PASSIVE LOCALIZATION OF AN ULTRASONIC EMITTER
IN HOMOGENEOUS AND HETEROGENEOUS MEDIA

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Zürich, March 25, 2009
Michael Flückiger
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Zusammenfassung


Lokalisierungsmethode die Position von einem Ultraschallsender in einem heterogenen Medium findet und dabei eine bessere Auflösung und Genauigkeit erzielt als zwei konventionelle Methoden.

Die Präzision bei der Lokalisierung von einem Objekt aufgrund seiner ausgesendeten Schallwellen hängt in erster Linie von genauen Zeit- und/oder Phasenmessungen ab. Die Frequenz der ausgesendeten Ultraschallwelle und die Anzahl, sowie die Anordnung von den Empfängern im Sensor-Array beeinflussen vor allem die Auflösung der Objekt-Lokalisierung. In unserer Methode benutzen wir eine optimale Empfängeranordnung, so dass die bestmögliche Auflösung bei der Objekt-Lokalisierung erzielt werden kann. Das Optimierungsverfahren der Empfängeranordnung wurde für Ultraschallanwendungen entwickelt. Es ist jedoch bei allen Methoden anwendbar, die Objekte aufgrund ausgesendeter Wellen lokalisieren. Der Optimierungsprozess verwendet die Frequenz des ausgesendeten Signals um eine Unsicherheitsanalyse durchzuführen. Aufgrund dieser Analyse können wir trotz relativ geringen Ultraschallfrequenzen von maximal 300 kHz eine hohe Präzision und Auflösung bei der Lokalisierung von Objekten garantieren. In einem homogenen Medium erhalten wir einen Fehler von $810 \pm 280 \mu m$ in zwei Dimensionen ($16 \times 16 \text{ cm}^2$) und einen Fehler von $1240 \pm 550 \mu m$ in drei Dimensionen ($16 \times 16 \times 16 \text{ cm}^3$). Diese Fehler gehen in einem heterogenen Medium auf $1030 \pm 600 \mu m$ und $1.70 \pm 1.11 \text{ mm}$ herauf.
Abstract

For the localization of minimally invasive medical components, such as camera pills or microrobots, in the human body, ultrasound combines good resolution, minimal adverse health effects, high speed, adequate frame rates, and low cost. In the case of miniaturized untethered devices, small onboard ultrasonic emitters with minimal power requirements have the potential to provide significantly enhanced localization.

We demonstrate for the first time acoustic emission in the kHz range using a wireless, untethered emitter based on the actuation principle of the wireless resonant magnetic microactuator recently developed in our group. The key features of the presented device are its small size, its simple design and fabrication, and its low cost. We present the theory for the magneto-acoustic transduction principle and obtain good agreement with the experimental data.

In addition, we propose a novel passive localization method, which can be used to accurately locate ultrasound emitters in homogeneous and heterogeneous media. The localization method is based on transmission ultrasound and time difference of arrival measurements, where emitted ultrasonic pulse trains are detected by a receiver array. Our method takes variations in the speed of sound, and the refraction, reflection, and interference of the ultrasound wave at boundary layers in heterogeneous media into account. We demonstrate the ability of the our localization method to estimate an ultrasound emitter position within heterogeneous media, yielding a better resolution and accuracy than conventional methods.

The accuracy in passive localization of an object from its emission primarily depends on accurate time and/or phase measurements. The frequency of the
emission and the number and arrangement of the receivers mainly effect the resolution of the emitter localization. We propose optimal receiver positions for passive localization methods, resulting in a maximal resolution for the emitter location. The optimization technique has been developed specifically for ultrasound signals obtained from omnidirectional emitters, although the results apply for other applications using passive localization techniques. In the optimization process the uncertainty of the emitted signal, including its frequency is analyzed. Thus, high localization accuracy and resolution can be achieved with relatively low ultrasound frequencies of maximal 300 kHz. In a homogeneous media a mean spatial error of $810 \pm 280 \mu m$ and $1240 \pm 550 \mu m$ is obtained in an area of $16 \times 16 \text{cm}^2$ and in a volume of $16 \times 16 \times 16 \text{cm}^3$, respectively. In a heterogeneous media these errors are increased slightly to $1030 \pm 600 \mu m$ and $1.70 \pm 1.11 \text{mm.}$
Notation

Roman Letters

- \( a \) Fitting coefficient
- \( A \) Area
- \( \hat{A} \) Amplitude
- \( b \) Width
- \( c \) Speed of sound
- \( C_R \) Reflection coefficient
- \( C_T \) Refraction coefficient
- \( d \) Distance
- \( D \) Damping coefficient
- \( [D] \) Damping matrix
- \( E \) Young’s modulus
- \( e \) Position vector to the emitter location \( E \)
- \( f \) Frequency
- \( \Delta f \) Error on the frequency
- \( F \) Force
- \( h \) Thickness
- \( H \) Magnetic field
- \( i \) Current
- \( I \) Intensity
- \( k \) Spring constant
- \( [k] \) Stiffness matrix
- \( K \) Polynomial coefficient
- \( l \) Length
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<thead>
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<td>$m$</td>
<td>Mass</td>
</tr>
<tr>
<td>$[m]$</td>
<td>Mass matrix</td>
</tr>
<tr>
<td>$M$</td>
<td>Magnetization</td>
</tr>
<tr>
<td>$n$</td>
<td>Demagnetization factor</td>
</tr>
<tr>
<td>$N$</td>
<td>Number, amount</td>
</tr>
<tr>
<td>$p$</td>
<td>Acoustic pressure</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>Normalized, time independent pressure waveform</td>
</tr>
<tr>
<td>$p$</td>
<td>Position vector to point $P$</td>
</tr>
<tr>
<td>$r, R$</td>
<td>Radius</td>
</tr>
<tr>
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</tr>
<tr>
<td>$r$</td>
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</tr>
<tr>
<td>$\text{Re}$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time difference of arrival</td>
</tr>
<tr>
<td>$T$</td>
<td>Cycle, period</td>
</tr>
<tr>
<td>$u$</td>
<td>Deflection, displacement</td>
</tr>
<tr>
<td>$u$</td>
<td>Displacement vector</td>
</tr>
<tr>
<td>$\dot{u}$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$\dot{u}$</td>
<td>Velocity vector</td>
</tr>
<tr>
<td>$\ddot{u}$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$\ddot{u}$</td>
<td>Acceleration vector</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>$Z$</td>
<td>Acoustic impedance</td>
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</tbody>
</table>

**Greek Letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha, \beta, \theta$</td>
<td>Angles</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Dirac delta function</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>Frequency shift</td>
</tr>
<tr>
<td>$\Delta \delta_f$</td>
<td>Error on the frequency shift</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Distance</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Error, or accuracy</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Dynamic viscosity</td>
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</tbody>
</table>
Abstract

\[\Delta \eta\] Error on the dynamic viscosity
\[\lambda\] Wavelength
\[\mu_0\] Permeability of free space
\[\nu\] Poisson ratio
\[\rho\] Density
\[\sigma\] Standard deviation, or resolution
\[\phi\] Velocity potential
\[\Phi\] Scalar impulse response
\[\varphi\] Phase
\[\chi\] Attenuation coefficient
\[\omega\] Angular frequency
\[\Omega\] Angular frequency of excitation

Abbreviations & Acronyms

2D Two-dimensional
3D Three-dimensional
AF\textsubscript{max} Maximum amplitude factor
AOA Angle of arrival
AR Amplitude ratio
ARES Assembling reconfigurable endoluminal surgical system
CAD Computer aided design
CAT Computed axial tomography
CCD Charge-coupled device
CT Computed tomography
DOF Degrees of freedom
DAQ Data acquisition
det Determinant
FE Finite element
FEM Finite element method
FFT Fast fourier transform
GI Gastrointestinal
GL Glycerin

xvii
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>Water</td>
</tr>
<tr>
<td>LDV</td>
<td>Laser doppler vibrometer</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro electro mechanical systems</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum likelihood</td>
</tr>
<tr>
<td>MR</td>
<td>Magnetic resonance</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnetic resonance imaging</td>
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<tr>
<td>PET</td>
<td>Positron emission tomography</td>
</tr>
<tr>
<td>PS</td>
<td>Phase shift</td>
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<tr>
<td>PZT</td>
<td>Lead zirconium titanate</td>
</tr>
<tr>
<td>rem</td>
<td>Remainder</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
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<tr>
<td>S350</td>
<td>Silicon oil 350</td>
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<tr>
<td>SAFT</td>
<td>Synthetic aperture focusing technique</td>
</tr>
<tr>
<td>sgn</td>
<td>Signum function</td>
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<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>SPECT</td>
<td>Single photon emission computed tomography</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time difference of arrival</td>
</tr>
<tr>
<td>TOF</td>
<td>Time of flight</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>UWB</td>
<td>Ultra wide band</td>
</tr>
<tr>
<td>WRMMA</td>
<td>Wireless resonant magnetic microactuator</td>
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Chapter 1

Introduction

Robotics and microrobotics is an inherent part of medical surgery and biological exploration. Microrobotics is an emerging field that combines the established theory and techniques of robotics with the exciting new tools provided by micro electro mechanical systems (MEMS) technology in order to create intelligent machines that operate at micron scales. The tools and processes required to fabricate and assemble micron sized robots are developed. Many of these systems are used for robotic exploration within biological domains, such as in the investigation of molecular structures, cellular systems, and complex organism behavior. A new research area is building autonomous microrobiotic machines that can explore the interior of organisms without being physically connected to the outside world. A microrobotic system like this will dramatically change our ability to explore the human body, gather information on its inner workings and, possibly, perform delicate microsurgery and deliver drugs in difficult to reach locations, such as in the eye, the brain, or other organs.

There is a clear trend toward the miniaturization of medical devices for minimally invasive medical procedures ranging from diagnosis and targeted drug delivery to complex surgical interventions. One commercially successful device is the M2A capsule endoscope, a camera pill with a length of 26 mm and diameter of 11 mm that is swallowed and delivers images as it moves through the gastrointestinal (GI) tract. Its essentially non-invasive nature allows for less painful diagnosis than traditional endoscopy, as well
as imaging of the small intestine, which is not possible with conventional endoscopes. Since the launch of the M2A capsule in 2001, more companies (see Fig. 1.1) have introduced camera pills with new functionalities such as multiple cameras or even locomotion abilities (rotation along the axis of the capsule) \[2, 3, 4, 5\], making the capsule endoscopy market a multimillion dollar business. For the state of the art in capsule endoscopy research, see \[6\].

Our current bio-microrobotics research focus is on building sub-millimeter sized, untethered devices for in vivo medical applications. Together with the design process of the minimally invasive medical devices, controlling and localization of them are the main challenges. This is also the case for the capsule endoscopes, where accurate localization during its motion through the GI tract remains difficult. Localization methods include measurement of the magnetic field of a permanent magnet mounted inside the capsule \[7, 8\] or extracting position information from the radio frequency (RF) signal used for sending the images \[9\], the spatial resolution of both methods being in the cm range. The drawback is that in case of the magnet method a relatively large magnet is needed for a good signal, as the magnetic field is proportional to the volume of the magnet, and for the RF method complex algorithms are required.
1.1 Localization of Miniaturized Devices

In the present work we examine the localization of such minimally invasive medical devices. Localization in general, comes in many forms, such as endoscopy, magnetic resonance (MR), computed tomography (CT), positron emission tomography (PET), x-ray, and ultrasound. However, only ultrasound among these techniques, combines good resolution, minimal adverse health effects, high speed, and low cost \[10, 11\]. In addition, there are several situations where only ultrasound is adequate. For instance in intravascular analysis or off-bypass cardiac surgery MR, CT, and x-rays have inadequate frame rates or involve high levels of harmful radiation, and endoscopy can not be used, due to the opaque blood flow \[12, 13\]. Additionally, ultrasonic methods are extensively used for many other applications beyond localization, or medical imaging and surgery \[14, 15, 16, 17\]. For instance ultrasound is applied for motion localization \[18, 19, 20, 21\], distance and flow measurements \[22, 23, 24\], servoing \[25, 26\], nondestructive testing \[27\], and structural health monitoring \[28\].

Most of the ultrasound applications require time of flight (TOF) measurements between transmitted ultrasonic bursts (pulse trains) and echoes reflected by a target. Therefore, great efforts are being made to measure the TOF as accurately as possible, ranging from extracting echoes with Kalman filtering \[29\] to applying aspects of bat behavior \[23\] in order to achieve improved TOF measurements. However, the spatial accuracy and resolution in ultrasonic imaging, localization, and distance measurements not only depends on accurate TOF measurements but also on precise knowledge of the speed of sound and the wave propagation path. Conventional ultrasound systems typically assume, that the speed of sound in human soft tissue is constant with an averaged value close to that for water \[14, 30\], and refraction and reflection at tissue boundary layers are neglected. It has long been known that these simplifications result in a serious spatial accuracy loss \[31\] and ray tracing based modelling was proposed by several authors \[32, 33, 34, 35\] to improve the aforementioned problems.

We propose a new ultrasonic localization algorithm that can be used to accurately locate any kind of ultrasound sources, such as ultrasonically marked
medical devices or tumors, in homogeneous and heterogeneous media. Other application areas of the localization algorithm are in the localization of our bio-microrobots, and the self-assembly modules of the assembling reconfigurable endoluminal surgical system (ARES) project [36] at our institute, or in targeted drug delivery in cancer therapies [37], where anticancer agents must be guided precisely through the human body. The new algorithm is based on transmission ultrasound and pulse propagation time measurements, where emitted ultrasonic pulses from a sound source are detected by several receivers located around the area of interest, as shown in Fig. 1.2.

With our algorithm high resolution localization to 250 µm is possible with relatively low ultrasound frequencies of maximal 300 kHz.

Our new algorithm uses finite element methods (FEM) and shows improved spatial accuracy and resolution over existing localization methods [38]. The algorithm accurately finds the position of an ultrasound emitter located in heterogeneous media by measuring the absolute TOF or the time difference of arrival (TDOA) between several receivers. The various speed of sound in different regions, and the refraction, reflection, and interference of the ultrasound wave at boundary layers is thereby taken into account.
1.2 Miniaturized Wireless Acoustic Emitter

For applicability of the new algorithm miniaturized acoustic emitters are required, which can be embedded into the minimally invasive surgical devices that are localized. Recently, the wireless resonant magnetic microactuator (WRMMA) was developed in our group and implemented into the Magmite microrobotic system [39, 40]. The core component of these microparticles is their magnetomechanical transducer that harvests the energy of an oscillating magnetic field and transforms it directly into an oscillating mechanical motion, thus allowing for high frequency wireless actuation at the microscale. A similar resonating structure has shown to be adequate to send out ultrasonic pressure waves [41, 42], resulting in the wireless acoustic emitter shown in Fig. 1.3.

The advantages of the wireless acoustic emitter are its untethered nature, its comparatively simple design and fabrication, and low cost, as the wireless resonator does not require on-board energy storage and wiring. In addition, only soft magnetic material is used instead of permanent magnets. A further advantage is the small size of the resonator, as the space is limited on which the emitter must be mounted, and an increasing need for miniaturized sensors can be observed in general [43].

To drive such an emitter a multitude of excitation schemes besides the WRMMA have been introduced in the past, including electrostatic, piezoelectric,
electromagnetic, and electrothermal. A variety of corresponding detection schemes, such as capacitive, piezoelectric, piezoresistive, and optic have also been proposed besides ultrasonic [44]. Thermal and piezoelectric excitations require high power or voltage. Electromagnetic actuation usually requires the integration of permanent magnets, which complicates the production process, and electrostatic excitation shows often nonlinear behavior (pull-in) [45, 46]. On the other hand, capacitive, piezoelectric, and piezoresistive detection requires wiring, and optic sensing can not be used in opaque media. Thus, the actuation principle based on the WRMMA with the ultrasonic detection scheme provides obvious advantages for our localization applications.

In addition, resonating structures are widely used as sensing elements to measure a variety of physical parameters such as mass, mechanical force and torque, acceleration, concentration, deposition rate, fluid flow, viscosity, humidity, temperature and density [47, 43]. The resonating structure is excited to resonance, characterized by the vibration mode, the resonance frequency, the vibration amplitude, the phase, and the $Q$ factor. Upon change of the physical parameter in question, one or more of these characteristics change and are detected. We experimentally demonstrated the potential for viscosity sensing with our wireless emitter [42] and, thus, an entire new application area presents itself.
Chapter 2

Sensing and Imaging Techniques

A wide variety of different \textit{in vivo} sensing or imaging techniques exists, which can all be used for the localization of minimally invasive medical devices inside of the human body. In this section the characteristics, advantages and disadvantages of these techniques are presented to obtain a broader view of ultrasonic localization.

2.1 X-rays

X-rays are used to image bones, teeth and the thorax (namely the lungs and breast). X-rays have high energy and short wavelengths enabling them to pass through tissue. On their propagation through the body, denser tissues (e.g. bones, or the medical device) will block more of the rays than soft tissues (e.g. lungs). As the x-rays are converted into light using a special film, the denser tissues will appear light and the soft tissues dark. X-ray is fast and gives a clear and detailed view of bones and teeth, but it does not provide equal information about soft tissues. In addition, harmful ionizing radiation is involved in this technique.
2.2 Computed Tomography

X-rays have led to the development of CT. CT or computed axial tomography (CAT) is used to image all parts of the body and is particularly good at testing the brain. The CT scanner emits several x-rays simultaneously from different angles, instead of sending out a single beam through the body. The relative density of the tissues, based on the scan information, is calculated. CT images are far more detailed than those obtained by x-ray, and can nicely distinguish many types of tissues in the same region. Nevertheless, CT has several drawbacks: x-ray ionizing radiation is involved in the technique, and occasionally contrast material must be injected into the body of the patient.

2.3 Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) provides remarkably clear and detailed pictures of internal organs and tissues. The technique has proven very efficient for the diagnosis of a broad range of pathologic conditions in all parts of the body, including cancer, heart and vascular disease, stroke, joint and musculoskeletal disorders. In MRI a high magnetic field is applied to the human body, and the hydrogen atoms (which represent 63% of the human body) emit a nuclear magnetic resonance signal. This signal changes during a lapse of time when photons from the magnetic field are absorbed by the protons from the hydrogen nuclei. Due to the magnetic field, the spins of hydrogen protons are aligned parallel to the field. When the protons absorb the photon energy (emitted in a RF), the net magnetization rotates about the z-axis; when the energy is emitted it returns to equilibrium. This lapse of time — called relaxation time — can be determined using Bloch equations \[ \text{[48]} \]. The nuclear magnetic relaxation time of tissues differs, causing this technique to develop into an imaging technique. Radiation is not involved in MRI, which offers very good spatial and temporal resolution of the brain and soft tissues. Variations in the pulse sequences can emphasize different aspects of the tissue. Furthermore, three-dimensional (3D) images can be achieved. Nevertheless, bone is better imaged by x-rays, and CT is preferred for patients with severe bleeding. In addition, the examina-
2.4 POSITRON EMISSION TOMOGRAPHY

PET scans are performed on the whole body and are most often used to detect cancer and to examine the effects of cancer therapy by characterizing biochemical changes in the cancer. PET scans can also be used to detect brain diseases such as memory disorders, tumors and seizure disorders, as well as heart diseases such as heart attack and blood flow disorders. A low dose of radiopharmaceutical (pharmaceutical radioactive isotope) is injected into the patient and integrated into the organs and tissues. Gamma rays are emitted and the scanner detects the spatial and temporal distribution of the radiopharmaceutical, thereby allowing a dynamic series or static images to be obtained. Coating the medical device with a radiopharmaceutical would make it possible to localize the device using PET. Although gamma radiation exposure is very low, radiation from the radionuclides is involved in this technique and the spatial resolution is relatively low.

2.5 Single Photon Emission Computed Tomography

PET have led to the development of Single photon emission computed tomography (SPECT). A radiopharmaceutical is administered internally. The radiopharmaceutical decays, resulting in the emission of single photon gamma rays. A gamma camera collects the gamma rays emitted within the patient, and reconstructs a picture of where the gamma rays originated. From this, 3D images are obtained to visualize functional information about the patients internal body parts. SPECT enables visualization of functional information about specific organs or body systems. SPECT can help diagnose diseases such as tumors, stroke, cardiac and coronary diseases as well as
bone fractures. Again, coating the medical device with a radiopharmaceuti-
cal would make it possible to localize the device using SPECT. As for PET,
pharmaceutical radioactive isotopes that emit gamma rays are administered
to patients and the spatial resolution is low.

2.6 Ultrasound

Ultrasound scanning is a real-time technique used for imaging internal body
parts as well as for fetus examinations. An ultrasound transducer can func-
tion, as both, a generator and a detector of sound. When the transducer is
in contact with soft tissue, it transmits high-frequency (up to GHz) sound
waves into the body. The sound waves propagate through the body and
hit a boundary between tissues, or a target. The sound waves are partly
reflected back to the transducer, and partly transmitted further until they
reach another boundary. The reflected and/or transmitted waves are picked
up by a transducer and an image of the body is calculated. If the medi-
cal device sends out ultrasound waves, the point of origin of the acoustic
emission can be determined and, thus, the device can be localized.

2.7 Conclusion

The choice of the best localization or imaging technique to help solve any
particular biomedical problem is based on factors such as resolution, contrast
mechanism, high speed, convenience, safety, and acceptability [10]. For
applications in soft tissues, ultrasound obtains high significance in all of
these factors.

