DEMOCRATIC CONSTITUTIONS,
POLITICAL PARTIES AND
PUBLIC-PROJECT PROVISION

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Preface

This thesis originated during my work as a research assistant at the chair of Prof. Dr. Hans Gersbach at the Swiss Federal Institute of Technology (ETH) Zurich. I want to briefly acknowledge several people who considerably helped me with my work:

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Contents

Thesis Summary ................................................................. 6

1 Introduction ................................................................. 10
  1.1 Motivation and Overview ............................................. 10
  1.2 The Structure of the Thesis .......................................... 11
  1.3 Summary .............................................................. 14

I Rules vs. Discretion ....................................................... 16

2 Constitutional Design: Separation of Financing and Project Decision 17
  2.1 Introduction .......................................................... 17
  2.2 Relation to the Literature ........................................... 19
  2.3 Model ........................................................................ 20
    2.3.1 Set-up .............................................................. 20
    2.3.2 Constitutions ....................................................... 22
    2.3.3 The Legislative Period ......................................... 23
    2.3.4 Socially Efficient Solutions ................................... 24
    2.3.5 Evaluation Criteria .............................................. 24
  2.4 Arbitrary Tax Code and Arbitrary Subsidy Scheme ............... 26
  2.5 Uniform Tax Code and Arbitrary Subsidy Scheme ............... 28
  2.6 Arbitrary Tax Code and Uniform Subsidy Scheme ............... 30
  2.7 Uniform Tax Code and Uniform Subsidy Scheme ............... 31
  2.8 Welfare Comparison .................................................. 32
3 Constitutional Design: Separation of Financing and Project Decision Revisited

3.1 Introduction .............................................. 40
3.2 Model .................................................. 41
   3.2.1 Set-up ........................................... 41
   3.2.2 Constitutions ...................................... 42
   3.2.3 The Legislative Period .............................. 43
   3.2.4 Socially Efficient Solutions ....................... 44
   3.2.5 Evaluation Criteria ................................. 45
3.3 A General Discussion .................................... 46
3.4 Analysis With an Ex-Ante Optimal Selection Device .......... 48
   3.4.1 Arbitrary Tax Code and Arbitrary Subsidy Scheme ... 49
   3.4.2 Uniform Tax Code and Arbitrary Subsidy Scheme .... 50
   3.4.3 Arbitrary Tax Code and Uniform Subsidy Scheme .... 50
   3.4.4 Uniform Tax Code and Uniform Subsidy Scheme ..... 51
   3.4.5 Conclusions ......................................... 51
3.5 Analysis With a Fixed Selection Device ..................... 52
   3.5.1 Arbitrary Tax Code and Arbitrary Subsidy Scheme ... 53
   3.5.2 Uniform Tax Code and Arbitrary Subsidy Scheme .... 54
      3.5.2.1 Individual Voting Behavior ..................... 54
      3.5.2.2 Best-response Function of $L$ ................. 56
      3.5.2.3 Optimal Proposals $\pi^W$ ..................... 62
   3.5.3 Arbitrary Tax Code and Uniform Subsidy Scheme .... 66
3.5.4 Uniform Tax Code and Uniform Subsidy Scheme . . . . . . . . 67
3.6 Welfare Comparison . . . . . . . . . . . . . . . . . . . . . . . . . . . 68
3.7 Endogenous Project Characteristics . . . . . . . . . . . . . . . . . .. 70
3.8 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 72

II Impact of Parties 74

4 Parties as Fairness and/or Commitment Devices 75
4.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 75
4.2 Relation to the Literature . . . . . . . . . . . . . . . . . . . . . . . . . 76
4.3 Set-up . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 77
  4.3.1 General Set-up . . . . . . . . . . . . . . . . . . . . . . . . . . . . 77
  4.3.2 The Game . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 80
  4.3.3 Parties and Devices . . . . . . . . . . . . . . . . . . . . . . . . . . 80
4.4 Fairness Device . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 83
4.5 Commitment Device . . . . . . . . . . . . . . . . . . . . . . . . . . . . 86
4.6 Fairness and Commitment Device . . . . . . . . . . . . . . . . . . . . . 89
4.7 Multiplier Effects . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 92

5 Comparison 96
5.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 96
5.2 Set-up . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 96
5.3 Fairness vs Absence of Fairness . . . . . . . . . . . . . . . . . . . . . 101
5.4 Commitment vs Absence of Commitment . . . . . . . . . . . . . . . . 112
5.5 Fairness and Commitment vs Absence of Fairness and Commitment . . 125
5.6 Other Comparisons . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 136

A Proofs for Chapter 2 137

B Proofs for Chapter 3 145
Thesis Summary

The present thesis is a contribution towards a better understanding of democratic institutions and political parties. Its purpose is to illustrate (i) how the constitutional design of transfer schemes influences the performance of a direct democracy, and (ii) how different roles of political parties impact on the supply of public projects and on social inequality.

The thesis has two parts: The first one examines how constitutional rules on taxation and subsidization affect the provision of public projects and the distribution of subsidies.

We obtain the following main results: If a society votes on a single proposal, rules on taxation exhibit several advantages over arbitrary taxation. First, they discipline the agenda-setter with respect to excessive subsidization. Second, tax rules induce the agenda-setter to exert effort in order to enhance a public project. Third, they prevent the adoption of socially extremely undesirable projects. Finally, they eliminate the possibility of endless cycles of project adoption and project reversal.

If a society votes on two competing proposals, the advantageous properties of tax rules only prevail if additional rules regarding the treatment of agenda-setters are imposed by the constitution. In the absence of special treatment rules for agenda-setters, only a constitution imposing strict rules on taxes and subsidies allows for project adoption. All other constitutions permanently prevent the implementation of public projects.

The second part of the thesis investigates how the roles fairness and commitment in political parties affect the provision of public projects, social welfare and individual utility.

The analysis supports the following main conclusions: Fairness in parties has no impact on public-project provision, but reduces social welfare. Moreover, only members of the proposal-making party benefit from fairness, while all other individuals suffer from it.
Commitment in parties has an ambiguous effect on project provision. However, in the case where parties are dominated by small groups of individuals (or a single leader), commitment yields higher acceptance rates of public projects. Hence, in this case, beneficiaries of public projects benefit from commitment in parties. The group of non-beneficiaries of public projects is divided into two subgroups. One of these subgroups benefits from commitment, while the other suffers from it. Consequently, the effect of commitment in political parties on social welfare remains ambiguous. In the opposite case, where parties are controlled by large groups of individuals, results are the opposite of those in the previous case.

Finally, we are able to show that the effect of fairness and commitment in political parties is a combination of the separate effects outlined above. Thus, our findings regarding the provision of public projects, social welfare, and individual utility are highly ambiguous.
Kurzfassung

Die vorliegende Doktorarbeit ist ein Beitrag zum besseren Verständnis demokratischer Institutionen und politischer Parteien. Ziel der Arbeit ist es aufzuzeigen, (i) wie sich die verfassungsmässige Ausgestaltung von Umverteilungsregeln auf die Ergebnisse einer direkten Demokratie auswirkt, und (ii) wie unterschiedliche Rollen von politischen Parteien die Bereitstellung öffentlicher Güter und die soziale Ungleichheit beeinflussen.

Konzeptionell umfasst die Dissertation zwei Teile: Im ersten Teil wird untersucht, wie sich Verfassungsregeln bezüglich Steuern und Subventionen auf die Bereitstellung öffentlicher Projekte und die Verteilung von Subventionen auswirken.


Im zweiten Teil der Dissertation wird untersucht, wie sich Fairness und Parteidisziplin in politischen Parteien auf die Bereitstellung öffentlicher Projekte, soziale Wohlfahrt und individuellen Nutzen auswirken.

Die Analyse führt zu folgenden Erkenntnissen: Fairness in Parteien hat keine Auswirkung auf die Bereitstellung öffentlicher Projekte, aber sie senkt die soziale Wohlfahrt. Darüber
hinaus profitieren nur jene Individuen von Fairness, die Mitglieder derjenigen Partei sind, die einen Vorschlag macht.


Letztlich kann gezeigt werden, dass der Effekt von Fairness und Parteidisziplin zusammen eine Kombination der jeweiligen Einzeleffekte ist. Daher erhalten wir für diese Parteirolle auch keine eindeutigen Resultate.
Chapter 1

Introduction

1.1 Motivation and Overview

“As democracy has swept the world, the euphoria of freedom has given way to more practical concerns.”

Terry M. Moe and Michael Caldwell (1994)

Starting from the premise that institutions matter, economists have been analyzing how democratic institutions perform and how they could be improved for several years. Despite a growing body of literature on these issues, the main questions - “What should democratic government look like?” and “What institutions will work best?” (Moe and Caldwell 1994) have only been answered partly.

This dissertation is a contribution towards a better understanding of democratic institutions and the role of political parties.

The present dissertation has two parts: First, we examine constitutional rules regarding taxation and subsidization. We will show that the welfare of democracy strongly depends on how tax and subsidy distributions are shaped by constitutional rules.

Second, we investigate the influence of different roles of political parties on democratic

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1 Early contributions include the classical works of Wicksell (1896), Buchanan and Tullock (1962), Harsanyi (1953), and Harsanyi (1955).

2 Previous studies have mainly focused on executive-legislative relations (see Moe and Caldwell 1994 and Persson, Roland, and Tabellini 2000 among others), electoral rules (see Lizzeri and Persico 2001, for instance), voting rules (see for example Aghion and Bolton 2003, Buchanan and Tullock 1962, Gersbach 2004, and Gersbach 2009a), and government organization (see Oates 1972 and Besley and Coate 2003 in particular).
performance. In particular we examine how party devices - such as fairness or commitment - affect public good provision. We will show that welfare and inequality strongly depend on the type of role political parties adopt in a democracy.

1.2 The Structure of the Thesis

Part I: Rules vs. Discretion

Constitutional Design: Separation of Financing and Project Decision
(Chapter 2)

In Chapter 2, which is based on Gersbach, Hahn, and Imhof (2009), we are interested in normative justifications of tax and subsidy rules. Moreover, we explore justifications of the common procedure to separate financing and project decision.

More specifically, we consider a polity with risk-neutral citizens who decide collectively about public project provision. A public project divides the society into two distinct groups: (i) the project winners (individuals who derive a high utility from the public project) and (ii) the project losers (individuals who derive a small or negative utility from the public project).

A project winner can make a proposal which comprises (i) the project decision, (ii) a tax and (iii) a subsidy distribution. This proposal is put to a vote and is adopted if it is supported by a simple or supermajority of individuals. We assume taxation to be distortionary, which implies that any kind of pure redistribution is socially harmful.

We examine four different constitutions, which vary in their degree of restrictiveness regarding the allocation of taxes and subsidies: In the most liberal one, the agenda-setter distributes taxes and subsidies at his own discretion. In less liberal constitutions, his hands are tied by a uniform criterion on either the tax scheme, the subsidy scheme, or the tax scheme and the subsidy scheme. Hence, a tax rule implies that all individuals have to be treated identically with respect to taxation, and analogously, a subsidy rule means that all individuals have to be treated identically with respect to subsidization.

The model identifies the following advantages of tax rules:

(i) Tax rules discipline the agenda-setter with respect to excessive subsidization.

(ii) Tax rules induce the agenda-setter to improve the project.
(iii) Tax rules prevent the adoption of socially extremely undesirable projects.

(iv) Tax rules (in combination with subsidy rules) prevent cycling, that is, sequences of project adoption - project reversal - renewed adoption, and so forth can be ruled out.

From a normative perspective, these advantages support constitutions with well-defined tax codes. Moreover, they rationalize the separation of financing and project decision.

Constitutional Design: Separation of Financing and Project Decision Revisited

(Chapter 3)

In Chapter 3, we extend the model of Chapter 2 and examine the robustness of the previous findings. In particular, we analyze whether the properties of tax rules prevail in a more competitive environment with competing proposals.

We start with the model outlined in the previous chapter. We then modify the legislative process in the following way: While in Chapter 2, only one agenda-setter can make a proposal, we now consider two proposal-makers. Then, the sequence of events is as follows:

First, a project winner makes a proposal. After observing it, a project loser is allowed to make a counter-proposal. All individuals observe both proposals and decide which one to accept. The proposal receiving a larger share of votes is implemented.

For the analysis, we reexamine the four constitutions of Chapter 2.

First, we find that in all constitutions but one, equilibrium outcomes depend on how the second mover selects among a continuum of best responses. Thus, we focus on two specific selection devices:

- Ex-ante efficient:
  The second mover is forward-looking and adapts a selection device that guarantees desirable outcomes for himself.

- Fixed:
  The second mover is not forward-looking.

Under both selection devices, we find that constitutions with tax rules exhibit desirable properties that constitutions without tax rules do not:
Under the ex-ante efficient device, there is only one constitution that allows project adoption. This constitution involves rules on taxes and subsidies. All other constitutions prevent the adoption of any kind of project.

Under the fixed selection device, the model identifies similar advantages of tax rules as in Chapter 2.

**Part II: Impact of Parties**

*Parties as Fairness and/or Commitment Devices and Comparison (Chapters 4 and 5)*

The following two chapters investigate the impact of different roles of political parties on the agenda-setter’s proposal-making. We are particularly interested in their consequences on social welfare and individual utility.

We consider three roles that can be adopted by parties

- **Fairness Device:**
  All party members have to be treated identically.

- **Commitment Device:**
  Members of a party have to comply to a position chosen by the party. We assume this position to be taken by a party-internal $n$-majority rule ($0 < n \leq 1$).\(^3\)

- **Fairness and Commitment Device:**
  The combination of the first two roles.

For the analysis, we apply the basic model of Chapter 2, that is the society consists of project winners and project losers. In accordance with the two roles of parties, we assume that all project winners, and all project losers each form their own party. Moreover, we assume again that a project winner can make a proposal which comprises a project decision, a tax and a subsidy distribution. In contrast to the previous chapters, the constitution is now fixed. We consider that it comprises a uniform rule on taxes and discretion regarding subsidies. Again, we suppose taxation to be distortionary.

We find that fairness and commitment are complementary in their impact on social welfare and individual utility. Compared to the absence of parties, fairness in parties

\(^3\)\(n\) can be interpreted as a measure of how intense commitment within a party is: If \(n\) is small, the party majority has to bend to a minority position, which requires strong commitment. However, if \(n\) is large, a minority has to bend to the majority position, which requires less commitment.
affects the subsidy distribution within project winners, whereas commitment in parties alters the subsidy distribution within project losers. More precisely, we obtain the following results:

In comparison with the absence of parties, fairness has the following impact on social welfare and individual utility:

(i) It does not have any influence on project adoption.

(ii) It causes higher or equal aggregate subsidies, which implies that social welfare is never higher if parties (with fairness) are present.

(iii) The accruing welfare losses of parties as fairness device are exclusively borne by the project losers. Thus, they do not benefit from fairness in parties.

(iv) Except the agenda-setter, project winners benefit from fairness in parties.

Compared to the absence of parties, commitment in parties has the following impact on social welfare and individual utility:

(i) If $n$ is small, the project is provided more often in the presence of parties. The opposite occurs if $n$ is large.

(ii) Whether commitment in parties is socially desirable or not depends on the characteristics of the public project.

(iii) If $n$ is small, all project winners benefit from parties with commitment, while the opposite occurs if $n$ is large.

Independent of the size of $n$, some project losers benefit from commitment in parties, while others do not.

Finally, the joint influence of fairness and commitment in parties on social welfare and individual utility is the combination of both effects above.

1.3 Summary

The main questions of the present thesis are:
1. What constitutional rules are best-suited for the financing of public projects?
2. How do different roles of parties impact on the outcome of the political process?
The structure of the thesis is:

![Structure of the thesis](image)

Figure 1.1: Structure of the thesis.

The main conclusions are:

Our analyses in Chapters 2 and 3 have illuminated the rich pattern of advantages and disadvantages arising from constitutional design with regard to transfer schemes. Our models recommend the application of strict tax rules and a separation of financing and project decisions.

Our analyses in Chapters 4 and 5 are a first step towards a better understanding of the impact of party roles on democratic performance. We find that if parties exhibit fairness, exploitation of non-party members can occur. Moreover, social welfare declines. Interestingly, fairness within a party does not help raise the frequency of project adoption. In contrast, party commitment yields higher acceptance rates of public projects, provided that commitment is strong. However, exploitation of non-party members through increased transfer activity does not occur.
Part I

Rules vs. Discretion
Chapter 2

Constitutional Design: Separation of Financing and Project Decision

2.1 Introduction

Democratic societies are characterized by separation between the financing of government expenditures and public project cum subsidy decisions. The financing of government expenditures is determined by detailed tax laws. Separately, legislatures decide on public projects and associated subsidy payments to members of the polity. Usually there are no legal constraints on the chosen subsidy scheme for a specific project.

This separation can be illustrated by various examples. Consider first public housing programs. The government builds houses and subsidizes rents so that a particular group in society particularly benefits from the program. The program is financed through tax revenues raised according to a separate tax law and independently of the specific project. Second, the government may invest in public transportation and simultaneously subsidize tickets for specific subgroups, such as the elderly. The subsidies are taken from government revenues generated by direct and indirect taxation. Third, a government may decide to foster growth in a particular region by investing in public infrastructure while simultaneously subsidizing entrepreneurs willing to move to this region. Again, financing occurs through tax revenues, independently of who benefits from the specific project.

In this chapter we provide a political-economy rationale for the separation between the financing of government expenditures and public project cum subsidy decisions. We also explore the limitations of this widespread procedure.
In standard models of mechanism design, tying the benevolent mechanism designer’s hands by imposing restrictions on possible tax schemes can never be welfare improving.\textsuperscript{1} By contrast, our model assumes an agenda-setter who pursues his own interests. This opens up a potential role for restrictions on taxes to improve welfare.

We consider a society with a continuum of citizens where an agenda-setter can make a proposal about the adoption of a public project and the distribution of taxes and subsidies. The proposal is adopted if it is supported by a majority of voters. Our model involves three incentive problems. First, the agenda-setter may want to provide a public project if it is beneficial to her, although the project may be undesirable from a utilitarian perspective. Second, the agenda-setter may want to raise more taxes than necessary in order to pay out subsidies to herself or to other citizens. Third, the agenda-setter may not want to look for the most efficient variant of the public project.

We investigate the types of rules that are suitable for limiting the distortions arising from these incentive problems. We obtain five major findings.

First, we find that tax rules may prevent the agenda-setter from securing the necessary majority of voters for socially inefficient projects that benefit only a small lobby group. By contrast, the absence of tax rules enables the proposer to enforce any project irrespective of its characteristics.

Second, tax rules reduce wasteful subsidies to a minimum, that is only redistribution-efficient proposals are made. The intuition for this finding is that under tax rules a large amount of total subsidies also implies high taxes for the agenda-setter.

Third, a constitution with rules on both taxes and subsidies is robust to counter-proposals, whereas the other constitutions under consideration are prone to cycles of project adoption and project reversal.

Fourth, we find that in combination with arbitrary taxation subsidy rules yield high welfare losses. Such a constitution is inferior to one with no rules on taxes and subsidies.

Fifth, an additional rationale for tax rules materializes when project characteristics are endogenous. We show that only constitutions involving tax rules induce the agenda-setter to enhance project efficiency.

Overall, this chapter provides a rationale suggesting that if incentive problems for the agenda-setter are taken into account decisions on projects cum subsidies should be

\textsuperscript{1}For a survey of the literature on mechanism design see Jackson (2001).
Chapter 2. Constitutional Design: Separation of Financing and Project Decision

separated from decisions on tax rules. Tax rules encoded in tax laws separated from public project cum subsidy decisions have several advantages over a scheme in which financing, project decision, and subsidies are jointly put to a vote.²

The chapter is organized as follows: We review the related literature in Section 2.2. Section 2.3 develops the basic framework. Sections 2.4-2.7 examine the outcomes for constitutions that differ with respect to their rules on taxes and subsidies. In Section 2.8 we examine the welfare implications of different constitutions. Section 2.9 discusses socially optimal constitutional rules for different categories of projects. We analyze endogenous project design in Section 2.10, and Section 2.12 concludes.

2.2 Relation to the Literature

There are no other studies inquiring why a polity may adopt strict tax rules but allow flexibility in using subsidies in public-project provision. Our model is a contribution to constructive constitutional economics, as outlined in the seminal study by Buchanan and Tullock (1962). Using the veil-of-ignorance device (see Rawls (1971)), Buchanan and Tullock (1962) have examined the costs and benefits of majority rules.³ Aghion and Bolton (2003) have refined and expanded this approach. When a society faces deadweight costs of redistribution, simple or supermajority rules are preferred to unanimity in order to overcome vested interests.⁴ Gersbach (2004) and Gersbach (2009a) show that increasingly sophisticated agenda and decision rules further improve the efficiency of public-project provision when all admissible rules have to fulfill the liberal democracy constraint.

In this chapter we focus on the efficiency properties of the simple or supermajority rule when it is coupled with tax or subsidy rules. Our main insight is that tax rules exhibit a variety of advantages and can rationalize the separation of taxation from public-project provision and subsidies.⁵

²In our model, tax rules imply that all citizens are treated identically with respect to taxation. This may be reminiscent of the literature on decentralization vs. centralization (see Oates (1972)), where it is sometimes assumed that centralized decision-making is associated with equal treatment of all citizens with respect to public good provision (see Besley and Coate (2003) for a critique of this assumption).

³Closely related ideas have been developed by Rae (1969) and Taylor (1969).

⁴In a complete contract framework, Romer and Rosenthal (1983) have established that a unanimity rule may implement the full-information efficient solution when payoffs are private information but no deadweight costs of transfers arise.

⁵In our model, subsidies can be used to ensure the majority necessary for the adoption of a proposal.
There is an extensive body of literature on optimal mechanisms for providing public goods when income taxes are a source of public-goods finance.\textsuperscript{6} \textsc{Hellwig}(2004) shows that when both income taxation and public-sector pricing are plagued by incentive considerations it is desirable to use a combination of income taxation and admission fees to finance public goods. We disregard incentive considerations at the citizen level and assume that financing is achieved by a simple, possibly personalized, lump-sum tax scheme. Our focus is on the incentive problem of the agenda-setter.\textsuperscript{7}

\section*{2.3 Model}

\subsection*{2.3.1 Set-up}

We consider a society facing the standard problem of public-project provision and financing. Citizens are indexed by $j$ and are uniformly arranged on the unit interval. The provision of a public project yields utility $v_j \forall j \in [0, 1]$ and involves per-capita costs $k \geq 0$. If the project is not adopted, the status quo will prevail; we normalize the utility under the status quo to zero for all citizens.

For simplicity of exposition, we assume $v_j \in \{V_w, V_l\}$ with $V_w \geq 0$ and $V_w > V_l$. Accordingly, we refer to individuals obtaining $V_w$ from the public project provision as “project winners” and to individuals receiving benefits $V_l$ as “project losers.” Without loss of generality, we assume the project winners to be located on the interval $[0, p]$ and the project losers to be located on the interval $[p, 1]$.

One particular individual can make a proposal $\pi$, which comprises a tax and a subsidy distribution as well as the project decision. The proposal is put to a vote; it is adopted if at least a fraction $m$ of all voters support it. Our framework thus corresponds to the government form of direct democracy (as practiced e.g. in California (US) or Switzerland) or parliamentary democracy with perfect representation of its citizens.\textsuperscript{8}
There are different ways of modeling which citizen has the right to set the agenda. We adopt the view that in a democracy it is impossible to deter beneficiaries of public projects from making proposals.\textsuperscript{9} Hence we directly assume that the agenda-setter is a project winner. Without loss of generality, we assume that the agenda-setter is $j = 0$.

Subsidies and taxes are constrained to be non-negative. Moreover, there is some maximal level of subsidies denoted by $\hat{s}$ with $\hat{s} \geq V_w - V_l$.\textsuperscript{10} Let $\mathcal{S}$ be the set of all non-negative Lebesgue-measurable functions on the unit interval that do not exceed $\hat{s}$. Thus each subsidy scheme involved by a proposal $\pi$ is a function $s(\pi) \in \mathcal{S}$. Accordingly, let $\mathcal{T}$ be the set of all non-negative Lebesgue-measurable functions on the unit interval. Then each tax scheme can be written as a function $t(\pi) \in \mathcal{T}$. Moreover, we use $g(\pi) \in \{0, 1\}$ as a variable indicating whether the project will be adopted ($g(\pi) = 1$) or not ($g(\pi) = 0$) according to proposal $\pi$.\textsuperscript{11}

We consider distortionary taxes, that is for each unit of taxes that is paid by a particular individual, only a fraction $1/(1 + \lambda)$ ($\lambda > 0$) can be used to finance the project or subsidies. There are various interpretations of $\lambda$. It may represent resources used for collecting and transferring funds from citizens to the state. The deadweight costs $\lambda$ may also represent the disincentive to work if wages are taxed. The assumption of linear deadweight costs can be justified by the relationship between taxes paid for the public project and individual income, the former being sufficiently smaller than the latter. Now the society’s budget constraint is

$$
(1 + \lambda) \left[ g(\pi)k + S(\pi) \right] = T(\pi),
$$

where we have introduced total subsidies $S(\pi) := \int_0^1 s_j(\pi) \, dj$ and total tax revenues $T(\pi) := \int_0^1 t_j(\pi) \, dj$ implied by proposal $\pi$. We assume $V_w - (1 + \lambda)k > 0$ and $V_l - (1 + \lambda)k < 0$.

Now we are in a position to give a formal description of the general set of possible proposals.

$$
\Pi := \{\pi \in \{0, 1\} \times \mathcal{S} \times \mathcal{T} \mid (1 + \lambda) \left[ g(\pi)k + S(\pi) \right] = T(\pi)\}.
$$

\textsuperscript{9}Another approach commonly applied is random selection of agenda-setters (see Gersbach (2009a)).

\textsuperscript{10}The assumption $\hat{s} \geq V_w - V_l$ simplifies the exposition, but does not qualitatively affect our findings.

\textsuperscript{11}Lizzeri and Persico (2001) consider a model where the agenda-setter must choose between redistribution and public-project provision. In our model, it is possible to combine public projects with redistribution.
2.3.2 Constitutions

We adopt the standard “veil of ignorance” procedure for constitutional design. The social choice problem is reduced to a two-period setting. The first period is the constitutional period and the second the legislative period. In the constitutional period all citizens are assumed to be identical and do not know whether they will be project winners or project losers. Moreover, the project’s parameters $V_w, V_l, k,$ and $p$ are not known with certainty. Citizens design a constitution or an incomplete social contract governing the supply of public goods, given a commonly known distribution of the project parameters.\(^{12}\)

Under the incomplete contract perspective, rules cannot depend on project characteristics, as those characteristics are not verifiable in court. Consequently, the rules that can be adopted in the constitutional period are those constraining the tax and subsidy schemes the agenda-setter is allowed to use. In this chapter we adopt the perspective that in the constitutional period the decision rule that will be used later is given. In particular, we assume that a proposal will be adopted if it receives at least a fraction $m$ of all votes. The only assumption we make is that \( \frac{1}{2} \leq m < \min \{ \frac{1}{1+\lambda} + p, 1 \} \).\(^{13}\) As we show later this assumption will significantly simplify our analysis.

In our model a constitution is simply a set of rules that constrain the set of proposals the agenda-setter can make. Accordingly, $\Pi$ represents a particular constitution, namely one without any further rules. In the course of the article we will consider less discretionary constitutions and impose rules on tax and/or on subsidy distribution. These constitutions represent subsets of $\Pi$.

It will be useful to define the set of all possible projects. It comprises all quadruples $(V_w, V_l, k, p)$ that satisfy the assumptions we have introduced so far. Formally, it is given by

\[
P := \{(V_w, V_l, k, p) \in \mathbb{R}^+ \times \mathbb{R} \times \mathbb{R}^+ \times [0; 1[ : p > m - 1/(1+\lambda), V_w > (1+\lambda)k, V_l < (1+\lambda)k, s \geq V_w + V_l \},
\]

where $\mathbb{R}$ and $\mathbb{R}^+$ denote the sets of real numbers and non-negative real numbers re-

\(^{12}\)Incomplete social contracts have been studied by Aghion and Bolton (2003) and Gersbach (2009a), among others.

\(^{13}\)Plausible estimates of $\lambda$ lie between 0.2 and 0.5 (see Stuart (1984), Ballard, Shoven, and Whalley (1985), and Browning (1987)). For these estimates and the simple majority rule ($m = 1/2$) the assumption \(1/2 \leq m < \min \{1/(1+\lambda) + p, 1\}\) is always fulfilled.
Chapter 2. Constitutional Design: Separation of Financing and Project Decision

respectively. The prior distribution of the project parameters can now be described by a joint probability density function on $\mathcal{P}$. At this stage, we do not specify a particular form for this density function.

### 2.3.3 The Legislative Period

In the legislative period, each individual observes $v_j$ and $k$, and all individual valuations become common knowledge.\(^{14}\) The agenda-setter makes a proposal that must obey the constitutional rules, otherwise the proposal is declared to be unconstitutional and the status quo prevails.

If proposal $\pi$ is adopted, the utility of individual $j \in [0; 1]$ will be\(^ {15}\)

$$u_j(\pi) = g(\pi)v_j + s_j(\pi) - t_j(\pi). \quad (2.4)$$

We assume that each individual will vote in favor of the proposal if and only if $u_j(\pi) \geq 0$. It will be useful to define the indicator function $I(\pi)$, which adopts a value of 1 if the proposal is implemented and of zero otherwise.

$$I(\pi) := \begin{cases} 
1 & \text{if } u_j(\pi) \geq 0 \text{ for at least a fraction } m \text{ of voters} \\
0 & \text{otherwise.} 
\end{cases} \quad (2.5)$$

Thus we can write the expected utility of individual $j$ as $U_j(\pi) = I(\pi)u_j(\pi)$ given proposal $\pi$ has been made. In addition, the utilitarian welfare measure for a particular proposal $\pi$ amounts to

$$W(\pi) := I(\pi) [(pV_w + (1-p)V_l - k(1+\lambda))g(\pi) - \lambda S(\pi)]. \quad (2.6)$$

For the sake of simplicity, we introduce the following tie-breaking rule: If the agenda-setter is indifferent between several proposals, she will choose a proposal with the highest $u_0(\pi)$, that is a proposal that if implemented would yield the highest utility for her.

\(^{14}\)An interesting variant of our model would involve citizens having private information about their types $v_j \in \{V_w, V_l\}$, while the value of $p$ is commonly known. This variant leads to results similar to those in this chapter.

\(^{15}\)We assume that the income of individuals is sufficient to pay taxes under any proposal considered in the chapter.
2.3.4 Socially Efficient Solutions

As a starting point it is instructive to consider socially optimal proposals. Consider a social planner who maximizes the utilitarian welfare measure by choosing and implementing a proposal \( \pi \in \Pi \) for a given realization of the project parameters \( V_w, V_l, k, \) and \( p \). It is obvious that the following lemma holds:

**Lemma 2.1**

A socially optimal proposal \( \pi \) has the following characteristics:

\[
g(\pi) = \begin{cases} 
1 & \text{for } pV_w + (1 - p)V_l \geq (1 + \lambda)k \\
0 & \text{for } pV_w + (1 - p)V_l < (1 + \lambda)k
\end{cases}
\]  

(2.7)

\[S(\pi) = 0.\]  

(2.8)

In particular, the social planner will never choose a positive level of total subsidies because of the losses caused by distortionary taxation. We note that the socially optimal solution is not normally unique because for \( g(\pi) = 1 \) the social planner is indifferent with respect to all possible tax schemes raising the revenues necessary to finance the project.

If the project parameters \( p, V_w, V_l, \) and \( k \) were verifiable, it would be straightforward to characterize a constitution guaranteeing the optimal level of welfare. However, we assume in the following that constitutional rules cannot depend on project characteristics, as even for perfectly observable costs and benefits of projects it is plausible that the project characteristics are not verifiable in court.

2.3.5 Evaluation Criteria

In the following we establish several desirable properties of constitutions. For this purpose it will be useful to define the following concept:

**Definition 1**

For a given constitution \( \tilde{\Pi} \subseteq \Pi \), a proposal \( \pi \in \tilde{\Pi} \) with \( I(\pi) = 1 \) is redistribution-efficient if no \( \pi' \in \tilde{\Pi} \) exists with \( S(\pi') < S(\pi) \), \( g(\pi) = g(\pi') \) and \( I(\pi') = 1 \). A proposal with \( I(\pi) = 0 \) is always redistribution-efficient.

For example, we refer to a proposal \( \pi \) that ensures the adoption of the project as redistribution-efficient if no alternative proposal exists that would guarantee the adopt-
tion of the project while involving strictly lower total subsidies. It is obvious that redistribution-efficiency is a desirable property of proposals, as it keeps wasteful redistribution to a minimum.

**Definition 2**
We refer to a constitution under which only redistribution-efficient proposals are made as a constitution satisfying GREP (guarantees redistribution-efficient proposals).

Now we turn to further desirable property of constitutions. While it is plausible that designing socially desirable projects is difficult, it may be much easier to conceive of socially harmful projects that benefit only a small lobby group. Thus one important feature of a constitution may be that it prevents the adoption of projects of this kind.

To be more precise, we define the set of lobby projects $LP(\varepsilon) \subset P$ for $\varepsilon < 1/2$ as the set of all projects in $P$ with parameters $V_w, V_l, k$, and $p$ such that $|V_w - k(1 + \lambda)| < |V_l - k(1 + \lambda)|$ and $p \leq \varepsilon$. Note that condition $|V_w - k(1 + \lambda)| < |V_l - k(1 + \lambda)|$ can be interpreted as the net benefits of project winners being lower than the net losses of project losers. For $p < 1/2$ this obviously implies $pV_w + (1 - p)V_l < (1 + \lambda)k$.

**Definition 3**
A constitution satisfies the property of “protection against lobby projects” (henceforth PALP) if a value for $\varepsilon \in ]0; 1/2[$ exists such that all projects in the set $LP(\varepsilon)$ are never adopted in equilibrium.

As a consequence, citizens would agree on a constitution satisfying PALP under a veil of ignorance if sufficient significance were attached to bad lobby projects in the prior distribution of project characteristics.

The reversal of some projects, like the construction of public buildings or infrastructure, may be prohibitively costly compared to the benefits involved. But in other cases project reversal may be relatively easy. Examples are a reform of penal law or changes to the tax system. For these cases, we cannot rule out the eventuality of one of the project losers proposing to reverse the project after a proposal has been adopted. It is obvious that a sequence of project adoption, reversal, renewed project adoption, and so forth is not desirable. Thus we propose robustness against counter-proposals (henceforth RACP) as another criterion for evaluating constitutions. More specifically, we assume that potentially reversible projects involve negligible costs $k$, that is $k = 0$.  

25
Chapter 2. Constitutional Design: Separation of Financing and Project Decision

So if the original project involves \( p = p_0 \), \( V_l = V_{l0} \), and \( V_w = V_{w0} \), reversal of the project can be characterized by \( p = 1 - p_0 \), \( V_l = -V_{w0} \), and \( V_w = -V_{l0} \).

In order to consider the reversal of projects, we have to specify a game involving a sequence of legislative stages. For simplicity of exposition we assume that agenda-setters and voters are short-sighted when making a proposal or voting. For example, when a decision is to be taken, voters do not take into account the eventuality of the project being reversed in the future. This assumption does not qualitatively affect our results. To sum up, a potentially reversible project can be reversed if both itself and the reversal of the project can be adopted in an equilibrium of our basic model.

We are now in a position to define \( RACP \) as follows:

**Definition 4**

A constitution displays robustness against counter-proposals (\( RACP \)) if no potentially reversible project can be reversed by a respective counter-proposal.

Obviously, it may be possible to rule out project reversal directly in the constitution. However, in a richer framework with project costs and benefits that are uncertain before implementation, such a constitutional rule may be disadvantageous as it eliminates the possibility to reverse projects that have turned out to be much less desirable than expected. Then constitutions displaying \( RACP \) may be desirable.

2.4 Arbitrary Tax Code and Arbitrary Subsidy Scheme

In our first scenario we impose no additional rules on taxes and subsidies, that is the agenda-setter can choose any proposal \( \pi \in \Pi \).

**Proposition 2.1**

For constitution \( \Pi \) the agenda-setter will always choose a proposal \( \pi^* \) with \( g(\pi^*) = 1 \), \( t_0(\pi^*) = 0 \), \( s_0(\pi^*) = \hat{s} \), and \( I(\pi^*) = 1 \).

**Proof**

The agenda-setter solves the following problem:

\[
\max_{\pi \in \Pi} \{ (g(\pi)V_w + s_0(\pi) - t_0(\pi))I(\pi) \}.
\]

It is obvious that \( g(\pi) = 1 \), \( t_0(\pi) = 0 \), and \( s_0(\pi) = \hat{s} \) guarantee maximum utility for the agenda-setter, provided that the proposal is actually adopted. Importantly, a proposal
with \( g(\pi) = 1 \), \( t_0(\pi) = 0 \), and \( s_0(\pi) = \hat{s} \) that entails \( I(\pi) = 1 \) always exists. For example, the agenda-setter can impose zero taxes on all individuals from the interval \([0; m]\) and tax all individuals from the interval \([m, 1]\) identically to cover the costs for the project \( k \) and the subsidies \( S(\pi) \) that may be necessary to gain support from all members in \([0; m]\).

