Sequential aggregation of verifiable information

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Sequential Aggregation of Verifiable Information

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Sequential Aggregation of Verifiable Information*

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Abstract

We introduce the notion of verifiable information into a model of sequential debate among experts who are motivated by career concerns. We show that self-censorship may hamper the efficiency of information aggregation, as experts withhold evidence contradicting the conventional wisdom. In this case, silence is telling and undermines the prevailing view over time if this view is incorrect. As a result, withholding arguments about the correct state of the world is only a temporary phenomenon, and the probability of the correct state of the world being revealed always converges to one as the group of experts becomes large. For small groups, a simple mechanism the principal can use to improve decision-making is to appoint a devil’s advocate.

Keywords: experts, committees, career concerns, verifiable information, information aggregation.

JEL: D71, D82.

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1 Introduction

According to the well-known Jury Theorem established by Condorcet (1785), groups are more likely to reach correct decisions than individuals because idiosyncratic errors of individuals wash out when votes are aggregated. By contrast, in a highly influential book Janis (1972) presents case-study evidence on foreign-policy committees to bolster his case that concurrence-seeking in groups may induce self-censorship in the sense that arguments contradicting the prevailing opinion are withheld. As a consequence, wrong decisions may be taken, even if group members have strong objections privately. He has coined the term “groupthink” for this psychological drive for consensus. In this paper we present a model of sequential debate among experts that reconciles these seemingly opposing views on the efficiency of information aggregation in groups.

Our framework is based on Ottaviani and Sørensen (2001). Building on the foundational work of Scharfstein and Stein (1990), they propose a model of sequential debate among experts who are motivated by career concerns. Ottaviani and Sørensen (2001) show that herding phenomena and informational cascades create a bound to the amount of information that can be aggregated if experts are uninformed about their own ability.

While Scharfstein and Stein (1990) and Ottaviani and Sørensen (2001) consider cheap talk, we introduce the assumption of verifiable information into a model of sequential debate.\(^1\) The assumption of verifiable information is particularly plausible with regard to experts. Experts may be able to present data or hard facts to bolster their case. Moreover, they may invest in making information hard by providing a detailed explanation for their views.\(^2\) The essence of our model is that, even if information is verifiable, it may be possible for experts to withhold it. This will be in an expert’s interest if the evidence is detrimental to his reputation.

\(^1\)Visser and Swank (2007) consider a simultaneous exchange of non-manipulable messages behind closed doors before experts vote. However, they do not allow for the possibility of experts withholding information. Moreover, in our framework information can also be verified by outside observers.

\(^2\)See Dewatripont and Tirole (2005) for a model in which senders and receivers can invest in making soft information hard.
More specifically, we consider a model populated by a principal who has to choose between two actions with uncertain outcomes and a number of experts who engage in a sequential debate about which decision is correct. Experts are privately endowed with pieces of verifiable information about the state of the world. Like in Ottaviani and Sørensen (2001), they are interested in creating the impression of expertise. This can be justified by the observation that a public perception of high competence may enable an expert to achieve more prestigious positions in the future. A favorable assessment of his ability may also enable him to earn a higher wage when moving on to another position.

Our model provides us with three main insights. First, for a large set of parameters we show that experts practice self-censorship. In order to preserve their reputation, they may withhold arguments that do not concur with conventional wisdom and present only evidence in favor of it.

Second, withholding arguments about the correct state of the world is only a temporary phenomenon in large groups of experts. This is a consequence of the observation that experts’ silence is telling in our model. Although initially experts will present only evidence supporting an incorrect view about the state of the world if the prior beliefs are sufficiently biased towards this view, the amount of evidence in favor of the incorrect view will be comparably meager. The scarcity of evidence in favor of conventional wisdom will induce all players to revise their assessments of the correct state of the world over time. At some point, these assessments will have shifted so much that experts will dare to present arguments in favor of the correct state of the world.

Third and consequently, we establish that the probability of the sequential debate of experts leading to a correct decision of the principal will converge to one if the number of experts becomes large. This contrasts with the finding in Ottaviani and Sørensen (2001) that herding problems pose a limit to the amount of information that can be aggregated (see their Lemma 1).

It is well-known that in models of cheap talk typically a large number of equilibria exist. As a consequence, attention is frequently restricted to a particular equilibrium.
like the most informative one, although it may be unclear why this equilibrium may be chosen. By contrast, all of our results do not depend on which perfect Bayesian Nash equilibrium is selected.

Moreover, we derive a range of parameter values under which equilibrium is unique and entails an efficient aggregation of information. Intuitively, this obtains if the mere fact that experts possess evidence is conducive to their reputation and thus induces them to present their arguments. In this case, the accuracy of the information released by experts is circumstantial for their reputation.

We also study several model variants. We use the special case of a single expert to illustrate the severity of self-censorship in our model by demonstrating that a single expert may withhold information that would affect the decision of the principal. By contrast, in a model where communication is cheap talk, a single expert would suppress his private information only in cases where this information would be immaterial to the principal’s decision. A simple mechanism the principal could use to improve decision-making is to appoint an advocatus diaboli. More specifically, we show that the quality of decision-making can be improved if an expert is only allowed to challenge the consensus view but not to present evidence in its favor. Finally, we prove that our findings about information aggregation in large groups of experts extend to (i) a model where experts propose their arguments simultaneously and (ii) the case where the state of the world cannot be observed directly after the principal has made her decision.

Our paper is organized as follows. In Section 2, we review additional papers related to ours. Section 3 outlines the model. The optimal behavior of experts is derived in Section 4. We characterize all perfect Bayesian Nash equilibria in Section 5. The efficiency of information aggregation is analyzed in Section 6. We consider extensions to our model in Section 7. Section 8 concludes.
We have already mentioned that our model is related to the literature on the aggregation of private information by voting, which goes back to Condorcet (1785). More recent treatments of the subject pursue a game-theoretic approach that takes into account the fact that it may not be in the agents’ interests to vote in line with their private information (see Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997), Wit (1998)).

Dekel and Piccione (2000) analyze sequential voting in a game-theoretic framework. They find that equilibria of a simultaneous voting game are also equilibria under sequential voting. Consequently, they argue that the results found in the literature on herd behavior and informational cascades do not immediately extend to models of voting because, in the latter models, voters condition their action on being pivotal.

Their model has been extended to include the possibility of abstention (see Battaglini (2005)) and the desire to vote for the winning candidate (see Callander (2007)). Glazer and Rubinstein (1998) examine different mechanisms to elicit private information from experts who are interested in the outcome of decision-making or wish their own recommendation be accepted. In contrast with these models, we assume that agents care about their reputation for being highly competent.

Thus our framework belongs to the literature on experts with career concerns (see Ottaviani and Sørensen (2001), Ottaviani and Sørensen (2006), Visser and Swank (2007), Gersbach and Hahn (2008), and Hahn (2008)). Visser and Swank (2007) show that in such a model experts may vote for the a priori unconventional decision. In addition, there are incentives to show a united front. As mentioned before, our paper is also related to the work of Ottaviani and Sørensen (2001), who examine the optimal order of speech for a group of experts. In our paper, we introduce verifiable information into a model of experts with career concerns.

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3 The Condorcet Jury Theorem was generalized to correlated votes by Ladha (1992).
4 For a lucid review of the literature on information aggregation and communication in committees see Austen-Smith and Feddersen (2009).
5 For the literature on herd behavior and informational cascades, see Scharfstein and Stein (1990), Banerjee (1992), Bikhchandani et al. (1992), and Ali and Kartik (2010). In these models, decision makers with similar preferences may suppress their private information if the actions of other agents suggest the opposite action is correct. This precludes efficient information aggregation.
The assumption of verifiable information was introduced into a sender-receiver game by Milgrom (1981), who determines conditions in terms of the sender’s and the receiver’s ideal actions that guarantee the existence of separating equilibria. Seidmann and Winter (1997) find more general conditions under which all equilibria are separating (see also Mathis (2008)). Giovannoni and Seidmann (2007) compare the amount of information conveyed in games of cheap talk to the respective amount in a framework with verifiable information. In the present paper, we study a game of verifiable information with an arbitrary number of senders who are not interested in the action chosen by the principal but in their reputation for being competent.

3 Model

We consider a model inhabited by a principal and \( N \geq 1 \) experts, indexed by \( i = 1, \ldots, N \). Experts engage in a sequential debate about the correct state of the world. Subsequently, the principal makes a decision, based on the information revealed in the debate. After the decision has been taken, all players observe the correct state of the world, and the ability of experts is assessed. In the following, we give a detailed account of the informational structure and the motivations of players.

Two different states of the world are possible, which we label \( s = -1 \) and \( s = 1 \). The prior probabilities of the states are \( \pi_s \in [0,1] \) (\( \pi_1 + \pi_{-1} = 1 \)). There are two types of experts, highly competent (\( H \)) and less competent ones (\( L \)). The ability of an individual expert is unknown both to the expert himself and to all other players. All players assign the common probability \( \kappa_i \) to the event of expert \( i \) being highly competent.

A few words are in order regarding our assumption that experts do not have private information about their own ability. It has been demonstrated in the literature that

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\(^{6}\)Crawford and Sobel (1982) developed the canonical model of partisan advise, in which conflicts of interest hamper a complete transmission of non-verifiable information.

\(^{7}\)Wolinsky (2003) proposes a model of verifiable messages in which the receiver is uncertain about the sender’s preferences. Gersbach and Keil (2005) examine the reallocation of budgets and tasks in a public organization as a means of eliciting verifiable information about productivity improvements from agents.
this assumption aggravates herding problems compared to the case where experts are privately informed about their competence. This follows from the observation that experts who have obtained private information about their own ability being high have strong incentives to choose an action that is in line with their private signal (see Trueman (1994), Ottaviani and Sørensen (2001), and Hahn (2008)). Despite the assumption of unobservable own ability, which makes herding problems particularly severe in our model, we are able to establish the result that information aggregation always leads to the state of the world being revealed if the number of experts is large.

Each expert $i$ receives an argument $A_i \in \{-1, 1\}$ about the state of the world with probability $q_H$ if he is highly competent and $q_L$ if he is less so. With probability $1 - q_H$ or $1 - q_L$ respectively, the expert receives no argument ($A_i = 0$). We assume $1 \geq q_H > q_L \geq 0$, i.e. the probability of receiving an argument is higher for a highly competent expert than for a less competent one. All events of experts receiving arguments are independent. The argument of a highly competent expert is correct with probability $p_H$ ($p_H \geq 1/2$). Hence, with probability $p_H$ a highly competent expert who observes an argument receives $A_i = s$. With probability $1 - p_H$ he receives $A_i = -s$. For less competent experts, the probability of obtaining a correct argument is $p_L$ ($p_L \geq 1/2$).\(^9\) Conditional on the state of the world, the arguments of all experts are independent. We assume $p_L < p_H$, which implies that arguments are more likely to be correct for highly competent experts than for less competent ones.

There is a sequential debate among experts, where the order of speech is exogenously given and is assumed to be $i = 1, 2, ..., N$, without loss of generality. We use $\bar{a} \in \{-1, 0, 1\}^N$ to denote the complete vector of arguments raised by all experts. In particular, the argument revealed by expert $i$ is denoted by $a_i \in \{-1, 0, 1\}$, where $a_i = 0$ means that expert $i$ announces no argument. Upon observing $A_i \in \{-1, 1\}$, expert $i$ can choose between truthful revelation ($a_i = A_i$) and withholding the information ($a_i = 0$). If expert $i$ has not observed an argument ($A_i = 0$), he faces a singleton

\(^8\)In Ottaviani and Sørensen (2001) the first case is considered in their Lemma 1 and the second one in their Lemma 4.

