Doctoral Thesis

Energy policies, price shocks, and economic consequences

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Publication Date:
2010

Permanent Link:
https://doi.org/10.3929/ethz-a-006139347

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ENERGY POLICIES, PRICE SHOCKS, AND ECONOMIC CONSEQUENCES

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ETH ZURICH
for the degree of
Doctor of Sciences
presented by
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2010
Preface

Theories should be as simple as possible.  
But not simpler.  
(Albert Einstein)

The challenges of obtaining Ph.D. are various, and I am thankful that many people have supported me to cope with them. I would like to thank my supervisor Lucas Bretschger for his kind support and for giving me the opportunity to write this thesis. I am also grateful to my co-supervisor Tom Rutherford for his helpful comments. This thesis would not have been possible without the friendly support of the Federal Office of Energy, and I would especially like to thank Nicole Mathys, Andreas Gysler, and Matthias Gutzwiller for constructive discussions. Truly helpful was also the skillful support of Frank Vöhringer within the modeling part. Likewise, I would also like to thank Roger Ramer for many discussions and comments during our collaboration on the project.

The RESEC group has been a source of many sociable events, and I would like to thank everyone for their support while I was writing my thesis. A strong support came from Karen Pittel and Simone Valente, who helped me with very constructive comments. Karen has been a great friend and I would like to thank her especially for all the private talks we had during the last years. I would also like to thank Björn Plaschnick for his help with the "big presentation", the magazine, and the LaTeX-problems. Special thanks go to my great friend Lisa Leinert for all the discussions of Ph.D. challenges and beyond.

The biggest source of strength and support has been my family. It is only thanks to them that I succeeded in writing this thesis. Their patience and devoted help gave
me the motivation to face all the challenges. Thanks to my parents and my sister Johanna for all the talks and their unflinching trust in me. My husband Bastian gave me day-to-day guidance through these turbulent waters, and I thank him for all the loving care and encouragement he gave me.

Zurich, April 2010

Florentine Schwark
# Contents

List of Figures .................................................. VIII
List of Tables .................................................. IX
Thesis Summary .................................................. XI
Zusammenfassung .................................................. XIII

1 Introduction .................................................. 1
   1.1 Energy and the economy .................................. 1
   1.2 Challenges in energy use .................................. 3
   1.3 International energy accords ............................. 6
   1.4 Models ..................................................... 7
      1.4.1 Simulation of policy measures ....................... 7
      1.4.2 Disruptions of steady energy supply ................ 8
   1.5 Contributions of this thesis ............................ 9
   1.6 Overview .................................................. 9

2 The theoretical foundation of the CITE model ............. 15
   2.1 Introduction ............................................. 16
      2.1.1 Switzerland’s energy consumption .................. 17
      2.1.2 Carbon emissions ................................... 18
2.1.3 Political instability ........................................ 19
2.1.4 Models of Swiss energy use .................................. 20
2.1.5 Growth and the role of variety ............................... 21
2.2 An overview of the CITE model ................................. 23
2.3 Final goods production of regular goods ......................... 26
2.4 Production of energy goods .................................... 28
2.5 Production of oil goods ....................................... 30
2.6 Intermediate composite production ............................ 31
2.7 Intermediate goods ........................................... 32
  2.7.1 Capital accumulation ................................... 32
  2.7.2 Production of the intermediate goods .................. 35
2.8 Representative household .................................... 38
2.9 Trade ...................................................... 41
2.10 Equilibrium .................................................. 42
2.11 Growth dynamics ............................................. 44
2.12 Specialties of Calibration .................................... 48
  2.12.1 Different growth rates .................................. 49
  2.12.2 Depreciation rate ...................................... 53
  2.12.3 Changes in energy efficiency and development of labor .. 53
  2.12.4 Capital demand, kappa, investments .................... 54
  2.12.5 Prices ................................................ 56
  2.12.6 Terminal condition .................................... 57
2.13 Conclusion .................................................. 57
2.14 Appendix - Utility functions ................................ 59

3 A comparison of growth dynamics: the CITE model vs. a model with
CONTENTS

homogeneous capital

3.1 Introduction ........................................ 62

3.1.1 CGE models with purely exogenous growth .... 64

3.1.2 Empirical evidence for induced innovation .... 65

3.2 CGE models with endogenous growth mechanisms .... 66

3.2.1 Learning-by-doing as the main driver of growth .... 67

3.2.2 Growth resulting from R&D ...................... 68

3.2.3 Comparison of impact of LbD and R&D .......... 70

3.2.4 Related approaches .............................. 71

3.3 Structure of the models ............................ 72

3.3.1 Final output .................................... 73

3.3.2 Intermediate goods composite and intermediate goods .... 75

3.3.3 Consumption .................................... 79

3.3.4 Demand for the intermediate good ............. 80

3.3.5 Capital formation and incentives to invest .... 84

3.3.6 Growth indices .................................. 86

3.4 Calibration of the models .......................... 87

3.5 Influence of the size of the capital stock .......... 90

3.6 Dynamic of the CITE and a HK growth model .... 92

3.6.1 Effects on production, capital, and consumption .... 93

3.6.2 Effects on endowments .......................... 97

3.7 Sensitivity analysis .................................. 100

3.8 Conclusion .......................................... 105

4 Energy price shocks and medium-term business cycles .... 107

4.1 Introduction ........................................ 108
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2 Empirical background</td>
<td>109</td>
</tr>
<tr>
<td>4.2.1 Output</td>
<td>109</td>
</tr>
<tr>
<td>4.2.2 Energy prices</td>
<td>111</td>
</tr>
<tr>
<td>4.2.3 Endogenous growth and persistence of shocks</td>
<td>114</td>
</tr>
<tr>
<td>4.3 Model</td>
<td>116</td>
</tr>
<tr>
<td>4.3.1 Home country</td>
<td>116</td>
</tr>
<tr>
<td>4.3.2 Energy producing country</td>
<td>124</td>
</tr>
<tr>
<td>4.3.3 Equilibrium conditions</td>
<td>125</td>
</tr>
<tr>
<td>4.4 Simulation results</td>
<td>129</td>
</tr>
<tr>
<td>4.4.1 Model calibration</td>
<td>129</td>
</tr>
<tr>
<td>4.4.2 Energy price shocks and medium-term fluctuations</td>
<td>129</td>
</tr>
<tr>
<td>4.5 Conclusions</td>
<td>135</td>
</tr>
<tr>
<td>References</td>
<td>137</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Swiss GDP and final energy use. ........................................... 2
1.2 Oil price fluctuations and US recessions, in US dollars per barrel. . . . 3
1.3 Economic consequences of undamped climate change. .................... 4
1.4 Final energy use in 2006. .................................................. 5

2.1 Production of $B_r$. .......................................................... 24
2.2 Production of energy. ....................................................... 24
2.3 Production of oil. ............................................................ 25
2.4 Investment nesting. .......................................................... 25
2.5 Consumption nesting. ....................................................... 26

3.1 Sectoral outputs in the HK model without VK. ......................... 90
3.2 Sectoral outputs in the HK model. ...................................... 91
3.3 Capital stocks in the HK model without VK. ............................ 92
3.4 Sectoral capital stocks in the HK model. ................................ 93
3.5 Sectoral outputs in the CITE model. .................................... 94
3.6 Sectoral capital stocks in the CITE model. ............................... 95
3.7 Import of fossil fuels in the CITE model. ............................... 96
3.8 Import of fossil fuels in the HK model. .................................. 97
3.9 Consumption in the CITE model. ........................................ 98
3.10 Consumption in the HK model. ................................. 98
3.11 Labor demand in the CITE model. ............................. 99
3.12 Labor demand in the HK model. ............................... 100
3.13 Demand for labor in research in the CITE model. .......... 100
3.14 Demand for labor in research in the HK model. .......... 101
3.15 Wage in research in the CITE model. ......................... 101
3.16 Wage in research in the HK model. ......................... 102
3.17 Output in the CITE model. ..................................... 103
3.18 Output in the HK model. ..................................... 103
3.19 Output in the CITE model. ..................................... 104
3.20 Output in the HK model. ..................................... 104
4.1 Fluctuations of nonfarm business output. .................... 110
4.2 Real oil price, indexed in 1992 US-Dollars per barrel. .... 112
4.3 Fluctuations of oil price. ...................................... 113
4.4 Oil import shares. ............................................ 114
4.5 Impulse-response functions after an energy price shock. ... 131
List of Tables

3.1 Specific relations in the models. ................................. 89

4.1 Standard deviations. ............................................. 115

4.2 Standard deviations after oil price shock: Data and model. . . . . 132

4.3 Standard deviations after supply shock: Data and model. . . . . . 134
Thesis Summary

Energy is a vital factor to economies worldwide. Nevertheless, despite its positive contribution to economic development, the use of energy also entails risks. Possibly the most dangerous consequences in the long run are pollution and climate change. Political attempts aim at limiting global warming by lowering emissions from energy conversion. This can be achieved by either consuming less energy or by substituting technologies via more efficient innovations. Due to better employment of resources, new inventions might even foster economic growth. However, opponents of strict energy policies fear that a reduction of energy use may impede nations’ competitiveness and thus their economic development.

Another risk that nations face is the prospect that their great dependency on energy may lead to economic turbulence after sudden changes in the energy supply. Production and transportation are based on energy input to a considerable degree. A shock to prices cannot simply be absorbed by quickly adjusting technologies because innovation takes time.

This thesis addresses these aspects of the relationship between economic development and energy prices. The first two chapters analyze the question of the impact of a carbon tax on the Swiss economy. Our CITE (Computable Induced Technical change and Energy) model, a new computable general equilibrium (CGE) model, is a sectoral representation of the Swiss economy and simulates the economic effects of energy policies. Growth dynamics are based on new growth theory and assume that innovations are endogenous to the model. The specific microeconomic foundation distinguishes this model from other CGE models. The CITE model is the first based on heterogeneous capital and gains from specialization as the main driver for growth.
The first chapter explains in detail how the growth dynamics are modeled and how this relates to new growth theory. All production functions and the optimization problems of market participants are presented. The maximization of consumption over time and the composition of the consumption good from sectoral outputs are described.

The particular specification of investment incentives in the CITE model results in different reactions to an energy policy compared to models with exogenous growth assumptions. In these models, productivity is assumed to increase exogenously, i.e., without efforts of any market participant ("manna from heaven"). The second chapter builds on these differences and analyzes reactions of the CITE model on a carbon tax compared to those of a model with exogenous growth. The results show that sectoral responses are more accentuated, and some sectors behave differently in the CITE model. Consequently, the structural composition of the Swiss economy changes more than in models with exogenous growth. This distinctness of the consequences of an energy policy is of interest for policy makers when they are considering introducing new political measures.

As opposed to anticipated energy policies, energy price shocks pose unexpected challenges to economies. The third chapter examines how strongly the U.S. economy has reacted to sudden changes in the oil price over the last decades. We focus mainly on the relationship of high-frequency (up to 8 years) and medium-frequency (8 to 50 years) business cycles. This chapter extends the analysis of energy price shocks by enlarging the time horizon to 50 years. Literature has thus far only taken the impact on "conventional" business cycles (the high-frequency component) into consideration. However, data shows that oil price shocks are very persistent and influence the economy for years afterwards.

We model this persistence by also applying endogenous growth mechanisms in this model. In the nested production structure we can describe mechanisms for the high- and medium-frequency components of the medium-term cycles. The model can display the general patterns of the data and shows that medium-frequency components outweigh high-frequency components of economic variables after an oil price shock.
Zusammenfassung


ZUSAMMENFASSUNG


Schocks sehr langlebig sind und die Wirtschaft noch Jahre später beeinflusst.

Wir modellieren diese Beständigkeit ebenfalls durch die Anwendung von endogenen Wachstumsmechanismen in diesem Modell. Mit der verschachtelten Produktionsstruktur können wir Mechanismen für die hohen und mittleren Frequenzen von mittelfristigen Zyklen beschreiben. Dieses Modell ist in der Lage, das allgemeine Muster der Daten wider zu geben, und zeigt, dass nach einem Ölpreis-Schock mittlere Wirtschaftszyklusfrequenzen die Entwicklung von wirtschaftlichen Variablen stärker beeinflusst haben als hohe Frequenzen.
Chapter 1

Introduction

Energy use and macroeconomic development are closely related. The central role of energy reveals its importance in political and academic discussions about recent developments. For example, climate change as one major challenge calls into question the autonomy of mankind to freely choose energy sources for industrial production. Pollution from different gases, such as carbon dioxide and methane, destroys the ozone layer as natural barrier against heat from the sun. As a result, modern economies face the difficulty of choosing among energy sources, which emit different levels of greenhouse gases during processing. Politicians of all nations fear that governmental intervention in market outcomes may negatively impact their economies.

1.1 Energy and the economy

Historically, there is little doubt that natural resources were important to the evolution of economic welfare over the centuries. Major societal and economic developments, such as the agricultural transition and the industrial revolution were based on the exhaustive use of resources and thus also of energy extracted from resources (cf. Barbier, 2005). One reason for the outstanding importance of energy for economic development its direct impact on innovation. Research and development as well as the implementation of new technologies are often energy-intensive (Jorgenson, 1984; Barbier, 1999). An increase in the use of final energy therefore seems beneficial for growth at first glance.
Figure 1.1: Swiss GDP and final energy use.

(Stern, 1993). According to this rationale, a decrease in energy use, possibly due to the depletion of resources, would lead to economic stagnation in the long run (Meadows et al., 1972).

Recent development shows, however, that the growth of GDP and of final energy use was partly decoupled. In the case of Switzerland, production increased during the last decade more than did the use of final energy (Figure 1.1). The gap between output and energy use can be explained by enhanced energy productivity as well as a more efficient mix of fuels (cf. for example Stern, 1993). Process and product innovations, e.g., better isolation for houses and machines for production that used less energy, were the major driving forces for higher energy efficiency. The relationship of GDP to energy use evolved similarly in other countries. This important fact sheds a different light on the importance of innovations and the consequences of energy policies in an economy.

A different view of the importance of energy for economic development comes from considering the affects of oil price shocks on macroeconomic fluctuations. The oil price has followed a very discontinuous movement during the last decades. In periods

\footnote{Data from Gesamtenergiestatistik Schweiz (2008) and from Federal Statistical Office Switzerland.}
after large oil price increases, economies worldwide suffered from lower growth or even stagnation. The United States economy, for example, went through many recessions after periods of high oil prices (Figure 1.2)\(^2\). The shaded areas indicate recessions as defined by the National Bureau of Economic Research (NBER). Many authors agree that there exists a causal relationship between both movements (e.g., Hamilton, 1983; 1996; 2003; Hall, 1988).

![Figure 1.2: Oil price fluctuations and US recessions, in US dollars per barrel.](image)

### 1.2 Challenges in energy use

Besides the positive characteristics of energy use mentioned above, the use of energy entails negative consequences. Probably the largest challenge is the emission of greenhouse gases and their effects on the environment. Recent studies (such as IPCC, 2007b) show that greenhouse gases emitted since the Industrial Revolution have had a detri-

---

\(^2\)Data from Federal Reserve Bank St. Louis.
mental effect on ecological equilibrium and have led to climate change. As environmental responses are worldwide, they are considered harmful not only for industrialized, but also for developing countries. Consequently, nations face the ethical and economic challenge of unequal burdens of ecological impact on developed and developing countries.

Additionally, ecological changes degrade the basis for productive activities of countries. The Stern report (Stern, 2007) tries to identify the economic costs of undamped climate change. The effects would lead to a large loss in productivity compared to 2000. Although the extended time frame involves much uncertainty, the range of scenarios of economic damage ranges from -5.3% to -13.8% until 2020 (Figure 1.3).

![Figure 1.3: Economic consequences of undamped climate change.](image)

These losses could be avoided by limiting emissions of greenhouse gases. Environmental pollution, however, is a negative external effect, and there exist too few incentives to diminish greenhouse gas emissions. A reduction of emissions would be effective if three measures are combined: first, an amelioration of energy efficiency; second, a decrease in energy use; and third, a change in the composition of primary energy resources.

---

1.2. CHALLENGES IN ENERGY USE

The current mix of energy types is dominated by fossil fuels with a small mixture of renewables (Figure 1.4)\textsuperscript{4}. The International Energy Agency (IEA) estimates that without political measures, the world’s primary energy demand will structurally not change until 2030 (IEA, 2009).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{energy_use.png}
\caption{Final energy use in 2006.}
\end{figure}

Fossil fuels are not uniformly distributed around the world. Most are located within a limited geographical area, particular the Middle East and Russia, the so-called energy ellipse. These countries with large fossil fuel resources are not necessarily also politically stable. Being import dependent upon them leaves other countries with great uncertainty concerning the reliability of the supply. If primary energy use continues to evolve as forecasted above, oil and gas import dependencies will increase for most countries, e.g., the US and the EU (Schwark, 2006; AER, 2008). Certainly, this development jeopardizes the political and economic independence of countries that import much of their energy.

Another fact that impedes a further increase of energy use is that it is becoming increasingly expensive to extract fossil fuels. Although proved reserves do not necessarily shrink (those of oil decline, but gas reserves grow), costs for oil and gas extraction have

\textsuperscript{4}Data from Gesamtenergiestatistik Schweiz, 2006, and from EU Energy and Transport Figures, 2006.
risen over the last years. It has become technically increasingly difficult to extract fossil fuels (AER, 2008). Partially based on this trend, the share of net imports spent on oil and gas is expected to rise in the future, according to the IEA. From the current 1% (average 1971-2008), expenses will double up to 2% (average 2008-2030) if energy demand evolves as forecasted (IEA, 2009).

1.3 International energy accords

The international community reacted to the threats of climate change by forming the United Nations Framework Convention on Climate Change (UNFCCC) in 1992. The UNFCCC aims at stabilizing greenhouse gas concentrations in the atmosphere at a level that will minimize negative impacts of climate change on humans' lives (UNFCCC, 2009).

In 1997, the Kyoto Protocol as a legally binding treaty united 37 industrialized countries and the European Community to agree on a common target for emission reductions. To reduce 5% of their emissions by 2012 compared to 1990 levels, the parties need to focus on domestic actions, but can also employ international mechanisms, such as the Clean Development Mechanism (CDM). In the context of CDM, industrialized countries can earn emission credits through investments in emission reduction projects in developing countries (UNFCCC, 2009).

The target of the Kyoto protocol for the cutback of emissions is the year 2012. Thereafter, there exists indeed an international agreement on limiting global climate change to 2°C. However, at the international conference in Copenhagen in 2009, nations did not sign a binding contract because the question of burden sharing between countries stopped them from agreeing on clear national targets.

Preceding the Copenhagen summit, the European Union passed the "20-20-20 by 2020" agreement in 2008. The EU aspires to assume global leadership in tackling climate change. It set legally binding standards for member states, which are to reduce 20% of greenhouse gas emissions, to increase the share of renewable energies in energy consumption to 20%, and to lower primary energy use compared to projected levels by 20% by improving energy efficiency by 2020. In case an international agreement with other
developed countries can be achieved in which they also agree on comparably reduce emissions, the EU was even willing to increase the target to an emission reduction of 30% (EU, 2010).

In Switzerland, the CO2-Law of 2000 aimed at reducing carbon emissions from fossil fuels by 10% until 2010 compared to 1990. Those measure with the highest priority in the implementation of this goal have been voluntary measures of firms and households. Only if the reduction goals cannot be attained should a carbon tax be imposed. Emissions trading and other flexible mechanisms of the Kyoto Protocols were included as possible measures to foster the goal (FOEN, 2009a).

For the time until 2020, two proposals currently exist in Switzerland. A public initiative demands a domestic reduction of greenhouse gases by at least 30% until 2020. In answer to this proposal, the Federal Council advises a revised CO2 law, which includes emission goals comparable to those of the European Union (FOEN, 2009b).

1.4 Models

1.4.1 Simulation of policy measures

The economic consequences of energy policy measures are never easy to predict. Complex economic structures impede solutions by simple calculations. All sectors in an economy follow different production functions that vary, for example, in the composition of inputs, the possibility of substituting for them, and the size of the sector. Furthermore, the sectoral interconnections by trade and supply of intermediate goods strongly influence production possibilities and sectoral dynamics.

Industries respond to energy policy measures by changing either the composition of inputs to production or by adapting the output level, or possibly by both. In doing so, they follow optimization conditions that relate the quantity and price of inputs to their output. The level of production, in turn, must correspond to the amount demanded by other sectors and households.

If an input, such as energy, becomes more expensive because of energy policies, producers invest in capital to increase their productivity. Their goal to produce the same
quantity with a smaller quantity of energy motivates them to enhance their productivity with investments. This can be done in two ways: First, they can augment the quality or quantity of physical capital by, e.g., exchanging old machines for new and more efficient ones. A second possibility is to invest in better production structures by changing characteristics of their products.

In an economy, the interplay of production and consumption decisions evokes macroeconomic reactions. Computable general equilibrium (CGE) models represent one way to model this complex behavior. The advantage of CGE models is that they can show economic behavior over long time horizons with the assumption that utility- or profit-maximizing behavior can be ensured.

1.4.2 Disruptions of steady energy supply

The evolution of energy supply has been very unsteady in the past decades. In particular, oil supply has fluctuated heavily and often unexpectedly, and prices of fossil fuels have seen large changes. Economies worldwide have needed to adapt to new situations. Although it is widely agreed that oil price shocks were largely responsible for major recessions, the exact extent and channels of impact are still subject to research.

CGE models are based on the assumption of perfect foresight over the modeled time horizon. However, oil price shocks occur unexpectedly. CGE models are thus not suitable for a detailed analysis of the effects of shocks.

Real Business Cycle (RBC) models, however, are suitable for such an analysis. They mirror macroeconomic behavior when expectations about economic variables, such as prices, are not met, since an economy is subject to an exogenous shock. RBC models are calibrated to a balanced growth path. This equilibrium gets disturbed by a shock, which gradually diminishes over time. Consequently, the economy returns to a new balanced growth path at the end of the time horizon.
1.5 Contributions of this thesis

This dissertation contributes to current research in three regards, each covered in an individual chapter. The first chapter combines a CGE model with endogenous growth theory. It describes the implementation of horizontal innovation as the basic driver for growth in a CGE model. Horizontal innovations thereby refer to the assumption that an economy grows by inventing more and more different products. The model is the first CGE model, which assumes that endogenous growth dynamics from horizontal innovations are the main reason for an increase in production.

The second chapter compares our CGE model with a CGE model with exogenous growth dynamics. The results illustrate that sectoral output responses are more amplified in our endogenous CGE model. The disparity of the reactions is attributable to different investment incentives. This suggests that CGE models with exogenous growth may underestimate sectoral reactions to energy policies.

In the third chapter, another novelty is introduced, the analysis of the effects of energy shocks on business cycles in the medium-term. Conventional business cycle analysis refers to fluctuations of up to eight years. This time frame seems, especially in the light of endogenous transmission of dynamics, too short to include all consequences of energy shocks. Thus, the third chapter addresses dynamics from energy shocks up to 50 years. The results show that energy shocks influence economic variables more than during the first eight years, and fluctuations are stronger in the medium-term than in the short-term.

1.6 Overview

The first two chapters emerged from a project with the Swiss Federal Office of Energy (SFOE). The aim of the project was to develop a new simulation model for the Swiss economy, which includes the newest insights of economic theory, especially of growth theory. When energy-saving policies are advocated, the macroeconomic effects and specific effects on different sectors need to be thoroughly evaluated. The intention of this project was to create an alternative to existing Computable General Equilibrium
(CGE) models for Switzerland. These usually build on the assumption of exogenous growth mechanisms. Although they might be enhanced by some modules that introduce endogenous dynamics, the basic driver for growth was exogenous and therefore invariant to changes of policies.

Our CITE model (Computable Induced Technological change and Energy model) builds on the assumption that growth evolves from incentives that are endogenous to the economy. It is based on the insight that dynamics from inside the economy are responsible for increased production and welfare. Investment incentives and the possibility of making profits for individual firms are the foundations for improved productivity. They induce a rise in the output level given a constant amount of endowments such as labor.

The ambitious goal of the project was to model the effects of the 2000 Watts Society introduced by ETH Zurich and promoted by the ETH board. The vision of this scenario is to reduce per capita energy consumption by two-thirds to 2000 watts. This amount was not arbitrarily chosen, but rather corresponds to the average world energy consumption. Nevertheless, it is not meant as an exact quantity target but as a vision of a modern low-energy society. Besides a reduction of energy and hence of carbon emissions, a balance between industrialized and developing countries is one of the goals.

The scenario entailed further political discussions when it was included in the "energy perspectives" of the Swiss Office of Energy as Scenario IV. Being the most stringent scenario, it was subject to many discussions and dismissed by critics as not being realistic. Nevertheless, promoters of ambitious energy policies continued to use the scenario in political discussions. The citizens of Zurich voted in 2008 that the 2000 Watts Society is a long-term goal of the city together with the one ton carbon emissions per capita. This vote confirmed that they were even more interested in the 2000 Watts Society than was expected.

While the technical feasibility of the 2000 Watts Society was subject to extensive inquiry of ETH Zurich (cf. Jochem et al., 2004), economic consequences had not yet been fully evaluated. Against this background, our project with the SFOE arose.

The first chapter introduces the theoretical background of the CITE model. It gives an overview of the status of Swiss energy consumption and the expected effects of
1.6. OVERVIEW

undamped climate change on the Swiss ecology and economy. The chapter clearly distinguishes our CITE model from other CGE models for Switzerland. There exist some top-down as well as bottom-up models that estimate consequences of energy policies, such as Schulz et al. (2008), different studies from Ecoplan, and Scieia et al. (2009a, 2009b). The basic growth assumptions of these models are exogenous.

The CITE models differ from these models by applying endogenous growth dynamics, specifically gains from specialization. Different authors have referred to increasing productivity by dividing work into smaller parts. Smith (1776), Romer (1987, 1990), and Grossman & Helpman (1991) are some representatives of the view that a high degree of specialization allows for learning and better application of knowledge. Profit incentives that result from higher efficiency and productivity cause investors to invent new products and thereby foster economic growth.

