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State of the cognitive interference channel: a new unified inner bound

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Abstract—The capacity region of the interference channel in which one transmitter non-causally knows the message of the other, termed the cognitive interference channel, has remained open since its inception in 2005. A number of subtly differing achievable rate regions and outer bounds have been derived, some of which are tight under specific conditions. In this work we present a new unified inner bound for the discrete memoryless cognitive interference channel. We show explicitly how it encompasses all known discrete memoryless achievable rate regions as special cases. The presented achievable region was recently used in deriving the capacity region of the linear high-SNR deterministic approximation of the Gaussian cognitive interference channel. The high-SNR deterministic approximation was then used to obtain the capacity of the Gaussian cognitive interference channel to within 1.87 bits.

I. INTRODUCTION

The cognitive interference channel (CIFC)[1] is an interference channel in which one of the transmitters - dubbed the cognitive transmitter - has non-causal knowledge of the message of the other - dubbed the primary - transmitter. The study of this channel is motivated by cognitive radio technology which allows wireless devices to sense and adapt to their RF environment by changing their transmission parameters in software on the fly. One of the driving applications of cognitive radio technology is secondary spectrum sharing: currently licensed spectrum would be shared by primary (legacy) and secondary (usually cognitive) devices in the hope of improving spectral efficiency. The extra abilities of cognitive radios may be modeled information theoretically in a number of ways - see [6], [11] for surveys - one of which is through the assumption of non-causal primary message knowledge at the secondary, or cognitive, transmitter.

The two-dimensional capacity region of the CIFC has remained open in general since its inception in 2005 [7]. However, capacity is known in a number of channels:

• **General deterministic CIFCs.** Fully deterministic CIFCs in the flavor of the deterministic interference channel [1] are being considered in [24, Ch.3], where new inner and outer bounds are shown to meet in certain classes of channels. A special case of the deterministic CIFC is the deterministic

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[1]Other names for this channel include the cognitive radio channel [8], interference channel with degraded message sets [14], [29], the non-causal interference channel with one cognitive transmitter [4], the interference channel with one cooperating transmitter [19] and the interference channel with unidirectional cooperation [13], [20].

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linear high-SNR approximation of the Gaussian CIFC, whose capacity region, in the spirit of [2], was obtained in [22].

• **Semi-deterministic CIFCs.** In [4] the capacity region for a class of channels in which the signal at the cognitive receiver is a deterministic function of the channel inputs is derived.

• **Discrete memoryless CIFCs.** First considered in [7], [8], its capacity region was obtained for very strong interference in [13] and for weak interference in [29]. Prior to this work and the recent work of [4], the largest known achievable rate regions were those of [8], [9], [14], [19]. The recent and independently derived region of [4] was shown to contain [14], [19], but was not conclusively shown to encompass [8] or the larger region of [9].

• **Gaussian CIFC.** This capacity region under weak interference was obtained in [15], [29], while that for very strong interference follows from [13]. Capacity for a class of Gaussion MIMO CIFCs is obtained in [28].

• **Z-CIFCs.** Inner and outer bounds when the cognitive-primary link is noiseless are obtained in [3], [18]. The Gaussian causal case is considered in [4], and is related to the general (non Z) causal CIFC explored in [26].

• **CIFCs with secrecy constraints.** Capacity of a CIFC in which the cognitive message is to be kept secret from the primary and the cognitive wishes to decode both messages is obtained in [17]. A cognitive multiple-access wiretap channel is considered in [27].

We focus on the discrete memoryless CIFC (DM-CIFC) and propose a new achievable rate region and show explicitly how it encompasses or reduces to all other known achievable rate regions. The best known outer bounds for the DM-CIFC are those of [19]. The new unified achievable rate region has been shown to be useful as: 1) specific choices of random variables yield the capacity region of the linear high-SNR approximation of the Gaussian CIFC [22], 2) specific choices of random variables yield capacity in certain regimes of the deterministic CIFC [24] and 3) specific choices of Gaussian random variables have resulted in an achievable rate region which lies within 1.87 bits, regardless of channel parameters, of an outer bound [25]. Numerical simulations indicate the actual gap is smaller.

