REFLECTION OF SEISMIC WAVES FROM ATTENUATING AND ANISOTROPIC OCEAN BOTTOM SEDIMENTS

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Abstract

There is an increasing trend towards recording marine seismic data directly on the seafloor. This acquisition strategy is mainly motivated by the possibility to simultaneously measure the three components of particle motion in addition to the pressure in the water column immediately above the seabed. Such four-component (4C) seismic recordings thus allow for the recording of S-waves in marine environments and offer the prospect of decomposing the wavefield into its up- and down-going P- and S-wave constituents.

The assumptions for acquisition and processing of 4C data is today based on the ocean bottom model as a welded acoustic-elastic contact at the seabed with a homogeneous acoustic layer overlying a homogeneous elastic half-space. This may not hold in wide areas of the oceans where the seafloor typically consists of soft, water-saturated sediments characterized by having strong to very strong seismic attenuation. Moreover, cyclically changing sedimentation processes lead to layering in the sediments, thus introducing macroscopic seismic anisotropy, and overburden pressure and associated compaction effects are likely to result in a strong velocity gradients.

The primary objective of this thesis was to gain a deeper understanding of the fundamental physical processes governing seismic wave propagation in such environments and to evaluate the implications arising for the acquisition and processing of 4C seabed seismic data. To this end, the effects of strong attenuation, anisotropy due to very fine layering, and pronounced velocity gradients have been investigated using advanced seismic modelling techniques.

In the first part of this thesis reflection coefficients obtained with the frequency slowness method, using a pseudo-spectral solution of the anisotropic visco-elastic wave equation, are compared with analytic reflection coefficients, calculated using the plane-wave approach. The ocean bottom is modeled as a specular interface separating a viscoacoustic medium (water) and a viscoelastic solid (sediments) characterized by transverse isotropy with a vertical symmetry axis (VTI). The algorithm uses one grid for the fluid layer and another grid for the solid half-space and employs Fourier and Chebyshev differential operators in the horizontal and the vertical directions, respectively. The visco-elastic stress-strain relation is described by a Zener model. Special attention is given to the boundary conditions at the ocean bottom. For this purpose, a special domain-decomposition technique for wave propagation at the fluid/anelastic-anisotropic-solid interface was further developed. The examples considered in this study cover a water-steel interface, for which experimental data is available, a soft water/sediment interface and a stiff water/crustal rock interface. An additional finding was that the analytical plane-wave reflection coefficients may exhibit non-physical jumps and discontinuities the reasons for which were unknown.

I therefore decided to investigate these non-physical jumps in more detail. To this end, the modeling code was further extended to the more general case of two VTI viscoelastic solids in welded contact and the numerical results for the reflection coefficient were compared to the corresponding analytical plane-wave solutions. The numerical solution was used to identify the most probable plane-wave solution. It could be shown that the non-physical jumps were related to non-continuous evaluation of the complex square roots associated with the vertical slowness in the analytic plane-wave reflection coefficients in attenuating media. I found that this problem can be effectively alleviated by following the
appropriate path along the Riemann surface of the evaluated complex square roots. For certain well known pathological cases where continuous evaluation of the complex square roots still leads to wrong phases a simple solution suitable for practical purposes could be found.

In the last part of the thesis simple end-member type seafloor models of sand and clay as well as the corresponding inter-bedding are considered and their effects on seismic wave propagation are assessed. The wave speeds for the isotropic sand and clay end-member models were taken from the literature. Equivalent media theory was then used to describe the apparent velocity and anisotropy for dominant frequencies typically between 10 and 100 Hz in seismic exploration. Introduction of a stochastic distribution of the layer thickness allows an estimate of the variability of the resolved parameters. My results indicate that anisotropy is most pronounced for SH-waves and more moderate for P-waves. Numerical calculation of reflection coefficients as a function of angle shows that the additional complexity introduced through attenuation, VTI, anisotropy and velocity gradients lead to significant changes mainly in the vicinity of the elastic equivalent critical (EEC) angle. Interestingly, this part of the reflection coefficient tends to be used by wavefield decomposition algorithms to determine the S-wave speed of the ocean bottom. Finally, I show that strong attenuation and intrinsic anisotropy in the clay layers lead to an ocean bottom that is strongly transmitting the incident wavefield.

The agreement between the analytical plane-wave solutions and the results of numerical modeling suggests that the former may also be applicable to more complicated environments, which in turn opens up the possibility to efficiently invert 4C seismic seabed measurements for attenuation and anisotropy. Moreover, the elimination of the non-physical discontinuities in the plane-wave reflection coefficients should also allow for the use of this technique in ray-tracing algorithms that account for the effects of attenuation and anisotropy. Finally, the results from numerical modelling of seismic wave propagation for a finely layered ocean bottom sediments with realistic velocity gradients indicate that the conventional approach of characterizing the shallow seabed as an isotropic elastic half-space is likely to be inadequate. My results further show that the corresponding errors which arise are likely to be relatively minor for P-waves, but rather dramatic for S-waves. Although there may be additional causes responsible, such as the incorrect calibration of the 4C sensors, this finding is consistent with the observation that the elastic decomposition of P-waves is generally successful and robust, whereas to date few, if any, truly convincing decompositions of S-waves have been reported.
Zusammenfassung

Seismische Daten werden zunehmend direkt am Meeresboden erfasst. Dies ist hauptsächlich darin begründet, dass zusätzlich zum konventionellen Wasserdruck auch die drei Komponenten der räumlichen Partikelbewegung gemessen werden können. Das Messen dieser vier Komponenten, das auch als 4C-Messung bezeichnet wird, erlaubt es in mariner Umgebung S-Wellen zu registrieren. Dies eröffnet die Perspektive auf eine Wellenfeldzerlegung in auf- und absteigende P- und S-Wellen.


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Chapter 1

Introduction

1.1 History of Reflection Seismology

Prior to the advent of reflection seismology entailing man-made (active or controlled) sources, experiments were mainly driven by natural (passive) sources like earthquakes which existed long before people were concerned by their destructive by-products. Even if the existence of earthquakes was commonly known due to their devastating potential, people failed to create an intuitive handling of this natural hazard. This was certainly the case for example regarding avalanches in alpine regions. The large scale of the events together with their long repetition rates mean they are outside the scope of direct human observations. Today advanced tools are instead used for monitoring and more detailed information about how earthquakes occur and seismic records around the globe are available. It is also now possible to use explosives or impact sources to produce earthquake-like wave fields and use these to resolve structures of the subsurface. While the surface is today closely monitored using airborne and satellite measurement methods, the subsurface even just a few meters below the surface is barely known. This region contains essential supplies such as raw materials, energy and fresh water. These resources are becoming increasingly important and at the same time obtain direct access is becoming more challenging. Geophysical methods are a key tool for exploring this part of our environment.

1.1.1 Seismology before the invention of the seismometer

Greek philosophers had various ideas about the origin of earthquakes. Some suspected the origins of earthquakes in enclosed ‘pneuma’ (compressed air or vaporizing water) that erupted (e.g. Demokrit (460-370 BC) and Plato(427-347 BC)) or ignited (Aristotle (384-322 BC)). Some also considered shrinking and cracking of the Earth due to drying-up processes (e.g. Anaximander (611-646 BC)) or collapse of cavities arising from subsurface burning processes (Empedokles (495-435 BC)). But also bodily changes of the Earth as a living creature that has become sick or the God Poseidon’s anger were blamed. Poseidonis (135-57 BC) realized that earthquakes must have been present on Earth long before men and Gods activities. He also differentiated vertical and horizontal tremors.

The Romans adopted the Greek theories and the philosopher and writer Seneca (4 BC-65 AD) is one of the most important sources for the records of Greek earthquake theories (Seneca & Schönberger, 1990). With fading scientific ambitions and realizing that none of
the suggested causes fully explained the occurrence of earthquakes the Romans reverted to the explanation that an enraged God giving vent to his anger was the origin of earthquakes. The difficulty was then to investigate which God was irritated and might be abated by oblation (Aulus Gelius (130-200)). A more detailed summary of earthquake theory in the ancient world can be found in Wilsdorf & Schmidt (1979).

Due to the inconsistency of seismological theory at the time and the strong position of religious exponents, theological arguments also gathered momentum in the Byzantine Empire. Nevertheless, Agathias (ca.536-582) describes an experiment by the technician and architect of the Hagia Sophia, Anthemius of Tralles (485-536), in which he simulated an earthquake. Even if the intention and circumstances of the experiment remain unclear it marks the beginning of a new approach in the treatment of earthquakes.

In the middle age Seneca and Aristotle where mandatory reading for the scholars of the time, but scholastic discussions did not lead to new findings in seismology. However, Roger Bacon (1214-1294) and Witelo (ca.1230-ca.1275) achieved progress in optics and wave propagation around this time.

It is believed that the Islamic Golden Age between the 8th and the 13th century was induced by the invention of paper and wealth due to successful trading. At the time paper was a tightly guarded secret of the Chinese that may have been obtained from Chinese prisoners captured at the Battle of Talas (751). Trading was a by-product of frequent traveling of pilgrims to Mecca which led to exchange of ideas and goods. Subsequently, the Muslim world became an intellectual center of science, philosophy, medicine and education. Modern scientific methods were developed and Ibn al-Haytam (ca.965-ca.1040), who is probably the best known scientist from this time, made substantial advances in Optics concerning reflection and refraction. He also discovered a result similar to Snell’s law of sines but did not quantify it or derive it mathematically. The Islamic Golden Age ended with the Crusades and the Mongol invasion.

With the beginning of the Renaissance at the end of the 14th century and its most famous exponent, Leonardo da Vinci (1452-1519), the advances in natural science in general and physics in particular became faster, more complex and are much better documented. A detailed description of the insights that physics delivered from the time of Leonardo da Vinci to now and the coherence between physics and development of the society can be found in Morus (2005). It should be noted that while the theory of wave propagation and mathematical principles to describe modern seismologic phenomenon were developed mostly during the 18th and 19th century it was not then possible to use this knowledge to obtain information of the Earth’s interior due to the lack of instrumentation to measure ground motion adequately. In 1880 the first seismometer was constructed at the Imperial College of Engineering in Tokyo, heralding the start of the era of observational seismology.

1.1.2 From passive to active seismology

The use and development of refraction and reflection seismology was based mainly on the development of sensitive and precise measuring instruments as the artificial (active) sources release considerably less energy than the perceptible natural (passive) events. The investigation of local structures also requires the ability to measure the traveltimes very precisely.
1.1. HISTORY OF REFLECTION SEISMOLOGY

The first measuring instrument in the literature is Chang Heng’s seismoscope from 132 AD (see Figure 1.1). For a contemporary observer it might look more like a piece of art than a measurement device as it consisted of dragons that where positioned in a circle, each holding a ball in its throat. The arriving wavefront from an earthquake would cause one of these balls to fall indicating that an earthquake occurred and also the direction of the first motion.

In 1845 the Irish engineer Robert Mallet (1810-1881) tried to measure seismic velocities of different surface materials. To do this he detonated buried gunpowder charges and measured the arrival times with an electronic chronometer. To indicate the arriving waves he used a spotlight that was reflected by a bowl of mercury. The pioneering experiment failed because the weak P-wave arrivals were unable to be recorded.

In 1897 the British Association for the Advancement of Science started to establish a coordinated network of 27 seismometers operating on all continents; these were in operation by 1899. It became clear that the measured signal was mainly dependent on the location of the earthquake and the station and not the earthquake itself. As the instruments improved, P-, S- and surface-waves could be distinguished. In 1909 Andrijan Mohorocičić discovered refractions from a strong impedance contrast that was named the Moho after him and was interpreted as the interface between crustal and mantle rocks.

The insight that it was possible to locate geological structures by artificial sources and microphones came during World War I. The first to use methods resembling early seismic networks were the French troops including Captain Conrad Schlumberger of later electrical well-logging fame. The methods were used to locate heavy enemy artillery. English
and later American troops adopted the system with modified equipment. German troops also developed their own system to locate enemy gunfire. As a byproduct reflections from geological structures were observed and after the war German and American artillery locaters founded independent seismic exploration companies to provide services to the booming oil industry in the USA. Further discussion of the recent history of exploration geophysics is described in some detail in Lawyer et al. (2001).

1.2 Marine seismics

1.2.1 Conventional streamer seismics

Conventional reflection seismic measurements in a marine environment involve generating acoustic waves and measuring the excess pressure wavefield (time series) at several locations immediately after the acoustic wave has been excited. The source and receivers are usually towed by a ship with the source close to the boat and hundreds of receivers placed inside an oil filled hose which is called a streamer or eel.

The source is designed to radiate a repeatable signature with adjustable frequency and energy content. This is often achieved by an array of air guns that are fired in a special manner to compensate for the reflection of the water surface and the bubble pulse problem. The airgun itself consists of a submerged pressurized air chamber that opens suddenly and releases compressed air bubbles which in turn radiate a pulse of acoustic energy, similar to an explosion, when they collapse (implosion).

The streamers are typically two to three kilometers long and receiver spacing is usually between 12.5m and 25 m. For 3D surveys it is convenient to tow more than one streamer simultaneously behind the ship. Common configurations are four to eight streamers, but surveys with up to twelve streamers have also been recorded (Yilmaz, 2001). A typical problem of streamer recording is the so called feathering which occurs because the streamers are not aligned behind the boat but drift due to wind and currents. As it is important to know the exact location of the receivers, the bearing of the streamers is monitored with several compasses. Like in other technical fields where exact locations are mandatory, Global Positioning System (GPS) measurements are becoming increasingly used.

1.2.2 Shear wave measurements

Shear waves properties of seismic media usually differ substantially from compressional wave properties. The wave-speed of shear waves is often only a fraction (sometimes less than a fifth) of the compressional wave speed. For this reason S-waves have shorter wavelengths than compressional waves of the same frequency. This means they provide a better resolution of the subsurface. For an equivalent resolution with compressional waves the dominant frequency has to be higher and the waves would be more attenuated in low seismic Q sub-surface (Barton, 2007).

An other important difference is in the response of P- and S-waves to changes in porosity. Compressional waves tend to respond very dramatically to changes in porosity while shear waves seem to be almost unaffected (Sheriff & Geldart, 1995; Caldwell, 1999). This is useful in hydrogeology where shear waves can be used to map also targets below
1.2. MARINE SEISMICS

Figure 1.2: Geometry of a shear wave survey in marine environment. As shear waves do not travel through water the recording sensors have to be deployed on the seafloor. Today this is accomplished by placing a number of individual seismometers like in the figure as well as with so-called ocean bottom cables (OBC) which incorporate the sensors in intervals. Conventional sources are used to produce converted (P to S) waves. Adapted from Barton (2007).

the water-table, while for compressional waves the water-table is often a distinct barrier (Haines et al., 2007). In the oil industry this property is often used to map targets below a gas cap or gas clouds which have a very strong acoustic impedance for compressional waves so that they reflect most of the P-wave energy. Again in this case shear waves are affected to a much lesser extent by porosity changes than by the lithological impedance contrast and so can offer illumination about what lies beneath such structures. Another application of shear wave measurements is monitoring of production in oilfields. The differing response to porosity of P- and S-waves makes it possible to trace the origin of the produced oil, which allows adjustment of the production parameters and optimization of recovery from the field.

Acquisition of shear wave surveys is more costly than conventional acoustic seismic surveys. To generate shear waves special sources may be needed which, although they have been available for some decades, are far more complicated than conventional P-wave sources. Alongside the technical problems it is not immediately clear in which direction the shear wave should be excited. The source problem is even more difficult when surveys are conducted in marine environments where shear wave sources have to be placed on the seafloor, due to the zero rigidity of water which prevents shear wave propagation.

Today, the shear source problem is usually bypassed by measuring converted (P to S) shear waves. The geometry of such a survey is illustrated in Figure 1.2. This method uses conventional sources and evaluates the shear waves that are generated at impedance
CHAPTER 1. INTRODUCTION

Figure 1.3: An incident P-wave on the seafloor is converted into a reflected and a transmitted P-wave as well as a transmitted S-wave. In the presence of attenuation the transmitted waves are in general inhomogeneous.

contrasts from incident compressional waves. This effect is closely related to the common observation that light is refracted and reflected at a water surface. Light as an electromagnetic wave can be mathematically described in the same way as SH-wave propagation (Carcione & Cavallini, 1995). In elastic 2D wave propagation SH-waves, which is the shear wave that has its particle motion (polarization) perpendicular to the plane, can be described and is decoupled from P-SV-modes that have their polarization within the plane. In the case of P-SV-wave propagation the two wave modes are connected to each other and an incident wave on an interface is converted in a reflected P- and SV-wave as well as a transmitted P- and SV-wave. Figure 1.3 shows how an incident P-wave is split up into a reflected P-wave, a transmitted P-wave as well as a transmitted S-wave on the seafloor. No conversion to a reflected S-wave occurs because this mode does not exist in water due to its zero rigidity. In the case of attenuation, generally inhomogeneous waves are generated. This means that propagation and attenuation vectors point in different directions. In contrast, most of the mode-converted S-wave energy in exploration surveys is converted at depth and not at the seafloor.

Conventional recording apparatus is usually not sufficient to record shear waves. Special efforts including three component geophones on land surveys are needed in addition to the conventional vertical component, to measure motion in the horizontal direction. Therefore two additional horizontal geophones oriented perpendicular to each other are added. In marine environments the conventional pressure sensor is supplemented by three motion sensors that measure particle movement in the vertical and horizontal directions. Additional complications come from the fact that the receivers have to be deployed at the ocean bottom. This makes measurements more elaborate and expensive than conventional streamer seismic.

Sensors in the form of ocean bottom seismometers (OBS) have been used for several decades in global seismology and refraction seismics (Schneider & Backus, 1964). For exploration purposes where a larger number of receivers are needed, shear wave recording sensors have been arranged in ocean bottom cables (OBC). These cables have first been
used in transition regions between land and marine seismic (Rigsby et al., 1987), but later proved to be useful in regions where conventional seismic produces poor results due to large or weak P-wave impedances (Caldwell, 1999).

