Geometric and Dynamic Evaluation and Optimization of Machining Centers

A dissertation submitted to the
ETH ZURICH

for the degree of
Dr. sc. ETH Zürich

presented by
SERGIO BOSSONI
Dipl. Masch.-Ing. ETH
born February 11th 1976
citizen of Italy and Switzerland

accepted on the recommendation of
Prof. Dr. K. Wegener, examiner
Prof. Dr. P. Hora, co-examiner
Dr. W. Knapp, co-examiner
Dr. S. Weikert, co-examiner

2009
Acknowledgements

This dissertation was realized during my time at the Institute for Machine Tools and Manufacturing (IWF) of the ETH Zurich. Many people contributed to my work. Their support ultimately made this dissertation possible.

First of all, I would like to thank Prof. Dr. Wegener, head of the IWF and supervisor of this thesis for his generous support, for his trust, for his constructive questions and suggestions, for his openness and for his patience.

Special gratitude I owe to my co-examiners Dr. Wolfgang Knapp and Dr. Sascha Weikert. Dr. Sascha Weikert was my group leader at the IWF and has supported me since my master thesis. I owe him most of my knowledge in the dynamic analysis of machine tools and especially in rigid body dynamics. Dr. Wolfgang Knapp is the leader of the metrology group at the IWF. The knowledge that I needed in the field of metrology to write the first part of this thesis I owe to his patient explanations. I want to thank both of them for their extraordinary personal effort and their patience.

I am also grateful to Prof. Dr. Hora for the inputs and for acting as a co-examiner.

Many more colleagues at the IWF contributed to this work: Dr. Fredy Kuster with his general support and his knowledge and great experience in experimental modal analysis. Dr. Bernhard Bringmann with his help for the geometrical simulations; Dr. Michael Hadorn with his inputs from the control point of view; Thomas Liebrich with his help with measurements; Josef Cupic and Josias Wacker with their help in the early test piece development as students; Nicolas Jochum, Martin Suter, Rolf Schroeter, Albert Weber and Marcel Schmid with their help with machining; Pascal Maglie with his knowledge in Finite Element Modeling; Günter Graf with his help with illustrations and my office mate Markus Steinlin that was always available for help. I would also like to thank my other colleagues and friends of the IWF for the great time passed together at the institute. In particular I want to thank, Angelo Gil Boeira, Sherline Wunder, Fabio Wagner Pinto, Guilherme Vargas, Michael Gull, Sascha Jaumann, Christian Jäger, Josef Mayr, Jérémie Monnin, Zoltan Sarosi, Marije van der Klis, Ewa Grob, Marianne Kästli and Roman Glaus.
I dedicate this work to my family. To my wife for her patience and all her support. To my parents and my sister who have always supported and encouraged me.

Sergio Bossoni
May 2009
# Contents

Symbols and abbreviations .......................................................... X

Abstract ......................................................................................... XIII

Zusammenfassung .............................................................................. XV

1 Introduction ................................................................................ 1
  1.1 Outline of the thesis .............................................................. 1
    1.1.1 Part I ........................................................................ 1
    1.1.2 Part II ...................................................................... 2

2 State of the art in evaluation of the geometric properties of 5-axis machining centers ............................................... 4
  2.1 Workpieces ........................................................................... 4
  2.2 Measuring techniques ............................................................ 8
  2.3 Current research ................................................................... 10
  2.4 Historic background of 5-axis machine tools ....................... 10
  2.5 Deficiencies of the state of the art ......................................... 11

3 Test piece for testing simultaneous 5-axis machining ........... 13
  3.1 Goals and requirements of the test piece .............................. 13
  3.2 Geometry of the test piece ..................................................... 14
  3.3 Calculation of the tool path ................................................... 15
    3.3.1 Analyzed machine setups ............................................. 15
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3.2</td>
<td>Parameters of a right circular cone</td>
<td>16</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Parameters of an inclined right circular cone</td>
<td>18</td>
</tr>
<tr>
<td>3.3.4</td>
<td>General transformation for 5-axis machines</td>
<td>19</td>
</tr>
<tr>
<td>3.3.5</td>
<td>Rotary machine axes</td>
<td>20</td>
</tr>
<tr>
<td>3.3.6</td>
<td>Linear machine axes</td>
<td>23</td>
</tr>
<tr>
<td>3.4</td>
<td>Interpretation of the movements of the rotary axes</td>
<td>31</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Cone angle $\alpha$ larger than inclination angle $\beta$</td>
<td>32</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Cone angle $\alpha$ smaller than inclination angle $\beta$</td>
<td>33</td>
</tr>
<tr>
<td>3.4.3</td>
<td>Cone angle $\alpha$ equal to inclination angle $\beta$</td>
<td>37</td>
</tr>
<tr>
<td>3.4.4</td>
<td>Interpretation of the velocities of the rotary axes for constant feed $f$</td>
<td>40</td>
</tr>
<tr>
<td>3.5</td>
<td>Interpretation of the movements of the linear machine axes</td>
<td>43</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Machines with both rotary axes on the tool side</td>
<td>43</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Machines with rotary axes on the tool and on the workpiece side</td>
<td>45</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Machines with both rotary axes on the workpiece side</td>
<td>48</td>
</tr>
<tr>
<td>3.6</td>
<td>Proposed test piece parameters for specific machine designs</td>
<td>49</td>
</tr>
<tr>
<td>3.6.1</td>
<td>Recommended angular parameters</td>
<td>49</td>
</tr>
<tr>
<td>3.6.2</td>
<td>Machines with both rotary axes on the tool side</td>
<td>49</td>
</tr>
<tr>
<td>3.6.3</td>
<td>Machines with rotary axes on the tool and on the workpiece side</td>
<td>50</td>
</tr>
<tr>
<td>3.6.4</td>
<td>Machines with both rotary axes on the workpiece side</td>
<td>51</td>
</tr>
<tr>
<td>3.7</td>
<td>Modeling of location errors</td>
<td>62</td>
</tr>
<tr>
<td>3.7.1</td>
<td>Orientation errors</td>
<td>62</td>
</tr>
<tr>
<td>3.7.2</td>
<td>Position errors</td>
<td>62</td>
</tr>
<tr>
<td>3.7.3</td>
<td>Modeling of orientation errors</td>
<td>63</td>
</tr>
<tr>
<td>3.8</td>
<td>Modeling of component errors</td>
<td>63</td>
</tr>
<tr>
<td>3.8.1</td>
<td>Straightness error motions</td>
<td>63</td>
</tr>
<tr>
<td>3.8.2</td>
<td>Positioning errors</td>
<td>66</td>
</tr>
<tr>
<td>3.8.3</td>
<td>Tilt error motions</td>
<td>66</td>
</tr>
<tr>
<td>3.8.4</td>
<td>Radial error motions of the rotary axes</td>
<td>70</td>
</tr>
<tr>
<td>3.8.5</td>
<td>Axial error motions of the rotary axes</td>
<td>71</td>
</tr>
</tbody>
</table>
3.9 Theoretical influence of geometric machine errors on a workpiece ........ 72
3.10 Effects of single geometric machine errors on the cone form ............ 72
  3.10.1 Components of the normal direction ............................. 73
  3.10.2 Radial representation of the position of the machine axes ........ 74
  3.10.3 Resulting cone forms ........................................... 74
  3.10.4 Typical cone form deviations .................................... 79
  3.10.5 Geometric errors with negligible effects on an end milled cone form 80
  3.10.6 Summary of results of errors with an influence on an end milled cone surface ........................................... 81
3.11 Effects of combinations of geometric machine errors on the cone form ... 83
3.12 Verification of the presented calculations and assumptions ............... 85
3.13 Summary ................................................................. 95

4 Conclusion of part I ...................................................... 98

5 State of the art in evaluation of the dynamic behavior of machining centers 100
  5.1 Experimental evaluation of the dynamic behavior of machining centers . 100
  5.2 Finite Element Method (FEM) ...................................... 101
  5.3 Rigid Body Simulation ................................................ 102
  5.4 Combination of Rigid Body and Finite Element Method .................. 103
  5.5 Uncertainty of the stiffness and damping parameters ...................... 103
  5.6 Simulation of movements .............................................. 104
  5.7 Axes Construction Kit (ACK) ...................................... 104
  5.8 Simulation in the early development stage ................................ 105
  5.9 Deficiencies of the state of the art .................................. 106

6 Exemplary evaluation of dynamic behavior of a machining center ......... 108
  6.1 Modeling of the machining center .................................... 108
    6.1.1 Bodies .......................................................... 108
6.1.2 Connecting elements ........................................... 110

6.2 Discussion and comparison of calculation and measurement results .... 114
6.2.1 Frequency-Response Function (FRF) .............................. 114
6.2.2 Mode shapes ..................................................... 117
6.2.3 Cross-talk in FRF ............................................... 118

6.3 Summary .................................................................. 124

7 Relative evaluation of alternative configurations of a dynamic machine tool using rigid body simulation 125

7.1 Machine configurations .............................................. 125
7.1.1 Original version .................................................. 125
7.1.2 Alternative configuration ....................................... 126

7.2 Flexible bodies ....................................................... 127

7.3 Dynamic load case ................................................... 127

7.4 Visual analysis of the effect of accelerations ....................... 128
7.4.1 Acceleration in X-direction .................................... 128
7.4.2 Acceleration in Y-direction .................................... 129
7.4.3 Acceleration in Z-direction .................................... 131

7.5 Numerical results .................................................... 132
7.5.1 Position 1 ............................................................ 132
7.5.2 Influence of the axes position ................................. 134

7.6 Schematic comparison ............................................... 136
7.7 Summary .................................................................. 137

8 Conclusion of part II .................................................... 139

Appendix ..................................................................... 142

A Practical aspects of cutting a Test Piece on a specific machine type 142
A.1 Test piece fixture ..................................................... 142
A.2 Top and bottom position ........................................... 143

VIII
A.3 Manufacturing process ........................................... 144
  A.3.1 Approach ..................................................... 144
  A.3.2 Further geometric elements ............................... 144
  A.3.3 Feed rate ..................................................... 145
  A.3.4 Stock allowance ............................................ 145
  A.3.5 Tool ............................................................ 146
A.4 Measurement of the test piece ..................................... 146
  A.4.1 Workpiece coordinate system ............................... 146
  A.4.2 Different aspects of measuring devices ................. 147
A.5 Encountered difficulties for face milling ............................ 148
  A.5.1 End mills in CAM systems .................................. 149
  A.5.2 Hollow cut of end mills .................................... 149
  A.5.3 Approach ....................................................... 150
  A.5.4 Tool offset .................................................... 152
A.6 Concluded suggestions for manufacturing ......................... 153

Bibliography .......................... 155

Curriculum Vitae .................. 166
## Symbols and abbreviations

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>cone angle (half apex angle of the cone)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>inclination angle</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>positioning angle of the inclined cone</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>radial angle of the cone</td>
</tr>
<tr>
<td>$\sum_i$</td>
<td>coordinate system $i$ (frame $\sum_i$)</td>
</tr>
<tr>
<td>$\kappa'_r$</td>
<td>minor cutting edge angle</td>
</tr>
<tr>
<td>A</td>
<td>apex of the cone</td>
</tr>
<tr>
<td>B</td>
<td>angle of the tilting axis B</td>
</tr>
<tr>
<td>$\Delta B$</td>
<td>range of the angle $B$ (total angular movement)</td>
</tr>
<tr>
<td>C</td>
<td>angle of the rotary table C</td>
</tr>
<tr>
<td>$\Delta C$</td>
<td>range of the angle $C$ (total angular movement)</td>
</tr>
<tr>
<td>d</td>
<td>physical damping</td>
</tr>
<tr>
<td>f</td>
<td>feed during the manufacturing of the cone geometry</td>
</tr>
<tr>
<td>$g_n$</td>
<td>standard acceleration due to gravity</td>
</tr>
<tr>
<td>h</td>
<td>height of the cone</td>
</tr>
<tr>
<td>$H_x/F_y$</td>
<td>dynamic compliance in X-direction due to a force $F_y$ in Y-direction</td>
</tr>
<tr>
<td>$H_x/\ddot{y}$</td>
<td>dynamic compliance in X-direction due to acceleration $\ddot{y}$ in Y-direction</td>
</tr>
<tr>
<td>k</td>
<td>stiffness</td>
</tr>
<tr>
<td>l</td>
<td>distance from the TCP to the crossing $Q$ of the two rotary axes</td>
</tr>
<tr>
<td>M</td>
<td>center of the circle formed by the points $Q$</td>
</tr>
<tr>
<td>$n_x$</td>
<td>X-component of the normal direction of the cone surface</td>
</tr>
<tr>
<td>O</td>
<td>center of the base of the cone</td>
</tr>
<tr>
<td>(center of tool path in the workpiece coordinate system)</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>point on the perimeter of the base of the cone</td>
</tr>
<tr>
<td>$P_0$</td>
<td>singular position</td>
</tr>
</tbody>
</table>
$iP$  
point $P$ in frame $∑_i$

$i p_x$  
X-component of $iP$

$Q$  
crossing of the two rotary axes

$r$  
radius of the base of the cone

$r_Q$  
radius of the circle formed by the points $Q$

$r_{TCP}$  
radius of the path of the TCP in the frame of the workpiece

$i R_j$  
Rotation Matrix describing the rotational part of the 
relative displacement of frame $∑_j$ from frame $∑_i$

$R_β$  
Rotation Matrix describing the influence of the inclination angle $β$

$R_γ$  
Rotation Matrix describing the influence of the positioning angle $γ$

$R_{table}$  
Rotation Matrix describing the rotational displacement 
due to the rotation of the rotary table

$R_{tilt}$  
Rotation Matrix describing the rotational displacement 
due to the rotation of the tilting axis

$s$  
length of the slant of the cone

$X_i$  
X-direction of frame $∑_i$

$X_M$  
Machine axis $X$

$X_{M_{min}}$  
minimal value of the X-axis of the machine 
throughout the entire movement

$X_{M_{max}}$  
maximal value of the X-axis of the machine 
throughout the entire movement

$ΔX_M$  
nominal length of total movement of the X-axis

**Abbreviations**

ACK  Axes Construction Kit

AIA  Aerospace Industries Association

ASME  American Society of Mechanical Engineers

CMM  Coordinate Measuring Machine

DBB  Double Ball Bar (also known as “telescopic magnetic ball-bar”)

EMA  Experimental Modal Analysis

FEM  Finite Element Method

FRF  Frequency Response Function

HSC  High Speed Cutting

ISO  International Organization of Standardization

IWF  Institut für Werkzeugmaschinen und Fertigung 
\(\text{(Institute of Machine Tools and Manufacturing)}\)

MIT  Massachusetts Institute of Technology
NAS National Aerospace Standards
RHR Roughness Height Reading (unit micro inches)
TCP Tool Center Point

**Geometric machine errors** (examples)

- $A0B$ Orientation error in $A$-direction of the $B$-axis
- $X0C$ Location error in $X$-direction of the $C$-Axis
- $EXX_{lin}$ Component error in $X$-direction of the motion in $X$-direction with linear dependency of the $X$-position (linear positioning error)
- $EYY_{hyst}$ Hysteresis of the positioning movement in $Y$-direction
- $EXY_{1h}$ Component error in $X$-direction of the motion in $Y$-direction modeled as a first harmonic (straightness error)
- $EXB_{2h}$ Component error in $X$-direction of the motion in $B$-direction modeled as a second harmonic (radial error motion)
Abstract

The geometric and dynamic properties of machining centers are both very important for productive precision manufacturing.

The first part of this thesis addresses the fact that no standardized test piece is available for testing of simultaneous 5-axis machining, which is directly applicable to different machine tool designs.

The cone geometry is derived to best meet the defined requirements of an ideal 5-axis test piece. How the resulting movements of the machine axes depend on the parameters of the test piece is derived in an analytical way and interpreted for practical usage. The resulting movements of the rotary axes are shown to be the same for any machine setup with three orthogonal linear axes and two orthogonal rotary axes (the nominal rotary axes are assumed to cross in one point). The movements of the linear axes on the other hand depend on whether the rotary axes are on the tool or on the workpiece side.

Parameters of proposed test pieces are derived for the three types of machine setups. A specific example of a test piece setup is presented for a characteristic machining center with a tilting rotary table setup. The calculation of the influence of geometric machine errors on the form error of the test piece geometry is explained and the resulting influences are interpreted for the presented example.

For the interpretation of the influence of the test piece parameters on the movements of the machine axes and for the analysis and interpretation of the influences of geometric errors of the test piece geometry novel types of visualization are presented. The entire analysis is also applicable for direct measurements (i.e. without test pieces) of the errors of the machine movements.

In the second part of this thesis the evaluation and optimization of the dynamic properties of machining centers is addressed for the early development phase.

The modeling process of a machining center with the rigid body approach is demonstrated with illustrations from a custom simulation environment. Through a comparison with
experimental results the calculation method is verified and the possibilities and limitations of the method are shown.

Evaluations of the dynamic reactions of machining centers to external forces and to inertial loads are both presented. The importance and possibilities of an analysis of the reactions of the displacements of a machine not only parallel but also orthogonal to the direction of the excitation (cross-talk) is demonstrated.

For the modeling process in the early development phase different recommendations can be derived. In the presented comparison of two alternative machine setups, the possibilities and advantages of being able to calculate the dynamic properties with a very efficient rigid body model become clear. Among other things, it allows to analyze the calculated properties in multiple positions of the tool center point throughout the working range of a machine. The analysis of the distribution with the presented form of visualization is shown to give important additional information for the evaluation process.

The general conclusion of the second part of this thesis is that a rigid body analysis is very well suited for the analysis of the dynamic properties of machining centers when looking at the machine setup and not at local details of components. This is especially the case in the early development phase.

This work was also published as “Fortschritt-Berichte VDI, Reihe 2, Nr. 672” [1].
Zusammenfassung

Sowohl geometrische als auch dynamische Eigenschaften von Werkzeugmaschinen sind sehr wichtig für produktive Präzisionsfertigung.

Im ersten Teil dieser Arbeit wird die Tatsache angegangen, dass bis zum heutigen Zeitpunkt kein normiertes Prüfwerkstück für simultane 5-Achs Fräsbearbeitung existiert, das für verschiedene Maschinenarten direkt anwendbar ist.

Um den Anforderungen an ein ideales 5-Achs Prüfwerkstück gerecht zu werden wird die Kegelform als geeignetste Geometrie hergeleitet. Was für Achsbewegungen der Maschine dafür notwendig sind wird analytisch berechnet und für die praktische Anwendung interpretiert. Es kann gezeigt werden, dass die erforderlichen Bewegungen der Drehachsen für alle Maschinenarten mit drei orthogonalen Linearachsen und zwei sich nominell kreuzenden, orthogonalen Drehachsen gleich sind. Die erforderlichen Bewegungen der Linearachsen anderseits sind abhängig davon ob die Rotationsachsen Werkzeug- oder Werkstückseitig angeordnet sind.

Die vorgeschlagenen Prüfwerkstückparameter werden für die drei Maschinentypen hergeleitet. Ein konkretes Beispiel eines Prüfwerkstückaufbaus wird für eine Werkzeugmaschine mit Dreh-/Schwenktisch Kinematik dargestellt. Die Bestimmung der Einflüsse geometrischer Maschinenfehler auf die Formfehler des Prüfwerkstückes wird erläutert und die resultierenden Zusammenhänge für das gewählte Beispiel interpretiert.

Für die Interpretation der Einflüsse verschiedener Parameter des Prüfwerkstückes auf die erforderliche Bewegung der Maschinenachsen und für die Interpretation der Einflüsse geometrischer Maschinenfehler auf die Formfehler des Prüfwerkstückes werden neuartige Darstellungsformen präsentiert. Die Überlegungen zum Prüfwerkstück sind auch für direkte Messungen (d.h. ohne Prüfwerkstück) auf der Werkzeugmaschine anwendbar.

Im zweiten Teil der Arbeit wird die Beurteilung und Optimierung der dynamischen Eigenschaften von Werkzeugmaschinen in der frühen Entwicklungsphase behandelt.

Der Modellierungsprozess von Werkzeugmaschinen mittels Starrkörperansatz mit einer e-
genen Simulationsumgebung wird dargestellt. Anhand des Vergleichs mit Resultaten einer experimentellen Modalanalyse werden die Resultate der Starrkörperberechnungen verifiziert sowie die Möglichkeiten und Grenzen der Methode gezeigt.

Auswertungen der dynamischen Reaktionen von Werkzeugmaschinen auf externe Kräfte und Beschleunigungslasten werden behandelt. Die Bedeutung und die Möglichkeiten einer zusätzlichen Analyse der Reaktionen in die Richtungen quer zur Anregungsrichtung (cross-talk) wird verdeutlicht.


Die allgemeine Schlussfolgerung des zweiten Teiles dieser Arbeit ist, dass der Starrkörperansatz sehr gut geeignet ist für die Beurteilung der dynamischen Eigenschaften von Werkzeugmaschinen mit einem Fokus auf den Gesamtaufbau und nicht auf die Details einzelner Komponenten. Dies ist vor allem in der frühen Entwicklungsphase der Fall.

Diese Arbeit wurde auch publiziert als “Fortschritt-Berichte VDI, Reihe 2, Nr. 672” [1].
Chapter 1

Introduction

More and more 5-axis machine tools are used in industry, but today a standardized test piece for 5-axis machine tools, directly applicable to different machine tool designs, is still missing. This deficiency is addressed in the first of two main parts of this dissertation (chapters 2–4).

The accuracy of efficiently produced workpieces depends not only on the geometric properties of the machining center (addressed in the first part of this dissertation) but also on the stability and accuracy of a dynamic movement of the tool center point (TCP) of a machine relative to the workpiece. The evaluation and optimization of the responsible dynamic properties needs to be started in the early stage of machine development because they depend mostly on the kinematic setup of the machine. How this can be done is addressed in the second part of this thesis (chapters 5–8).

1.1 Outline of the thesis

1.1.1 Part I

Part one of two main parts of this thesis deals with one aspect of the evaluation of geometric properties of 5-axis machining centers.

In chapter 2 the state of the art in the evaluation of the geometric properties of 5-axis machining centers is discussed. Existing test pieces are described in section 2.1 and available measuring techniques are analyzed with the focus of measurement of the accuracy of simultaneous movements of machine axes (section 2.2). The current research is discussed in section 2.3 and a brief overview of the historic background of 5-axis machining centers is given. Finally, in section 2.5 the deficiencies of the state of the art are presented.
In chapter 3 different aspects of a test piece for testing of simultaneous 5-axis machining is presented. The goals and requirements of an ideal test piece are analyzed in the first section 3.1 and in section 3.2 the most appropriate test piece geometry is derived from the discussed requirements.

The calculation of the tool path required for a cone geometry is derived in section 3.3 for any machine setup with three orthogonal linear axes and two orthogonal rotary axes (the nominal rotary axes are assumed to cross in one point). The general parameters of the cone geometry of the required tool path are described in sections 3.3.2 and 3.3.3. The resulting movements are derived in section 3.3.5 for the rotary axes and in section 3.3.6 for the linear axes. An interpretation of the movements follows in section 3.4 for the rotary axes and in section 3.5 for the linear axes.

In section 3.6, test piece parameters are derived for different machine designs. For machines with tilting rotary table setup one specific machine is taken as an example and the resulting machine movements are analyzed more in detail for the proposed test piece in section 3.6.4.

In the sections 3.7 to 3.11 the influence of geometric errors on the form errors of the test piece are analyzed. How the different geometric machine errors of the machine are modeled is described in section 3.7 for the location errors and in section 3.8 for the component errors. Section 3.9 describes the theoretical influence of geometric machine errors on a workpiece and how it can be calculated. The effect of single geometric errors on the cone form is analyzed in section 3.10 for one specific example. The effects of combinations of geometric machine errors on the cone form follows in section 3.11 and in section 3.12 results of such a Monte Carlo simulation are compared with measurement results as a verification.

Section 3.13 summarizes the content of chapter 3 and the conclusions that can be drawn are described in chapter 4.

In the appendix A, some practical aspects of cutting a test piece on a specific machine type are described.

1.1.2 Part II

Part two, dealing with the evaluation and optimization of the dynamic properties of machining centers, starts with a description of the state of the art in chapter 5. It describes possibilities of the finite element method (section 5.2) and of rigid body simulations (section 5.3). Possibilities of combinations of both methods are addressed in section 5.4. The uncertainties of the stiffness and damping parameters are discussed in section 5.5 with a special focus on the connecting elements and on the setup of the machines. Ways of simulating the path accuracy of movements of machining centers are described in section 5.6.
All dynamic evaluations presented in this thesis are done with a rigid body simulation environment. This so called Axis Construction Kit (ACK) is introduced in section 5.7. The different aspects of evaluation of dynamic properties of machining centers in the early development stage are presented in section 5.8 and finally in section 5.9 the deficiencies of the state of the art are described.

In chapter 6 the evaluation of the dynamic behavior of a machining center is presented. Results of calculations with the ACK are compared with results from an experimental modal analysis (EMA) as verification of the simulation environment. In section 6.1, the process of modeling a machining center with the ACK is shown. The focus is on how the rigid body model is defined and how the different connecting elements are modeled. In section 6.2, an evaluation procedure of the dynamic properties is shown. The cross-talk in the frequency response function (FRF) is analyzed in section 6.2.3. Section 6.3 summarizes the content of chapter 6 and the conclusions that can be drawn from such an analysis are described in chapter 8.

In chapter 7 a relative evaluation of two alternative configurations of a dynamic machine tool, using rigid body simulation, is described. In section 7.1, the two configurations to be compared are described. How flexible bodies can be integrated in rigid body models is shown in section 7.2. The description of the dynamic load case used for the comparison is given in section 7.3. The so called dynamic error budgeting can be used to analyze errors due to inertial loads. In section 7.4, the effects of accelerations are analyzed in a visual way. The comparison of the visualizations allows recognizing the main sources of errors. In order to have a quantitative comparison of the resulting errors at the TCP, numerical values are compared in section 7.5. In a first step of the comparison, only one critical position of the TCP is analyzed (section 7.5.1). In a further step, the advantages of analyzing multiple positions of the TCP throughout the working envelope of the machine are presented (section 7.5.2). Finally, in section 7.6 a schematic comparison of the two setups is shown that explains the main reason of the different dynamic properties of the two configurations. Section 7.7 summarizes the content of chapter 7 and the conclusions that can be drawn from this analysis are described in chapter 8.
Chapter 2

State of the art in evaluation of the geometric properties of 5-axis machining centers

2.1 Workpieces

In machine tool industry it is common to have at least one test piece manufactured when buying a new machine tool to prove the accuracy and performance of the machine. This type of test piece is not used to calibrate the machine tool but rather for acceptance purpose and for periodic re-verification.

In this section, test pieces standardized for milling purposes that can be used for acceptance tests are discussed shortly.

The International Organization of Standardization (ISO) has published an international standard describing test conditions for machining centers. Part seven of ISO 10791 [2] defines the accuracy of finished test pieces. Two types of test pieces are considered in this standard, each of them in two sizes. The first one is a positioning and contouring test piece (see figure 2.1 for the top view of the small test piece) whereas the second one is a face milling test piece. For checking of the geometric properties of machining centers, the first one is used. The maximum numbers of axes that need to act simultaneously to cut the test piece are two.

The German NC-Gesellschaft published a recommendation for workpieces for high speed cutting (HSC) [3]. The described test piece includes the elements described in the previously mentioned ISO standard [2]. The novelties in the NCG recommendation are features to test the influence of different feed rates of the machine tool and the cutting with three
2.1 Workpieces

Figure 2.1: Top view of the small size contouring test piece of ISO 10791-7 [2].

simultaneous translational axes. A drawing of the smaller one of two versions is shown in figure 2.2. The elements requiring a simultaneous movement of three axes are cut using a ball nose end mill. At the trade fair EMO Hannover 2005 the NC-Gesellschaft presented another recommendation, describing a test piece to be machined with up to five simultaneous axes [4]. The interpretation of the quality of the test piece is not by means of quantitative measurements and comparison with tolerances as in the other described test pieces but rather by qualitative - mostly optical - interpretation of features of various form elements. The test piece is distributed in a package including different interpretation aids and a detailed description of how the different features can be analyzed. Furthermore, suggestions are included on the causes of possible errors. The usage of this test piece is well suited for the periodic testing of machines by an expert and for tests after a crash. Nevertheless, it does not represent a test piece giving comparable and quantitative results. A picture of such a test piece is shown in figure 2.3.

The Aerospace Industries Association of America, Inc. (AIA) published a National Aerospace Standard (NAS) describing uniform cutting tests about 40 years ago [5]. In this specification, various cutting tests are described that were intended for use in evaluating machines procured for the aerospace manufacturing industry. Some cutting tests include elements similar to the ones described by ISO [2]. In addition to that, and of special interest for this work, it includes a profile cone frustum cutting test that can be performed
2. State of the art

Figure 2.2: Test piece for high speed cutting recommended by the German NC-Gesellschaft [3].