Interestingly, we obtain the following lemma:

**Lemma 2.2**

For constitution \( \Pi \) a proposal chosen by the agenda-setter may be redistribution-inefficient. Therefore constitution \( \Pi \) does not satisfy GREP.

**Proof**

The proof of this lemma is straightforward. Suppose a redistribution-efficient proposal \( \pi^* \) exists that maximizes the agenda-setter’s utility. It is obvious that for \( \pi^* \) some project losers exist who receive no subsidies \( (s_j(\pi^*) = 0) \). Now we can modify \( \pi^* \) by introducing positive subsidies for these individuals, which are financed by additional taxes for these very persons. The resulting proposal would also be adopted, but is clearly not redistribution-efficient. Thus for each redistribution-efficient proposal we can find a multitude of proposals that are not redistribution-efficient.

\[ \square \]

In addition, as all projects are adopted, it is obvious that the following lemma holds:

**Lemma 2.3**

Constitution \( \Pi \) does not satisfy PALP.

Intuitively, the high degree of flexibility for the proposer enables her to adopt any project, independently of its characteristics. Hence socially detrimental lobby projects are always implemented.

Finally we note

**Lemma 2.4**

Constitution \( \Pi \) does not satisfy RACP.

As any project can be adopted by a suitable tax-subsidy scheme, it is obvious that any project can be reversed by a respective counter-proposal.
2.5 Uniform Tax Code and Arbitrary Subsidy Scheme

Now we impose the requirement that all individuals have to be treated identically with respect to taxation. Recall that individuals have the same income, so equal taxation is the only tax rule that is non-discriminatory. As a consequence, we consider the following constitution:

\[ \Pi_{T} := \{ \pi_{T} \in \Pi \mid t_{j}(\pi_{T}) = t_{i}(\pi_{T}) \ \forall i, j \in [0, 1] \}. \]  \hspace{1cm} (2.9)

Here the agenda-setter can only choose proposals from \( \Pi_{T} \subset \Pi \). For notational convenience we define

\[ V_{w}^{*} := (1 + \lambda) \frac{k - (m - p)V_{l}}{1 - (1 + \lambda)(m - p)}. \]  \hspace{1cm} (2.10)

Note that, for \( p < m \), \( V_{w}^{*} > 0 \) follows from the assumption \( m < \min \{ \frac{1}{1 + \lambda}, p, 1 \} \). It is straightforward to verify \( V_{w}^{*} - V_{l} > 0 \), which follows from \( V_{l} - (1 + \lambda)k < 0 \).

**Proposition 2.2**

For constitution \( \Pi_{T} \), a unique equilibrium proposal \( \pi_{T}^{*} \) exists.

1. For \( V_{w} \geq V_{w}^{*} \) and \( p < m \), \( \pi_{T}^{*} \) is given by \( g(\pi_{T}^{*}) = 1 \), \( t_{j}(\pi_{T}^{*}) = V_{w}^{*} \ \forall j \in [0, 1] \), and

\[
s_{j}(\pi_{T}^{*}) = \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, p] \\ V_{w}^{*} - V_{l} & \text{for } j \in [p, m] \\ 0 & \text{for } j \in [m, 1]. \end{cases} \]  \hspace{1cm} (2.11)

2. For \( V_{w} < V_{w}^{*} \) and \( p < m \), \( \pi_{T}^{*} \) is given by \( g(\pi_{T}^{*}) = 0 \), \( t_{j}(\pi_{T}^{*}) = 0 \ \forall j \in [0, 1] \), and

\[
s_{j}(\pi_{T}^{*}) = \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, 1]. \end{cases} \]  \hspace{1cm} (2.12)

3. For \( p \geq m \), \( \pi_{T}^{*} \) is given by \( g(\pi_{T}^{*}) = 1 \), \( t_{j}(\pi_{T}^{*}) = (1 + \lambda)k \ \forall j \in [0, 1] \), and

\[
s_{j}(\pi_{T}^{*}) = \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, 1]. \end{cases} \]  \hspace{1cm} (2.13)

The equilibrium proposal is always adopted.

\(^{16}\)More precisely, the proposal is unique up to relabeling individuals and redistribution within masses of Lebesgue measure zero. We will use “unique” in this sense throughout the chapter.
The proof of Proposition 2.2 is given in Appendix A. According to Proposition 2.2, the agenda-setter will always choose the maximum level of subsidies for herself, which is plausible. In the following, we discuss the three cases mentioned in the proposition separately.

For $p \geq m$, the project winners alone can enforce the adoption of the project. As a consequence, a proposal will secure the necessary majority, even if it involves zero total subsidies, that is subsidies only to a group of Lebesgue measure zero.

For $p < m$, it is necessary to subsidize some project losers to induce them to accept the project. In the proof we show that $V^*_w$ represents the level of taxes that is necessary to finance these subsidies. For $V_w \geq V^*_w$, the project winners’ gains $V_w$ from the project exceed this tax level. However, for $V_w < V^*_w$ the benefits of the project winners are so low that they are not willing to finance the subsidies necessary to induce some project losers to support the proposal. Thus the agenda-setter will choose a proposal $\pi_T$ with $g(\pi_T) = 0$.

It is important to note that the agenda-setter always chooses a redistribution-efficient proposal. A proposal with a higher level of total subsidies $S(\pi)$ would entail a higher level of taxes, which would be harmful to the agenda-setter. We summarize this observation in the following lemma:

**Lemma 2.5**

*Constitution* $\Pi_T$ satisfies GREP, that is proposal $\pi_T^*$ is always redistribution-efficient.

In Appendix A we also show

**Lemma 2.6**

*Constitution* $\Pi_T$ satisfies PALP.

We note that projects with $k = 0$, $p < m$, and $V_w \geq V^*_w$ are susceptible to counter-proposals. As a consequence, we obtain

**Lemma 2.7**

*Constitution* $\Pi_T$ violates RACP.
2.6 Arbitrary Tax Code and Uniform Subsidy Scheme

Now we consider a constitution that allows for arbitrary tax schemes. However, we limit the subsidy schemes to those that treat all citizens identically. Hence we restrict our attention to the set of proposals $\Pi_S \subset \Pi$ with

$$\Pi_S := \{\pi_S \in \Pi \mid s_j(\pi_S) = s_i(\pi_S) \quad \forall i, j \in [0, 1]\}.$$  \hspace{1cm} (2.14)

For this case we obtain

**Proposition 2.3**

For constitution $\Pi_S$ each equilibrium proposal $\pi^*_S$ can be characterized by $g(\pi^*_S) = 1$, $s_j(\pi^*_S) = \hat{s} \forall j \in [0, 1]$, $t_0(\pi^*_S) = 0$, and $I(\pi^*_S) = 1$.

**Proof**

The agenda-setter solves the following problem:

$$\max_{\pi \in \Pi} \{g(\pi)V_w + s(\pi) - t_0(\pi))I(\pi)\}.$$  

It is obvious that $t_0(\pi) = 0$, $s(\pi) = \hat{s}$, $g(\pi) = 1$ guarantee the highest possible payoff for the agenda-setter, provided that she can induce enough voters to support such a proposal. It is always possible to secure the necessary majority by taxing only the individuals from the interval $[m; 1]$. In this case, non-taxed project winners will vote in favor of $\pi$ (as $V_w + \hat{s} > 0$) as well as non-taxed project losers (as $V_l + \hat{s} > 0$), which implies $I(\pi) = 1$.

As each proposal under $\Pi_S$ involves the maximum amount of total subsidies $S(\pi) = \hat{s}$ and a proposal would also be accepted for slightly lower total subsidies, we obtain

**Lemma 2.8**

Under constitution $\Pi_S$ the equilibrium proposal is never redistribution-efficient. As a consequence, $\Pi_S$ violates GREP.

As constitution $\Pi_S$ enables the proposer to implement all projects, even very poor ones, it is obvious that

**Lemma 2.9**

Constitution $\Pi_S$ does not satisfy PALP.
Additionally, it is straightforward to see

**Lemma 2.10**

*Constitution* \( \Pi_S \) does not satisfy RACP.

### 2.7 Uniform Tax Code and Uniform Subsidy Scheme

Finally we consider a constitution that stipulates that all citizens be treated equally with respect to subsidies and taxes. Hence the set of feasible proposals reduces to \( \Pi_{ST} \subset \Pi \), where

\[
\Pi_{ST} := \Pi_T \cap \Pi_S. \tag{2.15}
\]

For this constitution we obtain

**Proposition 2.4**

For constitution \( \Pi_{ST} \) the equilibrium proposal \( \pi_{ST}^* \) is unique with \( s_j(\pi_{ST}^*) = 0 \), \( t_j(\pi_{ST}^*) = (1 + \lambda)k \forall j \in [0, 1] \), and \( g(\pi_{ST}^*) = 1 \). For this proposal \( I(\pi_{ST}^*) = 1 \) iff \( p \leq m \).

**Proof**

The agenda-setter solves the following problem:

\[
\max_{\pi \in \Pi} \{ (g(\pi)(V_w - (1 + \lambda)k) + s(\pi) - (1 + \lambda)s(\pi))I(\pi) \}.
\]

Under constitution \( \Pi_{ST} \), introducing subsidies is not worthwhile for the agenda-setter as the taxes necessary to finance them are always higher. Thus a positive level of subsidies makes the proposal less attractive to all citizens, including the agenda-setter herself. As the agenda-setter always prefers project adoption, she will always propose implementing the project.

\[\square\]

We note that, for \( p \geq m \), the project winners are sufficiently numerous to enforce the project. For \( p < m \) the project losers can block the project.

Because the proposal never involves subsidies, we can conclude

**Lemma 2.11**

Proposal \( \pi_{ST}^* \) is always redistribution-efficient. Thus \( \Pi_{ST} \) satisfies GREP.
Our finding that projects with \( p < m \) are never adopted immediately implies

**Lemma 2.12**

Constitution \( \Pi_{ST} \) satisfies PALP.

Interestingly, under constitution \( \Pi_{ST} \) a project will be adopted if and only if \( p > m \), which immediately implies

**Lemma 2.13**

Constitution \( \Pi_{ST} \) satisfies RACP.

### 2.8 Welfare Comparison

In this section we compare social welfare. In Appendix A we derive the expressions for welfare for each constitution under a specific realization of project parameters \( V_w, V_l, k, \) and \( p \).

For constitution \( \Pi \) welfare cannot be pinned down exactly, because a multitude of redistribution-inefficient proposals exist in addition to the redistribution-efficient proposals. However, it is possible to compute an upper boundary for welfare by computing welfare for redistribution-efficient proposals. In addition, the proposal with the highest possible level of total subsidies \( \hat{s} \) yields a lower boundary for welfare under \( \Pi \).

\[
W(\pi^*) \leq pV_w + (1-p)V_l - (1+\lambda)k - \begin{cases} 
\lambda(m-p)\max\{0,-V_l\} & \text{if } p < m \\
0 & \text{if } p \geq m
\end{cases} \tag{2.16}
\]

\[
W(\pi^*) \geq pV_w + (1-p)V_l - (1+\lambda)k - \lambda \hat{s} \tag{2.17}
\]

Under constitution \( \Pi_T \) the project is adopted if \( p \geq m \) or if \( p < m \) and \( V_w \geq V_w^* \). Subsidies are only paid in the latter case. Hence the utilitarian welfare measure amounts to

\[
W(\pi_T^*) \begin{cases} 
 pV_w + (1-p)V_l - (1+\lambda)k - \lambda(m-p)(V_w^*-V_l) & \text{if } p < m \text{ and } V_w \geq V_w^* \\
0 & \text{if } p < m \text{ and } V_w < V_w^* \\
pV_w + (1-p)V_l - (1+\lambda)k & \text{if } p \geq m.
\end{cases} \tag{2.18}
\]

Under constitution \( \Pi_S \) the project will always be adopted. Moreover, the agenda-setter will choose the maximum level of subsidies for herself and, because of the uniform
subsidy rule, for all other citizens as well.

\[
W(\pi_S^*) = pV_w + (1 - p)V_l - (1 + \lambda)k - \lambda \hat{s}
\]  

(2.19)

Under constitution \( \Pi_{ST} \) no subsidies occur. Thus the project will be adopted if and only if it is beneficial to a majority.

\[
W(\pi_{ST}^*) = \begin{cases} 
0 & \text{if } p < m \\
 pV_w + (1 - p)V_l - (1 + \lambda)k & \text{if } p \geq m. 
\end{cases}
\]  

(2.20)

We note that constitutions \( \Pi \) and \( \Pi_S \) both yield project adoption for any admissible combination of the exogenous variables. However, \( \Pi_S \) entails a higher level of total subsidies in general. As a consequence, constitution \( \Pi_S \) is inferior to constitution \( \Pi \) and thus never represents the socially optimal constitution. Intuitively, the desire of the agenda-setter to receive high subsidies together with the rule that all other citizens are also required to receive the same level of subsidies induces excessive redistribution under \( \Pi_S \). Consequently, \( \Pi_S \) would never be adopted under a veil of ignorance.

For the other three constitutions \( \Pi, \Pi_T, \) and \( \Pi_{ST} \) no general ranking with respect to welfare can be established that would hold for all admissible values of the exogenous variables. Which one of these would be selected would depend on the distribution of project parameters \( V_w, V_l, k, \) and \( p \) in general.

We can rank constitutions \( \Pi, \Pi_T, \) and \( \Pi_{ST} \) according to their degree of restrictiveness, with \( \Pi \) the least restrictive and \( \Pi_{ST} \) the most restrictive constitution. Note that the less restrictive the constitution is, the larger the set of parameter values will be for which the project is adopted. This is intuitive, as less restrictive constitutions grant the agenda-setter higher flexibility in designing a proposal that will secure the majority of votes. In particular, the least restrictive constitution \( \Pi \) yields project adoption for any combination of parameters. The most restrictive constitution \( \Pi_{ST} \) entails project adoption for \( p \geq m \) only.

The most restrictive constitution \( \Pi_{ST} \) has the advantage of eliminating any redistribution activity. However, for some parameter constellations this may involve costs, as projects are never adopted if \( p < m \), although they may be socially desirable.
2.9 Examples

In the following we consider the implications of our model for different categories of projects. Two arguments support this approach. First, it may be known at the constitutional stage that a specific class of projects presents the major challenge facing the polity. Second, and perhaps more importantly, while it may not be possible to write constitutional rules dependent on project characteristics, it is plausible for different constitutional rules to be designed for different categories of projects. Project categories are likely to be verifiable, while the exact project parameters $V_l$, $V_w$, $p$, and $k$ are not. Accordingly, in the following we examine the optimal constitutional rules for different project categories. First we focus on the case of economic reform projects, then we examine locally beneficial projects.

2.9.1 Economic Reforms

We focus here on the special case of economic reforms, which represent a subset of $\mathcal{P}$. One important characteristic of economic reforms, such as labor-market reforms or product-market reforms leading to more intense competition, is that they are unlikely to involve substantial direct costs $k$. Thus we set $k = 0$. Moreover, it is plausible to assume that economic reforms will differ in the effect they have on small interest groups and the large majority of the population. More specifically, we distinguish between socially beneficial economic reforms and socially detrimental reforms.

Socially beneficial reforms, such as the liberalization of the agricultural sector, are harmful to a small interest group, that is those working in this sector. However, they are beneficial to the rest of society as they stand to gain from lower prices or lower subsidies, which in turn imply lower taxes. For this class of reforms we assume that $p$ is larger than $m$ and that the total benefits are positive, that is $pV_w + (1 - p)V_l > 0$.

Socially detrimental reforms, like measures leading to lower competition in a specific sector, benefit only a small interest group, for example the shareholders of the firms in the specified sector. As a consequence, for these reforms $p < m$ and $pV_w + (1 - p)V_l < 0$ hold.

Interestingly, for economic reforms constitution $\Pi_{ST}$ will always implement the first-best. All socially desirable projects are adopted, and socially harmful projects are
never implemented. Moreover, there are no losses from redistribution. We summarize this finding in the following proposition:

**Proposition 2.5**  
*Constitution* $\Pi_{ST}$ *always leads to the first-best outcome.*

We note that constitutions $\Pi$ and $\Pi_S$ are definitely inferior to $\Pi_{ST}$, as under the first two constitutions all reforms are adopted, including the socially detrimental ones. Constitution $\Pi_T$ may only lead to a welfare level identical to the one implied by constitution $\Pi_{ST}$ if $V_w < V_w^*$ holds for all socially detrimental reforms. Unless this is the case, $\Pi_T$ is strictly inferior to $\Pi_{ST}$ from an aggregate welfare perspective.

Hence, as far as economic reforms are concerned, highly restrictive rules maximize citizen utility from an ex-ante perspective under a veil of ignorance.

### 2.9.2 Locally Beneficial Projects

Next we study the case of locally beneficial projects, such as hospitals, bridges, kindergartens, or theaters. These projects yield benefits to some of the citizens who live in the vicinity, but largely leave the utility for the majority of citizens unchanged. Accordingly, we assume $p < 1 - m$ and $V_l = 0$. For simplicity we assume in the following that costs $k$ are uniformly distributed on the interval $[0; \bar{k}]$ and that $V_w$ and $p$ are drawn from a degenerate distribution.

In Appendix A we show

**Proposition 2.6**  
*For locally beneficial projects there exists a critical value of $\bar{k}$, denoted by $\hat{k}$, such that*

1. if $\bar{k} < \hat{k}$, then citizens will prefer $\Pi_T$ to $\Pi_{ST}$ from an ex-ante perspective;
2. if $\bar{k} > \hat{k}$, then citizens will prefer $\Pi_{ST}$ to $\Pi_T$ from an ex-ante perspective;
3. if $\bar{k} = \hat{k}$, then citizens will be indifferent with respect to $\Pi_{ST}$ and $\Pi_T$ from an ex-ante perspective.

To sum up, whether citizens would choose $\Pi_{ST}$ or $\Pi_T$ under a veil of ignorance depends on the distribution of the project’s costs. If expected project costs are low, which corresponds to a low value of $\bar{k}$, then citizens will prefer $\Pi_T$ because this constitution...
will enable some projects to be adopted. However, it also involves losses due to the taxes that need to be levied in order to subsidize some of the project losers. Conversely, for high expected costs (or high $k$) citizens would prefer $\Pi_{ST}$, as this constitution eliminates the implementation of locally beneficial projects completely.

### 2.10 Endogenous Project Characteristics

So far, we have discussed which proposal will be chosen by the agenda-setter for given characteristics of the project. However, it seems reasonable to assume that the project parameters $V_w$, $V_l$, $k$, and $p$ are not exogenously given, but can be influenced by the agenda-setter to some extent. While it is plausible to assume that the proposer will attempt to design a project with high levels of $V_w$, which is to her own benefit, the interesting question arises as to the circumstances under which she may also affect project parameters $k$, $p$ and $V_l$ in a desirable way. An improvement of the project along these lines does not make the project more valuable to the agenda-setter directly. Instead, it increases its benefits for other citizens.

More specifically, we assume that the agenda-setter can exert effort before she makes the proposal. This effort creates costs $c > 0$ for her. These costs are assumed to be so small that they have no bearing on welfare.\(^{17}\) We consider three different scenarios:

1. Improvement of the project for project losers:

   $$V_l = \begin{cases} V_l & \text{if the agenda-setter does not exert effort} \\ \frac{V_l}{V_l > V_l} & \text{if the agenda-setter exerts effort.} \end{cases}$$

   (2.21)

2. Increase of the fraction of project winners:

   $$p = \begin{cases} p & \text{if the agenda-setter does not exert effort} \\ \frac{p}{p > p} & \text{if the agenda-setter exerts effort.} \end{cases}$$

   (2.22)

3. Reduction of the project’s costs:

   $$k = \begin{cases} k & \text{if the agenda-setter does not exert effort} \\ k < k & \text{if the agenda-setter exerts effort.} \end{cases}$$

   (2.23)

\(^{17}\)Thus our model has the potential to explain why inefficient projects are chosen. For an interesting paper that provides an explanation why inefficient redistribution policies may occur in equilibrium see Drazen and Limão (2008).
Proposition 2.7

1. Under constitutions $\Pi$ and $\Pi_S$ the agenda-setter has no incentive to enhance the project under all three scenarios.

2. Under constitution $\Pi_{ST}$ the agenda-setter may enhance the project by increasing $p$ and by decreasing project costs $k$. She will never improve $V_l$.

3. Under constitution $\Pi_T$ the agenda-setter may enhance the project under all scenarios.

Under constitutions $\Pi$ and $\Pi_S$ the agenda-setter can always achieve project adoption and does not pay any taxes under her equilibrium proposal. Consequently, her utility does not depend on parameters $V_l$, $p$, and $k$. Thus there are no incentives to incur the costs necessary for the improvement of the project under all scenarios. Similarly, the agenda-setter would never facilitate an increase in $V_l$ under constitution $\Pi_{ST}$. Exerting effort does not reduce taxes for her, nor does it increase the likelihood of the project being adopted.

There are, however, several cases where the agenda-setter may profit from exerting effort. This applies to constitutions involving tax rules, that is for $\Pi_T$ and $\Pi_{ST}$. Tax rules may induce agenda-setters to exert effort for two reasons. First, exerting effort may secure the adoption of a project that would otherwise be rejected. For example, if $p < m$ and $p \geq m$, exerting effort to increase $p$ will be optimal for the agenda-setter for sufficiently small $c$ under constitution $\Pi_{ST}$. Second, the agenda-setter may want to improve the project, as this lowers the subsidies necessary to gain support for the proposal, which in turn lowers her tax burden.

2.11 Robustness and Extensions

In this section we discuss possible extensions and modifications to our model in order to check the robustness of our results.

Finite number of voters In this chapter, we considered a continuum of voters. As a consequence, the subsidies paid to an individual citizen, in particular the agenda-setter, do not affect the society’s budget constraint. If we relaxed this assumption, the
subsidies received by the agenda-setter would result in additional taxes for citizens. However, as long as the number of citizens is sufficiently large, the tax burden for individual citizens arising from the agenda-setter’s subsidies are small. As a consequence, our results would carry over to a model with a finite but large number of voters.

**Inequity** In our model inequity does not affect welfare. This is a consequence of our assumption that utility is linear in wealth and the gains from the project. For concave utility functions, redistribution might be welfare-enhancing. However, individual voters’ utility changes from a single public project are relatively small compared to the utility from private consumption and other public goods. A linear approximation to citizens’ utility functions can be justified in such cases.

**Special treatment of agenda-setter** One could consider alternative tax and subsidy rules that would apply to all citizens except for the agenda-setter; the agenda-setter could propose arbitrary taxes and subsidies for himself.

Subsidy rules modified along these lines would enable the agenda-setter to reap high subsidies without forcing him to propose a high level of total subsidies. This might reduce the disadvantages of subsidy rules.

Tax rules that exclude the agenda-setter from uniform taxation would not have the disciplining effect tax rules have in our framework; if the agenda-setter does not have to carry the tax burden of the ordinary layman, he will have less incentives to reduce distortionary taxation. To sum up, introducing exemptions for the agenda-setter would strengthen our finding that tax rules are desirable.

### 2.12 Conclusions

In this chapter we have examined four constitutions with different restrictions on taxes and subsidies. We have shown that a constitution that imposes only the restriction of identical treatment with respect to subsidies is always inferior to a constitution that imposes no restrictions on taxes and subsidies. Thus constitution Π₅ would never be chosen at the constitutional stage.

Moreover, we have identified four advantages of tax rules. First, they always lead to redistribution-efficient proposals. As the agenda-setter has to pay the same amount of
taxes as any other citizen, she avoids excessive subsidies. Second, tax rules may induce the agenda-setter to exert effort in order to improve the project. Exerting effort may reduce the subsidies required to enlist the support of sufficiently many voters, which also reduces taxes for the agenda-setter. Moreover, under tax rules the likelihood of project adoption is higher for more favorable projects. Third, constitutions without tax rules grant a high degree of flexibility to the agenda-setter, which enables her to gain support for any project, irrespective of its character. By contrast, constitutions with tax rules prevent the adoption of extremely bad projects that benefit only a small minority $p$, involve high costs $k$, and bring low benefits $V_l$ for losers. Fourth, a constitution with rules both on taxes and subsidies displays the desirable feature of robustness against counter-proposals. To sum up, our analyses provides a rationale for the observation that decisions on project cum subsidies are usually made independently of decisions on rules that determine how government expenditures are financed.
Chapter 3

Constitutional Design: Separation of Financing and Project Decision Revisited

3.1 Introduction

In this chapter, we extend the model of Chapter 2 by allowing for counter-proposals. More precisely, we modify the legislative process in the following way: First, a project winner makes a proposal. After observing it, a project loser has the opportunity to make a counter-proposal. Finally, voters decide whether to accept the first or the second proposal. Because both interest groups (project winners and losers) are able to make a proposal, the modified set-up can be regarded as being more competitive than the one considered in Chapter 2.

We are interested to know whether the advantageous properties of tax rules prevail in the more competitive setting. Moreover, by comparing the current findings with those of the previous chapter, we shed some light on the influence of political competition on democratic performance.\(^1\)

We show that the story is somewhat more complicated if political competition is present. However, constitutions with tax rules still entail advantageous properties that constitutions absent of tax rules do not.

\(^1\)There exists an extensive body of literature dealing with the question of whether political competition exhibits similar advantages as economic competition (see for example Bardhan and Yang (2004) and Besley, Persson, and Sturm (2005)).
Chapter 3. Constitutional Design: Separation of Financing and Project Decision Revisited

3.2 Model

3.2.1 Set-up

We consider a society facing the standard problem of public-good provision and financing. Citizens are indexed by \( j \) and are uniformly arranged on the unit interval. The provision of a public project yields \( v_j \) for all \( j \in [0, 1] \) and involves per-capita costs \( k \geq 0 \). For simplicity of exposition, we assume \( v_j \in \{V_w, V_l\} \) with \( V_w > 0 \) and \( V_l < 0 \). Accordingly, we refer to individuals obtaining \( V_w \) from the public project as “project winners” and to individuals receiving \( V_l \) as “project losers”. Without loss of generality, we assume the project winners to be located on the interval \([0, p]\) and the project losers on the interval \([p, 1]\).

Two individuals can make a proposal \( \pi \), which comprises a tax and a subsidy scheme as well as the project decision. First, a project winner can suggest a proposal denoted by \( \pi^W \). Second, a project loser can propose a counter-proposal denoted by \( \pi^L \). Finally, individuals vote on the two proposals. More precisely, citizens have to decide whether to accept \( \pi^W \) or \( \pi^L \). The proposal receiving more votes is adopted. Similar to Chapter 2, this setting corresponds to the legal system of direct democracy or parliamentary democracy with perfect representation. Without loss of generality, we assume the individual making the first proposal to be located at \( j = 0 \) and the individual making the second proposal to be located at \( j = 1 \). Moreover, we will refer to \( j = 0 \) as \( W \) and to \( j = 1 \) as \( L \).

Subsidies and taxes are constrained to be non-negative. Moreover, there is some maximal level of subsidies denoted by \( \hat{s} < \infty \), and some maximal level of taxes denoted by \( \hat{t} < \infty \).\(^2\) Let \( S \) be the set of all non-negative Lebesgue-measurable functions on the unit interval that do not exceed \( \hat{s} \). Thus, each subsidy scheme involved by a proposal \( \pi \in \{\pi^W, \pi^L\} \) is a function \( s(\pi) \in S \). Accordingly, let \( T \) be the set of all non-negative Lebesgue-measurable functions on the unit interval that do not exceed \( \hat{t} \). Then each tax scheme can be written as a function \( t(\pi) \in T \).

We use \( g(\pi) \in \{0, 1\} \) as a variable indicating whether the project will be suggested \((g(\pi) = 1)\) or not \((g(\pi) = 0)\) under a certain proposal \( \pi \).

\(^2\) The assumptions \( \hat{s} < \infty \) and \( \hat{t} < \infty \) simplify the exposition, but do not qualitatively affect our findings. Because it is meaningless to define an upper bound, we refer to this assumptions by saying “\( \hat{s} \) (or \( \hat{t} \)) is sufficiently high”.

41
We consider distortionary taxes, that is for each unit of taxes that is paid by a particular individual, only a fraction \(1/(1 + \lambda)\) \((\lambda \in [0, 1])\) can be used to finance the project or subsidies. There are various interpretations of \(\lambda\). It may represent resources used for collecting and transferring funds from citizens to the state. The deadweight cost \(\lambda\) may also represent the disincentive to work if wages are taxed. The assumption of linear deadweight costs can be justified by the relationship between taxes paid for the public project and individual income, the former being sufficiently smaller than the latter. So, the society’s budget constraint is

\[
(1 + \lambda) [g(\pi)k + S(\pi)] = T(\pi),
\]

where we have introduced \(S(\pi) = \int_0^1 s_j(\pi)d\pi\) and \(T(\pi) = \int_0^1 t(\pi)d\pi\), \(\pi \in \{\pi^W, \pi^L\}\).

Similar to Chapter 2, we assume \(V_w - (1 + \lambda)k > 0\) (\(V_l - (1 + \lambda)k < 0\) obviously holds because we assumed \(V_l < 0\)).

Now we are in a position to give a formal description of the general set of possible proposals:

\[
\Pi := \{\pi \in \{0, 1\} \times S \times T \mid (1 + \lambda)[g(\pi)k + S(\pi) = T(\pi)]\}
\]

### 3.2.2 Constitutions

We adopt the standard “veil of ignorance” procedure for constitutional design. The social choice problem is reduced to a two-period setting. The first period is the constitutional period and the second the legislative period. In the constitutional period all citizens are assumed to be identical and do not know whether they will be project winners or project losers. Moreover, the project’s parameters \(V_w\), \(V_l\), \(k\), and \(p\) are not known with certainty. Citizens design a constitution or an incomplete social contract governing the supply of public goods, given a commonly known distribution of the project parameters.\(^4\)

Under the incomplete contract perspective, rules cannot depend on project characteristics, as those characteristics are not verifiable in court. Consequently, the rules that

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\(^3\)The range of \(\lambda\) is well in line with the empirical literature where plausible estimates of \(\lambda\) lie between 0.2 and 0.5 (see Stuart (1984), Ballard, Shoven, and Whalley (1985), and Browning (1987)).

\(^4\)Incomplete social contracts have been studied by Aghion and Bolton (2003) and Gersbach (2009a), among others.
can be adopted in the constitutional period are those constraining the tax and subsidy schemes the agenda-setters are allowed to use. In this chapter we adopt the perspective that in the constitutional period the decision rule that will be used later is given. In particular, we assume that citizens have to vote for one of the two proposals $\pi^W$ and $\pi^L$, and the proposal receiving more votes to be adopted.

In this model a constitution is simply a set of rules that constrain the set of proposals the agenda-setters can make. Accordingly, $\Pi$ represents a particular constitution, namely one without any further rules. In the course of the article we will consider less discretionary constitutions and impose rules on tax and/or on subsidy distribution. These constitutions represent subsets of $\Pi$.

It will be useful to define the set of all possible projects. It comprises all quadruples $(V_w, V_l, k, p)$ that satisfy the assumptions we have introduced so far. Formally, it is given by

$$\mathcal{P} := \{(V_w, V_l, k, p) \in \mathbb{R}^+ \times \mathbb{R}^- \times \mathbb{R}^+ \times ]0; 1[; 1[ : V_w > (1 + \lambda)k\},$$

where $\mathbb{R}^-$ and $\mathbb{R}^+$ denote the sets of negative real numbers and positive real numbers respectively. The prior distribution of the project parameters can now be described by a joint probability density function on $\mathcal{P}$. At this stage, we do not specify a particular form for this density function.

### 3.2.3 The Legislative Period

In the legislative period, each individual observes $v_j$ and $k$, and all individual valuations become common knowledge. $W$ makes a proposal that has to obey the constitutional rules (otherwise the proposal is declared unconstitutional and a new proposal has to be made). After observing proposal $\pi^W$, $L$ makes a counter-proposal that has to obey the constitutional rules (otherwise, again, the proposal is declared unconstitutional and a new proposal has to be made).\(^5\) Finally, the society decides whether to accept $\pi^W$ or $\pi^L$.

---

\(^5\)The assumption that agenda-setters are obligated to make constitutional proposals does not affect our findings but simplifies the analysis.
Adoption of proposal $\pi \in \{\pi^W, \pi^L\}$ yields

$$u_j(\pi) = g(\pi)v_j + s_j(\pi) - t_j(\pi), \text{ for } j \in [0; 1].$$ \hspace{1cm} (3.4)

In order to avoid open-set nonexistence problems, we impose the following tie-breaking rules. These rules do not affect our results qualitatively.

**Tie-breaking Rule 3.1**
If $u_j(\pi^W) = u_j(\pi^L)$, individual $j$ votes for the counter-proposal $\pi^L$.

**Tie-breaking Rule 3.2**
If one half of the society votes in favor of $\pi^W$ and the other half in favor of $\pi^L$, the counter-proposal $\pi^L$ is implemented.

At that time, it will be useful to define the indicator function $I(\pi^W, \pi^L)$, which adopts a value of 1 if proposal $\pi^L$ is implemented and 0 if proposal $\pi^W$ is implemented:

$$I(\pi^W, \pi^L) = \begin{cases} 1 & \text{if } u_j(\pi^W) \geq u_j(\pi^L) \text{ holds for at least half of the society} \\ 0 & \text{otherwise.} \end{cases}$$ \hspace{1cm} (3.5)

**Tie-breaking Rule 3.3**
If $L$ is indifferent between a proposal involving $g(\pi^L) = 1$ and a proposal involving $g(\pi^L) = 0$, he always suggests the one including $g(\pi^L) = 1$.

### 3.2.4 Socially Efficient Solutions

As a starting point it is instructive to consider socially optimal proposals. Consider a social planner who maximizes the utilitarian welfare measure by choosing and implementing a proposal $\pi \in \Pi$ for a given realization of the project parameters $V_w, V_l, k,$ and $p$. It is obvious that the following lemma holds:

**Lemma 3.1**
A socially optimal proposal $\pi$ has the following characteristics:

$$g(\pi) = \begin{cases} 1 & \text{for } pV_w + (1-p)V_l \geq (1+\lambda)k \\ 0 & \text{for } pV_w + (1-p)V_l < (1+\lambda)k \end{cases}$$ \hspace{1cm} (3.6)

$$S(\pi) = 0.$$ \hspace{1cm} (3.7)

\^6 We assume that the income of individuals is sufficient to pay taxes under any proposal considered in the chapter.
In particular, the social planner will never choose a positive level of total subsidies because of the losses caused by distortionary taxation. We note that the socially optimal solution is not normally unique because for \( g(\pi) = 1 \) the social planner is indifferent with respect to all possible tax schemes raising the revenues necessary to finance the project.

If the project parameters \( p, V_w, V_l, \) and \( k \) were verifiable, it would be straightforward to characterize a constitution guaranteeing the optimal level of welfare. However, we assume in the following that constitutional rules cannot depend on project characteristics, as even for perfectly observable costs and benefits of projects it is plausible that the project characteristics are not verifiable in court.

### 3.2.5 Evaluation Criteria

In the following, we take the evaluation criteria from Chapter 2 and adapt them in a way that they can be applied to the model at hand. First, we define the concept of redistribution-efficiency:

**Definition 5**
For a given constitution \( \Pi \subsetneq \Pi \), a counter-proposal \( \pi^c \in \Pi \) with \( I(\pi^w, \pi^c) = 1 \) is redistribution-efficient if to an arbitrary proposal \( \pi^w \) no \( \pi' \in \Pi \) exists with \( S(\pi'c) < S(\pi^c) \), \( g(\pi') = g(\pi) \) and \( I(\pi^w, \pi') = 1 \).

Note that we constrain the concept of redistribution-efficiency to proposals \( \pi^c \). The reason is that proposal \( \pi^c \) is always implemented in equilibrium. We will prove this fact later in the chapter.

**Definition 6**
We refer to a constitution under which only redistribution-efficient proposals are implemented as a constitution satisfying GREP (guarantees redistribution-efficient proposals).

Another desirable property of constitutions might be that it prevents the adoption of lobby projects. A lobby project is defined as a socially inefficient project that only benefits a small lobby group \( (\varepsilon < \frac{1}{2}) \).

As the creation of such socially detrimental (lobby) projects seems to be much easier for an agenda-setter than the creation of socially valuable projects, it is important that
a constitution avoids the adoption of all kind of lobby projects. Following the outline of Chapter 2, we give a formal description of the set of lobby projects:

The set of lobby projects $LP(\varepsilon) \subset P$ for $\varepsilon < \frac{1}{2}$ is the set of all projects in $P$ for which $|V_w - (1 + \lambda)k| < |V_l - (1 + \lambda)k|$ and $p \leq \varepsilon$. It is obvious that such projects are socially detrimental, that is $pV_w + (1 - p)V_l - (1 + \lambda)k < 0$.

