Inverse analysis in road geotechnics

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INVERSE ANALYSIS IN ROAD GEOTECHNICS

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ABSTRACT

This research work had the aim of developing a procedure for back-calculating accurate and precise parameter values, describing the mechanical behaviour of the materials built in an existing road structure. After reviewing the existing testing techniques, a new device was designed and assembled at the IGT, Institute for Geotechnical Engineering (ETH Zürich) for measuring the three dimensional deflection bowl under a standard axle load (SAL). Particular attention was paid for obtaining precise and accurate significant measurements for inverse analysis. Three field tests on different locations and road structures were carried out: a flexible pavement type built in a concrete pit (indoor facility) at the EPFL (Ecole Polytechnique Federale de Lausanne), a semirigid type in Hinwil (Switzerland) and a flexible type in Bellinzona (Hinwil). The tests results show that the measured road displacements under a SAL, for relatively low temperatures, are generally reversible and time independent. Laboratory tests (uniaxial compression) were carried out on cores obtained from field samples. The strain measurements of the loaded samples were carried out with strain gages, and validated against devices with different technology (LVDT). The analysis of the test results showed that the materials have different bulk and deviatoric stress-strain behaviour. A new thermodynamical framework for non linear viscoelasticity (hyperviscoelasticity) was developed. Experimentally validated hyperviscoelastic and hyperelastic constitutive laws were adopted respectively for describing the mechanical behaviour of asphalt and cement stabilized mixtures. The inverse analysis of the field tests results was carried out with two different optimization algorithms (Levenberg Marquardt and Mesh Adaptative Direct Search), the FE program ABAQUS, and the developed user defined models. The results demonstrate the accuracy and precision of the parameter values obtained with the proposed inverse analysis procedure, demonstrating a potential for application of the developed technique for non destructing testing of real road structures.
SOMMARIO

Il presente lavoro di ricerca ha l’intento di sviluppare una procedura con la quale e’ possibile ottenere valori precisi e accurati per i parametri che descrivono il comportamento dei materiali impiegati nelle costruzioni stradali. Dopo un attento studio della letteratura sulle tecniche esistenti, un’apparecchiatura di nuova concezione è stata progettata e assemblata all’IGT, per la misura del bacino di deflessione sotto un’asse normale di carico. Particolare attenzione e’ stata prestata per ottenere valori precisi, accurati e naturalmente significativi per la procedura di analisi inversa. Tre prove insitu sono state effettuate in località e strutture differenti: una pavimentazione sperimentale di tipo flessibile costruita in un cassone di prova in calcestruzzo all’EPFL, una di tipo semirigido a Hinwil (Svizzera) e un tipo standard flessibile a Bellinzona (Svizzera). I risultati delle prove sperimentali, effettuate a temperature relativamente basse, mostrano che gli spostamenti verticali della pavimentazione sotto carico assiale standard costante sono generalmente reversibili e non variano nel tempo. Prove di laboratorio (carico monoassiale) sono state effettuate su provini estratti dai campi di prova. La misura delle deformazioni dei provini sotto carico sono state effettuate con estensimetri elettrici e validate mediante un’apparecchiatura di tecnologia differente (LVDT). L’analisi dei risultati sperimentali mostra un differente comportamento in compressione e taglio dei materiali. Una nuova struttura concettuale e’ stata sviluppata per la teoria della viscoelasticità non lineare (iperviscoelasticità). Leggi di comportamento iperviscoelastiche e iperelastiche validate sperimentalmente sono state adottate per descrivere il comportamento meccanico di miscele d’asfalto e di granulare stabilizzato. L’analisi inversa dei risultati delle prove insitu e’ stata effettuata con due algoritmi di ottimizzazione differenti (Levenberg Marquardt e Mesh Adaptative Direct Search), il programma agli elementi finiti ABAQUS, e i legami costitutivi sviluppati dall’autore. I risultati mostrano l’accuratezza e la precisione dei valori dei parametri ottenuti con la procedura di analisi inversa proposta e le potenzialità della tecnica sviluppata per prove non distruttive di reali strutture stradali.
Chapter 1: Inverse Analysis in Road Geotechnics

1.1 Introduction (a)

Inverse analysis in engineering is a tool of diagnostics. In road geotechnics this is related to the design of a new pavement structure, especially because nowadays overlays are more frequent than new roads. Overlays need to be designed according to the mechanical properties of the existing layers, which should be checked in the way a medical doctor does with his patient. After this analysis we should be able to define qualitatively and quantitatively the health status of the built in materials, to give a prognosis on its evolution, and prescribe a new overlay based on a mechanistic approach.

Fig. 1-1: Asphalt Concrete (AC) overlay (Italy, Milano ≈1950)
1.2 Introduction (b)

The circular tests track “Rundlauf” (Fig. 1-2) was for approximately 30 years (1977 - 2005†) one of the most important devices adopted for the PMS (Pavement Management System) in Switzerland. The results achieved with the research projects carried out on this testing facility highly influenced the Swiss standards specifications for road construction materials (Rossner et al. 1977, Jacot and Scazziga 1985, Horat et al. 1994, Beligni et al. 1995, Oekzul et al. 1999, Rabaiotti and Caprez 2007).

The test track did reproduce in real scale the structure of a road construction with a length of 100 m. The load consisted of three arms connected to one half of a real axle load, which could be loaded from 40 to 60 kN (corresponding to 80 and 120 kN of an entire real axle load). Each arm was driven by an electric motor. The load was applied by a twin-wheel like in a conventional truck. The wheels had a diameter of 1.1 m and a pressure of 700 kPa by 50 kN axle load and 750 kPa by 60 kN. The width of the track way was 1.4 m.

The transversal distribution of load was synchronized by an electronic random generator, in order to simulate the rutting development, like in real conditions. The road embankment was built in a trapezoidal concrete tank, and it had a depth of 2 meters.
In most of the research projects carried out on this testing facility, the analysis of the pavement structure performance was based on empirical and visual methods, combined with readings from strain gages embedded in the road structure (Rabaiotti and Caprez 2007, Beligni et al. 1995). In particular the PSI (Present Serviceability Index) based on the ASSHO (American Association of State Highway Organizations) road test rating panel (1961, AASHO interim guide for the design of rigid and flexible pavements, ASSHO road test) was for a long time the most influential indicator describing the state of serviceability of a pavement structure. The PSI is defined by empirical relationships (eq. 1-1, 1-2) which give an index calculated from road profile surface measurements (rut and longitudinal evenness) and visual observation (cracks). It allows defining the serviceability status of the pavement structure, but cannot describe the performance of the single layer materials. According to the AASHTO, American Association of State Highway and Transportation Officials, (AASHTO is the name of AASHO after 1973) road test rating panel, the index is different for flexible for rigid and flexible pavements (Hadley 1994):
Flexible pavements:

\[ PSI = 5.03 - 1.91 \log_{10}(1 + SV) - 0.01\sqrt{C + P} - 1.38RD^2 \]  
\[ 1-1 \]

Rigid pavements:

\[ PSI = 5.03 - 1.80 \log_{10}(1 + SV) - 0.9\sqrt{C + P} \]  
\[ 1-2 \]

Where:

- \( PSI \) present serviceability index
- \( SV \) mean of the slope variance for the two wheel paths
- \( \sqrt{C + P} \) measure of cracking and patching in the pavement surface
- \( RD^2 \) measure of rutting in the wheel paths

The PSI values are defined in a range between 0 (worst) and 5 (excellent). The end service life values are usually dependent on the type of road, ranging from 3.0 (very important) and 1.5 (low traffic).

The 1986 and 1993 editions of the guide made major steps in moving from an empirical towards an empirical-mechanistic based pavement design guide. Nevertheless, the mechanistic analysis introduced in these last editions is only a “way to go” or a “brief overview”, and the new edition “2002” is still under development (not yet implemented). The following picture (Fig. 1-3), which is found in the AASHTO guide 1993, shows how a road overlay can be designed according to a mechanistic-empirical analysis. It is important to notice how non-destructive testing is the basis of the proposed procedure, also representing the link between assessment of the pavement condition (mechanical properties) and the design.
Fig. 1-3: Overlay design according to the mechanistic-empirical analysis (ASSHTO 1993)
Pavement design based on mechanistic analysis carried out in the past produced lower quality results than those obtained with the empirical methods, due to technical limitations of computer capabilities and lack of reliable testing devices. An example of extensive analysis of Rundlauf test results, in particular inverse analysis based on simplified mechanistic analysis, can be found in Beligni et al. (1995). Static and dynamic measurements (Schwinger and electro optical measurements of the road deflection under rolling wheel) were used for back-calculation of elastic moduli of road layer materials. The mechanistic analysis was based on the multilayer theory (Burmister, 1945), which was implemented in the programs Vesys and Bisar. The pavement layers are assumed to be linear elastic, isotropic, without a mass, and homogeneous. The accuracy of the inverse analysis was tested comparing the strains calculated with the programs to those measured in the fields with the strain gages.

The assumption on the material and the boundary value problem were very simplified, and the results can still be considered, though based on a mechanistic analysis, qualitative.

This kind of accuracy test was also adopted by Uzan (2004) for validating the inverse analysis based on FWD (Falling Weight Deflectometer, see paragraph 1.3.2). In this case, by comparing the calculated and measured strains in the pavement layers, it was found that they still differ by a factor of 1.5, even if the forward analysis was carried out using advanced methods, i.e. FE (finite element) model and non linear elastic constitutive models for the built in materials.
1.3 Testing devices for inverse analysis

1.3.1 Introduction

Material properties of soils or materials used in road construction are normally analyzed with laboratory tests under well defined loading and boundary conditions. The samples are usually reconstituted in the laboratory, and their characteristics claim to be very similar to those of the field materials. Compaction standards, like Proctor standard or modified (SN 670 330b), for soils, the Marshall compaction (SN 671 969c) or more recently the superpave mix design standards for asphalt mixtures (Superpave 1994) are all efforts aimed to reproduce material in insitu conditions. Advanced numerical analysis techniques (i.e. FE method) allow nowadays modeling more complex boundary value problems, and therefore the interpretation of the insitu test results based on mechanistic analysis. Testing the material insitu would avoid, between others, the problems and the costs linked to the production or extraction of the samples. As already mentioned for the research projects carried out in the Rundlauf, several insitu testing procedures were developed in the past. The conceptual design of existing methods is to reproduce on the field almost real loading conditions (standard axle load), but with well defined loading and boundary for simplifying the interpretation of the test results.

It has to be stressed that the boundary and loading conditions which are reproduced in the laboratory tests are somewhat arbitrary, and can only partially model the stress state of a real axle load. This is a big limitation since the mechanical behaviour of the built in materials can be different under complex stress states (more details are found in Chapter 6).

The existing field tests, which will be illustrated in the next paragraphs, were developed in the past, as the computational capabilities were limited. For this reason the test procedure could not involve too much complexity in the interpretation of the results, making the calculation too time consuming.
1.3.2 Falling Weight Deflectometer (FWD)

The aim of the Falling Weight Deflectometer (see also ASTM D4694-96(2003)) is to reproduce under well defined boundary conditions the dynamic loading of an axle load on a road structure (Fig. 1-4). The load is applied to the surface by the impact of a weight on a steel plate, which is in contact to the pavement through a rubber membrane. The pavement deflection is recorded at several (7) positions from the point of load application by geophones, LVDT or accelerometers (Fig. 1-5). Usually only the amplitude of maximal deflection is analyzed.

The load is measured by a load cell, and its variation may be used to evaluate the stress dependency of the layer modulus.

Fig. 1-4: Falling weight Deflectometer (Dynatest Model 8000 Falling Weight Deflectometer)
The inverse analysis is carried out by comparing the measured deflection bowl to the one calculated according to Burmister multilayer theory or to more advanced techniques, such as the finite element method.

The procedure is called "basin matching" and it is carried out by minimizing the Sum of Squared Errors, (SSE) between measured and calculated displacements (a detailed description is found in chapter 6).

It is nowadays still matter of discussion if the measured deflection can be analyzed by adopting a quasi static analysis, ignoring the effects of wave propagation induced by the dynamic (pulse) loading, or by full dynamic analysis, taking into account all the loading/deflection time history. Uzan (1994a) did a complete overview on the most adopted forward-calculation techniques for back-calculation based on FWD, and concluded that dynamic phenomena can be neglected only if bedrock is located deeper than 6 meter from the surface (see also Chang et al., 1992, Chatti and Kim 2000). The same author (Uzan 2004) also found that road materials should be modeled as non linear elastic for improving the fit between calculated and measured deflection bowl. Another
important issue is represented by the frequency dependency of the asphalt mechanical properties (Chatti and Kim 2000), that are neglected by linear elastic constitutive modeling. The FWD only tests the material for certain loading frequency (40 ms), and this represents a big limitation for the mechanical properties characterization. Uzan (1994b) also showed that back-calculated moduli for unbound materials obtained with dynamic and static analysis are very similar, while are different for asphalt mixtures. This is a logic result, because asphalt is a high viscous material.

It appears that some issues related to the FWD test results interpretation are not completely solved, and this is due partially to testing procedure itself. Another device, which tries to overcome the difficult interpretation of the test results, and represents a purely dynamic testing and analyzing procedure, is described in the next paragraph.

1.3.3 Pavement Seismic Analyzer (PSA)

The pavement seismic analyzer (Nazarian et al. 1993), (Fig. 1-6) is a device which applies a load on the pavement through a vibrating source, and records the rate of wave propagation in the pavement. The procedure is based on the well known issue of mathematical physics, that properties (velocity and frequency) of waves in an elastic body are related through mathematical, theoretical relationships with the Young’s and shear modulus. Surface waves method is adopted for assessing the value of the shear modulus, compression waves method for the Young’s modulus, both belonging to the top paving layer (Nazarian et al. 1993). The impulse response method is adopted for measuring the modulus of subgrade reaction and of the foundation layers. The shear modulus of all the layers can be obtained with the Spectral-Analysis-of-Surface-Waves (SASW) Method, developed by Nazarian and Desai (1993).
Delamination and voids in the pavements can also be detected, according to the developers of the device, with the Echo and Impact Response methods. (Nazarian et al. 1993)

As for the FWD, the mechanical properties of the asphalt layer are still measured at one frequency, and analysis of the test results seems to focus only on linear elastic properties. Unfortunately it is well known (Uzan 2004) that most of the mechanical properties of road materials are non linear (elastic), and in particular are dependent on the rate and duration of load application for asphalt mixtures. The important assumption that material are homogeneous is also not true for asphalt mixtures, which are in reality made of two separate components (aggregate and bitumen), with very different mechanical properties.

Fig. 1-6: PSA, close view of the vibrating source and the geophones for measuring wave propagation
1.3.4 Benkelman beam

The Benkelman beam test developed at the American Association of State Highway Organizations (AASHO) Road Test in 1958 (Fig. 1-8), is carried out with a simple device that operates on the lever arm principle. The Benkelman Beam (Fig. 1-7) is used with a loaded truck (typically 100 kN) on a single axle with dual tires inflated up to 800 kPa. Measurement is made by placing the tip of the beam between the dual tires and measuring the pavement surface displacement rebound as the truck is moved away. The deflection bowl is measured indirectly by plotting the rebound displacement vs. the location of the load application point. The major advantages of this device compared to the dynamic tests, are the static loading, which allow filtering out the dynamic effects on the surface displacement (structural damping and wave propagation), and the loading conditions which are much closer to the reality (real axle load) than the ones applied by the FWD or the PSA. The elastic rebound is measured waiting for the viscous phenomena to disappear (pavement displacement is stable in time), therefore the long time (static) material mechanical behaviour is measured.
Benkelman beam tests were extensively carried out on the Rundlauf facility by the author (Rabaiotti and Caprez, 2007). The main source of inaccuracy related to this testing procedure are the length of the deflection bowl, which can significantly influence the results if the feet of the beam are not placed on a field with 0 displacement, and the fact that weaker pavements can heave between the dual tires. Analytical solutions for the results interpretation of this test are also not possible, due to the three dimensional boundary value problem, and the non uniform contact stresses. The analysis of the test results can be carried out nowadays with numerical methods, i.e. the Finite Element Method.
Rabaiotti and Caprez (2007) performed back-calculation of pavement layer material properties, using the deflection bowl measured with the BB. Test. The forward analysis was based on a 3D finite element model and linear elastic constitutive models were adopted for the built-in materials. The authors could back-calculate with a convergent procedure only one modulus value, fixing the parameters for all other layers. It was observed that inverse analysis based on these measurements has low rate of convergence (more details in chapter 6), due to the limited number and low precision of the measured displacements. The back-calculated values were nevertheless acceptable from an engineering point of view. Previous inverse analysis carried out by Beligni et al. (1995) showed that is nearly impossible to back-calculate layer material properties with only the maximal displacement measured with the Benkelman beam.
1.4 Conclusions

Two main families of existing testing techniques for inverse analysis in road geotechnics are found in the literature, based on static or dynamic loading of a pavement structure. The main interpretation challenge related to dynamic loading of the structure is that the pavement dynamic displacement is combined with wave propagation phenomena. Three different classes of analysis are found in the literature for back-analyzing the effect (displacements or shear wave propagation) of dynamic loading:

- Basin (deflection bowl) matching quasi static analysis which neglects the effect of dynamic phenomena, like side effects due to wave propagation and reflection (in case of bedrock near to the surface), the damping, and rate-time dependent material properties.

- A basin matching fully dynamic analysis, which takes into consideration all the loading/displacement time history (time or frequency domain fitting).

- The methods based on measuring the velocity of wave propagation, which is related to material (linear elastic) mechanical properties. In this case the loading is not represented by an impact loading but by a vibrating source.

These techniques, though being widely adopted internationally, still need improvements, and cannot completely characterize the mechanical properties of the pavement layer materials.

An inverse analysis technique based on the static deflection bowl measured with the Benkelman beam was developed by Rabaiotti and Caprez (2007), and inverse analysis was carried out using 3D finite element model. The results show good accuracy of the back-calculated elastic moduli but also the need of measuring an increased number of displacements (points) and the precision of
the measurements, for increasing the accuracy and the convergence of inverse analysis.
Chapter 2: ETH Delta Design

2.1 ETH Delta

2.1.1 Motivation

The ETH Delta non destructing testing device was designed for obtaining information on the present serviceability of a road construction and useful data for a consistent improvement in back-calculation of mechanical properties of the built in pavement materials.

Rabaiotti and Caprez (2007) observed that inverse analysis based on static deflection bowl measurement is a good indicator for describing the road service condition; in particular the surface displacement is strictly related to the mechanical properties of the single layers. However the existing deflection bowl measuring technique, based on the Benkelman beam, does not provide good quality results for inverse analysis. An effort was therefore made in order to measure the three dimensional form of the deflection bowl generated on a road structure by a standard axle load (100 kN), implementing state of the art contactless measuring technology. A new device, called ETH Delta, was designed and manufactured at the IGT, institute for geotechnical engineering at the ETH Zürich.

The main goal was to overcome the problems or simply improve the limitations of the Benkelman beam and of the Falling Weight Deflectometer, already described in the first chapter.
First, the device should be based outside the deflection bowl, whose extension can vary from 1 to more than 6 meters depending on many factors: loading conditions, temperature, layer stiffness and thickness, interface characteristics between the layers.

Second, the device should not deform during measurement, due to temperature changes and wind.

Third, an adequate testing procedure has to be developed.

From these preliminary observations, several hypotheses on the design of the device were discussed and analyzed.

2.2 Structural Design

The structural design of the device was developed in order to meet the following requirements:

1. The measuring should not be affected by the width of the deflection bowl
2. The sensors measuring the deflection should be mounted on an high stable frame
3. The possible movement of the frame should be monitored and measured by an independent measuring device.

Two different types of designs were studied in order to achieve good frame stiffness: a heavy frame with massive elements or a light construction, easy to transport and completely demountable. The second option was preferred due to the major flexibility.
The designed frame has a Delta shape consisting of four interconnected aluminum rectangular tubes, (30x100x3250 mm) jointed on one end, and on the other end connected by a circular tube (Fig. 2-2). During the prototype test it was observed that the frame was very unstable due to the length and the relative small transversal section of the beams. In particular it was found that the horizontal frame deformation represented a problem for the measurement, since the roughness of the asphalt is higher than the measured surface vertical displacement!

For this reason the frame was re-designed and stiffened with tensioned steel cables anchored to V-shaped frame (Fig. 2-1), placed at two thirds of the beam. The cable configuration is shown in Fig 2-2.

Fig. 2-1: Details of the V frame (section A-A)
Fig. 2-2: ETH delta frame, top view
A finite element calculation on an equivalent structure was carried out in order to study, qualitatively, the behavior of the frame under gravity and lateral loading.

The FE simulation shows the frame deformation under gravity and lateral load (Fig. 2-4). Lateral load unfortunately produces a "small" tilt of the frame around the longitudinal axes (Fig. 2-4), due to the steel cables configuration and the low value of polar moment of inertia of the frame section. It was found that the tilt is very small for moderate lateral wind load (normal operating condition). It can become larger only if different tension levels are applied to the vertical sails. The main advantage installing steel cables is to increase both the vertical and horizontal stiffness.

The vertical steel cables are also equipped with load cells, which allows for equally setting the tension level, and thus preventing torsion of the frame. The higher steel cables also increase the lateral frame damping when subjected to dynamic loading.
Fig. 2-3: Mesh of the FE model for the ETH DELTA Frame

Fig. 2-4: Maximal principal stresses in the cables when subjected to gravity and lateral loading, the deformation is 3000 times exaggerated.
Different angulations of the steel cables were tested, and it was found that 45° (Fig. 2-5) allows obtaining the highest stiffness and best damped response to dynamic loading.

The vertical positioning of the frame on the ground can be also set with screw feet.
2.3 Measuring sensors

2.3.1 Introduction

The displacement of the road surface under load and the eventual movement of the beam itself are measured using contactless laser technology. This allows measuring the displacements without the contact problems, caused by rough surfaces that can normally occur with other devices as, i.e., the LVDTs.

2.3.2 Laser for the road surface deflection

The maximal road surface deflection caused by a SAL (standard axle load = 100 kN) for road constructions in Switzerland and Europe is usually smaller than 1 mm for flexible, and can be in the order of 1/100 of millimeter for rigid pavements, and thus difficult to measure with enough precision. The triangulation laser technology is in laboratory conditions capable to precisely measure deformations within this order of magnitude. A preliminary study was then carried out for testing the precision of the sensors in field conditions.

2.3.2.1 Laser triangulation

Laser triangulation sensors determine the position of a target by measuring reflected light from the target surface. A “transmitter” (laser diode) projects a spot of light to the target, and its reflection is focused via an optical lens on a light sensitive device or “receiver”.

If the target changes its position from the reference point, the position of the reflected spot of light on the detector changes as well.

The signal conditioning electronics of the laser detects the spot position on the receiving element and, following linearization and additional digital or analogue signal conditioning, provides an output signal proportional to target position.
The most critical element in this arrangement is the receiver, which can be a position sensitive device (PSD) or charge coupled device (CCD) (Fig. 2-6). The PSD uses the light quantity distribution of the entire beam spot entering the light receiving element to determine the beam spot center and identifies this as the target position. However, the distribution of light quantity is affected by the surface condition of the target, causing variations in measured values.

The CCD detects the peak value of the light quantity distribution of the beam spot for each pixel and identifies this as the target position. Therefore, the CCD enables stable highly accurate displacement measurement, regardless of the light quantity distribution of the beam spot.

CCD triangulation sensor based lasers were chosen for measuring the vertical displacement of the road construction.
PSD based laser sensors, which only detect the position of the laser beam on their surface (without triangulation) were chosen for measuring the frame deformation or displacement, since they can measure at much higher distances.

### 2.3.2.2 Laser choice

CCD based triangulation laser sensors from three manufacturers, meeting the precision requirement for the new device, (at least according to the technical specifications declared by the manufacturer), were tested. The accuracy of three different products was additionally tested by the author and by Mr. Ernst Bleiker in field and in laboratory conditions.

The laser sensor and a LVDT were fixed at the end of a modified Benkelman beam (BB), and positioned between the twin wheels (Fig. 2-7).

In the standard procedure the surface deflection is measured indirectly: the beam is tilting around a pin, with one end which is in direct contact with the pavement surface and the other which is free to move in vertical direction. The pavement displacement is measured with a LVDT by the displacement of the free end of the beam, which is not in direct contact with the surface, by means of the lever arm principle. Unlike the standard BB test, the modified beam is fixed and the measurement is carried out by the laser and the LVDT directly on the measured point. The deflection bowl is obtained according to the standard analysis of BB test results plotting the vertical displacement vs. the wheel location (detailed description is found in chapter 3).

The measured points were adequately prepared in order to have smooth contact conditions, so that the lasers and LVDTs measurements are comparable.

The three compared CCD laser products were:

- Micro-epsilon ILD 1400-10
- Acuity Laser AR200
- Keyence LK-G 37
Fig. 2-7: Modified Benkelman-beam during test

Fig. 2-8: Deflection bowl measured with LVDT and Micro Epsilon ILD 1400-10
The results show that the highest accuracy and precision (resolution) was obtained with the Keyence LK-37 (Fig. 2-8, Fig. 2-9, Fig. 2-10).

In the following table the technical specifications given by the manufacturer and measured in the laboratory are presented:
Table 2-1: Laser sensors specifications

<table>
<thead>
<tr>
<th>Linearity (manufacturer) (max)</th>
<th>Linearity (measured) 0…4mm</th>
<th>Noise (manufacturer) static and dynamic</th>
<th>Noise (measured) 50 sample/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>ILD 1400-10</td>
<td>+20 ( \mu m )</td>
<td>10 ( \mu m )</td>
<td>1-5 ( \mu m )</td>
</tr>
<tr>
<td></td>
<td>+0.2%</td>
<td></td>
<td>3 ( \mu m )</td>
</tr>
<tr>
<td>LK-G 37</td>
<td>+5 ( \mu m )</td>
<td>2 ( \mu m )</td>
<td>0.03-4 ( \mu m )</td>
</tr>
<tr>
<td></td>
<td>+0.05%</td>
<td></td>
<td>0.4 ( \mu m )</td>
</tr>
<tr>
<td>AR200</td>
<td>+50 ( \mu m )</td>
<td>20 ( \mu m )</td>
<td>7.5 ( \mu m )</td>
</tr>
<tr>
<td></td>
<td>+0.2%</td>
<td></td>
<td>15 ( \mu m )</td>
</tr>
</tbody>
</table>

2.3.2.3 Laser supports

The lasers are attached on special supports, which are connected to the transversal tubular beam. The longitudinal position (Fig. 2-11, A) and transversal position (Fig. 2-11, B) of the lasers can be set according to the required position of the surface measurements (Fig. 2-11). The angle positioning (Fig. 2-11, C) of the sensor can be set for leveling the laser beam perpendicular to the surface.

Fig. 2-11: special laser support construction
2.3.3 Reference lasers

2.3.3.1 Introduction

The deformations of the frame induced by the wind and temperature changes, combined with the eventual deformation of the ETH Delta frame required the design of an independent device which measures the movement of the laser supports.

