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Author(s):
Balmer, Michael; Vogel, Arnd; Nagel, Kai

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Michael Balmer, IVT, ETH Zurich
Arnd Vogel, VSP, TU-Berlin
Kai Nagel, VSP, TU-Berlin

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Michael Balmer  
IVT  
ETH Zurich  
Switzerland  
Phone: +41-1-633 27 80  
Fax: +41-1-633 10 57  
email: balmer@ivt.baug.ethz.ch

Arnd Vogel  
VSP  
TU Berlin  
Germany  
Phone: +49-30-314-29522  
Fax: +49-30-314-26269  
email: vogel@vsp.tu-berlin.de

Kai Nagel  
VSP  
TU Berlin  
Germany  
Phone: +49-30-314-23308  
Fax: +49-30-314-26269  
email: nagel@vsp.tu-berlin.de

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Abstract

In a traffic network, capacities of parts of the network restrict the amount of transport units which can be handled by this network. The capacity of a given traffic network element is not fixed but influenced by parameters such as number of lanes, maximum speed, weather, view horizon and so on. These parameters also define the maximum capacity of roundabouts. Special shapes of roundabouts, particularly in urban regions, may further increase or decrease their capacity. This paper investigates how the capacity of such special roundabouts can be estimated with only the curbsides of a roundabout as an input. It is also of interest to see that changes to the shape decrease the amount of space “wasted” for the traffic roundabout while the capacity remains unchanged.

In this case study one special roundabout is examined: “Central” in downtown Zurich, Switzerland. The particularity of this roundabout is that it partially behaves like a roundabout but also contains two uncontrolled intersections. Due to its central position in the city, the roundabout is very busy with both individual cars and public transport vehicles.

In the first part of this paper, a simulation model is described which is able to produce realistically behaving vehicles only by using information about the curb side locations of the roundabout. In the second part of the paper, the simulation changes the topology of the scenario based on the observed behavior of the vehicles. Using a feedback loop allows one to optimize the capacity of the roundabout while its spatial extents are minimized.

Keywords


Citation

1. Motivation

Traffic simulation methods are widely accepted in transportation research. They are used to answer a large variety of questions like traffic demand (i.e. VISEM, http://www.ptv.de) capacity and breakdown estimation (Bernard, 2005), large scale traffic analysis based on microscopic demand (i.e. MATSIM, http://www.matsim.org) and so on. On the scope of a single intersection, simulation methods are used to optimize signalization (Shelby, 2004), to estimate intersection capacities using agent based intersection simulation (Manikonda et al., 2001), etc. On the other hand, the question about the shape of an intersection is typically an issue of constructors and designers. Capacity reduction caused by the shape of an intersection is normally not considered. This paper shows how to simulate the interaction between the capacity and the shape of a single roundabout.

Typically, the area of a roundabout is limited by existing buildings, necessary pedestrian ways, governmental rules and other additional constraints. To find a shape for a roundabout using as little space as possible while providing the required traffic capacity is therefore an optimization problem (Campbell et al., 1999, Dijkstra and Timmermans, 2002).

In this paper we demonstrate an agent based approach (see Ferber, 1999) solving this optimization problem. For this, we need to calculate the capacity of a given roundabout by only using the shape (the curbs) of this roundabout as input data. Other characteristics like lanes, traffic signs, etc. are not considered. Section 2 describes how a roundabout is defined. It also gives a brief description about the roundabout chosen for this study.

Section 3 describes how an agent based car driver simulation is being developed and applied in order to calculate traffic indicators like congestion or capacity. The simulation model developed in this section produces realistic agent behaviour although only the curbs of the roundabout are given as input data.

With the knowledge of the calculated indicators of Section 3, a method is shown in Section 4 that uses those to change the shape of the roundabout such that the calculated indicators will be optimized.

Section 5 describes some first results of the case study of the “Central” roundabout of Zurich. The paper finishes with a summary and a brief outlook on further works.
2. Modelling the Roundabout

We define a roundabout by directed street segments. A street segment consists of one or more driving lanes with the same driving direction and does not have any kind of junctions. An incoming street segment of the roundabout additionally holds a defined area inside the street segment shape where car driving agents are allowed to enter the roundabout. Each chosen roundabout holds $n_{in}$ incoming and $n_{out}$ outgoing street segments. For an outgoing street segment, a similar area is defined where agents are allowed to leave the system. For simplicity we define such an area by a circle $C(\bar{c}, r)$ with centre $\bar{c}$ and radius $r$. Entering circles are denoted by $C_{i}^{\text{start}}$ and leaving circles by $C_{j}^{\text{end}}$, while $i = 1 \ldots n_{in}$ and $j = 1 \ldots n_{out}$. The upper picture of Figure 1 shows an example traffic system.

The model of the roundabout provides the following additional information:

- Description of the curbs. The curbs are represented as geometric primitives like geometric nodes and links.
- Description of the driving routes through the roundabout. The driving routes are necessary for simulating the driver agents. Each car driving agent holds information about the streets where he is going to enter and to leave the roundabout. Each possible combination of entering and leaving street segments has to be associated with a route through the roundabout.