Definition 7

A constitution satisfies the property of “protection against lobby projects” (henceforth $PALP$) if a value for $\varepsilon \in [0, 1/2]$ exists such that all projects in the set $LP(\varepsilon)$ are never adopted in equilibrium.

In Chapter 2 we also investigate under which constitutions cycling is possible. As an endless sequence of project adoption, reversal, renewed adoption,..., is not socially desirable, an additional feature of a good constitution is robustness against project reversal.\footnote{Note that in Chapter 2 we defined the property of “robustness against counter-proposals”. As this terminology might be confusing in this model (we already allow for counter-proposals) we rename the property. Nevertheless, the meaning remains the same.}

To keep things simple, we consider the same procedure as in the previous chapter. First, we assume that potentially reversible projects involve negligible costs, that is $k = 0$. Moreover, if the original projects involves $p = p^0$, $V_l = V_l^0$, and $V_w = V_w^0$, a reversal of the project can be characterized by $p = 1 - p^0$, $V_l = -V_w^0$, and $V_w = -V_l^0$.

We also assume that individuals are myopic when voting. Hence, when a decision is to be taken, voters do not take into account the eventuality of the project being reversed in the future.

Definition 8

A constitution displays robustness against project reversal ($RAPR$) if no potentially reversible project can be reversed.

3.3 A General Discussion

It is obvious that an optimal counter-proposal, from now on denoted by $\pi^*L$, always constitutes a best response to the first proposal $\pi^W$. Thus, it is self-evident to apply the concept of subgame-perfect Nash equilibria. In our model, similar to the Stackelberg-model of duopoly, the choice of the first-mover (agenda-setter $W$) impacts on the choice
of the second mover (agenda-setter $L$). Thus, $W$ chooses the first proposal strategically, taking the reaction of $L$ into account.

Because we consider a model involving a continuum of citizens, we face the problem of non-uniqueness of optimal counter-proposals. More precisely, if taxes and subsidies are not constrained by constitutional rules, there exists a continuum of optimal counter-proposals because redistributions within masses of Lebesgue-measure zero can be conducted without affecting the outcome.

In the course of the text, we will show that the device according to which $L$ selects among the continuum of optimal counter-proposals, heavily impacts on the outcome of the game.

Before we start with the analysis, we present a result that applies to all constitutions considered in the chapter and substantially simplifies the subsequent evaluations:

Lemma 3.2

To any proposal $\pi^W \in \tilde{\Pi} \subseteq \Pi$, there exists a counter-proposal $\pi^L \in \tilde{\Pi} \subseteq \Pi$ for which

(i) $u_L(\pi^L) = u_L(\pi^W)$

(ii) $I(\pi^W, \pi^L) = 1$.

Proof

Suppose that an arbitrary proposal $\pi^W \in \tilde{\Pi} \subseteq \Pi$ was made by $W$. Then, the counter-proposal $\pi^L$, with

$$\left\{ g(\pi^L) = g(\pi^W), (s_j(\pi^L) = s_j(\pi^W), t_j(\pi^L) = t_j(\pi^W))_{j \in [0,1]}, (s_W(\pi^L), t_W(\pi^L)) \right\},$$

where $(s_W(\pi^L), t_W(\pi^L))$ do not violate constitutional constraints, is feasible under the constitution considered, that is $\pi^L \in \tilde{\Pi} \subseteq \Pi$.

Since proposal $\pi^L$ is “almost” identical to proposal $\pi^W$, we immediately obtain

$$u_j(\pi^L) = u_j(\pi^W), \forall j \in [0,1].$$

By Tie-breaking Rule 3.1, all individuals in $[0,1]$ vote for $\pi^L$ which yields $I(\pi^W, \pi^L) = 1$.

Moreover, we have $u_L(\pi^L) = u_L(\pi^W)$, which completes the proof of Lemma 3.2. 

---

Proposals $\pi^W$ and $\pi^L$ only differ with respect to subsidies and taxes for $W$. 

---
Lemma 3.2 makes clear that there always exists at least one counter-proposal for $L$ which is adopted in equilibrium and yields a utility not lower than under $\pi^W$. This proposal is a “copy” of proposal $\pi^W$. Hence, by making a counter-proposal that is implemented in equilibrium, $L$ is never worse off than with proposal $\pi^W$. Of course, there might exist more sophisticated counter-proposals than a simple “copy” of $\pi^W$, which yield $u_L(\pi_L) > u_L(\pi^W)$ and $I(\pi^W, \pi_L) = 1$. The investigation of such counter-proposals will be part of the subsequent analysis. However, these considerations imply:

**Corollary 3.1**

An optimal counter-proposal $\pi^*_L$ always yields

(i) $I(\pi^W, \pi^*_L) = 1$

(ii) $u_L(\pi^*_L) \geq u_L(\pi^W)$.

Moreover, there always exists at least one counter-proposal that satisfies conditions (i) and (ii).

For the subsequent analysis, Corollary 3.1 points out that (a) $L$ only chooses proposals that yield $I(\pi^W, \pi_L) = 1$, and (b) among the proposals that yield $I(\pi^W, \pi_L) = 1$ there exists at least one that yields $u_L(\pi^*_L) \geq u_L(\pi^W)$. As a consequence, $\pi^*_L$ always corresponds to the equilibrium outcome. We will therefore call it *equilibrium proposal*.

### 3.4 Analysis With an Ex-Ante Optimal Selection Device

As a starting point, it is suggestive to investigate a selection device that is ex-ante optimal for $L$. In our model, ex-ante means: “before $W$ made his proposal $\pi^W$”. Note that after proposal $\pi^W$ is made (that is, from an ex-post perspective), $L$ is indifferent among all proposals that constitute a best response to $\pi^W$. We already argued that there exists a continuum of such best responses.

However, the subsequent analysis will show that, dependend on the selection device, the outcome is more or less desirable for $L$. Of course, a forward-looking agenda-setter $L$ would apply a device that guarantees a desirable outcome for himself, that is an ex-ante optimal device. We will now explain the basic idea:

Because $L$ always suffers a utility loss from project adoption (remember that $V_l < 0$) and from positive personal taxes, we will call $\pi^W$ a *good* proposal for $L$, if a best response to it neither involves project adoption nor positive taxes for $L$. 

48
Accordingly, we will call \( \pi^W \) a bad proposal for \( L \), if a best response to it involves either project adoption or positive taxes for \( L \).

The ex-ante optimal selection device is as follows:

- Reward \( W \) with high subsidies and/or low taxes if he made a good proposal in the first stage.
- Punish \( W \) with low subsidies and/or high taxes if he made a bad proposal in the first stage.

Because the exact design of the device depends on constitutional principles, we will specify it more precisely in the analysis of each constitution.

Two remarks are necessary: First, note that this selection device is optimal for \( L \) but not necessarily for the society. Second, it applies to the set of subgame-perfect Nash equilibria and does not involve an incredible threat to \( W \). Thus, derived equilibria are subgame-perfect Nash.

### 3.4.1 Arbitrary Tax Code and Arbitrary Subsidy Scheme

In the first scenario considered we impose no additional rules on taxes and subsidies, that is both agenda-setters can choose any proposal from the set \( \Pi \).

**Proposition 3.1**

*In equilibrium, the project is never adopted.*

The formal proof of Proposition 3.1 is given in Appendix B. The intuition runs as follows: Because neither the tax code nor the subsidy scheme are constrained by constitutional principles, \( L \) is able to impose explicit subsidies and taxes for \( W \). An ex-ante optimal selection device in this constitution could be the following:

- Rise high subsidies and low taxes for \( W \) if he suggested a good proposal.
- Rise high taxes and low subsidies for \( W \) if he suggested a bad proposal.

Since \( L \) only fixes taxes and subsidies for \( W \) (which is of measure zero), he keeps the flexibility to attain optimal outcomes (given \( \pi^W \)). If applied, \( W \) is deterred from making a bad first proposal, because he prefers to renounce the project and receive high net-transfers in return.\(^9\)

For \( L \) this selection is ex-ante optimal since he does not have to bear the utility loss caused by the project.

\(^9\)In the course of the chapter, we show that under any constitution \( \tilde{\Pi} \subseteq \Pi \), \( W \) would be able to propose a sophisticated \( \pi^W \) such that the best response to it involves project adoption, for example.
3.4.2 Uniform Tax Code and Arbitrary Subsidy Scheme

Now we impose the requirement that all individuals have to be treated identically with respect to taxation. As a consequence, we consider the following constitution:

\[ \Pi_T := \{ \pi \in \Pi \mid t_i(\pi) = t_j(\pi), \forall i, j \in [0, 1] \} \]  

(3.8)

Both agenda-setters can only choose proposals from \( \Pi_T \subset \Pi \).

**Proposition 3.2**

*In equilibrium, the project is never adopted.*

Again, the formal proof is shifted to Appendix B. The intuition is quite similar to the intuition for Proposition 3.1: In contrast to Section 3.4.1, an ex-ante optimal selection device cannot involve explicit taxes for \( L \), since the tax code is predetermined by the constitution. Hence:

- Rise high subsidies for \( W \) if he suggested a *good* proposal.
- Rise low subsidies for \( W \) if he suggested a *bad* proposal.

Again, the application of this kind of selection device is credible. Similar to before, \( W \) prefers receiving high subsidies and therefore renounces the project.

This device is ex-ante optimal for \( L \) since he does not have to bear utility losses from the project and taxation.

3.4.3 Arbitrary Tax Code and Uniform Subsidy Scheme

Now we consider a constitution that allows for arbitrary tax schemes. However, we limit the subsidy schemes to those treating all citizens equally. Hence, we restrict our attention to the set of proposals \( \Pi_S \subset \Pi \) with

\[ \Pi_S := \{ \pi \in \Pi \mid s_i(\pi) = s_j(\pi), \forall i, j \in [0, 1] \} \]  

(3.9)

**Proposition 3.3**

*In equilibrium, the project is never adopted.*

The proof is complementary to the proof of Proposition 3.2 and given in Appendix B. The intuition is similar to the one issued in Section 3.4.2, with the only exception that the selection is regulated through the individual tax and not the individual subsidy for \( W \).
3.4.4 Uniform Tax Code and Uniform Subsidy Scheme

Finally, we consider a constitution stipulating that all citizens be treated identically with respect to subsidies and taxes. Hence, the set of feasible proposals reduces to \( \Pi_{ST} \subset \Pi \), where

\[
\Pi_{ST} := \Pi_T \cap \Pi_S. \tag{3.10}
\]

**Proposition 3.4**

If \( p > \frac{1}{2} \), the equilibrium proposal \( \pi^L \) comprises \( g(\pi^L) = 1 \).

If \( p \leq \frac{1}{2} \), the equilibrium proposal \( \pi^L \) comprises \( g(\pi^L) = 0 \).

The formal proof is shifted to Appendix B. In contrast to the other constitutions, \( L \) cannot choose a reward-punishment scheme as outlined in the previous sections, because the distribution of taxes and subsidies is predetermined by the constitution itself. As a consequence, \( W \) is not deterred from making a bad first proposal. This means that he makes a first proposal \( \pi^W \) which results in project adoption whenever possible. In the proof we show that the project is only adopted if \( p > 1/2 \).

3.4.5 Conclusions

We have shown that if \( L \) applies an ex-ante optimal selection device, the project is never adopted under constitutions \( \Pi, \Pi_T \), and \( \Pi_S \). Only constitution \( \Pi_{ST} \) allows for project implementation.\(^{10}\)

Because a permanent rejection of public projects is undesirable for a society, the analysis has risen a strong argument for the application of tax rules in combination with subsidy rules.

However, results are heavily driven by the selection device itself: Because \( L \) is able to combine specific taxes and/or subsidies for \( W \) with a certain proposal \( \pi^W \), that is to condition subsidies and/or taxes on \( \pi^W \), he is able to corrupt agenda-setter \( W \).

Hence, one might claim that if \( L \) did not have this opportunity (that is, he applied an alternative selection device), outcomes would differ.\(^{11}\)

---

\(^{10}\)To put it differently, the second mover has the possibility to enforce his policy preferences under constitutions \( \Pi, \Pi_T \), and \( \Pi_S \), but not under \( \Pi_{ST} \). **BERNHEIM, RANGEL, AND RAYO** (2006) show that in policy spaces where a Condorcet winner does not exist (as it is the case in constitutions \( \Pi, \Pi_T \), and \( \Pi_S \)), the last proposer is effectively a dictator. Our findings make clear that whether the last proposer (agenda-setter \( L \)) can really exhibit dictatorial power or not depends on how restrictive the distribution of taxes and subsidies is regulated.

\(^{11}\)Indeed, one can show that many outcomes could be sustained as an equilibrium outcome: \( L \) only had to connect the reward-punishment scheme (as outlined in Section 3.4) with different proposals \( \pi^W \).
Taking these considerations into account, one can argue that an ex-ante optimal selection device naturally favors restrictive constitutions. For that reason, we now present an analysis using another selection device.

### 3.5 Analysis With a Fixed Selection Device

In this section, we consider a fixed selection device. The basic idea is as follows: if taxes and/or subsidies are not constrained by constitutional rules, $\mathcal{L}$ chooses a fixed tax and/or subsidy for $\mathcal{W}$, independent of whether $\pi^W$ is good or bad for him. As we will see, such a selection device helps to overcome the “prevention” result of Section 3.4. Consequently, this device is not ex-ante optimal for $\mathcal{L}$.

Finally, we emphasize that a fixed selection device could be differently designed. We concentrate on the following one:

**Assumption 3.1**

*If no rules are imposed on the subsidy scheme, $\mathcal{L}$ chooses $s_W(\pi^L) = 0$. If no rules are imposed on the tax scheme, $\mathcal{L}$ chooses $t_W(\pi^L) = 0$.***

Recall that this selection device yields subgame-perfect equilibria.

Again, a remark is necessary: While it seems reasonable for a rational individual to choose an ex-ante optimal selection device, it appears less plausible to choose an ex-ante suboptimal device. We therefore provide two justifications for its application:

1. $\mathcal{L}$ might use a fixed selection device because he is forced to do so, that is, the constitution itself determines to what extent agenda-setters can be subsidized or taxed. Thus, constitutions might impose special treatment-rules on agenda-setters in case the tax code and/or the subsidy scheme is not well specified.

   Our analysis in Section 3.4 and the resulting “prevention” result raises a strong argument for special treatment of agenda-setters. Because the exact structuring of the agenda-setter rules is unimportant, as long as they ensure fixed taxes and/or subsidies for an agenda-setter, an alternative formulation of Assumption 3.1 could be:

   **For example, $\mathcal{L}$ could always enforce project adoption by rewarding bad proposals $\pi^W$ and punishing good proposals.**

   **Gersbach (2009a) and Gersbach (2009b) investigate similar voting models and show that in order to obtain first-best outcomes, it is necessary to apply special treatment rules for an agenda-setter.**

52
Assumption 3.2
If no rules are imposed on the subsidy scheme, both agenda-setters are obligated to choose \( s_i(\pi^j) = 0 \), \( i, j \in \{W, L\} \).

If no rules are imposed on the tax scheme, both agenda-setters are obligated to choose \( t_i(\pi^j) = 0 \), \( i, j \in \{W, L\} \).

2. Another justification makes use of a behavioral argument: \( L \) might use a fixed selection device due to negative reciprocity. Since \( W \) and \( L \) are political opponents, one might argue that \( L \) intends to harm \( W \) and therefore chooses the lowest subsidy and the highest tax for \( W \) whenever possible. Thus, an alternative formulation of Assumption 3.1 could be:

Assumption 3.3
If no rules are imposed on the subsidy scheme, \( L \) chooses \( s_W(\pi^L) = 0 \).
If no rules are imposed on the tax scheme, \( L \) chooses \( t_W(\pi^L) = \hat{t} \).

3.5.1 Arbitrary Tax Code and Arbitrary Subsidy Scheme

With a fixed selection device, we obtain the following result for constitution \( \Pi \):

Proposition 3.5
If \( p > \frac{1}{2} \), the equilibrium proposal \( \pi^{*L} \) comprises

\[
g(\pi^{*L}) = 1 \text{ and } 0 < S(\pi^{*L}) \leq \hat{s}.
\]

If \( p \leq \frac{1}{2} \), the equilibrium proposal \( \pi^{*L} \) comprises

\[
g(\pi^{*L}) = 0 \text{ and } 0 \leq S(\pi^{*L}) \leq \hat{s}.
\]

The proof of the proposition is shifted to Appendix B. The intuition runs as follows:
Because \( s_W(\pi^L) = t_W(\pi^L) = 0 \) in any case, \( W \) is only interested in project implementation and does not care about other properties (such as the distribution of subsidies and taxes) of the counter-proposal \( \pi^L \).
If \( p \geq \frac{1}{2} \), \( W \) is able to make a proposal such that \( I(\pi^W, \pi^L) = 1 \) can only hold if \( L \) choose project adoption in his proposal.\(^{13}\) If \( p < \frac{1}{2} \), any optimal counter-proposal by \( L \) involves \( g(\pi^L) = 0 \).

\(^{13}\)Recall Lemma 3.2, where we have shown that it is always optimal for \( L \) to choose a counter-proposal that yields \( I(\pi^W, \pi^L) = 1 \).
Because $\mathcal{L}$ is always able to choose $s_\mathcal{L}(\pi^\mathcal{L}) = \hat{s}$ and $t_\mathcal{L}(\pi^\mathcal{L}) = 0$, he does not have an incentive to reduce detrimental subsidization, and hence an optimal counter-proposal $\pi^*\mathcal{L}$ might comprise more subsidies than required for $I(\pi^W, \pi^\mathcal{L}) = 1$. Thus, we immediately conclude that

**Lemma 3.3**

*Constitution $\Pi$ does not satisfy GREP.*

The fact that only projects with $p > \frac{1}{2}$ are implemented means that

**Lemma 3.4**

*Constitution $\Pi$ satisfies PALP.*

**Lemma 3.5**

*Constitution $\Pi$ satisfies RAP R.*

### 3.5.2 Uniform Tax Code and Arbitrary Subsidy Scheme

Because a complete characterization of equilibrium outcomes is very extensive, we restrict our analysis to the derivation of necessary conditions for project adoption. The insights from this analysis are sufficient to evaluate constitution $\Pi_T$ according to our qualitative criteria and to compare it with other constitutions. Roughly, we proceed as follows:

- First, we investigate individual voting behavior for arbitrary proposals $\pi^W$, $\pi^\mathcal{L}$.
- Second, we characterize the subsidy scheme in an optimal counter-proposal for arbitrary $\pi^W$, that is $s(R(\pi^W))$.
- Third, we derive necessary conditions for $g(R(\pi^W)) = 1$.
- Fourth, we investigate proposals $\pi^W$ and provide some concrete examples.

#### 3.5.2.1 Individual Voting Behavior

An arbitrary proposal $\pi^W \in \Pi_T$ yields

$$u_j(\pi^W) = s_j(\pi^W) + g(\pi^W)(v_j - (1 + \lambda)k) - (1 + \lambda)S(\pi^W), \quad j \in [0, 1].$$

(3.11)
Chapter 3. Constitutional Design: Separation of Financing and Project Decision Revisited

An arbitrary counter-proposal \( \pi^c \in \Pi_T \) yields

\[
u_j(\pi^c) = s_j(\pi^c) + g(\pi^c)(v_j - (1 + \lambda)k) - (1 + \lambda)S(\pi^c), \quad j \in [0, 1]. \tag{3.12}\]

Individual \( j \) votes in favor of proposal \( \pi^c \) if \( u_j(\pi^c) \geq u_j(\pi^w) \), which can be rewritten as

\[
s_j(\pi^c) + g(\pi^c)(v_j - (1 + \lambda)k) - (1 + \lambda)S(\pi^c) \geq u_j(\pi^w). \tag{3.13}\]

Solving for \( s_j(\pi^c) \) yields

\[
s_j(\pi^c) \geq u_j(\pi^w) - g(\pi^c)(v_j - (1 + \lambda)k) + (1 + \lambda)S(\pi^c) := \sigma_j(g(\pi^c)). \tag{3.14}\]

Note that, given \( \pi^w \), \( \sigma_j(g(\pi^c)) \) represents the smallest subsidy that must be distributed to individual \( j \) such that he votes for proposal \( \pi^c \). The size of \( \sigma_j(g(\pi^c)) \) depends on whether the project is suggest or not (that is on \( g(\pi^c) \)) and on how subsidies are distributed under \( \pi^c \) (that is on the aggregate amount of subsidies \( S(\pi^c) \)).

If \( \sigma_j(g(\pi^c)) < 0 \), individual \( j \) votes for proposal \( \pi^c \) irrespective of the size of the subsidy he receives under \( \pi^c \). Conversely, if \( \sigma_j(g(\pi^c)) > s \), individual \( j \) votes for \( \pi^w \) irrespective of the size of the subsidy \( s_j(\pi^c) \).

It is important to note that \( \sigma_j(g(\pi^c)) \) can be arranged according to size for all individuals in \([0, 1]\) independent of the distribution of subsidies. The reason is that the aggregate amount of subsidies affects the utility of all individuals identically. Thus, while the absolute size of \( \sigma_j(g(\pi^c)) \) depends on \( g(\pi^c) \) and \( S(\pi^c) \), the relative size depends on \( g(\pi^c) \) solely. Let us illustrate this fact formally. Consider two individuals \( h, i \in [0, 1] \):

\[
\sigma_h(g(\pi^c)) - \sigma_i(g(\pi^c)) = u_h(\pi^w) - u_i(\pi^w) - g(\pi^c)(v_h - v_i) = s_h(\pi^w) - s_i(\pi^w) + (g(\pi^w) - g(\pi^c))(v_h - v_i). \tag{3.15}
\]

This difference is well defined for any \( h, i \in [0, 1] \), which allows to arrange \( \sigma_h(g(\pi^c)) \) and \( \sigma_i(g(\pi^c)) \) according to size. Thus, we are able to generate a complete ordering of all \( \sigma_j(g(\pi^c)) \) according to size.

Summary: We introduced \( \sigma_j(g(\pi^c)) \) as the minimal subsidy required to attain the support of individual \( j \) for proposal \( \pi^c \). This minimal subsidy plays an important role in the subsequent analysis, since \( \mathcal{L} \) only compensates individuals for which \( \sigma_j(g(\pi^c)) \) is relatively low. Thus, it is important to know that \( \sigma_j(g(\pi^c)) \) can be ranked for all individuals in \([0, 1]\) and that this ranking only depends on \( g(\pi^c) \) (for any given \( \pi^w \).
3.5.2.2 Best-response Function of $\mathcal{L}$

The best-response function of $\mathcal{L}$ is a mapping from and into $\Pi_T$, that is $R: \Pi_T \rightarrow \Pi_T$.\(^{14}\)

\[ R(\pi^W) = \arg\max_{\pi^T \in \Pi_T} \left\{ s_L(\pi^L) + g(\pi^L)(V_i - (1 + \lambda)k) - (1 + \lambda)S(\pi^L) \right\}, \quad \text{s.t. } I(\pi^W, \pi^L) = 1. \quad (3.17) \]

For the analysis it will be useful to define the following sets:

- Let $Q_{ns}$ be the set of all project losers, except $\mathcal{L}$, who receive zero subsidies under the first proposal $\pi^W$, that is

  \[ Q_{ns} := \{ j \in \]p, 1[ | s_j(\pi^W) = 0 \}. \quad (3.18) \]

Let $q_{ns} := |Q_{ns}|$ be the cardinality of the set $Q_{ns}$.

- Let $\tilde{Q}_{ns}$ be the set of all individuals, except $\mathcal{W}$ and $\mathcal{L}$, who do not belong to $Q_{ns}$, that is

  \[ \tilde{Q}_{ns} := \{ j \in \]0, 1[ | j \notin Q_{ns} \}. \quad (3.19) \]

It is obvious that $|\tilde{Q}_{ns}| = 1 - q_{ns}$.

Moreover, if $q_{ns} < \frac{1}{2}$, we split the set $\tilde{Q}_{ns}$ into two subsets:

- Let $Q_{gs}(R(\pi^W))$ be the set of $\frac{1}{2} - q_{ns} > 0$ individuals in $\tilde{Q}_{ns}$ for which $\sigma_j(g(R(\pi^W)))$ is smallest. It is obvious that $|Q_{gs}(R(\pi^W))| = \frac{1}{2} - q_{ns}$.

- Let $Q_{others}(R(\pi^W))$ be the set of all other individuals in $\tilde{Q}_{ns}$. Again, it is obvious that $|Q_{others}(R(\pi^W))| = |\tilde{Q}_{ns}| - |Q_{gs}(R(\pi^W))| = 1 - q_{ns} - \left(\frac{1}{2} - q_{ns}\right) = \frac{1}{2}$

Now we are in a position to describe the optimal subsidy scheme $s(R(\pi^W))$:

(i) $s_L(R(\pi^W)) = \hat{s}$,

(ii) $s_W(R(\pi^W)) = 0$,

(iii) $s_j(R(\pi^W)) = 0, \quad \forall j \in Q_{ns}$,

\(^{14}\) $R$ is unique because we do not allow for relabeling of individuals and redistribution within masses of Lebesgue-measure zero.

\(^{15}\) In Corollary 3.1, we pointed out that it is always optimal for $\mathcal{L}$ to choose a counter-proposal that yields $I(\pi^W, \pi^L) = 1$. 

56
(iv) Only if \( q_{ns} < \frac{1}{2} \): \( s_j(R(\pi^W)) = \max\{0, \sigma(g(R(\pi^W)))\} \), for \( j \in Q_{g(R(\pi^W))} \).

(v) \( s_j(R(\pi^W)) = 0 \), for all other individuals.

Item (i) directly follows from inspection of the maximization problem\(^{16}\) and item (ii) is a consequence of Assumption 3.1. The findings in items (iii)-(v) require some explanation:

Because \( u_{\mathcal{L}}(\pi^L) \) is decreasing in the aggregate amount of subsidies, \( \mathcal{L} \) has a strict incentive to form the “cheapest” coalition such that \( I(\pi^W, \pi^L) = 1 \). Thus, he only pays subsidies to the cheapest 50\% of the society and he pays them exactly the amount so that they just vote for \( R(\pi^W) \).

In Lemma 3.6 we show that project losers who do not receive a positive subsidy under proposal \( \pi^W \), that is all \( j \in Q_{ns} \), vote for \( R(\pi^W) \) in equilibrium, irrespective of the size of the subsidy they receive under \( R(\pi^W) \). Thus, in order to save taxes, it is optimal for \( \mathcal{L} \) to choose \( s_j(R(\pi^W)) = 0, \forall j \in Q_{ns} \).

Because \( \mathcal{L} \) knows the votes of individuals in \( Q_{ns} \) for sure, he needs the support of another \( \frac{1}{2} - q_{ns} \) individuals to fulfill the constraint \( I(\pi^W, R(\pi^W)) = 1 \). Note that if \( q_{ns} \geq \frac{1}{2} \), \( \mathcal{L} \) does not need the support of other individuals in order to implement \( R(\pi^W) \). However, if \( q_{ns} < \frac{1}{2} \), \( \mathcal{L} \) takes a fraction of \( \frac{1}{2} - q_{ns} \) individuals from the set \( \bar{Q}_{ns} \) into its coalition. Of course, it is profitable for him to take those individuals for which \( \sigma_j(g(R(\pi^W))) \) is smallest, since these individuals require the lowest subsidies for compensation, which lowers uniform taxes. Moreover, it is obvious that it is not profitable to pay higher subsidies than \( \max\{0, \sigma_j(g(R(\pi^W)))\} \). Thus, he pays \( s_j(R(\pi^W)) = \max\{0, \sigma_j(g(R(\pi^W)))\} \) for \( j \in Q_{g(R(\pi^W))} \).

Finally, the above subsidy scheme guarantees \( \mathcal{L} \) the support of \( \frac{1}{2} \) individuals. Thus, he does not need to distribute further subsidies. Again, to save taxes he sets \( s_j(R(\pi^W)) = 0 \) for all other individuals. These considerations make the statements in items (iii)-(v).

It remains to prove item (iii):

**Lemma 3.6**

\( R(\pi^W) \) comprises \( s_j(R(\pi^W)) = 0, \forall j \in Q_{ns} \).

The formal proof is shifted to Appendix B. The intuition runs as follows:

\(^{16}\)Remember that adoption of proposal \( R(\pi^W) \) does not depend on the size of \( s_{\mathcal{L}}(R(\pi^W)) \). Thus, \( \mathcal{L} \) is free to choose his preferred subsidy level without affecting the constraint \( I(\pi^W, \pi^L) = 1 \).
Roughly, $u_L(R(\pi^W))$ can be decomposed into two components: the self-subsidy and the “rest”:

$$u_L(R(\pi^W)) = s_L(R(\pi^W)) + g(R(\pi^W))(V_l - (1 + \lambda)k) - (1 + \lambda)S(R(\pi^W)),$$

(3.20)

Because $L$ seeks to maximize his own utility and we already know that $s_L(R(\pi^W)) = \hat{s}$, it remains to maximize the “rest”. It is obvious that the “rest” corresponds to the utility of project losers who are not subsidized under proposal $R(\pi^W)$.

Because $L$ can always “copy” a first proposal $\pi^W$ (see Lemma 3.2), the “rest” is bounded from below by $g(\pi^W)(V_l - (1 + \lambda)k) - (1 + \lambda)S(\pi^W)$ which corresponds to the utility of non-subsidized project losers in proposal $\pi^W$. More precisely

"rest" $\geq g(\pi^W)(V_l - (1 + \lambda)k) - (1 + \lambda)S(\pi^W) = u_j(\pi^W), j \in Q_{ns}$.

Hence, in equilibrium, it is not necessary to subsidize project losers who were not subsidized in proposal $\pi^W$, because they vote for $R(\pi^W)$ anyway. The reason is that by increasing his own utility, $L$ contemporaneously increases the utility of other (non-subsidized) project losers.

Furthermore, these considerations imply that

$$u_L(R(\pi^W)) = u_j(R(\pi^W)) + \hat{s}, \forall j \in Q_{ns}. \quad (3.21)$$

Now, we present the optimal counter proposal if $q_{ns} \geq \frac{1}{2}$:

**Lemma 3.7**

If $q_{ns} \geq \frac{1}{2}$, $R(\pi^W) = \pi^{*L}$ is given by

$$\left\{ g(\pi^{*L}_1) = 0, (s_j(\pi^{*L}_1) = 0)_{j \in [0,1]}, s_L(\pi^{*L}_1) = \hat{s}, t(\pi^{*L}_1) = 0 \right\}.$$

**Proof**

$\pi^{*L}_1$ yields for all $j \in Q_{ns}$:

$$u_j(\pi^{*L}_1) = 0 \geq g(\pi^W)(V_l - (1 + \lambda)k) - (1 + \lambda)S(\pi^W) = u_j(\pi^W).$$

(3.22)

Thus, all $j \in Q_{ns}$ vote for $\pi^{*L}_1$ which yields $I(\pi^W, \pi^{*L}_1) = 1$ because $q_{ns} \geq \frac{1}{2}$.

\[\square\]
Lemma 3.7 makes clear that the project is never adopted if \( q_{ns} \geq \frac{1}{2} \). Next, we concentrate on the case \( q_{ns} < \frac{1}{2} \):

Given the subsidy scheme \( s(R(\pi^W)) \), two alternatives for the best response \( R(\pi^W) \) remain: Either it is optimal to choose \( g(R(\pi^W)) = 0 \), or \( g(R(\pi^W)) = 1 \).

If \( g(R(\pi^W)) = 0 \), \( L \) distributes positive subsidies to individuals in \( Q_0 \). Recall that \( Q_0 \) is the set of \( \frac{1}{2} - q_{ns} \) individuals in \( \bar{Q}_{ns} \) for which \( \sigma_j(0) \) is smallest. From Equation 3.16, we know that \( \sigma_j(0) \) is smallest for those individuals for which \( s_j(\pi^W) - g(\pi^W)v_j \) is smallest. All other individuals (except \( L \)) receive zero subsidies.

We denote such a proposal by \( \tilde{\pi}^L \):\(^{17}\)

\[
g(\tilde{\pi}^L) = 0, \quad s_j(\tilde{\pi}^L) = \begin{cases} 
0 & \text{for } j = 0 \\
0 & \text{for } j \in Q_{ns} \\
\max\{0, \sigma_j(0)\} & \text{for } j \in Q_0 \\
0 & \text{for } j \in Q_0^{others} \\
\tilde{s} & \text{for } j = 1 
\end{cases}
\]

\( t(\tilde{\pi}^L) = (1+\lambda) \int_{j \in Q_0} \max\{0, \sigma_j(0)\} \, dj \).

If \( g(R(\pi^W)) = 1 \), \( L \) distributes positive subsidies to individuals in \( Q_1 \). Remember that \( Q_1 \) is the set of \( \frac{1}{2} - q_{ns} \) individuals in \( \bar{Q}_{ns} \) for which \( \sigma_j(1) \) is smallest. From Equation 3.16, we know that \( \sigma_j(1) \) is smallest for those individuals for which \( s_j(\pi^W) + (g(\pi^W) - 1)v_j \) is smallest. Again, all other individuals, with the exception of \( L \), receive zero subsidies.

We denote such a proposal by \( \tilde{\tilde{\pi}}^L \):\(^{18}\)

\[
g(\tilde{\tilde{\pi}}^L) = 1, \quad s_j(\tilde{\tilde{\pi}}^L) = \begin{cases} 
0 & \text{for } j = 0 \\
0 & \text{for } j \in Q_{ns} \\
\max\{0, \sigma_j(1)\} & \text{for } j \in Q_1 \\
0 & \text{for } j \in Q_1^{others} \\
\tilde{s} & \text{for } j = 1 
\end{cases}
\]

\( t(\tilde{\tilde{\pi}}^L) = (1+\lambda) \left[k + \int_{j \in Q_1} \max\{0, \sigma_j(0)\} \, dj \right]. \)

Thus, in equilibrium, either proposal \( \tilde{\pi}^L \) or \( \tilde{\tilde{\pi}}^L \) is suggested by \( L \) and implemented. It is important to note that both proposals, \( \tilde{\pi}^L \) and \( \tilde{\tilde{\pi}}^L \), are feasible, that is: Both proposals entail a balanced budget, a uniform tax, a feasible project decision as well as a feasible subsidy scheme. Note that the subsidy scheme is feasible because \( s_j(R(\pi^W)) \in [0, \tilde{s}] \) for

\(^{17}\)Thus, \( \tilde{\pi}^L \) represents the optimal counter-proposal if \( g(R(\pi^W)) = 0 \).

\(^{18}\)Thus, \( \tilde{\tilde{\pi}}^L \) represents the optimal counter-proposal if \( g(R(\pi^W)) = 1 \).
all $j \in [0, 1]$. More precisely, it is obvious that both, $\max\{0, \sigma_j(0)\}$ and $\max\{0, \sigma_j(1)\}$, are not lower than zero. However, it is more complicated to show that they are smaller than $\hat{s}$:

**Lemma 3.8**

For sufficiently high $\hat{s}$, the condition $\max\{0, \sigma_j(g(R(\pi^W)))\} < \hat{s}$ holds for all $g(R(\pi^W)) \in \{0, 1\}$.

The proof is given in Appendix B.

**Summary:** So far, we derived the optimal subsidy scheme dependent on $g(R(\pi^W))$. Out of it, we were able to construct two proposals, $\tilde{\pi}^L$ and $\tilde{\pi}^E$, which are the only candidates for the equilibrium proposal. Moreover, we showed that these proposals are always feasible. However, it is still unclear under what conditions it is optimal for $L$ to choose $g(R(\pi^W)) = 0$ or $g(R(\pi^W)) = 1$, respectively.

The derivation of these conditions will be the subject of the following analysis.

The following two lemmas simplify the later analysis:

**Lemma 3.9**

For all $j \in Q_{ns}$, it is true that $\max\{u_j(\tilde{\pi}^L), u_j(\tilde{\pi}^E)\} \geq u_j(\pi^W)$.

The proof of Lemma 3.9 is given in Appendix B. The intuition is simple:

If $g(\pi^W) = 0$, then $u_j(\tilde{\pi}^L) \geq u_j(\pi^W)$ for all $j \in Q_{ns}$, because $g(\tilde{\pi}^L) = g(\pi^W)$ and the aggregate amount of subsidies under proposal $\tilde{\pi}^L$ is not higher than under $\pi^W$.

If $g(\pi^W) = 1$, then $u_j(\tilde{\pi}^E) \geq u_j(\pi^W)$ for all $j \in Q_{ns}$, because $g(\tilde{\pi}^E) = g(\pi^W)$ and the aggregate amount of subsidies under proposal $\tilde{\pi}^E$ is not higher than under $\pi^W$.

Consequently, it is always true that $\max\{u_j(\tilde{\pi}^L), u_j(\tilde{\pi}^E)\} \geq u_j(\pi^W)$, for all $j \in Q_{ns}$.

**Lemma 3.10**

$g(R(\pi^W)) = 1$ holds if and only if $u_L(\tilde{\pi}^L) \geq u_L(\tilde{\pi}^E)$.