The movement is measured by two PSD sensors attached to two ends of the circular tube, whose position is tracked by laser beams generated by diodes placed behind beam (Fig. 2-14). The laser diode is fixed to a heavy steel tripod (8 kg), which can be leveled using three screw feet (Fig. 2-12). This ensures a very high stability thanks to the weight and the low center of gravity of the device.

Fig. 2-12: Reference laser tripod
The sensor is additionally fixed to a steel support for adjusting the angle positioning of the laser beam (Fig. 2-12).

2.3.3.2 PSD technology

The reference lasers make use of simpler technology compared to the triangulation lasers. These sensors are simpler because they do not measure a distance but only the position of the laser beam on the PSD sensor (no triangulation). The PSD sensor (SITEK-PSD 2L10-SP) is shown in Fig. 2-14 and 2-15.
Fig. 2-14: PSD sensor attached to the circular tube

The precision of the PSD is lower than the one of the triangulation lasers. The precision of the ETH Delta reduces during long term measurement to the one of the PSD sensors, which is about 1-2/100 mm.

2.3.4 Distance laser

The distance to the truck is measured with a distance laser DIMETIX DLS-B15, (Fig. 2-15). The measuring principle is based in this case on the phase shift method: a laser beam with sinusoidally modulated optical power is sent to a target; the reflected light (from diffuse or specular reflections) is monitored, and the phase of the power modulation is compared with that of the sent light. The phase-shift measurement between the optical signal reflected from the target and the reference signal emitted by the laser diode allows the determination of distance \( d \), as:
\[ \Delta \varphi = 2\pi f_0 \tau_d = 2\pi f_0 \frac{2d}{c} \]  

\[ d = \frac{c\Delta \varphi}{4\pi f_0} \]

Where:

- \( \Delta \varphi \) phase shift
- \( \tau_d \) propagation time
- \( f_0 \) modulation frequency

Typical measuring accuracy is ±1.5 mm, with a very high distance range up to 500 m.

Fig. 2-15: Laser diode tripod set up and the distance laser.
2.4 Test procedure

The transversal form of the deflection bowl is measured at maximum 11-12 surface points (Fig 2-16), together with the relative distance from the wheel during loading and unloading. The three dimensional form of the deflection bowl is obtained after plotting the measured displacement vs. the location of the applied load. This technique allows measuring indirectly the longitudinal extent of the deflection bowl, with the assumption that time dependent deformation does take place during loading or unloading phase. Plastic deformations can be measured subtracting the deflection bowl measurements in loading and unloading, as a result the elastic recovered deflection can be obtained. More details on the testing procedure are found in chapter 3.

Additionally, the measurements are recorded continuously during the constant loading and after the truck removal in order to measure eventual creep or time dependent rebound of the pavement surface.

Fig. 2-16: Measured points position
2.5 Optimized device design for inverse analysis

The number of measured points and their position is selected for increasing the overdetermination of the inverse analysis, increasing the probability and the rate of convergence of the solution, and the accuracy of the back-calculated parameters (see chapter 6).

In order to optimize the design, the road structure and the loaded section were modeled three dimensionally with the commercial finite element software ABAQUS. The modeled pavement consisted of 4 different layers: wearing course, base, sub-base, subgrade layer. A linear elastic constitutive model was adopted for the layer materials, and Mohr Coulomb failure criterion for the interface, defined by cohesion (maximal shear resistance) and friction.

The vertical displacements on fictitious points, calculated with hypothetical input elastic moduli, were then assumed as measured in a real experiment (Fig. 2-17).

A noise distribution was generated using MATLAB by a matrix containing pseudo-random values drawn from a normal distribution with \( \mu \) (mean) = 0 and \( \sigma^2 \) (variance) = 1 (Fig. 2-18). The noise distribution was normalized by its highest or lowest value, so that its values vary between 0 and 100% and afterwards it was added to the measured values. The noise simulation was introduced in order to have conditions closer to real measurements (roughness and other external factors), and therefore to study the effects of precision and number of sensors on the testing procedure (Fig. 2-19, Fig. 2-20, Fig. 2-21).
Fig. 2-17: Measured points position and number

Fig. 2-18: Gauss distribution of the generated random noise
Fig. 2-19: FE Simulated deflection bowl and 5% noise added

Fig. 2-20: Effect of number of sensors on the accuracy (coefficient of variation) of the back-calculated moduli
The Levenberg Marquardt optimization algorithm implemented in MATLAB was adopted for running the inverse analysis (more details on the procedure are found in chapter 6).

The accuracy of the back-calculated parameters is measured by a coefficient of variation $CV$ between calculated and “real” (the one used for generate the “measured” deflection bowl) moduli, described by eq. 2-3.

$$CV = \sqrt{\frac{\sum_{i=1}^{n}(E_{I,i} - E_{O,i})^2}{E_{I}}}$$  \hspace{1cm} 2-3

Where:

$CV =$ coefficient of variation  
$E_{I,i} =$ input original parameters (to be obtained with the inverse analysis)  
$E_{O,i} =$ optimized parameters
\( n \) = total number of parameters \\
\( \bar{E}_i \) = average value of the input original parameters

The results shown in Fig. 2-20 and 2-21 indicate that 7 laser sensors allow for an accurate estimation of the elastic moduli and, very important factor, this number reduces significantly the importance of measurement precision. The beam was designed very conservatively with 12 laser sensors. It is very interesting to notice that the FWD measurements are also carried out with 7 sensors! In that case the three dimensional form of the defection bowl is not needed due to the axial symmetry of the boundary value problem.
2.6 Calibration procedure

2.6.1 Reference lasers correction

The reference lasers allow for the tilt and vertical displacement of the beam to be corrected. Since the PSD sensors are placed ca. 80 cm behind the laser, a calibration of the reference laser readings should be done in order to account for the different level arm (Fig. 2-22).

The standard calibration consists on placing stepwise three small weights on one end of the front beam in three different times, causing a displacement and a tilting of the beam. The parameter $k_1$ and $k_2$ can be then obtained (Fig. 2-23).
The rotation of the laser sensors $\alpha$ is not significant: if a vertical displacement $r_1 = 1$ mm, $r_2 = -1.5$ mm are measured, corresponding to a tilting of $0.0597^\circ$, for $L_{2i} = 1400$ mm, $L_{21} = 2400$ mm, $k_1 = 1.6$, $k_2 = 1.5$, then

$$x_i = \frac{L_{2i}(r_1 \cdot k_1 - r_2 \cdot k_2)}{L_{21}} + r_2 \cdot k_2 = -0.0042 \text{ mm}$$

While

$$h_i^* = h_i \cdot \frac{1}{\cos \alpha} = 1.6000009 \approx h_i$$

It is important to calibrate the ETH Delta before each test, since the calibration parameters can vary considerably, depending on the climatic conditions and tension level on the steel cables.
2.6.2 Calibration tests

The tests for the calibration of the ETH Delta were carried out indoors, at constant temperature and no wind, and in the field (Fig. 2-24). In these preliminary tests the calibration was not carried out inducing the tilting of the beam with the weights, instead the natural tilting and deformation of the beam was measured for a long period of time. The laser readings are corrected with the reference laser measurements. The indoors test shows that without external light sources, the accuracy is still 1/1000 mm after 500 sec (Fig. 2-25).

Fig. 2-24: Calibration test obtained from a long time zero measurement

\[ y = 1.5867x \]
\[ R^2 = 0.9921 \]
Figure 2-25: Test at 0 load carried out indoors

Figure 2-26: Test at 0 load carried out in the field
In the field conditions this value unfortunately increases by an order of magnitude, reaching 1/100 mm (Fig. 2-26). A significant source of inaccuracy was the sunlight directly heating the PSD sensor, which induced a stochastic error that is quite impossible to filter out. For this reason a special protection was designed for protecting it from direct light.
Chapter 3: ETH Delta Field Tests

3.1 Introduction

Three field tests were carried out on different pavement types, with the aim of testing the accuracy of the ETH delta and of providing data for validating the inverse analysis procedure. The field tests were carried out on the following test tracks:

- The indoor APT Halle Fosse at the EPFL (Ecole Polytechnique Federal de Lausanne, LAVOC, Switzerland) (flexible pavement).
- Hinwil (Switzerland), combined with APT accelerated pavement testing carried out with the MLS10 device (Rabaiotti 2008) (semirigid pavement).
- Bellinzona (flexible pavement) (Switzerland).
3.2 Field tests at Halle Fosse (LAVOC, EPFL)

3.2.1 Introduction: the Halle Fosse testing facility

Fig. 3-1: Halle Fosse facility without the cooling/heating system

Fig. 3-2: Halle Fosse facility with the cooling/heating system
The “Halle Fosse” testing facility is an indoor accelerated pavement tester, located at EPFL, Ecole Polytechnique Federal de Lausanne. Five to six test tracks can be built inside a concrete pit, with dimensions: 5.4 x 19 m and the depth of 2 m. The facility applies the load as a real standard axle load, which can be driven on the surface at different speeds. The main specifications of the testing facility are:

- five different speeds
- pass-by frequency up to 2000 per hour (fifth speed)
- maximum speed: ≈10 km/h
- maximum axle load: 140 kN
- type of tire: single, super single or dual wheel
- tire pressure: between 7 and 9 bars
- total rolling length: 4.5 m
- constant speed rolling length: 2 m
- transverse movement (wandering): +/- 0.4 m (not used during these tests)
- load application: hydraulic
- breaking system: electrical

An insulated hall covering the tested fields is used to control the air temperature conditions provided by a cooling system (three ventilators) (Fig. 3-2).

### 3.2.2 Test track specifications

The test track was built in April 2006 for a research project at the EPFL (Lavoc), “Development of high performance underlayers with low cost materials and high percentage of re-use” (Bueche and Vanelstraete, 2006). Five road structures with different base course layer materials were built and tested for approximately 300,000 load repetitions.
The test field, where the ETH Delta test was carried out, had the following specifications:

Base course layer: 8 cm, M499, + 40% recycled material, 5.8% binder (HMA)
Base layer: 44 cm, gravel 0/32
Subgrade: 145 cm, fine Sand

The wearing course layer was not laid down in order to apply the load directly on the base course and try to induce fatigue cracks (Bueche and Vanelstraete, 2006).

The density of the insitu material was measured with a nucleodensimeter. A total of about 60 points was measured on the whole pavement. An average value of 2060 kg/m$^3$ was measured and the variation from this average value was less than 9 % for all the measured points.

The test track was also instrumented with PT100 for temperature measurements, at depths: 0, -3, -11 cm.

Fig. 3-3: Theoretical and measured grain size distribution and binder content for the bituminous mixture in the base course layer.

![Graph showing theoretical and measured grain size distribution and binder content for the bituminous mixture in the base course layer.](image)
At the time of the tests with the ETH Delta the pavement was still in good conditions, even if the test track had already been loaded for approximately 300,000 overruns.

### 3.2.3 Field test description

The ETH Delta configuration had to be modified for fitting into the facility, since the standard axle load is connected to two steel beams for facilitating the shift in transversal direction. The ETH Delta was put underneath the beams (Fig. 3-4), allowing transversal measurement of the deflection bowl. Unfortunately, due to limitations of the existing facility, it was technically impossible to precisely shift the wheel longitudinally with the beam in transversal position (Fig. 3-5). The measurements in longitudinal and transversal position were therefore carried out with the wheel in the same position, by quick loading and unloading the tire on the middle of the test track.

The measurements in longitudinal position were carried out with the same procedure and also applying the load with the rolling wheel. During the measurements with rolling wheel, the sensors were positioned at different distances from the tire shoulder in order to measure the lateral expansion of the deflection bowl (Fig. 3-5, 3-7).
Fig. 3-4: Test with load applied by the wheel in the same position (transversal section)

Fig. 3-5: ETH Delta positioning for the transversal and longitudinal measurement of the deflection bowl (wheel rolls only when the beam is in longitudinal position, otherwise the load is applied with the wheel in the same position). The lasers are shifted in transversal direction in order to measure the lateral extension of the deflection bowl.
Fig. 3-6: Load applied with the wheel in the same position (longitudinal section)

Fig. 3-7: Test with rolling wheel (longitudinal section). Note the lasers in the center of the beam, which are shifted for measuring the transversal expansion of the deflection bowl.
The tests were carried out at different loads, inflating pressures, temperatures and speeds: 36 longitudinal and transversal tests with the wheel in the same position, and 60 tests with rolling wheel, summarized in table 3-1, 3-2 and 3-3. It should be noticed that the tire footprint varies significantly within load and inflating pressure (Perret, 2003): the footprint measured with the tire having 800 kPa inflating pressure and an axle load of 115 kN is shown in Fig. 3-8.

![Footprint diagram](image)

Fig. 3-8: Measured and idealized (rectangle) tire footprint (800 kPa inflating pressure, load=115 kN), courtesy of LAVOC.

<table>
<thead>
<tr>
<th>Tests</th>
<th>Load [kN]</th>
<th>Pressure [kPa]</th>
<th>Temperature [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1/A2/A3/A4</td>
<td>60/80/100/120</td>
<td>650</td>
<td>24</td>
</tr>
<tr>
<td>A5/A6/A7/A8</td>
<td>60/80/100/120</td>
<td>750</td>
<td>24</td>
</tr>
<tr>
<td>A9/A10/A11/A12</td>
<td>60/80/100/120</td>
<td>850</td>
<td>24</td>
</tr>
</tbody>
</table>
### Table 3-2: Longitudinal tests with wheel in the same position

<table>
<thead>
<tr>
<th>Test</th>
<th>Load [kN]</th>
<th>Pressure [kPa]</th>
<th>Temperature [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1/B2/B3/B4</td>
<td>60/80/100/120</td>
<td>650</td>
<td>24</td>
</tr>
<tr>
<td>B5/B6/B7/B8</td>
<td>60/80/100/120</td>
<td>750</td>
<td>24</td>
</tr>
<tr>
<td>B9/B10/B11/B12</td>
<td>60/80/100/120</td>
<td>850</td>
<td>24</td>
</tr>
</tbody>
</table>

### Table 3-3: Rolling wheel tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Load [kN]</th>
<th>Pressure [kPa]</th>
<th>Temperature [°C]</th>
<th>Speed [km/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D16/D17/D18/D19/D20</td>
<td>80</td>
<td>650</td>
<td>24</td>
<td>2/6/4/8/10</td>
</tr>
<tr>
<td>D31/D32/D33/D34/D35</td>
<td>100</td>
<td>650</td>
<td>24</td>
<td>2/6/4/8/10</td>
</tr>
<tr>
<td>D36/D37/D38/D39/D40</td>
<td>100</td>
<td>750</td>
<td>24</td>
<td>2/6/4/8/10</td>
</tr>
<tr>
<td>D41/D42/D43/D44/D45</td>
<td>100</td>
<td>850</td>
<td>24</td>
<td>2/6/4/8/10</td>
</tr>
<tr>
<td>D46/D47/D48/D49/D50</td>
<td>120</td>
<td>650</td>
<td>24</td>
<td>2/6/4/8/10</td>
</tr>
<tr>
<td>D51/D52/D53/D54/D55</td>
<td>120</td>
<td>750</td>
<td>24</td>
<td>2/6/4/8/10</td>
</tr>
<tr>
<td>D56/D57/D58/D59/D60</td>
<td>120</td>
<td>850</td>
<td>24</td>
<td>2/6/4/8/10</td>
</tr>
</tbody>
</table>
3.2.4 Field test results

3.2.4.1 Tests with loading wheel in the same position (2D Measurements)

Transversal and longitudinal two dimensional measurements of the deflection bowl were carried out loading and unloading the wheel in the same position, as shown Fig. 3-4 and Fig. 3-6. Before measuring the deflection bowl, one test was carried out for calibrating the reference lasers (Fig. 3-9). The reference laser measurements, corrected with the calibration coefficients, as described in Chapter 2, are later used to correct the displacements measured with the lasers.

![Calibration](image)

Fig. 3-9: ETH Delta Calibration test

The measured vertical displacement did not increase in time under constant load (which lasted approximately 150 sec.) and after unloading also no irreversible displacements were measured, as it can be observed in Fig. 3-10.

The transversal and longitudinal measurement of the deflection bowl under different loadings is shown in Fig. 3-11, to 3-14. In Fig. 3-15 the transversal and
longitudinal measurements for two loading conditions, 60 and 120 kN are compared, and a very good correlation is found.

Fig. 3-10: Transversal section, long term measurement during loading and unloading.
Fig. 3-11: 2D transversal deflection bowl, test with wheel loading in the same position at 4 different axle loads, and inflating tire pressure of 650 kPa

Fig. 3-12: 2D transversal deflection bowl, test with wheel loading in the same position, at 4 different axle loads, and inflating tire pressure of 850 kPa
Fig. 3-13: 2D longitudinal deflection bowl, test with wheel loading in the same position at 4 different axle loads, and inflating tire pressure of 650 kPa

Fig. 3-14: 2D longitudinal deflection bowl, test with wheel loading in the same position at 4 different axle loads, and inflating tire pressure of 850 kPa
Fig. 3-15: Transversal compared to longitudinal section. The spatial position of the measurements is given by the section F-F and D-D.

Two qualitative remarks on the field test results can be made, while for the mechanical interpretation of the results the reader is addressed to Chapter 6.

- The (small) temperature difference between the tests on the transversal and longitudinal section (the indicated temperature is the average temperature in the base course layer) did not remarkably affected the form and the depth of the deflection bowl (Fig. 3-16).

- The tire pressure affected the total depth of the deflection bowl, but not its shape (Fig. 3-17).
Chapter 3: ETH Delta Field Tests

Fig. 3-16: Transversal section at different temperatures

Fig. 3-17: Transversal section at different tire inflating pressures
3.2.4.2 Rolling wheel tests

Rolling wheel tests were carried out for measuring the effects of rate of load application to the total displacements, since the (expected) time dependent displacements were not measured during the tests carried out with on place loading. Also in this case no effect of viscosity was measured: varying the rate of load application the deflection bowl does not change either the shape or the depth (Fig. 3-18, Fig. 3-19, Fig. 3-20).

![Graph showing displacement over time](image)

Fig. 3-18: Tests with rolling wheel load tests: the sensors, at different distances from the wheel path, show that the total vertical displacement is almost the same for all speeds (red line).

The three dimensional form of the deflection bowl was obtained plotting the vertical deflection vs. the location of the applied load, as in the standard analysis of the Benkelman beam test results. In this case, the wheel location is obtained multiplying the time by the speed, corrected by the longitudinal offset of the lasers (Fig. 3-7).

Fig. 3-19 and Fig. 3-20 show the isocontours of the displacements (3D deflection bowl) measured during the rolling wheel tests at two different speeds of 2.8 and 10.8 km/h.
Fig. 3-19: 3D deflection bowl obtained with rolling wheel, 120kN axle load, low speed (2.8 km/h)

Fig. 3-20: 3D deflection bowl obtained with rolling wheel, 120kN axle load, high speed (10.8 km/h), the dashed line represents the measurement position for the displacements plotted in Fig. 3-21.
The transversal and the longitudinal deflection bowl, measured on the same longitudinal spatial coordinates were compared, and also in this case no significant differences in the displacements were observed (Fig. 3-21).

![Diagram](A12 vs. D60)

Fig. 3-21: Transversal deflection bowl measured during loading wheel in the same position (static) and rolling (dynamic). During the test with rolling wheel the ETH Delta is positioned in longitudinal direction, therefore only the right part of the bowl is measured (section F-F). The measured point located near the wheel was slightly closer to the tire shoulder for the dynamic tests.
3.3 Field tests at Hinwil (MLS10)

3.3.1 Introduction: the MLS10

The MLS10 is a unique new full-scale APT machine with a total weight of 34 t. It consists of a noise protected steel frame where four dual tire loading wheels (Fig. 3-22), each loading the pavement up to 65 kN by a hydro-pneumatic suspension system, are mounted on wheel bogies that are running in one direction on guide rails in a vertical closed loop chain at a speed of maximal 22 km/h. The bogies are driven by 24 linear induction motors LIMS. The guide rails are built in such a way that the freely revolving wheels touch down smoothly to the pavement before loading the pavement over a path length of 4.2 m. In order to monitor the loading during trafficking, the dynamic movements of the suspension system are registered electronically and transferred via Bluetooth to a computer. The machine has four massive tubes providing stiffness of the frame. Three of them are used as diesel tanks with a capacity of 1400 liter thus providing additional ballast and allowing operating the machine about 300 hr without refueling.

The MLS10 is equipped with four transport wheels which can be raised and lowered hydraulically (Fig. 3-23), thus lifting up the whole frame by about 1m and giving room for maintenance work, such as checking tire pressures and pavement sensors, as well as measurements of profiles, crack and damage inspection or measurements with more elaborated devices such as the ETH Delta. Lifting the machine allows also for maintenance work, such as checking tire pressures and pavement sensors. The transport wheels allow to maneuver the machine around the test side and to drive the machine to or off low bed trucks for long distance transport.

The MLS10 is operational since April 2006 and has already been used for extensive testing in a project in Mozambique (De Vos et al. 2007) before being shipped to Switzerland for being used for this specific test campaign.
Fig. 3-22: Technical scheme of the MLS10

Fig. 3-23: MLS10 positioning on the test track
3.3.2 Test track specifications

The tests were carried out on a test track located in Hinwil (Switzerland). The pavement section was built approximately 20 years ago and never trafficked by motorized vehicles, since it was located on a highway dead end. The geometry of the cross section has been investigated with core borings and Geo-radar measurements. The construction is a semi rigid pavement type consisting of four layers:

Wearing Course layer: AC, 4 cm
Base course layer: HMA, 8 cm
1\textsuperscript{st} base layer: Cement stabilized layer, about 18 cm
2\textsuperscript{nd} base layer: Cement stabilized layer, about 18 cm (not known exactly)

Unfortunately it was not possible to exactly determine the thickness of the second base layer and the composition of the subgrade
3.3.3 Field test description

The static deflection tests were carried out with a conventional truck, applying a standard axle load of 100 kN (total truck load of 149 kN), axle wheel distance of 123 cm (Fig. 3-24) and inflating tire pressure was about 800 kPa, on the section which was previously loaded by the MLS10. The dynamic deflection bowl produced by the MLS10 was measured only at 1000.000 load repetitions, since the main test could not be delayed (maintenance works took place on the MLS10 during the ETH Delta tests).

The foot print was measured spraying the ground around the tire on its contour and reproduced schematically in Fig. 3-25.
The tests were carried out moving the truck toward the ETH Delta, waiting for measuring eventual time dependent displacements, and after few minutes the truck was removed in straight forward direction.

During the constant loading and unloading phase, no time dependent displacements were measured, meaning that these were eventually smaller than the total device precision for long term measurements, which is approximately 2/100 mm.

Irreversible deformations were measured after unloading at the end of the main test (1000,000 load repetitions).
3.3.4 Field test results

![Calibration graph](image)

Fig. 3-26: Calibration test

Calibration tests (Fig. 3-26) were carried out before each measurement, using the procedure described in Chapter 2, in order to correct the displacement laser readings for their eventual vertical movement.

3.3.4.1 Testing procedure

The tests were carried out periodically for assessing the effect of the repeated loading on the road structure. Two distinct measurements were carried out during each test, one on the loaded section, and one on an unloaded section nearby the test track. The displacements on the loaded and on the unloaded section allow comparing in real time the mechanical response of the damaged with the similar undamaged section. This procedure allows separating the changing in stiffness due to temperature changes and moisture content of the layers, from the effect of load repetition. Time dependent displacements were not measured during the
tests, while irreversible deformations took place especially towards the end of the test (>600,000 load repetitions).

The deflection bowls plotted in Fig. 3-27 to 3-30 are obtained using the measurements taken during unloading. The eventual irreversible deformations were filtered out comparing the deflection bowl measured during loading and unloading.

The shape of deflection bowl after different number of overruns leads to the following preliminary observations:

1. 0 loading repetitions (test carried out at $1 \times 10^5$ load repetitions on the nearby section): the deflection bowl is very wide in longitudinal direction (> 6m) and has a very low depth (0.1 mm, 10 cm offset from the tire shoulder). The deflection bowl is deeper in the middle of the axle load as close to the wheel (Fig. 3-27).

2. $1 \times 10^5$ load repetitions: the form of deflection bowl changes substantially. If the test on the unloaded section and the one at $1 \times 10^5$ are compared, a localization of a damaged zone is detected under the wheel path (Fig. 3-27, Fig. 3-28).

3. $6 \times 10^5$ load repetitions: The longitudinal extension and the depth of the deflection bowl significantly increases. The damage is localized mainly on the MLS10 wheel path (Fig. 3-29).

4. $1 \times 10^6$ load repetitions: The longitudinal extension and the depth of the deflection bowl still increase. 2 main longitudinal cracks seem to appear on the top layers (Fig. 3-30). The structure changes: from a multilayer to a completely fractured structure.
After 1 million load repetitions, a forensic investigation was performed by trenching and taking a slab from under the wheel track (Fig. 3-31). It was found that beneath the wheel path the whole asphalt pavement was punched at constant whole thickness (arrows), down into the cement stabilized 1st base layer which was heavily deteriorated and broken in a way that it was even impossible to take cores. On each side of the wheel path, at locations A-A and B-B, full depth vertical cracks occurred and the top layer suffered severe horizontal delamination from the base course layer.

Fig. 3-27: Recovered deflection bowl at 0 load repetitions (10^6 load repetitions on the loaded section)
Chapter 3: ETH Delta Field Tests

Fig. 3-28: Recovered deflection bowl at $10^5$ load repetitions

Fig. 3-29: Recovered deflection bowl at $6 \times 10^5$ load repetitions

Temperatures
Air = 2.31°C
-5 cm = 1.34°C
-10 cm = 1.25°C
-15 cm = 1.23°C

Temperatures
Air = 8.56°C
-5 cm = 8.41°C
-10 cm = 8.08°C
-15 cm = 8.15°C
Fig. 3-30: Recovered deflection bowl at $1 \times 10^6$ load repetitions

Fig. 3-31: Detected cracks after trenching the wheel path
3.3.4.2 MLS10 tests

A series of tests, where the load was applied directly with the MLS10, were carried out on the test section after $10^6$ load repetitions. The ETH Delta was modified for allowing measurements directly under the MLS10 (Fig. 3-32). In particular the front beam was placed underneath the frame of the facility, modifying the steel cables configuration.

![Fig. 3-32: ETH Delta new configuration for measuring the MLS10 induced deflection bowl](image)

The tests were performed at two different wheel speeds, 2.2 and 4.4 m/s. It was found that the speed of load application in this small range has no effect on the total displacements (Fig. 3-33, 3-34).

In laboratory conditions, time and rate dependent deformations of the asphalt mixtures are measured even at low temperatures (6°C), as it will be shown in Chapter 4. The viscosity effects can be appreciated only in the laboratory tests, on one hand due to the higher accuracy, on the other because the backcalculation of the (Hinwil) base course layer are much less sensitive to parameters (Chapter 6), which means that the deflection bowl is not much affected by the mechanical properties of the base course layer (in Hinwil).
Fig. 3-33: Displacements at 2.2 m/s (7.9 km/h)

Fig. 3-34: Displacements at 4.4 m/s (15.84 km/h)
3.3.4.3 Long term measurements and data filtering

The long term measurements were corrected with the reference sensors, which have worse accuracy than the displacement lasers.

