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Beamforming in Interference Networks: Multicast, MISO IFC and Secrecy Capacity

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Abstract—Motivated by the two-user beamforming in multi-antenna interference channels, we characterize the upper boundary of the achievable single-user gain-region. The eigenvector corresponding to the maximum eigenvalue of the weighted sum of Hermitian forms of channel vectors is shown to achieve all points on the boundary in some given direction. Thereby, we solve three different beamforming problems, namely the multicast beamforming problem, the beamforming optimization in MISO interference channels, and beamforming in MISO systems with secrecy constraints for arbitrary number of users. We are confident that the framework can be applied to beamforming problems in other interference networks as well. Numerical simulations illustrate the achievable gain-region.

I. INTRODUCTION

Interference channels are one of the basic elements of complex networks. Future wireless communication systems will suffer from interference since the number of subscribers as well as the required data rate increases. Therefore, it is important to exploit carefully the spatial dimension by using multiple transmit or receive antennas. In the current work, we focus on a generic $K$-user multiple-input single-output (MISO) interference channel [1]. Information-theoretic studies of the IFC have a long history [2], [3], [4], [5]. These references have provided various achievable rate regions, which are generally larger in the more recent papers than in the earlier ones. However, the capacity region of the general IFC remains an open problem. For certain limiting cases, for example when the interference is weak or very strong, respectively, the sum capacity is known [6]. If the interference is weak, it can simply be treated as additional noise. For very strong interference, successive interference cancellation (SIC) can be applied at one or more of the receivers. Multiple antenna interference channels are studied in [1]. Multiple-input multiple-output (MIMO) interference channels have also recently been studied in [7], from the perspective of spatial multiplexing gains. In [8], the rate region of the single-input single-output (SISO) IFC was characterized in terms of convexity and concavity.

The linear combination of the egoistic and altruistic beamformers is proved to be Pareto optimal in the 2-user MISO interference channel [9]. In [10], this idea is extended to the MIMO interference channel. Their proposed egoism and altruism balancing beamforming algorithm has connections with some important works such as rate optimization [11], [12] and interference alignment [13], [14]. In [15], the term coordinated beamforming is coined, and the optimal transmit beamforming and receive combining vectors under a zero inter-user interference constraint are derived for a two-user interference system in the context of two-cell coordination. Using ideas from game theory, the multi-antenna interference channel is studied in [16], [17], [18].

The contribution and outline of the paper is as follows:

1) In Section II, we define the MISO single-user gain-region and show that it is convex.
2) Then, the main result is a characterization of the boundary of the single-user gain-region in a given direction $\epsilon$ which follows from the convexity of the gain-region.
3) In Section III, the result is applied in order to completely characterize the Pareto boundary of

\begin{itemize}
  \item a) the achievable rate region of multi-cast transmission (e.g., broadcast phase of two-way relaying),
  \item b) the achievable rate region of the MISO interference channel with $K$ users,
  \item c) and the achievable secrecy and eavesdropping rates in MISO wiretap channels.
\end{itemize}

The characterization of the Pareto boundary for the MISO interference channel with $K$ users improves a former result in [9]. The characterization of the Pareto boundary of the achievable secrecy and equivocation rates contains the optimum beamforming derived in [19]. The theoretical results are illustrated by a numerical simulation and the paper is concluded in Section IV.

II. BOUNDARY OF THE SINGLE-USER GAIN-REGION

A. Preliminaries

Consider a multiple-antenna user $k$ in a $K$-user interference network and denote the flat-fading vector channels from user $k$ to single-antenna receiver $\ell$, $1 \leq \ell \leq K$ as $h_{k,\ell}$. Define the channel gain as a function of the beamforming vector $w$ as $x_\ell(w) = \|h_{k,\ell}^H w\|^2$ for $1 \leq \ell \leq K$. Define the achievable gain-region for user $k$ as

$$\Omega_k = \bigcup_{\|w\|=1} (x_1(w),...,x_K(w)).$$

(1)
The operational meaning of the gain-region $\Omega_k$ will be discussed in Section III. Before we illustrate the region and its boundary, we note the following important property.

**Lemma 1:** The gain region $\Omega_k$ is always convex, i.e., for $x, y \in \Omega_k$ it follows that $x(t) = tx + (1 - t)y \in \Omega_k$ for $t, 0 \leq t \leq 1$.

