measured as a function of magnetic field and potential of QD1. The data is measured in the same regime as shown in Fig. 9.11(a), with $V_{\text{QPC-SD}} = 500 \ \mu\text{V}$ and $\mu_2 = 110 \ \mu\text{eV}$. The dashed line marks the position of $\mu_1$ for the data shown in Fig. 9.11(a). At the position of the black horizontal line, electron counting was not possible due to strong increase in background noise from a fluctuating charge trap.

Below this line, the count rate is due to processes involving excitations of QD2. As for the case of photon absorption in QD1, the absorption rate varies strongly with B-field [see Fig. 9.8(a)]. Similar to the case of excitations in QD1, only the intensity of the absorption changes with magnetic field; the width of the shoulder remains constant. Also, the peak due to equilibrium fluctuations ($\mu_2 = 0$) shows only minor changes with applied field.

In Fig. 9.11(b), we plot traces corresponding to horizontal cross-section at the position of the dashed line in Fig. 9.11(a). The different traces are taken with increasing bias over the QPC. The huge increase in count rate occurring at the position of the peaks when going from $V_{\text{QPC-SD}} = 250 \ \mu\text{V}$ to $V_{\text{QPC-SD}} = 350 \ \mu\text{V}$ is striking. At higher QPC bias voltages we see an overall increase of the absorption rate which is less dependent on magnetic field.

Finally, we fix $\mu_2 = 110 \ \mu\text{eV}$ and measure the absorption as a function of $\mu_1$ and magnetic field. The results are shown in Fig. 9.12, with the dashed line marking the position of $\mu_1$ used for the measuring the data shown in Fig. 9.11. Similar to the cases of photon absorption in QD1 (see Fig. 9.6 and Fig. 9.9), the peaks in absorption rate moves with electrochemical potential of QD1. Again, the evolving pattern looks completely different compared to the measurements performed in the other configurations.
9.5 Conclusion and outlook

In this chapter, we have investigated single QD excitations driven by the current in the nearby quantum point contact. Although parts of the data agree relatively well with the picture of transitions driven by photon emission from the QPC, there are several open questions. In the remaining part of this chapter we summarize the results and make some speculations about the physical origin of the unexpected features.

Sharp peaks in magnetic field

The sharp peaks in the absorption rate occurring with external magnetic field are as spectacular as unexpected from the simple model of single-QD excitations considered in Fig. 9.1(a). What sets the width of these peaks? The typical magnetic field scale associated with ring structures or quantum dots corresponds to one flux quantum penetrating either the ring or one of the QDs, giving much larger field scales ($\Delta B > 100$ mT). Some of the features seen in Fig. 9.6 and Fig. 9.9 are sharper than 1 mT in width. However, we must not forget that it is not the QD energy that changes on such small field scales. The energy of the absorbed radiation is set by the excited state in the QD, and this stays fairly constant with magnetic field [see, for example Fig. 9.11(a)]. Instead, the magnetic field may change the efficiency of the absorption, most likely by tuning the spatial overlap between the ground and the excited states.

This makes this experiment different compared to a standard transport experiment where the measured current is given by the overlap of the QD wavefunction and the electron wavefunctions in the leads. This overlap is exponentially small everywhere except for a small region around the tunnel barriers. For the excitation processes investigated here, we must consider the spatial overlap over the whole quantum dot structure. In this sense, the measurement provides a sensitive probe of the similarities of the wavefunctions in the ground and in the excited state. Unfortunately, the shapes of the wavefunctions are hard to calculate since neither the confining potential nor the electron number in the DQD is exactly known.

The strong dependence on the electrochemical potential of the non-active QD together with the relatively strong intradot coupling indicate that we are dealing with states that are distributed over both QDs. In that case, the ring shape of the DQD could be responsible for the unexpected B-field features. For example, one could imagine that the sharp features are due to interference of trajectories taking several turns around the ring. For the ring geometry, it is likely that the external magnetic field is more efficient in tuning the shape of the wavefunction than would have been the case for a standard double quantum dot. It would therefore be interesting to perform similar experiments in a DQD without the ring.
QPC bias dependence

Finally, we come back once more to the strong increase of the absorption rate with QPC bias. As mentioned before, the almost exponential behavior [Fig. 9.3(b)] measured at low QPC voltages is not expected from the emission spectrum of the quantum point contact. In configurations where the magnetic field is tuned to maximize the absorption rates, we quickly reach a regime of saturation where the absorption and stimulated emission dominate over intrinsic QD relaxation processes. The relaxation time in a QD is typically of the order of nanoseconds, meaning that the rates for absorption and stimulated emission must be in the GHz regime.

With this in mind, we may draw a parallel between our QD-QPC system and the operation of a laser. In the laser system, light with a specific wavelength is kept in an optical cavity containing a gain medium. An external source of energy takes the atoms of the gain medium from the ground state to an excited state. When light already present inside the cavity passes through the medium, the photons stimulate the excited atoms to emit additional photons of the same frequency, phase and direction. When the pumping gain becomes larger than the cavity losses, the power of the light in the cavity quickly rises.

In the QD-QPC system, the broad-band pumping source would be the bias over the quantum point contact. The QD acts as the gain medium, by only allowing absorption to occur at a specific frequency. If one includes a second excited state at a higher energy and fast relaxation from the second to the first excited state, one could even think of reaching population inversion similar to the operation of a three-level laser. The difference compared to the laser is that there is no well-defined cavity around the structure designed to hold the light mode. The wavelength of the emitted light would be of the order of $\lambda = c/f = c/(150 \mu\text{eV}/h) \approx 1$ cm; one would need to calculate the impedances between the QPC and its environment to check if the electrical circuit could possibly support standing modes at these frequencies. If this were the case, the fast increase measured for the absorption rate could be due to the build-up of a light field inside a cavity, similar to the case of a real laser. The above discussion is highly speculative and perhaps completely incorrect, but it opens up ideas for new experiments to be performed to study the interaction between radiation and electron transport in semiconductor quantum dots.
Chapter 10

Counting electrons in a nanowire quantum dot

In this chapter we look at single-electron detection for a quantum dot defined in an InAs nanowire. The charge detector is realized by putting the nanowire on top of a quantum point contact defined in a GaAs heterostructure containing a two-dimensional electron gas. The small dimensions of the system lead to strong capacitive coupling between the quantum dot and the quantum point contact. This enables us to increase the detector bandwidth to a regime where we can simultaneously perform charge detection and measure the dot current with a conventional current meter. In this configuration the device can be thought of as a self-calibrated current meter. Moreover, by passing a high current in the quantum point contact we are able to drive transitions in the quantum dot. Since the nanowire quantum dot and the detector are fabricated in different material systems, we attribute the interactions between the two systems to photons rather than phonons.