The data were filtered using the subroutine “filtfilt” built in MATLAB. This filter performs zero-phase digital filtering by processing the input data in both the forward and reverse directions. After filtering in the forward direction, it reverses the filtered sequence and runs it back through the filter. The result has precisely zero-phase distortion; the magnitude is the square of the filter's magnitude response.

If e.g. a 1-second duration signal sampled at 100 Hz, composed of two sinusoidal components at 3 Hz and 40 Hz, has to be filtered so that the 40 Hz signal should be eliminated, the function filtfilt will filter it without shifting it, like the other built in function “filter”. This characteristic is very important since the displacement data are plotted against the location of the truck, and this would produce wrong deflection bowls if the data are shifted in time!

The following program produces the graph in Fig. 3-35 (source: MATLAB signal processing toolbox manual).

```matlab
fs = 100;
t = 0:1/fs:1;
x = sin(2*pi*t*3)+.25*sin(2*pi*t*40);
b = ones(1,10)/10;             % 10 point averaging filter
y = filtfilt(b,1,x);           % Noncausal filtering
yy = filter(b,1,x);            % Normal filtering
plot(t,x,t,y,'--',t,yy,':')
```
Fig. 3-35: Comparison of filtering functions

Particular attention has also to be paid, in order that the filter does not affect the precision of the measurement by altering too much the signal (Fig. 3-36, 3-37).

Fig. 3-36: Signal from the laser sensors corrected with the reference sensors
3.4 Field tests in Bellinzona

3.4.1 Introduction

The field tests at the EPFL and in HINWIL were carried out on structures, which are not very representative for real roads in Switzerland: the EPFL test track is an indoor facility built in a concrete pit; the test track in Hinwil is a semirigid pavement, which aged for 20 years without being loaded.

Additional tests were therefore carried out on a real road structure in Ticino (Switzerland), which should represent a standard flexible pavement type. In reality it turned out that the structure was not exactly the same as the one drawn in the plans: the real asphalt overlay was found to be much thicker.

Fig. 3-37: Filtered signal
3.4.2 Test Track specifications

The test field was located on a major road (Strada principale 13, Arbedo-Lumino-to Grigioni border) near the municipality of Arbedo-Castione (Bellinzona, Switzerland).

The road structure was re-built in June 2005. The road section (according to the design project) was built with the following materials:

- Wearing course: 3.5 cm, AC 11H and Leca© aggregate.
- Base course: 6.5 cm HMA 22S + 7cm HMA 22N
- Sub-base: 10 cm gravel 0-63 mm
- Subgrade: silty sand (?)

Additionally, cores were extracted from the pavement for checking the effective thickness and for obtaining samples for the laboratory tests. The measured thicknesses did not agree with the design project, since the layers were found much thicker: the base and wearing course layers had an average total thickness.
of 27 cm instead of the planned 17 cm. The contractor probably built instead of
the gravel an additional 10 cm asphalt layer.

Fig. 3-39: Test field (Section 2, direction Lumino).

3.4.3 Field test description

The deflection tests were carried out applying an axle load of 125 kN, the total
load of the truck was 160 kN, and the transversal wheel location 120 cm.
The footprint, which was measured spraying the ground around the tire, is
reproduced schematically in Fig. 3-40.
The tire inflating pressure was approximately 800 kPa. The tests were carried out by moving the truck toward the ETH Delta to the rest position. After approximately 300 sec the truck was removed. The temperature was measured with a standard PT100 in direct contact to the surface.

### 3.4.4 Field test results

The tests with the ETH Delta were carried out on both the tracks: section 1, direction Bellinzona and section 2, direction Lumino. The beam was set in a short configuration (free length: 3 meters), considering that the width of the deflection bowl in flexible pavements is usually smaller than 2 meters (Rabaiotti and Caprez 2007). A Calibration test was also carried out (Fig. 3-41). The calibration coefficients are in this case smaller than for the other tests ($\approx 1$), because of the shorter beam configuration.
The main results of the long term measurements can be summarized as follows:

1. In section 1 (direction Bellinzona), no time dependent displacements and no irreversible deformation took place on the track, during loading (constant load) and after unloading (Fig. 3-42, 3-43). In the unloading phase a heaving of the material near the footpath was measured (blue ellipse, Fig. 3-42).

2. In section 2 (direction Lumino), time dependent displacement took place during constant loading and after unloading, but no irreversible deformations were measured (Fig. 3-44, 3-45).
Fig. 3-42: Long term measurement, section 1. Blue circle = surface heave near the footpath

Fig. 3-43: Long term measurement, section 1
Chapter 3: ETH Delta Field Tests

Fig. 3-44: Long term measurement, section 2

Fig. 3-45: Long term measurement, section 2
The following pictures (Fig. 3-46 to 3-49) show the displacements during the loading and unloading phase. The width of the deflection bowl is quite large for a flexible pavement type and exceeds 3 meters. This phenomenon is triggered by peculiarities of the structure, which consists of a thick stiff layer on a soft subgrade. Indeed the measurements look quite similar. However, as it will be demonstrated in Chapter 6, the inverse analysis leads to different conclusions on the stiffness of the layers in the two sections.

**SECTION1**

![Deflection Bowl](image)

Fig. 3-46: Deflection bowl measured in loading (truck removed)
Fig. 3-47: Deflection bowl during unloading (truck removed)

Fig. 3-48: Deflection bowl during loading (truck moving towards the beam)
3.5 Conclusions

The deflection bowl width and depth depends mainly on the stiffness and the stiffness ratio of the layers, and varies from 1.5, for standard flexible pavement types, to 5-6 meters for semirigid pavements.

The form of the deflection bowl can also reveal the formation of cracks in cement stabilized layers.

In spite of the fact that time dependent deformations are measured for the asphalt mixtures, even at low temperatures, in the laboratory tests, their effect was measured only in one test section in Bellinzona. The reason is due to the low contribution of the asphalt layers to the total displacement, and to the even smaller time dependent part of the total asphalt layer deformation. This phenomenon becomes more consistent only for thick asphalt overlays, and at higher temperatures, when the deformations of the asphalt layers are higher.
Chapter 4: Laboratory Tests

4.1 Introduction

A series of laboratory tests on the sample materials collected during the field tests was carried out for assessing their mechanical properties, under well defined boundary and loading conditions. One goal of this small research program was also to provide experimental data for developing or adopting already existing constitutive models, which will be used in inverse analysis of field tests.

As it was observed during the field tests, the road structure subjected to real axle load undergoes very small displacements that are, in most of the cases, much smaller than 1 mm. In particular, the stiffer layers undergo very small deformations (strains), which are very difficult to measure, even in laboratory conditions.

On this account, it is very well known from the literature (Jardine et al., Symes and Burland 1984, Jovicic and Coop 1997), that geo-materials have different mechanical properties at small and large strains, therefore an effort was made for precisely measuring the deformation in the small strain region ($\varepsilon < 0.001 \%$).

Special attention was also devoted to the measurement of the horizontal strains, within the aim of exploring the mean pressure dependency of the bulk modulus, also predicted by soil mechanics theories, e.g. critical state soil mechanics. It is also important to notice that horizontal strains are more difficult to measure than the verticals, since they can be significantly smaller, depending on the soil
compressibility. Another relevant aspect which should not be neglected, especially when testing asphalt mixtures, is the environmental temperature. A special climatic chamber was designed and built in order to reproduce in the laboratory tests the same temperature measured in the field.

4.2 Strain gage measurements of small strains

Small horizontal and vertical strains of cylindrical specimens are typically measured using LVDTs or strain gages. The use of strain gages on soil, rock samples, dates back to early seventies (Franklin and Hoek 1970, Attinger and Köppel 1983), while LVDTs are normally employed for soft soils instead of strain gages (Cuccovillo and Coop 1997), because of their lower stiffness (Fig.4-1, 4-2). The materials which were tested for the research project have a relatively high stiffness, which allows for the strain gage to be glued on the specimen without increasing the local stiffness.

The vertical and horizontal strain gages length was determined for measuring on a representative portion of the core, almost the total vertical height and one half of the perimeter, due to the large maximal grain size (32 mm) of the mixture. The strain gage hence measures an average strain on the vertical and horizontal length of the specimen, without the effect of local concentration of grains distribution.

From preliminary tests it was observed that the specimens under uniaxial loading have the tendency to deform not perfectly vertical. In order to account for this phenomenon it was necessary to glue three vertical strain gages on the lateral surface of the specimen (Fig. 4-3), so that the magnitude of the vertical deformation is uniquely defined. It can be clearly observed (Fig. 4-3) that strains measured in the vertical direction are absolutely not homogeneous: in the rest of this work the total vertical strain is taken as the average value between the three measurements.
Fig. 4-1: Triaxial specimen equipped with strain gages (Franklin and Hoek, 1970)

Fig. 4-2: Triaxial specimen equipped with resistance wires (Attinger and Köppel, 1983)

Fig. 4-3: Measured vertical strains histories on the same core specimen
4.3 Triaxial stress-strain space

The triaxial stress-strain space is a very useful way of representing results from triaxial tests. It allows defining deviatoric (shear) and volumetric (bulk) behaviour, which are often different in granular and bituminous materials.

A triaxial test consists of a cylindrical specimen loaded on the top by a varying load $\sigma_1$ (or strain $\epsilon_1$), and on the side by a constant load $\sigma_2 = \sigma_3$, being $\sigma_1, \sigma_2, \sigma_3$ and $\epsilon_1, \epsilon_2, \epsilon_3$ the principal stresses or strains (Fig. 4-4).

Once the principal stress or strains are known, the volumetric and the deviatoric stress or strain tensor invariants can be calculated, and the according to them the geotechnical stress tensor invariants:

\[ p = \frac{\sigma_1 + 2\sigma_3}{3} = \frac{J_1}{3} \quad 4-1 \]

\[ q = \sigma_1 - \sigma_3 = \sqrt{3J_{2D}} \quad 4-2 \]

where $p$ and $q$ are the volumetric and deviatoric stress tensor invariants and

\[ J_{2D} = \frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \quad 4-3 \]

\[ J_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad 4-4 \]

where $J_1$ and $J_{2D}$ are the first stress tensor invariant and the second deviatoric stress tensor invariant, respectively.
And for the principal strains the following geotechnical strain tensor invariants can be obtained:

\[
\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 = I_1
\]

\[
\varepsilon_s = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) = \frac{2}{3}\sqrt{3I_{2D}}
\]

Where \( I_1 \) and \( I_{2D} \) are identical to 4-3 and 4-4 with \( \varepsilon \) instead of \( \sigma \).

### 4.4 Laboratory tests on EPFL materials

The laboratory tests on the materials built in the EPFL test section were extensively carried out for the research project “Development of high performance underlayers with low cost materials and high percentage of re-use” (Bueche and Vanelstraete, 2006). The results from two-point bending tests (EN12697-26) on the base course layer materials and plate load test results (SN 670 317b) on the unbound layers are presented in Chapter 6 for the validation of the inverse analysis.
4.5 Laboratory tests on Hinwil cores

4.5.1 Introduction

Cores from the base layers were collected before the field test with the MLS10 started. The vertical lengths of the cores also give the thickness of the top layers:

- Wearing course layer: AC, 4 cm
- Base course layer: HMA, 8 cm
- 1st Base layer: Cement stabilized layer, about 18 cm
- 2nd Base layer: Cement stabilized layer, about 18 cm (not exactly known)

Unfortunately it was not possible to exactly determine the thickness of the second base layer and the composition of the subgrade. As it was already mentioned, the cores were instrumented for measuring the strains in a very small range. The technical details are described in the following paragraph.
4.5.2 Base layer cores

4.5.2.1 Test set up

The samples are composed by a brittle cement stabilized gravel material, which was very difficult to cut in cylindrical specimens (Fig. 4-5).

Fig. 4-5: Specimen is cut and prepared, the top surface is very fragmented

Fig. 4-6: The surface are repaired with gypsum

Fig. 4-7: The strain gages are glued to the specimen and uniaxial tests are carried out

Only some samples remained intact after coring, and many were too damaged for being tested after the cutting.
Three intact cores were cut and the holes on the top and bottom surface filled with a gypsum mixture, in order to have uniform contact with the loading plate (Fig. 4-6).

The choice of Gypsum instead of stiffer materials, like epoxy resin, was dictated by the need of not altering the mechanical properties of the material. The strain gages, two for measuring the radial and three for the vertical strains, were glued to the surface with a special epoxy resin (Fig. 4-7).

### 4.5.2.2 Test specifications and setup

Triaxial tests at 0 confining pressure (uniaxial tests) were carried out under strain controlled conditions (Fig. 4-7). The load cell was controlled by an external LVDT that was unfortunately not capable to trigger the load cell with exact deformation rate and overestimated the total displacement of the specimen. The external LVDT measurement did not correlate with those from the strain gages, which are hereafter interpreted as the right measurement. The strain gages readings were also validated with another independent measurement technique, as it will be described demonstrated in paragraph 4.5.2.

The two tested cores had the following dimensions

Core “A”: height: 107 mm, diameter 99 mm  
Core “B”: height: 112 mm, diameter 99 mm

As it can be observed in Fig. 4-3, the tests were carried out at very small strains (<0.005 %). It has to be mentioned that the samples have a small ratio height/diameter (1:1), thus the friction between the specimen and the loading plate can affect the measurement of the vertical strains: the cylindrical core can develop a “barrel” shape, leading to an underestimation of the vertical strains. However it should be observed that the strain gages readings have only slightly underestimated the vertical displacements, as it will be demonstrated in Chapter 5.
4.5.2.3 Test results

The experimental results for the tests carried out on the two cores are similar (Fig. 4-8, 4-9)

![Graph showing deviatoric stress versus deviatoric strain tensor invariant for Core A and Core B.](image)

**Fig. 4-8:** Uniaxial test, deviatoric stress versus deviatoric strain tensor invariant (blue circle: after unloading)

![Graph showing volumetric stress versus volumetric strain tensor invariant for Core A and Core B.](image)

**Fig. 4-9:** Uniaxial test, volumetric stress versus volumetric strain tensor invariant

The results, plotted in geotechnical stress-strain tensor invariants, show that the deviatoric behaviour is linear and almost no plastic deformations were measured after unloading (Fig. 4-8). The volumetric stress is slightly non linear (Fig. 4-9), with volumetric stress rate increasing with increasing volumetric strains.
4.5.3 Base course layer cores

4.5.3.1 Material specifications

The material density of the cores was measured with wet weighting. The results (Table 4-1) show that the materials were homogeneously compacted.

Table 4-1: Core density

<table>
<thead>
<tr>
<th>Core</th>
<th>Mass [g]</th>
<th>Volume [cm³]</th>
<th>Density [g/cm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1263.1</td>
<td>511.48</td>
<td>2.47</td>
</tr>
<tr>
<td>B</td>
<td>1259</td>
<td>508.91</td>
<td>2.47</td>
</tr>
<tr>
<td>C</td>
<td>1298.8</td>
<td>530.95</td>
<td>2.45</td>
</tr>
<tr>
<td>D</td>
<td>1180.2</td>
<td>478.06</td>
<td>2.47</td>
</tr>
<tr>
<td>E</td>
<td>1124.6</td>
<td>452.99</td>
<td>2.48</td>
</tr>
<tr>
<td>F</td>
<td>1262</td>
<td>507.37</td>
<td>2.49</td>
</tr>
</tbody>
</table>

4.5.3.2 Test specifications and setup

Uniaxial unconfined compression tests were carried out on two cores from the base course layer: (A) and (B), both having a diameter of 100 mm and a height of 67 mm.

The cores were equipped with strain gages for measuring the vertical and the horizontal deformation (Fig. 4-10). The specimens were sealed before the tests with glycerin (Fig. 4-11), since they were put in a conditioning chamber that contains water (Fig. 4-12).

The testing temperature was set the same as for the field tests, in order to account for the temperature dependent mechanical properties of the bituminous mixtures, and therefore making the field and laboratory test results comparable.

The conditioning chamber was built expressly for this test: a copper spiral was fixed on the interior border of a triaxial test cell and connected with a conditioning device. The cell was then completely insulated with foam material.
4.5.3.3 Test program

The uniaxial unconfined compression tests were carried out with two different temperatures at constant strain rate, after which the load was suddenly removed and the recovery stress path was measured for observing the eventual development of plastic deformations during the loading. Unfortunately, as it was for the tests on the base layer, the external LVDT which controls the strain rate
could not trigger the loading cell with a constant strain rate. It was also impossible to reproduce the same strain rate at different temperatures. The tests which were carried out can be approximately described by Table 4-2

Table 4-2: Test program

<table>
<thead>
<tr>
<th>Test</th>
<th>Core</th>
<th>Temperature [°C]</th>
<th>Strain rate (*) [ ( \frac{\mu \varepsilon}{s} )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A6a</td>
<td>A</td>
<td>6°</td>
<td>1.5 (a)</td>
</tr>
<tr>
<td>A6b</td>
<td></td>
<td></td>
<td>2.4 (b)</td>
</tr>
<tr>
<td>A15a</td>
<td></td>
<td>15°</td>
<td>2.4 (a)</td>
</tr>
<tr>
<td>A15b</td>
<td></td>
<td></td>
<td>5.2 (b)</td>
</tr>
<tr>
<td>B6a</td>
<td>B</td>
<td>6°</td>
<td>1.5 (a)</td>
</tr>
<tr>
<td>B6b</td>
<td></td>
<td></td>
<td>2.4 (b)</td>
</tr>
<tr>
<td>B15a</td>
<td></td>
<td>15°</td>
<td>2.4 (a)</td>
</tr>
<tr>
<td>B15b</td>
<td></td>
<td></td>
<td>5.2 (b)</td>
</tr>
</tbody>
</table>

(*) these values are approximated, since the strain rate was not constant, especially during the initial loading phase

4.5.3.4 Test results

The stress-strain paths measured during the tests are shown in Fig. 4-13 to 4-16. As it should be expected the material exhibits a higher deviatoric stiffness (higher stress for the same strain level) for higher strain rates, due to the viscous mechanical behaviour of the asphalt mixture (Fig. 4-13, 4-15). It is interesting to

\[ \mu \varepsilon = \frac{\lambda \mu}{m} \]
observe that the volumetric behaviour is almost rate independent (Fig. 4-14, 4-16). It can be observed that the temperature have a high influence on the mechanical properties of the tested asphalt mixtures, as it can be expected (Fig. 4-13, 4-14, 4-15, 4-16).

Fig. 4-13: Core A, deviatoric stress vs. strain tensor invariant at two different strain rates and temperatures
Fig. 4-14: Core A, Volumetric stress vs. strain tensor invariant at two different strain rates and temperatures

Fig. 4-15: Core B, deviatoric stress vs. strain tensor invariant at two different strain rates and temperatures
Chapter 4: Laboratory Tests

4.6 Laboratory tests on Bellinzona cores

4.6.1 Introduction

The core samples, collected in the test field, have the following thickness:

Core “A”: 27 cm (4 cm wearing course + 23 cm base layer, HMA)
Core “B”: 27 cm (4 cm wearing course + 23 cm base layer, HMA)
Core “C”: 24 cm (4 cm wearing course + 20 cm base layer, HMA)

The specimen obtained from the cores had a diameter of 100 mm and a height of 200 mm, conform to the recommendations ASTM D2166-06 and EN 12697-26, and allow then for the effect of the friction on the contact surfaces to be minimized.
Core A and B were bored on the road track in direction Lumino, Core C in direction Bellinzona. A wet weighting test was carried out for measuring the core density. The results are shown in Table 4-3.

Table 4-3: Core density

<table>
<thead>
<tr>
<th>Core</th>
<th>Mass [g]</th>
<th>Volume [cm³]</th>
<th>Density [g/cm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3719.8</td>
<td>1551.8</td>
<td>2.40</td>
</tr>
<tr>
<td>B</td>
<td>3213.2</td>
<td>1339.1</td>
<td>2.40</td>
</tr>
<tr>
<td>C</td>
<td>3755.7</td>
<td>1554.7</td>
<td>2.42</td>
</tr>
</tbody>
</table>

4.6.2 Test set up

Strain gages were glued to the specimen surface, as for the samples collected during the field tests in Hinwil. Two LVDTs (Linear Variable Differential Transformer) were attached to the specimen lateral surface close to the gages, (Fig. 4-17, 4-18) in order to validate the accuracy of the readings of the strain gages. The strain gages length covers almost the entire vertical length of the sample, so that an average vertical strain of the mixture, without the effect of localized concentration of the grains, is measured.
Chapter 4: Laboratory Tests

Fig. 4-17: Sealed specimen with LVDTs and strain gages (core C)

Fig. 4-18: Comparison between LVDT and DMS test results

The strains measured by the LVDTs match those measured by the strain gages (Fig. 4-18). The main advantage of the gages is that they measure the strains...
directly\(^2\), not a displacement over a length, and their fixation on the surface is much simpler.

### 4.6.3 Test program

Three uniaxial compression tests at the temperature of 6°C and different constant stress rates \(\dot{\sigma}_1\) were carried out on each of the two specimens A and B (Table 4-4). In each test, after reaching the specified axial stress \(\sigma_{1\text{ max}}\), the stress was kept constant for a period of time of about 200 sec, allowing the sample to creep. Following the creep stage, the sample was unloaded to determine, if any permanent strains took place. From the recovery (unloading) strain path it was found that non-negligible irrecoverable strains take place only for the tests with highest creep stresses in each series (i.e., at \(\sigma_{1\text{ max}} = 318\) kPa for core A and \(\sigma_{1\text{ max}} = 453\) kPa for core B). For other tests in the series the strains are fully recoverable in time, i.e., the behavior is purely visco-elastic.

<table>
<thead>
<tr>
<th>Test</th>
<th>Core</th>
<th>(\dot{\sigma}_1) (kPa/sec)</th>
<th>(\sigma_{1\text{ max}}), (kPa)</th>
<th>(p_{\text{ max}}), (kPa)</th>
<th>(q_{\text{ max}}), (kPa)</th>
<th>(\varepsilon_{\text{irrecoverable}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>A</td>
<td>0.85</td>
<td>63</td>
<td>21</td>
<td>63</td>
<td>-</td>
</tr>
<tr>
<td>A2</td>
<td>A</td>
<td>2.5</td>
<td>189</td>
<td>63</td>
<td>189</td>
<td>-</td>
</tr>
<tr>
<td>A3</td>
<td>A</td>
<td>5.15</td>
<td>318</td>
<td>106</td>
<td>318</td>
<td>+</td>
</tr>
<tr>
<td>B1</td>
<td>B</td>
<td>3.82</td>
<td>153</td>
<td>51</td>
<td>153</td>
<td>-</td>
</tr>
<tr>
<td>B2</td>
<td>B</td>
<td>4.72</td>
<td>315</td>
<td>105</td>
<td>315</td>
<td>-</td>
</tr>
<tr>
<td>B3</td>
<td>B</td>
<td>8.23</td>
<td>453</td>
<td>151</td>
<td>453</td>
<td>+</td>
</tr>
</tbody>
</table>

\(^2\) The supports of the LVDT can represent a source of error, if the fixation point locations are not clearly defined and thus also the length over which the strains are calculated.
4.6.4 Experimental results

The tests were carried out with a rest period between two loadings, which lasts the time necessary for the deformations to become stable in time. The fact that deformations were not totally recovered after the rest phase does not exclude that some plastic deformations occurred, especially at higher strains. The volumetric and deviatoric strain histories are plotted in Fig. 4-19.

The test results show that the relationship between deviatoric and volumetric strains in time is slightly non linear (Fig. 4-20).

Unfortunately the test on the core C did not produce reliable experimental results. The experimental results presented here are the basis for the development and the validation of the new constitutive model described in Chapter 5.
Fig. 4-19: Volumetric and deviatoric strain history versus time (core A and B)
4.7 Conclusions

The laboratory tests, which were carried out in the present work, only represent a preliminary investigation for the assessment of the mechanical behaviour of the materials tested in the in situ field tests described in Chapter 3. The experimental results from the cement stabilized material from the Hinwil test site and the asphalt mixtures from the Bellinzona test site will serve as a basis for the development of a new constitutive model in Chapter 5. A special effort was made to measure the vertical and horizontal strains with accuracy in a very small strain region. Particular attention was paid on measuring the vertical strains on at least three independent locations and on a representative vertical length.
5.1 Introduction

Since the establishment of the Critical State Soil Mechanics, the non-linearity of the elastic component of soil behaviour (in particular, the pressure dependency of the bulk stiffness) has been a common feature of many rate independent soil models. However, when it comes to modelling the rate dependent elastic component, the majority of the existing models are linear visco-elastic, (e.g., Biot, 1962; Vgenopoulou and Beskos, 1992; Bardet, 1992; Di Benedetto and Tatsuoka, 1997). In visco-elasto-plastic soil models, the rate dependency was mainly associated with plastic, rather than elastic components of soil behaviour (e.g., Di Benedetto et al., 2002; Van Bang et al., 2007). Non-linear (reversible) viscoelastic models were mainly used for the constitutive modelling of composites, such as asphalts and polymers (e.g., Schapery, 1969; Hiel, 1984; Schapery, 2000) and, most recently, biomaterials (e.g., Fung, 1993; Miller and Chinzei, 1997; 2002; Kohandel et al., 2006; Oza et al., 2006; Drapaca et al., 2006a;b).

The widely used framework for non linear reversible viscoelasticity is based on the principles of quasi linear viscoelasticity (Fung, 1993), where reversible non linear and time dependent behaviours are decoupled. When such a model can be derived from an energy function, it is normally called “hyperviscoelastic”, supposedly satisfying the Laws of Thermodynamics (e.g., Miller and Chinzei, 1997; 2002, Kohandel et al., 2006). Built-in hyperviscoelastic constitutive models are widely used in commercial FE software (e.g., ABAQUS, 2006). The problem is that many of them actually violate the Laws of Thermodynamics!
Consider, for example, a one-dimensional case, where a visco-elastic relationship is formulated via the following convolution integral (e.g., Drapaca et al., 2006a):

\[
\sigma(\varepsilon, t) = \frac{1}{E_0} \int_{-\infty}^{t} E(t - \tau) \frac{\partial}{\partial \tau} \left( \frac{\partial f(\varepsilon)}{\partial \varepsilon} \right) d\tau \tag{5-1}
\]

where \( \sigma(\varepsilon, t) \) and \( \varepsilon(t) \) are the time dependent stress and strain, respectively; \( f(\varepsilon) \) is the Helmholtz free energy function for an instantaneously elastic material; \( E_0 \) is the instantaneous elastic modulus;

\[
E(t) = E_0 \left( 1 - \sum_{k=1}^{n} g_k \left( 1 - e^{-t/\tau_k} \right) \right) \tag{5-2}
\]

is the relaxation modulus presented using the Prony series with \( n \) terms, relaxation times \( \tau_k \) and coefficients \( g_k \).