The chapter describes the setup of the CITE model in great detail. Each agent in the economy is introduced with the corresponding maximization problem and the optimal solutions. It gives an overview how the agents are connected and what incentives lead to investments. Equilibrium conditions and an analysis of growth dynamics give an extensive overview of the model. Subsequently, specialities of the calibration are explained. The endogenous dynamics of the model obliged us to make a number of particular changes in the computer code. The aim of the chapter is to give future scholars the foundation for completely understanding the theoretical model and the numerical application.

The second chapter compares the dynamics of the CITE model with a model with exogenous growth and homogeneous capital (the so-called HK model). The HK model represents basic dynamics of CGE models, which depend on exogenous growth of endowments. The main difference between the two classes of models is the role of capital. In typical CGE models, capital is one input among others to production. It can be accumulated with investments and substitutes for energy if an energy policy is introduced. The extent of how well capital can substitute for energy is, however, limited by the production function, i.e., by the elasticity of substitution. If the elasticity is low, capital can increase productivity only to a small extent.

In the CITE model, capital accumulation can enhance productivity in a Hicks-neutral
manner. By investing in capital, energy efficiency can be increased without assuming that basic characteristics of production are automatically changed. The amount of capital in a sector is a measure for productivity and can, depending on investment incentives, be raised independently of technological constraints. This allows for enduring economic growth without the assumption that endowments must be increased exogenously over time.

In addition to these growth dynamics, production can also change characteristics by substituting endowments for energy. By using this assumption, we take into consideration the inevitability of technological change.

The comparison of the models demonstrates to what extent investment incentives influence the development of the Swiss economy over time. The results show that they are stronger in the CITE model, which generates more pronounced effects of a carbon tax. Production levels of the sectors change more in the CITE model than in the HK model. These amplified responses are attributable to the change in the sectoral capital stocks. Consequently, this comparison of the models demonstrates how important it is for CGE modelers to direct attention to underlying dynamics. The inclusion of endogenous growth and gains from specialization reveals that models with exogenous growth dynamics may underestimate the sectoral responses to energy policies.

The third chapter also applies insights of new growth theory. The main focus is on oil price shocks and their effects on business cycles in the medium-term.

In the literature, oil price shocks over a period of 2 to 32 quarters have been analyzed extensively by RBC literature (e.g., by Rotemberg & Woodford, 1996; Leduc & Sill, 2004; Aguiar-Conraria & Wen, 2007). These authors have partially explained the economic responses to an energy shock. The main challenge in this strand of literature is to explain the relatively strong reactions of economic variables after an increase in the price of energy, which constitutes only a small share of inputs to the economy.

The chapter differs from the existing RBC literature in that it analyzes economic fluctuations after an energy shock over a longer time horizon. Based on the model of Comin & Gertler (2006) we additionally include oscillations of 32 to 200 quarters (8 to 50 years) in the study. The underlying assumption is that energy shocks provoke not only high-frequency reactions (up to 32 quarters, or 8 years), but also medium-
frequency reactions (between 8 and 50 years). Although energy shocks might be short-lived, endogenous growth mechanisms transmit macroeconomic responses over a longer time horizon.

Growth dynamics are also discussed in this chapter based on gains from specialization and an expanding variety of products. However, the discussion differs from that in the first two chapters in three important areas: First, the nature of the disturbance of the economy considered in the third chapter is an unforeseen shock and could not have been anticipated before the shock arrived. The analysis of economic consequences is therefore ex post. In comparison, the energy policy of the CITE model is included in the rational expectations of market participants and economic responses are forecasted in the simulations.

Second, the model in the third chapter differs in the representation of the economy. It is a two-sector model with a consumption good and a capital good. The CITE model comprises 12 different sectors that are all related by trade. This complex structure yields a more inelastic behavior to a disturbance of the economy. Additionally, the elasticities of substitution in production are lower in the CITE model than in the third chapter. Again, the economy is less flexible in reacting to a change in the energy price than the economy described in the third chapter.

The third difference of the models is the production structure of the goods that contribute to innovation and growth. In the third chapter, innovations have the same production structure as the consumption good. In the CITE model, growth dynamics result from a sector with a different production function than the consumption good. Due to this important distinction, changes in energy prices result in different dynamics.

In the third chapter, the innovation process is modeled thus: The two sectors comprise independent research and development sections to create new products. The resulting blueprints go through a process of adaption before they can enter the production process of the final outputs within each sector. The outcome of the adoption section contributes to an increase in the number of goods. Consequently, it takes time for each new blueprint to become usable. A variation in these R&D and adoption expenditures leads to fluctuations in the medium-frequency.

The high-frequency fluctuations result from the assumption of free entry and exit to the
market of final goods. This gives each entrepreneur the opportunity to enter the market and provide goods, depending on the markup they can charge. Since the markup relates negatively to the number of firms on the market, firms leave the market as soon as it becomes unprofitable to produce.

The conjunction of both mechanisms enables the model to display similar standard deviations of economic variables compared to the data. The medium-term fluctuations are for most variables very close to the data in qualitative and quantitative terms when a shock to the oil price is assumed. Furthermore, the high- and medium-frequency components show values that are structurally comparable values to those the U.S. economy over the last decades. In a second case, we assume a shock to the supply of energy. The results are qualitatively also very sound. The quantitative magnitude differs, however, from the data, due to the simplified modeling of the energy producing country. We therefore conclude that energy shocks have effects in the high- as well as in the medium-frequencies and that medium-term cycles may be initiated by energy shocks.
Chapter 2

The theoretical foundation of the CITE Model

This chapter describes the details of our new CITE model, a computable general equilibrium (CGE) model with endogenous growth dynamics. This is the first CGE model that applies heterogeneous capital and gains from specialization as the main driver for growth. These dynamics are assumed to be present in the benchmark case without political measures as well as in the policy scenarios, which yields consistent technologies with and without an energy policy. The incentives for innovations are microeconomically based and differ from most other CGE models that assume exogenous growth mechanisms. All production and consumption functions as well as the optimality conditions are presented in the chapter. We particularly focus on the channels for incentives to innovate.
2.1 Introduction

Energy is undisputably a major input factor for economies around the globe. Securing its supply and ensuring an efficient use of it were crucial for continuing prosperity worldwide in the past years. However, coping with the negative consequences of climate change to preserve our environment is arguably one of the largest challenges humanity faces in the next decades. Within this challenge, public policy makers have a key role in identifying appropriate energy strategies with levers on both the supply and the demand side. Mankind is well advised to replace hydrocarbon energy resources with renewables, but trimming demand is still the ultimate goal en route to a sustainable energy usage.

For a conceivable amount of time, the world has enough accessible fossil reserves to secure its current pattern of supply. Although oil resources have a shorter range, natural gas resources will last at least until the end of the 21st century, and coal could even cover human energy demand for centuries. Concerning the demand side, by far the largest degree of energy consumption in the last decades has been in Western countries. Currently, the net growth in energy consumption mainly comes from the rapidly industrializing non-OECD economies like China, India, and Brazil. China alone accounts for nearly three-quarters of growth in global demand. This soaring demand resulted in non-OECD countries outpacing OECD countries, which are no longer the major contributor. This reversed energy demand has not only influenced global markets but also changed the composition of energy sources. For a sixth consecutive year, coal is the fastest-growing fossil fuel (BP, 2009). Given that coal has a comparatively high content of carbon, this is an alarming tendency and will strongly affect global carbon emissions.

Policy makers of all countries are aware of this situation and seek to find energy strategies that both secure energy supply and offer better energy efficiency. Economic models help estimate the consequences of political measures on the different industries and residents. The CITE (Computable Induced Technical change and Energy) model aims at estimating the effects of energy policies on Switzerland. It is a multi-sector growth model that includes endogenous growth dynamics in all sectors and all scenarios, including the benchmark (business-as-usual) case. This chapter provides the theoretical
2.1. INTRODUCTION

background for the model.

The CITE model is based on the work of Romer (1990) and Grossman and Helpman (1991) and exhibits endogenous growth dynamics based on research and development (R&D) for Hicks-neutral technical progress with the assumption of an expanding variety of intermediate goods (i.e., horizontal innovations). By including these modules both in the benchmark case and in the policy scenarios, the growth dynamics are greatly amplified compared to a neoclassical growth model. As all dynamics that arise from monopolistic competition and of gains of specialization are consistent in the cases with and without policies, dynamics are consistent among all scenarios.

The remainder of this chapter is organized in three parts. First, we give a general introduction of the environmental and political background for the energy debate. Second, we introduce the CITE model and describe markets and agents are described in some detail in Chapters 2.3 to 2.11. This part also includes an explanation of the growth dynamics. In the third part (Chapter 2.12), we explain some specialties of the calibration of our model. The focus is on the challenges posed by the inclusion of different growth rates in the GAMS code.

2.1.1 Switzerland's energy consumption

The energy consumption of a Swiss resident is comparable to a continuing performance of about 5000 watts per year. In comparison to the global average, this is about 2.5 times higher than the average energy a human being consumes, i.e., 2000 watts per capita. In Western Europe, the average energy demand equals 6000 watts, and in the United States, the figure corresponds to 12000 watts per capita. The calls in Switzerland for regaining a consumption of 2000 watts per capita, the so-called "2k Watts Society", are ambitious, but not unachievable. In the past, Switzerland last had a consumption of 2000 watts in 1960 (Novatlantis, 2005).

Of the current 6000 watts, about 60% are produced from oil and gas products (Spreng & Semadeni, 2001; Novatlantis, 2005). Regarding the need to produce one unit of gross domestic product (GDP), carbon intensity is relatively low in Switzerland, a fact that is, among others, due to the specific composition of output (Stern, 2007). In
comparison, the United States requires 50% more than the EU, and China even uses 500% more energy than the EU. Those benchmark numbers are not static; they may increase or decrease over time. When we look at the efforts of countries to reduce energy intensity, France, for example, has nowadays an energy intensity that is 30% lower than in the 1970s (EC, 2005).

Switzerland faced the global discussion about the reduction of energy consumption and carbon emissions by joining the Kyoto protocol and committing to reducing carbon emissions by 8% by the year 2012. In 2007, the Swiss progress towards this goal was still about 4% below the scheduled reductions. Only in April 2009 did the Swiss Federal Office for the Environment include effects from woods and foreign emission certificates and conclude that Switzerland will be able to comply with the Kyoto protocol (ETS, 2009). Additionally, Switzerland decided to join the official emission reduction targets of the EU to reduce emissions by 20% by the year 2020. To sum up, there is still a need for action to change the prevailing consumption pattern within the country.

2.1.2 Carbon emissions

Due to the increased demand for energy, the carbon dioxide concentration in the earth’s atmosphere has increased about 35% until 2005 compared to the preindustrial level (IPCC, 2007a). Although there is still significant uncertainty about the precise consequences for the ecosystem, some effects for the European climate can already be estimated. It should be emphasized that these estimates have only been corrected for the worse in the course of accumulating more knowledge about ecological consequences.

For Europe, it is assumed that rainfall will increase by about 10% in winter and decrease by about 20% in summer. In addition, the pattern of rainfall within the seasons will be altered. Extreme rainfall is expected to become more common, which results in an increase of floods. The general increase in temperature leads to fewer cold waves in winter. On the contrary, in summer when rain will become rarer, heat waves and droughts are expected to occur more frequently. As a result, the ecosystem in Switzerland will change in the long run (OcCC, 2007).

These ecological effects will soon have economic consequences. Tourism, for instance,
might both benefit and suffer from climatic changes. Hot summers can make domestic destinations more attractive, especially at lakes in the Alps. On the other hand, as the weather will also be warmer during winter times, the rising snow line will make ski areas in the foothills of the Alps unprofitable. By the year 2050, most of the smaller glaciers are expected to have disappeared. In total, higher frequencies of tourist visits in summer will not compensate for losses of revenue in winter (OcCC, 2007, IPCC, 2007b).

Besides the effects from energy transformation to the climate there will be also a reverse effect from the changed ecosystem on energy transformation. For example, hydro and nuclear power will be negatively affected as reduced water drainage and warmer rivers will result in a smaller cooling effect. This effect will become particularly visible in summer (OcCC, 2007).

### 2.1.3 Political instability

Climate change is clearly the most important reason for the strengthened effort to reduce Switzerland’s dependency on fossil fuels as well as to reduce overall energy consumption. Besides climate change, political instability in the countries that produce fossil fuels is increasingly perceived as a threat. This problem especially arises as the energy supply in Switzerland has shifted from coal to oil during the last seventy years (SFÖE, 2008).

Although the IEA (2007) considers Switzerland to have a well diversified oil and gas supply in terms of the countries it imports from as well as the import routes, critical voices have emerged. In general, criticism is based on the fact that easily accessible reserves are geographically concentrated within politically unstable countries (Proclim, 2007). For both oil and gas, about 80% of all reserves are each located within three areas or countries. About 60% of all oil reserves are situated in the Middle East, another 10% in South and Central America, and another 10% in Africa. Global reserves in gas also concentrate on limited areas, but are less focused in the Middle East. About 40% are located in the Middle East, 34% in Europe and Eurasia, and about 8% in Africa (BP, 2009).
2.1.4 Models of Swiss energy use

The Swiss Federal Office of Energy (SFOE) reacted to these challenges and formulated four distinct energy scenarios for Switzerland for the time frame until 2050. Two scenarios focus on political measures, while two focus on ecological and economic aims. All display very detailed technical analysis of energy sources and their contribution to energy supply in Switzerland (SFOE, 2007).

Schulz et al. (2008) analyze the most stringent scenario of the SFOE, the goal of a 2000 Watts Society, based on the Swiss MARKAL model. Their study includes a Reference Energy System (RES) that considers both currently available and possible future energy technologies and energy carriers. The model comprises the RES to find the least-cost energy system to saturate energy demand. Based on this analysis, if primary energy per capita consumption is given as a constraint to the model, the authors determine that carbon emissions can be reduced to an equivalent of 5% per decade at maximum. Consequently, a 3500 watts society is feasible by the year 2050 and a 2000 watts society can be maintained as the long-term aim thereafter. Comparing different kinds of objectives, primary energy per capita consumption targets yield higher costs than carbon reduction targets.

Moreover, the SFOE commissioned another set of scenarios that estimated the effects of high oil prices and drastic cuts in supply through an increase of world population, with peaks in 2010 and 2020, respectively. Interactions with international markets are quantitatively simulated and discussed, but none of these scenarios include political measures. This analysis resulted in the insight that long-term price development will possibly be less dramatic than is often projected. Nonetheless, the economic implications of high energy prices are considerable (Ecoplan, 2007).

The political debate in Switzerland focuses increasingly on two scenarios for a successive international treaty after the Kyoto protocol. Switzerland has already committed to decreasing carbon emissions by 20% by 2020 compared to 1990 and even announced that it would lower the target to 30%, depending on reduction goals of other countries. Ecoplan (2009) analyzes these scenarios in a CGE model with the assumption that no further climate relevant policies would be launched by other countries than Switzerland. The results show that the losses for welfare are negligible. Other models (e.g.,
2.1. INTRODUCTION

Kumbaroglu & Madlener, 2001; Ecoplan, 2008; Sceia et al., 2009a; Sceia et al., 2009b) similarly show that carbon taxes have only limited effects on welfare.

In Switzerland as well as in other countries, increasing oil prices during the last decades have augmented incentives for fast efficiency measures only in the short run. A lack of sound structural measures, a deficit of information, and the absence of financial instruments could not stabilize the incentives to increase energy efficiency for a longer period (EC, 2005). To accomplish structural changes and a transition to a less CO2-intensive economy, a "fundamental change in the innovation system (e.g., research policy, education, standards, incentives, intermediates and entrepreneurial innovation" (Jochem et al., 2004) is required.

2.1.5 Growth and the role of variety

Since the seminal work of Solow (1956), economists have seen technology as the main driver of innovation and growth (Niosi, 2008). However, decreasing returns to capital complicated the explanation of endogenous growth, so for a considerable length of time, economic models included exogenous growth mechanisms. However, the inability to identify the right drivers that might possibly enable an economy to grow was not caused by a knowledge gap. It was rather the immense difficulty of including the insight in mathematical models.

One possibility to endogenize growth dynamics is to assume gains from specialization, either in consumption or production. This explanation is also used in the CITE model we describe in this chapter. Different authors have referred to gains from specialization. By observing production in a pin factory, Smith reported as early as 1776 that specialization immensely increases the efficiency of the workers and therefore contributes to an augmented output. The increase of specialization led larger firms to have higher output per worker and lower average cost per pin than a small pin factory.

Also, Allyn Young (1928) stated that increased specialization may generate a higher output due to externalities that arise in production. He concluded that a larger market size would lead to more steps in production by a greater number of specialized firms (Sandilands, 2000). In 1963, Rosenberg noticed that in the automobile industry, many
small specialized firms each construct a limited number of tooling devices for specific mass-production processes. He concluded that this high degree of specialization not only permitted a learning process that was more effective, but also a better application of the knowledge. Historically, Rosenberg states, an important reason for innovation has been improvements in the efficiency of capital goods production.

The first attempt to include these gains from specialization in economic models was made by Spence (1976). He modeled consumer preferences that were enhanced if the amount of consumer goods rose. Dixit and Stiglitz (1977) and Grossman and Helpman (1991) refined Spence’s approach. The first to combine specialization with production was Ethier (1982), who assumed that an increasing number of inputs to production would raise output (Barro & Sala-i-Martin, 2004). Romer (1987, 1990) followed Ethier (1982) and assumed that output is an increasing function of intermediate goods. Using this specification, growth can continue indefinitely.

The incentive to specialize or to invent new products in these models is always the existence of monopolistic power and therefore the possibility for an inventor to make a profit with a new product. This, in turn, leads to the situation that the market equilibrium always generates too little research, i.e., too few new product varieties, compared to the social optimum (Bretschger, 1999). This suboptimal solution can be augmented by applying a generalized production function, which yields either the social optimum or even an outcome with too much R&D (Benassy, 1998).

The empirical extent of specialization in the European Union has been estimated by Mangâni (2007), who analyzes the correlation of economic (in terms of GDP) and technological (i.e., R&D aggregate expenditure or the number of patents granted) sizes. She asserts a positive correlation between the two. She distinguishes between two technological dimensions: the intensity of technological activities (intensive margin) and their variety (extensive margin). The technological variety is hereby defined as the number of technological fields in which a country is active. Both dimensions are positively correlated with the country size, i.e., larger countries have a wider spectrum of technological fields and produce a larger number of patents in each technological field. In Mangâni’s estimation, technological variety accounts for about 40% of the difference in patent application between larger and smaller economies and is therefore
2.2 An overview of the CITE model

The model displays a small open economy. It consists of \( n \) different regular sectors, an energy sector, and an oil sector, each with similar intrasectoral setups.

There are three types of agents in each sector. First, producers of final output \( Y_i \) use a sector-specific intermediate composite \( Q_i \) and Armington goods of regular sectors \( n \) that go to sector \( i \), \( A_{ni} \), in their production. These Armington goods consist of domestically produced goods and imported goods that are combined with an elasticity of substitution below unity (Armington, 1969). The assumption of imperfectly substitutable goods ensures that trade can be modeled realistically in the sense that the same sectors can import and export (for a more detailed description of trade and Armington goods, cf. Chapter 2.9). Second, a producer of the sector-specific intermediate composite \( Q_i \) assembles intermediate goods \( x_i \). And third, firms produce intermediate goods and sell them to the producer of the intermediate goods composite. This setup, illustrated in

extremely important in explaining the different technological standards. Many other empirical studies exist on gains from specialization in relation to trade. These models analyze the impact of specialization in exportation and importation on the productivity of the involved countries. Most of the papers find a close connection between technological innovation (growth possibilities) and export specialization patterns (Mangàni, 2007). Hummels and Klenow (2005) are close to Mangàni’s (2007) results. They find that the variety of goods accounts for about 60% of the greater exports of larger economies. They also find a distinctive correlation between the size of an economy and the degree of specialization. Their results are in line with those of others, such as Hummels et al. (2001) and Furman et al. (2002).

In addition to specialization, the capital stock plays an important role in determining the growth rate of an economy. Ezcurra et al. (2008) estimate for the EU that the capital stock per worker is relevant in explaining the level of technical efficiency. The latter is defined as the relation between the amount of a firm combination of inputs and output. Ezcurra et al. find a statistically significant relation between capital and improvements in technical efficiency, i.e., growth.
THEORETICAL FOUNDATION OF THE CITE MODEL

Intermediate composite

\[ B_i \]

\[ Q_i \]

Armington good of other sectors

\[ A_{ni} \]

Capital

\[ K_i \]

Intermediates

\[ x_i \]

Labor

\[ L_i \]

Armington good of the energy sector

\[ A_{ex} \]

Figure 2.1: Production of \( B_i \).

Figure 2.1, is analogous in all sectors.

In the regular sectors, \( B_i \) corresponds to the final output, \( Y_n \). The production of the energy sector requires as additional input imported gas and the output of the oil sector (Figure 2.2).

Similarly to the production of the energy sector, the oil sector also needs an additional input, namely crude oil, which is imported (Figure 2.3).

Final good

\[ Y_e \]

\[ B_e \]

Gas

Armington good of the oil sector

\[ A_o \]

Figure 2.2: Production of energy.
The dynamics in the model stem from the assumption that the variety of intermediate goods expands over time and generates gains of specialization (cf. Romer, 1987; Grossman & Helpman, 1991). These new varieties are invented by firms by investing in a capital composite consisting of physical and non-physical capital. The assumption that knowledge (and thus the capital stock) is sector-specific reflects the supposition that one kind of knowledge can only be used for a particular combination of inputs (cf. Basu and Weil, 1998). Investments in new capital are nested according to Figure 2.4.

A representative household maximizes intertemporal utility from a consumption good that consists of a final goods composite and energy, as depicted in Figure 2.5. The final goods composite includes the Armington goods of regular sectors. Agents are assumed to have perfect foresight.

Due to the small size of the economy, it faces exogenously given world-market prices for crude oil and gas. Domestically, the markets for final goods, for the intermediate goods composite, and for labor are perfectly competitive, whereas the market for intermediate goods is monopolistic. All markets clear, and the allocation and price vectors constitute
a competitive equilibrium. The supply of labor and of a so-called policy invariant capital that is introduced for reasons of calibration (for further details, see Chapter 2.12.4) is assumed to be inelastic and perfectly mobile across all firms and sectors.

### 2.3 Final goods production of regular goods

The producer of final good $Y_n$, $n \in N$, produces in a CES framework using two types of inputs: a sector specific intermediate composite $Q_n$ and Armington goods from regular sectors $n'$, $A_{n'n}$, $n' \in N$. Thus, all regular final goods are used in the production of all goods, including their own production. This reflects the intersectoral connections of the sectors in the Swiss economy as reported in the input-output table (Nathani & Wickart, 2006).

\[
Y_{n,t} = \left[ \alpha_{X,n} \frac{\sigma_{Y,n}^{-1}}{\sigma_{Y,n}} + (1 - \alpha_{X,n}) \left( \min \left( \left\{ \frac{A_{n'n,t}}{a_{n'n}} \right\}_{n' \in N} \right) \right) \frac{\sigma_{Y,n}^{-1}}{\sigma_{Y,n}} \right]^{-\sigma_{Y,n}} (2.1)
\]

The elasticity of substitution between $Q_n$ and $Y_{n'}$, $\sigma_{Y,n}$, is assumed to be smaller than unity to reflect limited substitution possibilities between the intermediate composite and final goods as inputs. The value share of the intermediate composite is $\alpha_{X,n}$. The
activity coefficients \( a_{n'n} \) give the amount of each Armington good \( A_{n'n} \) required for one unit of output in the Leontief function.

In the CITE model, the production structures are not described using production functions, but with cost functions. These have a similar structure as the production functions but are not as intuitive. Therefore, the analysis in this chapter is based on production functions. However, to give an idea how the cost functions are built in the CITE model, we also show the corresponding cost function to the described production function. It consists of the costs for the intermediate composite, \( p_{Q_n} \) and the prices of the Armington goods \( p_{A_{n'n}} \). These are combined very similarly to the CES production function:

\[
p_{Y_{n,t}} = \left[ \alpha_{X,n} p_{Q_{n,t}}^{\frac{1-\sigma_{Y,n}}{\sigma_{Y,n}}} + (1 - \alpha_{X,n}) \left( \sum_{n'' \in N} \alpha_{z_{n'n''},n'} p_{A_{n''}} \right)^{\frac{1-\sigma_{Y,n}}{1-\sigma_{Y,n}}} \right]^{\frac{1}{1-\sigma_{Y,n}}}\]

The parameter \( \alpha_{z_{n'n}} \) hereby denotes the share of the costs of the Armington good \( A_{n'n} \) in the Leontief cost function (all \( \alpha_{z_{n'n}} \) add up to unity).

Using the production functions, each sectoral producer of a final good maximizes profit

\[
\max_{Q_{n,t},A_{n'',t}} p_{Y_{n,t}} Y_{n,t} - p_{Q_{n,t}} Q_{n,t} - \sum_{n'' \in N} p_{A_{n''}} A_{n'',t}
\]

subject to (2.1) with \( p_H \) denoting the price for variable \( H \). Because the market for final goods is perfectly competitive, profits are zero and the inverse demand functions are given by

\[
p_{Q_{n,t}} = p_{Y_{n,t}} \alpha_{X,n} \left( \frac{Y_{n,t}}{Q_{n,t}} \right)^{\frac{1}{\sigma_{Y,n}}}\]
\[ p_{A_{n',t}} = p_{Y_{n,t}}(1 - \alpha_{X,n}) \left( \frac{Y_{n,t}}{\min\left(\left\{ \frac{A_{n',t}}{a_{n'n}} \right\}_{n' \in N} \right)} \right)^{\frac{1}{\sigma_{Y,n}}} D_{n'n} \]

with \( D_{n'n} \) being the derivative of the Leontief production structure w.r.t. the input of sector \( n' \) to sector \( n \). For a better understanding, the example of \( D_{11} \) can be helpful:

\[ D_{11} = \min \left( \left\{ \frac{1}{a_{11}}, A_{21}, \ldots, A_{n'1} \right\}_{n' \in N} \right) \]

\section{2.4 Production of energy goods}

Energy goods are produced very similarly to regular goods with the exception that fossil fuels are also required. Imported natural gas (\( GAS \)) and refined oil (\( A_o \)) are first combined in a Cobb-Douglas manner. This aggregate is then combined applying an elasticity of substitution smaller than unity with the function already used for regular goods \( B_e \)

\[ Y_{e,t} = \left[ \alpha_{TFF} \left( GAS_t^{\alpha_{FF,gas}} A_{o,t}^{\alpha_{FF,o}} \right)^{\frac{\sigma_{E}-1}{\sigma_{E}}} + (1 - \alpha_{tff}) B_{e,t}^{\frac{\sigma_{E}-1}{\sigma_{E}}} \right]^{\frac{\sigma_{E}}{\sigma_{E}-1}} \tag{2.2} \]

\[ B_{e,t} = \left[ \alpha_{X,e} Q_{e,t}^{\frac{\sigma_{Y,e}-1}{\sigma_{Y,e}}} + (1 - \alpha_{X,e}) \left( \min \left( \left\{ \frac{A_{n,e,t}}{a_{ne}} \right\}_{n \in N} \right) \right) \right]^{\frac{\sigma_{Y,e}-1}{\sigma_{Y,e}}} \tag{2.3} \]

Fossil fuels enter the upper nest exclusively, and the total amount of gas imported in the economy and the complete output of the oil sector are directed toward the energy sector. \( B_e \) contains only the inputs \( Q_e \) and the final outputs of other regular sectors, \( A_{ne} \). In our model, energy is interpreted as a composite of different usages of energy,
such as electricity, combustibles, and fuels. The lower nest might be interpreted as all types of energy that do not contain fossil fuels, such as renewable energies or nuclear energy. Oil and gas are mainly used to produce combustibles and fuels.