II. CHANNEL MODEL

The Discrete Memoryless Cognitive InterFerence Channel (DM-CIFC), as shown in Fig. 1, consists of two transmitter-
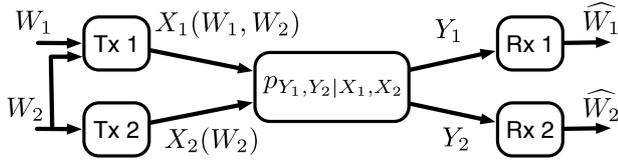
Fig. 1. The Cognitive Interference Channel.

receiver pairs that exchange independent messages over a common channel. Transmitter i, $i \in \{1, 2\}$, has discrete input alphabet $\mathcal{X}_i$ and its receiver has discrete output alphabet $\mathcal{Y}_i$. The channel is assumed to be memoryless with transition probability $p_{Y_1, Y_2 | X_1, X_2}$. Encoder i, $i \in \{1, 2\}$, wishes to communicate a message W_i uniformly distributed on $\mathcal{M}_i = [1 : 2^{NR_i}]$ to decoder i in N channel uses at rate R_i. Encoder 1 (i.e., the cognitive user) knows its own message W_1 and that of encoder 2 (the primary user), W_2. A rate pair (R_1, R_2) is achievable if there exist sequences of encoding functions

$$X_1^N = f_1^N(W_1, W_2), \quad f_1 : \mathcal{M}_1 \times \mathcal{M}_2 \to \mathcal{X}_1^N,$$
$$X_2^N = f_2^N(W_2), \qquad f_2 : \mathcal{M}_2 \to \mathcal{X}_2^N,$$

with corresponding sequences of decoding functions

$$\widehat{W}_1 = g_1^N(Y_1^N), \quad g_1 : \mathcal{Y}_1^N \to \mathcal{M}_1,$$
$$\widehat{W}_2 = g_2^N(Y_2^N), \quad g_2 : \mathcal{Y}_2^N \to \mathcal{M}_2.$$

The capacity region is defined as the closure of the region of achievable (R_1, R_2) pairs [5]. Standard strong-typicality is assumed; properties may be found in [16].

III. A NEW UNIFIED ACHIEVABLE RATE REGION

As the DM-CIFC encompasses classical interference, multiple-access and broadcast channels, we expect to see a combination of their achievability proving techniques surface in any unified scheme for the CIFC:

• **Rate-splitting.** As in [12] for the interference-channel and [8], [14], [19] for the CIFC, rate-splitting is not necessary in the weak [29] and strong [13] interference regimes.
• **Superposition-coding.** Useful in multiple-access and broadcast channels [5], the superposition of private messages on top of common ones [14], [19] is proposed and is known to be capacity achieving in very strong interference [13].
• **Binning.** Gel'fand-Pinsker coding [10], often referred to as binning, allows a transmitter to "cancel" (portions of) the interference known to it at its intended receiver. Related binning techniques are used by Marton in deriving the largest known DM-broadcast channel achievable rate region [21].

We now present a new achievable region for the DM-CIFC which generalizes all best known achievable rate regions including [8], [14], [19], [29] as well as [4].

Theorem 1: Region $\mathcal{R}_{RTD}$. A rate pair (R_1, R_2) such that

$$R_1 = R_{1c} + R_{1pb}, \tag{1}$$
$$R_2 = R_{2c} + R_{2pa} + R_{2pb} \tag{2}$$

is achievable for a DM-CIFC if $(R_0', R_1', R_2', R_{1c}, R_{1pb}, R_{2c}, R_{2pa}, R_{2pb}) \in \mathbb{R}_+^8$ satisfies (3a)–(3j) for some input distribution $p_{X_1, X_2, U_{1c}, U_{2c}, U_{2pa}, U_{1pb}, U_{2pb}}$.