In order to ensure that such a model is thermomechanically consistent, its stress-strain behaviour has to be derived using strain energy potential \( F(\varepsilon, t) \):

\[
\sigma(\varepsilon, t) = \frac{\partial F(\varepsilon, t)}{\partial \varepsilon} \tag{5-3}
\]

For the constitutive relationship (5-1), the corresponding Helmholtz free energy function is often given in the form (see, e.g., Miller and Chinzei, 1997; 2002; ABAQUS, 2006):

\[
F(\varepsilon, t) = \frac{1}{E_0} \int_{-\infty}^{t} E(t - \tau) \frac{\partial f(\varepsilon)}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \tau} d\tau \tag{5-4}
\]

which, indeed, upon substitution into equation (5-3) yields equation (5-1).
The problem, however, is that the true specific stored energy for the model (5-1) is different (as will be demonstrated in the following section) to that given by equation (5-4). Unfortunately, this correct energy function fails upon its substitution into equation (5-3) to produce the correct equation (5-1). This “paradox” will be also explained below.

Thermodynamic instability of many existing hyperviscoelastic models has been demonstrated in numerous studies (e.g., Kwon and Leonov, 1993; Kwon and Soo Cho, 2001) and a number of thermomechanically consistent hyperviscoelastic models is available (e.g., Schapery, 1969; Hutter, 1977). Nevertheless, the thermomechanically inconsistent models keep being used in the literature and computer codes, which can probably be attributed to the complexity of the existing thermomechanical formulations and of their finite element implementation.

The purpose of this Chapter is to develop a simple thermomechanically consistent framework describing a sufficiently general family of non-linear hyperviscoelastic materials, allowing for an easy implementation into FE codes. The proposed framework is then used to develop a hyperviscoelastic model with pressure-dependent bulk stiffness. While being based on common assumptions for the elastic component of soil behaviour, the proposed model is shown to be particularly useful for describing the behaviour of asphalts, which exhibit no irrecoverable strains in a much larger stress range than soils.

### 5.2 Background

Let us assume for simplicity that the instantaneous behaviour of the constitutive model (5-1) is linear elastic, with the corresponding instantaneous Helmholtz free energy:

\[
 f(\varepsilon) = \frac{1}{2} E_0 \varepsilon^2
\]
which upon substitution into (5-1) and (5-4) produces, respectively, the constitutive relation
\[ \sigma(\epsilon, t) = \int_{-\infty}^{t} E(t - \tau) \frac{\partial \epsilon(\tau)}{\partial \tau} d\tau \] 5-6

and the corresponding energy function
\[ F(\epsilon, t) = \frac{1}{2} \int_{-\infty}^{t} E(t - \tau) \frac{\partial^2 \epsilon^2(\tau)}{\partial \tau^2} d\tau \] 5-7

Next, the constitutive model (5-6) is subjected at \( t = 0 \) to an instantaneous step loading of the magnitude \( \epsilon_0 \) (Fig. 5-1) so that
\[ \epsilon(t) = \epsilon_0 H(t) \] 5-8

![Fig. 5-1: Instantaneous step loading at \( t = 0 \)]

where \( H(t) \) is the Heaviside step function. Substitution of equation (5-8) into (5-6) and (5-7) gives
where \( \delta(t) \) is the Dirac delta function.

The relaxation modulus (5-2) can be rewritten as

\[
E(t) = E_\infty + \sum_{k=1}^{n} E_k e^{-t/\tau_k}
\]

where \( E_k = E_0 g_k \) and \( E_\infty = E_0 - \sum_{k=1}^{n} E_k \) is the long term modulus. Substitution of the expression (5-11) into equations (5-9) and (5-10) produces the final equations for the constitutive relation:

\[
\sigma(\varepsilon_0, t) = E_\infty \varepsilon_0 + \sum_{k=1}^{n} E_k \varepsilon_0 e^{-t/\tau_k}
\]

and the corresponding energy function

\[
F(\varepsilon_0, t) = \frac{1}{2} E_\infty \varepsilon_0^2 + \frac{1}{2} \sum_{k=1}^{n} E_k \varepsilon_0^2 e^{-t/\tau_k}
\]

which are related via the equation:

\[
\sigma(\varepsilon_0, t) = \frac{\partial F(\varepsilon_0, t)}{\partial \varepsilon_0}
\]
It can be demonstrated that constitutive relationship (5-12) describes the behaviour of the generalized series-parallel Maxwell model shown in Fig. 2. Parameter $E_k$ represents here the elastic coefficient of the $k$-th linear elastic spring, while parameter $\eta_k = \tau_k E_k$ is the viscosity coefficient of the $k$-th linear viscous dashpot; $E_\infty$ is the spring coefficient of the zero-th element (without a dashpot). The total strain $\varepsilon$ in all the elements is the same, while the total stress in the model is the sum of the stresses in all the springs (because in each element the stress in the spring and the dashpot is the same):

$$\sigma(\varepsilon,t) = \sigma_0 + \sum_{k=1}^{n} \sigma_k = E_\infty \varepsilon + \sum_{k=1}^{n} E_k (\varepsilon - \alpha_k(t))$$ 5-15

where $\alpha_k$ is the strain in the $k$-th dashpot.

![Fig. 5-2: Generalized Maxwell model](image)

When this model is subjected to the step loading (5-8) in Fig. 5-1, the total strain remains constant $\varepsilon(t) = \varepsilon_0$, but because in each element the stress in the spring and in the dashpot should remain the same, $\alpha_k$ will change in time according to the following differential equations:
\[ \sigma_k = E_k (\varepsilon_0 - \alpha_k(t)) = \eta_k \dot{\alpha}_k \quad k = 1, \ldots, n \]

Solving equations (5-16) with the initial conditions \( \alpha_k(t = 0) = 0 \) (for instantaneous straining), we obtain:

\[ \alpha_k(t) = \varepsilon_0 \left( 1 - e^{-E_k t/\eta_k} \right) = \varepsilon_0 \left( 1 - e^{-t/\tau_k} \right) \quad k = 1, \ldots, n \]

which upon substitution together with \( \varepsilon(t) = \varepsilon_0 \) into equation (5-15) gives

\[ \sigma(\varepsilon_0, t) = E_x \varepsilon_0 + \sum_{k=1}^{n} E_k \varepsilon_0 e^{-t/\tau_k} \]

identical to equation (5-12), which confirms the equivalence of the two models.

Advantage of the mechanical representation, however, is that it has a clear physical meaning and allows for a straightforward calculation of the mechanical energy stored in the springs (i.e., the strain energy):

\[ F(\varepsilon_0, t) = \frac{1}{2} E_x \varepsilon_0^2 + \frac{1}{2} \sum_{k=1}^{n} E_k (\varepsilon_0 - \alpha_k(t))^2 = \frac{1}{2} E_x \varepsilon_0^2 + \frac{1}{2} \sum_{k=1}^{n} E_k \varepsilon_0^2 e^{-2t/\tau_k} \]

which is by no means identical to the popular strain energy formulation (5-13) – note the factor of two in the exponent. Finding this correct strain energy function (5-19), however, does not by itself solve the problem: its substitution into the hyperelastic relationship (5-14) does not lead to the correct stress-strain formulation (5-12)!

As a result of this simple mechanical interpretation of the relationship (5-12), it follows that within the popular formulation (5-1)-(5-4), the correct stress strain equation (5-12) is obtained by an incorrect differentiation of the incorrect energy function! Explanation of this "paradox" is given in the following section.
5.3 Thermomechanical framework for hyperviscoelasticity

The seemingly confusing result demonstrated above can be easily clarified by using a thermomechanically consistent framework with kinematic internal variables proposed by Houlsby and Puzrin (2002) and Puzrin and Houlsby (2003), based on the work of Ziegler (1977, 1983). Any model developed within this framework is fully defined by two scalar potential functions and automatically satisfies the First and the Second Laws of Thermodynamics. For example, the mechanical model in Fig. 5-2 is defined by two scalar potential functions with independent multiple kinematic internal variables $\alpha_k$:

the Helmholtz free energy potential:

$$F(\varepsilon, \alpha_1...\alpha_n, t) = \frac{1}{2} E_\infty \varepsilon^2 + \frac{1}{2} \sum_{k=1}^{n} E_k (\varepsilon - \alpha_k(t))^2$$  

and either the dissipation function:

$$d(\dot{\alpha}_1...\dot{\alpha}_n, t) = \sum_{k=1}^{n} \eta_k \dot{\alpha}_k^2$$  

or the force potential:

$$z(\ddot{\alpha}_1...\ddot{\alpha}_n, t) = \frac{1}{2} \sum_{k=1}^{n} \eta_k \ddot{\alpha}_k^2$$
The constitutive relationship is then obtained from the hyperelastic condition:

$$\sigma(\varepsilon, \alpha_1...\alpha_n, t) = \frac{\partial F(\varepsilon, \alpha_1...\alpha_n, t)}{\partial \varepsilon} = E_\infty \varepsilon + \sum_{k=1}^{n} E_k (\varepsilon - \alpha_k (t))$$ \hspace{1cm} 5-23

with the evolution law for the kinematic variables obtained from Ziegler's orthogonality condition (Ziegler, 1977; 1983), implying equality between the generalized and dissipative generalized stresses (Houlsby and Puzrin, 2006):

$$- \frac{\partial F(\varepsilon, \alpha_1...\alpha_n, t)}{\partial \alpha_k} = \frac{\partial z(\alpha_1...\alpha_n, t)}{\partial \alpha_k}$$ \hspace{1cm} 5-24

leading to the differential equations:

$$E_k (\varepsilon - \alpha_k (t)) = \eta_k \dot{\alpha}_k \hspace{1cm} k = 1,...,n$$ \hspace{1cm} 5-25

Solution of these equations for the loading $\varepsilon(t) = \varepsilon_0$ in Fig. 5-1 and initial conditions $\alpha_k (t = 0) = 0$ is given by equations (5-17), which being substituted into equation (5-20) produce the correct energy function (5-26) for the model in Fig. 5-2:

$$F(\varepsilon_0, t) = \frac{1}{2} E_\infty \varepsilon_0^2 + \frac{1}{2} \sum_{k=1}^{n} E_k (\varepsilon_0 - \alpha_k (t))^2 = \frac{1}{2} E_\infty \varepsilon_0^2 + \frac{1}{2} \sum_{k=1}^{n} E_k \varepsilon_0^2 e^{-\frac{2t}{\tau_k}}$$ \hspace{1cm} 5-26

Advantage of the thermomechanical formulation, however, is that it allows for the confusion described in the previous section to be avoided. Indeed, in spite of the fact that the evolution of the kinematic internal variables $\alpha_k (t)$ appeared to be dependent on the total strain $\varepsilon(t) = \varepsilon_0$, in derivation of the constitutive equation (5-23) they should always be treated as independent. That is, their evolution equations should only be substituted after the partial differentiation of the energy function with respect to $\varepsilon$. This is the proper way to obtain the correct constitutive
relationship (5-12) from the correct energy function (5-19). In the following it is
demonstrated how non-linear hyperviscoelastic models can be derived using the
above thermomechanical framework: first for a particular case of a power law
non-linearity, and then for a generalized family of non-linear models.

5.4 Energy function for a power law viscoelastic model

Derivation of the correct energy function for the constitutive relationship (5-1) for
a non-linear visco-elastic material appears to be even more challenging.
Consider the following instantaneous Helmholtz free energy function:

$$f(\varepsilon) = \frac{1}{m+1} E_0 \varepsilon^{m+1}$$  \hspace{1cm} 5-27

which upon substitution into (5-1) produces a non-linear constitutive relationship

$$\sigma(\varepsilon, t) = \int_{-\infty}^{t} E(t-\tau) \frac{\partial e^m(\tau)}{\partial \tau} d\tau$$  \hspace{1cm} 5-28

where $E(t) = E_\infty + \sum_{k=1}^{n} E_k e^{-t/\tau_k}$, $n$ is the model parameter.

Mechanical interpretation of this model is not trivial. Indeed, considering in Fig. 5-2
the power law non-linear elastic springs

$$\sigma_k = E_k (\varepsilon - \alpha_k)^m$$  \hspace{1cm} 5-29

in series with linear dashpots $\sigma_k = \eta_k \dot{\alpha}_k$, leads to the evolution equations for $\alpha_k$:
which cannot be solved analytically without recourse to special functions.

This severe complication can be circumvented by assuming that the viscous
dashpots in Fig. 5-2 are also non-linear in a form of the power law (known in
rheology as the Norton Law) with the same exponent \( m \):

\[ \sigma_k = \eta_k \dot{\alpha}_r \left( \frac{\dot{\alpha}_k}{\dot{\alpha}_r} \right)^m \quad 5-31 \]

where \( \dot{\alpha}_r \) is a reference strain rate. This leads to the evolution equations for \( \alpha_k \):

\[ E_k (\varepsilon - \alpha_k(t))^m = \eta_k \dot{\alpha}_k \left( \frac{\dot{\alpha}_k}{\dot{\alpha}_r} \right)^m \quad k = 1, \ldots, n \quad 5-32 \]

Solution of these equations for the general strain history loading \( \varepsilon = \varepsilon(t) \) and
initial conditions \( \varepsilon(t \leq 0) = \alpha_k(t \leq 0) = 0 \) is given by

\[ \alpha_k(t) = \int_0^t \varepsilon(\tau) e^{-(t-\tau)/\tau_k} = \varepsilon(t) - \int_0^t \left[ e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon(\tau)}{\partial \tau} \right] d\tau \quad 5-33 \]

where \( \tau_k = \frac{1}{\dot{\alpha}_r} \left( \frac{\eta_k \dot{\alpha}_r}{E_k} \right)^{1/m} \).

The total stress in the model is then

\[ \sigma(\varepsilon, t) = E_\alpha \varepsilon^m + \sum_{k=1}^n E_k (\varepsilon - \alpha_k(t))^m = E_\alpha \varepsilon^m + \sum_{k=1}^n E_k \left( \int_0^t e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \right)^m \quad 5-34 \]

For \( m = 1 \), i.e. linear springs, this expression becomes
\[ \sigma(\varepsilon, t) = \int_0^t \left( E_\infty + \sum_{k=1}^n E_k e^{-\frac{(t-\tau)}{\tau_k}} \right) \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \quad \text{with} \quad \tau_k = \eta_k / E_k \quad 5-35 \]

which is equivalent to the Prony series expression (5-28) for an arbitrary strain history \( \varepsilon = \varepsilon(t) \).

For \( m \neq 1 \), however, expression (5-34) becomes equivalent to the Prony series expression (5-28) only for an instantaneous step loading \( \varepsilon(t) = \varepsilon_0 H(t) \), in which case:

\[ \alpha_k(t) = \varepsilon_0 - \int_0^t e^{-\frac{(t-\tau)}{\tau_k}} \varepsilon_0 \delta(\tau) d\tau = \varepsilon_0 \left( 1 - e^{-\frac{t}{\tau_k}} \right) \quad k = 1, \ldots, n \quad 5-36 \]

so that

\[ \sigma(\varepsilon_0, t) = E_\infty \varepsilon_0^m + \sum_{k=1}^n E_k (\varepsilon_0 - \alpha_k(t))^m = \left( E_\infty + \sum_{k=1}^n E_k e^{-\frac{t}{\tau_k}} \right) \varepsilon_0^m \quad 5-37 \]

Note, however, that the relaxation times in the above expressions

\[ \tau_k = \frac{1}{\dot{\alpha}_m \left( \frac{\eta_k \dot{\alpha}_r}{E_k} \right)^m} \quad k = 1, \ldots, n \]

\[ \tau_k = \eta_k / E_k \quad 5-38 \]

differ from those of the linear model: \( \tau_k = \eta_k / E_k \).

Within the thermomechanical framework this mechanical model can be defined by two scalar potential functions:

- the Helmholtz free energy potential:
\[ F(\varepsilon, \alpha_1, ..., \alpha_n, t) = \frac{1}{m+1} E_\infty \varepsilon^{m+1} + \frac{1}{n+1} \sum_{k=1}^{n} E_k (\varepsilon - \alpha_k(t))^{m+1} \quad 5-39 \]

and either the dissipation function:

\[ d(\dot{\alpha}_1, ..., \dot{\alpha}_n, t) = \sum_{k=1}^{n} \eta_k \dot{\alpha}_r \left( \frac{\dot{\alpha}_k}{\dot{\alpha}_r} \right)^{m+1} \quad 5-40 \]

or the force potential:

\[ z(\dot{\alpha}_1, ..., \dot{\alpha}_n, t) = \frac{1}{m+1} \sum_{k=1}^{n} \eta_k \dot{\alpha}_r \left( \frac{\dot{\alpha}_k}{\dot{\alpha}_r} \right)^{m+1} \quad 5-41 \]

The constitutive relationship is then obtained from the hyperelastic condition:

\[ \sigma(\varepsilon, \alpha_1, ..., \alpha_n, t) = \frac{\partial F(\varepsilon, \alpha_1, ..., \alpha_n, t)}{\partial \varepsilon} = E_\infty \varepsilon^m + \sum_{k=1}^{n} E_k (\varepsilon - \alpha_k(t))^m \quad 5-42 \]

with the evolution law for the kinematic variables obtained from Ziegler’s orthogonality condition (5-24) leading to the differential equations (5-32) with solutions given by (5-33).

The resulting strain energy function is then given by substituting (5-33) into (5-39):
\[ F(\varepsilon, t) = \frac{1}{m+1} E_\infty \varepsilon^{m+1} + \frac{1}{m+1} \sum_{k=1}^{n} E_k \left( \int_0^t e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \right)^{m+1} \]  

5-43

Only for an instantaneous step loading \( \varepsilon(t) = \varepsilon_0 H(t) \), this relaxation function can be reduced to the Prony series form

\[ F(\varepsilon, t) = \left( E_\infty + \sum_{k=1}^{n} E_k e^{-t/\tau_k} \right) \frac{\varepsilon_0^{m+1}}{m+1} \]  

5-44

The relaxation times here, however,

\[ \tau_k = \frac{1}{\dot{\varepsilon}_r (m+1) \left( \frac{\eta_k \dot{\varepsilon}_r}{E_k} \right)^{m}} \quad k = 1, \ldots, n \]  

5-45

are again different to those for the stresses in equation (5-38).

### 5.5 A generalized family of hyperviscoelastic models

Consider now the following thermomechanical formulation, with the Helmholtz free energy potential:

\[ F(\varepsilon, \alpha_1 \ldots \alpha_n, t) = \frac{\sigma_r^2}{E_\infty} f \left( \frac{E_\infty}{\sigma_r} \varepsilon \right) + \sum_{k=1}^{n} \frac{\sigma_r^2}{E_k} f \left( \frac{E_k}{\sigma_r} (\varepsilon - \alpha_k) \right) \]  

5-46

and the force potential:

\[ z(\dot{\alpha}_1 \ldots \dot{\alpha}_n, t) = \sum_{k=1}^{n} \frac{\sigma_r^2}{\eta_k} f \left( \frac{\eta_k \dot{\alpha}_k}{\sigma_r} \right) \]  

5-47
where \( f(x) \) is a positive non-dimensional function of \( x \), with a monotonous smooth first derivative; \( \sigma_r \) is a reference stress.

The constitutive relationship is then obtained from the hyperelastic condition:

\[
\frac{\sigma(\varepsilon, \alpha_1 \ldots \alpha_n, t)}{\sigma_r} = \frac{1}{\sigma_r} \frac{\partial F(\varepsilon, \alpha_1 \ldots \alpha_n, t)}{\partial \varepsilon} = f\left( \frac{E_{\infty}}{\sigma_r} \varepsilon \right) + \sum_{k=1}^{n} f\left( \frac{E_k}{\sigma_r} (\varepsilon - \alpha_k) \right) \tag{5-48}
\]

where \( f'(x) = \frac{df(x)}{dx} \)

The evolution law for the kinematic variables is obtained from Ziegler’s orthogonality condition (5-24)

\[
\frac{\partial F(\varepsilon, \alpha_1 \ldots \alpha_n, t)}{\partial \alpha_k} = \frac{\partial z(\dot{\alpha}_1 \ldots \dot{\alpha}_n, t)}{\partial \dot{\alpha}_k}
\]

leading to the differential equations

\[
\frac{\sigma_k}{\sigma_r} = f\left( \frac{E_k}{\sigma_r} (\varepsilon - \alpha_k) \right) = f\left( \frac{\eta_k}{\sigma_r} \dot{\alpha}_k \right) \quad k = 1, \ldots, n \tag{5-49}
\]

Equation (5-49) implies that the springs and dashpots in Fig. 5-2 have the same type of non-linearity (with a particular case for a power law being considered in the previous section in equation (5-31)). This is the key assumption for the proposed framework. Obviously, it restricts the family of models that can be developed within this framework. This restricted class of models, however, still provides enormous flexibility for modeling of virtually any kind of non-linearity, thanks to the following features: (a) no restriction has been placed on the functional form of \( f \), apart from it being positive, with a monotonous smooth first derivative, and (b) elastic and viscous components of the behavior are both presented in form of series with arbitrary number of terms and independent coefficients \( E_k \) and \( \eta_k \).
The mathematical simplifications, however, provided by the key assumption (5-49) are immense. Thanks to this assumption, for a monotonous function \( f''(x) \), equations (5-49) lead to the familiar differential equations
\[
E_k (\varepsilon - \alpha_k (t)) = \eta_k \alpha_k \quad k = 1, \ldots, n
\]
5-50
with solutions given for initial conditions \( \varepsilon(t \leq 0) = \alpha_k (t \leq 0) = 0 \) by:
\[
\alpha_k (t) = \int_0^t \varepsilon(\tau) e^{-(t-\tau)/\tau_k} d\tau = \varepsilon(t) - \int_0^t e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \quad \text{with} \quad \tau_k = \frac{\eta_k}{E_k} \quad 5-51
\]
The resulting strain energy function and constitutive relationship can then be obtained in the closed form by substituting (5-51) into (5-46) and (5-48), respectively:
\[
F(\varepsilon, t) = \frac{\sigma_r^2}{E_\infty} f \left( \frac{E_\infty \varepsilon}{\sigma_r} \right) + \sum_{k=1}^n \frac{\sigma_r^2}{E_k} f \left( \frac{E_k}{\sigma_r} \int_0^t e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \right) \quad 5-52
\]
\[
\frac{\sigma(\varepsilon, t)}{\sigma_r} = f' \left( \frac{E_\infty \varepsilon}{\sigma_r} \right) + \sum_{k=1}^n f' \left( \frac{E_k}{\sigma_r} \int_0^t e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \right) \quad 5-53
\]
For the relaxation test \( \varepsilon(t) = \varepsilon_0 H(t) \) and the constant strain rate test \( \varepsilon(t) = \dot{\varepsilon}_0 t \), the corresponding constitutive equations (5-53) are obtained analytically, as:
\[
\frac{\sigma(\varepsilon, t)}{\sigma_r} = f' \left( \frac{E_\infty \varepsilon_0}{\sigma_r} \right) + \sum_{k=1}^n f' \left( \frac{E_k \varepsilon_0}{\sigma_r} e^{-t/\tau_k} \right) \quad 5-54
\]
and

\[ \frac{\sigma(t)}{\sigma_r} = f\left(\frac{E_\infty}{\sigma_r} t\right) + \sum_{k=1}^{n} f\left(\frac{\eta_k}{\sigma_r} \left(1 - e^{-t/\tau_k}\right)\right) \]

\[ 5-55 \]

respectively. These relationships can be used to find the model parameters by fitting the experimental stress-strain relations from the corresponding tests.

For an arbitrary strain history, the integral in equation (5-53) can be easily integrated numerically. After denoting

\[ A_k(t) = \int_{0}^{t} e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon}{\partial \tau} d\tau \]

\[ 5-56 \]

one can write

\[ A_k(t + \Delta t) = \int_{0}^{t+\Delta t} e^{-(t+\Delta t-\tau)/\tau_k} \frac{\partial \varepsilon}{\partial \tau} d\tau \]

\[ = e^{-\Delta t/\tau_k} \int_{0}^{t} e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon}{\partial \tau} d\tau + \int_{t}^{t+\Delta t} e^{-(t+\Delta t-\tau)/\tau_k} \frac{\partial \varepsilon}{\partial \tau} d\tau \]

\[ 5-57 \]

leading to the following useful property:

\[ A_k(t + \Delta t) \approx e^{-\Delta t/\tau_k} A_k(t) + \varepsilon(t + \Delta t) - \varepsilon(t) \]

\[ 5-58 \]

with \( A(0) = 0 \), so that only the strain at the previous time step has to be remembered.

In the FE codes, it is required to calculate the stiffness matrix for each time step. Within the thermomechanical formulation, the stiffness at the time \( t \) is derived from the following expression:
\[ d\sigma(\varepsilon, \alpha_1 \ldots \alpha_n, t) = \frac{\partial^2 F(\varepsilon, \alpha_1 \ldots \alpha_n, t)}{\partial \varepsilon^2} d\varepsilon + \sum_{k=1}^{n} \frac{\partial^2 F(\varepsilon, \alpha_1 \ldots \alpha_n, t)}{\partial \varepsilon \partial \alpha_k} d\alpha_k = E(\varepsilon, t) d\varepsilon \quad 5-59 \]

Substituting equation (5-46) into (5-59) we obtain:

\[ d\sigma(\varepsilon, \alpha_1 \ldots \alpha_n, t) = E_\infty f^*(E_\infty \frac{\varepsilon}{\sigma_r} \varepsilon) d\varepsilon + \sum_{k=1}^{n} E_k f^*(E_k \frac{\varepsilon - \alpha_k}{\sigma_r}) d(\varepsilon - \alpha_k) = E(\varepsilon, t) d\varepsilon \quad 5-60 \]

where \( f^*(x) = \frac{d^2 f(x)}{dx^2} \). Substituting \( \alpha_k(t) \) from equation (5-51) gives

\[ E(\varepsilon, t) = E_\infty f^*(E_\infty \frac{\varepsilon}{\sigma_r} \varepsilon) + \sum_{k=1}^{n} E_k \left(1 - \frac{A_k(t)}{\tau_k \frac{\partial \varepsilon(t)}{\partial t}} \right) f^*(E_k \frac{\varepsilon}{\sigma_r} A_k(t)) \quad 5-61 \]

where \( A_k(t) = \int_{0}^{t} e^{-(t-\tau)/\tau_k} \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \) and \( \tau_k = \eta_k / E_k \).