The proof of this lemma follows from direct computation of $w(t) = tw_x + (1 - t)w_y$ where $w_x$ achieves $x$ and $w_y$ achieves $y$. The complete proof can be found in [20].

In Figure 1, an example gain-region is shown for two users $K = 2$. The operational meaning of the gain-region and the directions $e_1, e_2, e_3$ in Figure 1 will be discussed in Section III. The arrows in Figure 1 correspond to interesting directions $e_1 = [1, 1]$, $e_2 = [1, -2]$, $e_3 = [-1, 1]$.

**B. Main result**

Following the discussion on the boundary of $\Omega_k$, we formalize the upper boundary following the definitions in [21]. There, this definition was used to derive the solution of a monotonic optimization problem [22].

**Definition 1:** A point $y \in \mathbb{R}^n_+$ is called upper boundary point of a convex set $C$ if $y \in \overline{C}$ while the set

$$K_y = y + \mathbb{R}^n_+ = \{ y' \in \mathbb{R}^n_+ | y' > y \} \subset \mathbb{R}^n_+ \setminus \overline{C}. \quad (2)$$

The set of upper boundary points of $C$ is called the upper boundary of $C$ and it is denoted by $\partial^+ C$.

The straightforward extension to include also the right boundary of a convex set $C$ is to define the upper boundary of $C$ in direction $e$.

**Definition 2:** A point $y \in \mathbb{R}^n_+$ is called upper boundary point of a convex set $C$ in direction $e$ if $y \in C$ while the set

$$K_y(e) = \{ y' \in \mathbb{R}^n_+ | y'e_\ell \geq y'e_\ell \forall 1 \leq \ell \leq n \} \subset \mathbb{R}^n_+ \setminus C \quad (3)$$

where the inequality has at least one strict inequality and directional vector $e \in \{-1, +1\}^n$. We denote the set of upper boundary points in direction $e$ as $\partial^e C$.

For the choice $e = 1$ the upper boundary in direction $e$ is the usual upper boundary, i.e., $\partial^1 C = \partial^+ C$.

In the following, we omit the index $k$ when considering only one user for convenience. For efficient operation the boundary points of $\Omega$ in all directions (except $e = -1$) are of interest. Define the set $E = \{ -1, 1 \}^n \setminus \{ -1 \}^n$. The following result is the main theorem of the paper. Interestingly, it follows easily from the convexity of the gain-region.

**Theorem 1:** All upper boundary points of the convex set $\Omega$ in direction $e \in E$ can be achieved by

$$w(\lambda) = v_{\max}\left( \sum_{\ell=1}^K \lambda_\ell e_\ell h_{k,\ell}^H \right) \quad (4)$$

with $v_{\max}(Z)$ denoting the eigenvector which belongs to the maximum eigenvalue of the Hermitian matrix $Z$, $\lambda = [\lambda_1, \ldots, \lambda_K, 1, \lambda_1, \ldots, \lambda_K]$ with $0 \leq \lambda_\ell \leq 1$, $1 \leq \ell \leq K - 1$ and $\lambda_K = 1 - \sum_{\ell=1}^{K-1} \lambda_\ell$.

**Proof:** We provide the sketch of the proof. The complete proof can be found in [20]. The boundary points in direction $e$ of the convex set $\Omega$ can be achieved by maximization of the weighted sum gain, i.e.,

$$\max_{w \mid \|w\|^2 = 1} \sum_{\ell=1}^K \lambda_\ell e_\ell \|w^H h_{k,\ell}\|^2. \quad (5)$$

The objective function in (5) can be rewritten as

$$y(w) = \sum_{\ell=1}^K \lambda_\ell e_\ell \|w^H h_{k,\ell}\|^2 = w^H \left( \sum_{\ell=1}^K \lambda_\ell e_\ell h_{k,\ell}^H h_{k,\ell} \right) w. \quad (6)$$

Note that the matrix $Z$ in (6) is not necessarily positive semidefinite because the directional vector $e$ can contain negative components. However, it is Hermitian and therefore, the solution to (5) is the eigenvector which corresponds to the maximum eigenvalue of $Z$.