1.3 Seismic Modeling

The earth contains complicated geological structures which all have their own influence on seismic recordings. Sometimes the superimposed effects of these structures make it difficult to interpret the origin of an observation. Seismic modeling provides a possibility to overcome this complexity by connecting a model of the subsurface with expected observations.

There exist a great many different methods to numerically calculate wave propagation, all with their advantages and disadvantages to varying degrees. An overview and further references are given in Carcione et al. (2002). A very popular method of seismic modeling is time domain finite differences, which is described in detail by Moczo et al. (2006).

There are different ways in which seismic modeling is used to improve the understanding of seismic wave propagation. A widely used technique is known as seismic tomography, which is usually formulated as an inverse problem. It is used to better resolve the subsurface in an iterative way. Starting from an initial assumption of the subsurface the corresponding seismograms are calculated. The difference between the modeling results and the real measurements then allows one to adjust the model of the subsurface. Through a number of subsequent generations of synthetic seismograms and their evaluation against the measurements, the subsurface model will gradually improve and the synthetic results will converge to the measurements. It has to be noted that several models may produce the same seismic response and that an inverted model does not necessarily coincide with the true subsurface; this is called model non-uniqueness.

Today the state of the art is the development of so called full-waveform inversion algorithms, which are distinct from conventional seismic inversion algorithms in that they consider not only the first arrival travel times or amplitude of a seismic wave, but also its subsequent coda. Today the prediction of the first arrivals is still a matter of ongoing research especially in the presence of attenuation, anisotropy and porosity. Therefore wave inversion algorithms have not yet reached regular production status.

1.4 Outline of this thesis

In a large part of this thesis a rather simple direct theoretical approach to calculate the partitioning of an incident wave at an interface, as described in Figure 1.3, is compared to the results of an advanced numerical modeling code for various reasons. In Chapter 2 the goal is mainly to test the simpler method for its validity in the presence of attenuation and anisotropy and in Chapter 3 the comparison is used to resolve a problem for the analytic method.

The theoretical approach uses the assumption of plane-waves. The advantage of using this sort of waves is their relatively simple mathematical description, even though the character of a wave is already considerably complex. For such waves it is possible to calculate the ratio between the amplitude of particle motion of the incident wave and the
subsequently reflected and transmitted waves in an analytical way, which is described in more detail in section 3.3. It is possible to extend this technique to a stack of layers involving multiple interfaces and to calculate the reflection coefficients at the first interface, to follow the paths of the reflected and converted waves and calculate the reflection and transmission at the subsequent interfaces. This then allows one to calculate the seismic response of a subsurface of some complexity and would therefore be called a ray-tracing seismic modeling algorithm.

The advanced numerical algorithm is a pseudo-spectral modeling code. It is characterized by time stepping in the time domain, but the lateral differences in the wavefields are calculated in the frequency domain. The transformation between the two domains is conducted by the use of fast Fourier transformations (FFT). The method is very precise and handles also dispersion effects correctly. The use of Chebyshev operators also enables an accurate handling of the boundary conditions, which is especially useful for modeling interfaces. The extra use of computational power through the domain transformation is partly compensated with the additional accuracy that allows one to use as few as two nodes per wavelength. As multimedia applications make frequent use of FFT’s, todays commodity computers often have special optimization for these operations. The additional precision and its subsequent sparse sampling is also beneficial in terms of memory usage. The method is described in detail by Tessmer (1990), Carcione et al. (2002) and Carcione (2007)

Chapter 4 is an evaluation of the seismic parameters that can be expected from an ocean bottom. A simple model is presented using data from the literature. The methods evaluated in the first two parts of the thesis along with equivalent media theory and stochastic variables are then used to evaluate the presented model. The results confirm the limitations found in wavefield decomposition.
Chapter 2

Wave reflection at an anelastic transversely isotropic ocean-bottom

Rolf Sidler and José M. Carcione
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2.1 Abstract

We study the reflection of waves at the ocean bottom, which is modeled as a plane interface separating a viscoacoustic medium (water) and a viscoelastic transversely isotropic solid whose axis of rotational symmetry is perpendicular to the bottom. We compute the plane-wave reflection coefficient (including the phenomenon known as the Rayleigh window) both numerically – by AVO analysis of synthetic seismograms generated using a domain decomposition method – and analytically. A first simulation considers the water-steel interface, for which experimental data is available. Then, we consider soft sediments and stiff crustal rocks for various values of the anellipticity parameter \( \delta \).

The domain-decomposition technique relies on one grid for the fluid and another grid for the solid and uses Fourier and Chebyshev differential operators. The anelastic and anisotropic stress-strain relation is described by the Zener model. Special attention is given to modeling the boundary conditions at the ocean bottom. For this purpose, we further develop the technique for wave propagation at fluid/anelastic-anisotropic-solid interfaces. AVO slowness-frequency domain analysis is used to compute the reflection coefficient and phase angle from the synthetic seismograms. This allows us to verify the domain decomposition algorithm, which is shown to model with high accuracy the Rayleigh window for varying \( \delta \). The comparison also verifies the calculation of the analytical plane-wave reflection coefficient, since a wrong choice of the sign of the vertical slowness of the reflected wave may cause non-physical discontinuities in the coefficient. Moreover, the pseudo-spectral modeling code allows a general material variability and a complete and accurate characterization of the seismic response of an anisotropic ocean bottom.
2.2 Introduction

The problem of reflection, refraction and propagation at a plane boundary separating a viscoacoustic medium (lossy fluid) and a viscoelastic anisotropic solid has practical application in seismic exploration, seismology, foundation engineering and non-destructive testing of materials. The ocean-bottom interface may separate the water column from a finely-layered formation whose strata are parallel to the interface. In this case, the formation can be replaced by a homogeneous transversely isotropic (TI) medium, whose symmetry axis is perpendicular to the bottom. This situation occurs when the wavelength of the seismic pulse is much larger than the thickness of the single layers (Postma, 1955).

In seismic exploration, the characterization of the ocean bottom is useful for data processing of multi-component seismic surveys acquired at the sea-floor. Knowledge of S-wave velocities is required for static corrections and imaging of mode-converted PS waves. Shear velocity is also important for multiple removal and amplitude-variations with offset (AVO) analysis.

To our knowledge, the viscoelastic problem has been addressed only in the isotropic case. In the case of isotropic porous media, the problem is somewhat similar due to the presence of a viscous fluid within the pores (e.g., Wu et al., 1990; Santos et al., 1992; Denneman et al., 2002). Borcherdt et al. (1986) present theoretical and experimental results corresponding to a water/steel interface, where the phenomenon called the Rayleigh window occurs. This viscoelastic effect implies that the energy incident on the boundary at angles within that window is substantially transmitted.

The phenomenon is associated with the presence of inhomogeneous body waves, only present in anelastic media. It cannot be predicted by using reflection coefficients based on the elasticity theory (Brekhovskikh, 1980, p. 34). The amplitude of minimum reflection depends on the shear-wave loss, while the position of the window depends mainly on the shear-wave velocity. These two effects are largely independent. Moreover, the effect is important for hard ocean bottoms, when the crustal shear velocity is greater than that of the incident P wave. The problem has been further investigated by Carcione & Helle (2004) and Carcione (2006) using numerical simulation.

In this article, we propose a new domain-decomposition technique for a fluid/TI solid interface to accurately model the ocean-bottom boundary conditions. There exists no analytical solution for this problem. Therefore, the domain-decomposition based algorithm is a novel tool for computing the response of the ocean bottom in the presence of anisotropy. A similar simulation algorithm, based on a domain-decomposition method to model wave propagation through a crack in a TI medium, is given in Carcione (1998).

We then show how to compute the reflection coefficient analytically. The analytical result serve as a cross-check of the domain decomposition algorithms. However, the choice of the correct sign of the vertical slowness of the various waves is essential, otherwise non-physical discontinuities may appear. As indicated by Krebes (1984), special care is needed when choosing that sign, since a wrong choice may lead to discontinuities of the vertical slowness as a function of the incidence angle. Unlike the elastic case, the amplitude of the scattered waves can grow exponentially with distance from the interface (Richards, 1984). Thus, the condition of an exponentially decaying wave is not sufficient to obtain the reflection and transmission coefficients. Instead, the signs of the real and imaginary parts of the vertical slowness should be chosen to guarantee a smooth variation
2.3. THE ELASTIC CONSTANTS

as a function of the incidence angle. This problem has recently been studied by Ruud (2006) who analyzes the different criteria to obtain the sign of the vertical slowness as a function of the incidence angle. Ruud (2006) concludes that the energy-velocity vector criterion should be used for sub-critical angles and the radiation condition should be used for supercritical angles, but the problem remains since these conditions have to be further verified, and the nature of critical angle in anelastic media is unclear.

The comparison between the plane-wave theory and the numerical calculations serves as a cross-check of the modeling algorithm, where we use an AVO inversion method to obtain the numerical plane-wave reflection coefficient. The cross-check also solves for ambiguities in the calculation of the reflection coefficients due to the choice of the sign of the vertical slowness, when computing the analytical plane-wave reflection coefficient.

The novel aspects of this work compared to that of Carcione & Helle (2004) are: i) the use of a viscoelastic TI stress-strain relation for the ocean bottom; ii) the generalization of the domain-decomposition technique to the TI case; iii) the generalization of the analytical plane-wave reflection coefficient to the TI case, and iv) the analysis of the behavior of the Rayleigh window for anisotropic media.

2.3 The elastic constants

Without loss of generality, we consider the 2-D qP-qSV case, where the relevant elastic constants in the \((x, z)\)-propagation plane are \(c_{11}, c_{33}, c_{13}\) and \(c_{55}\). These constants correspond to the unrelaxed (high-frequency) limit. We define \(c_{11} \equiv \rho v_P^2\) and \(c_{55} \equiv \rho v_S^2\), where \(\rho\) is the density and \(v_P\) and \(v_S\) are P- and S-wave velocities. The isotropic limit implies \(c_{11} = c_{33}\) and \(c_{13} = c_{11} - 2c_{55}\).

We quantify the degree anisotropy with

\[
\epsilon = \frac{c_{11} - c_{33}}{2c_{33}}
\]

(Thomson, 1986). While \(\epsilon\) is related to the fractional difference between the horizontal and vertical P-wave velocity, \(\delta\) is responsible for the angular dependence of the P-wave velocity in the vicinity of the vertical direction; the P-wave velocity increases away from the vertical if \(\delta\) is positive and decreases if \(\delta\) is negative (Tsvankin, 2005). In the isotropic case \(\epsilon = \delta = 0\).

Thus, given \(c_{11}, c_{33}, c_{55}\) and \(\delta\), we obtain

\[
c_{13} = \sqrt{2\delta c_{33}(c_{33} - c_{55}) + (c_{33} - c_{55})^2 - c_{55}}.
\]

Thus, varying \(\delta\) means changing \(c_{13}\). In general, when we vary \(\epsilon\), we keep constant \(c_{11}\) and vary \(c_{33} = c_{11}/(1 + 2\epsilon)\). Otherwise, we keep constant \(c_{33}\) and obtain \(c_{11} = (1 + 2\epsilon)c_{33}\). If \(c_{11} = c_{33}\) (cubic symmetry), \(\delta\) is the only parameter quantifying the anisotropy.
2.4 Modeling equations

The reference work is Carcione & Helle (2004), where the isotropic case is considered. Here, we extend the stress-strain relation to the TI case, with the symmetry axis perpendicular to the fluid/solid interface. The qP-qSV equations of motion are given by (Carcione, 2001):

i) Euler’s equations:
\[ \begin{align*}
\partial_x \sigma_{xx} + \partial_z \sigma_{xz} &= \rho \partial_t v_x, \quad (2.4) \\
\partial_x \sigma_{xz} + \partial_z \sigma_{zz} &= \rho \partial_t v_z, \quad (2.5)
\end{align*} \]

where \( v \) and \( \sigma \) denote particle velocity and stress, and \( \rho \) is the density.

ii) Stress-strain relations:
\[ \begin{align*}
\partial_t \sigma_{xx} &= c_{11} \partial_x v_x + c_{13} \partial_z v_z + \bar{K} e_1 + 2c_{55}e_2, \\
\partial_t \sigma_{zz} &= c_{13} \partial_x v_x + c_{33} \partial_z v_z + \bar{K} e_1 - 2c_{55}e_2, \quad (2.6, 2.7) \\
\partial_t \sigma_{xz} &= c_{55} \left( \partial_z v_x + \partial_x v_z + e_3 \right), \quad (2.8)
\end{align*} \]

where \( e_1, e_2 \) and \( e_3 \) are memory variables and
\[ \bar{K} = \bar{E} - 2c_{55}, \quad \bar{E} = \frac{1}{2}(c_{11} + c_{33}). \] (2.9)

iii) Memory-variable equations:
\[ \begin{align*}
\partial_t e_1 &= \frac{1}{\tau^{(1)}_\sigma} \left[ \left( \frac{\tau^{(1)}_\sigma}{\tau^{(1)}_\epsilon} - 1 \right) (\partial_x v_x + \partial_z v_z) - e_1 \right], \quad (2.10) \\
\partial_t e_2 &= \frac{1}{2\tau^{(2)}_\sigma} \left[ \left( \frac{\tau^{(2)}_\sigma}{\tau^{(2)}_\epsilon} - 1 \right) (\partial_x v_x - \partial_z v_z) - 2e_2 \right], \quad (2.11) \\
\partial_t e_3 &= \frac{1}{\tau^{(2)}_\sigma} \left[ \left( \frac{\tau^{(2)}_\sigma}{\tau^{(2)}_\epsilon} - 1 \right) (\partial_z v_x + \partial_x v_z) - e_3 \right], \quad (2.12)
\end{align*} \]

where \( \tau^{(\nu)}_\sigma \) and \( \tau^{(\nu)}_\epsilon \) are relaxation times related to dilatational (\( \nu = 1 \)) and shear (\( \nu = 2 \)) deformations. The frequency-domain stress-strain relations are obtained from the preceding equations by applying the Fourier transform (see Appendix 2.A). The stress-strain relations satisfy the condition that the mean stress depends only on the dilatational relaxation function in any coordinate system – the trace of the stress tensor should be invariant under coordinate transformations. Moreover, the deviatoric stresses solely depend on the shear relaxation function (see Carcione, 2001).

The equations for the fluid (viscoacoustic medium) are obtained from 2.4 -2.12 as a limiting case. The equations of motion read
\[ \begin{align*}
\rho \partial_t v_x &= \partial_x \sigma, \quad (2.13) \\
\rho \partial_t v_z &= \partial_z \sigma. \quad (2.14)
\end{align*} \]
2.5. MODELING ALGORITHM AND DOMAIN DECOMPOSITION

where

\[ \partial_t \sigma = \lambda (v_{x,x} + v_{z,z} + e_1) + s_f, \]  \hspace{1cm} (2.15)

together with the memory-variable equation 2.10, where \( \lambda \) is the bulk modulus and \( s_f \) is the source.

2.5 Modeling algorithm and domain decomposition

The numerical solution is obtained by generalizing to the anisotropic case the algorithm used by Carcione & Helle (2004). The boundary conditions at the ocean bottom require the continuity of the normal particle velocity \( v_z \) and stress component \( \sigma_{zz} \), while \( \sigma_{xz} = 0 \) (e.g., Carcione, 2001). Two grids model the fluid and solid sub-domains. The solution on each grid is obtained by using the Runge-Kutta method as time stepping algorithm and the Fourier and Chebyshev differential operators to compute the spatial derivatives in the horizontal and vertical directions, respectively (Carcione & Helle, 2004). In order to combine the two grids (domain decomposition), the wave field is decomposed into incoming and outgoing wave modes at the interface between the solid and the fluid. The inward propagating waves depend on the solution exterior to the sub-domains and therefore are computed from the boundary conditions, while the behavior of the outward propagating waves is determined by the solution inside the sub-domain. The approach is adapted here for the anisotropic case, and involves the equations given for updating the field variables at the grid points defining the fluid/solid interface.

The fluid is denoted by the subscript 1 and the solid by the subscript 2. The symbol P indicates the compressional wave in the fluid or the qP wave in solid, and S denotes the qS wave in this medium. The boundary equations at the fluid/solid interface are generalizations to the TI case (the interface perpendicular to the symmetry axis) of the equations given in Carcione & Helle (2004):

\[
\begin{align*}
v_x^{\text{new}}(1) &= v_x^{\text{old}}(1), \\
v_z^{\text{new}}(1) &= [Z_P(1) + Z_P(2)]^{-1} [Z_P(2)v_z^{\text{old}}(2) + Z_P(1)v_z^{\text{old}}(1) + \sigma^{\text{old}}(1) - \sigma_{zz}^{\text{old}}(2)], \\
\sigma_{zz}^{\text{new}}(1) &= \frac{Z_P(1)Z_P(2)}{Z_P(1) + Z_P(2)} \left[ v_z^{\text{old}}(1) - v_z^{\text{old}}(2) + \frac{\sigma^{\text{old}}(1)}{Z_P(1)} - \frac{\sigma_{zz}^{\text{old}}(2)}{Z_P(2)} \right], \\
\epsilon_1^{\text{new}}(1) &= \frac{e_1^{\text{old}}(1)}{\lambda} + \phi_1(1) \left[ \sigma_{zz}^{\text{new}}(1) - \sigma_{zz}^{\text{old}}(1) \right], \\
v_x^{\text{new}}(2) &= v_x^{\text{old}}(2) - \sigma_{xz}^{\text{old}}(2)/Z_S(2), \\
\sigma_{xx}^{\text{new}}(2) &= 0, \\
\epsilon_2^{\text{new}}(2) &= \frac{e_2^{\text{old}}(2)}{c_{33}} + \frac{\phi_2(2)/c_{33}}{c_{33}} \left[ \sigma_{zz}^{\text{new}}(2) - \sigma_{zz}^{\text{old}}(2) \right], \\
\epsilon_3^{\text{new}}(2) &= \frac{e_3^{\text{old}}(2)}{c_{55}} - \frac{\phi_2(2)/c_{55}}{c_{55}} \left[ \sigma_{zz}^{\text{new}}(2) - \sigma_{zz}^{\text{old}}(2) \right], \\
\end{align*}
\]  \hspace{1cm} (2.16)

where \( \phi_{\nu} = 1/\tau_{\sigma}^{(\nu)} - 1/\tau_{e}^{(\nu)} \), \( Z_P(1) = \sqrt{\rho_1 \lambda} \), \( Z_P(2) = \sqrt{\rho_2 c_{33}} \), and \( Z_S(2) = \sqrt{\rho_2 c_{55}} \).