10.1 Charge detection in a nanowire quantum dot

We start by describing the nanowire QD-QPC structure investigated in this chapter. InAs nanowires are catalytically grown by metal-organic vapor phase epitaxy (the detailed procedure is described in Ref. [128]). An InAs nanowire is deposited on top of a shallow (37 nm) AlGaAs/GaAs heterostructure based two-dimensional electron gas (2DEG). The QD in the InAs nanowire and a QPC in the underlying 2DEG are defined in a single etching step using patterned electron beam resist as an etch mask. This method guarantees perfect alignment as well as strong electrostatic coupling between the two devices [129].

Figure 10.1(a) shows a scanning electron microscope (SEM) image of a device similar to the one used in the measurements (structure D in appendix A). The QD is defined by the etched constrictions in the nanowire between S and D. The QPC is formed between the two etched trenches that separates it from the remaining 2DEG. The regions marked by L and R are used as side gates to control the QD population.
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Figure 10.1: (a) SEM image of the device. The quantum dot is formed in the nanowire, with the quantum point contact located in the 2DEG directly beneath the QD. (b) Typical time trace of the QPC conductance, showing a few electrons tunneling into and out of the QD. The upper level corresponds to a situation with $n$ electrons on the QD. (c) Time resolution of the detector, defined as the time needed for the current to cross the midline between current levels belonging to the $(n)$ and $(n+1)$ electron states.

and to tune the tunnel coupling between the QD and the source and drain leads. A voltage was applied to the 2DEG on both sides of the QPC to compensate for the shift in QPC potential when changing the voltages on gates L and R. For the results presented in the next two sections, the QPC was biased with a DC voltage of $V_{\text{QPC-SD}} = 1$ mV. The bias is set smaller than the single-level spacing of the QD to avoid excitations due to photon absorption (see chapter 9); we postpone the investigation of the QPC influence to section 10.3. The measurements were performed at a temperature of 1.7 K, but we have tested that the setup produces similar results at $T = 4$ K.

The charge detector is implemented by operating the QPC at the slope below the first plateau and continuously monitoring its conductance [115]. Due to strong electrostatic coupling between the QD and the QPC, an electron entering or leaving the QD will shift the QPC potential and thereby change its conductance. Figure 10.1(b) shows a typical example of QPC current trace. Coulomb blockade prohibits the QD to hold more than one excess electron. The two current levels in the figure cor-
responds to \((n)\) and \((n + 1)\) electrons on the QD, respectively. As described in chapter 3, transitions between the two levels relate directly to an electron tunneling into or out of the QD [8, 62, 104]. The times \(\tau_{\text{in}}\) and \(\tau_{\text{out}}\) describe the times needed to tunnel into and out of the QD.

In terms of charge detection, the structure in Fig. 10.1(a) has two major advantages compared to the AFM-defined samples considered in earlier chapters. First, the capacitive coupling between the QPC and the QD is stronger due to their small separation (<100 nm). This increases the change \(\Delta G_{\text{QPC}}\) in QPC conductance when adding an electron to the QD; for the nanowire sample, the ratio \(\Delta G_{\text{QPC}}/G_{\text{QPC}}\) can be greater than 50% (see chapter 3). Second, the small size of the nanowire QD gives a large charging energy \((E_C \approx 10 \text{ meV})\) and a large single-level spacing \((\Delta E \approx 2-3 \text{ meV})\). Corresponding values for an AFM-defined structure are \(E_C \approx 2-3 \text{ meV}\) and \(\Delta E \approx 200-300 \mu\text{eV}\). This makes it possible to increase the QPC bias voltage without driving charge transitions in the QD (see chapter 9). These two advantages result in a stronger measurable signal \(\Delta I_{\text{QPC}}\), which in turn allows us to increase the detection bandwidth. Figure 10.1(c) shows a magnification of one of the charge transitions in Fig. 10.1(b), indicating the rise time of the detector. Setting the threshold for event detection to the middle between the two levels, we find that the detector has a time resolution of \(\tau_{\text{det}} = 4 \mu\text{s}\). In the present setup, the bandwidth is limited by the low-pass filter formed by the capacitance of the cables and the room-temperature amplifier. The bandwidth can be greatly enhanced by using a cold amplifier [10] or an rf-QPC setup [11–13].

To characterize the system, we start by tuning the tunneling rates of the QD to be well below the time resolution of the detector. Figure 10.2(b) shows Coulomb diamonds for the QD, measured by counting electrons from traces such as the one shown in Fig. 10.1(b). In the regime of single-level transport, the tunneling times are expected to follow an exponential distribution

\[
p_{\text{in/out}}(t) \, dt = \Gamma_{\text{in/out}} e^{-\Gamma_{\text{in/out}} t} \, dt. \tag{10.1}
\]

Figure 10.2(b) shows the measured distribution of tunneling times, taken at the point marked by I in Fig. 10.2(a). The solid lines in Fig. 10.2(b) are fits to Eq. (10.1), with \(\Gamma_{\text{in}} = 640 \text{ Hz}\) and \(\Gamma_{\text{out}} = 220 \text{ Hz}\).