This type of production function shows similar dynamic behavior to that of regular final goods because $B_e$, which is to a great extent responsible for dynamic growth effect, is identical. Growth dynamics mainly occur in the sector-specific product $Q_e$. Therefore, the structure of this production function results in the fact that most dynamics are transmitted to non-fossil forms of energy first. Fossil energy is affected by growth dynamics via the upper CES nest.

The producer of energy goods maximizes profit assuming perfect competition according to

$$\max_{GAS_t, A_o,t, Q_e,t, A_{ne,t}} p_{Y_e,t} Y_{e,t} - p_{GAS_t} GAS_t - p_{Q_e,t} Q_{e,t} - p_{A_o,t} A_{o,e,t} - \sum_{n \in N} p_{A_n,t} A_{ne,t}$$

subject to (2.2) and (2.3), which results in the following first-order conditions for fossil fuels and $Q_e$:

$$p_{GAS_t} = p_{Y_e,t}^{\alpha_{TFF} \alpha_{FF,gas}} \left( \frac{Y_{e,t}}{GAS^{\alpha_{FF,gas}} A_{o,t}^{\alpha_{FF,o}}} \right)^{\frac{1}{\sigma_E}} GAS^{\alpha_{FF,gas} - 1} A_{o,t}^{\alpha_{FF,o}}$$

$$p_{A_o,t} = p_{Y_e,t}^{\alpha_{TFF} \alpha_{FF,o}} \left( \frac{Y_{e,t}}{GAS^{\alpha_{FF,gas}} A_{o,t}^{\alpha_{FF,o}}} \right)^{\frac{1}{\sigma_E}} GAS^{\alpha_{FF,gas} A_{o,t}^{\alpha_{FF,o}-1}}$$

$$p_{Q_e,t} = p_{Y_e,t}(1 - \alpha_{TFF})^{\frac{1}{\sigma_E}} \frac{\sigma_{E}^{-1}}{\sigma_{Y,e}^{-1}} \frac{\sigma_{Y,e}^{-1}}{\sigma_{Y,e}^{-1}} Q_{e,t}$$

The optimal input prices of $A_{ne}$ are denoted using the abbreviation $D_{ne}$, which is
explained above.

\[ p_{A_{ne,t}} = p_{Y_{e,t}}(1 - \alpha_{TFF})(1 - \alpha_{x,e})Y_{e,t}^{\frac{1}{\gamma - 1}} B_{e,t}^{\frac{\gamma - 1}{\gamma}} \left( \min \left( \left\{ \frac{A_{ne,t}}{a_{ne}} \right\}_{n \in N} \right) \right)^{-\frac{1}{\gamma_x}} D_{ne} \]

2.5 Production of oil goods

The production of (refined) oil requires a large amount of crude oil as input that is not substitutable by labor or other inputs. Therefore, crude oil is used as essential input in a Leontief production function. All other inputs are again sector-specific inputs identical to the input function of regular final goods.

\[ Y_{o,t} = \min \left( \frac{CRU_t}{a_{cru}}, \frac{B_{o,t}}{a_{noncru}} \right) \tag{2.4} \]

\[ B_{o,t} = \left[ \alpha_{X,o} Q_{o,t}^{\frac{\gamma_{Y,o}}{\gamma_{X,o}}} + (1 - \alpha_{X,o}) \left( \min \left( \left\{ \frac{A_{no,t}}{a_{no}} \right\}_{n \in N} \right) \right)^{\frac{\gamma_{Y,o} - 1}{\gamma_{Y,o}}} \right]^{\frac{\gamma_{Y,o}}{\gamma_{Y,o} - 1}} \tag{2.5} \]

Although the total output of the oil sector goes to the energy sector, we do not assume market power of the energy sector but perfect competition. Therefore, the optimization problem of the producer of oil looks like

\[
\max_{Q_{o,t}, A_{no,t}, CRU_t} p_{Y_{o,t}} Y_{o,t} - p_{Q_{o,t}} Q_{o,t} - \sum_{i \in I} p_{A_{i,t}} A_{no,t} - p_{CRU,t} CRU_t
\]

subject to (2.4) and (2.5). Due to the complex Leontief structure, the inverse demand functions are presented only for \( Q_o \) and \( CRU \) and not for \( A_{no} \). However, it is straightforward to calculate.
$p_{Q_o,t} = p_{Y_o,t} \min \left( \frac{CRL_t}{a_{cru}}, \frac{\alpha_{x,o}}{a_{noncru}} \left( \frac{B_{o,t}}{Q_{o,t}} \right)^{\frac{1}{\sigma_{Y,o}}} \right)$

$p_{CRU,t} = p_{Y_o,t} \min \left( \frac{1}{a_{cru}}, \frac{B_{o,t}}{a_{noncru}} \right)$

### 2.6 Intermediate composite production

The growth rate of the economy depends on the growth rates of the sectors. These, in turn, result from an increase of the varieties of sectoral intermediate goods, which is reflected in the production of the sectoral intermediate composite $Q_i$, $i \in \{N, E, O\}$. It is produced with a Dixit-Stiglitz production function

$$Q_{i,t} = \left[ \sum_{j=1}^{K_{i,t}} x_{ij,t}^{K_{i,t}} \right]^{\frac{1}{\kappa}}$$

(2.6)

with $x_{ij}$ being the employment of the $j$th type of specialized intermediate good and $K_i$ being the size of the sector specific capital stock, which equals the number of intermediates available in sector $i$. Based on the assumption that diversification in production increases productivity (gains of diversification), an increase in the number of intermediates available enhances the production of the intermediate composite disproportionately (cf. Baldwin et al., 2001). The share of inputs to the production of $x_{ij}$ other than capital, $\kappa$, is a measure for the substitutability of the intermediate goods:

$$\kappa = \frac{\sigma_{Q} - 1}{\sigma_{Q}}$$
with $\sigma_Q > 1$ being the elasticity of substitution between the intermediate goods. Technological progress takes the form of an increase in the capital stock of sector $i$.

Acting on a competitive market, the intermediate composite producer minimizes cost according to

$$\max_{x_{ij,t}} p_{Q_{i,t}} Q_{i,t} - \sum_{j=1}^{K_{i,t}} p_{x_{ij,t}} x_{ij,t}$$

subject to (2.6) which yields her optimal demand for intermediate good $x_{ij}$

$$x_{ij,t} = \left( \frac{p_{Q_{i,t}}}{p_{x_{ij,t}}} \right)^{1/\kappa} Q_{i,t} \quad (2.7)$$

2.7 Intermediate goods

In each sector, intermediate goods $x_{ij}$ are invented and produced, each good by a single firm. One can think of these firms as in-house R&D and in-house production of intermediate goods in each sector. However, for the purpose of this chapter, the institutional structure is irrelevant.

Each new variety of an intermediate good in the horizontal innovation process is represented by a new unit of capital. Two types of capital exist in the model: Physical capital in terms of machinery, buildings etc., and non-physical capital, such as blueprints and patents. The assumption that knowledge (and thus the capital stock) is sector-specific reflects the supposition that one kind of knowledge can only be used for a particular combination of inputs (cf. Basu and Weil, 1998).

2.7.1 Capital accumulation

Intermediate firms conduct research and development. By investing in a capital stock composite, intermediate firms $j$ invest in two types of capital, physical capital ($I_{P_{ij}}$)
and non-physical capital \( I_{NP_{ij}} \). Both types of investment can only be intra-sectoral, i.e., investments in new capital is sector-specific. Non-physical capital is produced by investments in R&D, \( I_{R&D_{ij}} \), and labor in research, \( RL_{ij} \). The formulation of \( I_{R&D_{ij}} \) stems from the fact that we apply data for R&D investments from the Swiss input-output table for \( I_{R&D_{ij}} \). It should not be confused with R&D as an activity in the production process.

\[
I_{NP_{ij},t} = \left[ \gamma_{N,i} \frac{I_{RL_{ij},t}}{\sigma_{N,i}} + (1 - \gamma_{N,i}) \frac{I_{R&D_{ij},t}}{\sigma_{N,i}} \right] \frac{\sigma_{N,i}}{\sigma_{N,i} - 1}
\]

Together with investments in physical capital, \( I_{P_{ij}} \), non-physical capital is then used in the production of a capital composite \( K_{ij} \).

\[
K_{ij,t+1} = \left[ \frac{\sigma_{I,i} - 1}{\sigma_{I,i}} \right] + (1 - \gamma_{i}) \left[ \frac{\sigma_{I,i} - 1}{\sigma_{I,i}} \right] \frac{\sigma_{I,i}}{\sigma_{I,i} - 1} + (1 - \delta) K_{ij,t}
\]

\( \delta \) depicts the depreciation rate of capital. Consequently, the increase of capital per sector is equal to

\[
K_{i,t+1} - K_{i,t} = \Delta K_{i,t+1} = \left[ \gamma_{i} \frac{\sigma_{I,i}^{-1}}{\sigma_{I,i}} + (1 - \gamma_{i}) \frac{\sigma_{I,i}^{-1}}{\sigma_{I,i}} \right] \frac{\sigma_{I,i}}{\sigma_{I,i} - 1} - \delta K_{i,t}
\]

The growth index \( g_{K} \) of the capital composite of the sector amounts to

\[
g_{K_{i}} = \frac{K_{i,t+1}}{K_{i,t}} = \left[ \gamma_{i} \frac{\sigma_{I,i}^{-1}}{\sigma_{I,i}} + (1 - \gamma_{i}) \frac{\sigma_{I,i}^{-1}}{\sigma_{I,i}} \right] \frac{\sigma_{I,i}}{\sigma_{I,i} - 1} + (1 - \delta)
\]

This relation implies several aggregations. First, investments in physical capital and
in R&D of the \( j \) producers of intermediate goods in each sector \( i \) can be added up to investments of the sector in physical and non-physical capital and in R&D according to

\[
I_{P_i,t} = \sum_{j=1}^{K_i,t} I_{P_{ij},t}
\]

\[
I_{NP_{i},t} = \sum_{j=1}^{K_i,t} I_{NP_{ij},t}
\]

\[
I_{R&D_{i},t} = \sum_{j=1}^{K_i,t} I_{R&D_{ij},t}
\]

The same must hold for the capital composite \( K_i \), assuming \( K_{ij,t} = 1 \):

\[
K_{i,t} = \sum_{j=1}^{K_i,t} K_{ij,t}
\]

Given that each new blueprint is produced with labor and investments in physical capital and in R&D, each intermediate firm minimizes research cost according to

\[
\max_{RL_{ij}, I_{P_{ij},t}, I_{R&D_{ij},t}} p_{K_{ij},t+1} \Delta K_{ij,t+1} - w_{R,t} RL_{ij,t} - p_{A,t}(I_{P_{ij},t} + I_{R&D_{ij},t})
\]

Optimization w.r.t. \( RL_{ij}, I_{P_{ij}} \) and \( I_{R&D_{ij}} \) leads to the inverse demand functions for labor and investments in the R&D process according to
2.7. INTERMEDIATE GOODS

\[ w_{R,t} = p_{K_{ij,t+1}}(1 - \gamma_i)\gamma_{N,i} \left( \frac{\Delta K_{ij,t+1}}{I_{NP_{ij,t}}} \right)^{\frac{1}{\sigma_{I,i}}} \left( \frac{I_{NP_{ij,t}}}{RL_{ij,t}} \right)^{\frac{1}{\sigma_{N,i}}} \]

\[ p_{A_{i,t}} = p_{K_{ij,t+1}}(1 - \gamma_i)(1 - \gamma_{N,i}) \left( \frac{\Delta K_{ij,t+1}}{I_{NP_{ij,t}}} \right)^{\frac{1}{\sigma_{I,i}}} \left( \frac{I_{NP_{ij,t}}}{IR_{kD_{ij,t}}} \right)^{\frac{1}{\sigma_{N,i}}} \]

\[ p_{A_{i,t}} = p_{K_{ij,t+1}}\gamma_i \left( \frac{\Delta K_{ij,t+1}}{I_{P_{ij,t}}} \right)^{\frac{1}{\sigma_{I,i}}} \]

where \( w_R \) denotes the wage rate of labor in research.

Intermediate firms need to finance their research activities in advance. More specifically, labor and investments that are necessary to invent a new intermediate good for period \( t + 1 \) need to be paid in period \( t \). To do so, intermediate firms borrow \( p_{K,t}I_{ij,t} \) from the representative household:

\[ p_{K,t}I_{ij,t} = w_{R,t}RL_{ij,t} + p_{A_{i,t}}(I_{P_{ij,t}} + IR_{kD_{ij,t}}) \]

After the discovery of a new blueprint, they re-pay the household the remaining profits from the production of the intermediate (see subsection 5.2).

2.7.2 Production of the intermediate good

After obtaining a perpetual patent on each new type of intermediate varieties, i.e., on each newly produced unit of the capital composite, intermediate firms produce the newly invented goods. Each firm faces a CES production function for good \( x_{ij} \) with inputs labor \( L_{ij} \) (which is different from labor in research), policy invariant capital \( VK_{ij} \), and energy \( A_{ex_{ij}} \):
The optimization of the intermediate good producer consists of two steps. First, cost minimization leads to the optimal correlation between inputs and output. The monopolist uses an imaginary output price $p^i_{x_{ij}}$, which is the price that would be charged under perfect competition:

$$\max_{L_{ij,t}, V_{K_{ij,t}}, A_{ex_{ij,t}}} p^i_{x_{ij,t}} x_{ij,t} - w_t L_{ij,t} - p_{V_{K_{ij,t}}} V_{K_{ij,t}} - p_{A_{ex_{ij,t}}} A_{ex_{ij,t}}$$

subject to (2.11). The resulting inverse demand functions are ($w$ denotes the wage rate)

$$w_t = \alpha_{L,i} p^i_{x_{ij,t}} \left( \frac{x_{ij,t}}{L_{ij,t}} \right)^{\frac{1}{\sigma_X}}$$

(2.12)

$$p_{V_{K_{ij,t}}} = \alpha_{V_{K,i}} p^i_{x_{ij,t}} \left( \frac{x_{ij,t}}{V_{K_{ij,t}}} \right)^{\frac{1}{\sigma_X}}$$

(2.13)

$$p_{A_{ex_{ij,t}}} = (1 - \alpha_{L,i} - \alpha_{V_{K,i}}) p^i_{x_{ij,t}} \left( \frac{x_{ij,t}}{A_{ex_{ij,t}}} \right)^{\frac{1}{\sigma_X}}$$

(2.14)

Since the production of a specific intermediate good requires the rights according to a patent, each intermediate good is produced by only one firm, which can charge monopoly prices. Profit maximization consequently takes account of the demand function of the $Q_i$ producer according to
\[ \max_{p_{x_{ij,t}}} p_{x_{ij,t}} x_{ij,t} - p_{x_{ij,t}}^i x_{ij,t} \]

subject to (2.7). This yields the relation between the market price of each \( x_{ij} \) and the imaginary price \( p_{x_{ij}}^i \)

\[ p_{x_{ij,t}} = \frac{1}{\kappa} p_{x_{ij,t}}^i \]

Inserting this in equations (2.12), (2.13), and (2.14) yields

\[ \frac{1}{\kappa} w_t = \alpha L,i p_{x_{ij,t}} \left( \frac{x_{ij,t}}{L_{ij,t}} \right) \]

\[ \frac{1}{\kappa} p_{VK,t} = \alpha V K,i p_{x_{ij,t}} \left( \frac{x_{ij,t}}{V K_{ij,t}} \right) \]

\[ \frac{1}{\kappa} p_{Ae,t} = (1 - \alpha L,i - \alpha V K,i) p_{x_{ij,t}} \left( \frac{x_{ij,t}}{A_{ex_{ij,t}}} \right) \]

where \( \kappa \) denotes the markup over marginal costs. Profits from the production of the intermediate goods amount for each intermediate firm

\[ \pi_{ij,t} = (1 - \kappa) p_{x_{ij,t}} x_{ij,t} \]

In equilibrium, the sum of the discounted profits need to be equal to the funds borrowed from the household (\( r \) denotes the interest rate).
Investments in capital need to satisfy the no-arbitrage condition

\[ p_{K,t} K_t = \sum_{\tau=t}^{\infty} (1 + r_\tau)^{t-\tau} \Pi_\tau \]

These two equations are not explicitly modeled in the CITE model. However, the calibration of the model complies with these assumptions.

### 2.8 Representative household

A representative, infinitely-lived household allocates income between consumption and investment in accordance with intertemporal utility maximization with perfect foresight. It faces an intertemporal utility function \( U \) with a consumption good \( C \) as only variable yielding utility and \( \rho \) denoting the discount rate of the household.

\[ U = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \frac{C_t^{1-\theta} - 1}{1 - \theta} \]

This additively separable utility function has the same intertemporal characteristics as the linearly homogeneous utility function included in the CITE model (cf. Rutherford, 2004):

\[ \hat{U} = \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t C_t^{1-\theta} \right]^{\frac{1}{1-\theta}} \]

Therefore, the intertemporal maximization of utility in the CITE model is in accordance
with the results shown in the following theoretical calculations.

The consumption good is composed of a final goods composite $C_S$ and energy $A_{ec}$:

$$C_t = \left(1 - \beta\right)C_{S,t}^{\frac{\sigma_C - 1}{\sigma_C}} + \beta A_{ec,t}^{\frac{\sigma_C - 1}{\sigma_C}}$$  \hspace{1cm} (2.15)

The final goods of all sectors go into the final goods composite according to

$$C_{S,t} = \prod_{n \in N} A_{NE,n}^{\beta_{NE,n}}$$

with $A_{nC}$ denoting the part of regular final good $n$ that goes into consumption and $\beta_{NE,n}$ being the share of final good $n$ in the final goods composite. We further assume constant returns to scale: $\sum_{n \in N} \beta_{NE,n} = 1$. The household faces the budget constrain

$$p_{V,t+1}V_{t+1} = (1 + r_{t+1})p_{V,t}V_t + w_tL_t + w_{R,t}RL_t + p_{VK,t}VK_t - p_{A_{ec,t}} - \sum_{n \in N} p_{A_{n,t}}A_{nc,t} - \sum_{i \in I} p_{V,t}I_{i,t}$$

Hereby, $V$ denotes the assets of the household. It holds that total investments of the household must equal investments in physical and in non-physical capital, $I_{i,t} = I_{P,t} + I_{NP,t}$. As energy is assumed to be owned by the consumer, income from energy also goes to the consumer as factor income. The budget constraint is explicitly modeled in CITE to ensure that income from endowments and capital are equal to consumption plus investments.

To indicate the dynamic behavior of consumption, i.e., the Euler equation, depending on the price of the consumption good, the cost functions for the consumption good and the final goods composite are applied for the calculation:
\[ p_{C,t} = [(1 - \beta)^{\sigma_C} p_{C,t}^{1-\sigma_C} + \beta^{\sigma_C} p_{A,t}^{1-\sigma_C}]^{\frac{1}{1-\sigma_C}} \]

\[ p_{C_S,t} = \prod_{n \in N} \left( \frac{p_{A_n,t}}{p_{A_n,t}^{\beta_{NE,n}}} \right)^{\beta_{NE,n}} \]

Accordingly, the budget constraint can be formulated differently to include the price of consumption according to:

\[ p_{V,t+1} V_{t+1} = (1 + r_{t+1}) p_{V,t} V_t + w_t L_t + w_{R,t} R L_t + p_{V K,t} V K_t \]
\[ - p_{C,t} C_t - \sum_{i \in I} p_{V,t} I_{i,t} \]

(2.16)

The augmented Lagrangian for the household is

\[ \mathcal{L} = \frac{C_t^{1-\theta} - 1}{1 - \theta} + \frac{1}{1 + \rho} \lambda_{t+1}(1 + r_{t+1}) p_{V,t} V_t + w_t L_t + w_{R,t} R L_t \]
\[ + p_{V K,t} V K_t - p_{C,t} C_t - \sum_{i \in I} p_{V,t} I_{i,t}) - \lambda_t p_{V,t} V_t \]

and the optimization yields

\[ C_t^{-\theta} = \frac{1}{1 + \rho} \lambda_{t+1} p_{C,t} \]

\[ \lambda_t = \frac{1}{1 + \rho} \lambda_{t+1}(1 + r_{t+1}) \]

with \( \lambda > 0 \) being the shadow price of consumption.
From the FOCs of household optimization, we can calculate the growth index of consumption $g_C$

$$g_C = \left[ \frac{1 + r_{t+2}}{1 + \rho} \frac{p_{C,t}}{p_{C,t+1}} \right]^{\frac{1}{\theta}}$$ (2.17)

### 2.9 Trade

The basic assumptions for trade are, first, that final goods of every sector are both imported and exported, and, second, that foreign goods differ in some way from domestically produced goods (cf. Armington, 1969). This implies that foreign and domestically produced goods are not simply exchangeable but have an elasticity of substitution smaller than unity. Imported foreign goods, $A_{imp,i}$, and domestically produced goods that remain inland, $Y_{dom,i,t}$, are assembled in a CES function and can then be demanded by sectors and the household. Thus, every final good demanded in the economy needs to be an Armington aggregate and is produced according to

$$A_{i,t} = \left( \alpha_{M,i} A_{imp,i,t}^{\frac{\sigma - 1}{\sigma M}} + (1 - \alpha_{M,i}) Y_{dom,i,t}^{\frac{\sigma - 1}{\sigma M}} \right)^{\frac{\sigma M}{\sigma - 1}}$$

Because outputs of all sectors remain inland and are also used abroad, every $Y_i$ is transformed by a constant elasticity of transformation (CET) function to exports ($Y_{ex,i}$) and domestic deployment ($Y_{dom,i}$, which is used in the composition of the Armington aggregate above).

$$Y_{i,t} = \left( \alpha_{EX,i} Y_{ex,i,t}^{\frac{\sigma y + 1}{\sigma r}} + (1 - \alpha_{EX,i}) Y_{dom,i,t}^{\frac{\sigma y + 1}{\sigma r}} \right)^{\frac{\sigma y}{\sigma r - 1}}$$

As all markets clear, the value of domestically produced goods that remain inland must equal the value of the share of Armington goods that is not imported:
Likewise, the foreign trade balance needs to be satisfied. The value of the exported goods must equal the value of the goods imported by our domestic economy (all in world market prices \( p_{FX} \) that reflect the terms of trade).

\[
\sum_{i \in I} p_{FX,i,t} Y_{ex,i,t} = \sum_{i \in I} p_{FX,i,t} A_{imp,i,t} + p_{FX,GAS,t} GAS_t + p_{FX,CRU,t} CRU_t
\]

### 2.10 Equilibrium

The competitive equilibrium is characterized by a system of prices at which all markets clear. The final output of each sector is divided among four uses. It can either be used in a sector as input, invested in R&D, invested in the accumulation of physical capital, or it can be consumed. Thus, it must hold that

\[
p_{A_{i,t}} A_{i,t} = \sum_{i' \in I} p_{A_{i,t}} A_{i',t} + p_{A_{i,t}} I_{R&D_{i,t}} + p_{A_{i,t}} I_{P_{i,t}} + p_{A_{i,t}} A_{ic,t}
\]

The markets for endowments clear, and thus it must hold that

\[
L_t = \sum_i \sum_j L_{ij,t} = \text{constant}
\]

\[
RL_t = \sum_i \sum_j RL_{ij,t}
\]
2.10. EQUILIBRIUM

\[ VK_t = \sum_i \sum_j VK_{ij,t} = \text{constant} \]

The endowments of labor and policy invariant capital stay constant over time, and the endowment of labor in research grows. This stems from the fact that we assume labor in research to be an effective input, meaning that it constitutes the labor force size augmented by human capital, which is assumed to grow over time (cf. also Davis, 2008).

As all intermediate goods in each sector are symmetric, i.e., \( x_{ij} = x_i \), the production function of \( Q_i \), (2.6), can be rewritten as

\[ Q_{i,t} = K_{i,t}^{\frac{1}{\kappa}} x_{i,t} = K_{i,t}^{\frac{1}{\kappa}} X_{i,t} \quad (2.18) \]

with

\[ X_i = K_i x_i \quad (2.19) \]

Because the prices for all sectoral intermediate goods must also be equal, it must hold that the inputs to the production of the intermediate composite have the same value as the output.

\[ p_{Q_i,t} Q_{i,t} = p_{x_i,t} X_{i,t} \]

The capital stock \( K = \sum_{i \in I} K_i \) is equal to the assets of the household \( V \). In a symmetric equilibrium it will therefore emerge that \( p_V = p_K \). Accordingly, the budget constraint of the household (2.16) changes to
THEORETICAL FOUNDATION OF THE CITE MODEL

\[ p_{C,t}C_t = w_tL_t + w_{RL}RL_t + p_{VK,t}VK_t + \Pi_t - \sum_{i \in l} p_{V,t}I_{i,t} \]

Also, the Euler equation (from (2.17)) can be reformulated as

\[ \frac{C_{t+1}}{C_t} = \left[ \frac{1 + r_{t+2} + p_{C,t+1}}{1 + \rho \frac{p_{C,t+1}}{p_{K,t}}} \right]^{\frac{1}{\rho}} \]

Additionally, the markets of both types of labor, L and RL, and policy invariant capital, VK, clear when their prices are the same across sectors.