It is now possible to visualize details in biological material at the subcellular
level with ultrasound. With ultrasound frequencies up to 1 GHz, the resolu-
tion approaches less than 1 µm at a penetration depth of about 80 µm [49],
and a scanning acoustic microscope can provide about the same spatial res-
olution as an optical microscope [16, 17, 50, 51]. Soft tissue contrast is good
and can be even enhanced by using contrast agents. Ultrasound can be
used in real-time applications. In addition, it is convenient to use, highly
acceptable to patients, and it is apparently safe. It can also be a relatively
inexpensive technology compared to other non invasive imaging methods, such as MR, CT, and x-ray.

The major disadvantages of ultrasound are that its images are affected by the presence of bone or gas, and an operator usually needs a high level of skill in both image acquisition and interpretation. Often, ultrasonic speckle and ultrasound reflections of small objects are indistinguishable from each other. These disadvantages are mainly a problem by using the pulse-response method and can be partially overcome by measuring the TOF or the TDOA.
Chapter 3

Acoustics and Ultrasound

The purpose of this ultrasound survey is to summarize the fundamental acoustic properties and equations. In the first section the basics of acoustics are presented and in the second section the wave phenomena are discussed.

3.1 Basics of Acoustics

Sound is a wave motion within a gaseous, liquid, or solid medium. A fundamental property of all wave motions is that the medium itself does not travel with the wave. The wave rather represents a periodic disturbance or modification of the medium and it is this disturbance that propagates away from the source. In sound propagation the periodic disturbance is an alteration in the ambient pressure of the medium in the same direction in which the wave is propagating. Ultrasound is therefore a longitudinal pressure wave.

The time elapsed for a point to return to the same vibration state (equal phase) is the period $T$. The frequency $f$ of the wave is given by

$$f = \frac{1}{T}. \quad (3.1.1)$$

In the time $T$ the wave covers a distance of one wavelength $\lambda$. Hence, the speed of sound $c$ of the propagating wave is given by

$$c = \frac{\lambda}{T}. \quad (3.1.2)$$
3.1. BASICS OF ACOUSTICS

Fig. 3.1: Classification of ultrasound in the sound frequency spectrum.

The acoustic impedance $Z$ is a material property of a medium in which a wave is propagating and can be written as

$$Z = c\rho,$$  \hfill (3.1.3)

where $\rho$ is the density of the medium. If a wave passes an interface between two media which have a high impedance mismatch, then the wave is mostly reflected. If the impedance mismatch is low, the wave is mostly refracted (see section 3.2).

The frequency span in acoustics range from about 1 Hz to more than $10^{13}$ Hz. Ultrasound describes the part of the sound frequency spectrum with a frequency greater than the upper limit of human hearing (approximately 20 kHz) and goes up to 1 GHz. The rest of the frequency spectrum is partitioned into infrasonic, audiosonic, hypersonic, and acoustic phonon regions, as shown in Fig. 3.1.

Mathematically the sound propagation in 3D space is described by the acoustic wave equation (3.1.4), also known as the Helmholtz equation. This equation is obtained by simplifying the Navier-Stokes equations of fluid momentum and the flow continuity equations \[52\] and can be formulated as

$$\frac{\partial^2 p}{\partial t^2} - c^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right) = 0$$  \hfill (3.1.4)

in Cartesian coordinates, where $p$ denotes the acoustic pressure at time $t$. A one dimensional solution of (3.1.4) for a pressure wave propagating in the $x$-direction is given by

$$p = p_0 \sin(\omega t \pm \frac{2\pi}{\lambda} x + \varphi),$$  \hfill (3.1.5)

where $p_0$ is the ultrasound pressure amplitude, $\omega$ is the angular frequency.
of the wave obtained from
\[ \omega = 2\pi f, \]  
(3.1.6)

2π/λ is its angular wave number, and \( \varphi \) is the phase. The phase of a wave is the fraction of a complete cycle corresponding to an offset in the displacement from a specified reference point at time \( t = 0 \).

### 3.2 Wave Phenomena

In this section we describe the wave characteristics of sound, which are reflection, refraction, diffraction, interference, and absorption.

#### 3.2.1 Reflection

Reflection is the change in direction of a wavefront at an interface between two different media, such that the wavefront returns into the medium from which it originated. The reflection depends significantly on the ratio of the sound wavelength to the size of the interface roughness. The specular reflection, as illustrated in Fig. 3.2, applies only to a totally planar, rigid interface with dimensions much larger than the wavelength. In this case the angle of incidence \( \alpha_I \) is equal to the angle of reflection \( \alpha_R \), that is the reflection that obeys Snell’s law:

\[ \alpha_R = \alpha_I. \]  
(3.2.1)

The intensity \( I_R \) of the reflected wave relative to the intensity \( I_I \) of the incident wave is described by the reflection coefficient [53]

\[ C_R = \frac{I_R}{I_I} = \left( \frac{Z_1 \cos \beta - Z_2 \cos \alpha_I}{Z_1 \cos \beta + Z_2 \cos \alpha_I} \right)^2, \]  
(3.2.2)

where \( \beta \) is the angle of refraction (see following section), and \( Z_1 \) and \( Z_2 \) are the acoustic impedances of medium 1 and medium 2, respectively.

When a sound wave reaches an interface with a roughness in the order of its wavelength the wave is scattered, that is the sound is reflected in a diffuse way. Scattering of ultrasound not only occurs because of rough interfaces
3.2. WAVE PHENOMENA

![Fig. 3.2: Incident, reflected and refracted wave at a boundary layer between two media.](image)

but also because of the finite size of a surface.

### 3.2.2 Refraction

In optics Fermat’s principle states that every wave propagates on the fastest path between two points — not on the shortest. This principle applies to acoustics as well. If the medium is homogeneous then the wave speed is constant and the fastest path is also the shortest, resulting in a propagation along a straight line. If a wave passes from one medium to another with a different speed of sound, the direction of the wave changes due to a change in speed, as shown in Fig. 3.2. This direction change is called refraction. The refraction at the boundary layer between two media is calculated according to Snell’s law, as

\[
\frac{c_1}{c_2} = \frac{\sin \alpha_1}{\sin \beta},
\]

(3.2.3)

with the speed of sound \(c_1\) and \(c_2\) in medium 1 and 2, respectively. The intensity \(I_T\) of the refracted wave relative to the incident wave is described by the refraction or transmission coefficient

\[
C_T = \frac{I_T}{I_I} = 1 - C_R.
\]

(3.2.4)

Depending on the roughness of the interface, the refracted wave is scattered similarly to the reflected wave.
3.2.3 Diffraction

Diffraction is the local change in the direction of the propagation of ultrasound waves passing the edge of an obstacle. This is one of the most important acoustic phenomena and also one of the most complex to solve. The diffraction phenomenon depends significantly on the ratio of the sound wavelength to the size of the obstacle. While diffraction occurs whenever propagating waves encounter obstacles in their paths, its effects are generally most pronounced for waves where the wavelength is in the order of the size of the objects. When a sound wave encounters an obstacle with a size in the order of its wavelength, the wave passes around it, almost as if it did not exist, and almost no acoustic shadow is formed. But if the sound frequency is sufficiently high, and the wavelength is sufficiently short compared to the obstacle and a significant acoustic shadow is obtained.

Diffraction effects can be divided into three groups: diffraction by a barrier, by an aperture, and by an edge. These three groups are illustrated in Fig. 3.3 where the ideal cases for low (a) and high frequencies (b) compared to the obstacle size are shown. The most obvious diffraction effect by a barrier is the presence of sound behind it. On the other hand, diffraction by an edge causes diffusion or scattering of sound and diffraction by an aperture is a
3.2. WAVE PHENOMENA

A general cause of energy loss.

3.2.4 Interference

If two or more ultrasound waves are propagating through the same medium, they interact with each other. This interaction of multiple waves, or one wave with itself after its reflection, is called interference. The resulting alteration of the ambient pressure at any point and time is the sum of all individual ultrasound waveforms. The interference can therefore be calculated by the superposition of all waves. For instance, if two waves with the same amplitude $p_0$, angular wave number and angular frequency are propagating in the same direction along the $x$-axis, shifted about the phase $\varphi$, the resulting pressure wave is given by

$$p = p_0 \left( \sin(\omega t \pm \frac{2\pi}{\lambda} x) + \sin(\omega t \pm \frac{2\pi}{\lambda} x + \varphi) \right).$$ (3.2.5)

3.2.5 Absorption or Attenuation

Absorption or attenuation refers to the reduction in intensity $I$ of the ultrasound wave as a function of propagation distance $x$ through a medium, calculated as

$$I(x) = I_0 e^{-\chi x},$$ (3.2.6)

where $I_0$ is the wave intensity at $x = 0$. $\chi$ is the frequency dependent attenuation coefficient, which is usually determined experimentally. To some extent, attenuation is the material property that changes acoustic energy into (usually) heat energy, which results in the absorption of the ultrasound waves. Accounting for attenuation effects in ultrasound is important because a reduced signal amplitude can affect the quality of the localization or ultrasonic imaging. By knowing the attenuation that an ultrasound beam experiences while travelling through a medium, one can adjust the input signal amplitude to compensate for any loss of energy at the desired imaging depth.
3.3 Conclusion

The physical phenomena involved in sound wave propagation are both numerous and complex, making overall analytical modelling difficult. In ultrasonic localization only an approximation of the physical characteristics is usually possible, due to the large number of parameters which must be taken into account in the description of a realistic environment. That is the reason for the existence of all present and applied approximations in the evaluation of wave propagations.
Chapter 4

Ultrasonic Localization Techniques

Ultrasonic localization is a wireless localization technique that can be broadly classified into two categories: angle of arrival (AOA) localization, and distance related localization. Reviews of wireless localization techniques can be found in [54, 55, 56, 57].

4.1 Angle of Arrival Localization

The AOA localization techniques, which are also called continuous wave methods, can be further divided into two subclasses: those making use of the receiver’s amplitude response (beamforming) and those making use of the receiver’s phase response (phase shift method).

4.1.1 Beamforming

The use of anisotropy in the receiving pattern of a receiver is called beamforming, and it is the basis of one category of AOA measurement techniques [58]. A receiver, which is suited for beamforming has an unidirectional sensitivity, i.e. the sensitivity of the receiver is dependent on its orientation. One can imagine that the beam of the receiver is rotated electronically or mechanically, and the direction corresponding to the maximum
signal strength is taken as the direction of the emitter. Relevant receiver parameters are, therefore, the sensitivity and the beam width of the receiver.

A technical problem to be faced and overcome arises when the emitted signal has a varying signal strength, i.e. if the emitted pressure wave is unidirectional as well. The receiver cannot differentiate the signal strength variation due to the varying amplitude of the emitted signal and the signal strength variation caused by the anisotropy in the receiving pattern. One approach to dealing with the problem is to use a second omnidirectional receiver. By normalizing the signal strength received by the rotating unidirectional receiver with respect to the signal strength received by the omnidirectional receiver, the impact of varying signal strength can be removed.

Another widely used approach to cope with the varying signal strength problem is to use a minimum of two (but typically at least four) stationary receivers with known, unidirectional patterns. Overlapping of these patterns and comparing the signal strength received from each receiver at the same time yields the emitter direction, even when the signal strength changes. Coarse tuning is performed by measuring which receiver has the strongest signal, and it is followed by fine tuning which compares amplitude responses. Small errors in measuring the received power can lead to a large AOA measurement error. A typical measurement accuracy for four receivers is $10^{-15}^\circ$, what can be improved to about $2 - 5^\circ$ with eight receivers [59].

### 4.1.2 Phase Shift Method

The second category of measurement techniques, known as phase interferometry [60] or phase shift (PS) method [58], derives the AOA measurements from the measurements of the phase differences in an arriving wave. The choice of the PS method, which was first introduced by Sutherland [61], introduce ambiguity into the measurements. It is pointed out that this method can only measure relative distance changes [62]. To measure absolute distances, one needs to know the starting point of the emitter and then keep track of the finite distance changes. Other ways of measuring absolute positions are to combine this method with TOF measurements (see time of flight method in section 4.2), or to consider the PSs between many receivers,
which results in high computational complexity.

Another problem is the effect that the signal, which is received, is often the sum of the direct path signal and one or more reflected signals. With changes in the position of the emitter, these multi-path reflections cause drastic and unpredictable variations in the received phase, thus making it hard to successfully implement this method.

The accuracy of AOA measurements is limited by the directivity of the receiver, by shadowing and by multi-path reflections. How to obtain accurate AOA measurements in the presence of multi-path and shadowing errors has been a subject of intensive research. AOA measurements rely on a direct line of sight path from the emitter to the receiver. However, a multi-path component may appear as a signal arriving from an entirely different direction and can lead to very large localization errors in AOA measurements.

Multi-path problems in AOA localization can be addressed by using the maximum likelihood (ML) algorithms [60]. Different ML algorithms have been proposed in the literature which make different assumptions about the statistical characteristics of the incident signals [63, 64, 65]. They can be classified into deterministic and stochastic ML methods. Typically ML methods will estimate the AOA of each separate path in a multi-path environment. The implementation of these methods is computationally intensive and requires complex multidimensional search. The dimensionality of the search is equal to the total number of paths taken by all the received signals [60]. The problem is further complicated by the fact that the total number of paths is not known a priori and must be estimated.

Different from the earlier ML methods, which assume the incoming signal is an unknown stochastic process, another class of ML methods is using subspace based algorithms. The most well known methods in this category are multiple signal classification [66, 67, 68, 69] and estimation of signal parameters by rotational invariance techniques [70, 71]. These eigenanalysis based direction finding algorithms utilize a vector space formulation, which takes advantage of the underlying parametric data model for the receiver array problem. Readers may refer to [72, 73, 74] for a detailed technical discussion on AOA localization techniques.
4.2 Distance Related Localization

Distance related localizations are geometrical methods that follow the high frequency approximations based on geometrical propagation paths. In principle, they consider the propagation of sound through the media in straight lines neglecting the wave nature of sound. While reflection, refraction, and absorption are modelled, diffraction and interference are usually not considered [75]. Currently, most known acoustic localization systems are based on distance related localization techniques [18].

Distance related localization include acoustic ray or beam tracing, propagation time based localization, which are the time of flight and pulse response method, and passive or hyperbolic localization. All these methods require in one or another way TOF measurements between emitted and received signals. In the following paragraphs we provide further details on these techniques.

4.2.1 Propagation Time Based Localization

Propagation time based localization is also known as time of arrival based localization. Distances between an emitter or a target and a receiver can be estimated from propagation time measurements. The propagation time based localization is classified in methods using transmission ultrasound (time of flight method), and pulse echo ultrasound (pulse response method).

Time of Flight Method

In the TOF method the target emitter generates an ultrasound pulse [76,77]. The pulse is detected by one or more receivers. The propagation TOF $t_i$ of the pulse from the emitter to the $i^{th}$ receiver is measured. This requires the local time at the emitter and the local time at the receivers to be accurately synchronized. This requirement may add to the cost of receivers and emitters by demanding a highly accurate clock and/or increase the complexity of the sensor network by demanding a sophisticated synchronization mechanism. With the speed of sound $c$ the distance $d_i$ between the $i^{th}$ receiver and the
4.2. DISTANCE RELATED LOCALIZATION

emitter can be calculated as

\[ d_i = c t_i. \quad (4.2.1) \]

If at least three receivers are used to detect the pulse, and the exact positions of the receivers are known, the position of the sound source in 3D space can be calculated by triangulation, as shown in Fig. 4.1. From equation (4.2.1), the distance \( d_i \) to each receiver is known. If we imagine a sphere with radius \( d_i \) around each receiver, then the emitter location estimate \( E(x_E, y_E) \) is found by solving the following system of equations with e.g. a least squares method

\[ \sqrt{(x_E - x_i)^2 + (y_E - y_i)^2} = d_i. \quad (4.2.2) \]

A major advantage of the TOF method over pulse response method (see next paragraph), like B-scan imaging, is in positive identification of minimally invasive medical devices (like endoscopes, catheters, or microrobots). Other advantages of the TOF method can be found in applications that already rely on ultrasonic keyhole surgery, such as intravascular imaging. The ultrasonic pulse, sent out for imaging purposes, can be directly used to track the device in the body.

A first system using ultrasonic TOF measurements as a localization method for biomedical application was mentioned by Breyer et al. [78]. They de-
scribed active and passive catheter localization techniques. The passive localization technique used a catheter transducer as a receiver in an ultrasonic field. The TOF between emission from the imaging transducer and its reception on the catheter transducer was recorded and used to inject a marker signal at the appropriate image location. Vilkomerson et al. presented a system for ultrasonic beacon-guidance of catheters and other minimally invasive medical devices [11]. With an omnidirectional receiver mounted on a device, the receiver’s location in the ultrasound image was deduced by determining which acoustic ray struck the receiver, and by measuring the TOF from the emitter to the receiver. The primary limitation of the previously described localization techniques was the requirement that the catheter tip is positioned in a two-dimensional (2D) scanner’s image plane.

A method for locating a transducer, which was mounted on a catheter, in a 3D ultrasonic imaging field is described by Merdes and Wolf [19]. The distance from the imaging transducer to the catheter transducer is measured by TOF, while the angular position is determined by a spatial cross-correlation of the received signals with stored receiver profiles. In vitro testing showed a resolution of $0.23 \pm 0.11 \text{mm}$ at a range of 75 mm and a resolution of $0.47 \pm 0.47 \text{mm}$ at a range of 97 mm. Although this approach produces relatively high resolution, it requires the instrument to be integrated electronically with the ultrasound scanner, which adds cost and complexity.

In a study by Aoyagi et al. [79] a system measuring 3D position and orientation of a robot hand using ultrasonic waves is proposed. In this system, three ultrasonic emitters are attached to the tip of a robot arm and several receivers are placed around the robot’s work space. The newly proposed principle of measurement is a triangulation using distances between the three emitters and more than three receivers.

A method to precisely determine the position of a sound source placed at a sea floor under a condition that the acoustic velocity distribution is not known is discussed by Isshiki et al. [80] [81] [82]. Not only the position of the emitter but also the unknown underwater parameters can be determined at the same time with their method, if the position of the surface transducers and ranges between the surface and bottom transducers are known.
4.2. DISTANCE RELATED LOCALIZATION

**Pulse Response Method**

The pulse response or pulse echo method depends on TOF measurements, as the time of flight method. Ultrasound pulses are directed to the region of interest at known angles $\alpha$, where the pulses are reflected at target interfaces along the propagation path. The echoes were detected by the same acoustic transducer, which emitted the pulses, or by a receiver located close to the emitter. Since the same clock is used to compute the round trip propagation time, there is no synchronization problem. The round trip time $t$ of the pulse between the transducers and the target is measured to predict the distance $d$ between them as

$$d = \frac{ct}{2}. \quad (4.2.3)$$

The target location is then predicted to be at a distance $d$ at an angle $\alpha$ from the acoustic transducer.

A high resolution system based on the pulse response method was presented by Culjat et al. [15]. The system provided a complete circumferential scan of a human tooth and its interior with an axial and lateral resolution of approximately 100 $\mu$m and 750 $\mu$m, respectively.

The big challenge of the pulse response method is to measure the propagation time as accurately as possible. Time measurement is a relatively mature field. The most widely used method for obtaining time delay measurement is the generalized cross-correlation method [77, 83, 84]. A detailed discussion on the cross-correlation method is given in appendix A.1. A recent trend in propagation time measurements is the use of ultra wide band (UWB) signals for accurate distance estimation [85, 86, 87]. A UWB signal is a signal whose bandwidth to center frequency ratio is larger than 0.2 or a signal with a total bandwidth of more than 500 MHz. UWB can achieve higher accuracy because its bandwidth is very large and therefore its pulse has a very short duration.

The major disadvantage of the pulse response method is that the pulses have to propagate twice through the media before they get detected. As a result the penetration depth is cut in half, which is particularly adverse at high ultrasound frequencies (as usually used in UWB signals) with higher attenuation. Another problem are the phase changes, that occur at inter-
faces between two media. The disregard of echoes with phase changes may result in a systematic error of the estimated round trip time. In addition, ultrasonic speckle and reflections of small objects can be indistinguishable from each other, such that small objects (as endoscope pills or microrobots) are hardly detectable. For example, with the pulse response method any section of a medical device will appear as the device tip, if the medical device crosses the scanning plane obliquely. If the medical device does not cross the scanning plane, it does not appear at all. Researchers have tried to overcome this problem by means of marking the device tip. However, the medical device still has to be located in the imaging plane [11]. Therefore this method is inadequate as a localization method for small untethered structures.

There are also ultrasonic localization systems, that combine TOF and phase shift measurements. The first such system was presented by Carson et al. [88]. More recently, Kuang and Morris [89] designed an integrated ultrasonic robot tracker, which uses, both, a TOF and a Doppler localization system to achieve better accuracy and resolution in 3D dynamic robot localization. The mean measurement error of this system was estimated to be in sub-mm range.

4.2.2 Acoustic Ray or Beam Tracing

Ray tracing based approaches, which have been first proposed by Greenleaf and Johnson [32, 90], are often used in localization and to determine the shape of a wavefront after the wave is reflected and refracted. Some ray tracing approaches are described in [33, 34, 35, 91]. The roots of ray tracing are located in geometrical optics, where the refraction at the boundary layer between two media is calculated according to Snell’s law (3.2.3).