These equations indicate how the wave field at the boundary grid points has to be updated (new) as a function of the previous values (old) at each time step.
The lower boundary of sub-domain 2 (the solid) satisfies the non-reflecting conditions:

\[
\begin{align*}
\nu_z^{(\text{new})} &= \frac{1}{2} \left( \nu_z^{(\text{old})} - \sigma^{(\text{old})}/Z_P \right), \\
\sigma^{(\text{new})} &= \frac{1}{2} \left( \sigma^{(\text{old})} - Z_P v_z^{(\text{old})} \right), \\
e_1^{(\text{new})} &= e_1^{(\text{old})} - \left[ \phi_1/(2\lambda) \right] \left( \sigma^{(\text{old})} + Z_P v_z^{(\text{old})} \right).
\end{align*}
\] (2.17)

The lower boundary of sub-domain 2 (the solid) satisfies the non-reflecting conditions:

\[
\begin{align*}
\nu_x^{(\text{new})} &= \frac{1}{2} \left( \nu_x^{(\text{old})} + \sigma_x^{(\text{old})}/Z_S \right), \\
v_z^{(\text{new})} &= \frac{1}{2} \left( v_z^{(\text{old})} + \sigma_z^{(\text{old})}/Z_P \right), \\
\sigma_{xx}^{(\text{new})} &= \sigma_{xx}^{(\text{old})} - (c_{13}/(2c_{33})) (\sigma_{zz}^{(\text{old})} - Z_P v_z^{(\text{old})}), \\
\sigma_{zz}^{(\text{new})} &= \frac{1}{2} (\sigma_{zz}^{(\text{old})} + Z_P v_z^{(\text{old})}), \\
\sigma_{xx}^{(\text{new})} &= \frac{1}{2} (\sigma_{zz}^{(\text{old})} + Z_S v_x^{(\text{old})}), \\
e_1^{(\text{new})} &= e_1^{(\text{old})} - \left[ \phi_1/(2c_{33}) \right] (\sigma_{zz}^{(\text{old})} - Z_P v_z^{(\text{old})}), \\
e_2^{(\text{new})} &= e_2^{(\text{old})} + \left[ \phi_2/(2c_{33}) \right] (\sigma_{zz}^{(\text{old})} - Z_P v_z^{(\text{old})}), \\
e_3^{(\text{new})} &= e_3^{(\text{old})} - \left[ \phi_2/(2c_{33}) \right] (\sigma_{xx}^{(\text{old})} - Z_S v_x^{(\text{old})}).
\end{align*}
\] (2.18)

In addition to the non-reflecting conditions, absorbing strips are used to further attenuate the wave field at non-physical boundaries (Carcione & Helle, 2004).

### 2.6 Analytical plane-wave reflection coefficients

The expression of the reflection and transmission coefficients are a generalization of equations (6.187)-(6.193) in Carcione (2001) to the TI case. \( R_{PP} \), \( T_{PP} \) and \( T_{PS} \) denote the reflection and transmission coefficient of the compressional and the compressional to shear converted wave respectively. The boundary conditions require continuity of

\[
v_z, \ \sigma_{zz}, \ \text{and} \ \sigma_{xx} = 0.
\] (2.19)

These conditions generate the following matrix equation for the reflection and transmission coefficients:

\[
\begin{pmatrix}
\xi_{P1} & \xi_{P2} & \xi_{S2} \\
Z_{P1} & -Z_{P2} & -Z_{S2} \\
0 & W_{P2} & W_{S2}
\end{pmatrix}
\begin{pmatrix}
R_{PP} \\
T_{PP} \\
T_{PS}
\end{pmatrix}
= \begin{pmatrix}
\xi_{P1} \\
-Z_{P1} \\
0
\end{pmatrix},
\]

where

\[
\begin{align*}
Z_{P1} &= ps = \sqrt{\rho p}, \quad p = \lambda M(\omega), \quad s = \sqrt{\rho/p}, \\
W_{P2} &= p_{55}(\xi_{P2} s_x + \beta_{P2} s_{z_{P2}}), \\
Z_{P2} &= \beta_{P2} p_{13} s_x + \xi_{P2} p_{33} s_{z_{P2}}, \\
W_{S2} &= p_{55}(\xi_{S2} s_x + \beta_{S2} s_{z_{S2}}), \\
Z_{S2} &= \beta_{S2} p_{13} s_x + \xi_{S2} p_{33} s_{z_{S2}},
\end{align*}
\] (2.21)

with the relaxed material constants \( p_{ij} \) and the complex modulus of the fluid \( M \) having the form 2.28 and 2.29.
The horizontal slowness is the same for all the waves (Snell’s law) and it is given by

\[ s_x = \sin \theta \sqrt{\frac{\rho}{\rho}}, \tag{2.22} \]

where \( \theta \) is the incidence propagation angle and \( \rho \) is the fluid density.

The polarization components for the fluid are

\[ \beta_{P_1} = \frac{s_x}{s}, \quad \xi_{P_1} = \frac{s_z}{s}, \quad s_z = \text{pv} \sqrt{s^2 - s_x^2}, \tag{2.23} \]

where \( \text{pv} \) denotes the principal value. (For the principal value, the argument of the square root lies between \(-\pi/2\) and \(+\pi/2\)).

The vertical slownesses \( s_{zP_2} \) and \( s_{zS_2} \) are computed as

\[ s_z = \pm \frac{1}{\sqrt{2}} \sqrt{K_1 \mp \text{pv} \sqrt{K_1^2 - 4K_2K_3}}, \tag{2.24} \]

where

\[ K_1 = \rho \left( \frac{1}{p_{55}} + \frac{1}{p_{33}} \right) + \frac{1}{p_{55}} \left[ \frac{p_{13}}{p_{33}}(p_{13} + 2p_{55}) - p_{11} \right] s_x^2, \]

\[ K_2 = \frac{1}{p_{33}}(p_{11}s_x^2 - \rho), \quad K_3 = s_x^2 - \frac{\rho}{p_{55}}. \]

If the \( z \)-axis points downward, the signs in \( s_z \) corresponds to

\( (+, -) \) downward propagating \( qP \) wave
\( (+, +) \) downward propagating \( qS \) wave
\( (-, -) \) upward propagating \( qP \) wave
\( (-, +) \) upward propagating \( qS \) wave.

whose incorrect choice may cause non-physical discontinuities in the reflection coefficients.

The polarization components for the solid are

\[ \beta = \text{pv} \sqrt{\frac{p_{55}s_x^2 + p_{33}s_x^2 - \rho}{p_{11}s_x^2 + p_{33}s_z^2 + p_{55}(s_x^2 + s_z^2) - 2\rho}}, \tag{2.25} \]

and

\[ \xi = \pm \text{pv} \sqrt{\frac{p_{11}s_x^2 + p_{55}s_z^2 - \rho}{p_{11}s_x^2 + p_{33}s_z^2 + p_{55}(s_x^2 + s_z^2) - 2\rho}}, \tag{2.26} \]

respectively. In general, the \( + \) and \( - \) signs correspond to the \( qP \) and \( qS \) waves, respectively. However one must choose the signs such that \( \xi \) varies smoothly with the propagation angle.
2.7 Numerical plane-wave reflection coefficients. AVO method

In order to compute the PP reflection coefficient versus incidence angle from the synthetic seismograms obtained with the domain-decomposition based modeling, we use the AVO inversion technique applied by Carcione & Helle (2004) to an isotropic and viscoelastic ocean-bottom interface. Denoting frequency and horizontal slowness by \( f = \frac{\omega}{2\pi} \) and \( s_x \), respectively, the method consists on the following:

1. Generate a synthetic seismogram of the pressure field \( \sigma \), placing a line of receivers at each grid point above the interface. This record contains the incident and reflected fields.

2. Compute the synthetic seismogram without interface at the same location (set the properties of the lower medium to those of the upper medium). The seismogram contains the incident field only.

3. Perform the difference between the first and second seismograms. The difference contains the reflected field only.

4. Perform an \((f, s_x)\)-transform of the incident field to obtain \( \sigma_I(f, s_x) \).

5. Perform an \((f, s_x)\)-transform of the reflected field to obtain \( \sigma_R(f, s_x) \).

6. The ratio \( |\sigma_R(f, s_x)|/|\sigma_I(f, s_x)| \) is the reflection coefficient, and the phase angle is calculated by the fraction of the reflected and the incident field (angle of \( \sigma_R(f, s_x)/\sigma_I(f, s_x) \)). Transform \( s_x \) to incidence angle by using \( \sin \theta = v_{P_1} s_x \), where \( v_{P_1} \) is the sound velocity in the upper medium.

2.8 Examples

Firstly, we show the type of non-physical discontinuity to be expected in the reflection coefficient when the wrong sign in the vertical slowness (eqn. 2.24) is chosen. In this case, and in all the examples shown in this article, the frequency is chosen as the center frequency of the relaxation peaks, i.e., \( \omega = 1/\tau_0 \). The properties of water are taken \( \lambda = 2.25 \text{ GPa}, \rho = 1040 \text{ kg/m}^3 \) and \( Q = 10000 \). The properties of the solid are \( v_P = 4323 \text{ m/s}, v_S = 1449 \text{ m/s}, \delta = 0.1, \rho = 2760 \text{ kg/m}^3, Q_1 = 40 \) and \( Q_2 = 100 \). Figure 2.1 shows the reflection coefficient as a function of the incidence angle. The solid line is the correct coefficient, while the dashed line corresponds to the wrong sign, which causes a discontinuity at nearly 50°. The symbols represent the numerical calculation applying the AVO method to the synthetic seismograms at different frequencies. The mesh for the fluid has \( 375 \times 81 \) points, while the mesh of the solid has \( 375 \times 41 \) points. The horizontal grid spacing is 5 m for both meshes and the vertical size of each grid is 260 m. The source is a Ricker wavelet located at 138 m above the interface, and has a dominant frequency of 35 Hz, which is the same as the central frequency of the relaxation peaks. The time step of the Runge-Kutta method is 50 \( \mu \text{s} \).

The mismatch between the theoretical and numerical phase angles is due to the fact that the receivers are located at the penultimate grid row of the upper grid, above the interface. Then, there is a phase shift between the incident wave and the reflected wave.
Figure 2.1: Absolute value (a) and phase angle (b) of the reflection coefficient as a function of the incidence angle. The solid line is the correct coefficient, while the dashed line corresponds to a wrong choice in the sign of the vertical slowness. The symbols represent the numerical calculations applying the AVO method to the synthetic seismograms.
In the next sections, we apply the theory and the numerical modeling to a water-steel interface. The associated reflection coefficient was measured experimentally by Becker & Richardson (1970). Their ultrasonic experiments were verified with an anelastic model in a later paper (Becker & Richardson, 1972), in particular the Rayleigh window that cannot be predicted by using reflection coefficients based on the elasticity theory. The isotropic case has been investigated by Borcherdt et al. (1986), who found that the Rayleigh window should be observable in appropriate sets of wide-angle reflection data and it is useful to estimate the shear-wave quality factor of the ocean bottom. Finally, the theory is used for computing the reflection coefficients of stiff and soft ocean bottoms for various values of the parameter $\delta$.

### 2.8.1 Water-steel interface

The properties of water are $v_f = 1490$ m/s, $\rho = 1000$ kg/m$^3$ and $Q = 10000$ at $f_0 = 10$ kHz ($f_0 = 1/2\pi\tau_0$). Steel belongs to the cubic crystal class (e.g., Auld, 1990); the unrelaxed compressional and shear velocities are $v_P = \sqrt{c_{11}/\rho} = \sqrt{c_{33}/\rho} = 5740$ m/s, and $v_S = \sqrt{c_{55}/\rho} = 3142$ m/s, respectively, the density is $\rho = 7932$ kg/m$^3$ and the dissipation factors at 10 kHz are $Q_1 = 140$ and $Q_2 = 80$. We assume $\delta = 0.5$. The slowness and the wavefront at the high-frequency limit are shown in Figure 2.2, where one quadrant is displayed. Note the cuspidal triangles corresponding to the qS wave. The thin lines indicate the polarizations.

Figure 2.3 represents the reflection and transmission coefficients, with the dashed line corresponding to the isotropic and elastic reflection coefficient. Moreover, we perform a numerical evaluation of the reflection coefficient versus incidence angle from the synthetic seismograms. The mesh for the fluid has $375 \times 161$ points, and that of the solid has $375 \times 41$ points, with both meshes having a horizontal grid spacing of 5 cm; the vertical sizes of the two grids are 3.1 m and 1.9 m. The source is a Ricker wavelet located at 1.2 m above the interface, and has a dominant frequency of 10 kHz, which is the same as the central frequency of the relaxation peaks. The time step of the Runge-Kutta method is 25 $\mu$s. The symbols in Figure 2.3 correspond to the numerical evaluation of the reflection coefficient. The perfect agreement between analytical and numerical results emphasize the accuracy of the modeling method.
Figure 2.2: Normalized slownesses (a) and group velocities (b) in steel. The normalization constant is the shear-wave velocity along the Cartesian axes.
Figure 2.3: The Rayleigh window for a water-steel interface. P-wave reflection coefficient (a) and phase angle (b) versus incidence angle. The dashed line corresponds to the isotropic and elastic case, and the symbols to the numerical evaluation of the AVO response.
2.8. EXAMPLES

2.8.2 Ocean bottom

We now consider stiff and soft ocean bottoms. The parameters of the meshes are the same as those used to simulate the case of Figure 2.1; the only difference is that the mesh for the soft ocean bottom has 121 vertical grid points. The source has a central frequency of 35 Hz and is located at 138 m above the interface. The time step of the Runge-Kutta method is 50 $\mu$s for the stiff ocean bottom and 25 $\mu$s for the soft ocean bottom. Figure 2.4 compares the analytical and numerical reflection coefficients for a stiff bottom and various values of $\delta$ (indicated in the figure), where $v_p = 4000$ m/s, $v_S = 1920$ m/s, $\epsilon = 0.1$, $\rho = 2460$ kg/m$^3$, $Q_1 = 60$ and $Q_2 = 30$. The equivalent curves for a soft bottom and various values of $\delta$ (indicated in the figure) are shown in Figure 2.5, where $v_p = 1800$ m/s, $v_S = 450$ m/s, $\epsilon = 0$, $\rho = 1600$ kg/m$^3$, $Q_1 = 4$ and $Q_2 = 4$. As can be seen, the change in the reflection coefficient is not significant. This means that an AVO analysis cannot discriminate the value of $\delta$. Figure 2.6 shows the curves for $\delta = 0.1$, $\epsilon = 0.3$ (lower curve) and for $\delta = 0$, $\epsilon = 0$ (upper curve; the dashed line is the isotropic and elastic case). In this case, the reflection coefficients are dissimilar enough to allow the AVO algorithm to distinguish the two values of $\epsilon$.

Let us consider the Rayleigh window for three values of $\delta$ and $\epsilon = 0$. Figure 2.7 shows the reflection coefficient for the oceanic crust defined by $v_p = 4850$ m/s, $v_S = 2800$ m/s, $\rho = 2600$ kg/m$^3$, $Q_1 = 1000$ and $Q_2 = 10$. As can be seen, the modeling algorithm simulates accurately the Rayleigh window, i.e., the magnitude of the reflection coefficient and the phase angle. With increasing $\delta$ the window is shifted towards larger incident angles. If we set $\delta = 0$ and vary $\epsilon$, we obtain the curves displayed in Figure 2.8. Curves A and B correspond to changes in $c_{33}$ and $c_{11}$, respectively, with respect to the case $\epsilon = 0.5$. In this case, the location of the window does not change significantly, but there is a big difference regarding the phases.

The modeling algorithm allows us to model heterogeneous models, such as for instance, a stratified ocean bottom and lateral variations of the material properties. Moreover, using generalized coordinates (e.g., Carcione, 1994), ocean-bottom topography can be modeled. In the case of stratified media, more general algorithms are needed to obtain the reflection coefficient from the seismograms. Possible approaches are given by Schoenberg (1978) and Frisk et al. (1980), which could be generalized to the anisotropic case.
Figure 2.4: P-wave reflection coefficient (a) and phase angle (b) versus incidence angle for a stiff ocean bottom. The symbols correspond to the numerical evaluation of the AVO response. Curves for various values of $\delta$ are shown.
Figure 2.5: P-wave reflection coefficient (a) and phase angle (b) versus incidence angle for a soft ocean bottom. Curves for various values of $\delta$ are shown. The dashed line is the isotropic and elastic case.
Figure 2.6: P-wave reflection coefficient (a) and phase angle (b) versus incidence angle. The curves correspond to $\delta = 0.1$, $\epsilon = 0.3$ (lower curve) and $\delta = 0$, $\epsilon = 0$ (upper curve). The dashed line is the isotropic and elastic case, and the symbols represent the numerical evaluation of the AVO response.
Figure 2.7: The Rayleigh window for a stiff ocean bottom and various values of $\delta$. P-wave reflection coefficient (a and b) and phase angle (c) versus incidence angle. The dashed line is the isotropic case, and the symbols represent the numerical evaluation of the AVO response.
Figure 2.8: The Rayleigh window for a stiff ocean bottom. P-wave reflection coefficient (a) and phase angle (b) versus incidence angle. The curves correspond to $\epsilon = 0$, $\epsilon = 0.5$ and $c_{33} = \rho v_P^2/(1 + 2\epsilon)$ (A) and $\epsilon = 0.5$ and $c_{11} = (1 + 2\epsilon)\rho v_P^2$ (B). The dashed line is the isotropic case.
2.9 Conclusions

We have analyzed and simulated the seismic reflection of the ocean bottom for an anisotropic-anelastic formation. To our knowledge, the reflection problem, i.e., the calculation of plane-wave reflection coefficients and the numerical simulation in the anisotropic-anelastic case has not been addressed before. Special attention is given to modeling the boundary conditions at the fluid/solid interface. This is accurately performed by using pseudospectral differential operators and a domain-decomposition method, adapted for fluid/anelastic-anisotropic-solid interfaces.

