Doctoral Thesis

Parametric optimization and constrained optimal control for polynomial dynamical systems

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Parametric Optimization and Constrained Optimal Control for Polynomial Dynamical Systems

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presented by

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Abstract

Constrained optimal control is a very active research area with broad attention from industry. Not without reason: It is among the few control methodologies providing a systematic way to perform nonlinear control synthesis while handling at the same time design specifications posed as system constraints. It is with certainty thanks to this capability that Model Predictive Control (MPC) enjoys so widespread and successful deployment in practice. However, one of the main challenges in the application of MPC is the inherent computational complexity of the underlying optimization problem, which generally restricts the applicability of MPC to relatively slow dynamical processes.

This problem has been tackled to some extent by the recent development of parametric programming, where the optimal control problem is solved off-line for all possible initial states. For linear and piece-wise affine (PWA) systems, the optimal control law can be precomputed in closed form, as a PWA state feedback law, which can be then on-line implemented by the use of look-up tables.

Motivated by the recent research in the field of parametric programming, this thesis focuses on extending the aforementioned framework for linear and PWA systems, to the class of polynomial dynamical systems, i.e. discrete-time systems whose state update equation comprises a polynomial vector field. The constrained optimal control problem for polynomial dynamical systems, gives rise to a nonlinear parametric program. In contrast to linear and PWA systems, the optimal feedback control law for polynomial systems cannot be computed in closed form, as a function of the initial state. However, we show that it is still possible to achieve a parametrization of the optimal control law, by combining a precomputation stage involving algebraic techniques with an on-line stage involving numerical computations.

This is achieved by developing three novel approaches, which constitute the main contribution of this thesis.

- We develop a new approach for nonlinear parametric optimization, based on the Cylindrical Algebraic Decomposition (CAD). The approach solves the nonlinear parametric program by dividing the associated computations in two parts; the off-line part, where the CAD of the problem is constructed; and the on-line part, where given the value of the parameter, the optimal solution is obtained by solving a sequence of univariate polynomial equations.

- We present a second approach for nonlinear parametric optimization, which is based on the Möller-Stetter method for the solution of polynomial equations. In its off-line part, the approach precomputes certain generalized companion matrices and in the
on-line part, given the value of the parameter, it finds the optimum by calculating the eigenvalues of the companion matrices.

- We propose an approach for the constrained optimal control of polynomial dynamical systems, based on the homotopy continuation method for solving systems of polynomial equations. This approach, like the previous two, subdivides the computations into two parts. An off-line, where a generic system of polynomial equations is solved, and an on-line part where, given the parameter value, the solution of the generic system is used to obtain the optimal control input.

The presented approaches are exact, in that they do not involve any approximation of the problem at hand, or its solution. Moreover, they are guaranteed to find the global optimum.

Furthermore, we apply all the developed approaches to the control design problem of the boost and buck DC-DC converters. The control objective is to achieve a regulated (constant) output dc voltage, despite changes in the unregulated input voltage and in the face of output power demand changes over time. Certain constraints pertaining to the safe operation of the device have also to be respected. We demonstrate the applicability and limitations of the developed approaches for constrained optimal control, through the various simulation results for real operation scenarios of the DC-DC converters.

The main drawback of the approaches is that they have high computational complexity. As such, their practical relevance is restricted to problems of relatively small size. Therefore, they are to be understood as a conceptual extension of the parametric programming framework to the class of polynomial systems, and not as ready-to-use computational tools.

Nonetheless, the computational characteristics of the approach based on homotopy continuation are attractive enough, so that there is a certain promise regarding applicability of the latter to real control design problems.
Zusammenfassung

Optimale Regelung ist ein sehr aktiver Forschungsbereich mit grosser Bedeutung in der industriellen Praxis. Der wohl bedeutenste Grund hierfür ist das systematische Vorgehen zur Reglersynthese für nichtlineare Systeme unter Beachtung verschiedener Restriktionen. Dank dieser Fähigkeit ist die Modellbasierte Prädiktive Regelung (MPC) sehr erfolgreich und in der Praxis weit verbreitet.

Ein wesentliches Problem von MPC ist die inhärente Komplexität der notwendigen Berechnungen. Dies beschränkt die Anwendungsmöglichkeit auf relativ langsam ablaufende Prozesse. Dieser Nachteil wurde zu einem gewissen Grad mit der Einführung der parametrischen Programmierung verringert. In einigen Fällen kann die Lösung des optimalen Regelungsproblems nun offline berechnet werden, indem das entsprechende Programm parametrisch, d.h. für alle möglichen Anfangszustände, gelöst wird. Für lineare und abschnittsweise affine (PWA) Systeme ist das Ergebnis eine explizite Formel, die die Lösung des Programms (also den optimalen Eingang) als eine Funktion der Problemparameter (gemessene Zustände) angibt. Die Lösung kann dann in Echtzeit effizient mittels einer Look-Up Tabelle implementiert werden.


Drei neue Ansätze auf diesem Gebiet bilden den Kern dieser Dissertation.

- Es wird einen neuen, auf zylindrischer algebraischer Dekomposition (CAD) basierenden Ansatz für nichtlineare, parametrische Optimierung vorgestellt. Der Ansatz löst das nichtlineare parametrische Programm durch die Gliederung der Berechnungen in zwei Teile: den Offline-Algorithmus, welcher eine CAD des Problems konstruiert und den Online-Algorithmus, der die gemessenen Parameterwerte benutzt, um die optimale Lösung zu finden. Damit ist online lediglich noch die Lösung univariater Polynome erforderlich.

- Ein zweiter Ansatz für das parametrische Lösen optimaler Regelungsprobleme für
polynomiale Systeme mit Beschränkungen basiert auf Gröbner Basen und der Möller-Stetter Methode. Während offline sogenannte parametrische Kompanionsmatrizen konstruiert werden, muss in der online Phase nur noch eine Eigenwertberechnung und ein einfacher Sortieralgorithmus abgearbeitet werden.


Die vorgestellten Ansätze sind exakt, da sie keinerlei Approximationen des Optimierungsproblems und dessen Lösung benötigen. Desweiteren ist garantiert, dass das globale Optimum gefunden wird.