\(^9\)According to our assumptions $p_H \geq 1/2$ and $p_L \geq 1/2$, all arguments are more likely to be correct than wrong. If arguments were correct with a probability less than one half, then one would simply have to relabel correct arguments as incorrect and vice versa.
choice set \((a_i \in \{0\})\). Thus we assume that experts cannot prove that they possess no argument. In this sense, we study a game with partially verifiable information (see Lipman and Seppi (1995) and Forges and Koessler (2005)).

Having observed all arguments \(\tilde{a}\) raised in the debate, the principal chooses an action \(\sigma \in \{-1, 1\}\). This action delivers utility 1 to the principal if it corresponds to the correct state of the world \(s\). The principal’s utility is 0 otherwise.\(^{10}\) After the principal has made her decision, the correct state of the world can be observed by all players.

In line with Ottaviani and Sørensen (2001), experts’ utility is equal to the probability outside observers (the market) assign to the event of the expert being highly competent, after observing the complete debate \(\tilde{a}\) and the correct state of the world \(s\). This can be motivated in several ways. For example, highly competent experts may earn higher wages in the future.\(^{11}\) Moreover, a favorable perception of their competency may enable experts to reach more prestigious positions. Finally, they may draw direct utility from being perceived as highly competent.

In the following, we derive all perfect Bayesian Nash equilibria of this game of sequential debate. In equilibrium, all experts behave optimally in the debate, given the strategies of the other experts and the way the market forms its assessment of experts’ abilities. The principal makes an optimal decision, based on the information revealed in the debate. Moreover, the market forms Bayesian updates of the probabilities of experts being highly competent.

4 Behavior of Experts

We begin our analysis with a characterization of experts’ behavior in equilibrium. Because each expert \(i\)’s behavior may depend on the choices of experts \(1, \ldots, i - 1\), we introduce \(\tilde{a}(i)\) as the \((i - 1)\)-dimensional vector \((a_1, \ldots, a_{i-1})\). This vector represents the arguments that can be observed by expert \(i\) when he makes his decision.

\(^{10}\)Note that the values 0 and 1 only represent convenient normalizations. These normalizations do not affect our results.

\(^{11}\)The incentives to signal a high level of ability in order to achieve higher wages in the future were first modeled explicitly by Holmström (1999).
Now consider the optimal behavior of an expert $i$, for a given $\vec{a}(i) \in \{-1, 0, 1\}^{i-1}$. If the expert has observed no argument ($A_i = 0$), his decision problem is trivial because his only possible choice is $a_i = 0$. Hence we focus on $A_i \in \{-1, 1\}$ in the following.

In order to derive expert $i$’s optimal behavior, we have to compute the competence the market assigns to him in all possible contingencies. We use $\kappa_i(a_i, s)$ to denote the market’s assessment of expert $i$’s competence if he has chosen $a_i \in \{-1, 0, 1\}$ and the correct state of the world is $s$.\textsuperscript{12}

Next we introduce $\lambda_i(A_i)$, which gives the probability of expert $i$ proposing his argument if he has obtained $A_i \in \{-1, 1\}$. Consequently, if expert $i$ has received argument $A_i$, he remains silent with probability $1 - \lambda_i(A_i)$.\textsuperscript{13} It will be convenient to define the following variables:

$$
\begin{align*}
\rho_H &:= q_H p_H & (1) \\
\rho_L &:= q_L p_L & (2) \\
\overline{\rho}_i &:= \kappa_i \rho_H + (1 - \kappa_i) \rho_L & (3) \\
\mu_H &:= q_H (1 - p_H) & (4) \\
\mu_L &:= q_L (1 - p_L) & (5) \\
\overline{\mu}_i &:= \kappa_i \mu_H + (1 - \kappa_i) \mu_L & (6)
\end{align*}
$$

We observe that $\rho_H$ ($\rho_L$) gives the probability of a highly competent (less competent) expert observing an argument and this argument being correct. Similarly, $\overline{\rho}_i$ is the probability of an expert of unknown level of expertise receiving an argument that is correct. The $\mu$’s denote the respective probabilities for wrong arguments. For example, $\mu_L$ is the probability that a less competent expert observes an argument but that this argument is wrong. Some useful properties of the variables defined above are stated in Appendix A.

\textsuperscript{12}It is important to keep in mind that $\kappa_i(\ldots)$ also depends on the preceding part of the debate $\vec{a}(i)$. However, to keep notation simple, we do not make this dependence explicit. Moreover, we note that $\kappa_i(\ldots)$ is independent of the strategies of the remaining experts $j = i + 1, \ldots, N$.

\textsuperscript{13}Like $\kappa_i(\ldots)$, $\lambda_i(A_i)$ will in general depend on the arguments raised by experts $j = 1, \ldots, i - 1$. However, because we consider the behavior of expert $i$ for a fixed pattern of previous arguments, we again abstain from making this dependency explicit.
Now we are in a position to derive expressions for $\kappa_i(., .)$, which pinpoint expert $i$’s utility:

$$\kappa_i(1, 1) = \kappa_i(-1, -1) = \frac{\rho_H}{\mu_i} \kappa_i$$  \hspace{1cm} (7)

$$\kappa_i(1, -1) = \kappa_i(-1, 1) = \frac{\mu_H}{\mu_i} \kappa_i$$  \hspace{1cm} (8)

$$\kappa_i(0, 1) = \frac{1 - \rho_H \lambda_i(1) - \mu_H \lambda_i(-1)}{1 - \overline{p}_i \lambda_i(1) - \mu_i \lambda_i(-1)} \kappa_i$$  \hspace{1cm} (9)

$$\kappa_i(0, -1) = \frac{1 - \rho_H \lambda_i(-1) - \mu_H \lambda_i(1)}{1 - \overline{p}_i \lambda_i(-1) - \mu_i \lambda_i(1)} \kappa_i$$  \hspace{1cm} (10)

These expressions can be explained as follows. Recall that $\kappa_i(1, 1)$ is the probability of expert $i$ being highly competent, given that he has proposed argument 1 and that 1 is actually correct. It is given by the ratio of two terms. $\kappa_i \rho_H$ corresponds to the probability of an expert being highly competent and receiving a correct argument. $\mu_i$ is the sum of the probability of an expert being highly competent and receiving a correct argument and the respective probability for a less competent expert. Equation (8) can be explained in a similar way.

The interpretations of (9) and (10) are somewhat more intricate. $\kappa_i(0, 1)$ stands for the probability of expert $i$ being highly competent if he has not announced an argument and the correct state of the world is 1. It can also be computed as the ratio of two expressions. First, it depends on $(1 - \rho_H \lambda_i(1) - \mu_H \lambda_i(-1)) \kappa_i$, which is the probability of expert $i$ being highly competent and announcing no argument. Here we have used the observation that the probability of a highly competent expert announcing an arbitrary argument is $\rho_H \lambda_i(1) + \mu_H \lambda_i(-1)$, which is the sum of the probabilities of his announcing 1 and −1. The denominator in (9) is the sum of $(1 - \rho_H \lambda_i(1) - \mu_H \lambda_i(-1)) \kappa_i$, which we have already discussed, and the probability of expert $i$ being less competent and announcing no argument, which is $(1 - \rho_L \lambda_i(1) - \mu_L \lambda_i(-1))(1 - \kappa_i)$. The explanation for (10) is analogous.

There is a conspicuous difference between (7) and (8) on the one hand and (9) and (10) on the other. (7) and (8) do not depend on $\lambda_i(1)$ and $\lambda_i(-1)$, i.e. the strategy chosen by expert $i$ in equilibrium. This is an immediate consequence of the fact that (7) and (8) correspond to the verifiable pieces of information $A_i = 1$ and $A_i = -1$. By contrast, for (9) and (10) expert $i$ claims to be of type $A_i = 0$, which is not verifiable.
We also note that $\kappa_i(0, A_i)$ is a decreasing function of $\lambda_i(A_i)$ for $A_i \in \{-1, +1\}$. We will see that, as a consequence, multiple equilibria may exist. Suppose the market believes that $\lambda_i(A_i) = 1$ for some $A_i$. Then it will assign a low level of competence to expert $i$ when he announces no argument and the state of the world turns out to be $s = A_i$. As a result, the expert will find it attractive to announce argument $A_i$. By contrast, if $\lambda_i(A_i) = 0$, then the market will ascribe a comparably high probability to $i$ being of type $H$ if $s = A_i$ and the expert has announced no argument. This, in turn, makes it attractive for expert $i$ to withhold argument $A_i$.

We introduce $\pi_1(\bar{a}(i))$ and $\pi_{-1}(\bar{a}(i))$ to denote the updated probability that 1 and $-1$ are the correct states of the world, conditional on $\bar{a}(i)$, the complete pattern of arguments raised by experts $j = 1, \ldots, i-1$. Using Bayes’ formula, it is straightforward to derive formal expressions for these probabilities. We will take up this point later.

For the sake of brevity, we consider only pure strategies.\footnote{All major findings in this paper extend to equilibria in mixed strategies. An analysis is available upon request.} The following lemma, which is proved in Appendix B, identifies all possible kinds of behavior that may occur in equilibrium:

**Lemma 1**

Consider a fixed $\bar{a}(i) \in \{-1, 0, 1\}^{i-1}$.

1. If the market believes that $i$ always withholds his argument upon observing $\bar{a}(i)$, then it will not be optimal for $i$ to do so.

2. Suppose the market believes that $i$ will withhold his argument for $A_i = \alpha$ and will present it for $A_i = -\alpha$, where $\alpha$ is fixed with $\alpha \in \{-1, +1\}$. Then this behavior is optimal for expert $i$ iff

$$\frac{\pi_\alpha(\bar{a}(i))}{\pi_{-\alpha}(\bar{a}(i))} \leq C_1 C_2, \tag{11}$$

3. Suppose the market believes that expert $i$ will always present his argument. Then this behavior is optimal for expert $i$ iff

$$\min \left\{ \frac{\pi_1(\bar{a}(i))}{\pi_{-1}(\bar{a}(i))}, \frac{\pi_{-1}(\bar{a}(i))}{\pi_1(\bar{a}(i))} \right\} \geq C_2. \tag{12}$$
We have utilized the following definitions:

\[
C_1 := \frac{1 - \rho_i}{1 - \mu_i} > 1 \tag{13}
\]

\[
C_2 := \frac{\mu_L(1 - \rho_H) - \mu_H(1 - \rho_L)}{\rho_H(1 - \mu_L) - \rho_L(1 - \mu_H)} < 1 \tag{14}
\]

Lemma 1 has several implications. The first part states that the constellation \( \lambda_i(1) = \lambda_i(-1) = 0 \) can be ruled out. This observation represents an important step towards our finding that sequential debate reveals the true state of the world in the long run, as it guarantees that in each equilibrium there is always a positive probability of each expert revealing some information.

Another consequence of Lemma 1 is that the sign of \( C_2 \) is crucial for the characterization of equilibria. If it is weakly negative, then \( \lambda_i(1) = \lambda_i(-1) = 1 \) is the only possible behavior of expert \( i \) in equilibrium. By contrast, if it is strictly positive, then which type of behavior may occur in equilibrium depends on \( \frac{\pi_{\alpha(a(i))}}{\pi_{-\alpha(a(i))}} \).