Figure 2.3: 5-axis test piece recommended by the German NC-Gesellschaft [4].

to verify accuracies of five combined axes of motion (three linear and two rotary) for 5-axis machines under finishing loads (diagram shown in figure 2.4). This standard will be discussed more in detail in section 2.5.

The Boeing Company has its own Equipment Design and Asset Acquisition Standards [6], which includes the profile cone frustum cutting test from NAS 979 [5]. The description of the test is not enhanced but the typical tolerance requirements are defined ten times
2.1 Workpieces

Figure 2.4: Diagram from profile - cone frustum test shown on sheet 34 of NAS 979 [5].

smaller than in the NAS 979. No tolerance is specified for the surface as it is in NAS 979 where a value is stated for the maximum finish in RHR, which stands for “Roughness Height Reading” and has the unit of micro inches (RHR of 35 micro inches corresponds to a total height of the profile $R_t$ [7] of 0.9µm). The defined parameters are summarized in table 2.1.

Table 2.1: Parameters of the cone frustum test described in NAS 979 [5].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>feed rate</td>
<td>m/min</td>
<td>0.635</td>
</tr>
<tr>
<td>spindle speed</td>
<td>1/min</td>
<td>1800</td>
</tr>
<tr>
<td>axial depth of cut</td>
<td>mm</td>
<td>25.4</td>
</tr>
<tr>
<td>finish maximum $RHR$</td>
<td>micro inches</td>
<td>35</td>
</tr>
<tr>
<td>roundness error</td>
<td>µm</td>
<td>100</td>
</tr>
<tr>
<td>concentricity</td>
<td>µm</td>
<td>100</td>
</tr>
<tr>
<td>angular accuracy</td>
<td>µm/m</td>
<td>600</td>
</tr>
</tbody>
</table>

The American Society of Mechanical Engineers (ASME) also takes reference to the NAS 979 for simultaneous 5-axis machined test pieces in one of their publications [8]. It does not add anything else to the testing of simultaneous 5-axis machining with test pieces.
2.2 Measuring techniques

To measure simultaneous movements of at least two linear and one rotary axis often telescoping magnetic ball-bars are used (also called “double ball bar” (DBB)).

Quite often geometric errors are identified through simulation and measurements using telescoping magnetic ball-bars \[9, 10, 11, 12\]. The principle of the ball-bar method was presented already in 1982 by Bryan \[13\].

In \[14\] a measurement setup is presented using three laser ball bars. Recent approaches use more compact measurement devices able to measure displacements in three directions simultaneously \[15, 16, 17, 18, 19, 20\]. The main idea is to measure the relative displacement of a precision sphere with three individual probes simultaneously to get the values in all three directions. This type of test has been included in the most recent draft version of ISO 230-1 \[21\], which is under revision. With this system, only one measurement is necessary in order to measure radial, tangential and axial relative displacements with respect to the movement of the rotary axis.

A detailed review of identification and calibration methods using different measuring devices is given in \[22\].

Kinematic tests for 5-axis machining centers analyzing a simultaneous movement of all five axes have not been published in any ISO Standard yet. A committee of ISO is elaborating a draft for ISO 10791-6 \[23\]. In the annex of this document, kinematic tests for “circular interpolation motion by simultaneous five-axis control” are being worked out for machines with three different types of setups. These are:

- Machines with a double pivot spindle head (figure 2.5).
- Machines with a tilting rotary table (figure 2.6).
- Machines with a swivel head and a rotary table (figure 2.7).

The main idea is to check the circularity of the circular path generated by simultaneous 5-axis movement (three linear, two rotary) of the machine. How this would look like on each of the three types of machines is shown in figures 2.5, 2.6 and 2.7. The path of the spindle axis relative to the workpiece mounting corresponds to a cone with an apex angle of the cone of 30°.

The limiting of the apex angle to only 30° is due to the configuration of the measurement. Most DBB are designed for measurements where the tool axis is perpendicular to the axis of the telescopic bar. In order to measure the radial deviation of the circular path of the described simultaneous 5-axis movement the angle between the telescopic bar and the tool
2.2 Measuring techniques

Figure 2.5: Diagram from working draft ISO 10791-6(WD) [23] showing the setup for kinematic test with simultaneous interpolation of five axes for machines with a double pivot spindle head.

Figure 2.6: Diagram from working draft ISO 10791-6(WD) [23] showing the setup for kinematic test with simultaneous interpolation of five axes for machines with a tilting rotary table.

Figure 2.7: Diagram from working draft ISO 10791-6(WD) [23] showing the setup for kinematic test with simultaneous interpolation of five axes for machines with a swivel head and rotary table.
axis has to be 90° minus half of the apex angle of the cone. In the case in discussion at ISO, it would be 75°. Much smaller angles can not be achieved with most standard DBB measurement devices.

If on the other hand instead of measuring the radial deviation, the normal deviation of the cone surface is measured, the angle between the spindle axis and the telescopic bar is always 90° and therefore the measuring device would not be a limiting factor. This approach is also under discussion in the ISO working group for testing machining centers (ISO/TC 39/SC 2/WG 3).

See chapter 4 starting on page 98 for conclusions on direct measurement of the 5-axis movement for a cone geometry.

2.3 Current research

Currently a great part of the research published in the field of evaluation of simultaneous 5-axis movements of machine tools comes from Japan. The idea of the cone frustum from [5] is analyzed and most tests are described using a DBB.

For machining centers with a double pivot head, the identification of geometric errors is addressed in [12, 24]. For machining centers with a tilting rotary table, similar analysis is described in [10, 11, 25].

In [26, 27] also results of manufactured test pieces are described to compare with the DBB measurements of simultaneous 5-axis movements on machining centers with a tilting rotary table. Very recently [28] presented experimental verifications of predictions and compensations of geometric machine errors using a cone frustum which was manufactured with simultaneous 5-axis movements of the machining center.

At the Institute of Machine Tools and Manufacturing (IWF) of the ETH Zurich, Switzerland, research on appropriate test piece geometries has been continued. The manufacturing of a cone with the cone axis not parallel to any axis of the machining center has been evaluated since 2006 [29, 30, 31, 32] and experiences are described in the next chapter.

2.4 Historic background of 5-axis machine tools

The aim of this section is to give some background information to the motives of the cone frustum cutting test described by the AIA in 1969 [5].

In the German speaking community, the reference work for questions concerning NC-
2.5 Deficiencies of the state of the art

More and more 5-axis machine tools are used in industry, but today a standardized test piece for a 5-axis machine tool, directly applicable to different machine tool designs, is still missing.

The profile cone frustum cutting test described by the NAS [5] already presented the idea of using the cone geometry as a 5-axis test but it is not directly applicable to different machine tool designs. The standard specifies the workpiece location as follows: “workpiece to be centrally located on work surface, except that workpiece to spindle ratio must be such that all five axes must actuate in the performance of this test.” No further specification of the work location is given. The milling procedure is described as follows: “profile mill the frustum of a 10 inch diameter cone with an apex angle of 30 degrees. The Z axis shall rise 1.000 inch min. while traversing 180° of circumference and fall 1.000 inch min. while traversing the remaining 180°. The Z axis motion to be measured along the center line of
cutter.” The description is well suited for profiler machines common in the aerospace sector addressed by the standard. Profiler machines are still built today with a limited angular range in the rotary axes. For such configurations, the description requires a simultaneous 5-axis movement without any further definition of the setup. On the contrary, for other types of machine designs, the setting conditions have to be defined more precisely in order to require a simultaneous 5-axis movement. This shows that a modernized standard of a test piece for 5-axis machine tools is required.

Furthermore, no detailed analysis of the adequacy of a cone geometry and possible combinations with other elements on a test piece has been published. Moreover, no research has been found about the effects of geometric machine errors on such test piece geometry. In a very recent publication [28] 11 location errors (defined as kinematic errors in that publication) are identified on a 5-axis machine. The influence of these errors on a cone frustum according to [5] is modeled and verified with experiments. No component errors of the motions of the machine axes are modeled.
Chapter 3

Test piece for testing simultaneous
5-axis machining

3.1 Goals and requirements of the test piece

The ideal test piece for acceptance tests of 5-axis machining should have the following features:

- easy and fast to manufacture;
- easy and fast to measure;
- easy to evaluate measuring results;
- Set up in a way that it must be manufactured using 5 simultaneous axes on different types of 5-axis machining centers.
- Give comparable and quantitative results.
- Show the influence of the machine and not of the tool and the tool set-up.
- Show the influence of errors relevant for the analyzed manufacturing strategy (e.g. end milling) with sufficient sensitivity.

The goals of being able to manufacture in an easy and fast way can be reached by including only few elements to be manufactured. The simpler these elements are the better. To be able to measure and interpret the results of the cutting tests the features should consist of simple geometrical elements. The parameters of the single geometrical elements have to be identifiable in a mathematical way from a cloud of points (resulting from the measurements) in a unique way. The tolerated parameters of the test piece should not depend on deviations of the used tool that can not be identified easily when measuring the test piece. In order to give comparable results for different types of 5-axis machining
centers the test piece and its manufacturing procedure should be designed in a way that on all kinds of 5-axis kinematics the part is manufactured using five axes simultaneously. The goal and a main advantage of a test piece manufactured with five simultaneous axes, is the possibility to see the influence of geometric errors of all five machine axes relevant for the analyzed manufacturing strategy (e.g. end milling).

3.2 Geometry of the test piece

The interaction of the tool and the workpiece (creating the workpiece geometry) depends on the tool type used. When using a ball end cutter the interaction can be approximated as a point (especially in the finishing process). When using an end mill on the other hand, the interaction is more extended. It can be approximated as a line or an arc and therefore allows generating a surface in one closed movement.

Using an end mill the surface of the workpiece is cut with the nominal cutter axis either perpendicular or parallel to the nominal surface. Therefore, the geometry of the surface features of the test piece should have normal and tangent vectors not all parallel to each other, so that the path of the tool relative to the workpiece is not purely translational but also rotational.

Geometries that satisfy all these criteria are a sphere, a cone and a toroid. They can all be described mathematically and standard best fit algorithms are available. Furthermore they can all be measured and identified on standard coordinate measuring machines (CMM).

Between these three geometries, the cone is the only surface, where straight lines can be used to generate it. This has the advantage that it can be manufactured with an end mill with only one closed path of the tool relative to the workpiece. Both the sphere and the toroid require much longer tool path for the same surface and therefore longer manufacturing time. Thus, the cone surface is selected because it is best for the first three criteria described in section 3.1.

Cone is defined as (1) A (conical) surface generated by a line, one point (the vertex) of which is fixed and one point of which describes a fixed plane curve. Any line lying in the surface is called generator. (2) A solid bounded by a conical surface and a plane (the base), which cuts the surface. If the fixed curve is a circle the cone is called a circular cone, and if the vertex is also perpendicularly over the center of the circle the cone is called a right circular cone [35].
3.3 Calculation of the tool path required for a cone geometry

When analyzing a required movement of a 5-axis machine a series of tool positions and directions have to be analyzed. For the case of a tool path for a cone geometry, the set of tool directions also describe a cone. The type and position of this cone depends on the cone geometry, the position of the workpiece and on the milling strategy.

For end milling the cone formed by the tool directions is parallel to the cone of the workpiece at a distance of the tool radius from the workpiece. For face milling on the other hand the tool vectors are perpendicular to the cone surface of the workpiece.

3.3.1 Analyzed machine setups

The simultaneous 5-axis tool path required for cone geometry depends not only on the milling strategy and the test piece parameters and setup but also on the machine setup.

This work is focused on serial kinematic setups with three orthogonal linear axes and two orthogonal rotary axes. The nominal rotary axes are assumed to cross in a point. That way compensational movements can be minimized (see section 3.3.6 starting on page 23). Machines with these types of setup are most common in high speed and high precision, simultaneous 5-axis milling.

For the calculation of the tool path, it is relevant to know if a rotary axis is on the tool side or on the workpiece side. Therefore, three general setups are distinguished:

- Both rotary axes on the tool side (known as double pivot spindle head, universal head or tilting rotary head setups).
- Both rotary axes on the workpiece side (known as tilting rotary table setups).
- One rotary axis on the workpiece side (usually a rotary table) and another rotary axis on the tool side (swivel or tilting head).

Specific machine example used throughout the work

The analysis in this work is mainly focused on simultaneous 5-axis end milling test pieces. A machining center with a tilting rotary table setup, more specific a machine with a t-(C)-Z-Y-b-X-B-C-w kinematic chain (structure code according to [23]) is used as a main example (see figure 3.1). In this structure code, the upper case letters stand for the
machine axes according to ISO 841 [36]. An upper case letter in brackets indicates an axis without positioning control. The lower case letters represent the following:

- t: tool
- b: bed
- w: workpiece

![Diagram of machine axes](image)

Figure 3.1: Illustration of the machine with a structure code t-(C)-Z-Y-b-X-B-C-w used for the test piece analysis.

Therefore, t-(C)-Z-Y-b-X-B-C-w is the code for a machine structure where the tool t is held by a rotary axis C without positioning ability (spindle of milling machine). From the tool t and the spindle (C) two linear positioning axes Z and Y follow to the bed b. A linear positioning axis X and two rotary positioning axes B and C follow from the bed b to the workpiece w. This setup is a version of a machine type commonly known as 5-axis machines with tilting rotary table.

### 3.3.2 Parameters of a right circular cone (figure 3.2)

The form of a right circular cone (figure 3.2) is defined only by the cone angle $\alpha$ (also referred to as half apex angle). The position and orientation in a specific frame can be defined either by definition of the position of the apex A and the direction of the cone axis or by definition of a circular section of the cone with the center O, the radius r and the direction of the cone axis toward the apex.

When analyzing the cone describing the tool path it makes sense to use the circular base represented by the nominal tool positions instead of the apex.
3.3 Calculation of the tool path

Figure 3.2: Example of a right circular cone ($\alpha = 30^\circ$) in the frame $\Sigma_0$ attached to it.

- $\alpha$: cone angle (half apex angle of the cone)
- $r$: radius of the base of the cone
- $O$: center of the base of the cone (center of tool path in the workpiece coordinate system)
- $A$: apex of the cone (crossing of the tool vectors)

With the cone angle $\alpha$ and the radius $r$ of the circular base of the cone the length $s$ of a slant and the height $h$ of the cone can be calculated (equations 3.1 and 3.2).

- $s$: length of the slant of the cone that corresponds to the distance from any point on the tool path to the apex $A$ of the cone (equation 3.1)
- $h$: height of the cone (equation 3.2)

\[
\begin{align*}
  s &= \frac{r}{\sin \alpha} \quad (3.1) \\
  h &= r \cot \alpha \quad (3.2)
\end{align*}
\]

For the definition of a point $P$ on the perimeter of the cone (representing a point of the tool path), a radial angle $\varphi$ of the cone is needed.

- $P$: point on the perimeter of the base of the cone (point of the tool path)
- $\varphi$: radial angle of the cone

In a coordinate system (frame $\Sigma_0$) attached to the cone with its origin in $O$ the coordinates of the point $P$ result according to equation 3.3 and the apex $A$ according to equation 3.4.

An example of a cone and the frame $\Sigma_0$ attached to it is shown in figure 3.2.
3.3.3 Parameters of an inclined right circular cone (figure 3.3)

The orientation of a cone depends on the inclination angle $\beta$. The cone is inclined relative to a frame $\sum_1$ with the angle $\beta$ around $Y_1$ as shown in figure 3.3 (Y-direction of frame $\sum_1$ that coincides with $Y_0$). The Rotation Matrix $^1R_0$ describes the rotational part of the relative displacement of frame $\sum_0$ from frame $\sum_1$ [37]. For simplification, it will be referred to as $R_\beta$ (equation 3.5).

![Diagram of a right circular cone inclined by an angle $\beta$](image)

- $\beta$: inclination angle
- $^1R_0$: Rotation Matrix describing the rotational part of the relative displacement of frame $\sum_0$ from frame $\sum_1$ (equation 3.5)
- $R_\beta$: Rotation Matrix describing the influence of the inclination angle $\beta$ (equation 3.5)

$$^0P = r \begin{bmatrix} \cos \varphi \\ \sin \varphi \\ 0 \end{bmatrix} \quad (3.3)$$

$$^0A = \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix} = r \begin{bmatrix} 0 \\ 0 \\ \cot \alpha \end{bmatrix} \quad (3.4)$$

$$^1R_0 = R_\beta = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \quad (3.5)$$
3.3 Calculation of the tool path

With the Rotation Matrix \( R_0 \) coordinates of any point expressed in frame \( \sum_0 \) can be transformed into coordinates of frame \( \sum_1 \) by matrix multiplication from the left \([37]\). The results of the transformation for the points \( P \) and the apex \( A \) are shown in the equations 3.6 and 3.7.

\[
\begin{align*}
1P &= \begin{bmatrix} \, 1p_x \\ \, 1p_y \\ \, 1p_z \end{bmatrix} = R_0 \begin{bmatrix} \cos \beta \cos \varphi \\ \sin \varphi \\ -\sin \beta \cos \varphi \end{bmatrix} \\
1A &= \begin{bmatrix} \, 1a_x \\ \, 1a_y \\ \, 1a_z \end{bmatrix} = R_0 \begin{bmatrix} \sin \beta \cot \alpha \\ 0 \\ \cos \beta \cot \alpha \end{bmatrix}
\end{align*}
\] (3.6)

3.3.4 General transformation for 5-axis machines

The transformation into the coordinates system of the machine (positions of the machine axes) is not unique. In general, any position and tool direction can be reached in at least two different ways if the range of the tilting axis is large enough. Figure 3.4 shows an illustration of two possible axes configurations for the same tool position and direction for machines with tilting rotary table setups.

![Diagram](image1.png)

Figure 3.4: Illustration of the two possible axes configurations for the same tool position and direction.

For the analyzed machine kinematics shown in figure 3.1 the rotary axes have to turn in a way that the vector describing the required tool direction at the given point is parallel
to the Z-direction (tool axis). This requires that the table axis C rotates until the vector describing the required tool direction be in a plane perpendicular to the tilting axis B. This can be done in two different ways as shown in figure 3.4. The range of the B-axis of the analyzed setup allows both solutions. For reasons of better visual control, the solution with negative angle $B$ was selected.

Both angles $B$ and $C$ depend only on the required tool direction and therefore they are independent of the required translational position of the tool.

### 3.3.5 Resulting movements of the rotary machine axes

The movements of the rotary axes of a 5-axis machine, required to manufacture a cone geometry, depend only on the cone form (angle $\alpha$) and the inclination (angle $\beta$) of the cone axis relative to the nominal direction of the rotary axis of the machine nearest to the workpiece (seen in the kinematic chain; corresponds to rotary axis with fixed direction relative to the cone axis). The movement of the rotary axes of all 5-axis machining centers with a serial kinematic setup with two orthogonal rotary axes is the same for a given combination of cone angle $\alpha$ and inclination angle $\beta$.

In case of the analyzed tilting rotary table machine setup (figure 3.1), the inclination angle $\beta$ is used to describe the angle between the C-axis (rotary table axis) and the cone axis. Therefore, the Z-axis of frame $\sum_1$ is defined to be parallel to the nominal direction of the axis C of the rotary table. The angle $\beta$ between this rotary axis and the cone axis is constant during any movement of the machine.

- $Z_1$: direction parallel to the nominal direction of the rotary axis of the machine closer to the workpiece

The orientation of the frame $\sum_1$ relative to the angle $C$ of the rotary table has an influence only on the starting angle of the rotary table. This is of reduced importance since the rotary table usually has an unlimited angular range and therefore an arbitrary point of origin. The zero of the C-axis is defined to be equal to the zero of the radial cone angle $\varphi$.

The translational position of the cone has an influence only on the movement of the linear axes as seen in the previous section 3.3.4.

### Required angles of tilting axis B

The origin of the angle $B$ of the tilting axis ($B = 0^\circ$) is defined to be the position where the machine table is square to the working spindle; therefore where the axis C is parallel
3.3 Calculation of the tool path

to the Z-axis of the machine tool.

The angle of the tilting axis $B$ corresponds to the angle between the tool direction and the direction of the rotary table axis $C$. This corresponds to the angle between a slant $s$ and the Z-axis of frame $\sum_1$ (see figure 3.3). Equation 3.8 results, which can be simplified with equations 3.1, 3.7 and 3.6 to equation 3.9.

- $B$: angle of the tilting axis

\[
\cos B = \frac{1a_z - 1p_z}{s} \quad (3.8)
\]

\[
B = \arccos (\cos \alpha \cos \beta + \sin \alpha \sin \beta \cos \varphi) \quad (3.9)
\]

Two important facts can be concluded from equation 3.9:

- The sign of the angle $B$ is not defined.
- The extreme values of the angle of the tilting axis $B$ for a given cone and setup ($\alpha$ and $\beta$ are constant) are at $\varphi = 0^\circ$ and $\varphi = 180^\circ$. This means that the extreme values of the tilting axis $B$ are in the XZ-plane of frame $\sum_1$.

Further interpretations will follow in section 3.4 on page 31.

**Required angles of rotary table $C$**

The angle $C$ of the rotary table can be derived when analyzing the projection of the cone in the plane of the rotary table (plane perpendicular to the axis of the rotary table, which corresponds to the XY-plane of frame $\sum_1$). The angle $C$ corresponds to the angle between the projection of the height of the cone (in this case in positive X-direction of frame $\sum_1$) and the projection of the slant (see figure 3.5). Thus, equation 3.10 results for the angle $C$.

\[
C = \arctan \left( \frac{1p_y}{1p_x - 1a_x} \right)
= \arctan \left( \frac{\sin \varphi}{\cos \beta \cos \varphi - \sin \beta \cot \alpha} \right) \quad (3.10)
\]
3. Test piece for testing simultaneous 5-axis machining

Figure 3.5: Example of a right circular cone ($\alpha = 30^\circ$) inclined by an angle $\beta = 20^\circ$ with the two frames $\sum_0$ and $\sum_1$ and the angle $C$ of the rotary table for an example of a radial angle $\varphi$ of the cone.

**Resulting velocities of the rotary axes**

The resulting velocities of the rotary axes of the machine are analyzed relative to the change of the radial angle $\varphi$ of the cone. When manufacturing a cone geometry with a constant feed $f$ the change of the radial angle of the cone with time $\dot{\varphi}$ is a linear function of the feed $f$ and the radius $r$ of the tool path (equation 3.11).

- $f$: feed during the manufacturing of the cone geometry (constant)

$$\frac{\partial \varphi}{\partial t} = \dot{\varphi} = \frac{f}{2\pi r}$$ \hspace{1cm} (3.11)

**Velocity of tilting axis B:** the velocity of the tilting axis B relative to the change of the radial cone angle $\varphi$ results from the partial derivative of $B$ with respect to $\varphi$ (equation 3.12).

$$\frac{\partial B}{\partial \varphi} = \frac{\sin \alpha \sin \beta \sin \varphi}{\sqrt{1 - \left(\cos \alpha \cos \beta + \cos \varphi \sin \alpha \sin \beta\right)^2}}$$ \hspace{1cm} (3.12)

**Velocity of rotary table C:** the velocity of the rotary table axis C relative to the change of the radial cone angle $\varphi$ results from the partial derivative of $C$ with respect to $\varphi$ (equation 3.13).

$$\frac{\partial C}{\partial \varphi} = \frac{\cos \beta - \cos \varphi \cot \alpha \sin \beta}{\cos^2 \beta \cos^2 \varphi - 2 \cos \beta \cos \varphi \cot \alpha \sin \beta + \cot^2 \alpha \sin^2 \beta + \sin^2 \varphi}$$ \hspace{1cm} (3.13)
3.3 Calculation of the tool path

3.3.6 Linear machine axes

The movements of the linear axes of 5-axis machines can be divided into a component needed for the relative translational displacement of the tool and a second component, which is needed in order to change the tool direction. This second compensational component consists of the linear displacement of the rotary axes in order to change the tool direction without changing the location of the tool center point (TCP). It is proportional to the distance between the rotary axes and the TCP. No compensational movement of linear axes is needed for rotations of axes, which have no distance from the TCP.

- **Compensational movement of linear machine axes**: movement of the linear machine axes needed in order to change the tool direction without changing the position of the TCP.

The distance between the TCP and the first tool sided rotary axis is constant for any movement of the machine. It changes only for different tool lengths. The distance between the TCP and an eventual second tool sided rotary axis depends on the angle of the first tool sided rotary axis. The translational position of the TCP has no influence on the distance between the TCP and a tool sided rotary axis. Therefore, the compensational movements of the linear axes for the rotation of tool sided rotary axes do not depend on the translational position of the workpiece.

The distance between the TCP and the workpiece sided rotary axes depends on the position of the workpiece. Therefore, the compensational movements of the linear axes for a rotation of a workpiece sided rotary axis depend on the position and orientation of the workpiece. A change of tool length has no influence on the compensational movement for the rotation of a workpiece sided rotary axis.

**Movement of the linear machine axes of 5-axis machines with both rotary axes on the tool side (double pivot spindle head) for the tool path required for a cone geometry**

For the analysis of the movement of the linear machine axes of 5-axis machines with both rotary axes on the tool side (double pivot spindle head), for the tool path required for a cone geometry, the cone can be analyzed with the two frames $\sum_0$ and $\sum_1$ as shown in figure 3.3. The axis $Z_1$ is defined to be pointing in the direction parallel to the rotary axis farther away from the tool. For a machine with a t-(C)-B-C-Z-Y-b-X-w setup (example shown in figure 3.6) $Z_1$ would be parallel to the Z-axis of the machine. To define the
relative orientation of the directions $X_1$ and $Y_1$ of frame $\sum_1$ relative to the machine frame (in this case $\sum_2$) of a double pivot head machine an angle $\gamma$ is introduced (see figure 3.7).

- $\gamma$: angle from the machine axis $X_2$ to the $X$-axis $X_1$ of the frame $\sum_1$ of the inclined cone (figure 3.7)

![Diagram of a vertical 5-axis machining center with a double pivot spindle head](image)

Figure 3.6: Example of a vertical 5-axis machining center with a double pivot spindle head (t-(C)-B-C-Z-Y-b-X-w setup) [23].

The movement of the linear axes of the machine correspond to the movement of the crossing $Q$ of the two rotary axes. For machines with both rotary axes on the tool side, the path of the point $Q$ is a circle, which is coaxial to the tool path and part of the cone formed by the tool directions. If the distance from the TCP to the crossing $Q$ of the two rotary axes is $l$, the center of the circle formed by the points $Q$ can be derived using figure 3.7. The coordinates of the center of the base of the cone $M$ in the machine frame $\sum_2$ depend on the length $l$ and the angles $\alpha$, $\beta$ and $\gamma$ (equation 3.16). The radius $r_Q$ of the circle formed by the crossings $Q$ of the rotary axes depends on the length $l$, the radius $r$ of the tool path and the cone angle $\alpha$ (equation 3.17).

- $Q$: crossing of the two rotary axes
• \( l \): distance from the TCP to the crossing \( Q \) of the two rotary axes

• \( M \): center of the circle formed by the points \( Q \) (figure 3.7)

• \( r_Q \): radius of the circle formed by the points \( Q \) in any frame attached to the cone (figure 3.7)

Figure 3.7: Example of a right circular cone \((\alpha = 30^\circ)\) inclined by an angle \( \beta = 20^\circ \) and rotated with an angle \( \gamma = 60^\circ \) relative to frame \( \sum_2 \).

\[
^0 M = \begin{bmatrix} 0 \\ 0 \\ l \cos \alpha \end{bmatrix} \tag{3.14}
\]

\[
^2 R_1 = R_\gamma = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.15}
\]

\[
^2 M = R_\gamma R_\beta^0 M = l \cos \alpha \begin{bmatrix} \sin \beta \cos \gamma \\ \sin \beta \sin \gamma \\ \cos \beta \end{bmatrix} \tag{3.16}
\]

\[
r_Q = r - l \sin \alpha \tag{3.17}
\]

The movements of the linear machine axes of 5-axis machining centers with double pivot spindle head result from the coordinates of \(^2Q\) (equation 3.19).
\[ 0Q = \begin{bmatrix} r_Q \cos \varphi \\ r_Q \sin \varphi \\ l \cos \alpha \end{bmatrix} \]  

(3.18)

\[ 2Q = R_\gamma R_\beta 0Q = 2M + r_Q \begin{bmatrix} \cos \beta \cos \gamma \cos \varphi - \sin \gamma \sin \varphi \\ \cos \beta \sin \gamma \cos \varphi + \cos \gamma \sin \varphi \\ -\sin \beta \cos \varphi \end{bmatrix} \]

\[ = \begin{bmatrix} l \cos \alpha \cos \beta \cos \gamma + r_Q (\cos \beta \cos \varphi \cos \gamma - \sin \varphi \sin \gamma) \\ l \cos \alpha \sin \beta \sin \gamma + r_Q (\cos \beta \cos \varphi \sin \gamma + \sin \varphi \cos \gamma) \\ l \cos \alpha \cos \beta - r_Q \sin \beta \cos \varphi \end{bmatrix} \]

(3.19)

Movement of the linear machine axes of 5-axis machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table) for the tool path required for a cone geometry

The analysis of the movements of the linear axes for machines with rotary axes of the tool and on the workpiece side (swivel head and rotary table) for the tool path required for a cone geometry is more complicated than for machines with both rotary axes on the tool side because all points of the workpiece (except for the points on the axis of rotation of that rotary axis) are moved when the rotary axis on the workpiece side (usually a rotary table) is turned. As an example of a vertical 5-axis machining center with a swivel head and a rotary table a machine with a t-(C)-B-Z-Y-b-X-C-w setup is shown in figure 3.8.

