A New Methodology for the Placement of reinforcement Doublers on Composite Space Structures

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A NEW METHODOLOGY FOR THE PLACEMENT OF REINFORCEMENT DOUBLERS ON COMPOSITE SPACE STRUCTURES

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Abstract

The occurrence of compliances near cut-outs of space structures demand for local reinforcement with laminate doublers in order to satisfy the specifications concerning the eigenfrequency. The placement of such layers can be found in an automated optimization routine. A new methodology named the Ghost Layer Concept is proposed. Ghost Layers are virtual laminate layers with defined material and orientation angle but no thickness. These layers influence neither stiffness- nor mass matrix and thus do not change the structural behaviour of the model. However, a Sensitivity Analysis can be performed for both, Ghost Layers and regular layers, by deriving the eigenfrequency with respect to layers thicknesses. The gradient indicates which layers of which elements have to be thickened in order to increase the eigenfrequency of the overall structure. Since the mass depends linearly on the element thicknesses, the gradient provides information how to increase the eigenfrequency with smallest mass growth possible. Introducing an arbitrary number of Ghost Layers and thickening the layers with maximal gradient value in an iterative procedure generates doublers. Tests have shown that good solutions can be found in short time.

1 Introduction

Spacecraft structures as satellites must be extremely light weight. Additionally, eigenfrequencies must exceed a given threshold in order to prevent dynamic coupling between launcher and spacecraft. Cut-outs, necessitated by man holes in payload carriers or openings for measuring devices on satellites as well as load introduction points reduce the eigenfrequency. Such weakening calls for mitigation by local laminate reinforcement. The placement of such reinforcements, also called doublers, is not trivial at all since eigenfrequency is not only depending on stiffness but also on mass distribution. The combination with the added complexity of multidirectional anisotropic composite laminates may cause human intuition to fail at finding good design solutions. Automated design processes can help improving solutions quality and reduce time-costs.

The possibility to tailor the properties of laminated composites to local requirements at different areas of the structure is an advantage. Varying laminates locally implies the problem of distributing material in design spaces. If void is accepted as a virtual material, topology- and material optimization takes place simultaneously. The Free Material Approach (FMA), which is based on homogenization methods, has been proposed in [1][2]. The method includes a direct search method that generates optimal material distribution in terms of virtual homogenized material properties. Lund et al. [3],[4],[5],[6] proposed a voxel-based parameterization concept named Discrete Material Optimization (DMO) which is inspired by the SIMP method. Laminates can be varied locally in terms of orientation angle. However, patches with uniform orientation angles have to be predefined and are not result of the optimization. The method of Hansel and Becker [7] works with a similar parameterization scheme. The initial implementation bases on an iterative heuristic search. In later approaches, Evolutionary Algorithms (EA) are employed [8].

A group of methods partitions the structure into regions where independent parameterization schemes can be defined. Some layers have to cover several sections in order to ensure connectivity of the structure [9][10]. Locally varying laminates can be achieved adding attributes associated with each ply, i.e. the regions it covers on the structure [11]. The concept of global plies ([12]) understands a varying laminate as the result of the application of fabric patches onto the structure. The overlappings lead to sections of different laminates in different regions. The geometry of every global ply is parameterized with a geometric CAD-environment in [13]. Patch shape, materials and orientations are then optimized by an EA. In order to reintroduce more freedom in design, the concept of global plies has been brought back to a voxel-based representation [14]. The usage of stochastic algorithms (e.g. EA) needs many evaluations of the structural model, which is normally performed by commercial software. Therefore, an application of these algorithms may be problematic in an industrial environment since the number of licenses is restricted.

The here proposed method tries to reduce the number of necessary structural evaluations to a minimum. A Sensitivity Analysis (SA) is performed on element level which provides information where doublers should be placed. Together with the proposed *Ghost Layer Concept*, iterative process leads to a propagation of these doublers. Thus, not only material and orientation but also the doublers geometry is a result of the process.