The encoding scheme used in deriving this achievable rate region is shown in Fig.2. The key aspects of our scheme are the following, where we drop n for convenience:

• We **rate-split** the independent messages W_1 and W_2 uniformly distributed on $\mathcal{M}_1 = [1 : 2^{nR_1}]$ and $\mathcal{M}_2 = [1 : 2^{nR_2}]$ into the messages W_i, $i \in \{1c, 2c, 1pb, 2pb, 2pa\}$, all independent and uniformly distributed on $[1 : 2^{nR_i}]$, each encoded using the random variable U_i, such that

$$W_1 = (W_{1c}, W_{1pb}), \qquad R_1 = R_{1c} + R_{1pb},$$
$$W_2 = (W_{2c}, W_{2pb}, W_{2pa}), \quad R_2 = R_{2c} + R_{2pa} + R_{2pb}.$$

• **Tx2 (primary Tx):** We **superimpose** U_{2pa}, which encodes the private ("p" for private, "a" for alone) message of Tx2 on top of U_{2c}, which encodes the common ("c" for common) message of Tx2. Tx2 sends X_2 over the channel.
• **Tx1 (cognitive Tx):** The common message of Tx1, encoded by U_{1c}, is **binned** against (U_{2pa}, X_2) conditioned on U_{2c}. The private message of Tx2, encoded by U_{2pb} ("b" for broadcast) and a portion of the private message of Tx1, encoded as U_{1pb}, are **binned** against each other and X_2 as in Marton's region [21] conditioned on U_{1c}, U_{2c}, U_{2pa} and U_{1c}, U_{2c} respectively. Tx1 sends X_1 over the channel. The incorporation of a Marton-like scheme at the cognitive transmitter was initially motivated by the fact that in certain regimes, this strategy was shown to be capacity achieving for the linear high-SNR deterministic CIFC [22]. It is also, independently, a key feature of the region in [4].

The codebook generation, encoding and decoding as well as the error event analysis are provided in [24, Ch.2].

IV. COMPARISON WITH EXISTING ACHIEVABLE REGIONS

We now show that the region of Theorem 1 contains all other known achievable rate regions for the DM-CIFC. We note that showing inclusion of the rate regions [4, Thm.2] and [9] is sufficient to demonstrate the largest known DM-CIFC region, since the region of [4] is shown to contain those of [19, Th.1] and [14]. However we include the independently derived inclusions of the regions of [19, Th.1], [14] and [21, Thm. 2] in our region $\mathcal{R}_{RTD}$ for completeness.

A. Maric et al.'s region [19, Th.1]

Note that, given the encoding and decoding scheme of [19, Th.1], rate splitting of message 2 does not enlarge the region, and hence $X_{2a} = \emptyset$ WLOG. This derivation is included in the Appendix of the long version of this work, found in [23]. To prove inclusion of [19, Th.1] in $\mathcal{R}_{RTCD}$ consider the following

$$R'_0 \geq I(U_{1c}; U_{2pa}, X_2 | U_{2c}) \tag{3a}$$

$$R'_0 + R'_1 + R'_2 \geq I(U_{1c}; U_{2pa}, X_2 | U_{2c}) + I(U_{1pb}; U_{2pa}, U_{2pb}, X_2 | U_{2c}, U_{1c})$$
$$+ I(U_{2pb}; X_2 | U_{2c}, U_{2pa}, U_{1c}) \tag{3b}$$

$$R_{2c} + R_{1c} + R_{2pa} + R_{2pb} + R'_0 + R'_2 \leq I(Y_2; U_{1c}, U_{2c}, U_{2pa}, U_{2pb}) + I(U_{1c}; U_{2pa} | U_{2c}) \tag{3c}$$

$$R_{1c} + R_{2pa} + R_{2pb} + R'_0 + R'_2 \leq I(Y_2; U_{1c}, U_{2pa}, U_{2pb} | U_{2c}) + I(U_{1c}; U_{2pa} | U_{2c}) \tag{3d}$$

$$R_{2pa} + R_{2pb} + R'_2 \leq I(Y_2; U_{2pa}, U_{2pb} | U_{2c}, U_{1c}) + I(U_{1c}; U_{2pa} | U_{2c}) \tag{3e}$$

$$R_{1c} + R_{2pb} + R'_0 + R'_2 \leq I(Y_2; U_{1c}, U_{2pb} | U_{2c}, U_{2pa}) + I(U_{1c}; U_{2pa} | U_{2c}) \tag{3f}$$

$$R_{2pb} + R'_2 \leq I(Y_2; U_{2pb} | U_{2c}, U_{1c}, U_{2pa}) + I(U_{2pa}; U_{1c} | U_{2c}) \tag{3g}$$

$$R_{2c} + R_{1c} + R_{1pb} + R'_0 + R'_1 \leq I(Y_1; U_{2c}, U_{1c}, U_{1pb}) \tag{3h}$$

$$R_{1c} + R_{1pb} + R'_0 + R'_1 \leq I(Y_1; U_{1c}, U_{1pb} | U_{2c}) \tag{3i}$$