The movement of the linear machine axes of 5-axis machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table) for the tool path required for a cone geometry corresponds to the movement of the point \( Q \) relative to the axis of the rotary axis on the workpiece side (axis C of the rotary table).

For the description of the position and orientation of the workpiece relative to the rotary axis C, an additional frame \( \mathbf{\Sigma}_3 \) is introduced, which is attached to the rotary table (figure 3.9). The origin \( O_2 \) is defined to have a distance \( x_0 \) from the origin of frame \( \mathbf{\Sigma}_3 \) in X-direction of frame \( \mathbf{\Sigma}_3 \) (equation 3.20) as shown in figure 3.9. The coordinates of the points \( M \) and \( Q \) can be expressed in frame \( \mathbf{\Sigma}_3 \) according to the equations 3.21 and 3.22 using equation 3.19 for the coordinates of \( Q \) in frame \( \mathbf{\Sigma}_2 \).
### 3.3 Calculation of the tool path

Figure 3.8: Example of a vertical 5-axis machining center with a swivel head and rotary table (t-(C)-B-Z-Y-b-X-C-w setup).

Figure 3.9: Example of a right circular cone \((\alpha = 30^\circ)\) inclined by an angle \(\beta = 20^\circ\) in the frame \(\sum_4\) with the frames \(\sum_3, \sum_2, \sum_1\) and \(\sum_0\).

\[
^3O_2 = \begin{bmatrix} x_0 \\ 0 \\ 0 \end{bmatrix} \quad (3.20)
\]

\[
^3M = ^2M + ^3O_2 = l \cos \alpha \begin{bmatrix} \sin \beta \cos \gamma + x_0 \\ \sin \beta \sin \gamma \\ \cos \beta \end{bmatrix} \quad (3.21)
\]
\[ 3Q = 2Q + 3O_2 = 2Q + \begin{bmatrix} x_0 \\ 0 \\ 0 \end{bmatrix} \]

\[ \begin{bmatrix} l \cos \alpha \sin \beta \cos \gamma + r_Q (\cos \beta \cos \varphi \cos \gamma - \sin \varphi \sin \gamma) + x_0 \\ l \cos \alpha \sin \beta \sin \gamma + r_Q (\cos \beta \cos \varphi \sin \gamma + \sin \varphi \cos \gamma) \\ l \cos \alpha \cos \beta - r_Q \sin \beta \cos \varphi \end{bmatrix} \]

(3.22)

For the influence of the angle \( C \) of the rotary table a frame \( \Sigma_3 \) is introduced according to figure 3.9. The direction of the angle \( C \) is defined as shown in figure 3.9 because the axis is moving the workpiece and therefore it has an inverse sign (as defined in [36]). The transformation of points expressed relative to frame \( \Sigma_3 \) into frame \( \Sigma_4 \) is done with \( 4R_3 \) that will be referred to as \( R_{table} \) (equation 3.23). The angle \( C \) for the radial angle \( \varphi = 0^\circ \) corresponds to the angle \( \gamma \) (equation 3.24). The points \( Q \) transformed into frame \( \Sigma_4 \) according to equation 3.25 represent the movement of the linear axes of machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table) for any combination of \( \alpha, \beta \) and \( x_0 \) at any angle \( \varphi \).

\[ 4R_3 = R_{table} = \begin{bmatrix} \cos C & \sin C & 0 \\ -\sin C & \cos C & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

(3.23)

\[ C(\varphi = 0^\circ) = \gamma \]

(3.24)

\[ 4Q = 4R_3 3Q \]

(3.25)

Movement of the linear machine axes of 5-axis machines with both rotary axes on the workpiece side (tilting rotary table) for the tool path required for a cone geometry

For the analysis of the movements of the linear machine axes of 5-axis machines with both rotary axes on the workpiece side (tilting rotary table) the position of the workpiece relative to the crossing of the two rotary axes is relevant. The inclined cone with the frames \( \Sigma_0 \) and \( \Sigma_1 \) is defined as in the previous sections but the frame \( \Sigma_2 \) is defined in a different way. A frame \( \Sigma_2 \) attached to the rotary table is introduced with its origin \( O_2 \) at the nominal crossing \( Q \) of the axis \( C \) of the rotary table with the tilting axis \( B \).

- frame \( \Sigma_2 \): coordinate system attached to the rotary table with its origin \( O_2 \) at the nominal crossing of the axis \( C \) of the rotary table with the tilting axis \( B \) (figure 3.10)
3.3 Calculation of the tool path

The relation between frame $\sum_1$ and $\sum_2$ is purely translational. Therefore, any point $^1P$ in frame $\sum_1$ can be transformed into frame $\sum_2$ by adding the vector $^2O_1$ of the origin of frame $\sum_1$ expressed in frame $\sum_2$. The components of $^2O_1$ are defined in equation 3.26. The coordinates of $^2P$ result from equation 3.27, which is also expressed as a function of the cone parameters using equation 3.6.

\[
^2O_1 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \quad \text{(3.26)}
\]

\[
^2P = ^1P + ^2O_1 = \begin{bmatrix} r \cos \beta \cos \varphi + x_0 \\ r \sin \varphi + y_0 \\ -r \sin \beta \cos \varphi + z_0 \end{bmatrix} \quad \text{(3.27)}
\]

For the analysis of the rotation of the rotary table, a frame $\sum_3$ is attached to the tilting axis B. Both origins $O_2$ and $O_3$ are defined to be at the crossing of the two rotary axes, the relation between frame $\sum_2$ and $\sum_3$ is purely rotational. It can be described by a rotation matrix $^3R_2$, which will be referred to as $R_{\text{table}}$ (equation 3.28). The situation with the coordinate systems is shown in figure 3.11.

- frame $\sum_3$: coordinate system attached to the tilting axis with its origin $O_3$ in $O_2$, which is at the nominal crossing of the axis C of the rotary table with the tilting axis B (figure 3.11)
- $R_{\text{table}}$: Rotation Matrix $^3R_2$ describing the rotational displacement of frame $\sum_2$ from frame $\sum_3$ due to the rotation of the rotary table (equation 3.28)
3. Test piece for testing simultaneous 5-axis machining

\[ ^3R_2 = R_{\text{table}} = \begin{bmatrix} \cos C & \sin C & 0 \\ -\sin C & \cos C & 0 \\ 0 & 0 & 1 \end{bmatrix} \] (3.28)

Figure 3.11: Example of a right circular cone \((\alpha = 30^\circ)\) inclined by an angle \(\beta = 20^\circ\) in frame \(\sum_3\) with the frames \(\sum_0, \sum_1\) and \(\sum_2\).

For the analysis of the rotation of the tilting axis B a frame \(\sum_4\) is attached to the non rotating part of the tilting axis. It can be interpreted as the machine frame. The origin \(O_4\) is chosen to be the same as for the frames \(\sum_2\) and \(\sum_3\) (at the crossing of the two rotary axes). The displacement of the frame \(\sum_3\) relative to the machine frame \(\sum_4\) is purely rotational and can be described by a rotation matrix \(^4R_3\), which will be referred to as \(R_{\text{tilt}}\) (equation 3.29). The situation with the coordinate systems is shown in figure 3.12.

- frame \(\sum_4\): coordinate system attached to the machine with its origin \(O_4\) in \(O_3\), which is at the nominal crossing of the axis C of the rotary table with the tilting axis B (figure 3.12)
- \(R_{\text{tilt}}\): Rotation Matrix \(^4R_3\) describing the rotational displacement of frame \(\sum_3\) from the machine frame \(\sum_4\) due to the rotation of the tilting axis B (equation 3.29)

\[ ^4R_3 = R_{\text{tilt}} = \begin{bmatrix} \cos B & 0 & -\sin B \\ 0 & 1 & 0 \\ \sin B & 0 & \cos B \end{bmatrix} \] (3.29)
3.4 Interpretation of the movements of the rotary axes

For the interpretation of the resulting movements of the rotary axes of the machine three situations are distinguished depending on the relation of the cone angle $\alpha$ and the inclination angle $\beta$.

The described machine setup with tilting rotary table will be taken as an example for a better illustration but the interpretations are valid for any other serial kinematic machine setup with rectangular, intersecting rotary axes.
3.4.1 Cone angle $\alpha$ larger than inclination angle $\beta$

Angle $C$ of the rotary table

For $\alpha > \beta$ the projection of the apex $A$ into the XY-plane of frame $\Sigma_1$ is inside the projection of the base as shown in figure 3.3 on page 18. Therefore, an entire revolution of the axis $C$ of the rotary table is needed for the path of the cone since the angle $C$ depends on the direction of the projection of the slants in the XY-plane of frame $\Sigma_1$ as described in section 3.3.5.

- $\Delta C$: range of the angle $C$ of the rotary table or in other words the total angular movement of the axis $C$ of the rotary table for the path of the cone

$$\Delta C = 360^\circ \quad \text{for } \alpha > \beta$$

Angle $B$ of the tilting axis

Two solutions with different signs exist for the angle $B$ (see equation 3.9 in section 3.3.5 and figure 3.4). Therefore, minimal and maximal values will refer to the absolute values of $B$.

- $B_{\min}$: minimum of the absolute value of the angle of the tilting axis $B$
- $B_{\max}$: maximum of the absolute value of the angle of the tilting axis $B$

The extreme values of the angle of the tilting axis $B$ are at the angles $\varphi = 0^\circ$ and $\varphi = 180^\circ$ as derived from equation 3.9. The values can be derived geometrically by analyzing the cone in the XZ-plane of frame $\Sigma_1$ as shown in figure 3.13. The minimal angle of the tilting axis $B_{\min}$ results to be the difference between the cone angle $\alpha$ and the inclination angle $\beta$ (equation 3.32) whereas the maximal angle of the tilting axis $B_{\max}$ is equal to the sum of $\alpha$ and $\beta$ (equation 3.33). Thus, the total range $\Delta B$ of the tilting axis results as twice the inclination angle $\beta$ (equation 3.34) and is therefore independent of the cone angle $\alpha$.

- $\Delta B$: range of the angle $B$ of the tilting axis or in other words the difference between the two extreme values
3.4 Interpretation of the movements of the rotary axes

Figure 3.13: Example of a cone configuration with $\alpha > \beta$ for the geometrical analysis of the extreme values $B_{\text{min}}$ and $B_{\text{max}}$ of the absolute values of the tilting axis B ($\alpha = 30^\circ$, $\beta = 20^\circ$).

\[ B_{\text{min}} = \alpha - \beta \quad \text{for } \alpha > \beta \quad (3.32) \]
\[ B_{\text{max}} = \alpha + \beta \quad (3.33) \]
\[ \Delta B = B_{\text{max}} - B_{\text{min}} = 2\beta \quad \text{for } \alpha > \beta \quad (3.34) \]

An example of positions of the rotary machine axes along the radial angle $\varphi$ of the cone for a configuration with $\alpha = 30^\circ$ and $\beta = 20^\circ$ ($\alpha > \beta$) is shown in figure 3.14. See table 3.1 on page 40 for comparison of the ranges of the rotary axes B and C depending on the relation of the cone angle $\alpha$ and the inclination angle $\beta$.

Generally for $\alpha > \beta$ the movement of the tilting axis B has a reversal but the movement of the rotary axis C has no reversal during the path of the cone.

3.4.2 Cone angle $\alpha$ smaller than inclination angle $\beta$

Angle C of the rotary table

For $\alpha < \beta$ the projection of the apex A into the XY-plane of frame $\sum_1$ is outside the projection of the base as shown in figure 3.15. The range of the angles $\Delta C$ of the rotary table is equal to the angle between the two tangents to the ellipse of the projection of the
Figure 3.14: Positions of the rotary machine axes along the radial angle of the cone $\varphi$ for a configuration with $\alpha = 30^\circ$ and $\beta = 20^\circ$ ($\alpha > \beta$).

tool path into the XY-plane of frame $\Sigma_1$ through the projection of the apex $A$ into the XY-plane of frame $\Sigma_1$ (figure 3.16). Therefore, $\Delta C$ must be smaller than $180^\circ$ for $\alpha < \beta$. The minimum range $\Delta C$ of the angle of the rotary table is twice the cone angle $\alpha$. This is the case if the inclination angle $\beta$ is $90^\circ$.

Figure 3.15: Example of a cone configuration with $\alpha < \beta$ for the geometrical analysis of the range of the angles of the rotary table $C$ ($\alpha = 30^\circ$, $\beta = 50^\circ$).

The radial angle $\varphi$ of the cone where the angle $C$ has an extreme value is equal to the value of $\varphi$ where $\partial C/\partial \varphi$ is zero (equation 3.35 derived from equation 3.11).
3.4 Interpretation of the movements of the rotary axes

Figure 3.16: View of the projection into the XY-plane of frame $\Sigma_1$ of an example of a cone configuration with $\alpha < \beta$ with the tangent needed for the calculation of the range of the angle $C$ of the rotary table ($\alpha = 30^\circ$, $\beta = 50^\circ$).

$$\frac{\partial C}{\partial \varphi} = 0 \quad \text{if} \quad \cos \beta - \cos \varphi \cot \alpha \sin \beta = 0 \quad (3.35)$$

With the help of equation 3.35, the values of the radial cone angle $\varphi$ where the angle $C$ has an extreme value can be derived according to equation 3.36.

- $\varphi(C_{\max})$: values of the radial cone angle $\varphi$ where the angle $C$ has an extreme value

$$\varphi(C_{\max}) = \arccos \left( \frac{\tan \alpha}{\tan \beta} \right) \quad (3.36)$$

In the XY-plane of frame $\Sigma_1$ the projection of the slant at $\varphi(C_{\max})$ must be tangent to the projection of the circular base (ellipse with semimajor axis $r$ and semiminor axis $r \cos \beta$) as shown in figure 3.16. The direction of the tangent is equal to the partial derivative of the coordinates of $^1P(C_{\max})$ (equation 3.37 derived from equation 3.6). With the direction of the tangent the angle $C_{\max}$ (between the tangent and $X_1$) results according to equation 3.38 (simplified using equation 3.36).
\[
\frac{\partial}{\partial \varphi} P_{xy}(C_{\max}) = r \varphi \begin{bmatrix} -\cos \beta \sin(\varphi(C_{\max})) \\ \cos(\varphi(C_{\max})) \end{bmatrix}
\]

(3.37)

\[
C_{\max} = \arctan \left( \frac{\cos(\varphi(C_{\max}))}{\cos \beta \sin(\varphi(C_{\max}))} \right)
= \arctan \left( \frac{\cos(\arccos(\tan \alpha \cot \beta))}{\cos \beta \sin(\arccos(\tan \alpha \cot \beta))} \right)
= \arctan \left( \frac{\tan \alpha}{\sin \beta \sqrt{1 - (\tan \alpha \cot \beta)^2}} \right)
\]

(3.38)

The range \( \Delta C \) of the axis \( C \) is twice the angle \( C_{\max} \) (equation 3.39).

\[
\Delta C = 2C_{\max}
= 2 \arctan \left( \frac{\tan \alpha}{\sin \beta \sqrt{1 - (\tan \alpha \cot \beta)^2}} \right)
\]

(3.39)

**Angle \( B \) of the tilting axis**

For \( \alpha < \beta \) the extreme values of the angle of the tilting axis \( B \) are derived geometrically by analyzing the cone in the XZ-plane of frame \( \sum_1 \) as shown in figure 3.17. The minimal angle of the tilting axis \( B_{\min} \) results to be the difference between the cone angle \( \alpha \) and the inclination angle \( \beta \) whereas the maximal angle of the tilting axis \( B_{\max} \) is equal to the sum of \( \alpha \) and \( \beta \). An example of positions of the rotary machine axes along the radial angle \( \varphi \) of the cone for a configuration with \( \alpha = 30^\circ \) and \( \beta = 50^\circ \) (\( \alpha < \beta \)) is shown in figure 3.18. See table 3.1 on page 40 for comparison of the ranges of the rotary axes \( B \) and \( C \) depending on the relation of the cone angle \( \alpha \) and the inclination angle \( \beta \).

\[
B_{\min} = \beta - \alpha \quad \text{for } \alpha < \beta
\]

(3.40)

\[
B_{\max} = \alpha + \beta
\]

(3.41)

\[
\Delta B = 2\alpha \quad \text{for } \alpha < \beta
\]

(3.42)

Generally for \( \alpha < \beta \) the movements of both rotary axes have reversal points.
3.4 Interpretation of the movements of the rotary axes

3.4.3 Cone angle $\alpha$ equal to inclination angle $\beta$

If the cone angle $\alpha$ is equal to the inclination angle $\beta$ the slant at $\varphi = 0^\circ$ is parallel to the Z-direction $Z_1$ of frame $\sum_1$ and the projection of the apex $A$ into the XY-plane of frame $\sum_1$ is on the projection of the base as shown in figure 3.19.

Since the angle $B$ of the tilting axis corresponds to the angle between a slant of the cone and the direction $Z_1$, $B$ must be zero for $\varphi = 0^\circ$ for $\alpha = \beta$ (spindle axis (tool axis) parallel to the axis C of the rotary table). The angle $C$ of the rotary table is not defined if the projection of the slant in the XY-plane of frame $\sum_1$ has no length and therefore $P_0$ is a singular position.

When moving across the singular position $P_0$ two options exist: either the TCP stays at the singular position $P_0$ while the axis C of the rotary table turns $180^\circ$ and the movement of the tilting axis B continues afterwards with the same sign of the angle $B$ of the tilting axis (figure 3.20) or the sign of the angle $B$ is changed at the singular position $P_0$ and a continuous movement is possible (figure 3.21).

The first option (figure 3.20) does not allow a steady movement of the TCP along the perimeter of the cone. Therefore, it is not an option for the manufacturing of the test piece. The angle $B$ of the tilting axis will have to change sign when passing the singular position $P_0$ for a continuous movement.

For the second option figure 3.21 shows that with one rotation of the rotary table the cone geometry is surrounded twice. By selecting the starting point, the range of the B-axis can
Figure 3.18: Positions of the rotary machine axes along the radial angle of the cone \( \varphi \) for a configuration with \( \alpha = 30^\circ \) and \( \beta = 50^\circ \) \((\alpha < \beta)\).

Figure 3.19: Example of a cone configuration with \( \alpha = \beta = 30^\circ \) with the two frames \( \Sigma_0 \) and \( \Sigma_1 \) and the singular position \( P_0 \).

be chosen.

**Start at singular position** \( P_0 \)

If the starting point is the singular position \( P_0 \), the starting angle \( C \) of the rotary table has to be selected depending on whether the path should be clockwise or counter clockwise.
3.4 Interpretation of the movements of the rotary axes

Figure 3.20: Positions of the rotary machine axes along the radial angle of the cone $\varphi$ for a configuration with $\alpha = \beta = 30^\circ$ without changing sign of the angle $B$ at the singular position $P_0$ and therefore requiring a movement of the axis $C$ of the rotary table of $180^\circ$ while staying at the singular position $P_0$.

This way the range of the angle $C$ of the rotary table is $180^\circ$ and the range of the angle $B$ of the tilting axis is twice the cone angle $\alpha$. An example of the positions of the rotary machine axes along the radial angle $\varphi$ of the cone for a configuration with $\alpha = \beta = 30^\circ$ when starting at the singular position of $P_0$ is shown in figure 3.22 (corresponds to part of figure 3.21).

**Start at $\varphi = 180^\circ$**

If the starting point is the position $\varphi = 180^\circ$ (opposite of the singular position $P_0$) the positions of the rotary axes for a continuous movement are according to figure 3.23 (corresponds to part of figure 3.21). The range of the angle $C$ of the rotary table is $180^\circ$ and the range of the angle $B$ of the tilting axis is four times the cone angle $\alpha$ (corresponds to maximum). See table 3.1 for a comparison of the ranges of the rotary axes $B$ and $C$ depending on the relation of the cone angle $\alpha$ and the inclination angle $\beta$. 
3. Test piece for testing simultaneous 5-axis machining

Figure 3.21: Positions of the rotary machine axes along the radial angle $\varphi$ of the cone for a configuration with $\alpha = \beta = 30^\circ$ for an entire rotation of the rotary table starting at $\varphi = 0^\circ$ and changing sign of the angle $B$ at the singular position $P_0$.

Table 3.1: Comparison of the ranges of the rotary axes B and C depending on the relation of the cone angle $\alpha$ and the inclination angle $\beta$.

<table>
<thead>
<tr>
<th>$\alpha$ relation</th>
<th>$B_{\text{min}}$</th>
<th>$B_{\text{max}}$</th>
<th>$\Delta B$</th>
<th>$\Delta C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha &gt; \beta$</td>
<td>$\alpha - \beta$</td>
<td>$\alpha + \beta$</td>
<td>$2\beta$</td>
<td>$360^\circ$</td>
</tr>
<tr>
<td>$\alpha &lt; \beta$</td>
<td>$\beta - \alpha$</td>
<td>$\alpha + \beta$</td>
<td>$2\alpha$</td>
<td>$2\alpha &lt; \Delta C &lt; 180^\circ$</td>
</tr>
<tr>
<td>$\alpha = \beta$ start at $\varphi = 0^\circ$</td>
<td>0</td>
<td>$2\alpha$</td>
<td>$2\alpha$</td>
<td>$180^\circ$</td>
</tr>
<tr>
<td>$\alpha = \beta$ start at $\varphi = 180^\circ$</td>
<td>$-2\beta$</td>
<td>$2\beta$</td>
<td>$4\alpha$</td>
<td>$180^\circ$</td>
</tr>
</tbody>
</table>

3.4.4 Interpretation of the velocities of the rotary axes for constant feed $f$

$\alpha = \beta$

For $\alpha = \beta$ the velocity of the rotary table is almost constant if the start and end point of the movement is the singular position $P_0$ or if a change of the sign of the angle $B$ of the tilting axis occurs at the singular position $P_0$ (see lower part of figure 3.21). If the singular position $P_0$ is passed without a change of the sign of the angle $B$ of the tilting
3.4 Interpretation of the movements of the rotary axes

Figure 3.22: Positions of the rotary machine axes along the radial angle of the cone $\varphi$ for a configuration with $\alpha = \beta = 30^{\circ}$ when starting and ending at the singular position $P_0$.

Figure 3.23: Positions of the rotary machine axes along the radial angle of the cone $\varphi$ for a configuration with $\alpha = \beta = 30^{\circ}$ when starting and ending at $\varphi = 180^{\circ}$ (opposite the singular position $P_0$).
axis an infinite velocity of the angle $C$ would be required at $P_0$ for a constant feed $f$ as can be seen in the lower part of figure 3.20.

$\alpha \neq \beta$

If the difference between the cone angle $\alpha$ and the inclination angle $\beta$ is small, the velocity of the rotary axis becomes very high. Figure 3.24 shows the required velocity of the angle $C$ of the rotary table at $\varphi = 0^\circ$ for a cone angle $\alpha = 45^\circ$ depending on the inclination angle $\beta$. It shows very clearly that very large velocities are needed for small differences between the cone angle $\alpha$ and the inclination angle $\beta$.

![Figure 3.24: Required velocity of the angle $C$ of the rotary table at $\varphi = 0^\circ$ for a cone angle $\alpha = 45^\circ$ depending on the inclination angle $\beta$.](image)

In cutting tests where the goal is to test the geometry of the machine a near to singular configuration should be avoided. If the goal of the test is to get information about the dynamic behavior of the rotary axes of the machine, the difference between $\alpha$ and $\beta$ should be chosen in such a way that the desired maximal velocity of the rotary axis is required for a given constant feed $f$ (see equation 3.11 on page 22 for dependence of $f$ from $\varphi$ and $r$).
3.5 Interpretation of the movements of the linear machine axes

3.5.1 Machines with both rotary axes on the tool side (double pivot spindle head)

The movements of the linear machine axes of 5-axis machining centers with double pivot spindle head result from the coordinates of $2Q$ (equation 3.19 on page 26).

The range of the linear machine axes of such machines can be interpreted in the following ways depending on the cone angle $\alpha$ and the relation between the length $l$ and the length $s$ of the slant (the length $l$ corresponds to the distance between the TCP and the crossing $Q$ of the two rotary axes, $s$ according to equation 3.1, see also figures 3.2 and 3.7 on the pages 17 and 25):

- $0^\circ < \alpha < 90^\circ$: if the cone angle $\alpha$ is between $0^\circ$ and $90^\circ$ the vector from $M$ to $A$ has a component in tool direction. In manufacturing terms this can be achieved e.g. when end milling a cone frustum with the apex of the cone in tool direction (the apex might be virtual).
  
  - $l = s = r/\sin \alpha$: if the length $l$ is equal to the slant length $s$, the apex $A$ coincides with the crossing $Q$ of the rotary axes and no movement of any linear axis is required for the tool path of the cone geometry (independent of the inclination angle $\beta$).
  
  - $l < 2s$: if the length $l$ is smaller than twice the slant length $s$, the radius $r_Q$ of the path of the linear axes is smaller then the radius $r$ of the TCP (independent of the inclination angle $\beta$).
  
  - $l > 2s$: if the length $l$ is larger the twice the slant length $s$, the radius $r_Q$ of the path of the linear axes is larger then the radius $r$ of the TCP (independent of the inclination angle $\beta$). The absolute maximum of $r_Q$ is $l - r$ for very flat cones ($\alpha$ approaching $90^\circ$).

- $0^\circ > \alpha > -90^\circ$: if the cone angle $\alpha$ is negative (between $-90^\circ$ and $0^\circ$), the apex of the vector from $M$ to $A$ has a component in the opposite tool direction. In manufacturing terms this can be achieved e.g. when end milling an upside down cone frustum or when face milling a cone frustum with its apex pointing in tool direction. With such a configuration the radius $r_Q$ of the linear machine axes is larger then the radius $r$ of the TCP. It can reach a maximal value of $l + r$ for cone angles approaching $90^\circ$. 
The length $l$ between the TCP and the crossing of the two rotary axes $Q$ is defined by the machine and the tool with its holder. It can be changed only to some extent by the tool length. The slant length $s$ on the other hand is a function of the size of the workpiece and of the cone angle $\alpha$ (equation 3.1).

If a large path of the linear axes is desired, without a large radius $r$ of the path of the TCP (size of the workpiece and milling strategy), a negative cone angle $\alpha$ between $0^\circ$ and $-90^\circ$ should be chosen (see figure 3.25 on page 50 for one example).

If only the synchronized movement of the rotary axes is of interest, the length $s$ of the slant should be selected to be equal to the length $l$ between the TCP and the intersection $Q$ of the rotary axes. This can be done by varying the cone angle $\alpha$ between $0^\circ$ and $90^\circ$ for a given radius $r$ of the path of the TCP. If $r$ is increased, the angle $\alpha$ has to be decreased towards $0^\circ$ and if $r$ is decreased, the angle $\alpha$ has to be increased towards $90^\circ$ in order to keep $s = l$. With these variations, the dynamics of the rotary axes can be influenced.

A summary of the relations of the radius $r_Q$ of the path of the linear axes of machines with both rotary axes on the tool side and the radius $r$ of the path of the TCP for given ranges of the cone angle $\alpha$ at special relations between the length $l$ (from the TCP to the crossing of the rotary axes $Q$) and the slant length $s$ is given in table 3.2.

The velocities of the linear machine axes relative to the change of the radial cone angle $\varphi$ result from the partial derivative of $2Q$ (equation 3.19 on page 26) with respect to $\varphi$ (equation 3.43). The angles $\varphi$ of the extreme values of the linear axes can be found by calculating the roots of the partial derivative of $2Q$ with respect to $\varphi$.

