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Simple Procedure for Seismic Analysis of Liquid-Storage Tanks

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Summary

This paper provides the theoretical background of a simplified seismic design procedure for cylindrical ground-supported tanks. The procedure takes into account impulsive and convective (sloshing) actions of the liquid in flexible steel or concrete tanks fixed to rigid foundations. Seismic responses – base shear, overturning moment, and sloshing wave height – are calculated by using the site response spectra and performing a few simple calculations. An example is presented to illustrate the procedure, and a comparison is made with the detailed modal analysis procedure. The simplified procedure has been adopted in Eurocode 8.

Introduction

Large-capacity ground-supported cylindrical tanks are used to store a variety of liquids, e.g. water for drinking and fire fighting, petroleum, chemicals, and liquefied natural gas. Satisfactory performance of tanks during strong ground shaking is crucial for modern facilities. Tanks that were inadequately designed or detailed have suffered extensive damage during past earthquakes [1–7].

Earthquake damage to steel storage tanks can take several forms. Large axial compressive stresses due to beam-like bending of the tank wall can cause "elephant-foot" buckling of the wall (*Fig. 1*). Sloshing liquid can damage the roof and the top of tank wall (*Fig. 2*). High stresses in the vicinity of poorly detailed base anchors can rupture the tank wall. Base shear can overcome friction causing the tank to slide. Base uplifting in unanchored or partially anchored tanks can damage the piping connections that are incapable of accommodating vertical displacements, rupture the plate-shell junction due to excessive joint stresses, and cause uneven settlement of the foundation.

Initial analytical studies [8, 9] dealt with the hydrodynamics of liquids in rigid tanks resting on rigid foundations. It was shown that a part of the liquid moves in long-period sloshing motion, while the rest moves rigidly with the tank wall. The latter part of the liquid – also known as the impulsive liquid – experiences the same acceleration as the ground and contributes predominantly to the base shear and overturning moment. The sloshing liquid determines the height of the free-surface waves, and hence the freeboard requirement.

It was shown later [10–12] that the flexibility of the tank wall may cause the impulsive liquid to experience accelerations that are several times greater than the peak ground acceleration. Thus, the base shear and overturning moment calculated by assuming the tank to be rigid can be non-conservative. Tanks supported on flexible foundations, through rigid base mats, experience base translation and rocking, resulting in longer impulsive periods and generally greater effective damping. These changes may affect the impulsive response significantly [13, 14]. The convective (or sloshing) response is practically insensitive to both the tank wall and the foundation flexibility due to its long period of oscillation.

Tanks analysed in the above studies were assumed to be completely anchored at their base. In practice, complete base anchorage is not always feasible or economical. As a result, many tanks are either unanchored or only partially anchored at their base. The effects of base uplifting on the seismic response of partially anchored and unanchored tanks supported on rigid foundations were therefore studied [15]. It was shown that base uplifting reduces the hydrodynamic forces in the tank, but increases significantly the axial compressive stress in the tank wall.

Further studies [16, 17] showed that base uplifting in tanks supported directly on flexible soil foundations does not lead to a significant increase in the axial compressive stress in the tank wall, but may lead to large foundation penetrations and several cycles of large plastic rotations at the plate boundary. Flexibly supported unanchored tanks are therefore less prone to elephant-foot buckling damage, but more prone to uneven settlement of the foundation and fatigue rupture at the plate-shell junction.

In addition to the above studies, numerous other experimental and numerical studies have provided valuable insight into the seismic behaviour of tanks [18–27]. This paper deals only with the elastic analysis of fully anchored, rigidly supported tanks. The effects of foundation flexibility and base uplifting on the tank response may be found elsewhere [13–17].