Since we do not want to include any information but the curbs into the model, also the driving routes have to be defined by a set of curb segments.
Figure 1 An example of modelling a traffic system with five street segments
2.1 Modelling the Curbs

Modelling of the curbs is a straightforward process. Instead of using the real shape of the roundabout, we simplify the curbs as line segments. This generic approach has the advantage of allowing the description of any kind of roundabout including any intersections and also larger networks without the need to handle different kinds of geometric primitive types. Furthermore, the degree of detail can be easily increased by creating more and therefore shorter line segments for the curbs. The following shows the curb modelling process:

1. Given a shape and the driving direction of a roundabout and an arbitrary origin of a Euclidian coordinate system

2. Define a set $N$ of nodes $\vec{n} = \left( \begin{array}{c} n_x \\ n_y \end{array} \right)$ along all curbs of the given shape

3. Define a set $L^{vis}$ of directed links $\vec{L}^{vis}(\vec{u}^{start}, \vec{u}^{end}) = \left( \begin{array}{c} n_x^{end} - n_x^{start} \\ n_y^{end} - n_y^{start} \end{array} \right)$ along the curbs, each connected by a start and an end node from the set $N$. The direction of the link must follow the driving direction in this street segment.

Note that a link vector has a defined location, direction and length in the coordinate system given by its start and end node. The middle picture of Figure 1 shows the three steps of modelling the curbs in a graphical way.

2.2 Modelling the Driving Routes (“Tunnels”)

In order to let the agents travel through the roundabout, a route through it needs to be assigned to each agent. Since we only want to use the curbs as given information about the roundabout, those driving routes (now also called “tunnels”) must be described as a set of curb segments (see Helbing et al., 1997 for other ways of describing routes).

A tunnel $T_j = T_j(L_j, C_{j}^{start}, C_{j}^{end})$ describes the area in which an agent is allowed to drive. The area is described by a set of links $L_j$. Since a tunnel has exactly one entrance area $C_{j}^{start}$ and one leaving area $C_{j}^{end}$, there are no “holes” in the tunnel, meaning that a tunnel is defined by exactly two curb sides, a left one and a right one. To provide this, we need to add additional “invisible” links to differentiate street segments which are part of a tunnel from those which are not. The following describes the modelling process for all possible tunnels of the given roundabout:

1. Given a set of links $L^{vis}$ as described in Section 2.1
2. Add a set of “invisible” links $L^{\text{invis}}$ (an invisible link is denoted as $\tilde{e}^{\text{invis}} = \tilde{e}^{\text{invis}}(\overline{n}^{\text{start}}, \overline{n}^{\text{end}})$) for each intersection area, such that each street segment is separated from the intersection area. The total set of links is therefore $L = L^{\text{invis}} \cup L^{\text{vis}}$.

3. For each pair of incoming and outgoing street segments of the roundabout, define one tunnel described by a set of visible and invisible links $L_{ij} \subseteq L$ such that each tunnel holds exactly two curb sides, a left and a right one, starting from the incoming street segment and ending at the outgoing one.

A link element of the set of visible or invisible links of tunnel $T_{ij}$ is denoted as $\tilde{e}_{ij} \in L_{ij}$. With this model, exactly $n_{\text{tunnel}} = n_{\text{in}} \cdot n_{\text{out}}$ tunnels are modelled. The bottom of Figure 1 shows an example of modelling the tunnels. Note that only two of the four possible tunnels are drawn.

### 2.3 “Central” Roundabout of Downtown Zurich

This paper uses the “Central” roundabout of downtown Zurich as a case study. There are several reasons for choosing this roundabout:

- It is one of the bottlenecks of the Zurich street network.
- Even if its shape correlates to a roundabout; it still holds two intersections.
- The number of lanes varies inside the roundabout. Therefore, the amount of space used by a street segment also varies within the roundabout.
- A major reconstruction was done during summer 2004.

This study was done before the reconstruction of the Central roundabout. So, part of the analysis of Section 5 can be done by comparing the results with the situation after the reconstruction.
Figure 2 shows the special shape of the “Central”. Note that the middle road (where left turns are not allowed) is a “short-cut” for leaving the “Central” towards the bridge over the river. Therefore the “Central” is not a “real” roundabout anymore. It holds five incoming and five outgoing street segments. This leads us to 25 different tunnels as described in Section 2.2.
3. Car Driver Simulation

Given a model of a roundabout as described in Section 2 we build an agent based car driver simulation. The guiding design principle for that simulation is to use a modelling approach that uses as little input data as possible. The model described in the following works without lanes, signals, turn priorities, etc. It is clear that such a model cannot reach the same levels of realism as, say, CORSIM (2005), MITSIM (2005), or VISSIM (2005), but it will turn out that the results are useful enough for the purposes of overall intersection layout. In addition, the approach does not only reduce the amount of data collection, but it also makes it unnecessary to potentially change intersection prioritization as a result of the intersection geometry adaptation process described in Sec. 4. The car driver simulation is designed according to the following specifications:

1. Every agent is assigned to a specific tunnel and is not allowed to change into another tunnel during the simulation.
2. Given a tunnel, an agent must not drive over the tunnel’s curb sides.
3. An agent must start at the incoming street segment and end at the outgoing one of the given tunnel.
4. An agent is not allowed to drive “unrealistically” through the given tunnel. He must not drive backwards and he must not drive extremely apart from the driving direction given by the tunnel (i.e. right-angled or driving in opposite direction).
5. An agent must respect the physical rules of acceleration.
6. An agent tries to drive through the system with a “desired driving speed”.
7. An agent can not steer more than a given “maximum steering” constant (otherwise cars could change directions right in place).
8. An agent must respect other agents in the system. He has to decelerate or overtake if a slower agent drives in front of him.