Table 3.2: Summary of the relations between $r_Q$ and $r$ for given ranges of $\alpha$ at special relations between $l$ and $s$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$l$, $s$</th>
<th>$r_Q$, $r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$-$90^\circ$</td>
<td>$l = s$</td>
<td>$r_Q = 0$</td>
</tr>
<tr>
<td></td>
<td>$l &lt; 2s$</td>
<td>$r_Q &lt; r$</td>
</tr>
<tr>
<td></td>
<td>$l &gt; 2s$</td>
<td>$r_Q &gt; r$</td>
</tr>
<tr>
<td></td>
<td>$l = 2s$</td>
<td>$r_Q = r$</td>
</tr>
<tr>
<td></td>
<td>$l = 0$</td>
<td>$r_Q = r$</td>
</tr>
<tr>
<td>$90^\circ$-$180^\circ$</td>
<td>$l &gt; 0$</td>
<td>$r_Q &gt; r$</td>
</tr>
<tr>
<td></td>
<td>$l = s$</td>
<td>$r_Q = 2r$</td>
</tr>
<tr>
<td>$s = r/\sin \alpha$</td>
<td></td>
<td>$r_Q = r - l \sin \alpha$</td>
</tr>
</tbody>
</table>
3.5 Interpretation of the movements of the linear machine axes

\[
\frac{\partial^2 Q}{\partial \varphi^2} = r_Q \begin{bmatrix}
- \cos \varphi \sin \gamma - \sin \varphi \cos \gamma \cos \beta \\
\cos \varphi \cos \gamma - \sin \varphi \sin \gamma \cos \beta \\
\sin \varphi \sin \beta 
\end{bmatrix}
\] (3.43)

For the X- and Y-axis of the machine the angles \( \varphi \), where the axis has an extreme value, are calculated according to the equations 3.44 and 3.45.

\[
\varphi(X_{\text{max}}) = \arctan \left( -\frac{\tan \gamma}{\cos \beta} \right) \\
= -\arctan(\sec \beta \tan \gamma) 
\] (3.44)

\[
\varphi(Y_{\text{max}}) = \arctan \left( \frac{1}{\tan \gamma \cos \beta} \right) \\
= \arctan(\cot \gamma \sec \beta) 
\] (3.45)

The angles \( \varphi \) where the Z-axis has an extreme value are 0° and 180° since the position of the Z-axis at a given angle \( \varphi \) is independent of the angle \( \gamma \) (see figure 3.7 on page 25 for the angle \( \gamma \)). The range of the Z-axis results to be according to equation 3.46, which can be expressed as a function of \( r \) and \( l \) using equation 3.17 on page 25.

\[
\Delta Z_M = 2r_Q \sin \beta \\
= 2 \sin \beta (r - l \sin \alpha) 
\] (3.46)

The ranges of the X- and Y-axis can be obtained by combination of equation 3.19 with equations 3.44 and 3.45. The range of the X-axis results to be according to equation 3.47 and the range of the Y-axis according to equation 3.48.

\[
\Delta X_M = 2r_Q \frac{\cos \beta \cos \gamma + \sec \beta \sin \gamma \tan \gamma}{\sqrt{1 + (\sec \beta \tan \gamma)^2}} 
\] (3.47)

\[
\Delta Y_M = 2r_Q \frac{\cos \beta \sin \gamma + \sec \beta \cos \gamma \cot \gamma}{\sqrt{1 + (\sec \beta \cot \gamma)^2}} 
\] (3.48)

A summary of the ranges of the linear axes \( \Delta X_M \), \( \Delta Y_M \) and \( \Delta Z_M \) relative to the diameter of the tool path is given in table 3.3 for different special combinations of \( \beta \) and \( \gamma \).

3.5.2 Machines with rotary axes on the tool and on the work-piece side (swivel head and rotary table)

As described in section 3.3.6 on page 26 the position of the linear axes of machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table) is described
Table 3.3: Summary of the ranges of the linear axes for machines with both rotary axes on the tool side (double pivot spindle head) for \( r_Q = 0.5 \) for different special combinations of \( \beta \) and \( \gamma \) in the first quadrant of \( \Sigma_2 \).

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \Delta X_M )</th>
<th>( \Delta Y_M )</th>
<th>( \Delta Z_M )</th>
<th>( \sin \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>( \gamma )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( \sin \beta )</td>
</tr>
<tr>
<td>30°</td>
<td>0°</td>
<td>( \sqrt{3}/2 )</td>
<td>1</td>
<td>1/2</td>
<td>( \sin \beta )</td>
</tr>
<tr>
<td>30°</td>
<td>45°</td>
<td>( \sqrt{7}/8 )</td>
<td>( \sqrt{7}/8 )</td>
<td>1/2</td>
<td>( \sin \beta )</td>
</tr>
<tr>
<td>30°</td>
<td>90°</td>
<td>1</td>
<td>( \sqrt{3}/2 )</td>
<td>1/2</td>
<td>( \sin \beta )</td>
</tr>
</tbody>
</table>

The component in \( Z \)-direction is given in equation 3.51. The rotation of the rotary table (angle \( C \)) does not influence the position of the Z-axis. In other words, the component of \( Q \) in frame \( \Sigma_4 \) is the same as in frame \( \Sigma_2 \) (see figures 3.7 and 3.9 and equations 3.19 and 3.51). Therefore, the range of the Z-axis of machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table) is the same as for machines with both rotary axes on the tool side (equation 3.46).

\[
{\begin{align*}
4q_z &= \cos C(l \cos \alpha \sin \beta \cos \gamma + r_Q(\cos \beta \cos \varphi \cos \gamma - \sin \varphi \sin \gamma) + x_0) \\
&\quad + \sin C(l \cos \alpha \sin \beta \sin \gamma + r_Q(\cos \beta \cos \varphi \sin \gamma + \sin \varphi \cos \gamma)) \quad (3.49) \\
4q_y &= -\sin C(l \cos \alpha \sin \beta \cos \gamma + r_Q(\cos \beta \cos \varphi \cos \gamma - \sin \varphi \sin \gamma) + x_0) \\
&\quad + \cos C(l \cos \alpha \sin \beta \sin \gamma + r_Q(\cos \beta \cos \varphi \sin \gamma + \sin \varphi \cos \gamma)) \quad (3.50)
\end{align*}}
\]

The component in \( Z \)-direction is given in equation 3.51. The rotation of the rotary table (angle \( C \)) does not influence the position of the Z-axis. In other words, the component of \( Q \) in frame \( \Sigma_4 \) is the same as in frame \( \Sigma_2 \) (see figures 3.7 and 3.9 and equations 3.19 and 3.51). Therefore, the range of the Z-axis of machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table) is the same as for machines with both rotary axes on the tool side (equation 3.46).

\[
{\begin{align*}
4q_z &= l \cos \alpha \cos \beta - r_Q \sin \beta \cos \varphi \\
&\quad (3.51)
\end{align*}}
\]

The position of the Y-axis (axis parallel to the tilting axis) is equal to the Y-coordinate of
the apex of the cone. The apex \( A \) of the cone has a constant distance \( r_A \) from the axis \( C \) of the rotary table, which can be calculated according to equation 3.52 (height \( h \) of the cone according to equation 3.2). Therefore, the range of the Y-axis corresponds to the diameter of the apex \( A \) (equation 3.53).

\[
\begin{align*}
\Delta Y_M &= 2r_A \\
&= \begin{cases} 
2(x_0 + h \sin \beta) & \text{for } \alpha > \beta \\
0 & \text{for } x_0 = -h \sin \beta 
\end{cases} \text{(3.53)}
\end{align*}
\]

**Orientation angle** \( \gamma = 0^\circ \)

If the orientation angle \( \gamma \) is chosen to be zero, the movements of the X- and Y-axis of the machine are symmetric to the XZ-plane (perpendicular to the tilting axis B). In this case, the radius \( r_A \) of the apex can be simplified according to equation 3.54, which gives equation 3.55 for the range of the Y-axis.

\[
\begin{align*}
\Delta Y_M(\gamma = 0^\circ) &= \begin{cases} 
2(x_0 + h \sin \beta) & \text{for } \alpha > \beta \\
0 & \text{for } x_0 = -h \sin \beta 
\end{cases} \text{(3.55)}
\end{align*}
\]

The positions of the X-axis at the angles \( \varphi = 0^\circ \) and \( 180^\circ \) are given in equations 3.56, 3.57 and 3.58. These are the top and bottom positions of the cone. Depending on the position and orientation of the cone, the X-axis can have its minimal or maximal value.

The difference of the position of the X-axis at \( \varphi = 0^\circ \) and \( \varphi = 180^\circ \) (\( \Delta X_{Mtb} \): t for top or \( \varphi = 180^\circ \) and b for bottom or \( \varphi = 0^\circ \)) follows from equations 3.56 to 3.58 for \( \gamma = 0^\circ \) according to equation 3.59 depending on the relation between \( \alpha \) and \( \beta \).

\[
\begin{align*}
4q_x(\gamma = 0^\circ, C = \varphi = 0^\circ) &= r_Q \cos \beta + l \cos \alpha \sin \beta + x_0 \text{ (3.56)} \\
4q_x(\gamma = 0^\circ, C = \varphi = 180^\circ) &= r_Q \cos \beta - l \cos \alpha \sin \beta - x_0 \text{ for } \alpha > \beta \text{ (3.57)} \\
4q_x(\gamma = 0^\circ, C = 0^\circ, \varphi = 180^\circ) &= r_Q \cos \beta - l \cos \alpha \sin \beta + x_0 \text{ for } \alpha < \beta \text{ (3.58)}
\end{align*}
\]

\[
\begin{align*}
\Delta X_{Mtb} = 4q_x(\varphi = 0^\circ) - 4q_x(\varphi = 180^\circ) &= \begin{cases} 
2(x_0 + l \cos \alpha \sin \beta) & \text{for } \alpha > \beta \\
2l \cos \alpha \sin \beta & \text{for } \alpha < \beta
\end{cases} \text{(3.59)}
\end{align*}
\]
3. Test piece for testing simultaneous 5-axis machining

The special case where $\Delta X_{M} = 0$ for $\alpha > \beta$ and $\gamma = 0^\circ$ occurs when $x_0 = -l \cos \alpha \sin \beta$, which means that the center $M$ of the points $Q$ is on the axis $C$ of the rotary table (equation 3.21 and figure 3.9 on page 27).

**Apex $A$ on axis $C$**

If the apex $A$ of the cone is positioned on the axis $C$ of the rotary table, the movement of the linear axis depends only on the tool sided rotary axis. It is the same as for machines with both rotary axes on the tool side (see previous section 3.5.1).

- If $l = s$ or in other words $l = r/\sin \alpha$ the apex $A$ is on the axis of rotation of the tool sided rotary axis $C$. This has the following consequences:
  - $r_Q = 0$ (equations 3.17 and 3.1)
  - $\Delta Z_M = 0$
  - $\Delta X_M = \Delta Y_M = 2r_A$ (equation 3.52)

**3.5.3 Machines with both rotary axes on the workpiece side (tilting rotary table)**

The expression of equation 3.30 as a function of the cone parameters with the radial angle $\varphi$ (instead of the angles of the rotary axes) becomes very complex.

Instead, some special configurations and setups are mentioned:

- If the apex $A$ of the cone formed by the tool directions coincides with the crossing of the two rotary axes of the machine, no movement of the linear axes is required. (2-axis movement)

- If the cone axis crosses the point $O_2$ (figure 3.10, crossing of the axes B and C), the $Z$-axis of the machine has no movement. (4-axis movement)

- The range of the $Y$-axis (axis parallel to the tilting axis) corresponds to twice the distance of the apex $A$ from the C-axis.

- If the cone axis crosses the C-axis ($y_0 = 0$) the movement of the machine axes have a symmetry to the XZ-plane of frame $\sum_2$ (figure 3.10). This corresponds to the effect of an orientation angle $\gamma = 0^\circ$ described in the previous section 3.5.2.
3.6 Proposed test pieces parameters for specific machine designs

3.6.1 Recommended angular parameters for all machine setups

The range of the rotary axes should be the same on all types of machines in order to have a fair comparison. Therefore, the cone angle $\alpha$ and the inclination angle $\beta$ are recommended to be the same, independent of the machine design.

In order to have a full rotation of the rotary axis C, the cone angle $\alpha$ has to be larger than the inclination angle $\beta$. The test piece should be applicable on machines with a range of the tilting axis of 90° or more. The range required during the simultaneous movement for the cone geometry is chosen to be $2/3$ of the minimal range of 90° ($2/3$ of the range according to the experiences with testing of linear axes [38]). This is obtained with an inclination angle $\beta = 30^\circ$ (according to equation 3.34 on page 33). The range of the tilting axis is selected to be centered between 0° and 90°. For an inclination angle $\beta = 30^\circ$ a cone angle of $\alpha = 45^\circ$ results (equations 3.32 and 3.33).

- $\Delta C = 360^\circ$
  $\rightarrow \alpha > \beta$ (section 3.4.1)
- $\Delta B = 2/3 \cdot 90^\circ = 60^\circ$
  $\rightarrow \beta = 30^\circ$ (equation 3.34)
- Range of $B$ centered between 0° and 90°
  $\rightarrow B_{\min} = 45^\circ - \Delta B / 2 = 15^\circ = \alpha - \beta$ (equation 3.32)
  $\rightarrow \alpha = 45^\circ$

3.6.2 Machines with both rotary axes on the tool side (double pivot head)

With the definition of the angular parameters of the workpiece according to the previous section 3.6.1 ($\alpha = 45^\circ$, $\beta = 30^\circ$) the movements of the rotary axes are defined. A larger movement of the linear machine axes can be obtained for the same test piece with an upside down setup (see example shown in figure 3.25 for end milling). In other words, a setup where the apex is pointing away from the tool as described in section 3.5.1. Such a setup corresponds to a negative cone angle $\alpha = -45^\circ$.

- Apex of the cone pointing away from the tool for larger path of the linear axes
Figure 3.25: Visualization of the effect of an upside down configuration (left) compared to a configuration with the apex pointing in tool direction (right) for the example of end milling on a machine with double pivot head.

\[ \rightarrow \alpha = -45^\circ \]
\[ \rightarrow r_Q = r_{TCP} - l \sin \alpha = r_{TCP} + l/\sqrt{2} \]

With these parameters and the size of the workpiece, the range of the Z-axis is defined according to equation 3.46 on page 45.

\[ \rightarrow \Delta Z = r_{TCP} + l/\sqrt{2} \]

For machines with comparable ranges of the X- and Y-axis the angle \( \gamma \) of the positioning should be chose to be 45° in order to have the same range required for the path of the cone (see table 3.3 and equations 3.47 and 3.48 in section 3.5.1).

\[ \cdot \gamma = 45^\circ \]
\[ \rightarrow \Delta X = \Delta Y = r_Q \sqrt{7/2} = (r_{TCP} + l \sin \alpha) \sqrt{7/2} \]

The translational position of the test piece has no influence on the range of the machine axes for machines with both rotary axes on the tool side. It should be positioned at a typical manufacturing position and the parameters (like e.g. position, tool and process parameters) should be noted according to ISO 10791-7 [2].

A radius of the tool path of \( r_{TCP} = 40mm \) is proposed as will be described in section 3.6.4.

3.6.3 Machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table)

Since the test goal of the test piece is to show the relevant geometric errors of the machine no unnecessary machine dynamics is wanted for the movement. Therefore, the angle \( \gamma \) of the positioning is recommended to be zero in order to avoid unnecessary accelerations of the linear axes due to an asymmetric movement (section 3.5.2).
• No unnecessary accelerations of the linear machine axes
  \[ \gamma = 0^\circ \]

For a given machine design the size and position of the test piece is limited by the ranges of the machine axes and by collisions. With a rotary axis on the workpiece side, the limits of the position and size of the workpiece become more complex. To avoid collisions the test piece is recommended to be placed within the surface of the workpiece pallet on the rotary table. The height above should be chosen according to a typical manufacturing height. A radius of the tool path of \( r_{TCP} = 40 \text{mm} \) is proposed as will be described in section 3.6.4. With \( r_{TCP} \) and the angular parameters of the test piece (cone angle \( \alpha = 45^\circ \) and inclination angle \( \beta = 30^\circ \)) the movement of the Z-axis is defined for a given machine (section 3.5.2). Next to the Z-axis, also the other axis perpendicular to the swivel axis requires compensational movements. In this case, it corresponds to the X-axis. Therefore, the test piece should be placed in a way that the movement of the X-axis is larger than the one of the Y-axis because in Y-direction only the range of the rotary table is required. By positioning the test piece all the way to the front, these goals are obtained. The proposed value for \( x_0 \) results according to equation 3.60. In this case, the extreme positions of the X-axis correspond to the top and bottom positions mentioned in section 3.5.2 and therefore the range of the X-axis results to be according to equation 3.59.

\[
x_0 = r_{\text{Table}} - r_{TCP} \cos \beta
\]  \hspace{1cm} (3.60)

Next to machine setups with a portal (like the one shown in figure 3.8) also machines with only one column are common for setups with swivel head and rotary table (figure 2.7). These types of machines are usually limited in the range of the linear axis perpendicular to the swivel axis. If the ranges of the axes are limited in a way that a configuration with cone angle \( \alpha = 45^\circ \) and inclination angle \( \beta = 30^\circ \) is not possible a setup with an inclination angle \( \beta = 60^\circ \) instead of 30° is proposed. That way the range of the swivel axis on the tool side is the same but the required range of the linear axis perpendicular to the swivel axis is only on one half of the diameter of the rotary table. The range of the axis C of the rotary table however is reduced to about 110° according to equation 3.39.

### 3.6.4 Machines with both rotary axes on the workpiece side (tilting rotary table)

The proposed test piece for machines with both rotary axes on the workpiece side (tilting rotary table) is presented for the analyzed vertical machining center shown in figure 3.1
Since the test goal of the test piece is to show the relevant geometric errors of the machine no unnecessary machine dynamics is wanted for the movement. Therefore, the lateral displacement $y_0$ is recommended to be zero to avoid unnecessary dynamics of the linear axes due to an asymmetric movement.

- No unnecessary accelerations of the linear machine axes
  $\rightarrow$ symmetric movement
  $\rightarrow y_0 = 0$

The diameter of the cone limits the range of the linear axes because with an increase of the diameter, the cone has to be positioned closer to the crossing of the two rotary axes (point $O_2$ in figure 3.10). In order to be still well manageable (fixation, measurement) the radius of the tool path is proposed to be $r_{TCP} = 40mm$.

- Test piece small but still well manageable
  $\rightarrow r_{TCP} = 40mm$

To avoid collisions the test piece is recommended to be placed within the surface of the workpiece pallet on the rotary table as mentioned in the previous section. In this case, the radius of the table is $r_{Table} = 100mm$. The movement of the linear axis parallel to the tilting axis (Y-axis) is not influenced by the motion of the tilting axis B. Thus, it only needs a range over the surface of the workpiece pallet. The other two axes (X and Z) need a larger range of movement for the compensational movements. Therefore, the range of the linear axes should be larger for the X- and Z-axis than for the Y-axis. This can be reached by placing the cone at a negative distance $x_0 = -r_{Table} + r_{TCP} \cos \beta = -65mm$ and at a maximal height above the tilting axis B.

- Maximize movement of X-axis
  $\rightarrow x_0 = -r_{Table} + r_{TCP} \cos \beta = -65mm$

For the given ranges a height $z_0 = 150mm$ is chosen in order to stay within the limits of the ranges of the X- and Z-axes.

- Maximize movement of Z-axis within its range for typical total tool length of 150mm
  $\rightarrow z_0 = 150mm$

The resulting movements of the machine axes are shown in figure 3.26 as a function of the radial angle $\varphi$. To give a better idea of the path of the TCP, figure 3.27 shows the path relative to the machine frame.
Figure 3.26: Positions of the machine axes (linear in $mm$ and rotary in $^\circ$) as a function of the radial cone angle $\varphi$ for the position $^2M = [-65, 0, 150]$. 
Figure 3.27: Illustration of the cone position in the machine coordinate system (mm) when the TCP is at the bottom position of the cone ($\varphi = 0^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
3.6 Proposed test piece parameters for specific machine designs

Figure 3.28: Three views of the illustration of the cone position in the machine coordinate system (mm) when the TCP is at the bottom position of the cone (\(\varphi = 0^\circ\)). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
Figure 3.29: Illustration of the cone position in the machine coordinate system (mm) when the TCP is at $Y_{M_{\text{max}}}$ ($\varphi = 54.7^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
3.6 Proposed test piece parameters for specific machine designs

Figure 3.30: Three views of the illustration of the cone position in the machine coordinate system (mm) when the TCP is at \( Y_{M_{\text{max}}} \) (\( \phi = 54.7^\circ \)). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
Figure 3.31: Illustration of the cone position in the machine coordinate system (mm) when the TCP is at the top position of the cone ($\varphi = 180^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
3.6 Proposed test piece parameters for specific machine designs

Figure 3.32: Three views of the illustration of the cone position in the machine coordinate system (mm) when the TCP is at the top position of the cone ($\varphi = 180^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
Figure 3.33: Illustration of the cone position in the machine coordinate system (mm) when the TCP is at $Y_{M_{\text{min}}}$ ($\varphi = -54.7^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
3.6 Proposed test piece parameters for specific machine designs

Figure 3.34: Three views of the illustration of the cone position in the machine coordinate system (mm) when the TCP is at $Y_{M_{\text{min}}}$ ($\varphi = -54.7^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
3.7  Modeling of location errors of the machine axes

The location errors represent the geometric errors of the position and orientation of the least squares directions of the machine axes at a functional point. A set of location errors can be set to zero depending on the chosen coordinate system of a machine. How the location errors are modeled is described for the analyzed machine configuration with structure code t-(C)-Z-Y-b-X-B-C-w shown in figure 3.1 on page 16.

3.7.1  Orientation errors

The first main direction is defined by the direction of the longest linear axis on the bed, which corresponds to the X-axis. Therefore, the X-axis has no errors of orientation:

- $B_0X = 0$
- $C_0X = 0$

The second main direction is defined by the nominal direction of the second linear axis on the bed, which corresponds to the Y-axis. It defines the nominal XY-plane and the nominal direction of the Z-axis. Therefore, the orientation of the Y-axis relative to the Z-axis has no error:

- $A_0Y = 0$

The remaining 7 errors of orientation are:

- $C_0Y$: squareness between the Y- and X-axis
- $A_0Z$: squareness between the Z- and Y-axis
- $B_0Z$: squareness between the Z- and X-axis
- $A_0B$: parallelism between the B- and the Y-axis (angle A)
- $C_0B$: squareness between the B- and X-axis
- $A_0C$: squareness between the C- and Y-axis
- $B_0C$: squareness between the C- and X-axis

3.7.2  Position errors

For the definition of the origin of the coordinate system of the machine, one position errors can be set to zero for each direction. In this case the following errors were set to zero:
• $X0X = 0$
• $Y0C = 0$
• $Z0B = 0$
• $B0B$ directly influences $B0C$ and is therefore not looked at separately.
• $C0B$ has no influence on this machine, as long as no reference slots on the rotary table have to be considered; therefore $C0C$ is not looked at separately.

3.7.3 Modeling of orientation errors

A squareness error changes the direction of a machine axis. The squareness error $C0Y$ of the Y-axis relative to the X-axis changes the direction of the Y-axis. This has an influence on the X- and Y-coordinates of the TCP. In this case, the reference is the X-axis and therefore the influence on the position in Y-direction is negligible for small errors. The error of the position of the X-axis $\Delta X(C0Y)$ of the machine due to a squareness error $C0Y$ follows according to equation 3.61. Errors of parallelism are modeled the same way.

$$\Delta X(C0Y) = -Y_M \cdot C0Y$$  (3.61)

3.8 Modeling of component errors of the machine

Component errors describe motion errors of the individual machine axes. The motion errors of a machine axis are described with 6 component errors. An example of the 6 component errors is shown for a horizontal X-axis in figure 3.35 [21] and for the vertical C-axis in figure 3.36 [21].

3.8.1 Straightness error motions

Straightness of the motion in X- and Z-direction

The motion in X- and Z-direction is not symmetric to the origin of the machine coordinate system. To analyze the effects of a straightness error of the motion in X- or Z-direction, the straightness is modeled as a first harmonic between the minimal and the maximal value of the path (figure 3.37). Equation 3.63 shows how the Y-coordinates are changed due to a horizontal straightness error of the motion in X-direction.
Figure 3.35: Illustration of the component errors of the motion in X-direction [21].

Figure 3.36: Illustration of the component errors of the motion of the rotary axis C [21].

- $\Delta X_M$: nominal length of total movement of the X-axis
- $EYX_{1h}$: amplitude of the horizontal straightness deviation of the movement in X-direction
- $X_{M_{\text{min}}}$: minimal value of the X-axis of the machine throughout the entire movement
- $X_{M_{\text{max}}}$: maximal value of the X-axis of the machine throughout the entire movement
### 3.8 Modeling of component errors

\[
\Delta X_M = X_{M_{\text{max}}} - X_{M_{\text{min}}} \quad (3.62)
\]

\[
\Delta Y(EYX_{1h}) = EYX_{1h} \cos \left( \pi \frac{X_M - X_{M_{\text{min}}} - \Delta X_M/2}{\Delta X_M} \right) \quad (3.63)
\]

Figure 3.37: Straightness error motion \( EYX \) modeled as a first harmonic between the minimal and the maximal value of the path.

#### Straightness of the motion in Y-direction

The motion in Y-direction is symmetric to the origin of the machine coordinate system. To analyze the effects of a straightness error of the motion in Y-direction, the straightness is modeled as a first harmonic between the minimal and the maximal value of the path (figure 3.38). Equation 3.65 shows how the X-coordinates are changed due to a horizontal straightness error of the motion in Y-direction.

- \( \Delta Y_M \): nominal length of total movement of the Y-axis
- \( EYX_{1h} \): amplitude of the horizontal straightness deviation of the movement in Y-direction
- \( Y_{M_{\text{min}}} \): minimal value of the Y-axis of the machine throughout the entire movement
- \( Y_{M_{\text{max}}} \): maximal value of the Y-axis of the machine throughout the entire movement

\[
\Delta Y_M = Y_{M_{\text{max}}} - Y_{M_{\text{min}}} = 2Y_{M_{\text{max}}} \quad (3.64)
\]

\[
\Delta X(EYX_{1h}) = EYX_{1h} \left( \cos \left( \pi \frac{Y_M - Y_{M_{\text{min}}} - \Delta Y_M/2}{\Delta Y_M} \right) - 1 \right) \quad (3.65)
\]
3.8.2 Positioning errors

To analyze the effects of a positioning error it is modeled with a linear dependency from the position. Equation 3.66 shows how the X-coordinates are changed as a function of the motion in X-direction (see also figure 3.39). Linear positioning errors are typical for rotary axes with indirect measurement systems.

- \( EXX_{\text{lin}} \): amount of positioning error in X-direction on the length of the movement in X-direction (linear dependency).

\[
\Delta X( EXX_{\text{lin}} ) = EXX_{\text{lin}} \frac{X_M}{\Delta X_M} \tag{3.66}
\]

3.8.3 Tilt error motions

To analyze the effects of the tilt error motions of the linear machine axes, the errors are modeled with a linear component.

The offsets are relative to the origin of the machine frame \( \sum_M \) in order not to introduce additional errors already included in the other geometric errors.
Motion of the workpiece sided X-axis

**EAX** The tilt error motion \( EAX \) corresponds to a roll error of a movement of the workpiece sided machine axis \( X \) (tilt around the axis of movement \( X \)).

For the analysis of the effect of \( EAX \), the error is modeled with linear dependency of the X-position of the machine.

An offset in the horizontal Y-direction causes an error mainly in Z-direction (\( \Delta Z(EAX_{lin}) \)) with a linear dependence of the X-position for the assumption of a linear error dependency (equation 3.68).

An offset in the vertical Z-direction causes an error mainly in Y-direction (\( \Delta Y(EAX_{lin}) \)) with a linear dependence of the X-position for the assumption of a linear error dependency (equation 3.67).

\[
\Delta Y(EAX_{lin}) = Z_M \cdot EAX_{lin} \cdot X_M \quad (3.67)
\]
\[
\Delta Z(EAX_{lin}) = Y_M \cdot EAX_{lin} \cdot X_M \quad (3.68)
\]

**EBX** The tilt error motion \( EBX \) corresponds to a pitch error of a movement of the workpiece sided machine axis \( X \) (tilt around the horizontal Y-axis).

For the analysis of the effect of \( EBX \), the error is modeled with linear dependency of the X-position of the machine.

Since the X-axis of the machine is on the workpiece side a tilt error motion causes an error in the vertical Z-direction (\( \Delta Z(EBX_{lin}) \)) with a quadratic dependence of the X-position for the assumption of a linear error dependency (linear for the amount of the error and linear for the offset as shown in equation 3.70).

An offset in the horizontal Y-direction has no influence.

An offset in the vertical Z-direction causes an error mainly in X-direction (\( \Delta X(EBX_{lin}) \)) with a linear dependence of the X-position for the assumption of a linear error dependency (equation 3.69).

\[
\Delta X(EBX_{lin}) = Z_M \cdot EBX_{lin} \cdot X_M \quad (3.69)
\]
\[
\Delta Z(EBX_{lin}) = -X_M \cdot EBX_{lin} \cdot X_M \quad (3.70)
\]
**ECX** The tilt error motion \( ECX \) corresponds to a yaw error of a movement of the workpiece sided machine axis \( X \) (tilt around the vertical \( Z \)-axis).

For the analysis of the effect of \( ECX \), the error is modeled with linear dependency of the \( X \)-position of the machine.