Using ray tracing for the localization of an emitter $E$ inside a medium 1 with several receivers $R_i$ located in a medium 2, is illustrated in Fig. 4.2. An estimate $E_{ij}$ of the target emitter position for a measured TOF $t_i$ from the emitter $E$ to the $i^{th}$ receiver $R_i$ can be found by solving the following equation

$$t_i = \frac{|e_{ij} - p_j|}{c_1} + \frac{|p_j - r_i|}{c_2},$$

(4.2.4)
4.2. DISTANCE RELATED LOCALIZATION

Fig. 4.2: Illustration for acoustic ray tracing.

where $|e_{ij} - p_j|$ is the norm of the vector from the possible emitter location to any point $P_j$ on the boundary layer, and $|p_j - r_i|$ is the norm of the vector from point $P_j$ to the receiver location $R_i$. The directions of these vectors can be calculated from

$$
\frac{c_1}{c_2} = \frac{\sin \alpha_{ij}}{\sin \beta_{ij}}. \tag{4.2.5}
$$

If we calculate $e_{ij}$ for all boundary points $P_j$ we get all possible emitter locations for a given TOF $t_i$ to one receiver $R_i$. The resulting emitter location $E$ can then be calculated by finding the smallest intersection area of all found emitter locations from at least three different receiver positions.

As described in [92], ray tracing in acoustics is a high frequency (or short wavelength) approximation of the exact integral representation of the acoustic pressure field. For applicability of this high frequency approximation for $c_1 > c_2$, it is necessary that the location of the target emitter is sufficiently far from the boundary in comparison with the wavelength. On the other hand, when $c_1 < c_2$, it is necessary that the receivers are sufficiently far from the boundary. Furthermore, the boundary must be broken up into small planar segments, where the dimensions have to be in the order of the ultrasonic wavelength. The breaking up of the boundary into small segments advise to use FEM. General limits of the ray tracing approximation for acoustics are discussed in detail in [92, 93, 94].

Beam tracing is a derivative of the ray tracing algorithm that replaces rays, which have no thickness, with beams. Beams are shaped like unbounded
CHAPTER 4. ULTRASONIC LOCALIZATION TECHNIQUES

pyramids, with polygonal cross sections. For some applications of beam tracing we refer to [95, 96, 97]. In the field of acoustic modeling, beams are used as an efficient way to track deep reflections from a sound source to a receiver, where ray tracing is prone to sampling errors.

As the TOF is measured in ray or beam tracing, it is required to accurately synchronize the local time at the emitter and the local time at the receivers, as in the time of flight method.

4.2.3 Passive or Hyperbolic Localization

Passive or hyperbolic localization [98, 99, 100, 101, 102] refers to the problem of fixing the spatial position of an emitter based on the TDOA of its emission at different receivers. The solutions are based on the hyperbolic trajectory described by

\[
\sqrt{(x_E - x_1)^2 + (y_E - y_1)^2} - \sqrt{(x_E - x_2)^2 + (y_E - y_2)^2} = c\Delta t, \quad (4.2.6)
\]

of an emitter \(E(x_E, y_E)\) which is emitting with a constant time difference of arrival \(\Delta t\) to two receivers at positions \(R_1(x_1, y_1)\) and \(R_2(x_2, y_2)\) in 2D. With three receivers three hyperbolas can be constructed, the intersection of which localizes the emitter, as shown in Fig. 4.3. In 3D the curves must be replaced by hyperboloid surfaces. The resulting system of equations is not linear and generally does not have a direct analytical solution.

Measuring the TDOA of a signal at two receivers at separate locations is a relatively mature field [103]. The most widely used method to determine the TDOA is the cross-correlation method, as in time delay measurements.

The advantage of hyperbolic localization over the time of flight method is that it does not depend on absolute TOF measurements, and, thus, no knowledge about the starting time of the pulse and no synchronization of the emitter with the receivers is required. Only the TDOA is needed, and therefore, it requires only very accurate synchronization among the receivers.

Fang [98] gave an exact solution to the hyperbolic localization problem for the case in which the number of TDOA measurements is equal to the number of unknown emitter coordinates in (4.2.6). However, his solution cannot
make use of extra measurements. Other techniques that can deal with the more general situation with extra measurements include the spherical interpolation method introduced by Smith and Abel [104] and the divide and conquer method by Abel [105]. The spherical interpolation method is derived from least-squares minimization of equation errors, and the divide and conquer method is formed by combining the ML estimates using possibly overlapping subsections of the measurement TDOA data. Chan et al. [99] developed a closed form solution valid for an arbitrary number of TDOA measurements and arbitrarily distributed emitters. The solution is an approximation of the ML estimator when the TDOA measurement errors are small. Chan's method performs significantly better than the spherical interpolation method and is more robust against noise than the divide and conquer method. The computational complexity of Chan's method is comparable to the spherical interpolation method but substantially less than the Taylor-series method [106]. Doğançay and Drake developed a closed form solution for localization of distant emitters based on triangulation of hyperbolic asymptotes [107, 108], where the hyperbolic curves are approximated by linear asymptotes. The solution exhibits some performance degradation with respect to the ML estimator at low noise levels but outperforms the ML estimator at medium to high noise levels.
4.3 Conclusion

Distance related localization works fine for acoustic systems, but not very well for RF and optical systems, as sound travels relatively slow compared to radio or light waves. This results in a significant time difference between the arrival of the direct path signal and the first reflection signal. A striking criterion of ultrasonic distance related systems over AOA systems is that multi-path reflections are of minor importance.

The need for synchronization of the clocks, which is required for the time of flight method and acoustic ray tracing, makes them a less attractive option than the pulse response method or passive localization. The pulse response method has a limited penetration depth due to the doubling of the propagation path and only a limited feasibility in positive identification of small objects. Thus, passive localization is the most suitable method for localizing minimally invasive medical devices.
Chapter 5

Wireless Acoustic Emitter

When localizing a device using acoustic emission, a miniaturized sound source must be embedded into the device. Available ultrasonic emitters include piezoelectric crystals excited with a harmonic voltage of up to several hundred volt, or MEMS devices that still require up to 25V driving voltage \[109\] in addition to the wiring and energy storage inside the miniaturized medical device.

The advantages of the wireless acoustic emitter presented here are its untethered nature, small size, simple design, and low cost, as the wireless resonator does not require on-board energy storage and wiring, and only soft magnetic material is used instead of permanent magnets.

5.1 Design, Fabrication, and Microassembly

The wireless emitter consists of two parallel soft magnetic (nickel) square dies (facing area \(A\), thickness \(h_{Ni}\)) separated by a plastic shim (thickness \(h_{PS}\)). One of the dies, the resonator, is suspended on two gold beam springs (thickness \(h_{Au}\)); the other one is fixed to the gold base frame of the device as shown in Fig. 5.1(a). The dimensions of the resonator (gold springs with attached nickel die) is shown in Fig. 5.1(b). The exact dimensions of the structure can be found in the framework plan shown in Fig. B.4 in the appendix and in Table 5.1. The resonator and the base frame are microfabricated separately, and then glued together in our microassembly.
CHAPTER 5. WIRELESS ACOUSTIC EMITTER

Gold frame with springs
Nickel die (resonator)
Plastic shim
Nickel die
Gold base frame

(a) Schematic design

(b) Dimensions

Fig. 5.1: Schematic design and dimensions of the resonator system with the plastic shim between them.

5.1.1 Microfabrication of the Resonator and Base Frame

The microfabrication of the resonator is shown in Fig. 5.2. The process is the same for the base frame. Initially, wafers are cleaned using a piranha solution in order to remove native oxides as well as organic and ionic substances from the surface. Additionally, the wafers are cleaned with distilled water and finally dried in a spin rinse dryer. Electron-beam evaporation in a high vacuum chamber is then used to apply a good-adhesion titanium layer (thickness: 25 nm), followed by a sacrificial copper layer (thickness: 500 nm), and again a final titanium layer (thickness: 25 nm). The titanium-copper-titanium layer on top of the wafer is shown in Fig. 5.2(a). A negative photoresist is applied by spin coating and shaped by ultraviolet (UV) photolithography [5.2(b)]. The top titanium layer is removed using a solution of 0.5% HF and 5.5% NH$_4$F and subsequent cleaning with distilled water. The native oxide on the copper surface is then etched by rinsing with a 10% solution of H$_2$SO$_4$. Afterwards the main gold structure is electrodeposited using a commercial sulphite gold electrolyte and the photoresist is removed using acetone [5.2(c)]. The same procedure of applying and patterning photoresist, as well as pre-electroplating surface treatment is repeated again for the nickel dies [5.2(d)]. The electroplating of nickel using a sulphamate nickel electrolyte and removal of the photoresist with acetone [5.2(e)] fol-
5.1. DESIGN, FABRICATION, AND MICROASSEMBLY

(a) Titanium-copper-titanium layer on top of wafer

(b) Photoresist is added and developed

(c) Top titanium layer is removed, gold structure is added and photoresist is removed

(d) Photoresist is added and developed

(e) Nickel is added and photoresist is removed

(f) Etching of copper layer released the resonator

Fig. 5.2: Schematic fabrication process flow.

Finally, selective etching of the copper layer using a 3.5% solution of \( \text{NH}_4\text{S}_2\text{O}_8 \) and 7.3% \( \text{NH}_3 \) for 48 hours releases the parts from the wafer (5.2(f)).

5.1.2 Microassembly of the Wireless Emitter

Despite the relatively large size of the elements, manual assembly has proven to be inefficient. Therefore, the assembly of the microfabricated components and the plastic shim is performed using our microassembly system, which is a serial system with full six degrees of freedom for high precision microassembly [110, 111].

The microassembly process is shown in Fig. 5.3. First, resonator, base frame, and plastic shim are placed on the workbench and held in place by a vacuum. Small droplets of UV-curable glue are deposited on two opposing sides of the base frame using a miniature pipette (5.3(a)). Next, the plastic shim is picked in a corner (5.3(b)), and exactly placed and aligned on the base frame (5.3(c)). UV illumination is used to fixate these parts. Next, the resonator is gripped at one of its two springs first (5.3(d)), and, again, small droplets of UV glue are applied to two opposing corners (5.3(e)) of the plastic
shim. The resonator is then precisely placed on the existing structure. The gripper fingers are used to compress the sandwich (5.3(f)) in order to squeeze out excessive amounts of glue, thus achieving design dimensions. Finally, intense UV illumination is used to cure the glue and to fixate the entire wireless resonator.

The microassembly system is also used for a first function check of the assembled wireless emitter. For this purpose the gripper is used to push down the suspended resonator as shown in Fig. 5.4. The visual analysis of the shown device yields a maximum range of the resonator movement on the order of 100 µm, which is the upper limit for efficient functionality. Emitters with higher displacements can be rejected directly after the microassembly without further testing. With the microassembly station it is also possible to reject emitters, where the resonator is glued to the base frame in a way that no oscillation can be achieved.

Fig. 5.3: Microassembly of the acoustic emitter. See text for details.
5.2 Magneto-Acoustic Transduction Theory

Figure 5.5 shows the working principle of the wireless emitter. When it is placed in a magnetic field $H$, the soft magnetic dies become magnetized and an attractive or repulsive force $F$ between the dies is generated, causing the suspended nickel die to move and the gold springs to deflect. Exciting the structure at resonance results in a continuous vibration, acting as a source for pressure waves $p$ inside the surrounding fluid.

5.2.1 Magnetic Force

In general, the magnetization of the parts will mutually interact, and the determination of the resulting magnetization is a complex problem. As an approximation, we neglect this interaction and assume the same uniform magnetization $M$ in both nickel dies. Because the dies are very thin, $M$ will lie in the plane of the dies for a large range of applied angles $\theta$. For $\theta$
## Variable Description Value Unit

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<th>Variable</th>
<th>Description</th>
<th>Value</th>
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**Tab. 5.1:** Variables and material properties of different media.
5.2. MAGNETO-ACOUSTIC TRANSDUCTION THEORY

![Diagram showing magnetic field and forces](image)

**Fig. 5.5:** Working principle of the wireless acoustic emitter.

close to 90°, the magnetization will rotate and align with the z-axis. For the two limiting cases $\theta = 0^\circ$ ($H$ in the $x$ direction) and $\theta = 90^\circ$ ($H$ in the $z$ direction), the resulting forces $F^x$ and $F^z$ in the $z$ direction between the two dies are computed as

$$F^x = \frac{3\mu_0 V^2 |M^x|^2}{4\pi d_d^4} e_z \quad \text{and} \quad F^z = -\frac{3\mu_0 V^2 |M^z|^2}{2\pi d_d^4} e_z,$$

(5.2.1)

where $d_d$ is the distance between the dipole centers, $V$ is the volume of the dies, and $\mu_0$ is the permeability of free space. The magnetization $|M^x|$ and $|M^z|$ in the $x$ and $z$ direction can be approximated as

$$|M^x| \approx \frac{1}{n_x} |H| \quad \text{and} \quad |M^z| \approx \frac{1}{n_z} |H|,$$

(5.2.2)

with $n_x$ and $n_z$ being the demagnetization factors in the $x$ and $z$ direction. With $d_d$ and $V$ being equal in both cases of (5.2.1), we estimate

$$\frac{|F^x|}{|F^z|} \approx \frac{1}{2} \left( \frac{n_x}{n_z} \right)^2.$$

(5.2.3)

Using $n_x = 0.0595$ and $n_z = 0.8810$ computed from (5.1.4), we conclude that the force $F^x$ is about 110 times bigger than $F^z$.

The magnetic field $H$ generated by a coil with radius $R$ and $N$ turns is given
by
\[ |H(z_0, t)| = \frac{NR^2}{2\left(R^2 + z_0^2\right)^{3/2}}i(t) \] (5.2.4)
where \(z_0\) is the position of interest above the coil and \(i(t)\) is the excitation current. When an oscillating current \(i(t)\) with frequency \(f\) is applied, an oscillating magnetic field and an oscillation force with the same frequency is generated, and no phase shift occurs as the magnetization settles within a few µs.

Instead of a sinusoidal excitation current we use an on/off signal with amplitude \(i_0\) and angular frequency \(\Omega = 2\pi f\), given by
\[ i(t) = i_0 \cdot \Gamma(t), \] (5.2.5)
where
\[ \Gamma(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin[(2n - 1)\Omega t]}{2n - 1} \] (5.2.6)
is a normalized on/off signal. The magnetic force due to a sinusoidal current would act against the spring force during the whole actuation cycle, thereby reducing the restoring force. This is not the case using this on/off current.

### 5.2.2 Oscillation of the Resonator

Since the moving die is much stiffer than the springs, the vertical motion of the resonator is described by the one dimensional (in the \(z\) direction) forced mass-spring-damper system
\[ m\ddot{z}(t) + D\dot{z}(t) + kz(t) = |F| \cdot \Gamma(t)^2, \] (5.2.7)
where \(m = A(\rho_{Au}h_{Au} + \rho_{Ni}h_{Ni})\) is the total mass of the resonator. The damping \(D\) is approximated from the drag of circular disks \([115]\)
\[ D = -16\eta\sqrt{\frac{A}{\pi}}, \] (5.2.8)
with the dynamic fluid viscosity \(\eta\). Tab. \([5.2]\) summarizes the damping coefficient \(D\) in water (H₂O), silicon oil 350 (S350), and glycerin (GL). The total

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<table>
<thead>
<tr>
<th>fluid</th>
<th>( \eta ) [Pa s]</th>
<th>( D ) [Ns/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(_2)O</td>
<td>0.0010</td>
<td>9.03 \times 10^{-6}</td>
</tr>
<tr>
<td>S350</td>
<td>0.35</td>
<td>3.16 \times 10^{-3}</td>
</tr>
<tr>
<td>GL</td>
<td>1.41</td>
<td>1.27 \times 10^{-2}</td>
</tr>
</tbody>
</table>

**Tab. 5.2:** Damping coefficient \( D \) in water (H\(_2\)O), silicon oil 350 (S350), and glycerin (GL) with different dynamic viscosities \( \eta \).

<table>
<thead>
<tr>
<th>( h_{Au} ) [( \mu \text{m} )</th>
<th>( m ) [mg]</th>
<th>( k ) [N/m]</th>
<th>( f_n ) [kHz]</th>
<th>( k_{AN} ) [N/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.733</td>
<td>90.7</td>
<td>1.77</td>
<td>107</td>
</tr>
<tr>
<td>20</td>
<td>0.829</td>
<td>215</td>
<td>2.56</td>
<td>249</td>
</tr>
<tr>
<td>25</td>
<td>0.926</td>
<td>420</td>
<td>3.38</td>
<td>476</td>
</tr>
</tbody>
</table>

**Tab. 5.3:** Analytically calculated mass \( m \), spring constant \( k \), spring constant \( k_{AN} \) obtained in ANSYS (see chapter 5.3.2), and natural frequency \( f_n \) for different gold springs thicknesses \( h_{Au} \).

The spring constant \( k \) of the suspension gold beams is given by

\[
k = \frac{4E_{Au}bh_{Au}^3}{l^3},
\]

where \( l \) is the length, \( b \) the width (see Fig. 5.1(b) and Tab. 5.1), and \( h_{Au} \) the thickness of the gold beam, and \( E_{Au} \) is the Young’s modulus of gold. In Tab. 5.3 the analytically calculated mass \( m \) and the spring constant \( k \) are summarized for different gold springs thicknesses \( h_{Au} \). \(|\text{F}^j|\) is the maximal value of the magnetic force \( \text{F}^j \) in the \( j \) direction (\( j = x, z \)) given by

\[
|\text{F}^x| = \frac{3\mu_0}{16\pi} \left( \frac{VR^2}{z_0} \right)^2 \left( \frac{n^2}{d^2} \right)^2 \text{ and } |\text{F}^z| \approx 2 \left( \frac{n^2}{n^2} \right)^2 |\text{F}^x|.
\]

As \( \Gamma(t) \) is an on/off signal with amplitude one, we can use the identity \( \Gamma(t)^2 = \Gamma(t) \) to simplify (5.2.7), and the forced mass-spring-damper system becomes

\[
m\ddot{z}(t) + D\dot{z}(t) + kz(t) = |\text{F}^j| \cdot \Gamma(t).
\]

The natural frequency \( f_n \) of the system is given by

\[
f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]
with $\omega_n$ being the natural angular frequency. The resonant frequency $f_R$ of (5.2.11) is calculated as

$$f_R = \frac{1}{2\pi} \sqrt{\frac{k}{m} - \frac{D^2}{4m^2}} = \frac{1}{2\pi} \sqrt{\frac{k}{m} - \frac{64A\eta^2}{\pi m^2}}.$$  \hspace{1em} (5.2.13)

### 5.2.3 Pressure Field

To determine the acoustic pressure field from a single planar resonator the impulse response method can be used, where the pressure field is related to the resonator geometry and the surface velocity of the resonator \[116, 117, 118\]. The impulse response method is based on the Rayleigh equation \[119\], which states that the time-dependent velocity potential, $\phi(p, t)$ is (see Fig. 5.6)

$$\phi(p, t) = \frac{1}{2\pi} \iint \frac{\dot{z}(t - \Delta/c)}{\Delta} dA,$$  \hspace{1em} (5.2.14)

where $c$ is the speed of sound in the fluid, $\dot{z}(t - \Delta/c)$ is the normal velocity across the area of the resonator within the plane in which it moves. The vector $p$ defines the point $P(x, y, z)$ in a non-dissipative and homogeneous medium at which $\phi(p, t)$ occurs at time $t$, and $\Delta$ is the distance from $P(x, y, z)$ to the infinitesimal surface element $dA$ of the resonator.

The velocity potential is related to the pressure by

$$p(p, t) = \rho \frac{\partial \phi(p, t)}{\partial t},$$  \hspace{1em} (5.2.15)

with the fluid density $\rho$. Using the convolution operator $\ast$, the problem
(5.2.15) can be expressed as
\[ p(p, t) = \rho \frac{\partial \hat{z}(t)}{\partial t} \ast \Phi(p, t), \tag{5.2.16} \]
where \( \hat{z}(t) \) denotes the velocity waveform at the surface of the resonator, which corresponds to the velocity steady-state solution of (5.2.11). \( \Phi(p, t) \) is the scalar impulse response according to (5.2.14) for the resonator geometry as generated by an impulsive front surface motion in the form of a Dirac delta function \( \delta \):
\[ \Phi(p, t) = \frac{1}{2\pi} \int \int \frac{\delta(t - \Delta/c)}{\Delta} dA. \tag{5.2.17} \]
The Dirac delta function \( \delta \) is defined as
\[ \delta(t - \Delta/c) = \begin{cases} \infty, & t - \Delta/c = 0 \\ 0, & t - \Delta/c \neq 0 \end{cases}. \tag{5.2.18} \]
The \( \delta \) function obeys the scaling property
\[ \delta(\kappa x) = \frac{\delta(x)}{|\kappa|}, \tag{5.2.19} \]
for \( \kappa \neq x \), and the sifting property
\[ \int_{x_0 - \xi}^{x_0 + \xi} f(x) \delta(x - x_0) dx = f(x_0), \tag{5.2.20} \]
for \( \xi \in \mathbb{R}^+ \). It is clear from (5.2.17), (5.2.18), and Fig. 5.6 that for a given point \( P \) at time \( t \), only the surface element of the resonator which is located at a distance of \( \Delta = ct \) from \( P \) contributes to the scalar impulse response \( \Phi(p, t) \), as \( \delta(t - \Delta/c) = 0 \) for \( \Delta \neq ct \). In principle (5.2.17) states that the scalar impulse response is the area of the emitter at a distance \( \Delta = ct \) from \( P \) divided by \( 2\pi \Delta \).

