GENERALIZED LEAST SQUARES MULTIPLE 3D SURFACE MATCHING

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ABSTRACT:

A method for the simultaneous co-registration and georeferencing of multiple 3D pointclouds and associated intensity information is proposed. It is a generalization of the 3D surface matching problem. The simultaneous co-registration provides for a strict solution to the problem, as opposed to sequential pairwise registration. The problem is formulated as the Least Squares matching of overlapping 3D surfaces. The parameters of 3D transformations of multiple surfaces are simultaneously estimated, using the Generalized Gauss-Markoff model, minimizing the sum of squares of the Euclidean distances among the surfaces. An observation equation is written for each surface-to-surface correspondence. Each overlapping surface pair contributes a group of observation equations to the design matrix. The parameters are introduced into the system as stochastic variables, as a second type of (fictitious) observations. This extension allows to control the estimated parameters. Intensity information is introduced into the system in the form of quasisurfaces as the third type of observations. Reference points, defining an external (object) coordinate system, which are imaged in additional intensity images, or can be located in the pointcloud, serve as the fourth type of observations. They transform the whole block of “models” to a unique reference system. Furthermore, the given coordinate values of the control points are treated as observations. This gives the fifth type of observations. The total system is solved by applying the Least Squares technique, provided that sufficiently good initial values for the transformation parameters are given. This method can be applied to data sets generated from aerial as well as terrestrial laser scanning or other pointcloud generating methods.

1. INTRODUCTION

The early approach for the multiple pointclouds registration is to sequentially apply pairwise registrations until all views are combined. Chen and Medioni (1992) propose a method, which registers successive views incrementally with enough overlapping area. Each next view is registered and merged with the topological union of the former pairwise registrations. Later, this approach is equipped with a coarse-to-fine mesh hierarchy (Turk and Levoy, 1994), and the least median of squares (LMS or LMedS) estimator with random sampling (Masuda and Yokoya, 1995). The shortcomings of the incremental solution were recognized early. The registration of a view does not change once it has been added to the integrated model. However, it is possible that a following view brings information that could have improved the registration of previously processed views (Bergevin et al., 1996; Pulli, 1999). Bergevin et al. (1996) propose a solution in which every view is sequentially matched with all other overlapping views. The procedure is iteratively executed over all views. The iteration is stopped when the registration converges. For each view a separate transformation is calculated, and they are applied simultaneously before the next run of iteration. Although it confines the registration errors evenly among all views, slow convergence is the main disadvantage. Benjemaa and Schmitt (1997) accelerate the method by applying the new transformation as soon as it is calculated (like the Gauss-Seidel method) and employ a multi-z-buffer technique which provides a 3D space partitioning. Pulli’s (1999) solution performs pairwise registrations between every overlapping view pairs. Subsequently, these pairwise registrations are incrementally treated as constraints in a global registration step. However, these constraints do not imply functional constraints in the optimization scheme. Rather, it is a set of virtual points that uniformly subsample the overlapping areas, called as virtual mate. This approach has the capability to handle large data sets, since using the virtual mates from pairwise alignments does not require loading the entire data set into memory. A concrete mate version of this method, in which a set of corresponding points themselves rather than the virtual points is used as constraint, is proposed for robot navigation (Lu and Milios, 1997). The subsequent global registration is achieved by employing a sequential estimation procedure.

Alternatively, some works carry out the multiview registration task in the sensor coordinate system. In Blais and Levine (1995) couples of images are incrementally registered. It is based on reversing the range finder calibration process, resulting in a set of equations which can be used to directly compute the location of a point in a range image corresponding to an arbitrary location in the three dimensional space. Another multiview registration method based on inverse calibration, developed independently, called Iterative Parametric Point (IPP), is given in Jokinen (1998). Differently, it simultaneously registers all views using the Levenberg-Marquardt non-linear optimization technique. Although the reverse calibration method, also called point-to-projection technique, provides fast access mechanisms for the point correspondence, it is performed on 2.5D range maps. It is not suitable for truly 3D applications.