It is instructive to examine the factors determining the sign of \( C_2 \) more closely. Due to \( \rho_H(1 - \mu_L) > \rho_L(1 - \mu_H) \) (see (16)), \( C_2 > 0 \) iff

\[
\mu_L(1 - \rho_H) > \mu_H(1 - \rho_L). \tag{15}
\]

In order to give an intuition for (15), we compare two formal expressions. First, we state the probability of an expert being of type \( H \) if he has not received a correct argument, which implies that he has either received a wrong argument or no argument. This probability is given by \( (\kappa_i(1 - \rho_H))/(\kappa_i(1 - \rho_H) + (1 - \kappa_i)(1 - \rho_L)) \). Second, we consider the probability of an expert being of high competency, given that he has received a wrong argument. This probability is \( (\kappa_i \mu_H)/(\kappa_i \mu_H + (1 - \kappa_i) \mu_L) \). It is easy to see that (15) is equivalent to the statement that the first probability is larger than the second. As a consequence, (15) has the interpretation that the information that an expert has not observed the correct argument would be more favorable to this expert’s reputation than the information that the expert has observed the wrong signal. So releasing wrong signals is particularly harmful to an expert’s reputation, which may induce experts to withhold information if this information does not conform to the consensus view.
An alternative interpretation can be given to (15) by noting that it can be rewritten as

\[ q_L > \mu_H / ((1 - p_L)(1 - \rho_H) + p_L \mu_H) \]

which is tedious but straightforward to show. Accordingly, (15) stipulates that \( q_L \) be sufficiently large. If \( q_L \) is lower, less competent experts will receive an argument with very low probability. Consequently, the mere fact that an expert has observed an argument suggests a high probability of his being highly competent, irrespective of the accuracy of his argument. As a result, expert \( i \) will always present his argument in equilibrium if (15) does not hold.

The findings of Lemma 1 in the case where (15) holds are displayed in Figure 1, where we adopt the convenient convention to display the conditions stated in the lemma in log-likelihoods.\textsuperscript{15} The figure makes it clear that, for sufficiently low and sufficiently high values of \( \ln (\pi_1(\vec{a}(i)) / \pi_{-1}(\vec{a}(i))) \), herding always occurs, as only one type of argument is presented by expert \( i \), namely the one consistent with the consensus view. Evidence challenging the consensus view is withheld by the expert because it is probably incorrect and thus would damage his reputation.

\[ \begin{align*}
-|\ln C_2| &+ \ln C_1 &|\ln C_2| - \ln C_1 \\
-|\ln C_2| &0 &|\ln C_2| \ln \left( \frac{\pi_1(\vec{a}(i))}{\pi_{-1}(\vec{a}(i))} \right) \\
\lambda_i(1) = 0, \lambda_i(-1) = 1 & &\lambda_i(1) = 1, \lambda_i(-1) = 0 \\
\lambda_i(1) = \lambda_i(-1) = 1 & &
\end{align*} \]

Figure 1: Overview over the pure strategies that may be chosen in equilibrium.

The findings of Lemma 1 in the case where (15) holds are displayed in Figure 1, where we adopt the convenient convention to display the conditions stated in the lemma in log-likelihoods.\textsuperscript{15} The figure makes it clear that, for sufficiently low and sufficiently high values of \( \ln (\pi_1(\vec{a}(i)) / \pi_{-1}(\vec{a}(i))) \), herding always occurs, as only one type of argument is presented by expert \( i \), namely the one consistent with the consensus view. Evidence challenging the consensus view is withheld by the expert because it is probably incorrect and thus would damage his reputation.

5 Equilibria

In the following we characterize all perfect Bayesian Nash equilibria. For this purpose, we need to give a detailed account of the updating process for probabilities \( \pi_s(\vec{a}(i)) \),

\textsuperscript{15}Mixed strategies may be profitable for \(- \ln C_1 \leq \ln (\pi_1(\vec{a}(i)) / \pi_{-1}(\vec{a}(i))) \leq - \ln C_1 + |\ln C_2| \) and \( \ln C_1 - |\ln C_2| \leq \ln (\pi_1(\vec{a}(i)) / \pi_{-1}(\vec{a}(i))) \leq \ln C_1 \) respectively.
which is delegated to Appendix C. There we demonstrate that the updating process is most conveniently formalized in terms of log-likelihoods $\ln \left( \pi_s(\vec{a}(i))/\pi_{-s}(\vec{a}(i)) \right)$. We show that the release of arguments $+1$ and $-1$ always makes the respective states more likely. Because arguments $+1$ and $-1$ correspond to the verifiable types, the magnitude of these shifts in beliefs is independent of the strategies chosen by the expert in equilibrium. By contrast, the impact of $a_i = 0$ on $\ln \left( \pi_s(\vec{a}(i+1))/\pi_{-s}(\vec{a}(i+1)) \right)$ does depend on expert $i$’s equilibrium strategy. For example, if the expert makes both types of argument public in equilibrium, $a_i = 0$ will reveal no information about the state of the world and hence $\pi_s(\vec{a}(i+1)) = \pi_s(\vec{a}(i))$ or $\ln \left( \pi_s(\vec{a}(i+1))/\pi_{-s}(\vec{a}(i)) \right)$ equivalently. If the expert releases only $+1$ but not $-1$ in equilibrium, then $a_i = 0$ will make state $-1$ appear more probable. This effect has the intuitive explanation that an expert who does not present an argument may withhold evidence in favor of $s = -1$ if $\lambda_i(-1) = 0$ and $\lambda_i(1) = 1$. In this sense, silence is telling in our model.

The principal’s optimal choice of $\sigma$ can be specified in a particularly simple manner:

**Lemma 2**

The principal chooses $\sigma = +1$ if $\ln \left( \pi_1(\vec{a})/\pi_{-1}(\vec{a}) \right) > 0$. She chooses $\sigma = -1$ if $\ln \left( \pi_1(\vec{a})/\pi_{-1}(\vec{a}) \right) < 0$.

The proof of the lemma is obvious. If $\ln \left( \pi_1(\vec{a})/\pi_{-1}(\vec{a}) \right) > 0$ holds, then it is more likely that $s = 1$ is the correct state of world than $s = -1$. Consequently, the principal’s expected utility is maximized by $\sigma = 1$. Analogously, $\ln \left( \pi_1(\vec{a})/\pi_{-1}(\vec{a}) \right) < 0$ implies that $-1$ is more likely to be correct than $1$, which induces the principal to opt for $\sigma = -1$.

In line with Lemma 1, a unique equilibrium obtains if (15) fails to hold (and $C_2 \leq 0$ accordingly). In this equilibrium, all experts present their arguments, irrespective of the arguments’ types. We summarize this important finding in the following proposition:

**Proposition 1**

Suppose (15) does not hold. Then a unique perfect Bayesian Nash equilibrium exists. All experts $i = 1, ..., N$ always choose $a_i = A_i$. The principal behaves according to Lemma 2.
This equilibrium describes the case of efficient information aggregation because the principal obtains all private information from the experts. From our previous discussion, we know that (15) does not hold in situations where an expert who is known to have received a wrong argument has a higher probability of being highly competent than an expert who has not observed the correct signal. Therefore, releasing one’s argument is sufficiently attractive to experts. For even if this argument turns out to be wrong, its release will be comparably beneficial to an expert’s reputation.

If (15) does hold, the behavior of expert $i$ will depend on $\ln \left( \frac{\pi_1(\vec{a}(i))}{\pi_{-1}(\vec{a}(i))} \right)$ and thus on the arguments presented by colleagues $j = 1, ..., i - 1$. Collecting our findings about the optimal behavior of experts and the principal as well as the updating procedure concerning beliefs about the correct state of the world (see Appendix C), we can characterize all perfect Bayesian Nash equilibria if (15) is satisfied:

**Proposition 2**

Suppose (15) holds. Then Lemmas 1 and 2 and Equations (34)-(37) (see Appendix C) jointly describe all perfect Bayesian Nash equilibria.

### 6 Information Aggregation

In the previous section we have demonstrated that, if $\ln \left( \frac{\pi_1(\vec{a}(i))}{\pi_{-1}(\vec{a}(i))} \right)$ is sufficiently large, expert $i$ will always withhold arguments indicating that the state of the world is $s = -1$ in the case where (15) holds. Conversely, if $\ln \left( \frac{\pi_1(\vec{a}(i))}{\pi_{-1}(\vec{a}(i))} \right)$ is sufficiently low, expert $i$ will suppress arguments in favor of the view that $s = 1$.

Hence one might conjecture that, even for a large number of experts, the principal will never learn the correct state of the world if there is a sufficiently strong but incorrect prior belief about the state of the world. This is, however, untrue. Suppose, for example, that the correct state of the world is $-1$, but that $\ln \left( \frac{\pi_1}{\pi_{-1}} \right)$ is large. Although, at the beginning of the debate, experts will present only arguments supporting $s = 1$, they are not likely to find many arguments in line with this view. As a consequence, experts will remain silent frequently, which will shift the common assessment of the state of the world towards $-1$. At some point, $\ln \left( \frac{\pi_1(\vec{a}(i))}{\pi_{-1}(\vec{a}(i))} \right)$ will be sufficiently
low such that some expert $i$ will find it worthwhile to present an argument supporting $s = -1$. Therefore large groups of experts will always enable the principal to make correct decisions.

To make these arguments more precise, we have to specify a procedure how new members are added to an existing group of experts. We start from a particular equilibrium of the game with $N$ experts and a vector of competencies $(\kappa_i)_{i=1}^N$. Then we introduce an additional expert $N+1$ (with a level of competence $\kappa_{N+1} \in ]0, 1[$). Importantly, the strategies of experts $i = 1, \ldots, N$ in the equilibrium of the original game correspond to an equilibrium in the game extended in this way. Thus we can consider the same strategies of experts $i = 1, \ldots, N$ as before. Finally, possible equilibrium strategies of expert $N + 1$ can be identified by Lemma 1. In Appendix D we show

**Proposition 3**

*If $N \to \infty$, the probability of the principal choosing the correct option converges to 1.*

The proposition is a variant of the Condorcet Jury Theorem. For very large groups of experts, the correct state of the world is perfectly revealed despite the fact that arguments contradicting the conventional view may be withheld by experts. By contrast, for non-verifiable information, a similar proposition does not hold. As shown by Ottaviani and Sørensen (2001) in their Lemma 1, experts would always pool for sufficiently informative priors and no additional information would be revealed.

The main idea of the proof is that, in each equilibrium, information aggregation can be described by a stochastic process $X_i$ for the log-likelihoods of the updated beliefs about the state of the world. One has to show that the probability of this process suggesting the wrong state of the world converges to zero, as the size of the committee goes to infinity. The stochastic process fails to have convenient properties such as the Markov property. The Markov property does not hold because the actions chosen by a particular member $i$ may not only depend on the probability that a specific state of the world is correct, updated for the arguments raised in the debate (with corresponding log-likelihood $X_{i-1}$), but in addition can be a function of the pattern of arguments
raised by colleagues $j$ with $j < i$. In the course of the proof, we construct another stochastic process $Y_i$ with more convenient properties, which enables us to find an upper bound to the probability of $X_i$ suggesting that the wrong state of the world is correct. This bound can be shown to converge to zero for large committees, which establishes the claim of the proposition.

Additionally, we obtain a corollary, which is proved in Appendix H:

**Corollary 1**

Independent of $\pi_1$ and $\pi_{-1}$, the probability of expert $N$ suppressing correct arguments converges to zero as $N \to \infty$.

In particular, the corollary holds if experts always withhold information about the correct state of the world at the beginning of the game because the priors $\pi_{-1}$ and $\pi_1$ are sufficiently biased. Therefore herding behavior where experts withhold correct arguments is always a temporary phenomenon. This distinguishes our model from standard models of herding in which informational cascades for the incorrect state of the world continue indefinitely (see Scharfstein and Stein (1990), Banerjee (1992), and Bikhchandani et al. (1992)).

### 7 Model Variants

In this section, we consider several variants of our basic model. We demonstrate that in general first-best decision-making cannot be attained, even if there is only a single expert. A simple mechanism is proposed and shown to improve decision-making: the appointment of a devil’s advocate. Finally, we prove that our findings about information aggregation in large committees continue to hold (i) if all experts announce their arguments simultaneously, (ii) if the state of the world cannot be observed directly, and (iii) if the probability of obtaining arguments is identical for types of high and low competence.