To simplify (5.2.17) the surface element \( dA \) is taken into account as \( \Delta \) increases to \( \Delta + d\Delta \). \( dA \) can be found by considering a sphere of radius \( \Delta \) centered at point \( P(x, y, z) \). The plane of the emitter will dissect this sphere, and the line of dissection will be a circle of radius
\[ \Delta' = \sqrt{\Delta^2 - z^2} \tag{5.2.21} \]
centered at a new location $P'(x, y, 0)$. A second sphere of radius $\Delta + d\Delta$ centered at point $P(x, y, z)$ results in a second line of dissection, which is a circle of radius
\[
\Delta' + d\Delta' = \sqrt{(\Delta + d\Delta)^2 - z^2}
\]
(5.2.22)
centered at $P'(x, y, 0)$. Assuming $d\Delta$ to be very small we obtain from (5.2.21) and (5.2.22) that
\[
\Delta' d\Delta' = \Delta d\Delta.
\]
(5.2.23)
Hence, the infinitesimal surface element $dA$ can then be expressed as
\[
dA = \beta(p, \Delta/c)\Delta' d\Delta' = \beta(p, \Delta/c)\Delta d\Delta,
\]
(5.2.24)
where the angle $\beta(p, \Delta/c)$ refers to the angle in the plane of the emitter as a function of vector $p$ and time $\Delta/c$. Finally, the surface integral in (5.2.17) can be expressed as an integral with respect to $d\Delta$ over the infinite range of $\Delta$:
\[
\Phi(p, t) = \frac{1}{2\pi} \int_{0}^{\infty} \delta(t - \Delta/c)\beta(p, \Delta/c) \frac{\Delta}{\Delta} d\Delta
\]
(5.2.25)
Making the substitution $\tau = \frac{\Delta}{\tau}$ and $\frac{d\tau}{d\Delta} = \frac{1}{\tau}$, respectively, results in
\[
\Phi(p, t) = \frac{c}{2\pi} \int_{0}^{\infty} \delta(t - \tau)\beta(p, \tau) d\tau
\]
(5.2.26)
Using the scaling property (5.2.19), and the sifting property (5.2.20) of the Dirac delta function results in
\[
\Phi(p, t) = \frac{c}{2\pi} \int_{0}^{\infty} \delta(-\tau - t)\beta(p, \tau) d\tau
\]
(5.2.27)
where the angle $\beta(p, t)$ refers to the angle in the plane of the resonator as a function of vector $p$ and time $t$. The pressure waveform (5.2.16) can finally
5.2. MAGNETO-ACOUSTIC TRANSDUCTION THEORY

be calculated as
\[ p(\mathbf{p}, t) = \frac{\varepsilon \rho}{2\pi} \frac{\partial [\ddot{z}(t)]}{\partial t} * \beta(\mathbf{p}, t). \]  

(5.2.28)

Figure 5.7 and Fig. 5.8 show the predicted normalized peak-to-peak pressure waveform in the \(xz\)-plane for two square emitters of different side lengths. The emitters are driven by a tone-burst signal of 20 cycles centered at \(f = 3.38 \text{ kHz}\). The pressure field is shown for clarity in terms of, both, a three-dimensional plot (5.7(a) and 5.8(a)), and a contour plot (5.7(b) and 5.8(b)). The emitter, marked with the gray area in the 3D plot, lies in the \(xy\)-plane with its center located at the origin of the coordinate system.

In Fig. 5.7 the emitter has a side length of \(4\lambda\). It can be seen, that a transducer of this size has an unidirectional pressure field, with a finite beam width and a focal point at \(x = 0\) and \(z \approx 5.5\lambda\). The emitter of Fig. 5.8 has a side length of \(\lambda/2\). It can be seen that the pressure field in this case is point symmetric about the origin of the coordinate system, that is the field is omnidirectional. Such an omnidirectional acoustic pressure field is a key requirement for efficient operation of the device as an acoustic emitter for passive localization applications, as the acoustic signal must be detectable regardless of the resonator orientation.
5.2.4 Potential for Viscosity Determination

As (5.2.13) relates the viscosity of a fluid to the resonant frequency of the system, we can use (5.2.13) to determine an unknown viscosity. For this, the resonant frequency $f_1$ in a fluid with known viscosity $\eta_1$ is used. $f_1$ correspond to the frequency of the acoustic pressure waveform emitted in the known fluid. The unknown viscosity $\eta_2$ of a second fluid is then related to the measured resonant frequency $f_2$ in this fluid, and to $f_1$, and $\eta_1$ as

$$\eta_2 = \sqrt{\frac{\pi^3 m^2}{16 A}} (f_1^2 - f_2^2) + \eta_1^2. \quad (5.2.29)$$

With the frequency shift $\delta f = f_1 - f_2$ the relative viscosity error $\Delta \eta_2 / \eta_2$ of (5.2.29) can be calculated as

$$\frac{\Delta \eta_2}{\eta_2} = \frac{\partial \eta_2}{\partial f_1} \frac{\Delta f_1}{\eta_2} + \frac{\partial \eta_2}{\partial \delta f} \frac{\Delta \delta f}{\eta_2} + \frac{\partial \eta_2}{\partial \eta_1} \frac{\Delta \eta_1}{\eta_2} \quad (5.2.30)$$

$$= \frac{1}{\eta_2^2} \left( \frac{\pi^3 m^2}{16 A} (f_1 \Delta \delta f + \delta f \Delta f_1 - \delta f \Delta \delta f) + \eta_1 \Delta \eta_1 \right), \quad (5.2.31)$$

with the absolute errors $\Delta f_1$, $\Delta \delta f = 2 \Delta f_1$ (for $\Delta f_2 = \Delta f_1$), and $\Delta \eta_1$ of $f_1$, $\delta f$, and $\eta_1$, respectively. Solving (5.2.30) for $\delta f$, such that $\Delta \eta_2 / \eta_2 \leq \varepsilon$, with the maximal relative error $\varepsilon$, results in the minimal frequency difference $\delta f_{\text{min}}$ for which (5.2.29) is valid:

$$\delta f_{\text{min}} = f_1 + \frac{\Delta f_1}{2 \varepsilon} - \sqrt{\left( f_1 - \frac{\Delta f_1}{2 \varepsilon} \right)^2 + \frac{16 A \eta_1}{\pi^3 m^2} \left( \frac{\eta_1 - \Delta \eta_1}{\varepsilon} \right)}, \quad (5.2.32)$$
5.3 Mechanical Simulation

Finite element (FE) simulations in ANSYS are used to create a design tool for the wireless acoustic emitter. For this purpose input text files with macros were created for the fast changing of the simulation environment.

The following sections explain how the mechanical properties of the wireless emitter are characterized. In the first part a modal and in the second part a structural analysis is performed.

5.3.1 Modal Analysis

To design a wireless emitter with specific natural modes a FE simulation is used. The emitter model for the FE simulation is a 50 $\mu$m thick nickel die (grey) mounted on a gold frame (black) of variable thickness $h_{Au}$, which acts as a spring, as shown in Fig. 5.9. The nickel die has an active acoustic area of $1 \times 1 \text{mm}^2$ size. The gold springs are 300 $\mu$m wide and arranged in a way that the first resonant mode results in the appropriate movement of the nickel die in its normal direction.

For the FE modal analysis in ANSYS the SOLID186 element type is chosen. This is a higher order 3D solid element that exhibits quadratic displacement...
behavior. The element is defined by 20 nodes having three degree of freedom (DOF) per node: translations in the nodal x, y, and z directions. The wireless emitter is meshed with this tetrahedral-shaped element. The material properties (Young’s modulus $E$, Poisson’s ratio $\nu$, and density $\rho$) defined in Tab. 5.1 for gold and nickel are assigned to the elements. The meshed FE model can be seen in Fig. 5.10 where the lower resonant modes of the emitter are shown.

If the geometry, the element type, and the material properties are defined the natural frequency and mode shape of the structure can be determined with a mode-frequency analysis. This analysis type is usually used for undamped systems. It solves an eigenvalue and eigenvector problem which has the form of:

$$\text{det}([k] - \omega_n^2[m]) = 0,$$  \hspace{1cm} (5.3.1)

where det is the determinant, $[k]$ denotes the structure stiffness matrix, $[m]$ the mass matrix, and $\omega_n$ the natural angular frequencies of the undamped system. As a solution the natural modes can be displayed and the natural frequencies are calculated.

The resonant modes of the wireless emitter are shown in Fig. 5.10. The first mode is a translation of the nickel die along the z-axis (normal direction to the die) at a frequency of 3.41 kHz. This is the required motion to generate the omnidirectional acoustic pressure field. The second mode (b) is a rotation of the nickel die about the x-axis at $f = 4.68$ kHz and the third mode (c) is a rotation about the y-axis at $f = 11.9$ kHz. The fifth mode, shown in (d), is again a translation along the z-axis, but at $f = 46.9$ kHz. Frequencies in the lower kHz range are not adequate for localization applications. However, for the first prototype of a wireless emitter the useful mode was tuned to a frequency below 10 kHz, as the magnetic driving system has a limited frequency range.

More information about solving a modal analysis or an eigenvalue and eigenvector problem in ANSYS can be found in the chapters “Mode-Frequency Analysis” and “Eigenvalue and Eigenvector Extraction” in the release 11.0 documentation for ANSYS.
5.3. MECHANICAL SIMULATION

Fig. 5.10: First (a), second (b), third (c) and fifth (d) mode of a wireless emitter with $h_{\text{Au}} = 25 \, \mu m$.

5.3.2 Structural Static Analysis

A static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia and damping effects, such as those caused by time-varying loads. With a static analysis we can determine the displacement vector $\mathbf{u}$ of a structure caused by an externally applied load $\mathbf{F}^a$, from the linear system of equations

$$[\mathbf{k}]\mathbf{u} = \mathbf{F}^a.$$

The SOLID186 element type is chosen for the FE structural static analysis as well. In Fig. 5.11 the deflection of the gold spring calculated in ANSYS can be seen. Applying an external load $|\mathbf{F}^a| = 70 \, \text{mN}$ on the nickel die in its normal direction, results in an absolute displacement of the die of about $u = 150 \, \mu m$ for $h_{\text{Au}} = 25 \, \mu m$, as shown in (a). The displacements $u$ for different values of $|\mathbf{F}^a|$ are shown in (b) for different spring thicknesses $h_{\text{Au}}$. The inverse slopes of the lines in (b) correspond to the spring constants $k_{\text{AN}}$ obtained in ANSYS, which are summarized in Tab. 5.3. Compared to the analytical results, we achieved an error of less than 20%.
CHAPTER 5. WIRELESS ACOUSTIC EMITTER

(a) Deflection of the resonator. (b) Deflection against the applied force for different spring thicknesses.

Fig. 5.11: Determination of the spring constant.

5.4 Conclusion

Omnidirectionality of the ultrasonic pressure field from the emitter is a key requirement in passive localization. To depict the pressure fields from a single plane transducer the impulse response method was used. It was found that emitters of a side length less than half a wavelength are well suited for being treated as omnidirectional. Hence, a maximum size of 2.5 mm is required at $f = 300$ kHz in water at 20°C. In addition, good agreement is found between the theoretical results and the numerical results in ANSYS.
Chapter 6

Localization in Homogeneous Media

In many applications from sonar and radar to nondestructive testing and medical ultrasound, it is often required to localize an object from its emission. Localization can be done by spatially separated receivers combined in an array where the receivers capture the emitted signal. The emitter location relative to the receiver locations can then be calculated by estimating the TDOA or PS for different groups of receivers.

To improve the accuracy of pulse response methods and passive localization systems using TDOA or PS measurements, significant effort has been made to measure the time and phase as accurately as possible, ranging from applying Kalman filters [29, 120] to mimicking bat behavior [24]. However, the spatial accuracy and, notably, the resolution of emitter localization methods not only depend on accurate time and phase measurements, but also on the number and the arrangement of the receivers as well as the emitted frequency.

If the TDOA or the PS is defined, the emitter location can be determined from the intersection point of several hyperbolas (see hyperbolic localization in section 4) in the 2D case or hyperboloids in the 3D case. The hyperbolic curves or surfaces are defined by a nonlinear system of equations obtained from the TDOA or PS data.
Several authors proposed solutions to this nonlinear system of equations assuming ideal signals without considering the uncertainty due to more realistic sinusoidal signals. Fang [98], and Chan [99] gave solutions in closed form. Militello [102] proposed a new exact linear solution to the emitter localization problem based on TDOA. Additionally, the receivers were positioned in suboptimal, linear, or arbitrarily arrangements, for example in diamond or square configurations [101].

We proposed optimal receiver positions for both TDOA and PS measurements by analyzing the uncertainty of the emitted signals and taking the frequency into account [121]. The uncertainty analysis depends on an inverse approach using a method similar to the synthetic aperture focussing technique (SAFT).

For illustration purposes the derivation in the next section is in a 2D plane. Extension to 3D is straightforward and shown in section 6.2.

### 6.1 Passive Localization in 2D

#### 6.1.1 Time Difference of Arrival Measurements

In hyperbolic localization the delay between the arrival time of an emitted ultrasound signal at two spatially separated receivers $R_i$ ($i = 1, 2$) defines a hyperbolic curve on which the ultrasound emitter must be located. Assuming two receivers positioned on the $x$-axis at $R_1(x_1, y_1 = 0)$ and $R_2(x_2, y_2 = 0)$, respectively, the emitter location $E(x, y)$ is defined by the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where $a^2 = \frac{(c\Delta t_{2,1})^2}{4}$ and $b^2 = \frac{(|x_1| + |x_2|)^2}{4} - a^2$ with the TDOA $\Delta t_{2,1}$ and $|x_i|$ as the absolute value of $x_i$. The hyperbola (6.1.1) is shown in Fig. 6.1 for $\Delta t_{2,1} = \pm 50 \mu s$, $x_1 = -0.1 \text{ m}$, $x_2 = 0.1 \text{ m}$, and $c = 1482 \text{ m/s}$. The small circles denote the receiver locations.

In this analytical solution, the emitter location does not depend on the ultrasound frequency. In addition the signals are assumed to be optimal, akin to the waveform shown in Fig. 6.2a. Using more realistic, sinusoidal
6.1. PASSIVE LOCALIZATION IN 2D

Fig. 6.1: Emitter location hyperbola for two receivers positioned on the x-axis at $x_1 = -0.1 \, \text{m}$ and $x_2 = 0.1 \, \text{m}$, respectively with an assumed TDOA $\Delta t_{2,1} = \pm 50 \, \mu\text{s}$.

![Hyperbola with two receivers](image)

(a) Optimal signal  
(b) Sinusoidal signal

Fig. 6.2: Having a sharp line on which the emitter location must lie, requires optimal signals akin to (a), which have a sharp peak value. More realistic, sinusoidal signals (b) lead to probability maps for the emitter location.

signals, as in Fig. 6.2(b), leads to probability distributions for the emitter location. Instead of a hyperbolic curve on which the emitter must lie, a probability map for the emitter location is obtained, where the probability not only depends on the TDOA and the speed of sound, but also on the ultrasound frequency.

Derivation

To obtain the probability distribution for the emitter location, a similar approach to array focussing [122] and SAFT [123] is considered. The SAFT algorithm is an imaging technique based on the assumption that at a certain point in the area of interest a reflector is located [124]. The propagation path from the emitter to this assumed point and back to the receiver is calculated. Based on the propagation path length, a time shift is computed that is added
to each received waveform. All waveforms are summed, and if the assumed point is the real reflector location constructive interference (a larger signal) occurs. Taking each point in the area of interest into account builds up a probability map for the reflector location.

We transform the SAFT technique in an inverse approach. It is assumed that the receivers act as emitters. From the receiver locations circular wavefronts with their corresponding, measured TDOA are emitted. At each point $P(x, y)$ in the area of interest the interference of the waves is calculated. The waves constructively interfere at the location, from where the correct TDOA at the receivers is obtained.

First the distance

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

(6.1.2)

from $P$ to each receiver $R_i$ is calculated, including the $y$-coordinates of the receivers to get a general solution for 2D situations. Then the wave propagation time $t_i$ from each receiver $R_i$ to $P$ is calculated with the TDOA $\Delta t_{i,1}$ at receiver $R_i$ with respect to the first receiver:

$$t_1 = \frac{d_1}{c}$$

(6.1.3)

and

$$t_i = \frac{d_i}{c} - \Delta t_{i,1}.$$  

(6.1.4)

The phases $\varphi_i$ of the waves at point $P$ are calculated by

$$\varphi_i = 2\pi \frac{\text{rem}(t_i, T)}{T},$$

(6.1.5)

where rem$(t_i, T)$ denotes the remainder of the TOF after division with the cycle $T$. The solution $p_i$ for an acoustic pressure waveform, normalized with the ambient pressure $p_0$ is given by

$$\frac{p_i(t_i)}{p_0} = e^{j(\omega t_i + \varphi_i)} = e^{j\omega t_i} e^{j\varphi_i},$$

(6.1.6)

with $j = \sqrt{-1}$. If point $P$ is the right emitter location, then the waves must constructively interfere at any TOF $t_i$. Therefore, the solution must be independent of the wave propagation time. Thus, in (6.1.6) only the phase
dependent part must be considered and the time independent, normalized waveforms $\hat{p}_i$ emitted at the receiver locations can be written as

$$\hat{p}_i = e^{j\phi_i}. \quad (6.1.7)$$

The amplitude $\hat{A}$ of the resulting wave after the interference, is obtained from

$$\hat{A} = |\sum_{i=1}^{N} \hat{p}_i|, \quad (6.1.8)$$

with the number of receivers $N$.

**Discussion**

For a TDOA $\Delta t_{2,1} = \pm 50 \mu s$, Fig. 6.3(a) and (b) are obtained, if the amplitude $\hat{A}$ at any point in the area of interest is calculated. These two images represent the ultrasound emitter location probability maps for $c = 1482 \text{ m/s}$, and a frequency $f = 50 \text{ kHz}$ and $f = 300 \text{ kHz}$, respectively. The white points are the two receiver locations. The white and black colored regions represent the amplitude values for their corresponding points in the $xy$-plane. The black colored regions represent the positions with a low amplitude value and, therefore, a low probability for the emitter location. On the other hand, the white colored main lobe represents the positions with a high amplitude value and, thus, a high probability for the emitter location. It can be seen that the ML regions for the emitter location diverge (rather wide main lobe) with distance from the receivers and converge (narrower main lobe) with higher frequencies. The main lobe is also narrower if the two receivers are further apart from each other. In Fig. 6.3(c) the receivers are at a distance $d = 0.04 \text{ m}$, and the main lobe is much wider than in Fig. 6.3(a), where $d = 0.2 \text{ m}$, although the frequency is the same for both.

Taking the algebraic sign of the TDOA $\Delta t_{2,1}$ into account eliminates one of the two main lobe parts in Fig. 6.3 according to the sign. For $\Delta t_{2,1} > 0$ the right lobe part is eliminated, and for $\Delta t_{2,1} < 0$ the left lobe part is eliminated. This is due to the linear dependence of $t_2$ from $\Delta t_{2,1}$ in (6.1.4). In the analytical solution (6.1.1) the two parts of the main lobe are always received, as the TDOA is squared.
CHAPTER 6. LOCALIZATION IN HOMOGENEOUS MEDIA

(a) $f = 50$ kHz, $d = 0.2$ m

(b) $f = 300$ kHz, $d = 0.2$ m

(c) $f = 50$ kHz, $d = 0.04$ m

Fig. 6.3: ML estimates of the emitter location for two receivers positioned on the $x$-axis at $x_1 = -0.1$ m and $x_2 = 0.1$ m in (a) and (b) and at $x_1 = -0.02$ m and $x_2 = 0.02$ m in (c). The ultrasound frequency is 50 kHz in (a) and (c) and 300 kHz in (b). The TDOAs $\Delta t_{2,1}$ are in (a) and (b) $\pm 50$ µs and in (c) $\pm 10$ µs.

6.1.2 Phase Shift Measurements

For a continuous sinusoidal wave emission it is not possible to measure the TDOA; instead only the PS is known between incoming waves measured at each receiver. With continuous waveforms one can expect much higher amplitudes and a better signal-to-noise ratio (SNR). Compared to TDOA measurements, where it is potentially possible to detect a time delay between two receivers from different pulses which would result in large spatial localization errors, the same periodic signal is constantly present in the PS case.

The equations for PS measurements are similar to those presented in section
The difference is that multiple lobes exist rather than just one main lobe. This is due to the ambiguity of sinusoidal signals. If a PS $\varphi$ between the first and second receiver is measured, any multiple of $2\pi$ results in correct solutions as well. Therefore, the PSs $\varphi_{i,1}$ at the receiver $R_i$ with respect to the first receiver are

$$\varphi_{i,1} = \varphi + 2n\pi,$$

(6.1.9)

where $n$ is an integer.

**Derivation**

To obtain the probability distribution for the emitter location, the distance $d_i$ from any point $P$ in the area of interest to each receiver $R_i$ is first calculated from (6.1.2). Then the wave propagation time $t_i$ from each receiver $R_i$ to $P$ is calculated from (6.1.3) and

$$t_i = \frac{d_i}{c},$$

(6.1.10)

The phases $\varphi_i$ of the waves at point $P$ are obtained from

$$\varphi_1 = 2\pi \frac{\text{rem}(t_1, T)}{T},$$

(6.1.11)

for $R_1$, and

$$\varphi_i = 2\pi \frac{\text{rem}(t_i, T)}{T} - \varphi_{i,1}$$

(6.1.12)

for $R_i$.

If point $P$ is the real emitter location, then the waves must constructively interfere at $P$ at any TOF. Thus, the time independent, normalized waveforms $\hat{p}_i$ emitted at the receiver locations are obtained from (6.1.7) and the amplitudes $\hat{A}$ of the resulting waves after interference are obtained from (6.1.8), similar to the TDOA measurements.