In Fig. 10.2(c, d) we plot the separate tunneling rates \(\Gamma_{\text{in/out}}\) for the upper part of the middle diamond in Fig. 10.2(a). Going upwards along the dashed line in Fig. 10.2(c, d), the Fermi level of the source lead is raised while the potential of the drain and the QD is kept constant. At \(V_L = 1 \text{ mV}\) (marked by an arrow in the figure), there is a distinct step as the Fermi level of the source is raised above an excited state of the QD. At the same time the rate for tunneling out measured along the same line stays constant. We attribute this to fast relaxation of the excited state, so that the tunneling-out process always occurs through the QD ground state (see also section 4.5). The situation is depicted in the inset of Fig. 10.2(d). The results shown in Fig. 10.2 demonstrate the stability and tunability of the system and makes us confident that the electron tunneling detected by the QPC originate
10.2 Measuring current by counting electrons

An envisioned application of the single-electron detector is to use it as a metrology standard for current [130]. Bylander et al experimentally verified the fundamental relation $I = e f$ by relating the highly-correlated current $I$ through an array of tunnel junctions to the frequency response $f$ of a single-electron transistor [22]. In this section we investigate the possibility to use the nanowire QD-QPC system for...
measuring current. The bandwidth and stability of the setup enables time-resolved operation of the detector in a regime where we can simultaneously measure the QD current with a conventional current meter. In this way, we can count electrons one by one and make direct comparison to the measured current.

We start by estimating what magnitude of currents we expect to be able to measure using a detector with time resolution $\tau_{\text{det}}$. In the regime of single-level transport, the distribution of tunneling times $p_{\text{in}/\text{out}}(t)$ is given by Eq. (10.1). However, the detector will only be able to detect the part of the distribution having $t > \tau_{\text{det}}$. We can thus estimate the probability for measuring a tunneling electron by integrating the distribution function

$$p_{\text{detect}}^{\text{in}/\text{out}} = \int_{t=\tau_{\text{det}}}^{\infty} p_{\text{in}/\text{out}}(t) \, dt = e^{-\Gamma_{\text{in}/\text{out}} \tau_{\text{det}}}. \quad (10.2)$$

The current measured by the charge detector is given by the actual number of electrons passing the QD multiplied with the detection probability. To simplify the expression we only consider the fastest of the two tunneling processes $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$. This is reasonable since the detection probability in Eq. (10.2) falls off exponentially with increased tunneling rates.

$$I_{\text{QD}}^{\text{meas.}} \approx e^{-\Gamma_{\text{in}} \Gamma_{\text{out}} / \Gamma_{\text{in}} + \Gamma_{\text{out}}} \times \min[p_{\text{in}}^{\text{detect}}, p_{\text{out}}^{\text{detect}}] = e^{\frac{\Gamma_{\text{in}} \Gamma_{\text{out}}}{\Gamma_{\text{in}} + \Gamma_{\text{out}}}} e^{-\max[\Gamma_{\text{in}}, \Gamma_{\text{out}}] \tau_{\text{det}}}. \quad (10.3)$$

The exact obtainable precision depends on the details of the algorithm used for finding the transitions between the two states in the experimental QPC conductance traces; the result of Eq. (10.3) should be considered as a rough estimate of the dynamic range of the QPC when operated as a current meter.

Figure 10.3 shows the result of Eq. (10.3), plotted for two different symmetries of the tunneling barriers. The detector time resolution was set to $\tau_{\text{det}} = 4 \mu s$. For
low currents, the detector response increases linearly with QD current. Deviations from linear behavior start to appear as the time scale for the tunneling electrons becomes comparable to the time resolution of the detector. However, the charge detector can still be operated as a current meter since it is possible to compensate for the electrons missed due to the finite detector bandwidth [20]. When increasing the QD current even further, we see an abrupt decrease in the detector response with increased QD current. This is due to the exponential fall-off of the detection probability in Eq. (10.2). In this regime, it is not possible to compensate for missed events and the detector is no longer usable for measuring current. For the symmetric case ($\Gamma_{\text{in}} = \Gamma_{\text{out}}$), the transition occurs around $I_{\text{QD}} \approx 20\, \text{fA}$. For the asymmetric configuration ($\Gamma_{\text{in}} = \Gamma_{\text{out}}/10$), the limit is reached for much lower QD currents. This is easily understood by considering the dynamics of the QD and the detector; the limitations of the charge detector are set by the fast tunneling rate, while the actual current flowing through the QD is limited by the slow tunneling rate. For optimal operation of the charge detector as a current meter we need to tune the QD to a symmetric configuration.

The results of Fig. 10.3 show that there should indeed exist a regime where the QD current is measurable with both the charge detector ($I_{\text{QD}} \lesssim 10\, \text{fA}$) and the conventional current meter ($I_{\text{QD}} \gtrsim 1\, \text{fA}$). In the following, we try to reach this configuration by opening the barriers between the QD and the leads compared to the measurements shown in Fig. 10.2. Figure 10.4(a) shows the count rate for the positive bias part of a Coulomb diamond. In the measured configuration, the ground state of the QD is weakly coupled to the source lead. The measurement shows equilibrium fluctuations between the QD and the drain lead [region I in Fig. 10.4(a)], but almost no counts inside the region marked by II. As the bias is further increased, the first excited state is available for transport and the count rate is increased (region III). Figure 10.4(b) shows the current through the QD for the same region as in (a), measured with a conventional current-to-voltage (I-V) converter. We only see current well inside the region corresponding to case III in Fig. 10.4(a). This is expected, since the current measurement in contrast to the charge detection is directional; charge fluctuations between the QD and the drain lead as depicted in region I of Fig. 10.4(a) will not contribute to a net current flow. The discrepancies between the counting and current signal in the upper-right corner of Figs. 10.4(a) and (b) are due to the limited time resolution of the detector; a second excited state entering the bias window makes the tunneling-in rate exceed the detector bandwidth. This leads to a decrease in the detected count rate, in agreement with the results of Fig. 10.3.