Since the \( X \)-axis of the machine is on the workpiece side a tilt error motion causes an error in the horizontal \( Y \)-direction \( (\Delta Y(\text{ECX}_{\text{lin}})) \) with a quadratic dependence of the \( X \)-position for the assumption of a linear error dependency (linear for the amount of the error and linear for the offset as shown in equation 3.72).

An offset in the horizontal \( Y \)-direction causes an error mainly in \( X \)-direction \( (\Delta X(\text{ECX}_{\text{lin}})) \) with a linear dependence of the \( X \)-position for the assumption of a linear error dependency (equation 3.71).

An offset in the vertical \( Z \)-direction has no influence.

\[
\begin{align*}
\Delta X(\text{ECX}_{\text{lin}}) &= -Y_M \cdot \text{ECX}_{\text{lin}} \cdot X_M \quad (3.71) \\
\Delta Y(\text{ECX}_{\text{lin}}) &= X_M \cdot \text{ECX}_{\text{lin}} \cdot X_M \quad (3.72)
\end{align*}
\]

**Motion of the tool sided \( Y \)- and \( Z \)-axis**

The position of the workpiece in the horizontal \( X \)-direction has no effect on the error caused by the tilt error motions of the tool sided axes \( Y \) and \( Z \).

**EAY** The tilt error motion \( EAY \) corresponds to a tilt error around the \( X \)-axis depending on the motion in \( Y \)-direction.

For the analysis of the effect of \( EAY \), the error is modeled with linear dependency of the \( Y \)-position of the machine.

An offset in the vertical \( Z \)-direction causes an error mainly in \( Y \)-direction \( (\Delta Y(\text{EAY}_{\text{lin}})) \) with a linear dependence of the \( Y \)-position for the assumption of a linear error dependency (equation 3.73).

For small tool diameters, the change in \( Y \)-direction is negligible. Therefore, the error in \( Z \)-direction is of second order.

\[
\Delta Y(\text{EAY}_{\text{lin}}) = Z_M \cdot \text{EAY}_{\text{lin}} \cdot Y_M \quad (3.73)
\]
EBY  The tilt error motion EBY corresponds to a tilt error around the axis of motion Y. For the analysis of the effect of EBY, the error is modeled with linear dependency of the Y-position of the machine.

An offset in the vertical Z-direction causes an error mainly in X-direction ($\Delta X(EBY_{lin})$) with a linear dependence of the Y-position for the assumption of a linear error dependency (equation 3.74).

For small tool diameters, the change in X-direction is negligible. Therefore, the error in Z-direction is of second order.

$$\Delta X(EBY_{lin}) = Z_M \cdot EBY_{lin} \cdot Y_M$$  \hspace{1cm} (3.74)

ECY  The tilt error motion ECY corresponds to a tilt error around the Z-axis depending on the motion in Y-direction.

With the Z-axis corresponding to the tool axis a tilt error motion around the tool axis has no influence on the path of the TCP.

Tilt error motion of the workpiece sided tilting axis B

The tilt error motions EAB and ECB of the workpiece sided tilting axis B changes the direction of the tilting axis B depending on its angle B.

EAB  The tilt error motion EAB of the workpiece sided tilting axis B changes the angle A of the tilting axis B depending on its angle B.

For the analysis of the effect of EAB the error magnitude is modeled with a first harmonic dependency of its angle B and is therefore abbreviated as $EAB_{1h}$.

The resulting erroneous angle A of the axis B as a function of the angle B is given in equation 3.75 and shown in figure 3.40 for the proposed setup with the tilting angle B from $B_{min} = -75^\circ$ to $B_{max} = -15^\circ$.

$$\Delta AB(EAB_{1h}) = EAB_{1h} \cos \left( \frac{2\pi B}{\Delta B} \right)$$  \hspace{1cm} (3.75)
The tilt error motion $E_{CB}$ of the workpiece sided tilting axis $B$ changes the angle $C$ of the tilting axis $B$ depending on its angle $B$. It is modeled in the same way as the previously described tilt error motion $E_{AB}$.

**Tilt error motion of the workpiece sided axis $C$ of the rotary table**

The tilt error motions $E_{AC}$ and $E_{BC}$ of the workpiece sided axis $C$ of the rotary table changes the direction of the axis $C$ depending on its angle $C$.

$E_{AC}$ The tilt error motion $E_{AC}$ of the workpiece sided tilting axis $C$ changes the angle $A$ of the axis $C$ depending on its angle $C$. It is modeled in the same way as the previously described tilt error motion $E_{AB}$.

$E_{BC}$ The tilt error motion $E_{BC}$ of the workpiece sided tilting axis $C$ changes the angle $B$ of the axis $C$ depending on its angle $C$. It is modeled in the same way as the previously described tilt error motion $E_{AB}$.

### 3.8.4 Radial error motions of the rotary axes

Radial error motions of the rotary axes describe errors motions of the rotary axes in the directions perpendicular to the axes of rotation.

$E_{XB}$ The radial error motion $E_{XB}$ describes the error motion of the rotary axis $B$ in the direction $X$.

For the analysis of the effect of $E_{XB}$ the error magnitude is modeled with a second harmonic dependency of its angle $B$ and is therefore abbreviated as $E_{XB_{2h}}$. The resulting offset $\Delta XB$ in X-direction of the rotary axis $B$ is described according to equation 3.76.
\( \Delta XB(\Delta XB_{2h}) = \Delta XB_{2h} \cos \left( 4\pi \frac{B}{\Delta B} \right) \) (3.76)

**EZB** The radial error motion \( EZB \) describes the error motion of the rotary axis B in the direction Z. It results in an offset \( \Delta ZB \) in Z-direction of the rotary axis B depending on its angle \( B \). It is modeled the same way as the previously described error motion \( EXB \).

**EXC** The radial error motion \( EXC \) describes the error motion of the rotary axis C in the direction X.

For the analysis of the effect of \( EXC \) the error magnitude is modeled with a second harmonic dependency of its angle \( C \) and is therefore abbreviated as \( EXC_{2h} \). The resulting offset \( \Delta XC \) in X-direction of the rotary axis C is described according to equation 3.77.

\( \Delta XC(EXC_{2h}) = \Delta XC_{2h} \cos \left( 4\pi \frac{C}{\Delta C} \right) \) (3.77)

**EYC** The radial error motion \( EYC \) describes the error motion of the rotary axis C in the direction Y. It results in an offset \( \Delta YC \) in Y-direction of the rotary axis C depending on its angle \( C \). It is modeled the same way as the previously described error motion \( EXC \).

### 3.8.5 Axial error motions of the rotary axes

Axial error motions of the rotary axes describe error motions of the rotary axes in the directions parallel to the axis of rotation.

**EYB** The axial error motion \( EYB \) describes the error motion of the rotary axis B in the direction Y parallel to the axis of rotation.

For the analysis of the effect of \( EYB \) the error magnitude is modeled with a first harmonic dependency of its angle \( B \) and is therefore abbreviated as \( EYB_{1h} \). The resulting offset \( \Delta YB \) in Y-direction of the rotary axis B is described according to equation 3.78.

\( \Delta YB(EYB_{1h}) = \Delta YB_{1h} \cos \left( 2\pi \frac{B}{\Delta B} \right) \) (3.78)
EZC The axial error motion EZC describes the error motion of the rotary axis C in the direction parallel to the axis of rotation. It is modeled the same way as the previously described error motion EYB.

3.9 Theoretical influence of geometric machine errors on a workpiece

For the simulation of the geometric errors and identifying the resulting test piece errors, three basic steps are carried out [22, 39]:

1. Transform points of the workpiece geometry into the nominal kinematic model of the machine.

2. Transform points back into the workpiece coordinate system with a general kinematic model containing all potential geometric errors.

3. Identify the resulting cone form:

   (a) Least squares plane and circle through the actual coordinates of the points.
   
   (b) Cone axis through center of least squares circle normal to least squares plane.
   
   (c) Least squares circle evaluation of the distances of points from cone axis in direction normal to cone surface results in measure for cone form deviation (equivalent cone form).

For the proposed example with a cone angle $\alpha = 45^\circ$ the special case shown in figure 3.41 results for the identification of the equivalent cone form. In this case, the point $C$ has a distance from the center of the least squares circle along the cone axis equivalent to the least squares radius $r$ of the actual coordinates of the points. By evaluating the distances of the actual coordinates of the points from the point $C$, the equivalent cone form is obtained.

3.10 Effects of single geometric machine errors on the cone form

In this section, the effect of single geometric errors are analyzed for the one example described in section 3.6.4 (end milled cone frustum with cone angle $\alpha = 45^\circ$, inclination angle $\beta = 30^\circ$ for the machine with tilting rotary table shown in figure 3.1 on page 16).
3.10 Effects of single geometric machine errors on the cone form

The effect of an individual error depends on the error magnitude and on the direction of the error. The component of the direction of the error in the sensitive direction (i.e. normal to the cone surface) affects the form of the cone. However, only changing effects can be seen as a form deviation since constant effects just change the cone angle or the cone position.

3.10.1 Components of the normal direction

When analyzing the components of the normal directions \((n_x, n_y, n_z)\) of the cone surface in the machine frame as a function of the radial cone angle \(\varphi\), the following points can be seen (figure 3.42):

- \(n_x\): the component of the normal direction of the cone surface in X-direction is always positive and has a sensitivity between 1 and 0.7.
- \(n_y\): the component of the normal direction of the cone surface in Y-direction has a sensitivity between \(-0.7\) and 0.7.
- \(n_z\): the normal direction of the cone surface has no component in Z-direction since it corresponds to the tool direction (sensitivity = 0). Therefore, any error acting only in Z-direction has no effect on the cone form.

Figure 3.41: Special case with cone angle \(\alpha = 45^\circ\) for the evaluation of the resulting cone form.
3. Test piece for testing simultaneous 5-axis machining

3.10.2 Radial representation of the position of the machine axes

For the analysis of the effects of geometric machine errors with linear dependencies from the position of the machine axes, the analysis of a radial representation of the position of the machine axes can be used.

Figure 3.43 shows the radial representation of the positions of the machine axes as a function of the radial cone angle $\varphi$ for the analyzed example (compare with the linear representation as a function of the radial cone angle $\varphi$ shown in figure 3.26 on page 53). The lines in figure 3.43 represent the extreme values of the positions of the axes. All except the Y-axis $Y_M$ have their extreme values either at the top or bottom position.

3.10.3 Resulting cone forms

Constant geometric errors acting in X- or Y-direction

The different sensitivities of the components of the normal vectors transform geometric errors accordingly (section 3.10.1). For a constant geometric error in X-direction (e.g. $X_0X$), the deviations from the cone depend only on the sensitivity of the normal direction of the cone surface in X-direction. The result can be analyzed with a radial evaluation of...
Figure 3.43: Radial representation of the positions of the machine axes as a function of the radial cone angle $\varphi$ for the analyzed example.
the normal direction $n_x$. The results are shown in figure 3.44 for a positive error (red) and a negative error (blue). The manufactured test piece can only be evaluated in respect to its circular form, which transforms the deviations shown in figure 3.44 (a) to the deformations shown in figure 3.44 (b).

![Figure 3.44: Effect of constant error in X-direction for positive error (red) and for negative error (blue): (a) radial deviation, (b) circular form evaluation.](image)

For constant errors acting in positive or negative Y-direction the same evaluation is shown in figure 3.45.

![Figure 3.45: Effect of constant error in Y-direction for positive error (red) and for negative error (blue): (a) radial deviation, (b) circular form evaluation.](image)

**Hysteresis of the positioning movement**

The effect of a hysteresis of a positioning movement depends on the sensitivity of the normal direction of the cone surface in direction of movement at the position of reversal.
of the movement.

The movement of the X-axis reverses at the top ($\varphi = 180^\circ$) and bottom ($\varphi = 0^\circ$) position of the cone as seen in figure 3.43. The normal direction of the cone surface is in positive X-direction at both positions as can be seen in the figures 3.42, 3.28 and 3.32. A cone form error resulting from a hysteresis of the positioning of the X-axis $E_{XX_{hyst}}$ is shown in figure 3.46 (a).

The movement Y-axis is the only one that does not reverse at the top or bottom position as shown in figure 3.43 (see also figures 3.30 and 3.34). A cone form error resulting from a hysteresis of the positioning of the Y-axis $E_{YY_{hyst}}$ is shown in figure 3.46 (b).

Figure 3.46: Examples of circular form evaluations resulting from a hysteresis of the positioning of the: (a): X-axis, (b): Y-axis.

**Too long movement in X-direction**

The effect of tool long movements in X-direction is influenced by the sensitivity of $n_x$ and by the dependency of the error magnitude. Figure 3.47 shows the effect of too long (red) and too short (blue) movement in X-direction with linear dependency from the X-position (e.g. $E_{XX_{lin}}$ or $E_{BX_{lin}}$). The manufactured test piece can only be evaluated in respect to its circular form, which transforms the deviations shown in figure 3.47 (a) to the deformations shown in figure 3.47 (b).

**Too long movement in Y-direction**

The effect of tool long movements in Y-direction is influenced by the sensitivity of $n_y$ and by the dependency of the error magnitude. Figure 3.48 shows the effect of too long
Figure 3.47: Effect of too long (red) and too short (blue) movement in X-direction with linear dependency from the X-position (e.g. $EXX_{lin}$ or $EBX_{lin}$): (a) radial deviation, (b) circular form evaluation.

Figure 3.48: Effect of too long (red) and too short (blue) movement in Y-direction with linear dependency from the Y-position (e.g. $EYY_{lin}$ or $EAY_{lin}$). The manufactured test piece can only be evaluated in respect to its circular form, which transforms the deviations shown in figure 3.48 (a) to the deformations shown in figure 3.48 (b).

Figure 3.49: Effect of a non square movement of the Y-axis relative to the X-axis (e.g. $COY$ or $EBY_{lin}$) is influenced by the sensitivity of $n_x$ and by the dependency on the error magnitude. Figure 3.49 shows the effect of a non square movement of the Y-axis relative to the X-axis.
3.10 Effects of single geometric machine errors on the cone form
to the X-axis with linear dependency from the Y-position. The red curve represents an
eexample for \( +C0Y \) and the blue one for \( -C0Y \). The manufactured test piece can only
be evaluated in respect to its circular form, which transforms the deviations shown in
figure 3.48 (a) to the deformations shown in figure 3.48 (b).

![Figure 3.49](image)

Figure 3.49: Effect of a non square movement of the Y-axis relative to the X-axis with linear dependency
from the Y-position (red: \( +C0Y \), blue: \( -C0Y \)): (a) radial deviation, (b) circular form evaluation.

3.10.4 Typical cone form deviations
For many of the analyzed groups of errors two types of cone form deviations result that
are analyzed more in detail.

Type 1: top-bottom
As form error type “top-bottom” the circular form evaluations shown in part (b) of fig-
ures 3.44, 3.47 and 3.48 is identified. The location errors and the linear component errors
that result in a form error type “top-bottom” are the following: \( B0Z \), \( B0C \), \( X0B \), \( X0C \),
\( EXX_{lin} \), \( EBX_{lin} \), \( EYY_{lin} \), \( EAY_{lin} \), \( ECC_{lin} \).

Type 2: asymmetric
As form error type “asymmetric” the circular form evaluations shown in part (b) of fig-
ures 3.45 and 3.49 is identified. It results for errors that either act in Y-direction or depend
linearly from the Y-position. The following location errors and linear component errors
result in a form error type “asymmetric”: \( C0Y \), \( EBY_{lin} \), \( A0Z \), \( A0B \), \( A0C \), \( Y0Y \), \( ECX_{lin} \),
\( EAX_{lin} \). They all act in Y-direction or depend linearly on the Y-position (\( C0Y \)). For
errors that act in Y-direction and depend linearly on the Y-position, the form error type “top-bottom” results as seen for $EY_Y\text{lin}$. Figure 3.50 shows a direct comparison of the two identified form error types.

![Figure 3.50: Comparison of two identified form error types: (1): form error type 1 (top-bottom), (2): form error type 2 (asymmetric).](image)

**Other types** Especially if analyzing higher order dependencies of the magnitudes of the geometric errors other cone form errors can result. The presented analysis with one specific example shows an approach for a better understanding of the effects of geometric errors on the cone form.

### 3.10.5 Geometric errors with negligible effects on an end milled cone form

The errors that show no relevant effect on the cone form are the following:

- Linear errors acting in tool direction $EZX$, $EZY$, $EZ\bar{Z}$, $Z0Z$, $EZB$:
  - The vertical straightness error motions ($EZX$, $EZY$), the positioning errors of the Z-axis ($EZ\bar{Z}$), the zero position of the Z-axis ($Z0Z$) and the vertical component of the radial error motion of the B-axis ($EZB$) represent geometric errors that act only in Z-direction. For the analyzed setup, the displacements due to these errors are in tool direction and therefore tangent to the cone surface. Therefore, these errors have no influence on an end milled cone surface nor on any end milled surface for machines with tilting rotary table setup.

- Hysteresis of the positioning error of the rotary table $ECC_{hyst}$
3.10 Effects of single geometric machine errors on the cone form

- The axis C of the rotary table has no reversal point for setups with cone angles $\alpha$ larger than the inclination angle $\beta$. Therefore, the hysteresis of the positioning error of the rotary table cannot have any influence. If an analysis of the influence of $ECC_{hyst}$ on the cone form is desired, a setup with $\alpha < \beta$ needs to be chosen.

- Tilt error motions of the Z-axis around the TCP $EAZ, EBZ$

- Tilt error motions of the Z-axis (parallel to tool axis) around the TCP have negligible influence on the cone form for a small cone height of 5mm.

- Tilt error motions of tool sided linear axes around the tool axis $ECY, ECZ$

- Y yaw error motion $ECY$ and Z roll error motion $ECZ$ are rotations around the tool axis and have no additional influence on the workpiece geometry (additional to straightness error motions) since both axes are on the tool side.

- Orientation $C0B$

- The squareness error between the B- and the X-axis ($C0B$) has negligible influence on the cone surface because $B_{\text{max}}$ and $B_{\text{min}}$ are reached at $Y_M = 0$ where the sensitive direction is in X and not in Y direction.

- Tilt error motions $EAC, EBC, ECB$:

- The C tilt error motions $EAC$, $EBC$ and B tilt error motion $ECB$ have no additional influence to radial error motion of the axis C, respectively B for small cone heights.

3.10.6 Summary of results of errors with an influence on an end milled cone surface

In table 3.4 the numeric values of the simulation of the effect of individual geometric errors are summarized for the described example. It includes the assumed magnitudes of the individual errors and the resulting equivalent cone form errors. All errors with an effect larger than $0.5\mu m$ on the equivalent cone form are included in the table. The differences in the resulting effects give information about the sensitivity of the resulting equivalent cone form errors to the individual geometric machine errors.
Table 3.4: Summary of the effects of singular geometric errors with considerable effect on the cone form for the example test piece.

<table>
<thead>
<tr>
<th>Error</th>
<th>Equivalent cone form errors (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C0Y 30 µm/m</td>
<td>1.0</td>
</tr>
<tr>
<td>A0Z 30 µm/m</td>
<td>3.5</td>
</tr>
<tr>
<td>B0Z 30 µm/m</td>
<td>1.0</td>
</tr>
<tr>
<td>A0B 30 µm/m</td>
<td>3.5</td>
</tr>
<tr>
<td>A0C 30 µm/m</td>
<td>2.0</td>
</tr>
<tr>
<td>B0C 30 µm/m</td>
<td>1.0</td>
</tr>
<tr>
<td>X0B 10 µm</td>
<td>3.3</td>
</tr>
<tr>
<td>Y0Y 10 µm</td>
<td>5.8</td>
</tr>
<tr>
<td>X0C 10 µm</td>
<td>3.3</td>
</tr>
<tr>
<td>EXX_{hyst} 10 µm</td>
<td>9.9</td>
</tr>
<tr>
<td>EYY_{hyst} 10 µm</td>
<td>10.0</td>
</tr>
<tr>
<td>EBB_{hyst} 50 µm/m</td>
<td>13.3</td>
</tr>
<tr>
<td>EXX_{lin} 10 µm over ΔX_M</td>
<td>2.0</td>
</tr>
<tr>
<td>EYY_{lin} 10 µm over ΔY_M</td>
<td>4.0</td>
</tr>
<tr>
<td>EBB_{lin} 50 µm/m over ΔB</td>
<td>0.6</td>
</tr>
<tr>
<td>ECC_{lin} 50 µm/m over ΔC</td>
<td>0.7</td>
</tr>
<tr>
<td>EYX_{1h} 10 µm over ΔX_M</td>
<td>8.1</td>
</tr>
<tr>
<td>EXY_{1h} 10 µm over ΔY_M</td>
<td>8.4</td>
</tr>
<tr>
<td>EXZ_{1h} 10 µm over ΔZ_M</td>
<td>8.1</td>
</tr>
<tr>
<td>EYZ_{1h} 10 µm over ΔZ_M</td>
<td>3.9</td>
</tr>
<tr>
<td>EAX_{lin} 30 µm/m over ΔX_M</td>
<td>0.8</td>
</tr>
<tr>
<td>EBY_{lin} 30 µm/m over ΔY_M</td>
<td>5.7</td>
</tr>
<tr>
<td>EBX_{lin} 30 µm/m over ΔX_M</td>
<td>2.8</td>
</tr>
<tr>
<td>ECX_{lin} 30 µm/m over ΔX_M</td>
<td>0.9</td>
</tr>
<tr>
<td>EAY_{lin} 30 µm/m over ΔY_M</td>
<td>4.2</td>
</tr>
<tr>
<td>EXB_{2h} 10 µm over ΔB</td>
<td>9.8</td>
</tr>
<tr>
<td>EXC_{2h} 10 µm over ΔC</td>
<td>11.1</td>
</tr>
<tr>
<td>EYC_{2h} 10 µm over ΔC</td>
<td>16.7</td>
</tr>
<tr>
<td>EYB_{1h} 10 µm over ΔB</td>
<td>8.6</td>
</tr>
<tr>
<td>EZC_{1h} 10 µm over ΔC</td>
<td>1.2</td>
</tr>
<tr>
<td>EAB_{1h} 30 µm/m over ΔB</td>
<td>5.7</td>
</tr>
</tbody>
</table>
3.11 Effects of combinations of geometric machine errors on the cone form

A machine has always a combination of geometric errors. To analyze what errors to expect on a machine type with given ranges of geometric errors, random combinations of geometric errors are assumed and the resulting errors are calculated according to [39]. The ranges of the geometric errors were determined mainly according to the tolerances stated in the acceptance protocol of the manufacturer. Moreover, assumptions based on knowledge about the errors of the specific machine were used for the definition. The distributions of the geometric errors are assumed to be uniform. Next to the linear dependencies of errors from the nominal axes position, also higher order dependencies were modeled. Table 3.9 show a list of all errors with the numerical values of the lower and the upper limit of the uniform ranges (starting on page 92).

Figure 3.51 shows the distribution of resulting errors of the test piece due to 1000 random combinations of geometric errors. It represents a visualization of the distribution in form of Box Plots, which are a widely used tool in exploratory data analysis and in preparing visual summaries for statisticians and nonstatisticians alike [40].

Five values from the calculated data are visualized:

- Upper and lower extremes: the two lines on top and on the bottom of each box plot represent the extremes of the data.
- Median: the red line inside the box represents the median of the data.
- Upper and lower quartiles: the upper and lower limits of the blue boxes represent the quartiles of the data distributions (also known as hinges).

The numerical values of these parameters of the distribution are summarized in table 3.5.

**Median** is defined as *the central value in a set of observations ordered by value, dividing the ordered set into two equal parts; the argument of the cumulative distribution function of a random variable corresponding to a probability of one-half [35].*

**Quartile** is defined as *the argument of the cumulative distribution function corresponding to a probability of either 1/4 (first or lower quartile) or 3/4 (third or upper quartile); (of a sample) the value below which occurs a quarter (first or lower quartile) or three-quarters (third or upper quartile) of the observations in the ordered set of observations. A measure of the variability of a distribution or data set [35].*

The following conclusions can be drawn from this analysis:
Figure 3.51: Box Plot of the errors resulting from 1000 random combinations of geometric machine errors.

Table 3.5: Numerical values of the distribution of the errors resulting from 1000 random combinations of geometric machine errors.

<table>
<thead>
<tr>
<th></th>
<th>Circular form error of cylinder</th>
<th>Equivalent cone form error</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum</td>
<td>µm 13.6</td>
<td>µm 23.9</td>
</tr>
<tr>
<td>upper quartile</td>
<td>µm 6.7</td>
<td>µm 11.0</td>
</tr>
<tr>
<td>median</td>
<td>µm 5.2</td>
<td>µm 8.8</td>
</tr>
<tr>
<td>lower quartile</td>
<td>µm 4.1</td>
<td>µm 6.9</td>
</tr>
<tr>
<td>minimum</td>
<td>µm 1.3</td>
<td>µm 2.6</td>
</tr>
</tbody>
</table>

- The distributions represent what form errors of the test piece can be expected due to machines with geometric errors that are within the expected range.
- If errors result that are larger than the calculated maximum the causes can be the following:
  - Other errors that were not modeled (e.g. for the tool or the approach) caused an additional error on the test piece.
  - The machine has geometric errors that are outside the assumed range.
3.12 Verification of the presented calculations and assumptions

- The combination of geometric errors of the machine was not analyzed and results in a larger form error on the test piece.

- 75% of the machines should be able to manufacture test pieces with form errors smaller than the upper quartile if no additional errors influence the test piece form.

3.12 Verification of the presented calculations and assumptions

For the verification of the presented calculations and assumptions, a comparison of calculations and measurements is presented in this section for the setup shown in figure 3.52.

Figure 3.52: Illustration of position of test piece for comparison between calculations and measurement results.

The illustration represents the setup in which test pieces have been cut at the IWF before the proposed (optimized) setup had been defined. The angular parameters are the same as for the setup proposed in section 3.6 (cone angle $\alpha = 45^\circ$, inclination angle $\beta = 30^\circ$). Therefore the resulting movements of the rotary machine axes result to be the same (compare the lower parts of the figures 3.53 and 3.26). The translatory position of the test piece however is different, which results in different movements of the linear machine axes. The different setup parameters and the resulting required ranges of the machine axes are compared in table 3.6. It shows that for the version used for comparison, the required ranges of the X- and Z-axis of the machine are much smaller than the range of the Y-axis. As described in section 3.6 the contrary is wanted: the required ranges of the X- and Z-axis should be larger than the range of the Y-axis since the axes perpendicular to the swivel axis B are needed for compensational movements.
Table 3.6: Comparison of setup parameters and resulting required movements of the machine axes for two setups.

<table>
<thead>
<tr>
<th>parameter</th>
<th>Version for comparison</th>
<th>Proposed version (section 3.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone angle ( \alpha )</td>
<td>30°</td>
<td>30°</td>
</tr>
<tr>
<td>Inclination angle ( \beta )</td>
<td>45°</td>
<td>45°</td>
</tr>
<tr>
<td>( x_0 )</td>
<td>48.9mm</td>
<td>-65mm</td>
</tr>
<tr>
<td>Translational setup parameters ( y_0 )</td>
<td>0mm</td>
<td>0mm</td>
</tr>
<tr>
<td>( z_0 )</td>
<td>66.1mm</td>
<td>150mm</td>
</tr>
<tr>
<td>Radius of tool path ( r_{TCP} )</td>
<td>44.8mm</td>
<td>40.0mm</td>
</tr>
<tr>
<td>Required range of X-axis ( \Delta X )</td>
<td>36.0mm</td>
<td>185.7mm</td>
</tr>
<tr>
<td>Required range of Y-axis ( \Delta Y )</td>
<td>142.6mm</td>
<td>90.0mm</td>
</tr>
<tr>
<td>Required range of Z-axis ( \Delta Z )</td>
<td>13.2mm</td>
<td>185.7mm</td>
</tr>
<tr>
<td>Required range of B-axis ( \Delta B )</td>
<td>60°</td>
<td>60°</td>
</tr>
<tr>
<td>Required range of C-axis ( \Delta C )</td>
<td>360°</td>
<td>360°</td>
</tr>
</tbody>
</table>

When comparing figures 3.53 to 3.57 with figures 3.26 to 3.28 (starting on page 53) additional differences become clear:

- The X-axis reverses four times instead of only twice for the setup used for comparison.
- The linear axes start in the opposite direction for the setup used for comparison.

For a verification of the model however, the setup used for comparison is also very well suited.

The process parameters used to manufacture the test piece used for comparison are listed in table 3.7. The measurement results of the test piece (circular form error of the cylinder and normal circular form error of the cone geometry) are shown in figure 3.58. A description of the measurement procedure follows in the appendix in section A.4 on page 146.

For the verification of the presented calculations and assumptions the measurement results are compared with the expected distributions of the test piece errors.

Figure 3.59 shows the distribution of resulting errors of the test piece due to 1000 random combinations of geometric errors. The numerical values of these parameters of the distribution are summarized in table 3.8.