To ensure that the optimal path is always followed, the transversality condition must also hold. It requires that the present value of capital must converge to zero as the planning horizon approaches infinity. It must therefore hold that

\[ \lim_{\tau \to \infty} \left( \frac{1}{1 + \rho} \right)^{\tau} p_{K,\tau}K_\tau = 0 \]

Given this transversality condition, the optimal path must always be maintained, and no alternative path, for which capital deviates from the optimum at each time and increases discounted utility, is possible.

2.11 Growth dynamics

Economic growth is determined by the endogenously determined growth rate of the capital stock, which reflects investment decisions of the representative household. If \( g_H = \frac{H_{t+1}}{H_t} \) denotes the growth index of variable \( H \), the growth index of consumption can be derived by dividing consumption in period \( t + 1 \) through consumption in period \( t \) (cf. 2.15):
\[ g_c = \left( \frac{(1 - \beta) C_{S,t+1}^{\sigma_C^{-1}} + \beta A_{ec,t+1}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}} + \beta A_{ec,t}^{\sigma_C^{-1}}} \right)^{\sigma_C^{-1}} \]

Using \( \bar{g}_H = \left( \frac{H_{t+1}}{H_t} \right)^{\sigma_C^{-1}} \) for an adjusted growth index the above equation can be simplified to

\[ \bar{g}_C = \frac{(1 - \beta) C_{S,t+1}^{\sigma_C^{-1}} + \beta A_{ec,t+1}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}} + \beta A_{ec,t}^{\sigma_C^{-1}}} \]  

(2.20)

To determine which variables influence the growth index of consumption, (2.20) can be decomposed in the two terms before and after the "plus" symbol on the right hand side according to

\[ \bar{g}_C = \frac{(1 - \beta) C_{S,t+1}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}}} + \frac{\beta A_{ec,t+1}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}}} \]  \( (2.21) \)

After rearranging and inserting the growth index for \( C_S \), the first term on the right hand side of (2.21) can be rewritten as

\[ \frac{(1 - \beta) C_{S,t+1}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}}} + \frac{\beta A_{ec,t+1}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}}} \]

(2.21)

Analogously, the second term on the right hand side of equation (2.21) can be arranged as

\[ \frac{(1 - \beta) C_{S,t+1}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}}} = \frac{1 - \beta A_{ec,t}^{\sigma_C^{-1}}}{(1 - \beta) C_{S,t}^{\sigma_C^{-1}}} \]

(2.21)
\[
\frac{(1 - \beta)A_{ec,t}^{\sigma_C^{-1}}}{(1 - \beta)C_S^{\sigma_C^{-1}} + \beta A_{ec,t}^{\sigma_C^{-1}}} = \frac{\beta}{(1 - \beta)\left(\frac{C_{S,t}}{A_{ec,t}}\right)^{\sigma_C^{-1}} + \beta} \frac{\sigma_C^{-1}}{g_{A_{ec}}}
\]

Applying these terms in (2.21), the adjusted growth rate of consumption can be reformulated as

\[
\overline{g_C} = \psi_{C_S} \overline{g_C} + \psi_{A_{ec}} \overline{g_{A_{ec}}}
\]

with

\[
\psi_{C_S} = \frac{1 - \beta}{(1 - \beta) + \beta \left(\frac{A_{ec,t}}{C_{S,t}}\right)^{\sigma_C^{-1}}} \quad (2.22)
\]

\[
\psi_{A_{ec}} = \frac{\beta}{(1 - \beta)\left(\frac{C_{S,t}}{A_{ec,t}}\right)^{\sigma_C^{-1}} + \beta} \quad (2.23)
\]

It can be shown that \(\psi_{A_{ec}} = 1 - \psi_{C_S}\). Therefore, the adjusted growth index of consumption is equal to

\[
\overline{g_C} = \psi_{C_S} \overline{g_C} + (1 - \psi_{C_S}) \overline{g_{A_{ec}}} \quad (2.24)
\]

This equation implies that the demand for both inputs to this CES function with an elasticity of substitution smaller than unity \((\sigma_C < 1)\) must grow at the same rate on a balanced growth path. The proof is done by contradiction: Assuming \(g_{A_{ec}} < g_{C_S}\), it follows that
\[
\lim_{t \to \infty} \left( \frac{C_{S,t}}{A_{ec,t}} \right)^{\frac{\sigma_C - 1}{\sigma_C}} = 0
\]

and

\[
\lim_{t \to \infty} \left( \frac{A_{ec,t}}{C_{S,t}} \right)^{\frac{\sigma_C - 1}{\sigma_C}} = \infty
\]

Therefore, the shares of \(C_S\) and of \(A_{ec}\) must evolve according to (cf. (2.22) and (2.23)) like

\[
\lim_{t \to \infty} \psi_{C_S} = 0
\]

\[
\lim_{t \to \infty} \psi_{A_{ec}} = 1
\]

Inserting in (2.24) yields

\[
\lim_{t \to \infty} g_C = g_{A_{ec}}
\]

From this it follows that the use of \(C_S\) must grow at the same rate as \(C\) and \(A_{ec}\) on a balanced growth path to avoid excess demand.

Applying similar calculations as above, it can be shown that

\[
g_{C_S} = \prod_{n \in N} (g_{Y_{nc}})^{\beta_{N,E,n}}
\]
The growth index of $Q_i$ can be calculated straightforwardly. Knowing its production function from (2.18) and the growth index of capital from (2.10), and applying a similar calculation as above, $g_{Q_i}$ is equal to

$$g_{Q_i} = \left[ \frac{\frac{\sigma_{I,i}^{-1}}{\gamma_i I_{P,i}^{\sigma_{I,i}}} + (1 - \gamma_i I_{N_{P,i}}^{\sigma_{I,i}})}{K_{i,t}} + (1 - \delta) \right] \frac{1}{\kappa} + (1 - \gamma_i) I_{N_{P,i}}^{\sigma_{I,i}} - \sigma_{I,i}^{-1}$$

As the growth of the intermediate goods equals zero and therefore the growth index $g_{x_i}$ must be one, (2.25) is equal to

$$g_{Q_i} = g_{K_i}^{\frac{1}{\kappa}}$$

The above equation is the central relationship in the model as it ensures endogenous growth through gains of specialization in the production of intermediate goods. The economy is able to grow even without growth of the inputs to intermediate goods, i.e., labor and energy.

### 2.12 Specialties of Calibration

The CITE model differs from other CGE models by the calibration of the benchmark scenario. The main difference is that we use the microfoundation of the model and calibrate it in the benchmark to a balanced growth path including gains of specialization. These gains and the resulting difference between growth of inputs and of outputs represent a challenge to the realization in GAMS.
2.12. SPECIALties OF CALIBRATION

2.12.1 Different growth rates

As can be seen from equation (2.26), there are three different steady state growth rates in the economy: the growth rate of labor and policy invariant capital, which is zero (the growth index equals one), the growth rate of capital, and the growth of outputs, both being larger than zero (the growth indices are larger than one). To implement the model in GAMS, we used an approach that takes this specialty into account.

As we know from equation (2.19) that \( x_i \) grows at the rate of capital \( K_i \), we can relate the two growth indices as

\[
g_{x_i} = g_{K_i}
\]

Thus, we know that if we model \( X_i \) instead of \( x_i \), we only need to take two different growth rates into account. We can model the growth indices of labor, policy invariant capital, and intermediate goods like the growth index of capital.

For the calibration we proceed as follows. First, we introduce a growth rate for sectoral outputs \( Y_i \) and sectoral intermediate composites \( Q_i \) and call it \((1 + gr)\). Second, we introduce a growth rate for capital \( K_i \), \((1 + grk)\). The relation of both growth rates can be derived using the production function for \( Q_i \):

\[
Q_{i,t} = \left[ \frac{K_{i,t}}{\sum_{j=1}^{\kappa} x_{ij,t}^\kappa} \right]^{\frac{1}{\kappa}}
\]

As the sum of intermediate goods continues to grow, we assume the equilibrium version with identical production of intermediate goods (2.18):

\[
Q_{i,t} = K_{i,t}^{\frac{1-n}{n}} X_{i,t}
\]
To analyze the intertemporal dynamics we can rewrite the above equation like

\[
\frac{Q_{t+1}}{Q_t} = \left( \frac{K_{t+1}}{K_t} \right)^{\frac{1-\kappa}{\kappa}} \frac{X_{t+1}}{X_t}
\]

As we know that

\[
\frac{Q_{t+1}}{Q_t} = 1 + gr
\]  
(2.28)

\[
\frac{K_{t+1}}{K_t} = 1 + grk
\]

\[
\frac{X_{t+1}}{X_{t,t}} = 1 + grk
\]  
(2.29)

we can relate the two growth rates according to

\[
1 + gr = (1 + grk)^{\frac{1}{\kappa}}
\]  
(2.30)

with \((1 + gr)\) being larger than \((1 + grk)\) (as \(\kappa\) is smaller than 1). The difference in the two growth indices has its origin in gains from specialization. Other CGE models do not consider gains of specialization as driving force for endogenous growth for the whole economy, so they do not consider two different growth rates in the benchmark. In our model, however, we include them, which leads to growth dynamics that are structurally identical to the dynamics of policy scenarios. We regard this attribute as an outstanding advantage of our model.

How can we make sure that \(X_i\) behaves exactly like \(K_i\)? We define the development of
2.12. SPECIALIES OF CALIBRATION

$X_i$ (expressed by the appendix "L" for "level" after a variable) as equal to that of $K_i$:

$$X_i.L = K_i.L$$

The input-output table of the Swiss economy gives clear indications for the size of $X_i$. But since $Q_i$ represents a theoretical variable we cannot assign a value for it from the Swiss input-output-table. Fortunately, we do not need to find out the size of $Q_i$ because it can be replaced after some calculations:

For the purpose of differentiating between these theoretical calculations and GAMS notation, a specific syntax is introduced. Values from the Swiss input-output table are labeled with a bar. For instance, $\bar{X}_i$ denotes the value of intermediate goods that is added up from labor input, energy input, and input of policy invariant capital. We also know that $\bar{X}_i$ grows with the growth rate $(1 + grk)$ due to the fact that $X_i = K_i \bar{x}_i$. $\bar{X}_i$ does not change, neither in the benchmark case nor in the policy scenarios.

Each value is then combined with a variable of the same name, in our case $\tilde{X}_i$, which denotes an activity index. The variables are calibrated to unity in the benchmark case, but they can change in the policy scenarios. The combination of the activity index with the value of a variable, $\tilde{X}_i \bar{X}_i$, yields the possibility of calculating with original data and the flexibility to change the resulting value in policy scenarios.

Rewriting (2.27) in GAMS notation we get

$$\tilde{Q}_{i,t} \bar{Q}_{i,t} = \tilde{K}_{i,t}^{\frac{1}{\kappa}} \tilde{X}_{i,t} \bar{X}_{i,t}$$

Relation (2.27) must also hold in the benchmark case, so we know that

$$\bar{Q}_{i,t} = \bar{X}_{i,t} ((1 + grk)^t)^{\frac{1}{\kappa}}$$

(2.32)

Inserting (2.32) in (2.31) and rearranging yields the development of $Q_i$. 
\[ \tilde{Q}_{i,t} = \tilde{X}_{i,t} \left( \frac{\tilde{K}_{i,t}}{(1 + grk)^t} \right)^{\frac{1-a}{a}} \] (2.33)

All variables in GAMS are calibrated to unity, so we face an obstacle in this equation. \( \tilde{K}_i \) does not grow and therefore cannot yield any growth dynamics. We thus introduce an auxiliary variable \( \tilde{N}_i \), which grows at the rate of capital, and apply it to equation (2.33):

\[ \tilde{Q}_{i,t} = \tilde{X}_{i,t} \left( \frac{\tilde{N}_{i,t}}{(1 + grk)^t} \right)^{\frac{1-a}{a}} \]

To be sure that \( N_i \) behaves exactly like \( K_i \) we proceed threefold. First, we define the development of \( N_i \) equal to \( K_i \) augmented by the growth rate of capital:

\[ \tilde{N}_i.L = \tilde{K}_i.L(1 + grk)^t \]

Second, we make sure that the price for \( N_i \) equals the rental price of capital \( rk_i \):

\[ p_{N_i,t} = rk_{i,t} \]

Third, we introduce an equation that equals the stock of capital to the size of \( N0_i \):

\[ RK_{i,t} \tilde{K}_{i,t} \tilde{K}_{i,t} = p_{N_i,t} \tilde{N}_{i,t} \tilde{N}_{i,t} \]
2.12.2 Depreciation rate

Since sectoral investments need to grow at the same rate as consumption and thus like sectoral output (with $1 + gr$) and the fact that capital grows at a lower rate $(1 + grk)$, there is a disequilibrium between the two growth rates. Consequently, the calibration of capital accumulation (cf. (2.9)) needs an auxiliary assumption for a balanced growth path (BGP). To calibrate capital growth, we assume an increasing depreciation rate over time that compensates for higher growth of investments than growth of capital. This assumption appears realistic because capital in a higher developed economy is highly specialized and therefore subject to a higher depreciation than generally is applicable to capital in a lesser developed economy.

Starting with the equation for capital accumulation

$$K_{i,t+1} = I_{i,t} + (1 - \delta(t))K_{i,t}$$

we assume a discount rate that acts as adjustment factor. We can calculate the development of the depreciation rate applying GAMS relations.

$$\delta_{i,t} = \left( \frac{1 + gr}{1 + grk} \right)^t \delta_{i,0} + grk \left( \left( \frac{1 + gr}{1 + grk} \right)^t - 1 \right)$$

We see that the depreciation rate depends on its initial value, both growth rates and the value of the growth index of capital, $grk$. This growth rate ensures a constant growth rate of capital, $gK_i$.

2.12.3 Changes in energy efficiency and development of labor

Energy is used in the production of intermediate goods. Since energy is a sectoral output it grows at a higher rate than intermediate goods. To ensure a balanced growth path, the difference between both growth rates needs to be adjusted by an efficiency index, $enef_i$, that is introduced in the production of intermediate goods:
\[ X_{i,t} = \left[ \alpha_{L,i} \frac{X_{i,t}}{X_{i,t}} + \alpha_{V,K,i} V K_{i,t} \frac{X_{i,t}}{X_{i,t}} + (1 - \alpha_{L,i} - \alpha_{V,K,i}) \text{enef}_{i,t} A_{eX_{i,t}} \right] \]

This efficiency index behaves according to

\[ \text{enef}_{i,t} = \left( \frac{1 + gr}{1 + grk} \right)^t \]

Thus, the amount of energy demanded by the \( X_i \)-producer can also be interpreted as energy services that show a lower growth rate than energy itself.

Labor is used both in the production of intermediate goods and also in the production of capital. Since labor in the production of \( X_i \) goods does not grow and labor in research needs to grow at the rate of investments, \((1 + gr)\), we introduced two types of labor to allow for this difference. This results in a timely change in the ratio of both, i.e., labor in research increases in relation to labor in production. This can be due to an exogenous increase in human capital, which is not modeled in more detail. Reasons for this, for example, can be migration, better schooling, a higher level of university education, or a sophisticated system of continuing education.

### 2.12.4 Capital demand, kappa, investments

The growth rate of outputs, \((1 + gr)\), depends on the size of \( \kappa \) (cf. (2.30)). On a balanced growth path, all sectors in the economy must grow at the same rate. Therefore, \( \kappa \) must be identical across all sectors. At the same time, \((1 - \kappa)\) describes the capital intensity of the sector, i.e., the share capital in the production of intermediate goods. Since the capital intensity is not the same across sectors in the input-output matrix, the original capital stock from each sector from the input-output matrix, \( K_{IO,i} \), need to be divided into two capital stocks. One serves as endogenous driver for growth and equals the share \((1 - \kappa)\), which is \( K_i \), and the other one includes the remaining capital, \( V K_i \).
2.12. SPECIALties of Calibration

\[ K_{i,t} = K_{i,t-1} + VK_{i,t}; \]

\[ VK_i \text{ cannot be accumulated by the household; instead, it grows exogenously at the rate of the other inputs, } (1+grk), \text{ and serves as input in the production of intermediate goods (cf. (2.11)). Only } K_i \text{ is exposed to investment decisions by the household and is thus the driver of endogenous growth. We chose } VK_i \text{ to be a sectoral independent factor (similar to labor) because a sector-specific and in its quantity invariant factor would constitute a fixed cost to each sector. It must hold that} \]

\[ \sum_{i \in I} VK_{i,t} = VK_t \]

Demand for capital by intermediate good producers, i.e., the rental price they pay each period for capital, is in constant relation to the interest rate \( r \), the depreciation rate \( \delta_i \), the sectoral capital stock \( K_i \), and sectoral investments in steady state \( I_i \) (cf. Paltsev, 2004). We calculate it for the first period as

\[ K_{i,0} = \frac{I_{i,0}}{\delta_{i,0} + grk} \]

and

\[ K_{i,0} = \frac{r k_{i,0} K_{i,0}}{r + \delta_{i,0}} \]

From the above equations we know that the following relation must hold:

\[ I_{i,0} = \frac{(\delta_{i,0} + grk)r k_{i,0} K_{i,0}}{r + \delta_{i,0}} \]
Policy invariant capital $VK$ is supplied by the household independently of these relations.

### 2.12.5 Prices

Due to different growth rates of capital and outputs and the fact that all benchmark values of quantities are assumed to be unity, the initialization of prices needs to be adjusted. All prices for goods that grow with the higher growth rate $(1 + gr)$ remain unity in the benchmark case. The prices of the goods that grow as fast as capital $(1 + grk)$ must adjust so the value of inputs always equals the value of outputs in each production function. The exact dynamics can be derived from the production function of $Q_i$ because the value of outputs must equal the value of inputs:

$$p_{Q_i,t}Q_{i,t} = p_{x_i,t}X_{i,t}$$

As the dynamics can be analyzed according to

$$\frac{p_{Q,t+1}Q_{t+1}}{p_{Q,t}Q_t} = \frac{p_{x_i,t+1}X_{i,t+1}}{p_{x_i,t}X_{i,t}}$$

and we know that (2.28), (2.29) and

$$\frac{p_{Q,t+1}}{p_{Q,t}} = 1$$

it must hold that

$$p_{x_i,t} = \left(\frac{1 + gr}{1 + grk}\right)^t$$
2.13. CONCLUSION

Accordingly, the prices of inputs to $X_i$ (labor, policy invariant capital, the rental price of capital, and therefore also the price for $N_i$) also must grow at this rate.

2.12.6 Terminal condition

The terminal condition regulates the state of the model in the last period $T$. Since the model is assumed to be on a balanced growth path in the last period, the growth rate of investments in each sector must equal the growth rate of production of that sector:

$$\frac{I_{nv_i,T}}{I_{nv_i,T-1}} = \frac{Y_{i,T}}{Y_{i,T-1}}$$

This equation ensures that the dynamic behavior of the model is not uncontrolled but complies with certain assumptions. The most important is that investments do not suddenly drop to zero in the last time period, but stay on a level that would ensure continued growth of the economy. If this condition was missing, an optimizing household would decrease investments toward the end of the time horizon to zero and would use up all capital. This would obviously strongly influence all other periods in an unrealistic way. Hence, the terminal condition ensures that the economy exists also after the last period of simulation.

2.13 Conclusion

The CITE model can be used to run policy scenarios and to estimate influences of political measures on different sectors of the Swiss economy. The microfoundations of the model ensure the propinquity to up-to-date endogenous growth theory. All growth dynamics that are applied in policy scenarios also influence the benchmark case. Gains of specialization ensure that the model economy grows without growth of endowments (aside from $RL$), and it can therefore be said that our model is a major advance in CGE modeling. It features endogenous growth dynamics that exclude the dependence on growth on input factors.
The specialty of our model is clearly in the implementation of microeconomically based endogenous growth theory on the basis of Grossman and Helpman (1991). Aside from this strength, there are several possibilities for enhancing the model's performance. First and foremost, the existing energy sector is a greatly simplified version and can be modeled in more detail. In this regard, it might also be interesting to combine the CITE model with a bottom-up model that gives more information about different sources and uses of energy. This would also strongly improve the acceptability of the model in political discussions because practitioners might criticize on the simplification of the energy sector.

The model also shows interesting opportunities for enlargement in the area of knowledge spillovers from abroad. The version of the model at hand assumes that the stock of knowledge is exclusively increased by research and development effected in Switzerland. However, it needs to be taken account the fact that Switzerland is a small economy and that a large portion of knowledge comes from abroad. These additions to knowledge could also be modeled in a more detailed way.

Third, the rest of the world might be the center of interest for further enhancement of the model. Policies that are undertaken abroad, such as climate policies in the EU, might strongly affect the terms of trade for Switzerland and therefore its international competitiveness. If foreign policies could be included more explicitly in the model, scenarios with different combinations of Swiss and foreign energy policies would greatly contribute to the determination of appropriateness of Swiss energy policies.

Fourth, the basic setup of the model can also be used to build CGE models for other countries. The general growth dynamics could be maintained, but energy and energy-related inputs would need to be redesigned. This universality of the growth mechanisms could even lead to a multi-country-model that shows Switzerland and its most important trade partners.
2.14 Appendix - Utility functions

In Section 2.8 we stated that the additively separable utility function

\[
U = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t C_t^{1-\theta} - 1 \frac{1}{1 - \theta}
\]

and the linearly homogeneous utility function

\[
\hat{U} = \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t C_t^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

have the same intertemporal characteristics. To prove this, we refer to Rutherford (2004) and recall that a monotonic transformation of utility has no influence on the underlying preference ordering. If we assume that \(\hat{U}\) is a function \(V(U)\) and that the first derivative is larger than zero \((V' > 0)\), the optimization of \(U\) and \(\hat{U}\) yield identical demand functions.

To show this, we assume that

\[
\hat{U} = V(U) = [aU + \omega]^{\frac{1}{\theta}}
\]

where

\[
\omega = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t = \frac{1 + \rho}{\rho}
\]

and
\[ \mu = 1 - \theta \] 

(2.36)

If we insert \( \omega, \mu, \) and \( U \) in \( \hat{U} \) we get

\[
\hat{U} = \left[ (1 - \theta) \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \frac{C_t^{1-\theta} - 1}{1 - \theta} \right] + \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \right]^{\frac{1}{1-\theta}} \]

(2.37)

which exactly equals the notation of \( \hat{U} \) above.

Alternatively, the equivalence of dynamics can be shown by regarding the marginal rate of substitution. It is in both models identical to

\[
\frac{\partial U}{\partial C_{t+1}} = \frac{1}{1 + \rho} \left( \frac{C_t}{C_{t+1}} \right)^\theta \]

(2.38)

As this defines the preference orderings, both utility functions show the same intertemporal characteristics.
Chapter 3

A comparison of growth dynamics: the CITE model vs. a model with homogeneous capital

The simulation of economic reactions on energy policies depends strongly on the assumption of underlying technologies. This chapter compares the outcomes of the new CITE model with those of a model with exogenous growth and emphasizes the differences between both approaches to innovation. The results show that the CITE model unveils dynamics that cannot be obtained with the model based on exogenous growth. Reactions are stronger in the CITE model, and sectors show different responses than in the model with exogenous growth. Additionally, industries need more time to approach the new balanced growth path in the CITE model.
3.1 Introduction

Mankind’s impact on the natural environment and the resulting consequences for climate change are acknowledged by most scientific reports, such as the latest IPCC report. The increasing atmospheric greenhouse gas concentration will severely impact the global climate with tremendous effects on ecological and economic systems (Kemfert, 2002).

Consequently, a major task for the world’s economies in the next decades will be to decrease total emissions of greenhouse gases (GHG) and to change the carbon intensity of production. Since a significant part of carbon dioxide emissions results from the usage of energy, two major options can be envisaged for GHG reduction. The direct way would be to proportionately reduce energy use and GHG emissions by substituting labor and capital for energy (e.g., public transport instead of privately owned vehicles). However, the essential role of energy for production yields only limited possibilities for this option. Generally, energy and other inputs to production can yield only insufficient abatement levels and have hence only moderate effects (Löschel, 2002; Gerlagh et al., 2004; Gerlagh and Lise, 2005). The second possibility is to transform energy production by using fossil fuels with less carbon or to reduce energy intensity in production. This form of induced technical change typically needs more time and can therefore become significant rather in the medium- to long-term (Knapp, 1999). The short-term effects of an energy price change can therefore be distinct from the allocation of inputs in the long run (Bretschger, 2007).

Generally, changes in energy use are subject to supply and demand decisions in markets. Demand, in turn, is affected by changes in the energy price based on the price elasticity that induces factor substitution (Löschel, 2002). Public policies thus have a central role as they affect prices and also incentives that can modify energy use. As a result, new energy services or the amelioration of existing ones can emerge (Kverndokk et al., 2004). However, political and regulatory measures can result in a variety of societal and welfare outcomes (Jaffe et al., 2002; Otto et al., 2007). It is therefore crucial to analyze the characteristics of technologies to understand the reaction of technology on the economic incentives created by politicians.
Many relevant issues concerning the consequences of energy policies cannot be addressed entirely by theoretical models or on the basis of empirical research. The methodological foundation to approach these questions needs to include multiple viewpoints (Jaffe et al., 2002). A remedy to approximate the consequences of energy policies is the concept of computable general equilibrium (CGE) models that combine theoretical economic models with actual economic data. A multitude of CGE models aim at estimating the effects of energy policies on different industries in an economy and on the rate and direction of technical change. These Arrow-Debreu models involve the analysis of the interaction of consumers and producers in markets. The requirement of a general equilibrium yields a framework to analyze price-dependent interactions between the energy system and the other sectors (Loschel, 2002; Jaffe et al., 2003).

Scenarios produced by CGE models help balance advantages and disadvantages of policies and emphasize the importance of technical change.