2 Sensitivity Analysis

The new designs generated with the proposed process should have maximal first eigenfrequency and minimal mass. The eigenfrequency itself is not only depending on the stiffness of the structure but also on its mass and on its mass distribution in particular. Usually, the eigenfrequencies of a structure drop when increasing the mass. Thus, placing reinforcements on a structure can cause also a drop of eigenfrequencies. Generally, maximization of eigenfrequency and minimization of mass is a trade-off and a clever placement of reinforcement doublers is not trivial at all. The problem becomes even more complex when doublers are made of anisotropic materials such as unidirectional carbon fiber reinforced composites. A sensitivity analysis provides the basis for a efficient placement of doublers. The first eigenvalue of the system (which is proportional to the square of the first eigenfrequency) is differentiated with respect to the thicknesses of all shell elements of the structure. This provides a gradient with as many entries as there are elements. The impact of mass and stiffness of a structure on its dynamic behaviour is expressed with equation (1).

$$\left(\mathbf{K} - \lambda \mathbf{M}\right) \mathbf{\Phi} = 0 \tag{1}$$

There, **K** stands for the stiffness- and **M** for the mass matrix. λ and $\mathbf{\Phi}$ are the eigenvalues and the eigenvectors of the system. Differentiating with respect to the element thicknesses t_i and making some rearrangements leads to the known sensitivity equation of the eigenvalues (2), which is described and employed in several publications [3],[15],[16][17]. The index n emphasizes that only one eigenvalue and its eigenvector are considered.

$$\frac{\partial \lambda_n}{\partial t_i} = \frac{\mathbf{\Phi}_n^T \left(\frac{\partial \mathbf{K}}{\partial t_i} - \lambda \frac{\partial \mathbf{M}}{\partial t_i} \right) \mathbf{\Phi}_n}{\mathbf{\Phi}_n^T \mathbf{M} \mathbf{\Phi}_n} \tag{2}$$

This equation delivers a value for every element expressing the eigenvalue increase when thickening an element by an infinitesimal increment. The higher the value the higher the eigenfrequency increase. By comparing these values with each other, it can be predicted where reinforcement doublers should be placed preferably. This equation can be employed to structures with homogeneous material properties over the shell thickness. For inhomogeneous materials like fiber reinforced laminates it cannot be used without making some modifications since stiffness and density can vary over thickness. Thus, the eigenvalue is not differentiated with respect to the shell thickness but with respect to the thickness of every single ply of the composite. The sensitivity equation forms to

$$\frac{\partial \lambda_n}{\partial h_{i,j}} = \frac{\mathbf{\Phi}_n^T \left(\frac{\partial \mathbf{K}}{\partial h_{i,j}} - \lambda \frac{\partial \mathbf{M}}{\partial h_{i,j}} \right) \mathbf{\Phi}_n}{\mathbf{\Phi}_n^T \mathbf{M} \mathbf{\Phi}_n} \tag{3}$$

where i is the number of elements and j the number of layers. Thus, the adapted equation provides a gradient with as many entries as elements times layers. Therefore, it is not only a measure of the preferred areas but also of the preferred material choice. Assuming that the structure consists of a laminated composite whose single layers are identical and only material orientation angles differ, the gradient proposes which layers with which orientations should be placed where.
It may seem at first that the determination of sensitivities is computationally intensive. Nevertheless, an efficient calculation results from taking advantage of the underlying problem simplicity. The design variables, namely the thickness values of layers, are defined for each finite element. The global stiffness and mass matrices are, loosely speaking, the separable sum of all the individual element matrices. Thus, the matrix composed of the non-zero entries of the derivation of the global matrix with respect to the design variable is equal the derivative of the one finite element on which the design variable is defined.

$$\frac{\partial \mathbf{K}_{el}}{\partial h_{i,j}} = \left. \frac{\partial \mathbf{K}_{glob}}{\partial h_{i,j}} \right|_{non-zeros} \tag{4}$$

The following computational step prescribed in (3) is multiplication of the matrix sensitivities with the eigenvector from the right and from the left. It is consistent with (4) to extract from the eigenvector only those entries which correspond with the local finite element degrees-of-freedom and calculate the matrix products only with these because all other contributions are equal to zero. The great numerical savings obtained from performing the operations on element level are further enhanced if the derivatives of the matrices are calculated analytically.