Using the property (5-58) and \( \frac{\partial \varepsilon(t + \Delta t)}{\partial t} = \frac{\varepsilon(t + \Delta t) - \varepsilon(t)}{\Delta t} \), the stiffness matrix \( E(\varepsilon(t + \Delta t), t + \Delta t) \) can be easily updated numerically, with only the strain at the previous time step required to be remembered. This greatly facilitates implementation of the above model in FE codes.
5.6 A hyperviscoelastic model for granular materials with pressure dependent bulk stiffness

As an example of the application of the above framework to modeling non-linear visco-elastic granular materials, consider the following energy function defined in the triaxial strain space:

\[
F(e_v, e_s, \alpha_{v1}, ..., \alpha_{vn_v}, \alpha_{s1}, ..., \alpha_{sn_s}, t) =
\frac{1}{m+1} \frac{p_r^2}{K_\infty} (K_\infty e_v)^{m+1} + \frac{1}{m+1} \sum_{k=1}^{n_v} \frac{p_r^2}{K_k} (K_k (e_v - \alpha_{vk}))^{m+1} + \frac{3}{2} \xi_v e_s^2 + \frac{3}{2} \sum_{k=1}^{n_s} G_k (e_s - \alpha_{sk})^2
\]

5-62

\[
z(\dot{\alpha}_{v1}, ..., \dot{\alpha}_{vn_v}, \dot{\alpha}_{s1}, ..., \dot{\alpha}_{sn_s}, t) = \frac{1}{m+1} \sum_{k=1}^{n_v} \frac{p_r^2}{\eta_{vk}} (\eta_{vk} \dot{\alpha}_{vk})^{m+1} + \frac{1}{2} \sum_{k=1}^{n_s} \eta_{sk} \dot{\alpha}_{sk}^2
\]

5-63

Where:

\(F\)  
strain energy function

\(e_v, e_s\)  
total volumetric and deviatoric strain tensor invariants

\(n_v, n_s\)  
total number of volumetric and deviatoric Maxwell elements

\(\alpha_{vk}, \alpha_{sk}\)  
k-th viscous volumetric and deviatoric strain tensor invariant

\(\dot{\alpha}_{vk}, \dot{\alpha}_{sk}\)  
k-th viscous volumetric and deviatoric strain tensor invariant velocity

\(t\)  
total time

\(m\)  
non linear parameter (for the volumetric behaviour)

\(K_\infty, K_k\)  
infinite and k-th Bulk modulus

\(p_r\)  
reference pressure

\(G_\infty, G_k\)  
infinite and k-th shear modulus

\(z\)  
force potential

\(\eta_{vk}\)  
k-th volumetric viscosity coefficient

\(\eta_{sk}\)  
k-th deviatoric viscosity coefficient
The hyperelastic relationships for the mean and deviatoric stresses follow:

\[
\frac{p(\varepsilon_v, \alpha_v, \ldots, \alpha_n), t}{p_r} = \frac{1}{p_r} \frac{\partial F}{\partial \varepsilon_v} = \left( \frac{K_{\infty}}{p_r} \varepsilon_v \right)^m + \sum_{k=1}^{n} \left( \frac{K_k}{p_r} (\varepsilon_v - \alpha_v) \right)^m 
\]

5-64

\[
q(\varepsilon_s, \alpha_s, \ldots, \alpha_n), t = \frac{\partial F}{\partial \varepsilon_s} = 3G_\alpha \varepsilon_s + \sum_{k=1}^{n} 3G_k (\varepsilon_s - \alpha_s) 
\]

5-65

With the evolution equations for internal variables derived from Ziegler’s orthogonality condition

\[
\alpha_vk(t) = \varepsilon_v(t) - \int_0^t e^{-(t-\tau)/\tau_v} \frac{\partial \varepsilon_v(\tau)}{\partial \tau} d\tau 
\]

5-66

\[
\alpha_{sk}(t) = \varepsilon_s(t) - \int_0^t e^{-(t-\tau)/\tau_s} \frac{\partial \varepsilon_s(\tau)}{\partial \tau} d\tau 
\]

5-67

where \( \tau_v = \eta_v / K_k \) and \( \tau_s = \eta_s / 3G_k \).

Substituting equations (5-66) and (5-67) into (5-64) and (5-65), respectively, we obtain non-linear hyperviscoelastic constitutive equations

\[
\frac{p(\varepsilon_v), t}{p_r} = \left( \frac{K_{\infty}}{p_r} \varepsilon_v \right)^m + \sum_{k=1}^{n} \left( \frac{K_k}{p_r} \int_0^t e^{-(t-\tau)/\tau_v} \frac{\partial \varepsilon_v(\tau)}{\partial \tau} d\tau \right)^m 
\]

5-68

\[
q(\varepsilon_s), t = 3G_\alpha \varepsilon_s + \sum_{k=1}^{n} 3G_k \int_0^t e^{-(t-\tau)/\tau_s} \frac{\partial \varepsilon_s(\tau)}{\partial \tau} d\tau 
\]

5-69

which are guaranteed to satisfy the First and the Second Laws of Thermodynamics.

Constitutive equations (5-68) and (5-69) imply a power dependency of the instantaneous elastic bulk moduli on the mean effective stress and linearity of the instantaneous deviatoric behavior, respectively. These are the widely used...
assumptions in many rate independent soil models, such as the Modified Cam Clay. In soils, however, plastic strains become significant already at early loading stages, obscuring purely visco-elastic behavior (e.g., Puzrin and Burland, 1998). In asphalt, in contrast, the purely visco-elastic region is known to be larger, while the instantaneous (dynamic) modulus highly depends on the confining pressure (Uzan, 2003). Therefore, it would be reasonable to assume that the rigorous hyperviscoelastic model (5-68)-(5-69) could be applicable to asphalts as well. In the following sections this assumption is validated against the experimental data obtained from the uniaxial compression tests on asphalt cores taken from a real road structure.

5.7 Validation of the hyperviscoelastic constitutive model

The developed hyperviscoelastic constitutive model is validated against the test results obtained in Chapter 4 on the cores from the Bellinzona test field. The corresponding volumetric and deviatoric strain histories measured during the constant stress rate and creep tests from Table 5-1 are shown in Fig. 5-3. These strain histories can be fed as an input into the constitutive equations (5-68) and (5-69) to predict the measured volumetric and deviatoric stress histories shown in Fig. 5-4, 5-5 and 5-6, provided the model parameters are known. Inversely, the model parameters can be back calculated by fitting the predicted stress histories to the measured ones, using some kind of an inverse analysis.

In this study, the built in MATLAB subroutine “fmincon”, with the option medium scale optimization algorithm (MATLAB, 2007), was adopted for changing the guessed model parameters by minimizing the objective function (the mean squared error between measured and calculated stresses).
Table 5-1: Experimental Program

<table>
<thead>
<tr>
<th>Test</th>
<th>Core</th>
<th>$\sigma_1$ ,[kPa/sec]</th>
<th>$\sigma_{1\text{max}}$ ,[kPa]</th>
<th>$p_{\text{max}}$ ,[kPa]</th>
<th>$q_{\text{max}}$ ,[kPa]</th>
<th>$\varepsilon_{\text{irrecoverable}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>A</td>
<td>0.85</td>
<td>63</td>
<td>21</td>
<td>63</td>
<td>-</td>
</tr>
<tr>
<td>A2</td>
<td>A</td>
<td>2.5</td>
<td>189</td>
<td>63</td>
<td>189</td>
<td>-</td>
</tr>
<tr>
<td>A3</td>
<td>A</td>
<td>5.15</td>
<td>318</td>
<td>106</td>
<td>318</td>
<td>+</td>
</tr>
<tr>
<td>B1</td>
<td>B</td>
<td>3.82</td>
<td>153</td>
<td>51</td>
<td>153</td>
<td>-</td>
</tr>
<tr>
<td>B2</td>
<td>B</td>
<td>4.72</td>
<td>315</td>
<td>105</td>
<td>315</td>
<td>-</td>
</tr>
<tr>
<td>B3</td>
<td>B</td>
<td>8.23</td>
<td>453</td>
<td>151</td>
<td>453</td>
<td>+</td>
</tr>
</tbody>
</table>

Fig. 5-3: Deviatoric and volumetric strain histories measured during constant stress rate and creep test (cores A and B).

Objective of the validation is to demonstrate that the developed constitutive model is able to fit the experimental data in the stress range where no permanent
strains occur. Therefore only the tests A1, A2, B1 and B2 were used for the inverse analysis procedure. For each core A and B a unique set of parameters has been obtained.

For the volumetric behaviour, described by equation (5-68), two different models were evaluated: LVE (linear viscoelastic with: \( m = 1, \ n_v = 1 \)) and HVE (hyperviscoelastic with: \( m \neq 1, \ n_v = 1 \)). The best fit achieved with the help of the inverse analysis for the cores A (tests A1 and A2) and B (tests B1 and B2) is shown in Fig. 5-4, with the corresponding model parameters given in the first four rows of Table 5-2. The better fit provided by the hyperviscoelastic models can be clearly observed. The predicted stress history curves appear to be rather wavy, as a reaction to significant variations in the measured input strain rates.

Increasing the number of elements in the both models (\( n_v = 2 \)) did not produce significant improvement of the fit of the experimental stress histories.

An interesting phenomenon has been observed, when the same models with the same set of parameters for each core were applied to model the volumetric behaviour in higher stress tests A3 and B3 (Fig. 5-5). As expected, none of the models provided a good fit of the experimental stress histories, due to the irrecoverable plastic strains, which are not accounted for by the models. As a result, the models were expected to produce a stiffer response than the one measured. For the hyperviscoelastic HVE model this was indeed the case, but not for the LVE model, which produced a softer response, due to its inability to capture bulk stiffness increase with the confining pressure.

The deviatoric behaviour in all the tests was modeled by a linear viscoelastic model (5-69) with \( n_s = 1 \) and parameters given in the last two rows of Table 5-2. The LVE provided a good fit for all three tests for both cores (Fig. 5-6), confirming the assumption that the deviatoric behaviour remains linear visco-elastic with pressure independent stiffness in a wider range of stresses than the volumetric behaviour.

The resulting model non-linear hyperviscoelastic model has 7 parameters: 4 for volumetric and 3 for deviatoric stress-strain behaviour, and is
thermomechanically consistent, i.e., satisfies the First and the Second Laws of Thermodynamics.

Fig. 5-4: Linear viscoelastic (LVE) and Hyperviscoelastic (HVE) modeling of the constant stress rate and creep tests for the volumetric stress behaviour (cores A and B).

Fig. 5-5: Mean stress prediction for the tests A3 and B3.
Fig. 5-6: Linear viscoelastic (LVE) modeling of the constant stress rate and creep tests for the deviatoric stress behaviour (cores A and B).

Table 5-2: Parameter estimation for the constant stress rate and creep tests

<table>
<thead>
<tr>
<th>Tests</th>
<th>$K_\infty$ [GPa]</th>
<th>$K_1$ [GPa]</th>
<th>$m$ [-]</th>
<th>$\eta_1$ [GPa*s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVEA (p)</td>
<td>0.6749</td>
<td>1.2352</td>
<td></td>
<td>37.59</td>
</tr>
<tr>
<td>LVEB (p)</td>
<td>1.0359</td>
<td>1.5485</td>
<td></td>
<td>37.82</td>
</tr>
<tr>
<td>HVEA (p)</td>
<td>0.2178</td>
<td>0.7197</td>
<td>1.4729</td>
<td>14.3619</td>
</tr>
<tr>
<td>HVEB (p)</td>
<td>0.4885</td>
<td>1.1137</td>
<td>1.2205</td>
<td>20.8073</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tests</th>
<th>$G_\infty$ [GPa]</th>
<th>$G_1$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVEA (q)</td>
<td>0.2078</td>
<td>0.7991</td>
</tr>
<tr>
<td>LVEB (q)</td>
<td>0.2887</td>
<td>0.8673</td>
</tr>
</tbody>
</table>
5.8 Hyperelastic modeling of cement stabilized base material

The constitutive model developed for the asphalt mixture can also be adopted, without rate dependency, for modelling the mechanical behaviour of other granular mixes, such as the cement stabilized material from the Hinwil test site.

![Graph](image_url)

Fig. 5-7: deviatoric behaviour and modelling of cement stabilized material, Core A and B

![Graph](image_url)

Fig. 5-8: volumetric behaviour and modelling of cement stabilized material, Core A and B
The deviatoric behaviour is in this case perfectly linear (Fig. 5-7), while the volumetric stress-strain history (Fig. 5-8) can be well described by the power law model (eq. 5-68) without the rate dependent term.

The parameter values calculated for core A and B (see Chapter 4) are given in Table 5-3.

Table 5-3: Parameter estimation for the uniaxial compression test (stabilized material)

<table>
<thead>
<tr>
<th>Tests</th>
<th>$K$ [GPa]</th>
<th>$m$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HVEA (p)</td>
<td>3.21</td>
<td>1.399</td>
</tr>
<tr>
<td>HVEB (p)</td>
<td>3.19</td>
<td>1.338</td>
</tr>
<tr>
<td>LVEA (q)</td>
<td>5.92</td>
<td></td>
</tr>
<tr>
<td>LVEB (q)</td>
<td>6.54</td>
<td></td>
</tr>
</tbody>
</table>

5.8.1 Accuracy

The analytical solution, described by eq. 5-68 and 5-69 (without the time dependent part), is capable describing accurately the triaxial test results only if there is no friction on the contact or when the ratio between height and diameter is > 2 (ASTM D2166-06, EN 12697-26). The tested cores have on the contrary a ratio equal to one. In this case the effect of the friction on contact surfaces cannot be neglected. Fig. 5-9 shows the vertical strain distribution calculated analytically for a 100 mm high specimen under constant load with the parameter values from Table 5-3 (A), and calculated by ABAQUS, with full bound on the contact surfaces. It can be observed that the strain gage readings are very close to the assumed average values, even for the unrealistic assumption of full bound between loading plate and specimen.
5.9 General stress-strain state formulation and index notation

The constitutive model formulated with stress and strain tensor invariants has the advantage of being easily generalized to general stress-strain state. The geotechnical strain tensor invariants correspond to the following general strain formulation:

\[ \varepsilon_v = \varepsilon_{ii} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \]
\begin{align*}
\varepsilon_s &= \frac{2}{3} \sqrt{3I_{2D}} = \frac{2}{3} \sqrt{3 \left( I_2 - \frac{I_1^2}{6} \right)} = \frac{2}{3} \sqrt{3 \left( \frac{1}{2} \varepsilon_{ij} \varepsilon_{ij} - \frac{\varepsilon_u^2}{6} \right)} \quad 5-71 \\
I_{2D} &= \frac{1}{6} \left[ (\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2 \right] + \varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2 \quad 5-72
\end{align*}

The general stress state can be derived as:
\[ \sigma_{ij} = \frac{\partial f}{\partial \varepsilon_{ij}} = \frac{\partial f}{\partial \varepsilon_s} \frac{\partial \varepsilon_s}{\partial \varepsilon_{ij}} + \frac{\partial f}{\partial \varepsilon_v} \frac{\partial \varepsilon_v}{\partial \varepsilon_{ij}} \quad 5-73 \]

This formula can also be re-written as:
\[ \sigma_{ij} = \frac{\partial f}{\partial \varepsilon_{ij}} = q \frac{\partial \varepsilon_s}{\partial \varepsilon_{ij}} + p \frac{\partial \varepsilon_v}{\partial \varepsilon_{ij}} \quad 5-74 \]

\[ \frac{\partial \varepsilon_v}{\partial \varepsilon_{ij}} = \frac{\partial \varepsilon_{ij}}{\partial \varepsilon_{ij}} = \frac{\partial (\varepsilon_{ij} \delta_{ij})}{\partial \varepsilon_{ij}} = \delta_{ij} \quad 5-75 \]

\[ \frac{\partial \varepsilon_s}{\partial \varepsilon_{ij}} = \frac{2}{3} \frac{1}{2} \frac{1}{\sqrt{3 \left( \frac{1}{2} \varepsilon_{ij} \varepsilon_{ij} - \frac{\varepsilon_u^2}{6} \right)}} \left( 3 \varepsilon_{ij} - 3 \cdot \frac{2 \cdot \varepsilon_{mm} \delta_{ij}}{6} \right) = \frac{1}{3} \frac{1}{3 \varepsilon_s} \left( 3 \varepsilon_{ij} - \varepsilon_{mm} \delta_{ij} \right) = \frac{2}{3 \varepsilon_s} \left( \varepsilon_{ij} - \frac{\varepsilon_{mm} \delta_{ij}}{3} \right) \quad 5-76 \]

\[ \frac{\partial \varepsilon_S}{\partial \varepsilon_{ij}} = \frac{2 \cdot \varepsilon'_{ij}}{3 \cdot \varepsilon_s} \quad 5-77 \]

The stress tensor is therefore obtained:
\[ \sigma_{ij} = q \cdot \varepsilon_{ij}^s + p \delta_{ij} \]  

5-78

The stress tensor consists then of two parts: a volumetric and a deviatoric part. Using now eq. 5-68 and 5-69 the viscohyperelastic function for general stress states is obtained:

\[
\sigma_{ij} = \left[ 3G_x \varepsilon_s + \sum_{k=1}^{n} 3G_k \int_{0}^{t} e^{-(t-\tau)/\tau_d} \frac{\partial \varepsilon_{s}(\tau)}{\partial \tau} d\tau \right] \frac{2\varepsilon_{ij}'}{3\varepsilon_s} + \\
+ \left[ p_r \left( \frac{K_x}{p_r} \varepsilon_v \right)^m + p_r \sum_{k=1}^{n} \left( \frac{K_k}{p_r} \int_{0}^{t} e^{-(t-\tau)/\tau_d} \frac{\partial \varepsilon_{v}(\tau)}{\partial \tau} d\tau \right)^m \right] \delta_{ij} 
\]

5-79

The stiffness matrix for the hyperelastic strain increment can also be easily derived:

\[
D_{ijkl} = 4 \frac{\varepsilon_{ij}'}{\varepsilon_s^2} \frac{\varepsilon_{kl}'}{\varepsilon_s^2} \left( \frac{\partial^2 f}{\partial \varepsilon_s} - \frac{1}{\varepsilon_s} \frac{\partial f}{\partial \varepsilon_s} \right) + \frac{2}{3} \frac{\partial f}{\partial \varepsilon_s} \delta_{ik} \delta_{jl} + \left( \frac{\partial^2 f}{\partial \varepsilon_s^2} - \frac{2}{9} \frac{\partial f}{\partial \varepsilon_s} \right) \cdot \delta_{ij} \delta_{kl}  
\]

5-80

5.10 The finite element implementation

There are two possible methods for implementing a hyperviscoelastic constitutive model in the finite element program ABAQUS: through the user subroutines UMAT or UHYPER, which have to be written in FORTRAN. The user subroutine UMAT is called for each material point at each iteration of every increment: it describes how the stresses (Cauchy) have to be updated at the end of the increment and the stiffness matrix formulation. The stiffness matrix is not used for stress calculations but for achieving quadratic convergence of the Newton-Raphson algorithm (or quasi-Newton) for the equilibrium equations (ABAQUS 2006).
As stated in the manual, “this matrix will also depend on the integration scheme used if the constitutive model is in rate form and is integrated numerically in the subroutine”, in the present work with the scheme defined by eq. 5-61 and implemented in eq. 5-80.

The numerical integration is possible thanks to the state variable (“statev”), which allows storing for the last iteration, variables with iteration dependent value (i.e. $\mathcal{A}_k$ in eq. 5-56).

An initial zero bulk stiffness for zero strain increment, as defined by the constitutive equations 5-68, would produce a numerical error in ABAQUS, which performs a test with zero strain-stress at the beginning of the calculations. The formulation of the general stress tensor 5-78 would also produce a singularity at $\varepsilon_s = 0$. A fictitious small initial value for the deviatoric strain and initial stiffness should therefore be given.

The second method for implementing a hyperviscoelastic model is through the subroutine called “UHYPER”. In this program the Helmholtz energy function and its partial derivatives have to be implemented. The viscous component is applied using the quasi linear viscoelastic postulation of uncoupled time/strain behaviour. Therefore it cannot be adopted for the implementation of the present model. In the annexes the UMAT and the translation between geotechnical and ABAQUS stress and strain tensor invariants is given, so that any hyperelastic model formulated with the geotechnical strain tensor invariants can be easily implemented with the UHYPER subroutine.
5.11 Validation of the implemented subroutine UMAT

In order to validate the implementation of the user subroutine UMAT in the finite element program ABAQUS a uniaxial constant vertical strain rate compression test was simulated; The stress tensor invariants calculated by ABAQUS were after compared to the analytical solution integrated numerically according to eq. 5-68 and 5-69 with the following parameter values:

\[
\begin{align*}
G_x &= 1.00\times10^6 \text{ kPa} \\
K_x &= 5.00\times10^7 \text{ kPa} \\
m &= 1.2 [-] \\
G_1 &= 2.00\times10^6 \text{ kPa} \\
G_2 &= 3.00\times10^6 \text{ kPa} \\
\eta_s1 &= 3.00\times10^6 \text{ kPa}\cdot\text{s} \\
\eta_s2 &= 9.00\times10^7 \text{ kPa}\cdot\text{s} \\
K_1 &= 6.00\times10^6 \text{ kPa} \\
K_2 &= 3.00\times10^6 \text{ kPa} \\
\eta_v1 &= 3.00\times10^7 \text{ kPa}\cdot\text{s} \\
\eta_v2 &= 6.00\times10^7 \text{ kPa}\cdot\text{s}
\end{align*}
\]

Fig. 5-10: Validation test for the hyperviscoelastic model: deviatoric stress – strain plot calculated by ABAQUS and analytical solution integrated numerically

Fig. 5-11 Validation test for the hyperviscoelastic model: volumetric stress – strain plot calculated by ABAQUS and analytical solution integrated numerically

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5.12 Conclusions

In spite of a lot of attention devoted in the literature to derivation of thermomechanically consistent non-linear hyperviscoelastic models, this subject proves to be a source of confusion. This confusion can be avoided if the models are derived within Ziegler’s thermomechanical framework, where both the energy and flow potentials depend on kinematic internal variables. However, the necessity to satisfy Ziegler’s orthogonality conditions can lead to the evolution equations for internal variables, which cannot be solved in the closed form, making the entire formulation extremely cumbersome and inconvenient for finite element applications.

This Chapter offers a formulation, based on the thermomechanical analysis of generalized Maxwell models, where the springs and the dashpots are assumed to exhibit a similar kind of non-linearity. While placing no significant restrictions on the type of non-linearity of the elastic and viscous components, the proposed framework allows for both the energy functions and stress-strain relationships to be expressed in a closed form. This particular form allows for an easy update of stiffness in FE calculations.

The proposed framework has been used to develop a simple thermomechanically consistent hyperviscoelastic model with pressure dependent bulk stiffness. While being based on common assumptions for the elastic component of soil behaviour, the proposed model could be particularly useful for describing the behaviour of asphalts, which exhibit no irrecoverable strains in a much larger stress range than soils. Evaluation of the model against the laboratory tests on asphalts confirmed this assumption.

The hyperelastic model with pressure dependent bulk modulus can also very well describe the mechanical behaviour of cement stabilized granular mixtures.
The hyperviscoelastic constitutive model has been implemented in the finite element program ABAQUS, to be used in the inverse analysis in the following Chapter.
Chapter 6: Field Tests Validation

6.1 Introduction

In Chapter 1 a review of tests and inverse analysis techniques used in practice showed the advantages and the limitations of the existing procedures. A new inverse analysis technique (Rabaiotti and Caprez, 2007) based on the deflection bowl measured with the Benkelman beam, showed the feasibility of estimating specific road material properties. Unfortunately this technique did not allow obtaining more than one parameter value. This limitation was due to the poor accuracy and precision of the device and to the low amount of experimental data (displacements) for the convergence of the optimization algorithm. Then again, from preliminary studies, it was observed (Chapter 2) that knowing a series of displacements defined by a raster of points on the surface would allow at least 4 parameters to be back-calculated within realistic values. Therefore a new device for measuring the deflection bowl under a real axle load in three dimensions was developed.

In Chapter 5 advanced constitutive models for materials used in road construction were developed and implemented in a FE code. They were derived for modelling the mechanical behaviour of pavement materials in laboratory uni- and triaxial compression tests. However, the fact that the derived models can describe the mechanical behaviour of these materials under simple loading conditions does not automatically guarantee that they are reliable for the field test results. The standard laboratory tests only apply one or two principal stresses in compression or extension and therefore cannot completely characterize the mechanical behaviour of the material, which is often considered to be isotropic and with
equal tension and compression stiffness. On the other hand, it is well known that compacted materials are generally cross anisotropic and that in particular bituminous mixtures have different mechanical behaviour in tension and in compression, also called in the literature bimodularity (Mamlouk and Witczak, 2002).

It is still an open question to what extent these aspects are relevant for describing the mechanical behaviour of pavements under real traffic conditions. Field tests, if adequately modeled, represent a very good basis for checking the validity of the developed constitutive models and the adopted simplifications of isotropy and homogeneity for their use in the praxis.

The objectives of this research are defined below:

- To find the constitutive model that allows achieving the best fit between measured and calculated deflection bowl.

- To assess the maximum number of parameters that the inverse analysis procedure allows back-calculating within an acceptable computational timeframe.

- To determine if the parameter values are close or at least in the same range of those obtained from inverse analysis of laboratory test results.
6.2 Introduction on inverse analysis

6.2.1 Nonlinear least squares optimization: an overview

Inverse analysis procedure based on deflection bowl (also called basin) matching is a standard optimization problem in road geotechnics, as already mentioned in Chapter 1. The parameter values of the constitutive models that describe the mechanical behaviour of the built in materials are obtained with an algorithm which minimizes the difference between analytically (or numerically) predicted and measured deflections. The most frequent parameter used to describe the difference between estimated and observed deflections is the sum of the squared errors also called residuals.

\[
SSE = \sum_{i=1}^{n} (f(x_i, \beta_i) - y_i)^2 = \sum_{i=1}^{n} r_i^2
\]

6-1

Where:

- \(f(x_i, \beta_i)\) modeled response
- \(x_i\) evaluated points
- \(\beta_i\) parameter values
- \(y_i\) observed values
- \(n\) total number of evaluated points
- \(r_i\) residuals

The methods based on the minimization of the sum of the squared errors (residuals) are called linear or nonlinear least squares (Nocedal and Wright 2006). They are called linear if the model function \(f(x_i, \beta_i)\) is a linear combination, nonlinear in case of nonlinear combination of the parameters \(\beta_i\)
respectively. Since the function $f(x_i, \beta)$ for the basin matching inverse analysis is nonlinear, a nonlinear least squares method has to be adopted. The simplest nonlinear least squares minimization algorithm is the Newton method.