In Fig. 10.4(d), we plot the measured QD current together with the electron count rate for fixed bias on the QD ($V_{\text{QD}} = 7.1\, \text{mV}$). The count rate has been converted to current using $I = e/\langle \tau_{\text{in}} + \tau_{\text{out}} \rangle$. Even though the two curves show qualitatively the same behavior, the charge detector registers a current which is $\sim 30\%$ lower than the one measured by the IV-converter. We attribute the difference to the limited bandwidth of the charge detector. As already seen in Fig. 10.3, tunneling events
10.2. Measuring current by counting electrons

Figure 10.4: (a) Electron count rate, measured with the QD in a more open regime compared to Fig. 10.2. (b) QD current for the same region as in (a), measurement with a conventional current meter. (c) Energy level diagrams for the three regions marked in (a). In I, the count rate is due to equilibrium fluctuations between the QD and the drain lead. In region II, transport is blocked due to weak coupling between the QD ground state and the source lead. In III, a more strongly coupled excited state is available for transport and the current through the QD is strongly increased. (d) Cross-sections of the graphs in (a) and (b), taken at $V_{QD} = 7.1 \text{ mV}$. The black curve is the current measured with current meter, while the red one is measured by counting electrons. The dashed line is the counting signal when compensating for limited bandwidth of the detector.

occurring on a timescale on the order of or faster than the time resolution of the detector are less likely to be detected and will modify the measured statistics [20, 131]. However, knowing the detection time and assuming that Eq. (10.1) correctly describes the distribution of tunneling times, we can estimate the number of electrons missed by the detector. Following the ideas of Naaman and Aumentado [20], we find
Chapter 10. Counting electrons in a nanowire quantum dot

that the current is given by

\[ I = \frac{e}{\left(\tau_{\text{in}}^* + \tau_{\text{out}}^*\right) \left(1 - \tau_{\text{det}} \frac{\tau_{\text{in}}^* + \tau_{\text{out}}^*}{\tau_{\text{in}}^* \tau_{\text{out}}^*}\right)} \tag{10.4} \]

Here, \( \tau_{\text{in}}^* \) and \( \tau_{\text{out}}^* \) are average tunneling times extracted from the measurement. The result of Eq. (10.4) is shown as the dashed line in Fig. 10.4(d), with \( \tau_{\text{det}} = 4 \, \mu s \) as extracted from Fig. 10.1(c). The current calculated taking the finite bandwidth into account agrees very well with the current measured with the I-V converter. We emphasize that the curve does not include any free parameters, since the detector rise time is determined separately using the method shown in Fig. 10.1(c). It should be noted that the current measured by counting is determined with much higher precision than with conventional methods. The signal of the I-V converter was integrated for 10 s at each point, yielding a resolution of \( \sim 1 \, \text{fA} \). Also, the signal had to be carefully compensated for amplifier drift. On the other hand, the counting signal was measured for 0.2 s, giving a standard deviation of only 70 aA. The insensitivity to drift and the high precision of the counting procedure demonstrate big advantages of electron counting compared to conventional current measurement techniques.

10.3 Excitations driven by the quantum point contact

In this section we study quantum dot transitions driven by the current flowing in the quantum point contact. Such excitations were already studied in chapter 9 for the case of a quantum dot defined in a 2DEG. The main difference between the nanowire and the AFM-defined structures is that for the nanowire sample, the QD and the QPC are fabricated in different material systems. This allows us to make a statement about the physical processes involved in transmitting energy between the QD and the QPC. Since the two systems sit in separate crystals with different lattice constants and given that the systems hardly touch each other, we can assume that phonons only play a minor role as a coupling mechanism [103]. Furthermore, the phonon spectra of the two materials are different. Instead, we assume the quantum dot transitions to be driven by radiation emitted from the quantum point contact. Another advantage of the nanowire system is that the QD energy scales are an order of magnitude larger compared to the AFM-defined QDs. This allows us to investigate radiation at much higher frequencies, reaching almost into the THz regime.

We first discuss the QD configuration used for probing the radiation of the QPC. Since we do not have a double quantum dot, we can only drive transitions at fixed frequencies corresponding to excited states in the single QD. Figure 10.5(a) shows the level configuration of the system, with the QD electrochemical potential \( \mu \) below the Fermi level of the leads. The tunneling barriers are highly asymmetric, with the
10.3. Excitations driven by the quantum point contact

barrier connecting the QD to the drain lead being almost completely pinched off. We do not apply any bias voltage to the QD. The system is in Coulomb blockade, but by absorbing a photon the QD may be put into an excited state with electrochemical potential above the Fermi energy of the leads. From here, the electron may leave to the source contact, the QD is refilled and the cycle may be repeated. The situation is completely analogous to the single-QD excitations seen in Fig. 9.1 in chapter 9.

In Fig. 10.5(b) we plot the electron count rate versus QD potential and QPC bias. The main peak at $\mu = 0$ is due to equilibrium fluctuations between the QD and the source contact, with the width set by the electron temperature in the lead. As the QPC bias is increased above $\gtrsim 2.5 \text{ mV}$, a shoulder appears in the region of $\mu < 0$. This is consistent with the picture in Fig. 10.5(a); we need to apply a QPC bias larger than the single-level spacing for the photon-assisted tunneling to become possible. The width of the shoulder is set by $\Delta E \approx 2.5 \text{ meV}$ and is therefore expected to be independent of QPC bias; we will see later in this section that the apparent smearing of the features in Fig. 10.5(b) are due to temperature and tuning of the tunneling rates. The picture is symmetric with respect to $V_{\text{QPC-SD}}$, meaning that the emission and absorption processes do not depend on the direction of the QPC current. The lack of data points around $V_{\text{QPC-SD}} = 0$ are due to the fact that the low QPC bias prevents the operation of the QPC as a charge detector (see chapter 3). Due to asymmetric coupling of the QD to the source and drain lead, we could not make a direct confirmation of the existence of an excited state with $\Delta E = 2.5 \text{ mV}$ using finite bias spectroscopy. However, the value is consistent with excited states found in Coulomb diamond measurements in regimes where the tunnel barriers are more symmetric [see, for example Fig. 10.2(a)].