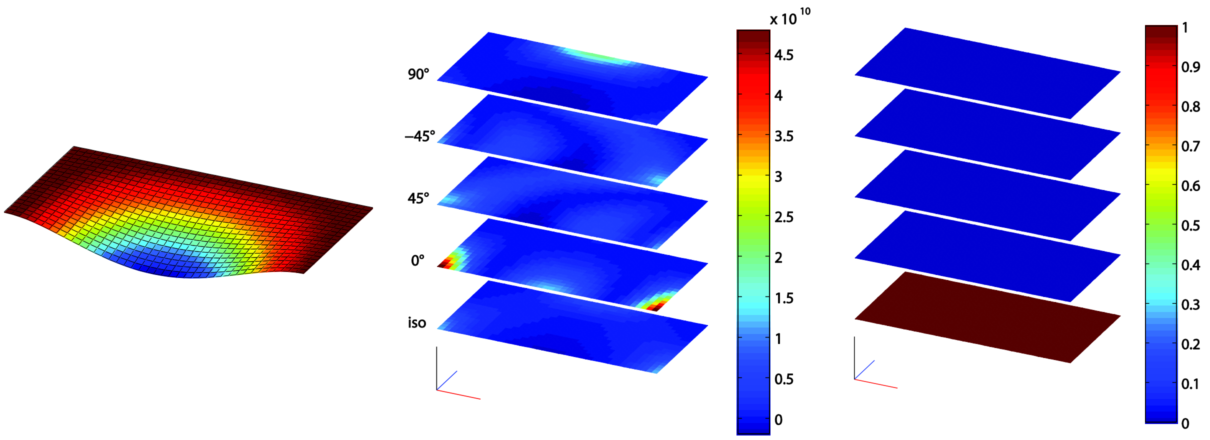
3 The Ghost Layer Concept

A *Ghost Layer* carries information on material properties and material orientation but its thickness is specified to zero. It has thus no influence on the structural behavior of the current design. Nevertheless, the gradient (3) with respect to a change of thickness of *Ghost Layers* can be calculated as explained in Section 2 and be used to provide information in which part of the structure's domain the *Ghost Layer* should be transformed to a real layer by assigning a finite thickness value to it. The zero-thickness property of *Ghost Layers* allows inserting any number of them at the interface between two real layers. The gradients with respect to all of those provide the additional information which material properties and which orientation angle the real layer should have preferably.

Illustration Experiment Part 1

The first eigenfrequency of a rectangular, isotropic plate is to be maximized by attaching doublers of unidirectional carbon fiber composites. The plate is fully clamped at the two short sides and at one long side. The other long side is free (see Figure 1a). An invariant base sheet is made from an isotropic material. The set of feasible orientation angles, foreseen for the doublers, includes 0, 45, -45 and 90 degrees. The set is reflected by four *Ghost Layers* each of which represents the same unidirectional reinforced material aligned with the respective fiber orientation. The internal calculation of the model guaranties a symmetrical layup by always attaching the doublers on both sides of the plate.

The gradient field is shown in Figure 1b refers to the initial design and points to a distribution of doublers leading to performance improvement next to it. The numbers indicate that adding of a real 0°-layer at the corners between the clamped and free edges will best increase the eigenfrequency whilst adding the least amount of mass. A scheme for developing the plate design with optimum placement of doublers is discussed in the following section.



a) b) c)

Figure 1. a) First eigenmode of a three side clamped, rectangular plate. b) Gradient distribution for isotropic layer and four Ghost Layers. c) Thickness distribution for isotropic layer and four Ghost Layers.