**Discussion**

For a PS of $\varphi_{2,1} = \frac{5}{3}\pi + 2n\pi$ Fig. 6.4 is obtained. The ultrasound emitter location probability maps can be seen for $f = 50$ kHz in (a), (c), and (d),
CHAPTER 6. LOCALIZATION IN HOMOGENEOUS MEDIA

(a) $f = 50\, \text{kHz}, \; d = 0.2\, \text{m}$
(b) $f = 300\, \text{kHz}, \; d = 0.2\, \text{m}$
(c) $f = 50\, \text{kHz}, \; d = 0.04\, \text{m}$
(d) $f = 50\, \text{kHz}, \; d = \frac{3}{4} \lambda \approx 0.022\, \text{m}$

Fig. 6.4: Probability distribution for the emitter location for two receivers positioned on the $x$-axis at $x_1 = -0.1\, \text{m}$ and $x_2 = 0.1\, \text{m}$ in (a) and (b), at $x_1 = -0.02\, \text{m}$ and $x_2 = 0.02\, \text{m}$ in (c), and at $x_1 = \frac{3}{2} \lambda \approx -0.011\, \text{m}$ and $x_2 = \frac{3}{4} \lambda \approx 0.011\, \text{m}$ in (d). The ultrasound frequency is $50\, \text{kHz}$ in (a), (c), and (d), and $300\, \text{kHz}$ in (b). The PS $\varphi_{2,1}$ is $\frac{5}{2} \pi + 2n\pi$ in (a) to (d).

and $300\, \text{kHz}$ in (b), respectively. Again the lobes are rather wide, if the frequency is low. As the frequency increases the lobes become narrower, but the number of lobes increases. In Fig. 6.4(c) it can be seen that the number of lobes decreases for receivers close together, but these fewer lobes become wider. To obtain one main lobe from the receiver pair $R_1R_2$, the distance $d$ between these receivers must be smaller than $\lambda$. However, it is not possible to eliminate one part of the main lobe as in the TDOA case. In Fig. 6.4(d) the receivers are at a distance of $d = \frac{3}{4} \lambda$, thus only one main lobe for the emitter location exists. Though, if the receivers are close together these regions become very broad resulting in a poor spatial resolution.
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(a) Distance $d = 28 \lambda$ between receiver $R_1$ and $R_2$, and $R_2$ and $R_3$.

(b) Distance $d = 6 \lambda$ between receiver $R_1$ and $R_2$, and $R_2$ and $R_3$.

Fig. 6.5: Probability distribution for two different receiver arrangements in (a) and (b). The assumed emitter location is at point $\left(x_E, y_E\right) = \lambda \left(9, 6\right)$.

6.1.3 Measurements with Multiple Receiver Pairs

If more than two receivers are considered, the receivers can be used as multiple pairs and the TDOA or PS between each pair can be taken into account. For the case of three receivers, it is possible to consider only two pairs, for example, pair $R_1R_2$ and pair $R_2R_3$ as in Fig. 6.6(a) for TDOA, or to consider all three combinations ($R_1R_2$, $R_1R_3$, and $R_2R_3$) of three receivers to pairs, as in Fig. 6.5 and Fig. 6.6(b) for TDOA and Fig. 6.15 for PS. In these figures the probability distributions are shown for clarity in terms of a 3D plot and a gray-scale image. The $x$- and $y$-axis are scaled with $\lambda$ and, therefore, dimensionless.
TDOA Method with Multiple Receivers

In Fig. 6.5 the probability distribution is shown for an assumed emitter location at point \( (x_E, y_E) = \lambda \left( \frac{9}{6}, \frac{6}{4} \right) \). The three receivers are placed orthogonally to each other at the points \( (x_1, y_1) = \lambda \left( -\frac{14}{6}, -\frac{14}{6} \right) \), \( (x_2, y_2) = \lambda \left( \frac{14}{6}, -\frac{14}{6} \right) \), and \( (x_3, y_3) = \lambda \left( \frac{14}{6}, \frac{14}{6} \right) \) in Fig. 6.5(a), and at \( (x_1', y_1') = \lambda \left( -\frac{3}{3}, -\frac{3}{3} \right) \), \( (x_2', y_2') = \lambda \left( \frac{3}{3}, -\frac{3}{3} \right) \), and \( (x_3', y_3') = \lambda \left( \frac{3}{3}, \frac{3}{3} \right) \) in (b). From the assumed emitter location \( (x_E, y_E) = \lambda \left( \frac{9}{6}, \frac{6}{4} \right) \) the TOFs to each receiver are calculated, and the corresponding TDOAs \( \Delta t_{2,1}, \Delta t_{3,1} \) and \( \Delta t_{3,2} \) for each receiver pair are determined. With these TDOAs the probability distribution for the assumed emitter location can be calculated for each receiver pair. Combining the three parts from the mainlobes obtained from the three pairs results in the final probability distribution for the assumed emitter location. It can be seen in (a), that the region with the highest probability for the emitter location is a sharp peak. If the receivers are closer together, as in (b), the area of this region is increased and the resolution is degraded.

Fig. 6.6 shows the probability distribution for another assumed emitter location at point \( (x_E, y_E) = \lambda \left( -\frac{4}{12}, -\frac{4}{12} \right) \). The three receivers are placed orthogonally to each other at the same positions as in Fig. 6.5(a). Again, the TOFs from the emitter location to each receiver are calculated, and the corresponding TDOAs for each receiver pair are determined. With the TDOAs the probability distribution for the assumed emitter location is then calculated. In Fig. 6.6(a) only the two receiver pairs \( R_1R_2 \) and \( R_2R_3 \) with their corresponding TDOAs \( \Delta t_{2,1}, \Delta t_{3,2} \) are considered, while all three pairs are taken into account in (b). It can be seen in (a) that two receiver pairs are adequate for uniqueness of the emitter position estimate, but the consideration of the remaining pair in (b) slightly reduces the region with the ML estimates for the emitter location. In (b) the peak in the emitter location probability distribution is sharper and the ML estimate becomes tighter. So the probability distribution can be improved by considering more than two lobes or even more than three receivers.
6.1. PASSIVE LOCALIZATION IN 2D

(a) Consideration of two receiver pairs.

(b) Consideration of three receiver pairs.

Fig. 6.6: Probability distribution for the assumed emitter location at point $E(-4\lambda, -12\lambda)$. In (a) the three receivers are handled as two pairs: $R_1R_2$ and $R_2R_3$. In (b) all three receiver pair combinations are considered.

**Optimization of the TDOA method**

From Fig. 6.5 and Fig. 6.6, it is clear that the resolution of the TDOA method can be improved, if the receiver arrangement and the number of receivers is optimized. Therefore, we first analyze the resolution of the TDOA method, if the receiver amount is increased. Then the resolution depending on the receiver arrangement is studied.

**Number of receivers:** The area with the ML estimate for the emitter location is used as an indicator for the resolution. $R^*$ denotes the radius of that area above a certain threshold $AF_{max}$ (maximum amplitude factor). The radius $R^*$ depends on $AF_{max}$, on the number of receivers $N$, and on the emitter position, but it does not depend on the frequency of the emission and
can, therefore, be scaled with \( \lambda \). To find the optimum number of receivers for the highest resolution, nine emitter locations \( E_j, j = 1,...,9 \) are defined, as shown in Fig. 6.7. For a configuration with \( N \) receivers \( R_i, i = 1,...,N \) distributed equally around a circle of radius \( r = 14 \lambda \), the emitters are located on the \( x \)-axis and on a line such that \( \alpha = \frac{\pi}{N} \). \( E_1 \) and the circle center are coincident. \( E_5 \) is on the \( x \)-axis directly within the circle at \( x_{E_5} = 13.99 \lambda \). \( E_9 \) is at \((x_{E_9}, y_{E_9}) = 13.99 \lambda \left(\cos \alpha, \sin \alpha\right)\). The remaining emitter locations lie in between on the \( x \)-axis and on the line from \( E_9 \) to the circle center, respectively.

The ratio \( R^*/\lambda \) is shown in Fig. 6.8 for the nine emitter locations if \( N \) increases from 3 to 50. The maximum amplitude factor is chosen as \( AF_{\text{max}} = 0.90 \) in (a), \( AF_{\text{max}} = 0.95 \) in (b), \( AF_{\text{max}} = 0.96 \) in (c), and \( AF_{\text{max}} = 0.98 \) in (d). It can be seen that the radius \( R^* \) reaches a constant value independent of the emitter location \( E_j \) for about \( N \geq 11 \). The most interesting emitter locations are \( E_5 \) and \( E_9 \). For \( E_5 \) the radius \( R^* \) decreases exponentially with increasing \( N \) until a constant level is reached. For \( E_9 \) the radius \( R^* \) decreases only for about \( N < 5 \), and then starts to increase slightly before a constant level is reached. These two positions show the lower limits for the resolution. For less than 11 to 15 receivers the position \( E_5 \) limits the resolution, and for more receivers the position \( E_9 \) results in the lowest resolution. If 11 receivers are taken into account the maximal ratio \( R^*/\lambda \) becomes 0.16 for \( AF_{\text{max}} = 0.90 \) and can be reduced to \( R^*/\lambda = 0.07 \) for \( AF_{\text{max}} = 0.98 \). Therefore, at a frequency \( f = 300 \text{kHz} \) the expected resolution based on \( R^* \).
6.1. PASSIVE LOCALIZATION IN 2D

![Fig. 6.8: Radius $R^*$ against the number of receivers $N$ for $AF_{\text{max}} = 0.90$ to $AF_{\text{max}} = 0.98$ in (a) to (d).](image)

lies in the range of 800 $\mu$m for $AF_{\text{max}} = 0.90$ and it can be improved to about 350 $\mu$m for $AF_{\text{max}} = 0.98$. Considering more than 11 receivers does not significatively improve the resolution further. For the case of 11 receivers the dependence of $R^*/\lambda$ from $AF_{\text{max}}$ can be seen in Fig. 6.9. It shows how the resolution can be improved with a better data acquisition system. If it would be possible to detect a signal higher than $AF_{\text{max}} = 0.998$ the ratio $R^*/\lambda$ could be reduced to 0.021 and a resolution of almost 100 $\mu$m could be obtained at $f = 300$ kHz.

**Receiver arrangement:** The handling of 11 receivers is inconvenient and computationally intensive, as computational complexity in TDOA and PS measurements is generally exponential in the number of receivers [54]. It would be advantageous if fewer receivers could be used to obtain a com-
CHAPTER 6. LOCALIZATION IN HOMOGENEOUS MEDIA

Fig. 6.9: Radius $R^*$ against $AF_{\text{max}}$ if $N = 11$ receivers are considered on a circle of $r = 14\,\lambda$.

Fig. 6.10: The ratio $R^* / \lambda$ at position $E_5$ for $N = 3$ receivers distributed equally around a circle of increasing radius $r$.

parable resolution. It is clear from Fig. 6.5 that the resolution strongly depends on the distance between the receivers. Therefore, the resolution of the TDOA method is analyzed in a next step if the receiver arrangement is optimized instead of the receiver number. Again, the radius $R^*$ of the area with the ML estimate for the emitter location above $AF_{\text{max}}$ is used as an indicator for the resolution. If $N \leq 11$ the emitter position $E_5$ limits the resolution. Therefore, the emitter is positioned at point $E_5$ in the following analysis.

In Fig. 6.10 the ratio $R^* / \lambda$ for the emitter location $E_5$ can be seen if three receivers are distributed equally around a circle of increasing radius $r$: $1.8\,\lambda \leq r \leq 60\,\lambda$. The ratio $R^* / \lambda$ reaches a constant level for, both, $AF_{\text{max}} = 0.90$ and $AF_{\text{max}} = 0.98$ for about $r \geq 35\,\lambda$. For three receivers distributed equally around a circle of $r = 40\,\lambda$ the ratio $R^* / \lambda$ becomes $0.17$.
6.1. PASSIVE LOCALIZATION IN 2D

![Diagram of orthogonal arrangement of 3 receivers at a distance d.](image)

**Fig. 6.11:** Orthogonal arrangement of 3 receivers at a distance $d$.

![Graph showing the ratio $R^*/\lambda$ at position $E_5$ against the distance $d$ between 3 orthogonal receivers.](image)

**Fig. 6.12:** The ratio $R^*/\lambda$ at position $E_5$ against the distance $d$ between 3 orthogonal receivers.

and the expected resolution lies in the range of 840 µm for $AF_{\text{max}} = 0.90$ or in the range of 480 µm for $AF_{\text{max}} = 0.98$ with $R^*/\lambda = 0.097$.

Instead of distributing the receivers equally on a circle, the receiver can be arranged in an orthogonal arrangement, as shown in Fig. 6.11. $R_1$ and $R_3$ are on the circle forming the diagonal and $R_3$ is on the circle at a distance $d$ to the other receivers. The radius of the circle is $r = \sqrt{2}d$. The radius $R^*$ at the emitter position $E_5$ is the indicator for the resolution, if $d$ is increased. $R^*/\lambda$ against the distance $d$ is shown in Fig. 6.12. The ratio $R^*/\lambda$ reaches a constant level for about $d \geq 45\lambda$ (according to about $r \geq 30\lambda$). At a distance of $45\lambda$ the ratio $R^*/\lambda$ becomes $0.12$ for $AF_{\text{max}} = 0.90$ and $0.06$ for $AF_{\text{max}} = 0.98$. Therefore, at a frequency of 300 kHz a resolution of less than 310 µm can be expected, if three receivers are arranged orthogonal to each other at a distance of at least $45\lambda$ and if a signal higher than $AF_{\text{max}} = 0.98$.
Fig. 6.13: Radius $R^*$ against the maximum amplitude factor $AF_{\text{max}}$ if $N = 3$ orthogonal receivers are considered.

Fig. 6.14: as Fig. 6.13, but if four orthogonal receivers are considered in (a) and if 11 receivers are considered on a circle of $r = 32 \lambda$ in (b).

can be detected. Comparing these results with Fig. 6.9 shows that the resolution with three orthogonal receivers at a distance of 45 \( \lambda \) is comparable to the resolution obtained with 11 receivers on a circle of \( r = 14 \lambda \).

It is shown in Fig. 6.13 how the resolution can be improved with a better data acquisition system if three orthogonal receivers are used. The ratio $R^*/\lambda$ is shown for the nine emitter positions if the receivers are at a distance of 45 \( \lambda \). At $f = 300 \text{ kHz}$ and $AF_{\text{max}} = 0.90$ a resolution of less than 610 \( \mu \text{m} \) can be expected with $R^*/\lambda = 0.123$. If it would be possible to detect a signal higher than $AF_{\text{max}} = 0.998$ the ratio $R^*/\lambda$ could be reduced to 0.023 and a resolution of about 110 \( \mu \text{m} \) could be obtained.
6.1. PASSIVE LOCALIZATION IN 2D

The results for the resolution that can be expected for different receiver arrangements is summarized in Tab. 6.1. A rectangular arrangement of three receivers improves the resolution compared to the circular arrangement. The resolution can be further improved by increasing the distance \( d \) or radius \( r \) between the receivers and/or the receiver amount \( N \). At reasonable computational complexity the best resolution is obtained with four receivers located on the corners of a square with a side length of at least 45 \( \lambda \). Using

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \frac{R^*}{\lambda} ) for ( AF_{max} = )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ( r = 40 \lambda )</td>
<td>840 480 370</td>
</tr>
<tr>
<td>3 ( d = 45 \lambda )</td>
<td>610 310 110</td>
</tr>
<tr>
<td>4 ( d = 45 \lambda )</td>
<td>570 260 100</td>
</tr>
<tr>
<td>11 ( r = 14 \lambda )</td>
<td>800 350 105</td>
</tr>
<tr>
<td>11 ( r \approx 32 \lambda )</td>
<td>( \leq 570 ) ( \leq 260 ) ( \leq 100 )</td>
</tr>
</tbody>
</table>

Tab. 6.1: Summary of the expected resolution based on radius \( R^* \) with different receiver arrangements where the TDOA is measured at \( f = 300 \text{kHz} \).

In Fig. 6.14(a) the same situation as in Fig. 6.13 is shown but an additional fourth receiver is taken into consideration. The fourth receiver is placed in a way that all receivers form a square with side length \( d = 45 \lambda \). This arrangement corresponds to the arrangement of four receivers distributed equally around a circle of radius \( r \approx 32 \lambda \). Compared to the situation with three orthogonal receivers the resolution can be improved slightly to about \( 570 \mu m \) at \( f = 300 \text{kHz} \), if a signal higher than \( AF_{max} = 0.90 \) can be detected, and about \( 100 \mu m \) if \( AF_{max} = 0.998 \). Thus, the arrangement of four receivers located at the corners of a square results in a better resolution than the arrangement of 11 receivers on a circle of \( r = 14 \lambda \). In addition, this arrangement not only improves the resolution slightly but also reduces the influence of the emitter location on the resolution. In contrast to Fig. 6.13 there is a smaller difference in the resolution for the different emitter locations \( E_j \). Considering more than four receivers on a circle with a large radius does not continuously improve the resolution, but it slightly reduces the influence of different emitter locations on the resolution further, as shown in Fig. 6.14(b). Fig. 6.14(b) shows the dependence of \( R^*/\lambda \) from the maximum amplitude factor if 11 receivers are distributed equally around a circle of \( r \approx 32 \lambda \).
more receivers or increasing the distance between the receivers further does not significantly improve the resolution. Hence, if it is possible to place the receivers at a distance of about $45\lambda$ (which corresponds to about 22 cm at $f = 300$ kHz in water) from each other, four receivers are adequate to obtain a very high resolution with TDOA measurements in 2D.

It is actually not surprising that the best accuracy and resolution is obtained by TDOA measurements if four receivers are positioned on the corners of a square as far apart as possible from each other. If the receivers are orthogonal to each other the lobes usually intersect at approximately $\pi/2$ and, thus, result in a minimal area with the ML estimate for the emitter location, and therefore achieve the highest possible resolution. In addition the further apart the receivers are, the narrower the lobes, which results in an even smaller area with the ML estimate for the emitter location. Increasing the distance between the receivers is more effective than increasing the receiver number. The slight reduction of the emitter location influence on the resolution does not compensate the additional computational costs, due to an increased receiver number.

**Phase Shift Method with Multiple Receivers**

If the PS is measured, the situation becomes more complicated than in the TDOA case. In TDOA measurements, only one main lobe part per receiver pair is obtained, while multiple lobes occur for the PS method. More than one receiver pair is required to obtain a probability distribution with a unique peak for the emitter location. In Fig. 6.15 the probability distribution is shown for the same situation as in Fig. 6.5 but instead of measuring the TDOA the PS between three receiver pairs is measured. Again the assumed emitter location is at point $(x_E, y_E) = \lambda (9, 6)$. It can be seen that the assignment of only three receiver pairs is inadequate for uniqueness of PS measurements, even though it is sufficient in the TDOA case. Numerous small regions with the ML estimates for the emitter location occur, if the receivers are at a distance of $28\lambda$, as in Fig. 6.15(a). Fewer, but larger ML regions occur, if the receivers are closer together at a distance of $6\lambda$, as in Fig. 6.15(b). It is clear that definitely more than three receivers are required.

---

1 Actually only two pairs are sufficient for uniqueness in the TDOA case in 2D.
6.1. PASSIVE LOCALIZATION IN 2D

Fig. 6.15: Probability distribution for the same receiver arrangement as in Fig. 6.5, if the PS is measured. The emitter is located at \((x_E, y_E) = \lambda \left(\frac{9}{6}\right)\).

to obtain a unique receiver location estimate. This leads to two fundamental questions, which must be addressed for PS measurements with multiple receiver pairs:

1. What is the minimum number of receivers or receiver pairs that are required in order to obtain a unique solution for the emitter location?

2. Is it best to have receivers that are close together, and, therefore, produce one wide main lobe, or is it better to have further separated receivers, and thus more than one narrower lobe?
Optimization of the Phase Shift Method

**Minimum Number of Receivers:** For the PS method the number of receivers required depends on the size of the area of interest, the receiver arrangement, and on the (unknown) position of the emitter. The influence of the unknown emitter location on the number of receivers can be reduced if the receivers are very far apart from each other, compared to the wavelength and the size of the area of interest. If the receivers are at an infinite distance to each other and to the area of interest, then the distance between the receivers and the emitter does not change if the emitter location changes and the influence of the emitter location is minimized. Therefore, the receivers are positioned equidistant in a circular array of infinite diameter in the following analysis to evaluate the minimum number of receivers required for uniqueness in a certain area of interest.

If the receivers are at an infinite distance to the emitter, it is assumed that regardless of the distance from the receivers to the emitter, a signal can be measured. This has some inherent assumptions, firstly omnidirectionality of the emitter. To calculate the acoustic pressure fields from a single emitter the impulse response method is used as described in section 5.2.3. Emitters of a side length less than half a wavelength can be considered as omnidirectional. Thus, a frequency of 300 kHz results in a maximum emitter size of approximately 2.5 mm. Secondly, it is assumed there is no dissipative attenuation, and thirdly, no diffraction. Clearly, these last two assumptions are both invalid. However, despite the practical implications, we include this situation as it depicts the ideal case.

In Fig. 6.16 and Fig. 6.17 the probability maps for a square area of 30 λ side length can be seen. The maps are shown for clarity in terms of both a projection on the xz-plane and the xy-plane. In each arrangement, the assumed emitter location is at the origin of the coordinate system, but could be elsewhere without a change in the map structure. In Fig. 6.16(a) the probability distribution for 11 receivers, and in (b) for 12 receivers is shown. Figure 6.17(a) and (b) show the probability distributions for 13 and 14 receivers, respectively. The emitter localization with 12 receivers is ambiguous, while it is unique with 11, 13 and 14 receivers. Fewer than 11 receivers is not adequate for unique results, as they yield unclear probability maps.
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With 14 receivers, there are multiple locations with high probabilities for the emitter location next to the ML estimate. With 11 and 13 receivers the probability map shows only one high probability peak. For this arrangement the model shows that at least 11 receivers must be used to obtain unique results, although 12 receivers provide ambiguous results. 13 or more receivers produce unique results.