The main idea of this agent based car driver simulation follows the principle of particle simulations with discrete time steps $\Delta t$ used in various topics in computational science. Assume each tunnel defines a current which flows in the direction of the given directed curbs and assume that a car driving agent is one particle of fluid in that current, then the constraints 1, 2, 3, 5 and 8 of the above list are fulfilled (for laminar flow).

By adding more, partially overlapping currents representing the other tunnels, and by fulfilling constraint number 4, 6 and 7, those particles become agents (Ferber, 1999). This idea will be formalized in the following subsections.
3.1 Defining a Car Driving Agent

As mentioned above, each car driving agent $a_k$ is assigned to a tunnel $T'_{ij}$ of the given roundabout (denoted as $T'_{ij}$). $T'_{ij}$ defines the path of $a_k$ through the system. For each point in time $t$, each agent holds a certain amount of information about his current state:

- At time $t$, each agent’s position is defined as $\bar{a}_k(t) = (x_k(t), y_k(t))$.
- At time $t$, each agent holds his current driving speed denoted as $s_k(t)$.
- At time $t$, each agent holds his current driving direction denoted as $\phi_k(t)$.

Each agent also holds some predefined constant parameters:

- Each agent has a desired driving speed $s_k^{des}$.
- Each agent has a defined maximum acceleration $acc_k^{max}$.
- Each agent has a defined shape. For simplicity the shape is defined as a circle with radius $r_k$.
- Each agent has a maximum steering limit, denoted as $\rho_k^{max}$.
- Each agent holds a mass $m_k$.
- Finally, each agent holds a maximum allowed angle with respect to a given flow force vector, called $\theta_k^{max}$. The flow force will be described in details in Section 3.2.

Note that in the following sections, we do not take account of the mass $m_k$. Therefore we just define $m_k = 1, \forall a_k$.

With the two scalars $s_k(t)$ and $\phi_k(t)$, the agent’s velocity vector $\bar{v}_k(t)$ is defined by a polar coordinate system, denoted as $\bar{v}_k(t)_{\phi,\rho}$. The conversion into the global Euclidian coordinate system is therefore $\bar{v}_k(t)_{\phi,\rho} = (s_k(t) \cdot \cos(\phi_k(t)), s_k(t) \cdot \sin(\phi_k(t)))$. It is important to notice that we will use the polar representation, since it has one main advantage: An agent who stopped ($s_k(t = t_0) = 0$) still has a direction ($\phi_k(t = t_0) \in [-\pi, \pi]$) of his car. In Euclidian coordinates we would lose that information.

To follow the idea of a particle simulation each agent in the system reacts to an external force field (similar approaches in Gloor et al., 2003). For each point in time $t$ during the simulation and on each agent’s position $\bar{a}_k(t)$ a force $\bar{F}_{tot}(\bar{a}_k(t))$ needs to be calculated which influences the agent. This force consists of three components, a “flow force” $\bar{F}_{flow}(\bar{a}_k(t))$, a
“curb repulsion force” $\vec{F}_{\text{curb}}(\vec{a}_{k}^{(T_i)})$ and a “neighbour agent repulsion force” $\vec{F}_{\text{neighbour}}(\vec{a}_{k}(t))$.

Therefore, the force which influences an agent at a given position at a given time is

$$\vec{F}_{\text{tot}}(\vec{a}_{k}(t)) = \vec{F}_{\text{flow}}(\vec{a}_{k}^{(T_i)}(t)) + \vec{F}_{\text{curb}}(\vec{a}_{k}^{(T_i)}) + \vec{F}_{\text{neighbour}}(\vec{a}_{k}(t)).$$

The following sections define the three forces in detail.

### 3.2 Flow Force

The flow force field defines the flow of a tunnel $T_{ij}$. It pushes an agent $a_{k}^{(T_i)}$ in the right driving direction through his given tunnel. It is also responsible for letting an agent drive with his desired speed.