4 Iteration procedure

The loop for the generation of the doublers is illustrated in Figure 2. The process starts with a specified initial design with laminate layup whose finite element model includes boundary conditions and loads. Along with real layers, also *Ghost Layers* are defined. With that information, element matrices are generated. After assembly and solution of the system equations the obtained solutions are used for termination criteria. If these are not met, the sensitivities are calculated using equation (3). These results are used to modify the layer thickness values within the finite element sub domains. To keep the solutions of the simulation close to real-world designs, the thickness of a layer is changed in discrete steps. The steps are given by the thickness of the materials. Afterwards, the element matrices of the modified elements have to be recalculated again. The remaining element matrices can be recalled from memory, saving much computing time.

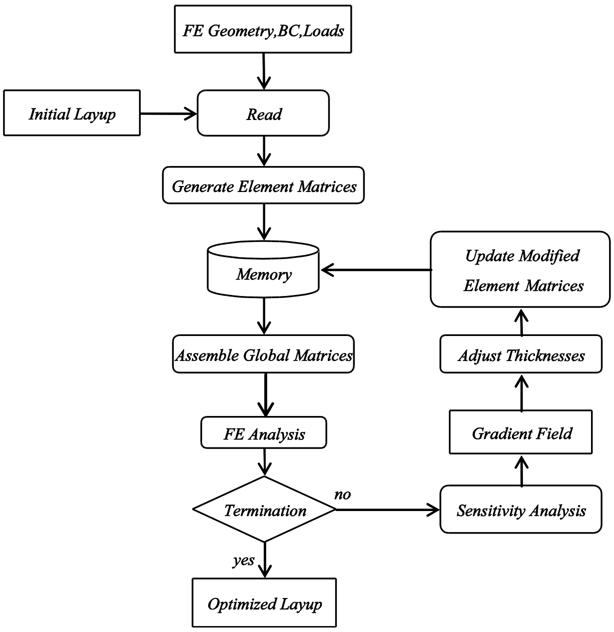


Figure 2. Iteration procedure based on a Sensitivity Analysis with the Ghost Layer Concept

The first-order search methods, i.e. those using gradient information, generate a sequence of line searches along search directions depending on gradient information or, in addition, previous search directions. The line search determines the step length from a reference design along the current search direction to a point where the objective cannot be further improved. This is the point where the local gradient is orthogonal to the current search direction. The first-order search methods must assume that the variables be smooth. A modified scheme for design improvement has to be applied in order to take into account that, in engineering reality, the composite material layers have discrete thickness values. The scheme must balance between a version for step-by-step accuracy and one for low numerical cost. The former would transform a *Ghost Layer* into a real layer in only one finite element per iteration, requiring a large number of iterations. The latter would apply the transformation to a larger number of finite elements, requiring much less iterations. A guideline for developing the design improvement scheme could be that both versions of it should eventually produce the same optimized design solutions.

Illustration Experiment Part 2

The plate, shown in Section 3, will now be reinforced with doublers using the *Ghost Layer Concept* and the proposed iterative process. Layers of elements, whose gradient value differ less than one percent of the maximal value of all *Ghost Layers* will be turned into real layers. In order to illustrate the solution quality during the process, no termination criteria concerning frequency or mass will be applied. Thus, the structure is reinforced until all four *Ghost Layers* have completely turned to real layers. Figure 3 shows the normalized increase of the eigenfrequency over the normalized increase of the mass. The configuration with 0% mass increase is the initial plate without reinforcement while 100% mass increase is the final solution with just real layers.