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In a previous work, we proposed an algorithm for the least squares matching of overlapping 3D surfaces, called least squares 3D surface matching (LS3D). The LS3D method estimates the transformation parameters of one or more fully 3D search surfaces with respect to a template one, using the Generalized Gauss-Markoff model, minimizing the sum of squares of the Euclidean distances between the surfaces (Gruen and Akca, 2005). The mathematical model is a generalization of the Least Squares image matching method, in particular the method given by Gruen (1985).

In order to optimize the run-time, a rapid method for searching the correspondence is added. It is a space partitioning method, called 3D boxing (Akca and Gruen, 2005b). False correspondences with respect to outliers and occlusions are detected and eliminated using a weighting scheme adapted from Robust Estimation methods (Akca and Gruen, 2005c).

When the object surface lacks sufficient geometric information, i.e. homogeneity or isotropy of curvatures, the basic algorithm will either fail or find a side minimum. We propose an extension of the basic algorithm in which available attribute information, e.g. intensity, color, temperature, etc., is used to form quasisurfaces in addition to the actual ones. The matching is performed by simultaneous use of surface geometry and attribute information under a combined estimation model (Akca and Gruen, 2005a).

When more than two point clouds with multiple overlaps exist, we adopt a two step solution (Akca and Gruen, 2005b). First, pairwise LS3D matchings are run on every overlapping pairs and a subset of point correspondences are saved to separate files, similar to Lu and Milios’s approach (1997). In the global registration step, all these files are passed to a block adjustment by independent models procedure (Ackermann et al., 1973), which is a well known orientation procedure in photogrammetry.

In some applications georeferencing is needed, which is the procedure to transform the spatial data from a local system to an external object coordinate system. We also provide for an integrated solution for this problem.

1.1 Our proposed method

Terrestrial laser scanning companies (e.g. Z+F, Leica, Riegl) commonly use special kind of targets for the registration of point clouds. However such a strategy has several deficiencies with respect to fieldwork time, personnel, equipment costs, and accuracy. In a recent study, Sternberg et al. (2004) reported that registration and geodetic measurement parts comprise 10-20% of the whole project time. In another study a collapsed 1000-car parking garage was documented in order to assess the damage and structural soundness of the structure. The scanning took 3 days, while the conventional survey of the control points required 2 days (Greaves, 2005). In a recent project conducted by our group at Pinchango Alto (Peru) two persons set the targets to the field and measured with Real-Time Kinematic GPS in 1½ days.

Not only fieldwork time but also accuracy is another important concern. The target-based registration methods cannot exploit the full accuracy potential of the data. The geodetic measurement naturally introduces some errors, which might exceed the internal error of the scanner instrument. In addition, the targets must be kept stable during the whole scanning campaign. This might be inconvenient with the scanning works stretching over more than one day. On the other hand, target-based registration techniques can provide immediate georeferencing to an object coordinate system.

Surface-based registration techniques stand as efficient and versatile alternative to the target-based techniques. They simply bring the strenuous additional fieldwork of the registration task to the computer in the office while optimizing the project cost and duration and achieving a better accuracy. However, they do not provide the georeferencing option.
This work proposes a method which combines the advantageous parts of both techniques based on the least squares matching framework. The proposed method is a (truly) simultaneous one step solution for the matching and georeferencing of multiple 3D surfaces with their intensity information. The mathematical model is a hybrid system which contains different type of observations. The proposed method is an algorithmic extension of our previous work given in Gruen and Akca (2005). It generalizes the 3D surface matching problem in the sense that multiple 3D surfaces with their intensity information are globally matched and simultaneously georeferenced. Multiple primitives, surface information (geometry and intensity) and the (reference) point features, are co-registered together.

The paper is structured as follow. The next chapter introduces the mathematical model with the execution aspects. The third chapter presents the experimental results.

2. MATHEMATICAL MODELLING

2.1 Least Squares Multiple 3D Surface Matching

Assume a set of observations. The proposed method is an algorithmic extension of the least squares matching of the reference (template) surface and the search (search) surface. They can be replaced by another type of surface, whereas the system, whereas the transformation parameters, $\{t_x, t_y, t_z\}$ is the translation vector, $m_i$ is the uniform scale factor, and $(x_0, y_0, z_0)$, stand for the initial location of the surface.