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16This follows from the fact that, for specified prior probabilities about the states of the world, different behaviors may occur in equilibrium (compare Figure 1). Which behavior occurs may depend on the pattern of previous arguments.
7.1 Single expert

In their analysis of sequential debate with non-verifiable information, Ottaviani and Sørensen (2001) show in their Lemma 2 that the first-best is implemented when the last expert is the only one to be herding with positive probability. The reason for their finding is that the last expert herds exactly in situations where his signal would not affect the principal’s decision anyway.

Now we examine whether this result extends to our framework of verifiable information. Equivalently, we consider whether a single expert entails the first-best, i.e. whether the decision taken by the principal for $N = 1$ always corresponds to the decision she would take if she obtained the expert’s private information. In Appendix I, we show

Proposition 4

Suppose $N = 1$, (15), and $\bar{\rho}_1/\bar{\mu}_1 > \pi_{\alpha}/\pi_{-\alpha} > \max\{C_1, 1/C_2\}$ for some $\alpha \in \{-1, 1\}$. Then it is impossible to reach the first-best.

Proposition 4 highlights the severity of herding problems in our model. An expert may withhold verifiable information that contradicts conventional wisdom even if, given this information, the expert believes that the conventional wisdom is more likely to be wrong than correct. Notably, the expert keeps information secret precisely in situations where this information would be valuable to the principal because it would affect her decision. Conversely, the expert releases information that strengthens an already existing bias. This information is worthless to the principal because it does not influence her choice. In this sense, herding problems are more severe in our model than in Ottaviani and Sørensen (2001).

7.2 Mechanisms to extract more information

The finding of the previous subsection raises the question how the principal could improve decision-making. One plausible approach is to delegate the role of the advocatus diaboli to an expert (see Janis (1972)). In the case of a single expert, this means

\textsuperscript{17}An interesting alternative would be to offer rewards to experts who propose arguments, in particular to those experts who challenge the consensus view.
that the expert is only allowed to raise the argument that corresponds to the a-priori less likely view but must not raise evidence in line with conventional wisdom.

**Proposition 5**

Suppose in addition to \( N = 1 \), (15), and \( \frac{p_1}{\mu_1} > \frac{\pi_{\alpha}}{\pi_{-\alpha}} > \max\{C_1, 1/C_2\} \) for some \( \alpha \in \{-1, 1\} \) (i.e. the conditions stated in Proposition 4), \( C_1(\rho_H - \rho_L)/(\mu_L - \mu_H) > \frac{\pi_{\alpha}}{\pi_{-\alpha}} \) holds. Then the advocatus-diaboli mechanism ensures the first-best.

Hence assigning the role of an advocatus diaboli to an expert can actually improve decision-making. Intuitively, if the expert must not announce arguments that are in line with the a-priori more likely view, the only way he can signal his competence is by challenging this view.\(^{18}\) These arguments are particularly valuable to the principal because they may affect her decision, whereas arguments that strengthen the prevailing opinion about the state of the world have no effect on her choice.

**7.3 Simultaneous debate**

It is an interesting question whether the simultaneous release of arguments guarantees an optimal decision of the principal if the group of experts is sufficiently large. We find

**Proposition 6**

Suppose all experts exchange their arguments simultaneously. Then the probability of the principal choosing the correct option converges to one as the committee size approaches infinity.

The proof is delegated to Appendix K. The proposition holds even if prior beliefs about the state of the world are so strongly tilted towards the incorrect state of the world that all experts withhold evidence in support of the correct state of the world. Intuitively, information that is in line with the incorrect state of the world is released

\(^{18}\)According to (10), \( \kappa_1(0, -1) = \frac{1-\rho_H \lambda_1(-1) - \mu_H \lambda_1(1)}{1-\rho_L \lambda_1(-1) - \mu_L \lambda_1(1)} \). As can be verified easily, this expression is increasing in \( \lambda_1(1) \) if (15) holds. As a consequence, conditional on \( s = -1 \), withholding \( A_i = -1 \) is less harmful to the expert if \( \lambda_1(1) = 1 \) than in the case where \( \lambda_1(1) = 0 \). This explains why an expert is more inclined to raise an argument challenging the consensus view if he must not announce arguments in line with this view.
by all experts in this case. However, if only few experts find this kind of evidence, the principal will learn that the consensus view is wrong.

7.4 State of the world unobservable

We have assumed in this paper that the state of the world is perfectly revealed after the principal has made her decision. This has the convenient implication that each expert does not have to consider the consequence of his action for the future course of the debate. If we made the assumption that the state of the world is not revealed by the principal’s decision, the model would be intractable in general. However, for very large committees, the strategy profiles representing equilibria in our model would also correspond to equilibria in this model variant. This is a direct consequence of the fact that these strategy profiles reveal the state of the world perfectly, even if a single expert deviates from his equilibrium strategy. Thus the strategy profiles remain individually optimal. To sum up, for very large committees, there are always equilibria in which the state of the world is revealed perfectly by the debate among experts. This conclusion holds for both sequential and simultaneous debate.

7.5 Case with \( q_H = q_L \)

So far, we have assumed \( q_H > q_L \). This may raise the question how our results would be affected if we considered \( q_H = q_L \). This assumption would have the consequence that an additional behavior of experts is consistent with equilibrium in the knife-edge case where \( \pi_1(\bar{a}(i)) = \pi_{-1}(\bar{a}(i)) = 1/2 \). In addition to the constellations described in the second and third part of Lemma 1, it is possible that an expert \( i \) presents no argument (to see this, examine the analysis of \( \lambda_i(1) = \lambda_i(-1) = 0 \) in Appendix B and note that \( q_H = q_L \) entails \( (\mu_L - \mu_H)/(\rho_H - \rho_L) = 1 \)). Our findings about information aggregation in large committees would continue to hold if (i) we excluded the possibility that the priors about the two different realizations of the state of the world are perfectly balanced or (ii) we introduced the tie-breaking rule that experts announce their arguments when indifferent between withholding and announcing them.
8 Conclusions

In this paper, we have proposed a model of sequential debate among experts who have private verifiable information about the state of world and are motivated by career concerns. We have demonstrated that self-censorship may hamper the aggregation of information and may lead to wrong decisions by small groups, even if experts have objections privately. These findings are in line with the pessimistic view of group decision-making held by Janis (1972).

An expert may withhold evidence that conflicts with conventional wisdom, even if, based on this information, he believes the prevailing opinion to be wrong and even if the evidence would induce the principal to revise her decision. In this case, decision-making can be improved by assigning the role of a devil’s advocate to an expert.

Despite the problems created by self-censorship, there is no upper bound to the amount of information that can be aggregated in the debate if the group of experts becomes large. Intuitively, it may be the case that experts present only evidence in line with common wisdom initially. However, if common wisdom is wrong, only few experts will be able to provide evidence in its support. As more and more experts remain silent, players revise their beliefs about the state of the world. In this sense, experts’ silence is telling and will encourage other experts to challenge the prevailing view after some time. In the end, this will lead to a correct assessment of the state of world, which lends support to the Condorcetian perspective that decision-making guarantees an optimal decision for large groups.
A Useful Properties

Our assumptions $q_H > q_L$ and $p_H > p_L \geq 1/2$ and definitions (1)-(6) immediately imply $\rho_H > \rho_L$, $\rho_H + \mu_H > \rho_L + \mu_L$, $\rho_H > \mu_H$, $\rho_L \geq \mu_L$, $\overline{p_i} > \overline{p_i}$. In addition, we obtain

$$\rho_H (1 - \mu_L) > \rho_L (1 - \mu_H).$$

(16)

This claim can be verified by applying definitions (1), (2), (4), and (5), which yields $q_H p_H \rho_H (1 - \mu_L) > q_L p_L (1 - \mu_H)$ or equivalently $q_H p_H (1 - \mu_H) > q_L p_L (1 - \mu_H)$. The latter inequality results from $q_L < q_H$ and $p_L < p_H$. (16) has the intuitive interpretation that the probability of an expert being highly competent is higher if he has observed a correct argument than in the case where he has not observed the wrong argument.\(^{19}\)

\[\square\]

B Proof of Lemma 1

Derivation of a condition guaranteeing that it is profitable for $i$ to announce $A_i$

Suppose expert $i$ has observed $A_i \in \{-1, 1\}$. Then the probability of $A_i$ corresponding to the correct state of the world is $$(\pi_{A_i}(\overline{a}(i)))/(\pi_{A_i}(\overline{a}(i)) \overline{p_i} + \pi_{-A_i}(\overline{a}(i)) \overline{p_i}),$$ which takes into account the facts that the probability of $i$ observing a correct argument is $\overline{p_i}$ and that the probability of his observing a wrong argument is $\overline{p_i}$. In addition, we have applied our assumption that the event of $i$ observing a correct argument is independent of whether other experts observe a correct or incorrect argument (or no argument at all). In a similar vein, expert $i$ estimates the probability of $-A_i$ being the correct state of the world to be $$(\pi_{-A_i}(\overline{a}(i)) \overline{p_i})/(\pi_{A_i}(\overline{a}(i)) \overline{p_i} + \pi_{-A_i}(\overline{a}(i)) \overline{p_i}).$$

After these preparations we can state a condition guaranteeing that it is advantageous

\[\text{Notice that the probability of an expert being highly competent, conditional on his having observed the correct argument, is } (\kappa_i \rho_H)/(\kappa_i \rho_H + (1 - \kappa_i) \rho_L). \text{ Given that an expert has not observed a wrong argument, his probability of being of type } H \text{ amounts to } (\kappa_i(1 - \mu_H))/(\kappa_i(1 - \mu_H) + (1 - \kappa_i)(1 - \mu_L)). \text{ Comparing both expressions yields (16).}\]
for expert $i$ to announce his argument $A_i$:

$$
\frac{\pi_A_i(\bar{a}(i))\bar{p}_i}{\pi_A_i(\bar{a}(i))\bar{p}_i + \pi_{-A_i}(\bar{a}(i))\bar{p}_i} \kappa_i(A_i, A_i) + \frac{\pi_{-A_i}(\bar{a}(i))\bar{p}_i}{\pi_A_i(\bar{a}(i))\bar{p}_i + \pi_{-A_i}(\bar{a}(i))\bar{p}_i} \kappa_i(A_i, -A_i)
$$

(17)

If the condition is violated, it is profitable for $i$ to withhold $A_i$. With the help of (7)-(10), (17) can be equivalently stated as

$$
\frac{\pi_A_i(\bar{a}(i))\bar{p}_i}{\pi_A_i(\bar{a}(i))\bar{p}_i + \pi_{-A_i}(\bar{a}(i))\bar{p}_i} \geq 0
$$

(18)

In the following, we evaluate this condition for the three cases mentioned in the lemma.

Analysis of $\lambda_i(-1) = 0$ and $\lambda_i(1) = 0$

Suppose expert $i$ has observed argument $A_i$. Then he will remain silent ((18) does not hold) if

$$
\frac{\pi_A_i(\bar{a}(i))\bar{p}_i}{\pi_A_i(\bar{a}(i))\bar{p}_i + \pi_{-A_i}(\bar{a}(i))\bar{p}_i} \leq \frac{\rho_H - 1}{1 - \rho_H \lambda_i(A_i) - \mu_H \lambda_i(-A_i)}
$$

(19)

where we have used $\lambda_i(-1) = 0$ and $\lambda_i(1) = 0$. Condition (19) can be rewritten as

$$
\pi_A_i(\bar{a}(i)) (\rho_H - \bar{p}_i) \leq \pi_{-A_i}(\bar{a}(i)) (\bar{p}_i - \mu_H).
$$

(20)

Because $\rho_H - \bar{p}_i = \rho_H - \kappa_i \rho_H - (1 - \kappa_i) \rho_L = (1 - \kappa_i)(\rho_H - \rho_L)$ and $\bar{p}_i - \mu_H = \kappa_i \mu_H + (1 - \kappa_i) \mu_L - \mu_H = (1 - \kappa_i)(\mu_L - \mu_H)$, (20) is equivalent to

$$
\frac{\pi_A_i(\bar{a}(i))}{\pi_{-A_i}(\bar{a}(i))} \leq \frac{\mu_L - \mu_H}{\rho_H - \rho_L}.
$$

(21)

We note that the right-hand side of this inequality is strictly lower than 1, which is a consequence of $\mu_L - \mu_H < \rho_H - \rho_L$, which follows from $\rho_H + \mu_H = q_H$, $\rho_L + \mu_L = q_L$, and $q_H > q_L$.