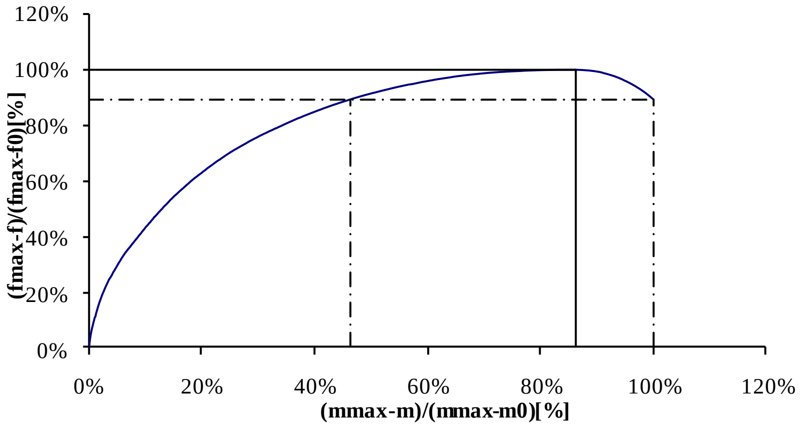


Figure 3. Normalized frequency increase over normalized mass increase

Obviously, the increase of eigenfrequency with respect to mass increase in the first few steps is maximal: The additional material can be placed at locations where it increases the eigenfrequency most effectively. Subsequently, the curve slope decreases until the maximal frequency is reached. Not all layers have turned to real layers and the mass takes only 86.38% of the fully developed solution. Continuing the process, the remaining material can only be placed at locations where it decreases the eigenfrequency. In the end, the eigenfrequency drops to 89.25% of the maximum value which has been reached with a design whose mass is 46.25% of the maximum. This implies that the mass of the fully developed plate could be reduced significantly by removing parts of layers without having a drop in eigenfrequency.
The shown curve is not valid for general structures. It depends on the initial design with layup and boundary conditions. In some cases, maximal frequency is only reached with the fully developed design. It is up to the user to define a termination criterion. Either the process is run until a given mass is reached or until the frequency exceeds a given value.

5 Simulation Experiment

The potential of the proposed process is illustrated on an eigenfrequency optimization for a panel with cut-outs (see Figure 4) that could be a common part of a satellite structure.