Figure 10.5(c) shows the separate rates for electrons tunneling into and out of the QD at horizontal cross-sections of (b), measured at four different QPC bias voltages. Around the resonance [$\mu = 0$, case I in Fig. 10.5(c, d)], the tunneling is due to equilibrium fluctuations and the rates for tunneling into and out of the QD are roughly equal. By lowering the electrochemical potential $\mu$ the rate for electrons leaving the QD first falls off exponentially due to the thermal distribution of the electrons in the lead. By continuing to case II of Fig. 10.5(c), we come into the regime of QD excitations. Here, the rate $\Gamma_{\text{out}}$ is directly related to the photon absorption process sketched in Fig. 10.5(a), while the rate $\Gamma_{\text{in}}$ corresponds to the refilling of an electron from the lead. Consequently, $\Gamma_{\text{out}}$ shows a strong QPC bias dependence, while $\Gamma_{\text{in}}$ shows only weak variations.

In case III, the excited state goes below the Fermi level of the source lead and the photon absorption rate drops quickly. At the same time, $\Gamma_{\text{in}}$ increases as the refilling of an electron into QD1 may occur through either the ground state or the excited state. Similar to the situation for the AFM-defined QD, this provides a way to determine the tunnel coupling between the source contact and the excited state in the QD ($\Gamma_{\text{ES}}^S$). From the data in Fig. 10.5(c), we find $\Gamma_{\text{ES}}^S \approx 90 \text{ kHz} - 20 \text{ kHz} = 70 \text{ kHz}$. The change of tunnel coupling with gate voltage makes the exact determination of $\Gamma_{\text{ES}}^S$ difficult, the value given here should only be considered as a rough estimate.
Chapter 10. Counting electrons in a nanowire quantum dot

Figure 10.5: (a) Energy level diagram describing the absorption process. The electron in the QD is excited due to photon absorption, which allows it to tunnel out to the lead. (b) Single quantum dot excitations, measured for different QPC bias voltage. The main peak is due to equilibrium fluctuations between the source lead and the QD. As the gate voltage $V_L$ is increased, the electrochemical potential of the QD drops below the Fermi level of the lead and only tunneling processes involving QD excitations become possible. The absorption process is only possible for QPC bias voltages higher than the QD level separation $\Delta E$, giving rise to the shoulder-like features appearing at high $V_{\text{QPC-SD}}$. The data was extracted from QPC conductances traces taken at $G_{\text{QPC}} \approx 0.4 \times 2e^2/h$, filtered at 50 kHz. The data taken at low QPC bias $|V_{\text{QPC-SD}}| < 0.4 \text{ mV}$ was filtered at a lower bandwidth (15 kHz) to allow counting in this regime. (c) Cross-sections of (b) taken at four different QPC bias voltages [position of arrows in (a)], showing the rates for electrons entering (dashed lines) and leaving (solid lines) the QD. (d) Energy level diagrams for the three configurations marked in (c).

The tunneling rates within the region of photon-assisted tunneling show much stronger gate voltage dependence than the AFM-defined QD [for comparison, see Fig. 9.3(a)]. There are several possible explanations for the differences. First, the QD level spacing and therefore the corresponding changes in gate voltage are much larger for the nanowire sample. This could lead to a stronger tuning of the tunneling barriers [29] as discussed in section 4.6. Another difference concerns the properties
of the electronic states in the leads. For the AFM-defined QDs, the leads consist of a two-dimensional electron gas where the ideal density of states (DOS) is independent of energy. For the nanowire QD, the leads are also parts of the nanowire and the corresponding electron DOS may show strong variations with energy due to the quasi-one dimensionality and finite length of the wire. Within the region of photon-assisted tunneling in Fig. 10.5(c), we shift the electrochemical potential of the QD and thereby change the energy of the tunneling electrons relative to the Fermi level in the lead. The measured tunneling rates could therefore show variations due to changes in the DOS in the lead. If one would apply a compensation voltage to a second gate to make sure that the coupling strength of the barrier stays constant during the measurement, the method could actually be used to probe the density of states in the lead.

However, it turns out that the behavior seen in region II of Fig. 10.5(c) is not compatible with either of the two explanations discussed in the previous paragraph. The rate $\Gamma_{\text{in}}$ is directly related to the tunnel coupling $\Gamma_{S}$ between the source lead and the QD ground state, while the rate $\Gamma_{\text{out}}$ depends on the coupling $\Gamma_{S}^{\text{ES}}$ between source and the excited state in the QD. For arguments based on barrier tuning and varying electron DOS, we would expect both $\Gamma_{S}$ and $\Gamma_{S}^{\text{ES}}$ to change in the same way with gate voltage. This is in disagreement with the results of Fig. 10.5(c); $\Gamma_{\text{in}}$ increases while $\Gamma_{\text{out}}$ decreases with gate voltage. Instead, we speculate that the observed behavior is due to non-resonant processes involving energy relaxation in the leads. Focusing on the energy level configuration pictured in Fig. 10.5(a), we see that there are a large number of states in the lead with energy higher than the electrochemical potential of the QD. Elastic tunneling can only occur for electrons with energy equal to $\mu$ relative to the Fermi level, but electrons at higher energy may contribute to the measured rate in terms of processes involving relaxation. As we lower $\mu$, the amount of initial states available for the inelastic process increases and would therefore explain the increase in $\Gamma_{\text{in}}$ with decreased $\mu$. Inelastic tunneling is also possible for electrons leaving the QD excited state to empty states in the lead. Here, the number of empty states available for the inelastic processes goes down when the QD potential $\mu$ is lowered. This is in agreement with the measured decrease in $\Gamma_{\text{out}}$ with decreased $\mu$.