The flow force $\vec{F}_{\text{flow}}(\vec{a}_{k}^{(T_i)}(t))$ of an agent $a_{k}^{(T_i)}$ with tunnel $T_{ij}$ at a position $\vec{a}_{k}$, desired speed $s_{k}^{\text{des}}$ and velocity vector $\vec{v}_{k}(t)$ is defined by

$$\vec{F}_{\text{flow}}(\vec{a}_{k}^{(T_i)}(t)) = \alpha \cdot (s_{k}^{\text{des}} \cdot \vec{v}_{\text{flow}}(\vec{a}_{k}) - \vec{v}_{k}(t))$$

The proportion parameter $\alpha > 0$ describes a similar effect as the viscosity of a fluid. For a small $\alpha$ the flow force does not have much influence to the agent and therefore acceleration of an agent is small. For a big $\alpha$, agents would accelerate faster. The time independent flow velocity vector $\vec{v}_{\text{flow}}(\vec{a}_{k})$ of the tunnel $T_{ij}$ at position $\vec{a}_{k}$ is

$$\vec{v}_{\text{flow}}(\vec{a}_{k}) = \sum_{l_{ij}} \left( \frac{\vec{a}(l_{ij}, \vec{a}_{k})}{\vec{v}(l_{ij})} \right)^{\beta} \vec{v}(l_{ij}).$$

Note, that the tunnel flow speed is $|\vec{v}_{\text{flow}}(\vec{a}_{k})| = 1$. The “real” tunnel flow speed is different for each agent because their desired speeds vary. The parameter $\beta \geq 0$ defines the influence of links far away. If it’s big, such a link gets less important. If $\beta = 0$, then each link of the tunnel is weighted equally for calculating the flow velocity. The distance vector $\vec{d}(l_{ij}, \vec{a}_{k})$ between a link of tunnel $T_{ij}$ and the position of agent $\vec{a}_{k}$ is defined as:
Assume that the flow force is the only one which influences an agent; it is possible to drive over the curbs of his tunnel. He also will not respect other driving agents, meaning he just drives through them. The following two forces prevent these.

### 3.3 Curb Repulsion Force

The curb repulsion force field pushes agents away from the curb sides of a tunnel in order to prevent the agents from driving across them (see Stucki, 2003 for a similar approach). The closer an agent gets to a curb, the stronger he will get pushed away from it.

The curb repulsion force \( \vec{F}_{\text{curb}}(\vec{a}_k^{(r_j)}) \) of an agent \( a_k \) with tunnel \( T_j \) at a position \( \vec{a}_k \), desired speed \( s_k^{\text{des}} \) and car radius \( r_k \) is defined by

\[
\vec{F}_{\text{curb}}(\vec{a}_k^{(r_j)}) = \sum_{l_j} \vec{F}_{\text{curb}}(\vec{a}_k^{(r_j)}, \vec{l}_j),
\]

while the repulsion force of one link \( \vec{l}_j \in L_j \) of tunnel \( T_j \) is calculated as

\[
\vec{F}_{\text{curb}}(\vec{a}_k^{(r_j)}, \vec{l}_j) = s_k^{\text{des}} \cdot \left( \| \vec{d}(\vec{l}_j, \vec{a}_k) \| - r_k \right)^\gamma \cdot \frac{\vec{d}(\vec{l}_j, \vec{a}_k)}{\| \vec{d}(\vec{l}_j, \vec{a}_k) \|},
\]

The distance vector \( \vec{d}(\vec{l}_j, \vec{a}_k) \) is already defined in Section 3.2. Since an agent’s car has a certain extent \( n_k \), this has to be subtracted from the distance. With parameter \( \gamma > 0 \) the repulsion force of a link near to an agent is larger than the one of a link far away.

Figure 3 shows a graphical interpretation of the curb repulsion force \( \vec{F}_{\text{curb}}(\vec{a}_k^{(r_j)}, \vec{l}_j) \). As we can see, agents who are nearer to the given link are receiving a stronger repulsion force than an agent far away. Note that an agent who touches the link tangentially receives an infinite repulsion force.
Adding this force to the flow force described in Section 3.2 will guarantee that an agent is driving through his tunnel without crossing the tunnel’s curb side.

### 3.4 Neighbour Agent Repulsion Force

By adding the neighbour agent repulsion force field to the total force of an agent in the simulation system, the agents respect their counterparts (Helbing et al., 2000).

The neighbour agent repulsion force $\vec{F}_{\text{neighbour}}(\vec{a}_k(t))$ of an agent $a_k$ at position $\vec{a}_k$ at time $t$ with speed $s_k$ and car radius $r_k$ is defined by

$$\vec{F}_{\text{neighbour}}(\vec{a}_k(t)) = \sum_{\vec{a}_m \neq \vec{a}_k} \vec{F}_{\text{neighbour}}(\vec{a}_k(t), \vec{a}_m(t)),$$

where $A_{\text{from}}(\vec{a}_k(t))$ is the area in front of agent $a_k$. The position of an agent $\vec{a}_m(t)$ is part of area $A_{\text{from}}(\vec{a}_k(t))$, only if $\vec{v}_k(t) \cdot (\vec{a}_m(t) - \vec{a}_k(t)) \geq 0$. The repulsion force given by an agent $a_m$ on an agent $a_k$ is

$$\vec{F}_{\text{neighbour}}(\vec{a}_k(t), \vec{a}_m(t)) = s_k \cdot \left( \frac{\vec{a}_k(t) - \vec{a}_m(t)}{r_k - r_m} \right)^\delta \cdot \frac{\vec{a}_k(t) - \vec{a}_m(t)}{||\vec{a}_k(t) - \vec{a}_m(t)||}.$$
With parameter $\delta \geq 0$ the neighbour agent repulsion force of an agent $a_k$ near to agent $a_m$ is higher than the one of another agent far away. Figure 4 shows a graphical interpretation of force $\vec{F}_{\text{neighbour}}(\vec{a}_k(t), \vec{a}_m(t))$. It also points out the area in which other agents have influence on agent $a_k$.