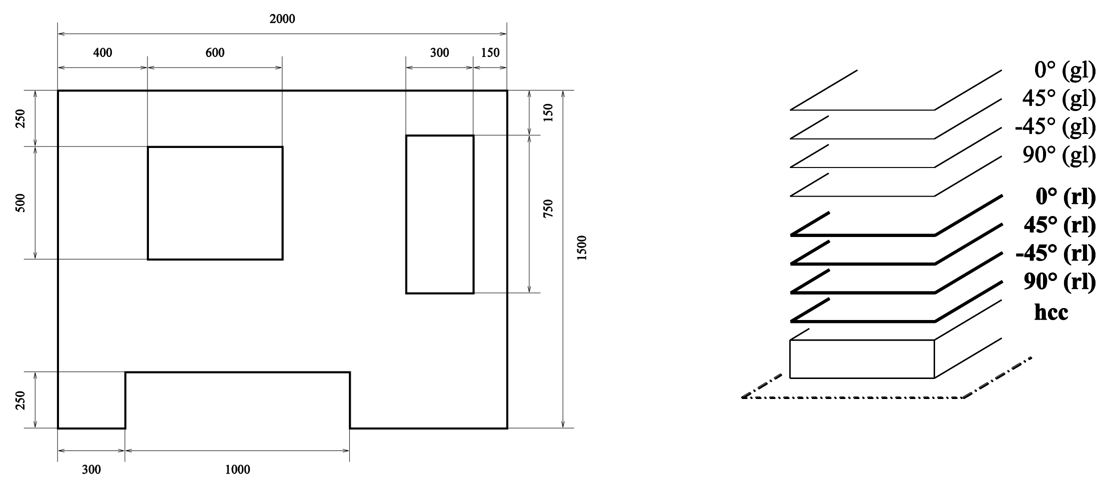


Figure 4. Geometry and layup of the space panel

The panel is simply supported at all outer edges of the enclosing rectangle. It consists of a sandwich layup with aluminum honey-comb core (Table 2) ten millimeters thick. The facesheets are composed of four layers of a unidirectional carbon fiber ply (Table 2) with orientation angles 0, 45, -45 and 90 degrees (Figure 4: (rl)) and a thickness of *0.25mm*.
At first, the plate is considered without the cut-outs which results in a mass of *11.10kg* and a first eigenfrequency of *47.566Hz*. The cut-outs reduce the mass to *8.23kg* and the eigenfrequency to *36.411Hz*. The plate with cut-outs is the initial design to be improved by adding doublers. The generation of these is foreseen with four *Ghost Layers*, consisting of the same material as the base plate and with orientation angles 0, 45, -45 and 90 degrees, which are stacked onto the panel (Figure 4: (gl)). Both, basic laminate and *Ghost Layers* are symmetrically arranged. Transforming *Ghost Layers* to real layers will add mass but we restrict the total mass to be not more than that of the basic plate without cut-outs. With our above mentioned current design improvement scheme, the process needs 614 iteration steps (which takes about 2 hours with a common workstation) to reach the mass constraint. The terminal solution with doublers has a lowest eigenfrequency of *44.586Hz* and a mass of *11.08kg*. Figure 5 shows the geometry of the reinforcement doublers for the four different orientations.