### 10.3.1 QPC bias dependence

Next, we investigate how the QPC bias influences the efficiency of the photon absorption process. For this purpose we apply the model of single-QD excitations developed in section 9.2. For convenience, we repeat the main result of that section:

$$\Gamma_{\text{out}} = \frac{\Gamma_{\text{abs}}}{\Gamma_{\text{abs}} + \Gamma_{\text{rel}} + \Gamma_{\text{em}}}. \quad (10.5)$$

Here, $\Gamma_{\text{out}}$ is the measured rate for electrons leaving the QD, $\Gamma_{\text{ES}}^{\text{S}}$ is the tunnel coupling of the excited state and the source lead, $\Gamma_{\text{abs}}$ is the photon absorption rate,
Chapter 10. Counting electrons in a nanowire quantum dot

Figure 10.6: (a) Electron tunneling due to photon absorption measured versus QPC bias voltage. The data was taken at three different positions of the shoulder seen in Fig. 10.5. As soon as the QPC bias voltage exceeds the QD level separation $\Delta E$, the absorption rate increases linearly with $V_{\text{QPC-SD}}$. (b) Same data as in (a), but plotted in logarithmic scale. The absorption rate shows exponential decay for $eV_{\text{QPC-SD}} < \Delta E$, with the slope of the decay set by the electron temperature in the QPC. The solid lines are fits to Eq. (10.7) in the text, while the dashed lines are the corresponding results assuming zero temperature.

$\Gamma_{\text{rel}}$ is an intrinsic relaxation rate of the excited state in the absence of the QPC and $\Gamma_{\text{em}}$ is a rate due to stimulated emission driven by the QPC. In the regime of weak absorption ($\Gamma_{\text{abs}} \ll \Gamma_{\text{rel}}$, $\Gamma_{\text{em}} \ll \Gamma_{\text{rel}}$), the expression simplifies to

$$\Gamma_{\text{out}} = \Gamma_S^{\text{ES}} \frac{\Gamma_{\text{abs}}}{\Gamma_{\text{rel}}}.$$  \hspace{1cm} (10.6)

Under these conditions the measured rate $\Gamma_{\text{out}}$ is expected to scale linearly with the absorption rate. If we assume that the excitations are driven by photon emission from the QPC, we can combine Eq. (10.6) with the emission spectrum of the point contact (see chapter 7):

$$\Gamma_{\text{out}} \propto \Gamma_S^{\text{ES}} S_I(\Delta E/\hbar) = \Gamma_S^{\text{ES}} \frac{4e^2}{\hbar} D(1 - D) \frac{eV_{\text{QPC}} - \Delta E}{1 - e^{-(eV_{\text{QPC}} - \Delta E)/k_B T}}.$$  \hspace{1cm} (10.7)

Note that Eq. (10.7) only gives us the proportionality between $\Gamma_{\text{out}}$ and $S_I(\omega)$; to make quantitative predictions for the absorption rate we need to determine the overlap between the ground and the excited state. For the double QD, this coupling can be extracted from charge localization measurements (see chapter 7). However, it is not as straightforward to estimate the coupling for single-QD excitations. One would need to know the shape of the wavefunctions for the different QD states, which is a formidable task to calculate.

In Fig. 10.6 we plot the tunneling rate related to absorption ($\Gamma_{\text{out}}$) versus bias on the QPC, measured for three different electrochemical potentials of QD1. The
traces correspond to vertical cross-sections for positive $V_{\text{QPC-SD}}$ in Fig. 10.5(b). Figure 10.6(a) shows the rates plotted on a linear scale; the rates taken at all three positions increase linearly with QPC bias as soon as $eV_{\text{QPC-SD}} > \Delta E$. The solid lines are fits to Eq. (10.7) with $T = 2\,\text{K}$ and assuming $\Delta E = 2.5\,\text{meV}$ to be the same for all three traces. As described in the previous section, we attribute the difference in slope for the three cases to be due to changes in effective tunnel coupling with gate voltage [see Fig. 10.5(c)]. Figure 10.6(b) shows the same data plotted on a logarithmic scale. Here, we see a clear exponential decay for $eV_{\text{QPC-SD}} < \Delta E$; this is due to the thermal distribution of electrons in the QPC. The dashed lines in Fig. 10.6 show the rates expected for the case of zero temperature. The weak but non-zero count rate occurring at low QPC bias voltages ($V_{\text{QPC-SD}} < 1\,\text{mV}$) for the data taken at $\mu = -1.8\,\mu\text{eV}$ is due to non-photon induced thermal fluctuations between the QD ground state and the lead.

To quantify the efficiency of the absorption process we compare the rates $\Gamma_{\text{out}}$ and $\Gamma_{\text{S}}^{\text{ES}}$ with the result of Eq. (10.6). Due to the strong change of $\Gamma_{\text{S}}^{\text{ES}}$ with gate voltage, we can only make quantitative comparisons between $\Gamma_{\text{out}}$ and $\Gamma_{\text{S}}^{\text{ES}}$ in the region where we can determine $\Gamma_{\text{S}}^{\text{ES}}$ (around $-\mu = \Delta E$). This corresponds to the red circles in Fig. 10.6. For this data set, the measured rate goes up to around 5kHz for $V_{\text{QPC-SD}} = 4\,\text{mV}$, so that we still have $\Gamma_{\text{out}} \ll \Gamma_{\text{S}}^{\text{ES}}$. This confirms that we are in a regime of weak absorption where the relative population of the excited state is much smaller than the population of the ground state. Note that the same is most likely true also for the data sets taken at $\mu = -1.8\,\mu\text{eV}$ and $\mu = -3.2\,\mu\text{eV}$ in Fig. 10.6; however, we can not make a quantitative comparison with $\Gamma_{\text{S}}^{\text{ES}}$ since we do not have an independent measurement of $\Gamma_{\text{S}}^{\text{ES}}$ for those regions.

The results are qualitatively very different from the exponential increase and saturation of $\Gamma_{\text{out}}$ with QPC bias found for the case of single-QD excitations in the AFM-defined double QD (see chapter 9). Also, for the nanowire QD we found nothing of the striking magnetic field behavior seen for the DQD. Apart from differences in sample geometry and design, the dissimilarities between the two situations could possibly be due to differences in radiation frequency. For the excitations in the DQD sample, the level spacing was around $\Delta E \approx 150\,\mu\text{eV} \approx h \times 36\,\text{GHz}$, which corresponds to a wavelength of $\sim 10\,\text{nm}$. For the nanowire QD, the excited state is found at $\Delta E = 2.5\,\text{meV} \approx h \times 0.6\,\text{THz}$, with a corresponding wavelength of $\sim 0.5\,\text{mm}$. It may be that the shorter-wavelength photons interact differently with the environment and thus give a different effective coupling to the electron in the QD. More experiments are needed to illuminate the issue.