As explained in section 3.11 the ranges of the geometric errors were determined mainly according to the tolerances stated in the acceptance protocol of the manufacturer. Moreover, assumptions based on knowledge about the errors of the specific machine were used for the definition. The distributions of the geometric errors are assumed to be uniform. Next to a uniform variation of the amplitude of the error between two limits also the phase
3.12 Verification of the presented calculations and assumptions

Figure 3.53: Positions of the machine axes (linear in \textit{mm} and rotary in \textdegree) as a function of the radial cone angle $\phi$ for the test piece setup used for comparison between calculations and measurement results.
Figure 3.54: Illustration of test piece setup used for comparison in the machine coordinate system (mm) when the TCP is at the bottom position of the cone ($\varphi = 0^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.

Figure 3.55: Illustration of test piece setup used for comparison in the XZ-plane of the machine coordinate system (mm) when the TCP is at the bottom position of the cone ($\varphi = 0^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
3.12 Verification of the presented calculations and assumptions

Figure 3.56: Illustration of test piece setup used for comparison in the YZ-plane of the machine coordinate system (mm) when the TCP is at the bottom position of the cone ($\varphi = 0^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.

Figure 3.57: Illustration of test piece setup used for comparison in the XY-plane of the machine coordinate system (mm) when the TCP is at the bottom position of the cone ($\varphi = 0^\circ$). The red curve corresponds to the path of the TCP in the machine coordinate system and represents the movement of the linear machine axes.
3. Test piece for testing simultaneous 5-axis machining

Figure 3.58: Measurement results of test piece used for comparison: (a): circular form error of the cylinder (5.8\textmu m total error), (b): normal circular form error of the cone geometry (8.4\textmu m total error).

Figure 3.59: Box Plot of the errors resulting from 1000 random combinations of geometric machine errors.
3.12 Verification of the presented calculations and assumptions

Table 3.7: Summary of the used process parameters of the test piece used for comparison.

<table>
<thead>
<tr>
<th>Workpiece material</th>
<th>Aluminum EN AW-6082 (AlMgSi1) minimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lubrification</td>
<td></td>
</tr>
<tr>
<td>Tool</td>
<td>Diameter $d_{tool}$</td>
</tr>
<tr>
<td>Number of teeth $z$</td>
<td></td>
</tr>
<tr>
<td>Coating</td>
<td>none</td>
</tr>
<tr>
<td>Material</td>
<td>solid carbide</td>
</tr>
</tbody>
</table>

| Spindle speed $s$ | rough | 30,000 rpm |
| finish           |       | 8,333 rpm  |
| Feed per tooth $f_z$ | rough | 0.03 mm |
| finish           |       | 0.03 mm   |
| Resulting cutting speed | rough | $v_c = s \pi d_{tool}$ 1,131 m/min |
| finish           |       | 314 m/min  |
| Resulting contouring speed | rough | $v_f = s f_z z$ 1,800 mm/min |
| finish           |       | 500 mm/min |

Table 3.8: Numerical values of the distribution of the errors resulting from 1000 random combinations of geometric machine errors.

<table>
<thead>
<tr>
<th>Circular form error of cylinder</th>
<th>Equivalent cone form error</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum $\mu m$</td>
<td>11.2</td>
</tr>
<tr>
<td>upper quartile $\mu m$</td>
<td>6.4</td>
</tr>
<tr>
<td>median $\mu m$</td>
<td>5.0</td>
</tr>
<tr>
<td>lower quartile $\mu m$</td>
<td>4.0</td>
</tr>
<tr>
<td>minimum $\mu m$</td>
<td>1.2</td>
</tr>
</tbody>
</table>

of the error is randomly chosen for errors with higher order dependencies. All errors and the corresponding limits are summarized in table 3.9.

When comparing the values of the measurements shown in figure 3.58 with the values of the calculated distribution of resulting test piece errors (figure 3.59 and table 3.8) it can be seen that the measured errors are nicely between the calculated median and the upper quartile. This corresponds to a good correlation of calculations and experiment. For a better verification of the calculations and assumptions, many test pieces have to be manufactured on different machining centers of the same type in order to compare a distribution of measured errors with the distribution of calculated errors.
Table 3.9: List of geometric machine errors with the limits used in the calculation of the expectable distribution of test piece errors due to random combinations of machine errors (Monte Carlo).

<table>
<thead>
<tr>
<th>Error</th>
<th>lower limit</th>
<th>upper limit</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>X0X</td>
<td>0.0</td>
<td>0.0</td>
<td>µm</td>
</tr>
<tr>
<td>B0X</td>
<td>0.0</td>
<td>0.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>COX</td>
<td>0.0</td>
<td>0.0</td>
<td>µm</td>
</tr>
<tr>
<td>EXX_{lin}</td>
<td>-3.0</td>
<td>3.0</td>
<td>µm over ∆X_M</td>
</tr>
<tr>
<td>EXX_{hist}</td>
<td>0.0</td>
<td>4.0</td>
<td>µm</td>
</tr>
<tr>
<td>EXX_{cyclic}</td>
<td>-1.0</td>
<td>1.0</td>
<td>µm over 30 mm</td>
</tr>
<tr>
<td>EYY_{lin}</td>
<td>-3.0</td>
<td>3.0</td>
<td>µm over ∆Y_M</td>
</tr>
<tr>
<td>EXX_{2harm}</td>
<td>-1.5</td>
<td>1.5</td>
<td>µm over ∆X_M</td>
</tr>
<tr>
<td>EYY_{2harm}</td>
<td>-1.5</td>
<td>1.5</td>
<td>µm over ∆Y_M</td>
</tr>
<tr>
<td>EYY_{3harm}</td>
<td>-1.5</td>
<td>1.5</td>
<td>µm over ∆Y_M</td>
</tr>
<tr>
<td>EAX_{lin}</td>
<td>-35.0</td>
<td>35.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>EAX_{1harm}</td>
<td>-15.0</td>
<td>15.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>EAX_{2harm}</td>
<td>-7.0</td>
<td>7.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>EBX_{lin}</td>
<td>-35.0</td>
<td>35.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>EBX_{1harm}</td>
<td>-15.0</td>
<td>15.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>EBX_{2harm}</td>
<td>-7.0</td>
<td>7.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>ECX_{lin}</td>
<td>-35.0</td>
<td>35.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>ECX_{1harm}</td>
<td>-15.0</td>
<td>15.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>ECX_{2harm}</td>
<td>-7.5</td>
<td>7.5</td>
<td>µm/m</td>
</tr>
<tr>
<td>Y0Y</td>
<td>-7.0</td>
<td>7.0</td>
<td>µm</td>
</tr>
<tr>
<td>A0Y</td>
<td>0.0</td>
<td>0.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>C0Y</td>
<td>-50.0</td>
<td>50.0</td>
<td>µm/m</td>
</tr>
<tr>
<td>EYY_{lin}</td>
<td>-4.0</td>
<td>4.0</td>
<td>µm over ∆Y_M</td>
</tr>
<tr>
<td>EYY_{hist}</td>
<td>0.0</td>
<td>4.0</td>
<td>µm</td>
</tr>
<tr>
<td>EYY_{cyclic}</td>
<td>-1.0</td>
<td>1.0</td>
<td>µm over 30 mm</td>
</tr>
<tr>
<td>EXY_{1harm}</td>
<td>-3.0</td>
<td>3.0</td>
<td>µm over ∆Y_M</td>
</tr>
<tr>
<td>EXY_{2harm}</td>
<td>-1.5</td>
<td>1.5</td>
<td>µm over ∆Y_M</td>
</tr>
<tr>
<td>EXY_{3harm}</td>
<td>-1.5</td>
<td>1.5</td>
<td>µm over ∆Y_M</td>
</tr>
<tr>
<td>EZY_{1harm}</td>
<td>-3.0</td>
<td>3.0</td>
<td>µm over ∆Y_M</td>
</tr>
</tbody>
</table>

Continued on next page
3.12 Verification of the presented calculations and assumptions

<table>
<thead>
<tr>
<th>Error</th>
<th>lower limit</th>
<th>upper limit</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EZY_{2\text{har}}$</td>
<td>-1.5</td>
<td>1.5</td>
<td>$\mu m$ over $\Delta Y_M$</td>
</tr>
<tr>
<td>$EZY_{3\text{har}}$</td>
<td>-1.5</td>
<td>1.5</td>
<td>$\mu m$ over $\Delta Y_M$</td>
</tr>
<tr>
<td>$EAY_{\text{lin}}$</td>
<td>-35.0</td>
<td>35.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAY_{1\text{har}}$</td>
<td>-15.0</td>
<td>15.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAY_{2\text{har}}$</td>
<td>-7.0</td>
<td>7.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBY_{\text{lin}}$</td>
<td>-35.0</td>
<td>35.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBY_{1\text{har}}$</td>
<td>-15.0</td>
<td>15.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBY_{2\text{har}}$</td>
<td>-7.0</td>
<td>7.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECY_{\text{lin}}$</td>
<td>-35.0</td>
<td>35.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECY_{1\text{har}}$</td>
<td>-15.0</td>
<td>15.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECY_{2\text{har}}$</td>
<td>-7.0</td>
<td>7.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$Z0Z$</td>
<td>-11.6</td>
<td>11.6</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$A0Z$</td>
<td>-34.0</td>
<td>34.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$B0Z$</td>
<td>-34.0</td>
<td>34.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EZZ_{\text{lin}}$</td>
<td>-4.0</td>
<td>4.0</td>
<td>$\mu m$ over $\Delta Z_M$</td>
</tr>
<tr>
<td>$EZZ_{\text{hist}}$</td>
<td>0.0</td>
<td>4.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EZZ_{\text{cyclic}}$</td>
<td>-1.0</td>
<td>1.0</td>
<td>$\mu m$ over 30mm</td>
</tr>
<tr>
<td>$EXZ_{1\text{har}}$</td>
<td>-3.0</td>
<td>3.0</td>
<td>$\mu m$ over $\Delta Z_M$</td>
</tr>
<tr>
<td>$EXZ_{2\text{har}}$</td>
<td>-1.5</td>
<td>1.5</td>
<td>$\mu m$ over $\Delta Z_M$</td>
</tr>
<tr>
<td>$EXZ_{3\text{har}}$</td>
<td>-1.5</td>
<td>1.5</td>
<td>$\mu m$ over $\Delta Z_M$</td>
</tr>
<tr>
<td>$EZZ_{1\text{har}}$</td>
<td>-3.0</td>
<td>3.0</td>
<td>$\mu m$ over $\Delta Z_M$</td>
</tr>
<tr>
<td>$EZZ_{2\text{har}}$</td>
<td>-1.5</td>
<td>1.5</td>
<td>$\mu m$ over $\Delta Z_M$</td>
</tr>
<tr>
<td>$EZZ_{3\text{har}}$</td>
<td>-1.5</td>
<td>1.5</td>
<td>$\mu m$ over $\Delta Z_M$</td>
</tr>
<tr>
<td>$EAX_{\text{lin}}$</td>
<td>-35.0</td>
<td>35.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAX_{1\text{har}}$</td>
<td>-15.0</td>
<td>15.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAX_{2\text{har}}$</td>
<td>-7.0</td>
<td>7.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBZ_{\text{lin}}$</td>
<td>-35.0</td>
<td>35.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBZ_{1\text{har}}$</td>
<td>-15.0</td>
<td>15.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBZ_{2\text{har}}$</td>
<td>-7.0</td>
<td>7.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECZ_{\text{lin}}$</td>
<td>-35.0</td>
<td>35.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECZ_{1\text{har}}$</td>
<td>-15.0</td>
<td>15.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECZ_{2\text{har}}$</td>
<td>-7.0</td>
<td>7.0</td>
<td>$\mu m/m$</td>
</tr>
</tbody>
</table>

Continued on next page
### Table 3.9 – continued from previous page

<table>
<thead>
<tr>
<th>Error</th>
<th>lower limit</th>
<th>upper limit</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X0B$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$Z0B$</td>
<td>0.0</td>
<td>0.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$B0B$</td>
<td>0.0</td>
<td>0.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$A0B$</td>
<td>-45.0</td>
<td>45.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$C0B$</td>
<td>-50.0</td>
<td>50.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBB_{lin}$</td>
<td>-25.0</td>
<td>25.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBB_{hist}$</td>
<td>0.0</td>
<td>20.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBB_{1harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBB_{2harm}$</td>
<td>-5.0</td>
<td>5.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAB_{1harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAB_{2harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECB_{1harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECB_{2harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EXB_{2harm}$</td>
<td>-2.0</td>
<td>2.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EXB_{3harm}$</td>
<td>-1.0</td>
<td>1.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EZB_{2harm}$</td>
<td>-2.0</td>
<td>2.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EZB_{3harm}$</td>
<td>-1.0</td>
<td>1.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EYB_{1harm}$</td>
<td>-2.5</td>
<td>2.5</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EYB_{2harm}$</td>
<td>-1.0</td>
<td>1.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$X0C$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$Y0C$</td>
<td>0.0</td>
<td>0.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$C0C$</td>
<td>0.0</td>
<td>0.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$A0C$</td>
<td>-45.0</td>
<td>45.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$B0C$</td>
<td>-50.0</td>
<td>50.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECC_{1harm}$</td>
<td>-15.0</td>
<td>15.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$ECC_{2harm}$</td>
<td>-5.0</td>
<td>5.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAC_{1harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EAC_{2harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBC_{1harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EBC_{2harm}$</td>
<td>-10.0</td>
<td>10.0</td>
<td>$\mu m/m$</td>
</tr>
<tr>
<td>$EXC_{2harm}$</td>
<td>-2.0</td>
<td>2.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EXC_{3harm}$</td>
<td>-1.0</td>
<td>1.0</td>
<td>$\mu m$</td>
</tr>
</tbody>
</table>

Continued on next page
3.13 Summary

In this chapter, different aspects of a test piece for testing of simultaneous 5-axis machining have been discussed.

In the first section 3.1 the goals and requirements of an ideal test piece are analyzed. In the following section 3.2, the geometry of a right circular cone is derived to be the most appropriate test piece geometry to meet the discussed requirements.

The calculation of the tool path required for a cone geometry is derived in section 3.3 for any machine setup with three orthogonal linear axes and two orthogonal rotary axes (the nominal rotary axes are assumed to cross in one point). The general parameters of the cone geometry of the required tool path are described in sections 3.3.2 and 3.3.3. The form and size is described by the cone angle $\alpha$ and by the radius of the tool path $r$. Furthermore the inclination angle $\beta$ is introduced in section 3.3.3.

The resulting movements of the rotary axes are derived in section 3.3.5. It could be demonstrated that they depend only on the cone angle $\alpha$ and on the inclination angle $\beta$. Furthermore they are the same for all analyzed machine setups if $\alpha$ and $\beta$ are the same.

The movement of the linear machine axes is derived in section 3.3.6. It depends not only on the angular parameters of the cone but also on the position, size and orientation of the workpiece. The compensational movements of the linear machine axes depend on the setup of the machine. Therefore, the required movement of the linear machine axes is derived for the following three setups separately:

- Machines with both rotary axes on the tool side (double pivot spindle head).
- Machines with rotary axes on the tool and on the workpiece side (swivel head and rotary table).

Table 3.9 – continued from previous page

<table>
<thead>
<tr>
<th>Error</th>
<th>lower limit</th>
<th>upper limit</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EYC_{2\text{harm}}$</td>
<td>-2.0</td>
<td>2.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EYC_{3\text{harm}}$</td>
<td>-1.0</td>
<td>1.0</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EZC_{1\text{harm}}$</td>
<td>-2.5</td>
<td>2.5</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>$EZC_{2\text{harm}}$</td>
<td>-1.0</td>
<td>1.0</td>
<td>$\mu m$</td>
</tr>
</tbody>
</table>
• Machines with both rotary axes on the workpiece side (tilting rotary table).

An interpretation of the required movements of the machine axes follows in section 3.4 for the rotary axes and in section 3.5 for the linear axes. For the interpretation of the resulting movements, the following three situations are distinguished depending on the relation of the cone angle $\alpha$ and the inclination angle $\beta$:

- $\alpha > \beta$ (section 3.4.1)
- $\alpha < \beta$ (section 3.4.2)
- $\alpha = \beta$ (section 3.4.3)

The required velocities of the rotary axes are interpreted in section 3.4.4. The effect of a setup close to the singular configuration with an inclination angle $\beta$ close to the cone angle $\alpha$ can be seen very nicely when looking at the required maximal angular velocity of the rotary table as a function of the inclination angle $\beta$ for a given cone angle $\alpha$ (figure 3.24).

In section 3.6, test piece parameters are derived for different machine designs. A cone angle $\alpha = 45^\circ$ combined with an inclination angle $\beta = 30^\circ$ is derived as the proposed angular parameters for all analyzed machine setups. For machines with a tilting rotary table setup one specific machine is taken as an example and the resulting machine movements are analyzed more in detail for the proposed test piece (section 3.6.4). A new kind of illustration that allows the visualization of the cone position in the machine coordinate frame relative to movement of the linear machine axes is introduced (figures 3.27–3.34).

In sections 3.7 to 3.12 the influences of geometric errors on the form errors of the test piece are analyzed. How the different geometric machine errors are modeled is described in section 3.7 for the location errors and in section 3.8 for the component errors. Section 3.9 describes the theoretical influence of geometric machine errors on a workpiece. The effect of single geometric errors on the cone form is analyzed in section 3.10 for one specific example. The influence of single geometric errors is interpreted for the analyzed example for end milling but the presented procedure is applicable in general. For end mills with large diameters additional influences would need to be taken into account. This case was not analyzed because such end mills are not common in high speed 5-axis milling. In section 3.10.1, the points that can be seen from analyzing the components of the normal directions of the cone surface in the machine frame as a function of the radial cone angle $\varphi$ are shown. The radial representation of the position of the machine axes is introduced in section 3.10.2. Resulting cone forms are described in section 3.10.3 and in section 3.10.4 two typical cone form deviations are identified. The errors with negligible effects on an
end milled cone form are discussed in section 3.10.5. The numerical values of the assumed geometric errors that have an influence on the cone form are summarized in table 3.4 in section 3.10.6. The effects of combinations of geometric machine errors on the cone form follows in section 3.11 and in section 3.12 results of such a Monte Carlo simulation are compared with measurement results as a verification.

The conclusions that can be drawn from this first part of the thesis are described in chapter 4.

In the appendix A, some practical aspects of cutting a test piece on a specific machine type are described.
Chapter 4

Conclusion of part I

The conclusions that can be drawn from this first part of the dissertation are discussed in this chapter.

The main achievement regarding the deficiencies of the state of the art (see section 2.5) is that with this part of the thesis the foundation has been laid for a standardized test piece for the testing of simultaneous 5-axis machining.

The parameters of the test piece and its setup have been analyzed for any machining center with three orthogonal linear axes and two orthogonal rotary axes (the nominal rotary axes are assumed to cross in one point). Angular parameters of the test piece (cone angle $\alpha$ and inclination angle $\beta$) could be deduced that are proposed for all analyzed machine setups.

The influence of the position of the test piece relative to the intersection of the rotary axes of the machines is derived for the three analyzed types of machine setups. Recommended parameters are derived and justified. Their influence depends on whether the rotary axes are on the tool or on the workpiece side. The method of the selection of the test piece parameters is explained and can be adopted for any other goal.

Novel visualizations have been developed to illustrate positions of the cone geometry relative to the tool path. These are very helpful for the understanding of the dependencies of the 5-axis movement. Furthermore they support the interpretation of the influences of geometric machine errors on the errors of the test piece. For this purpose, another novel visualization form has been presented that consists of a radial representation of the positions of the machine axes.

The influences of geometric machine errors on measurable errors of the test piece geometry have been shown for a specific example. Circularity deviations on the conical test piece show typical forms. However, always several geometric errors of the machine tool cause circularity deviations of similar forms. This leads to the following conclusions:
• The identification of an individual geometric machine error is not possible from the analysis of the measured cone form.
• If the machined conical test piece shows a typical error form, the number of potential geometric errors can be narrowed down to a group of a few machine tool errors.
• Combinations of errors exist that neutralize their effect on the cone form.

The entire analysis is applicable also for direct measurements (i.e. without test pieces) of the errors of the machine movements. It can be concluded that the results of a measurement setup with a double ball bar (DBB) [41] square to the cone axis do not represent the errors of the cone form because errors parallel to the cone surface are also measured as form errors. For a more realistic representation of the form error, the DBB should be set up square to the cone surface. This way also collision problems are avoided. On a machine with a tilting rotary table setup, an alternative would be to use an R-Test [16] and to position the precision sphere at the apex of the cone.

In any case, the measurement cannot replace the manufacturing of a test piece but it is very well suited for a study of parameter influences, such as changing contouring feed.

As a verification of the presented calculations and assumptions, the results of calculated expectable test piece errors distributions are compared with measurement results, showing good correlations.

From the practical experiences described in the appendix of this thesis can be concluded that a combination of a cone frustum and a cylinder represents a test piece that allows identifying possible errors that are caused by the approach strategy of the machining process of the test piece.
5. State of the art in evaluation of the dynamic behavior of machining centers

Chapter 5

State of the art in evaluation of the dynamic behavior of machining centers

The dynamic properties of machining centers can be evaluated either by experimental measurements or by simulation. In this section, different existing methods to evaluate the dynamic behavior of machining centers will be discussed. A wider overview of virtual machine tool technology is presented in [42]. The focus of this second part is set on the comparative evaluation of the dynamic behavior of machining centers, more specifically on the evaluation of concepts or simplified representations of existing machining centers.

5.1 Experimental evaluation of the dynamic behavior of machining centers

The evaluation of the dynamic behavior of available machining centers can be done in an experimental way.

Modal Analysis is defined as a vibration analysis method that characterizes a complex structural system by its modes of vibration, i.e. its natural frequencies, modal damping and mode shapes, and based on the principle of superposition [43].

When the bases of the analysis are experimental measurements, the method is referred to as Experimental Modal Analysis (EMA) [44]. In this work the results of an EMA will be presented as verification of a simulation model in section 6.2 (starting on page 114).
5.2 Finite Element Method (FEM)

Modeling a machining center using the finite element method (FEM) for the analysis of their stiffness properties is the most common approach today. The most known field of FEM is the calculation of deformations and stresses due to static forces. In the field of machine tools, the stresses are usually not critical but the goal is mostly to minimize weight without compromising the stiffness. This optimization can be done in an automated way for critical components of machine tools [45, 46, 47, 48].

The calculation of dynamic properties of machining centers with FEM requires a lot of computation time. While the calculation of the effects of a static load on one component of a machine tool might be possible in a few minutes, the calculation time for the mode shapes of an entire machine tool is stated in the range of a few hours in recent publications [49]. The harmonic analysis of a machine (which results in the frequency dependent deformations of a system) can easily take more than 10 hours of computation time for a model with 100000 nodes [50]. The calculation time can be shortened by reducing the number of elements as it can be seen in older publications [51] or by model reduction techniques [52]. A detailed mathematical comparison of model reduction techniques is presented in [53]. A comparison for a calculation of a part modeled with FEM is presented in [54] showing advantages of the different methods depending on the frequency range of interest. Early applications on the evaluation of the dynamic properties of machining centers can be found in [55, 56].

To consider the variation of the stiffness properties depending on the position of the TCP within the working range, the calculations have to be repeated for every position of interest. In [57] a simulation of a pick and place machine is presented. It shows results of configuration-dependent dynamics with three positions of the TCP calculated with FEM. In [58] “precalculated structures” and movable joints are introduced for a modular synthesis of the static and dynamic behavior of machine tools at several positions in the workspace. In section 7.5.2 on page 134, an analysis of 400 positions using a rigid body simulation will be shown.

The fact that standardized interfaces exist between CAD and FEM software and that FEM is very diffused in industry for structural evaluations makes it a widespread tool also for the evaluation of dynamic properties of machining centers.
5.3 Rigid Body Simulation

Rigid Body Simulations are an effective way of modeling the dynamic interaction of systems that consist of multiple rigid bodies connected by elastic elements. In this context, rigid bodies are elements that have mass and inertia properties but cannot deform [59].

Machining centers consist of multiple bodies connected by linear guide ways, rotary bearing systems and drives. Using a rigid body approach, the bodies are modeled as rigid and all the compliances are concentrated in the connecting points. The connection with the inertial system is also modeled with compliances.

Different commercial software packages exist in the field of rigid body simulation. Nevertheless, no specialized software package exists for the field of machine tools with a diffusion and acceptance close to that of the leading FEM software packages. A comparison of different packages for rigid body and finite element calculations has been presented in [60]. Probably the most known multi-body simulation package is Adams™ from MSC Software. It is widely used in vehicle design for which specialized toolboxes are available. For machine tools, on the other hand no such commercial toolbox is available at the moment.

Some publications describing the usage of Adams™ for the evaluation of the dynamic properties of machining centers exist [61, 62, 63]. In [61] the implementation of user written subroutines is described and in [62] the possibilities of integrating Adams™ in simulation environments are presented. The advantage of using such a commercial product over programming an own environment seems not given, especially in an academic environment [64]. The usage of an environment programmed in Matlab without the usage of any toolbox, has been developed at the Institute of Machine Tools and Manufacturing (IWF) of the ETH Zurich [65, 66]. The main advantages of using an own environment are the following:

- Possibility to optimize evaluations for the specific application.
- Usage of well known environment used in other fields.
- Flexibility to integrate resent research results.
- Knowing the applied algorithms in detail.

A description of how a specific machining center can be modeled with this Axis Construction Kit (ACK) and a comparison with measurement results follows in the next chapter. Recently the usage of the toolbox SimMechanics from Matlab has been used for the modeling of mechanical systems. Not much experience is available yet in the field of machining centers but first steps in that direction have been made [67].
Machine parts that can not be simplified as rigid bodies (like for example extended beams) can be divided into multiple rigid bodies connected by resilient elements in order to represent the properties of the structure [68, 59, 69, 70]. Very good results have been shown for applications with dominant bending loads [71, 72].

5.4 Combination of Rigid Body and Finite Element Method

To consider the flexibility of individual machine components more in detail, combinations of the Rigid Body and Finite Element Method in one model are possible. Siedl recently presented applications showing good results in the simulation of dynamic positioning motions of machining centers [60, 73]. The basis for such combination is the coupling of substructures for dynamics analyses (often using the Craig-Bampton method [74]). An application in the optimization of a hybrid kinematics has been presented by [75]. In [76] an application is show for the simulation of the dynamic positioning of a machine tool manipulator with a linear and a rotary axis.

5.5 Uncertainty of the stiffness and damping parameters

The uncertainty of the stiffness and damping parameters is a difficulty of all approaches. Especially the damping parameters have large uncertainties [77].

The difficulty being that the damping of the system depends not only on the nominal machine design but also largely on the setup of the machine [78, 79, 80, 81]. Furthermore it does not have to be constant in time and can vary depending on the environmental conditions and on the state of the components (e.g. influence of wear). For an absolute prediction of expectable amplitudes of different vibrations, the damping properties are crucial.

For a relative evaluation of the dynamic properties of different machine concepts, the exact value of the damping and stiffness properties is not required. This is especially true if the difference between the concepts is mainly the kinematic setup and not the individual linking components as will be shown in chapter 7.
5.6 Simulation of movements

When simulating a movement of a machining center, the interaction between the structure, the drives and the control has to be modeled. Different publications can be found that use FEM to simulate such movements [82, 83, 84]. One of the leading users of such technologies in industry is the machine tool company Heller, which also regularly presents its work [48, 85, 86].

Applications using rigid body simulations [87] or combinations of rigid body and FEM [76] also exist but are still less common. In the automotive sector for example, the multi body approach is more diffused [88].

5.7 Axes Construction Kit (ACK)

At the Institute of Machine Tools and Manufacturing (IWF) of the ETH Zurich in Switzerland, a Rigid Body Simulation Environment has been developed using Matlab® [59, 66]. This Axes Construction Kit (ACK) is designed as a tool for the numerical analysis of machine tools as rigid body systems, especially for the early conception phase of the machine tool development, where different ideas of setups and configurations are to be analyzed. The other scope of application of the ACK is the analysis of vibration problems where effects of design modifications have to be evaluated.

The goal is to reach as accurate results as possible with as little data as possible. With the ACK, the machine structure is modeled with simple elements. In case of machine tools, these would be the different bodies, the linear and rotary axes, the drives and the setup relative to the ground. This way it can be used to model machine concepts before knowing any constructive details. It allows giving an estimation considering the most important parameters affecting the dynamic behavior of machine tools. These would be:

- Comparison of alternative axes configurations of machine tools (kinematic setup).
- Effects of the stiffness and arrangement of the coupling elements.
- Effects of masses and inertia of the individual bodies.

The results of the first analysis are:

- static and dynamic stiffness (evaluated at the TCP, between the tool and the work-piece side).
- natural frequencies and modes of vibration.
5.8 Simulation in the early development stage

• dominant frequencies and corresponding mode shapes.