Figure 4  Neighbour agent repulsion force on an agent

3.5 Acceleration and Steering

As already mentioned at the beginning of Section 3 an agent at position $\vec{a}_k(t)$ at time $t$ reacts in two ways to a given force $\vec{F}_{\text{tot}}(\vec{a}_k^{(v)}(t))$. In each time step $\Delta t$, he accelerates (decelerates, resp.) and he changes the driving direction by steering. This section describes the update rules for the agent’s speed and direction by the calculated total force. The update is done in the following two steps.

Step I:

Let us first define an angle $\phi$ as the one between the velocity vector $\vec{v}_k(t),\phi$ and the total force $\vec{F}_{\text{tot}}(\vec{a}_k^{(v)}(t))$ with

$$\phi = \angle(\vec{v}_k(t), \vec{F}_{\text{tot}}(\vec{a}_k^{(v)}(t))), \phi = [-\pi, \pi].$$
According to the physical rules of motion, an agent at position $\vec{a}_k(t)$ with speed $s_k(t)$, direction $\phi_k(t)$ and force $\vec{F}_{tot}(\vec{a}_k(t))$ reacts like the following

\begin{align*}
s'_k(t) &= s_k(t) + acc_k(t) \cdot \Delta t \\
\phi'_k(t) &= \phi_k(t) + \rho_k(t) \cdot s_k(t) \cdot \Delta t
\end{align*}

The acceleration $acc_k(t)$ of the agent induced by the total force can be calculated with the formula

\[ acc_k(t) = \begin{cases} 
acc_k^{\text{max}} & \text{if} \quad \left| \vec{F}_{tot}(\vec{a}_k(t)) \right| \cdot \cos(\phi) > acc_k^{\text{max}} \\
\left| \vec{F}_{tot}(\vec{a}_k(t)) \right| \cdot \cos(\phi) & \text{else}
\end{cases} \]

and the steering $\rho_k(t)$ induced by the total force is

\[ \rho_k(t) = \begin{cases} 
\rho_k^{\text{max}} & \text{if} \quad \left| \vec{F}_{tot}(\vec{a}_k(t)) \right| \cdot \sin(\phi) > \rho_k^{\text{max}} \\
-\rho_k^{\text{max}} & \text{if} \quad \left| \vec{F}_{tot}(\vec{a}_k(t)) \right| \cdot \sin(\phi) < -\rho_k^{\text{max}} \\
\left| \vec{F}_{tot}(\vec{a}_k(t)) \right| \cdot \sin(\phi) & \text{else}
\end{cases} \]

Figure 5 shows the calculation of these two scalars. Therefore, $acc_k(t)$ and $\rho_k(t)$ are the values of the abscissa, ordinate resp. of the total force in the local coordinate system given by the velocity vector. To ensure that the calculated values are inside the defined range given by the maximum acceleration and maximum steering, they have to be reduced to those limits in case that they are out of range.
Figure 5  Calculation of acceleration and steering of an agent

Therefore, the velocity vector after step I is 

\[
\vec{v}_k(t)_{s,\theta} = \vec{v}_k\left(s_k(t), \phi_k(t)\right)_{s,\theta}. 
\]

**Step II:**

Let us first define an angle \( \theta \) as the one between the velocity vector \( \vec{v}_k(t)_{s,\theta} \) and the flow force \( \vec{F}_{\text{flow}}(\vec{a}_k(t))_{s,\theta} \), with

\[
\theta = \angle(\vec{v}_k(t)_{s,\theta}, \vec{F}_{\text{flow}}(\vec{a}_k(t))_{s,\theta}), \quad \theta = [-\pi, \pi].
\]

Since an agent is still allowed to drive backwards (negative speed) and to drive extremely apart from the given flow direction of his tunnel, we need to correct the velocity vector \( \vec{v}_k(t)_{s,\theta} \) such that it respects these constraints. The correction is defined as

\[
s_k(t + \Delta t) = s_k(t) + \begin{cases} 0, & \text{if} \quad s_k(t) < 0 \\ s_k(t), & \text{else} \end{cases}
\]

\[
\phi_k(t + \Delta t) = \phi_k(t) + \begin{cases} \phi_{\text{flow}}(\vec{a}_k(t)) + \theta_k^{\text{max}}, & \text{if} \quad \theta > \theta_k^{\text{max}} \\ \phi_{\text{flow}}(\vec{a}_k(t)) - \theta_k^{\text{max}}, & \text{if} \quad \theta < \theta_k^{\text{max}} \\ \phi_k(t), & \text{else} \end{cases}
\]
The speed is therefore reset to zero if it is negative and the direction of the velocity vector is turned towards the flow force vector if the angle between those two vectors is too large. The final updated velocity vector is therefore

\[ \vec{v}_k(t + \Delta t) = \vec{v}_k^\text{II}(t) = \vec{v}_k^\text{II}(s_k^\text{II}(t), \phi_k^\text{II}(t)), \]

with speed

\[ s_k(t + \Delta t) = s_k^\text{II}(t), \]

and direction

\[ \phi_k(t + \Delta t) = \phi_k^\text{II}(t). \]