Finally, we have to take into account that (21) has to hold for both $A_i = 1$ and $A_i = -1$. Hence both $\frac{\pi_1(\bar{a}(i))}{\pi_{-1}(\bar{a}(i))} \leq \frac{\mu_L - \mu_H}{\rho_H - \rho_L} < 1$ and $\frac{\pi_{-1}(\bar{a}(i))}{\pi_1(\bar{a}(i))} \leq \frac{\mu_L - \mu_H}{\rho_H - \rho_L} < 1$ must hold, which establishes a contradiction. To sum up, no equilibrium exists in which expert $i$ never proposes his argument.
Analysis of $\lambda_i(\alpha) = 0$ and $\lambda_i(-\alpha) = 1$ for $\alpha \in \{-1, +1\}$

We consider the case with $\alpha = 1$ where expert $i$ always announces his argument if it amounts to $A_i = -1$ and where he remains silent for $A_i = 1$. Obviously, the analysis of the other case is completely analogous. The expert’s behavior must be optimal both for $A_i = 1$ and $A_i = -1$. We consider the case $A_i = -1$ first. Utilizing $\lambda_i(1) = 0$ and $\lambda_i(-1) = 1$, condition (18) can be stated as

$$\pi_{-1}(\vec{a}(i))\rho_{\vec{H}}(1 - \mu_{\vec{L}}) - \rho_{\vec{L}}(1 - \mu_{\vec{H}}) \leq \pi_{1}(\vec{a}(i))\mu_{\vec{L}}(1 - \rho_{\vec{H}}) - \mu_{\vec{H}}(1 - \rho_{\vec{L}})$$

or equivalently

$$\pi_{-1}(\vec{a}(i))\rho_{\vec{H}}(1 - \mu_{\vec{L}}) - \rho_{\vec{L}}(1 - \mu_{\vec{H}}) \leq \pi_{1}(\vec{a}(i))\mu_{\vec{L}}(1 - \rho_{\vec{H}}) - \mu_{\vec{H}}(1 - \rho_{\vec{L}}).$$

Applying $\rho_{\vec{H}} - \rho_{\vec{L}} = \rho_{\vec{H}} - \kappa_i\rho_{\vec{H}} - (1 - \kappa_i)\rho_{\vec{L}} = (1 - \kappa_i)(\rho_{\vec{H}} - \rho_{\vec{L}})$ and $\mu_{\vec{L}} - \mu_{\vec{H}} = \kappa_i\mu_{\vec{H}} + (1 - \kappa_i)\mu_{\vec{L}} - \mu_{\vec{H}} = (1 - \kappa_i)(\mu_{\vec{L}} - \mu_{\vec{H}})$, this inequality can be reformulated as

$$\frac{\pi_{-1}(\vec{a}(i))}{\pi_{1}(\vec{a}(i))} \frac{\rho_{\vec{H}} - \rho_{\vec{L}}}{1 - \rho_{\vec{L}}} \leq \frac{\mu_{\vec{L}} - \mu_{\vec{H}}}{\rho_{\vec{H}} - \rho_{\vec{L}}}, \quad \frac{1 - \mu_{\vec{L}}}{1 - \rho_{\vec{L}}} = \frac{\mu_{\vec{L}} - \mu_{\vec{H}}}{\rho_{\vec{H}} - \rho_{\vec{L}}}, \quad \frac{1}{C_1}.$$ (22)

If (22) holds, then it is optimal for $i$ to present argument $A_i = -1$.

As a next step, we identify the circumstances under which it is optimal for $i$ to withhold argument $A_i = 1$. This is the case if (18) is violated for $A_i = 1$ or

$$\pi_{1}(\vec{a}(i))\rho_{\vec{H}}(1 - \mu_{\vec{H}}) - \rho_{\vec{L}}(1 - \mu_{\vec{H}}) \leq \pi_{-1}(\vec{a}(i))\mu_{\vec{L}}(1 - \rho_{\vec{H}}) - \mu_{\vec{H}}(1 - \rho_{\vec{L}}),$$

which can be re-arranged as follows:

$$\pi_{1}(\vec{a}(i))\left(\frac{\rho_{\vec{H}}(1 - \mu_{\vec{H}}) - \rho_{\vec{L}}(1 - \mu_{\vec{H}})}{1 - \rho_{\vec{L}}}\right) \leq \pi_{-1}(\vec{a}(i))\left(\frac{\mu_{\vec{L}}(1 - \rho_{\vec{H}}) - \mu_{\vec{H}}(1 - \rho_{\vec{L}})}{1 - \rho_{\vec{L}}}\right).$$

With the help of (3) and (6), we obtain

$$\pi_{1}(\vec{a}(i))\left(\frac{\rho_{\vec{H}}(1 - \kappa_i\mu_{\vec{H}} - (1 - \kappa_i)\mu_{\vec{L}}) - (\kappa_i\rho_{\vec{H}} + (1 - \kappa_i)\rho_{\vec{L}})(1 - \mu_{\vec{H}})}{1 - \rho_{\vec{L}}}\right)$$

$$\leq \pi_{-1}(\vec{a}(i))\left(\frac{(\kappa_i\mu_{\vec{H}} + (1 - \kappa_i)\mu_{\vec{L}})(1 - \rho_{\vec{H}}) - \mu_{\vec{H}}(1 - \kappa_i\rho_{\vec{H}} - (1 - \kappa_i)\rho_{\vec{L}})}{1 - \rho_{\vec{L}}}\right),$$

which can be simplified to

$$\pi_{1}(\vec{a}(i))\frac{\rho_{\vec{H}}(1 - \mu_{\vec{L}}) - \rho_{\vec{L}}(1 - \mu_{\vec{H}})}{1 - \rho_{\vec{L}}} \leq \pi_{-1}(\vec{a}(i))\frac{\mu_{\vec{L}}(1 - \rho_{\vec{H}}) - \mu_{\vec{H}}(1 - \rho_{\vec{L}})}{1 - \rho_{\vec{L}}}.$$ (23)
We can re-arrange (23) and obtain
\[
\frac{\pi_1(\vec{a}(i))}{\pi_{-1}(\vec{a}(i))} \leq \frac{\mu_L(1 - \rho_H) - \mu_H(1 - \rho_L)}{\rho_H(1 - \mu_L) - \rho_L(1 - \mu_H)} \cdot \frac{1 - \overline{\mu}_i}{1 - \mu_i} = C_1C_2. \tag{24}
\]

Finally, we show that (24) implies (22). For this purpose, we note that (24) implies
\[
\mu_L(1 - \rho_H) > \mu_H(1 - \rho_L), \text{ or equivalently } C_1C_2 > 0, \text{ and hence}
\frac{\pi_{-1}(\vec{a}(i))}{\pi_1(\vec{a}(i))} \geq \frac{1}{C_1C_2}. \tag{25}
\]

Drawing on \(\rho_H + \mu_H > \rho_L + \mu_L\), (15), and (16), we observe
\[
\frac{1}{C_2} = \frac{\rho_H(1 - \mu_L) - \rho_L(1 - \mu_H)}{\mu_L(1 - \rho_H) - \mu_H(1 - \rho_L)} > 1
\]
and
\[
\frac{\mu_L - \mu_H}{\rho_H - \rho_L} < 1,
\]
which establishes that (22) follows from (25) and thus from (24).

**Analysis of** \(\lambda_i(1) = \lambda_i(-1) = 1\)

Now we evaluate (18) for \(\lambda_i(1) = \lambda_i(-1) = 1\):
\[
\pi_{A_1}(\vec{a}(i))\overline{\rho}_i\left(\frac{\rho_H}{\overline{\mu}_i} - \frac{1 - q_H}{1 - \overline{\mu}_i - \overline{\mu}_i}\right) \geq \pi_{-A_1,\overline{\mu}_i}\left(\frac{1 - q_H}{1 - \overline{\rho}_i - \overline{\mu}_i} - \frac{\mu_H}{\overline{\mu}_i}\right),
\]
which, in turn, is readily shown to be equivalent to
\[
\pi_{A_1}(\vec{a}(i)) \left(\rho_H(1 - \overline{\rho}_i - \overline{\mu}_i) - \overline{\rho}_i(1 - q_H)\right) \geq \pi_{-A_1,\overline{a}(i)} \left(\overline{\rho}_i(1 - q_H) - \mu_H(1 - \overline{\rho}_i - \overline{\mu}_i)\right).
\tag{26}
\]

As a next step, we show
\[
\rho_H(1 - \overline{\rho}_i - \overline{\mu}_i) - \overline{\rho}_i(1 - q_H)
= \rho_H(1 - \overline{\rho}_i - \overline{\mu}_i) - \overline{\rho}_i(1 - \rho_H - \mu_H)
= \rho_H(1 - \overline{\rho}_i) - \overline{\rho}_i(1 - \mu_H)
= \rho_H(1 - \kappa_i \mu_H - (1 - \kappa_i) \mu_L) - (\kappa_i \rho_H + (1 - \kappa_i) \rho_L)(1 - \mu_H)
= (1 - \kappa_i) \left(\rho_H(1 - \mu_L) - \rho_L(1 - \mu_H)\right),
\tag{27}
\]
where we have applied \(q_H = \rho_H + \mu_H\). Similarly, it is straightforward to check
\[
\overline{\rho}_i(1 - q_H) - \mu_H(1 - \overline{\rho}_i - \overline{\mu}_i) = (1 - \kappa_i) \left(\mu_L(1 - \rho_H) - \mu_H(1 - \rho_L)\right).
\tag{28}
\]
Now (27) and (28) can be used to rewrite (26) as
\[
\frac{\pi_{A_i}(\vec{a}(i))}{\pi_{-A_i}(\vec{a}(i))} \geq \frac{\mu_L(1 - \rho_H) - \mu_H(1 - \rho_L)}{\rho_H(1 - \mu_L) - \rho_L(1 - \mu_H)} = C_2.
\] (29)

This condition has to be satisfied for \(A_i = 1\) and \(A_i = -1\). Hence the proposed behavior is optimal if (12) holds. □

C Beliefs about the State of the World

In this Appendix, we describe the updating process for probabilities \(\pi_s(\vec{a}(i))\). Trivially, we obtain \(\pi_s(\vec{a}(1)) = \pi_s\) because \(\vec{a}(1)\) is empty. As a next step, we consider \(\pi_s(\vec{a}(2)) = \pi_s(a_1)\), which is the probability of the state of the world being \(s\), conditional on expert 1’s choice \(a_1 = \vec{a}(2)\). Recall that the prior probability of \(A_1\) being correct is \(\pi_{A_1}\). Then the posterior probability of \(s \in \{-1, 1\}\) corresponding to the correct state of the world is
\[
\pi_s(a_1) = \frac{\pi_s Pr(a_1|s)}{\pi_s Pr(a_1|s) + (1 - \pi_s) Pr(a_1|-s)},
\] (30)
where \(Pr(a_1|s)\) is the probability of expert 1 choosing \(a_1\) conditional on \(s\) being the correct state of the world. Equation (30) can be rewritten as
\[
\frac{\pi_s(a_1)}{\pi_{-s}(a_1)} = \frac{\pi_s}{\pi_{-s}} \cdot \frac{Pr(a_1|s)}{Pr(a_1|-s)}.
\] (31)

It is useful to follow Ottaviani and Sørensen (2001) in considering log-likelihoods of beliefs:
\[
\ln \left( \frac{\pi_s(a_1)}{\pi_{-s}(a_1)} \right) = \ln \left( \frac{\pi_s}{\pi_{-s}} \right) + \ln \left( \frac{Pr(a_1|s)}{Pr(a_1|-s)} \right). \tag{32}
\]

This representation has the advantage that Bayesian updating is additive.