From Fig. 6.16 and Fig. 6.17 it is assumed that generally clearer results are obtained with an odd number of receivers than with an even number. In Fig. 6.18 the amplitude ratio AR between the correct peak $\hat{A}_{max}$ and the highest false positive $\hat{A}'_{max}$ is expressed in terms of the logarithmic decibel

Fig. 6.16: Probability distribution for 11 and 12 receivers in (a) and (b), respectively. The assumed emitter location is in the origin of the coordinate system.
The receivers are again assumed to be at an infinite distance to each other. It can be seen that the AR is always higher for odd receiver numbers and can be increased with more receivers. For an odd number of receivers, higher than approximately 43, a constant amplitude ratio $\text{AR} \approx 7.3$ is reached. The amplitude ratio AR is an indication of the clarity of the probability maps. The higher the amplitude ratio, the clearer the localization results. A value of $\text{AR} = 0$ shows that the result for the emitter location is not unique and for $\text{AR} < 1$ the probability maps are very unclear.

$$\text{AR} = 20 \log \left( \frac{\hat{A}_{\text{max}}}{\hat{A}'_{\text{max}}} \right).$$  \hspace{1cm} (6.1.13)
6.1. PASSIVE LOCALIZATION IN 2D

Fig. 6.18: Amplitude ratio AR between the correct peak and the highest false positive as a function of the receiver number.

(a) $AF_{\text{max}} = 0.90$

(b) $AF_{\text{max}} = 0.98$

Fig. 6.19: Radius $R^*$ against the number of receivers $N$ for $AF_{\text{max}} = 0.90$ in (a) and $AF_{\text{max}} = 0.98$ in (b).

Optimal Number of Receivers: Again the nine emitter locations $E_j$, $j = 1, ..., 9$ shown in Fig. 6.7 and the radius $R^*$ are used for the characterization of the probability map as a function of $N$. In Fig. 6.19 the ratio $R^*/\lambda$ is shown for the nine emitter locations if the PS between $N = 3, ..., 50$ receivers is measured. The ratio is shown for $AF_{\text{max}} = 0.90$ in (a) and for $AF_{\text{max}} = 0.98$ in (b). It is not surprising that these figures are similar to Fig. 6.8(a) and (d) where the TDOA was measured. There are only slight differences. The ratio $R^*/\lambda$ reaches a constant value for more than 15 to 20 receivers. Again, the most interesting emitter locations are $E_5$ and $E_9$ which show the lower limits for the resolution. For $E_5$ the radius $R^*$ decreases exponentially with increasing $N$ until its constant level is reached. For $E_9$ the radius $R^*$ decreases only for $N < 6$, and then starts to increase slightly
before its constant level is reached. For $N < 13$, position $E_5$ limits the resolution, and for $N \geq 13$, position $E_9$ results in the lowest resolution. The best resolution is obtained with 13 receivers. As for the TDOA measurements for $A F_{\text{max}} = 0.90$ one obtains $R^*/\lambda = 0.16$ and for $A F_{\text{max}} = 0.98$ the ratio $R^*/\lambda$ becomes 0.07. The results for the resolution based on the radius $R^*$, which can be expected for different receiver arrangements is summarized in Tab. 6.2.

Despite the higher amplitudes and a better SNR with PS measurements, one can not expect a higher resolution compared to TDOA measurements. In addition, the implementation of at least 11 or 13 receivers to get unique results is inconvenient and very computational intensive.

### 6.2 Passive Localization in 3D

PS measurements are not appropriate in 3D, as an unreasonable number of receivers would be required for unique localization results. In addition, the resolution with PS measurements in 2D was not higher than with TDOA measurements. Therefore, we consider only TDOA measurements for passive localization in 3D.

Figure 6.20 shows the probability distributions in 3D for two receivers $R_1$ and $R_2$, and $\Delta t_{2,1} = \pm 35\mu s$. The distributions are shown for clarity in color, where light colored regions represent high probabilities. Instead of a probability map in the shape of a hyperbola a probability volume in shape of a hyperboloid is obtained.

In 3D at least four receiver pairs are required to obtain a unique localization

<table>
<thead>
<tr>
<th>$N$</th>
<th>$R^*$ [µm] for $A F_{\text{max}} = 0.90$</th>
<th>$A F_{\text{max}} = 0.98$</th>
<th>$A F_{\text{max}} = 0.998$</th>
</tr>
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<tbody>
<tr>
<td>11</td>
<td>805</td>
<td>355</td>
<td>100</td>
</tr>
<tr>
<td>13</td>
<td>800</td>
<td>350</td>
<td>105</td>
</tr>
<tr>
<td>25</td>
<td>810</td>
<td>455</td>
<td>110</td>
</tr>
<tr>
<td>50</td>
<td>830</td>
<td>420</td>
<td>275</td>
</tr>
</tbody>
</table>

Tab. 6.2: Summary of the expected resolution based on radius $R^*$ with different receiver arrangements, if the PS is measured at $f = 300$ kHz.
6.2. PASSIVE LOCALIZATION IN 3D

(a) Lobe part for $\Delta t_{2,1} = 35 \mu$s.

(b) Lobe part for $\Delta t_{2,1} = -35 \mu$s.

Fig. 6.20: Probability volume in 3D in shape of a hyperboloid.

Therefore at least four receivers are needed, which can form six different pairs. Nevertheless, the probability distribution with four receivers combined to six pairs is inaccurate for certain locations. For example, an inaccurate prediction for the emitter position is obtained if the emitter is located close to the plane of three receivers. Figure 6.21 shows the probability distribution of four receivers located in the corners of a cube of $20\lambda$ side length combined to six pairs. In Fig. 6.21(a) the assumed emitter location is at point $x_E = 0$, $y_E = -9\lambda$, and $z_E = 3\lambda$, which is close to the plane of the receivers $R_1$, $R_2$, and $R_4$. In this case multiple broad regions with the highest probability for the emitter location are obtained. Therefore, the emitter location prediction is not clear. In Fig. 6.21(b) the assumed emitter location is at point $x_E = 5\lambda$, $y_E = 5\lambda$, and $z_E = -\lambda$, which results in an accurate prediction for the emitter location.

To overcome the inaccuracy of four receivers, the number of receivers can be increased. Our experimental setup has four receiver channels (see section 8.2). If we want to use more than four receivers in an experiment, we have to perform the experiment twice and position the four receivers at different locations in each case. This results in eight receiver positions, which can be used for the emitter localization. Figure 6.22 shows the probabil-

2Depending on the arrangement, it can be possible that three pairs are sufficient for unique results.
(a) Inaccurate location prediction.  
(b) Accurate location prediction.

Fig. 6.21: Probability distribution for 4 receivers combined to 6 pairs in 3D.

Fig. 6.22: Accurate prediction of the emitter location from probability distribution of 2 × 4 receivers combined to 12 pairs. In both Fig. 6.22(a) and 6.22(b) only one small region with the highest probability for the emitter location exists and, thus, an accurate prediction for the emitter location is achieved.

In Fig. 6.23 the ratio $R^*/\lambda$ for an emitter located in the origin of the co-
ordinate system can be seen when eight receivers are placed at the corners of a cube of increasing side length $d$. In the 3D case, $R^*$ denotes the radius of the volume with the ML estimate for the emitter location above the threshold $AF_{\text{max}}$. The maximum amplitude factor $AF_{\text{max}}$ is chosen as 0.90 and 0.98. The ratio $R^*/\lambda$ reaches a constant level for about $d \geq 25 \lambda$, which is almost half of the distance, where a constant level is reached in the 2D case. For eight receivers located in the corner of a cube of $d = 25 \lambda$ the ratio $R^*/\lambda$ becomes 0.15 and 0.12 for $AF_{\text{max}} = 0.90$ and $AF_{\text{max}} = 0.98$, respectively. Thus, a resolution of 600 to 650 $\mu$m can be expected at a frequency of 300 kHz, which is in accordance with the expected resolution in 2D.

### 6.3 Conclusion

For unique localization results it is sufficient to consider only three receivers in a 2D environment if the TDOA is measured, while in the PS case at least 11 or 13 receivers are required. Despite the higher receiver number required for PS measurements, the expected resolution is not significantly higher than in the TDOA case. Increasing the distance between a few receivers and their arrangement effects the resolution much more than increasing the receiver number. At reasonable computational cost, the best resolution is obtained by measuring the TDOA between four receivers in a square arrangement with $d \geq 45 \lambda$. In this case a resolution of less than 600 $\mu$m can be expected if $AF_{\text{max}} \geq 0.9$, and if an omnidirectional emitter with a center frequency
at 300 kHz is assumed.

In the 3D case only the TDOA measurements are considered. To get accurate localization results more than four receivers must be considered. If $2 \times 4$ receivers are combined to $2 \times 6$ pairs and positioned in the corners of a cube with $d \geq 25\lambda$ side length, a resolution of $600 - 650 \, \mu m$ can be expected for $AF_{\text{max}} \geq 0.9$ at 300 kHz.
Chapter 7

Localization in Heterogeneous Media

In a homogeneous environment the wave propagation is circular in 2D and spherical in 3D. However, if the environment is heterogeneous the wave propagation becomes more complex. An important challenge in acoustics is to represent the shape of waveforms after the waves are reflected and refracted at obstacles, as the spatial accuracy in ultrasonic imaging, localization, and distance measurements not only depends on accurate time or phase measurements but also on precise knowledge of the wave propagation path. In conventional ultrasound systems reflection and refraction at tissue boundary layers, as well as the interference of multiple waves are typically neglected. It has long been known that these simplifications result in a spatial accuracy loss \[24, 31, 125\]. To improve the aforementioned problems, geometrical approaches like ray tracing have been proposed, though these approaches still neglect some wave characteristics of ultrasound.

7.1 Mechanical Simulation

To overcome the errors made by neglecting refraction, reflection, interference, and absorption the acoustic wave propagation is calculated with FE simulations in ANSYS. For this purpose input text files with macros were created for the fast changing of the simulation environment. The follow-
CHAPTER 7. LOCALIZATION IN HETEROGENEOUS MEDIA

Tab. 7.1: Young’s modulus $E$, Poisson’s ratio $\nu$, density $\rho$, speed of sound $c$, attenuation coefficient $\chi$ at 300 kHz, and acoustic impedance $Z$ used in the FE simulation of the wave propagation.

<table>
<thead>
<tr>
<th>material</th>
<th>$E$ [MPa]</th>
<th>$\nu$ [-]</th>
<th>$\rho$ [kg/m$^3$]</th>
<th>$c$ [m/s]</th>
<th>$\chi$ [dB/mm]</th>
<th>$Z$ [Ns/m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GL</td>
<td>—</td>
<td>—</td>
<td>1260</td>
<td>1904</td>
<td>90</td>
<td>2.34·10$^6$</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>—</td>
<td>—</td>
<td>998.2</td>
<td>1482</td>
<td>50</td>
<td>1.48·10$^6$</td>
</tr>
<tr>
<td>PZT</td>
<td>63000</td>
<td>0.35</td>
<td>7500</td>
<td>2500</td>
<td>0.56</td>
<td>33·10$^6$</td>
</tr>
<tr>
<td>silicone</td>
<td>2.05</td>
<td>0.45</td>
<td>900</td>
<td>948</td>
<td>0.84</td>
<td>0.24·10$^6$</td>
</tr>
</tbody>
</table>

7.1.1 Finite Element Model

A tube filled with a fluidic medium and surrounded by a second different fluidic medium is a common configuration to represent a blood-vessel surrounded by soft tissue [126]. Thus, our FE model consists of a first medium (GL) of radius $r = 40.5$ mm, surrounded by a second medium (H$_2$O), separated by a thin cylindrical membrane (silicone tube). The emitter is situated within the first medium. The arrangement for the FE simulation must be adapted, if other conditions are of interest. The FE model is based on the localization setup shown in Fig. 8.7 in section 8.2. A cubic stack made of lead zirconium titanate (PZT) acts as the emitter $E$. All media have different acoustic properties. In particular the speed of sound in GL and H$_2$O is not the same: $c_1 \neq c_2$. The material properties of GL, H$_2$O, PZT, and silicone, which are used in the wave propagation simulation, are summarized in Tab. 7.1.

Instead of a FE simulation in 3D, which would be time-consuming and inaccurate, two FE simulations in 2D are performed, as shown in Fig. 7.1. The FE simulations are implemented in the two sectional views along the $xy$-plane and the $yz$-plane, which are the two geometrical limiting cases.

The arrangements in Fig. 7.1 can be further reduced for the FE analysis. Regardless of the location of the emitter within medium 1 the situation in Fig. 7.1(a) is axially symmetric along the line from $E$ to the center of medium 1. Therefore, it can be assumed that the emitter lies on the $y$-axis.
7.1. MECHANICAL SIMULATION

**Fig. 7.1:** Sectional views of the FE simulation of an emitter $E$ in heterogeneous media.

(a) Sectional view of the $xy$-plane  
(b) Sectional view of the $yz$-plane

**Fig. 7.2:** Reduced model for the FE analysis of the wave propagation in (a) and close-up view of the elements in (b).

and only half of the emitter and the half-plane satisfying $x \geq 0$ can be considered without loss of information. This reduction applies similarly for Fig. 7.1(b).

The infinite half-plane is simulated by a semicircular area with an infinite boundary on the outside. The reduced arrangement for Fig. 7.1(a) is shown in Fig. 7.2. In 7.2(a) the four areas with different materials properties can be seen. In 7.2(b) the quadrangular finite elements are shown. A general guideline for the element size is to have at least 20 elements per wavelength along the propagation direction. With the frequency $f = 300$ kHz of the emitted pulses and the minimal speed of sound in the fluidic media
(1482 m/s), the maximal element side length results in about 250 µm. In addition, Fig. 7.2(b) shows the following three element types used for the simulation of the 2D wave propagation:

1. The FLUID29/1 element is used for the fluid medium and the FLUID29/0 element for the interface in fluid/structure interactions. In our case the FLUID29/0 is used for GL and H₂O elements, which are in contact with a solid structure, and FLUID29/1 is used for the rest of medium 1 and 2. The FLUID29 element is defined by four corner nodes with three DOF per node: translations in the nodal $x$ and $y$ directions, and pressure. The translations, however, are applicable only at nodes that are in contact with a solid structure. These nodes must be marked with a fluid/structure interaction flag to denote the interface surface between the fluid and structure part of the model. All other elements have only pressure as one DOF.

2. The infinite boundary is meshed with FLUID129 elements. This element is a companion element to FLUID29. It is intended to be used as an envelope to a structure made of FLUID29 elements. It simulates the absorbing effects of a fluid domain that extends to infinity beyond the boundary of the FLUID29 FE domain. FLUID129 realizes a second-order absorbing boundary condition, in a way that an outgoing pressure wave which reaches the boundary of the model, is absorbed with minimal reflections back into the fluid domain. FLUID129 is a line element with two nodes and one pressure DOF per node.

3. The PLANE42 element is a 2D structural solid element. It is defined by four nodes having two DOF at each node: translations in the nodal $x$ and $y$ directions. In our case the PLANE42 element is used to mesh the emitter (PZT) and the silicone membrane.

### 7.1.2 Finite Element Simulation of the Wave Propagation

In acoustical fluid-structure interaction problems, the structural dynamics equation, which can be formulated as

$$[m][\ddot{u}] + [D][\dot{u}] + [k][u] = F^a,$$

(7.1.1)
7.1. MECHANICAL SIMULATION

is considered along with acoustic wave equation (3.1.4). In \( \mathbf{D} \) denotes the structural damping matrix, \( \mathbf{\ddot{u}} \) the nodal acceleration vector, \( \mathbf{\dot{u}} \) the nodal velocity vector, \( \mathbf{u} \) the nodal displacement vector, and \( \mathbf{F}^a \) the applied load vector.

At the nodes of the PLANE42 elements (7.1.1) is solved to obtain the motion of these nodes. At the nodes of the FLUID29 and FLUID129 element, (3.1.4) is approximated for the acoustic pressure \( p(x_n, y_n, t) \) over the TOF \( t \) where \( x_n \) and \( y_n \) are the node coordinates. Equation (7.1.1) and (3.1.4) are coupled at the nodes of the fluid/structure interface, which have, both, displacement and pressure DOFs. As a result the displacement of the solid structures over the time \( t \) and the pressure as a function of \( t \) for the fluid nodes is obtained.

With the pressure distribution over the TOF \( t \), the wave propagation can be displayed, as shown in Fig. 7.3 and Fig. 7.4. In these figures the symmetric expansion mode in ANSYS is used to show the full model, instead of the reduced one. Hence, the full contour lines of equal pressure (isobars) at time \( t \) can be seen. Figure 7.3(a) and Fig. 7.4(a) show the wavefronts of the emerging pressure waves after 1.2 \( \mu s \), which corresponds to approximately \( T/3 \). The omnidirectional pressure field can be seen in Fig. 7.3(b) and Fig. 7.4(b). The upper half circular isobar corresponds to a positive pressure and the lower half circular isobar to a negative pressure. The refraction and reflection at the silicone interface can be seen in Fig. 7.3(c) to 7.3(f) and Fig. 7.4(c) to 7.4(f). The reflected waves interfere with the incident wave inside of the tube resulting in complex patterns of the isobars.

More information about an acoustic analysis, and solving (7.1.1) and (3.1.4) in ANSYS can be found in the chapters “Acoustics”, “Acoustic Fluid Fundamentals”, “Fluid-Structure Interaction”, and “Transient Analysis” in the release 11.0 documentation for ANSYS.
Fig. 7.3: Wave propagation in ANSYS in the $xy$-plane for time steps of 8 $\mu$s starting at 1.2 $\mu$s in (a), and ending after 41.2 $\mu$s in (f).
7.1. MECHANICAL SIMULATION

Fig. 7.4: Wave propagation in ANSYS in the $yz$-plane for time steps of 8 $\mu$s starting at 1.2 $\mu$s in (a), and ending after 41.2 $\mu$s in (f).
7.2 FE-Based Localization

7.2.1 Emitter Location at a Certain Time of Flight

The shape of the wavefront in ANSYS after a certain TOF $t$ can be used for the localization of the sound source. The wavefront shape is found by searching all node coordinates where the corresponding acoustic pressure exceeds a certain threshold value. This value is higher than the static pressure in the model (the static pressure is usually very small compared to the acoustic pressure).

For the wave propagation time $t_i$ it does not make any difference whether an ultrasonic wave is emitted at the emitter location $E$ and received at the location of receiver $R_i$, or emitted at the location of receiver $R_i$ and received at $E$. Therefore, we can find all possible locations of $E$ from a measured time $t_i$ by calculating the wavefront shape of a wave, which is emitted at the known location of receiver $R_i$, and then propagates through the media over a period of time $t_i$. The different possible locations of $E$ (corresponding to the waveform shapes) for different TOF, thus obtained, are shown as the curved lines in Fig. 7.5. Figure 7.5(a) and Fig. 7.5(b) show the wavefront shapes in the $xy$- and the $yz$-plane, respectively.

Polynomials of degree four fit the wavefront shapes inside medium 1 well.
7.2. FE-BASED LOCALIZATION

<table>
<thead>
<tr>
<th>plane</th>
<th>(a_1^\text{plane})</th>
<th>(a_2^\text{plane})</th>
<th>(a_3^\text{plane})</th>
<th>(a_4^\text{plane})</th>
<th>(a_5^\text{plane})</th>
<th>(a_6^\text{plane})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(xy)</td>
<td>300</td>
<td>-1.40</td>
<td>-9.72</td>
<td>1.20</td>
<td>-0.552</td>
<td>-0.820</td>
</tr>
<tr>
<td>(yz)</td>
<td>25.0</td>
<td>-0.503</td>
<td>-9.72</td>
<td>6.49</td>
<td>-1.59</td>
<td>-0.820</td>
</tr>
</tbody>
</table>

Tab. 7.2: Fitting coefficients \(a_j^\text{plane}\) of the polynomial coefficients \(K_{i,1}^\text{plane}\) and \(K_{i,2}^\text{plane}\) for the \(xy\)- and \(yz\)-plane, respectively.

The odd polynomial coefficients are zero, as the wavefront shapes are axisymmetric along the \(y\)-axis. Hence, the wavefront shapes can be described with the emitter location polynomials

\[ y_i^{xy} = K_{i,1}^{xy} x_i^4 + K_{i,2}^{xy} x_i^2 + K_{i,3} \quad \text{and} \quad y_i^{yz} = K_{i,1}^{yz} z_i^4 + K_{i,2}^{yz} z_i^2 + K_{i,3}, \]  

for the \(i\)th receiver in the \(xy\)- and \(yz\)-plane. In (7.2.1) the \(z\)-axis is aligned with the cylinder axis of medium 1 and the coordinate system is always turned such that the receiver \(R_i\) lies on the \(y\)-axis: \(r_{i,xy} = (0, y_{R_i})\) in the \(xy\)-plane, or \(r_{i,yz} = (y_{R_i}, 0)\) in the \(yz\)-plane. The polynomial coefficients are functions of the speed of sound \(c_1\) and \(c_2\), the distance \(d_i = y_{R_i} - r\), the measured TOF \(t_i\), and the radius \(r\) of medium 1. From the FE simulations we determined that the coefficients \(K_{i,1}^{xy}, K_{i,2}^{xy}, K_{i,1}^{yz},\) and \(K_{i,2}^{yz}\) for \(c_1 = 1904\) m/s, \(c_2 = 1482\) m/s, and \(r = 40.5\) mm are described well by functions of \(t_i\) and \(d_i\). The coefficients are calculated as

\[ K_{i,1}^{xy} = a_{1,xy}^{} e^{a_{2,xy} (\frac{t_i}{T})} e^{a_{3,xy} (\frac{d_i}{r})} \quad \text{and} \quad K_{i,2}^{xy} = a_{4,xy}^{} \left(\frac{t_i}{T}\right)^{a_{5,xy}} e^{a_{6,xy} (\frac{d_i}{r})}, \]  

and

\[ K_{i,1}^{yz} = a_{1,yz}^{} e^{a_{2,yz} (\frac{t_i}{T})} e^{a_{3,yz} (\frac{d_i}{r})} \quad \text{and} \quad K_{i,2}^{yz} = a_{4,yz}^{} \left(\frac{t_i}{T}\right)^{a_{5,yz}} e^{a_{6,yz} (\frac{d_i}{r})}, \]  

where the fitting coefficients \(a_{j,xy}\) and \(a_{j,yz}\) \((j = 1, 2, \ldots, 6)\) are different for both planes. The TOF \(t_i\) and the distance \(d_i\) from the \(i\)th receiver to the interface between medium 1 and 2 are normalized with the cycle \(T\) and the radius \(r\), respectively. The coefficients \(a_{j,xy}\) and \(a_{j,yz}\), which are determined numerically \([38]\), are summarized in Tab. 7.2.