Because Equations (1) are non-linear, they are linearized by Taylor series expansion.

\[
-e_i(x, y, z) = g_i^0(x, y, z) + \frac{\partial g_i^0}{\partial x} d x_j + \frac{\partial g_i^0}{\partial y} d y_j + \frac{\partial g_i^0}{\partial z} d z_j
\]

\[
+ \frac{\partial g_j^0}{\partial x} d x_i - \frac{\partial g_j^0}{\partial y} d y_i - \frac{\partial g_j^0}{\partial z} d z_i
\]

\[
d x = dt_x + a_{10} dm + a_{11} do + a_{12} d \phi + a_{13} d \kappa
\]

\[
d y = dt_y + a_{20} dm + a_{21} do + a_{22} d \phi + a_{23} d \kappa
\]

\[
d z = dt_z + a_{30} dm + a_{31} do + a_{32} d \phi + a_{33} d \kappa
\]

with $a_{pq}$ as the coefficient terms.

Using the notation

\[
g_x = \frac{\partial g_i^0}{\partial x}, \quad g_y = \frac{\partial g_i^0}{\partial y}, \quad g_z = \frac{\partial g_i^0}{\partial z} (5)
\]

and substituting Equations (4), Equation (3) results in:

\[
-e_i(x, y, z) = g_{ij} dt_{ij} + g_{ij} dt_{ij} + g_{ij} dt_{ij}
\]

\[
+ (g_{ij} a_{10} + g_{ij} a_{20} + g_{ij} a_{30}) dm
\]

\[
+ (g_{ij} a_{11} + g_{ij} a_{21} + g_{ij} a_{31}) do
\]

\[
+ (g_{ij} a_{12} + g_{ij} a_{22} + g_{ij} a_{32}) d \phi
\]

\[
+ (g_{ij} a_{13} + g_{ij} a_{23} + g_{ij} a_{33}) d \kappa
\]

\[
-g_{ij} dt_{ij} - g_{ij} dt_{ij} - g_{ij} dt_{ij}
\]

\[
- (g_{ij} b_{10} + g_{ij} b_{20} + g_{ij} b_{30}) dm
\]

\[
- (g_{ij} b_{11} + g_{ij} b_{21} + g_{ij} b_{31}) do
\]

\[
- (g_{ij} b_{12} + g_{ij} b_{22} + g_{ij} b_{32}) d \phi
\]

\[
- (g_{ij} b_{13} + g_{ij} b_{23} + g_{ij} b_{33}) d \kappa
\]

\[
(6)
\]

where $a_{pq}$ and $b_{pq}$ are the coefficient terms for the differentiation of the transformation equations of the $i$-th and $j$-th surface, respectively. The terms $g_x$, $g_y$, and $g_z$ are the numerical derivatives of the object surface function $g(x, y, z)$. They are defined as the elements of the local surface normal vectors at the exact surface correspondence locations (Gruen and Akca, 2005). The linearized observation Equations (6) are written for each element of the $i$-th surface.
Equations (6) result in the following linear systems in matrix/vector form

\[-e_1 = A_1 x - l_1 , \quad P_1 \]
\[-e_2 = A_2 x - l_2 , \quad P_2 \]
\[\vdots \]
\[-e_m = A_m x - l_m , \quad P_m \]

Equations (7) consist of \( m \) groups of observation equations. They can be combined in one sub-system as

\[-e = A x - l , \quad P \]

where \( A \) is the design matrix, \( x \) is the parameter vector which contains \( n \) sets of transformation parameters, \( P = P_0 \) is the a priori weight matrix, \( l = g'(x, y, z) - g'(x, y, z) \) is the discrepancies vector that consists of the Euclidean distances between the corresponding elements of the overlapping surfaces. The calculation of the discrepancy vector \( l \) and the numerical derivative terms \( g_x, g_y, g_z \) requires an appropriate correspondence search procedure (Akca and Gruen, 2005b).

Provided that \( m \geq n \) is satisfied, the sub-system (of the design matrix) consisting of \( m \) Equations (7) implicitly contains the multiple overlap conditions. The normal equation matrix explicitly shows all the spatial relationships by non-zero off-diagonal elements (see Chapter 2.3.1).