Changes in $\delta$ have no significant effect on the reflection coefficients of a soft ocean bottom. In contrast, they are more sensitive to changes in $\epsilon$. On the other hand, the location of the Rayleigh window, which is observed in stiff bottoms, is sensitive to changes in $\delta$.

The cross-check between the plane-wave analysis and the modeling algorithm confirms the accuracy of both methods and provide an accurate tool for a correct characterization of the ocean-bottom interface. The cross-check is also necessary to test the modeling algorithm, since there is no known transient analytical solution in the lossy anisotropic case.

One of the features of the analysis is that non-physical discontinuities may arise from a wrong chosen sign of the vertical slowness when computing the reflection coefficient. The cross-check with the modeling algorithm allows us to identify this problem and ensure a correct physical analysis of the ocean-bottom reflection event.

2.10 Acknowledgments

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Appendix

2.A Frequency-domain stress-strain relation

Transforming the memory-variable equations 2.10, 2.11 and 2.12 to the (frequency) $\omega$-domain (e.g., $\partial_t e_1 \rightarrow i\omega e_1$), and substituting the memory variables into equations 2.6, 2.7 and 2.8, we obtain the stress-strain relation:

$$i\omega \begin{pmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{pmatrix} = \begin{pmatrix} p_{11} & p_{13} & 0 \\ p_{13} & p_{33} & 0 \\ 0 & 0 & p_{55} \end{pmatrix} \begin{pmatrix} \partial_x v_x \\ \partial_z v_z \\ \partial_z v_x + \partial_z v_z \end{pmatrix},$$

(2.27)

where

$$p_{11} = c_{11} - \tilde{\epsilon} + \tilde{K} M_1 + c_{55} M_2$$
$$p_{33} = c_{33} - \tilde{\epsilon} + \tilde{K} M_1 + c_{55} M_2$$
$$p_{13} = c_{13} - \tilde{\epsilon} + \tilde{K} M_1 + c_{55} (2 - M_2)$$
$$p_{55} = c_{55} M_2$$

(2.28)
are the complex stiffnesses, and

\[ M_\nu = \frac{\tau^{(\nu)}_{\sigma}}{\tau^{(\nu)}_{\epsilon}} \left( \frac{1 + i\omega \tau^{(\nu)}_{\epsilon}}{1 + i\omega \tau^{(\nu)}_{\sigma}} \right), \quad \nu = 1, 2 \quad (2.29) \]

are the Zener complex moduli (Zener, 1948; Carcione, 2001). Note that when \( \omega \to \infty \), \( p_{1J} \to c_{1J} \).

The relaxation times can be expressed as

\[ \tau^{(\nu)}_{\epsilon} = \frac{\tau_0}{Q^{(\nu)}} \left( \frac{1}{\sqrt{Q^{2}_{\nu} + 1}} + 1 \right) \quad \text{and} \quad \tau^{(\nu)}_{\sigma} = \frac{\tau_0}{Q^{(\nu)}} \left( \sqrt{Q^{2}_{\nu} + 1} - 1 \right), \quad (2.30) \]

where \( \tau_0 \) is a relaxation time such that \( 1/\tau_0 \) is the center frequency of the relaxation peak and \( Q_\nu \) are the minimum quality factors.
Chapter 3

On the evaluation of the plane-wave reflection coefficients in anelastic media

Rolf Sidler, José M. Carcione and Klaus Holliger

3.1 Abstract

Analytical evaluations of the reflection coefficients in anelastic media inherently suffer from ambiguities related to the complex square roots contained in the expressions of the vertical slowness and polarization. This leads to a large number of mathematically correct but physically unreasonable solutions. To identify the physical solution, we compute full-waveform synthetic seismograms and use a frequency-slowness method for evaluating the amplitude and phase of the corresponding reflection coefficient. We perform this analysis for transversely isotropic media. The analytical-solution space and its ambiguities are explored by analyzing the paths along the Riemann surfaces associated with the square roots. This analysis allows us to choose the correct sign. Although this approach is generally effective, there are some cases which require an alternative solution because the correct integration path for the vertical slowness does not exist on the corresponding Riemann surface. Closer inspection then shows that these "pathological" cases, which are essentially characterized by a higher attenuation layer overlying a lower-attenuation layer, can readily be resolved through an appropriate change of direction on the Riemann sheet. The thus resulting recipe for the analytical evaluation of plane-wave reflection coefficients in anelastic media is conceptually simple and robust, and provides correct solutions beyond the equivalent elastic critical (EEC) angle.

3.2 Introduction

Earth media are generally both attenuating and anisotropic and hence conventional elastic isotropic approximations prove to be inadequate for fully exploiting the information contained in modern seismic data (Tsvankin & Thomsen, 1994); (Carcione, 2007). It is
in part for this reason that the problem of effectively evaluating seismic reflection coefficients for layered attenuating media has recently received a significant amount of attention. Although the underlying mathematics is well understood, the problem per se must be regarded as unresolved (Nechtschein & Hron, 1996; Červený & Pšenčík, 2005; Ruud, 2006; Krebes & Daley, 2007). This is primarily due to the ambiguities related to the signs of the complex-valued square roots involved in the expression of the vertical slownesses, which result in a set of mathematically correct but physically unreasonable plane-wave reflection coefficients.

There are a number of approaches attempting to solve these ambiguities. Convergence tests with regard to the well constrained elastic case represent one option. In this case, the reflection coefficient is expected to change smoothly from the purely elastic case to the weakly anelastic case. Although there has been doubt as to the validity of this approach, it is by now widely accepted (Richards, 1984; Krebes, 1984; Hearn & Krebes, 1990). However, Krebes (1984) found that the introduction of little attenuation may lead to substantial phase changes.

Ruud (2006) analyzed different criteria to obtain the correct sign of the square roots by comparing seismograms based on analytical reflection coefficients to seismograms computed with a reflectivity algorithm. The presence of a phase difference seems to be related to a particular choice of the anelastic properties for which the amplitudes of reflected/transmitted waves grow exponentially with distance from the interface (Cooper, 1967; Richards, 1984). The criteria for isotropic media, which are physically relevant, but not universally valid, can be summarized as follows:

1. Impose continuity/smoothness of the reflection coefficient as a function of the incidence angle.

2. Choose the signs of the square roots according to the direction of the energy-velocity vector: upwards for reflected waves and downward for transmitted waves.

3. Apply the radiation condition, meaning that the attenuation vector must point upwards for reflected waves and downward for transmitted waves thus implying that the amplitudes of the waves cannot grow exponentially with increasing distance from the interface.

Ruud (2006) concludes that the energy-velocity vector criterion should be used for pre-critical angles and the radiation condition should be used for post-critical angles, where, in principle, the location of the critical angle is that of the elastic case. He discards the continuity criterion and claims that the coefficients obtained with his approach tend to the elastic coefficients in particular the phase angle when the attenuation tends to zero, even if there are discontinuities. In a recent study, Krebes & Daley (2007) compare SH-wave reflection coefficients of anelastic media to the elastic equivalent, localize the incidence angles where non-physical jumps or discontinuities occur and explore three different approaches for choosing the sign of the vertical slowness, which they also apply to the P-SV case.

The objective of this study is to extend and complement previous work on this problem by providing an effective way to evaluate the reflection coefficients in attenuating anisotropic media. The method consist in the use of full-wave numerical modeling to
verify the results and comprehensive rules that allow us to determine the signs of the complex-valued square roots in agreement with the numerical simulations.

We begin with a brief review of the theory for calculating the plane-wave reflection coefficients in attenuating anisotropic media, followed by an overview of the algorithm for the numerical simulation. The modeling algorithm is based on a domain-decomposition technique – one grid for the upper solid and another grid for the lower solid – and the Fourier and Chebyshev differential operators. The anelastic and anisotropic stress-strain relation is based on the Zener model. Special attention is given to modeling the boundary conditions. For this purpose, we further develop the technique for wave propagation in transversely isotropic media. We then present the frequency-slowness approach that allows us to compute the reflection coefficients from the computed seismograms. In the last section, we explore and attempt to resolve the ambiguities present in the analytical solution on the basis of the physically correct solution obtained by numerical modeling.

3.3 Reflection coefficients

Without loss of generality, we consider the 2-D P-SV-wave case where the relevant elastic constants in the \((x, z)\)-propagation plane are \(c_{11}, c_{33}, c_{13}\) and \(c_{55}\). These constants correspond to the unrelaxed, high-frequency limit. We define \(c_{11} \equiv \rho v_{P}^2\) and \(c_{55} \equiv \rho v_{S}^2\), where \(\rho\) is the density and \(v_{P}\) and \(v_{S}\) are the P- and S-wave velocities.

Following Tsvankin (2005), we quantify the degree of anisotropy as

\[
\epsilon = \frac{c_{11} - c_{33}}{2c_{33}} \quad (3.1)
\]

and the anisotropy parameter \(\delta^*\)

\[
\delta^* = \frac{1}{2c_{33}^2} \left[ 2(c_{13} + c_{44})^2 - (c_{33} - c_{44})(c_{11} + c_{33} - 2c_{44}) \right]. \quad (3.2)
\]

While \(\epsilon\) is related to the fractional difference between the horizontal and vertical P-wave velocity, \(\delta^*\) is responsible for the angular dependence of the P-wave velocity in the vicinity of the vertical direction with the P-wave velocity increasing away from the vertical if \(\delta^*\) is positive and decreasing if \(\delta^*\) is negative (Tsvankin, 2005). The isotropic limit implies \(c_{11} = c_{33}\) and \(c_{13} = c_{11} - 2c_{55}\) and \(\epsilon = \delta^* = 0\). For the transversely isotropic case \(c_{44}\) is equal to \(c_{55}\).

Given \(c_{11}, c_{33}, c_{55}\) and \(\delta^*\), we obtain

\[
c_{13} = \sqrt{\frac{2c_{33}^2\delta^* + (c_{33} - c_{44})(c_{11} + c_{33} + 2c_{44})}{2}} - c_{44}. \quad (3.3)
\]

Therefore, varying \(\delta^*\) means changing \(c_{13}\). In the case of cubic symmetry \((c_{11} = c_{33})\), \(\delta^*\) is the only parameter quantifying the anisotropy. We consider \(\epsilon = 0\) in this work.

Reflection and transmission of a wave on a planar interface between two anelastic transversely isotropic media can be estimated using the plane-wave approximation which has been investigated by several authors, amongst others Zoeppritz (1919) and Aki & Richards (1980) for isotropic elastic solids, Daley & Hron (1977) and Graebner (1992).
for elastic transversely isotropic solids, and Carcione (1997) for anelastic transversely isotropic solids. A general plane-wave solution for the particle velocity field is

\[ \mathbf{v} = i\omega U \exp[i\omega(t - s_x x - s_z z)], \]

where \( t \) is the time, \( i = \sqrt{-1} \) the imaginary unit, \( \omega \) the angular frequency, \( s_x \) and \( s_z \) are the horizontal and vertical components of the complex slowness vector, respectively, and

\[ U = U_0 \left( \begin{array}{c} \beta \\ \xi \end{array} \right), \]

(3.5)

where \( U_0 \) is a constant amplitude.

The polarization components can be calculated as (Carcione, 2007)

\[ \beta = \sqrt{\frac{p_{55} s_x^2 + p_{33} s_z^2 - \rho}{p_{11} s_x^2 + p_{33} s_z^2 + p_{55}(s_x^2 + s_z^2) - 2\rho}}, \]

(3.6)

and

\[ \xi = \pm \sqrt{\frac{p_{11} s_x^2 + p_{55} s_z^2 - \rho}{p_{11} s_x^2 + p_{33} s_z^2 + p_{55}(s_x^2 + s_z^2) - 2\rho}}, \]

(3.7)

where, following the standard sign convention, the + sign corresponds to the qP-wave and the − sign corresponds to the qS-wave; moreover, \( p_{IJ} \) are components of the stiffness tensor and \( \rho \) is the mass density (see Appendix 3.A).

According to Snell’s law, the horizontal slowness is the same for all the waves and can be computed for the incident wave as

\[ s_x = \frac{\sin \theta}{v_c(\theta)}, \]

(3.8)

where \( \theta \) is the propagation angle measured with respect to the \( z \)-axis and \( v_c \) is the complex velocity, solution of the Kelvin-Christoffel equation (Carcione, 2007). The vertical slowness can be obtained from the dispersion relation

\[ s_z = \pm \frac{1}{\sqrt{2}} \sqrt{K_1 \pm \sqrt{K_1^2 - 4K_2K_3}}, \]

(3.9)

where

\[ K_1 = \rho \left( \frac{1}{p_{55}} + \frac{1}{p_{33}} \right) + \frac{1}{p_{55}} \left[ \frac{p_{13}}{p_{33}} (p_{13} + 2p_{55}) - p_{11} \right] s_x^2, \]

\[ K_2 = \frac{1}{p_{33}} (p_{11} s_x^2 - \rho), \quad K_3 = s_x^2 - \frac{\rho}{p_{55}}. \]

Following the standard sign convention (Carcione, 2007), the signs in \( s_z \) correspond to

\[(+, -) \text{ downward propagating qP-wave,} \]
\[(+, +) \text{ downward propagating qS-wave,} \]
\[(-, -) \text{ upward propagating qP-wave,} \]
\[(-, +) \text{ upward propagating qS-wave.} \]
For a qP-wave incident from above, the particle velocities above and below the interface are given by
\[
\begin{align*}
\mathbf{v}_1 &= \mathbf{v}_{Pl} + \mathbf{v}_{Pr} + \mathbf{v}_{Sr}, \\
\mathbf{v}_2 &= \mathbf{v}_{Pl} + \mathbf{v}_{Sr},
\end{align*}
\]
where the subscripts 1 and 2 refer to the upper and lower media and the subscripts \(I, R\) and \(T\) denote incident, reflected and transmitted plane waves defined as
\[
\begin{align*}
\mathbf{v}_{Pl} &= i\omega \left( \begin{array}{c} \beta_{P1} \\ \xi_{P1} \\ \xi_{P1} \\ W_{P1} \end{array} \right) \exp[i\omega(t - s_x x - s_z P_1 z)], \\
\mathbf{v}_{Pr} &= i\omega R_{PP} \left( \begin{array}{c} \beta_{P1} \\ -\xi_{P1} \\ \xi_{P2} \\ -Z_{P1} \end{array} \right) \exp[i\omega(t - s_x x + s_z P_1 z)], \\
\mathbf{v}_{Sr} &= i\omega R_{PS} \left( \begin{array}{c} \beta_{S1} \\ -\xi_{S1} \\ \xi_{S2} \\ -Z_{S1} \end{array} \right) \exp[i\omega(t - s_x x + s_z S_1 z)], \\
\mathbf{v}_{Pt} &= i\omega T_{PP} \left( \begin{array}{c} \beta_{P2} \\ \xi_{P2} \\ \xi_{P2} \\ W_{P2} \end{array} \right) \exp[i\omega(t - s_x x - s_z P_2 z)], \\
\mathbf{v}_{St} &= i\omega T_{PS} \left( \begin{array}{c} \beta_{S2} \\ \xi_{S2} \\ \xi_{S2} \\ W_{S2} \end{array} \right) \exp[i\omega(t - s_x x - s_z S_2 z)].
\end{align*}
\]

The amplitude of the incident wave \(U_0\) is set to unity so that the amplitudes of the reflected and transmitted waves \((R_{PP}, R_{PS}, T_{PP} \text{ and } T_{PS})\) directly correspond to the complex-valued reflection and transmission coefficients.

The boundary conditions on a welded solid-solid interface require continuity of the horizontal and vertical particle velocities \(v_x\) and \(v_z\) as well as the components of the stress tensor \(\sigma_{xz}\) and \(\sigma_{zz}\). The boundary conditions then lead to the following matrix equation for the reflection and transmission coefficients
\[
\begin{pmatrix}
\beta_{P1} & \beta_{S1} & -\beta_{P2} & -\beta_{S2} \\
\xi_{P1} & \xi_{S1} & \xi_{P2} & \xi_{S2} \\
Z_{P1} & Z_{S1} & -Z_{P2} & -Z_{S2} \\
W_{P1} & W_{S1} & W_{P2} & W_{S2}
\end{pmatrix}
\begin{pmatrix}
R_{PP} \\
R_{PS} \\
T_{PP} \\
T_{PS}
\end{pmatrix}
= \begin{pmatrix}
-\beta_{P1} \\
\xi_{P1} \\
-\xi_{P1} \\
-\xi_{P1}
\end{pmatrix},
\]
\[
(3.12)
\]
where
\[
W = p_{55}(\xi s_x + \beta s_z) \quad \text{and} \quad Z = \beta p_{13} s_x + \xi p_{33} s_z.
\]
\[
(3.13)
\]
In the elastic case, the stiffnesses \(p_{11}\) correspond to the elastic constants \(c_{11}\) and have real values. In the viscoelastic case, however, the stiffnesses are complex-valued. It has to be noted that in the anelastic case a critical (and post-critical) angle does not exist (Cooper, 1967; Krebes, 1983; Carcione, 2007) as it refers to the situation where the transmitted wave travels parallel to the interface which usually does not occur if at least one of the media is anelastic. Therefore, we refer here to the equivalent elastic critical (EEC) angle. The stiffnesses for a transversely isotropic medium are
\[
\begin{align*}
p_{11} &= c_{11} - \tilde{\epsilon} + \bar{\kappa} M_1 + c_{55} M_2, \\
p_{33} &= c_{33} - \tilde{\epsilon} + \bar{\kappa} M_1 + c_{55} M_2, \\
p_{13} &= c_{13} - \tilde{\epsilon} + \bar{\kappa} M_1 + c_{55}(2 - M_2),
\end{align*}
\]
\[
(3.14) \quad (3.15) \quad (3.16)
\]
and
\[
p_{55} = c_{55} M_2.
\]
\[
(3.17)
\]
with
\[
\bar{\varepsilon} = \frac{1}{2}(c_{11} + c_{33}), \quad \bar{\kappa} = \bar{\varepsilon} - c_{55},
\] (3.18)
and the complex moduli
\[
M_\nu = \sqrt{Q^{(\nu)}^2 + 1} - 1 + iQ^{(\nu)} \sqrt{Q^{(\nu)}^2 + 1 + iQ^{(\nu)}}
\] (3.19)
where \(\nu = 1\) refers to dilatational deformations and \(\nu = 2\) to shear deformations, and \(Q^{(\nu)}\) denotes the corresponding quality factors.