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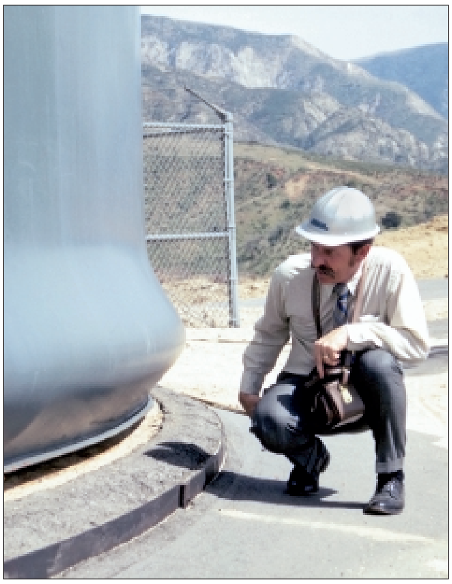

Fig. 1: Elephant-foot buckling of a tank wall (courtesy of University of California at Berkeley)

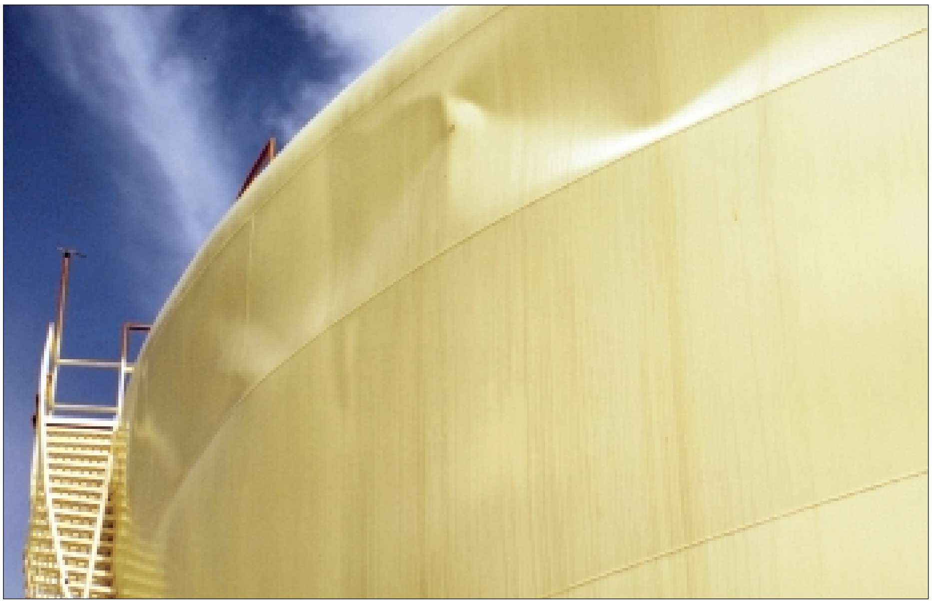

Fig. 2: Sloshing damage to upper shell of tank *(courtesy of University of California at Berkeley)*

Method of Dynamic Analysis

The dynamic analysis of a liquid-filled tank may be carried out using the concept of generalised single-degree-of-freedom (SDOF) systems representing the impulsive and convective modes of vibration of the tank-liquid system. For practical applications, only the first few modes of vibration need to be considered in the analysis (*Fig. 3*). The mass, height and natural period of each SDOF system are obtained by the methods described in [10–14]. For a given earthquake ground motion, the response of various SDOF systems may be calculated independently and then combined to give the net base shear and overturning moment.

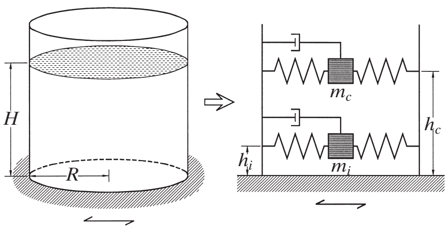

Fig. 3: Liquid-filled tank modelled by generalised single-degree-of-freedom systems

For most tanks (0.3 < H/r < 3, where H is the height of water in the tank and r the tank radius), the first impulsive and first convective modes together account for 85–98% of the total liquid mass in the tank. The remaining mass of the liquid vibrates primarily in higher impulsive modes for tall tanks (H/r > 1), and higher convective modes for broad tanks ($H/r \leq 1$). The results obtained using only the first impulsive and first convective modes are considered satisfactory in most cases. There is, however, some merit in slightly adjusting the modal properties of these two modes to account for the entire liquid mass in the tank.