We can now calculate the position of the agent at time \( t + \Delta t \):

\[ \vec{a}_k(t + \Delta t) = \vec{a}_k(t) + \vec{v}_k(t + \Delta t). \]

### 3.6 Congestion

The above described simulation model produces car driving agents who “realistically” drive through a roundabout inside a defined tunnel. They can overtake or follow other agents in the system. Especially slow driving agents can produce tailbacks. But this does not mean that the simulation produces congestion in terms of capacity constraints of a street network. Typically entering lanes and crossroads normally are the cause of occurring congestion. The simulation developed in this paper is able to reproduce that. Figure 6 gives an example of a congested situation in the simulation. In this example we define a crossroad with two tunnels. One agent uses tunnel \( T_{12} \), the other tunnel \( T_{34} \). Because of the short distance between the two agents, the neighbour agent repulsion force has the largest contribution to the total force of each agent. Therefore the total force is directed more or less in the opposite directions of the desired directions of the agents, which means that they have to decelerate and finally stop driving (speed equals zero).

This is a typical “Deadlock” situation, which has to be prevented. Fortunately, it also indicates “difficult” intersection topologies and therefore we can use that information for changing the topology (details in Section 4). Nevertheless, we need to resolve this deadlock situation, which is done in quite a simple way. If an agent’s velocity is zero, he starts counting the number of time steps he doesn’t go on driving. The higher this number is the more probable it becomes that he will just drive on in the next time step. That means that the agents then do not
respect the other agents anymore for the next time step. Therefore they will just drive across each other.

Of course this is not realistic anymore, but on the other hand, in uncontrolled intersections (and sometimes also in controlled intersections) similar situations occur. Cars are getting stuck similar to a deadlock situation and then, they try to find a gap and “squeeze” themselves through it without respecting driving rules anymore. The way the simulation handles this is just a simplification.

Figure 6 Congestion in the simulation

The probability of driving on in a deadlock situation is calculated as followed:

\[
p(\text{drive on}) = \begin{cases} 
\frac{\text{numberOfStepsBeingStuck}}{100}, & \text{if } \text{numberOfStepsBeingStuck} < 100 \\
1, & \text{else}
\end{cases}
\]

With this simple approach occurring deadlocks can be resolved.
4. **Shape Morphing of the Roundabout**

The main idea of morphing the shape of a roundabout can be described by the following statement:

\[
\text{Congestion occurs because the street segment (or intersection segment) is too small.}
\]

This simple statement gives us the idea how we could morph the shape of a roundabout. Everywhere congestion happens, the roadside corners (nodes) should move away. As we already described in Section 3.6 an agent is in a congested area when he has stopped because of a deadlock situation and then just drives on without respecting the other agents anymore. In other words, a car driver simulation as described in Section 3 can produce “drive-on” events. In the following, we will use the position of the agent which produces such an event (denoted as the agent’s position \( \bar{a}_k \)) in order to modify the shape of the roundabout.

Morphing of the shape is done iteratively. This iteration process is presented in the following section which is based on iterative learning processes like described in Raney and Nagel (2004).

### 4.1 Iteration Process

The iteration process is done by the following steps:

1. Given an initial shape of the roundabout (modelled as described in Section 2).
2. Run the agent based car driver simulation for a defined time period with simulation step \( \Delta t \) (as described in Section 3) and keep track of the positions of all the “drive-on” events.
3. Change the shape of the roundabout by using the information of the “drive-on” events from the previous simulation.
4. Rerun the simulation with the changed roundabout, and so on.

The advantage of this process is that the morphing algorithm can be easily replaced by another. The following section describes one possible algorithm to morph the shape on iteration step 3.

### 4.2 Morphing Model

The model should provide the following feature: In congested areas the streets should get wider while in non-congested areas the streets should shrink. For that we just need to move the roadside corners (nodes \( \bar{n} \)), since the roadside links \( \bar{I} \) are defined by its start and end
node. But to move the nodes we need to know the movement direction. A simple but robust way is to calculate the centroid of each disjoint “non-street” area $A^\text{non-street}_i(N_i)$ (defined by the set of its border nodes $\bar{n} \in N_i$) and to move the nodes toward or away from these centroids. Figure 7 gives an example of those centres. It is calculated as the arithmetic average of the border nodes of this area:

$$\hat{c}_i = \frac{\sum_{n \in N_i} \bar{n}}{\|N_i\|}$$

The normalized moving direction for each node $\bar{n} \in N_i$ of area $A^\text{non-street}_i(N_i)$ is therefore