### 3.4 Numerical modeling

#### 3.4.1 Governing equations

We consider the stress-strain relation corresponding to the case of two welded transversely isotropic media with the symmetry axis of both media perpendicular to the solid/solid interface. The qP-qSV equations of motion for each medium are given by (i) equations of motion, (ii) the stress-strain relations, and (iii) the memory-variable equations (Carcione, 2007), which we outline in the following.

i) Equations of motion:
\[
\partial_x \sigma_{xx} + \partial_z \sigma_{xz} = \rho \partial_t v_x,
\] (3.20)
\[
\partial_x \sigma_{xz} + \partial_z \sigma_{zz} = \rho \partial_t v_z.
\] (3.21)

ii) Stress-strain relations:
\[
\partial_t \sigma_{xx} = c_{11} \partial_x v_x + c_{13} \partial_z v_z + \bar{\kappa} e_1 + 2c_{55} e_2 + S,
\] (3.22)
\[
\partial_t \sigma_{zz} = c_{13} \partial_x v_x + c_{33} \partial_z v_z + \bar{\kappa} e_1 - 2c_{55} e_2 + S,
\] (3.23)
\[
\partial_t \sigma_{xz} = c_{55}(\partial_x v_x + \partial_z v_z + e_3),
\] (3.24)
where \(S\) is the source term corresponding to an isotropic perturbation, such as an explosion, in the upper medium, \(e_1, e_2\) and \(e_3\) are memory variables and
\[
\bar{\kappa} = \bar{\varepsilon} - c_{55}, \quad \bar{\varepsilon} = \frac{1}{2}(c_{11} + c_{33}).
\] (3.25)

iii) Memory-variable equations:
\[
\partial_t e_1 = \frac{1}{\tau_\sigma^{(1)}} \left[ \left( \frac{\tau_\sigma^{(1)}}{\tau_\epsilon^{(1)}} - 1 \right) (\partial_x v_x + \partial_z v_z) - e_1 \right],
\] (3.26)
\[
\partial_t e_2 = \frac{1}{2\tau_\sigma^{(2)}} \left[ \left( \frac{\tau_\sigma^{(2)}}{\tau_\epsilon^{(2)}} - 1 \right) (\partial_x v_x - \partial_z v_z) - 2 e_2 \right],
\] (3.27)
3.4. NUMERICAL MODELING

\[
\frac{\partial_t e_3}{e_3} = \frac{1}{\tau^{(2)}_{\sigma}} \left( \left( \frac{\tau^{(2)}_{\sigma}}{\tau^{(2)}_\epsilon} - 1 \right) \left( \partial_z v_x + \partial_x v_z \right) - e_3 \right),
\]

(3.28)

where \( \tau^{(\nu)}_{\sigma} \) and \( \tau^{(\nu)}_\epsilon \) are relaxation times. The frequency-domain stress-strain relations are obtained from the preceding equations by applying the Fourier transform (see Appendix 3.A). The stress-strain relations satisfy the condition that the mean stress depends only on the dilatational relaxation function in any coordinate system, implying that the trace of the stress tensor should be invariant under coordinate transformations. Moreover, the deviatoric stresses solely depend on the shear relaxation function (Carcione, 2007).

3.4.2 Boundary conditions and modeling algorithm

The boundary conditions at a solid/solid interface require the continuity of the particle velocities \( v_x \) and \( v_z \) and normal stress components \( \sigma_{xz} \) and \( \sigma_{zz} \). Two grids model the sub-domains above and below the interface. The solution on each grid is obtained by using a Runge-Kutta method for the time stepping and the Fourier and Chebyshev differential operators to compute the spatial derivatives in the horizontal and vertical directions, respectively (Carcione, 2007)). In order to combine the two grids, the wave field is decomposed into incoming and outgoing wave modes at the interface. The inward propagating waves depend on the solution outside the subdomains and therefore are computed from the boundary conditions, while the behavior of the outward propagating waves is determined by the solution inside the subdomain (e.g., Sidler & Carcione, 2007). The approach, is adapted here for the anelastic transversely isotropic solid-solid case (axis of symmetry perpendicular to the interface), and involves the equations given in Appendix 3.B for updating the field variables at the grid points defining the interface.

3.4.3 The frequency-slowness method

In order to compute the numerical reflection coefficient as a function of the incidence angle, we use the technique developed by Kindelan et al. (1989) for elastic media and extended by Carcione & Helle (2004)) to the case of a viscoelastic seafloor overlain by an acoustic water layer. The method consists on the following steps, where the frequency is denoted by \( f = \omega/(2\pi) \):

1. Generate synthetic seismograms of the pressure field \( \sigma_{xx} + \sigma_{zz} \), placing a line of receivers at each grid point above the interface. These records contain the incident and reflected fields.

2. At the same location compute the synthetic seismogram without interface by setting the properties of the lower medium to those of the upper medium. These seismograms contain the incident field only.

3. Take the difference between the wave fields calculated in steps (1) and (2) to isolate the reflected wavefield.

4. Perform an \((f,s_x)\)-transform of the incident field to obtain \( \sigma_I(f,s_x) \).

5. Perform an \((f,s_x)\)-transform of the reflected field to obtain \( \sigma_R(f,s_x) \).
Table 3.1: Material properties for the examples.

<table>
<thead>
<tr>
<th>Layer</th>
<th>(v_P) (m/s)</th>
<th>(v_S) (m/s)</th>
<th>(\rho) (kg/m(^3))</th>
<th>(Q_1)</th>
<th>(Q_2)</th>
<th>(\delta^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1500</td>
<td>0.1</td>
<td>1040</td>
<td>10000</td>
<td>10000</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>4323</td>
<td>1449</td>
<td>2760</td>
<td>40</td>
<td>100</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Layer</th>
<th>(v_P) (m/s)</th>
<th>(v_S) (m/s)</th>
<th>(\rho) (kg/m(^3))</th>
<th>(Q_1)</th>
<th>(Q_2)</th>
<th>(\delta^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2500</td>
<td>1000</td>
<td>2100</td>
<td>25</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>5000</td>
<td>3000</td>
<td>2200</td>
<td>40</td>
<td>20</td>
<td>0.1</td>
</tr>
</tbody>
</table>

6. The ratio \(|\sigma_R(f, s_x)|/|\sigma_I(f, s_x)|\) corresponds to the absolute value of the reflection coefficient, while its phase angle is calculated from the fraction of the reflected and the incident field (angle[\(\sigma_R(f, s_x)/\sigma_I(f, s_x)\)]). Transform \(s_x\) to incidence angle \(\theta\) using \(\sin \theta = \frac{v_P}{v_{s_x}}\), where \(v_P\) is the P-wave velocity of the upper medium.

As we locate the receivers close to the interface we do not need to correct for the amplitude difference between the incident and the reflected waves.

### 3.5 Solving the ambiguities in the evaluation of the reflection coefficient

If anelasticity is involved, the analytic expressions for the plane-wave reflection and transmission coefficients contain a number of square roots of complex numbers due to the vertical slownesses (see equation 3.9). The square root is the inverse of the square of a function of the form \(\sqrt{\lambda^2} = \lambda\). For instance, if \(\lambda = -\alpha i\), where \(\alpha\) is real, \(\sqrt{(-\alpha i)^2}\) can also be written as \(\sqrt{(-1)^2(\alpha i)^2}\) so that \(\alpha i\) as well as \(-\alpha i\) are valid solutions.

Therefore, the analytical evaluation of reflection coefficients does not result in a unique solution, but due to the nested structure of the complex square-root expressions, gives rise to many mathematically correct, but physically unreasonable solutions. Figure 3.1, shows the set of all possible solutions for the fluid-solid example given in Table 3.1. Even if some of these solutions can be discarded based on physical considerations, the remaining subset is still vast. Only one of these solutions corresponds to the physical one. One way of identifying the physical solution is to use numerical modeling. This solution is represented by open circles.

The inherently high computational cost of obtaining numerical solutions does, however, make this approach rather unattractive for many practical applications. One would therefore rather find ways to identify the correct signs in the corresponding analytical expressions. The standard sign convention in equations 3.7 and 3.9 is generally valid for incident angles smaller than the EEC angle, but may fail for larger angles. This is illustrated in Figure 3.2, which compares the analytical solution based on the standard sign convention to the corresponding numerical solution. We see that the two solutions agree well for incidence angles smaller than 53°, whereas for larger angles the analytical solution is characterized by a seemingly non-physical discontinuity.
Figure 3.1: Set of possible solutions for the absolute value (a) and phase angle (b) of the reflection coefficient corresponding to the fluid-solid model given in Table 3.1 (solid lines). The numerical solution is represented by open circles.
Figure 3.2: Analytical (dashed and solid lines) and numerical (symbols) solutions for the absolute value of the reflection coefficient for the fluid-solid case given in Table 3.1. The symbols denote numerical results for different frequencies. The dashed line represents the solution corresponding to the standard sign convention, whereas the solid line corresponds to the solution constrained by enforcing continuity of the vertical slowness by following the Riemann surface of the square root.
In complex analysis, elementary real functions, such as exponentials, square roots, logarithms and trigonometric functions, are expanded into the complex domain and conditions are specified so that the complex functions maintain certain properties of their real-valued counterparts. A particularly common and desirable property of many functions used to describe a physical phenomena is their differentiability (Ablowitz & Fokas, 2003). To obtain differentiability on an open subset in the complex domain, the so-called Riemann sheets are defined for which the function is differentiable (Riemann, 1857). Such a function is called analytic and it can be shown that it is “infinitely differentiable”. Compared to other complex functions, the complex square root has a relatively simple Riemann surface, which is, however, inherently 4-D in nature and therefore needs two 3-D plots to be visualized (Fig. 3.3).

Given that the slowness vectors and polarizations of the incident, reflected and transmitted waves are expected to change continuously as a function of the incidence angle, it is reasonable to impose differentiability. This can be achieved by following the Riemann surface when evaluating the corresponding square roots, which assures continuity. In this context, it is important to note that a linear equation system, such as equation 3.12, yields continuous results if composed of continuous functions. This in turn ensures that non-physical discontinuities, such as those obtained by using the standard sign convention, can be avoided. Closer inspections shows that the abrupt step in Figure 3.2 arises if the square root at an incident angle larger than about 53° is evaluated in the third quadrant of the complex plane, where the standard sign convention indicates the use of the positive sign. Figure 3.3 illustrates that this interrupts the continuity of the vertical slowness, as its path does not follow the corresponding Riemann surface. Conversely, we see that the path of the vertical slowness resulting in an analytical solution that is in agreement with the numerical solution does indeed follow the Riemann surface in a continuous manner.

An additional problem may arise in the calculation of the vertical slowness of the transmitted P-wave, if the attenuation of the incident wave is higher than that of the transmitted wave. In such cases, the attenuation vector may point upwards in the vicinity of the EEC angle (Cooper, 1967; Richards, 1984; Ruud, 2006) and the path of the square-root expression to calculate the vertical slowness follows the Riemann surface anti-clockwise implying that in the second quadrant the positive solution has to be chosen. This is at odds with the radiation condition which postulates the choice of the negative solution for the vertical slowness. For post-critical angles of incidence, apart from the immediate vicinity of the EEC angle, the radiation condition is certainly a physically reasonable constraint, which, however, is not honored by simply ensuring that the path of the vertical slowness follows the Riemann surface. This in turn leads to important phase discrepancies as well as differences in the absolute value of the reflection coefficient compared to the corresponding numerical solution (Figures 3.4a and 3.4b).

Figure 3.5 schematically illustrates the difference between a problematic case and an unproblematic case for the calculation of the vertical slowness of the transmitted P-wave, regarding the behavior of the argument of the square root. The difference resides in the location of the intersection between the argument and the real axis. If this intersection is located on the negative real axis we have an unproblematic case. Conversely, if this intersection is located in the positive real axis, we have a problematic case and simply following the Riemann surface leads to substantial errors at large incidence angles. A possible way to address and alleviate this problem is to choose the negative solution for the
Figure 3.3: Riemann surface for a complex function of the form $\sqrt{\lambda}$. The 4-D surface is displayed in two 3-D plots, for the real (a) and imaginary (b) parts of the solution. The plots show the path corresponding to square-root expressions of the vertical P-wave slowness for the fluid-solid case given in Table 3.1. The red line denotes the path corresponding to the standard sign convention, whereas the green line denotes the path corresponding to the continuous solution following the Riemann surface. The slowness values are normalized for display purposes.
3.5. AMBIGUITIES IN THE REFLECTION COEFFICIENT

Figure 3.4: Absolute value (a) and phase angle (b) of the reflection coefficient for the isotropic solid-solid case given in Table 3.1 ($\delta^* = 0$). The solution corresponding to the standard sign convention (dashed line) exhibits two problems: abrupt steps in the reflection coefficient due to discontinuous root expressions and a wrong phase due to a “pathological” behavior of the attenuation vector. The solid line corresponds to the correct analytical solution and the symbols to numerical solutions for different frequencies ranging between 34 and 36 Hz.
Figure 3.5: Complex-valued argument of the square root to calculate the vertical slowness of the transmitted P-wave. It is shown how to distinguish a problematic case (dashed line) from an unproblematic (solid line) case by following the Riemann surface to obtain physically meaningful results. In the unproblematic case, where the Riemann surface has to be followed, the argument intersects the negative real axis. Conversely, in the problematic case the intersection occurs along the positive real axis. In this case, continuous solutions follow the positive Riemann sheet after the EEC angle (see Figure 3.3). On this Riemann sheet the imaginary part is positive thus corresponding to a reversed attenuation vector, which is, however, not likely to be the case outside the immediate vicinity of the EEC angle. Here, the Riemann surface should not be followed and the negative solution in the second quadrant should be chosen.

vertical slowness in the second quadrant of the complex plane irrespective of the path of the square root expressions in the Riemann surface. This approach is admittedly pragmatic and heuristic, but also highly effective for practical purposes and rather general in nature. As illustrated in Figure 3.4, it generates a solution that is consistent with the numerical one.

The isotropic solid-solid model corresponding to Figure 3.4 (Table 3.1) is taken from (Krebes & Daley, 2007). This example is of particular interest as it suffers from both problems discussed in this paper. The absolute value of the reflection coefficient and its phase exhibit considerable discontinuities at an incident angle of approximately 63°, which can be avoided by evaluating the root expressions following the Riemann surface. Doing so, does however, still result in a phase angle that is entirely inconsistent with that inferred by numerical modeling at angles larger than the EEC angle (of about 30° incidence angle). This is due to the fact that the attenuation is higher in the incident medium and therefore the argument of the square root corresponding to the transmitted P-wave intersects the positive real axis, which makes this case one of the exceptions discussed before.

Figure 3.6 shows the path of the square-root expressions of the vertical slowness of the transmitted S-wave corresponding to the isotropic solid-solid case. This slowness suffers from discontinuities when the standard sign convention is used and the Riemann-
surface criterion is not followed. The discontinuity is also visible at an incidence angle of approximately 63° in the corresponding absolute value of the reflection coefficient and it is even more pronounced in the phase angle (Figure 3.4). Figure 3.7, addressing the problematic case, shows the path of the square root expression of the vertical slowness of the transmitted P-wave. The red line corresponds to the path following the Riemann surface, whereas the green line shows the choice of the negative solution in the second quadrant. The latter solution corresponds to the heuristic approach proposed above and leads to a result that is not entirely correct in the vicinity of the EEC angle, but it is otherwise consistent with the numerical solution.

The last example has the same parameters as the first solid-solid case albeit with anisotropy added (Table 3.1). It is interesting and important to note that this rather minor amount of anisotropy leads to additional and rather dramatic complications with regard to the previously considered isotropic case (Figure 3.4). The result obtained by using the standard sign convention again yields seemingly non-physical effects beyond the EEC angle, whereas following the Riemann-surface criterion and choosing the negative solution for the vertical slowness of the transmitted P-wave, as indicated in Figure 3.5, leads to reasonable changes in comparison to the isotropic case and an analytical solution that is consistent with its numerical counterpart (Figure 3.8).