Simple Procedure for Seismic Analysis

The procedure presented here is based on the work of Veletsos and co-workers [10, 12, 14] with certain modifications that make the procedure simple, yet accurate, and more generally applicable. Specifically, these modifications include

- representing the tank-liquid system by the first impulsive and first convective modes only
- combining the higher impulsive modal mass with the first impulsive mode and the higher convective modal mass with the first convective mode
- adjusting the impulsive and convective heights to account for the overturning effect of the higher modes
- generalising the impulsive period formula so that it can be applied to steel as well as concrete tanks of various wall thicknesses.

The impulsive and convective responses are combined by taking their numerical sum rather than their root-mean-square value.

Model Properties

The natural periods of the impulsive (T_{imp}) and the convective (T_{con}) responses are

$$T_{imp} = C_i \frac{H\sqrt{\rho}}{\sqrt{h/r} \times \sqrt{E}} \tag{1}$$

$$T_{con} = C_c\sqrt{r} \tag{2}$$

where h is the equivalent uniform thickness of the tank wall, ρ the mass density of liquid, and E the modulus of elasticity of the tank material. The coefficients C_i and C_c are obtained from *Fig. 4* or *Table 1*. The coefficient C_i is dimensionless, while C_c is expressed in s/√m. For tanks with non-uniform wall thickness, h may be calculated by taking a weighted average over the wetted height of the tank wall, assigning the highest weight near the base of the tank where the strain is maximal.

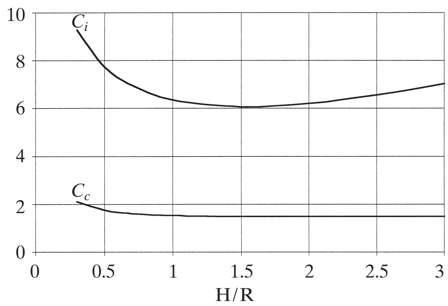

Fig. 4: Impulsive and convective coefficients C_i and C_c

H/r	C_i	C_c [s/√m]	m_i/m_l	m_c/m_l	h_i/H	h_c/H	h_i'/H	h_c'/H
0.3	9.28	2.09	0.176	0.824	0.400	0.521	2.640	3.414
0.5	7.74	1.74	0.300	0.700	0.400	0.543	1.460	1.517
0.7	6.97	1.60	0.414	0.586	0.401	0.571	1.009	1.011
1.0	6.36	1.52	0.548	0.452	0.419	0.616	0.721	0.785
1.5	6.06	1.48	0.686	0.314	0.439	0.690	0.555	0.734
2.0	6.21	1.48	0.763	0.237	0.448	0.751	0.500	0.764
2.5	6.56	1.48	0.810	0.190	0.452	0.794	0.480	0.796
3.0	7.03	1.48	0.842	0.158	0.453	0.825	0.472	0.825

Table 1: Recommended design values for the first impulsive and convective modes of vibration as a function of the tank height-to-radius ratio (H/r). All coefficients are based on an exact model of the tank-liquid system [10, 12, 14].

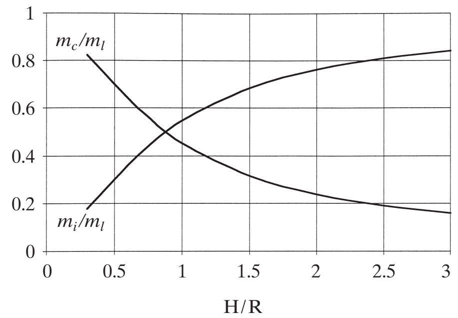

Fig. 5: Impulsive and convective masses as fractions of the total liquid mass in the tank

The impulsive and convective masses (m_i and m_c) are obtained from *Fig. 5* or *Table 1* as fractions of the total liquid mass (m_l).