**Proof**

$\Rightarrow$ If $u_L(\tilde{\pi}^L) \geq u_L(\tilde{\pi}^E)$, then $L$ prefers to implement proposal $\tilde{\pi}^L$. The reason is that there is no alternative proposal including $g(\pi^E) = 1$ that yields higher utility and is adopted. Moreover, there is no alternative proposal comprising $g(\pi^L) = 0$ that yields higher utility and is adopted.

Because $u_L(\tilde{\pi}^L) \geq u_L(\tilde{\pi}^E)$ it follows that $u_j(\tilde{\pi}^L) + \hat{s} \geq u_j(\tilde{\pi}^E) + \hat{s}$, for $j \in Q_{ns}$ (see Equation 3.21), which implies $u_j(\tilde{\pi}^L) \geq u_j(\tilde{\pi}^E)$, for $j \in Q_{ns}$.

\[19\text{Note that } \min\{u_j(\tilde{\pi}^L), u_j(\tilde{\pi}^E)\} \leq u_j(\pi^W) \text{ for } j \in Q_{ns}.\]
By Lemma 3.9, \( \max\{u_j(\tilde{\pi}^L), u_j(\bar{\pi}^L)\} = u_j(\bar{\pi}^L) \geq u_j(\pi^W) \) for \( j \in Q_{ns} \) which implies that all \( j \in Q_{ns} \) vote for \( \bar{\pi}^L \).

By construction, individuals in \( Q_1 \) vote for \( \tilde{\pi}^L \).

Hence, \( I(\pi^W, \tilde{\pi}^L) = 1 \) and so \( g(R(\pi^W)) = 1. \)

\( \Leftarrow \) If \( u_L(\tilde{\pi}^L) < u_L(\bar{\pi}^L) \), then \( L \) prefers to implement proposal \( \tilde{\pi}^L \). Again, the reason is that there exists neither a proposal including \( g(\pi^L) = 1 \) nor \( g(\pi^L) = 0 \) that yields higher utility for him and is adopted.

Because \( u_L(\tilde{\pi}^L) < u_L(\bar{\pi}^L) \) it follows that \( u_j(\tilde{\pi}^L) < u_j(\bar{\pi}^L) \), for \( j \in Q_{ns} \).

By Lemma 3.9, \( \max\{u_j(\tilde{\pi}^L), u_j(\bar{\pi}^L)\} = u_j(\bar{\pi}^L) \geq u_j(\pi^W) \) for \( j \in Q_{ns} \) which implies that all \( j \in Q_{ns} \) vote for \( \bar{\pi}^L \).

By construction, individuals in \( Q_0 \) vote for \( \tilde{\pi}^L \).

Hence, \( I(\pi^W, \tilde{\pi}^L) = 1 \) and so \( g(R(\pi^W)) = 0. \)

\( \square \)

**Summary:** In Lemmas 3.9 and 3.10 we showed that among the two candidates for the equilibrium proposal, \( \bar{\pi}^L \) and \( \tilde{\pi}^L \), \( L \) chooses the one that yields higher utility for him, which is plausible. However, it is important to note that this proposal is accepted by individuals in \( Q_{ns} \), which guarantees adoption of that very proposal. The condition \( u_L(\tilde{\pi}^L) \geq u_L(\bar{\pi}^L) \) can also be written as

\[
(1 + \lambda) \left( \int_{j \in Q_0} \max\{0, \sigma_j(0)\} dj - \int_{j \in Q_1} \max\{0, \sigma_j(1)\} dj \right) \geq (1 + \lambda)k - V_i. \tag{3.23}
\]

In the following we derive necessary conditions for \( u_L(\tilde{\pi}^L) \geq u_L(\bar{\pi}^L) \) to hold. It will show useful to introduce \( \hat{p} \in [0, 1] \) as the \( p \) for which

\[
\hat{p}V_w + (1 - \hat{p})V_l - (1 + \lambda)k = 0. \tag{3.24}
\]

**Lemma 3.11**

For \( L \) to choose proposal \( \tilde{\pi}^L \), that is \( g(R(\pi^W)) = 1 \), it is necessary that \( Q_1 \) consists of at least \( \frac{\hat{p}}{1 + \lambda} \) project winners.

The proof of Lemma 3.11 is given in Appendix B. The intuition runs as follows: *Ceteris paribus*, \( \sigma_j(1) > \sigma_j(0) \) for project winners and \( \sigma_j(1) < \sigma_j(0) \) for project losers. Thus, the difference in aggregate subsidies (as outlined on the left-hand side of Equation 3.23) can be increased if \( Q_1 \) and \( Q_0 \) consist of many project winners. It can be shown that \( \hat{p} \in [0, 1] \) holds because \( V_w - (1 + \lambda)k > 0, V_l - (1 + \lambda)k < 0 \) and \( pV_w + (1 - p)V_l - (1 + \lambda)k \) is linear in \( p \).
\( \hat{p}_{1+\lambda} \) represents a threshold value, such that if less project winners are in coalition \( Q_1 \), Condition 3.23 cannot hold. In contrast, if the share of project winners in \( Q_1 \) exceeds the threshold value, it might hold. Since further conditions have to be met to make 3.23 hold, we derived necessary and not sufficient conditions for \( g(R(\pi^W)) = 1 \).

From Lemma 3.11, the following corollary results immediately

**Corollary 3.2**

For \( g(R(\pi^W)) = 1 \) to hold, it is necessary that \( p \geq \frac{\hat{p}}{1+\lambda} \).

**Proof**

Suppose to the contrary that \( p < \frac{\hat{p}}{1+\lambda} \). In this case it is not possible to have a minimal share of \( \frac{\hat{p}}{1+\lambda} \) project winners in the set \( Q_1 \), because there do not exist enough project winners.

\[ \square \]

### 3.5.2.3 Optimal Proposals \( \pi^W \)

Now we turn to the first stage and explore optimal proposal-making of \( \mathcal{W} \):

\[
\pi^w = \arg\max_{\pi^W \in \Pi_T} u_{\mathcal{W}}(\pi^W, R(\pi^W)) = g(R(\pi^W))(V_w - (1+\lambda)k) - (1+\lambda)S(R(\pi^W)). \quad (3.25)
\]

It is obvious that \( \mathcal{W} \) tries to induce project adoption (that is \( g(R(\pi^W)) = 1 \)) and reduce aggregate subsidies. In the following, we provide three examples for different parameter constellations. In particular we show that it is possible to find proposals \( \pi^W \) that induce \( g(R(\pi^W)) = 1 \) and \( S(R(\pi^W)) = 0 \).

First, suppose that \( p \geq \frac{1}{2} + \frac{\hat{p}}{1+\lambda} \): For the subsequent analysis it will be useful to understand that \( \hat{p} \leq (1+\lambda)(p - \frac{1}{2}) \) implies the following relations:

<table>
<thead>
<tr>
<th>a</th>
<th>( (1+\lambda)p &gt; (1+\lambda)(p - \frac{1}{2}) ) implies the following relations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>( p &gt; \frac{1}{2} ).</td>
</tr>
</tbody>
</table>

---

\( ^{21} \)The derivation of sufficient conditions would be very extensive, too. Because it would not provide much additional insight for the subsequent qualitative analysis, it is omitted.

\( ^{22} \)Under this condition, \( p > \frac{\hat{p}}{1+\lambda} \) obviously holds which is necessary to attain \( g(R(\pi^W)) = 1 \) (see Corollary 3.2).

\( ^{23} \)This inequality holds because: \( (1+\lambda)p > (1+\lambda)(p - \frac{1}{2}) \geq \hat{p} \).

\( ^{24} \)This inequality holds because: \( (1+\lambda)(p - \frac{1}{2}) \geq \hat{p} > 0 \).
Fact 3.1
Proposal $\pi^{*W}$ with
\[
\left\{ g(\pi^{*W}) = 1, (s_j(\pi^{*W}) = 0)_{j \in [0,1]} \right\},
\]
yields $g(\pi^{*L}) = 1$ and $S(\pi^{*L}) = 0$.

Proof
Proposal $\tilde{\pi}^L$ is given by
\[
g(\tilde{\pi}^L) = 0, s_j(\tilde{\pi}^L) = \begin{cases} 0 & \text{for } j \in [0, \frac{1}{2} \cup ]p, 1[ \\ \hat{s} & \text{for } j = 1, \end{cases}
\]
for $j \in \left[ \frac{1}{2}, p \right]$, $t(\tilde{\pi}^L) = (1 + \lambda)k$. \[25\]

Proposal $\tilde{\pi}^L$ is given by
\[
g(\tilde{\pi}^L) = 1, s_j(\tilde{\pi}^W) = \begin{cases} 0 & \text{for } j \in [0, 1[ \\ \hat{s} & \text{for } j = 1 \end{cases}
\]
for $j \in \left[ \frac{1}{2}, p \right]$, $t(\tilde{\pi}^L) = (1 + \lambda)k$.

It yields $u_L(\tilde{\pi}^L) = \hat{s} + (V_i - (1 + \lambda)k)$.

It is easy to verify that $u_L(\tilde{\pi}^L) \leq u_L(\tilde{\pi}^L)$ because $(1 + \lambda)(p - \frac{1}{2}) \geq \hat{p}$.

Since proposal $\tilde{\pi}^L$ yields a higher utility for $L$ than proposal $\tilde{\pi}^L$, the equilibrium proposal equals $\tilde{\pi}^L$.

$I(\pi^{*W}, \tilde{\pi}^L) = 1$ holds because all individuals vote for $\tilde{\pi}^L$ when put to a vote against $\pi^{*W}$.

Now, suppose that $p < \frac{1}{2} + \frac{\hat{p}}{1+\lambda}$, but still $\frac{\hat{p}}{1+\lambda} \leq p$, and additionally $1 - (1 + \lambda)\frac{1}{2} > \hat{p}$: Again, it will be useful to understand that inequalities (a) $1 - (1 + \lambda)\frac{1}{2} > \hat{p}$, (b) $\hat{p} > (1 + \lambda)(p - \frac{1}{2})$, and (c) $(1 + \lambda)p \geq \hat{p}$ imply the following relations:

(i) From (b): $(1 + \lambda)(\frac{1}{2} - p + \frac{\hat{p}}{1+\lambda}) > 0$.

(ii) From (a): $1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1+\lambda}) > 0$.

\[25\text{Subsidies are non-negative due to the fact that } 1 - (1 + \lambda)(\frac{1}{2} - p) > 1 - (1 + \lambda)\frac{1}{2} > 0 \text{ and smaller than } \hat{s} \text{ (for sufficiently high } \hat{s} \text{).} \]
(iii) From (a) and (b): $1 - (1 + \lambda)p > 0.$

(iv) $1 > \frac{1}{2} + \frac{\hat{p}}{1 + \lambda}.$ \textsuperscript{26}

(v) $p + 1 - \left(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}\right) \geq \frac{1}{2}.$ \textsuperscript{27}

**Fact 3.2**

Proposal $\pi^W$ with $g(\pi^W) = 1,$

$$s_j(\pi^W) = \begin{cases} 
\frac{(1+\lambda)(\frac{1}{2} - p + \frac{\hat{p}}{1 + \lambda})}{(1 - (1 + \lambda)\left(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}\right)}} & \text{for } j \in [0, p] \\
\frac{(1 - (1 + \lambda)p)[V_w - V_l]}{1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda})} & \text{for } j \in [\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}, 1] \\
0 & \text{for } j \in [1 + \frac{\hat{p}}{1 + \lambda}, 1]
\end{cases}$$

t(\pi^W) = \frac{(1+\lambda)(\frac{1}{2} - p + \frac{\hat{p}}{1 + \lambda})[V_w - V_l]}{1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda})}$

yields $g(\pi^L) = 1$ and $S(\pi^L) = 0.$ \textsuperscript{28}

**Proof**

Proposal $\tilde{\pi}^L$ is given by

$$g(\tilde{\pi}^L) = 0, s_j(\tilde{\pi}^L) = \begin{cases} 
0 & \text{for } j \in [0, \frac{1}{2} \bigcup \frac{1}{2} + \frac{\hat{p}}{1 + \lambda}, 1] \\
\frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda})} & \text{for } j \in [\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}, 1] \\
\hat{s} & \text{for } j = 1,
\end{cases}$$

t(\tilde{\pi}^L) = \frac{\hat{s}(V_w - (1 + \lambda)k)}{1 - \hat{p}}$.

We obtain $u_L(\tilde{\pi}^L) = \hat{s} - \frac{\hat{p}V_w - (1 + \lambda)k}{1 - \hat{p}}.$ \textsuperscript{29}

Proposal $\tilde{\pi}^L$ is as defined in the proof of Fact 3.1.

It is easy to verify that $u_L(\tilde{\pi}^L) = u_L(\tilde{\pi}^L)$.

Since proposal $\tilde{\pi}^L$ yields a utility for $L$ that is not lower than the one under proposal $\pi^L$, the equilibrium proposal equals $\tilde{\pi}^L$.

$I(\pi^W, \tilde{\pi}^L) = 1$ holds because project losers not subsidized under $\pi^W$ (that is $j \in [\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}, 1]$) and all project winners vote for $\tilde{\pi}^L$ when put to a vote against $\pi^W$.

□

An alternative proposal of $W$ could be the following:

\textsuperscript{26}This inequality holds because: $1 + \lambda > 1 > (1 + \lambda)(\frac{1}{2} - \frac{\hat{p}}{1 + \lambda}),$ see (ii).

\textsuperscript{27}This inequality is equal to (c).

\textsuperscript{28}Note that all subsidies are non-negative due to (i), (ii), and (iii).

\textsuperscript{29}Subsidies in $\tilde{\pi}^L$ are non-negative because $\hat{p} < 1$ and smaller than $\hat{s}$ (for sufficiently high $\hat{s}$).
Fact 3.3
Proposal $\pi^W$ with $g(\pi^W) = 0,$

$$s_j(\pi^W) = \begin{cases} \frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda})} - (1 + \lambda)\frac{k}{1 + \lambda} & \text{for } j \in \left[0, \frac{1}{2} + \frac{\hat{p}}{1 + \lambda}\right], \\ 0 & \text{for } j \in \left[\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}, 1\right] \end{cases} \quad t(\pi^W) = \frac{(1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda})(V_w - (1 + \lambda)k)}{1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda})}$$

yields $g(\pi^W) = 1$ and $S(\pi^W) = 0.$

The proof follows the same lines as the proof of Fact 3.2 and is therefore omitted.

If neither $p \geq \frac{1}{2} + \frac{\hat{p}}{1 + \lambda}$, nor $1 - (1 + \lambda)\frac{1}{2} > \hat{p} > (1 + \lambda)(p - \frac{1}{2})$ hold, further research is required to specify equilibrium outcomes.

However, if $(1 + \lambda)p < \hat{p}$, the project is never implemented in equilibrium (see Lemma 3.11 and Corollary 3.2) which means that $W$ has to accept $g(R(\pi^W)) = 0$. Thus, he only intends to minimize $S(R(\pi^W))$. It is obvious that there always exists a proposal $\pi^W$ that yields $S(R(\pi^W)) = 0$. For example, $W$ could suggest the status quo as his proposal $\pi^W$. In this case, the best response of $L$ comprises

$$g(R(\pi^W)) = 0, \quad s_j(R(\pi^W)) = \begin{cases} 0 & \text{for } j \in \left[0, 1\right[^1, \\ \hat{s} & \text{for } j = 1. \end{cases} \quad t(R(\pi^W)) = 0,$$

which yields $S(R(\pi^W)) = 0$. Proposition 3.6 summarizes these findings:

**Proposition 3.6**
If $\hat{p} \leq (1 + \lambda)(p - \frac{1}{2}),$ the equilibrium proposal $\pi^*\mathcal{L}$ comprises

$$g(\pi^*\mathcal{L}) = 1 \quad \text{and} \quad S(\pi^*\mathcal{L}) = 0.$$

If $1 - (1 + \lambda)\frac{1}{2} > \hat{p} > (1 + \lambda)(p - \frac{1}{2})$ and $(1 + \lambda)p \geq \hat{p},$ the equilibrium proposal $\pi^*\mathcal{L}$ comprises

$$g(\pi^*\mathcal{L}) = 1 \quad \text{and} \quad S(\pi^*\mathcal{L}) = 0.$$

If $p(1 + \lambda) < \hat{p},$ the equilibrium proposal $\pi^*\mathcal{L}$ comprises

$$g(\pi^*\mathcal{L}) = 0 \quad \text{and} \quad S(\pi^*\mathcal{L}) = 0.$$

Recall that Proposition 3.6 does not provide a complete characterization of equilibrium outcomes. In particular, results under the parameter constellation

$$\min\{1 - (1 + \lambda)\frac{1}{2}, (1 + \lambda)(p - \frac{1}{2})\} > \hat{p} \quad \text{and} \quad (1 + \lambda)p \geq \hat{p}$$

are not specified.

\[\text{Note that all subsidies are positive due to (ii) and } V_w - (1 + \lambda)k > 0.\]
The previous considerations made clear that $L$ has a strict incentive to reduce subsidization to a minimum in order to decrease taxes. This implies:

**Lemma 3.12**

*Constitution $\Pi_T$ satisfies GREP.*

Since $(1 + \lambda)p \geq \hat{p}$ has to hold for project implementation, we can conclude that all projects with $p < \frac{\hat{p}}{1+\lambda}$ are not implemented. Hence, if we set $\varepsilon = \min\left\{\frac{\hat{p}}{1+\lambda}, \frac{1}{2}\right\} - \delta$ $(\delta \to 0)$, all projects with $p \leq \varepsilon$ are not implemented. It immediately follows:

**Lemma 3.13**

*Constitution $\Pi_T$ satisfies PALP.*

Because we did not derive a proper characterization of equilibrium proposals, it remains unclear whether constitution $\Pi_T$ satisfies $RAPR$ or not.

### 3.5.3 Arbitrary Tax Code and Uniform Subsidy Scheme

Applying the fixed selection device, we obtain

**Proposition 3.7**

If $p > \frac{1}{2}$, the equilibrium proposal comprises

$$g(\pi^*L) = 1 \text{ and } S(\pi^*L) = \hat{s}.$$  

If $p \leq \frac{1}{2}$, the equilibrium proposal comprises

$$g(\pi^*L) = 0 \text{ and } S(\pi^*L) = \hat{s}.$$  

The proof of Proposition 3.7 is given in Appendix B. The intuition is similar as for Proposition 3.5: If $p > 1/2$, $W$ can distribute subsidies and taxes in a way that $u_j(\pi^W) > \hat{s}$ for more than half of the society. This forces $L$ to propose $g(\pi^L) = 1$ in equilibrium. In contrast, if $p < 1/2$, $L$ cannot be induced to choose $g(\pi^L) = 1$.

However, because of the uniformity-rule on subsidies, the total amount of subsidies is unambiguously given as $\hat{s}$.

With respect to redistribution-efficiency, we obtain the following result:

**Lemma 3.14**

*Constitution $\Pi_S$ does not satisfy GREP.*
Proof
Suppose that \( p < \frac{1}{2} \). Then, \( \mathcal{W} \) knows that it is not possible to implement the project. So, a possible first proposal \( \pi^W \) could involve
\[
g(\pi^W) = 0 \text{ and } s_j(\pi^W) = t_j(\pi^L) = 0 \quad \forall j \in [0, 1].
\]
In this case, \( \mathcal{L} \) could suggest a proposal with
\[
g(\pi^L) = 0 \text{ and } s(\pi^L) \leq \hat{s},
\]
which yields \( S(\pi^L) < \hat{s} \) and is still adopted if taxes are distributed adequately. Hence, the equilibrium proposal \( \pi^*\mathcal{L} \) outlined in the second part of Proposition 3.7 (case \( p \leq \frac{1}{2} \)) is not redistribution-efficient.

\( \square \)

Similar to constitution \( \Pi \), the project is only implemented if \( p > \frac{1}{2} \). This yields the following results:

**Lemma 3.15**
Constitution \( \Pi_S \) satisfies PALP.

**Lemma 3.16**
Constitution \( \Pi_S \) satisfies RAPR.

3.5.4 Uniform Tax Code and Uniform Subsidy Scheme

Finally, we show that results under constitution \( \Pi_{ST} \) are not affected by the selection device:

**Proposition 3.8**
If \( p > \frac{1}{2} \), the equilibrium proposal comprises
\[
g(\pi^*\mathcal{L}) = 1 \text{ and } S(\pi^*\mathcal{L}) = 0.
\]
If \( p \leq \frac{1}{2} \), the equilibrium proposal comprises
\[
g(\pi^*\mathcal{L}) = 0 \text{ and } S(\pi^*\mathcal{L}) = 0.
\]
Chapter 3. Constitutional Design: Separation of Financing and Project Decision Revisited

The proof of Proposition 3.8 is already given in Appendix B. Since Assumption 3.1 does not impact on constitution $\Pi_{ST}$, results are similar as in Section 3.4.

Because the optimal counter-proposal by $L$ always comprises $S(\pi^L) = 0$, it is obvious that:

**Lemma 3.17**
Constitution $\Pi_{ST}$ satisfies $GREP$.

Again, because the project is only implemented if $p > \frac{1}{2}$ we obtain:

**Lemma 3.18**
Constitution $\Pi_{ST}$ satisfies $PALP$.

**Lemma 3.19**
Constitution $\Pi_{ST}$ satisfies $RAPR$.

### 3.6 Welfare Comparison

In this section we compare social welfare. The utilitarian welfare measure for a particular proposal $\pi$ amounts to

\[
W(\pi) := g(\pi)(pV_w + (1 - p)V_l - (1 + \lambda)k) - \lambda S(\pi). \tag{3.26}
\]

Constitutions $\Pi$, $\Pi_S$, and $\Pi_{ST}$ can be ranked according to social welfare because under all these constitutions the project is implemented if and only if $p > \frac{1}{2}$. For notational convenience, we denote an equilibrium proposal from constitution $\Pi$ as $\pi^*\ell$, from constitution $\Pi_T$ as $\pi_T^*\ell$, from constitution $\Pi_S$ as $\pi_S^*\ell$, and from constitution $\Pi_{ST}$ as $\pi_{ST}^*\ell$.

Propositions 3.5, 3.7, and 3.8 imply:

If $p \leq \frac{1}{2}$

\[
W(\pi_{ST}^*\ell) = -\lambda S(\pi_{ST}^*\ell), \quad \text{where } S(\pi_{ST}^*\ell) \in [0, \hat{s}]
\]

\[
W(\pi_T^*\ell) = -\lambda \hat{s}
\]

\[
W(\pi_S^*\ell) = 0.
\]

It is straightforward that

\[
W(\pi_{ST}^*\ell) \geq W(\pi_T^*\ell) \geq W(\pi_S^*\ell), \tag{3.27}
\]
If \( p \geq \frac{1}{2} \)

\[
W(\pi^*_{\text{CE}}) = pV_w + (1 - p)V_l - (1 + \lambda)k - \lambda S(\pi^*_{\text{CE}}), \quad \text{where } S(\pi^*_{\text{CE}}) \in [0, \hat{s}]
\]

\[
W(\pi^*_{\text{SE}}) = pV_w + (1 - p)V_l - (1 + \lambda)k
\]

\[
W(\pi^*_{\text{SE}T}) = pV_w + (1 - p)V_l - (1 + \lambda)k.
\]

It is straightforward that

\[
W(\pi^*_{\text{SE}T}) > W(\pi^*_{\text{CE}}) \geq W(\pi^*_{\text{SE}}),
\]  

(3.28)

From Equations 3.27 and 3.28, it follows immediately that constitution \( \Pi_{\text{ST}} \) never yields lower social welfare than constitution \( \Pi \), which in turn, never yields lower welfare than constitution \( \Pi_S \). Hence, constitutions \( \Pi \) and \( \Pi_S \) are inferior to constitution \( \Pi_{\text{ST}} \) and therefore never represent the socially optimal constitutions. Consequently, neither \( \Pi \) nor \( \Pi_S \) would be adopted behind a veil of ignorance.

However, a general welfare ranking of the two constitutions \( \Pi_T \) and \( \Pi_{\text{ST}} \) cannot be established because different constellations of exogenous parameters favor different constitutions.

To be more precise: On the one hand it might be possible that a socially inefficient project is adopted under constitution \( \Pi_T \) but not under constitution \( \Pi_{\text{ST}} \).\(^{31}\) In this case, constitution \( \Pi_{\text{ST}} \) would be preferred over constitution \( \Pi_T \) from a welfare perspective.

On the other hand, it might be possible that a socially efficient project is adopted under constitution \( \Pi_T \) but not under constitution \( \Pi_{\text{ST}} \).\(^{32}\) Obviously, constitution \( \Pi_T \) would be socially preferable over constitution \( \Pi_{\text{ST}} \) in this case.

Hence, a general welfare ranking which holds for all admissible values of exogenous variables cannot be established.

---

\(^{31}\) Such a project satisfies:

(i) Inefficiency: \( p < \hat{p} \)

(ii) Non-implementation under \( \Pi_{\text{ST}} \): \( p < \frac{1}{2} \)

(iii) Implementation under \( \Pi_T \): \( (1 + \lambda)p \geq \hat{p} \) and \( 1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}) \).

These equations hold for example if \( p = 0.1, \lambda = 0.5, \) and \( \hat{p} = 0.12 \).

\(^{32}\) Such a project satisfies:

(i) Efficiency: \( p > \hat{p} \)

(ii) Non-implementation under \( \Pi_{\text{ST}} \): \( p < \frac{1}{2} \)

(iii) Implementation under \( \Pi_T \): \( (1 + \lambda)p \geq \hat{p} \) and \( 1 - (1 + \lambda)(\frac{1}{2} + \frac{\hat{p}}{1 + \lambda}) \).

These equations hold for example if \( p = 0.4, \lambda = 0.5, \) and \( \hat{p} = 0.1 \).
3.7 Endogenous Project Characteristics

In Chapter 2 we show that tax rules exhibit additional desirable properties if the agenda-setter is able to enhance the project parameters $V_w$, $V_l$, $k$ and $p$. The intuition was as follows: If tax rules are absent, the agenda-setter can always implement the project and choose zero taxes for himself. Hence, there is no incentive for the agenda-setter to exert effort in order to enhance the project parameters $V_l$, $k$ and $p$. However, if tax rules are present, the agenda-setter might want to exert effort and enhance project efficiency in order to decrease taxes, or to enable project implementation.

Now, we want to investigate if similar results apply in the model at hand.

First of all, it is plausible that only $W$ can influence the project parameters because he is the one who suggests the proposal about which the society takes a vote.\textsuperscript{33}

Following Chapter 2, we assume that $W$ can exert effort before he makes a proposal. This effort creates small costs $c > 0$ for him. We distinguish three different scenarios:

1. Improvement of the project for project losers:

\[
V_l = \begin{cases} 
V_l & \text{if the agenda-setter does not exert effort} \\
\frac{V_l}{V_l > V_l} & \text{if the agenda-setter exerts effort.}
\end{cases} 
\] (3.29)

2. Increase of the fraction of project winners:

\[
p = \begin{cases} 
p & \text{if the agenda-setter does not exert effort} \\
\frac{p}{p} & \text{if the agenda-setter exerts effort.}
\end{cases} 
\] (3.30)

3. Reduction of the project’s costs:

\[
k = \begin{cases} 
k & \text{if the agenda-setter does not exert effort} \\
\frac{k}{k} & \text{if the agenda-setter exerts effort.}
\end{cases} 
\] (3.31)

\textsuperscript{33}If $L$ could also alter the project characteristics, individuals had to vote among two different public projects: the one in $\pi^W$ and the one in $\pi^L$. This would constitute a different modeling approach we do not consider. However, the investigation of such possibilities could be an interesting topic for further research.
Chapter 3. Constitutional Design: Separation of Financing and Project Decision Revisited

In Appendix B we show:

**Proposition 3.9**

1. Under constitutions $\Pi$ and $\Pi_S$, $W$ may enhance the project by increasing $p$. He will never improve $k$ or $V_l$.

2. Under constitution $\Pi_{ST}$, $W$ may enhance the project by increasing $p$ and by decreasing project costs $k$. He will never improve $V_l$.

3. Under constitution $\Pi_T$ the agenda-setter may enhance the project under all scenarios.

The results are similar to those in Chapter 2 insofar as tax rules still yield higher incentives for an agenda-setter to improve the project characteristics. An important difference, however, is that $W$ might have an incentive to improve the project under all constitutions. Recall that in the previous chapter, the agenda-setter has no incentive to enhance project efficiency in the two constitutions $\Pi$ and $\Pi_S$.

The reason for this difference lies in the higher degree of competition which allows project implementation only if $p$ is sufficiently high. Hence, in order to get the project implemented, $W$ may have to increase $p$.

For constitution $\Pi_{ST}$, the intuition is completely similar to the intuition in Chapter 2. The agenda-setter would never facilitate an increase in $V_l$ as this neither increases the likelihood of project adoption, nor does it reduce taxes for him. In contrast, an increase in $p$ enhances the probability of project adoption and a decrease in the project costs $k$ lowers uniform taxes.

Under constitution $\Pi_T$, $W$ might profit from exerting effort in all three scenarios (similar to Chapter 2). The reason for this behavior is that more efficient projects are more likely to be adopted. Moreover, $W$ also benefits from lower project costs $k$ in case of project implementation.

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34Recall that $\hat{p}$ decreases (and hence makes project adoption more likely under constitution $\Pi_T$) with higher $V_l$ and/or $p$ and with lower project costs $k$.
3.8 Conclusions

In this chapter we have reexamined the four constitutions of Chapter 2 in a more competitive setting. We have shown that in constitutions allowing for discretion on either taxes, subsidies, or both, equilibrium outcomes heavily depend on how agenda-setter \( L \) selects among a continuum of optimal counter-proposals.

By contrast, we found that outcomes in a constitution stipulating rules on taxes and subsidies are not affected by the selection device. Moreover, we have shown that outcomes are similar as in Chapter 2, which implies that this constitution is robust against more intense political competition.

For all other constitutions, we focused on two specific devices: \textit{Ex-ante efficient} and \textit{fixed}:

Under \textit{ex-ante efficient}, agenda-setter \( L \) is forward-looking and rewards good (punishes bad) behavior of agenda-setter \( W \) in the first stage. Using this kind of selection device, agenda-setter \( W \) is deterred from making proposals that yield project adoption in equilibrium. As a consequence, the project is never adopted under constitutions that allow for flexibility regarding the distribution of subsidies and/or taxes. Since permanent prevention of public projects is undesirable for a society, it is either reasonable to impose the very restrictive constitution \( \Pi_{ST} \), or an alternative selection device to \( L \).

Under \textit{fixed}, agenda-setter \( L \) chooses a fixed amount of subsidies and taxes for \( W \), irrespective of proposal \( \pi^W \). Applying such a device helps to overcome the problem of permanent project prevention. We have shown that tax rules exhibit similar advantageous properties as in Chapter 2: They lead to redistribution efficient proposals, motivate the agenda-setter to enhance project efficiency, and (in combination with subsidy rules) display the desirable feature of robustness against counter-proposals. Moreover, we were able to show that the welfare-maximizing constitution always comprises tax rules.

Finally, we found that more intense political competition (in combination with an appropriate selection device, such as \textit{fixed}) definitely exhibits desirable properties under constitutions \( \Pi, \Pi_T, \) and \( \Pi_S^{35} \).

Constitutions \( \Pi \) and \( \Pi_S \) now display the desirable features of \textit{protection against lobby projects}, and \textit{robustness against project reversal}, which was not the case in Chapter 2.

\footnote{Recall that \( \Pi_{ST} \) is unaffected.}
Under constitution $\Pi_T$, a project might now be implemented without detrimental subsidization. Moreover, the necessary condition for project implementation $((1+\lambda)p \geq \hat{p})$ exhibits an interesting property: If $\lambda$ is small, this condition approaches $p \geq \hat{p}$, which is equivalent to the definition of a socially desirable project.\footnote{Recall that $\hat{p}$ is defined as the minimal $p$ for which the public project is socially desirable, that is $\hat{p}V_w + (1 - \hat{p})V_l - (1 + \lambda)k = 0.$} Hence, if $\lambda \to 0$, project adoption requires an efficient project. However, efficiency is not sufficient for project implementation, that is there might exist efficient projects that are not adopted in equilibrium. By contrast, any adopted project is efficient.
Part II

Impact of Parties
Chapter 4

Parties as Fairness and/or Commitment Devices

4.1 Introduction

“Political parties are a hallmark of democracies.”¹

Nowadays, it is a commonplace of political economy that institutions heavily impact on political outcome, or to put it simply, that institutions matter (see for example MOE AND CALDWELL (1994), or LEVY (2004)). In modern democracy, the most important institutions are political parties. But what exactly is a party and what functions does it fulfill?

In political science, there is a broad consensus that political parties are more than ordinary coalitions of like-minded persons who cooperate to achieve common goals. Rather, they are seen as cohesive groups that have adopted party-internal rules, norms and procedures (see ALDRICH (1995)) and generated symbols of identification or loyalty (see CHAMBERS (1967)). KREHBIEL (1993) formalized these ideas and stated that parties should be interpreted as a phenomenon separate from simple rational behavior of individuals with similar preferences. He argues that one cannot draw conclusions about the impact of parties simply from observing that like-minded individuals cooperate, because they would be expected to behave like this anyway. Thus, the significance of parties must go beyond simple cooperation. This argument has an important implication: In any party model,

¹LEVY (2004)
it must be possible to distinguish between equilibrium outcomes absent of parties and equilibrium outcomes with parties. We will examine such differences in Chapter 5.

Despite this widespread insight, a proper investigation of party roles - that is its internal rules, norms and procedures - has not been pursued thoroughly. However, it seems plausible that such roles influence proposal-making behavior of a party, which in turn impact on the outcome in a democracy. This is the theme of Chapter 4 of the dissertation.

We apply the basic model of Chapter 2, where a society of risk-neutral individuals decides by majority voting whether to adopt or reject a proposal made by a specific individual. In contrast to Chapter 2, we incorporate political parties into this model. We adopt the view that a party is a coalition of like-minded voters trying to enforce common interests. In addition, the parties in our model are more than ordinary coalitions: we assume that they can play the following three roles:

(i) Fairness device: Under this role, members of a party have to be treated equally under a proposal made by this very party.

(ii) Commitment device: Under this role, members of a party have to bend to a certain party position and vote uniformly.

(iii) Fairness and commitment device: The combination of the first two roles.

4.2 Relation to the Literature

There are no other studies inquiring the impacts of fairness or commitment on political outcome. However, there is a huge body of literature dealing with the question how a group of agents shares the benefits of a coalition (see the classical contribution of Nash (NASH (1950)) and its characterization by Rubinstein (RUBINSTEIN (1982)), as well as the approach of Lloyd Shapley (SHAPLEY (1953))).

We ask how a specific party rule impacts on political outcomes.

There is also an extensive body of literature dealing with behavioral aspects of economic decision-making. In this strand of the literature, fairness or ambiguity aversion plays

\[ ^2 \text{Alternatively, parties could be modeled as coalitions of elites, seeking to win elections and control the governing apparatus (see DOWNS (1957)).} \]

\[ ^3 \text{An explicit definition of these two party devices is given in Section 4.3.3.} \]
an important part (see Camerer (2003) for a detailed overview). While we do not establish an explicit behavioral model of fairness, insights from this literature can be used to justify that people might care about preferences of other party members.\footnote{Calvert and Dietz (2005) present a model where legislators have preferences over the distributive allocations to others as well.}

There exists a well developed strand of literature dealing with party commitment: In recent years, the focus of economic coalition theory has shifted to the question whether economic agents have an incentive to form a coalition or not (see Carraro (2003)). The main assumption in this body of literature is that there exists a trade-off between the potential gains of cooperative group action and the sacrifice of individual freedom involved in group formation (see Eguia (2007)). In political economy models, the sacrifice of individual freedom is assumed to be caused by party commitment, that is, members of a party have to obligate themselves to a certain party position (see Dhillon (2003) for a recent survey).

To our knowledge, the existing literature on party commitment has mainly focused on party formation.\footnote{Levy (2004) provides a comprehensive overview of the literature on endogenous party formation.} However, once formed, parties stay together for a certain time\footnote{Chambers, for instance, defined a party as “[…] as a relatively durable social formation […]” (see Chambers (1967)).} and thus, commitment does not only affect party formation, but also impacts on this party’s behavior during a legislation. This behavior is the focus of this chapter.

\section{Set-up}

\subsection{General Set-up}

We use the basic structure of the model outlined in Chapter 2:

Individuals (indexed by \( j \)) are uniformly arranged on the interval \([0, 1]\). The provision of a public project yields utility \( v_j \in \{V_w, V_l\} \) for all \( j \in [0, 1] \) and involves per-capita costs \( k \geq 0 \).

We consider a constitution that imposes rules on taxes. For simplicity and in accordance with Chapters 2 and 3, we assume the tax rule to be the uniform one. Moreover, we suppose taxation to be distortionary. Let \( \lambda \) represent the shadow costs of public funds. This implies that to finance the public-project costs \( k \), a per-capita tax of
(1 + \lambda)k\) has to be levied. We assume that \(V_w - (1 + \lambda)k > 0\) and \(V_l - (1 + \lambda)k < 0\). Accordingly, we refer to individuals obtaining \(V_w\) from public project provision as project winners, and individuals obtaining \(V_l\) as project losers. Without loss of generality, we assume that project winners are located on the interval \([0, p]\) and project losers on \([p, 1]\).