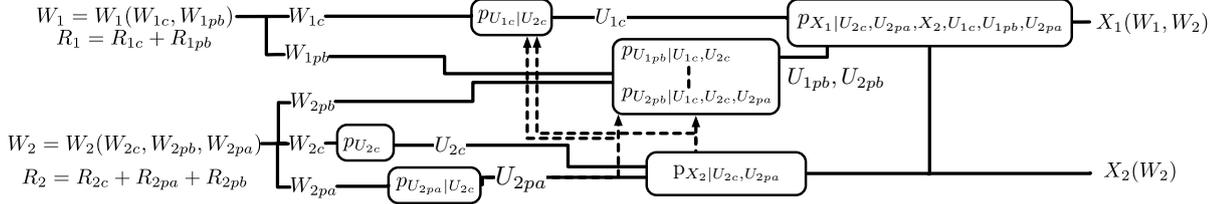$$R_{1pb} + R'_1 \leq I(Y_1; U_{1pb} | U_{2c}, U_{1c}) \tag{3j}$$

Fig. 2. The achievable encoding scheme of Thm 1. The ordering from left to right and the distributions demonstrate the codebook generation process. The dotted lines indicate binning. We see rate splits are used at both users, private messages $W_{1pb}, W_{2pa}, W_{2pb}$ are superimposed on common messages W_{1c}, W_{2c}. U_{1pb} and U_{2pb} are binned against each other and X_2 conditioned on U_{1c}, U_{2c} and U_{1c}, U_{2c}, U_{2pa} respectively

assignment in (3):

$$
\begin{array}{lll}
U_{2c} = Q & R_{2c} = 0 & \\
U_{1c} = U_{1c} & R_{1c} = R_{1c} & R'_0 = I(U_{1c}; U_{2pa} | U_{2c}) \\
U_{2pa} = X_{2b} & R_{2pa} = R_{2b} & \\
U_{1pb} = U_{1a} & R_{1pb} = R_{1a} & R'_1 = I(U_{1pb}; U_{2pa} | U_{1c}) \\
U_{2pb} = U_{2pa} & R_{2pb} = 0 & R'_2 = 0
\end{array}
$$

Moreover let X_1 and X_2 be deterministic functions, that is $X_2 = f_{X_2}(U_{2c}, U_{2pa})$ and $X_1 = f_{X_1}(U_{2c}, U_{2pa}, U_{1c}, U_{1pb}, U_{2pb})$. With this assignment note that we may drop (3g) and (3f) since incorrect decoding of U_{1c} at decoder 2 is not an error.

Also X_2 can be dropped from the binning rates since $I(X; Y | Z) = I(X; Y, g(Y, Z) | Z)$.

From this we conclude that the region of $[19] \subseteq \mathcal{R}_{RTD}$. The weak interference regions of [15], [29] are special cases of [19, Th.1] by [19, Thm. 3], and are also $\subseteq \mathcal{R}_{RTD}$.

B. Marton's region [21, Thm. 2]

One key ingredient that was missing in all previous regions, as also noted in [4] and first addressed in the context of the CIFC in [3], was the inclusion of a broadcast strategy from the cognitive Tx to both receivers. To remedy this obvious gap, we proposed a Marton-like [21] binning of U_{1pb} and U_{2pb}. Our region may be reduced to Marton's broadcast channel region,

using the notation of [21, Thm. 2] by the following assignment of random variables:

$$
\begin{array}{ll}
U_{1pb} = U & R_{1pb} = R_x, \\
U_{2pb} = V & R_{2pb} = R_z, \\
& R'_1 + R'_2 = I(U_{1pb}; U_{2pb} | U_{2c}) \\
U_{2c} = U_{1c} = U_{2pa} = W & R_{1c} = R_{2c} = R_{2pa} = 0 \\
& R'_0 = 0 \\
X_2 = f_{X_2}(U_{2c}). & X_1 = f_{X_1}(U_{1pb}, U_{2pb}, U_{2c})
\end{array}
$$