As a next step, we compute a formula for beliefs about \(s\), updated for a vector \(\vec{a}(i+1)\), which contains the first \(i\) statements of a general sequence of arguments \(\vec{a} \in \{-1, 0, 1\}^N\). Then, analogously to (32), the updated beliefs satisfy the following recursive formula for \(i = 1, \ldots, N\):
\[
\ln \left( \frac{\pi_s(\vec{a}(i+1))}{\pi_{-s}(\vec{a}(i+1))} \right) = \ln \left( \frac{\pi_s(\vec{a}(i))}{\pi_{-s}(\vec{a}(i))} \right) + \ln \left( \frac{Pr(a_i|s, \vec{a}(i))}{Pr(a_i|-s, \vec{a}(i))} \right), \tag{33}
\]
where $\Pr(a_i|s, \vec{a}(i))$ stands for the probability of expert $i$ choosing $a_i$, conditional on $s$ being the correct state of the world and conditional on $\vec{a}(i)$. Recursive iterations yield

$$\ln \left( \frac{\pi_s(\vec{a}(i + 1))}{\pi_{-s}(\vec{a}(i + 1))} \right) = \ln \left( \frac{\pi_s}{\pi_{-s}} \right) + \sum_{j=1}^{i} \ln \left( \frac{\Pr(a_j|s, \vec{a}(j))}{\Pr(a_j|-s, \vec{a}(j))} \right).$$

(34)

It remains to specify $\Pr(a_i|s, \vec{a}(i))$ for an arbitrary expert $i = 1, ..., N$. We note that $\Pr(s|s, \vec{a}(i)) = \lambda_i(s)\overline{\mu}_i$, $\Pr(s|-s, \vec{a}(i)) = \lambda_i(s)\overline{\mu}_i$, $\Pr(0|s, \vec{a}(i)) = 1 - \lambda_i(s)\overline{\mu}_i - \lambda(-s)\overline{\mu}_i$. At this stage, it is crucial to recall that $\lambda_i(s)$ depends on $\vec{a}(i)$. Therefore $\ln \left( \frac{\Pr(a_i|s, \vec{a}(i))}{\Pr(a_i|-s, \vec{a}(i))} \right)$ will depend on $\vec{a}(i)$ in general.

Using the expressions for $\Pr(a_i|s, \vec{a}(i))$ we have derived, we proceed by determining $\ln \left( \frac{\Pr(a_i|s, \vec{a}(i))}{\Pr(a_i|-s, \vec{a}(i))} \right)$ for all three possible actions $a_i \in \{-1, 0, 1\}$. We start with $a_i = 1$ and $a_i = -1$:

$$\ln \left( \frac{\Pr(s|s, \vec{a}(i))}{\Pr(s|-s, \vec{a}(i))} \right) = \ln \left( \frac{\overline{\mu}_i}{\overline{\mu}_i} \right) > 0 \quad \text{for } a_i = 1,$$

(35)

$$\ln \left( \frac{\Pr(-s|s, \vec{a}(i))}{\Pr(-s|-s, \vec{a}(i))} \right) = \ln \left( \frac{\overline{\mu}_i}{\overline{\mu}_i} \right) < 0 \quad \text{for } a_i = -1,$$

(36)

where we have utilized $\overline{\mu}_i < \overline{\mu}_i$ (see Appendix A). Hence if expert $i$ presents his argument $A_i \in \{-1, 1\}$, this will shift others’ beliefs towards the view that $s = A_i$ is correct. This statement holds independently of whether the expert behaves according to the second or third part of Lemma 1 and thus in every equilibrium.

For $a_i = 0$, we obtain

$$\ln \left( \frac{\Pr(0|s, \vec{a}(i))}{\Pr(0|-s, \vec{a}(i))} \right) = \ln \left( \frac{1 - \overline{\mu}_i\lambda_i(s) - \overline{\mu}_i\lambda_i(-s)}{1 - \overline{\mu}_i\lambda_i(-s) - \overline{\mu}_i\lambda_i(s)} \right).$$

(37)

Interestingly, $\ln \left( \frac{\Pr(0|s, \vec{a}(i))}{\Pr(0|-s, \vec{a}(i))} \right)$ depends on $\lambda_i(1)$ and $\lambda_i(-1)$, in contrast to the respective expressions for $a_i = 1$ or $a_i = -1$ (see (35) and (36)). It is instructive to examine this relationship in more detail.

Suppose $\lambda_i(-1) = 0$, $\lambda_i(1) = 1$, and $a_i = 0$. With the help of (37), it is easily verified that this involves $\ln \left( \frac{\Pr(0|s, \vec{a}(i))}{\Pr(0|-s, \vec{a}(i))} \right) = \ln \left( \frac{1 - \overline{\mu}_i}{1 - \overline{\mu}_i} \right) < 0$ (note that $\overline{\mu}_i > \overline{\mu}_i$). This has the important implication that, if expert $i$ presents no argument, this will induce an outside observer to attach a higher probability to the state of the world being $-1$ than before.
Finally, we consider \( \lambda_i(1) = \lambda(-1) = 1 \), in addition to \( a_i = 0 \). In this case, expert \( i \) would make both arguments public. As a result, silence \( (a_i = 0) \) is completely uninformative, as can be verified by inserting \( \lambda_i(1) = \lambda(-1) = 1 \) into (37), which results in

\[
\ln \left( \frac{\Pr(0 \mid s, \vec{a}(i))}{\Pr(0 \mid -s, \vec{a}(i))} \right) = 0.
\]

\[\square\]

D Proof of Proposition 3

The proposition is obvious if (15) does not hold, which entails Proposition 1. Thus we assume (15) is satisfied. Without loss of generality, suppose the state of world is \(-1\). We introduce \( \mathcal{A} \) as the set of all possible combinations of arguments that \( N \) experts may obtain, i.e. \( \mathcal{A} := \{-1, 0, 1\}^N \). We use \( \bar{A} \in \mathcal{A} \) to denote a particular combination of arguments that experts may obtain (recall that \( A_i \) may differ from the argument \( a_i \) announced by expert \( i \) because the expert may withhold information). Let \( \mathcal{F} \) be the full power set of \( \mathcal{A} \). Then \((\mathcal{A}, \mathcal{F})\) is a measurable space and our assumptions about the arrival of arguments, conditional on the state of the world being \(-1\), define a probability measure \( P \) on \( \mathcal{F} \).

Moreover, we introduce the filtration \((\mathcal{F}_i)_{i=0,\ldots,N}\), where \( \mathcal{F}_i \) is the \( \sigma \)-algebra describing all information about \( A_1, \ldots, A_i \) for \( i = 0, \ldots, N \). Now each equilibrium of our game of sequential exchange of arguments represents a mapping from observed arguments \( \bar{A} \) to announced arguments \( \vec{a} \), i.e. from the set \( \mathcal{A} \) into \( \mathcal{A} \). As a consequence, each equilibrium specifies a stochastic process for the updated log-likelihoods \( \ln \left( \frac{\pi_i(\vec{a}(i+1))}{\pi_{-1}(\vec{a}(i+1))} \right) \) (see (33)). We use \( X_i := \ln \left( \frac{\pi_i(\vec{a}(i+1))}{\pi_{-1}(\vec{a}(i+1))} \right) \) to denote this stochastic process. Thus the claim of the proposition amounts to showing \( P(X_N \geq 0) \to 0 \) for \( N \to \infty \) because it can be optimal for the principal to choose 1 only if \( X_N \geq 0 \), according to Lemma 2.

The following lemma, which will be proved in Appendix E, will be useful:

**Lemma 3**

*The stochastic process \( X_i \) has bounded innovations, i.e. a positive constant \( K \) exists such that \( |X_i - E[X_i \mid \mathcal{F}_{i-1}]| < K \) holds for all \( \bar{A} \in \mathcal{A} \) and for all \( i > 0 \). In addition, a strictly positive constant \( k \) exists with \( E[X_{i+1} \mid \mathcal{F}_i] < -k + X_i \) for all \( i > 0 \).*
Now we define a stochastic process recursively by

\[ Y_0 := X_0 \]
\[ Y_i := Y_{i-1} + X_i - E[X_i | F_{i-1}] \quad \text{for } i > 0 \]

We note that \( Y_i \) is an \( F_i \)-adapted martingale. Lemma 3 implies that \( Y_i \) has bounded increments, i.e. \( |Y_i - Y_{i-1}| < K \) holds for all \( \tilde{A} \in \mathcal{A} \) and for all \( i > 1 \). Inserting recursively yields

\[ Y_i = X_i - \sum_{j=1}^{i} (E[X_j | F_{j-1}] - X_{j-1}). \]

Now the second part of Lemma 3 implies

\[ Y_i \geq X_i + ki. \]

Hence \( P(X_i \geq 0) \leq P(Y_i - ki \geq 0) = P(Y_i \geq ki) \). Consequently, by computing \( P(Y_i \geq ki) \), we can establish an upper bound to \( P(X_i \geq 0) \).

Finally, we apply the Azuma-Hoeffding inequality, which goes back to Hoeffding (1963) and Azuma (1967), and is presented in McDiarmid (1989) (see the proof of Lemma 4.1 on p. 160):

**Lemma 4**

*(Azuma-Hoeffding inequality)* Suppose \( \{Y_i\}_{i=0}^{N} \) is a martingale with \( |Y_i - Y_{i-1}| \leq c_i \) for each \( i \), for suitable constants \( c_i \). Then for any \( \chi > 0 \)

\[ P(Y_n \geq Y_0 + \chi) \leq \exp \left( -\chi^2 / (2 \sum_{j=1}^{n} c_j^2) \right). \] (38)

If we set \( \chi = kn - Y_0 \), which is positive for sufficiently high values of \( n \), and recall that \( |Y_i - Y_{i-1}| < K \) holds for all \( i > 1 \), we obtain that (38) implies

\[ P(Y_N \geq kN) \leq \exp \left( -(kN - Y_0)^2 / (2NK^2) \right). \] (39)

If we increase the number of experts, the right-hand side of (39) converges to zero. Thus \( P(Y_N \geq kN) \) converges to zero, which in turn implies \( P(X_N \geq 0) \rightarrow 0 \) as \( P(X_N \geq 0) \leq P(Y_N \geq kN) \). □
\[ A = 1 \quad A = -1 \quad A = 0 \quad \text{Exp. with respect to } F_{i-1} \]

<table>
<thead>
<tr>
<th>( \lambda_i(1) = 1, \lambda_i(-1) = 1 )</th>
<th>( \ln \left( \frac{\rho_i}{\mu_i} \right) )</th>
<th>( -\ln \left( \frac{\rho_i}{\mu_i} \right) )</th>
<th>0</th>
<th>( - (\overline{\rho}_i - \overline{\mu}_i) \ln \left( \frac{\rho_i}{\mu_i} \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_i(1) = 0, \lambda_i(-1) = 1 )</td>
<td>( \ln \left( \frac{1-\rho_i}{1-\mu_i} \right) )</td>
<td>( -\ln \left( \frac{1-\rho_i}{1-\mu_i} \right) )</td>
<td>( \ln \left( \frac{1-\rho_i}{1-\mu_i} \right) )</td>
<td>( -\overline{\rho}_i \ln \left( \frac{\rho_i}{\mu_i} \right) + (1 - \overline{\mu}_i) \ln \left( \frac{1-\rho_i}{1-\mu_i} \right) )</td>
</tr>
<tr>
<td>( \lambda_i(1) = 1, \lambda_i(-1) = 0 )</td>
<td>( \ln \left( \frac{\rho_i}{\mu_i} \right) )</td>
<td>( -\ln \left( \frac{1-\rho_i}{1-\mu_i} \right) )</td>
<td>( -\ln \left( \frac{1-\rho_i}{1-\mu_i} \right) )</td>
<td>( \overline{\rho}_i \ln \left( \frac{\rho_i}{\mu_i} \right) - (1 - \overline{\mu}_i) \ln \left( \frac{1-\rho_i}{1-\mu_i} \right) )</td>
</tr>
</tbody>
</table>

Table 1: \( X_i - X_{i-1} \) for the different possible behaviors described in Lemma 1 (rows) and different realizations of \( A_i \) (columns). The last column specifies \( E[X_i|F_{i-1}] - X_{i-1} \).