The definitions of this vocabulary according to [43] are:

• **stiffness**: ratio of change of force (or torque) to the corresponding change in translational (or rotational) deformation of an elastic element.

• **dynamic stiffness**: complex ratio of the force, taken at a point in a mechanical system, to the displacement, taken at the same or another point in the system.

• **natural frequency** (of a mechanical system): frequency of free vibration of an undamped linear vibration system.

• **natural mode of vibration**: mode of vibration assumed by a system when vibrating freely (also called eigenmode or eigen mode).

• **dominant frequency**: frequency at which a maximum value occurs in a spectral density curve.

• **mode shape**: shape of a given mode of vibration of a mechanical system, given by the maximum change in position, usually normalized to a specified deflection magnitude at a specified point, of its neutral surface (or neutral axis) from its mean value.

In a later step, the behavior of the control and drives can be considered more in detail to analyze the model in the time domain for an evaluation of the reachable dynamics and the path accuracy [87].

A great advantage of modeling machine tools with the ACK is that it allows getting results in a very short time. Especially in the first step (where the calculations are carried-out for static and dynamic loads in the frequency domain) results of different variants can be achieved within few seconds using standard computers. This allows analyzing dynamic properties in many points throughout the working volume of the machine (example in section 7.5.2 starting on page 134). Results of such dynamic analyses represent a breakthrough for many designers of machine tools that are not familiar with these aspects and therefore often neglect them up to now.

**5.8 Simulation in the early development stage**

In the early development stage, the following design variables have to be defined:

• kinematic setup of the machine.

• width and length of guide way systems.
5. State of the art in evaluation of the dynamic behavior of machining centers

- position of the drives.
- configuration and parameters of connecting elements in general.

A change of such a parameter in a later stage of the development would result in unwanted additional costs. The rigid body simulation method is a very powerful tool to answer these questions at a stage where few details are known about the machine. Next to helping in the definition of the above mentioned design variables, it helps to chose proper elements and to detect weak spots in the early design stages [42]. It is also used as a decision-making aid for the design of reconfigurable machine tools and even as a modeling tool for the generation of models for model based control systems [65, 66].

At this stage no detailed dimensions of the parts and about the drives and control are defined yet. Therefore, it seams much more appropriate to use a rigid body approach instead of a more detailed one using FEM. Only after the machine architecture has been optimized and defined, simulations using FEM should be addressed to optimize the structural behavior of the different machine parts in detail [75].

5.9 Deficiencies of the state of the art

As described above, different ways exist to evaluate the dynamic behavior of machining centers. While most works focus mainly on new mathematical approaches with the goal of improved accuracy or efficiency the main goals of this work are to show the following:

- Possibilities and limitations of modeling the dynamic behavior of machining centers with as little as four rigid bodies.
- A simple model has the advantages to allow a very efficient analysis of effects of:
  - Changes in the design parameters of the machine.
  - Changing position of the TCP.
- The main difficulty is not the degree of detail of the model but the estimation of the required damping and stiffness parameters.
- For a relative comparison of the dynamic behavior of different machine concepts no detailed knowledge of damping and stiffness parameters is required if the connecting elements are alike.
- Additional information that can be drawn from the following analysis:
  - Reactions of the machine in directions orthogonal to the direction of the excitation (cross-talk).
- Distribution of the dynamic properties at the TCP throughout the working range of the machine.

Furthermore a special focus will be given on the procedure of the analysis, on rules of thumb on how to select starting values for the required parameters, on how to select the appropriate load case, on what parameters to evaluate and on how to interpret the results.
Chapter 6

Exemplary evaluation of dynamic behavior of a machining center

In this section, the results of a rigid body calculation using only four bodies will be compared to measurement results from an experimental modal analysis of a machining center prototype. The example is used as verification of the simulation environment and to show how such modeling can be done. Parts of this comparison have been presented by the author in [89].

6.1 Modeling of the machining center

The modeling procedure is presented for a high performance machining center prototype with a vertical working spindle. Even if the measured prototype was equipped with a tilting rotary table or in other words configured as a 5-axis machine, the model represents the 3-axis version of the machine. The rotary axes could also be modeled, but the part of the rotary axes was not to be changed and the remaining design variables only applied to the 3-axis part. Therefore, only that part of the machine was modeled.

6.1.1 Bodies

Each body in a rigid body environment is defined by its position and its inertia properties. To define the position of a body, the position of its center of mass is defined.

The center of mass is defined as a point at which an object is in balance in a uniform gravitational field [43].
The actual dimensions of each body are not required for the calculations but they are helpful to avoid errors in the modeling process and necessary for the visualization of the results. In the mentioned Matlab environment ACK, different options to define bodies are implemented. The simplest way to model concepts is by definition of elements in form of a cuboid, prism or cylinder that can be united to one resulting body. Figure 6.1 shows a representation of the machine model with the centers of mass of each of the ten individual body elements (coordinate system according to manufacturer and not according to international standard [36]).

The inertia properties can be indicated explicitly during the definition of each element or can be calculated with the given parameters such as the estimated dimensions, weight and density.

The **moment of inertia** is defined as the sum (integral) of the product of the masses of the individual particles (elements of mass) of a body and the square of their perpendicular distances from the axis of rotation [43].

By combining different bodies to a single resulting rigid body, only the resulting body is used for further calculations. Figure 6.2 (a) shows a representation of the machine model with the centers of mass of the four combined bodies. The center of mass and its inertia properties can either be defined explicitly for the resulting body or it can be derived from the properties of its individual elements. For a better visibility of the bodies and for a visualization of the kinematic setup, in figure 6.2 (b) the bodies are colored according to the direction (X, Y, Z) that they are actuated.
6. Dynamic behavior of a machining center

6.1.2 Connecting elements

To model a connecting element, the position, the linked bodies and the stiffness and damping values have to be defined. Depending on the type of connection, different simplified forms of definition can be chosen or libraries can be used [66].

The visualization of the position of the different connecting elements is shown in figure 6.3. The direction of the stiffness components is represented by the color and form of the representation (* = X, ○ = Y, □ = Z). The visualization is helpful in order to check the modeled parameters.

Linear guide ways

The linear guide way systems are represented by two stiffness and damping components perpendicular to the guide way direction at the position of each carriage. Manufacturers of guide ways present more and more efforts in delivering dynamic properties of their products [77].

A very detailed analysis of bearings with strategies for their simulation is presented in [90].
6.1 Modeling of the machining center

Figure 6.3: Representation of the machine model with the different connecting elements as explained in section 6.1.2 (stiffness components directions: $\ast = X$, $\circ = Y$, $\square = Z$).

It looks at the reactions inside of the bearings of linear guide ways due to dynamic loads and due to friction. In [91] the possibility of modeling the bearings of the linear guide ways using solids is described for a finite element approach. Two measures are proposed to model the actual stiffness of the bearings:

- Modify the geometry of the bearing model such that it will deflect as much as the real bearing.
- Modify the material properties of the modeled bearing such that it will deflect as much as the real bearing.

The second one is adopted in that work to closely resemble the appearance of the actual design. A calculation of what the geometry of the bearing model should be in order to deflect as much as the real bearing visualizes very well how small the stiffness of the bearing is compared to the stiffness of the other machine parts.

**Example for visualization:** As an example, the vertical stiffness of the bearing shall be $k_v = 1000 N/\mu m$. The equivalent cuboid geometry shall have a height of $h = 10 mm$ and a square base with a side length $a$. With these assumptions a side length $a$ of less than $7 mm$ results for material properties of steel (Young’s modulus $E = 200 GPa$).
In rigid body models, the stiffness and damping properties of the linking elements represent not only the linking element itself but also part of the linked bodies. As a starting value for the stiffness of the linear guide ways, a value of one third of the catalogue value has shown good results. The division by the factor three is based on the assumption that each linked body contributes the same compliance to the link as the link itself and therefore a total stiffness of one third results, which is the optimum, when looking for maximum stiffness with minimal component stiffness.

**Drives**

In the early development phase, the drives can be modeled with one stiffness and damping component in drive direction. At this early stage, the position and its desired stiffness are the only available design parameter. Parameters as for example the diameter and lead of a ball screw drive are not defined yet.

Very detailed models of ball screw feed drive systems have been developed [92, 93]. Detailed modeling of direct drives requires the consideration of the control since the magnetic principle has no physical stiffness in drive direction without control [94]. In an early stage of the rigid body model it can be regarded as one stiffness component in drive direction that can be modeled as an idealized control stiffness given by the proportional gains in the position and velocity control loops.

**Direct measurement system**

To analyze the influence of a direct measuring system compared to an indirect measuring system from a dynamic point of view, a connecting element can be modeled at the position of the direct measurement system. It represents the influence of the control with a constant stiffness and damping value. This allows a very simple and effective way to evaluate the influence of different measurement system positions [87].

**Friction**

The fundamental effects of friction on the positioning behavior of machine tools can be demonstrated using rigid body simulation. Basic analytical relationships can be found in [95].
Setup relative to the ground

The setup of a machining center relative to the ground has a relevant influence on its dynamic behavior [96, 80, 81]. The correct modeling of the setup however is connected with many uncertainties since not only the setup elements have to be taken into account but also the foundation. In [96] the uncertainties of the material properties of the foundation are emphasized.

Best results have been achieved by modeling each setup element as a stiffness and damping element to be able to predict the influence of different setups. That way recommendations concerning the setup can be made. In [97] this approach was used to evaluate the appropriate stiffness values for best vibration isolation between a machine and the foundation.

The actual values of the stiffness and the physical damping, modeled in each setup element, have to include the properties of each connecting partner. On one side the base, which is usually very stiff and depending on the material can have more or less damping properties but at least it is more or less constant for each machine. On the other side the fundament, which has the most variation of all. Considerable changes have been reported within the same setup, when changing the position of the machine by only few meters. Special attention has to be paid when a machine is not setup on the lowest floor. Results of calculations and detailed analysis of the influence of the installation location of machine tools on their productivity and quality have been shown in [81].

Often the damping effects are modeled as loss factors and not as physical damping values. In an identification of equivalent stiffness and physical damping values of each setup elements of a machining center, setup on a floor having a cellar underneath, the values have resulted to be the following (index $v$ for vertical and index $h$ for horizontal):

- stiffness: $k_v = 450 \text{N}/\mu \text{m}$, $k_h = 600 \text{N}/\mu \text{m}$

- physical damping: $d_v = 900 \text{kN}/(\text{m/s})$, $d_h = 10 \text{kN}/(\text{m/s})$

This showed that the physical damping properties reduced into the setup elements were mainly acting in vertical direction (horizontal component only about 1% of the vertical component). The main reason is that the bending of the floor has a damping effect in vertical direction but not in horizontal direction.

The stiffness values are rather high because the connection of the measured machine with the floor consisted of large screws rather than of dedicated damping setup elements.
6.2 Discussion and comparison of calculation and measurement results

6.2.1 Frequency-Response Function (FRF)

When evaluating the dynamic properties of machining centers the main question is the following:

- What is the reaction of the machining center depending on the frequency of the excitation?

To answer this question for the analyzed milling machine an excitation near the TCP is introduced and the motion-response is evaluated relative to a point representing the workpiece.

The frequency-response function (FRF) is defined as frequency-dependent ratio of the motion-response Fourier transform to the Fourier transform of the excitation force of a linear system [43].

Depending on how the motion is expressed (velocity, acceleration or displacement) the corresponding frequency-response function designations are:

- **mobility**: complex ratio of the velocity, taken at a point in a mechanical system, to the force, taken at the same or another point in the system.
- **accelerance**: frequency-dependent ratio of the spectrum, or spectral density, of the acceleration to the spectrum, or spectral density, of the force.
- **dynamic compliance**: frequency-dependent ratio of the spectrum, or spectral density, of the displacement to the spectrum, or spectral density, of the force.

To get direct information about the possible expectable error on the workpiece, the motion is expressed as a displacement. In other words, the FRF is evaluated as a dynamic compliance. The measurements for the experimental modal analysis were made with piezoelectric accelerometers [98]. Therefore, the dynamic compliance results by integrating the measured accelerance twice, involving large errors in the low frequency spectrum where only very small acceleration were measured (typically below the frequency range of interest).

As an abbreviation for the dynamic compliance $H_i/F_j$ will be used in this work. The index $i$ describes the direction of the displacement (reaction) and $j$ the direction of the excitation. Thus, the FRF can be looked at as a matrix with the diagonal elements representing the displacements in the direction parallel to the excitation (e.g. $H_x/F_x$). In most cases,
the component of the response is greatest in direction parallel to the excitation. For this reason, the diagonal elements of the dynamic compliance are analyzed first. In a second step, the components of the displacement in the direction orthogonal to the direction of the excitation (e.g. $H_y/F_x$) can be analyzed (section 6.2.3). This effect, which is also known as cross-talk, can cause great difficulties in precision manufacturing.

The dynamic compliance gives information about what frequencies are expected to be critical. The dominant frequencies result from the highest peaks of the dynamic compliance. To analyze how a structure reacts at the different dominant frequencies, the corresponding mode shapes are analyzed (section 6.2.2).

For this specific case, the diagonal elements of the calculated dynamic compliance are shown in figure 6.4 for the frequency range between 0 and 200 Hz. Three dominant peaks in the range of $0.1\mu m/N$ can be seen.

The measurement results of the corresponding dynamic compliances are shown in figure 6.5. The first two peaks are very clear again but the peak in the dynamic compliance in Y-direction due to an excitation in Y-direction was measured with smaller amplitude at about $117 Hz$. For a better comparison, the results of the calculation (ACK) and the ones from the measurement (EMA) are all shown in 6.6.
6. Dynamic behavior of a machining center

Figure 6.5: Measured diagonal elements of the dynamic compliance.

Figure 6.6: Comparison of calculated (ABK) and measured (EMA) diagonal elements of the dynamic compliance.
The goal of the presented comparison is not to achieve the best possible identification of the model parameters with the measurement results. Different methods exist to obtain such identifications [99, 100, 101]. In this chapter, the goal is to show the correlation obtained with little adjustments and the interpretation of results of an exemplary dynamic analysis.

### 6.2.2 Mode shapes

To get information about how the machine reacts at the dominant frequencies, the mode shapes are analyzed.

At about 50 Hz, the curves of the diagonal elements of the dynamic compliance show a maximum in the vertical Z-direction (blue). The components in X- and Y-direction have a smaller peak in the same order of magnitude. The calculation results and the fitting methods of the EMA show that two natural frequencies are close to 50 Hz. One mode shape is responsible for the more critical peak in Z- and Y-direction and the other for the lower peak in X-direction (red). A representation of the more critical mode shape is shown in figure 6.7. The comparison shows the mode shape calculated with the four body model on the left (a) and on the right (b), the one identified with the experimental modal analysis.

![Figure 6.7: Comparison of the representations of the mode shapes corresponding to the dominant frequency at about 50 Hz: (a) calculated with four body model, (b) identified with experimental modal analysis.](image)

At about 90 Hz, a second dominant frequency is clearly recognizable in the dynamic compliance in the horizontal X-direction due to an excitation in X-direction (peak of red curve at about 90 Hz). It represents the highest peak in the measurements and results smaller in the rigid body simulations. Since the machining center has a configuration with a vertical
working spindle, the majority of the alternating process forces will be in the horizontal XY-plane (perpendicular to the tool axis Z). Therefore, this can be regarded as a mode shape, which could be limiting for the high performance cutting process and even lead to instabilities as chatter. Detailed analysis of the dynamics of the cutting process can be found in [102, 103, 104, 105]. A comparison of the corresponding mode shape resulting from the calculation and from the EMA is shown in figure 6.8. It consists of a distortion of the X- and the Z-axis around the Y-direction.

At about 120Hz, a third dominant frequency is recognizable. The dominant direction is the Y-direction (green) with a lower peak in Z-direction (blue). Here the calculations resulted in an amplitude a little larger than the measurements. A comparison of the corresponding mode shape resulting from the calculation and from the EMA is shown in figure 6.9. It consists of a similar mode shape than the one of the first dominant frequency (figure 6.7) but with an inverted phase between the tilt movement of the Z- and the X-axis.

### 6.2.3 Cross-talk in FRF

As described in section 6.2.1 the displacements in the direction orthogonal to the direction of the excitation are defined as cross-talk. The importance of this effect is often underestimated in the dynamic evaluation of machining centers. In this section, an evaluation of the cross-talk behavior is presented as an example.
6.2 Discussion and comparison of calculation and measurement results

Figure 6.9: Comparison of the representations of the mode shapes corresponding to the dominant frequency at about $120\,Hz$: (a) calculated with four body model, (b) identified with experimental modal analysis.

Calculations

Figure 6.10 shows the calculated dynamic compliances in the three directions due to an excitation in X-direction. The cross-talk in Y- and Z-direction due to excitation in X-direction results to be negligible since it is about two orders of magnitude smaller than the reaction in X-direction due to excitation in X-direction. Therefore, the reactions of the machine in Y- and Z-direction can be regarded as independent of the excitations in X-direction.

When exciting the machine in Y-direction the response is not only in Y-direction but there is a large cross-talk in Z-direction. Figure 6.11 shows the calculated dynamic compliances in the three directions due to an excitation in Y-direction. In the range of the three recognized dominant frequencies (between $50\,Hz$ and $120\,Hz$), the cross-talk in Z-direction due to excitation in Y-direction is in the same range as the direct reaction in Y-direction.

A similar behavior can be seen when exciting the machine in Z-direction. In the range of the three recognized dominant frequencies (between $50\,Hz$ and $120\,Hz$) the cross-talk in Y-direction due to excitation in Z-direction is also in the same range as the direct reaction in Z-direction (figure 6.12).

The cross-talk in X-direction is negligible. Just as an excitation in X-direction does not cause a relevant reaction in Y- or Z-direction (figure 6.10) no relevant reaction is caused in X-direction due to an excitation in Y- or Z-direction (figures 6.11 and 6.12).
Figure 6.10: Calculated dynamic compliance due to an excitation in X-direction.

Figure 6.11: Calculated dynamic compliance due to an excitation in Y-direction.
Measurements

When measuring the frequency response functions the structure of the machining center is excited at one point using an impulse hammer with integrated force capture. The motion response is measured in multiple points with triaxial piezoelectric accelerometers.

The calculated cross-talks in the dynamic compliances shown in figures 6.10, 6.11 and 6.12 correspond to a reaction to an excitation in the exact orthogonal direction. In the practical measurement, this can not be achieved perfectly. If the direction of the excitation has a component of only 1° in the direction measured as the cross-talk it results in 1.75% of the force acting in the parallel direction ($\sin(1°) = 0.0175$). Furthermore the three-axis piezoresistive accelerometers have cross-axis sensitivity in the range of about 2% [106]. To show the effect of this uncertainty, gray curves with 2% of the reaction parallel to the excitation are shown in the figures of the calculated results as an estimated measurement uncertainty (figures 6.10, 6.11 and 6.12). For the cross-talks $H_y/F_x$, $H_z/F_x$, $H_z/F_y$ and $H_x/F_y$ the estimated measurement uncertainty is larger than the calculated reaction. This explains why it is not realistic to interpret this measurement; improved metrology would be needed first.

When comparing the results of the calculated dynamic compliances due to an excitation in X-direction (shown in figure 6.10) with the measurement results (shown in figure 6.13)
the following points can be observed:

- The cross-talk in Y- and Z-direction results to be negligible also in the measurement results.
- The measured cross-talk is larger than the calculated.
- The difference between the measured cross-talk and the measured direct reaction is never as large as in the calculations.

![Figure 6.13: Measured dynamic compliance due to an excitation in X-direction.](image)

When looking at the negligible cross-talk in X-direction measured due to an excitation in Y- and Z-direction (red curve in figures 6.14 and 6.15) the same conclusions can be drawn. When analyzing the measurements of the cross-talk in Z-direction due to an excitation in Y-direction (blue curve in figure 6.14) and the cross-talk in Y-direction due to an excitation in Z-direction (green curve in figure 6.15) the following points can be observed:

- Large cross-talk effects can be measured very well with the described setup.
- The measured cross-talks in X-direction due to an excitation in Y- and Z-direction have a peak at about 90Hz. These peaks are clearly caused by a small component of the excitation in X-direction.
6.2 Discussion and comparison of calculation and measurement results

Figure 6.14: Measured dynamic compliance due to an excitation in Y-direction.

Figure 6.15: Measured dynamic compliance due to an excitation in Z-direction.
6.3 Summary

In this chapter, the evaluation of the dynamic behavior of a machining center has been presented. Results of calculations with the ACK are compared with results from an EMA as verification of the simulation environment.

In section 6.1, the procedure of modeling a machining center with the ACK is explained. The focus is on how the rigid body model is defined and how the different connecting elements are modeled. It shows the importance of the analysis of the FRF and the corresponding mode shapes for an example of a machining center prototype.

In section 6.2, an evaluation procedure of the dynamic properties is shown. It shows the importance of the analysis of the FRF and the corresponding mode shapes for an example of a machining center prototype. The comparison with results from an EMA serves as verification of the simulation environment. The comparison demonstrates how well the dynamic behavior of a machining center can be modeled with a model consisting of only four rigid bodies.

In section 6.2.3, the cross-talk in the FRF is analyzed. The interpretation of the calculation results shows the additional conclusions that can be drawn from the analysis of the cross-talk. The comparison with the measurement results from the EMA shows the difficulties of the measurement of cross-talk due to external excitation.

The conclusions that can be drawn from this analysis are described in chapter 8.
Relative evaluation of alternative configurations of a dynamic machine tool using rigid body simulation

In this section, a relative evaluation of alternative machine tool configurations is presented. Dynamic machine tools are taken as an example of the rigid body calculations. The presented results represent the basis for the comparison. The models use fictive parameters and the results do not represent values of real machines. By using the same parameters for both configurations, a relative evaluation is possible without knowing the exact physical values.

7.1 Machine configurations

The ACK (described in section 5.7, starting on page 104) was used to model two alternative machine setups. As a first position of the machine axes, it was chosen to analyze the critical position where the Y- and the Z-axis are in an extended position. This position will be referred to as position 1.

7.1.1 Original version

Figure 7.1 shows a representation of the machine model of the original version in position 1 with its bodies colored according to the direction that they are actuated (X, Y, Z). The bodies and the connecting elements are modeled as described in section 6.1.

In this original version, the cantilever arm of the machine (red in figure 7.1) is connected
with the base of the machine (gray in figure 7.1). It can be actuated in X-direction parallel to the length of the base. The Y-axis is represented by the green body. It can be actuated in Y-direction along the length of the cantilever arm. The last actuated axis consists of the Z-axis (blue in figure 7.1). The TCP is considered to be at the bottom of the Z-axis and the workpiece is on a table connected to the inertial system. Since the table and the workpiece are independent of the analyzed configuration change, they are not included in the model. The structure code (as introduced in section 3.3.1 on page 15) results to be t-Z-Y-X-b-w.

![Figure 7.1: Representation of the machine model of the original version in position 1 with its bodies colored according to the direction that they are actuated (structure code t-Z-Y-X-b-w).](image)

### 7.1.2 Alternative configuration

The structure code is the same for both versions (t-Z-Y-X-b-w). The main difference consists in the configuration of the cantilever arm. While in the original version the cantilever arm corresponds to the X-axis it corresponds to the Y-axis in the alternative configuration. Figure 7.2 shows the representation of the machine model of the alternative version in position 1 with its bodies colored according to the direction that they are actuated (X, Y, Z). It shows that the cantilever arm (green) is not directly connected to the base, as it is the case for the original version, but that an X-axis element is in between (red). The difference of the kinematic setup is visualized in figure 7.6 on page 135 where a comparison of two extreme positions is represented for the two kinematic setups.
7.2 Flexible bodies

The flexibility of the cantilever arm and of the Z-axis is modeled by division of the bodies into multiple rigid elements connected by resilient elements (see also section 5.3 on page 103).

For this relative analysis, five segments were chosen for each long structure (see figures 7.1, 7.2 and 7.6). The three central segments are chosen to have twice the extension in longitudinal direction than the two segments at the end. With such a model, the static deformation due to bending can be modeled with 1% uncertainty [69]. The first five natural frequencies can be approximated with an error smaller than 5% [59].

The properties of the elements and the connections were calculated based on the outer dimension of the extended structure, the thickness and equivalent density of the wall region and the equivalent density of the core of the beam. This way the inertial and stiffness properties of beam structures containing ribs can be approximated in an efficient way at a stage where details are not known yet.

7.3 Dynamic load case

For high performance milling machines (as discussed in section 6.2.1 on page 114) the load case to be analyzed is usually a dynamic force between the tool and the workpiece even if
the acceleration loads can not be neglected [107, 59].

For dynamic machine tools with negligible process loads on the other hand, the main load case to be analyzed is the one coming from the acceleration of the individual axes. This load is equivalent to a force acting at the driving point and reacting in the center of mass of the accelerated body. Consequently, the frequency response function to analyze is the response to a changing acceleration load rather than a changing external excitation as it occurs in the milling process. The response is again analyzed as a displacement and therefore it can be looked at as a special kind of dynamic compliance. The unit chosen for dynamic machining centers is $\mu m/g_n$ since common accelerations of dynamic machining centers are in the range of the standard acceleration due to gravity $g_n$ and the expectable deviations are commonly expressed in micrometers. The abbreviation used for a dynamic compliance in X-direction due to acceleration in Y-direction would be $H_{x/y}$.

The standard acceleration due to gravity is defined as value adopted in the International Service of Weights and Measures and confirmed in 1913 by the 5th CGPM as the standard for acceleration due to gravity. ($g_n = 9.80665 m/s^2$) [43]

This type of analysis is also known as dynamic error budgeting. An analytical analysis is presented in [59] for the 2-dimensional case and generalized for machining centers.

7.4 Visual analysis of the effect of accelerations

In this relative evaluation first a visual representation of the effect is compared for the two variants at one position and then numerical values are analyzed.

7.4.1 Acceleration in X-direction

Figure 7.3 shows a comparison of the representations of the effect of an acceleration in X-direction on the two structures. The amplitude can not be taken as a comparison since both represent a unitary deflection.

The basic effect is the same for both variants: when trying to accelerate in X-direction, the TCP lags behind due to the inertial load. Two main components contribute to this effect:

- The first one is due to the compliance of the drive system. The stiffness of the drive can be regarded as equal in a first step and the total moved mass in X-direction is in the same range. Therefore, this effect is expected to be similar.
The second main effect is that the cantilever arms are rotated around the vertical axis. When looking at the visualization in the XY-plane (right side of figure 7.3) it can be noted that the rotation is more evident for the alternative variant than for the original variant.

The effects of structural deformations result to be of second order for this load case. The exact influence on the position of the TCP will be analyzed in section 7.5.

![Figure 7.3: Comparison of the representations of the effect of an acceleration in X-direction on the two structures (original on top and alternative on bottom).](image)

### 7.4.2 Acceleration in Y-direction

The different effect of an acceleration in Y-direction of the two variants is much more evident than for the acceleration in X-direction. Figure 7.4 shows a comparison of the representations of the effect of an acceleration in Y-direction on the two structures.

The reaction of the two kinematic setups can be interpreted in the following way:

- The compliance of the drive system of the Y-axis leads to a displacement as it could be seen for the X-axis but the mass to be displaced in Y-direction is much smaller
for the original kinematic setup than for the alternative one. Therefore, this effect is much smaller for the original setup.

- The compliance in the guide way systems of the Y- and Z-axis and the offset of the center of mass of the moved bodies from the driving point (depending on the vertical position of the TCP) lead to a rotation of the moved bodies around the X-axis. The visualization shows that for the alternative configuration the effect depends also a lot on the connection of the machine relative to the ground. This can be seen as a disadvantage because of the large variations of the setup parameters and the resulting uncertainty.

The reason for the large difference is that in the original version, the cantilever arm is not moved for a displacement of the TCP in Y-direction but in the alternative kinematic setup it is.

![Comparison of the representations of the effect of an acceleration in Y-direction on the two structures (original on top and alternative on bottom).](image-url)
7.4.3 Acceleration in Z-direction

The effect of an acceleration in Z-direction is similar for the two setups because the accelerated body (Z-axis) is the same. Therefore, the effect of the drive system is also the same. Figure 7.5 shows a comparison of the representations of the effect of an acceleration in Z-direction on the two structures. The following differences can be noted:

- The reaction of the original concept is due to the compliance in the setup elements and of the guide way systems of the X-axis. For the alternative concept the compliance in the X- and Y-guide way system are clearly more involved than the one of the setup elements.

- The inertial force of the reaction acts centered on the cantilever arm of the alternative kinematic setup while in the original setup it acts on the side of the arm. This represents an advantage for the alternative setup because no relevant cross-talk results in X-direction (see table 7.1 in the following section 7.5).

Figure 7.5: Comparison of the representations of the effect of an acceleration in Z-direction on the two structures (original on top and alternative on bottom).
7.5 Numerical results

To be able to compare the two kinematic setups in a quantitative way the numerical values of the calculated displacements are analyzed.