C. Jiang and Xin's region [14]

We compare $\mathcal{R}_{RTD}$ with the region described by (11)-(12), (17)-(19) of [14]. Note that the indices 1 and 2 are switched. Our region may be reduced, with some manipulation, to that of [14] for the following choices of random variables:

$$
\begin{array}{lll}
U_{2c} = Q & R_{2c} = 0 & \\
U_{1c} = U & R_{1c} = R_{21} & R'_0 = I(U_{1c}; U_{2pa} | U_{2c}) \\
U_{2pa} = W & R_{2pa} = R_1 & \\
U_{1pb} = V & R_{1pb} = R_{22} & R'_1 = I(U_{1pb}; U_{2pa} | U_{2c}, U_{1c}) \\
U_{2pb} = (U_{2c}, U_{2pa}) & R_{2pb} = 0 & R'_2 = 0
\end{array}
$$

Note that we may again drop (3g) and (3f) since incorrect decoding of U_{1c} at decoder 2 is not an error.

D. Devroye et al.'s region [9, Thm. 1]

The comparison of the region of [9, Thm. 1] with that of [4] and [19] has been unsuccessfully attempted in the past. In

the Appendix of [23] we show that the region of [9, Thm. 1] $\mathcal{R}_{DMT}$, is contained in our new region $\mathcal{R}_{RTD}$ along the lines:

- We make a correspondence between the random variables and corresponding rates of $\mathcal{R}_{DMT}$ and $\mathcal{R}_{RTD}$.
- We define new regions $\mathcal{R}_{DMT} \subseteq \mathcal{R}_{DMT}^{out}$ and $\mathcal{R}_{RTD}^{in} \subseteq \mathcal{R}_{RTD}$ which are easier to compare: they have identical input distribution decompositions and similar rate equations.
- For any fixed input distribution, an equation-by-equation comparison leads to $\mathcal{R}_{DMT} \subseteq \mathcal{R}_{DMT}^{out} \subseteq \mathcal{R}_{RTD}^{in} \subseteq \mathcal{R}_{RTD}$.

E. Cao and Chen's region [4, Thm. 2]

The independently derived region in [4, Thm. 2] uses a similar encoding structure as that of $\mathcal{R}_{RTD}$ with two exceptions: a) the binning is done sequentially rather than jointly as in $\mathcal{R}_{RTD}$ leading to binning constraints (43)–(45) in [4, Thm. 2] as opposed to (3a)–(3b) in Thm.1. Notable is that both schemes have adopted a Marton-like binning scheme at the cognitive transmitter, as first introduced in the context of the CIFC in [3]. b) While the cognitive messages are rate-split in identical fashions, the primary message is split into 2 parts in [4, Thm. 2] ($R_1 = R_{11} + R_{10}$, note the reversal of indices) while we explicitly split the primary message into three parts $R_2 = R_{2c} + R_{2pa} + R_{2pb}$. In the Appendix of [23] we show that the region of [4, Thm.2], denoted as $\mathcal{R}_{CC} \subseteq \mathcal{R}_{RTD}$:

- We first show that we may WLOG set $U_{11} = \emptyset$ in [4, Thm.2], creating a new region R'_{CC}.
- We next make a correspondence between our random variables and those of [4, Thm.2] and obtain identical regions.

V. Conclusion

A new achievable rate region for the DM-CIFC has been derived and shown to encompass all known achievable rate regions. Of note is the inclusion of a Marton-like broadcasting scheme at the cognitive transmitter. Specific choices of this region have been shown to achieve capacity for the linear high-SNR approximation of the Gaussian CIFC [22], [24], and lead to capacity achieving points in the deterministic CIFC [24]. This region has furthermore been shown to achieve within 1.87 bits of an outer bound, regardless of channel parameters in [25]. Numerical evaluation of the region under Gaussian input distributions for the Gaussian CIFC, and further comparisons with the region of [4] are our short-term goals, while extensions of the CIFC to multiple users will be investigated in the longer term.

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