Seismic Responses

The total base shear is given by

$$Q = (m_i + m_w + m_r) \times S_e(T_{imp}) + m_c S_e(T_{con}) \quad (3)$$

where m_w is the mass of tank wall, m_r the mass of tank roof, $S_e(T_{imp})$ the impulsive spectral acceleration (obtained from a 2% damped elastic response spectrum for steel and prestressed concrete tanks, or a 5% damped elastic response spectrum for concrete tanks), and $S_e(T_{con})$ the convective spectral acceleration (obtained from a 0.5% damped elastic response spectrum).

The overturning moment above the base plate, in combination with ordinary beam theory, leads to the axial stress at the base of the tank wall. The net overturning moment immediately above the base plate (M) is given by Eq. (4), where h_i and h_c are the heights of the centroids of the impulsive and convective hydrodynamic wall pressures (*Fig. 6*, *Table 1*), and h_w and h_r are the heights of the centres of gravity of the tank wall and roof, respectively.

The overturning moment immediately below the base plate (M') is dependent

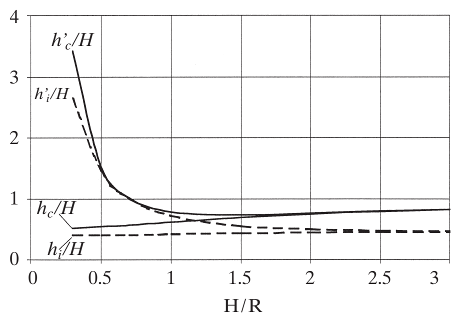

Fig. 6: Impulsive and convective heights as fractions of the height of the liquid in the tank

$$M = (m_i h_i + m_w h_w + m_r h_r) \times S_e(T_{imp}) + m_c h_c S_e(T_{con}) \quad (4)$$

$$M' = (m_i h_i' + m_w h_w + m_r h_r) \times S_e(T_{imp}) + m_c h_c' S_e(T_{con}) \quad (5)$$

$$d = R\frac{S_e(T_{con})}{g} \quad (6)$$

$$Q = \sqrt{(Q_i^1)^2 + (Q_i^2)^2 + (Q_i^3)^2} + \sqrt{(Q_c^1)^2 + (Q_c^2)^2 + (Q_c^3)^2} \quad (7)$$

on the hydrodynamic pressure on the tank wall as well as that on the base plate. It is given by Eq. (5), where the heights h_i' and h_c' are obtained from *Fig. 6* or *Table 1*.

If the tank is supported on a ring foundation, M should be used to design the tank wall, base anchors and the foundation. If the tank is supported on a mat foundation, M should be used to design the tank wall and anchors only, while M' should be used to design the foundation.

The vertical displacement of the liquid surface due to sloshing (d) is given by Eq. (6), where g is the acceleration due to gravity.

Comparison with Detailed Modal Analysis

Three steel tanks were selected for comparing the results obtained from the proposed procedure with those from a detailed modal analysis. Three impulsive and three convective modes were used in the detailed analysis. The modal analysis results were calculated using a combination of root-mean-square and algebraic-sum rules. The net impulsive and the net convective responses were calculated first, using the root-mean-square rule, then numerically added to give the overall response. The base shear, for example, was obtained using Eq. (7), where Q_i^1 and Q_c^1 are the base shear values for the first impulsive and first convective modes, respectively. The response spectra for the site are the same as those used in the given example (*Fig. 7*).