The Newton method is based on the properties of the minimum of a continuous function that is a stationary point: The function which has to be minimized is approximated with the first derivative on an evaluation point (eq. 6-2), it is derived one time, set equal to zero and solved iteratively (eq. 6-3, 6-4). For the one dimensional case it is obtained:

$$f(\beta_{i,k+1}) = f(\beta_{i,k}) + \frac{d^2f}{d\beta^2}(\beta_{i,k})(\beta_{i,k+1} - \beta_{i,k})$$  \hspace{1cm} 6-2

$$f'(\beta_{i,k+1}) = f'(\beta_{i,k}) + \frac{d^3f}{d\beta^3}(\beta_{i,k})(\beta_{i,k+1} - \beta_{i,k}) = 0$$  \hspace{1cm} 6-3

$$\beta_{i,k+1} = \beta_{i,k} + \left[\frac{d^3f}{d\beta^3}(\beta_{i,k})\right]^{-1} f'(\beta_{i,k})$$  \hspace{1cm} 6-4

In more dimensions this means calculating the Jacobian (eq. 6-5) and the Hessian (eq. 6-6) of the function:

$$\nabla f(\beta) = 2\sum_{i=1}^{n} r_i(\beta) \nabla r_i(\beta) = 2J(\beta)^{\top} r(\beta)$$  \hspace{1cm} 6-5

$$\nabla^2 f(\beta) = 2\sum_{i=1}^{n} \nabla r_i(\beta) \nabla r_i(\beta)^{\top} + 2\sum_{i=1}^{n} r_i(\beta) \nabla^2 r_i(\beta)^{\top} =$$

$$= 2J(\beta)^{\top} J(\beta) + 2\sum_{i=1}^{n} r_i(\beta) \nabla^2 r_i(\beta)^{\top}$$  \hspace{1cm} 6-6

A very important performance indicator for optimization algorithms is the rate of convergence $\rho$, defined in eq. 6-7. It defines the rate of the searching strategy to the convergent solution $\overline{\beta_i}$.  

6-146
\[ \| \beta_{i,k+1} - \bar{\beta}_i \| \leq c \| \beta_{i,k} - \bar{\beta}_i \|^p \] 6-7

Where:

- \( \beta_{i,k} \) : guess parameter
- \( \bar{\beta}_i \) : final convergent parameter value
- \( c \) : constant
- \( p \) : rate of convergence

Gauss made an important modification to this method, considering an approximation of the Hessian, as product of mixed derivatives of the function: in this technique the second term \( 2 \sum_{i=1}^{n} r_i(\beta) \nabla^2 r_i(\beta)^T \) in eq. 6-6 is disregarded. This simplification has many advantages:

- The Hessian is, for most of the practical applications, too time-consuming to be computed numerically.
- The convergence rate is comparable to the Newton method (quadratic).
- The search direction is a descent direction.
- The method reduces to a linear least square problem: the techniques here developed for the least square problems are still valid (Nocedal and Wright, 2006).

Marquardt (1944) and Levenberg (1963) proposed a correction for the Gauss Newton (GN) algorithm (called hereafter LM method), introducing a damped term in the approximated Hessian (eq. 6-9). The algorithm allows achieving
convergence when GN fails: this happens for instance when the approximation of the Hessian in the GN is nearly singular (Fan and Yuan, 2005), or rank deficient (Nocedal and Wright, 2006).

Hereafter the search strategy for the GN and the LM methods are described.

The guess parameter $\beta_i$ at the step $k + 1$, if $SSE = \frac{1}{2} \sum_{i=1}^{n} r_i^2$, is defined by the GN and the LM method by equation (6-8) and (6-9) respectively:

**Gauss Newton**

$$\beta_{i,k+1} = \beta_{i,k} - \left( J_r^T J_r \right)^{-1} J_r^T r$$  \hspace{1cm} 6-8

**Levenberg Marquardt**

$$\beta_{i,k+1} = \beta_{i,k} - \left( J_r^T J_r - \lambda \text{diag}(J_r^T J_r) \right)^{-1} J_r^T r$$  \hspace{1cm} 6-9

Where:

- $\beta_i = (a, b, c, d)$ vector of input guess parameters $a, b, c, d$
- $r(\beta_i) = y_i - f(x_i, \beta)$ vector of the errors
- $y_i$ observed values
- $f(x_i, \beta)$ calculated values
- $J_r$ Jacobian of $r(\beta)$
- $\lambda$ damping parameter

One critical point of the original LM method is the choice of $\lambda$, which is found using a heuristic algorithm (Moré, 1978). A more efficient and theoretically robust...
formulation for the algorithm which dictates the choice of \( \lambda \) was developed by Moré (1978). In the modern formulation the LM algorithm, if it does not degenerate into the GN method (\( \lambda = 0 \)) when approaching the solution, is the first example of the so-called trust region methods (Nocedal and Wright 2006). Trust region methods define a region around the current iterate, within which they trust the local objective function approximation and then choose the step to be the approximate minimizer of the model in this region. A deterministic framework for the LM algorithm based on the trust region theory can be found in Nocedal and Wright (2006). The LM algorithm adopted in this work is the one which is implemented in MATLAB (2008): a description of the robust updating procedure of the parameter \( \lambda \), which refers to the Moré formulation, can be found in the MATLAB optimization toolbox manual.

The LM and GN searching methods can be described from a more practical point of view by means of a simple example: let the nonlinear function
\[
y = \frac{\beta_1 x}{\beta_2 + x}
\]
be the model which describes an experimental data set.

The Jacobian of the residuals becomes:
\[
J_r = \begin{bmatrix}
-x & \frac{\beta_1 x}{(\beta_2 + x)^2} \\
\end{bmatrix}
\]

The parameter \( \beta_{1,k+1} \) for the next iteration is calculated by solving eq. 6-8 or 6-9.

The iteration is stopped according to a specific user-defined criterion (i.e. tolerance values for the objective function). The iteration procedure described by eq. 6-8 and 6-9 can easily be implemented in an electronic data sheet, e.g. Microsoft Excel. If the matrix \( J_r^T J_r \) is more complex than in the example and therefore difficult to invert, eq. 6-8, 6-9 can be solved in the implicit form using Cholesky decomposition.

The experimental results can be fictitiously generated by considering \( \beta_1 = 3 \) and \( \beta_2 = 5 \) at the evaluation points \( x \) from 0 to 10. It is observed that submitting initial guess parameter \( \beta_1 = 1 \), and \( \beta_2 = 50 \), only the LM algorithm converges to the exact
solution. The convergence is obtained by choosing adequate value for the damping parameter, in this case $\lambda_1 = 20, \lambda_2 = 5$ for the first and the second step (Fig.6-1).

![Comparison of gradient algorithms](image)

Fig.6-1: Comparison of the convergence properties of GN and LM optimization algorithms on a specific problem

For GN and LM algorithms the rate of convergence is between linear and quadratic (as in the Newton Method), this means that the parameter $p$ in equation 6-7 has values between 1 and 2.

The damping parameter was adjusted manually in order to achieve a convergent solution, and it is therefore heuristic.

The LM algorithm usually converges for standard nonlinear optimization problems. Nevertheless it could not converge for highly nonlinear problems: i.e. when the objective function is discontinuous, nondifferentiable, or the results are stochastically distributed.
Inverse analysis of complex boundary value problems, because of sophisticated constitutive models and complex geometries, may produce non-smooth objective functions, and therefore Jacobian-based methods could fail in finding a convergent solution.

Methods which do not require Jacobian calculations (also called derivative-free) are common in inverse analysis based on basin matching: e.g. the genetic algorithms (GA) (Reddy et al., 2004), or neural networks (ANN) (Meier and Rix, 1994). In these works a derivative-free method which belongs to another family of derivative-free methods is adopted, the MADS (Mesh Adaptative Direct Search) developed by Audet and Dennis (2004). These algorithms are already implemented in the program MATLAB (2008). The MADS method belongs to the family called Pattern Search (PS), (Nocedal and Wright, 2006). It is worth to observe that PS methods have a solid mathematical formulation, while GA and ANN are heuristic, and are not even cited in the state of the art book of Nocedal and Wright (2006).

The main advantage of the PS methods is that they perform a search using a “pattern” of points that is independent of the objective function and global convergence is guaranteed (Torczon, 1997; Lagarias et al., 1998; Abramson and Audet, 2005).

Detailed theoretical derivation of the method can be found in the literature (Abramson and Audet, 2005, Audet and Dennis, 2006; Audet and Orban, 2006). A simplified explanation is given in the following paragraph.

MADS consists of a two-step optimization process, called “search step” and “poll step”. During the “search step”, the objective function is evaluated within different set of parameters, whose values must belong to “the mesh”. The mesh, defining the allowable trial parameter value set, is constructed from a finite set of directions $n_D$ and scaled by mesh size parameter $\Delta_k^m$.

$$\Delta_{k+1}^m = \tau^{w_k} \Delta_k^w$$ for some $w_k \in \{(a) \{0,1,\ldots,w^+\} \cup (b)\{w^-, w^- + 1,\ldots,-1\}\}$
Where:

\[ \Delta_k^m \quad \text{mesh size parameter} \]

\[ \tau \quad \text{fixed rational number} \]

\[ w^- \quad \leq -1 \]

\[ w^+ \quad \geq 0 \]

If the search step is successful, (a) produces a mesh expansion, otherwise (b) produces a mesh contraction (refinement).

The main task of the search step is to find a new point (parameter set) that has a lower objective function value than the best current solution, called “incumbent”. When the incumbent is replaced, so the objective function is \( f(x_{k+1}) < f(x_k) \), then \( x_{k+1} \) is said to be an “improved mesh point” (parameter value set).

Whenever the search step fails to generate an improved mesh point, then the mesh is refined and the poll step is invoked. The set of trial points during the poll step is called the “frame” \([p_i]\).

A new parameter \( \Delta_k^p \neq \Delta_k^m \) dictates the magnitude of maximal feasible distance from the last incumbent solution for the frame on the new generated finer mesh. The mesh size is always smaller than the poll size \( \Delta_k^m \leq \Delta_k^p \) and the important following condition should be satisfied:

\[
\lim_{k \in K} \Delta_k^m = 0 \quad \text{if and only if} \quad \lim_{k \in K} \Delta_k^p = 0 \quad \text{for any infinite subset of indices} \quad k.
\]

(Audet and Dennis, 2006)

In the original GPS method, the mesh and the poll size are equal, having the disadvantage of reduced polling directions.

Fig. 6-2 (a, b, c) shows the poll step for a two dimensional problem, after the search step has failed and the mesh is refining. The mesh size \( \Delta_k^m = 1 \) and
Chapter 6: Field Tests Validation

\[ n_D = 8 \], defines a raster of 8 points with the following coordinates: \([1, 0]; [1, 1]; [0, 1]; [-1, 1]; [0, -1]; [-1, -1]; [0, -1]; [1, -1]\). The poll step size is given by: \[ \Delta^p_k = n\sqrt{\Delta^n_k} \], \[ n = 2 \) (2 dimensional mesh) and the frame is equal to \([p_1, p_2, p_3]\). It can be observed that the poll size defines a boundary region (dark contour) related to the last improved mesh size, and it activates only when the mesh cannot be improved (search step fails, Fig. 6-2a, poll size=dark contour in Fig. 6-2b). The trial points (3) are chosen randomly on the new finer mesh, inside the boundary. If the poll step fails, it dictates a new mesh refinement and a new poll size (Fig. 6-2c); the mesh becomes then asymptotically dense till tolerance values are reached, and the optimization stops. The algorithm is still under development (Abramson et al., 2008), but the results achieved during the simulation tests (Fig.6-4) have proven its efficiency for the proposed inverse analysis technique.

\[ \Delta^m_k = 1, \Delta^p = 2 \quad \Delta^m_k = \frac{1}{4}, \Delta^p = 1 \quad \Delta^m_k = \frac{1}{16}, \Delta^p = \frac{1}{2} \]

Fig.6-2 (a, b, c): Search strategy of the MADS optimization algorithm: the mesh (the raster of points, parameter value set, defined by the crossing of the lines) and the poll size (dark contour) during the search for a incumbent solution (after Audet and Dennis, 2006). \[ \Delta^m = \text{mesh size}, \Delta^p = \text{poll size}, \Delta^m_k = n\sqrt{\Delta^n_k}, n_D = 8, n = 2, \text{ frame } p = [p_1, p_2, p_3] \]
6.2.2 Implementation and linking subroutines

Pattern search and Jacobian algorithms are implemented in the commercial software MATLAB. MATLAB (Matrix laboratory) at the time of this work (2008) is one of the most widely used programming languages for mathematical applications, and its optimization toolboxes are steadily growing. The implemented subroutine starts the FE calculation with input parameters optimized by the strategy according to the chosen algorithm (Fig.6-3). The results (vertical displacements) used for the inverse analysis are extracted by a FORTRAN subroutine from an encrypted file generated by the FE tool ABAQUS (Ozan, 2008).

Fig.6-3: Optimization routine main scheme
6.2.3 Choice of the optimization algorithm: preliminary study

The LM and MADS, together with another PS algorithm, the Nelder Mead (NM) (Lagarias, 1998) were tested on the inverse analysis procedure by comparing their convergence rate on a simulated basin matching optimization. The deflection bowl was generated with an arbitrary set of parameters and hereafter considered as real measurements. The convergence rate and the accuracy of the solution (SSE for the displacements and the searched moduli values) for different algorithms are shown in Fig.6-4.

It was found that the MADS algorithm converges after a lower number of iterations than the LM and the NM. A LM routine implemented in FORTRAN, from the repository library MINPACK (Ozan, 2008), was also adopted, showing very similar results to the one implemented in MATLAB (Fig. 6-4). Nevertheless the MADS algorithm achieved the best correlation with the values adopted for the simulation of the measured results (Fig.6-4, z axis).
The MADS algorithm can be very slow if the number of function evaluations for a successful iteration becomes too large. Therefore MADS and LM were used in all the calculations in this chapter.

6.3 Inverse analysis of the EPFL (LAVOC) test site results

6.3.1 Introduction

Two tests (A9 and A12) were chosen for the inverse analysis: two different loading conditions, 60 and 120 kN axle load, tire inflating pressure of 850 kPa at constant base course layer temperature of 24°C. Unfortunately, due to limitations of the existing facility, it was technically impossible to precisely move the wheel while the beam was in transversal position (Fig. 6-5). The entire transversal form of the deflection was therefore measured in only 2 dimensions (Fig.6-5).

Fig.6-5: ETH Delta positioning for the transversal and longitudinal measurement of the deflection bowl (wheel rolls only when the beam is positioned longitudinal, otherwise the load is applied by the wheel which stays in the same position)
The three dimensional measurement was carried out with the beam in longitudinal position but only for the external part of the deflection bowl, which is in this case not symmetric.

The load was applied by quick loading and unloading in the middle of the test field for the transversal and also with rolling wheel, for the longitudinal measurement.

### 6.3.2 Forward modelling

![Fig.6-6: Boundary value problem](image)

The boundary value problem is completely defined, thanks to symmetries of the real problem, by one fourth of the structure only (Fig.6-6). This allows for the inverse analysis being completed in a reasonable time, depending mostly on the
number of parameters to be optimized. The footprint geometry of the tire was modeled as a rectangle, very close to the one measured for each loading amplitude and inflating pressure (Perret, 2003). The contact tire/pavement was modeled with the simplification of uniformly distributed pressure. The total load was then divided by the footprint surface and the contact pressure obtained. The calculated contact pressures, for an inside inflation of 850 kPa, are: 600 kPa for 60 kN axle load, and 850 kPa for 120 kN axle load. The total number of elements is 4712, 8-node linear brick, hybrid, constant pressure (C3D8H) (Fig.6-7). Hybrid elements are recommended by ABAQUS for nonlinear hyperelastic constitutive models.

Fig.6-7: Mesh generated for the test field at EFPL (the different colors show the 3 modeled layers)

The layer interface was modeled with Mohr Coulomb failure criterion, with cohesion $c$ =200 KPa and a friction angle $\phi$ = 38° and no slip between the layers till failure occurs.

It is a known fact from the literature (Romanoschi and Metcalf, 2003) that a wrong constitutive model for the interface could affect the accuracy of back-
calculated moduli, at least for inverse analysis of FWD test results. In this work it is assumed that the interface between layers behaves according to the Mohr-Coulomb criterion, which is, of course, only a rough approximation.

The constitutive model for the bound and unbound materials is the nonlinear hyperelastic model derived in Chapter 5. Since no viscosity effects were measured, no time dependent component was introduced in the constitutive models. The stress formulation for the hyperelastic model is defined by the following stress invariants:

\[
\frac{p(\varepsilon_v)}{p_r} = \left( \frac{K}{p_r} \varepsilon_v \right)^m \quad 6-13
\]

\[
q(\varepsilon_s) = 3G\varepsilon_s \quad 6-14
\]

where:

- \(p\)  volumetric stress invariant
- \(q\)  deviatoric stress invariant
- \(\varepsilon_v, \varepsilon_s\)  total volumetric and deviatoric strain
- \(K\)  bulk modulus
- \(p_r\)  reference pressure
- \(G\)  shear modulus
- \(m\)  nonlinear parameter (for the volumetric stress, eq. 6-13)
6.3.3 Inverse Analysis

The three-dimensional measurements missed one important part of the deflection bowl, the one located between the wheels. The inverse analysis was therefore carried out with the two dimensional measurements. The transversal measurements of the deflection bowl for the two different axle loads 60 kN and 120 kN and inflating pressure 850 kPa, which correspond to contact pressure of 600 and 850 kPa respectively, and temperature of 24°C, were used (test A9 and A12).

The same set of parameter values was optimized for the tests under different loading conditions. The idea was to evaluate the effect of nonlinear mechanical properties of the material on the shape of the deflection bowl: if the material behaviour is not linear the calculation carried out with linear elastic constitutive models would over or underestimate the deflection bowl in one out of two loading conditions. Another important issue is given by the increase in the overdetermination of the inverse analysis problem, due to the optimization based on two different tests, which allow obtaining convergence for a large number of parameters, even with two dimensional measurements.

The 2 dimensional longitudinal form of the deflection bowl, measured under the same loading conditions (the wheel stays in the same position), was then compared to the predicted one. In this case the parameters values used for the FE calculation were the values obtained from inverse analysis of the transversal measurements (Fig.6-9). The interface parameter values were constant for all the layers, with a cohesion $c = 200$ KPa and a friction angle $\phi = 38^\circ$. 

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In Table 6-1 the back-calculated parameters values for the linear elastic and nonlinear hyperelastic constitutive modelling are summarized.

Table 6-1: Back calculated layer pavement moduli

<table>
<thead>
<tr>
<th>Layer</th>
<th>Parameter</th>
<th>Linear elastic model</th>
<th>Nonlinear hyperelastic model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base course</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$G$</td>
<td>$2.09 \times 10^6$ [kPa]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>$7.87 \times 10^6$ [kPa]</td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>$G$</td>
<td>$1.68 \times 10^5$ [kPa]</td>
<td>$1.67 \times 10^5$ [kPa]</td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>$3.29 \times 10^5$ [kPa]</td>
<td>$2.02 \times 10^5$ [kPa]</td>
</tr>
<tr>
<td></td>
<td>$m$</td>
<td>$1.6954$ [-]</td>
<td>$1.6954$ [-]</td>
</tr>
<tr>
<td>Subgrade</td>
<td>$G$</td>
<td>$8.2 \times 10^4$ [kPa]</td>
<td>$7.18 \times 10^4$ [kPa]</td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>$21.35 \times 10^4$ [kPa]</td>
<td>$7.05 \times 10^4$ [kPa]</td>
</tr>
<tr>
<td></td>
<td>$m$</td>
<td>$1.5$ [-]</td>
<td></td>
</tr>
</tbody>
</table>

*) Value obtained with full bound layer interface

In general the fit between experimental and calculated deflection bowl is very good and there is no improvement by adopting a nonlinear hyperelastic constitutive model (Fig.6-8, 6-9).
Fig. 6-8: Transversal measured and calculated deflection bowl for axle loads equal to 60 and 120 kN. The calculations are carried out with linear elastic (LE) and nonlinear hyperelastic (HE) constitutive models.

Fig. 6-9: Longitudinal measured and calculated deflection bowl for two axle loads equal to 60 and 120 kN. The calculations are carried out with linear elastic (LE) and nonlinear hyperelastic (HE) constitutive models.
6.3.3.1 Precision

The precision of the values obtained with the LM and MADS optimization algorithms, described in Table 6-2, is at least as important as the adoption of sophisticated constitutive models. The precision is described by a confidence interval, defined a maximum and a minimum feasible parameter value. The defined range is obtained for parameter combinations which give nearly the same values of the objective functions, optimized by two different algorithms.

It has to be noticed that each variation is dependent from the other values, meaning that there is only one combination of the parameters giving the values for the plotted objective function. This is the reason why the values obtained by the MADS and the LM algorithms are different, since they produce different combinations. The MADS algorithm converged after 108, the LM after 63 function evaluations assuming the same initial guess parameters.

In general the precision is lower for the top layers, which is a logical consequence of the FE calculation (Fig.6-10, 6-11), which is not very sensitive to the variation of the base course material parameters (see also Fig. 6-16, 6-17). The precision of the value of the Poisson’s ratio is rather low for the base course and the base layer and is very high for the subgrade (±12%) (Table 6-2, Fig.6-11, 6-13, 6-15). The precision of the values of the shear moduli is high for the base course layer, very high for the base and subgrade layer (±15%) (Table 6-2, Fig.6-10, 6-12, 6-14).
Table 6-2: Precision

<table>
<thead>
<tr>
<th>Layer</th>
<th>Confidence</th>
<th>( G ) [kPa]</th>
<th>( \nu ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base course</td>
<td>Min</td>
<td>( 0.8 \times 10^6 )</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>( 2.3 \times 10^6 )</td>
<td>0.42</td>
</tr>
<tr>
<td>Base</td>
<td>Min</td>
<td>( 1.6 \times 10^5 )</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>( 2.2 \times 10^5 )</td>
<td>0.35</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Min</td>
<td>( 6 \times 10^4 )</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>( 8.3 \times 10^4 )</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Fig. 6-10: Precision of the shear modulus, base course layer

Fig. 6-11: Precision of the Poisson’s ratio, base course layer

Fig. 6-12: Precision of the shear modulus, base layer

Fig. 6-13: Precision of the Poisson’s ratio, base layer
6.3.3.2 Accuracy

Extensive laboratory tests were carried out by the EPFL (LAVOC) and their partners for the research project “Development, assessment, and application of innovations for interurban infrastructures” (Bueche and Vanelstraete, 2006). Particularly interesting for this project are the two points bending tests carried out with the bituminous mixture of the base course layer and the plate load test results on the unbound layers (base and subgrade). The stiffness modulus was determined according to EN12697-26 by the two-point bending method on trapezoidal specimens. The measurements were performed at different temperatures (between -20 and 30°C) and frequencies (between 1 and 30 Hz).

For the linear elastic constitutive model the back-calculated shear $G$ and bulk modulus $K$ are equivalent to an elastic modulus $E = 5758$ MPa and Poisson’s ratio $\nu = 0.378$. This value agrees very well to the elastic modulus (Bueche and Vanelstraete, 2006) $E(24^\circ C)=6010$ MPa, which was measured with the two point bending test at 24°C and with a low frequency cyclic loading of 1Hz. The elastic moduli for the base and subbase layer were determined from compression modulus obtained from plate load tests (Perret 2003), and calculated according to the isotropic elastic half space theory (Boussinesq). The back-calculated($I$), and from the plate load tests($P$) obtained linear elastic moduli for the unbound
materials differ by a factor of 2: $E^I_{(base)}=432\text{ MPa}$, $E^P_{(base)}=270\text{ MPa}$ (ratio =1.6) and $E^I_{(subgrade)}=218\text{ MPa}$, $E^P_{(subgrade)}=90\text{ MPa}$ (ratio = 2.4).

Bulk and shear moduli for the base and the subbase calculated from the elastic moduli\(^3\) and from inverse analysis can be also compared: $K^P_{(base)}=300\text{ MPa}$, $K^I_{(base)}=329\text{ MPa}$; (ratio = 1.1); $K^P_{(subgrade)}=100\text{ MPa}$, $K^I_{(subgrade)}=213\text{ MPa}$; (ratio = 2.13); $G^P_{(base)}=100\text{ MPa}$, $G^I_{(base)}=169\text{ MPa}$, (ratio = 1.69); $G^P_{(subgrade)}=33\text{ MPa}$, $G^I_{(subgrade)}=82\text{ MPa}$, (ratio = 2.5). The difference in the linear elastic moduli for the unbound materials can be explained after taking into account the following factors:

- Post compaction occurred under repeated loading, since the measurements were carried out at the top of the unbound layers just after their construction.

- The stress dependency of the moduli: the compression modulus is in general nonlinear, and it is considered constant since it is calculated as secant modulus on a defined stress region (SN 670 317b).

- The value of the Poisson’s ratio was assumed by Bueche and Vanelstraete (2006) and not calculated. The values of the bulk modulus for the base layer obtained with inverse analysis correlate better with the plate load test results: $\frac{K^I_{(base)}}{K^P_{(base)}}=1.1$.

- The precision of the inverse analysis, summarized in the accuracy table (Table 6-3):

\(^3\) (from the plate load test, defined by $p=\text{plate}$, with $\nu=0.35$ from Bueche and Vanelstraete, 2006)
Table 6-3: Accuracy of the inverse analysis, $I = \text{inverse analysis of the deflection bowl}$, $T = \text{two point bending and plate load test}$.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Confidence</th>
<th>$G^I$ [kPa]</th>
<th>$I^I$ [-]</th>
<th>$G^T$ [kPa]</th>
<th>$T^I$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base course</td>
<td>Min</td>
<td>$0.8\cdot10^6$</td>
<td>0.2</td>
<td>$2.14\cdot10^6$</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>$2.3\cdot10^6$</td>
<td>0.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>Min</td>
<td>$1.6\cdot10^5$</td>
<td>0.15</td>
<td>$1.0\cdot10^5$</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>$2.2\cdot10^5$</td>
<td>0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subgrade</td>
<td>Min</td>
<td>$6\cdot10^4$</td>
<td>0.34</td>
<td>$3.3\cdot10^4$</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>$8\cdot10^4$</td>
<td>0.42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.3.3.3 Sensitivity analysis (parametric study)

The influence (sensitivity) of the different parameters on the form of the deflection bowl was determined with a parametric study. A standard parameter set was chosen and the influence of each parameter on the form of the deflection bowl was studied (Fig.6-16 to 6-25).

In Table 6-4 the parameter values and their respective coding are summarized. The results show that

- Due to the relatively small thickness of the layer, the only sensitive parameter is the shear modulus (Fig.6-16 to 6-18).

- The parameters of the layers become more important with increasing depth

- The nonlinear parameter $m$ is very important for the subgrade, which becomes less compressible and produces also heaving of the surface (Fig.6-24).
- Setting cohesion \( c = 200 \text{ KPa} \) and a friction angle \( \phi = 38^\circ \) or no friction and no cohesion between the layers produce similar vertical displacements. On the contrary if full bound between the layers is modeled, the displacements decrease significantly (Fig.6-25).