Technical change can be modeled in diverse ways. Differences in model structures may have considerable effects on the outcome of policy scenarios (Buonanno et al., 2003). The aim of this chapter is to identify some approaches to model induced technical change and to emphasize the differences between the CITE model (based on endogenous growth dynamics) and a model with homogeneous capital and exogenous growth. The CITE model can be considered an expansion of the group of models with endogenous technical change by including gains from specialization as driver for growth. These gains are assumed to be present in the benchmark case without political measures as well as in the policy scenarios.

The CITE model is a top-down model that describes of the energy sector in a highly aggregated way. Like other top-down models, it characterizes industries by means of neoclassical production functions that include possibilities to substitute production inputs. Technical change in these models amends the cost of production at the industry level. As opposed to the top-down approach, bottom-up models use detailed specifications of energy systems and typically do not comprise a detailed modeling of the overall macroeconomy. They are usually employed to "compute the least-cost method of meeting a given demand for final energy or energy services subject to various system constraints, such as exogenous emission reduction targets" (Loschel, 2002).

Bottom-up models illustrate the diffusion of new technologies on the basis of costs and
performance characteristics. Technical change is present in these models if one technology is substituted for another (Löschel, 2002). The difference in the description of induced technological change appears to be the main reason for the different results of top-down and bottom-up models in assessing economic costs of energy policies (Carraro & Galeotti, 1997).

3.1.1 CGE models with purely exogenous growth

The first CGE models were based on the assumption of exogenous growth and the autonomous amelioration of energy efficiency. They ignored interconnections between technological change and policy measures. Changes in energy prices due to political actions only resulted in substitution of other factors for energy, leaving the rate of growth in energy efficiency unchanged. As energy policies have yet an impact on the price of fuels and therefore on the incentives to invest R&D, they are strongly linked to technological change. Such policies might cause research efforts to concentrate on the discovery of new production methods or of entirely new products that depend less on energy. Moreover, energy policies can influence the accumulation of knowledge via learning-by-doing (LbD) related to experience with alternative energy fuels or energy-conserving processes. Including these endogenous growth mechanisms increases the intertemporal connections and therefore more strongly connects the cost of emission reduction in the future and measures taken today (Dasgupta & Heal, 1974; Boyd & Uri, 1991; Goulder & Mathai, 2000).

In the first generation of CGE models, the autonomous increase of energy efficiency was defined by the "autonomous energy efficiency index" (AEEI), which is a heuristic measure of all non-price driven enhancements in energy technology, including structural change in the economy and sector-specific technological change. It is a separate coefficient in the production or cost functions and represents either factor-augmenting or price diminishing technical change. The basic assumption of the AEEI is that an energy efficiency increase results from a large number of minor innovations that evolve mainly from a common stock of knowledge that grows gradually (Löschel, 2002). The main difficulty with applying an AEEI is to identify the difference in the influence of technical progress and long-term price effects (Jones, 1994).
3.1. INTRODUCTION

Nordhaus (1992) first introduced a top-down model and estimated the consequences of energy policies on the economy. The DICE (Dynamic Integrated Climate-Energy) model comprised very simplified growth dynamics. In the RICE (Regional Integrated model of Climate and the Economy) model, Nordhaus and Yang (1996) expanded the DICE model and included a number of regions to reflect the world economy. Their main focus, however, was not on an amelioration of growth dynamics. Rather, they analyzed national strategies with three different levels of international cooperation and their effects on emission reduction. Another example of a model that used an AEEI parameter is the CETA (Carbon Emissions Trajectory Assessment) model of Peck and Teisberg (1992), which is based on Manne and Richels (1992) and focuses on the path of an optimal carbon tax. For a more detailed overview of models with AEEI parameters, see also Jorgenson and Wilcoxen (1993).

3.1.2 Empirical evidence for induced innovation

The empirical evidence for the effects of energy price changes on innovation is relatively univocal and builds on Hicks' induced innovation hypothesis. Hicks (1932) proposes that changes in relative factor prices should result in innovations that diminish the demand for the relatively expensive factor. Popp (2001) confirms this hypothesis empirically and finds that the effects of a price change can be attributed by about two-thirds to factor substitution, and around one-third result from induced innovation. Also Newell et al. (1999) find that increasing energy prices have an observable effect on the types of products offered in stores. Likewise, Popp (2002) finds evidence of a positive impact of energy prices and the stock of knowledge on innovation.

One of the leading views on the positive impact of energy prices on innovation is ascribed to Porter and van der Linde (1995), who argue that environmental regulation can result in innovations and economic gains. This effect is even more amplified if the innovations can be exported to other regions or countries. They argue that a country that adopts stricter environmental policies than others will have a boost in innovations and become a net exporter of the newly developed environmental technologies. On the firm's individual level, Porter and van der Linde (1995)'s claim means that environmental regulations have a "net beneficial effect on firm's competitiveness" (Sue Wing,
2003). However, econometric tests at the firm level have yielded mixed results (Sue Wing, 2003).

At the country level, however, Porter and van der Linde (1995)’s hypothesis was affirmed by Bretschger (2007), who finds that lower energy input induces investments in physical, human, and knowledge capital. This, in turn, fosters the growth rate of these countries. Bretschger shows that for OECD countries, the simple correlation between energy use and growth is negative. Jaffe and Palmer (1997) show that lagged environmental compliance expenditures have a significant positive effect on R&D expenditures. Carraro and Galeotti (1997) find similar results.

3.2 CGE models with endogenous growth mechanisms

New growth theory is based on the observation that technological innovation is an economic activity. Profit-maximizing agents optimize their behaviors according to profit incentives. Endogenous growth theory thus builds on innovation theory, which states that Schumpeterian profit incentives account for a major source of technological change (Weyant & Olavson, 1999; Löschel, 2002).

Technology growth is specified in endogenous growth theory as an endogenous process that focuses on increasing productivity. If climate policy is introduced, the role of innovations becomes more complicated as they are directed toward reducing energy use. Such a alteration changes the growth trajectory of overall productivity. These effects may not be negligible, and therefore a good perception of the relationship of the energy system and the economy-wide productivity is substantial (Azar & Dowlatabadi, 1999).

This section provides an overview of CGE models that introduced two specific forms of endogenous growth mechanisms, specifically learning-by-doing and research. These growth assumptions differ crucially from the endogenous growth dynamics of the CITE model, which are based on gains from specialization in production.
3.2.1 Learning-by-doing as the main driver of growth

Learning is a major driving force of technological change because it improves the relation of cost and performance of technologies. A learning curve describes the declining cost of a technology as a function of cumulative capacity, which can be seen as an approximation for accumulated experience (Barreto & Kypreos, 2004). Today, LbD is among the best empirically analyzed phenomena that lead to technological change (Messner, 1997). Concerning energy, the evidence is in clear favor of the fact that production costs depend on cumulative experiences (McDonald & Schrattenholzer, 2001).

The first paper about learning effects was written by Wright (1936). He described the relation of costs and quantity in the aircraft industry and concluded that there exists a interrelation between accumulated experience and the performance in the production processes. His observations were generalized by Arrow (1962), who became known as a pioneer for LbD.

The first to introduce LbD in a bottom-up model was Messner (1997) in the model MESSAGE, which is a systems-engineering approach that included systematic technological learning; that is, it linked the cost of developing a new technology to the already installed cumulative capacity. The model minimized the discounted costs of supplying energy, subject to an exogenously given level of final energy demand and assumptions on costs, efficiencies, and market penetration constraints. She evaluates the effects of including LbD and thus induced technical change (ITC) and concludes that ITC lowers the necessary investment level and that investments in expensive technologies start earlier.

Based on a later version of the model (MESSAGE III from Messner & Strubegger, 1995), Grübler and Messner (1998) combine the LbD dynamics with a carbon cycle component and include research and development to analyze the time path of a carbon tax for a given carbon concentration stabilization limit. They find that global emissions rise initially but stabilize later and eventually decline. ITC in this model results in a higher level of abatement than in a model with exogenous technology.

Both these models yield the "typical pattern of a discrete and complete switch" (Löschel, 2002) toward an alternative and environmentally friendlier technology as soon as the
technology becomes competitive. Yet the diffusion of new technologies in reality is never a discrete switch, but typically follows an S-shaped curve (Löschel, 2002). Gerlagh et al. (2004) face this criticism and combine the MESSAGE (1997) model with the DEMETER model from van der Zwaan et al. (2002) and Gerlagh and van der Zwaan (2003) in a top-down approach with LbD. They model niche markets for new technologies, which enables them to gradually become competitive. Their results yield a low level of carbon taxes needed to accomplish a given reduction compared to the DICE model.

Less optimistic regarding the difference between ITC and exogenous growth models in bottom-up setups are Manne and Richels (2004). They conclude that LbD does not significantly change the results. The only considerable impact is on the cost of emission abatement, which is, however, substantial.

Another bottom-up model with LbD as main driving force is the one of Barreto and Kypreos (2004), which is based on the MARKAL (MARKet ALlocation by Fishbone and Abilock, 1981) from the Energy Technology Systems Analysis Programme (ET-SAP) of the International Energy Agency. It is a multi-regional model of the global energy system and includes LbD across sectors so that there are linkages between the cost of technologies across regions. Their results show that LbD only slightly reduces emissions compared to the case without LbD.

### 3.2.2 Growth resulting from R&D

Models that include investments in R&D are inspired by macroeconomic models of endogenous growth, such as those of Lucas (1988), Romer (1990), and Grossman and Helpman (1994). In these models, a carbon tax has two opposite effects on the economy. First, it causes higher factor prices and therefore diminishes knowledge accumulation and the rate of technical progress. This development results in a decline in income and output. At the same time, the price effect triggers substitution of inputs and a reallocation of knowledge to sectors that offer the largest gains from research. Because this changes the technological structure of the economy, it can adjust more elastically to price changes. This leads to an augmentation of gross substitutability on the supply side, which eventually mitigates the loss incurred by the tax. The total effect of the
tax can be even positive (Sue Wing, 2003).

Based on his DICE model, Nordhaus (2002) introduced R&D (R&DICE) into the analysis and adds a carbon-intensive energy input to the production function. He includes two forms of technological change, an economy-wide technological change (Hicks neutral and exogenous) and a carbon-energy-saving technological change (CESTS), which reduces carbon emissions per unit of output. The model also accounts for diminishing returns to research (cf. also Popp (2002) for an empirical analysis on diminishing returns). Thus, the rate of energy efficiency varies with research, but there is no possibility of developing a carbon-free energy source to reduce carbon emissions independently of energy use. This setup might explain why Nordhaus finds that direct substitution of capital and labor for energy has a much greater influence on the results than ITC (Bretscher, 2005).

Buonanno et al. (2001) enhance the RICE model of Nordhaus and Yang (1996) and include ITC in their ETC-RICE model. A world wide stock of knowledge that all countries can use in their production has a reducing effect on their emission-output ratios (cf. also Böhringer & Rutherford (2002) on the welfare implications of international spillovers). These spillovers take into account international entanglement through trade, capital flows, and technology transfers. They analyze the pros and cons of introducing ceilings on emission trading (i.e., of having a maximum fraction of emissions that can be traded via international emission trading systems) and find that ITC has limited influence on growth. In a later paper (Buonanno et al., 2003), they additionally introduce sectoral spillovers within countries and human capital, which result in increasing returns to scale. Their paper suggests that ITC does reduce costs and that incentives to invest in R&D are smaller if international knowledge spillovers are accounted for due to free-riding incentives.

One key difference between Nordhaus (2002) and Buonanno et al. (2003) is their assumptions about the potential opportunity costs of R&D. Nordhaus assumes a fixed amount of total R&D spending in the economy and therefore crowding-out of R&D in other industries by the energy sector. On the contrary, Buonanno et al. model a single R&D stock that simultaneously enhances total factor productivity and lowers the carbon intensity of the economy. Consequently, ITC increases overall productiv-
ity, which is in accordance with Popp (2004). However, Popp (2004) finds that both assumptions are extreme presuppositions and states that only partial crowding out is realistic. In his ENTICE model (ENdogenous Technical change in the DICE model), he incorporates 50% crowding out and finds that ITC does reduce opportunity costs of lowering fossil fuel emissions.

Goulder and Schneider (1999) also introduce in their model the possibility of crowding out of research and deal with the criticism of the need to include different kinds of energy sources by including fossils and renewables (with limited substitutability). They find that carbon abatement policies have very different impacts on research across sectors and do not necessarily increase the economy-wide rate of technological progress. Although ITC yields lower cost for achieving a given target (as net benefits increase), gross costs (i.e., costs before netting out environmentally related benefits) of a given carbon tax are higher. Other papers that include research as the main driver for growth are, for example, Kemfert (2002) with WIA GEM (World Integrated Assessment General Equilibrium Model) and Gerlagh (2007) with the DEMETER-1CCS model.

Another focus of research is the timing of climate policy. Wigley et al. (1996) argued that a "wait and see" strategy should be realized by politicians to have the time to develop new technologies and to "reoptimize" the capital stock. This point stirred up intense debate. For example, Gerlagh et al. (2009) assume that the abatement sector grows according to Romer (1990) with positive spillovers to innovation from the previous stock (standing on shoulders) and additionally with negative externalities from aggregate current research (fishing out) (on the importance of R&D spillovers see also Griliches, 1992). Their result is that the timing of optimal emission reduction crucially depends on the policy instrument applied (i.e., R&D targeting or carbon taxes).

### 3.2.3 Comparison of impact of LbD and R&D

Some models compare the impact of LbD and research. Goulder and Mathai (2000), for instance, use a partial equilibrium model of knowledge accumulation in which a firm decides upon the time paths of abatement and R&D investment to minimize the costs corresponding to a particular emission target. They compare two different forms of endogenous technical change analytically and numerically. First, they apply LbD while
assuming that the stock of knowledge is a function of the level of abatement. Second, they include R&D in the analysis. For both forms, they assume that the stocks of capital are sector-specific. For the LbD simulation, they include the MESSAGE model from Messner (1997) in energy production. Their analysis is centered on the application of two different criteria on the evaluation of LbD and R&D: cost-effectiveness (Given a specific target for the atmospheric concentration, what would be the abatement profile at minimum cost?) and cost-benefit (What would be the optimal concentration target from maximizing the benefits from avoided climate change damages minus abatement cost?). They find that given a cost-effective setting ITC always leads to a lower time profile of optimal carbon taxes. In a cost-benefit analysis the results are similar as long as damages are convex in the atmospheric carbon concentration. The impact of ITC on the optimal abatement path is equivocal: If R&D is applied, abatement is shifted in the future. On the contrary, if LbD is present in the model, the timing is analytically ambiguous.

In the DEMETER-2E model, Gerlagh and Lise (2005) compare the two channels of innovation and also two types of energy (fossil fuels and a carbon-free energy). This model considers only the energy sector of the DEMETER model, i.e., total energy demand is exogenous. Their results are congruent to the folk theorem, which states that cheap (easily accessible) resource reserves are exhausted before more expensive energy sources are exploited. The continuous exhaustion of cheap reserves creates a shadow price or resource rent, which is comparable to the Hotelling resource rent. They also conclude that ITC has a positive impact on costs.

### 3.2.4 Related approaches

Apart from these models, other approaches cover similar research questions. For instance, Böhringer (1998) and Böhringer and Rutherford (2008) demonstrate how to integrate bottom-up activity analysis into top-down models (hybrid approach).

Another type of model focuses on the role of uncertainty for an optimal carbon tax. Kolstad (1996) and Ulph and Ulph (1997) both analyze the impact of the possibility of gaining more knowledge about the damage caused by global warming. Their results show that current abatement should be lower if the possibility existed of having better
information in the future.

Dowlatabadi (1998) and Dowlatabadi and Oravetz (2006) argue that historic trends are inconsistent with an AEEI factor and include an endogenous formulation in their models, which relates to the influence of price expectations (contrary to price changes ex post) on the energy intensity of the economy. They call this form of technical change price-induced efficiency (PIE). The result is that energy price changes modify expectation and the overall energy efficiency of the economy with a time lag.

On the question of the timing of the optimal carbon or energy tax, a number of authors have reported contradictory results (cf. Nordhaus, 1982; Sinclair, 1994; Farzin & Tahvonen, 1996; Ha-Duong et al., 1997). Azar (1998) consolidated the opinions by showing that the optimal time path depends on the carbon stabilization target chosen. In the case of low targets (for example smaller than 450 ppm), then early abatement is cost efficient. If high stabilization targets are chosen (e.g., larger than 600 ppm), then abatement should be conducted later.

3.3 Structure of the models

The two models compared in this chapter are the CITE model and a model with homogeneous capital and exogenous growth. Both have a decentralized structure, and each contains a Ramsey optimizer that maximizes utility by deciding about the extent of investments in different time periods. Therefore, the savings rates are endogenous in the two models. Both models represent open economies with trade modeled with Armington goods. The principal production structures are identical. The main difference is in the inclusion of gains from specialization and therefore of heterogeneous capital in the CITE model. The incentives to accumulate capital generate endogenous growth dynamics. In comparison, in the model with homogeneous capital, growth is assumed to come from endowments that grow by an exogenously defined rate in each period. This growth comes at no cost ("manna from heaven").

The policy scenario is based on a carbon tax that is based on the carbon intensity of the output of the oil sector and of imported gas. The tax on gas amounts to 8% of the price in the first period, and thereafter the absolute value remains constant over
3.3. STRUCTURE OF THE MODELS

... time. As oil contains 34% more carbon than gas, the tax is initially 10.72% and then stays constant. The revenues from the tax are redistributed with a subsidy on R&D that goes to all sectors except the oil sector. The amount each sector gets is optimized during the simulations.

In the theoretical calculations in this section, we abstract from Armington goods to keep the analysis as simple as possible.

3.3.1 Final output

The production of final output is nearly identical in both models. Regular goods as well as final goods in the oil and in the energy sector have the same production structure. The main difference is attributed to the intermediate goods. In the model with homogeneous capital (HK model), intermediate goods enter the production of final goods directly, whereas in the CITE model, the intermediate goods are first combined to an intermediate composite (cf. Chapter 3.3.2) and then enter the production of the final goods.

Starting with the CITE model, we can see that output of regular goods \( Y_n \) are produced using a sector-specific intermediate composite \( Q_n \) and the final goods of all regular sectors \( n' \) that go to sector \( n \), \( Y_{n',t,n} \).

\[
Y_{n,t} = \left[ \alpha_{X,n} Q_{n,t}^{\sigma_{Y,n}^{-1}} + (1 - \alpha_{X,n}) \left( \min_{n' \in N} \left( \frac{Y_{n',t,n}}{y_{n',n}} \right) \right) \right]^{\sigma_{Y,n}^{-1}}
\]  \hspace{1cm} (3.1)

The production functions for energy goods and oil goods resemble the production function above. In the production of energy goods, the above shown production structure is additionally combined with a Cobb-Douglas function of imported gas and the final good of the oil sector.

\[
Y_{e,t} = \left[ \alpha_{TFF} \left( GAS_t^{\sigma_{FF,\text{gas}}} Y_{o,t}^{\sigma_{FF,o}} \right)^{\sigma_{E}^{-1}} + (1 - \alpha_{TFF}) B_{e,t} \right]^{\sigma_{E}^{-1}}
\]
\[ B_{e,t} = \left[ \alpha_{X,e} Q_{e,t}^{\frac{\sigma_{Y,e}}{\sigma_{Y,e} - 1}} + (1 - \alpha_{X,e}) \left( \min_{n \in N} \left( \frac{Y_{ne,t}}{y_{ne}} \right) \right)^{\frac{\sigma_{Y,e}}{\sigma_{Y,e} - 1}} \right]^{\frac{\sigma_{Y,e}}{\sigma_{Y,e} - 1}} \]

The production in the oil sector also resembles the production of regular goods, and it uses as additional input crude oil, which is added with a Leontief function.

\[ Y_{o,t} = \min \left( \frac{CRU_t}{y_{cru}}, \frac{B_{o,t}}{y_{noncru}} \right) \]

\[ B_{o,t} = \left[ \alpha_{X,o} Q_{o,t}^{\frac{\sigma_{Y,o}}{\sigma_{Y,o} - 1}} + (1 - \alpha_{X,o}) \left( \min_{n \in N} \left( \frac{Y_{no,t}}{y_{no}} \right) \right)^{\frac{\sigma_{Y,o}}{\sigma_{Y,o} - 1}} \right]^{\frac{\sigma_{Y,o}}{\sigma_{Y,o} - 1}} \]

In comparison, in the HK model, no intermediate goods composite exists because gains from specialization are completely absent. As a result, it is abstracted from a number of different intermediate goods, but we assume that only one intermediate good per sector, \( \tilde{X}_i \), exists. Consequently, instead of an intermediate composite, the intermediate goods \( \tilde{X}_i \) enter the production functions of the sectors. The production function of regular goods then changes to

\[ \tilde{Y}_{n,t} = \left[ \alpha_{X,n} \tilde{X}_{n,t}^{\frac{\sigma_{Y,n}}{\sigma_{Y,n} - 1}} + (1 - \alpha_{X,n}) \left( \min_{n' \in N} \left( \frac{\tilde{Y}_{n',n,t}}{\tilde{y}_{n'}} \right) \right)^{\frac{\sigma_{Y,n}}{\sigma_{Y,n} - 1}} \right]^{\frac{\sigma_{Y,n}}{\sigma_{Y,n} - 1}} \]

We choose to display variables that are distinct in the HK model compared to corresponding variable in the CITE model with an tilde. The production of energy goods equals
3.3. STRUCTURE OF THE MODELS

\[
\dot{Y}_{e,t} = \left[ \alpha_{TFF} \left( GAST_{t,FF,gas} \cdot Y_{o,t}^{\alpha_{FF,o}} \right)^{\frac{\sigma_y}{\sigma_e}} + (1 - \alpha_{tff}) \dot{B}_{e,t}^{\frac{\sigma_y}{\sigma_e}} \right]^\frac{\sigma_e}{\sigma_y - 1}
\]

\[
\dot{B}_{e,t} = \left[ \alpha_{X,e} \dot{X}_{e,t}^{\frac{\sigma_y}{\sigma_e}} + (1 - \alpha_{X,e}) \left( \min_{n \in N} \left( \frac{\dot{Y}_{ne,t}}{y_{ne}} \right) \right) \right]^\frac{\sigma_y}{\sigma_e - 1}
\]

The production of oil in the HK model corresponds to

\[
\dot{Y}_{o,t} = \min \left( \frac{CRIU_t}{y_{cru}}, \frac{\dot{B}_{o,t}}{y_{ncru}} \right)
\]

\[
\dot{B}_{o,t} = \left[ \alpha_{X,o} \dot{X}_{o,t}^{\frac{\sigma_y}{\sigma_o}} + (1 - \alpha_{X,o}) \left( \min_{n \in N} \left( \frac{\dot{Y}_{no,t}}{y_{no}} \right) \right) \right]^\frac{\sigma_y}{\sigma_o - 1}
\]

3.3.2 Intermediate goods composite and intermediate goods

This section illustrates the main differences in both models. As already explained above, the intermediate goods are combined in the CITE model to an intermediate composite, whereas in the HK model, the intermediate good directly enters the production function of the final good. In this section, the aggregation to the intermediate composite in the CITE model as well as the resulting differences for the optimization problems of the intermediate good producers are explained.

In the CITE model, the intermediate goods composite \( Q_i \) is produced from a number of different intermediate goods \( x_{ij} \). The amount of intermediate goods is assumed to equal the capital stock in each sector.
\[ Q_{i,t} = \left[ \frac{K_{i,t}}{\sum_{j=1}^{\infty} x_{ij,t}^{\kappa}} \right]^{\frac{1}{\kappa}} \]

As all intermediate goods \( x_{ij} \) are produced symmetrically with the same production structure, we can assume \( x_{ij} = x_i \) and the above production function can be rewritten as

\[ Q_{i,t} = K_{i,t}^{\frac{1}{\kappa}} x_{i,t} = K_{i,t}^{\frac{1-n}{\kappa}} X_{i,t} \]

with

\[ X_i = K_i x_i \]

The intermediate goods are assumed to be produced with labor, \( L_{ij} \), policy invariant capital, \( VK_{ij} \), and energy, \( Y_e \), according to

\[
x_{ij,t} = \left[ \alpha_{L,i} L_{ij,t}^{\frac{\sigma X_i - 1}{\sigma X_i}} + \alpha_{V,K,i} VK_{ij,t}^{\frac{\sigma X_i - 1}{\sigma X_i}} + (1 - \alpha_{L,i} - \alpha_{V,K,i}) Y_e^{\frac{\sigma X_i - 1}{\sigma X_i}} \right]^{\frac{1}{\sigma X_i}}
\]

The optimization of each \( x_{ij} \) producer yields the inverse demand functions

\[
\frac{1}{\kappa} w_i = \alpha_{L,i} p_{x_{ij,t}} \left( \frac{x_{ij,t}}{L_{ij,t}} \right)^{\frac{1}{\sigma X_i}}
\]
3.3. STRUCTURE OF THE MODELS

\[ \frac{1}{\kappa} p_{VK,i,t} = \alpha_{VK,i} p_{x_{ij},t} \left( \frac{x_{ij,t}}{VK_{ij,t}} \right)^{\frac{1}{\sigma}} \]

\[ \frac{1}{\kappa} p_{Ye,t} = \left( 1 - \alpha_{L,i} - \alpha_{VK,i} \right) p_{x_{ij},t} \left( \frac{x_{ij,t}}{Ye_{x_{ij},t}} \right)^{\frac{1}{\sigma}} \]

where \( \frac{1}{\kappa} \) denotes the markup over marginal costs. It is important to note that each \( x_{ij} \) has a constant quantity of production over time. Thus, the growth index of \( x_{ij} \) equals one.