$$\hat{m}(\bar{n} \in N_i) = \hat{c}_i - \bar{n} \over |\hat{c}_i - \bar{n}|.$$

Figure 7 Example of four independent non-street areas and their centres

Now, we need to calculate an influence parameter $\kappa(\bar{n}, \bar{a}_k)$ of a node $\bar{n} \in N_i$ by a given “drive-on” event $\bar{a}_k$. The calculation is done inversely proportional to the distance between the node and the event:
\[ \kappa(\vec{n}, \vec{a}_k) = \begin{cases} 0, & \text{if } |\vec{n} - \vec{a}_k| > r^\text{max}_\kappa \\ \frac{1}{r^\text{max}_\kappa} |\vec{n} - \vec{a}_k| + 1, & \text{else} \end{cases}, \quad \kappa(\vec{n}, \vec{a}_k) = [0,1] \]

The parameter \( r^\text{max}_\kappa \) defines the maximum radius of influence of an event. If a node is located outside of the influence area of an event, \( \kappa(\vec{n}, \vec{a}_k) \) is zero. Since there can be more than just one event during the agent based simulation, we sum up the calculated influences for each node:

\[ \kappa(\vec{n}) = \sum_{a_i} \kappa(\vec{n}, \vec{a}_k) \]

Since we can’t control the absolute number of events which occur during a simulation, the range of \( \kappa(\vec{n}) \) can vary a lot from between iterations. By normalizing it, we calculate an appropriate value which describes the offset by which a node should be moved towards its corresponding centroid \( \vec{c}_i \). Additionally we also want to allow that streets can shrink, which means, that the nodes with minimal influence should move away from its centre. Therefore the “moving length” \( l(\vec{n}) \) of a node \( \vec{n} \in N_j \) of area \( A^\text{non-street}_j(N_j) \) with centre \( \vec{c}_i \) can be calculated as

\[ l(\vec{n}) = \frac{l^\text{max} - l^\text{min}}{\max(\kappa(\vec{n})) - \min(\kappa(\vec{n}))} \cdot (\kappa(\vec{n}) - \max(\kappa(\vec{n}))) + l^\text{max}. \]

The parameter \( l^\text{max} \geq 0 \) defines the maximal length a node is allowed to move towards its centre, while \( l^\text{min} \leq 0 \) defines the maximal length a node is allowed to move away from its centre. If \( l^\text{max} = 0 \) the street segments do not grow, while with \( l^\text{min} = 0 \), they do not shrink.
5. Setup and First Results

In general it is not that easy to verify the results using the above described model. On the other hand, qualitative comparisons can give us some good indications about the benefits of such a model. The major reconstruction work on the “Central” roundabout which ended in autumn 2004 enable us to evaluate the results of the shape morphing against some real world experiences.

5.1 Setup

The Roundabout Model

Figure 8 shows the roundabout model of the “Central”. It consists of 108 nodes, 98 visible links and 26 invisible links. There are five incoming and five outgoing street segments and therefore five start and five end circles. This results in by 25 different tunnels. twelve disjoint non-street areas are set.

Figure 8  Setup of the central roundabout (5 of 25 Tunnels are highlighted)
The Agents

The following parameters are equal for each agent in the system:

- Radius \( r_k = r = 1.3 \ m \)
- Maximum steering limit \( \rho_k^{\text{max}} = \rho^{\text{max}} = \pi/4 \)
- Maximum acceleration \( acc_k^{\text{max}} = acc^{\text{max}} = +\infty \ \frac{m}{s^2} \) (no upper boundary)
- Maximum allowed angle to flow force vector \( \theta_k^{\text{max}} = \theta^{\text{max}} = \pi/12 \)

The desired speed is set differently for each agent:

- Desired speed \( s_k^{\text{des}} \) is uniformly distributed in the range \([20 \ m, 50 \ m]\) \(\frac{3.6 \ s}{3.6 \ s}\)

Injecting the agents into the system is done by the following rule: If there is no agent inside the entering area \( C_i^{\text{start}} \), add a new one at the centre of the entering area with a random tunnel \( T_g \). The radius of \( C_i^{\text{start}} \) is set to 30 meters.

The Forces

- Proportion parameter of the flow force \( \alpha = 5 \)
- Distance influence parameter of flow force \( \beta = 3 \)
- Distance influence parameter of curb repulsion force \( \gamma = 3 \)
- Distance influence parameter of neighbour agent repulsion force \( \delta = 3 \)

The Time Step and Simulation Time

The time step is set to \( \Delta t = 0.05 \ \text{sec} \). Note that it has to be chosen carefully, because an inadequate combination of time step duration and maximum allowed angle to the flow direction can cause inconsistency. If the time step is set too big, it could happen that an agent “jumps” over a curb just in one time step.

The simulation time is set to 240 seconds. Therefore we simulate 4800 time steps starting with an empty roundabout (no agents in the system at time equals zero). Since the parameters which are used by the morphing model are normalized, it is not that important how many steps are simulated. We only need to make sure that there is enough time for a substantial number of agents to leave the roundabout before the simulation ends. Otherwise, we only would simulate a scenario where the roundabout is getting “filled up”.