Table 6-4: Sensitivity analysis, coding for the tests: A1 means 0 + parameter change only for G, etc.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Test</th>
<th>( G ) [kPa]</th>
<th>( \nu ) [-]</th>
<th>Test</th>
<th>( m ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base course</td>
<td>0</td>
<td>( 2.09 \times 10^6 )</td>
<td>0</td>
<td>0.3781</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>( 3.09 \times 10^6 )</td>
<td>A3</td>
<td>0.2781</td>
<td>A5</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>( 4.09 \times 10^6 )</td>
<td>A4</td>
<td>0.4781</td>
<td>A6</td>
</tr>
<tr>
<td>Interface(*)</td>
<td>0</td>
<td>200</td>
<td>38</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D1</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>( \infty )</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0</td>
<td>( 1.68 \times 10^5 )</td>
<td>0</td>
<td>0.28</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>( 2.18 \times 10^5 )</td>
<td>B3</td>
<td>0.18</td>
<td>B5</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>( 2.68 \times 10^5 )</td>
<td>B4</td>
<td>0.38</td>
<td>B6</td>
</tr>
<tr>
<td>Subgrade</td>
<td>0</td>
<td>( 9.22 \times 10^3 )</td>
<td>0</td>
<td>0.27</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>( 14.22 \times 10^3 )</td>
<td>C3</td>
<td>0.17</td>
<td>C5</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>( 29.22 \times 10^3 )</td>
<td>C4</td>
<td>0.37</td>
<td>C6</td>
</tr>
</tbody>
</table>

(*) The same interface has been modeled between the base and the subgrade.
Chapter 6: Field Tests Validation

Fig. 6-16: Shear modulus

Fig. 6-17: Poisson's ratio

Fig. 6-18: Nonlin. parameter

Fig. 6-19: Shear modulus

Fig. 6-20: Poisson's ratio

Fig. 6-21: Nonlin. parameter
Fig. 6-22: Shear modulus

Fig. 6-23: Poisson’s ratio

Fig. 6-24: Nonlin. parameter

Fig. 6-25: Interface

(*) The experimental results are given by the plotted points: $\Delta = 60 \text{ kN}$, $\square = 120 \text{ kN}$. 

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6.3.4 Discussion

The EPFL test site represents the ideal link between laboratory and field test conditions, due to the very well defined boundaries and the real loading conditions. The inverse analysis results are not only highly precise, especially for the unbound layers, but also accurate. The elastic moduli calculated with the inverse analysis agreed very well to those obtained from the two point bending test. It is worth noticing that in this test the load was applied on the same location and at a relatively low frequency (1 per 10-15 sec).

The elastic moduli calculated from the plate load test results are slightly smaller than the back-calculated, which is realistic since these measurements were carried out just after the layer compaction. Moreover the ETH Delta tests were carried out after the test section was loaded by 300,000 load repetitions.

The moduli obtained from the plate load tests are calculated using the theory of homogeneous elastic half space. This is a strong assumption, especially for the tests on the base layer, which lies on a subgrade with different mechanical properties.

Less important (but probably still present!) is the effect of nonlinear behaviour of the bulk modulus for the unbound materials. On the other hand the value of the linear elastic bulk modulus (compressibility) agrees very well for both calculation procedures.

The adoption of constitutive nonlinear elastic models was found unnecessary for achieving a better fit between measured and back-calculated displacements. In particular linear elasticity allows predicting vertical displacements with different loading conditions within the same set of parameter values. The inverse analysis test results lead to the conclusion that, for these particular test results, the assumption of linear elasticity is sufficient for describing the deflection bowl.
6.4 Inverse analysis of the Hinwil test site results

6.4.1 Introduction

Inverse analysis was carried out only for two tests, at 0 and 100,000 load repetitions (overruns) respectively. The measurements were made on the section loaded by the overruns and on an intact section nearby. This allowed comparing the results for a similar damaged and undamaged section under comparable temperature and moisture conditions. The load applied to the surface was a real axle load of 100 kN.

The beam was positioned in transversal direction to the loading path (Fig. 6-26) and the deflection bowl was measured during loading and unloading. The inverse analysis was carried out using the measurements obtained during unloading. Only the reversible displacements were adopted for the inverse analysis procedure.

Fig.6-26: ETH Delta positioning for the measurement of the deflection bowl
6.4.2 Forward modelling

In general, the field tests carried out in Hinwil (Switzerland) represented a considerable challenge for the measurements, especially due to the very small displacements and the very wide deflection bowl.

The geometry and the boundary conditions of the finite element model are shown in Fig.6-27. The wearing and the base course layers have small thicknesses 8 and 4 cm respectively, therefore they were considered as one layer, also as a consequence of the lesson learned from the inverse analysis of the EPFL test results. The parameters of the constitutive models become then enough sensitive for the optimization by the LM and MADS algorithms.

![Diagram of the boundary value problem](image-url)
The boundary value problem is completely defined, thanks to symmetries of the real problem, by modelling one fourth of the structure only (Fig.6-27). The elements (6427) are 6-node linear triangular, hybrid, constant pressure prisms (C3D6H) (Fig.6-28). The footprint geometry was measured on site, and reproduced in the model with the simplification of full contact and uniformly distributed pressure, equal to 340 kPa. Note that the contact pressure of a twin tire is much lower than the one applied by the supersingle tire: i.e. in the EPFL test with similar loads the super single contact tire pressure was between 600 and 800 kPa. Two main longitudinal cracks were introduced in order to reproduce the form of the deflection bowl, as it will be explained further below (Fig.6-33).

The cracks were modeled considering the pavement structure made by blocks, connected by an interface, defined by Mohr Coulomb failure criterion having the following arbitrary values: cohesion $c = 100$ kPa, friction angle $\phi = 38^\circ$.

The interface between the layers was modeled with "realistic" arbitrary values: between base course and 1$^{\text{st}}$ base layer, $c = 100$ kPa, friction angle $\phi = 38^\circ$; between 1$^{\text{st}}$ base and 2$^{\text{nd}}$ base layer, $c = 100$ kPa, friction angle $\phi = 27^\circ$ and between base and subgrade layer, $c = 100$ KPa, friction angle $\phi = 17^\circ$. No slip between the layers takes place before failure occurs.
The hyperelastic constitutive model described by eq. 6-13, and 6-14 was implemented for describing the mechanical properties of the built in road materials.

### 6.4.3 Inverse Analysis

The inverse analysis carried out on the field test results at 0 load repetitions was carried out with the MADS and the LM algorithm. The initial guess was determined before starting the inverse analysis in order to have guess values in the range of the convergent solution. The MADS and the LM algorithm found different local minima of the objective function, as for the inverse analysis carried out on the EPFL test results. The parameter optimization for the test field results after 100,000 load repetitions was manually adjusted, since the problem was too complex to be handled by the algorithms.
The inverse analysis carried out on the track after 100,000 overruns showed that a simple reduction of the moduli values does not result in a reasonable fit for the deflection bowl (Fig.6-31). Therefore cracks were introduced in the FE model. A forensic investigation, which consists of trenching the pavement and extracting a slab, confirmed that the cracks effectively developed (Fig. 6-30). The results show that a good fit can be achieved only if both the crack formation and the reduction of the moduli values are enabled (Fig. 6-31, 6-32). In Table 6-5 the new values obtained for the moduli are summarized. The results from the inverse analysis show that the stiffness of the base layers and of the subgrade under the wheel path dramatically decreased.
The very low stiffness value of the subgrade layer obtained from the backcalculation (Table 6-5) is probably due to the formation of a void lens under the wheel path, caused by the repeated loading transmitted by the upper layer. Furthermore, the FEM results show that the formation of the two main cracks is
problematic for the road structure, since the main deformations are taking place in the subgrade (Fig. 6-33).

The inverse analysis for the measurements after 100,000 overruns does not produce reliable results, since the FE model is not able to model the measured deflections, mainly because of the influence of the inhomogeneous cracks in the base layers, which highly influence the form of the deflection bowl.

Fig. 6-31: Measured and calculated deflection bowl obtained from the modeling with deteriorated layer materials after 100,000 overruns
Fig. 6-32: Measured and calculated deflection bowl obtained from the modeling of longitudinal cracks and deteriorated layer materials after 100,000 overruns.

Fig. 6-33: Maximal principal strains and deflection bowl formation mechanism for 100,000 overruns.
6.4.3.1 Precision

The precision of the back-calculated parameters for the intact pavement is summarized in Table 6-6 (at 0 load repetitions). The precision is described by the maximum and minimum values of the back-calculated parameters, within small range values of the objective functions. The results show a relatively low variability, apart from the nonlinear parameter \( m \) in the first base layer material.

Fig. 6-34 to 6-41 show the search history for both the MADS and LM algorithm.

Table 6-6: Precision

<table>
<thead>
<tr>
<th>Layer</th>
<th>Confidence</th>
<th>( G ) [kPa]</th>
<th>( K ) [kPa]</th>
<th>( m ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base course</td>
<td>Min</td>
<td>1.72( \times )10^6</td>
<td>8.02( \times )10^6</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.85( \times )10^6</td>
<td>8.64( \times )10^6</td>
<td>1.1</td>
</tr>
<tr>
<td>Base1</td>
<td>Min</td>
<td>0.5( \times )10^6</td>
<td>1.7( \times )10^6</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.6( \times )10^6</td>
<td>4.8( \times )10^6</td>
<td>5</td>
</tr>
<tr>
<td>Base2</td>
<td>Min</td>
<td>1.27( \times )10^6</td>
<td>3.81( \times )10^6</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.37( \times )10^6</td>
<td>4.11( \times )10^6</td>
<td>1.3</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Min</td>
<td>1.55( \times )10^4</td>
<td>3.35( \times )10^4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>1.56( \times )10^4</td>
<td>3.38( \times )10^4</td>
<td>2</td>
</tr>
</tbody>
</table>
Chapter 6: Field Tests Validation

Fig. 6-34: Precision of the shear modulus, base course layer

Fig. 6-35: Precision of the bulk modulus, base layer

Fig. 6-36: Precision of the shear modulus, first base layer

Fig. 6-37: Precision of the bulk modulus, first base layer

Fig. 6-38: Precision of the shear modulus, second base layer

Fig. 6-39: Precision of the bulk modulus, second base layer
6.4.3.2 Accuracy

The parameters for the nonlinear hyperelastic constitutive model of the base layer materials were also determined with inverse analysis of the uniaxial compression test results (Chapter 5). The laboratory test results on the base course layer material were not used for the deriving parameters, since their values do not play a role in describing the development of the deflection bowl after repeated loading. The comparison between inverse analysis of laboratory and field test results is shown in Table 6-7.

Table 6-7: Accuracy for the stabilized layer material. I = inverse analysis of the deflection bowl, T = uniaxial compression test.

<table>
<thead>
<tr>
<th>Layer</th>
<th>( K^T ) [kPa]</th>
<th>( G^T ) [kPa]</th>
<th>( m^T ) [-]</th>
<th>Confidence</th>
<th>( K^I ) [kPa]</th>
<th>( G^I ) [kPa]</th>
<th>( m^I ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base 1(*)</td>
<td>3.2 ( \times 10^6 )</td>
<td>6.23 ( \times 10^6 )</td>
<td>1.369</td>
<td>Min</td>
<td>1.8 ( \times 10^6 )</td>
<td>0.5 ( \times 10^6 )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Max</td>
<td>4.8 ( \times 10^6 )</td>
<td>1.6 ( \times 10^6 )</td>
<td>5</td>
</tr>
</tbody>
</table>

(*) average values
It can be observed that the values for the bulk modulus are of the same order of magnitude, but the values for the shear modulus differ from one to one third of the order of magnitude.

There are 2 possible reasons for this discrepancy:

- The specimen tested in the laboratory was intact, while the material in the field was probably split in blocks already at 0 overruns. These main cracks have no effect on the compressibility of the material, but highly affect the shear stiffness for the total layer.

- The friction between loading plate and specimen cannot be neglected since the small ratio height/diameter (1:1) of the specimen: this has led to a slight underestimation of the total displacement due to the barrel effect, as it was shown in Chapter 5. This leads to an overestimation of the moduli.

6.4.4 Discussion

The measurements during Hinwil test results were very difficult to obtain, due to the very small displacements and the large extension of the deflection bowl. It must be stated that the device precision (2/100 mm) played also a role in the inverse analysis, and explains the relatively poor quality fit between calculated and measured displacement for the section at 0 load repetitions. Nonetheless the achieved inverse analysis results are quite interesting. The modelling of the deflection bowl allowed defining the nature of the damage, due to crack propagation in a well defined region along the loading path and the weakening of the cement stabilized layers and of the subgrade. These results were also confirmed by forensic investigations. The precision of the inverse analysis, which can be carried out with an optimization algorithm only for the section at 0 load repetitions, is quite high. The accuracy analysis shows that the stiffness of the cement stabilized materials obtained with inverse analysis of the laboratory tests
is highly overestimated. Due to the presence of major cracks in the structure, the stiffness in the field is significantly smaller. The inverse analysis of the real field tests allows therefore achieving more realistic parameter values.

6.5 Inverse analysis of the Bellinzona test site results

6.5.1 Introduction

The test field in Bellinzona (CH) consists of flexible pavement with a very thick asphalt overlay. The test results showed some interesting peculiarities: first, the footpath near the structure behaved like a rigid boundary and affected the shape of the deflection bowl; second, considering that the pavement is flexible, the length of the deflection bowl is very large (3 m), as a consequence of the thick base course layer (27 cm) (Fig.6-42).

Fig.6-42: Deflection bowl and ETH Delta positioning for the measurement of the deflection bowl at the Bellinzona test site
The beam was positioned in a transversal direction to the loading path (Fig.6-42) and the deflection bowl was measured during loading and unloading. The inverse analysis was carried out with the measurements obtained during the unloading and only the recovered displacements were taken into account.

Both track ways of the road were built at the same time and by the same contractor. However, the surface conditions of the track way (section 2) in direction Lumino are poor, while the other track way is still good. Because the cantonal authorities were concerned about the durability of the pavement and could not explain the different surface conditions of the track ways, the tests were carried out on the same test track but on different track ways. In particular, section 2, in direction Lumino, was of major concern.

One of the goals of this test is therefore to prove the capability of the developed testing and inverse analysis procedure to explain this paradox.
6.5.2 Forward modelling

Fig. 6-43: Boundary value problem: note that vertical relative displacements on the right side are prevented because of the rigidity of the nearby footpath

The boundary value problem is completely defined, thanks to symmetries of the real problem, only by one fourth of the structure (Fig. 6-43). Vertical relative displacements between pavement and footpath were prevented. The two layers modeled are a thick asphalt layer (27cm) and the subgrade (200cm).
Fig. 6-44: Mesh generated for the test field in Bellinzona (the different colors show the 2 modeled layers)

The total number of elements is 7476, 6-node linear triangular prism, hybrid constant pressure (C3D6H), (Fig 6-44).

The tire footprint geometry measured on field was reproduced in the FE model, with the simplification of full contact and uniformly distributed pressure. The interface between the asphalt layer and the subgrade was modeled as for the EPFL and Hinwil field tests with a Mohr Coulomb failure criterion, defined by a cohesion $c = 20$ kPa, and friction angle $\phi = 38^\circ$. The hyperelastic constitutive model described by eq. 6-13, and 6-14 was adopted for describing the mechanical properties of the modeled road materials. No slip between the layers occurs until failure.
**6.5.3 Inverse Analysis**

The inverse analysis was carried out with the LM and the MADS algorithm. The solution (with the same initial guess values) converged after 10 (LM) and 32 (MADS) iterations for section 1, and after 10 (LM) and 31 (MADS) iterations for section 2. The good quality of the fit between measured and calculated deflection bowl is shown in Fig.6-45 to 6-48.

In Table 6-8 the back-calculated parameter values for the hyperelastic constitutive model are summarized.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Section 1</th>
<th>Section 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>( G = 2.905 \times 10^5 ) [kPa]</td>
<td>( G = 1.80 \times 10^5 ) [kPa]</td>
</tr>
<tr>
<td></td>
<td>( K = 10.61 \times 10^6 ) [kPa]</td>
<td>( K = 4.8 \times 10^6 ) [kPa]</td>
</tr>
<tr>
<td></td>
<td>( m = 3.02 ) [-]</td>
<td>( m = 3.21 ) [-]</td>
</tr>
<tr>
<td>Subgrade</td>
<td>( G = 2.54 \times 10^4 ) [kPa]</td>
<td>( G = 1.9 \times 10^4 ) [kPa]</td>
</tr>
<tr>
<td></td>
<td>( K = 3.8 \times 10^4 ) [kPa]</td>
<td>( K = 6.45 \times 10^4 ) [kPa]</td>
</tr>
<tr>
<td></td>
<td>( m = 1 ) [-]</td>
<td>( m = 1.58 ) [-]</td>
</tr>
</tbody>
</table>

Inverse analysis was also carried out using the linear elastic constitutive models and the LM algorithm: it can be observed that unrealistic values for the Poisson's ratios are obtained (Table 6-9).

<table>
<thead>
<tr>
<th>Layer</th>
<th>Section 1</th>
<th>Section 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>( G = 7 \times 10^5 ) [kPa]</td>
<td>( G = 1.422 \times 10^5 ) [kPa]</td>
</tr>
<tr>
<td></td>
<td>( \nu = 0.46 ) [-]</td>
<td>( \nu = 0.499 ) [-]</td>
</tr>
<tr>
<td>Subgrade</td>
<td>( G = 5.68 \times 10^4 ) [kPa]</td>
<td>( G = 3.11 \times 10^4 ) [kPa]</td>
</tr>
<tr>
<td></td>
<td>( \nu = 0.035 ) [-]</td>
<td>( \nu = 0.39 ) [-]</td>
</tr>
</tbody>
</table>
Fig. 6-45: Section 1, modeled (line) and measured deflection bowl at different longitudinal distances from the point of load application.

Fig. 6-46: Section 1, modeled (line) and measured deflection bowl at different transversal distances from the point of load application.
Fig. 6-47: Section 2, modeled (line) and measured deflection bowl at different longitudinal distances from the point of load application.

Fig. 6-48: Section 2, modeled (line) and measured deflection bowl at different transversal distances from the point of load application.
6.5.3.1 Precision

The precision of the values obtained with the LM and MADS optimization is summarized in Table 6-10.

The precision is described by the confidence interval, the maximum and minimum values of the back-calculated parameters, within small range values of the objective functions.

Table 6-10: Precision

<table>
<thead>
<tr>
<th>Layer</th>
<th>Confidence</th>
<th>Section 1</th>
<th></th>
<th>Section 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$G$ [kPa]</td>
<td>$K$ [kPa]</td>
<td>$m$ [-]</td>
<td>$G$ [kPa]</td>
</tr>
<tr>
<td>Base course</td>
<td>Min</td>
<td>2.5 $\times$ 10^5</td>
<td>7 $\times$ 10^6</td>
<td>2.2</td>
<td>1.8 $\times$ 10^5</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>4.0 $\times$ 10^5</td>
<td>1.2 $\times$ 10^7</td>
<td>3</td>
<td>1.8 $\times$ 10^5</td>
</tr>
<tr>
<td>Subgrade</td>
<td>Min</td>
<td>1.5 $\times$ 10^4</td>
<td>3 $\times$ 10^4</td>
<td>1</td>
<td>2.0 $\times$ 10^4</td>
</tr>
<tr>
<td></td>
<td>Max</td>
<td>2.6 $\times$ 10^4</td>
<td>7 $\times$ 10^4</td>
<td>1.1</td>
<td>2.5 $\times$ 10^4</td>
</tr>
</tbody>
</table>

In general, the confidence interval of the parameter values is small, except for the bulk modulus of the base course. The subgrade moduli for the two track ways are very similar. Fig. 6-49 to 6-60 show the search history for both the MADS and LM algorithm.
Fig. 6-49: Section 1, precision of the shear modulus, base course

Fig. 6-50: Section 1, precision of the bulk modulus, base course

Fig. 6-51: Section 1, precision of the nonlinear parameter, base course

Fig. 6-52: Section 1, precision of the shear modulus, subgrade

Fig. 6-53: Section 1, precision of the bulk modulus, subgrade

Fig. 6-54: Section 1, precision of the nonlinear parameter, subgrade
**Chapter 6: Field Tests Validation**

Fig. 6-55: Section 2, precision of the shear modulus, base course

Fig. 6-56: Section 2, precision of the bulk modulus, base course

Fig. 6-57: Section 2, precision of the nonlinear parameter, base course

Fig. 6-58: Section 2, precision of the shear modulus, subgrade

Fig. 6-59: Section 2, precision of the bulk modulus, subgrade

Fig. 6-60: Section 2, precision of the nonlinear parameter, subgrade
6.5.3.2 Accuracy

The back-calculated parameter values for the base course layer material are compared (Table 6-11) to those obtained, under the same temperature conditions, with the inverse analysis of the uniaxial test results, obtained in Chapter 5. The results show that the shear parameter values are close to those obtained from the inverse analysis of the laboratory test results at infinite time. The main difference is found for the volumetric behaviour. The material volumetric behaviour is almost incompressible, as shown by the very high bulk modulus values and the much higher value for the non linear parameter $m$.

Table 6-11: Accuracy. I = inverse analysis, T = Uniaxial compression test results on cores (Section 2).

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Base course</td>
<td>Min(*)</td>
<td>$1.8 \cdot 10^5$</td>
<td>$3 \cdot 10^5$</td>
<td>3</td>
<td>$2.5 \cdot 10^5$</td>
<td>$3.5 \cdot 10^5$</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>Max(*)</td>
<td>$1.8 \cdot 10^5$</td>
<td>$1 \cdot 10^7$</td>
<td>4</td>
<td>$10.8 \cdot 10^5$</td>
<td>$12.7 \cdot 10^5$</td>
<td>1.34</td>
</tr>
</tbody>
</table>

(*) the minimum and the maximum average value of the moduli for the two tested cores, initial (max) and long term (min) value.

The mean pressure dependency of the asphalt layer material should be therefore better investigated in the laboratory tests, by applying confining stresses in a proper triaxial test.
6.5.4 Discussion

The inverse analysis of the Bellinzona test results showed the importance of adopting an advanced nonlinear constitutive model for the layer materials. The back-calculation carried out with linear elastic constitutive models was much slower (Fig. 6-61, 6-62) and delivered nearly unrealistic parameter values for the analyzed materials. In particular the nonlinear compressible behaviour was found to be very important for describing the shape of the deflection bowl. The accuracy analysis proves again that the inverse analysis of the field tests is able to indicate the relevant modelling issues needed for describing the in situ mechanical properties, once the right constitutive models, developed within the help of the laboratory tests, are chosen. Triaxial tests with confining pressure should be carried out for better analyzing the mean pressure dependency of bituminous mixture in the laboratory.

Fig. 6-61: Section 1, Convergence of LM for the linear elastic (LE) and hyperelastic (HE) constitutive model
The comparison between the mechanical properties of the two fields shows that:

- The values of the nonlinear parameter $m$ are much higher for the material in section 2 than in section 1.
- The shear modulus of the base course material in section 1 is larger by a factor of 2 than in section 2.

These results seem to justify the different characteristics measured on the surface (unevenness), which are related to both different base course layer and subgrade nonlinear volumetric compressibility.
6.6 Conclusions

The inverse analysis carried out on the experimental field test results demonstrates that the proposed procedure, is feasible to back-calculate a large number (8) of parameters, and to obtain accurate parameter values. Two different algorithms for inverse analysis are described paying attention to the quality of the results, defined by accuracy, precision, and sensitivity of the parameters.

The accuracy is obtained by the comparison of the results from the inverse analysis of the laboratory and field test results. Of course neither ones of them are the exact solution, because of the simplified boundary and loading conditions (in the laboratory) and of the worse precision of the measurements (in the field). However if their values are close to each other, they are accurate: this means both that the laboratory tests and materials are representative of the field conditions and that the inverse analysis of field tests provides results reproducible in laboratory tests. The mechanical properties of the materials are therefore clearly quantitatively defined. However if the parameter values obtained with inverse analysis of the field tests are significantly different from those of the laboratory tests, this does not necessarily means that the field test results are wrong. On the contrary, most probably the laboratory tests (material or boundary/loading conditions) are not representative of the real field test conditions.

An effort was made to assess the precision, defined by the confidence interval of the optimized parameter values, given by the maximum and minimum values of the parameters by similar values of the objective function. The confidence is furthermore directly associated to the correlation between measurement and parameter precision, and gives a region of possible values which are all in a small range of objective function values.
In general, the magnitudes of the values are quite in a good agreement with the values obtained from inverse analysis of the laboratory test results, in particular if the forward calculation is carried out with a model, which reliably reproduces the real test conditions and if the built in materials are homogeneous.

The following conclusions can be made with respect to precision:

- The precision of the parameters depends on the thickness and on the depth of the layer.

- In general the Poisson’s ratio has worse precision than the other parameter values.

- Nonlinear constitutive modelling is not always necessary for achieving a better fit, but is useful for obtaining more meaningful parameter values.

The sensitivity of the parameters was analyzed considering their influence on the shape of the deflection bowl.

The findings, which are complementary to those from the precision study, are:

- The top layer should have at least a thickness of 10 cm in order to make the forward calculation sensitive to its change.

- The deeper are the layers, the higher is their influence on shape of the deflection bowl.

- The interface parameters (cohesion and friction angle) in a range of “realistic” values does not significantly affect the displacements, while the full bound condition between the layers causes high underestimation of the parameter values.
Chapter 7: Summary and Conclusions

7.1 Summary

A review of the most widely adopted inverse analysis techniques showed that there is still a need for improvement in the existing testing devices, procedures and in the interpretation of the results. It was observed that these testing techniques try to apply in the field well defined loading conditions, allowing for a simplified interpretation of the results. This issue, which was very important in the past, becomes nowadays less important due to the tremendous progress made by computer technology. FE programs with large number of elements and sophisticated constitutive models are now available for inverse analysis. A source of inaccuracy for the existing inverse analysis techniques is that they often model a dynamic problem, such as the one generated by the FWD, using a quasi static analysis. This is not always a justified assumption. On the other hand the analysis of the complex dynamic problems cannot reveal the whole range of material mechanical properties, in particular the effect of rate dependency for the viscous material, such as the bituminous mixtures.

A novel testing technique was therefore introduced, based on the three dimensional measurements of the static deflection bowl: in this framework the quasi static approach for the interpretation of the results is completely justified. Preliminary tests had shown that knowing the vertical displacement of the road surface on a raster of points would allow significant material parameters being accurately back-calculated. The ETH Delta testing device was designed and built at the IGT, Institute for Geotechnical Engineering at the ETH Zürich, in order to measure the three dimensional form of the deflection bowl using a contactless measuring technology. The frame was designed for achieving high stiffness, and
good damping properties against dynamic loading. The number and position of
the sensors was determined by optimizing the degree of convergence for the
back-calculation procedure. This also led to a reduction of the effect of precision
in the measurements on the accuracy of the back-calculated parameters. A
calibration technique was developed in order to account for the deformation and
the tilting of the beam during the measurements.

Three major field tests were carried out in different locations and on
different pavement structures, so that the testing procedure is validated in a
representative range of in situ field conditions: the EPFL test track, located at the
EPFL (Lausanne, Switzerland), a semirigid road structure in Hinwil (Switzerland)
and a flexible road structure in Bellinzona (Switzerland). One major result of the
testing procedure was the measurement of the real extent of the deflection bowl,
which is underestimated by the Benkelman beam test, usually adopted for this
kind of measurements. The shape and size of the deflection bowl were found
dependent not only on the thickness and stiffness of the layers, but also on the
presence of cracks in the lower layers. The dynamic deflection bowl produced by
loading simulators MLS10 and the EPFL APT (accelerated pavement tester) was
measured in the Hinwil field test and in the EPFL test respectively. Surprisingly
the dynamic and static deflection bowls have almost the same depth and shape,
in the range of applied velocities.