In comparison, the HK model includes only one intermediate good per sector, \( \tilde{X}_i \), which grows at the rate of its inputs. Of these inputs, the growth rates of \( VK_i \) and \( L_i \) are exogenously given. Therefore, all growth dynamics depend on the growth rates of these inputs. The production function for the intermediate good is given as

\[ \hat{X}_{i,t} = \left[ \alpha_{L,i} \frac{\sigma_{X,i}^{-1}}{\alpha_{L,i} L_{it}^{\frac{1}{\sigma}}} + \alpha_{VK,i} VK_{it}^{\frac{1}{\sigma}} + \alpha_{K,i} K_{it}^{\frac{1}{\sigma}} + (1 - \alpha_{L,i} - \alpha_{VK,i} - \alpha_{K,i}) Ye_{x_{ij},t} \right]^{\frac{\sigma_{X,i}}{\sigma_{X,i} - 1}} \]  \hspace{1cm} (3.2)

The intermediate good \( \hat{X}_i \) of the HK model is comparable to \( X_i \) in the CITE model. However, there exist some crucial differences. First of all, capital is used directly in the production function of \( \hat{X}_i \). Second, the total amount of \( L_{i,t} \), \( VK_{i,t} \), and \( Ye_{x_{ij},t} \) in the HK model must equal the sum of each used by the intermediate good producers in the CITE model in the first period:

\[ L_{i,0} = \sum_j L_{ij,0} \]
\[ VK_{i,0} = \sum_j VK_{ij,0} \]

\[ Y_{\tilde{X},i,0} = \sum_j Y_{\tilde{X}ij,0} \]

The most important distinction between \( \tilde{X}_i \) and \( X_i \) lies in the dynamics. These are different because the inputs in the HK model exogenously grow over time whereas they remain constant in the CITE model.

The maximization problem for the intermediate good producer in the HK model is

\[
\max_{L_{i,t}, V_{K_{i,t}}, K_{i,t}, Y_{\tilde{X}_{i,t}}} \quad p_{\tilde{X}_{i,t}} \tilde{X}_{i,t} - w_t L_{i,t} - p_{VK_{i,t}} V_{K_{i,t}} - p_{K_{i,t}} K_{i,t} - p_{Y_{\tilde{X}_{i,t}}} Y_{\tilde{X}_{i,t}} \\
\text{s.t. (3.2). The resulting inverse demand functions are}
\]

\[
w_t = \alpha_{L,i} p_{\tilde{X}_{i,t}} \left( \frac{\tilde{X}_{i,t}}{L_{i,t}} \right)^{\frac{1}{\sigma_X}}
\]

\[
p_{VK_{i,t}} = \alpha_{VK,i} p_{\tilde{X}_{i,t}} \left( \frac{\tilde{X}_{i,t}}{V_{K_{i,t}}} \right)^{\frac{1}{\sigma_X}}
\]

\[
p_{K_{i,t}} = \alpha_{K,i} p_{\tilde{X}_{i,t}} \left( \frac{\tilde{X}_{i,t}}{K_{i,t}} \right)^{\frac{1}{\sigma_X}}
\]
### 3.3. Structure of the Models

\[ p_{Ye,t} = (1 - \alpha_{L,i} - \alpha_{V,i})p_{\bar{X}_i,t} \left( \frac{\bar{X}_{i,t}}{e_{X_i,t}} \right)^{1/\sigma} \]

Comparing the FOCs of the intermediate producer(s) in both models, it is striking that in the CITE model, each intermediate good producer can charge a mark-up on the marginal costs. As in the HK model, we assume competitive markets in which prices always equal marginal costs.

#### 3.3.3 Consumption

The intertemporal allocation of factors is optimized in both models by a representative household that maximizes utility having perfect foresight. Utility \( U \) is drawn from a consumption good \( C \) and is discounted at the rate \( \rho \) over time. The utility function of the household is identical in the models:

\[
U = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \frac{C_t^{1-\theta} - 1}{1 - \theta}
\]

Consumption consists of two goods, energy \( Y_{ec} \) and a final good composite \( C_S \) composed of the final goods of the regular sectors.

\[
C_t = \left[ (1 - \beta)C_{S,t}^{1/\sigma_C} + \beta Y_{ec,t}^{1/\sigma_C} \right]^{\sigma_C/\sigma_C-1}
\]

\[
C_{S,t} = \prod_{n \in N} Y_{nc,t}^{\beta_{NE,n}}
\]

Hereby, \( Y_{nc} \) denotes the part of the final output of sector \( n \) that is used for consumption. \( \beta_{NE,n} \) is the value share of final good \( n \) in the composite \( C_{S,t} \) and the sum of these
shares, \( \sum_{n \in N} \beta_{NE,n} \), adds up to one to assume constant returns to scale. The budget constraint of the household in both models equals

\[
p_{V,t+1}V_{t+1} = (1 + r_{t+1})p_{V,t}V_t + w_tL_t + w_{RL_t}RL_t + p_{V,K}VK_t \\
- p_{Y,c,t}Y_{ec,t} - \sum_{n \in N} p_{Y,n,t}Y_{nc,t} - \sum_{i \in I} p_{I,i,t}I_{i,t}
\]

The household is able to hold assets \( V \) on the capital market, which includes that total investments made by the household must equal investments in physical and in non-physical capital, \( I_{i,t} = I_{P,i} + I_{NP,i} \). Energy is assumed to be an endowment of the household, and therefore income from energy goes to the consumer.

#### 3.3.4 Demand for the intermediate good

To gain a better understanding of the growth dynamics, we need to consider the transmissibility of the models from the intermediate good(s) to the final output. The intuition behind this is that we want to know how and to what extent dynamics from intermediate good(s) influence the dynamics of the intermediate goods composite in the CITE model and then, for both models, the influence on final output. To keep the analysis as simple as possible, we only consider the output of regular sectors.

Because the producer of the intermediate goods composite in the CITE model needs intermediate goods to produce, the demand function for each intermediate good is

\[
x_{nj,t} = \left( \frac{p_{Q,n,t}}{p_{x_{nj,t}}} \right)^{\frac{1}{1-\kappa}} Q_{n,t}
\]

Since all intermediate goods are symmetric, we can assume that \( X_n = K_n x_n \), and thus the above equation equals

\[
\frac{X_{n,t}}{K_{n,t}} = \left( \frac{p_{Q,n,t}}{p_{x_{nj,t}}} \right)^{\frac{1}{1-\kappa}} Q_{n,t}
\]
Rearranging yields the inverse demand function for $X_n$ by the producer of the intermediate composite

$$p_{Q_n,t} = \left( \frac{X_{n,t}}{K_{n,t}Q_{n,t}} \right)^{1-\kappa} p_{x_n,t}$$

To make the above equation comparable to the corresponding one in the HK model, we also need to take account of the prices. Due to the relation $X_n = K_n x_n$, we know that $p_{X_n} = K_n p_{x_n}$ must also hold. This yields for $p_{Q_n,t}$

$$p_{Q_n,t} = \left( \frac{X_{n,t}}{K_{n,t}Q_{n,t}} \right)^{1-\kappa} \frac{p_{X_n,t}}{K_{n,t}} \quad (3.4)$$

The intermediate composite is demanded by the producer of the final good. The inverse demand function of the latter for $Q_n$ is

$$p_{Q_n,t} = p_{Y_n,t}^{\alpha_{X_n}} \left( \frac{Y_{n,t}}{Q_{n,t}} \right)^{\frac{1}{\alpha_{Y_n}}}$$ \quad (3.5)

By inserting (3.5) in (3.4) and by rearranging we get for $p_{X_n,t}$

$$p_{X_n,t} = p_{Y_n,t}^{\alpha_{X_n}} \left( \frac{Y_{n,t}}{Q_{n,t}} \right)^{\frac{1}{\alpha_{Y_n}}} K_{n,t} \left( \frac{K_{n,t}Q_{n,t}}{X_{n,t}} \right)^{1-\kappa}$$

The intermediate composite does not exist in the HK model. Accordingly, we need to replace $Q_n$ in the above equation by $Q_{n,t} = \frac{1-\kappa}{\kappa} X_{n,t}$. This yields for the inverse demand for $X_n$ by the final good producer $Y_n$

$$p_{X_n,t} = p_{Y_n,t}^{\alpha_{X_n}} \left( \frac{Y_{n,t}}{X_{n,t}} \right)^{\frac{1}{\alpha_{Y_n}}} K_{n,t} \left( \frac{X_{n,t}}{Q_{n,t}} \right)^{\frac{1-\kappa}{\kappa}} \frac{1}{\alpha_{x_n}} \quad (3.6)$$
A comparable relation can be derived in the HK model by the optimization of the final good producer. She maximizes

$$\max_{\tilde{X}_n,t, \tilde{Y}_n,t} p_{\tilde{Y}_n,t} \tilde{Y}_n,t - p_{\tilde{X}_n,t} \tilde{X}_n,t - \sum_{n' \in N} p_{Y_{n'},t} Y_{n',t}$$

subject to (3.1).

The resulting inverse demand function is

$$p_{\tilde{X}_n,t} = p_{\tilde{Y}_n,t} \alpha_{X,n} \left( \frac{\tilde{Y}_{n,t}}{\tilde{X}_{n,t}} \right)^{\frac{1}{\sigma_{Y,n}}} \tag{3.7}$$

If we compare equations (3.6) and (3.7), we can see that the demand for $\tilde{X}_n$ depends in the HK model only on the prices of $\tilde{Y}_n$ and of $\tilde{X}_n$, the parameter $\alpha_{X,n}$ and the quantity of output, $\tilde{Y}_n$. In the CITE model, however, the demand also depends on the capital stock, which renders the demand even more dynamic.

From these equations, we can see that the dynamics of $X_n$ are transmitted in both models to final output production. Although this seems to be similar, there are different dynamics hidden within. In the HK model, $\tilde{X}_n$ grows exogenously at a constant rate. In the CITE model, $X_n$ grows endogenously at the same rate as capital. Therefore, the stock of capital enters the demand function (3.6) twice, once directly via $K_n$ and again indirectly via $X_n$. To show this effect of the CITE model more explicitly, we consider the demand or the output producer for an intermediate good $x_n$. Therefore, we assume again $X_n = K_n x_n$, which yields for the inverse demand function

$$p_{X_n,t} = p_{Y_n,t} \alpha_{X,n} \left( \frac{Y_{n,t}}{x_{n,t}} \right)^{\frac{1}{\sigma_{Y,n}}} K_n \left(1 - \frac{1}{\sigma_{Y,n}} \right)^{-\frac{1}{\sigma_{Y,n}}} \tag{3.8}$$

In the above equation, $x_i$ does not grow and the influence of capital dynamics can directly be seen.
3.3. STRUCTURE OF THE MODELS

For a comparison with the HK model, we need to assume two artificial variables that do not exist in the model. First, we know that capital is an input to the production of the intermediate good with an elasticity of substitution smaller than 1. As a result, capital grows on a balanced growth rate that is always the same rate as the exogenously growing inputs. We therefore assume $D_t$ being the size of capital in the first period that grows at the exogenously given rate $grk$.

$$D_t = K_{n,0}(1 + grk)^t$$

Second, we assume that $\tilde{X}_{n,t}$ can be split in a number of $x_{n,t}$, which together equal $\tilde{X}_{n,t}$:

$$\tilde{X}_{n,t} = x_{n,t}D_t = x_{n,t}K_{n,0}(1 + grk)^t$$

If we insert this in (3.7) we get

$$p_{X_{n,t}} = p_{Y_{n,t}}\alpha_{X,n} \left(\frac{Y_{n,t}}{x_{n,t}}\right)^\frac{1}{\sigma_{Y,n}} D_t^{-\frac{1}{\sigma_{Y,n}}}$$  \hspace{1cm} (3.9)

The comparison of equations 3.8 and 3.9 shows exactly the point at which capital growth enters the demand functions. In the HK model, the exogenously growing capital stock enters the demand function with an exponent of $-\frac{1}{\sigma_{Y,n}}$ (the growth of the capital stock depends on the exogenously given growth rate of inputs on a balanced growth path). In the CITE model, the capital stock enters the demand function with a different exponent $\frac{1}{\kappa} \left(1 - \frac{1}{\sigma_{Y,n}}\right)$. Also, the size of the capital stock is subject to endogenous dynamics.
3.3.5 Capital formation and incentives to invest

This section shows how the demand functions calculated above translate to capital formation and to incentives to invest on a balanced growth path. Capital formation is identical in the models. Capital is produced through investments in physical and non-physical capital, the latter itself being a function of labor in research and investments in R&D.

\[
I_{NP_{ij},t} = \left[ \gamma_{N,i}^{\sigma_{N,i}^{-1}} RL_{ij,t}^{\sigma_{N,i}} + (1 - \gamma_{N,i})I_{R&D_{ij},t}^{\sigma_{N,i}} \right]^{\sigma_{N,i}}
\]

\[
K_{ij,t+1} = \left[ \gamma_{i}^{\sigma_{I,i}^{-1}} I_{P_{ij},t}^{\sigma_{I,i}} + (1 - \gamma_{i})I_{NP_{ij},t}^{\sigma_{I,i}} \right]^{\sigma_{I,i}} + (1 - \delta)K_{ij,t}
\]

After optimization for investments in physical capital (the same calculation can be done for investments in non-physical capital) we have

\[
p_{Y_{n},t} = p_{K_{n,t+1}} \gamma_{n} \left( \frac{\Delta K_{n,t+1}}{I_{P_{n},t}} \right) \frac{1}{\gamma_{n}}
\]

If we insert the above equation in (3.8) for the CITE model we arrive at the magnitude of optimal investments in physical capital in both models. In the CITE model, investments are equal to

\[
I_{P_{n},t} = p_{X_{n},t} \left( p_{K_{n,t+1}} \gamma_{n} \alpha_{X,n} K_{n,t}^{\frac{1}{\gamma_{n}}} \left( \frac{1}{\gamma_{n}} \right) \right)^{\frac{\sigma_{I,n}}{\gamma_{n}}} \left( \frac{Y_{n,t}}{X_{n,t}} \right)^{\frac{\sigma_{I,n}}{\gamma_{n}}} \Delta K_{n,t+1}
\]

Similarly, for the HK model we arrive at
$I_{P_n,t} = p_{X_n,t}^{-\sigma_{I,n}} \left( p_{K_n,t+1} \gamma_n \alpha_{X_n} D_t \frac{1}{\sigma_{Y,n}} \right)^{\sigma_{I,n}} \left( \frac{Y_{n,t}}{X_{n,t}} \right)^{\sigma_{I,n}} \sigma_{Y,n} \Delta K_{n,t+1}$

Investments depend on, among others, the aspired growth in capital. As the capital stock in the HK model evolves with an exogenously given growth rate on a balanced growth path, we can rewrite $\Delta K_{n,t+1}$ as

$\Delta K_{n,t+1} = K_{n,t+1} - K_{n,t} = K_{n,0} \left( (1 + grk)^{t+1} - (1 + grk)^t \right) = K_{n,0} (1 + grk)^t grk$

Using this in the optimal function for physical investments in the HK model, investment incentives become equal to

$I_{P_n,t} = p_{X_n,t}^{-\sigma_{I,n}} \left( p_{K_n,t+1} \gamma_n \alpha_{X_n} D_t \frac{1}{\sigma_{Y,n}} \right)^{\sigma_{I,n}} \left( \frac{Y_{n,t}}{X_{n,t}} \right)^{\sigma_{I,n}} \sigma_{Y,n} \Delta K_{n,t+1}$

In the HK model, capital enters as an exogenously defined variable. Investment incentives are given by the growth rate of inputs on a balanced growth path. In the CITE model, however, investment incentives are derived from the stock of capital and the absolute increase of capital, which are both derived endogenously. The investment decision in the CITE model evolves entirely from the optimization results from market participants. Depending on the monopolistic power of the producers of intermediate goods (thus, how large $\kappa$ is), the incentives to invest change. If the Chamberlinian large group assumption yields that each producer of an intermediate good can charge a large markup on the cost (thus, if $\kappa$ is small), the incentives to innovate and therefore to accumulate capital rise. If the markup on the cost is small (thus if $\kappa$ is large), the incentives to innovate and investments in capital fall.
3.3.6 Growth indices

Another way to look at the dependency of the growth rate of output on the model setups is to regard the growth indices. From the capital accumulation equation, we know that the growth index of capital is in both models equal to

\[
g_{K_i} = \frac{K_{i,t+1}}{K_{i,t}} = \left[ \frac{\sigma_{I,i}^{-1}}{\gamma_i I_{P_i,t} + (1 - \gamma_i) I_{NP_i,t}^\sigma_{I,i}^{-1}} \right] \frac{\sigma_{I,i}^{-1}}{\sigma_{I,i}^{-1}} + (1 - \delta)
\]

In the CITE model, the growth index of final goods equals to the growth index of the intermediate composite good, which in turn is

\[
g_{Y_i} = \frac{1}{\sqrt{g_{K_i}}}, \quad g_{x_i} = g_{K_i}^{\frac{1}{2}}
\]

as the production of each intermediate good stays constant over time. We can see that the growth of output in the CITE model completely depends on the growth of capital. In the HK model, the growth index of output equals the growth index of \( \tilde{X}_i \). We can therefore write

\[
g_{Y_i} = g_{\tilde{X}_i}
\]

Consequently, in the HK model, output grows at the same rate as capital, \((1 + grk)\), and in the CITE model, output grows at a higher rate than capital \((1 + gr) = (1 + grk)^{\frac{1}{\sigma}}\). In other words, the growth of the intermediate good in the HK model completely determines the growth rate of output.
3.4 Calibration of the models

The models are calibrated according to the input-output table of the Swiss economy. The latter provides information about the intermediate demands as well as the sectoral share in investments and consumption. Additionally, labor and capital as inputs to production are displayed.

This section gives some information about the calibration of the CITE model and of the HK model with which it is compared. Moreover, the calibration of the HK model is then modified and the capital stock is defined differently. This calibration of the HK model without policy invariant capital is then compared to the calibration with policy invariant capital in the next section. The comparison is done to give an intuition of the importance of capital calibration and contributes to a better understanding of the effects of the capital calibration in the CITE model.

The capital stocks of the CITE model and of the HK model with policy invariant capital are calibrated by dividing the capital stock from the input-output table in two parts. The definition of capital is based upon the assumption that the size of the capital stock has a significant influence on the growth rate in the CITE model. The policy invariant capital is the difference between the calibrated capital stock and the capital stock from the input-output table. In the calibration without policy invariant capital, the capital stocks in the HK model are fully congruent with the capital stock in the input-output table.

For the comparison of the relations in Table 3.1, it must be taken into account that $X_i$ is defined differently in the CITE model than in the HK model. In the CITE model, $X_i$ is produced using labor, energy, and policy invariant capital. The capital stock is needed exclusively as a precondition to produce $X_i$ and is thus not included in the calculation of $X_i$ from the input-output table. On the contrary, in the HK model, capital is a direct input to the production of the intermediate good and is therefore also included in the calculation of the intermediate good. This difference in the definitions is essential for interpreting the capital, energy, and labor shares.

If we regard the relation of energy to final output $Y_i$ in the sectoral production in Table 3.1, it is striking that the energy sector has by far the largest share (38%). The sectors
agriculture and transport follow only with a large gap (4% and 5%, respectively). The oil, banking, and insurance industries have the smallest share of energy in final output.

Comparing the sectoral shares of energy in the production of the intermediate good, a similar picture emerges. The different energy intensities of the sectors result in diverse sensitivities of the sectors to energy policies. However, the size and the dynamics of the capital stocks also strongly influence the economic consequences of policies.

Thus, the capital stocks are defined differently in the calibrations. The CITE model and the calibration with policy invariant capital have a smaller capital stock compared to the calibration without policy invariant capital (VK). This distinction is also reflected in the relations in Table 3.1. In the calibration with VK, the capital stock constitutes by definition 25% of the value of the intermediate good. Comparing this with the calibration without VK, we can see that the latter calibration has much higher shares of capital in $X_i$ in some sectors (e.g., oil, chemical, and agriculture), while in other sectors the shares remain almost the same (such as in construction and health). The difference, VK, therefore varies greatly (between 1% and 39%).

The accumulation of the capital stock drives growth and determines the dynamics of $X_i$ in the CITE model. Hence, a comparison of $X_i$ to final output gives an intuition to what extent the intra-sectoral growth dynamics contribute to the development of a sector. In the CITE model, the sectoral shares of $X_i$ to final output $Y_i$ varies considerably between 6% in the oil sector and 54% in the health sector. Accordingly, the health sector can rely on strong intra-sectoral dynamics, whereas the oil industry mainly depends on the dynamics of other sectors and of imported crude oil. In comparison, the share of the intermediate good in final output in the HK models is distributed even more unevenly. The share in the oil industry is as low as 9%, whereas the health sector needs the intermediate good for as much as 72% of final output.

The relation of the intermediates to final output also affects the share of capital to final output. In the CITE model and the calibration with VK, the shares are the highest in the sectors of energy, banking, health, and other services. A structurally different result emerges if we consider the calibration without VK. Here, the sectors of agriculture, chemical, banking, and other services have the highest capital share.

Concerning the shares of labor in $X_i$, the different calibrations have a similar distribu-
Table 3.1: Specific relations in the models. The sectoral abbreviations are AGR - agriculture; OIL - oil; CHM - chemistry - MCH - machinery; EGY - energy; CON - construction; TRN - transport; BNK - banking; INS - insurance; HEA - health; OSE - other services; OIN - other industries.
tion. The construction and the health sector have the largest shares. On the contrary, the energy sector has by far the smallest share of labor in the intermediate production.

3.5 Influence of the size of the capital stock

The CITE model as well as the calibration with VK of the HK model have capital stocks that are not congruent with the capital stock in the Swiss input-output table. The reason for adapting the capital stocks is that they must fulfill certain conditions in the CITE model. As a consequence, the capital stocks in the calibration with VK have also been adjusted to ensure the comparability of the dynamics of both models.

![Figure 3.1: Sectoral outputs in the HK model without VK.](image)

The modification of the capital stocks results in smaller capital stocks that are subject to investments. This change affects different dynamics. This section aims to give an intuition for the impact of a quantitative change in the capital stock. Two calibrations of the HK model are compared with the focus on dynamics. One calibration has the same capital stock as the CITE model, and the other incorporates a capital stock that is identical to the input-output table. These assumptions about the capital stocks are the only differences in the calibrations. Therefore, all changes in dynamics are the result of different calibrations of the capital stocks.
The dynamics of the sectoral outputs after the introduction of a tax on carbon follow different patterns in the two calibrations (Figures 3.1 and 3.2). In the calibration with policy invariant capital, the output of most sectors show only limited reactions. They vary by less than 1% compared to the benchmark. The two exceptions are the energy and the oil sector, which drop by 4.3% and 6.2%, respectively. Since the carbon tax is tied to the output of the oil sector and to imported gas, and since both are used in the energy sector, this result is not surprising.

In comparison to these effects, the response of the industries in the calibration without VK is more dynamic. Most sectors show a small sensitivity to the carbon tax (again with the exceptions being energy and the oil industry). However, the production of some industries needs a long time to adapt to the new situation and varies more than in the other calibration. For example, the chemical industry continues to grow over the total time horizon and has even then not yet reached a new balanced growth path. The effect of the tax on this sector is very strong, as it still grows by more than 2% in the last period. Another interesting industry is the machinery and equipment sector, which has a larger output in the periods after the introduction of the tax and then shrinks so much that at the end of the time horizon, output is smaller than in the benchmark.
The distinctive dynamics of the sectors in the two calibrations result from different evolutions of the capital stocks. Figures 3.3 and 3.4 depict the sectoral capital stocks and their development over time. The dynamics are comparable to those of final production. Because capital is an important input to the production of the intermediate good and is crucial for its dynamics, the behavior of output is also heavily influenced. Due to the larger capital stock in the calibration without policy invariant capital, the adaptation to the new balanced growth path needs more time for both the capital stock and the final output.

3.6 Dynamic of the CITE and a HK growth model

The dynamics of the CITE model are based on gains from specialization that rise with an increasing capital stock. This section compares these dynamics with those of a HK model with exogenous growth. The capital stock of this model is calibrated identically to that of the CITE model to ensure that the differences in the dynamics arise only from the general growth dynamics and not from the quantitative difference of the capital stocks. The center of interest is the effects of the different investment incentives on
3.6. DYNAMIC OF THE CITE AND A HK GROWTH MODEL

Figure 3.4: Sectoral capital stocks in the HK model.

intertemporal dynamics and structural change.

3.6.1 Effects on production, capital, and consumption

After the introduction of a carbon tax, all sectors in the economy must adjust to the new situation. The tax increases the prices of fossil fuels, which directly affects production and investments.

The reaction of final output in the CITE model differs considerably from that in the HK model in three regards (Figures 3.5 and 3.2). First, the effects of the tax are in most sectors very small in the HK model; their production changes only less than 1% compared to the benchmark case. Only the energy and the oil sector show a strong reduction in output (4.3% and 6.2%, respectively). On the contrary, in the CITE model, most of the sectors react in a more pronounced way, which leads to a larger spread in the figure. The production of the energy and the oil sector decreases even more, whereas the machinery sector gains and has an increased production of 2.6% in the last period. This positive reaction is at least partly ascribable to the small energy share of the machinery sector. The direct effects of the carbon tax are hence limited. Other sectors have large energy shares, such as the sectors of agriculture and other
industries and suffer more because of the tax. Additionally, the machinery sector has a high elasticity of substitution between energy and other inputs for the production of intermediate goods. The production of the agriculture industry decreases by 1.4%, and output of other industries drops by 1.7%.

The second between the two models is the speed in which the sectors approach a new balanced growth path after the introduction of a carbon tax. The adaptation lasts much longer in the CITE model than in the HK model with policy invariant capital (HKwVK); indeed, it takes decades for the sectors to adjust to the new situation. In the HKwVK, all industries reach the new balanced growth path very quickly and then remain there for the rest of the time horizon. This difference in the dynamics results from the fact that only labor and policy invariant capital can substitute for energy in the CITE model. This yields limited possibilities to adapt, so the industries need to invest in capital to increase their productivity. The slow adjustment is due to the fact that capital accumulation takes time. In the HKwVK, capital can additionally and immediately substitute for energy, which results in a fast change of the composition of inputs to production. As a consequence, the level of production adjusts very quickly to the tax. On the other hand, capital cannot, as in the CITE model, enhance total productivity of the intermediate good.