There is one particular individual who can propose a project/financing package \(\pi\) (also called a proposal). The suggested proposal is put to a general vote and adopted if at least a fraction \(m\) of individuals support it. We will go into details of the voting stage later in the text.\footnote{Recall that this framework corresponds to the legal system of direct democracy (as practiced for example in California (USA) or Switzerland) or parliamentary democracy with perfect representation. Perfect representation means that the parliament is a perfect image of the society and representatives vote according to the preferences of their electorate.} We assume the proposal-making individual (also called agenda-setter) to be a project winner.\footnote{There are different ways of modeling which citizen has the right to set the agenda. We adopt the view that in a democracy it is impossible to deter beneficiaries of public projects from making a proposal (see Chapter 2). Another approach is random selection of agenda-setters (see Gersbach (2009a)).} Without loss of generality, we assume the agenda-setter to be located at \(j = 0\).

A project/financing package \(\pi\) comprises a project decision, denoted by the indicator variable \(g(\pi)\). It takes a value of 1 if the project is suggested and 0 otherwise. Moreover, \(\pi\) includes a subsidy scheme \(s(\pi)\). We consider a constitution that imposes no rules on subsidies, that is, \(s(\pi)\) can be chosen arbitrarily on the unit interval.

However, subsidies are constrained to be non-negative and not higher than \(\hat{s}\) (so that \(\hat{s}\) represents the maximal per-capita subsidy).\footnote{Throughout the chapter, we assume \(\hat{s} \geq \max \left\{ \frac{(1 + \lambda)k - V_l}{1 + \lambda - \frac{1}{1 + \lambda}} \cdot \frac{1 - m}{1 - m + (1 - n)p}, \frac{1 - m}{1 - m + (1 - n)p} \right\} \). This assumption simplifies the exposition, but does not affect our findings qualitatively.} Thus, the set of all feasible subsidy schemes, denoted by \(S\), comprises all non-negative Lebesgue-measurable functions on the unit interval that do not exceed \(\hat{s}\).

Since taxes are constrained to be uniform and the budget has to be balanced, per-capita taxes are automatically given by:

\[
t(\pi) = g(\pi)(1 + \lambda)k + (1 + \lambda)S(\pi),
\]

where we have introduced total subsidies \(S(\pi) := \int_0^1 s_j(\pi) dj\).

A remark regarding the budget constraint is in order. As the agenda-setter makes a proposal that simultaneously determines revenues and expenditures, we have an in-
Chapter 4. Parties as Fairness and/or Commitment Devices

stance of soft budget constraint. The agenda-setter has no incentive to make proposals for which revenues and expenditures do not match.\footnote{If planned expenditures exceed revenues, a proposal could be declared unconstitutional. Conversely, if there are excess revenues, they could be refunded as lump sum transfers to citizens. With such rules, the agenda-setter only makes proposals for which (4.1) holds.} Nevertheless, she may collect more taxes than required for public project financing in order to pay out subsidies.

In summary, a feasible proposal comprises \( g(\pi) \in \{0, 1\}, s(\pi) \in S \) and a uniform tax \( t(\pi) \) that balances the society’s budget. Formally, the set of feasible proposals is defined as

\[
\Pi := \{ \pi \in \{0, 1\} \times S \mid (1 + \lambda) [g(\pi)k + S(\pi)] = t(\pi) \}.
\]  \( (4.2) \)

The agenda-setter makes a proposal that must obey the constitutional rules, otherwise the proposal is declared unconstitutional and the status quo prevails. If a feasible proposal \( \pi \) is adopted, utility of individual \( j \in [0, 1] \) is given by\footnote{Throughout this chapter we assume the income of individual \( j \) to be sufficiently high to pay taxes under any proposal considered.}

\[
u_j(\pi) = g(\pi)v_j + s_j(\pi) - t(\pi),
\]  \( (4.3) \)

and zero otherwise.

Decisions are taken by an \( m \)-majority rule, that is a feasible proposal \( \pi \) is implemented if at least a fraction \( m \) of the society votes in favor of \( \pi \). Regarding \( m \), we make the following assumption:

**Assumption 4.1**

\( \frac{1}{2} \leq m < \min \{ \frac{1}{1 + \lambda} + p, 1 \} \).\footnote{This constitutes the same assumption as in Chapter 2. We will show that it will significantly simplify our analysis. As argued in Chapter 2, the assumption is always fulfilled for the simple majority (\( m = 1/2 \)) if we consider plausible estimates of \( \lambda \). Studies by Stuart (1984), Ballard, Shoven, and Whalley (1985), and Browning (1987) have shown that \( \lambda \) most likely lies in the interval \([0.2; 0.5]\).}

We assume that each individual votes in favor of \( \pi \) if and only if \( u_j(\pi) \geq 0 \).

For notational convenience, we use the indicator function \( I(\pi) \) defined as

\[
I(\pi) = \begin{cases} 
1 & \text{if } u_j(\pi) \geq 0 \text{ for at least a fraction } m \text{ of voters} \\ 
0 & \text{otherwise.}
\end{cases}
\]  \( (4.4) \)

to indicate if a proposal is adopted \( (I(\pi) = 1) \), or not \( (I(\pi) = 0) \).
Thus, the expected utility of individual $j$, given an arbitrary proposal $\pi$ has been made, can be written as $U_j(\pi) = I(\pi)u_j(\pi)$.

In order to obtain proper equilibria, the agenda-setter breaks ties as follows:

**Tie-breaking Rule 4.1**

If the agenda-setter is indifferent between providing and not providing the public project in case $I(\pi) = 1$, she chooses $g(\pi) = 1$.

**Tie-breaking Rule 4.2**

If the agenda-setter is indifferent between raising and not raising subsidies for herself in case $I(\pi) = 1$ (that is if $\frac{\partial u_0(\pi)}{\partial s_0} = 0$), she chooses $s_0 = 0$.

### 4.3.2 The Game

The game described in the previous section is a simple two-stage game where the proposal is suggested, followed by a vote on it. The sequence of events is summarized as follows:

Stage 1: The agenda-setter suggests a project/financing package $\pi = \langle g(\pi), s(\pi), t(\pi) \rangle$.

Stage 2: Citizens decide simultaneously whether to accept $\pi$ ($\delta_j(\pi) = 1$) or not ($\delta_j(\pi) = 0$). If at least a fraction $m$ of all citizens vote for the proposal, it is adopted. It is rejected otherwise.

### 4.3.3 Parties and Devices

Since we are interested in the impact of different party roles on political outcomes, we have to specify what a party is and what functions it should fulfill.

There are different ways to model the composition of a party. Following Feddersen (1993) and Baron (1993), we adopt the view that a party is a coalition of like-minded voters. In our model, it is therefore sensible to define two parties:

(i) Party $W$: It equals the set of all project winners, that is $j \in [0, p]$.

(ii) Party $L$: It equals the set of all project losers, that is $j \in [p, 1]$. 

80
Chapter 4. Parties as Fairness and/or Commitment Devices

Having specified the composition of a party, we now define its roles. Roles are simply a set of rules that constrain the set of proposals the agenda-setter can make, or that constrain individual voting behavior. Accordingly, Π represents a particular role, namely one without further rules (we already investigated optimal proposal-making in the absence of party roles in Chapter 2). In the present chapter, we will focus on the two roles Fairness Device and Commitment Device. These roles will represent subsets of Π.

(i) Fairness Device:

In our model, we consider a rather pronounced concept of fairness: Fairness device requires equal treatment of all members of a party.13

As outlined in Section 4.3.1, only Party W makes a proposal. Moreover, party members can only be treated differently with respect to subsidies. Hence, in our model, fairness device requires that subsidies given to project winners have to be uniform. More formally, we can state:

Definition 9

Party W acts as fairness device if the agenda-setter is obligated to choose a proposal $\pi$ that comprises $s_i(\pi) = s_j(\pi)$ for all $i, j \in [0, p]$.

(ii) Commitment Device:

The role of party commitment is well specified in the literature.14 “A party is modeled as a binding agreement among its members to act as one player […]. That is, they can commit each to follow the same single action […].” (JACKSON AND MOSELLE (2002)). Accordingly, we will state that a party acts as a commitment device if all members of the party are forced to vote uniformly.

An important question is how the voting behavior of the party is determined. To answer this question, we apply a commonly-used procedure: If a coalition intends to act as a “voting bloc”, that is, to cast all its votes together, an internal decision-making rule is used to determine the voting behavior of the whole party. We assume the internal rule to be an $n$-majority rule, that is, if at least a

---

13 Note that equal treatment in a coalition also occurs under the Nash-bargaining approach (if all members of the coalition exhibit equal bargaining power) and under the Shapley value (if marginal contributions of each member to the coalition are identical). However, assuming a less pronounced concept of fairness would not affect our results qualitatively.

14 See for example EGUIA (2007) and JACKSON AND MOSELLE (2002)
fraction $n$ of party members supports a proposal, the whole party votes in favor of this very proposal. To simplify the analysis, we make the following assumption regarding $n$:

**Assumption 4.2**

$$n < \min \left\{ \frac{1}{(1+\lambda)(1-p)}, 1 \right\}. \quad (15)$$

Commitment device can be interpreted as a two-step procedure. After a proposal $\pi$ was made, in a first step, each party determines by internal majority voting whether to accept or to reject it. At the general voting stage, that is in a second step, all party members vote according to the party decision reached in step one. Hence, party members exhibit party discipline in the sense that they bend to the party decision and may vote against their own interests.\(^\text{16}\)

In contrast to fairness device, commitment device does not impact on the subsidy scheme, but it affects the citizens’ voting behavior. More formally, we can state:

**Definition 10**

Party $W$ acts as commitment device if, for any proposal $\pi$, members of Party $W$ are obligated to choose $\delta_j(\pi) = \delta_W(\pi)$ ($j \in [0, p]$). Similarly, Party $L$ acts as commitment device if, for any proposal $\pi$, members of Party $L$ are obligated to choose $\delta_j(\pi) = \delta_L(\pi)$ ($j \in [p, 1]$).

$\delta_W(\pi)$ and $\delta_L(\pi)$ are determined by party-internal majority voting, where

$$\delta_W(\pi) = \begin{cases} 
1 & \text{if } u_j(\pi) \geq 0 \text{ for at least a fraction } n \text{ of individuals in } [0, p] \\
0 & \text{otherwise,}
\end{cases}$$

$$\delta_L(\pi) = \begin{cases} 
1 & \text{if } u_j(\pi) \geq 0 \text{ for at least a fraction } n \text{ of individuals in } [p, 1] \\
0 & \text{otherwise.}
\end{cases}$$

\(^{15}\)Assumption 4.2 is equivalent to the second part of Assumption 4.1. However, we do not restrict $n$ to be higher than $1/2$. This allows the consideration of a small committee deciding on a party’s position. Note that Assumption 4.2 holds for $n \leq 1/2$ for plausible estimates of $\lambda$ (such estimates are in the range of $[0.2; 0.5]$, see Stuart (1984), Ballard, Shoven, and Whalley (1985), and Browning (1987)).

\(^{16}\)Under the legal system of direct democracy, it might be difficult to sustain party discipline if parties are coalitions of voters, that is, it might be difficult to monitor the voting behavior of each member of the party. To avoid this problem, double majority rules could be used. Under such rules, a proposal is adopted if and only if at least a fraction of $n$ project winners and a fraction of $n$ project losers support it. For a general discussion on double majority rules, see Gersbach (2009a) for example.
(iii) Fairness and Commitment Device:

The last role of parties we will consider in this chapter is the combination of the
first two roles. It can be defined by linking Definitions 9 and 10.

4.4 Fairness Device

We start our analysis by deriving equilibrium proposals for parties as fairness device.
By Definition 9, parties acting as fairness device are characterized by uniform subsi-
didization of their own party members. Thus, the agenda-setter can choose a proposal
from her set of feasible proposals $\Pi_F \subset \Pi$, given by

$$\Pi_F := \{\pi_F \in \Pi \mid s_i(\pi_F) = s_j(\pi_F) \ \forall i, j \in [0, p]\}.$$ (4.5)

For notational convenience we define

$$V^*_w := (1 + \lambda) \frac{k - (m - p)V_i}{1 - (1 + \lambda)(m - p)}.$$ (4.6)

**Proposition 4.1**

Suppose $p < m$.

(i) For $1 - (1 + \lambda)m > 0$ and $V_w \geq V^*_w$, the equilibrium proposal is given by

$$\pi^{p<m}_{F1} := \langle g(\pi^{p<m}_{F1}) = 1, s(\pi^{p<m}_{F1}), t(\pi^{p<m}_{F1}) \rangle,$$

where

$$t(\pi^{p<m}_{F1}) = (1 + \lambda) \left[ k + p\hat{s} + (m - p) \frac{(1 + \lambda)p\hat{s} + ((1 + \lambda)k - V_i)}{1 - (1 + \lambda)(m - p)} \right]$$

and

$$s_j(\pi^{p<m}_{F1}) = \begin{cases} \hat{s} & \text{for } j \in [0, p] \\ \frac{(1 + \lambda)p\hat{s} + ((1 + \lambda)k - V_i)}{1 - (1 + \lambda)(m - p)} & \text{for } j \in [p, m] \\ 0 & \text{for } j \in [m, 1]. \end{cases}$$

(ii) For $1 - (1 + \lambda)m > 0$ and $V_w < V^*_w$, the equilibrium proposal is given by

$$\pi^{p<m}_{F2} := \langle g(\pi^{p<m}_{F2}) = 0, s(\pi^{p<m}_{F2}), t(\pi^{p<m}_{F2}) \rangle,$$
where
\[ t(\pi_{p<m}^{F_2}) = (1 + \lambda) \left[ p\hat{s} + (m - p) \frac{(1 + \lambda)p\hat{s}}{1 - (1 + \lambda)(m - p)} \right] \]
and
\[ s_j(\pi_{p<m}^{F_2}) = \begin{cases} 
\hat{s} & \text{for } j \in [0, p] \\
\frac{(1 + \lambda)p\hat{s}}{1 - (1 + \lambda)(m - p)} & \text{for } j \in [p, m] \\
0 & \text{for } j \in [m, 1].
\end{cases} \]

(iii) For \(1 - (1 + \lambda)m \leq 0\) and \(V_w \geq V_w^*\), the equilibrium proposal is given by
\[ \pi_{p<m}^{F_3} := \left\langle g(\pi_{p<m}^{F_3}) = 1, s(\pi_{p<m}^{F_3}), t(\pi_{p<m}^{F_3}) \right\rangle, \]
where
\[ t(\pi_{p<m}^{F_3}) = (1 + \lambda) \left[ k + (m - p) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)} \right] \]
and
\[ s_j(\pi_{p<m}^{F_3}) = \begin{cases} 
0 & \text{for } j \in [0, p] \\
\frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)} & \text{for } j \in [p, m] \\
0 & \text{for } j \in [m, 1].
\end{cases} \]

(iv) For \(1 - (1 + \lambda)m \leq 0\) and \(V_w < V_w^*\), the status quo prevails.

The proof of Proposition 4.1 is given in Appendix C. The intuition runs as follows: Since \(p < m\), project winners need the support of project losers to implement a proposal. This means that compensatory payments have to be paid if the project is suggested, and if subsidies are distributed to project winners. In the proof, we show that \(V_w^*\) represents a threshold value for \(V_w\), such that it is only profitable to suggest the project if \(V_w \geq V_w^*\). If \(V_w < V_w^*\), the amount of taxes required to finance the compensatory payments for project adoption outweighs the utility gain \(V_w\).

Similarly, we show that \(1/(1 + \lambda)\) represents a threshold value for \(m\), such that subsidization of project winners is only profitable if \(m < 1/(1 + \lambda)\). In case \(m \geq 1/(1 + \lambda)\), the taxes needed to finance compensatory payments outweigh the gains involved in self-subsidization of project winners.

**Proposition 4.2**

Suppose \(p \geq m\).

(i) For \(1 - (1 + \lambda)p > 0\), the equilibrium proposal is given by
\[ \pi_{p \geq m}^{F_1} := \left\langle g(\pi_{p \geq m}^{F_1}) = 1, s(\pi_{p \geq m}^{F_1}), t(\pi_{p \geq m}^{F_1}) \right\rangle, \]
where

\[ t \left( \pi_{F1}^{p \geq m} \right) = (1 + \lambda) [k + p\hat{s}] \]

and

\[ s_j(\pi_{F1}^{p \geq m}) = \begin{cases} 
\hat{s} & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, 1].
\end{cases} \]

(ii) For \( 1 - (1 + \lambda)p \leq 0 \), the equilibrium proposal is given by

\[ \pi_{F2}^{p \geq m} := \left( g(\pi_{F2}^{p \geq m}) = 1, s(\pi_{F2}^{p \geq m}), t(\pi_{F2}^{p \geq m}) \right), \]

where

\[ t(\pi_{F2}^{p \geq m}) = (1 + \lambda)k \]

and

\[ s_j(\pi_{F2}^{p \geq m}) = 0 \text{ for } j \in [0, 1]. \]

**Proof**

The optimal proposal with \( I(\pi_F) = 1 \) comprises:

1. \( s_j(\pi_F) = 0 \text{ for } j \in [p, 1] \) for two reasons:
   
   (i) Project winners are sufficiently numerous to enforce any proposal, and thus no support from project losers is needed.
   
   (ii) Higher subsidies for project losers cause higher uniform taxes, which in turn lowers the agenda-setter’s utility.

2. \( g(\pi_F) = 1 \), because net benefits from project implementation are strictly positive, that is \( V_w - (1 + \lambda)k > 0 \).

3. \( s_j(\pi_F) = \hat{s} \text{ for } j \in [0, p] \) if and only if the net benefits from self-subsidization are strictly positive, that is if \( s^W > (1 + \lambda)p\hat{s}^W \).

\[ \text{17} \]

\[ \text{Recall that the fairness criterion requires uniform subsidization of project winners. This implies that for any dollar of self-subsidization, uniform taxes of } (1 + \lambda)p \text{ arise.} \]
These considerations constitute proposals $\pi_{F1}^{p \geq m}$ and $\pi_{F2}^{p \geq m}$. Note first that both proposals are adopted in equilibrium, since all project winners support them. Second, both proposals are feasible because $s(\cdot) \in S$, $g(\cdot) \in \{0, 1\}$, and the budget is balanced:

$$\left(1 + \lambda\right)k + \left(1 + \lambda\right)ps^{W} = \left(1 + \lambda\right)\left[k + ps^{W}\right],$$

where $s^{W} \in \{0, \hat{s}\}$.

Finally, it is obvious that the agenda-setter is strictly better off with each of these proposals than with any proposal that yields $I(\pi_F) = 0$. Thus, $\pi_{F1}^{p \geq m}$ and $\pi_{F2}^{p \geq m}$ are suggested by the agenda-setter.

\[\square\]

4.5 Commitment Device

Now, we turn to the second role of parties considered in this chapter. Recall that according to Definition 10, commitment device impacts on individual voting behavior, but imposes no further restrictions on the subsidy scheme. Thus, the set of feasible proposals is given by

$$\Pi_C := \Pi.$$  \hspace{1cm} (4.7)

For notational convenience, we define

$$V_{w^*} := (1 + \lambda)\frac{k - n(1 - p)V_i}{1 - (1 + \lambda)n(1 - p)}.$$  \hspace{1cm} (4.8)

Proposition 4.3

Suppose $p < m$ and $1 - p \geq m$.

(i) For $V_w \geq V_{w^*}$, the equilibrium proposal is given by

$$\pi_{C1}^{p < m} := \left(g(\pi_{C1}^{p < m}) = 1, s(\pi_{C1}^{p < m}), t(\pi_{C1}^{p < m})\right),$$

where

$$t(\pi_{C1}^{p < m}) = (1 + \lambda) \left[\frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)}\right]$$

86
and

\[ s_j(\pi^{p\leq m}_{C1}) = \begin{cases} 
\hat{s} & \text{for } j = 0 \\
0 & \text{for } j \in [0, p] \\
\frac{(1+\lambda)k-V_l}{1-(1+\lambda)n(1-p)} & \text{for } j \in [p, p+n(1-p)] \\
0 & \text{for } j \in [p+n(1-p), 1]. 
\end{cases} \]

(ii) For \( V_w < V_w^{**} \), the equilibrium proposal is given by

\[ \pi^{p\leq m}_{C2} = (g(\pi^{p\leq m}_{C2}) = 0, s(\pi^{p\leq m}_{C2}), t(\pi^{p\leq m}_{C2})) , \]

where

\[ t(\pi^{p\leq m}_{C2}) = 0 \]

and

\[ s_j(\pi^{p\leq m}_{C2}) = \begin{cases} 
\hat{s} & \text{for } j = 0 \\
0 & \text{for } j \in [0, 1]. 
\end{cases} \]

The proof of Proposition 4.3 is given in Appendix C. According to Proposition 4.3, the agenda-setter will always choose the maximum level of subsidies for herself, which is plausible in the absence of fairness. Since \( p < m \), the support of project losers is needed to implement the project. However, project losers only accept a proposal with \( g(\pi_C) = 1 \) if they are sufficiently compensated for incurred utility losses. In the proof, we show that \( V_w^{***} \) represents a threshold value of \( V_w \), such that it is only profitable to suggest the project if \( V_w \geq V_w^{***} \). Otherwise, the amount of taxes required to finance compensatory payments outweighs the utility gain \( V_w \).

Note that only the voting behavior of Party \( L \) decides on approval or rejection of the proposal (since \( p < m \) and \( 1-p \geq m \)). Thus, paying subsidies to project winners is not profitable for the agenda-setter at all.

However, if \( p < m \) and \( 1-p < m \), we obtain

**Proposition 4.4**

Suppose \( p < m \) and \( 1-p < m \).

(i) For \( V_w^{**} \geq V_w \), the equilibrium proposal is given by \( \pi^{p\leq m}_{C1} \).

(ii) For \( V_w^{**} < V_w \), the equilibrium proposal is given by \( \pi^{p\leq m}_{C2} \).
The proof of Proposition 4.4 is given in Appendix C. The results from Proposition 4.4 are equivalent to the results from Proposition 4.3. Nevertheless, the line of reasoning is different:

In the former case where \( p < m \) and \( 1 - p \geq m \), the agenda-setter did not need the support of Party \( W \). Consequently, she did not pay any subsidies to project winners. By contrast, in the case where \( p < m \) and \( 1 - p < m \), the agenda-setter needs the support of both parties to implement a proposal. Thus, she might have to pay subsidies to project winners in order to gain their support. However, we show in the proof that the agenda-setter chooses project adoption if and only if subsidies to project winners can be set to zero. Thus, the results are similar to the ones in Proposition 4.3.

**Proposition 4.5**

Suppose \( p \geq m \). Then the equilibrium proposal is given by

\[
\pi_{C}^{p \geq m} := \left(g(\pi_{C}^{p \geq m}) = 1, s(\pi_{C}^{p \geq m}), t(\pi_{C}^{p \geq m})\right),
\]

where

\[
t(\pi_{C}^{p \geq m}) = (1 + \lambda)k
\]

and

\[
s_j(\pi_{C}^{p \geq m}) = \begin{cases} 
\hat{s} & \text{for } j = 0 \\
0 & \text{for } j \in ]0, 1].
\end{cases}
\]

**Proof**

The optimal proposal with \( I(\pi_C) = 1 \) comprises:

1. \( s_j(\pi_C) = 0 \) for \( j \in [p, 1] \) and \( g(\pi_C) = 1 \) for the same reasons as in Proposition 4.2 (see the proof of Proposition 4.2, items 1. and 2.).

2. \( s_j(\pi_C) = 0 \) for \( j \in [0, p] \), because higher subsidies cause higher uniform taxes, which lowers the agenda-setter’s utility.

3. \( s_0 = \hat{s} \), since it is always optimal for the agenda-setter to collect the maximal amount of subsidies for herself.

This constitutes proposal \( \pi_{C}^{p \geq m} \). Note first that this proposal is always adopted in equilibrium because all project winners support it. Second, the proposal is feasible (that is, \( g(\pi_{C}^{p \geq m}) \in \{0, 1\} \), \( s(\pi_{C}^{p \geq m}) \in \mathbb{S} \), and the budget is balanced).
Finally, it is obvious that the agenda-setter is better off with such a proposal than with any other proposal that yields \( I(\pi_C) = 0 \). Thus, \( \pi_{C}^{p \geq m} \) is suggested by the agenda-setter.

\[
\square
\]

### 4.6 Fairness and Commitment Device

The last role of parties we want to consider in this chapter is the combination of the first two roles. Since commitment device imposes no further restrictions on the subsidy scheme, the set of feasible proposals is only affected by the fairness criterion. Thus, the agenda-setter can choose a proposal from the set \( \Pi_{FC} \) given by

\[
\Pi_{FC} := \Pi_F. \tag{4.9}
\]

For notational convenience, we will reuse the definition of \( V_{w}^{**} \) (see Section 4.5).

**Proposition 4.6**

Suppose \( p < m \) and \( 1 - p \geq m \).

(i) For \( 1 - (1 + \lambda) [n + (1 - n)p] > 0 \) and \( V_{w} \geq V_{w}^{**} \), the equilibrium proposal is given by

\[
\pi_{FC1}^{p \leq m} := \left\langle g(\pi_{FC1}^{p \leq m}) = 1, s(\pi_{FC1}^{p \leq m}), t(\pi_{FC1}^{p \leq m}) \right\rangle,
\]

where

\[
t(\pi_{FC1}^{p \leq m}) = (1 + \lambda) \left[ k + p \hat{s} + n(1 - p) \frac{(1 + \lambda)p \hat{s} + ((1 + \lambda)k - V_l)}{1 - (1 + \lambda)n(1 - p)} \right]
\]

and

\[
s_j(\pi_{FC1}^{p \leq m}) = \begin{cases} 
\hat{s} & \text{for } j \in [0, p] \\
\frac{(1 + \lambda)p \hat{s} + ((1 + \lambda)k - V_l)}{1 - (1 + \lambda)n(1 - p)} & \text{for } j \in [p, p + n(1 - p)] \\
0 & \text{for } j \in [p + n(1 - p), 1].
\end{cases}
\]

(ii) For \( 1 - (1 + \lambda) [n + (1 - n)p] > 0 \) and \( V_{w} < V_{w}^{**} \), the equilibrium proposal is given by

\[
\pi_{FC2}^{p \leq m} := \left\langle g(\pi_{FC2}^{p \leq m}) = 0, s(\pi_{FC2}^{p \leq m}), t(\pi_{FC2}^{p \leq m}) \right\rangle,
\]

89
where
\[ t(\pi_{FC2}^{<m}) = (1 + \lambda) \left[ p\hat{s} + n(1 - p) \frac{(1 + \lambda)p\hat{s}}{1 - (1 + \lambda)n(1 - p)} \right] \]

and
\[
s_j(\pi_{FC2}^{<m}) = \begin{cases} \hat{s} & \text{for } j \in [0, p] \\ \frac{(1 + \lambda)p\hat{s}}{1 - (1 + \lambda)n(1 - p)} & \text{for } j \in [p, p + n(1 - p)] \\ 0 & \text{for } j \in [p + n(1 - p), 1] \end{cases}.
\]

(iii) For \(1 - (1 + \lambda)[n + (1 - n)p] \leq 0\) and \(V_w \geq V_w^{**}\), the equilibrium proposal is given by
\[ \pi_{FC3}^{<m} : = \left\langle g(\pi_{FC3}^{<m}) = 1, s(\pi_{FC3}^{<m}), t(\pi_{FC3}^{<m}) \right\rangle, \]

where
\[ t(\pi_{FC3}^{<m}) = (1 + \lambda) \left[ k + n(1 - p) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)} \right] \]

and
\[
s_j(\pi_{FC3}^{<m}) = \begin{cases} 0 & \text{for } j \in [0, p] \\ \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)} & \text{for } j \in [p, p + n(1 - p)] \\ 0 & \text{for } j \in [p + n(1 - p), 1] \end{cases}.
\]

(iv) For \(1 - (1 + \lambda)[n + (1 - n)p] \leq 0\) and \(V_w < V_w^{**}\), the status quo prevails.

The proof of Proposition 4.6 is given in Appendix C. The intuition is similar as for Proposition 4.1: Since the support of project losers is required for the adoption of a proposal, compensatory payments have to be paid. In the proof, we show that \(V_w^{**}\) and \((1 - (1 + \lambda)p)/(1 + \lambda)(1 - p)\) represent threshold values for \(V_w\) and \(n\), respectively, that determine project adoption and self-subsidization of project winners.

Since \(p < m\) and \(1 - p \geq m\), only the voting behavior of Party \(L\) is decisive for proposal adoption or rejection. Nevertheless, the agenda-setter pays subsidies to project winners because of party fairness.

In the other case, where \(p < m\) and \(1 - p < m\), we obtain

**Corollary 4.1**

*Suppose \(p < m\) and \(1 - p < m\).*

(i) For \(1 - (1 + \lambda)[n + (1 - n)p] > 0\) and \(V_w \geq V_w^{**}\), the equilibrium proposal is given by \(\pi_{FC1}^{<m}\).
(ii) For \( 1 - (1 + \lambda) [n + (1 - n)p] > 0 \) and \( V_w < V_w^{**} \), the equilibrium proposal is given by \( \pi_{FC2}^{p < m} \).

(iii) For \( 1 - (1 + \lambda) [n + (1 - n)p] \leq 0 \) and \( V_w \geq V_w^{**} \), the equilibrium proposal is given by \( \pi_{FC3}^{p < m} \).

(iv) For \( 1 - (1 + \lambda) [n + (1 - n)p] \leq 0 \) and \( V_w < V_w^{**} \), the status quo prevails.

Proof
From the proof of Proposition 4.6, we know that proposals \( \pi_{FC1}^{p < m} - \pi_{FC3}^{p < m} \) yield utility higher than zero for members of Party \( W \). Hence, an internal majority vote on \( \pi_{FC1}^{p < m} \), \( \pi_{FC2}^{p < m} \), or \( \pi_{FC3}^{p < m} \) would ensure unanimous support from Party \( W \) for any of these proposals. Of course, also Party \( L \) still supports these proposals because a fraction of \( n \) party members is compensated, so that they vote for the proposals. Since both parties accept \( \pi_{FC1}^{p < m} \), \( \pi_{FC2}^{p < m} \) and \( \pi_{FC3}^{p < m} \), adoption is guaranteed.

Since \( \pi_{FC1}^{p < m} \), \( \pi_{FC2}^{p < m} \) and \( \pi_{FC3}^{p < m} \) are chosen optimally given the parameter constellation, there are no alternative proposals that yield higher utility for the members of Party \( W \).

Finally, if \( 1 - (1 + \lambda) [n + (1 - n)p] \leq 0 \) and \( V_w < V_w^{**} \), it is neither profitable to suggest the project nor to collect subsidies for project losers (see the proof of Proposition 4.6). Thus, the status quo prevails.

These considerations prove the corollary.

\[ \square \]

Proposition 4.7
Suppose \( p \geq m \).

(i) For \( 1 - (1 + \lambda)p > 0 \), the equilibrium proposal is given by

\[ \pi_{FC1}^{p \geq m} = \left( g(\pi_{FC1}^{p \geq m}) = 1, s(\pi_{FC1}^{p \geq m}), t(\pi_{FC1}^{p \geq m}) \right), \]

where

\[ t(\pi_{FC1}^{p \geq m}) = (1 + \lambda) [k + ps] \]

and

\[ s_j(\pi_{FC1}^{p \geq m}) = \begin{cases} \hat{s} & \text{for } j \in [0, p] \\ 0 & \text{for } j \in [p, 1] \end{cases}. \]
(ii) For $1 - (1 + \lambda)p \leq 0$, the equilibrium proposal is given by

$$
\pi_{FC2}^{m_{\geq}} := \left( g(\pi_{FC2}^{m_{\geq}}) = 1, s(\pi_{FC2}^{m_{\geq}}), t(\pi_{FC2}^{m_{\geq}}) \right),
$$

where

$$
t(\pi_{FC}^{m_{\geq}}) = (1 + \lambda)k
$$

and

$$
s_j(\pi_{FC2}^{m_{\geq}}) = 0 \text{ for } j \in [0,1].
$$

The proof of Proposition 4.7 is equivalent to the proof of Proposition 4.2 and therefore omitted.

### 4.7 Multiplier Effects

Sections 4.4-4.6 reveal insight regarding the amount of subsidies required to compensate project losers. We will now expand this issue. For that purpose, it will prove to be useful to introduce the following definition.

**Definition 11**

The direct utility loss of a project loser from an arbitrary proposal $\pi$, denoted by $D_j(\pi)$, amounts to

$$
D_j(\pi) := g(\pi)(V_i - (1 + \lambda)k) - (1 + \lambda) \int_0^p s_j(\pi) dj \leq 0.
$$

According to Definition 11, the direct utility loss of a project loser comprises two components: (i) the utility loss caused by public-project implementation, and (ii) the utility loss caused by the taxes necessary to finance the subsidies for project winners. The term direct is justified, since $D_j(\pi)$ comprises all components of proposal $\pi$ that yield a contemporaneous utility loss for project losers.

In our model, subsidies for project losers are used as compensatory payments that ensure the majority necessary for the adoption of a proposal. Thus, subsidies represent the institutionalized way of forming majorities and are therefore used by the agenda-setter. On the other hand, higher subsidies are reflected in higher uniform taxes that harm the agenda-setter. Thus, there is a strict incentive to keep the amount of subsidies as low as possible (this is one of several properties of uniform taxation; see Chapters 2 and 3 for a detailed discussion).
Because of uniform taxation, compensatory payments have to exceed the payments necessary to compensate a project loser for the direct utility loss. Let us illustrate this point by an example: Consider proposal $\pi_{F1}^{p<m}$. This proposal is suggested under parties as fairness device if the parameters are such that $p < m$, $1 - (1 + \lambda)m > 0$ and $V_w \geq V_W^*$ (see Proposition 4.1). Proposal $\pi_{F1}^{p<m}$ comprises $g(\pi_{F1}^{p<m}) = 1$ and $s_j(\pi_{F1}^{p<m}) = \hat{s}$ for $j \in [0, p]$.

According to Definition 11, the direct utility loss for a project loser amounts to

$$D_j(\pi_{F1}^{p<m}) = V_l - (1 + \lambda)k - (1 + \lambda)p\hat{s} < 0. \quad (4.10)$$

However, compensatory payments for a subsidized project loser amount to$^{18}$

$$s_j(\pi_{F1}^{p<m}) = \frac{(1 + \lambda)p\hat{s} + ((1 + \lambda)k - V_l)}{1 - (1 + \lambda)(m - p)} > (1 + \lambda)p\hat{s} + ((1 + \lambda)k - V_l) = |D_j|, \ j \in ]p, m].$$

Similar findings are observed for proposals $\pi_{F2}^{p<m}$, $\pi_{F3}^{p<m}$, $\pi_{C1}^{p<m}$, and $\pi_{FC1}^{p<m} - \pi_{FC3}^{p<m}$.

The reason for this observation is that project losers have to pay for their own subsidies, which can be interpreted as a multiplier effect.$^{19}$

We now examine the nature and the magnitude of these effects more technically. More precisely, we illustrate that the amount of subsidies given to a project loser can be derived stepwise, that is, variables in one step feed into variables in the next step. A constant rate of flow yields geometric series, which computes a multiplier (see for example Hegeland (1954)).

Let $D < 0$ denote the direct utility loss of a project loser from an arbitrary proposal $\pi$.$^{20}$

Step 1: Suppose an arbitrary proposal $\pi^1$ that does not involve compensations for project losers. Thus, utility of project losers from $\pi^1$, denoted by $u^1$, equals the direct utility loss, that is

$$u^1 = D < 0.$$
However, if the agenda-setter needs the support of project losers to implement a proposal, she has to compensate a certain fraction of project losers for their incurred utility losses. Let $x$ denote the share of subsidized project losers. If the agenda-setter compensates $x$ project losers for the losses incurred by $\pi^1$, each one obtains a subsidy $|u^1| = |D|$. The taxes necessary to finance these compensatory payments amount to $(1 + \lambda)x|D|$.

Step 2: Now, consider the new proposal $\pi^2$ that adapts proposal $\pi^1$ by the additional payments to project losers, that is subsidies $|D|$ for $x$ project losers. The utility of a subsidized project loser from $\pi^2$, denoted by $u^2$, equals

$$u^2 = u^1 + |D| - (1 + \lambda)x|D| = D + |D| - (1 + \lambda)x|D| = (1 + \lambda)xD < 0.$$ 

Because $u^2$ is still smaller than zero, subsidized project losers reject $\pi^2$. Consequently, proposal adoption requires additional compensations for the $x$ project losers. If the agenda-setter balances the losses from $\pi^2$, each subsidized project loser obtains an additional subsidy of $|u^2| = (1 + \lambda)x|D|$. In order to finance these transfers, taxes of $(1 + \lambda)x \cdot (1 + \lambda)x|D|$ are required.