It is important to note that this approach is essentially an alternative formulation of the recipe proposed by Krebes & Daley (2007), which is based on the enforcement of continuity of the vertical slowness in the pre-critical range and honoring the radiation condition in the post-critical range. While the two ways of arriving at this solution are evidently equivalent, and the formulation by Krebes & Daley (2007) favors physical intuition, we believe that our approach will prove to be quite suitable from an algorithmic point of view.
Figure 3.6: Real (a) and imaginary (b) parts of the square root involved in the vertical S-wave slowness of the lower half-space for the isotropic solid-solid case given in Table 3.1 ($\delta^* = 0$). The green line corresponds to the continuous solution obtained by following the Riemann surface and the red line to the solution resulting from the standard sign convention. Note the abrupt step in the path corresponding to the standard convention that is also present in the reflection coefficient and phase angle shown in Figure 3.4. The slowness values are normalized for display purposes.
3.5. AMBIGUITIES IN THE REFLECTION COEFFICIENT

Figure 3.7: Real (a) and imaginary (b) parts of the square root involved in the vertical P-wave slowness of the lower half-space for the isotropic solid-solid case given in Table 3.1. The red line corresponds to a continuous sign choice, which is problematic in this case (see also Figure 3.4 and 3.5) and leads to a wrong phase angle beyond the EEC angle. A simple but effective solution is to choose the negative sign for the square-root expression of vertical P-wave slowness of the lower solid in the second quadrant (green line). The slowness values are normalized for display purposes.
Figure 3.8: Absolute value (a) and phase angle (b) of the reflection coefficient for the anisotropic solid-solid case given in Table 3.1. The solid line corresponds to the physically correct solution whereas the dashed line is the solution obtained when choosing the standard sign convention for all the square roots. The symbols denote numerical solutions for frequencies ranging between 34 and 36 Hz. Note that even though the amount of anisotropy present is rather weak, the solution using the standard sign convention (dashed line) shows large differences compared to the corresponding isotropic solution shown in Figure 3.4.
3.6 Conclusions

We have used an accurate numerical technique, based on a frequency-slowness approach, to evaluate plane-wave reflection coefficients in anelastic anisotropic layered media to systematically explore the inherent ambiguities associated with the corresponding analytical solutions. Our results indicate that the continuity criterion based on continuous paths along the Riemann surfaces of the square-root expressions associated with the complex slownesses provide a convenient solution to resolve the ambiguities in the calculation of the reflection coefficients. However, there exist some cases for which a continuous path does not exist on the corresponding Riemann surface. These cases can be identified by the intersection of the argument of the square root with the positive real axis. Our results demonstrate that in these problematic, but well defined, cases an appropriate change of direction on the Riemann surface provides physically correct solutions for all practical applications beyond the vicinity of the EEC angle. The approach developed here is essentially equivalent to enforcing continuity of the vertical slowness in the pre-critical range and honoring the radiation condition in the post-critical range.

3.7 Acknowledgments

The authors thank Stewart Greenhalgh, editor Johan Robertsson and two anonymous reviewers for their constructive comments and suggestions that improved the paper. This project was funded by the European Commission’s Human Resources and Mobility Program. Marie Curie Research Training Network SPICE Contract MRTN-CT-2003-504267.

Appendix

3.8 Frequency-domain stress-strain relation

Transforming the memory-variable equations 3.26, 3.27 and 3.28 to the (frequency) \( \omega \)-domain (e.g., \( \partial_t e_1 \rightarrow i\omega e_1 \)), and substituting the memory variables into equations 3.22, 3.23 and 3.24, we obtain the stress-strain relation:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{zz} \\
\sigma_{xz}
\end{bmatrix} = 
\begin{bmatrix}
 p_{11} & p_{13} & 0 \\
p_{13} & p_{33} & 0 \\
0 & 0 & p_{55}
\end{bmatrix}
\begin{bmatrix}
\partial_x v_x \\
\partial_z v_z \\
\partial_z v_x + \partial_x v_x
\end{bmatrix},
\]

(3.29)

where

\[
\begin{align*}
p_{11} &= c_{11} - \tilde{E} + \tilde{K}M_1 + c_{55}M_2 \\
p_{33} &= c_{33} - \tilde{E} + \tilde{K}M_1 + c_{55}M_2 \\
p_{13} &= c_{13} - \tilde{E} + \tilde{K}M_1 + c_{55}(2 - M_2) \\
p_{55} &= c_{55}M_2
\end{align*}
\]

(3.30)

are the complex stiffnesses, and

\[
M_\nu = \frac{M_\nu^{(\nu)}}{\tau_\nu^{(\nu)}} \left( 1 + i\omega \tau_\nu^{(\nu)} \right)^{-1}, \quad \nu = 1, 2
\]

(3.31)
are the Zener complex moduli ((Zener, 1948); (Carcione, 2007). Note that for $\omega \rightarrow \infty$ we have $p_{1J} \rightarrow c_{1J}$.

The relaxation times can be expressed as

$$\tau_{e}^{(\nu)} = \frac{\tau_{0}}{Q_{\nu}} \left( \sqrt{Q_{\nu}^2 + 1} + 1 \right) \quad \text{and} \quad \tau_{\sigma}^{(\nu)} = \frac{\tau_{0}}{Q_{\nu}} \left( \sqrt{Q_{\nu}^2 + 1} - 1 \right), \quad (3.32)$$

where $\tau_{0}$ is a relaxation time such that $1/\tau_{0}$ is the center frequency of the relaxation peak and $Q_{\nu}$ are the minimum quality factors.

### 3.B Boundary equations

The upper solid is denoted by the subscript 1 and the lower solid by the subscript 2. The symbol $P$ indicates the compressional wave in the fluid or the q$P$ wave in solid, and $S$ denotes the q$S$ wave in this medium. The boundary equations at the solid/solid interface for the transversely-isotropic case (the interface perpendicular to the symmetry axis) are generalizations of the equations given in Tessmer et al. (1992) for the isotropic case:

$$v_{x}^{(\text{new})}(1) = [Z_{S}(1) + Z_{S}(2)]^{-1} [Z_{S}(2) v_{x}^{(\text{old})}(2) + Z_{S}(1) v_{x}^{(\text{old})}(1) + \frac{\sigma_{xx}^{(\text{old})}(1) - \sigma_{xx}^{(\text{old})}(2)}{Z_{S}(1) + Z_{S}(2)}]$$

$$\sigma_{xx}^{(\text{new})}(1) = \frac{Z_{S}(1) Z_{S}(2)}{Z_{S}(1) + Z_{S}(2)} v_{x}^{(\text{old})}(1) - v_{x}^{(\text{old})}(2) + \frac{\sigma_{xx}^{(\text{old})}(1)}{Z_{S}(1)} + \frac{\sigma_{xx}^{(\text{old})}(2)}{Z_{S}(2)}]$$

$$e_{1}^{(\text{new})}(1) = e_{1}^{(\text{old})}(1) + \frac{[\phi_{1}(1)/c_{33}(1)] [\sigma_{zz}^{(\text{new})}(1) - \sigma_{zz}^{(\text{old})}(1)]}{\sigma_{xx}^{(\text{new})}(1)}$$

$$e_{2}^{(\text{new})}(1) = e_{2}^{(\text{old})}(1) - \frac{[\phi_{2}(1)/c_{33}(1)] [\sigma_{zz}^{(\text{new})}(1) - \sigma_{zz}^{(\text{old})}(1)]}{\sigma_{xx}^{(\text{new})}(1)}$$

$$e_{3}^{(\text{new})}(1) = e_{3}^{(\text{old})}(1) + \frac{[\phi_{2}(1)/c_{55}(1)] [\sigma_{xx}^{(\text{new})}(1) - \sigma_{xx}^{(\text{old})}(1)]}{\sigma_{zz}^{(\text{new})}(1)}$$

$$v_{x}^{(\text{new})}(2) = v_{x}^{(\text{new})}(1)$$

$$\sigma_{xx}^{(\text{new})}(2) = \sigma_{xx}^{(\text{old})}(2) + [c_{13}(2)/c_{33}(1)] [\sigma_{zz}^{(\text{new})}(2) - \sigma_{zz}^{(\text{old})}(2)]$$

$$e_{1}^{(\text{new})}(2) = e_{1}^{(\text{old})}(2) + \frac{[\phi_{1}(2)/c_{33}(2)] [\sigma_{zz}^{(\text{new})}(2) - \sigma_{zz}^{(\text{old})}(2)]}{\sigma_{xx}^{(\text{new})}(2)}$$

$$e_{2}^{(\text{new})}(2) = e_{2}^{(\text{old})}(2) - \frac{[\phi_{2}(2)/c_{33}(2)] [\sigma_{zz}^{(\text{new})}(2) - \sigma_{zz}^{(\text{old})}(2)]}{\sigma_{xx}^{(\text{new})}(2)}$$

$$e_{3}^{(\text{new})}(2) = e_{3}^{(\text{old})}(2) + \frac{[\phi_{2}(2)/c_{55}(2)] [\sigma_{xx}^{(\text{new})}(2) - \sigma_{xx}^{(\text{old})}(2)]}{\sigma_{zz}^{(\text{new})}(2)}$$

(3.33)

where $\phi_{\nu} = 1/\tau_{e}^{(\nu)} - 1/\tau_{\sigma}^{(\nu)}$, $Z_{P} = \sqrt{\rho c_{33}}$ and $Z_{S} = \sqrt{\rho c_{55}}$.

The lower boundary of subdomain 2 (lower medium) satisfies the non-reflecting con-
ditions:

\[
v_x^{(\text{new})} = \frac{1}{2} \left( v_x^{(\text{old})} + \sigma_{xz}^{(\text{old})} \right) / Z_S,
\]

\[
v_z^{(\text{new})} = \frac{1}{2} \left( v_z^{(\text{old})} + \sigma_{zz}^{(\text{old})} \right) / Z_P,
\]

\[
\sigma_{xx}^{(\text{new})} = \sigma_{xx}^{(\text{old})} - \left( c_{13}/2c_{33} \right) \left( \sigma_{zz}^{(\text{old})} - Z_P v_z^{(\text{old})} \right),
\]

\[
\sigma_{zz}^{(\text{new})} = \frac{1}{2} \left( \sigma_{zz}^{(\text{old})} + Z_P v_z^{(\text{old})} \right),
\]

\[
\sigma_{xz}^{(\text{new})} = \frac{1}{2} \left( \sigma_{xz}^{(\text{old})} + Z_S v_x^{(\text{old})} \right),
\]

\[
e_1^{(\text{new})} = e_1^{(\text{old})} - \left[ \phi_1/(2c_{33}) \right] \left( \sigma_{zz}^{(\text{old})} - Z_P v_z^{(\text{old})} \right),
\]

\[
e_2^{(\text{new})} = e_2^{(\text{old})} + \left[ \phi_2/(2c_{33}) \right] \left( \sigma_{zz}^{(\text{old})} - Z_P v_z^{(\text{old})} \right),
\]

\[
e_3^{(\text{new})} = e_3^{(\text{old})} - \left[ \phi_2/(2c_{55}) \right] \left( \sigma_{xz}^{(\text{old})} - Z_S v_x^{(\text{old})} \right),
\]

where the index 2 has been omitted for brevity. The upper boundary of the upper medium also satisfies non-reflecting conditions. To obtain the equations for this boundary, the method requires the following substitutions: \( z \rightarrow -z \), which implies \( v_z \rightarrow -v_z \), \( \sigma_{xz} \rightarrow -\sigma_{xz} \), and \( e_3 \rightarrow -e_3 \).

In addition to the non-reflecting conditions, absorbing strips are used to further attenuate the wave field at non-physical boundaries (Carcione et al., 2002).
Chapter 4

On estimating ocean bottom seismic anisotropy and its effect on reflectivity

Rolf Sidler and Klaus Holliger
in preparation for submission to *Geophysics*

4.1 Abstract

We present a transversely isotropic ocean bottom model derived using equivalent media theory from an isotropic stochastic layer sequence of sand and clay layers, and then calculate the corresponding reflection coefficients (amplitude and phase). Sand-clay layer sequences are often encountered in marine sedimentation environments. The wave speed gradients for the sand and clay end-members used in our model were taken from the literature. The introduction of a stochastic distribution of the layer thickness offers the possibility to estimate a standard deviation of the resolved values as well as the fraction of a wavelength suitable for equivalent media theory. To evaluate the reflection coefficient of the transversely isotropic, attenuating medium with velocity gradients, we used the frequency-slowness technique on wavefields calculated with a pseudo-spectral modeling code. Comparison with plane-wave reflection coefficients calculated for an isotropic, elastic interface shows that the differences are most pronounced in the vicinity of the critical angle which controls S-wave velocity estimates in homogeneous, isotropic and elastic models. Intrinsic anisotropy in clay layers in the sand-clay layer sequence may lead to an ocean bottom that strongly absorbs the incident wavefield.

4.2 Introduction

Shear wave recording and processing in petroleum search has opened up new possibilities for subsurface characterization. These arise from smaller velocity contrasts, low sensitivity to porosity and the low velocity characteristics which means smaller wavelengths. These greater resolution is especially useful in the exploration of the near-surface (Dasios et al., 1999) and to resolve gas bearing sediments (Stewart et al., 2002). Even though
measurements using ocean bottom seismometers have been carried out in academia for several decades (Schneider & Backus, 1964) the recording of so called 4C seismic in industry has only been recently practised (Rigsby et al., 1987; Caldwell, 1999). Four-component (4C) recording involves measurements in water (mostly marine) that record besides the conventional pressure, also the particle motion vector (the vertical and the horizontal components).

The zero rigidity of water makes it impossible for shear waves to travel through sea water and 4C measurements therefore have to be accomplished directly on the water-solid interface, which is typically the ocean bottom. This makes the measurements more expensive than conventional recordings. Additional complexity comes from the fact that due to the different experimental geometry not only waves from below but also from above (water reverberations) are recorded. Automated wavefield decomposition algorithms have been developed to overcome this problem (Muijs et al., 2004), but the assumption of perfectly elastic and isotropic behavior does not fully account for the problem as the method tends to fail in particular conditions (Muijs, 2005). It is believed that the failure is due to an attenuating and anisotropic ocean bottom. The algorithm for the wavefield decomposition is based on estimates of wave speed and density parameters of the ocean bottom, which are obtained by inverting the reflection coefficient of the ocean bottom. As the reflection coefficient of attenuating and anisotropic ocean bottoms is thought to be substantially different to that of an elastic and isotropic equivalent (Carcione & Helle, 2004; Sidler & Carcione, 2007), estimates of wave speed and density will be wrong in such an environment.

In this paper we evaluate the actual amount of anisotropy that may be expected from a sedimentary ocean bottom and model its effects on the reflection coefficient. While there exists a significant literature on measurements of shear and compressional wavespeed as well as density (Hamilton, 1976, 1979, 1980; Bryan & Stoll, 1988; Bowles, 1997; Buckingham, 2005) and quite some information about attenuation (Hamilton, 1972; Stoll, 1989) measurements of ocean bottom anisotropy are rather rare (Berge et al., 1991; Bachman, 1983; Carlson et al., 1984). To estimate anisotropy, we specify wave speed end-member models for sediments based on the literature and then calculate the anisotropy due to layering of such an end-member sediment sequence.

### 4.3 Velocity end-member models

In this section we derive the velocity end-member models for sediments that we can later use as components for anisotropy estimation and reflection coefficient modeling. In defining the end-member models we follow mainly the suggestion of Bowles (1997) to describe sediments according to the standard sedimentary nomenclature as defined for example by Shepard (1954). We define sand, silt and clay as end-members as they are the dominant individual components.

The next step is to specify shear and compressional wavespeeds as well as density and attenuation for each end-member model. Bowles (1997) presents a compilation of these ocean bottom parameters from various authors. He cautions about the reliability of P- and S-wave attenuation and S-wave velocity models since the measuring methods and sediment description are not standardized. This makes it difficult to compare results from
different regions and investigators. This however does not much affect our choice for the end-member models, as we are interested more in magnitude and range of most probable values than exact specification for a defined medium.

Sand is known to have high P-wave speeds as well as high S-wave speeds. So we choose the equation for P-wave velocity from Hamilton (1979) and the equation for S-wave velocity from Hamilton (1987). The equations are also used by Buckingham (2005) for his comparison with theoretical geoacoustical models. As his model is based on an unconsolidated granular medium with the grains in contact but unbounded, it represents well our sandy end-member. Also the wave speeds are on the rather high side for unconsolidated sediments.

The end-member in the opposite direction is clay, which consists of fine-grained platy minerals glued together with complex ionic interactions. The P- and S-wave wavespeeds are low in such a material. Hamilton (1980) gives a velocity-depth function for terrigenous sediments in what he specifies as silt clays, turbidites and mudstone shales. This is not exactly clay, but somewhere between silt and clay. The P-wave wave speeds seem not to differ substantially between these two sediments. We therefore take this equation to define the P-wave wave speeds for the end-member clay. As we are concerned more with magnitude than exact numbers we omit the quadratic and cubic parts of the regression equation. The linear equation agrees also with laboratory measurements from Mondol et al. (2007).

The S-wave velocity for the clay end-member is taken from Lovell & Odgen (1984). Bowles (1997) claims the shear wave velocity to correlate well with grain size and designates two types of shear wave velocity-depth profile: the high-velocity group and the low-velocity group. It is convenient for our purposes to specify as simple as possible the end-member models. While the S-wave velocity profile we have chosen for sand belongs to the high velocity group, we choose a characteristic low-velocity profile for clay.

Finding acceptable attenuation-depth functions is problematic. Hamilton (1976) gives a range of puzzling equations and some rather lengthy explanations for what can be condensed to the statement that the subject is poorly known. At least he indicates that wave attenuation is influenced by porosity and therefore use of different attenuation functions for high-porosity sand and low-porosity silt and clay may be justified. Buckingham (2005), who restricts his investigations to sandy sediments, argues that the equation of Hamilton (1976) for sand does not fit the data. He provides a theoretical equation of his own, which does not fit the data any better. Bowles (1997) makes no concrete statement on the sandy sediment attenuation functions but claims the functions for silt-clay profiles from Hamilton (1976) show excessive attenuation. In return he presents a variety of regression equations from various authors, of which none represents the provided data even closely. For the uninitiated observer it seems that attenuation is not correlated at all to depth for silt-clay profiles.

Our numerical modeling experiments with attenuation profiles established from the literature (Hamilton, 1972, 1976; Spencer, 1981; Stoll, 1989) have shown that its influence on reflectivity in our sand-clay sequence ocean bottom model is rather low. To illustrate the influence of attenuation we present an example with very pronounced attenuation in the modeling section.

Our end-member models are described in tabular form in Table 4.1 and the corresponding values as a function of depth are shown in Fig. 4.1.
Table 4.1: Functions of ocean bottom parameters for end-member models.