**10.3.2 Changing the QPC operating point**

In this section, we will modify the operating point of the QPC to check how this influences properties of the emitted radiation. Since we use the same QPC both for emitting radiation as for performing charge detection, it is not possible to operate the device at the plateaus where the conductance is fully quantized. However, we can
tune the QPC conductance in a region between \(0.05 \times 2e^2/h < G_{QPC} < 0.8 \times 2e^2/h\) while still being able to detect the tunneling electrons.

In Fig. 10.7(a) we plot the electron count rate at the photon-absorption shoulder measured versus change in gate voltage on the 2DEG connected to both sides of the QPC. Figure 10.7(b) shows how the conductance of the QPC changes with gate voltage within the region of interest. Compensation voltages were applied to the gates L and R in order to keep the QD potential fixed while sweeping \(V_{2DEG}\). The data was taken with fixed \(V_{QPC-SD} = 2\ mV\) to make the photon absorption process possible. The strong peak at the top of Fig. 10.7(a) (\(\mu = 0\)) corresponds to equilibrium fluctuations between the QD and the source lead. In the region of photon-assisted tunneling [marked by the arrow in Fig. 10.7(a)], the shoulder appears with increased QPC conductance. Going above \(G_{QPC} = 0.5 \times 2e^2/h\), the strength of tunneling at the position of the shoulder decays slightly.

Assuming that the shoulder appears because of radiation emitted from shot noise fluctuations in the QPC current, we expect that the measured absorption rate should depend on the transmission of the QPC. From Eq. (10.7) we see that the
emission spectrum scales with \( D(1 - D) \), where \( D \) is the transmission coefficient of the channel. In Fig. 10.7(c) we plot the rate \( \Gamma_{\text{out}} \) related to the absorption process, measured at \( \mu = -1.9 \text{ meV} \) [position of the arrow Fig. 10.7(a)]. The dashed line shows the emission expected from the QPC, \( S_I \propto D(1 - D) \). For low \( G_{\text{QPC}} \), the measured rate follows the expected emission spectrum reasonably well, showing a maximum around \( G_{\text{QPC}} \approx 0.5 \times 2e^2/h \).

Still, the measured curve shows deviations compared to the predicted behavior. Suppression of noise close to \( G_{\text{QPC}} = 0.7 \times 2e^2/h \) has been reported \cite{132, 133} to be related to 0.7 anomaly \cite{134}. In our case the deviations are more likely to origin from an increase in background charge fluctuations triggered by the QPC current. As discussed in section 3.3.3, the noise in the system increases strongly with \( I_{\text{QPC}} \). This could not be attributed to the intrinsic QPC shot noise but was rather explained by fluctuations of trapped charges driven by the high QPC current. The QD is thus placed in an environment of fluctuating potentials, which may also lead to QD transitions. The strength of such transitions depends strongly on the number of fluctuators in the neighborhood of the QD. The charge traps also influence the count rate in the regime of tunneling due to equilibrium fluctuations [peak at \( \mu = 0 \) in Fig. 10.7(a)]. For \( G_{\text{QPC}} = 0.8 \times 2e^2/h \) (\( \Delta V_{\text{DEG}} = -20 \text{ mV} \)), the peak is considerably wider than for \( G_{\text{QPC}} = 0.05 \times 2e^2/h \). Again, this can be attributed to a fluctuating potential at the location of the QD.

To minimize the influence of the charge traps, one would prefer to decrease \( V_{\text{QPC-SD}} \) and operate the QPC at lower current levels. For the configuration used in Fig. 10.7 (\( V_{\text{QPC-SD}} = 2 \text{ mV} \)), the QPC current reaches values above 100 nA at \( G_{\text{QPC}} \approx 0.7 \times 2e^2/h \). However, \( V_{\text{QPC-SD}} \) can not be made too small; we need to make sure that \( eV_{\text{QPC-SD}} \) is on the same order of magnitude as the level spacing \( \Delta E \), otherwise the QPC will not emit radiation in the right frequency range. For future devices it will therefore be a great advantage if one could minimize the number of charge traps in the neighborhood of the nanostructure.
## Appendices

### A  List of samples

<table>
<thead>
<tr>
<th>Picture</th>
<th>Wafer</th>
<th>Structure</th>
</tr>
</thead>
</table>
| ![Sample A](image) | Depth 2DEG: 34 nm  
Grown by:  
D.C. Driscoll  
A.C. Gossard  
University California, Santa Barbara | Single quantum dot with a quantum point contact for charge read-out. Fabricated by local oxidation. |
| ![Sample B](image) | Depth 2DEG: 34 nm  
Grown by:  
M. Reinwald  
W. Wegscheider  
Universität Regensburg | Single quantum dot with a quantum point contact for charge read-out. Similar design to sample A. Fabricated by local oxidation. |
<table>
<thead>
<tr>
<th>Picture</th>
<th>Wafer</th>
<th>Structure</th>
</tr>
</thead>
</table>
| C       | Depth 2DEG: 34 nm  
         | Grown by:  
         | D.C. Driscoll,  
         | A.C. Gossard,  
         | University California,  
         | Santa Barbara  
         | Double quantum dot in a ring with two quantum point contacts.  
         | Fabricated by local oxidation. |
| ![C.png](image) |   |   |
| D       | Depth 2DEG: 37 nm  
         | Grown by:  
         | S. Schönh  
         | FIRST lab  
         | ETH Zurich  
         | InAs nanowire deposited on top of a GaAs heterostructure.  
         | The QD is formed in the nanowire, the QPC is in the 2DEG.  
         | Fabricated by etching. |
| ![D.png](image) |   |   |
B Statistics of electron tunneling