The results (*Table 2*) show that the values of base shear and moment obtained from the proposed procedure were 2–10% higher than those from

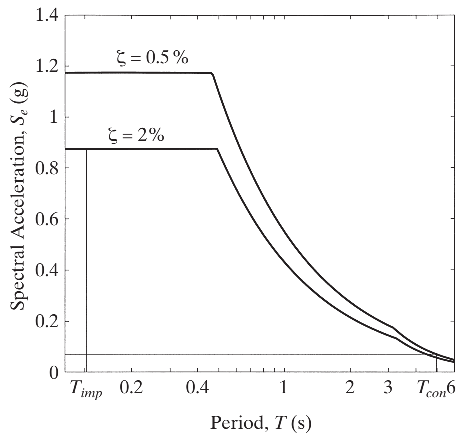

Fig. 7: Elastic design response spectra for 0.5% and 2% damping

r [m]	H [m]	h/r	Q [MN]	M [MNm]	M' [MNm]	D [cm]
15	7.5	0.001	15.7 (14.9)	49.3 (44.6)	167 (164)	57 (51)
15	15	0.001	53.6 (52.4)	346 (334)	577 (557)	75 (66)
7.5	15	0.001	18.3 (16.5)	127 (123)	140 (136)	79 (67)

Table 2: Comparison of results from proposed procedure with those from detailed analysis (values in parentheses are from the modal analysis)

Tank contents	Importance factor (γ_I) for		
	Class 1	Class 2	Class 3
Drinking water, non-toxic non-flammable chemicals	1.2	1.0	0.8
Fire-fighting water, non-volatile toxic chemicals, lowly flammable petrochemicals	1.4	1.2	1.0
Volatile toxic chemicals, explosive and highly flammable liquids	1.6	1.4	1.2

Table 3: Importance factor (γ_I) for tanks according to Eurocode 8 [28]

the detailed modal analysis. The values of sloshing wave height obtained from the proposed procedure were 12–18% higher than those from the detailed modal analysis. The results of the proposed procedure are therefore conservative but close to those from the detailed modal analysis.

Example

A steel tank with a radius r of 10 m and total height of 9.6 m is fully anchored to a concrete mat foundation. The tank is filled with water to a height H of 8 m (H/r = 0.8). The total mass of water in the tank (m_l) is 2.51 × 10^6 kg. The tank wall is made of four courses, each 2.4 m high. The lower two courses are 1 cm thick and the upper two courses 0.8 cm thick. The total mass of the tank wall (m_w) is 43 × 10^3 kg, and the height of its centre of gravity (h_w) is 4.53 m. The mass of the tank roof (m_r) is 25 × 10^3 kg and the height of its centre of gravity (h_r) is 9.6 m. The 0.5% and 2% damped elastic response spectra for the site are shown in *Fig. 7*.

Model Properties

First, the equivalent uniform thickness of the tank wall is calculated by the weighted average method. Using weights equal to the distance from the liquid surface

$$h = \frac{0.01\times 2.4\times 6.8 + 0.01\times 2.4\times 4.4 + 0.008\times 2.4\times 2 + 0.008\times 0.8\times 0.4}{2.4\times 6.8 + 2.4\times 4.4 + 2.4\times 2 + 0.8\times 0.4} = 0.00968 \text{ m}$$

For steel, $E = 2\times 10^{11}$ N/m^2. For water, $\rho = 1000$ kg/m^3. For H/r = 0.8, C_i = 6.77 and C_c = 1.57 s/m$^{0.5}$ (*Table 1*). Hence, from Eqs. (1) and (2),

$$T_{imp} = 6.77\times \frac{8\times\sqrt{1000}}{\sqrt{0.00968/10}\times\sqrt{2\times 10^{11}}} = 0.123 \text{ s}$$

$$T_{con} = 1.57\times\sqrt{10} = 4.96 \text{ s}$$

For H/r = 0.8, m_i/m_l = 0.459 and m_c/m_l = 0.541 (*Table 1*). Hence,

$$m_i = 0.459\times 2.51\times 10^6 = 1.15\times 10^6 \text{ kg}$$

$$m_c = 0.541\times 2.51\times 10^6 = 1.36\times 10^6 \text{ kg}$$

Also from *Table 1*, h_i/H = 0.404, h_c/H = 0.583, h_i'/H = 0.891, h_c'/H = 0.954. Hence, h_i = 3.23 m, h_c = 4.66 m, h_i'= 7.13 m, and h_c' = 7.63 m.