Step 3: Consider the new proposal $\pi^3$ that starts from proposal $\pi^2$, but additionally involves the compensation scheme outlined in step 2. The utility of a subsidized project loser from $\pi^3$, denoted by $u^3$, is given by

$$u^3 = u^2 + (1 + \lambda)x|D| - (1 + \lambda)x \cdot (1 + \lambda)x|D| = D + |D| - (1 + \lambda)x|D| + (1 + \lambda)x|D| - (1 + \lambda)x \cdot (1 + \lambda)x|D| = (1 + \lambda)^2x^2D < 0.$$ 

Again, the compensation scheme in $\pi^3$ is not sufficient to gain the support of subsidized project losers, since $u^3 < 0$. If the agenda-setter balances the losses incurred from proposal $\pi^3$, additional subsidies amount to $|u^3| = (1 + \lambda)^2x^2|D|$. Because the financing of these subsidies requires supplementary taxes, a new proposal involving such a compensation scheme would still be rejected.

\footnote{In our setting, $x$ is either equal to $m - p$ or $n(1 - p)$ (see the proofs of Propositions 4.1, 4.3, and 4.6).}
From step to step, the utility loss of a subsidized project loser declines, that is \(|u^1| > |u^2| > |u^3| > \ldots\). This implies that the additional amount of subsidies required for compensation becomes smaller from step to step, and converges to zero. Consequently, the amount of subsidies needed to compensate a project loser for the direct utility loss is equal to the sum of additional subsidies he receives in any step, and can be written as

\[
|D| + (1 + \lambda)x|D| + (1 + \lambda)^2x^2|D| + \ldots = \sum_{t=0}^{\infty} (1 + \lambda)^t x^t |D| = \frac{|D|}{1 - (1 + \lambda)x}. \tag{4.11}
\]

Replacing \(|D|\) and \(x\) by their respective expressions yields the amount of subsidies required for compensation of a project loser in a specific proposal. For example, in proposal \(\pi_{F1}^{p < m}\), the direct utility loss is \(D_j(\pi_{F1}^{p < m}) = V_I - (1 + \lambda)k - (1 + \lambda)p\hat{s}\) (see Equation 4.10) and \(x = m - p\). We obtain

\[
\frac{|D|}{1 - (1 + \lambda)x} = \frac{(1 + \lambda)p\hat{s} + ((1 + \lambda)k - V_I)}{1 - (1 + \lambda)(m - p)} = s_j(\pi_{F1}^{p < m}).
\]

The analysis makes clear that the multiplier is given by \(1/(1 - (1 + \lambda)x)\). It is obvious that the higher the share of subsidized project losers, the stronger the multiplier effects and the higher the required subsidies.

In general, the amount of subsidies needed for compensation of project losers depends on two variables: (i) the direct utility loss, (ii) the fraction of subsidized project losers.

\[\text{\footnotesize 22Recall that we assumed } (1 + \lambda)x < 1 \text{ for } x \in \{m - p, n(1 - p)\} \text{ (see Assumptions 4.1 and 4.2).}\]
Chapter 5

Comparison

5.1 Introduction

In this chapter we provide a thorough comparison of the four models of parties considered in the dissertation: fairness device (henceforth PF), commitment device (henceforth PC), fairness and commitment device (henceforth PFC), and absence of fairness and commitment device (henceforth P). We derived optimal proposals in the absence of fairness and commitment in Chapter 2 and optimal proposals under the three party devices PF, PC, PFC in Chapter 4.

5.2 Set-up

Since we do not draw a new model in this chapter, but compare the outcomes from Chapters 2 and 4, we reuse the notation introduced in Chapter 4 (see Section 4.3.1). In particular, we identify individuals by index $j$, where $j \in [0, 1]$. Recall that the agenda-setter is located at $j = 0$, project winners are located on the interval $[0, p]$ and project losers on $[p, 1]$.

Moreover, $\pi$ denotes a proposal suggested by the agenda-setter. Thereby, proposal $\pi$ comprises a project decision $g(\pi) \in \{0, 1\}$, a subsidy distribution $s(\pi)$ as well as a uniform tax $t(\pi)$.

Finally, the function $I(\pi)$ indicates whether a proposal is adopted by the society ($I(\pi) = 1$), or not ($I(\pi) = 0$).

\footnote{Remember that $g(\pi) = 1$ indicates that the proposal is suggested, and $g(\pi) = 0$ that it is not.}
Chapter 5. Comparison

At this point, it is also useful to recall that utilitarian welfare associated with a particular proposal $\pi$ amounts to

$$W(\pi) = I(\pi)[(pV_w + (1 - p)V_l - (1 + \lambda)k)g(\pi) - \lambda S(\pi)],$$

where $S(\pi) = \int_0^1 s_j(\pi)dj$.

Additionally, utility of an individual $j \in [0, 1]$ under a certain proposal $\pi$ amounts to

$$U_j(\pi) = I(\pi)[g(\pi)v_j + s_j(\pi) - t(\pi)],$$

where $v_j \in \{V_w, V_l\}$.

For notational convenience we reformulate the three proposals of Proposition 2.2 in the notation of Chapter 4. The first proposal is $\pi_1^{p < m}$:

$$\pi_1^{p < m} := \langle g(\pi_1^{p < m}) = 1, s(\pi_1^{p < m}), t(\pi_1^{p < m}) \rangle,$$

where

$$t(\pi_1^{p < m}) = (1 + \lambda) \left[ k + (m - p) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)} \right],$$

and

$$s_j(\pi_1^{p < m}) = \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, p] \\ \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)} & \text{for } j \in [p, m] \\ 0 & \text{for } j \in [m, 1]. \end{cases}$$

The second proposal is $\pi_2^{p < m}$:

$$\pi_2^{p < m} := \langle g(\pi_2^{p < m}) = 0, s(\pi_2^{p < m}), t(\pi_2^{p < m}) \rangle,$$

where

$$t(\pi_2^{p < m}) = 0,$$

and

$$s_j(\pi_2^{p < m}) = \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, 1]. \end{cases}$$

The third proposal is $\pi^{p \geq m}$:

$$\pi^{p \geq m} := \langle g(\pi^{p \geq m}) = 1, s(\pi^{p \geq m}), t(\pi^{p \geq m}) \rangle,$$
where \( s_j(\pi^{p\geq m}) = s_j(\pi_2^{p\geq m}) \) and \( t(\pi^{p\geq m}) = (1 + \lambda)k \).

In a next step, we derive social welfare and utilities \( U_j(\cdot) \) for any of the three proposals. We use insights from Proposition 2.2 to distinguish which proposal is suggested under what parameter constellation:

**If \( m > p \) and \( V_w \geq V_w^* \), proposal \( \pi_1^{p<m} \) is implemented. Using \( g(\pi_1^{p<m}) \) and \( s_j(\pi_1^{p<m}) \), we obtain social welfare
\[
W(\pi_1^{p<m}) = pV_w + (1 - p)V_i - (1 + \lambda)k - \lambda \left[ \frac{(m - p)}{1 - (1 + \lambda)(m - p)} \right]
\]
and individual utilities
\[
U_j(\pi_1^{p<m}) = \begin{cases} 
\hat{s} + V_w - (1 + \lambda)k - (1 + \lambda) \left[ \frac{(m - p)}{1 - (1 + \lambda)(m - p)} \right] V_i & \text{for } j = 0 \\
V_w - (1 + \lambda)k - (1 + \lambda) \left[ \frac{(m - p)}{1 - (1 + \lambda)(m - p)} \right] V_i & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, m] \\
V_i - (1 + \lambda)k - (1 + \lambda) \left[ \frac{(m - p)}{1 - (1 + \lambda)(m - p)} \right] V_i & \text{for } j \in [m, 1]. 
\end{cases}
\]

**If \( m > p \) and \( V_w < V_w^* \), proposal \( \pi_2^{p<m} \) is implemented. Using \( g(\pi_2^{p<m}) \) and \( s_j(\pi_2^{p<m}) \), we obtain \( S(\pi_2^{p<m}) = 0 \) and hence social welfare
\[
W(\pi_2^{p<m}) = 0,
\]
and individual utilities
\[
U_j(\pi_2^{p<m}) = \begin{cases} 
\hat{s} & \text{for } j = 0 \\
0 & \text{for } j \in [0, 1]. 
\end{cases}
\]

**If \( m \leq p \), proposal \( \pi^{p\geq m} \) is implemented. Using \( g(\pi^{p\geq m}) \) and \( s_j(\pi^{p\geq m}) \), we obtain \( S(\pi^{p\geq m}) = 0 \) and hence social welfare
\[
W(\pi^{p\geq m}) = pV_w + (1 - p)V_i - (1 + \lambda)k,
\]
and individual utilities
\[
U_j(\pi^{p\geq m}) = \begin{cases} 
\hat{s} + V_w - (1 + \lambda)k & \text{for } j = 0 \\
V_w - (1 + \lambda)k & \text{for } j \in [0, p] \\
V_i - (1 + \lambda)k & \text{for } j \in [p, 1]. 
\end{cases}
\]
To simplify the subsequent analysis, it will be useful to decompose social welfare and individual utilities. Consecutively, we will show that social welfare and individual utility can be divided into two distinct parts: a project part and a subsidy part. We will get specific about them after the formal decomposition.

Let \( \pi \in \{ \pi_1^{p<m}, \pi_2^{p<m}, \pi^{p\geq m} \} \). Moreover, let \( \sigma(\pi) \in \{0, 1\} \) denote the agenda-setter’s self-subsidy decision in a specific proposal \( \pi \). More precisely, \( \sigma(\pi) = 1 \) indicates that the agenda-setter levies positive subsidies for herself, whereas \( \sigma(\pi) = 0 \) indicates the opposite, that is, zero subsidies for the agenda-setter.

Finally, we introduce the indicator function \( J \) defined as
\[
J = \begin{cases} 
1 & \text{if } p < m \\
0 & \text{otherwise.} 
\end{cases}
\]

**Decomposition of Social Welfare**

For any \( \pi \in \{ \pi_1^{p<m}, \pi_2^{p<m}, \pi^{p\geq m} \} \), social welfare can be written as the sum of two distinct parts, that is
\[
W(\pi) = W(g(\pi)) + W(\sigma(\pi)).
\]

\( W(g(\pi)) \) represents the project part and \( W(\sigma(\pi)) \) the subsidy part of social welfare. The project part is given by
\[
W(g(\pi)) = g(\pi)[J \cdot GP_{W}^{p<m} + (1 - J) \cdot GP_{W}^{p\geq m}],
\]
where
\[
GP_{W}^{p<m} := pV_w + (1 - p)V_l - (1 + \lambda)k - \lambda \left[ (m - p) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)} \right],
\]
\[
GP_{W}^{p\geq m} := pV_w + (1 - p)V_l - (1 + \lambda)k.
\]

The subsidy part is given by
\[
W(\sigma(\pi)) = \sigma(\pi) \cdot SP_W,
\]
where \( SP_W := 0 \).

**Intuition:** Social welfare is the composite of two complementary parts:

The first part is called project part of social welfare. It is defined as the product of the project decision variable \( (g(\pi) \in \{0, 1\}) \) and a specific expression \( GP_{W}^{p<m} \) or \( GP_{W}^{p\geq m} \).
Chapter 5. Comparison

$GP_{W}^{p\geq m}$). An expression comprises all components of social welfare that result from project implementation. We can distinguish two types of components:

1. **Direct component:** It involves the social welfare characteristics of the public project, that is, it equals $pV_w + (1 - p)V_l - (1 + \lambda)k$.

2. **Indirect component:** It comprises the amount of welfare distortions that arises due to compensatory payments for project implementation.

The second part is called *subsidy part* of social welfare. Analogously to the *project part*, it is the product of a binary variable ($\sigma(\pi) \in \{0, 1\}$) and a specific expression ($SP_W$). Expression $SP_W$ combines a direct and an indirect component. More precisely:

1. **Direct component:** The direct component includes the amount of welfare distortions that arise due to self-subsidization of the agenda-setter.

2. **Indirect component:** It comprises the amount of welfare distortions caused by compensatory payments for self-subsidization of the agenda-setter.

**Decomposition of Individual Utility**

For any $\pi \in \{\pi_1^{p<m}, \pi_2^{p<m}, \pi^{p\geq m}\}$, utility of individual $j \in [0, 1]$ can be written as the sum of two distinct parts, that is

$$U_j(\pi) = U_j(g(\pi)) + U_j(\sigma(\pi)).$$

$U_j(g(\pi))$ represents the *project part* and $U_j(\sigma(\pi))$ the *subsidy part* of individual utility. The *project part* is given by

$$U_j(g(\pi)) = g(\pi)[J \cdot GP_j^{p<m} + (1 - J) \cdot GP_j^{p\geq m}],$$

where

$$GP_j^{p<m} := \begin{cases} V_w - (1 + \lambda)k - (1 + \lambda)(m - p) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)} & \text{for } j \in [0, p] \\ 0 & \text{for } j \in ]p, m] \\ V_l - (1 + \lambda)k - (1 + \lambda)(m - p) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)} & \text{for } j \in ]m, 1] \end{cases}$$

and

$$GP_j^{p\geq m} := \begin{cases} V_w - (1 + \lambda)k & \text{for } j \in [0, p] \\ V_l - (1 + \lambda)k & \text{for } j \in ]p, 1]. \end{cases}$$
The **subsidy part** is given by

\[ U_j(\sigma(\pi)) = \sigma(\pi) \cdot SP_j. \]

where

\[ SP_j := \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, 1]. \end{cases} \]

**Intuition:** Analogously to social welfare, the utility of an arbitrary individual \( j \) is the composite of two complementary parts:

The first part is called **project part** of individual utility. It is defined as the product of the project decision variable \((g(\pi))\) and a specific expression \((GP_{j}^{p < m} \text{ or } GP_{j}^{p \geq m})\).

An expression comprises all benefits and costs (taxes) for individual \( j \) that result from project implementation.

The second part is called **subsidy part** of individual utility. It equals the product of the binary variable \( \sigma(\pi) \) and a specific expression \((SP_j)\). This expression comprises all benefits and costs (taxes) for individual \( j \) that result from self-subsidization of the agenda-setter.

In the course of this chapter, we will show that social welfare and individual utility under PF, PC, and PFC can be decomposed according to the same procedure. More precisely, social welfare and individual utilities are always the sum of a **project part** and a **subsidy part**. Moreover, each part is the product of the binary variable \( g(\cdot) \), (or \( \sigma(\cdot) \), respectively) and a specific expression.

Thus, we developed a procedure to disassemble social welfare and individual utility which yields a consistent notation and helps to understand the differences in proposal-making under different party devices.

### 5.3 Fairness vs Absence of Fairness

In this section, we explore the effect of fairness in parties in terms of social welfare and individual utility under different proposals. We proceed as follows: First, for any proposal \( \pi_F \in \{\pi_{F1}^{p < m}, \pi_{F2}^{p < m}, \pi_{F3}^{p < m}, \pi_{F4}^{p = m}, \pi_{F5}^{p = m}, \pi_{F6}^{p = m}\}^2 \) we derive social welfare \( W(\pi_F) \)

\[ \text{Proposals } \pi_{F1}^{p < m} - \pi_{F5}^{p < m} \text{ are defined in Proposition 4.1 (Chapter 4), and proposals } \pi_{F1}^{p \geq m}, \pi_{F2}^{p \geq m} \text{ are defined in Proposition 4.2 (Chapter 4). We define proposal } \pi_{F4}^{p < m} \text{ to be equivalent to the status quo (that is to comprise } g(\pi_{F4}^{p < m}) = 0 \text{ and } s_j(\pi_{F4}^{p < m}) = 0 \text{ for all } j \in [0, 1]). \]
Chapter 5. Comparison

and individual utilities $U_j(\pi_F)$, $j \in [0, 1]$.

Second, we decompose welfare and utilities according to the procedure in Section 5.1. Third, a table summarizes the complete comparison of PF and P, and results are discussed.

If $\frac{1}{1+\lambda} > m > p$ and $V_w \geq V_w^*$, proposal $\pi_{F1}^{p,m}$ is implemented. Using $g(\pi_{F1}^{p,m})$ and $s_j(\pi_{F1}^{p,m})$, we obtain social welfare

$$W(\pi_{F1}^{p,m}) = pV_w + (1-p)V_i - (1+\lambda)k - \lambda \left[ p\hat{s} + (m-p) \frac{(1+\lambda)p\hat{s} + (1+\lambda)k - V_i}{1 - (1+\lambda)(m-p)} \right],$$

and individual utilities

$$U_j(\pi_{F1}^{p,m}) = \begin{cases} s + V_w - (1+\lambda)k - (1+\lambda) \left[ p\hat{s} + (m-p) \frac{(1+\lambda)p\hat{s} + (1+\lambda)k - V_i}{1 - (1+\lambda)(m-p)} \right] & \text{for } j \in [0, p] \\ 0 & \text{for } j \in [p, m] \\ V_i - (1+\lambda)k - (1+\lambda) \left[ p\hat{s} + (m-p) \frac{(1+\lambda)p\hat{s} + (1+\lambda)k - V_i}{1 - (1+\lambda)(m-p)} \right] & \text{for } j \in [m, 1]. \end{cases}$$

If $\frac{1}{1+\lambda} > m > p$ and $V_w < V_w^*$, proposal $\pi_{F1}^{p,m}$ is implemented. Using $g(\pi_{F1}^{p,m})$ and $s_j(\pi_{F1}^{p,m})$, we obtain social welfare

$$W(\pi_{F1}^{p,m}) = -\lambda \left[ p\hat{s} + (m-p) \frac{(1+\lambda)p\hat{s}}{1 - (1+\lambda)(m-p)} \right],$$

and individual utilities

$$U_j(\pi_{F1}^{p,m}) = \begin{cases} s - (1+\lambda) \left[ p\hat{s} + (m-p) \frac{(1+\lambda)p\hat{s}}{1 - (1+\lambda)(m-p)} \right] & \text{for } j \in [0, p] \\ 0 & \text{for } j \in [p, m] \\ -(1+\lambda) \left[ p\hat{s} + (m-p) \frac{(1+\lambda)p\hat{s}}{1 - (1+\lambda)(m-p)} \right] & \text{for } j \in [m, 1]. \end{cases}$$

If $m > p$, $m \geq \frac{1}{1+\lambda}$ and $V_w \geq V_w^*$, proposal $\pi_{F1}^{p,m}$ is implemented. Using $g(\pi_{F1}^{p,m})$ and $s_j(\pi_{F1}^{p,m})$, we obtain social welfare

$$W(\pi_{F1}^{p,m}) = pV_w + (1-p)V_i - (1+\lambda)k - \lambda \left[ (m-p) \frac{(1+\lambda)k - V_i}{1 - (1+\lambda)(m-p)} \right],$$

and individual utilities

$$U_j(\pi_{F1}^{p,m}) = \begin{cases} V_w - (1+\lambda)k - (1+\lambda) \left[ (m-p) \frac{(1+\lambda)k - V_i}{1 - (1+\lambda)(m-p)} \right] & \text{for } j \in [0, p] \\ 0 & \text{for } j \in [p, m] \\ V_i - (1+\lambda)k - (1+\lambda) \left[ (m-p) \frac{(1+\lambda)k - V_i}{1 - (1+\lambda)(m-p)} \right] & \text{for } j \in [m, 1]. \end{cases}$$

\[^{3}\text{We use insights from Propositions 4.1 and 4.2 to distinguish which proposal is suggested under what parameter constellation.}\]
If $m > p$, $m \geq \frac{1}{1+\lambda}$ and $V_w < V^*$, the status quo prevails. From Proposition 4.1 we know that the agenda-setter either proposes the status quo or any other proposal that yields $I(\pi) = 0$. For the sake of consistency we denoted the status quo as proposal $\pi_F^{p<m}$. Hence, social welfare is given by $W(\pi_F^{p<m}) = 0$ and individual utilities amount to $U_j(\pi_F^{p<m}) = 0$ for all $j \in [0,1]$.

If $\frac{1}{1+\lambda} > p \geq m$, proposal $\pi_F^{p \geq m}$ is implemented. Using $g(\pi_F^{p \geq m})$ and $s_j(\pi_F^{p \geq m})$, we obtain social welfare

$$W(\pi_F^{p \geq m}) = pV_w + (1-p)V_1 - (1+\lambda)k - \lambda \cdot \frac{p\hat{s}}{s(\pi_F^{p \geq m})},$$

and individual utilities

$$U_j(\pi_F^{p \geq m}) = \begin{cases} \hat{s} + V_w - (1+\lambda)k - (1+\lambda)p\hat{s} & \text{for } j \in [0,p] \\ V_1 - (1+\lambda)k - (1+\lambda)p\hat{s} & \text{for } j \in [p,1]. \end{cases}$$

If $p \geq m$ and $p \geq \frac{1}{1+\lambda}$, proposal $\pi_f^{p \geq m}$ is implemented. Using $g(\pi_F^{p \geq m})$ and $s_j(\pi_F^{p \geq m})$, we obtain $s(\pi_F^{p \geq m}) = 0$ and hence social welfare

$$W(\pi_F^{p \geq m}) = pV_w + (1-p)V_1 - (1+\lambda)k,$$

and individual utilities

$$U_j(\pi_F^{p \geq m}) = \begin{cases} V_w - (1+\lambda)k & \text{for } j \in [0,p] \\ V_1 - (1+\lambda)k & \text{for } j \in [p,1]. \end{cases}$$

For the decomposition, let again $g(\pi_F) \in \{0,1\}$ denote the agenda-setter’s project decision and $\sigma(\pi_F) \in \{0,1\}$ the agenda-setter’s self-subsidy decision in a specific proposal $\pi_F$. The indicator functions $I(\pi_F)$ and $J$ are as in Section 5.1.

**Decomposition of Social Welfare**

For any $\pi_F \in \{\pi_F^{p<m}, \pi_F^{p \leq m}, \pi_F^{p \leq m}, \pi_F^{p \geq m}, \pi_F^{p \geq m}, \pi_F^{p \geq m}\}$, social welfare can be written as the sum of two distinct parts, that is

$$W(\pi_F) = W(g(\pi_F)) + W(\sigma(\pi_F)).$$

$W(g(\pi_F))$ represents the project part and $W(\sigma(\pi_F))$ the subsidy part of social welfare. The project part is given by

$$W(g(\pi_F)) = g(\pi_F)[J \cdot GF_W^{p < m} + (1 - J) \cdot GF_W^{p \geq m}].$$
where
\[
GF_{W}^{p < m} := pV_{w} + (1 - p)V_{l} - (1 + \lambda)k - \lambda \left( m - p \right) \frac{(1 + \lambda)k - V_{l}}{1 - (1 + \lambda)(m - p)}
\]
\[
GF_{W}^{p \geq m} := pV_{w} + (1 - p)V_{l} - (1 + \lambda)k.
\]

The subsidy part is given by
\[
W(\sigma(\pi_{F})) = \sigma(\pi_{F})[J \cdot SF_{W}^{p < m} + (1 - J) \cdot SF_{W}^{p \geq m}],
\]
where
\[
SF_{W}^{p < m} := -\lambda \left[ p + (m - p) \frac{(1 + \lambda)p}{1 + (1 + \lambda)(m - p)} \right] \hat{s}
\]
\[
SF_{W}^{p \geq m} := -\lambda p \hat{s}.
\]

**Decomposition of Individual Utility**

For any \(\pi_{F} \in \{\pi_{F_1}^{p < m}, \pi_{F_2}^{p < m}, \pi_{F_3}^{p < m}, \pi_{F_4}^{p < m}, \pi_{F_1}^{p \geq m}, \pi_{F_2}^{p \geq m}\}\), utility of individual \(j \in [0, 1]\) can be written as the sum of two distinct parts, that is
\[
U_{j}(\pi_{F}) = U_{j}(g(\pi_{F})) + U_{j}(\sigma(\pi_{F})).
\]
\(U_{j}(g(\pi_{F}))\) represents the project part and \(U_{j}(\sigma(\pi_{F}))\) the subsidy part of individual utility.

The project part is given by
\[
U_{j}(g(\pi_{F})) = g(\pi_{F})[J \cdot GF_{j}^{p < m} + (1 - J) \cdot GF_{j}^{p \geq m}],
\]
where
\[
GF_{j}^{p < m} := \begin{cases} 
V_{w} - (1 + \lambda)k - (1 + \lambda)(m - p) \frac{(1 + \lambda)k - V_{l}}{1 - (1 + \lambda)(m - p)} & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, m] \\
V_{l} - (1 + \lambda)k - (1 + \lambda)(m - p) \frac{(1 + \lambda)k - V_{l}}{1 - (1 + \lambda)(m - p)} & \text{for } j \in [m, 1]
\end{cases}
\]

and
\[
GF_{j}^{p \geq m} := \begin{cases} 
V_{w} - (1 + \lambda)k & \text{for } j \in [0, p] \\
V_{l} - (1 + \lambda)k & \text{for } j \in [p, 1]
\end{cases}
\]

The subsidy part is given by
\[
U_{j}(\sigma(\pi_{F})) = \sigma(\pi_{F})[J \cdot SF_{j}^{p < m} + (1 - J) \cdot SF_{j}^{p \geq m}],
\]
Chapter 5. Comparison

where
\[
SF_{p}^{<m} := \begin{cases} 
s - (1 + \lambda) \left[ p + (m - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)(m - p)} \right] & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, m] \\
-(1 + \lambda) \left[ p + (m - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)(m - p)} \right] & \text{for } j \in [m, 1] 
\end{cases}
\]
and
\[
SF_{p}^{\geq m} := \begin{cases} 
s - (1 + \lambda)p\hat{s} & \text{for } j \in [0, p] \\
-(1 + \lambda)p\hat{s} & \text{for } j \in [p, 1] 
\end{cases}
\]

Comparison

In Table 5.1 we summarize the comparison between the outcomes if parties exhibit fairness and the outcomes if fairness is absent.

The table is constructed as follows: First, rows are divided into the two cases \( m > p \) and \( m \leq p \). Each of these cases is further divided into two subcases: If \( m > p \), these subcases are \( \frac{1}{1+\lambda} > m \) and \( \frac{1}{1+\lambda} \leq m \). If \( m \leq p \), we distinguish between \( \frac{1}{1+\lambda} > p \) and \( \frac{1}{1+\lambda} \leq p \). Columns separate the cases \( V_w \geq V^*_w \) and \( V_w < V^*_w \) where necessary.

The classification shown in Table 5.1 allows to compare social welfare and individual utility. Each cell in the table is linked with a corresponding proposal of parties with fairness and a corresponding proposal of parties without fairness. From now on, a specific cell is called a scenario.

For notational convenience, we will state that a specific statement is always true if it is true for all scenarios. Correspondingly, if a statement is not true for all scenarios, we will say that it is never true. Hence, we use always as a synonym for “in any scenario” and never as a synonym for “in no scenario”.

Proposals \( \pi \) and \( \pi_F \) split the society into different subgroups. We introduce the following notation to indicate their utility under a specific proposal \( \pi \) or \( \pi_F \):

If \( p < m \), we use
- \( U_0(\cdot) \) for the agenda-setter,
- \( U_{[0,p]}(\cdot) \) for project winners with the exception of the agenda-setter (from now on also called “other project winners”),
- \( U_{[p,m]}(\cdot) \) for subsidized project losers,
- \( U_{[m,1]}(\cdot) \) for non-subsidized project losers.

If \( p \geq m \), we use
- \( U_0(\cdot) \) for the agenda-setter,
- \( U_{[0,p]}(\cdot) \) for other project winners,
- \( U_{[p,1]}(\cdot) \) for project losers.
### Table 5.1: Comparison of PF with P

<table>
<thead>
<tr>
<th></th>
<th>$V_w \geq V_w^*$</th>
<th>$V_w &lt; V_w^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m &gt; p$</td>
<td>$W(\pi_{1}^{p &lt; m}) &gt; W(\pi_{F1}^{p &lt; m})$</td>
<td>$W(\pi_{2}^{p &lt; m}) &gt; W(\pi_{F2}^{p &lt; m})$</td>
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<td></td>
<td>$U_0(\pi_{1}^{p &lt; m}) &gt; U_0(\pi_{F1}^{p &lt; m})$</td>
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<td>$U_{[0,p]}(\pi_{1}^{p &lt; m}) &lt; U_{[0,p]}(\pi_{F1}^{p &lt; m})$</td>
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<td>$U_{[m,1]}(\pi_{1}^{p &lt; m}) &gt; U_{[m,1]}(\pi_{F1}^{p &lt; m})$</td>
<td>$U_{[m,1]}(\pi_{2}^{p &lt; m}) &gt; U_{[m,1]}(\pi_{F2}^{p &lt; m})$</td>
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<tr>
<td>$m \leq p$</td>
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<tr>
<td></td>
<td>$W(\pi_{1}^{p \geq m}) = W(\pi_{F3}^{p \geq m})$</td>
<td>$W(\pi_{2}^{p \geq m}) = W(\pi_{F4}^{p \geq m})$</td>
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<td>$U_0(\pi_{1}^{p \geq m}) &gt; U_0(\pi_{F3}^{p \geq m})$</td>
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<td>$U_{[m,1]}(\pi_{1}^{p \geq m}) = U_{[m,1]}(\pi_{F3}^{p \geq m})$</td>
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<td>$\frac{1}{1+\lambda} &gt; m$</td>
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<td>$\frac{1}{1+\lambda} &gt; p$</td>
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<td>$U_{[p,1]}(\pi_{p \geq m}) = U_{[p,1]}(\pi_{F2}^{p \geq m})$</td>
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</tbody>
</table>
The findings in Table 5.1 are unambiguous and have intuitive explanation.

**Fact 5.1**

Welfare is **never** lower under P than under PF.

The intuition for this result can be explained by considering the *project part* and the *subsidy part* of social welfare separately:

The *project part* of social welfare is always of similar size under PF and P, that is $W(g(\pi)) = W(g(\pi_F))$ holds in any scenario. This is the result of two observations:

First, welfare expressions resulting from project implementation are equal under PF and P:

$$GF_W^{p < m} = GP_W^{p < m} \quad \text{and} \quad GF_W^{p \geq m} = GP_W^{p \geq m}.$$ 

Second, the project decision is always the same under PF and P, that is $g(\pi) = g(\pi_F)$ holds in any scenario.

Both observations combined yield

$$W(g(\pi)) = g(\pi) \left[ J \cdot GP_W^{p < m} + (1 - J) \cdot GP_W^{p \geq m} \right] = g(\pi_F) \left[ J \cdot GF_W^{p < m} + (1 - J) \cdot GF_W^{p \geq m} \right] = W(g(\pi_F)),$$

which confirms our statement that $W(g(\pi)) = W(g(\pi_F))$ always holds.

We provide an intuition for these observations. The first one can be explained as follows: Expressions $GF_W^{p < m}$, $GP_W^{p < m}$, $GF_W^{p \geq m}$, and $GP_W^{p \geq m}$ combine a direct and an indirect component. The direct component equals social welfare of the public project. Since we consider implementation of the same project under PF and P, it follows that the direct component of the expressions $GF_W^{p < m}$, $GP_W^{p < m}$, and the direct component of the expressions $GF_W^{p \geq m}$, $GP_W^{p \geq m}$ is of similar size.

The indirect component comprises the welfare distortions that arise due to the compensatory payments required for project implementation. Let us explore the dimension of these distortions:

Compensatory payments are only needed in scenarios where $m > p$. In scenarios where $m \leq p$, project winners are sufficiently numerous to enforce a proposal without the support of project losers. Consequently, no subsidies are paid and no welfare distortions are involved, neither under PF nor under P. Hence, the indirect component of the expressions $GF_W^{p \geq m}$ and $GP_W^{p \geq m}$ is zero. Because both components in $GP_W^{p \geq m}$ and $GF_W^{p \geq m}$ are of equal size, we obtain $GF_W^{p \geq m} = GP_W^{p \geq m}$. 

107
In scenarios where $m > p$, project winners are not sufficiently numerous to enforce the project without the support of project losers. More precisely, under PF and P, a minimal fraction of $m - p$ project losers has to be offset for the utility losses incurred by project implementation. Because it is neither profitable to compensate more project losers than the minimal share nor to pay higher subsidies than required, the agenda-setter has to collect a similar amount of compensatory payments under PF and P.\textsuperscript{4}

Since welfare distortions are proportional to the aggregate amount of subsidies (more precisely, they equal factor $-\lambda$ of aggregate subsidies), the indirect component of the expressions $GP_{W}^{p < m}$ and $GF_{W}^{p < m}$ is identical. Because the size of both components in $GP_{W}^{p < m}$ and $GF_{W}^{p < m}$ is equal, we obtain $GP_{W}^{p < m} = GF_{W}^{p < m}$.

As a general result, we find that project implementation exhibits similar effects under PF and P.

For the second observation we explore the incentives of the agenda-setter:

Because project implementation exhibits similar effects under PF and P, we conclude that its costs and benefits are of equal size under PF and P.\textsuperscript{5} Hence, if it is profitable to implement the project under PF, it is also profitable under P. Moreover, if it is not profitable to implement it under PF, it is neither under P.

The \textit{subsidy part} of social welfare might vary across PF and P. More precisely, it is \textbf{never} higher under PF than under P, that is $W(\sigma(\pi)) \geq W(\sigma(\pi_F))$ holds in any scenario. Again, this is the result of two observations:

First, welfare expressions resulting from self-subsidization of the agenda-setter are lower under PF than under P:

\[ SP_{W} > SF_{W}^{p \geq m} \quad \text{and} \quad SP_{W} > SF_{W}^{p < m}. \]

Second, self-subsidization occurs more often under P than under PF. More precisely, $\sigma(\pi) \geq \sigma(\pi_F)$ holds in any scenario.

The combination of both observations yields

\[ W(\sigma(\pi)) = \sigma(\pi) \cdot SP_{W} \geq \sigma(\pi_F) \left[ J \cdot SF_{W}^{p < m} + (1 - J) \cdot SF_{W}^{p \geq m} \right] = W(\sigma(\pi_F)), \]

\textsuperscript{4}In Chapter 4, we have shown that the amount of subsidies given to a project loser depends on two expressions: (i) the direct utility loss from project implementation, and (ii) the fraction of subsidized project losers. The direct utility loss equals $V_i - (1 + \lambda)k$ and the share of subsidized project losers equals $m - p$ under PF and P. Thus, it immediately follows that the aggregate amount of compensatory payments is equal under PF and P.

\textsuperscript{5}This can be seen by the fact that $GP_{0}^{p < m} = GF_{0}^{p < m}$ and $GP_{0}^{p \geq m} = GF_{0}^{p \geq m}$. 

108
which confirms our statement that $W(\sigma(\pi)) \geq W(\sigma(\pi_F))$ always holds.

We present an intuition for these findings. The observation regarding the welfare expressions can be explained as follows:

Remember that the expressions $SP_W$, $SF^{p\geq m}_W$, and $SF^{p<m}_W$ combine a direct and an indirect component. First, we explore the dimension of the direct components:

Under PF, the fairness device requires that all members of Party $W$, that is all $j \in [0, p]$, receive the same amount of subsidies. The minimal share of subsidized project winners is therefore equal to $p$. By contrast, under P, the agenda-setter is not obligated to subsidize other project winners which implies that the minimal share of subsidized project winners is zero.\(^6\)

Because it is not profitable to distribute subsidies to more project winners than the minimal share, more project winners are subsidized under PF than under P. Consequently, the aggregate amount of subsidies resulting from self-subsidization of the agenda-setter and the welfare distortions involved are strictly higher under PF than under P. This yields the fact that the direct component of the expression $SP_W$ is bigger than the one of the expressions $SF^{p\geq m}_W$ and $SF^{p<m}_W$.

The indirect component comprises welfare distortions that arise due to compensatory payments for self-subsidization of the agenda-setter. Let us now explore the dimension of these distortions:

In scenarios where $m \leq p$, compensatory payments are required neither under PF nor under P, because project winners are sufficiently numerous to implement a proposal without the support of project losers.

By contrast, in scenarios where $m > p$, the votes of some project losers are needed. However, under P, the agenda-setter is the only project winner receiving a positive subsidy. Since no taxes arise from subsidization of measure zero groups, project losers do not incur a utility loss from self-subsidization of the agenda-setter under P and therefore do not demand compensations either. Consequently, the indirect component of the expression $SP_W$ is zero. Under PF, project losers incur a utility loss from self-subsidization of the agenda-setter because positive taxes are required to finance the subsidies to all project winners. This causes the need for compensatory payments, which further distorts social welfare. Consequently, the indirect component of the expression $SF^{p<m}_W$ is smaller than zero.

\(^6\)The agenda-setter itself is of Lebesgue measure zero.
All in all, the direct component is higher in $SP_W$ than in $SF_W^{p \geq m}$ and in $SF_W^{p < m}$. The indirect component is of equal size in $SP_W$ and in $SF_W^{p \geq m}$, but higher than in $SF_W^{p < m}$. This explains our observation regarding the welfare expressions.

As a general result, we find that self-subsidization of the agenda-setter exhibits different effects under PF than under P.

For the second observation, again, we explore the incentives of the agenda-setter: Since self-subsidization of the agenda-setter causes a higher amount of subsidies under PF than under P, it follows that costs associated with self-subsidization are higher under PF than under P.\textsuperscript{7} Consequently, it might be profitable for the agenda-setter to levy self-subsidies under P, but not under PF. Conversely, if it is profitable to levy self-subsidies under PF, so is it under P.