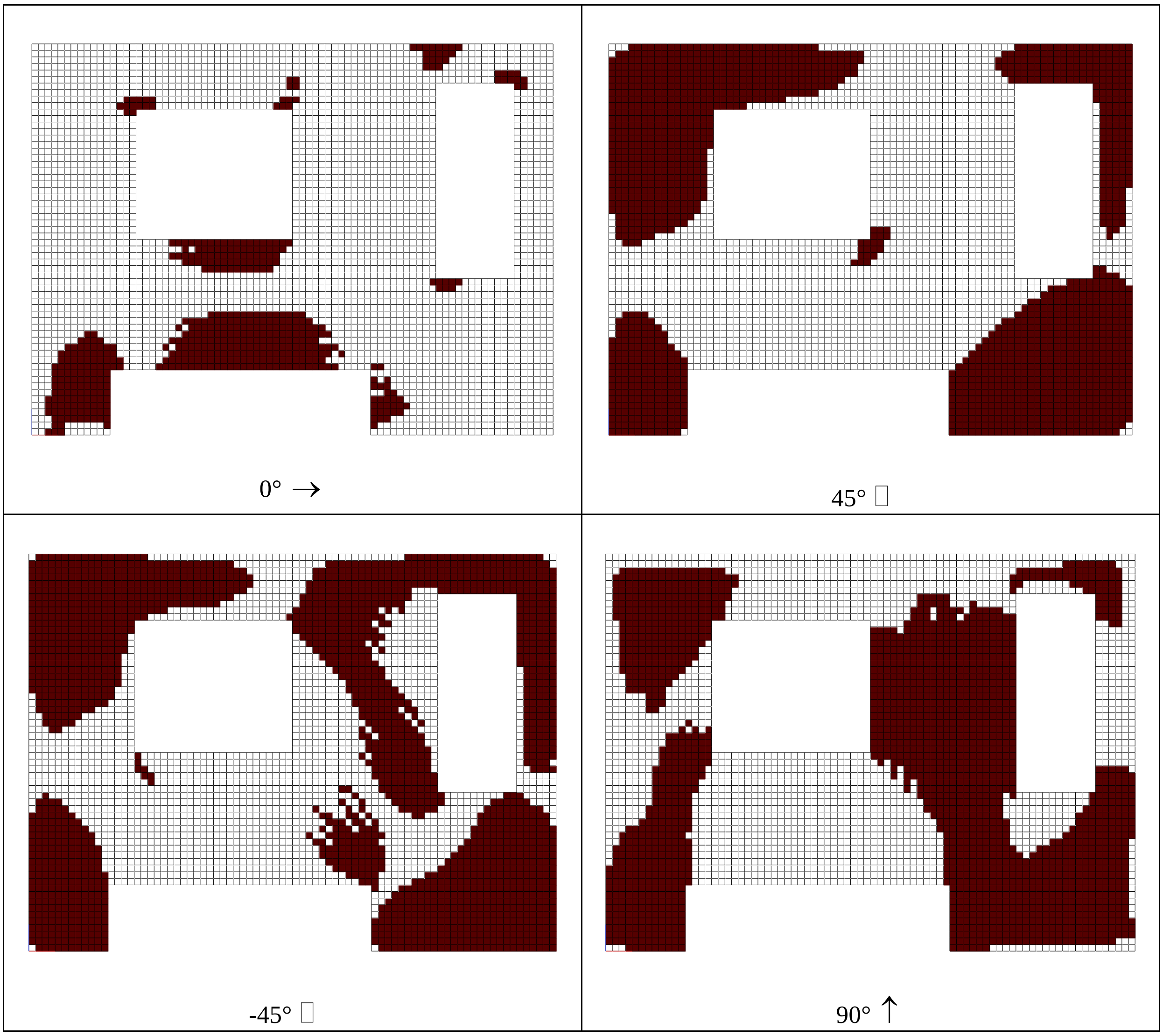


Figure 5. Doubler geometries for the 0°, 45°, -45° and 90° directions

Discussion:

The reinforced solution has a frequency increase of 22.45% and a mass increase of 34.63% with respect to the unreinforced solution. Obviously, the eigenfrequency of the panel configuration without cut-outs (that has equal mass) cannot be reached. Thus, the weakening due to the cut-outs cannot be compensated with doublers without a weight penalty. However, a solution where all *Ghost Layers* have completely turned to real layers (which means double facesheets) has an eigenfrequency of *39.711Hz* and a mass of *15.35kg*. This is 10.93% less eigenfrequency and 38.52% more mass referred to the configuration with doublers. Therefore, a systematic reinforcement with doublers leads to superior designs, concerning high eigenfrequencies and low mass, compared to solutions where layers are covering the full panel. The results of the four mentioned panel configurations are summarized in Table 1.
Apparently, there are regions where the doublers are scattered so that they cannot be manufactured without any modifications. The doublers have to be transferred manually to an appropriate design. However, the connected parts prevail so that it can be done with little effort.

Panel Configuration	**First Eigenfrequency [Hz]**	**Mass [kg]**
w/o cut-outs, single facesheet	47.566	11.10
cut-outs, single facesheet, no doubler	36.411	8.23
cut-outs, single facesheet, with doubler	44.586	11.08
cut-outs, double facesheet	39.711	15.35

Table 1. Result summary of different panel configurations

	UD CFRP	**Honey-comb-core**
E_{11} [MPa]	3.1E+5	1.0E-3
E_{22} [MPa]	6.0E+3	1.0E-3
ν_{12} [-]	.3	.001
G_{12} [MPa]	3.5E+9	1.0E-9
G_{1Z} [MPa]	3.5E+9	1.4E+2
G_{2Z} [MPa]	3.5E+9	3.0E+2
ρ [kg/m^3]	1600	50

Table 2. Material property table

6 Conclusion & Outlook

A new method for the placement of reinforcement doublers for laminated composite structures is introduced. *Ghost Layers* enable to perform a Sensitivity Analysis for virtual layers whose thickness is specified to zero. Within an iterative process, *Ghost Layers* turn to real layer and build doublers. Thus, not only principal material axis orientation but also doubler geometry are result of the process. It has been shown that a systematic placement of doublers leads to solutions with a higher eigenfrequency-to-mass ratio compared to solutions with fully covering layers. It can also be said that the shown solutions could not be found intuitively. Since doublers are stacked related to the sequence of the *Ghost Layers*, the stiffness contribution of an upper layer is also influenced by its subjacent layers. Thus, the solution is depending on the stacking sequence of the *Ghost Layers* and the obtained solutions are strongly depending on the initial configuration of layup and the *Ghost Layer* stacking sequence. The process is feasible to reinforce an existing design. The finding of global optimal solutions requires more research effort.

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