Seismic Responses

The impulsive spectral acceleration for T_{imp} = 0.123 s, obtained from the 2% damped elastic response spectrum (*Fig. 4*), is $S_e(T_{imp}) = 0.874\,g$. The convective spectral acceleration for T_{con} = 4.96 s, obtained from the 0.5% damped response spectrum in *Fig. 4*, is $S_e(T_{con}) = 0.07\,g$.

The base shear obtained from Eq. (3) is

$$Q = (1.15 + 0.043 + 0.025)\times 10^6\times 0.874\times 9.81 + 1.36\times 10^6\times 0.07\times 9.81 = 11 \text{ MN}$$

The overturning moment above the base plate, obtained from Eq. (4), is

$$M = (1.15\times 3.23 + 0.043\times 4.53 + 0.025\times 9.6)\times 10^6\times 0.874\times 9.81 + 1.36\times 10^6\times 4.66 = 40 \text{ MNm}$$

and the overturning moment below the base plate, obtained from Eq. (5), is

$$M' = (1.15\times 7.13 + 0.043\times 4.53 + 0.025\times 9.6)\times 10^6\times 0.874\times 9.81 + 1.36\times 10^6\times 7.63\times 0.07\times 9.81 = 81 \text{ MNm}$$

The maximum vertical displacement of the liquid surface due to sloshing, obtained from Eq. (6), is

$$d = 10\times 0.07 = 0.7 \text{ m}$$

Design According to Eurocode 8

The presented simple procedure was used in Eurocode 8 [28] and integrated in its limit state design concept. The serviceability and ultimate limit states have to be verified. The specification of the corresponding seismic actions is left to the national authorities. The level of seismic protection is established based on the risk to life and the economic and environmental consequences. This reliability differentiation is achieved by adjusting the return period of the design seismic event. Three tank reliability classes are defined corresponding to situations with high (Class 1), medium (Class 2) and low (Class 3) risk. Depending on the tank contents, an importance factor (γ_I) is assigned to each of the three classes (*Table 3*).

The seismic action effects have to be multiplied by the selected importance factor. For the reference case (γ_I = 1), the recommended return periods of the design seismic event are 475 years for the ultimate limit state and 50–70 years for the serviceability limit state. In the case of the largest importance factor (γ_I = 1.6), the return period of the design event for the ultimate limit state is about 2000 years. According to Eurocode 8, the analysis has to assume linear elastic behaviour, allowing only for localised non-linear phenomena without affecting the global response, and to include the hydrodynamic response of the fluid. Particularly, it should account for the convective and impulsive components of fluid motion as well as the tank shell deformation due to hydrodynamic pressure and interaction effects with the impulsive component. The proposed procedure satisfies these principles in a simple and efficient way for the design of fixed-base cylindrical tanks.

Future Research Needs

In regions of strong ground shaking, it is sometimes impractical to design tanks for forces obtained from elastic (no damage) response analysis. Elastic forces are so large that they are arbitrarily reduced by factors of 3 or more to obtain the design forces. When subjected to strong shaking, tanks therefore respond in a non-linear fashion and experience some damage. However, no generally acceptable methods exist to perform a non-linear seismic analysis of tanks. Therefore, the damage sustained by tanks subjected to ground motions of different intensities cannot be quantified easily. There is a need for practical methods of non-linear analysis and design of liquid-storage tanks.

Unlike ductile building systems, tanks lack a mechanism to dissipate large amounts of seismic energy in a ductile manner. Methods of improving the seismic performance of tanks by increasing their ability to dissipate seismic energy need to be examined. The tank could either be anchored to its foundation with energy dissipating devices [29] or seismically isolated by special bearings [30, 31].

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