Time dependent displacement can be measured during static loading; however
additional static tests are necessary in order to have a representative basis of
experimental data for running back-calculation with rate dependent constitutive
models. Rigid boundaries of the road structures affect the form of the deflection
bowl: this is particularly relevant in Switzerland, due to the presence of very stiff
and “well” built footpaths.

A series of laboratory tests was carried out in order to characterize the
mechanical material behaviour of the materials tested in the field, and to assess
parameter values for the validation of the inverse analysis. Uniaxial compression
tests were therefore carried out on cylindrical specimens from material samples (cores) collected in the field. The material tested in the IGT laboratory for this research work were: the asphalt mixture from the base layer of the Bellinzona test site, the asphalt base course mixture and the cement stabilized gravel of the base layers from the Hinwil test site.

Particular attention was devoted to characterize the small strain behaviour of these samples under compression, as measured in the field tests. Vertical and horizontal strain gages were glued to the sample surface, to allow for local deformation measurements. The gage dimensions were chosen to cover a representative specimen length. The strain gage readings were validated against measurements carried out with LVDTs. A major improvement was the precise measurement of the horizontal strains, otherwise very difficult to measure. The test results show for all the tested materials different bulk and deviatoric behaviours. Asphalt mixes have rate dependent mechanical behaviour even at relatively low temperatures (6°C). It was found that the cement stabilized gravel material did not develop plastic strains during the uniaxial compression test. For asphalt plastic strains become important only at high maximal deviatoric strains (> 200 microstrains).

A thermodynamic consistent framework for derivation of a class of nonlinear viscoelastic (hyperviscoelastic) models was developed. The theory showed that some existing non linear viscoelastic constitutive models that claim to be thermodynamically consistent, in reality are not. A hyperviscoelastic model with pressure dependent bulk stiffness for asphalt mixtures was proposed. The hyperviscoelastic constitutive modelling of the tests, carried out on the asphalt samples from the field, allowed for a very good fit of the experimental results with few meaningful parameters. The non linear constitutive model without the rate dependent part also reproduces very well the behaviour of cement stabilized gravel under compression. An attempt was made for studying the effect of friction on specimens with a low ratio height/diameter, which was found negligible in the
range of the back-calculated parameter values. The constitutive models were implemented in a finite element code (ABAQUS) for being used in inverse analysis of the field tests.

The EPFL test site represents the ideal link between laboratory and field test conditions. The results showed that for these particular conditions the deflection bowl is not significantly affected by the non linear properties of the building materials. The inverse analysis results are rather precise (reproducible), especially for the base and subgrade layers. The back-calculated parameter values for the asphalt mixture of the EPFL base course layer were rather accurate (as compared to the results from the two points bending test).

The inverse analysis of the Hinwil test site results showed that it is quite challenging to use the ETH delta on a semirigid structure, due to the large extension (6 meters) of the deflection bowl and the very small displacements (<0.1mm). A positive outcome is the quantitative explanation from a mechanical point of view of the deflection bowl formation mechanism, due to the presence of cracks. The accuracy analysis showed that, for this kind of structure, the parameter values obtained within the inverse analysis of the uniaxial compression tests highly overestimate the stiffness of the cement stabilized material.

The inverse analysis of the Bellinzona test site results highlighted the importance of non linear constitutive modelling for the road materials. Linear elastic constitutive models for the layer materials were found inadequate to model the experimental field test results. The accuracy analysis showed that the inverse analysis of the static deflection bowl, obtained with rolling wheel and slow unloading (60 sec), produced parameter values for the asphalt materials, which are comparable to long term parameter values from the uniaxial compression test results. The parameter values obtained for the two tested track ways helped explaining their different surface conditions after two years of service.
7.2 General conclusions

- The new device ETH Delta can be used for measuring the deflection bowl under a real axle load.
- Based on these measurements the inverse analysis procedure can be used for back-calculation of road material parameters.
- The obtained parameters correspond to the long term moduli obtained in the laboratory tests.
Literature


EN 12697-26 Test methods for hot mix asphalt - Part26: Stiffness.


SN 670 317b (Swiss Standard)
SN 670 330b (Swiss Standard)

SN 671 969c (Swiss Standard)


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NOTATION

AASHO American Association of State Highway Organizations
AASHTO American Association of State Highway and Transportation Officials
APT Accelerated Pavement Tester
c cohesion
CCD Charge Coupled Device
CV Coefficient of Variation
d dissipation function
$D_{ij}$ tangential stiffness matrix
$E$ elastic modulus
$E_0$ elastic modulus at initial value
$E_\infty$ elastic modulus at infinite time
$E_k$ k-th elastic modulus
EPFL École Polytechnique Fédérale de Lausanne
$F, f$ strain (Helmholtz) energy function
FWD Falling Weight Deflectometer
$g_k$ k-th Prony series normalized modulus
$G$ shear modulus
$G_0$ shear modulus at initial value
$G_\infty$ shear modulus at infinite time
$G_k$ k-th shear modulus
GN Gauss Newton
HMA Hot Mix Asphalt
$H$ Heaviside function
$K$ bulk modulus
$K_0$ bulk modulus at initial value
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>$K_\infty$</td>
<td>bulk modulus at infinite time</td>
</tr>
<tr>
<td>$K_k$</td>
<td>k-th bulk modulus</td>
</tr>
<tr>
<td>LAVOC</td>
<td>Laboratoire des Voies de Circulation</td>
</tr>
<tr>
<td>LM</td>
<td>Levenberg Marquardt</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear Variable Differential Transformer</td>
</tr>
<tr>
<td>MADS</td>
<td>Mesh Adaptative Direct Search</td>
</tr>
<tr>
<td>$m$</td>
<td>non linear parameter</td>
</tr>
<tr>
<td>$n$</td>
<td>total number of Maxwell elements</td>
</tr>
<tr>
<td>$n_v, n_s$</td>
<td>total number of volumetric and deviatoric additional Maxwell elements</td>
</tr>
<tr>
<td>$p, p_r$</td>
<td>volumetric stress tensor invariant (mean pressure), and reference mean pressure</td>
</tr>
<tr>
<td>PSA</td>
<td>Pavement Seismic Analyzer</td>
</tr>
<tr>
<td>PSD</td>
<td>Position Sensitive Device</td>
</tr>
<tr>
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<td>Present Serviceability Index</td>
</tr>
<tr>
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</tr>
<tr>
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</tr>
<tr>
<td>SAL</td>
<td>Standard Axle Load (10kN)</td>
</tr>
<tr>
<td>SSE</td>
<td>Sum of Squared Errors</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$z$</td>
<td>force potential</td>
</tr>
<tr>
<td>$\alpha_k, \dot{\alpha}_k$</td>
<td>k-th viscous strain and viscous strain velocity</td>
</tr>
<tr>
<td>$\dot{\alpha}_r$</td>
<td>reference viscous strain velocity</td>
</tr>
<tr>
<td>$\alpha_{sk}, \dot{\alpha}_{sk}$</td>
<td>k-th deviatoric viscous strain, k-th deviatoric strain velocity</td>
</tr>
<tr>
<td>$\alpha_{vk}, \dot{\alpha}_{vk}$</td>
<td>k-th volumetric viscous strain, k-th volumetric strain velocity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Dirac Function</td>
</tr>
<tr>
<td>$\Delta^m_k$</td>
<td>mesh size</td>
</tr>
<tr>
<td>$\Delta^p_k$</td>
<td>poll size</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>initial constant strain step</td>
</tr>
<tr>
<td>$\varepsilon, \dot{\varepsilon}$</td>
<td>total strain and strain velocity</td>
</tr>
<tr>
<td>$\varepsilon_v$</td>
<td>volumetric strain tensor invariant</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>deviatoric strain tensor invariant</td>
</tr>
<tr>
<td>$\eta_D$</td>
<td>directions set</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>k-th viscosity coefficient</td>
</tr>
<tr>
<td>$\eta_{sk}$</td>
<td>k-th deviatoric viscosity coefficient</td>
</tr>
<tr>
<td>$\eta_{vk}$</td>
<td>k-th volumetric viscosity coefficient</td>
</tr>
<tr>
<td>$\mu$</td>
<td>mean</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>total stress</td>
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<td>variance</td>
</tr>
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<td>Cauchy stress tensor</td>
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<tr>
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<td>stress in the k-th Maxwell element</td>
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<tr>
<td>$\sigma_r$</td>
<td>reference stress</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time (superposition)</td>
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<tr>
<td>$\tau_k$</td>
<td>k-th relaxation time</td>
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<tr>
<td>$\tau_{k}^{r}$</td>
<td>non linear k-th relaxation time</td>
</tr>
<tr>
<td>$\tau_{sk}$</td>
<td>k-th deviatoric relaxation time</td>
</tr>
<tr>
<td>$\tau_{vk}$</td>
<td>k-th volumetric relaxation time</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>friction angle</td>
</tr>
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</table>
UMAT

C -------------------------------
C UMAT FOR THERMODYNAMIC CONSISTENT HYPERSOFTIBLEASTICITY
C CANNOT BE USED FOR LARGE STRAINS
C AUTHO: CARLO RABAIOTTI ETHZ, 2008, COPYRIGHTED
C -------------------------------
C PROPS(1) - GINF
C PROPS(2) - KINF
C PROPS(3) - m
C PROPS(4) - G1
C PROPS(5) - G2
C PROPS(6) - etag1
C PROPS(7) - etag2
C PROPS(8) - K1
C PROPS(9) - K2
C PROPS(10) - etak1
C PROPS(11) - etak2
C

SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,
1 RPL,DDSDDT,DRPLDE,DRPLDT,STRAN,DSTRAN,
2 TIME,DTIME,TEMP,DTTEMP,PREDEF,DPRED,MATERL,NDI,NSHR,NTENS,
3 NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,CELENT,
4 DFGRD0,DFGRD1,NOEL,NPT,KSLAY,KSPT,KSTEP,KINC)

C
INCLUDE 'ABA_PARAM.INC'
C
CHARACTER*8 MATERL
DIMENSION STRESS(NTENS),STATEV(NSTATV),
1 DDSDDDE(NTENS,NTENS),DDPLDE(NTENS),DRPLDE(NTENS),
2 STRAN(NTENS),DSTRAN(NTENS),DFGRD0(3,3),DFGRD1(3,3),PS(6),PP(6),
3 TIME(2),PREDEF(1),DPRED(1),PROPS(NPROPS),COORDS(3),DROT(3,3)
DIMENSION DSTRESS(6), STRESSA(6), AN(3,3), STRESSP(6), SPRIME(6)

DIMENSION STRT(6), astrt(6), dstrt(6)
C
double precision GINF,
KINF, m, E, J, I2D, gi, tau, nd, E1P, E2P,K,G
a E3P, AES, AI2D, AEV, Q, P, DTIME,
K1,K2,K3,K4,G1,G2,G3,G4,tauk1,tauk2,taug1,taug2,knum
a geta1, geta2, geta3, geta4, keta1, keta2, keta3, keta4
C***********************************************************************
C ELASTIC PROPERTIES
C***********************************************************************
GINF=PROPS(1)
KINF=PROPS(2)
m=PROPS(3)
C***********************************************************************
C VISCO-ELASTIC PROPERTIES (SHEAR)
C***********************************************************************
G1 = PROPS(4)
G2 = PROPS(5)
G3 = PROPS(6)
G4 = PROPS(7)
geta1 = PROPS(6)
geta2 = PROPS(7)
geta3 = PROPS(10)
geta4 = PROPS(11)
taug1 = (geta1/G1);
taug2 = (geta2/G2);
taug3 = (geta3/G3);
taug4 = (geta4/G4);

C**************************************************************
C    VISCO-ELASTIC PROPERTIES (BULK, COMPRESSIBILITY)
C**************************************************************
K1 = PROPS(8)
K2 = PROPS(9)
K3 = PROPS(14)
K4 = PROPS(15)
keta1 = PROPS(10)
keta2 = PROPS(11)
keta3 = PROPS(18)
keta4 = PROPS(19)
tauk1 = (keta1/K1);
tauk2 = (keta2/K2);
tauk3 = (keta3/K3);
tauk4 = (keta4/K4);
Knum = 1000.

C**************************************************************
C     Knum is a numerical adjustemnt for avoiding initial 0 stiffness
C     which can cause numerical problems
C**************************************************************
C    STRAIN TENSOR (TRANSLATION LOG-ENG STRAINS)
C**************************************************************
strt(1:ndi) = \text{Exp}(stran(1:ndi)+dstran(1:ndi))-1.
strt(ndi+1:) = \text{Exp}((stran(ndi+1:)+dstran(ndi+1:))/2.)-1.
astrt(1:ndi) = \text{Exp}(stran(1:ndi))-1.
astrt(ndi+1:) = \text{Exp}((stran(ndi+1:))/2.)-1.
dstrt(1:ndi) = strt(1:ndi)-astrt(1:ndi)
dstrt(ndi+1:) = strt(ndi+1:)-astrt(ndi+1:)

C**************************************************************
C    CALCULATE GEOTECHNICAL STRAIN TENSOR INVARIANTS AND EPS PRIME
C    Numerical adjustemnt for ES = 0 (prevent inf. values)
C**************************************************************
 EV = strt(1)+strt(2)+strt(3)
 I2D = 1./6.*((strt(1)-strt(2))**2.+(strt(2)
 -strt(3))**2.+(strt(1)-strt(3))**2.)+strt(4)**2.+
astrt(5)**2.+astrt(6)**2.
 ES = -2./3.*(3.*I2D)**(1./2.)
 AEV = astrt(1)+astrt(2)+astrt(3)
 AI2D = 1./6.*((astrt(1)-astrt(2))**2.+(astrt(2)
 -astrt(3))**2.+(astrt(1)-astrt(3))**2.)+astrt(4)**2.+
astrt(5)**2.+astrt(6)**2.
 AES = -2./3.*(3.*AI2D)**(1./2.)
 E1P= strt(1)-1./3.*EV
 E2P= strt(2)-1./3.*EV
E3P=strt(3)-1./3.*EV

if (ES .le. 0.0000000000000000001) then
ES = ES+0.0000000000000000001
elseif (ES .gt. 0.0000000000000000001) then
ES = ES
endif

C*******************************************************************************
C CALCULATE THE DERIVATIVES OF THE STRAIN ENERGY FUNCTION IN RESPECT TO ES EV
*******************************************************************************

**************
C    Q = df/des
C    P = df/dev
C    3G = d**2f/des**2
C    K = d**2f/dev**2
C*******************************************************************************

C RECURSIVE COEFFICIENT Ak
C*******************************************************************************
A1=STATEV(1)*exp(-DTIME/tauk1)+
  (EV-AEV)*exp((-0.5*DTIME)/tauk1);
STATEV(1)=A1
A2=STATEV(2)*exp(-DTIME/tauk2)
  + (EV-AEV)*exp((-0.5*DTIME)/tauk2);
STATEV(2)=A2

*******************************************************************************
C p VOLUMETRIC STRESS TENSOR INVARIANT (FUNCTION OF STRAIN TENSOR INVARIANT)
C correction for extension necessary sign*modulus
*******************************************************************************

if (A1 .gt. 0 .and. A2 .gt. 0 .and. EV .gt. 0) then
    P=(KINF*(abs(EV)))**m+Knum*abs(EV)
elseif (A1 .le. 0 .and. A2 .le. 0 .and. EV .le. 0) then
    P=-(KINF*(abs(EV)))**m-Knum*abs(EV)
elseif (A1 .gt. 0 .and. A2 .le. 0 .and. EV .le. 0) then
    P=-(KINF*(abs(EV)))**m-Knum*abs(EV)
elseif (A1 .gt. 0 .and. A2 .gt. 0 .and. EV .le. 0) then
    P=-(KINF*(abs(EV)))**m-Knum*abs(EV)
elseif (A1 .gt. 0 .and. A2 .gt. 0 .and. EV .gt. 0) then
    P=(KINF*(abs(EV)))**m+Knum*abs(EV)
elseif (A1 .le. 0 .and. A2 .gt. 0 .and. EV .gt. 0) then
    P=(KINF*(abs(EV)))**m+Knum*abs(EV)
elseif (A1 .le. 0 .and. A2 .gt. 0 .and. EV .le. 0) then
    P=-(KINF*(abs(EV)))**m-Knum*abs(EV)
if (A1 .le. 0 .and. A2 .le. 0 .and. EV .gt. 0) then
P=+(KINF*(abs(EV)))**m+Knum*abs(EV)
endif

C*****************************************************************
C    RECURSIVE COEFFICIENT Ag
C*****************************************************************
B1=STATEV(3)*exp(-DTIME/taug1)
B1+(ES-AES)*exp((-0.5*DTIME)/taug1);
STATEV(3)=B1
B2=STATEV(4)*exp(-DTIME/taug2)
B2+(ES-AES)*exp((-0.5*DTIME)/taug2);
STATEV(4)=B2

******************************************************************
C    q DEVIATORIC STRESS TENSOR INVARIANT (FUNCTION OF STRAIN TENSOR INVARIANT)
******************************************************************
Q=3*GINF*(ES)
Q+3*G1*B1+3*G2*B2;

C*****************************************************************
C    3G SHEAR MODULUS (FUNCTION OF STRAIN TENSOR INVARIANTS)
C*****************************************************************
G =3*GINF
G+3*G1+3*G2;

C*****************************************************************
C    K BULK MODULUS (FUNCTION OF STRAIN TENSOR INVARIANTS)
C    NUMERICAL INTEGRATION
C*****************************************************************
K = m*(KINF*abs(EV))**(m-1.)*KINF+Knum
K+K1*m*(K1*abs(A1))**(m-1.)+K2*m*(K2*abs(A2))**(m-1.)

C*****************************************************************
C    J CROSS TERM (FUNCTION OF STRAIN TENSOR INVARIANTS)
C*****************************************************************
Ea = 0.

C*****************************************************************
C   STRESS TENSOR
C*****************************************************************
DO K1=1,NDI
STRESS(K1)=(Q)*2./(3.*ES)*(strt(K1)
STRESS(K1)=(Q)*2./(3.*ES)*(strt(K1)
END DO

C*****************************************************************
C    STIFFNESS MATRIX (FOR THE STRAIN INCREMENT)
C*****************************************************************
DDSDDE(1, 1)= (4./9.*E1P*E1P/(ES**2.)*(G-1./ES*Q)+
DDSDDE(2, 2)= (4./9.*E2P*E2P/(ES**2.)*(G-1./ES*Q)+
DDSDDE(3, 3)= (4./9.*E3P*E3P/(ES**2.)*(G-1./ES*Q)+
DDSDDE(1, 2)= (4./9.*E1P*E2P/(ES**2.)*(G-1./ES*Q)+
\( c(K-2./(9.*ES)*Q) \)

\[
\begin{align*}
DDSDDE(1, 3) &= (4./9.*E1P*E3P/(ES**2.)*(G-1./ES*Q) + \\
DDSDDE(2, 3) &= (4./9.*E2P*E3P/(ES**2.)*(G-1./ES*Q) + \\
DDSDDE(1, 4) &= 4./9.*E1P*strt(4)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(2, 4) &= 4./9.*E2P*strt(4)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(3, 4) &= 4./9.*E3P*strt(4)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(4, 4) &= (4./9.*strt(4)*c*strt(4)/(ES**2.)*(G-1./ES*Q) + \\
&2./(3.*ES)*Q) \\
\text{IF} (NSHR.EQ.3) \text{ THEN} \\
DDSDDE(1, 5) &= 4./9.*E1P*strt(5)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(2, 5) &= 4./9.*E2P*strt(5)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(3, 5) &= 4./9.*E3P*strt(5)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(1, 6) &= 4./9.*E1P*strt(6)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(2, 6) &= 4./9.*E2P*strt(6)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(3, 6) &= 4./9.*E3P*strt(6)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(5, 5) &= (4./9.*strt(5)*c*strt(5)/(ES**2.)*(G-1./ES*Q) + \\
&2.)/(3.*ES)*Q) \\
DDSDDE(6, 6) &= (4./9.*strt(6) \\
&c*strt(6)/(ES**2.)*(G-1./ES*Q) + \\
&2.)/(3.*ES)*Q) \\
DDSDDE(4, 5) &= 4./9.*strt(4)*strt(5)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(4, 6) &= 4./9.*strt(4)*strt(6)/(ES**2.)*(G-1./ES*Q) \\
DDSDDE(5, 6) &= 4./9.*strt(5)*strt(6)/(ES**2.)*(G-1./ES*Q) \\
\text{END IF} \\
\text{DO} K1=1, NTENS \\
\text{DO} K2=1, K1-1 \\
\quad DDSDDE(K1, K2)=DDSDDE(K2, K1) \\
\quad \text{END DO} \\
\text{END DO} \\
\text{RETURN} \\
\end{align*}
\]
RELATIONSHIPS BETWEEN GEOTECHNICAL AND ABAQUS TENSOR INVARIANTS

ABAQUS invariants:

\[ \bar{I}_1 = \text{tr}(\bar{B}) = \text{tr}(F \cdot F^T) \]  
(1)

\[ \bar{I}_2 = \frac{1}{2}(\bar{I}_1^2 - \text{tr}(\bar{B} \cdot \bar{B})) \]  
(2)

\[ J = \det(F) = (1 + \varepsilon_1) \cdot (1 + \varepsilon_2) \cdot (1 + \varepsilon_3) \]  
(3)

Where:

\[ F = \begin{bmatrix} (1 + \varepsilon_1) & 0 & 0 \\ 0 & (1 + \varepsilon_2) & 0 \\ 0 & 0 & (1 + \varepsilon_3) \end{bmatrix} \]  
(4)

\[ \bar{F} = \frac{F}{3\sqrt{J}} \]  
(5)

\[ B = F \cdot F^T \]  
(6)

\[ \bar{B} = \bar{F} \cdot \bar{F}^T \]  
(7)

\[ \bar{B} = \frac{1}{3\sqrt{J^2}}(\Delta U + \bar{I})(\Delta^T U + \bar{I}) = \frac{1}{3\sqrt{J^2}}(I + (\Delta U + \Delta^T U) + (\Delta U \cdot \Delta^T U)) \]  
(8)

\[ F \] deformation gradient

\[ \bar{F} \] deformation gradient (without volume change)

\[ B \] left Cauchy-Green tensor

From the condition of small strain behavior:
\[ \bar{B} = \frac{1}{3\sqrt{J^2}} \left[ \partial_{ij} + \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + \tilde{\sigma} \right] \]  

(9)

It follows that

\[ \bar{B} = \frac{1}{3\sqrt{J^2}} \begin{bmatrix} 1 + 2\varepsilon_1 & 0 & 0 \\ 0 & 1 + 2\varepsilon_2 & 0 \\ 0 & 0 & 1 + 2\varepsilon_3 \end{bmatrix} \]  

(10)

The ABAQUS invariants (eq. 1, 2, 3) translation follows:

\[ \bar{I}_1 = \frac{1}{3\sqrt{J^2}} (3 + 2(\varepsilon_1 + \varepsilon_2 + \varepsilon_3)) = \frac{1}{3\sqrt{J^2}} (3 + 2\varepsilon_i) \]  

(11)

\[ = \frac{1}{2} \left( \bar{I}_1 - tr(B \cdot B) \right) = \frac{1}{2} \left( \frac{1}{3\sqrt{J^4}} \left( (3 + 2(\varepsilon_1 + \varepsilon_2 + \varepsilon_3))^2 - (1 + 2\varepsilon_1)^2 - (1 + 2\varepsilon_2)^2 - (1 + 2\varepsilon_3)^2 \right) \right) = \]

\[ = \frac{1}{2} \left( \frac{1}{3\sqrt{J^4}} \left( (1 + 2\varepsilon_1) + (1 + 2\varepsilon_2) + (1 + 2\varepsilon_3) \right)^2 - (1 + 2\varepsilon_1)^2 - (1 + 2\varepsilon_2)^2 - (1 + 2\varepsilon_3)^2 \right) = \]

\[ = \frac{1}{2} \left( \frac{1}{3\sqrt{J^4}} \left( 2(1 + 2\varepsilon_1)(1 + 2\varepsilon_2) + 2(1 + 2\varepsilon_2)(1 + 2\varepsilon_3) + 2(1 + 2\varepsilon_1)(1 + 2\varepsilon_3) \right) \right) = \]

\[ \bar{I}_2 = \frac{4}{3\sqrt{J^4}} \left( \frac{3}{4} + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_3 \right) \]  

(12)

Let introduce the geotechnical deviatoric and the volumetric strain tensor invariants.

\[ \varepsilon_i = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \]  

(13)
\[ \varepsilon_s = \frac{\sqrt{2}}{3} \left( (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2 \right)^{0.5} = \]
\[ \frac{\sqrt{2}}{3} \left( 2\varepsilon_1^2 + 2\varepsilon_2^2 + 2\varepsilon_3^2 - 2\varepsilon_1\varepsilon_2 - 2\varepsilon_2\varepsilon_3 - 2\varepsilon_1\varepsilon_3 \right)^{0.5} = \]
\[ \frac{2}{3} \left( \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 - (\varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_3) \right)^{0.5} \]

From
\[ \varepsilon_v^2 = \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + 2 \left( \varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_3 \right) \]

It is obtained:
\[ \varepsilon_s = \frac{2}{3} \left( \varepsilon_v^2 - 3(\varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_3) \right)^{0.5} \]  
\[ (14) \]
\[ \varepsilon_s^2 = \frac{4}{9} \left( \varepsilon_v^2 - 3(\varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_3) \right) \]
\[ \varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_3 = -\frac{3\varepsilon_s^2}{4} + \frac{\varepsilon_v^2}{3} \]

It follows:
\[ I_2 = \frac{4}{3\sqrt{J^4}} \left( \frac{3}{4} + \varepsilon_v - \frac{3\varepsilon_s^2}{4} + \frac{\varepsilon_v^2}{3} \right) \]
\[ I_1 = \frac{1}{3\sqrt{J^2}} (3 + 2\varepsilon_v) \]

\[ \varepsilon_v = \frac{1}{2} \left( \sqrt[3]{J^2 I_1} - 3 \right) \]
\[ (15) \]
\[ \bar{I}_2 = \frac{4}{3\sqrt{J^4}} \left\{ \frac{3}{4} + \frac{1}{2} \left( \sqrt[3]{J^2 I_1} - 3 \right) - \frac{3\varepsilon_s^2}{4} + \frac{I}{4} \left( \sqrt[3]{J^2 I_1} - 3 \right)^2 \right\} \]
\[ -3\varepsilon_s^2 = \sqrt[3]{J^4 I_2} - 3 - 2 \left( \sqrt[3]{J^2 I_1} - 3 \right) - \frac{1}{3} \left( \sqrt[3]{J^2 I_1} - 3 \right)^2 \]

\[ \varepsilon_s = \sqrt[3]{\frac{1}{3} \left( 3 + 2 \left( \sqrt[3]{J^2 I_1} - 3 \right) + \frac{1}{3} \left( \sqrt[3]{J^2 I_1} - 3 \right)^2 - 3\sqrt[3]{J^4 I_2} \right)} \]
\[ (16) \]
# CV

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