Figure 3.5: Sectoral outputs in the CITE model.
The third main distinction concerns the structural composition of the economy. The sectoral reactions to the carbon tax are different in the models, which yields different relations of the sizes of the industries. In the CITE model, the machinery and equipment sector benefits most from the tax. This sector has a small energy share and also a relatively small share of capital in final output. Due to these conditions, the machinery sector can react very flexibly to the tax. The impact on costs is very limited because energy costs are only a small fraction of total costs. Additionally, the small share of capital enables the sector to proportionately increase its capital share with relatively small investments. These investments, in turn, contribute to higher productivity, which overcompensates for the increase in energy prices. This mechanism is not possible in the HKwVK. Here, the machinery sector can only substitute capital for energy, but cannot increase the total productivity of the intermediate good. As the elasticity of substitution is lower than 1, the possibility of using capital instead of energy is limited. As a result, the machinery sector does not increase production in the HKwVK as much as in the CITE model.

The consequences of the difference in the growth dynamics can also be seen in other sectors. The insurance industry, for example, decreases production initially in the CITE model and then intensifies production over time due to an increase of the capital
stock. At the end of the time period, output is slightly above the benchmark level. In the HKwVK, however, output rises directly after the introduction of the tax and remains high, so that at the end of the time frame, the insurance sector has the largest rise of output.

Since the development of production is tightly connected to the evolution of capital, all three dissimilarities can also be seen there (Figures 3.6 and 3.4). First, the spread of the capital stock is much larger in the CITE model. Second, the time until the capital stocks reach the new balanced growth path is longer in the CITE model. Finally, the structural composition of the sectoral capital stocks changes. Additionally, the evolution of the capital stock of the energy sector is very interesting. In the HKwVK, the capital stock behaves similarly to the output. Both strongly decrease compared to the benchmark. In the CITE model, however, we can see that the capital stock is even higher in the new balanced growth path than in the benchmark. This observation shows again the difference in the investment incentives in the two models.

The changing combination of inputs to production also influences on the demand for other inputs than capital, such as fossil fuels (Figures 3.7 and 3.8). The imports of crude oil and gas decrease more in the CITE model than in the HK model. Crude oil
drops in the CITE model by 7.5% at the end of the time horizon, which is 1.2% more than in the HKwVK. The difference in gas importation is smaller at about 0.5%.

After the introduction of a carbon tax, the production structure of the economy changes in both models. Despite these changes, the negative consequences for consumption and welfare remain very limited (Figures 3.9 and 3.10). Although consumption initially falls by around 0.4% and 0.16% in the CITE and the HKwVK, respectively, it quickly recuperates and is in the last period only slightly below the benchmark level. Welfare is very similar in both models and decreases only slightly (less than 1%) compared to the benchmark.

### 3.6.2 Effects on endowments

The consequences of a carbon tax for labor is structurally similar to output in both models (Figures 3.11 and 3.12). The spread of demand for labor is larger in the CITE model than in the HK model. The demand in both models needs some time to adapt, although the CITE model shows a longer time period for adaption in most sectors (except for the chemical industry and the construction sector). The main difference compared to output is the behavior of the energy sector in the CITE model, which has
a very high demand for labor.

Similarly to the demand for labor that almost duplicates the patterns of output, labor in research shows a similar behavior compared to the capital stock (Figures 3.13 and 3.14). This result is very intuitive, and at the same time it yields some insights into the
development of the labor market. The proportion of labor between the sectors changes compared to the benchmark, as does the intrasectoral relation of labor to labor in research.

The changes in demand are also reflected in the prices. If we regard the wage for labor in research in Figures 3.15 and 3.16, we find that it increases in both models, but it remains at a higher level in the CITE model. We can conclude that the marginal product of labor in research stays higher in the CITE model than in the HK model in the long run. This difference is also a consequence of the different roles of capital in the models. In both models, labor in research contributes to investments, which raises the capital stocks. However, capital has a larger effect on production and therefore a larger marginal product in the CITE model because it increases overall productivity of the intermediate composite good. In the HK model, an increase in capital can only enhance production of intermediates by substitution and not by a productivity factor. It has therefore a smaller possibility to influence output. This fact is reflected in the wage of labor in research.

![Figure 3.11: Labor demand in the CITE model.](image-url)
3.7 Sensitivity analysis

The influence of capital on the dynamics become in both models clearer if we regard the reactions of the models to a change in two important elasticities. As capital goes in the
3.7. SENSITIVITY ANALYSIS

Figure 3.14: Demand for labor in research in the HK model.

Figure 3.15: Wage in research in the CITE model.

HK model in the production of the intermediate good, the elasticity that connects $\tilde{K}_i$ with the other inputs in the production of $\tilde{X}_i$, $\sigma_{X,i}$, plays an important role. Moreover, the elasticity in the production of the final good that connects the intermediate good (the intermediate composite in the CITE model) and the inputs from other sectors,
\( \sigma_{Y,i} \) influences the outcomes. To find out how sensitively the models react to a change in the elasticities, we ran the counterfactual with elasticities that are doubled. Some of them then take very high values, and it must be kept in mind that these values are unrealistic.

Taking a closer look at the reaction of the CITE model to doubling \( \sigma_{X,i} \) in all sectors (i.e., policy invariant capital, energy, and labor are in the production of \( x_i \) more substitutable), it is clear that the model is fairly reactive (Figures 3.17 and 3.18). The machinery sector has a lower output of 0.4% compared to the policy scenario with the original value of \( \sigma_{X,i} \). At the same time, the production of the energy and oil sectors shrinks by 3.2% and 3.1%, respectively. This development seems to be an effect of the ability of the other sectors to better substitute labor and policy invariant capital for energy. Therefore, the demand for energy decreases and thus for the output of the oil sector. The production of the other sectors changes about as much as in the scenario with the original value of \( \sigma_{X,i} \).

If we compare these results with the reactions of the industries in the HK model, we find that they are less sensitive to a change in \( \sigma_{X,i} \). The energy and oil sectors’ production drops more than before (about 2.2%), but the other sectors show few deviations. Some

![Figure 3.16: Wage in research in the HK model.](image-url)
structural changes take place, but overall, the economy shows little sensitivity.

Another important elasticity for transmitting growth dynamics is \( \sigma_{Y,i} \). It connects the intermediate good (intermediate composite) with other inputs and therefore influences to what extent the dynamics of capital accumulation can drive growth of the final
output. It is thus interesting to take a look at the reactions of the models to a change in $\sigma_{Y,i}$ in Figures 3.19 and 3.20.

In the CITE model, we see that doubling $\sigma_{Y,i}$ affects on a number of industries. The machinery sector increases production, and the energy and oil sectors have a slightly
lower output. Also, the agriculture sector and the other industries produce less than before. In total, the effects are limited, but are still stronger than those in the HKwVK. There, the changes are negligible.

To sum up, the reactions of the models to changes in elasticities are quite small, and the models are very stable. The CITE model is slightly more reactive to changes than the HKwVK, which showed few effects of doubling the elasticities.

3.8 Conclusion

The comparison of the dynamics of the CITE model with a HK model with exogenous growth shows that the endogenous growth mechanism of the CITE model yields different reactions to a carbon tax. In the CITE model, capital growth generates gains from specialization and ensures endogenous growth dynamics. These dynamics are influenced by a carbon tax as the incentives to invest change. Investments target a substitution of energy in production and result in a higher productivity of the intermediate composite. This, in turn, contributes to a change in the production of the final outputs. In the HK model, capital accumulation can only contribute to a substitution for energy, but not to an increase in productivity. Accordingly, investment incentives are different compared to the CITE model.

The dissimilarity of investment incentives are reflected in the reactions of the sectors to a carbon tax. In the CITE model, most industries show a stronger sensitivity to the change in input costs than in the HK model. Three differences mainly emerge: First, the spread of output of the sectors is larger in the CITE model. Second, the speed in which the industries approach a new balanced growth path is lower in the CITE model. Finally, the structural composition of the economy is different in the two models. It can therefore be concluded that the endogenous growth mechanism in the CITE model uncovers dynamics triggered by a carbon tax that cannot be discovered with a HK model.
Chapter 4

Energy price shocks and medium-term business cycles

As opposed to anticipated energy policies, energy price shocks pose sudden challenges to economies. This chapter examines how oil price shocks have influenced the U.S. economy over the last decades. We extend existing literature by considering medium-term business cycles, which consist of high-frequency components ("conventional" business cycles, up to 8 years) and medium-frequency components (8 to 50 years). We find that the medium-frequency consequences of energy price shocks are considerable and dominate the medium-term business cycles.
4.1 Introduction

Scholars, politicians, and the general public are engaged in the discussion about possible effects of oil price shocks on macroeconomic variables. Of particular interest are consequences for economic behavior in the medium run. So far, there exists no paper that examines macroeconomic responses over the short and medium run. This chapter aims to fill this gap.

During the last decades, many industrialized countries have gone through periods of robust growth and of relative stagnation. In the case of the U.S. economy, the country achieved high-output growth during the early to late 1960s. Nevertheless, in the subsequent period from the early 1970s to the early 1980s, average output growth was low. This changed as late as the mid-1990s with a new surge in output growth. Europe and Japan have similar experiences with economic ups and downs. These oscillations usually occur over a longer time frame than conventionally considered in business cycle analysis. Accordingly, these kinds of fluctuations are often swept into the trend and thus excluded from a closer analysis (Blanchard, 1997; Comin & Gertler, 2006).

Business cycles over the medium-term horizon have been analyzed so far by only a small group of authors, including Blanchard (1997), Evans, Honkapohja and Romer (1998), Caballero and Hammour (1998), and Solow (2000). Recently Comin and Gertler (2006) have analyzed medium-term business cycles on the basis of endogenous growth theory. Their approach is different in that it not only limits the analysis to medium-frequency fluctuations in the data, but also includes both the conventional business cycles and medium-frequency variations. Their particular approach is that both types of frequencies are not treated as mutually independent but rather as interconnected by endogenous dynamics.

Similarly, the effects of energy price shocks have been examined only for conventional business cycles, i.e., over a time horizon of up to 8 years. However, it seems implausible that the consequences of an energy price shock die out after such a short period of time. The endogenous growth dynamics of new growth theory necessarily affect macroeconomic activity over a longer time horizon. Induced changes in research and development might lead to a new trajectory of output and other economic variables.
This chapter analyzes the impact of oil price shocks in the medium-term. We first analyze the average persistence of shocks in the U.S. economy over the last years and then examine with our model (based on Comin & Gertler) to what extent effects of oil price shocks mimic real macroeconomic behavior. Our results show that the structural behavior of variables after the shock and the data show similar patterns (confidence intervals are not regarded in this context, however). An evaluation of the quantitative effects depends on the assumption of the shocks and the calibration of the model.

4.2 Empirical background

In this section, we further analyze some facts about business fluctuations in the short- and medium-term for annual post-war data. This approach differs from conventional decompositions that regard frequencies between 2 and 32 quarters (0 to 8 years) (e.g., Long & Plosser, 1983; King, Plosser & Rebelo, 1988; Christiano & Eichenbaum, 1992; Fiorito & Kollintzas, 1994; Baxter & King, 1999; King & Rebelo, 1999). However, in these papers a strong variation around a linear trend can be observed (cf. Comin & Gertler, 2006). Hence, we can assume that a cyclical activity at the medium frequencies is present in the data but not further regarded in the analysis.

4.2.1 Output

Because we consider lower frequencies of fluctuations as interesting, especially in combination to oil price shocks, we follow Comin and Gertler (2006) in their approach and include frequencies of 2 to 200 quarters (0 to 50 years) in our analysis. The long-term trend then corresponds to frequencies of fifty years and less. The intuition behind this is that this trend is influenced by factors that change only slowly, such as demographics. Especially for the question of how high energy prices affect the economy in the medium-term, this filtering offers the possibility of including endogenous growth effects that have an impact only after some decades.

The medium-term cycle consists of a high-frequency component (2 to 32 quarters; the standard representation of business cycles) and a medium-frequency component of 32 to
200 quarters. It is important to keep in mind that these two components are not meant to be orthogonal, but rather that both frequencies have interrelations and affect each other’s behavior. High-frequency oscillations trigger medium-frequency fluctuations, which in turn influence high-frequency oscillations in their dynamics.

The data we use is annual data from 1953 to 2005 for nonfarm business output, hours worked, labor productivity, energy price, consumption (including nondurables and services), and nonfederally funded R&D. When applicable, we normalize all variables by the working-age population (ages 15 to 64). We use a band-pass filter close to Rotemberg (1999) which is a two-sided moving average filter, in which the moving average depends on the frequencies of the data that one wishes to isolate. This procedure is identical to Comin and Gertler (2006) to ensure comparability. An alternative would be to use the Hodrick-Prescott (HP) (1997) filter, which is often applied in business cycle literature, but the band-pass filter has the advantage that one can be more precise in frequency domain terms about the measurement of the cycle.

![Figure 4.1: Fluctuations of nonfarm business output.](image)

Figure 4.1 depicts the medium-term cycle of per capita output. The blue line shows the percent deviation of per capita output from trend for the medium-term cycle. The

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1Data from U.S. Department of labor; output is normalized by the working-age population (ages 15-64).
red line gives the medium-frequency component. Consequently, the difference between the two lines is the high-frequency component, which is the object of the conventional business cycle analysis.

Over the whole time horizon, the medium-frequency component seems to dominate the swings in economic activity. During the 1960s, it shows a sustained upward movement, while from the late 1960s to the early 1980s, output growth decreased and was small on average. Afterwards, an upward movement persisted until the late 1990s. The high-frequency component fluctuates around the medium-frequency component, but is small compared to the latter.

Additionally, the two frequency types show some interrelations. It appears that periods of stagnation (such as the 1970s and early 1980s) are often accompanied by significant high-frequency downturns. On the other hand, medium-term upward trends are interrupted by only modest high-frequency downturns. Conventional business cycle analysis tends to exclude these movements since these are included in the trend. Including them yields the possibility to analyze interrelations and to consider the impacts of shocks of low persistence for medium-term cycles. Business cycles may be more persistent than conventional research suggests.

4.2.2 Energy prices

Substantial evidence suggests that oil price increases are directly linked to economic activity or even to the onset of recessions (e.g., Hamilton, 1983; 1996; 2003; Hall, 1988; Blanchard & Quah, 1989; Mork, 1989). Several authors have contributed by partially explaining conventional business cycles after an energy shock (such as Rotemberg & Woodford, 1996; Finn, 2000; Barsky & Kilian, 2004; Leduc & Sill, 2004; Aguiar-Conrraria & Wen, 2007). Additionally, research on the impact of energy shocks on the great moderation (a period of remarkable macroeconomic stability) (e.g., Nakov & Pescatori, 2007) as well as on terms of trade (such as Backus & Crucini, 2000) has shown similar results.

Looking at the behavior of the real oil price in absolute levels (before taking logs and applying the filter) in Figure 4.2, we can see that three main surges occurred: The
first and the second happened in the 1970s and thus coincided with the two largest recessions in U.S. history (measured according to the loss of GDP; Temin, 1998). The third spike took place in the last few years.

Figure 4.2: Real oil price, indexed in 1992 US-Dollars per barrel.

Taking a closer look at the real oil price after detrending and filtering in the medium run, Figure 4.3\(^2\) shows that there have been large fluctuations during the last several decades. The medium-term frequencies decrease until around 1969, and then start a large increase in the next decade. Then again, they dropped until the late 1990 and started to rise afterwards. The medium-term cycle was at the beginning of the time period quite stable and started only in the early 1970s to fluctuate around the medium-frequency component. Over the whole time horizon, the largest increase occurred in the early and late 1970s, when the medium-term cycle passed the medium-frequency cycle twice from below from relatively low levels and then rose far above the medium-frequency component. Since the difference between both lines is the high-frequency component, this increase in the oil price was short-term in nature. Only once afterward did the medium-term cycle of the oil price pass the medium-frequency component so clearly from below, in the late 1990s. However, the difference between both lines, i.e., the high-frequency component, before its increase was smaller than before, which

\(^2\)Data from Federal Reserve Bank San Louis.
means that the increase in the high-frequency was relatively smaller than the increases in the 1970s.

Simultaneously to the two large oil price increases in the 1970, the U.S. economy passed through two large recessions. The analysis of the filtered series thus confirms the results of the above-mentioned literature that oil prices appear to have a strong impact on economic variables. Another interesting attribute of the data is that the medium-frequency component of output moves in the opposite way from that of the medium-frequency component of the oil price. During times when the medium-frequency component of output rises, the one corresponding to the oil price falls (1960-69, 1969-79, and the early 1980s until 1997). This again gives reason to support the assumption that the oil price significantly influences macroeconomic activity in the US.

Taking a closer look at the oil market, it is interesting to see that the Organization of Petroleum Exporting Countries (OPEC) changes its production quantities quite severely over time. Figure 4.4 shows the import quantities to the U.S. of OPEC and non-OPEC countries. The amount of petroleum from non-OPEC countries increased quite steadily over time from 1960 until 2005. Only in the early 1970s did the amount exported to the U.S. suddenly increase, and then it returned to a long-term trend. The

\[\text{Deviations from long-term trend} -0.5 0 0.5 1 1.5 1953 1958 1963 1968 1973 1978 1983 1988 1993 1998 2003\]

\[\text{Medium-term cycles} -\text{Medium-frequency component}\]

Figure 4.3: Fluctuations of oil price.

\[\text{Medium-term cycles} -\text{Medium-frequency component}\]

\[\text{Deviations from long-term trend} -0.5 0 0.5 1 1.5 1953 1958 1963 1968 1973 1978 1983 1988 1993 1998 2003\]

\[\text{Medium-term cycles} -\text{Medium-frequency component}\]

\[\text{Deviations from long-term trend} -0.5 0 0.5 1 1.5 1953 1958 1963 1968 1973 1978 1983 1988 1993 1998 2003\]

\[\text{Medium-term cycles} -\text{Medium-frequency component}\]

\[\text{Deviations from long-term trend} -0.5 0 0.5 1 1.5 1953 1958 1963 1968 1973 1978 1983 1988 1993 1998 2003\]

\[\text{Medium-term cycles} -\text{Medium-frequency component}\]

\[\text{Deviations from long-term trend} -0.5 0 0.5 1 1.5 1953 1958 1963 1968 1973 1978 1983 1988 1993 1998 2003\]

\[\text{Medium-term cycles} -\text{Medium-frequency component}\]

\[\text{Deviations from long-term trend} -0.5 0 0.5 1 1.5 1953 1958 1963 1968 1973 1978 1983 1988 1993 1998 2003\]

\[\text{Medium-term cycles} -\text{Medium-frequency component}\]
supply of oil from non-OPEC countries could thus be seen as a rather steady variable in the U.S. economy. However, the quantity of oil from OPEC countries was much more variable. It surged substantially during the 1970s and only returned to a longer upward trend in the early 1980s. Even afterward, the oil supply to the U.S. varied considerably. Consequently, the oil market share of OPEC countries suddenly rose during the 1970s, and political issues as well as production cost shocks in OPEC countries had a larger impact on the price of oil than before. On average, the share of OPEC to non-OPEC oil import to the U.S. was about 42% between 1953 and 2005.

Figure 4.4: Oil import shares.

4.2.3 Endogenous growth and persistence of shocks

Macroeconomic variables do not change during every period but show significant persistence over time. A model that aims at describing macroeconomic behavior needs to take account of this persistence. One possibility is to compare the relations of the standard deviations of the variables in the medium and the high-frequency. If those of the high-frequency component are smaller than those of the medium-frequency component, we can conclude that fluctuations are very persistent in the medium run. Table 4.1 confirms that an important sample of macroeconomic variables is not only short-term
in nature. In all cases, the medium frequencies show a larger variance than the high frequencies.

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>Medium-term cycle</th>
<th>High-frequency component</th>
<th>Medium-frequency component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>3.57</td>
<td>1.90</td>
<td>2.99</td>
</tr>
<tr>
<td>Hours</td>
<td>3.45</td>
<td>1.59</td>
<td>3.06</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>2.76</td>
<td>0.85</td>
<td>2.62</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.60</td>
<td>0.94</td>
<td>2.42</td>
</tr>
<tr>
<td>Investments</td>
<td>8.43</td>
<td>4.31</td>
<td>7.11</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>8.18</td>
<td>2.61</td>
<td>7.73</td>
</tr>
<tr>
<td>Oil price</td>
<td>40.29</td>
<td>16.56</td>
<td>39.56</td>
</tr>
</tbody>
</table>

Table 4.1: Standard deviations.


An alternative way to approach this issue was revealed by Rotemberg (2003), who proposed modeling the high-frequency and medium-frequency fluctuations separately. Accordingly, low persistence markup shocks result in high-frequency fluctuations, whereas variations in the medium frequencies result from exogenous diffusion of new technologies. Therefore, high and medium-frequency fluctuations are orthogonal (i.e., independent and without endogenous mutual impact) and also medium-term fluctuations are driven by exogenous shocks.

Both approaches rely on exogenous mechanisms to prolong the effects of shocks. However, data shows a strong positive correlation between long-term growth rates and the persistence of output fluctuations (Fatás, 2000). This fact cannot be taken account of if disturbances cannot produce persistent fluctuations.

A possibility of modeling the persistence of the impact of shocks on macroeconomic variables is the approach by Comin and Gertler (2003, 2006). They argue that high and medium-frequency oscillations are not orthogonal and that low persistence shocks are endogenously transmitted through the economy. Thereby, the design of endogenous growth mechanisms plays a central role. By applying endogenous productivity, cyclical
variations in both embodied and disembodied technological change can be modeled. The basis is Romer’s (1990) model of an expanding variety of intermediate goods. The model is enhanced by the inclusion of an endogenous rate of adoption of new technologies, which follows Rotemberg (2003). This setup creates the possibility of modeling realistic time lags between the creation and diffusion of new technologies.

Based on these endogenous growth mechanisms, the model is able to illustrate the strong co-movement between output and both disembodied and embodied technological change. R&D intensities vary endogenously, which leads to a pro-cyclical movement of productivity that can also be seen in reality (cf. also Barlevy, 2007). R&D is very variable at both frequencies and also co-moves strongly with output in the high-frequency (cf. Comin, 2009).

4.3 Model

Based on Comin and Gertler (2006), we additionally apply energy in the production of the home country and introduce a second country that produces energy. The rationale is that the supply of energy is exogenous to the households of the home country and that it is ensured that revenues from energy supply go abroad. This approximates energy supply in reality, although we are aware that some percentage of energy supply is domestic. We abstract from this share.

We compare the consequences of an exogenous price shock to those of an exogenous productivity shock of the energy producing country. The reason is that energy production cannot be modeled realistically in this simplified model of the energy country. To take account of the limitation this offers to the model, we compare the outcomes of both types of shocks.

4.3.1 Home country

The nestings of the production follow Comin and Gertler (2006). The motivation for this specific structure is to describe dynamics of both short- and medium-term cycles and to calibrate the results to U.S. data.
4.3. MODEL

The home country is a country with endogenous productivity dynamics based on Romer (1990) and Grossman and Helpman (1991). By investing in new blueprints of technology, the country can increase its Hicks-neutral productivity. Gains from specialization augment the efficiency in the economy and allow for growth.

Production

There are two sectors in the home economy. One produces a consumption good (c) and the other produces capital (k). The production in each sector is realized on several levels. A final output composite, \( Y_x, x = c, k \), is assembled from final goods \( Y^j_x \) of firms \( j \) in sector \( x \) according to

\[
Y_{x,t} = \left( \int_{N_{x,t}}^N (Y^j_x) \frac{1}{\mu_{x,t}} \frac{d}{dj} \mu_{x,t} \right)
\]

These final goods are produced by monopolistic competitors that can charge a time-variant markup \( \mu_x > 1 \). Individual firms take both the markup \( \mu_x \) as well as the number of final goods producers \( N_x \) as given. However, the number of firms is endogenous to the model. The entry of final goods producers to the market is free, but the markup is inversely related to the number of firms. Therefore, pro-cyclical competitive pressure ensures a balanced growth path of the economy.

The final output composite of the capital sector is used in the production of final goods in both sectors. Each final good producer maximizes over the production function

\[
Y^j_{x,t} = \left[ (E^j_{x,t})^{\alpha_e} (U^j_{x,t} K^j_{x,t})^{\alpha} (L^j_{x,t})^{1-\alpha-\alpha_e} \right]^{1-\gamma} [M^j_{x,t}]^\gamma
\]

and uses four inputs to production: utilized capital, \( U^j_{x,t} K^j_{x,t} \) with \( K^j_{x,t} \) being capital from firm \( j \) in sector \( x \) and \( U^j_{x,t} \) being the capital utilization rate; labor \( L^j_{x,t} \); energy \( E^j_{x,t} \); and an intermediate goods composite \( M^j_{x,t} \). The value share of the intermediate goods composite equals \( \gamma \) and the value shares of utilized capital and energy are \( \alpha \) and \( \alpha_e \), respectively. Using a Cobb-Douglas production function with an elasticity
of substitution of unity appears to be an optimistic assumption of the possibility of substituting labor and capital for energy. To take account of the lower substitutability in reality, we introduce adjustment cost for labor and adjust some parameters for a change in incentive decisions (cf. Chapters 4.3.1 and 4.4.1).

The sectoral intermediate goods composite $M^j_x$ is itself an aggregate of differentiated intermediate goods $M^{j,k}_x$: 

$$M^j_{x,t} = \left( \int_0^{A_{x,t}} (M^{j,k}_{x,t})^{\frac{1}{\vartheta}} \, dk \right)^{\vartheta}$$  (4.3)

with $\vartheta > 1$. The specialized intermediate goods have the same production function as the final output composite of the consumption sector.

Growth in the economy is driven by an increasing number $A_x$ of these intermediate goods, which is determined endogenously by investment decisions in R&D and adoption. Changes in these activities induce long pro-cyclical swings. Looking at a shorter time frame, the magnitude of fluctuations after a shock depends, among others, on the entry/exit behavior of the number of final good firms. This, in turn, is determined by the markup each final good firm can charge. The approach chosen by Comin and Gertler is based on Galí and Zilibotti (1995) and implies pro-cyclical net entry and therefore competitive pressure, which results in counter-cyclical changes in the markup. This relation stems from the fact that the elasticity of substitution among final good firms in each sector rises in $N_x$. As the markup equals the inverse of the elasticity of substitution, it must consequently fall (for further information cf. Galí & Zilibotti, 1995). Therefore, the relation must hold that

$$\mu_{x,t} = \mu(N_{x,t})$$  (4.4)

and
The markup is needed to liquidate per-period operation costs, $b_x \Psi_t$, that incur for each final goods producer. The costs consist of a sector-specific parameter $b_x$ and a time-varying component $\Psi_t$. Each individual firm takes these costs as given and ensures that profits are equal to operating costs. This zero-profit condition determines the markup and the number of final good firms in each sector. As the economy grows over time, it needs to be ensured that the markup does not become a negligible cost. Otherwise, a balanced growth path could not be ensured. Therefore, $\Psi_t$ drifts up over time in a proportionate manner to the value of the aggregate capital stock. Comin and Gertler propose as possible interpretation that operating costs correspond to the sophistication of an economy.