### E Proof of Lemma 3

The definition \( X_i := \ln \left( \frac{x_i(d(i+1))}{\pi_i(d(i+1))} \right) \) can be applied to (33) to establish \( X_i - X_{i-1} = \ln \left( \frac{\Pr(a_i|1,d(j))}{\Pr(a_i|1,d(j))} \right) \). In Table 1, we summarize the different values for \( X_i - X_{i-1} \) that may occur. The values can be verified by using (35)-(37). The last column gives \( E[X_i|F_{i-1}] - X_{i-1} \) for the three different possible behaviors specified in Lemma 1, where we utilize that, conditional on the state of the world being \( s = -1 \), \( A_i = 1 \) occurs with probability \( \overline{\rho}_i \), \( A_i = -1 \) with probability \( \overline{\mu}_i \), and \( A_i = 0 \) with probability \( 1 - \overline{\rho}_i - \overline{\mu}_i \).

In Appendix F, we show \( \ln \left( \frac{\rho_i}{\mu_i} \right) < \ln \left( \frac{\rho_i}{\rho_{L}} \right) \) and \( \ln \left( \frac{1-\rho_i}{1-\mu_i} \right) < \ln \left( \frac{\rho_{L}}{\mu_{L}} \right) \) \( \forall \kappa_i \in ]0,1[ \). As a consequence, it is obvious from Table 1 that \( K := \ln \left( \frac{\rho_{L}}{\mu_{L}} \right) = \ln \left( \frac{\rho_{L}}{\mu_{L}} \right) \) involves \( |X_i - X_{i-1}| < K \).

As a next step, we show a constant \( k > 0 \) exists such that \( E[X_i|F_{i-1}] - X_{i-1} < -k \) for each row in Table 1. For this purpose, we show in Appendix G that the derivatives of all entries in the last column of Table 1 with respect to \( \kappa_i \) are strictly negative \( \forall \kappa_i \in ]0,1[ \). Hence these entries are always strictly larger if they are evaluated at \( \kappa_i = 0 \), which entails \( \overline{\rho}_i = \rho_L \) and \( \overline{\mu}_i = \mu_L \), than for arbitrary \( \kappa_i \in ]0,1[ \). As a consequence, we obtain for the first entry in the last column of Table 1:

\[ -(\overline{\rho}_i - \overline{\mu}_i) \ln \left( \frac{\rho_i}{\mu_i} \right) < -(\rho_L - \mu_L) \ln \left( \frac{\rho_L}{\mu_L} \right) < 0, \]

where we have used \( \rho_L > \mu_L \).
For the second entry in the last column, we obtain

\[-\rho_i \ln \left( \frac{\overline{\rho}_i}{\mu_i} \right) + (1 - \rho_i) \ln \left( \frac{1 - \overline{\rho}_i}{1 - \rho_i} \right) < -\rho_L \ln \left( \frac{\rho_L}{\mu_L} \right) + (1 - \rho_L) \ln \left( \frac{1 - \mu_L}{1 - \rho_L} \right)\]

\[
= \rho_L \ln \left( \frac{\mu_L}{\rho_L} \right) + (1 - \rho_L) \ln \left( \frac{1 - \mu_L}{1 - \rho_L} \right) < \rho_L \left( \frac{\mu_L}{\rho_L} - 1 \right) + (1 - \rho_L) \left( \frac{1 - \mu_L}{1 - \rho_L} - 1 \right) = 0,
\]

where we have utilized the result derived in Appendix G about the derivative of 

\[-\rho_i \ln \left( \frac{\overline{\rho}_i}{\mu_i} \right) + (1 - \rho_i) \ln \left( \frac{1 - \overline{\rho}_i}{1 - \rho_i} \right) \text{ with respect to } \kappa_i \text{ and the fact that } \ln x < x - 1 \forall x \neq 1.
\]

Finally, for the last entry in the last column, we obtain

\[-\rho_i \ln \left( \frac{\overline{\rho}_i}{\mu_i} \right) - (1 - \overline{\rho}_i) \ln \left( \frac{1 - \overline{\rho}_i}{1 - \rho_i} \right) < \mu_L \ln \left( \frac{\rho_L}{\mu_L} \right) - (1 - \mu_L) \ln \left( \frac{1 - \mu_L}{1 - \rho_L} \right)\]

\[
= \mu_L \ln \left( \frac{\rho_L}{\mu_L} \right) + (1 - \mu_L) \ln \left( \frac{1 - \rho_L}{1 - \mu_L} \right) < \mu_L \left( \frac{\rho_L}{\mu_L} - 1 \right) + (1 - \mu_L) \left( \frac{1 - \rho_L}{1 - \mu_L} - 1 \right) = 0.
\]

Note that we have again used \(\ln x < x - 1 \forall x \neq 1\). Hence all entries in the last column of Table 1 are strictly smaller than a negative constant that does not depend on \(\kappa_i\). This establishes the claim of the lemma.

\[\Box\]

**F Proof of** \(\ln \left( \frac{\overline{\rho}_i}{\mu_i} \right) < \ln \left( \frac{\rho_H}{\mu_H} \right)\) and \(\ln \left( \frac{1 - \overline{\rho}_i}{1 - \rho_i} \right) < \ln \left( \frac{\rho_H}{\mu_H} \right)\)

\(\forall \kappa_i \in]0, 1[\)

First, we show \(\ln \left( \frac{\overline{\rho}_i}{\mu_i} \right) < \ln \left( \frac{\rho_H}{\mu_H} \right)\). This is equivalent to \(\frac{\overline{\rho}_i}{\mu_i} < \frac{\rho_H}{\mu_H}\). Inserting (3), this can be re-arranged as \((\kappa_i \rho_H + (1 - \kappa_i) \rho_L) \mu_H < (\kappa_i \mu_H + (1 - \kappa_i) \mu_L) \rho_H\), which is equivalent to \(\rho_H \mu_L > \rho_L \mu_H\). Using (1), (2), (4), and (5), this can be rewritten as 

\(q_H p_H q_L (1 - p_L) > q_L p_L q_H (1 - p_H)\), which follows from \(p_H > p_L\).

Second, we demonstrate \(\ln \left( \frac{1 - \overline{\rho}_i}{1 - \rho_i} \right) < \ln \left( \frac{\rho_H}{\mu_H} \right)\) \(\forall \kappa_i \in]0, 1[\). For this purpose we proceed as follows. We first demonstrate that the left-hand side of the inequality is strictly increasing in \(\kappa_i\). This has the consequence \(\ln \left( \frac{1 - \overline{\rho}_i}{1 - \rho_i} \right) < \ln \left( \frac{1 - \mu_H}{1 - \rho_H} \right)\), as \(\lim_{\kappa_i \to 1} \overline{\rho}_i = \mu_H\) and \(\lim_{\kappa_i \to 1} \overline{\rho}_i = \rho_H\). Finally, we will show \(\ln \left( \frac{1 - \mu_H}{1 - \rho_H} \right) < \ln \left( \frac{\rho_H}{\mu_H} \right)\), which establishes \(\ln \left( \frac{1 - \overline{\rho}_i}{1 - \rho_i} \right) < \ln \left( \frac{\rho_H}{\mu_H} \right)\).

31
The derivative of \( \ln \left( \frac{1 - \mu_i}{1 - \rho_i} \right) \) with respect to \( \kappa_i \) can be computed as follows:

\[
\frac{d}{d \kappa_i} \ln \left( \frac{1 - \mu_i}{1 - \rho_i} \right) = \frac{\mu_H - \mu_L}{1 - \mu_i} + \frac{\rho_H - \rho_L}{1 - \rho_i} \tag{40}
\]

\[
= \frac{1}{1 - \mu_i} \cdot \left( (\rho_H - \rho_L) \cdot \frac{1 - \mu_i}{1 - \rho_i} - (\mu_H - \mu_L) \right) \tag{41}
\]

\[
> \frac{1}{1 - \mu_i} \cdot ((\rho_H - \rho_L) - (\mu_H - \mu_L)) \tag{42}
\]

\[
= \frac{1}{1 - \mu_i} \cdot (q_H(2p_H - 1) - q_L(2p_L - 1)) > 0, \tag{43}
\]

where we have used \( \frac{d \rho_i}{d \kappa_i} = \mu_H - \mu_L \) (see (3)), \( \frac{d \mu_i}{d \kappa_i} = \rho_H - \rho_L \) (see (6)), (1)-(3), (4), (6), and \( \mu_i > \rho_i \). Consequently, we conclude \( \ln \left( \frac{1 - \mu_i}{1 - \rho_i} \right) > \ln \left( \frac{1 - \mu_H}{1 - \rho_H} \right) \).

Finally, we show \( \ln \left( \frac{1 - \mu_H}{1 - \rho_H} \right) < \ln \left( \frac{1 - \mu_L}{1 - \rho_L} \right) \). Obviously, this is equivalent to \( \frac{1 - \mu_H}{1 - \rho_H} < \frac{1 - \mu_L}{1 - \rho_L} \), which can be rewritten as \( \frac{1 - q_H(1 - p_H)}{1 - q_H p_H} < \frac{1 - q_L(1 - p_L)}{1 - q_L p_L} \) (see (1) and (4)). Rearranging yields \( (1 - q_H(1 - p_H))(1 - p_H) < p_H(1 - q_H p_H) \). Collecting terms, we obtain \( q_H(p_H^2 - (1 - p_H)^2) < 2p_H - 1 \) and thus \( q_H(2p_H - 1) < 2p_H - 1 \), which holds due to \( p_H > 1/2 \) and \( q_H < 1 \).