Figure 7.6 shows a comparison between the simulated values for the polynomial coefficients \(K_{i,1}^{xy}\) and \(K_{i,1}^{yz}\) and their numerically calculated fits against
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Fig. 7.6: First polynomial coefficients in the $xy$- and $yz$-plane for $c_1 = 1904 \text{ m/s}$, $c_2 = 1482 \text{ m/s}$, and $r = 40.5 \text{ mm}$.

Fig. 7.7: Second polynomial coefficients in the $xy$- and $yz$-plane for $c_1 = 1904 \text{ m/s}$, $c_2 = 1482 \text{ m/s}$, and $r = 40.5 \text{ mm}$.

The TOF $t_i$ and the distance $d_i$. In Fig. 7.7 a comparison between polynomial coefficients $K_{1,2}^{xy}$ and $K_{1,2}^{yz}$ obtained from the FE simulation and their numerically calculated fits is shown.

The polynomial coefficient $K_{i,3}$ denotes the $y$-axis intercept of the polynomials in, both, the $xy$- and $yz$-plane. It can be calculated analytically as

$$K_{i,3} = r - c_1 \left( t_i - \frac{d_i}{c_2} \right). \quad (7.2.4)$$

Very good agreement is found between the analytically calculated values for $K_{i,3}$ and the values obtained from the FE simulation, as shown in Fig. 7.8. $K_{i,3}$ is normalized with $r$ and shown against $t_i/T$ and $d_i/r$. 

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Fig. 7.8: Normalized polynomial coefficient $K_{i,3}$ in the $xy$- and $yz$-plane for $c_1 = 1904 \text{ m/s}$, $c_2 = 1482 \text{ m/s}$, and $r = 40.5 \text{ mm}$.

All possible emitter locations, defined by the vector $\mathbf{e}_i^{xy} = \left( x_i, y_i \right)$, in the $xy$-plane are calculated by solving (7.2.1) with the corresponding values for the polynomial coefficients $K_{i,1}^{xy}$, $K_{i,2}^{xy}$, and $K_{i,3}$, for a given TOF $t_i$. Similarly, all possible emitter locations in the $yz$-plane, defined by $\mathbf{e}_i^{yz} = \left( y_i, z_i \right)$, are calculated by solving (7.2.1) with $K_{i,1}^{yz}$, $K_{i,2}^{yz}$, and $K_{i,3}$ for $t_i$. The 2D shapes of the polynomials for the two limiting planes can be combined to a saucer-type area for the emitter location in 3D by an elliptic interpolation of the intermediate points. Figure 7.9 shows the emitter location areas in 3D for different TOFs to one receiver located on the $y$-axis at $y = 0.5 r$, if $x$, $y$, and $z$ are normalized with the radius $r$. 

Fig. 7.9: Combined areas for the emitter location in 3D at different TOFs to one receiver located on the $y$-axis at 0.5 $r$. 

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7.2.2 Combination of the Finite Element Method with Time Difference of Arrival Measurements

The areas obtained from the TOF measurements to two different receivers $R_1$ and $R_2$ can be combined to a single intersecting line, which corresponds to a certain TDOA $\Delta t_{2,1}$ between $R_1$ and $R_2$. Figure 7.10 and Fig. 7.11 show the resulting intersecting lines, and the shapes of the emitter location areas corresponding to $\Delta t_{2,1} = 4T, 5T, \ldots, 11T$. The emitter location area from to the first receiver corresponds to a TOF $t_1 = 6.5T$. With the TDOAs the emitter location areas from the second receiver refer to the TOFs $t_2 = 10.5T, 11.5T, \ldots, 17.5T$. Figure 7.10 shows the 3D view of the emitter location areas, the TDOA intersecting lines, and the cylindrical interface between the two media. In Fig. 7.11(a) and 7.11(b) the views on the $xz$- and $yz$-plane can be seen. The TDOA intersection lines must be located inside the first medium. Therefore, only the segments of the intersection lines which are located in the interval $-1 \leq x/\lambda, y/\lambda \leq 1$ are valid.

Considering the uncertainty inherent with real sinusoidal signals, probability volumes result instead of emitter location areas and intersecting lines.
7.2. FE-BASED LOCALIZATION

Fig. 7.11: TDOA intersection lines for multiple TOF measurements.

Fig. 7.12: 3D view of the probability volumes for multiple TDOA measurements.

Figure 7.12 shows the probability volumes for the intersection lines from Fig. 7.10. The interface between medium 1 and 2 is represented by the dashed lines. The probability volumes are only shown inside medium 1. The axes are normalized with the wavelength $\lambda$ in medium 2. Light colored
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Fig. 7.13: ML prediction of the emitter location in a heterogeneous media from probability distribution of $2 \times 4$ receivers combined to 12 pairs in 3D.

regions represent high probabilities, and dark colored regions low probabilities for the emitter location.

Taking $2 \times 4$ receivers located in the corners of a cube of $20\lambda$ side length, combined into $2 \times 6$ pairs as in section 6.2 results in the probability distribution, shown in Fig. 7.13. The interface between medium 1 and 2 is again represented by the dashed lines. In Fig. 7.13(a) the assumed emitter location inside medium 1 is at point $x_E = 0, y_E = 3\lambda$, and $z_E = -9\lambda$, and in Fig. 7.13(b) the assumed emitter location inside medium 1 is at point $x_E = 5\lambda, y_E = -5\lambda$, and $z_E = 2\lambda$. The probability distribution is calculated in both media, though, it is valid only inside of medium 1. Nevertheless, a correct ML estimation for the emitter location is obtained inside medium 1.
Chapter 8

Experimental Results

Several wireless emitters were fabricated. These emitters can be localized with our proposed method, and also used for viscosity sensing in different fluids. The characterization of the wireless emitters, and the potential for passive localization and viscosity sensing is described in the first section.

In the second section the localization of small piezo stacks is described. The localization results are shown in homogeneous and heterogeneous media in 2D and 3D. In addition, we compare our FE-based localization method with conventional localization methods, and we show improved results.

8.1 Wireless Emitter Characterization

We fabricated six wireless emitters (Ni1 – Ni6) with different gold spring thicknesses (Tab. 8.1). The motion of the emitters in air was characterized with a laser doppler vibrometer (LDV). Figure 8.1 shows the first three modes of the (nearly) undamped oscillation of an emitter with $h_{Au} = 25 \, \mu m$ (Ni6) as measured by the LDV in air (8.1(a)-8.1(c)) and simulated with a FEM modal analysis using ANSYS (8.1(d)-8.1(f)). Both methods indicate that the first mode is a translation of the resonator in its normal direction, the required motion to generate the omnidirectional acoustic pressure field.

In general, good agreement is found between the experimentally measured and simulated results for all resonance modes, with a maximal error < 25%. The difference in the resonant frequencies is due to an uncertainty in the
8.1.1 Experimental Setup

A schematic of the experimental setup is shown in Fig. 8.2. A coil with radius $R = 18\,\text{mm}$ and $N = 21$ turns is used to generate a magnetic field with field strength as low as $2.5\,\text{mT}$, which is roughly 50 times the earth’s magnetic field. The coil is driven by a custom built current amplifier (see appendix B.1) that controls the input current to the coil up to a maximal frequency of $10\,\text{kHz}$ (at $4\,\text{A}$). The amplifier input signal is an on/off voltage signal with tunable frequency $f$ and varying voltage amplitude proportional to the desired current. The on/off voltage signal is generated on a data acquisition (DAQ) card (NI-6110) in a PC. The resonator is glued to the wall of a container that is filled with S350 or GL. For the limiting cases (see section 5.2) the driving coil is placed below the container (A, corresponding to $\theta = 0^\circ$) or at the wall of the container (B, corresponding to $\theta = 90^\circ$), such that the wireless emitter lies on the axis of the coil at a distance of $5\,\text{mm}$. The magnetic field of the coil excites the emitter with the frequency $f$. The pressure wave generated by the emitter relative to the ambient pressure is picked up by a hydrophone, which is a cubic stack of $2\,\text{mm}$ side length...
8.1. WIRELESS EMITTER CHARACTERIZATION

Fig. 8.2: Experimental setup for the emitter characterization: the coil at position A (θ = 0°) or B (θ = 90°) excites the wireless emitter, which emits an acoustic pressure wave. The acoustic signal is picked up by a hydrophone and transmitted to the PC.

made from PZT. The hydrophone is sealed with silicone rubber and mounted to a motorized micromanipulator from Sutter Instruments (MP-285). The motorized micromanipulator is used to accurately position the hydrophone and to scan the acoustic pressure field emitted from the wireless emitter. The pressure change recorded with the hydrophone is bandpass filtered, amplified (BPF/Amp) and transferred back to the data acquisition card. For the bandpass filter and amplifier circuitry see appendix B.1.

8.1.2 Pressure Field

Figure 8.3 shows a typical result for the normalized pressure amplitude (●) in the xz-plane measured in GL compared to the theoretical prediction (mesh). The pressure field is symmetric, as explained in section 5.2.3 Therefore, only the points in the area x ≥ 0 and z ≥ 0 are scanned. The wireless emitter lies in the xy-plane with its center located at the origin of the coordinate system, according to Fig. 5.7 and Fig. 5.8.

As the emitter works at resonance, the direction of the excitation field can be arbitrary. This is an important feature for wireless devices operating in fluids, where accurate position and orientation control is difficult. In fact, it was shown in [5.2.3] that the force $F_x$ for $\theta = 0^\circ$ is more than 100 times larger than the force $F_z$ for $\theta = 90^\circ$. Nevertheless, the emitter sends out an acoustic signal at the desired frequency for both coil positions (A and B) in
both fluids, as shown in Fig. 8.4. The hydrophone is placed at a distance 5 mm from the resonator. The results are normalized to the pressure level at an applied current of 4 A. In addition, the measured pressure corresponds well to the theoretical prediction.

### 8.1.3 Passive Localization

A detectable pressure wave is sent out into the surrounding fluid, independent of the orientation of the excitation field, as shown in Fig. 8.4. This is an important feature for devices operating inside the human body, where accurate position and orientation control is difficult. With the proposed emitter, localization independent of the pose of the device is possible. In addition, its wireless nature considerably reduces the power and circuitry
8.1. WIRELESS EMITTER CHARACTERIZATION

Fig. 8.5: Design tradeoff between emitting frequency and pressure. The spring stiffness is varied from 200 to 1200 N/m, and the emitter-hydrophone distance from 0.2 to 2.0 cm, both in equal steps. The natural frequency $f_n$ and the area are normalized to the presented design (marked with ⊗).

requirement of the devices while their functionality is increased.

The tradeoff in the design of the wireless emitter is its natural frequency, which corresponds to the desired frequency of the acoustic signal, versus the signal strength (acoustic pressure). The larger the area of the nickel dies (for a given thickness), the higher the amplitude of the emitted pressure wave, and the further apart from the device a signal can be measured. Yet, at the same time its resonant frequency will decrease as its mass is increased. This tradeoff is illustrated in Fig. 8.5 with the spring constant and the emitter-hydrophone distance as parameters. The plot allows one to select the right area size and spring constant for the desired frequency and measuring range. The design of the presented wireless emitters is marked with ⊗. A frequency in the lower kHz-range is of course not sufficient for a localization with a high resolution. Nevertheless, a great potential for an applicability of these wireless emitters in passive localization is shown. Current work focuses on designing emitters with higher resonant frequencies and signal strengths to improve localization.

8.1.4 Viscosity Sensing

Sensing is another field of application for such wireless resonators, as their resonant frequencies depend strongly on, e.g. the viscosity of the fluid in
which the emitter is submerged. The use of the wireless emitters as viscosity sensors is explored in S350 and GL.

In S350 and GL a fast fourier transform (FFT) is performed on the recorded acoustic signal measured with the hydrophone, and on the velocity of the resonator recorded with the LDV. Table 8.1 shows the results for the six resonators Ni1 – Ni6 with different spring thicknesses $h_{Au}$. The velocity resonant frequency $f_{LDV}^R$ measured with the LDV is compared with the measured resonant frequency $f_{AS}^R$ of the acoustic signal. Additionally, a comparison of the fundamental frequency $f_n$ and the resonant frequency $f_{LDV}^1$ measured with the LDV in air is shown. Good agreement is found between $f_{AS}^R$ and $f_{LDV}^R$ in both fluids. The difference between $f_{LDV}^1$ and $f_{FEM}^i$ ($i = 1, 2, 3$).

With $\eta_{S350}$ as the reference viscosity $\eta_1$ we determine the viscosity $\eta_2$ of GL by (5.2.29), and the validity of the result is checked with the criterion (5.2.32). The term $\delta f / \delta f_{\text{min}}$ is calculated for an error $\Delta f_1 = 10$ Hz of $f_1$, $\Delta \delta f = 20$ Hz of $\delta f$, $\Delta \eta_1 = 0.01 \eta_1$ of the reference viscosity, and a maximal relative error of $\varepsilon = 5\%$. As shown in Table 8.1, the requirement $\delta f / \delta f_{\text{min}} \geq 1$ is satisfied by resonator Ni6. Figure 8.6 shows the relative change in viscosity $\delta \eta = (\eta_1 - \eta_2)/\eta_1$ due to a relative change in frequency $\delta f = \delta f/f_1$ computed by (5.2.29). The change in viscosity is shown for increasing frequencies $f_1$. The dashed line refers to Ni6 for $f_1 = f_{AS}^R$ in S350. For the obtained resonant frequencies $f_{AS}^R$ in S350 and GL we deter-

<table>
<thead>
<tr>
<th>$h_{Au}$ [µm]</th>
<th>$f_n$ [kHz]</th>
<th>$f_{LDV}^1$ [kHz]</th>
<th>$f_{LDV}^R$ [kHz]</th>
<th>$f_{AS}^R$ [kHz]</th>
<th>$\delta f / \delta f_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni1</td>
<td>1.77</td>
<td>0.72</td>
<td>0.68</td>
<td>0.73</td>
<td>0.69</td>
</tr>
<tr>
<td>Ni2</td>
<td>2.56</td>
<td>1.56</td>
<td>1.45</td>
<td>1.52</td>
<td>1.41</td>
</tr>
<tr>
<td>Ni3</td>
<td>2.56</td>
<td>1.94</td>
<td>1.81</td>
<td>1.92</td>
<td>1.81</td>
</tr>
<tr>
<td>Ni4</td>
<td>2.56</td>
<td>1.97</td>
<td>1.83</td>
<td>1.98</td>
<td>1.83</td>
</tr>
<tr>
<td>Ni5</td>
<td>3.38</td>
<td>2.07</td>
<td>1.92</td>
<td>2.05</td>
<td>1.89</td>
</tr>
<tr>
<td>Ni6</td>
<td>3.38</td>
<td>2.78</td>
<td>2.61</td>
<td>2.83</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Tab. 8.1: Comparison of fundamental frequency $f_n$, resonant frequency $f_{LDV}^1$ measured with the LDV in air, as well as the measured resonant frequency $f_{LDV}^R$ in the LDV, and the resonant frequency $f_{AS}^R$ of the acoustic signal in S350 and GL for six resonators. The resonators are valid for viscosity sensing, if $\delta f / \delta f_{\text{min}} \geq 1$. 

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8.2. LOCALIZATION RESULTS

Fig. 8.6: Change in viscosity $\delta \eta$ due to a change in frequency $\delta f$. The reference frequency $f_1$ increases from 350 Hz to 6.30 kHz.

mine a frequency change of 8.1% (marked with ⊘), which corresponds to $\delta \eta = -323\%$, or $\eta_2 = 1.48 \pm 0.07$ Pa s. This is in good agreement with $\eta_{GL} = 1.41$ Pa s. As the sensitivity increases with the frequency, it is again advantageous to use wireless emitters with resonant frequencies in the higher kHz-range.

8.2 Localization Results

We use commercial piezoelectric emitters from Physik Instrumente with a bandwidth of 500 kHz in the localization experiments. With our fabricated wireless emitters it is possible to send out an acoustic pressure wave with a frequency in the lower kHz range which is only capable of lower resolution localization.

8.2.1 Experimental Setup

To test the localization algorithm, the experimental setup illustrated in Fig. 8.7 is used. Voltage pulses with a frequency of 300 kHz were generated on a DAQ card (NI-6110) with a sampling frequency of 4 MHz on a PC. The pulses are amplified with a Krohn-Hite 7500 amplifier (KH 7500) and routed to the piezoelectric emitter, which is a cubic stack 2 mm on a side made from PZT. The emitter is placed inside of a silicone tube with 200 µm
wall thickness and radius $r = 40.5$ mm, which is filled with GL. To accurately control motion along a predefined path the emitter is clamped to the motorized micromanipulator from Sutter Instrument (MP-285) with a spatial resolution of 40 nm. The TDOAs to several receivers (PZT hydrophones of 2 mm side length) are measured along the path. The receivers are mounted on a rack at well defined positions at a distance of $6 \lambda = 29.64 \pm 0.01$ mm or $32 \lambda = 158.08 \pm 0.01$ mm (accuracy of the receiver position is less than 10 $\mu$m) and immersed in a water tank. The rack can be rotated around the tube along a circular path with $6 \lambda$ or $32 \lambda$ diameter. The angle of rotation of the rack is measured with a circular encoder with a resolution of 0.18°. The tube and the ring are eccentric by 11.5 mm to avoid a concentric special case. All PZT transducers are sealed with silicone rubber. The signal picked up at the receivers is transferred to a custom built bandpass filter and amplifier (BPF/Amp), and back to the DAQ card, where it is sampled with 4 MHz. For the bandpass filter and amplifier circuitry see appendix B.1.

The TDOA measurement is performed by cross-correlating the received signals first (see appendix A.1). Then, a Hilbert transform (see appendix A.2) is applied on the cross-correlated signal, and the absolute value of the Hilbert transform is calculated to extract the envelope of the cross-correlation. The TDOA can then be determined by searching the maximum value of the envelope of the cross-correlation. Figure 8.8 shows the cross-correlations.
8.2. LOCALIZATION RESULTS

(a) Homogeneous media
(b) Heterogeneous media

Fig. 8.8: Cross-correlated signals and their envelopes in a homogeneous and heterogeneous media for the TDOA determination with LabView.

of two measured signals and the envelopes of the cross-correlations in a homogeneous and heterogeneous media using LabView for the DAQ and data processing. The cross-correlation and, thus, the Hilbert transform produce clearer results in the homogeneous media, as the measured signals are stronger with a better SNR in this case. It can be seen that the amplitude in Fig. 8.8(a) is about 18 times bigger than the amplitude of the cross-correlation in Fig. 8.8(a). In addition, the cross-correlation can have multiple peaks in heterogeneous media, as shown in Fig. 8.8(b) and, thus, the TDOA measurements can be incorrect.

8.2.2 Passive Localization in Homogeneous Media

For the first passive localization experiments the emitter is placed in homogeneous media. That is the emitter is immersed directly in the water tank without using the silicone tube filled with GL. The passive localization algorithm described in chapter 6 is verified with these experiments, first in 2D and then in 3D.
Passive Localization in 2D

A rectangular arrangement of four receivers is used for the localization of the emitter in 2D. The emitter is moved along a 2.5 mm grid in a square of 25 mm side length using the motorized micromanipulator. At 121 grid nodes the TDOA is measured between all six combinations of four receivers to pairs. With the TDOA data the location of the emitter is determined and the obtained emitter location is compared with the reference position from the micromanipulator.

In Fig. 8.9 the probability map for the 121 emitter locations from the measured TDOAs is shown when the receivers are at a distance of $6 \lambda$ to each other. The reference position from the micromanipulator is marked with $\times$. The area above a threshold $A_{F_{\text{max}}} \geq 0.9$, which corresponds to our emitter location estimate, is colored green. In Fig. 8.9(a) all emitter locations are inside the receiver array, and in 8.9(b) the emitters are located outside of the array. In Fig. 8.9(b) the area above $A_{F_{\text{max}}} \geq 0.9$ is larger and usually further away from the reference position $\times$, than in 8.9(a). Hence, both, the resolution and the accuracy are higher if the receivers are located around the emitter such that the emitter lies inside the receiver array.

In Fig. 8.10 the probability maps are shown when the receivers are at a distance of $32 \lambda$ to each other. Again, the reference position from the micromanipulator is marked with $\times$, and the area above $A_{F_{\text{max}}} \geq 0.9$ is colored green. In Fig. 8.10(a) all emitter locations are inside the receiver array, and in 8.10(b) the emitters are located outside of the array. As the receivers are further apart from each other than in Fig. 8.9, the resolution and the accuracy are higher in this case. Again, better localization results are obtained, if the receivers are located around the emitter positions. However, due to the larger distance between the receivers, there is not a significant difference between (a) and (b), as in Fig. 8.9.