R&D and adoption

The process of research up until the introduction of a new technology takes place in two steps. First, R&D is conducted and blueprints are invented. In a second step, producers of specialized intermediate goods need to dedicate resources to convert the blueprints into a usable technology. This process allows taking into account the fact that technology adoption takes time.

Research is conducted by innovators in each sector that sell the rights to goods to adopters (producers of differentiated intermediate goods). To finance their activity, they need to borrow from households. The consumption good is used as input and the stock of innovations, $Z_x$, is accumulated according to

$$Z_{x,t+1} = \psi_{x,t} S_{x,t} + \phi Z_{x,t}$$  \hspace{1cm} (4.6)
parameter $\psi_x$ that each innovator takes as given. The optimal amount of R&D is determined by solutions to maximization problems of all innovators. The stock of innovations from the current period, $Z_x$, becomes obsolete with the probability $1 - \phi$ until the next period.

The technology coefficient $\psi_x$ depends on aggregate conditions, such as positive spillovers of the sectoral stock of innovations (i.e., it increases linearly in $Z_x$) and a sectoral scaling factor $\chi_x$ for calibration. Furthermore, Comin and Gertler introduce a congestion externality $[\Psi_t S_x^{1-\rho}]^{-1}$:

$$
\psi_{x,t} = \chi_x Z_{x,t} [\Psi_t S_x^{1-\rho}]^{-1}
$$

(4.7)

The congestion effect increases the cost of developing new products as soon as the amount of R&D raises. The inclusion of the congestion factor is supposed to ensure that the growth rate of innovations is stationary. As in equilibrium the elasticity of creating new blueprints becomes $\rho$, this parameter can be freely chosen to calibrate the model to data.

The process of adopting the newly developed blueprints (i.e., the conversion of $Z_x$ to $A_x$) takes time, and the variables are modeled to vary pro-cyclically (to be consistent with the data). Adopters need to buy the blueprints from the innovators and then convert the technology into a usable form. To finance this process, they obtain loans from the households. After a successful conversion, the adopters produce the goods and sell them to the producer of the intermediate goods composite.

The process of adopting a new technology is assumed to be successful with a probability $\lambda_x < 1$, which is increasing in adoption expenditures $H_x$:

$$
\lambda_{x,t} = \lambda \left( \frac{A_{x,t}}{\Psi_t} \cdot H_{x,t} \right)
$$

(4.8)

with $\lambda' > 0$ and $\lambda'' < 0$. The scaling factor $\frac{A_{x,t}}{\Psi_t}$ is again exogenous for each individual adopter and ensures a balanced growth path; otherwise adoption could increase
infinitely as the economy grows. Each adopter has the choice to invest more or less in the adoption process and thus influence the probability to be successful. Adoption expenditures are composed from the consumption good. If the conversion of a new technology to a usable technology has not been successful, an adopter may try it again in the next period. This all contributes to the slow diffusion of technologies on average. The adoption rate per sector therefore equals to

\[ A_{x,t+1}^q = \lambda_{x,t} \phi[Z_{x,t}^q - A_{x,t}^q] + \phi A_{x,t}^q \]  \hspace{1cm} (4.9)

The term in brackets is the stock of products the producers of intermediate goods own but did not yet convert. The above equation influences cyclical movements in two ways: First, pro-cyclical investments in R&D increase \( Z_{x,t}^q \) and, second, pro-cyclical adoption expenditures raise \( A_{x,t}^q \), which results in a pro-cyclical variation in adoption rates.

Each adopter calculates profits to find the optimal amount of adoption expenditures to be spent. In particular, if \( \Pi_{m,x,t} \) is profits in sector \( x \) earned on a differentiated intermediate good, \( V_{x,t} \) is the value of an intermediate good, and \( J_{x,t} \) is the value of an unadopted blueprint, than the optimal amount of adoption expenditures, \( H_{x,t} \), is calculated according to

\[ J_{x,t} = \max_{H_{x,t}} \left[-H_{x,t} + \phi E_t (\Lambda_{t+1} [\lambda_{x,t} V_{x,t+1} + (1 - \lambda_{x,t}) J_{x,t+1}]) \right] \]  \hspace{1cm} (4.10)

and

\[ V_{x,t} = \Pi_{m,x,t} + \phi E_t (\Lambda_{t+1} V_{x,t+1}) \]  \hspace{1cm} (4.11)

Hereby, \( \Lambda_{t+1} \) equals the discount factor and \( E_t \) denotes expectations.
Households

A number of households supply heterogeneous types of labor and therefore hold market power. This monopolistic structure allows us to include a wage markup, which provides the source for the main exogenous disturbances in Comin and Gertler (2006).

This approach is implemented thus: There is a continuum of households of measure unity. Total labor input $L_t$ is a CES aggregate of the different types of labor $L^h_t$ according to

$$L_t = \left( \int_0^1 (L^h_t)^{\frac{1}{\mu_{w,t}}} dh \right)^{\mu_{w,t}} \quad (4.12)$$

with $\mu_w$ being the elasticity of substitution between the types of labor. This corresponds to the markup each household can charge on household’s cost to supply labor. Therefore, the wages of all households, $W_t^h$, are assembled to the wage index $W_z$ according to

$$W_t = \left( \int_0^1 (W_t^h)^{\frac{1}{1-\mu_{w,t}}} dh \right)^{1-\mu_{w,t}} \quad (4.13)$$

and the demand for labor of type $h$ is

$$L^h_t = \left( \frac{W_t^h}{W_z} \right)^{-\frac{\mu_{w,t}}{1-\mu_{w,t}}} L_t \quad (4.14)$$

It is assumed that although households differ in the type of labor they supply, consumption is distributed symmetrically across households. The utility function then equals
4.3. MODEL

\[ \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t - \frac{(L_t^h)^{1+\zeta}}{1 + \zeta} L_t^h \right] \]  

(4.15)

We choose this function for the following reason. To assume that utility comes from \( \ln C_t \) and \( \frac{(L_t^h)^{1+\zeta}}{1+\zeta} \) is as standard assumption for a household that can choose the amount of labor it can supply and is also used by Comin and Gertler. We include \( \frac{L_t^h}{L_{t-1}^h} \) in the function to take account of costs to substitute labor for energy. Since the households are assumed to have disutility not only from a change in labor supply but also from the absolute level of labor supply, we add the term \( L_t^h \) to the utility function. We hereby follow a large strand of literature that applies adjustment costs for capital. Because energy is used in a Cobb-Douglas way in the production functions of \( Y_c \) and \( Y_k \), it is very easy to substitute labor for energy. By assuming that the households dislike changing the amount of energy used in production (for example, after a shock in the energy price), we can limit the amount that labor substitutes for energy.

The budget constraint of households contains several parts. All households are assumed to consume and to save. Savings are used to invest in firms that do research and that adopt new technologies. The investments in research firms are summarized in bonds \( B_{t+1} \). \( R_t B_t \) thus equals the revenues from loans that households make at \( t-1 \) and that are payable in \( t \). Additionally, households receive profits from monopolistic firms that convert blueprints into usable technologies (adopters) and then sell these intermediate goods. Since households hold equities of the firms (the intermediate good producers can only pay back their loans over time), they also have the right to earn the profits. Additionally, households own capital, so changes in the value of capital influence the resources of households. Therefore, the budget constraint of the households looks like

\[ C_t = W_t^h L_t^h + \Pi_t + [D_t + P_{t}^k]K_t - P_{t}^k K_{t+1} + R_t B_t - B_{t+1} - T_t \]  

(4.16)

The optimization problem of the household is therefore to maximize (4.15) subject to (4.16) as well as the labor demand curve (4.14). It will result that the wage equals the
marginal rate of substitution between labor and consumption enhanced by the markup:

\[ W_t^h = \mu_{w,t} \left( \frac{L_t^h}{L_{t-1}^h} \right)^\zeta C_t \]  
(4.17)

Due to the symmetrical utilization of labor in production, all households charge the same wage in the equilibrium and supply the same amount of labor.

**Government**

Government spending is financed with lump-sum taxes according to

\[ G_t = T_t \]  
(4.18)

### 4.3.2 Energy producing country

Energy is produced in a foreign country. In this country, households supply country-specific labor, \( L_e \). Energy is produced by labor as well as a productivity parameter \( Z_e \). This productivity parameter is supposed to describe the impact of OPEC oil production (cf. Chapter 4.2.2). Labor and the productivity parameter are combined in a Cobb-Douglas manner according to

\[ E_t = \left( \frac{1}{Z_{e,t}} \right)^\nu L_{e,t}^{(1-\nu)} \]  
(4.19)

with \( \nu \) being the share of OPEC oil production in total production of energy.

A representative household of the energy country maximizes utility according to

\[ E_0 \sum_{t=0}^{\infty} \beta^t \ln(C_{e,t} - \kappa L_{e,t}^e) \]  
(4.20)
The optimization of the energy producer and of the household yields for the equilibrium of the labor market in the energy country

\[ \kappa t L_{e,t}^{\tau -1} = p_{e,t}(1 - \nu) \frac{E_t}{L_{e,t}} \]  

(4.21)

Although energy is the only good in this economy, it cannot be consumed by the household. The consumption good of the home country needs to be imported to the energy country and as a trade balance, it must hold that

\[ C_{e,t} = p_{e,t} E_t \]  

(4.22)

### 4.3.3 Equilibrium conditions

The model contains three state variables, which are the aggregate capital stock, \( K_t \), the total stocks of sectoral blueprints, \( Z_c \) and \( Z_k \), and the total stock of adopted intermediate goods, \( A_c \) and \( A_k \). The optimization of the household as well as the production constraints characterize the equilibrium.

**Aggregate resources**

\( Y_t \) denotes aggregate net value added output, which equals the output of both sectors, \( Y_x \), minus expenditures on specialized intermediate goods, \( (A_x)^{1-\vartheta}M_x \), and operating costs, \( \psi_x \).

\[ Y_t = \sum_{x=c,k} [P_{x,t}Y_{x,t} - (A_{x,t})^{1-\vartheta}M_{x,t} - \psi_{x,t}] \]  

(4.23)

\( P_c \) is normalized to 1 in the following analysis. As intermediate goods are produced from the consumption good, their production costs must equal \( M_x \) normalized by the per-unit cost of producing, \( (A_x)^{\vartheta-1} \). This reflects the increase in productivity by gains.
from specialization according to their production function (4.3).

Once \( Y_t \) has been produced, it is used for consumption in the home country, \( C_t \), consumption in the energy producing country, \( C_e \), investment, \( P_k Y_k \), and the total costs of R&D and adoption, which equals \( \sum_{x=c,k} [S_{x,t} + (Z_{x,t-j} - A_{x,t})H_{x,t}] \). Thus, the deployment of \( Y \) adds up like

\[
Y_t = C_t + C_{c,t} + P_k Y_k + G_t + \sum_{x=c,k} [S_{x,t} + (Z_{x,t-j} - A_{x,t})H_{x,t}]
\] (4.24)

Moreover, capital accumulation equals to

\[
K_{t+1} = (1 - \delta(U_t))K_t + Y_{k,t}
\] (4.25)

Capital depreciation increases in the utilization rate of capital, i.e., for each firm, the depreciation rate is given by \( \delta(U_t) \) with \( \delta' > 0 \). This approach follows Greenwood et al. (1988).

**Factor markets and aggregated variables**

Some aggregations must hold in equilibrium. First, the total quantity of labor \( L_t \) is used in both sectors and it must hold that

\[
L_t = L_{c,t} + L_{k,t}
\] (4.26)

Additionally, total energy that is imported is also shared between sectors:

\[
E_t = E_{c,t} + E_{k,t}
\] (4.27)
4.3. **MODEL**

Using (4.26), the sectoral production functions in a symmetric equilibrium can be rewritten as

\[
Y_{x,t} = (N_{x,t})^{\mu_{x,t}-1} \left[ (E_{x,t})^{\alpha_e} \left( \frac{U_t K_t}{L_t} \right)^{\alpha} (L_{x,t})^{1-\alpha_e} \right] (M_{x,t})^\gamma
\]

where \((N_{x})^{\mu_{x,-1}}\) reflects the efficiency gains from specification implied by the final goods sector.

From the optimization calculations of the producer of the final output composite, the demand for labor, capital, and energy as well as the optimal utilization of capital must be equal to

\[
(1 - \alpha - \alpha_e)(1 - \gamma) \frac{P_{x,t} Y_{x,t}}{L_{x,t}} = \mu_{x,t} w_t
\]

\[
\alpha(1 - \gamma) \frac{P_{x,t} Y_{x,t}}{K_{x,t}} = \mu_{x,t} [D_t + \delta(U_t) P_{h,t}]
\]

\[
\alpha(1 - \gamma) \frac{P_{x,t} Y_{x,t}}{U_t} = \mu_{x,t} \delta'(U_t) P_{h,t} K_{x,t}
\]

\[
\alpha_e(1 - \gamma) \frac{P_{x,t} Y_{x,t}}{E_{x,t}} = \mu_{x,t} P_{e,t}
\]

The optimal use of the intermediate goods is

\[
\gamma \frac{P_{x,t} Y_{x,t}}{M_{x,t}} = \mu_{x,t} P^M_{x,t}
\]

(4.28)
where $P^M_x$ is the relative price of the intermediate goods composite in sector $x$.

It must additionally always hold that

$$\frac{K_{x,t}}{K_t} = \frac{L_{x,t}}{L_t} = \frac{E_{x,t}}{E_t}$$

(4.34)

The intertemporal Euler equation must equal

$$\mathcal{E}_t \left( \Lambda_{t+1} \frac{D_t + P_{k,t+1}}{P_{k,t}} \right) = \mathcal{E}_t(\Lambda_{t+1} R_{t+1})$$

(4.35)

and assuming that arbitrage between capital acquisition and loans to adopters and innovator is possible implies

$$\Lambda_{t+1} = \beta \frac{C_t}{C_{t+1}}$$

(4.36)

**Shock process**

We apply two types of shock in this section. First, the energy price is shocked, which decreases demand for energy in the home country. The shock process assumed equals

$$\ln P_{e,t+1} = 0.8626 \ln P_{e,t} + \epsilon_{p,t}$$

(4.37)

after estimating the autoregression parameter of the filtered oil price.

Furthermore, OPEC’s supply to the U.S. shows great variability compared to total oil imports. As this can be seen as one source of oil price shocks, we assume that OPEC supply decisions, i.e., the productivity parameter of the energy country, $Z_e$, are exogenous to all agents in the model. We thus apply the shock process
\[ \ln Z_{e,t+1} = 0.8626 \ln Z_{e,t} + \epsilon_{z,t} \] (4.38)

with \( \epsilon_t \) being the shock to the system.

4.4 Simulation results

In this section, we apply the model to exogenous shocks to the energy price \( P_e \) and the productivity parameter \( Z_e \). We first present the benchmark calibration and then show results of simulations.

4.4.1 Model calibration

The model calibration is based on the model of Comin and Gertler (2006), but we adjusted some parameters for a better calibration. The discount rate \( \beta \) is set at 0.95 and the capital share in production, \( \alpha \), equals 1/3. The value share of energy in production, \( \alpha_e \), equals 0.8 and of materials, \( \gamma \), is 0.5. The steady-state relation of government spending to output, \( G_t/Y_t \), is 0.2, the depreciation rate, \( \delta \), is 0.1, and the inverse of the Frisch elasticity of labor supply \( \zeta \), equal to 0.8. Furthermore, the change in the depreciation rate with respect to the utilization rate, \((\delta''/\delta')U\) is equal to 1/3. The steady-state markups \( \mu_c \) and \( \mu_k \) are 1.15 and 1.2, respectively. The elasticity of \( \mu_x \) with respect to \( N_x \) equals -1. The survival probability of new technologies, \( \phi \), is 0.97. As gross markup for specialized intermediate goods, \( \vartheta \), we assume 2 and the elasticity of new intermediate goods with respect to R&D, \( \rho \), equals 0.8. The diffusion possibility of innovations, \( \lambda_x \), is 0.1. The markup on wage, \( \mu_w \), equals 1.2.

4.4.2 Energy price shocks and medium-term fluctuations

Because we are interested in how the economy reacts in the medium-term to a shock to the energy price, we first look at the impulse-response functions and, in a later step, at the standard deviations that the model creates and compare them to the data.
Shocks to the oil price were considerable during the last decades, and we estimate the pertinence of a shock to the oil price to explain the patterns in the data. We assume that the shock to the oil price is the only driving force, but we are aware that this is a rather unrealistic assumption. In reality, a sum of different shocks may hit the economy, leading to a reaction that cannot be explored by the analysis of only one shock. However, by concentrating on the effects of one shock, we can get an intuition of how an economy reacts to exactly this impulse and find out whether the general patterns of the data could be explained by this specific type of shock.

**Impulse-response functions**

Before taking a closer look at the standard deviations, it is interesting to see the structural reactions of the model to an oil price increase. Figure 4.5 shows the impulse-response functions of several variables: output, consumption, investments, capital utilization, the capital stock, labor, total factor productivity, labor productivity, energy productivity, energy use, the oil price, and the inverse of the productivity of the energy producing country.

A rise in $p_e$ decreases demand for energy. As a consequence, the other inputs to production, labor and capital utilization, also decline, albeit not as much as energy. Final output producers optimize the amount of each input according to the input prices and substitute labor and capital utilization for energy. Therefore, the decline in these goods is smaller than that of energy. The resulting smaller amount of inputs to production leads to a decrease of output, both of the consumption good as well as of the investment good. The consumption good climbs back to trend over time, but does not reach the original level due to the endogenous permanent decline in productivity. It levels off at a new steady state.

This consequent drop in productivity stems from the fact that the initially lower amount of the consumption good is used as input to research and adoption. As fewer resources are devoted to both, research and adoption outcomes decline strongly. Accordingly, new technologies arrive at a lower rate, and total factor productivity decreases. These effects are in line with those of Comin and Gertler’s paper. Although this jump and the resulting long-term difference in levels appears to be strong, it must be kept in
mind that the households still have strong incentives to invest in capital to substitute for energy.

But research and adoption are not only meant to substitute for energy; labor is also
 supplied at a higher rate in the longer run. Households want to maximize their utility by consuming as much as possible. This is only attainable if their income is the maximal level, which they wish to achieve by increasing labor supply. As a consequence, labor productivity falls even further and reaches a lower steady state level than right after the shock. The endogenous dynamic effects result in a long-term decline of total factor productivity due to a temporal oil price shock.

However, if we regard the productivity of energy in production, we can see that it gradually approaches its initial level. This is only a logical consequence of the fact that final good producers apply an input to production according to its relative prices to other inputs. Since the price of energy is exogenously fixed and returns to its pre-shock level, the marginal product and therefore its productivity needs to be equal to the pre-shock level. The productivity in the energy producing country decreases (and $Z_{e,t}$ increases) due to the exogenously given price surge and the fact that the market must always clear.

**Standard deviations for an oil price shock**

The magnitude of the reactions of the variables to an oil price increase compared to the data shows that most deviations closely fit the data (Table 4.2). Generally, fluctuations of the model in the medium-term are larger than those of the medium-frequency component. Fluctuations in the high-frequency component are even smaller.

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>Medium-term cycle</th>
<th>High-frequency</th>
<th>Medium-frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-50</td>
<td>0-8</td>
<td>8-50</td>
</tr>
<tr>
<td>Output</td>
<td>data 3.57</td>
<td>model 3.52</td>
<td>data 1.90</td>
</tr>
<tr>
<td>Hours</td>
<td>3.45</td>
<td>1.51</td>
<td>1.59</td>
</tr>
<tr>
<td>Labor productivity</td>
<td>2.76</td>
<td>2.54</td>
<td>0.85</td>
</tr>
<tr>
<td>Consumption</td>
<td>2.60</td>
<td>2.63</td>
<td>0.94</td>
</tr>
<tr>
<td>Investments</td>
<td>8.43</td>
<td>12.68</td>
<td>4.31</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>8.18</td>
<td>8.31</td>
<td>2.61</td>
</tr>
<tr>
<td>Energy consumption</td>
<td>5.63</td>
<td>11.98</td>
<td>1.80</td>
</tr>
<tr>
<td>Oil price</td>
<td>40.29</td>
<td>40.29</td>
<td>16.56</td>
</tr>
</tbody>
</table>

Table 4.2: Standard deviations after oil price shock: Data and model.

Additionally, we can see that consumption smoothing results in standard deviations
are smaller than those of output. Furthermore, investments show an even stronger variability. It is crucial to verify these ratios before going into more detail, because they show the general behavior of the model.

Comparing the standard deviations of output shows the eligibility of the model to matching the underlying data. In particular, the medium-term cycle is well mirrored by the model. The high-frequency and medium-frequency components also are closely matched. Moreover, the fluctuations of consumption and labor are very similar to the data. Although the high-frequency component of labor varies more strongly in the model and the medium-frequency component shows a comparably smaller variability, the medium-term cycle of labor shows a suitable standard deviation.

The dynamic components of the model, i.e., R&D and investments, show a pertinent match to the U.S. data. Research activities vary similarly to the data in the medium-term cycle and a bit more in the high-frequency component. The medium-frequency component consequently varies slightly less than the data. Investments are relatively strong compared to the data, but they are still in a reasonable range. Fluctuations of the oil price are by definition the same in the medium-term cycle, since all standard variations are normalized according to its variations.

The only variable that varies much more strongly than the data is energy consumption. In all three frequencies, the standard deviations are higher than those in the data. We can conclude that the energy market, i.e., the conjunction of production (supply) and consumption (demand) is not fully modeled. This is a consequence mainly of two assumptions: First, the energy producing country is only imprecisely modeled. If we would like to show the exact characteristics of supply, we would need to model it in much more detail. Second, the demand of energy is modeled very variable. Energy is used with a Cobb-Douglas function, which means that other inputs can easily substitute for it. This assumption is unrealistic and can only provide an approximation of the realistic demand. As a consequence, the use of energy in the model fluctuates strongly if the oil price is shocked.
A shock to the supply of energy

These calculations show that a shock to the oil price and the resulting calibration can quite well replicate the data. An alternative way to estimate the influence of an oil price shock is to shock the productivity of the energy supplying country \( Z_e \). This mimics a shock to the production of oil in the OPEC countries. The model is then calibrated to match the energy consumption in the U.S. economy. The fluctuations in energy consumption are less strong than those in the oil price, and the model consequently fluctuates less strongly. The price fluctuations that would be necessary to trigger these variations in the energy consumption are much lower than in the case above. According to the data, this underestimates the price fluctuations of the last decades. The results can thus be seen as a lower bound of the reactions of the variables to a change in the oil price with the magnitude seen in the data.

<table>
<thead>
<tr>
<th>Frequencies</th>
<th>Medium-term cycle</th>
<th>High-frequency</th>
<th>Medium-frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>output</td>
<td>hours</td>
<td>labor productivity</td>
</tr>
<tr>
<td>0-50</td>
<td>3.57</td>
<td>3.45</td>
<td>2.76</td>
</tr>
<tr>
<td>0-8</td>
<td>1.65</td>
<td>0.71</td>
<td>1.19</td>
</tr>
<tr>
<td>8-50</td>
<td>1.90</td>
<td>1.59</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>0.78</td>
<td>0.30</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>2.99</td>
<td>3.06</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>1.63</td>
<td>0.63</td>
<td>1.07</td>
</tr>
</tbody>
</table>

Table 4.3: Standard deviations after supply shock: Data and model.

The shocks are conducted thus: We shock the productivity of the energy country, \( Z_e \), and calibrate the magnitude of the shock to the standard deviation of energy consumption. We use the autoregressive component of the energy price to account for the persistence of the shock. By using the autoregressive component of the oil import from OPEC countries, we would need to model the energy exporting country in more detail to account for the persistence of the shock.

The results show that the magnitude of a shock to the energy supply is much lower (Table 4.3). All variables show standard deviations that are about half of the standard deviations found in the data. However, the relations of the standard deviations are
structurally similar and show that an energy supply shock can mimic the fluctuations in the data qualitatively. Additionally, the relations of the standard deviations in the medium-term cycle and the two components are also maintained. We can conclude that, although mode cannot quantitatively match the data after a shock to energy supply, the structural consistency can still be maintained.

4.5 Conclusions

The influence of energy price shocks on the economy in the medium-term is a topic that matters enormously. This chapter examines the effects on the basis of the U.S. economy and shows that the consequences of energy price shocks can qualitatively be well modeled. The results show that energy price shocks strongly influence the economy. Particularly in the medium-term, oil price shocks show a large persistence and influence the long-run equilibrium via endogenous growth dynamics.

We differentiate between two components in business cycles. The high-frequency component mimics the fluctuations of 0 to 8 years. These variations are conventionally analyzed in real business cycle theory. Additionally, we analyze the medium-frequency component (8 to 50 years). Oscillations at these frequencies seem to result from the high-frequency fluctuations and show that macroeconomic oscillations are a persistent phenomenon.

Building on these assumptions, energy price shocks have effects not only on the few years that follow the shocks. Immediate changes in production, investment, and consumption behavior pass into medium-frequency oscillations. Accordingly, oil price shocks have consequences that are not taken into account in the literature so far.

Although the value share of energy in gross domestic product is fairly small, the consequences of a rise in its price are considerable. The medium-frequency outcomes confirm that relatively small disturbances in the high-frequency have prolonged effects in the period of 8 to 50 years. The medium-term cycle, which includes both the high- and medium-frequency component, shows strong effects of oil price shocks on the economy. When the model is calibrated to the energy consumption, the effects are smaller but qualitatively equally valuable. The latter results can be seen as a lower bound of reac-
tions, which leads to the implication that energy price shocks cannot have triggered the 
fluctuations in the data alone but can have had, added to other shocks, a non-negligible 
impact on the U.S. economy over the last decades.
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