In summary, we have shown that project implementation exhibits similar effects under PF and P. Thus, the project part of social welfare is always equal under these two party devices. By contrast, self-subsidization of the agenda-setter exhibits different effects under PF and P. These differences become manifest in the fact that the subsidy part of social welfare is never higher under PF than under P. Overall-welfare is therefore never higher when parties exhibit fairness than when fairness is absent. Thus, from a welfare perspective it is weakly desirable that parties do not exhibit fairness.

Let us now investigate how fairness impacts on individual utility. For project winners, we obtain the following results:

Fact 5.2

Utility of the agenda-setter is always higher under P than under PF.

Fact 5.3

Utility of other project winners is never lower under PF than under P.\textsuperscript{8}

The intuition for these findings follows from the previous considerations:

Because project implementation exhibits similar effects under PF and P, the project part of project winners’ utility is always the same under PF and P. Thus, in terms of the project, they are indifferent between PF and P. From now on, we will also call them to be indifferent between PF and P from a pure project point of view.

\textsuperscript{7}This can be seen by the fact that $SP_0 > SF_0^{p < m}$ and $SP_0 > SF_0^{p \geq m}$.

\textsuperscript{8}Recall that “other project winners” are defined as the set of project winners without the agenda-setter.
Since self-subsidization of the agenda-setter causes higher aggregate subsidies under PF than under P, the consequences for the agenda-setter are: (i) She has to pay higher taxes under PF, and (ii) it is less often profitable to levy self subsidies under PF. Consequently, the subsidy part of the agenda-setter’s utility is strictly higher under P than under PF. We will also say that the agenda-setter strictly prefers P over PF from a pure subsidy point of view. For other project winners, the consequence is: They might receive positive net transfers under PF. Because this is not the case under P, they benefit from fairness in some scenarios. These considerations make Facts 5.2 and 5.3.

For project losers we can state the following result:

**Fact 5.4**

*Utility of project losers is never lower under P than under PF.*

We use the concepts *form a pure project point of view* and *from a pure subsidy point of view* analogously for project losers.

*From a pure project point of view*, project losers are always indifferent between PF and P. Again, this is a consequence of the fact that project implementation exhibits similar effects under PF and P.

The analysis of the subsidy part of project losers’ utility is more extensive. We start by investigating scenarios where \( p < m \), that is, we analyze the upper part of Table 5.1 first.

If \( p < m \), members of Party \( \mathcal{L} \) are divided into subsidized \( (j \in [p, m]) \) and nonsubsidized project losers \( (j \in [m, 1]) \). It is suggestive to investigate preferences of each subgroup separately:

- **Project losers in \([p, m]\):** Because they are fully compensated for incurred utility losses under both party devices, they are not affected by the different effects arising from self-subsidization of the agenda-setter.
- **Project losers in \([m, 1]\):** Since taxes associated with self-subsidization of the agenda-setter are higher under PF than under P, they weakly prefer if fairness is absent.

The lower part of Table 5.1 comprises scenarios where \( m \geq p \). In this scenario, project losers are treated identically by a proposal and not divided into diverse subgroups. However, since taxes associated with self-subsidization of the
agenda-setter are again higher under PF than under P, they weakly prefer parties without fairness.

Summing up: *Form a pure project point of view*, project losers are indifferent between PF and P. *From a pure subsidy point of view*, they weakly prefer P over PF. Hence, project losers are never worse off with P than with PF.

**Conclusion**

In this section, we have shown that fairness in parties exhibits a *fairness effect* that affects the *subsidy part* of social welfare and individual utility but does not impact on the *project part*. This *fairness effect* results from the fact that the minimal share of subsidized project winners is higher under PF than under P. As a consequence, compared with P, the aggregate amount of subsidies increases and self-subsidization of the agenda-setter is less often profitable under PF.

The *fairness effect* is of such a kind that from a welfare perspective, it is **always** desirable that fairness is absent.

However, in any scenario, members of Party W, with exception of the agenda-setter, prefer fairness in parties over absence of fairness. By contrast, the agenda-setter **always** prefers a party without fairness.

While it is desirable for members of the proposal-making party to exhibit fairness, it causes non-positive spill-overs to the antagonistic party. Correspondingly, members of Party L would weakly prefer that Party W did not exhibit fairness.

### 5.4 Commitment vs Absence of Commitment

In this section, we explore the effect of commitment in parties in terms of social welfare and individual utility under different proposals. We proceed as follows: First, for any proposal $\pi_C \in \{\pi^{p< m}_{C1}, \pi^{p< m}_{C2}, \pi^{p= m}_{C}\}$ we derive social welfare $W(\pi_C)$ and individual utilities $U_j(\pi_C)$, $j \in [0,1]$.\(^9\) Second, we decompose welfare and utilities according to the procedure in Section 5.1. Third, three tables summarize the comparison of PC and P, and results are discussed.

\(^9\)Proposals $\pi^{p< m}_{C1}$ and $\pi^{p< m}_{C2}$ are defined in Proposition 4.3, proposal $\pi^{p= m}_{C}$ is defined in Proposition 4.5 (see Chapter 4).

\(^{10}\)We use insights from Propositions 4.3, 4.4, and 4.5 to distinguish which proposal is suggested under what parameter constellation.
If $p < m$ and $V_w \geq V_{w^*}$, proposal $\pi_{C_1}^{p < m}$ is implemented. Using $g(\pi_{C_1}^{p < m})$ and $s_j(\pi_{C_1}^{p < m})$, we obtain social welfare

$$W(\pi_{C_1}^{p < m}) = pV_w + (1 - p)V_i - (1 + \lambda)k - \lambda \left( n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \right),$$

and individual utilities

$$U_j(\pi_{C_1}^{p < m}) = \begin{cases} \hat{s} + V_w - (1 + \lambda)k - (1 + \lambda) \left( n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \right) & \text{for } j = 0 \\ V_w - (1 + \lambda)k - (1 + \lambda) \left( n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \right) & \text{for } j \in [0, p] \\ 0 & \text{for } j \in [p, p + n(1 - p)] \\ V_i - (1 + \lambda)k - (1 + \lambda) \left( n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \right) & \text{for } j \in [p + n(1 - p), 1] \end{cases}$$

If $p < m$ and $V_w < V_{w^*}$, proposal $\pi_{C_2}^{p < m}$ is implemented. Using $g(\pi_{C_2}^{p < m})$ and $s_j(\pi_{C_2}^{p < m})$, we obtain $S(\pi_{C_2}^{p < m}) = 0$ and hence social welfare

$$W(\pi_{C_2}^{p < m}) = 0,$$

and individual utilities

$$U_j(\pi_{C_2}^{p < m}) = \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, 1] \end{cases}.$$ 

If $p \geq m$, proposal $\pi_C^{p \geq m}$ is implemented. Using $g(\pi_C^{p \geq m})$ and $s_j(\pi_C^{p \geq m})$, we obtain $S(\pi_C^{p \geq m}) = 0$ and hence social welfare

$$W(\pi_C^{p \geq m}) = pV_w + (1 - p)V_i - (1 + \lambda)k,$$

and individual utilities

$$U_j(\pi_C^{p \geq m}) = \begin{cases} \hat{s} + V_w - (1 + \lambda)k & \text{for } j = 0 \\ V_w - (1 + \lambda)k & \text{for } j \in [0, p] \\ V_i - (1 + \lambda)k & \text{for } j \in [p, 1] \end{cases}.$$ 

For the decomposition, let $g(\pi_C) \in \{0, 1\}$ indicate the project decision and $\sigma(\pi_C) \in \{0, 1\}$ the self-subsidy decision of the agenda-setter in a specific proposal $\pi_C$. The indicator functions $I(\pi_C)$ and $J$ are as in Section 5.1.
**Decomposition of Social Welfare**

For any \( \pi_C \in \{\pi_{C1}^{p<m}, \pi_{C2}^{p<m}, \pi_{C}^{p\geq m}\} \), social welfare can be written as the sum of two distinct parts, that is

\[
W(\pi_C) = W(g(\pi_C)) + W(\sigma(\pi_C)).
\]

\(W(g(\pi_C))\) represents the *project part* and \(W(\sigma(\pi_C))\) the *subsidy part* of social welfare. The *project part* is given by

\[
W(g(\pi_C)) = g(\pi_C)[J \cdot GC_{W}^{p<m} + (1 - J) \cdot GC_{W}^{p\geq m}],
\]

where

\[
GC_{W}^{p<m} := pV_w + (1 - p)V_i - (1 + \lambda)k - \lambda \left[ n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \right]
\]

\[
GC_{W}^{p\geq m} := pV_w + (1 - p)V_i - (1 + \lambda)k.
\]

The *subsidy part* is given by

\[
W(\sigma(\pi_C)) = \sigma(\pi_C) \cdot SC_{W},
\]

where \(SC_{W} := 0\).

**Decomposition of Individual Utility**

For any \( \pi_C \in \{\pi_{C1}^{p<m}, \pi_{C2}^{p<m}, \pi_{C}^{p\geq m}\} \), utility of an individual \( j \in [0,1] \) can be written as the sum of two distinct parts, that is

\[
U_j(\pi_C) = U_j(g(\pi_C)) + U_j(\sigma(\pi_C)).
\]

\(U_j(g(\pi_C))\) represents the *project part* and \(U_j(\sigma(\pi_C))\) the *subsidy part* of individual utility. The *project part* is given by

\[
U_j(g(\pi_C)) = g(\pi_C)[J \cdot GC_{j}^{p<m} + (1 - J) \cdot GC_{j}^{p\geq m}],
\]

where

\[
GC_{j}^{p<m} := \begin{cases} 
V_w - (1 + \lambda)k - (1 + \lambda)n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, p + n(1 - p)] \\
V_i - (1 + \lambda)k - (1 + \lambda)n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} & \text{for } j \in [p + n(1 - p), 1]
\end{cases}
\]
and
\[
GC^{p \geq m}_j := \begin{cases} 
V_w - (1 + \lambda)k & \text{for } j \in [0, p] \\
V_l - (1 + \lambda)k & \text{for } j \in [p, 1].
\end{cases}
\]
The subsidy part is given by
\[
U_j(\sigma(\pi_C)) = \sigma(\pi_C) \cdot SC_j,
\]
where
\[
SC_j := \begin{cases} 
\hat{s} & \text{for } j = 0 \\
0 & \text{for } j \in [0, 1].
\end{cases}
\]

**Comparison**

For a thorough comparison, it is necessary to understand the following relations:

(i) If \( n = \frac{m-p}{1-p} \), then \( V_w^* = V_w^{**} \).

(ii) If \( n < \frac{m-p}{1-p} \), then \( V_w^* > V_w^{**} \).

(iii) If \( n > \frac{m-p}{1-p} \), then \( V_w^* < V_w^{**} \).

Tables 5.2 - 5.4 summarize the comparison between the outcomes when parties exhibit commitment and the outcomes when commitment is absent. They are arranged analogously to Table 5.1: Rows separate the cases \( m > p \) and \( m \leq p \). Columns indicate whether \( V_w \leq \max\{V_w^*, V_w^{**}\} \) and whether \( V_w \leq \min\{V_w^*, V_w^{**}\} \), where necessary.

**Relation \( n < \frac{m-p}{1-p} \)**

Proposals \( \pi \) and \( \pi_C \) split the society into different subgroups. We introduce the following notation to indicate their utility under a specific proposal \( \pi \) or \( \pi_C \). For notational convenience, we define \( \hat{m} := p + n(1-p) \).

If \( p < m \), we use
- \( U_0(\cdot) \) for the agenda-setter,
- \( U_{[0,p]}(\cdot) \) for other project winners,
- \( U_{[p,n]}(\cdot) \) for project losers that are subsidized under P and PC\(^{11}\),
- \( U_{[\hat{m},m]}(\cdot) \) for project losers that are subsidized under P but not PC,
- \( U_{[m,1]}(\cdot) \) for project losers that are subsidized neither under P nor PC.

If \( p \geq m \), we use
- \( U_0(\cdot) \) for the agenda-setter,
- \( U_{[0,p]}(\cdot) \) for other project winners,
- \( U_{[p,1]}(\cdot) \) for project losers.
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<tr>
<th>$m &gt; p$</th>
<th>$V_w \geq V^* &gt; V^{**}$</th>
<th>$V^* &gt; V_w \geq V^{**}$</th>
<th>$V^* &gt; V_w^{**} &gt; V_w$</th>
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<tr>
<td>$W(p_{\leq m}^1) &lt; W(p_{\leq m}^{1C_1})$</td>
<td>$W(p_{\leq m}^{2C_1}) \leq W(p_{\leq m}^{2C_1})$</td>
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<th>$U_0(p_{\geq m}^1) = U_0(p_{\geq m}^{1C_1})$</th>
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Table 5.2: Comparison of PC with P if $n < \frac{m-p}{1-p}$.
Table 5.2 summarizes the comparison of parties with commitment device with parties without commitment device if \( n < \frac{m-p}{1-p} \). The findings have intuitive explanation.

**Fact 5.5**
Welfare may be higher or lower under PC than under P.

The intuition for this result can be explained by analyzing the project part and the subsidy part of social welfare separately:

The subsidy part of social welfare is always of similar size under PC and P, that is\(^{12}\)
\[ W(\sigma(\pi)) = W(\sigma(\pi_C)) \]
holds in any scenario. This is the result of two observations:

First, welfare expressions resulting from self-subsidization of the agenda-setter are equal under PC and P, that is
\[ SP_W = SC_W. \]

Second, the self-subsidy decision is always the same under PC and P, that is \( \sigma(\pi) = \sigma(\pi_C) \) holds in any scenario.

The combination of both observations yields
\[ W(\sigma(\pi)) = \sigma(\pi) \cdot SP_W = \sigma(\pi_C) \cdot SC_W = W(\sigma(\pi_C)), \]
which confirms our statement that \( W(\sigma(\pi)) = W(\sigma(\pi_C)) \) always holds.

We present an intuition for these findings. The first one can be explained as follows:
Recall that the expressions \( SP_W \) and \( SC_W \) combine a direct and an indirect component.
The direct component involves welfare distortions that arise due to self-subsidization of the agenda-setter. Since fairness is absent under PC and P, the agenda-setter is not obligated to subsidize other project winners. Hence, the minimal share of subsidized project winners is zero under PC and P.\(^{12}\)
Because it is not profitable to distribute subsidies to more project winners than the minimal share, other project winners do not receive grants. Consequently, the aggregate amount of subsidies resulting from self-subsidization of the agenda-setter is zero and no welfare distortions arise. Moreover, since project losers do not incur a utility loss (no additional taxes are necessary to finance the subsidy of the agenda-setter), it is not necessary to pay out compensations. Thus, the indirect component is also zero, which immediately yields \( SP_W = SC_W = 0 \).

As a general result, we find that self-subsidization of the agenda-setter exhibits similar effects under PC as under P.

\(^{12}\)The agenda-setter itself is of Lebesgue measure zero.
Chapter 5. Comparison

For the second observation we explore the incentives of the agenda-setter:
Because self-subsidization of the agenda-setter exhibits similar effects under PC and P, we conclude that costs and benefits of self-subsidization are equal under PC and P. Consequently, if it is profitable to levy self-subsidies under PC, so is it under P. Moreover, if it is not profitable under PC, it is neither under P.

The project part of social welfare might vary across PC and P. More precisely, depending on the scenario, the project part of social welfare is higher, equal or lower under PC than under P, that is $W(g(\pi)) \leq W(g(\pi_C))$. Again, this is the result of two observations:
First, welfare expressions resulting from project implementation are not higher under P than under PC, that is

$$GP_W^{p<m} < GC_W^{p<m} \text{ and } GP_W^{p \geq m} = GC_W^{p \geq m}.$$  

Second, the project is more often implemented under PC than under P. More precisely, $g(\pi_C) \geq g(\pi)$ holds in any scenario.

Both observations together yield

$$W(g(\pi)) = g(\pi) \left[ J \cdot GP_W^{p<m} + (1 - J) \cdot GP_W^{p \geq m} \right]$$

$$\leq g(\pi_C) \left[ J \cdot GC_W^{p<m} + (1 - J) \cdot GC_W^{p \geq m} \right] = W(g(\pi_C)),$$

which confirms our statement that $W(g(\pi)) \leq W(g(\pi_C))$.

Let us provide an intuition for these findings. The observation regarding the welfare expressions can be explained as follows:
Expressions $GP_W^{p<m}$, $GC_W^{p<m}$, $GP_W^{p \geq m}$, and $GC_W^{p \geq m}$ are the composite of a direct and an indirect component. The direct component equals social welfare of the public project. Because we consider implementation of the same project under PC and P, it is obvious that the direct component of the expressions $GP_W^{p<m}$ and $GC_W^{p<m}$, and the direct component of the expressions $GP_W^{p \geq m}$ and $GC_W^{p \geq m}$ is of similar size.

The indirect component comprises welfare distortions that arise due to compensatory payments for project implementation. We explore the dimension of these distortions:
Compensations are only required in scenarios where $m > p$. In scenarios where $m \leq p$, project winners are sufficiently numerous to enforce a proposal without the support of project losers. Consequently, no subsidies are paid and no welfare distortions are

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13This can be seen by the fact that $SP_b = SC_0$. 

118
involved, neither under PC nor under P. Hence, the indirect component of the expressions $GC_{W}^{p \geq m}$ and $GP_{W}^{p \geq m}$ is zero. Because both components in $GC_{W}^{p \geq m}$ and $GP_{W}^{p \geq m}$ are of equal size, we obtain $GC_{W}^{p \geq m} = GP_{W}^{p \geq m}$.

In scenarios where $m > p$, project winners are not sufficiently numerous to enforce the project without the support of project losers. Under PC, the commitment device requires that all members of Party $\mathcal{L}$ vote uniformly. To obtain the votes of Party $\mathcal{L}$, a fraction $n$ of its members has to support the proposal. Consequently, at least $n(1 - p)$ project losers have to be offset for the utility losses incurred by project implementation. However, under P, project adoption requires the compensation of at least $m - p$ project losers. If $n < \frac{m - p}{1 - p}$, the minimal share of subsidized project losers is strictly lower under PC than under P.

Because it is neither profitable to compensate more project losers than the minimal share nor to pay higher subsidies than required, the agenda-setter has to collect a higher amount of compensatory payments under P than under PC.\textsuperscript{14} Since welfare distortions are proportional to the aggregate amount of subsidies (they equal factor $-\lambda$ of aggregate subsidies), the indirect component of the expression $GC_{W}^{p < m}$ is lower than the indirect component of $GP_{W}^{p < m}$. Taking into account that the direct component is of equal size, we obtain $GP_{W}^{p < m} < GC_{W}^{p < m}$.

As a general result, we find that project implementation exhibits different effects under PC than under P.

For the second observation we consider the incentives of the agenda-setter:

Since project implementation requires higher compensatory payments under P than under PC, we conclude that costs for project adoption are higher under P than under PC.\textsuperscript{15} Hence, it might be profitable to induce project adoption under PC, but not under P. Conversely, if it is profitable to implement a project under P, so is it under PC.

In summary: Self-subsidization of the agenda-setter exhibits similar effects under PC and P. Consequently, the subsidy part of social welfare is always equal under these two party devices. By contrast, project implementation exhibits different effects under PC and P.

\textsuperscript{14}In Chapter 4, we have shown that the amount of subsidies given to a project loser depends on two expressions: (i) the direct utility loss from project implementation, and (ii) the fraction of subsidized project losers. The direct utility loss equals $V_{l} - (1 + \lambda)k$ under PC and P. However, the share of subsidized project losers is higher under P than under PC, which implies that multiplier effects are stronger under P (see Chapter 4). Thus, it immediately follows that the aggregate amount of compensatory payments is (disproportionately) higher under P than under PC.

\textsuperscript{15}This can be seen by the fact that $GC_{0}^{p < m} > GP_{0}^{p < m}$ and $GC_{0}^{p \geq m} = GP_{0}^{p \geq m}$.
and P. These differences become manifest in the fact that a clear-cut comparison of the project part of social welfare is not obtainable. The combination of these findings yields that an overall-welfare comparison is ambiguous.

Let us now investigate how commitment impacts on individual utility. For project winners we obtain the following result:

**Fact 5.6**

Utility of all project winners is never lower under PC than under P.

The intuition for Fact 5.6 follows from our previous considerations: Because self-subsidization of the agenda-setter exhibits similar effects under PC and P, the subsidy part of project winners’ utility is always of equal size under PC and P. Since project implementation causes lower aggregate subsidies under PC than under P, the consequences for project winners are: (i) They have to pay lower taxes for project implementation under PC, and (ii) project implementation is more often profitable under PC. Consequently, the project part is never lower under PC than under P. Hence, from a pure subsidy point of view, project winners are indifferent between PC and P. From a pure project point of view, they weakly prefer PC over P. As a consequence, project winners weakly prefer PC over P.

For project losers, we can state the following result:

**Fact 5.7**

If \( p < m \), different subgroups of project losers exhibit different preferences on PC and P:

- Utility of project losers in \( [p, p + n(1 - p)] \) is always the same under PC as under P.
- Utility of project losers in \( [p + n(1 - p), m] \) is never lower under P than under PC.
- Utility of project losers in \( [m, 1] \) can be higher or lower under PC than under P, depending on the scenario.

If \( p \geq m \), project losers are indifferent between PC and P.

From a pure subsidy point of view, project losers are always indifferent between PC and P. This is a consequence of the similar effects arising from self-subsidization of the agenda-setter under PC and P.

The analysis of the project part of project losers’ utility is more extensive. We start the analysis by investigating scenarios where \( p < m \), that is we analyze the upper part of Table 5.2 first. There are three subgroups of project losers: those
subsidized under PC and P ($j \in [p, p + n(1 - p)]$), those subsidized under P but not PC ($j \in [p + n(1 - p), m]$), and those subsidized neither under PC nor under P ($j \in [m, 1]$).

It is suggestive to investigate preferences of each subgroup separately:

Project losers in $[p, p + n(1 - p)]$: Because they are fully compensated for incurred utility losses under both party devices, they are not affected by the different effects arising from project implementation.

Project losers in $[p + n(1 - p), m]$: Since they are compensated only under P, they are strictly better off with P than with PC if the project is implemented under both party devices. Moreover, if the project is provided under PC but not under P, they are again better off with P. Hence, they weakly prefer if commitment is absent.

Project losers in $[m, 1]$: Because taxes for project implementation are lower under PC, they prefer PC if the project is adopted under both party devices. However, if the project is implemented under PC but not under P, they are better off with P. Hence, they prefer PC in some scenarios and P in others.

Let us now turn to the lower part of Table 5.2, that is to the scenario $p \geq m$.

In the welfare comparison, we have shown that proposal-making under PC and P is equal in this scenario. Hence, project losers are indifferent between PC and P.

Generally, we find that different subgroups of project losers exhibit different preferences on PC and P.

Relation $n = \frac{m - p}{1 - p}$

For a coherent comparison, we slightly adapt the afore notation.

If $n = \frac{m - p}{1 - p}$, the share of subsidized project losers under P equals the share of subsidized project losers under PC, that is $m = \hat{m}$.\(^{16}\)

According to the previous notation, we use $U_x(\cdot)$, where $x$ is a line segment in on the unit interval, to indicate the utility of the respective subgroup of the society. More precisely, if $p < m$, $U_0(\cdot)$ stands for agenda-setter’s, $U_{[0,p]}(\cdot)$ for other project winners’, $U_{[p,m]}(\cdot)$ for subsidized project losers’, and $U_{[m,1]}(\cdot)$ for non-subsidized project losers’ utility. If $p \geq m$, the notations $U_0(\cdot)$, $U_{[0,p]}(\cdot)$, and $U_{[p,1]}(\cdot)$ are used similarly as in the previous analysis.

\(^{16}\)Recall that $\hat{m} := p + n(1 - p)$.
Chapter 5. Comparison

Table 5.3: Comparison of PC with P if \( n = \frac{m-p}{1-p} \).

<table>
<thead>
<tr>
<th>( m &gt; p )</th>
<th>( V_w \geq V^*_w = V^{**}_w )</th>
<th>( V_w &lt; V^*_w = V^{**}_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W(\pi_1^{p&lt;m}) = W(\pi_{C1}^{p&lt;m}) )</td>
<td>( W(\pi_2^{p&lt;m}) = W(\pi_{C2}^{p&lt;m}) )</td>
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<tr>
<td>( U_0(\pi_1^{p&lt;m}) = U_0(\pi_{C1}^{p&lt;m}) )</td>
<td>( U_0(\pi_2^{p&lt;m}) = U_0(\pi_{C2}^{p&lt;m}) )</td>
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<td>( U_{[0,p]}(\pi_1^{p&lt;m}) = U_{[0,p]}(\pi_{C1}^{p&lt;m}) )</td>
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<td>( U_{[p,m]}(\pi_2^{p&lt;m}) = U_{[p,m]}(\pi_{C2}^{p&lt;m}) )</td>
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<tr>
<td>( U_{[m,1]}(\pi_1^{p&lt;m}) = U_{[m,1]}(\pi_{C1}^{p&lt;m}) )</td>
<td>( U_{[m,1]}(\pi_2^{p&lt;m}) = U_{[m,1]}(\pi_{C2}^{p&lt;m}) )</td>
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</tbody>
</table>

Table 5.3 shows that if \( n = \frac{m-p}{1-p} \), social welfare and individual utilities are always equal under PC and P. The reason is that in any scenario the same proposal under PC and P is made.\(^{17}\)

The intuition follows from our previous considerations: Commitment impacts on the project part of social welfare and individual utility, but not on the subsidy part. The differences in the project part are caused by an unequal share of compensated project losers. However, if \( n = \frac{m-p}{1-p} \), the share of project losers needed to implement a project is equal under PC and P and therefore, the differences in the project part disappear.

Fact 5.8

For \( n = \frac{m-p}{1-p} \), proposal-making is equivalent under PC and P.

Relation \( n > \frac{m-p}{1-p} \)

If \( n > \frac{m-p}{1-p} \) and \( p < m \), the share of subsidized project losers is smaller under P than under PC, that is \( m < \tilde{m} \). We have to conform our notation to this fact. We use

- \( U_0(\cdot) \) for the agenda-setter, and \( U_{[0,p]}(\cdot) \) for other project winners,
- \( U_{[p,m]}(\cdot) \) for project losers that are subsidized under P and PC,
- \( U_{[m,\tilde{m}]}(\cdot) \) for project losers that are subsidized under PC but not P,
- \( U_{[m,1]}(\cdot) \) project losers that are subsidized neither under P nor PC.

If \( p \geq m \), the “standard” notation of the previous analyses is used.

\(^{17}\)Two proposals are called to be equal if the project decision and the chosen subsidy scheme are the same in both proposals.

122
<table>
<thead>
<tr>
<th>$m &gt; p$</th>
<th>$V_w \geq V^{**} &gt; V^*$</th>
<th>$V^{**} &gt; V_w \geq V^*$</th>
<th>$V^{**} &gt; V_w &gt; V^*$</th>
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<tr>
<td>$W(\pi_1^{p&lt;m}) &gt; W(\pi_1^{p&lt;m})$</td>
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<td>$U_0(\pi_1^{p&lt;m}) &gt; U_0(\pi_1^{p&lt;m})$</td>
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<tr>
<td>$U_{[m,\tilde{m}]}(\pi_1^{p&lt;m}) &lt; U_{[m,\tilde{m}]}(\pi_1^{p&lt;m})$</td>
<td>$U_{[m,\tilde{m}]}(\pi_1^{p&lt;m}) &lt; U_{[m,\tilde{m}]}(\pi_1^{p&lt;m})$</td>
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<td>$U_{[\tilde{m},1]}(\pi_1^{p&lt;m}) &gt; U_{[\tilde{m},1]}(\pi_1^{p&lt;m})$</td>
<td>$U_{[\tilde{m},1]}(\pi_1^{p&lt;m}) &lt; U_{[\tilde{m},1]}(\pi_1^{p&lt;m})$</td>
<td>$U_{[\tilde{m},1]}(\pi_2^{p&lt;m}) = U_{[\tilde{m},1]}(\pi_2^{p&lt;m})$</td>
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</tr>
</tbody>
</table>

| $m \leq p$ | $W(\pi^{p\geq m}) = W(\pi^{p\geq m})$ |
| $U_0(\pi^{p\geq m}) = U_0(\pi^{p\geq m})$ |
| $U_{[0,p]}(\pi^{p\geq m}) = U_{[0,p]}(\pi^{p\geq m})$ |
| $U_{[p,1]}(\pi^{p\geq m}) = U_{[p,1]}(\pi^{p\geq m})$ |

Table 5.4: Comparison of PC with P if $n > \frac{m-p}{1-p}$. 
Table 5.4 is the opposite of Table 5.2. Arguments for the findings are identical but reversed compared to the relation \( n < \frac{m-p}{1-p} \). We will therefore omit the analysis and simply summarize the facts:

**Fact 5.9**
Welfare may be higher or lower under PC than under P.

**Fact 5.10**
Utility of all project winners is **never** lower under P than under PC.

**Fact 5.11**
If \( p < m \), different subgroups of project losers exhibit different preferences on PC and P: 
Utility of project losers in \([p, m]\) is **always** the same under PC as under P. 
Utility of project losers in \([m, p + n(1 - p)]\) is **never** lower under PC than under P. 
Utility of project losers in \([p + n(1 - p), 1]\) can be higher or lower under PC than under P, depending on the scenario. 
If \( p \geq m \), project losers are indifferent between PC and P.

**Conclusion**
In this section we have shown that commitment in parties exhibits a **commitment effect** that affects the **project part** of social welfare and individual utility but does not impact on the **subsidy part**. This commitment effect results from the fact that the minimal share of subsidized project losers might be different under PC and P (depending on the relation of \( n \) and \( \frac{m-p}{1-p} \)). As a consequence, the aggregate amount of subsidies might differ among PC and P, and project implementation may be profitable under one party device but not under the other.

If \( n < \frac{m-p}{1-p} \), the commitment effect is of such a kind that desirability of commitment is ambiguous from a welfare perspective.
However, all members of Party \( W \) weakly prefer parties with commitment.
By contrast, depending on the scenario, subgroups of Party \( L \) might have opposing preferences on commitment in parties.

If \( n = \frac{m-p}{1-p} \), the commitment effect “disappears”, and commitment in parties does not have an impact on proposal-making.

If \( n > \frac{m-p}{1-p} \), the commitment effect is of such a kind that desirability of commitment remains ambiguous from a welfare perspective.

124
However, for any subgroup on the unit interval, desirability of commitment in parties is reversed compared to \( n < \frac{m-p}{1-p} \).

Comparing the \textit{fairness effect} and the \textit{commitment effect} makes clear that both are of different nature. While the \textit{fairness effect} impacts on the distribution of subsidies within Party \( \mathcal{W} \), the \textit{commitment effect} impacts on the distribution of subsidies within Party \( \mathcal{L} \). Thus, the \textit{fairness effect} affects the subsidy part, and the \textit{commitment effect} the project part of social welfare and individual utility. In this sense, these effects are of complementary nature.

As we will show in subsequently, the \textit{fairness effect} might or might not oppose the \textit{commitment effect}.

\section{5.5 Fairness and Commitment vs Absence of Fairness and Commitment}

In this section we the effect of fairness and commitment in parties in terms of social welfare and individual utility under different proposals. We proceed as follows: First, for any proposal \( \pi_{FC} \in \{ \pi_{FC1}^{p \leq m}, \pi_{FC2}^{p \leq m}, \pi_{FC3}^{p \leq m}, \pi_{FC4}^{p \leq m}, \pi_{FC1}^{p \geq m}, \pi_{FC2}^{p \geq m} \} \)\(^{18}\) we derive social welfare \( W(\pi_{FC}) \) and individual utilities \( U_j(\pi_{FC}), j \in [0,1] \).\(^{19}\) Second, welfare and utilities are decomposed according to our standard procedure. Third, several tables summarize the comparison of PFC and P, and results are discussed.

\begin{enumerate}
    \item If \( m > \nu, n < \frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} \) and \( V_w \geq V_w^{**} \), proposal \( \pi_{FC1}^{p \leq m} \) is implemented. Using \( g(\pi_{FC1}^{p \leq m}) \) and \( s_j(\pi_{FC1}^{p \leq m}) \), we obtain social welfare
    \[
    W(\pi_{FC1}^{p \leq m}) = pV_w + (1-p)V_l - (1+\lambda)k - \lambda \left[ p\hat{s} + n(1-p) \frac{(1+\lambda)p\hat{s} + (1+\lambda)k - V_l}{1 - (1+\lambda)n(1-p)} \right],
    \]
\end{enumerate}

\(^{18}\)Proposals \( \pi_{FC1}^{p \leq m} - \pi_{FC3}^{p \leq m} \) are defined in Proposition 4.6 (Chapter 4), and proposals \( \pi_{FC1}^{p \geq m}, \pi_{FC2}^{p \geq m} \) are defined in Proposition 4.7 (Chapter 4). We define proposal \( \pi_{FC4}^{p \geq m} \) to be equivalent to the status quo (that is to comprise \( g(\pi_{FC4}^{p \geq m}) = 0 \) and \( s_j(\pi_{FC4}^{p \geq m}) = 0 \) for all \( j \in [0,1] \)).

\(^{19}\)We use insights from Propositions 4.6, 4.7, and Corollary 4.1 to distinguish which proposal is suggested under what parameter constellation.
and individual utilities

\[
U_j(\pi_{FC1}^{p<m}) = \begin{cases} 
\hat{s} + V_w - (1 + \lambda)k - (1 + \lambda) \left[ p\hat{s} + n(1-p) \frac{(1+\lambda)p\hat{s} + (1+\lambda)k - V_l}{1-(1+\lambda)n(1-p)} \right] & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, \tilde{m}] \\
V_l - (1 + \lambda)k - (1 + \lambda) \left[ p\hat{s} + n(1-p) \frac{(1+\lambda)p\hat{s} + (1+\lambda)k - V_l}{1-(1+\lambda)n(1-p)} \right] & \text{for } j \in [\tilde{m}, 1]. 
\end{cases}
\]

If \( m > p, \quad n < \frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} \) and \( V_w < V_w^{**} \), proposal \( \pi_{FC2}^{p<m} \) is implemented. Using \( g(\pi_{FC2}^{p<m}) \) and \( s_j(\pi_{FC2}^{p<m}) \), we obtain social welfare

\[
W(\pi_{FC2}^{p<m}) = -\lambda \left( p\hat{s} + n(1-p) \frac{(1+\lambda)p\hat{s}}{1-(1+\lambda)n(1-p)} \right),
\]

and individual utilities

\[
U_j(\pi_{FC2}^{p<m}) = \begin{cases} 
\hat{s} - (1 + \lambda) \left[ p\hat{s} + n(1-p) \frac{(1+\lambda)p\hat{s}}{1-(1+\lambda)n(1-p)} \right] & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, \tilde{m}] \\
-(1 + \lambda) \left[ p\hat{s} + n(1-p) \frac{(1+\lambda)p\hat{s}}{1-(1+\lambda)n(1-p)} \right] & \text{for } j \in [\tilde{m}, 1]. 
\end{cases}
\]

If \( m > p, \quad n \geq \frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} \) and \( V_w \geq V_w^{**} \), proposal \( \pi_{FC3}^{p<m} \) is implemented. Using \( g(\pi_{FC3}^{p<m}) \) and \( s_j(\pi_{FC3}^{p<m}) \), we obtain social welfare

\[
W(\pi_{FC3}^{p<m}) = pV_w + (1-p)V_l - (1 + \lambda)k - \lambda \left[ n(1-p) \frac{(1+\lambda)k - V_l}{1-(1+\lambda)n(1-p)} \right],
\]

and individual utilities

\[
U_j(\pi_{FC3}^{p<m}) = \begin{cases} 
V_w - (1 + \lambda)k - (1 + \lambda) \left[ n(1-p) \frac{(1+\lambda)k - V_l}{1-(1+\lambda)n(1-p)} \right] & \text{for } j \in [0, p] \\
0 & \text{for } j \in [p, \tilde{m}] \\
V_l - (1 + \lambda)k - (1 + \lambda) \left[ n(1-p) \frac{(1+\lambda)k - V_l}{1-(1+\lambda)n(1-p)} \right] & \text{for } j \in [\tilde{m}, 1]. 
\end{cases}
\]

If \( m > p, \quad n \geq \frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} \) and \( V_w < V_w^{**} \), the status quo prevails. From Proposition 4.6 we know that the agenda-setter either proposes the status quo or any other proposal that yields \( I(\pi) = 0 \). For the sake of consistency we denoted the status quo as \( \pi_{FC4}^{p<m} \).

Hence, social welfare is given by \( W(\pi_{FC4}^{p<m}) = 0 \) and individual utilities amount to \( U_j(\pi_{FC4}^{p<m}) = 0 \) for all \( j \in [0, 1] \).