The spatial error $\varepsilon$ with the standard deviation $\sigma$ is shown in Fig. 8.11 for the measurements in 2D when all emitter locations are inside the receiver array. The errors $\varepsilon \pm \sigma$ for the arrangements of Fig. 8.9(a) and Fig. 8.10(a) are shown in 8.11(a) and in 8.11(b), respectively. As expected, the distribution is broader and the mean values for the spatial error and the standard deviation
8.2. LOCALIZATION RESULTS

Fig. 8.9: Rectangular arrangement of four receivers at a distance of 6\(\lambda\).

are higher for \(d_R = 6\lambda\), than for \(d_R = 32\lambda\). The mean and maximal values for \(\varepsilon\) and \(\sigma\) are summarized in Tab. 8.2. It can be seen, that the mean and maximal value for the accuracy and the resolution can be improved if the receivers are positioned further apart from each other.

Passive Localization in 3D

For localization in 3D the rectangular arrangement of four receivers at a distance of \(d_R = 32\lambda\) is used. Though, the TDOA measurements are per-
CHAPTER 8. EXPERIMENTAL RESULTS

Fig. 8.10: Rectangular arrangement of four receivers at a distance of $32\,\lambda$.

<table>
<thead>
<tr>
<th>$\varepsilon \pm \sigma$ [µm]</th>
<th>$d_R$</th>
<th>$\varepsilon_{\text{mean}}$</th>
<th>$\sigma_{\text{mean}}$</th>
<th>$\varepsilon_{\text{max}}$</th>
<th>$\sigma_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6 , \lambda</td>
<td>980 3060</td>
<td>530 1860</td>
<td>810 1990</td>
<td>280 340</td>
</tr>
</tbody>
</table>

Tab. 8.2: Mean and maximal values for the spatial error $\varepsilon$ and the standard deviation $\sigma$ for a square arrangement of four receivers in 2D at a distance $d_R$ to each other.
8.2. LOCALIZATION RESULTS

![Localization error plots](image)

Fig. 8.11: Localization error for the receiver arrangements of Fig. 8.9(a) with $d_R = 6\lambda$, and Fig. 8.10(a) with $d_R = 32\lambda$.

formed twice, and the four receivers are rotated to new positions for the second measuring cycle. This rotation results in an accuracy loss of the receiver position, as the resolution of the circular encoder is limited to 0.18°. Thus, the receiver position accuracy for the second measuring cycle is only about 230 µm, which influences the emitter localization accuracy.

Using the motorized micromanipulator the emitter is moved along a 2.5 mm grid in a cube of 25 mm side length. All emitter locations are inside the receiver array. The TDOA from the emitter to the four receivers combined into six pairs is measured two times at 1331 grid nodes. This corresponds to the TDOA measurements between 8 receivers combined into 12 pairs. The volume above the threshold $AF_{\text{max}} > 0.9$ is determined from the TDOA measurements. The center of mass of this volume (marked with a green □) corresponds to our emitter location estimate in 3D. The emitter location estimate is compared to the reference position from the micromanipulator, which is marked with the red ×. This experiment is repeated four times. Each time the motorized micromanipulator is moved to a different location, such that the emitter is located within the following cubic regions (see Fig. 8.12):

**A:** $0 \leq x_E, y_E, z_E \leq 25$ mm,

**B:** $-25$ mm $\leq x_E, y_E, z_E \leq 0$,
CHAPTER 8. EXPERIMENTAL RESULTS

Fig. 8.12: Different locations for the motorized micromanipulator.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\varepsilon_{\text{mean}}$ [mm]</th>
<th>$\sigma_{\text{mean}}$ [$\mu$m]</th>
<th>$\varepsilon_{\text{max}}$ [mm]</th>
<th>$\sigma_{\text{max}}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.25</td>
<td>550</td>
<td>2.71</td>
<td>1.85</td>
</tr>
<tr>
<td>B</td>
<td>1.24</td>
<td>540</td>
<td>2.77</td>
<td>1.91</td>
</tr>
<tr>
<td>C</td>
<td>1.22</td>
<td>540</td>
<td>2.70</td>
<td>1.82</td>
</tr>
<tr>
<td>D</td>
<td>1.26</td>
<td>560</td>
<td>2.63</td>
<td>1.81</td>
</tr>
</tbody>
</table>

Tab. 8.3: Mean and maximal values for the spatial error $\varepsilon$ and the standard deviation $\sigma$ for a cubic arrangement of $2 \times 4$ receivers in the 3D homogeneous media.

In Fig. 8.13 the results for region C can be seen. The spatial errors $\varepsilon$ and the standard deviations $\sigma$ for the 1331 measurements in all four regions in 3D are shown in Fig. 8.14 and summarized in Tab. 8.3 Over all measurements a mean spatial error of $1240 \pm 550 \mu$m was achieved. Even though the receivers are further apart form each other, the accuracy and the resolution is in the same range as in the 2D arrangement of four receivers at a distance of $6 \lambda$. This is due to the increased uncertainty of the receiver positions, as the receivers must be relocated for the 3D measurements.
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Fig. 8.13: Cubic arrangement of $2 \times 4$ receivers at a distance of $32 \lambda$. The emitter is located in the region C.

8.2.3 Passive Localization in Heterogeneous Media

To experimentally investigate passive localization in heterogeneous media, the silicone tube is filled with GL and immersed in the water tank. The emitter is then positioned in GL with the micromanipulator, and the receivers are located around the silicone tube in water.

Verification of the FE-Based Localization with Time of Flight Measurements in 2D

To verify the FE-based localization TOF measurements in 2D are performed, such that our method can be compared with the time of flight method (see section 4.2.1) with a mean value for the speed of sound $c = \frac{c_1 + c_2}{2}$, and acoustic ray tracing (see section 4.2.2). In this experiment the motorized micromanipulator is used to accurately control the emitter motion along a square path with 25 mm side length. Each mm along the path, the TOF $t_i$ to four receivers at a distance of $32 \lambda$ is measured to estimate the location of the emitter. In our case the synchronization of the emitter with the receivers for the TOF measurement is unproblematic, as the emitted and received signal is processed on the same DAQ card. Thus, the emitted and received signal can be directly compared.
The experimental results are shown in Fig. 8.15. It can be seen that the FE-based localization finds the best estimate for the emitter location compared to the reference positions. The spatial errors $\varepsilon_{\text{mean}} \pm \sigma_{\text{mean}}$ of the 100 measurements with the three different localization methods are summarized in Tab. 8.4. The mean spatial error over all 100 measurements with the FE-based localization is less than half of the spatial errors obtained with acoustic ray tracing and nearly one third of the spatial error derived from the time of flight method with a mean value for the speed of sound. Figure 8.15 and Tab. 8.4 collectively demonstrate the ability of the FE-based localization to estimate an ultrasound emitter position within heterogeneous media, yielding a better resolution and accuracy than conventional methods.

It is clear that the worst localization results are obtained with the time of
8.2. LOCALIZATION RESULTS

Fig. 8.15: Comparison between different localization algorithms.

<table>
<thead>
<tr>
<th>Localization technique</th>
<th>$\varepsilon_{\text{mean}}$ [µm]</th>
<th>$\sigma_{\text{mean}}$ [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FE-based localization</td>
<td>750</td>
<td>310</td>
</tr>
<tr>
<td>Acoustic ray tracing</td>
<td>1600</td>
<td>630</td>
</tr>
<tr>
<td>Time of flight method</td>
<td>2110</td>
<td>860</td>
</tr>
</tbody>
</table>

Tab. 8.4: Mean spatial errors $\varepsilon_{\text{mean}}$ and the standard deviation $\sigma_{\text{mean}}$ with our FE-based localization, acoustic ray tracing, and the time of flight method.

flight method, as this method neglects the relatively large difference in speed of sound of the two media. In Fig. 8.15 the emitter location estimates from the time of flight method are distorted and shifted compared to the reference positions. It appears that the time of flight method causes a systematic error on the localization results. Acoustic ray tracing causes spatial errors as well, as it is a high frequency (or small wavelength) approximation of the acoustic pressure field. This is in contrast to our case, as we are using relatively low frequencies of maximum 300 kHz and, thus, we obtain long wavelengths of almost 5 mm in water or more than 6 mm in GL.

Despite the heterogeneity, the mean error of the FE-based localization is smaller than the mean error obtained in section 8.2.2. However, these two errors are difficult to compare, as in section 8.2.2 the TDOA is measured, and not the TOF. In the TOF case the absolute propagation times to differ-
ent receivers are known, while in the TDOA case only the arrival times at different receivers are determined. Due to the additional information in the TOF case, it is possible to obtain a smaller localization error, even though an interface between the receivers and the emitter is present. The results of the TDOA method in a heterogeneous media are shown in the following paragraphs.

Passive Localization in 2D

A rectangular arrangement of four receivers is used for the localization of the emitter in a 2D heterogeneous media. The emitter is positioned inside the silicone tube, which is surrounded by the receivers. The emitter is moved along a 2.5 mm grid in a square of 25 mm side length using the motorized micromanipulator. As in the 2D homogeneous media, the TDOA is measured between all six combinations of four receivers to pairs at 121 grid nodes. The area above the threshold $AF_{\text{max}} > 0.9$ corresponds to our emitter location estimate.

The spatial error $\varepsilon$ with the standard deviation $\sigma$ is shown in Fig. 8.16 for the measurements in a 2D heterogeneous media. As expected, the spatial error is higher than in the 2D homogeneous media with TOF measurements. The emitter location is determined with an accuracy of 1.03 mm, and a resolution of 600 $\mu$m. However, there are a few emitter locations with poor localization results, and spatial errors of up to $4.13 \pm 3.36$ mm. The poor results at these emitter locations are due to the dropped signal strength if the emitter is located in the heterogeneous media (see Fig. 8.8). Therefore, it is possible to select an incorrect signal for the time estimate in the cross-correlation and, thus, an error in the TDOA measurement can be obtained.

Passive Localization in 3D

For localization in a heterogeneous media in 3D the motorized micromanipulator was used to move the emitter inside the tube along a 2.5 mm grid in a cube of 25 mm side length. The TDOA from the emitter to the four receivers combined to six pairs is measured two times at 1331 grid nodes. This corresponds to the TDOA measurements between $2 \times 4$ receivers com-
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**Fig. 8.16:** Localization error for four receivers at a distance of $d_R = 32\,\lambda$ in a heterogeneous media.

![Localization error graph](image)

<table>
<thead>
<tr>
<th>Region</th>
<th>$\varepsilon_{\text{mean}}$ [mm]</th>
<th>$\sigma_{\text{mean}}$ [mm]</th>
<th>$\varepsilon_{\text{max}}$ [mm]</th>
<th>$\sigma_{\text{max}}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.70</td>
<td>1.10</td>
<td>32.85</td>
<td>4.91</td>
</tr>
<tr>
<td>B</td>
<td>1.71</td>
<td>1.09</td>
<td>38.36</td>
<td>4.94</td>
</tr>
<tr>
<td>C</td>
<td>1.69</td>
<td>1.09</td>
<td>35.87</td>
<td>4.89</td>
</tr>
<tr>
<td>D</td>
<td>1.71</td>
<td>1.14</td>
<td>5.54</td>
<td>4.26</td>
</tr>
</tbody>
</table>

**Tab. 8.5:** Mean and maximal values for the spatial error $\varepsilon$ and the standard deviation $\sigma$ for a cubic arrangement of $2 \times 4$ receivers in the 3D heterogeneous media.

The volume above the threshold $AF_{\text{max}} > 0.9$ is determined from the TDOA measurements and the center of mass of this volume corresponds to our emitter location estimate. This experiment is repeated four times, such that the emitter is located again in the regions A to D, which are defined in the previous section.

The spatial errors $\varepsilon \pm \sigma$ for the four regions are shown in Fig. 8.17 and summarized in Tab. 8.5. In all four regions the emitter location is usually determined with an accuracy of about $1.7\,\text{mm}$, and a resolution in the range of $1.1\,\text{mm}$. However, there are again a few emitter locations with poor localization results and spatial errors of up to $40\,\text{mm}$. As in 2D the poor results at these emitter locations are due to the dropped signal strength in the heterogeneous media.
8.3 Summary: Accuracy and Resolution

8.3.1 Homogeneous Media

The best passive localization results are achieved in a 2D homogeneous media, if the receivers are positioned at the corners of a square with $158.08 \pm 0.01$ mm side length. In this case we obtain a mean spatial error of $810 \pm 260 \mu$m. In 3D localization, the accuracy and resolution are affected due to an increased uncertainty in the receiver arrangement. Therefore, the mean spatial error is increased to $1250 \pm 550 \mu$m where the receivers are positioned at the corners of a cube with $158.08 \pm 0.25$ mm side length.
8.3.2 Heterogeneous Media

In a heterogeneous media we first used TOF measurements between four receivers to compare our FE-based algorithm with two conventional localization methods. The receivers are again at a distance of $158.08 \pm 0.01$ mm to each other. In this case we obtained a mean spatial error of $750 \pm 310 \mu m$, which is in the same range as the localization results in the 2D homogeneous media using TDOA measurements. In addition, the results with our method clearly exceed the results obtained with two conventional localization methods.

In the 2D heterogeneous media using passive localization with TDOA measurements, the emitter location is determined with an accuracy of $1.03$ mm, and a resolution of $600 \mu m$. In 3D a mean spatial error of $1.70 \pm 1.11$ mm is obtained. The receivers are again positioned in the corners of a square with $158.08 \pm 0.01$ mm side length, or in the corners of a cube with $158.08 \pm 0.25$ mm side length. An additional error source in the heterogeneous media is the possibility for an imprecise TDOA measurement, which can cause very high localization inaccuracies with position errors of more than $4$ mm in 2D and more than $4$ cm in 3D.
Chapter 9

Conclusion and Future Work

9.1 Conclusion

9.1.1 Wireless Emitter

We demonstrated for the first time acoustic emission in the kHz range using a wireless emitter based on the actuation principle of the WRMMA. The untethered resonator is small and comparatively simple to fabricate, as the device is made from soft magnetic material and does not require on-board energy storage and wiring. Its wireless nature considerably reduces the power and circuitry requirement of the device while increasing its range of applications.

Our experiments show good agreement with the theoretical model. We demonstrated the potential for passive localization independent of the pose of the device, as the pressure field from the wireless emitter is omnidirectional. The measured pressure field in different fluids correspond well with the pressure fields, which were calculated with the impulse response method.

In addition, the capability for wireless viscosity sensing is shown, as a significant frequency change of the acoustic signal in different fluids can be measured. From the change in the resonant frequency a change in the viscosity from S350 to GL was determined with an error of less than 5%.
9.1. CONCLUSION

9.1.2 Passive Localization

An inverse analysis using a method similar to SAFT is presented, which can be used to optimize receiver configurations for systems using TDOA and PS measurements for passive localization of a sound source. It is shown that the localization accuracy primarily depends on accurate time or phase measurements. The configuration and number of receivers effect the accuracy in a subordinate way, but strongly effect the resolution. Optimal receiver arrangements for passive localization applications are given under the assumptions of omnidirectionality, no dissipative attenuation, and no diffraction. The receiver number and arrangement for TDOA and PS measurements are investigated to achieve uniqueness of the localization results. In the optimal theoretical case it is demonstrated that a resolution of approximately 600µm can be expected at a frequency of 300kHz.

We developed and tested a new algorithm for passive localization of an ultrasound source within homogeneous and heterogeneous media. The localization technique utilizes TDOA measurements between different groups of receivers, and, thus, it does not need absolute TOF measurements. Furthermore, synchronization of the emitter with the receivers is not required. In addition, unlike most conventional methods, our localization technique accounts for reflection and refraction at tissue boundaries, and for wave interference, as the wave propagation through the media is simulated using FEM.

In homogeneous media an accuracy of approximately 800µm with a resolution of almost 250µm was obtained with a relatively low frequency of 300kHz in a 2D square area of about 16 cm side length. Conventional ultrasound systems require a frequency of more than 2 MHz to achieve a similar resolution [19]. A lower frequency is advantageous for the actuation of the wireless emitter, as the bandwidth of the wireless system is limited. In a cubic volume of 16 cm side length an accuracy of less than 1.3 mm with a resolution of less than 600µm was achieved. The decreased resolution and accuracy in the cubic volume is due to an increased uncertainty in the receiver arrangement in 3D.

In a heterogeneous media the absolute TOF instead of the TDOA was mea-
 CHAPTER 9. CONCLUSION AND FUTURE WORK

sured first, to compare our localization method with acoustic ray tracing and the time of flight method using trilateration with a mean value for the speed of sound. With our method an accuracy of 750 µm with a resolution of almost 300 µm was obtained in square area of about 16 cm side length. This is a spatial localization error of less than half of the errors derived with acoustic ray tracing and the time of flight method.

Using the TDOA measurements in heterogeneous media an accuracy of 1030 ± 600 µm was obtained in a square area of 16 cm side length. In a cubic volume of 16 cm side length an accuracy of 1.7 mm with a resolution of about 1.1 mm was achieved. However, errors in the TDOA measurements in heterogeneous media and, thus, misinterpretations of the maximum values of the cross-correlation can result in high localization errors of more than 4 mm in 2D and 4 cm in 3D.

9.2 Future Work

9.2.1 Wireless Emitter

Due to the frequency dependence of the localization resolution and accuracy, as well as the sensitivity in viscosity sensing, better results are obtained with higher resonant frequencies. Therefore, current work focuses on designing emitters with higher resonant frequencies and signal strengths to improve localization and sensing. This work aims for the replacement of the resonator material with other materials such as cobalt-nickel with improved magnetic properties. In addition, the design of the wireless emitter can be improved to obtain a higher resonant frequency. Instead of suspending the resonator on two gold beam springs, it could be suspended on a spring akin to a membrane, and thus the spring stiffness could be increased resulting in a faster oscillation.

There is a tradeoff between the desired excitation frequency and the achievable frequency for the wireless actuation of the emitter. The bandwidth of our current amplifier is limited to about 10 kHz. Therefore, we also need to improve our current amplifier to obtain a higher bandwidth, which can support the desired frequencies of up to 300 kHz.
9.2.2 Passive Localization

An inaccuracy in the receiver arrangement affects the localization accuracy and resolution. Therefore, it is important to position the receivers as accurately as possible. As our data acquisition system has only four channels, we have to reposition four receivers for the localization experiments in 3D, which results in a serious error in the receiver positions and, thus, in an additional uncertainty in the emitter localization. In the future, a new localization setup is required that supports at least eight autonomous channels, such that eight receivers can be permanently integrated in the system at well known positions.

We examine the localization of an emitter within a cylindrical first medium, which is surrounded by a second medium. This is a common configuration to depict a blood vessel surrounded by soft tissue. In the future work it is important to consider also other areas of the human body, where it can be important to localize a minimally invasive medical device, and to examine the wave propagation in these new situations.

Our FE analysis includes reflection, refraction, interference, and absorption of the acoustic waves in the media and at tissue boundaries. In our configuration of the heterogeneous media these are the predominant wave phenomena. In other arrangements it can be possible that diffraction must be considered in addition. It is important to further explore the wave phenomena, which occur at tissue boundaries, and to implement all of them in the FE simulation of the wave propagation.
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Appendix A

Time Measurement

A.1 Cross-Correlation

In signal processing, cross-correlation is used to detect similarities in two signals. The cross-correlation \([f \ast t](t)\) of two signals \(f(t)\) and \(g(t)\) is defined by \([127]\)

\[
[f \ast t](t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau. \quad (A.1.1)
\]

As an illustration for the cross-correlation a windowed sinusoidal tone-burst signal of six cycles is used, as shown in Fig. A.1. The signal \(f(t)\) is sent out at \(t = 0\) µs and the signal \(g(t)\) is received after \(t = 20\) µs with a phase shift of \(-90^\circ\). The cross-correlation coefficient \([f \ast t]\) of these two signals reaches a minimum at \(t = 20\) µs, while there are two maxima at 19.5 µs and 20.5 µs.

A.2 Hilbert Transform

The Hilbert transform is a linear operator that takes a function \(u(t)\), and produces a function \(\mathcal{H}[u(t)]\), with the same domain. The Hilbert transform can be imagined as the convolution of \(u(t)\) with the function \(h(t) = \frac{1}{\pi t}\). Because \(h(t)\) is not integrable the integrals defining the convolution do not converge. Explicitly, the Hilbert transform of a function or signal \(u(t)\) is given by

\[
\mathcal{H}[u(t)] = \frac{1}{\pi} \lim_{\epsilon \to 0} \int_{\epsilon}^{\infty} \frac{u(t + \tau) - u(t - \tau)}{\tau} d\tau. \quad (A.2.1)
\]
A.2. HILBERT TRANSFORM

Fig. A.1: Illustration for the cross-correlation and the Hilbert transform.

Taking the absolute value of the Hilbert transform can be used to determine the envelope of the signal $u(t)$.

In the last plot of Fig. A.1 the envelope of the cross-correlation is shown. The maximum value of the envelope corresponds to the arrival time $t = 20 \mu s$ of the signal $g(t)$. 
Appendix B

Circuitry and Framework Plans

B.1 Amplifier/ Filter Circuitry

B.1.1 Current Amplifier

Fig. B.1: Control circuit for the current amplifier.
B.1. AMPLIFIER/ FILTER CIRCUITRY

Fig. B.2: Bridge circuit for the current amplifier.

B.1.2 Bandpass Filter and Amplifier

Fig. B.3: Preamplifier and filter circuit.
B.2 Framework Plans

Fig. B.4: CAD drawing of the wireless emitter with exact dimensions.
Curriculum Vitæ

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