B.1 Cumulants or central moments of a distribution

The full distribution function \( P_{t_0}(N) \) or the complete set of central moments \( \mu_i \) give a complete description of the current in a system. The moments and the distribution function contain the same information, making the two equivalent. Another way to represent the same information is in terms of the cumulants \( C_k \) and the cumulant generating function \( \mathcal{F}(\chi) \). The cumulants are defined as

\[
C_k = -(-i)^k \frac{\partial^k}{\partial \chi^k} \mathcal{F}(\chi) \bigg|_{\chi=0},
\]

with the cumulant generating function given by

\[
e^{-\mathcal{F}(\chi)} = \sum_N P_{t_0}(N) e^{iN\chi}.
\]

In terms of the central moments, we have for the first few cumulants

\[
C_1 = \mu_1, \quad C_2 = \mu_2, \quad C_3 = \mu_3, \quad C_4 = \mu_4 - 3\mu_2^2, \quad C_5 = \mu_5 - 10\mu_3\mu_2.
\]

The cumulants can be seen as an irreducible representation of the moments. Again, this means that the knowledge of either the moments \( \mu_i \), the cumulants \( C_k \) or the distribution function \( P_{t_0}(N) \) provide the same information.
C  LabVIEW programs

In this appendix we shortly describe the structure of the instrument drivers used in the Step&Log measurement program. The drivers are implemented as independent LabVIEW virtual instruments (VI) and need to provide a fixed set of input- and output parameters. During a measurement sequence, Step&Log controls the driver VIs using the input parameters, while the measured data is passed back through the output parameters. In the next sections we describe the operation requirements of an instrument driver and give a list of all input- and output parameters. The easiest way to implement a new instrument driver is to use an existing driver as a template. In this way one can be sure that the VI structure fulfills the requirements of Step&Log and that the data types for inputs and outputs are correctly defined.

C.1  Instrument driver operation cases

Depending on the stage of a measurement sequence, the instrument driver needs to perform different types of operations. This is implemented by defining several operation cases; the actual case to perform is controlled by the main Step&Log program. Here follows a list of cases that need to be implemented:

InitCfg  This case is called during measurement initialization. No actual hardware communication is performed, the case is rather used to initialize the instrument driver.

OpenInstr  Opens up communication with the measurement hardware. The case is normally called directly after driver initialization.

SetupInstr  Used to configure the measurement hardware. The case is called during the start-up phase of a measurement sequence, but may also be called at a later stage to change some instrument settings.

SetData  Outputs data to an instrument.

GetData  Reads data from an instrument.

CloseInstr  Used to close the instrument connections. The case also returns the last used instrument settings in the ”instrumentCfg” output.

GetCfg  This case returns the currently used instrument configuration.

Figure C.1 shows the LabVIEW block diagram of a Yokogawa voltage source instrument driver, depicting the code structure of the ”Get data”-case.
Figure C.1: LabVIEW block diagram of the Yokogawa instrument driver. The diagram shows the code structure of the "Get data" case.

C.2 Input parameters

Figure C.2 shows the VI icon of an instrument driver together with the required inputs and outputs. A general instrument VI must have the following input parameters:

quantity Quantity to be read or set during a driver call. Used by the "SetData" and "GetData" cases. Note that the quantity may differ from call to call.

inData Value to be set by an output instrument. Used by the "SetData" case.

instrOperations This input defines which case to perform during a call. See section C.1 for a list of possible values.

instrumentCfg Provides instrument settings at initialization of an instrument driver. Used by the "InitCfg" case.

errorIn Incoming error cluster.

instrData Used to control properties of output signals, like sweep rates, max/min limits, etc.
C.3 Output parameters

**outData** Data read from an input instrument or the actual value of a signal set by an output instrument. If the instrument handles scalar values, only the first element in the array is set. Used by the "SetData" and "GetData" cases.

**instrumentCfg** Outputs the current instrument configuration. The output is set by the "GetDefaultCfg" and "CloseInstr" cases.

**errorOut** Outgoing error cluster.

**measChn** Array of measurement channels, used by instruments configured to return several output parameters at a single call. The output is set by the "GetData" and "GetDefaultCfg" cases.
Publications

Time-resolved detection of single-electron interference
S. Gustavsson, R. Leturcq, M. Studer, T. Ihn, K. Ensslin, D. C. Driscoll, A. C. Gossard
Submitted

Frequency-selective single-photon detection with a double quantum dot
R. Leturcq, S. Gustavsson, M. Studer, T. Ihn, K. Ensslin, D.C. Driscoll and A.C. Gossard
Physica E, in press, corrected proof at: doi:10.1016/j.physe.2007.11.032

Measuring current by counting electrons in a nanowire quantum dot
S. Gustavsson, I. Shorubalko, R. Leturcq, S. Schön, K. Ensslin

Counting Statistics of Single Electron Transport in a Semiconductor Quantum Dot
S. Gustavsson, R. Leturcq, B. Simovic, R. Schleser, T. Ihn, P. Studerus, K. Ensslin, D. C. Driscoll, A. C. Gossard

Frequency-selective single photon detection using a double quantum dot
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Noise measurements in quantum dots using charge detection techniques
S. Gustavsson, R. Leturcq, T. Ihn, K. Ensslin, D.C. Driscoll, A.C. Gossard
Physica E 40, 103 (2007)

Conditional statistics of electron transport in interacting nanoscale conductors
E. V. Sukhorukov, A. N. Jordan, S. Gustavsson, R. Leturcq, T. Ihn, and K. Ensslin
Measurements of higher order noise correlations in a quantum dot with a finite bandwidth detector
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S. Gustavsson, D. Gunnarsson, P. Delsing

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Few-electron quantum dot fabricated with layered scanning force microscope lithography
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Study of the microwave-induced transport through a quantum dot inserted in a 35-GHz loop-gap resonator
B. Simovic, S. Gustavsson, R. Leturcq, P. Studerus, K. Ensslin, J. Forrer, A. Schweiger, and R. Schuhmann

Design of Q-Band loop-gap resonators at frequencies (34-36 GHz) for single electron spin spectroscopy in semiconductor nanostructures
B. Simovic, P. Studerus, S. Gustavsson, R. Leturcq, K. Ensslin, R. Schuhmann, J. Forrer, A. Schweiger
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