The Morphing Model
- Maximum radius of influence of an event $r_{e}^{\text{max}} = 10\ m$
- Maximal growing length $l_{\text{max}} = 1.0\ m$
- Maximal shrinking length $l_{\text{min}} = -0.2\ m$

5.2 Results

Car Driver Simulation

As the outcome of the case study shows, the car driver simulation produces the expected results. Agents find their ways through the system inside their tunnels. They do not drive over a curb side and they respect other agents in the system. They also overtake or follow slower agents depending on the width of the street segment. Congested situations occur and dissolve dependent on the amount of agents in the system. The “drive-on” rule resolves deadlock situations.

An important fact is that “drive-on” events occur only in congested areas, which is important for the morphing model.

Morphing Model

Also the morphing model shows the expected behaviour. Congested street areas are getting larger while free flow areas are shrinking. But the shape of the roundabout is getting more and more unrealistic. This happens because of the extremely simple morphing rules. From an engineering point of view, one could say that this result is not usable. Nevertheless the created shape of the roundabout gives us very good indication about areas where the system has too much capacity and vice versa.

Figure 9 shows us the result of the morphing process. The car driver simulation produces two main congestion areas (shown in iteration 0 of Figure 9). Those areas are expanding ($A_{1}^{\text{grow}}$ and $A_{2}^{\text{grow}}$, shown in iteration 10 of Figure 9). On the other hand there are several street segments that shrink. Interestingly, almost the whole left part of the roundabout is shrinking ($A_{4}^{\text{shrink}}$ and $A_{5}^{\text{shrink}}$), even though there are junctions. The streets on that area were built with two or three lanes (see also the schematic drawing in Figure 2). This could lead us to a conclusion that at least one lane can be closed.
Another interesting shrinking area is $A_i^{\text{shrink}}$. That street segment almost shrinks to the size of an agent (2.6 meter width). It also looks as if this street could be the cause of the two congested areas. Since agents who want to leave the roundabout at the outgoing street segment on the top ($C_i^{\text{end}}$ of Figure 8) could also drive along the right loop. So, it is possible to completely close this street segment.
shows that there are also some street segments which have more or less the proper capacity (equal to a two lane street segment). Other stable areas can be found at the incoming and outgoing street segments.

5.3 Qualitative Comparison to Reality

Until spring 2004 the shape of the “Central” roundabout was looking like the Figure 2. During the peak hours traffic policemen were used to control the traffic at the two major congestion areas shown in the upper picture of Figure 9. During the reconstruction process the street segment at area $A_{\text{shrink}}^1$ (see Figure 9) was closed for a long time and the drivers were redirected along the right loop. Figure 10 shows the shape of the “Central” after the reconstruction was finished. The shrinking and growing areas of Figure 9 are labelled for better orientation. As we can see in the two areas $A_{\text{shrink}}^4$ and $A_{\text{shrink}}^5$, there is a reduction from three to two driving lanes and the centre street segment is reopened again.

It is quite fascinating that the simulation shows the same changes. Even more interesting is that the simulation indicates that the middle street segment is causing too much problems and therefore should be closed in this special case. It would be of interest to measure the behaviour of the reconstructed “Central” if we would close this street again for a longer time period.
Figure 10 Some views on the “Central” roundabout after reconstruction
6. Future Work

The Car Driver Simulation

At the moment each agent calculates his present total force completely “from scratch” for each link of his tunnel and for all other agents in the system at every time step. This wastes a lot of the available computational performance. As the above defined formula already described, there are several issues where it would make sense to pre-compute forces at the beginning of each car driver simulation run (discretization of space; see also Nishinari et al., 2001 and Schadschneider, 2001). With a more appropriate data-structure (i.e. Quad-Trees), neighbour agents could also be found much faster.

It is also of interest to add other traffic participants, like trams and pedestrians. Especially for the “Central” the pedestrians influence the capacity of the roundabout a lot, because the direct way from the Zurich main station to the University goes through the “Central”. With this, during the morning and the evening peak the place is “flooded” with pedestrians.

The Morphing Model

The above described morphing model is a quite simple. The nodes change their position only along a given line. The model also does not respect geographical constraints like already existing buildings, pedestrian roads, etc. Last but not least the resulting shape of the scenario does not look like streets anymore.
7. Summary

This paper shows two approaches: First, a “realistic” agent based car driver simulation using only the curb side information of the scenario as an input and second, a morphing model for changing the shape of the given roundabout. With a simple iteration process it is shown that good indications can be found for optimizing the shape of the scenario. The iteration process allows us to replace the given morphing model by a more enhanced one.

Apart of the above, using iteration processes for optimization problems has at least one other great advantage: It allows us to separate the problem into pieces such that they are easier to understand, monitor and analyse.
8. References


Nishinari, K., A. Kirchner, A. Nazami and A. Schadschneider (2001). Extended floor field CA model for evacuation dynamics. In *Special Issue on Cellular Automata of IEICE Transactions on Information and Systems*, volume E84-D.