The interesting observation from Theorem 1 is that all upper boundary points of the $K$-dimensional gain-region can be achieved by a parameterization using $K - 1$ real parameters between zero and one, i.e.,

$$\lambda \in \Lambda = \{ \lambda \in [0, 1]^K : \sum_{\ell=1}^K \lambda_\ell = 1 \}. \quad (7)$$

Depending on the application context different directions or even certain operating points are to be optimized. The gain-region and its boundary are illustrated in Figure 2.

The four colors in Figure 2 correspond to the four directions $e_1 = [1, 1, 1]$, $e_2 = [1, -1, 1]$, $e_3 = [1, 1, -1]$, and $e_4 = [1, -1, -1]$. Inside the three nets (constructed by varying the parameter vector $\lambda$ on a grid with $100 \times 100$ points) there is the convex hull of 100,000 gain points achieved by random generated beamforming vectors. The channels $h_{11}$, $h_{12}$, $h_{13}$ are randomly generated with three transmit antennas.
III. APPLICATIONS

The result from Theorem 1 can be applied to an interference network in which (virtual) users are equipped with multiple antennas. We present three representatives for different applications. Needless to say that it can be applied to other scenarios as well.

A. Multicast beamforming

We start with a trivial example and consider the simple multicast beamforming scenario in which one transmitter sends common information to \( K \) receivers. The motivation for this illustrative scenario could be the second phase of a two-way relaying system with decode-and-forward relaying [23]. In the broadcast phase, one transmitter with multiple antennas can transmit the data to the terminals which then subtract the self-interference (analog network coding). Denote the channels from the relay to the terminals by \( h_1, ..., h_K \). In this simple scenario, the achievable rate of user terminal \( k \) is given by

\[
R_k(w) = \log \left( 1 + \frac{|w^H h_k|^2}{\sigma^2} \right).
\]  

The multicast beamforming rate region \( \mathcal{R} \) is defined as

\[
\mathcal{R} = \bigcup_{|w|=1} \left( R_1(w), ..., R_K(w) \right).
\]  

The next corollary follows from Theorem 1 since the upper boundary of \( \mathcal{R} \) corresponds exactly to the upper boundary of \( \Omega \) in direction \( e = 1 \).

**Corollary 1:** Any point on the upper boundary of the rate region \( \mathcal{R} \) in (9) can be achieved by

\[
w(\lambda) = w_{max} \sum_{\ell=1}^K \lambda_\ell h_\ell h_\ell^H
\]

with \( \lambda \in \Lambda \) in (7).

B. MISO interference channel with \( K \geq 2 \) users

The MISO interference channel with \( K \) users is studied in [9]. All base stations BS\(_k\) have \( N \) transmit antennas each, that can be used with full phase coherency. The mobiles MS\(_k\), however, have a single receive antenna each. We shall assume that transmission consists of scalar coding followed by beamforming, and that all propagation channels are frequency-flat. This leads to the following basic model for the matched-filtered, symbol-sampled complex baseband data received at MS\(_k\):

\[
y_k = h_k^T w_k s_k + \sum_{l=1,l\neq k}^K h_k^T w_l s_l + e_k,
\]

where \( s_l, 1 \leq l \leq K \) is the symbol transmitted by BS\(_l\), \( h_k \) is the (complex-valued) \( N \times 1 \) channel vector of BS\(_k\), MS\(_l\), and \( w_k \) is the beamforming vector used by BS\(_k\). The variables \( e_k \) are noise terms which we model as i.i.d. complex Gaussian with zero mean and variance \( \sigma^2 \).

We assume that each base station can use the transmit power \( P \), but that power cannot be traded between the base stations. Without loss of generality, we shall take \( P = 1 \). This gives the power constraints

\[
||w_k||^2 \leq 1, \quad 1 \leq k \leq K
\]

Throughout, we define the SNR as \( 1/\sigma^2 \). The precoding scheme that we will discuss requires that the transmitters (BS\(_k\)) have access to channel state information (CSI) for some of the links. However, at no point we will require phase coherence between the base stations. In [9], a characterization of the beamforming vectors that reach the Pareto boundary of the achievable rate region with interference treated as additive Gaussian noise is provided by a complex linear combination.