With the statistical expectation operator \( E \{ \} \), it is assumed that

\[ E \{ e \} = 0 , \quad E \{ e e^T \} = \sigma_e^2 P_e^{-1} \]

The parameters are introduced into the system as observables with the associated weight coefficient matrix \( P_e \) as

\[-e_b = I x - l_b , \quad P_b \]

where \( I \) is the identity matrix and \( l_b \) is the (fictitious) observation vector. The weight matrix \( P_b \) has to be chosen appropriately, considering a priori information of the parameters.

2.2 The Generalized Model with Intensity Matching and Georeferencing

When some surfaces lack sufficient geometric information, their intensity information, if available, is introduced to the system. The intensity information is used to form quasisurfaces in addition to the actual ones. The formation of quasisurfaces is given in Akca and Gruen (2005a). The quasisurfaces are treated like actual surfaces in the estimation model. They contribute observation equations to the design matrix, joining the system by the same set of transformation parameters

\[-e_c = A_c x - l_c , \quad P_c \]

where \( e_c, A_c, \) and \( P_c \) are the true error vector, the design matrix, and the associated weight coefficient matrix for the quasisurface observations, respectively, and \( l_c \) is the constant vector that contains the Euclidean distances between the corresponding quasisurface elements.

Reference points whose coordinates are defined in an external (object) coordinate system, which are imaged in additional intensity images, or can be located in the pointclouds, serve as the fourth type of observations. They are formulated as 3D similarity transformations from local pointcloud systems to the object coordinate system in linearized matrix form

\[-e_d = A_d x - l_d , \quad P_d \]

where \( A_d \) is the design matrix, \( P_d \) is the associated weight matrix, and \( l_d \) is the discrepancies vector which contains the coordinate value differences of the reference points between the transformed local system and object coordinate system. At least 7 coordinate elements of 3 control points are needed for georeferencing.

Actually, the coordinates of the control points are not error-free quantities. In a strict model, they are treated as observations with their associated weight matrices as

\[-e_c = A_c x - l_c , \quad P_c \]

where \( A_c, x, \) and \( P_c \) are the design matrix, the parameter vector, and the associated weight coefficient matrix for the observations of the control point coordinates, respectively, and \( l_c \) is the discrepancy vector that contains the differences between the observed and estimated coordinate values. Here, the vector \( x \) is extended to include the \( x-y-z \) coordinate values of the control points in addition to the transformation parameters.

Equations (12) eliminate the datum deficiency existing in Equations (8). Alternatively, the datum constraints can be imposed by fixing the minimal number of parameters in Equations (10).

The hybrid system of Equations (8), (10), (11), (12) and (13) is of the combined adjustment type that allows simultaneous matching of geometry and intensity and additionally georeferencing of multiple 3D surfaces. The Least Squares solution of the system gives the solution vector as

\[
\hat{x} = (A^T P A + A_c^T P_c A_c + A_d^T P_d A_d + A_e^T P_e A_e)^{-1} (A^T P l + P_b l_b + A_c^T p_c l_c + A_d^T p_d l_d + A_e^T p_e l_e)
\]

and the variance factor as

\[
\sigma_0^2 = \frac{v^T P v + v_b^T P_b v_b + v_c^T P_c v_c + v_d^T P_d v_d + v_e^T P_e v_e}{r}
\]

where \( r \) is the system redundancy, \( v, v_b, v_c, v_d \) and \( v_e \) are residual vectors for actual surface observations, parameter observations, quasisurface observations, reference point observations (for georeferencing) and control point coordinate observations, respectively.

The solution is iterative. At the end of each iteration all surfaces are transformed to their new states using the updated sets of transformation parameters, and the design matrices and discrepancy vectors are re-evaluated. The iteration stops if each element of the alteration vector in Equation (14) falls below a certain limit.
The estimation model is the Generalized Gauss-Markoff, which can accommodate any kind of functional constraint flexibly, e.g. concentric scans, certain rotational differences, parallel or perpendicular objects in the pointcloud data, etc.

2.3 Execution Aspects

2.3.1 Matrix Structures

Figure 1 shows the matrix structures of a hypothetical example of a data set with four pointclouds and three control points. For the sake of simplicity, the example does not cover the intensity matching case.