<table>
<thead>
<tr>
<th></th>
<th>Sand</th>
<th>Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_P$</td>
<td>$V_P(z) = 1800 \frac{1}{s} * z^{0.015}$</td>
<td>$V_P(z) = 1511 \frac{1}{s} + 1.304 \frac{1}{s} * z$</td>
</tr>
<tr>
<td></td>
<td>Buckingham (2005); Hamilton (1979)</td>
<td>Hamilton (1980)</td>
</tr>
<tr>
<td>$V_S$</td>
<td>$V_S(z) = 128 \frac{1}{s} * z^{0.28}$</td>
<td>$V_S(z) = 20.5 \frac{1}{s} * z^{0.4406}$</td>
</tr>
</tbody>
</table>

Figure 4.1: Wave speeds of the end-member models sand (red) and clay (green) plotted against depth. The solid line corresponds to P- and the dashed line to the S-wave.
4.4 Estimating anisotropy

Even if there is evidence for anisotropy in soft ocean bottom sediments (Bachman, 1979, 1983; Odom et al., 1996; Berge et al., 1991) only little is known of its extent and the reason for the anisotropy. Several possible causes for anisotropy have been proposed in other geological circumstances, including fracturing (Thomsen, 1995; Lynn, 2004), stress induction (Mavko et al., 1998), intrinsic anisotropy (Vega-Ruiz, 2003) or fine layering (Postma, 1955; Carlson et al., 1984). Here we try to estimate anisotropy on the seafloor by considering alternating layers with velocity gradients and a stochastic thickness sequence.

4.4.1 Sand-Clay Sequence

In sedimentary deposition environments the type of deposited sediment usually changes with time. This can be due to climatic changes in a great variety of time scales as well as topographical changes due to the sedimentation process. This results in layering which is commonly observed in sediments (Seibold & Berger, 1996). In marine deposit environments sand-clay sequences are often observed due to a variety of different sedimentation processes. Terwindt et al. (1968) describes sand-clay laminae in estuaries and tidal inlets with layer thicknesses between 1 and 30 millimeters. Fjord sedimentation is described by Hiemstra et al. (2004) to depend on season conditioned glacier deposit accumulation which results in a sand-silt-clay lamination with layer thicknesses between 1 and 20 millimeters. Sand-clay layering is also described in deltaic environments (Abam, 2004; Rotondo, 2004).

Analysis of borehole logs have shown that in addition to a smooth trend also residual velocity fluctuations can be observed that can be described by autocovariance functions corresponding to bandlimited self-affine stochastic processes with exponential decaying probability density functions (Holliger, 1996). In our model we consider this property by generating a stack of layers with the above mentioned stochastic thickness distribution and assign the layers elastic properties that alternate between sand and clay. The elastic properties are derived from the end-member models at the corresponding depth. We expect that the elastic properties inside a single layer to be homogeneous at the scale of the seismic wavelengths (typically $> 10$ m) that are significantly larger than a single layer (typically $\approx 1$ cm).

To calculate the stochastic layer sequence we use a second-order stationary realization of a random variable with given mean and auto-covariance function, where we use the band-limited "fractal" von Kármán function (von Kármán, 1948)

$$C(r) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(r/a)^\nu K_\nu(r/a). \quad (4.1)$$

Here $\Gamma$ is the gamma function, $K_\nu$ is the modified Bessel function of the second kind of order $0 \leq \nu \leq 1$, $r$ is the lag, and $\sigma$ is the variance. The media described by such correlation functions are self-affine for $0 \leq \nu \leq 0.5$ and self-similar for $0.5 \leq \nu \leq 1$, at distances considerably shorter than the correlation length $a$ (Klimeš, 2002; Carcione et al., 2003).

A straightforward and efficient way to perform such realizations is the spectral method (Christakos, 1992). The key point is the Wiener-Khintchine theorem (Wiener, 1949;
Khinchin, 1949) that connects the spectral density function with the autocorrelation of a variable. The stochastic variable $S(x)$ is calculated as

$$S(x) = IFFT[A(f)e^{i2\pi\varphi(f)}], \quad (4.2)$$

where $A(f)$ is the amplitude spectrum, $\varphi(f)$ a uniformly distributed random number between 0 and 1 and $f$ is frequency. The amplitude spectrum is the square root of the energy spectral density function which corresponds according to the Wiener-Khinchine theorem to the Fourier transform of the auto-covariance function. The Fourier transform of the von Kármán auto-covariance function (eq. 4.1) for $E$-dimensional space is given by (Goff & Jordan, 1988):

$$P_{hh}(\vec{k}) = \sigma^2 h (2\sqrt{\pi a})^E \Gamma(\nu + E/2) \Gamma(\nu)(1 + \vec{k}^2 a^2)^{\nu + E/2}, \quad (4.3)$$

For the one-dimensional case ($E = 1$) this yields:

$$P_{hh-1D}(\vec{k}) = \frac{2\sigma^2 \sqrt{\pi a} \Gamma\left(\frac{1}{2} + \nu\right)}{\Gamma(\nu)(1 + a^2 k^2)^{\nu + E/2}} \quad (4.4)$$

For our simple seafloor model we generate a stochastic variable for the layer thickness. So $S(x)$ is the layer thickness and $x$ is the layer number in the sequence. To achieve the desired property of self-similarity we calculate the variable for approximately thirty times more layers than we actually use for our model. The variable is calculated for a zero mean, normal distribution and variance of one. To adapt it to the layer thickness we transform the variable to a mean of 0.01 m (which corresponds to a layer thickness of ten millimeters) and expected logarithmic distribution.

We then start at an arbitrary point of the variable and use the following layers until we have a stack of 100 m thickness. For the correlation length we use the size of our sediment pile stack which is 100 m and $\nu = 0.1$ which is commonly observed in nature (References). We then assign the layers the given values, alternating between the elastic properties of the sand and clay end-member model for the corresponding depth in which the layer is situated.

### 4.4.2 Anisotropic equivalents

General velocity anisotropy is rather complex and can be described by a symmetrical $6 \times 6$ elasticity matrix

$$C = \begin{pmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\
c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\
c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\
c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66}
\end{pmatrix} \quad (4.5)$$

with at most twenty one independent elastic constants. Most geological models do not make use of such a great diversity of directional velocity differences. Therefore it is possible to assume symmetries to reduce the number of independent elastic constants. Popular symmetries to describe geological models are called monoclinic, orthorhombic and...
hexagonal. Orthorhombic symmetry has nine independent elastic constants and can represent a geological model characterized by two orthogonal sets of cracks or fine layering and one set of cracks that are perpendicular to each other. Monoclinic symmetry entails thirteen independent elastic constants and can represent the same geological model but with arbitrary dip. Hexagonal symmetry has a single axis of symmetry in arbitrary direction with isotropic behavior in the plane normal to this axis and is therefore often referred to as transverse isotropy (TI) in connection with seismic anisotropy. A special case of transverse isotropy is the vertical transverse isotropy (VTI) for which the symmetry axis corresponds to the vertical axis. In this case there are only 5 independent elastic constants. VTI can represent a horizontally layered medium (Carcione, 2001). It is very popular in modeling not only for the simpler mathematical handling, but also for a feasible comparison to measurements. It is mostly not possible to measure the elastic constants in place and even in laboratory environment it is difficult to measure elastic symmetries weaker than transverse isotropy. For vertically transverse isotropic media the elasticity matrix reduces to

\[
\mathbf{C}_{\text{VTI}} = \begin{pmatrix}
  c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
  c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\
  c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\
  0 & 0 & 0 & c_{55} & 0 & 0 \\
  0 & 0 & 0 & 0 & c_{55} & 0 \\
  0 & 0 & 0 & 0 & 0 & c_{66}
\end{pmatrix}, 
\]

\[2c_{66} = c_{11} - c_{12}. \tag{4.6}\]

As it is intuitively difficult to get an impression about the inherent anisotropy of a medium by examining the components of the elasticity matrix it is common to use Thomsen notation to describe transversely isotropic media. The idea is to separate the anisotropic influence on the isotropic P- and S-wave quantities. To achieve this Thomson (1986) uses the three dimensionless parameters

\[
\epsilon = \frac{c_{11} - c_{33}}{2c_{33}}, \tag{4.7}
\]

\[
\delta^* = \frac{1}{2c_{33}^2} \left[ 2(c_{13} + c_{55})^2 - (c_{33} - c_{55})(c_{11} + c_{33} - 2c_{55}) \right], \tag{4.8}
\]

\[
\gamma = \frac{c_{66} - c_{55}}{2c_{55}}, \tag{4.9}
\]

which go to zero for isotropic media. The parameter \(\epsilon\) is an indication of the difference between horizontal and vertical P-wave velocity. \(\gamma\) is the same for the SH-wave and \(\delta^*\) which is the second derivative of the P-wave phase-velocity function at vertical incidence describes the anisotropic behavior near vertical incidence angles which are often used in exploration seismics (Tsvankin, 2005). Thomson (1986) uses also an approximation for the parameter \(\delta^*\) which he calls \(\delta\).

It is well known that a stack of finely layered isotropic units can be replaced by an equivalent single transversely isotropic layer, for waves much longer than the equivalent layer thickness (Postma, 1955). The end-member models derived above have velocity gradients that are especially prominent at shallow depths. Also with increasing wave speed the wavelength increases with depth. This is important because the long wave equivalent approximation is valid only for waves substantially longer than the evaluated layer sequence.
Therefore we can define $R$ as

$$R = \frac{\lambda_0}{d}. \quad (4.10)$$

This ratio between dominant wavelength $\lambda_0$ and an equivalent layer thickness $d$ was estimated by Carcione et al. (1991) to be in the range between five to eight for a periodic layer sequence (Postma, 1955). For an arbitrary layer sequence equivalent media theory is described by Backus (1962) and further generalized by Schoenberg & Muir (1989). For the generalized effective media theory Carcione (2001) proposes to use as a rule of thumb that the wavelength must be more than eight times the layer thickness. For smaller ratios waves are scattered and dispersed, because shorter wavelengths get trapped inside the lower velocity layers.

To estimate the macroscopic characteristics of the above produced layer sequence on a wavefield which is typical for exploration seismics (frequencies of 10 Hz-100 Hz) we set up a substitute layer stack in which the layers have a thickness of a tenth of the dominant wavelength. We consider a frequency of 50 Hz and the velocity of clay, which corresponds to the lower velocity of the alternating layers. We then replace the sand and clay layers from our geological model inside such a substitute layer by effective media theory. Following Carcione (2001), for transversely isotropic media this is:

$$
\begin{align*}
\langle c_{11} \rangle &= \langle c_{11} \rangle - \langle c_{13} c_{33} \rangle + \langle c_{33}^{-1} \rangle^{-1} \langle c_{33}^{-1} c_{13} \rangle^2, \\
\langle c_{33} \rangle &= \langle c_{33}^{-1} \rangle^{-1}, \\
\langle c_{13} \rangle &= \langle c_{33}^{-1} \rangle^{-1} \langle c_{33}^{-1} c_{13} \rangle \\
\langle c_{55} \rangle &= \langle c_{55}^{-1} \rangle^{-1}, \\
\langle c_{66} \rangle &= \langle c_{66} \rangle,
\end{align*}
\quad (4.11)
$$

where the weighted average of a quantity $a$ is defined as

$$\langle a \rangle = \sum_{i=1}^{L} p_{li} a_{li} \quad (4.12)$$

in which $p_{li}$ is the proportion of the material $l$.

The above described process of creating a stochastic layer sequence, assigning material properties of alternating clay or sand to the layers and estimating the material properties of equivalent layers with a tenth of the wavelength at the current depth using effective media theory is visualized in Fig. 4.2.

An inherent property of a stochastic variable is that an infinite number of different realizations are possible. In this study we calculated the elastic equivalent of one hundred different realizations and exploited the means and standard deviations of the P- and S-wave velocities as well as the three Thomsen parameters $\epsilon$, $\delta$ and $\gamma$ to describe a transversely isotropic medium. The mean of all realizations corresponds to an alternating layer sequence with equally thick layers of mean size (here 1 centimeter).

An important point is that the variance estimate depends strongly on the scale length. Depending on wave speed, layer thickness and where exactly effective media theory starts, the resulting media is either strongly scattering or a medium with relatively smooth gradients. To illustrate this we calculated the anisotropic equivalent of our stack model for
4.4. ESTIMATING ANISOTROPY

Figure 4.2: Visualization of the process of creating a stochastic layer sequence, assigning material properties of alternating clay or sand to the layers and estimating the material properties of layers with a tenth of the current wavelength using effective media theory.

different layer fractions \((R = 1, 5, 10, 100)\). The resulting P-wave velocities for the equivalent medium are shown in Fig. 4.3. The representation for \(R = 100\) shows the geological details of our underlying model but does not well illustrate the influence of this model on the wavefield since the small scale perturbations do not have significant influence on the much longer wavelength of the assumed wavefield. On the contrary, representations for \(R = 1\) and \(R = 5\) exhibit smoother gradients and much less evidence from the complex geological settings. These models are oversimplified and do not account for the amount of scattering that would be expected from our model setting. Sams (1995) describes the effect of sparsely sampled borehole logs to underestimate the present anisotropy. By contrast, a too densely sampled borehole log will not improve the anisotropy estimate as equivalent layer theory has to be applied to estimate the effects on a significantly longer wavelength and locally very high anisotropy values are averaged out.

The mean and standard deviation of the wave speed of the equivalent medium are shown in Fig. 4.4a). The P- and SV-wave velocities of the transversely isotropic medium are calculated for vertically traveling waves as

\[
V_P = \sqrt{c_{33}/\rho}, \tag{4.13}
\]

\[
V_{SH} = \sqrt{c_{55}/\rho}. \tag{4.14}
\]

In directions other than vertical the velocities vary depending on the anisotropy.

Fig. 4.4b) shows the Thomsen parameters for the equivalent medium. The parameter \(\epsilon\) describes the relation between P-wave velocity in the horizontal and vertical directions. It is zero for all depths. In terms of reflection coefficient this parameter controls steeply incident waves and increases or decreases the reflection coefficient before any critical angle.

Figure 4.5 shows P- and S-wave velocities for a number of realizations. It is interesting to notice that even if the velocity difference between the two types of sediment and the
Figure 4.3: Ten realizations of the P-wave velocity for the same stochastic layer sequences are displayed with different $R$ for equivalent media theory. a) $R = 1$, b) $R = 5$, c) $R = 10$ and d) $R = 100$. The red line corresponds to the sand (fast) and the green line to the clay (slow) end-member model.
Figure 4.4: Mean (solid line) and standard deviation (dashed line) for the transversely isotropic equivalent with $R = 10$ of 100 different stochastic sand-clay layer stacks. P-wave and S-wave speeds in vertical direction are shown in a) and Thomsen parameters are shown in b).
thickness contrast is the same for the S-wave than for the P-wave the variability of the resulting equivalent is much lower for the S-wave than for the P-wave. So we expect scattering to be much less of a concern for S-waves than for P-waves.

The parameter $\delta$ describes velocity differences for P-waves traveling at an angle close to the vertical. In terms of reflection coefficient this parameter describes the zone prior to the first critical angle where its influence is rather moderate. After the first critical angle its influence seems to be more pronounced, especially in combination with attenuation ( Siddler & Carcione, 2007). The equivalent medium shows a slight increase of $\delta$ toward larger depths but also fluctuations that are significantly larger than its maximal mean value.

The most surprising parameter of the equivalent medium is certainly $\gamma$. It shows a large variation with depth. The anisotropy is high at the ocean bottom and rapidly decreases with depth. $\gamma$ is significantly larger than the other anisotropy parameters. The variation in different realization is large. Parameter $\gamma$ describes the relation between the elastic constants $c_{55}$ and $c_{66}$ which are in the 2D case responsible for elliptic wave propagation of the SH-wavefield. In the 2D case the SH wavefield is entirely decoupled from the qP-SV wavefield (Carcione, 2001). Therefore conversion of P-waves to SH-waves does not occur in 2D calculations. 2D modeling of marine seismic experiments with sources in water that radiate only P-waves do not account for the parameter $\gamma$. No SH-waves are ever excited in the calculations, neither from the source nor from wave conversion at media interfaces nor from layer boundaries. As far as we know at present it is not possible to accurately calculate the influence of the Thomsen parameter $\gamma$ on the reflection coefficient in the 3D case. So the impact of the most prominent parameter from our ocean bottom model on experimental geometries remains unclear.

While the intrinsic anisotropy of sand and silt is considered to be rather circumspect (Carlson et al., 1984) due to random grain orientation and predominantly round grain shape, the intrinsic anisotropy of clay is thought to correlate with porosity due to increasing orientation of the platy clay minerals (Wang, 2002) during compaction. In the first 100m of depth compaction is especially high and porosity decreases from more than 70% to roughly 50% (Mondol et al., 2007). As the anisotropic behavior of clay is rather complex and still a matter of ongoing research we refrain from constructing a depth dependent model. We use the constant Thomsen parameters derived by Bayuk et al. (2007) which are $\epsilon = 0.89$, $\gamma = 3.1$ and $\delta = -0.34$ to demonstrate the effect of intrinsic clay anisotropy on the macroscopic anisotropy of our layer stack.

Fig. 4.6 shows the anisotropy of the resulting layer stack of isotropic sand layers and intrinsically anisotropic clay layers. Some of the more interesting features are that anisotropy of $\delta$ and $\gamma$ becomes less erratic while the mean values are still in the same order of magnitude. But the distinct difference that comes with intrinsic anisotropy is the parameter $\epsilon$. In contrast to its lack of influence on purely isotropic layering of the stack it shows high absolute values as well as a strong dependence on layer thickness in the model with intrinsically anisotropic clay.

4.5 Model of reflection coefficients

The reflection coefficient is defined to be the ratio between incident and reflected wave pressure as a function of the incidence angle. In contrast to most seismic attributes, mea-
Figure 4.5: Velocities for (a) P- and (b) S-wave speeds in vertical direction for equivalent media \((R = 10)\) of ten realizations for different stochastic layer sequences are displayed. The red line corresponds to the sand (fast) and the green line to the clay (slow) end-member model.
Figure 4.6: a) P-wave and S-wave velocities in vertical direction and b) Thomsen parameters $\epsilon$, $\delta$ and $\gamma$ of the transversely isotropic medium derived with equivalent media theory from one hundred stochastic realizations of sand-clay layer stacks. The continuous lines correspond to the mean and the dashed lines correspond to the standard deviation. In contrast to Fig. 4.4 the clay layers in this model include constant intrinsic anisotropy $\epsilon = 0.89$, $\gamma = 3.10$ and $\delta = -0.34$. 
4.5. MODELING OF REFLECTION COEFFICIENTS

Measuring the reflection coefficient is relatively straightforward and can be extracted from conventional seismic measurements. Despite its simple definition and measurability, calculation of the reflection coefficient from the physical properties of the respective media with the plane-wave approximation is more sophisticated (Aki & Richards, 1980) and for visco-elastic and anisotropic media even cumbersome (Krebes & Daley, 2007; Sidler et al., 2008).