In what follows we will assume that all receivers treat co-channel interference as noise, i.e. they make no attempt to decode and subtract the interference. For a given set of beamforming vectors \( \{w_1, ..., w_K\} \), the following rate is then achievable for the link BS\(_k\) → MS\(_k\), by using codebooks approaching Gaussian ones:

\[
R_k(w_1, ..., w_K) = \log_2 \left( 1 + \frac{|w_k^H h_k|^2}{\sum_{l \neq k} |w_l^H h_k|^2 + \sigma^2} \right).
\]  

We define the achievable rate region to be the set of all rates that can be achieved using beamforming vectors that satisfy the power constraint:

\[
\mathcal{R} \triangleq \bigcup_{\{w_k||w_k||^2 \leq 1, 1 \leq k \leq K\}} \{R_1(w_1, ..., w_K), ..., R_K(w_1, ..., w_K)\} \subset \mathbb{R}^K.
\]

The outer boundary of this region is called the Pareto boundary, because it consists of operating points \( (R_1, ..., R_K) \) for which it is impossible to improve one of the rates, without simultaneously decreasing at least one of the other rates. More precisely we define the Pareto optimality of an operating point as follows.
Definition 3: A rate tuple \((R_1, \ldots, R_K)\) is Pareto optimal if there is no other tuple \((Q_1, \ldots, Q_K)\) with \((Q_1, \ldots, Q_K) \geq (R_1, \ldots, R_K)\) and \((Q_1, \ldots, Q_K) \neq (R_1, \ldots, R_K)\) (the inequality is component-wise).

Theorem 2: All points of the Pareto boundary of the achievable rate region of the MISO interference channel can be reached by beamforming vectors

\[
\mathbf{w}_k(\lambda_k) = \mathbf{v}_{\max} = \left( \sum_{\ell=1}^{K} \lambda_{k,\ell} e_{\ell} \mathbf{h}_{k,\ell} \mathbf{h}_{k,\ell}^H \right)
\]

with \(\lambda_k \in \Lambda\) defined in (7) and

\[
e_{\ell} = \begin{cases} +1 & \ell = k \\ -1 & \text{otherwise} \end{cases}
\]

Note that for two users \(K = 2\), the characterization in [9, Corollary 1] follows as a special case.

The proof of Theorem 2 follows from the observation that the Pareto boundary of the achievable rate region \(R\) in (14) corresponds for user \(k\) with the upper boundary of \(\Omega_k\) in direction of \(e_k = [-1, \ldots, -1, 1, -1, \ldots, -1]\) with a 1 at the \(k\)-th position. The complete proof is provided in [20].

C. Secrecy capacity in MISO systems

As a brief third example, consider the scenario where the transmitter called Alice has multiple antennas \(n_A\) to send a confidential message to the legitimate receiver called Bob with a single antenna while the eavesdropper Eve with single antenna overhears the message. This is the MISO wiretap channel for which the secrecy capacity for perfect information at Alice is computed in [19]. Denote the channel from Alice to Bob by \(\mathbf{h}_A\) and the channel from Alice to Eve by \(\mathbf{h}_E\). The secrecy rate achievable with beamforming vector \(\mathbf{w}\) is given by

\[
R_s(\mathbf{w}) = \log \left( 1 + \frac{\| \mathbf{w}^H \mathbf{h}_A \|^2}{\sigma^2} \right) - \log \left( 1 + \frac{\| \mathbf{w}^H \mathbf{h}_E \|^2}{\sigma^2} \right)
\]

By application of Theorem 1, the secrecy rate maximization is simply obtained as the solution in [19].

IV. CONCLUSIONS

The characterization of the Pareto boundary of the achievable rate regions in interference channels is a necessary prerequisite in order to develop efficient resource allocation strategies. Motivated by the simple characterization of the rate region of the two-user MISO interference channel, this paper develops a general theory for beamforming in interference networks. The idea is to study the problem for one terminal separately based on its gain-region. Since only operating points on the boundary are of interest, we characterize the beamforming vectors which achieve boundary points in a given direction in Theorem 1. Thus it is possible to obtain operating points which maximize the gain in one direction and minimize it in another direction. Finally, we apply the characterization to three representative scenarios: the multicast beamforming, the MISO interference channel rate region, and the secrecy capacity in MISO system. Currently, we study the extension to MIMO interference channels.