The design matrix (Figure 1a) consists of 4 sub-systems. The first sub-system includes the observations of surface geometry. Each overlapping pointcloud, i.e. \( g_1-g_2, g_2-g_3, g_3-g_4 \) and \( g_4-g_1 \), gives a group of observation equations. The second sub-system represents the fictitious observations of the unknown transformation parameters. The third sub-system contains the reference point observations for the georeferencing. The last sub-system consists of the \( x-y-z \) coordinate value observations of the control points. The columns stand for 4 sets of transformation parameters (28 elements) and the coordinates of 3 full control points (9 elements).

The normal equations matrix comprises 4 sets of unknown transformation parameters and 3 sets of control point coordinates. The non-zero 7x7 sub-parts show the spatial overlaps among the pointclouds.

2.3.2 Memory Efficiency

In a typical real-world example, the data set may contain 10-20, sometimes more than 100 pointclouds. It is not operable to load all pointclouds into the physical memory. Our software implementation loads at maximum two pointclouds into the memory at any instant of (processing) time. All the information, e.g. 3D coordinates, correspondences, elements of the 3D boxing for the space partitioning, etc. are kept in the files whose contents are loaded into memory only when needed.

3. EXPERIMENTAL RESULTS

Because of the 125 anniversary of the construction of the Gotthard Tunnel (Switzerland), Credit Suisse has decided to have an exhibition in Zurich about the life and person of Alfred Escher (1819-1882), Swiss politician, promoter of the Gotthard Tunnel, railroad entrepreneur, and founder of Credit Suisse as well as of ETH Zurich.

In Zurich, there is a monument of Alfred Escher, which is located in front of the main railway station and is approximately 5 meter in high (9.5 meter considering also the basement). The goal of the project is the production of ten physical replicas of the Escher monument, starting from a 3D computer model.

The digitization was done with a Faro LS880 HE80 laser scanner, placed on a cherry picker (Figure 2). Totally 36 scans were acquired during two nights of on-site work. The data set contains approximately 4.4 million points with an average point spacing of 5-10 millimetres.

The proposed algorithm was used for the co-registration of the point clouds. Only the surface geometry and parameter observations were used. The example does not include the geo-referencing and intensity matching extensions.

At the first step, 3-5 tie points per pointcloud were interactively measured. Initial approximations were calculated by use of the tie point coordinates in a chained 3D similarity transformation. The first pointcloud was defined as the datum by fixing its parameters to a unit transformation with zero translation and rotation elements.

The transformation parameters of the all pointclouds were simultaneously calculated with \( \sigma_0 = 2.7 \) mm for the accuracy of the surface observations. Any surface correspondence whose Euclidean distance exceeds 6 times the current \( \sigma_0 \) value was excluded from the design matrix. The final iteration of the adjustment used 20,442,040 surface correspondences. A high noise level in the data slowed down the convergence to 16 iterations.

The computation lasted more than 18 hours processing time on a laptop computer with Intel dual-core 2.16 GHz CPU and 2 GB physical memory. The main reason is the file-access oriented design of our software implementation. The file access
for reading and writing is within a few milliseconds, while the memory access is within some nanoseconds. On the other hand, the memory request of the software has never exceeded 300 MB during the entire calculation.

After the co-registration step, all pointclouds were merged, filtered for noise reduction, sub-sampled and triangulated for surface generation. The 3D modelling operations were carried out using Geomagic Studio 9. Note that no editing has been made on the final model, except for the cropping of the area of interest (Figure 3). An edited version of the 3D model was used for the replica production. Ten replicas were produced at a scale 1:2 (Figure 4).

4. CONCLUSIONS

A method for the simultaneous co-registration of multiple 3D pointclouds is presented. It is capable of georeferencing as well as matching of the intensity information when some parts of the object surface lack sufficient geometry information. The estimation model is the Generalized Gauss-Markoff which allows any kind of object space conditions to be formulated as functional constraints, e.g. co-centric scans, perpendicular or parallel objects in the pointclouds, etc.

A practical experiment shows the capability of the method. A successful solution has been achieved. However, the computation time is the main burden. A more efficient software implementation and a multi-resolution approach during the iterations can accelerate the procedure substantially. The future work will also include experimentations with the georeferencing and intensity matching approaches.

5. REFERENCES