Carcione & Helle (2004) and Sidler & Carcione (2007) showed that reflection coefficients calculated with the plane-wave approximation coincide well with reflection coefficients extracted from pseudo-spectral numerical modeling codes with a special treatment of the interface (Tessmer et al., 1992; Carcione, 1991, 1996). Calculation of plane-wave reflection coefficients is restricted to homogeneous layers and therefore suitable to only a limited extent for the above constructed ocean-bottom model with its strong dependency on depth and it is not possible to directly compare the reflection coefficients resulting from numerical modeling with a corresponding plane-wave solution.

To evaluate the effects of velocity and anisotropy gradients on the reflection coefficients, we compare the numerical reflection coefficients calculated with a gradient model with the corresponding plane-wave reflection coefficient for a homogeneous layer. To calculate the numerical reflection coefficient we use the method described in Sidler & Carcione (2007).

In the first example we investigate the influence of the velocity gradient in the medium on the reflection coefficient. In Figure 4.7 we show the reflection coefficient for a homogeneous sand layer with the elastic properties of the sand end-member model at the ocean bottom. For this model we can directly compute the corresponding plane-wave reflection coefficient. It agrees closely with the numerical solution. We also plot the plane-wave reflection coefficient for a viscoelastic sand with the specific quality factors at the ocean bottom of $Q_p = 34$ and $Q_S = 25$. The difference is rather small and it is not possible to resolve this attenuation from the numerical solution.

We then introduce a velocity gradient in the solid layer for the numerical solution and compare it to the homogeneous plane-wave solution. Except for the velocity gradient the model is isotropic. It has to be noted that the elastic parameters at the ocean bottom (depth $= 0$) are the same for figure a) and b). Interestingly the numerical solution is not frequency-dependent (the different signs correspond to the solutions at different frequencies) but shows a clear difference in the region of the critical angle where the reflection coefficient increases more sharply and at a lower angle than for the homogeneous layer. Also the sharp edge at the critical angle seems to be rounded. The decreasing reflection coefficient after the critical angle could be due to higher wave absorption of the solid due to the gradient, but also numerical effects can not be excluded at large incidence angles.

The effect of attenuation is rather minor as would be expected by the small difference it makes for the plane-wave solution. The effects of the gradient seem to be rather negated by attenuation.

In Figure 4.8 the ocean floor reflection coefficient (amplitude and phase) of the equivalent sand-clay sequences is shown. The elastic properties of this medium correspond to the values shown in Figure 4.4. The reflection coefficient lies somewhere in between the reflection coefficients for sand and clay, which are shown as dashed lines. Inversion for the best fitting elastic reflection coefficient yielded $V_P = 1640 \frac{m}{s}$ and $V_S = 90 \frac{m}{s}$. The corresponding plane-wave reflection coefficient is plotted as solid line. The estimated P-
Figure 4.7: Reflection coefficients for a water-sand interface. a) shows a homogeneous sand with constant velocities ($V_p = 1800 \text{ m/s}, V_s = 128 \text{ m/s}$); b) is an elastic model including the wave speed gradient of the sand end-member model and c) contains also the attenuation gradient. Symbols represent the numerical solution at 34Hz, 36Hz and 37Hz. The solid line corresponds to the elastic plane-wave reflection coefficient and the dashed line to the viscoelastic plane-wave reflection coefficient for $V_p = 1800 \text{ m/s}, V_s = 128 \text{ m/s}, Q_p = 34, Q_s = 25$. 
wave velocity corresponds well with the true P-wave velocity at the interface. The S-wave velocity however is considerably overestimated. The gradient effect, as for the sand example, can be observed here too. The reflection coefficient increases more dramatically and earlier, as for a medium without a velocity gradient.

The phase angle corresponding to the reflection coefficient of the layered sand-clay ocean bottom model shows a phase jump at an incidence angle which is equal to the critical angle. This phase jump is not predicted using plane-wave theory for homogeneous layers of elastic isotropic media. It is also not present in the isotropic sand gradient model investigated above. It might therefore be related to the transversely anisotropic velocity gradient.

To show the effect of attenuation we calculated the same model as shown in Figure 4.8 with very pronounced attenuation ($Q_P = Q_S = 15$) and show the results in Figure 4.9. We do not expect such a strong attenuation to be common in ocean bottom sediments but to be possible in certain regions. The amplitude of the reflection overall is only slightly less than for the elastic model to the point of the elastic equivalent critical angle, but significantly less at larger incidence angles. An interesting point is that the phase jump present in Figure 4.8b) vanishes in the presence of attenuation.

For the third example we introduce intrinsic anisotropy in the clay layers of the layered sand-clay sequence. The elastic properties of the ocean bottom then correspond to the values shown in Figure 4.6. The resulting amplitude and phase of the reflection coefficient is shown in Figure 4.10. For this configuration the ocean bottom seems to be very absorbing and only a small fraction of energy is reflected. Also the reflectivity seems to be only slightly dependent on the incidence angle. The reflection coefficients of the sand and the anisotropic clay are shown as the dashed lines. Inversion for the best fitting elastic parameters was difficult and the deduced parameters were $V_p = 1430 \text{ m/s}$ and $V_S = 210 \text{ m/s}$. Neither give a good fit to the reflection coefficient nor represent the true parameters at the interface. The corresponding reflection coefficient curve is shown in the figure as the solid line.
Figure 4.8: Reflection coefficient for the elastic equivalent ocean bottom model of a finely layered sand-clay sequence with stochastically distributed layer thickness. The properties of this medium corresponds to the values in Figure 4.4. Symbols represent the numerical solution at 34Hz, 36Hz and 37Hz. The dashed lines corresponds to the elastic sand and clay layers respectively at depth zero. The solid line corresponds to the solution of an inversion for elastic and isotropic plane-wave solutions ($V_P = 1640\, \text{m/s}$, $V_S = 90\, \text{m/s}$ and $\rho = 2100\, \text{kg/m}^3$).
4.5. MODELING OF REFLECTION COEFFICIENTS

Figure 4.9: Reflection coefficient for the equivalent ocean bottom model of a finely layered sand-clay sequence with stochastically distributed layer thickness with strong attenuation ($Q_P = Q_S = 15$). Symbols represent the numerical solution at 34Hz, 36Hz and 37Hz. The dashed lines corresponds to the elastic sand and clay layers respectively at depth zero.
Figure 4.10: Reflection coefficient for the elastic equivalent ocean bottom model of a finely layered sand-clay sequence with stochastically distributed layer thickness and intrinsic anisotropy for the clay layers. The properties of this medium corresponds to the values in Figure 4.6. Symbols represent the numerical solution at 34Hz, 36Hz and 37Hz. The dashed lines corresponds to the elastic sand and clay layers respectively at depth zero. The solid line is the solution of an inversion for isotropic and elastic plane-wave solutions \((V_P = 1430\, \text{m/s}, V_S = 210\, \text{m/s} \text{ and } \rho = 2100\, \text{kg/m}^3)\).
4.6 Discussion

A sequence of equally thick layers would be sufficient for anisotropy estimation. The stochastic distribution of the layer thickness introduces the possibility to estimate statistical information about the calculated values. In this study we use this information to estimate a standard deviation for the P-wave and S-wave speed and the anisotropy of the equivalent medium of the layer sequence. This gives an impression on how sensitive a parameter is to layer thickness and in what range changes of the parameter due to equivalent media theory must be expected. The statistical information also proved to be useful to intuitively confirm findings from Carcione et al. (1991) and Carcione (2001) regarding the boundary at which equivalent media theory becomes valid. Figure 4.3 shows that there is no sharp intersection and an exact value for $R$ does not exist. A value in the order of $R = 8$ is a good approximation. Wrong values such as $R = 1$ and $R = 100$ can be identified clearly as either oversimplifying the situation or being inefficient. Figure 4.3 also illustrates that a coarse sampling of fine layered sediments in boreholes leads to an underestimation of the anisotropy in place as was pointed out by Sams (1995). The results also shows that this underestimation will not be of much concern to seismic experiments, since the resulting simplification follows the main trend of the anisotropy.

Modeling of the reflectivity at the ocean bottom showed that gradients, anisotropy and attenuation do not have a strong influence on near normal incidence angles. The most pronounced changes compared to an elastic isotropic model occur in the vicinity of the critical angle. This part seems important for S-wavespeed estimations. Schalkwijk (2001), for example, describes decomposition of multicomponent ocean-bottom data into P- and S-waves and states that S-wave estimation is sensitive to small errors in the P-wave estimation as well as the noise level near the critical angle. She uses an algorithm based on the assumption of an elastic and isotropic ocean bottom for which such effects as velocity gradients, fine layering and attenuation may appear as recording noise. These effects have their strongest influence in the vicinity of the critical angle and affect mostly the S-wave estimate.

Accumulation of sediments at the ocean bottom presumably generates a large amount of porosity that will decrease with compaction and subsequent diagenesis. The overlying ocean makes it most likely that existing pores are filled with water. Therefore it is possible that the assumption of welded contact between the fluid and the solid as we assumed it in this study does not fully explain the prevailing conditions at the ocean bottom and it would be interesting to investigate the influence of poro-elasticity in the ocean bottom model.

4.7 Conclusions

Equivalent media theory offers the possibility to investigate the macroscopic anisotropic properties of a stack of isotropic layers that are relatively thin compared to the dominant wavelength of seismic waves. We constructed trial layer sequences such as are often found in marine sediments and investigated its effects on the reflectivity. These effects include velocity gradients, transverse anisotropy induced by fine layering, intrinsic anisotropy and attenuation.
Wave speeds and velocity anisotropy of an alternating sand-clay layering can be estimated by using equally thick layers. The introduction of random layer thicknesses with a given mean, distribution and covariance function offers additionally the possibility to estimate the variation that comes from irregular layer thicknesses as we find it in nature. At the same time it visualizes that layer-induced anisotropy is inherently wavelength dependent and the amount of anisotropy obtained by equivalent media theory depends partly to the choice of the ratio between the thickness of the equivalent layer and the wavelength.

The resulting anisotropy parameters show that anisotropy controlled by the Thomsen parameter $\epsilon$, which is commonly known as P-wave anisotropy, is not obtained by our model. In contrast SH-wave anisotropy which is controlled by $\gamma$ is very high. It might be of a lesser concern as SH-waves are not exited or converted from conventional marine seismic sources. The parameter $\delta$ is moderately pronounced but it controls the region which elastic and isotropic algorithms use to a large extent to determine the S-wave velocity. Small changes of the reflectivity at these incidence angles lead to distinct misinterpretation of the S-wave velocity.

Numerical modeling of the ocean bottom models with additional complexity like velocity gradients, intrinsic anisotropy in the clay layers and attenuation show that these effects too produce differences most pronounced in the vicinity of the critical angle. So an oversimplified model will overestimate the S-wave velocity. Ocean bottoms with intrinsic anisotropy in clay and strong attenuation tend to strongly absorb the incident wavefield. In such cases a critical angle can not be clearly identified.

4.8 Acknowledgments

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Chapter 5

Conclusions

Wave equation-based decomposition of 4C seafloor measurements into up-going and down-going P- and S-wavefields require à priori information about the elastic parameters just below the ocean bottom. Elastic and isotropic approaches have been able to decompose the compressional wavefields, but have failed to successfully decompose the shear wavefield. It is presumed that the elastic and isotropic assumptions may not hold on the ocean bottom. The purpose of this thesis was to investigate possible reasons for the additional complexity of the ocean bottom and to evaluate its effect on reflection coefficients.

Reflection coefficients as a function of incidence angle contain substantial information about the corresponding interface. Due to their close affinity with the wavefield decomposition, they provide a good physical vehicle for describing the complications due to complex wave propagation. Unlike other seismic attributes, the reflection coefficient is straightforward to measure and can be extracted from conventionally recorded seismic data. Being a complicated function of the incidence angle and the media parameters, it can also be computed by various theoretical or numerical methods. Cross-checks of different methods allow an estimate of the accuracy of the results.

In the first part of this thesis existing work comparing two methods to calculate reflection coefficients of a viscoelastic ocean bottom was extended to include seismic anisotropy of the hexagonal symmetry class. Thus for the so-called transversely isotropic case with a vertical axis of symmetry (VTI). This symmetry is often used to describe a horizontally layered medium. The two methods consist of the analytical plane-wave solutions and a numerical evaluation of seismograms obtained with a pseudo-spectral modeling code, which is particularly useful at modeling interfaces. For this purpose the pseudo-spectral code and its associate boundary condition at the interface was extended to VTI. Comparison to the analytic plane-wave solution revealed that the solutions coincide well in the presence of VTI, indicating the validity of the simpler analytical plane-wave solution. This shows that a successful wavefield decomposition can be carried out for an attenuating and VTI ocean bottom. The presented examples focus particularly on the Rayleigh window, which is a narrow window of incidence angles in which the incident wavefield is strongly transmitted. It exists only in presence of attenuation. The effects of the Thomsen parameters which define the degree of anisotropy are illustrated using a soft sediment and a solid rock ocean bottom interface. In addition, the analysis of the plane-wave solution turned out to be more involved than expected and a wrongly chosen sign of the involved complex square roots can lead to non-physical discontinuities in the reflection coefficients.
The origin of these discontinuities was unclear, and required further investigation.

The second part of the thesis considers these non-physical discontinuities arising from evaluation of viscoelastic plane-wave solutions for the reflection coefficient and presents a solution to this problem. Plane-wave reflection coefficients are used in ray-tracing algorithms and are also of interest for parameter inversion from deduced reflection coefficients. The pseudo-spectral algorithm was further developed to the more general case of two viscoelastic, VTI solids. It was used to determine the relevant physical solution from the many solutions for the plane-wave reflection coefficient. Their result from vertical slowness expression square roots become complex with the introduction of attenuation, and therefore have ambiguous solutions. The additional information towards the physical solution enabled the problem of non-physical discontinuities to be referred to non-continuous evaluation of the complex square roots. Additional complexity comes from pathological cases in which continuous evaluation does not result in a correct solution. These well known pathological cases can be solved by an adequate switch of direction of the path on the Riemann surface. This simple and admittedly heuristic solution provides results that can be used for practical purposes even beyond the vicinity of the elastic equivalent critical (EEC) angle.

The third part of this thesis focuses on a quantitative estimation of the seismic parameters at the ocean bottom. A simple geological model is presented and evaluated. The geological model consists of an alternating sequence of sand and clay layers. This sort of sediment is often found on the ocean bottom and results from sedimentation processes. The velocity gradients for the sand and clay end-member models are taken from the literature. The isotropic geological model is then investigated using equivalent media theory for its macroscopic VTI anisotropy. Fine layering in the ocean bottom sediments mostly affects the SH-wave anisotropy described by the Thomsen parameter $\gamma$, followed by the Thomsen parameter $\delta$ which characterizes P-wave anisotropy at near vertical incidence. Geological layer sequences usually do not show equally thick layers. Therefore stochastic variables are used to describe the layer thickness and estimate the variability of the resulting parameters. The impact of the ocean bottom model on seismic measurements is evaluated by computing the reflection coefficients at the ocean bottom for such a multi-layered medium. The effects of velocity gradients and anisotropy due to fine layering are most pronounced in the vicinity of the critical angle. In models with intrinsic anisotropy in the clay layers or strong attenuation, transmission is pronounced over all incident angles. These findings are consistent with the observation that decomposition of P-waves are generally successful, while decomposition of shear wavefields in its up- and down-going components usually fails.

The research carried out in the framework of this thesis clearly helps to improve our understanding of seismic wave propagation phenomena in the vicinity of a seabed underlain by soft, unconsolidated sediments. The results of this study may, for example, contribute to improvements in the decomposition of 4C seabed seismic recording into the up- and downgoing P- and S-wave constituents. Our results indicate that particularly S-wave recordings, for which conventional elastic seismic wavefield decomposition notoriously provides inadequate results, stand to profit from such improvements.

Due to the methodological nature of this study, these findings could, however, not be verified against pertinent measurements and observations. Therefore, it remains to be clarified, whether the canonical seismic models explored in this study capture the essential
physical phenomena at play or whether there are additional pertinent aspects that eluded our analysis. So far, for example, we have only a crude understanding of attenuation of P- and S-waves in the shallow seabed. It is also not clear whether it is physically sufficient and adequate to represent the water-sediment contact as a first-order seismic discontinuity of a visco-elastic half-space overlain by an acoustic water layer. There is, albeit rather circumstantial, evidence to suggest that the physical properties of shallow “mushy” seabed sediments may be closer to those of a solid-liquid suspension than to those of attenuating solid, which in turn would require a poro-visco-elastic rather than a visco-elastic description of the system.

The methods developed in this thesis may also prove to be useful for advancing the investigation of interface waves, such as for example Scholte waves. Scholte waves are guided waves propagating along a liquid-solid interface, with applications ranging from geophysics (Gusev et al., 1996; Brekhovskikh, 1980; Carcione & Helle, 2004) and engineering (Ayres & Theilen, 1999) to non-destructive testing (Lee & Cobly, 1977; Carcione & Helle, 2004). A primary objective of the analysis of Scholte waves for geophysical and engineering applications is the determination of S-wave velocity structure of the shallow seabed (Bohlen et al., 2004; Kugler et al., 2007). The results of this thesis show that the propagation of S-waves in the shallow ocean bottom is particularly strongly affected by the effects of attenuation and anisotropy. This seriously questions the standard inversion approaches for Scholte waves based on the assumption of an elastic and isotropic layered half-space.
Bibliography


Chapter 6

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