7.5.1 Position 1

First of all the dynamic compliances due to quasi static acceleration are analyzed for position 1 (Y- and Z-axis in extended position). This allows comparing the amplitudes of the displacements of the reactions shown in the figures 7.3, 7.4 and 7.5. Especially the reaction in the direction perpendicular to the acceleration can be evaluated, as it is more difficult to see in the visual representations. This effect is known as inertial cross-talk. A description with measurement possibilities is included in [108].

The inertial cross-talk is defined as displacements perpendicular to the intended direction of motion owing to a lateral offset between the driving force and the centre of mass, which lead to tilt motions during acceleration and deceleration [108].

Table 7.1 shows a comparison of the three components of dynamic compliances due to quasi-static acceleration of the axes at position 1 for each direction.

Table 7.1: Comparison of the magnitudes of the dynamic compliances due to quasi-static acceleration of the axes at position 1.

<table>
<thead>
<tr>
<th></th>
<th>original</th>
<th>alternative</th>
<th>change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_x/\ddot{x}$</td>
<td>$\mu m/g_n$</td>
<td>339</td>
<td>609</td>
</tr>
<tr>
<td>$H_y/\ddot{x}$</td>
<td>$\mu m/g_n$</td>
<td>10</td>
<td>33</td>
</tr>
<tr>
<td>$H_z/\ddot{x}$</td>
<td>$\mu m/g_n$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$H_x/\ddot{y}$</td>
<td>$\mu m/g_n$</td>
<td>1</td>
<td>75</td>
</tr>
<tr>
<td>$H_y/\ddot{y}$</td>
<td>$\mu m/g_n$</td>
<td>192</td>
<td>361</td>
</tr>
<tr>
<td>$H_z/\ddot{y}$</td>
<td>$\mu m/g_n$</td>
<td>7</td>
<td>87</td>
</tr>
<tr>
<td>$H_x/\ddot{z}$</td>
<td>$\mu m/g_n$</td>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>$H_y/\ddot{z}$</td>
<td>$\mu m/g_n$</td>
<td>18</td>
<td>61</td>
</tr>
<tr>
<td>$H_z/\ddot{z}$</td>
<td>$\mu m/g_n$</td>
<td>70</td>
<td>128</td>
</tr>
</tbody>
</table>
7.5 Numerical results

Acceleration in X-direction

The first three rows of table 7.1 include the reaction of the two structures due to acceleration in X-direction and correspond to the amplitudes of the reaction shown in figure 7.3. The following points can be concluded:

- $H_x/\ddot{x}$: The displacement in X-direction due to acceleration in X-direction is 80% larger for the alternative kinematic setup than for the original one. For both setups, this value represents the largest component of the reaction. This is because the largest mass is accelerated with a large offset. The larger magnitude of the alternative setup is because the moment of the inertial force acts on the compliance of the guide way system of the X- and the Y-axis and not only on the one of the X-axis as for the original version. See section 7.6 for more detail.

- $H_y/\ddot{x}$: The inertial cross-talk in Y-direction due to acceleration in X-direction is more than three times as large for the alternative configuration as for the original one.

- $H_z/\ddot{x}$: The inertial cross-talk in Z-direction due to acceleration in X-direction is small and the same for both setups.

Acceleration in Y-direction

The central three rows of table 7.1 include the reaction of the two structures due to acceleration in Y-direction and correspond to the amplitudes of the reaction shown in figure 7.4. The following points can be concluded:

- $H_x/\ddot{y}$: The inertial cross-talk in Y-direction due to acceleration in X-direction is negligible for the original setup but quite large for the alternative one. For the original setup, the moved mass is much smaller and the horizontal offset is small. For the alternative setup on the other hand, the moved mass in Y-direction is large because the entire cantilever arm needs to be displaced. Since the drive is modeled on one side of the arm (corresponding to a rack and pinion drive system) a horizontal and vertical offset result between the mass and the center of mass to be displaced.

- $H_y/\ddot{y}$: The displacement in Y-direction due to acceleration in Y-direction is almost twice as large for the alternative solution as for the original one. This is again mainly due to the large mass to be accelerated. It has a visible influence on the connecting elements of the base relative to the ground as can be seen in the lower right part of figure 7.4.
7. Relative evaluation of alternative configurations

- $H_{z/y}$: The inertial cross-talk in Z-direction due to acceleration in Y-direction is more than ten times larger for the alternative kinematic setup than for the original one. For the original setup it is small and is mainly due to a rotation of the Y- and the Z-axis relative to the cantilever arm. For the alternative setup on the other hand, it corresponds to the largest component of inertial cross-talk.

Acceleration in Z-direction

The last three rows of table 7.1 include the reaction of the two structures due to acceleration in Z-direction and correspond to the amplitudes of the reaction shown in figure 7.5. The following points can be concluded:

- $H_{x/z}$: The inertial cross-talk in X-direction due to acceleration in Z-direction is the only component of inertial cross-talk, which is smaller for the alternative solution than for the original one. As explained in section 7.4.3 this is because the inertial force of the reaction acts centered on the cantilever arm of the alternative kinematic setup while in the original setup it acts on the side of the arm.
- $H_{y/z}$: The inertial cross-talk in Y-direction due to acceleration in Z-direction represents the largest component for the original configuration. This is because of the large offset of the inertial force to the compliance of the guide way system of the X-axis and of the connection of the base relative to the ground. For the alternative solution, it is even larger because the same force acts with a similar offset also on the compliance of the Y-axis.
- $H_{z/z}$: The displacement in Z-direction due to acceleration in Z-direction represents the smallest direct inertial dynamic compliance for both setups because the lightest body is accelerated with a small offset to the driving point.

7.5.2 Influence of the axes position

In the previous sections the dynamic properties of the two concepts were analyzed for one position at the corner of the working range were the Y- and the Z-axis are in an extended position. The X-axis was left in the centered position. When looking at figure 7.6 it seems obvious that the analyzed parameters change depending on the position of the TCP.

The influence of the axes position can be analyzed very efficiently with the ACK. In this case, a grid of 20 by 20 points was analyzed in an YZ-section of the working range. As an example, the evaluation of the displacement in X-direction due to acceleration in X-direction ($H_{x/x}$) is discussed.
Figure 7.6: Comparison of two extreme positions represented for the two different kinematic setups.

Figure 7.7 shows a comparison of distributions of calculated quasi-static displacements in X-direction due to acceleration of the X-axis $H_x/\ddot{x}$ for the two machine models.

Figure 7.7: Comparison of distributions of calculated quasi-static displacements in X-direction due to acceleration of the X-axis ($H_x/\ddot{x}$) for the two machine models.

On the left side of the figure, the distributions of the values are shown. The following points can be noted from these two images:

- The analyzed position 1 (lower right corner of the images) represents the worst case for both configurations with the values stated in table 7.1.
- For both setups, the top left corner represents the position where $H_x/\ddot{x}$ is smallest.
• While the position of the Z-axis has the main influence on the reaction of the original setup, the influence of the Y-axis is dominant for the alternative setup.

Additional information can be drawn from the right side of figure 7.7. It represents a visualization of the distribution in form of Box Plots (see section 3.11 starting on page 83). The following conclusions for the distribution of the values of $H_x/\dot{x}$ for the two setups can be drawn from this representation:

• The values vary a lot more for the alternative setup than for the original one (difference between the two quartiles and between the extremes is much larger). This is a disadvantage because it can result in an undesired higher variation of machining results depending on the position of the workpiece within the working range. If the dynamic behavior changes a lot in the working envelope it makes it also difficult to setup the control parameters that are usually not model based or in other words the same for all positions [109].

• The upper quartile of the data of the original setup is smaller (in this case better) than the lower quartile of the alternative setup. Therefore, values at over 75% of the calculated positions are better for the original setup.

• The best value of the alternative setup (low extreme) is better than most values of the original setup. This shows the importance of an analysis of values at different positions because when comparing values at only one position, wrong conclusion can be drawn.

7.6 Schematic comparison

The main reason for the differences found in the dynamic behavior of the two setups can be understood when analyzing the setup of the machines. Figure 7.8 shows a schematic comparison of the machine setups.

In the original setup, the long and heavy cantilever arm (black) is connected directly to the base. The smaller and lighter Y-axis (green) is located between the cantilever arm and the Z-axis, which is the same for both configurations.

In the alternative setup, the cantilever arm (black) is not directly connected to the base but the X-axis (red) is in between. This has two negative consequences for the dynamic behavior of the machine:

• In addition to the compliances of the X-axis, also the compliances of the Y-axis are located between the base and the cantilever arm. Assuming the same tilt stiffness
for the X- and Y-axis guide way systems, the resulting combined stiffness between base and cantilever arm for the alternative concept is about one half of the original layout.

- The heavy cantilever arm is actuated not only in X-direction but also in Y-direction. This results in larger errors in Y-direction due to the compliance in the drive in Y-direction.

### 7.7 Summary

In this chapter, a relative evaluation of two configurations of a dynamic machine tool, using rigid body simulation, has been described.

In section 7.1, the two configurations are described.

How flexible bodies can be integrated in rigid body models is shown in section 7.2. Both presented models include two bodies each that have been divided into multiple rigid bodies in order to take into account the flexibility of the beam like components.

The description of the dynamic load case used for the comparison is in section 7.3. The so called dynamic error budgeting can be used to analyze errors due to inertial loads.

In section 7.4, the effects of accelerations are analyzed in a visual way. The comparison of the visualizations allows recognizing the main sources of errors.

In order to have a quantitative comparison of the resulting errors the numerical values
are compared in section 7.5. This allows also comparing the inertial cross talks that are not always clearly recognizable in the visualizations. In a first step of the comparison, only one critical position of the TCP is analyzed (section 7.5.1). In a further step, the advantages of analyzing multiple positions of the TCP throughout the working envelope of the machine are presented (section 7.5.2). The visual comparison of the alternative setups in two extreme positions (figure 7.6) shows the difference between the two setups very well. The comparison of the distribution of the dynamic properties is shown for one parameter as an example (figure 7.7). The advantages of analyzing the distribution of the analyzed parameters are shown and the relevance emphasized.

Finally in section 7.6 a schematic comparison of the two setups is shown that explains the main reason of the different dynamic properties of the two configurations.

The conclusions that can be drawn from this analysis are described in the following chapter 8.
Chapter 8

Conclusion of part II

The main scope of chapter 6 is to give a verification of the calculation results achieved with the ACK and to demonstrate a modeling process.

It can be concluded that with a rigid body model, consisting of only few bodies (in this case four), the dynamic behavior of a machining center can be approximated to the following extent:

- The critical frequencies with the corresponding mode shapes can be approximated with enough precision to be able to:
  - identify the mode shapes most critical for the manufacturing process.
  - understand the dynamic behavior of the machining center, which allows to plan critical machining operations accordingly (e.g. selection of direction of milling).
  - compare effects of design modifications (e.g. parameters of the connecting elements) on the behavior of the machining center.

- The FRF can be approximated within the accuracy of the estimation of the damping properties. The main difficulty of the estimation of the FRF in the frequency range critical for the manufacturing process is not the degree of detail of the model but the estimation of the equivalent stiffness and physical damping properties in the connecting elements. The analysis of the FRF represents an important tool for a first selection of dominant frequencies.

- The effect of different setup properties (stiffness and damping) can be compared.

The fact that a machining center can be modeled so well with a rigid body model leads to the conclusion that the relevant compliances are located in the connecting elements.

Results that can not be achieved with a rigid body approach include the following:
- Local effects of structural variation of components.
- Local zones of components with maximal stress.

Both of these aspects are not relevant in the early development phase. They can be analyzed in a later step with FEM when designing and optimizing the different components.

For the modeling process of a rigid body model the following conclusions can be drawn:

- The stiffness properties modeled in the connecting elements represent also the compliance of part of the linked bodies. For the consideration of this effect, a starting value for the stiffness of the linear guide way systems of one third of the catalogue value has shown good results.
- By modeling the damping effects as linear physical damping elements in the connecting points the effect of changes (such as in the configuration) can be modeled better than by assumption of a constant loss factor for the damping.
- An exact modeling of the damping behavior is very difficult but not needed for the relative analysis of changes.
- The properties of the drive systems can be modeled as a constant stiffness for the analysis of the dynamic properties at a given position of the TCP.

The analysis of the cross-talk in the FRF gives important information about the spatial dynamic behavior. With the described measurement method used for the EMA, the measured cross-talks include part of the component of the excitation in measurement direction.

The main scope of chapter 7 is to demonstrate how the dynamic properties of two machine tools can be compared at an early development phase.

A multi body approach is very well suited for a relative comparison of kinematic setups. It allows quantifying the expectable changes of static and dynamic stiffness properties. The possibility of integrating flexible bodies in rigid body models has been demonstrated.

By using the same parameters for both configurations, a relative evaluation is possible without knowing the exact physical values.

The importance of the visualization of the results has been shown with the numerous interpretations that could be drawn from the visualizations.

Next to being able to analyze machine properties at one specific position it allows to analyze the influence of the position of the machine in a very efficient way, which is much more time consuming with standard finite element approaches.

The additional information that can be drawn from analyzing the distribution of values at different positions of the TCP throughout the TCP has been demonstrated and a very
helpful way a visualizing the distributions for a comparison has been presented for this application.

In contrast to the example shown in chapter 6, where the analyzed load case is an external force between the TCP and the workpiece in the relative evaluation of chapter 7 the effects of acceleration loads were analyzed. This so called dynamic error budgeting represents a good way of analyzing the dynamic properties of machining centers where the external loads are negligible compared to the inertial loads.

For the comparison of the expectable path accuracy of dynamic movements, the analysis of inertial cross-talk results as very important.

As a final conclusion, the author would like to state that if the dynamic behavior of two concepts results to be very similar then this criterion should not be decisive for the selection of the concept. It is important to take into account multiple criteria already in the development phase [110, 111].
Appendix A

Practical aspects of cutting a Test Piece on a specific machine type

In this chapter, different practical aspects of cutting a specific test piece are described. The goal is to share experiences and encountered difficulties and solutions.

A.1 Test piece fixture

The simplest form of cutting a test piece is to use a cuboid block as fixture and blank. Especially if multiple test pieces are planed a manufacturing of a dedicated separate test piece fixture and blank should be considered.

Figure A.1: Illustration of the difference between the first test piece cut out of a cubic block and the second step with fixture and blank in form of a disk: (a) first test piece with the blank, (b) the fixture with an inclination angle $\beta = 30^\circ$, (c) the blank on the fixture, (d) finished test piece on the fixture.

The first test pieces were cut out of a cuboid block. In a second step a fixture was made
and the blank for the test piece was reduced to only a disk. An illustration is shown in figure A.1. It shows the first test piece with the blank (a), the fixture with an inclination angle $\beta = 30^\circ$ (b), the blank on the fixture (c) and finally the finished test piece on the fixture (d).

The use of such a fixture brings the following advantages:

- less raw material needed for each test piece
- machining
  - less rough cutting
    → few minutes instead of half an hour machining time
- measurement
  - simple alignment on measuring device
    → same measurement setup for different inclination angles $\beta$
- same blank for different test piece setups (only fixture changes)

### A.2 Top and bottom position

To simplify the interpretation of the movements of the machine axes and the measurements, a top and a bottom position of the test piece is defined according to figure A.2.

Figure A.2: Test piece on the mounting element ($\alpha = 45^\circ, \beta = 30^\circ$) with top and bottom positions marked.
A.3 Manufacturing process

The result of any test piece depends not only on the machine but also on the process parameters, tools used and the programming technique. These parameters as well as the size, the material and other conditions should be agreed upon and recorded with the measurement results [2]. Here some values are presented that showed good results for specific examples.

A.3.1 Approach

The approach is quite a critical point of the manufacturing process. The changeover for the approach movement to the cutting of the nominal geometry mostly implies a discontinuous movement of the machine axes. Furthermore the chip thickness is not constant during the approach movement. These factors are not only true when milling a cone geometry with five simultaneous axes but also when milling a cylinder with only two simultaneous axes.

In order to recognize whether an error on the test piece geometry at the point of approach is due to the machine (undesired displacement due to discontinuous movement) or due to the tool or the amount of allowance, the effects of the approach can be compared on the geometry of the cylinder and the cone. In order to have a good comparison the approach process should be as similar as possible for both geometries.

Difficulties encountered with the approach when face milling a test piece are described in section A.5.3.

A.3.2 Further geometric elements

With more than only one element on a test piece, relative evaluations can be made and by comparison of the results additional conclusions can be achieved. A cylinder element represents a well known geometry to be included in the test piece. End milling a cylinder can be achieved by circular interpolation of two linear machine axes. For the interpretation of measurements of an end milled cylinder a lot of experience exists from the field of circular tests [38, 112, 113].

The parameters needed to describe the nominal geometry of such a test piece are shown in figure A.3.
A.3.3 Feed rate

The contouring feed rate $f$ should be kept constant for the cylinder and the cone. For the cylinder it results in a sinusoidal movement of the X- and Y-axis. For the cone on the other hand all five axis need to perform a simultaneous movement, which depends on the setup of the test piece.

On an up to date machining center high speed cutting parameters are possible. Good experiences have been made with the parameters shown in table 3.7 on page 91.

A.3.4 Stock allowance

Stock allowance is defined as thickness of the material, which is to be removed by the machining process [114].

The amount of stock allowance has an effect on the force acting between the tool and the workpiece and therefore on the deformation and the resulting displacement between the tool and the workpiece. During the approach, the cutting depth and therefore the cutting force increase. Especially when the approach to a nominal surface occurs in a convex part of the surface the amount of stock allowance defines the zone in which the cutting force is not constant. This is the case for both: the cylinder and the cone geometry of the test piece. In the specific case of a round path, like of the cylinder and the cone, the zone of approach is also the zone of departure which means that in this zone the force decreases in the departure process and that therefore the cutting depth gradually decreases to zero.

With a small stock allowance (in this case of 0.2mm) and a tangential approach this zone
can be kept small.

A.3.5 Tool

Next to the strategy of approach also the tool has an influence on the resulting geometry in the approach and departure zone. The largest effect on the very local deviations at the zone of approach and departure were found as a function of the tool.

As a result of these tests it seems very appropriate to include not only a cone geometry but also a cylinder surface to be manufactured (see previous section A.3.2). To ensure that it can be manufactured on most machining centers in comparable ways it is recommended to end mill both surfaces in order to be able to compare and identify the errors due to the approach.

A.4 Measurement of the test piece

The elements on the test piece that can be tolerated in addition to the circular form of the circular form of the cone surface according to [115, 116] are shown in figure A.4. For a periodic examination of machine tools in order to detect changes in the performance of a machine the analysis of all these parameters can be helpful.

![Test geometry with tolerated elements](image)

Figure A.4: Test geometry with tolerated elements [115, 116]

A.4.1 Workpiece coordinate system (frame $\sum W$)

When measuring the test piece on a coordinate measuring machine (CMM) as a first step a workpiece coordinate system (frame $\sum W$) has to be defined. The elements needed for this task are highlighted in the figures A.4 and A.5. As a first element the reference plane A is measured (red in figure A.5) which defines the XY-plane of the frame $\sum W$ (or the Z-axis). As a second step the reference cylinder C (green in figure A.5) is measured and the intersection of the axis of the reference cylinder C and the reference plane A will
result in the origin of the frame $\sum_W$. The direction of either the X- or the Y-axis in the reference plane A can be chosen arbitrarily. To ensure repeatable measurement results and an angular reference, a groove can be included in the workpiece (gray in figure A.5). The left plane of this groove (blue in figure A.5) is measured and projected into the reference plane A to define the Y-direction. With that the frame $\sum_W$ is defined.

![Figure A.5: Test piece with workpiece coordinate system (frame $\sum_W$)](image)

**A.4.2 Different aspects of measuring on a coordinate measuring machine (CMM) or on a roundness tester**

Both, the CMM and the roundness tester have advantages and disadvantages for the measurement and evaluation of the test piece.

**Coordinate measuring machine (CMM)** On a CMM all tolerated elements of the test piece (shown in figure A.4) can be evaluated in one measurement with standard measurement software. Calculation of a best fit cone and the corresponding cone form error (normal deviation from the best fit cone) as well as relative squareness and concentricity errors are standard evaluations.

The disadvantages of a CMM compared to a roundness tester is the larger measurement uncertainty. A measurement uncertainty of about $1.2 \mu m$ for the circular form of a circle with a diameter of $100 \text{mm}$ is a realistic value for a standard CMM.
Roundness tester  The great advantage of a roundness tester is its small measurement uncertainty. A measurement uncertainty of only $0.25\mu m$ for a measured circular form error of $10\mu m$ is a typical value for an up to date roundness tester.

The disadvantage is that evaluations of all elements tolerated in figure A.4 can usually not be evaluated with a roundness tester.

A.5  Encountered difficulties for face milling

The first test pieces manufactured in this investigation at the IWF were face milled. The original idea was that when end milling a cone with a universal head kinematic the compensational movements of the linear axes would be a lot smaller than for a face milled version. Furthermore when face milling, angular error motions could be seen better on the geometry. Nevertheless, this is only the case for large tools and can be neglected for small tool diameters and the expectable machine errors.

To avoid an evanescent cutting velocity in the center of the tool the spindle axis was chosen with an offset to the conical surface as shown in figure A.6.

![Figure A.6: Position of the tool with respect to the cone for face milling.](image)

The very first test piece in aluminum was cut on a machining center of the 80’s. A picture with a detailed view of the part of the cone with the largest deviations is shown in figure A.7.

At first the sources of the largest error was thought to be on the machine side but it turned out that it was an error of the strategy. Details follow in the next sections.
A.5 Encountered difficulties for face milling

A.5.1 End mills in CAM systems

In CAM systems the representation of an end mill is usually a finite section of a right circular cylinder with its end closed to form a flat circular surface.

The tool was modeled ideally as a cylinder as it is in most CAM systems. The simplification of modeling the tool as a cylinder can cause an error when face milling a convex surface due to the hollow cut of the face of most end mills. A detailed description of the error caused for the chosen cone test piece setup follows in the next paragraph.

A.5.2 Hollow cut of end mills

Common end mills have a hollow cut face with a minor cutting edge angle $\kappa'_r$ [117] in the range of typically $1^\circ$ to $2^\circ$. The exact amount is usually not stated in the specifications of the tools because for most cutting tasks this parameter has little influence. The reason why end mills have a hollow cut is because that way, when face milling a plane surface the end mill cuts only with the circumferential major cutting edges and with the corner of the cutting edges but not with the minor cutting edges of the face of the tool. If a tool had no hollow cut face it would lead to undesired friction between the workpiece and the cutting edges on the face of the end mill when face milling. An image of the end mill used for face milling of the test piece shown in figure A.7 is shown in figure A.8. The hollow cut with the minor cutting edge angle $\kappa'_r$ of the of about $1.7^\circ$ can clearly be seen. For a

Figure A.7: Picture of the first manufactured test piece in aluminum with a detailed view of the part of the cone with the largest deviations.
tool with a diameter of 20\text{mm} \ (r = 10\text{mm}) \ this \ leads \ to \ a \ deviation \ dz \ in \ the \ center \ of \ the \ face \ of \ the \ tool \ in \ direction \ of \ the \ tool \ axis \ of \ almost \ 0.3\text{mm} \ (equation \ A.1).

\[ dz = r \tan(\kappa'_r) \] \quad (A.1)

When face milling a convex surface with a big enough curvature the surface is not cut with only the circumference of the end mill but with the cutting edges of the hollow cut edges of the face of the end mill (if tool axis is perpendicular to the surface). It corresponds to an intersection of a cone instead of a plane with the curved surface. In the specific case of the face milling of the test piece (setup according to figure A.6) the resulting errors are illustrated in figure A.9. As a consequence of the cutting edges of the face of the end mill not being perpendicular to the tool axis the resulting apex angle of the cone is too small by twice the minor cutting edge angle $\kappa'_r$.

The described effect can occur also when face milling convex sculptured surfaces. To avoid this (and for other reasons) usually the tool axis has a lead angle. The idea of a lead angle will be analyzed further on in this section.

**A.5.3 Approach**

Another error resulting from the hollow cut depends on the approach strategy chosen. For the first test pieces the approach strategy was tangent to the cone surface as illustrated in figure A.10.
A.5 Encountered difficulties for face milling

In this case, during the approach of the tool to the cone surface, the interaction between the tool and the workpiece is at the corner of the cutting edges of the tool. On the other hand during the cutting of the cone surface, the cutting edges of the face of the end mill are responsible for the cone surface (see figure A.9). In other words: when approaching the surface in a tangential way, the intersection of the tool and the workpiece is the tool circumference and not the bottom with the minor cutting edge angle $\kappa'$. This explains why there is a part with a large deviation at the point of the approach when using this strategy. Figure A.7 shows a detailed view of how this looks like.
A.5.4 Programming of tool offset with lead angle

To avoid the error caused by the hollow cut of the end mill another cutting strategy was chosen. In the first try the tool axis nominally intersected the cone axis during the manufacturing of the cone surface. The idea of the second approach was to have the leading axis of the tool intersect with the cone axis. This corresponds to an offset of the tool axis and the cone axis by the radius of the tool. How this would look like for a cylinder is shown in figure A.11.

![Figure A.11: Different face milling strategies: tool axis intersects cylinder axis (a), cutting edge in line with cylinder axis or in other words offset between tool axis and cylinder axis by the tool radius (b).](image)

In NC-programming the offset between the tool and the cone axis can not be defined directly in most cases. It can be obtained by definition of a lead angle of the tool. In the documentation of the used CAM software the “lead angle” is defined as follows:

The lead angle is the angle that the tool makes with the plane that is perpendicular to the current direction of motion. A positive angle corresponds to a Lead angle (i.e., the angle between the tool axis and the direction of motion is acute), while negative angle corresponds to a Lag angle (i.e., the angle between the tool axis and the direction of motion is obtuse) [118].

Sometimes it is also referred to as “inclination angle” [119] but the same author also uses “lead angle” in a later publication [120]. In ISO 3002-1 [117] “lead angle” is also used
for an angle of the tool itself and is given as the American English definition whereas in
British English “approach angle” is used. The Greek letter used in this document for the
representation of the “lead angle” is $\varphi$ (see figure A.12).

Figure A.12: Representation of the lead angle $\varphi$ for milling of a nominally plane surface (a) and for milling
of a convex surface (b): the angle $\phi$ is larger than $\varphi$ and does not represent the lead angle.

In this specific case the definition is not quite clear for a lead angle of zero degrees because
with or without offset the tool axis is always in the plane perpendicular to the current
direction of motion. This means that if programming no lead angle at all the tool axis
and the cone axis intersect and if programming an infinitesimal lead angle (e. g. 0.01°) the
tool axis and the cone axis have an offset of the tool radius.

Cutting with a the tool axis not in the plane perpendicular to the current direction of
motion has a drawback for cylindrical end mills: the resulting surface is concave. The
amount depends on the magnitude of the lead angle, on the relation of the tool diameter
to the cutting width and on the curvature of the surface to be manufactured. This ap-
proach was tested on a high speed cutting machine and therefore a tool with a smaller
diameter was used (12 mm instead of 20 mm as on the older machine). Nevertheless, for
this setup the resulting deviation of the generatrix of the cone frustum is only about one
micrometer. Furthermore it has no effect on the circular form of a circle measured in the
plane perpendicular to the cone axis.

A.6 Concluded suggestions for manufacturing

From the practical experience with cutting test pieces the following suggestions can be
concluded:
• Manufacture Test Piece with milling strategy of interest (e.g. end milling).
• Combination with cylinder element is helpful to separate errors resulting from approach.
• Use tangential approach.
• Use high precision set points for cone if possible.
Bibliography


[18] Siemens Product Lifecycle Management Software Inc. *Documentation to NX5 CAD/CAM software*.


# Curriculum Vitae

Sergio Umberto Bossoni, born February 11\textsuperscript{th}, 1976 in Zurich, Switzerland.

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983 - 1991</td>
<td>Elementary school in Zurich, Switzerland</td>
</tr>
<tr>
<td>1991 - 1997</td>
<td>Mathematical and natural scientific school in Zurich (MNG Rämibühl)</td>
</tr>
<tr>
<td></td>
<td>Graduation with Type C Matura</td>
</tr>
<tr>
<td>1993 - 1994</td>
<td>Exchange student in Elizabeth, Illinois, USA</td>
</tr>
<tr>
<td></td>
<td>High School graduation</td>
</tr>
<tr>
<td>1997 - 2003</td>
<td>Student at Faculty of Mechanical Engineering at ETH Zurich</td>
</tr>
<tr>
<td></td>
<td>Specialization in product development and lightweight construction</td>
</tr>
<tr>
<td></td>
<td>Graduation with degree of Dipl. Masch.-Ing. ETH</td>
</tr>
<tr>
<td>since 2003</td>
<td>Ph. D. student and research associate at the Institute of Machine</td>
</tr>
<tr>
<td></td>
<td>Tools and Manufacturing (IWF), ETH Zurich, Switzerland</td>
</tr>
</tbody>
</table>