**G Proof that the Derivatives of All Entries in the Last Column of Table 1 with Respect to \( \kappa_i \) are Negative**

It will prove useful to evaluate the following term first:

\[
\frac{\rho_H - \rho_L}{\mu_H - \mu_L} = \frac{\mu_H - \mu_L}{\rho_i} \tag{45}
\]

\[
= \frac{1}{\rho_i \mu_i} \left[ \kappa_i \mu_H \rho_H + (1 - \kappa_i) \mu_L \rho_H - \kappa_i \mu_H \rho_L \right. \\
- \left. (1 - \kappa_i) \mu_L \rho_H - \mu_H \kappa_i \rho_H - \mu_L \kappa_i \rho_H + \mu_L (1 - \kappa_i) \rho_L \right] \\
= \frac{1}{\rho_i \mu_i} [\rho_H \mu_L - \rho_L \mu_H] > 0,
\]

where the expression is strictly positive because \( \rho_H \mu_L > \rho_L \mu_H \), which is readily verified by using (1), (2), (4), and (5). In addition, recall that, according to (3) and (6),
\[ \overline{p}_i = \kappa_i \rho_H + (1 - \kappa_i) \rho_L \] and \[ \overline{\mu}_i = \kappa_i \mu_H + (1 - \kappa_i) \mu_L. \] This implies \[ \frac{\partial \overline{p}_i}{\partial \kappa_i} = \rho_H - \rho_L \] and \[ \frac{\partial \overline{\mu}_i}{\partial \kappa_i} = \mu_H - \mu_L. \]

After these preparations, we are in a position to compute the sign of the derivatives of the entries in the last column of Table 1 with respect to \( \kappa_i \). With the help of the expressions for \( \frac{\partial \overline{p}_i}{\partial \kappa_i} \) and \( \frac{\partial \overline{\mu}_i}{\partial \kappa_i} \) derived above, we obtain for the first entry in the last column:

\[
- \frac{d}{d\kappa_i} \left( (\overline{p}_i - \underline{p}_i) \ln \left( \frac{\overline{p}_i}{\underline{p}_i} \right) \right) \\
= - (\rho_H - \rho_L) \left( \ln \left( \frac{\overline{p}_i}{\underline{p}_i} \right) + \ln \left( \frac{1 - \overline{p}_i}{1 - \underline{p}_i} \right) \right) - \overline{p}_i \left( \frac{\rho_H - \rho_L}{\overline{p}_i} - \frac{\mu_H - \mu_L}{\overline{p}_i} \right) \\
+ (1 - \overline{p}_i) \left( - \frac{\mu_H - \mu_L}{1 - \overline{p}_i} + \frac{\rho_H - \rho_L}{1 - \overline{p}_i} \right) \\
= - (\rho_H - \rho_L) \ln \left( \frac{\overline{p}_i}{\underline{p}_i} \cdot \frac{1 - \overline{p}_i}{1 - \underline{p}_i} \right) + \overline{p}_i \left( \frac{\mu_H - \mu_L}{\overline{p}_i} \right) - (1 - \overline{p}_i) \left( \frac{\mu_H - \mu_L}{1 - \overline{p}_i} \right) \\
= (\rho_H - \rho_L) \ln \left( \frac{\overline{p}_i}{\underline{p}_i} \cdot \frac{1 - \overline{p}_i}{1 - \underline{p}_i} \right) + \frac{(\mu_H - \mu_L)(\overline{p}_i - \underline{p}_i)}{\overline{p}_i(1 - \overline{p}_i)} \\
< (\rho_H - \rho_L) \left( \frac{\overline{p}_i}{\underline{p}_i} \cdot \frac{1 - \overline{p}_i}{1 - \underline{p}_i} - 1 \right) + \frac{(\mu_H - \mu_L)(\overline{p}_i - \underline{p}_i)}{\overline{p}_i(1 - \overline{p}_i)} \\
= - (\rho_H - \rho_L) \frac{\overline{p}_i - \underline{p}_i}{\overline{p}_i(1 - \overline{p}_i)} + \frac{(\mu_H - \mu_L)(\overline{p}_i - \underline{p}_i)}{\overline{p}_i(1 - \overline{p}_i)} \\
= - \frac{\overline{p}_i - \underline{p}_i}{1 - \overline{p}_i} \left( \frac{\rho_H - \rho_L}{\overline{p}_i} - \frac{\mu_H - \mu_L}{\overline{p}_i} \right),
\]

where we have applied \( \frac{\partial \overline{p}_i}{\partial \kappa_i} = \rho_H - \rho_L \), \( \frac{\partial \overline{\mu}_i}{\partial \kappa_i} = \mu_H - \mu_L \), and \( \ln x < x - 1 \quad \forall x \neq 1 \). This expression is negative because the term in brackets is positive, which is in line with (45).
Finally, we turn to the derivative of the last entry in the last row of Table 1.

\[
\frac{d}{d\kappa_i} \left( \overline{\rho_i} \ln \left( \frac{\overline{\rho_i}}{\rho_i} \right) - (1 - \overline{\rho_i}) \ln \left( \frac{1 - \overline{\rho_i}}{1 - \rho_i} \right) \right)
\]

\[
= (\mu_H - \mu_L) \left( \ln \left( \frac{\overline{\rho_i}}{\rho_i} \right) + \ln \left( \frac{1 - \overline{\rho_i}}{1 - \rho_i} \right) \right) + \overline{\rho_i} \left( \frac{\rho_H - \rho_L}{\rho_i} - \frac{\mu_H - \mu_L}{\overline{\rho_i}} \right)
\]

\[
+ (1 - \overline{\rho_i}) \left( \frac{\mu_H - \mu_L}{1 - \overline{\rho_i}} - \frac{\rho_H - \rho_L}{1 - \rho_i} \right)
\]

\[
= (\mu_H - \mu_L) \ln \left( \frac{\overline{\rho_i} \cdot 1 - \overline{\rho_i}}{1 - \overline{\rho_i}} \right) + \overline{\rho_i} \left( \frac{\rho_H - \rho_L}{\rho_i} \right) - (1 - \overline{\rho_i}) \left( \frac{\rho_H - \rho_L}{1 - \overline{\rho_i}} \right)
\]

If \( \mu_H \leq \mu_L \), this expression is strictly negative (recall \( \overline{\rho_i} > \overline{\rho_i} \)). In the following, we therefore consider \( \mu_H > \mu_L \). Because \( \ln x < x - 1 \ \forall x \neq 1 \), we obtain

\[
(\mu_H - \mu_L) \ln \left( \frac{\overline{\rho_i} \cdot 1 - \overline{\rho_i}}{1 - \overline{\rho_i}} \right) - (\rho_H - \rho_L) \frac{\overline{\rho_i} - \overline{\rho_i}}{\overline{\rho_i}(1 - \overline{\rho_i})}
\]

\[
< (\mu_H - \mu_L) \left( \frac{\overline{\rho_i} \cdot 1 - \overline{\rho_i}}{1 - \overline{\rho_i}} - 1 \right) - (\rho_H - \rho_L) \frac{\overline{\rho_i} - \overline{\rho_i}}{\overline{\rho_i}(1 - \overline{\rho_i})}
\]

\[
= (\mu_H - \mu_L) \frac{\overline{\rho_i} - \overline{\rho_i}}{\overline{\rho_i}(1 - \overline{\rho_i})} - (\rho_H - \rho_L) \frac{\overline{\rho_i} - \overline{\rho_i}}{\overline{\rho_i}(1 - \overline{\rho_i})}
\]

\[
= - \frac{\overline{\rho_i} - \overline{\rho_i}}{1 - \overline{\rho_i}} \left( \frac{\rho_H - \rho_L}{\overline{\rho_i}} - \frac{\mu_H - \mu_L}{\overline{\rho_i}} \right).
\]

According to (45), the term in brackets is positive. As a result, \( \frac{d}{d\kappa_i} \left( \overline{\rho_i} \ln \left( \frac{\overline{\rho_i}}{\rho_i} \right) - (1 - \overline{\rho_i}) \ln \left( \frac{1 - \overline{\rho_i}}{1 - \rho_i} \right) \right) < 0 \) also for \( \mu_H > \mu_L \).

\[\square\]

**H Proof of Corollary 1**

The claim is trivial if (15) does not hold, which implies a revelation of all arguments, according to Proposition 1. Consequently, we assume (15) in the following. Suppose, without loss of generality, that \( s = -1 \).

Lemma 1 yields that expert \( N \) always presents \( A_i = -1 \) if the prevailing view is that \(-1\) is sufficiently more likely to be correct or, more precisely, if \( X_{N-1} < \min \{-|\ln C_2|, |\ln C_2| - |\ln C_1|\} \). Hence expert \( N \) presents \( A_i = -1 \) with probability one if \( N \) is sufficiently large because \( \lim_{N \to \infty} P (X_{N-1} < \min \{-|\ln C_2|, |\ln C_2| - \ln C_1\}) = 1 \), which follows from the proof of Proposition 3. \[\square\]
I Proof of Proposition 4

In order to demonstrate that it is possible to satisfy the conditions given in the proposition, we specify a respective parameter constellation. Suppose $N = 1$, $p_H = 1$, $q_H = 1/2$, $p_L = 1/2$, $q_L = 1/3$, $\kappa_1 = 3/4$, $\pi_{+1} = 7/9$, and $\pi_{-1} = 2/9$. Then $\rho_H = 1/2$, $\mu_H = 0$, $\rho_L = 1/6$, $\mu_L = 1/6$, $\bar{\pi}_1 = 5/12$, and $\bar{\mu}_1 = 1/24$. This implies $\rho_1 = 5/12$, $\mu_1 = 1/24$. Then $\pi_{+1}/\pi_{-1} > 1/2$, $\pi_{-1}/\pi_{+1} < 1/3$, and $\pi_{+1}/\pi_{-1} < 2/9$. For $\alpha = 1$, it can be immediately verified that all conditions stated in the proposition are fulfilled.

As a next step, we derive the equilibria if the conditions given in the proposition hold, assuming without loss of generality $\alpha = 1$. It is straightforward to verify that the conditions imply $\pi_{+1}/\pi_{-1} > C_1 C_2$, $\pi_{-1}/\pi_{+1} < C_1 C_2$, and $\min\{\pi_{+1}/\pi_{-1}, \pi_{-1}/\pi_{+1}\} < C_2$. In line with Lemma 1, there is a unique equilibrium in which $a_1 = 1$ for $A_1 = 1$ and $a_1 = 0$ for $A_1 = -1$.

Suppose $A_1 = -1$, which results in $a_1 = 0$. Then $\ln\left(\frac{\pi_{+1}(a_1)}{\pi_{-1}(a_2)}\right) = \ln\left(\frac{\pi_{+1}}{\pi_{-1}}\right) - \ln C_1 > 0$.\(^{20}\) In line with Lemma 2, the principal will choose $\sigma = 1$. However, suppose now that it was known to the principal that $A_1 = -1$. In this case, $\ln\left(\frac{\pi_{+1}(A_1)}{\pi_{-1}(A_2)}\right) = \ln\left(\frac{\pi_{+1}}{\pi_{-1}}\right) - \ln\left(\frac{\pi_1}{\bar{\mu}_1}\right) < 0$ (see (36)), which would lead to the decision $\sigma = -1$ (compare Lemma 2). Hence the expert withholds valuable information from the principal and the first-best is not achieved.

\[\Box\]

J Proof of Proposition 5

Suppose $\alpha = 1$, without loss of generality. We note that the conditions mentioned in the Proposition can be fulfilled for the set of parameter values introduced in Appendix I, for example.

According to Appendix I, there is a unique equilibrium in our basic model with $\lambda_1(1) = 1$ and $\lambda_1(-1) = 0$. However, the first-best would require that expert 1 announces $A_1 = -1$. Because $\pi_\alpha/\pi_{-\alpha} > C_1$ for $\alpha = 1$, $\alpha = 1$ corresponds to the a-priori more likely option. Accordingly, the advocatus-diaboli mechanism stipulates that the expert

\[^{20}\] $-\ln\left(\frac{1-\pi_1}{1-\bar{\mu}_1}\right) = -C_1$ corresponds to (37), evaluated at $\lambda_1(1) = 1$ and $\lambda_1(-1) = 0$. 

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must not propose \( A_1 = 1 \). The expert will announce \( A_1 = -1 \) in this case if (22) or

\[
\frac{\pi_1(\bar{a}(i))}{\pi_{-1}(\bar{a}(i))} \leq \frac{\rho_H - \rho_L}{\mu_L - \mu_H} \cdot C_1,
\]

which holds by assumption. Thus the mechanism ensures an optimal decision. \( \square \)

**K Proof of Proposition 6**

If (15) does not hold, a unique equilibrium exists, in which all arguments received by the experts are released. In this case it is obvious that the correct option is chosen with certainty for large committees.

Consequently, we assume in the following that (15) does hold. The behavior of experts can be described by Lemma 1 if we replace \( \pi_s(\bar{a}(i)) \) by \( \pi_s \). Suppose \( s = -1 \) and that the principal observed the arguments raised by the experts sequentially. Then it is straightforward to specify a process \( X'_i := \ln \left( \frac{\pi_1(\bar{a}(i))}{\pi_{-1}(\bar{a}(i+1))} \right) \) that describes all information about the state of the world that is contained in the arguments presented by experts 1, ..., \( i \). For this stochastic process \( P(X'_i \geq 0) \to 0 \) as \( N \to \infty \) because the proof in Appendix D can be applied to \( X'_i \). \( \square \)
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