\(^{20}\)Recall that \( \tilde{m} := p + n(1-p) \).
If \( \frac{1}{1+\lambda} > p \geq m \), proposal \( \pi_{FC1}^{p \geq m} \) is implemented. Using \( g(\pi_{FC1}^{p \geq m}) \) and \( s_j(\pi_{FC1}^{p \geq m}) \), we obtain social welfare

\[
W(\pi_{FC1}^{p \geq m}) = pV_w + (1 - p)V_i - (1 + \lambda)k - \lambda \cdot \frac{p\hat{s}}{S(\pi_{FC1}^{p \geq m})},
\]

and individual utilities

\[
U_j(\pi_{FC1}^{p \geq m}) = \begin{cases} 
\hat{s} + V_w - (1 + \lambda)k - (1 + \lambda)p\hat{s} & \text{for } j \in [0, p] \\
V_i - (1 + \lambda)k - (1 + \lambda)p\hat{s} & \text{for } j \in ]p, 1].
\end{cases}
\]

If \( p \geq m \) and \( p \geq \frac{1}{1+\lambda} \), proposal \( \pi_{FC2}^{p \geq m} \) is implemented. Using \( g(\pi_{FC2}^{p \geq m}) \) and \( s_j(\pi_{FC2}^{p \geq m}) \), we obtain \( S(\pi_{FC2}^{p \geq m}) = 0 \) and hence social welfare

\[
W(\pi_{FC2}^{p \geq m}) = pV_w + (1 - p)V_i - (1 + \lambda)k,
\]

and individual utilities

\[
U_j(\pi_{FC2}^{p \geq m}) = \begin{cases} 
V_w - (1 + \lambda)k & \text{for } j \in [0, p] \\
V_i - (1 + \lambda)k & \text{for } j \in ]p, 1].
\end{cases}
\]

For the decomposition, let again \( g(\pi_{FC}) \in \{0, 1\} \) denote the agenda-setter’s project decision and \( \sigma(\pi_{FC}) \in \{0, 1\} \) the agenda-setter’s self-subsidy decision in a specific proposal \( \pi_{FC} \). The indicator functions \( I(\pi_{FC}) \) and \( J \) are as in Section 5.1.

**Decomposition of Social Welfare**

For any \( \pi_{FC} \in \{\pi_{FC1}^{p < m}, \pi_{FC2}^{p < m}, \pi_{FC3}^{p < m}, \pi_{FC4}^{p < m}, \pi_{FC1}^{p \geq m}, \pi_{FC2}^{p \geq m}\} \), social welfare can be written as the sum of two distinct parts, that is

\[
W(\pi_{FC}) = W(g(\pi_{FC})) + W(\sigma(\pi_{FC})).
\]

\( W(g(\pi_{FC})) \) represents the *project part* and \( W(\sigma(\pi_{FC})) \) the *subsidy part* of social welfare. The *project part* is given by

\[
W(g(\pi_{FC})) = g(\pi_{FC})[J \cdot GFC_{W}^{p < m} + (1 - J) \cdot GFC_{W}^{p \geq m}],
\]

where

\[
GFC_{W}^{p < m} := pV_w + (1 - p)V_i - (1 + \lambda)k - \lambda \left[ n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \right]
\]

\[
GFC_{W}^{p \geq m} := pV_w + (1 - p)V_i - (1 + \lambda)k.
\]
Chapter 5. Comparison

The subsidy part is given by

\[ W(\sigma(\pi_{FC})) = \sigma(\pi_{FC})[J \cdot SFC^p_{W} + (1 - J) \cdot SFC^{p \geq m}_{W}], \]

where

\[ SFC^p_{W} := -\lambda \left[ p + n(1 - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)n(1 - p)} \right] \hat{s} \]
\[ SFC^{p \geq m}_{W} := -\lambda p \hat{s}. \]

**Decomposition of Individual Utility**

For any \( \pi_{FC} \in \{\pi_{FC1}, \pi_{FC2}, \pi_{FC3}, \pi_{FC4}, \pi_{FC5}, \pi_{FC6}\} \), utility of an individual \( j \in [0, 1] \) can be written as the sum of two distinct parts, that is

\[ U_j(\pi_{FC}) = U_j(g(\pi_{FC})) + U_j(\sigma(\pi_{FC})). \]

\( U_j(g(\pi_{FC})) \) represents the project part and \( U_j(\sigma(\pi_{FC})) \) the subsidy part of individual utility. The project part is given by

\[ U_j(g(\pi_{FC})) = g(\pi_{FC})[J \cdot GFC^p_{j} + (1 - J) \cdot GFC^{p \geq m}_{j}], \]

where

\[ GFC^p_{j} := \begin{cases} V_w - (1 + \lambda)k - (1 + \lambda)n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} & \text{for } j \in [0, p] \\ 0 & \text{for } j \in [p, \hat{m}] \\ V_i - (1 + \lambda)k - (1 + \lambda)n(1 - p) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} & \text{for } j \in [\hat{m}, 1] \end{cases} \]

and

\[ GFC^{p \geq m}_{j} := \begin{cases} V_w - (1 + \lambda)k & \text{for } j \in [0, p] \\ V_i - (1 + \lambda)k & \text{for } j \in [p, 1] \end{cases} \]

The subsidy part is given by

\[ U_j(\sigma(\pi_{FC})) = \sigma(\pi_{FC})[J \cdot SFC^p_{j} + (1 - J) \cdot SFC^{p \geq m}_{j}], \]

where

\[ SFC^p_{j} := \begin{cases} 1 - (1 + \lambda) \left[ p + n(1 - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)n(1 - p)} \right] \hat{s} & \text{for } j \in [0, p] \\ 0 & \text{for } j \in [p, \hat{m}] \\ -(1 + \lambda) \left[ p + n(1 - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)n(1 - p)} \right] \hat{s} & \text{for } j \in [\hat{m}, 1] \end{cases} \]
and
\[ SFC_{\geq m}^p := \begin{cases} \hat{s} - (1 + \lambda)p\hat{s} & \text{for } j \in [0, p] \\ -(1 + \lambda)p\hat{s} & \text{for } j \in [p, 1]. \end{cases} \]

**Comparison**

For a complete comparison, we need to distinguish the three relations

\[ n < \frac{m-p}{1-p}, \quad n = \frac{m-p}{1-p}, \quad n > \frac{m-p}{1-p}. \]

Tables 5.5 - 5.9 summarize the comparison between the outcomes when parties exhibit fairness and commitment and the outcomes when fairness and commitment are absent for all three relations. The tables are arranged analogously to the previous tables.\(^{21}\)

**Relation \( n < \frac{m-p}{1-p} \)**

Proposals \( \pi \) and \( \pi_{FC} \) split the society into different subgroups. Similar to the previous sections, these subgroups are (i) the agenda-setter, (ii) other project winners, (iii) subsidized project losers, and (iv) non-subsidized project losers if \( p < m \). As in Section 5.4, the fraction of subsidized project losers is higher under P than under PFC if \( n < \frac{m-p}{1-p} \), or equivalently \( \tilde{m} < m \).\(^{22}\) Thus, subsidized project losers have to be further divided into project losers that are subsidized under P and PFC (they are located at \( j \in [p, \tilde{m}] \)), project losers that are subsidized under P but not PFC (they are located at \( j \in [\tilde{m}, m] \)) and project losers that are subsidized neither under P nor PFC (they are located at \( j \in [m, 1] \)).

We reuse the notation introduced in Section 5.4, “Relation \( n < \frac{m-p}{1-p} \), because similar subgroups occur.

\(^{21}\)Due to the narrowness of the page, we have to split the comparisons into two tables. Nevertheless, the structure from the previous tables is preserved.

\(^{22}\)Recall that \( \tilde{m} := p + n(1-p) \).
| $\frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} > n$ | $m > p$ | $W(\pi_1^{p<m}) \leq W(\pi_{\text{FC}1})$ | $U_0(\pi_1^{p<m}) \leq U_0(\pi_{\text{FC}1})$ | $U_{[0,p]}(\pi_1^{p<m}) < U_{[0,p]}(\pi_{\text{FC}1})$ | $U_{[m,1]}(\pi_1^{p<m}) \leq U_{[m,1]}(\pi_{\text{FC}1})$ |
| | | $V_w \geq V_w^* > V_w^{**}$ | $V_w^* > V_w \geq V_w^{**}$ | $V_w^* > V_w^{**} > V_w$ |

| $\frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} \leq n$ | $m > p$ | $W(\pi_1^{p<m}) < W(\pi_{\text{FC}3})$ | $U_0(\pi_1^{p<m}) \leq U_0(\pi_{\text{FC}3})$ | $U_{[0,p]}(\pi_1^{p<m}) < U_{[0,p]}(\pi_{\text{FC}3})$ | $U_{[m,1]}(\pi_1^{p<m}) < U_{[m,1]}(\pi_{\text{FC}3})$ |

| | | $W(\pi_2^{p<m}) \leq W(\pi_{\text{FC}1})$ | $U_0(\pi_2^{p<m}) \leq U_0(\pi_{\text{FC}1})$ | $U_{[0,p]}(\pi_2^{p<m}) < U_{[0,p]}(\pi_{\text{FC}1})$ | $U_{[m,1]}(\pi_2^{p<m}) < U_{[m,1]}(\pi_{\text{FC}1})$ |

| | | $W(\pi_2^{p<m}) > W(\pi_{\text{FC}2})$ | $U_0(\pi_2^{p<m}) > U_0(\pi_{\text{FC}2})$ | $U_{[0,p]}(\pi_2^{p<m}) < U_{[0,p]}(\pi_{\text{FC}2})$ | $U_{[m,1]}(\pi_2^{p<m}) < U_{[m,1]}(\pi_{\text{FC}2})$ |

Table 5.5: Comparison of PFC with P if $m > p$, and $n < \frac{m-p}{1-p}$.
Tables 5.5 and 5.6 display multiple ambiguities in the comparison of PFC and P. The main intuition for these ambiguities is that both, the fairness and the commitment effect occur and that these effects might go into different directions. If the fairness effect opposes the commitment effect, it remains unclear which one dominates over the other.

The following analysis can be seen as a combination of the analysis in Section 5.3, where we explored the fairness effect, and the analysis in Section 5.4, where we investigated the commitment effect. For that reason, we contract the discussion and do not give a detailed intuition but instead refer to the corresponding sections.

**Fact 5.12**

Welfare may be higher or lower under PFC than under P.

The fairness effect implies that the subsidy part of social welfare is weakly higher under P than under PFC. More precisely, the fairness effect yields $SFC_W^{p< m} ( = SF_W^{p< m}) < 0$ and $SFC_W^{p\geq m} ( = SF_W^{p\geq m}) < 0$, which in combination with $\sigma(\pi_{FC}) \in \{0, 1\}$ implies that $W(\sigma(\pi_{FC})) \leq 0$. Because $SP_W = 0$, we obtain $W(\sigma(\pi)) = 0$ in any scenario and hence $W(\sigma(\pi_{FC})) \leq W(\sigma(\pi))$.\(^{23}\)

The commitment effect implies that the project part of social welfare might be higher or lower under PFC than under P. This is due to the fact that $GFC_W^{p< m} = (GC_W^{p< m}) \leq 0$ and $g(\pi_{FC}) \geq g(\pi)$.\(^{24}\)

These considerations illustrate that the fairness effect might decrease welfare under PFC compared to P, while the commitment effect may go in the opposite direction. Thus, we find many ambiguities in the comparison of social welfare.

\(^{23}\)A detailed explanation on why $SP_W = 0$, $SF_W^{p< m} < 0$ and $SF_W^{p\geq m} < 0$ is given in Section 5.3.

\(^{24}\)A detailed explanation on why $GC_W^{p< m} \leq 0$ and $g(\pi_{FC}) \geq g(\pi)$ is given in Section 5.4.
Chapter 5. Comparison

Fact 5.13
Utility of the agenda-setter may be higher or lower under PFC than under P. However, for high \( \hat{s} \), utility of the agenda-setter is always higher under P than under PFC.

Fact 5.14
Utility of other project winners is never lower under PFC than under P.

The fairness effect on the agenda-setter’s utility implies that the subsidy part of \( U_0(\cdot) \) is strictly higher under P than under PFC. The same effect on other project winners’ utility implies that the subsidy part of \( U_{[0,p]}(\cdot) \) is weakly higher under PFC than under P. The argumentation follows the same lines as in the comparison of PF with P. Hence, from a pure subsidy point of view, the agenda-setter prefers parties without fairness and commitment, whereas other project winners weakly prefer the opposite.

The commitment effect on project winners’ utility implies that the project part of \( U_0(\cdot) \) is weakly higher under PFC than under P. The argument is as in the comparison of PC with P. Hence, from a pure project point of view, project winners weakly prefers PFC over P.

Thus, we find opposing effects for the agenda-setter, which explains the ambiguities in the comparison. However, if \( \hat{s} \) was sufficiently high, the fairness effect would always dominate over the commitment effect, and the agenda-setter would always prefer P over PFC. By contrast, for other project winners, we find effects that go into the same direction. Hence, results are unambiguously given.

Fact 5.15
Utility of project losers may be higher or lower under PFC than under P.

As for project winners, the findings in Fact 5.15 are a combination of the analyses in Sections 5.3 and 5.4, that is they combine the fairness effect and the commitment effect. Since these effects might oppose each other, and because project losers have to be divided into different subgroups (if \( p < m \)), it is obvious no clear-cut results for members of Party \( \mathcal{L} \) can be derived.

Case \( n = \frac{m-p}{1-p} \)

The society is split into the same subgroups as in Section 5.4 in case \( n = \frac{m-p}{1-p} \). Hence, we reuse the notation introduced there.
\[
\begin{array}{c|c|c}
\text{Case} & V_w \geq V^* = V^{**} & V_w < V^* = V^{**} \\
\hline
m > p & \begin{align*}
W(\pi_1^{p<m}) & > W(\pi_{FC1}^{p<m}) \\
U_0(\pi_1^{p<m}) & > U_0(\pi_{FC1}^{p<m}) \\
U_{[0,p]}(\pi_1^{p<m}) & < U_{[0,p]}(\pi_{FC1}^{p<m}) \\
U_{[p,m]}(\pi_1^{p<m}) & = U_{[p,m]}(\pi_{FC1}^{p<m}) \\
U_{[m,1]}(\pi_1^{p<m}) & > U_{[m,1]}(\pi_{FC1}^{p<m})
\end{align*} & \begin{align*}
W(\pi_2^{p<m}) & > W(\pi_{FC2}^{p<m}) \\
U_0(\pi_2^{p<m}) & > U_0(\pi_{FC2}^{p<m}) \\
U_{[0,p]}(\pi_2^{p<m}) & < U_{[0,p]}(\pi_{FC2}^{p<m}) \\
U_{[p,m]}(\pi_2^{p<m}) & = U_{[p,m]}(\pi_{FC2}^{p<m}) \\
U_{[m,1]}(\pi_2^{p<m}) & > U_{[m,1]}(\pi_{FC2}^{p<m})
\end{align*} \\
\hline
\frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} > n & \begin{align*}
W(\pi_1^{p<m}) & = W(\pi_{FC3}^{p<m}) \\
U_0(\pi_1^{p<m}) & > U_0(\pi_{FC3}^{p<m}) \\
U_{[0,p]}(\pi_1^{p<m}) & = U_{[0,p]}(\pi_{FC3}^{p<m}) \\
U_{[p,m]}(\pi_1^{p<m}) & = U_{[p,m]}(\pi_{FC3}^{p<m}) \\
U_{[m,1]}(\pi_1^{p<m}) & = U_{[m,1]}(\pi_{FC3}^{p<m})
\end{align*} & \begin{align*}
W(\pi_2^{p<m}) & = W(\pi_{FC4}^{p<m}) \\
U_0(\pi_2^{p<m}) & > U_0(\pi_{FC4}^{p<m}) \\
U_{[0,p]}(\pi_2^{p<m}) & = U_{[0,p]}(\pi_{FC4}^{p<m}) \\
U_{[p,m]}(\pi_2^{p<m}) & = U_{[p,m]}(\pi_{FC4}^{p<m}) \\
U_{[m,1]}(\pi_2^{p<m}) & = U_{[m,1]}(\pi_{FC4}^{p<m})
\end{align*} \\
\hline
m \leq p & \begin{align*}
W(\pi_{p\geq m}) & > W(\pi_{FC1}^{p\geq m}) \\
U_0(\pi_{p\geq m}) & > U_0(\pi_{FC1}^{p\geq m}) = U_{[0,p]}(\pi_{p\geq m}) > U_{[0,p]}(\pi_{FC1}^{p\geq m}) \\
U_{[p,1]}(\pi_{p\geq m}) & > U_{[p,1]}(\pi_{FC1}^{p\geq m})
\end{align*} & \begin{align*}
W(\pi_{p\geq m}) & = W(\pi_{FC2}^{p\geq m}) \\
U_0(\pi_{p\geq m}) & > U_0(\pi_{FC2}^{p\geq m}) = U_{[0,p]}(\pi_{p\geq m}) = U_{[0,p]}(\pi_{FC2}^{p\geq m}) \\
U_{[p,1]}(\pi_{p\geq m}) & = U_{[p,1]}(\pi_{FC2}^{p\geq m})
\end{align*}
\end{array}
\]

Table 5.7: Comparison of PFC with P if \( n = \frac{m-p}{1-p} \).
The findings in Table 5.7 are unambiguous and have similar intuition as the findings in Table 5.1. This is a consequence of the fact that the commitment effect disappears if \( n = \frac{m-p}{1-p} \) while the fairness effect prevails. Hence, differences in proposal-making are solely due to the fairness effect. It implies:

**Fact 5.16**
Welfare is *never* lower under P than under PFC.

**Fact 5.17**
Utility of the agenda-setter is *always* higher under P than under PFC.

**Fact 5.18**
Utility of other project winners is *never* lower under PFC than under P.

**Fact 5.19**
Utility of project losers is *never* lower under P than under PFC.

**Case \( n > \frac{m-p}{1-p} \)**

In this case, we find the same subgroups as in Section 5.4 if \( n > \frac{m-p}{1-p} \). We therefore reuse the notation introduced there.

<table>
<thead>
<tr>
<th>( m \leq p )</th>
<th>( \frac{1}{1+\lambda} &gt; p )</th>
<th>( U_0(\pi^{p \geq m}) &gt; U_0(\pi_{FC1}^{p \geq m}) = U_{0,p}(\pi_{FC1}^{p \geq m}) &gt; U_{0,p}(\pi^{p \geq m}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{1+\lambda} \leq p )</td>
<td>( U_0(\pi^{p \geq m}) &gt; U_0(\pi_{FC2}^{p \geq m}) = U_{0,p}(\pi_{FC2}^{p \geq m}) = U_{0,p}(\pi^{p \geq m}) )</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8: Comparison of PFC with P if \( m \leq p \), and \( n > \frac{m-p}{1-p} \).
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
 & $V_w \geq V_{w}^{**} > V_{w}^{*}$ & $V_w^{**} > V_w \geq V_{w}^{*}$ & $V_{w}^{**} > V_{w} > V_{w}^{*}$ \\
\hline
$m > p$ & $W(\pi_1^{p<m}) > W(\pi_{FC1}^{p<m})$ & $W(\pi_1^{p<m}) \leq W(\pi_{FC2}^{p<m})$ & $W(\pi_2^{p<m}) > W(\pi_{FC2}^{p<m})$ \\
\hline
$\frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} > n$ & $U_0(\pi_1^{p<m}) > U_0(\pi_{FC1}^{p<m})$ & $U_0(\pi_1^{p<m}) > U_0(\pi_{FC1}^{p<m})$ & $U_0(\pi_2^{p<m}) > U_0(\pi_{FC2}^{p<m})$ \\
\hline
& $U_{[0,p]}(\pi_1^{p<m}) \leq U_{[0,p]}(\pi_{FC1}^{p<m})$ & $U_{[0,p]}(\pi_1^{p<m}) \leq U_{[0,p]}(\pi_{FC1}^{p<m})$ & $U_{[0,p]}(\pi_1^{p<m}) < U_{[0,p]}(\pi_{FC1}^{p<m})$ \\
& $U_{[p,m]}(\pi_1^{p<m}) = U_{[p,m]}(\pi_{FC1}^{p<m})$ & $U_{[p,m]}(\pi_1^{p<m}) = U_{[p,m]}(\pi_{FC1}^{p<m})$ & $U_{[p,m]}(\pi_1^{p<m}) = U_{[p,m]}(\pi_{FC1}^{p<m})$ \\
& $U_{[m,\tilde{m}]}(\pi_1^{p<m}) < U_{[m,\tilde{m}]}(\pi_{FC1}^{p<m})$ & $U_{[m,\tilde{m}]}(\pi_1^{p<m}) < U_{[m,\tilde{m}]}(\pi_{FC1}^{p<m})$ & $U_{[m,\tilde{m}]}(\pi_1^{p<m}) < U_{[m,\tilde{m}]}(\pi_{FC1}^{p<m})$ \\
& $U_{[\tilde{m},1]}(\pi_1^{p<m}) > U_{[\tilde{m},1]}(\pi_{FC3}^{p<m})$ & $U_{[\tilde{m},1]}(\pi_1^{p<m}) \leq U_{[\tilde{m},1]}(\pi_{FC2}^{p<m})$ & $U_{[\tilde{m},1]}(\pi_1^{p<m}) > U_{[\tilde{m},1]}(\pi_{FC3}^{p<m})$ \\
\hline
$\frac{1-(1+\lambda)p}{(1+\lambda)(1-p)} \leq n$ & $W(\pi_1^{p<m}) > W(\pi_{FC3}^{p<m})$ & $W(\pi_1^{p<m}) \leq W(\pi_{FC4}^{p<m})$ & $W(\pi_2^{p<m}) = W(\pi_{FC4}^{p<m})$ \\
\hline
& $U_0(\pi_1^{p<m}) > U_0(\pi_{FC3}^{p<m})$ & $U_0(\pi_1^{p<m}) > U_0(\pi_{FC4}^{p<m})$ & $U_0(\pi_2^{p<m}) > U_0(\pi_{FC4}^{p<m})$ \\
& $U_{[0,p]}(\pi_1^{p<m}) > U_{[0,p]}(\pi_{FC3}^{p<m})$ & $U_{[0,p]}(\pi_1^{p<m}) \geq U_{[0,p]}(\pi_{FC4}^{p<m})$ & $U_{[0,p]}(\pi_1^{p<m}) < U_{[0,p]}(\pi_{FC4}^{p<m})$ \\
& $U_{[p,m]}(\pi_1^{p<m}) = U_{[p,m]}(\pi_{FC3}^{p<m})$ & $U_{[p,m]}(\pi_1^{p<m}) = U_{[p,m]}(\pi_{FC4}^{p<m})$ & $U_{[p,m]}(\pi_1^{p<m}) = U_{[p,m]}(\pi_{FC4}^{p<m})$ \\
& $U_{[m,\tilde{m}]}(\pi_1^{p<m}) < U_{[m,\tilde{m}]}(\pi_{FC3}^{p<m})$ & $U_{[m,\tilde{m}]}(\pi_1^{p<m}) < U_{[m,\tilde{m}]}(\pi_{FC4}^{p<m})$ & $U_{[m,\tilde{m}]}(\pi_1^{p<m}) < U_{[m,\tilde{m}]}(\pi_{FC4}^{p<m})$ \\
& $U_{[\tilde{m},1]}(\pi_1^{p<m}) > U_{[\tilde{m},1]}(\pi_{FC3}^{p<m})$ & $U_{[\tilde{m},1]}(\pi_1^{p<m}) < U_{[\tilde{m},1]}(\pi_{FC4}^{p<m})$ & $U_{[\tilde{m},1]}(\pi_1^{p<m}) < U_{[\tilde{m},1]}(\pi_{FC4}^{p<m})$ \\
\hline
\end{tabular}
\caption{Comparison of PFC with P if $m > p$, and $n > \frac{m-p}{1-p}$.}
\end{table}
Tables 5.9 is more or less the opposite of Tables 5.5 and Table 5.8 is the same as Table 5.6. Arguments are identical but reversed compared to the case $n < \frac{m-p}{1-p}$. We will therefore omit the analysis.

**Fact 5.20**
Welfare may be higher or lower under PFC than under P.

**Fact 5.21**
Utility of the agenda-setter is always higher under P than under PFC.

**Fact 5.22**
Utility of other project winners may be higher or lower under PFC than under P.

**Fact 5.23**
Utility of project losers may be higher or lower under PFC than under P.

### 5.6 Other Comparisons

Other comparisons might suggest themselves. For example a comparison of PC with PFC, or a comparison of PF with PFC.

By using the concepts *subsidy effect* and *commitment effect*, comparisons of this kind can be reduced to the comparison of P with PF, or P with PC, respectively.

For example, in the comparison of PC with PFC, the *commitment effect* is unimportant since commitment is present under both party devices. Hence, the only difference in proposal-making is due to the *fairness effect*. Consequently, a table of the comparison of PC and PFC would be similar to Table 5.1 and results on welfare and individual utilities would be equal to the results in Section 5.3.

Analogously, in the comparison of PF with PFC, the fairness effect is unimportant since fairness is present under both party devices. Thus, the only difference in proposal-making results from the *commitment effect*. Consequently, tables of comparison would be similar to Tables 5.2-5.4 and corresponding results on welfare and individual utility would be the same as in Section 5.4.
Appendix A

Proofs for Chapter 2

Proof of Proposition 2.2

S1: Uniform taxes imply the following problem for the agenda-setter:

$$\max_{\pi \in \Pi} \{(g(\pi)(V_w - (1 + \lambda)k) + s_0(\pi) - (1 + \lambda)S(\pi)) I(\pi)\}.$$  

Recall that $u_0(\pi) = g(\pi)(V_w - (1 + \lambda)k) + s_0(\pi) - (1 + \lambda)S(\pi)$.

S2: We first construct the optimal proposal for the agenda-setter when the project is not proposed. We denote this proposal by $\tilde{\pi}$.

We claim that $\tilde{\pi}$ is given by $g(\tilde{\pi}) = 0, t_j(\tilde{\pi}) = 0, \forall j \in [0, 1]$ and

$$s_j(\tilde{\pi}) = \begin{cases} \hat{s} & \text{for } j = 0 \\ 0 & \text{for } j \in [0, 1]. \end{cases}$$

To prove our claim, we first argue that, in equilibrium, proposal $\tilde{\pi}$ will be adopted (that is $I(\tilde{\pi}) = 1$) as $u_j(\tilde{\pi}) \geq 0, \forall j \in [0, 1]$.

Second, any other proposal would yield a smaller $u_0(\pi)$. Hence proposal $\tilde{\pi}$ maximizes $u_0(\pi)I(\pi)$ under the restriction that $g(\pi) = 0$.

S3: We now consider proposals when the project is proposed. In contrast to Step 2, there is no unique proposal for all distributions of parameters $p, m, V_w, V_l$ and $k$. We therefore have to distinguish several cases.

S4: Consider the case $p \geq m$.

We claim that the optimal proposal for the agenda-setter is given by $g(\pi) = 1, t_j(\pi) = (1 + \lambda)k, \forall j \in [0, 1]$ and $s_j(\pi) = s_j(\tilde{\pi})$. 
To prove the claim, we first argue that, in equilibrium, proposal $\pi$ will be adopted (that is $I(\pi) = 1$) as $u_j(\pi) > 0$, \( \forall j \in [0, p] \).

Second, any other proposal would yield a smaller $u_0(\pi)$. Hence proposal $\pi$ maximizes $U_0(\pi) = u_0(\pi)I(\pi)$ under the restriction that $g(\pi) = 1$ and $p \geq m$.

S5: For the agenda-setter, proposal $\pi$ is preferable to proposal $\tilde{\pi}$, as

$$U_0(\tilde{\pi}) = \hat{s} < \hat{s} + V_w - (1 + \lambda)k = U_0(\pi).$$

Hence, in case $p \geq m$, proposal $\pi$ will be implemented.

S6: Consider the case $p < m$ and $V_w \geq V_w^*$.

We claim that the optimal proposal for the agenda-setter is given by $g(\pi) = 1$, $t_j(\pi) = (1 + \lambda)(m - p)s^L$, \( \forall j \in [0, 1] \) and

$$s_j(\pi) = \begin{cases} 
\hat{s} & \text{for } j = 0 \\
0 & \text{for } j \in [0, p] \\
s^L(\pi) & \text{for } j \in [p, m] \\
0 & \text{for } j \in [m, 1], 
\end{cases}$$

where $s^L(\pi) = \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)(m - p)}$.

To prove the claim, note first that proposal $\pi$ will be adopted in equilibrium (that is $I(\pi) = 1$) because $u_j(\pi) \geq 0$, \( \forall j \in [0, p] \) and $U_j(\pi) = 0$, $j \in [p, m]$.

Second, any other proposal $\pi'$ with $S(\pi') < S(\pi)$ would not be adopted. This follows directly from the fact that a smaller $S(\pi)$ implies that either the fraction of subsidized project losers is smaller than $m - p$ or the subsidy given to each subsidized project loser is smaller than $s^L(\pi)$, or both. However, the fraction of voters supporting $\pi'$ is smaller than $m$ and thus $I(\pi') = 0$.

Third, there is no other proposal $\pi'$ with $S(\pi') \geq S(\pi)$ that yields higher utility for the agenda-setter.

From these considerations it follows that proposal $\pi$ maximizes $U_0(\pi) = u_0(\pi)I(\pi)$ under the restriction that $g(\pi) = 1$ and $p < m$, $V_w \geq V_w^*$.

S7: For the agenda-setter, proposal $\pi$ is preferable to proposal $\tilde{\pi}$ as

$$U_0(\tilde{\pi}) = \hat{s} \leq \hat{s} + V_w - V_w^* = U_0(\pi).$$

Hence, in case $p < m$ and $V_w \geq V_w^*$, proposal $\pi$ will be implemented.
Appendix A. Proofs for Chapter 2

S8: Consider the case $p < m < \frac{1}{1+\lambda}$ and $V_w < V^*_w$.

We claim that the optimal proposal for the agenda-setter is given by $g(\pi) = 1$, $t_j = (1 + \lambda)[ps^W(\pi) + (m - p)s^C(\pi)]$, $\forall j \in [0,1]$ and

$$s_j(\pi) = \begin{cases} \hat{s} & \text{for } j = 0 \\ s^W(\pi) & \text{for } j \in [0, p] \\ s^C(\pi) & \text{for } j \in [p, m] \\ 0 & \text{for } j \in [m,1], \end{cases}$$

where

$$s^W(\pi) = \frac{1 - (1 + \lambda)(m - p)}{1 - (1 + \lambda)m}(V^*_w - V_w),$$

$$s^C(\pi) = \frac{(1 + \lambda)k - (1 + \lambda)pV_w - (1 - (1 + \lambda)p)V_i}{1 - (1 + \lambda)m}.$$

To prove the claim, we first argue that proposal $\pi$ will be adopted in equilibrium (that is $I(\pi) = 1$), as $u_j(\pi) = 0$, $\forall j \in [0, m]$.

Second, any other proposal $\pi'$ with $S(\pi') < S(\pi)$ would not be adopted. The reasons are the same as in Step 6.

Third, there is no other proposal $\pi'$ with $S(\pi') \geq S(\pi)$ that yields higher utility to the agenda-setter.

Again, we can conclude that proposal $\pi$ as stated above maximizes $U_0(\pi) = u_0(\pi)I(\pi)$ under the restriction that $g(\pi) = 1$ and $p < m$, $V_w < V^*_w$.

S9: For the agenda-setter, proposal $\tilde{\pi}$ is preferable to proposal $\pi$, as

$$U_0(\tilde{\pi}) = \hat{s} > \hat{s} + V_w - V^*_w > U_0(\pi).$$

Hence, in the case $p < m < \frac{1}{1+\lambda}$ and $V_w < V^*_w$, proposal $\tilde{\pi}$ will be implemented.

S10. Consider finally the last case $\max\{p, \frac{1}{1+\lambda}\} < m$ and $V_w < V^*_w$.

We claim that if $\frac{1}{1+\lambda} < m$ and $V_w < V^*_w$, there is no constitutional proposal for $I(\pi) = 1$.

From Step 8 we know that if $V_w < V^*_w$, it will be necessary to subsidize not only a fraction of $m - p$ project losers but also all project winners, that is $s^W(\pi) > 0$. This is due to the fact that $V_w$ is not high enough to compensate project winners for the utility loss incurred by tax $V^*_w$. In this case, the overall fraction of subsidized voters is
equal to $m$, so the costs for increasing all subsidies by one dollar are equal to $(1 + \lambda)m$ (that is, in order to increase subsidies by one dollar, taxes to the tune of $(1 + \lambda)m$ have to be paid). Otherwise the benefit from receiving one dollar of redistribution is equal to one. As $(1 + \lambda)m > 1$, the costs of redistribution are higher than the benefit from redistribution, so project losers cannot be compensated for their utility loss.\footnote{In order to compensate project losers for utility losses incurred by $g(\pi) = 1$, subsidies should become negative (note that $s^L(\pi)$ in Step 8 turns negative if $1 - (1 + \lambda)m < 0$). But as we do not allow for negative subsidies, there is no way to compensate project losers.}

\textbf{Proof of Lemma 2.6}

We show that an $\epsilon > 0$ exists such that $V_w < V_w^*$ holds for all $|V_w - k(1 + \lambda)| < |V_l - k(1 + \lambda)|$ and $p < \epsilon$. We note that

$$V_w^* - V_w = (1 + \lambda) \frac{k - (m - p)V_l}{1 - (1 + \lambda)(m - p)} - V_w$$

$$= (1 + \lambda)k - (1 + \lambda)(m - p)V_l - V_w + (1 + \lambda)(m - p)V_w$$

$$> (1 + \lambda)k + (1 + \lambda)(m - p) (V_w - 2k(1 + \lambda)) - V_w + (1 + \lambda)(m - p)V_w$$

$$> \frac{2(1 + \lambda)(m - p) - 1}{1 - (1 + \lambda)(m - p)} (V_w - k(1 + \lambda)),$$

where we have used $-V_l > V_w - 2k(1 + \lambda)$. Recall that for all projects $V_w - k(1 + \lambda) > 0$ holds. Moreover, for all $p < \frac{1 - \lambda}{2(1 + \lambda)}$ we have $2(1 + \lambda)(m - p) - 1 > 0$. Hence $PALP$ holds for constitution $\Pi_T$.

\textbf{Derivation of Welfare}

The utilitarian welfare measure for a particular proposal is given by

$$W(\pi) := I(\pi)[(pV_w + (1 - p)V_l - (1 + \lambda)k)g(\pi) - \lambda S(\pi)].$$

If proposals are not unique, only upper and lower bounds for welfare may be computed. By Definition 1, a redistribution-efficient proposal yields maximal welfare as deadweight loss from redistribution is minimized for all $\pi$ for which $g(\pi)I(\pi) = \text{const.}$
Constitution Π

(I.) Highest levels of welfare

(i.) $p \geq m$: The lowest level of $S(\pi)$ for $I(\pi) = 1$ is given by $S(\pi) = 0$. Note that, for this case, the tax scheme must be chosen such that $V_w - t_j \geq 0$ holds. Otherwise project winners would not support the proposal, and the required majority cannot be achieved.

Hence the highest level of welfare under constitution Π in case $p \geq m$ is given by

$$W = pV_w + (1 - p)V_l - (1 + \lambda)k.$$ 

(ii.) $p < m$: $S(\pi)$ is minimized if the smallest share of voters is subsidized with the smallest amount of subsidies such that $I(\pi) = 1$. The smallest share of subsidized voters occurs if a fraction of $(m - p)$ project losers is subsidized. The minimal subsidy that must be given to them is $\max\{0, -V_l\}$. Again, the tax scheme must be such that project winners and subsidized project losers will support the proposal.

The highest level of welfare under constitution Π in case $p < m$ is given by

$$W = pV_w + (1 - p)V_l - (1 + \lambda)k - \lambda(m - p)\max\{0, -V_l\}.$$ 

(II.) Lowest levels of welfare

No matter if $p \geq m$ or $p < m$, the lowest level of welfare occurs if every voter receives the maximal subsidy $\hat{S}$, that is $S = \hat{s}$. Hence the lower bound on welfare is given by

$$W = pV_w + (1 - p)V_l - (1 + \lambda)k - \lambda\hat{s}.$$ 

Constitutions Π\(_T\), Π\(_S\) and Π\(_ST\)

Under constitutions Π\(_T\), Π\(_S\), and Π\(_ST\), the total amount of subsidies $S(\pi)$ is uniquely given and hence welfare functions can be derived directly from Propositions 2.2-2.4.

- From Proposition 2.2

$$S(\pi_T^*) = \begin{cases} 
(m - p)(V_w^* - V_w) & \text{if } p < m \text{ and } V_w \geq V_w^* \\
0 & \text{otherwise}
\end{cases}$$
The project will be proposed and implemented if \( p < m \) and \( V_w \geq V_w^* \) or if \( p \geq m \). Hence the welfare level under constitution \( \Pi_T \) is given by

\[
W(\pi_T^*) = \begin{cases} 
 pV_w + (1 - p)V_l - (1 + \lambda)k - \lambda(m - p)(V_w^* - V_w) & \text{if } p < m \text{ and } V_w \geq V_w^* \\
 0 & \text{if } p < m \text{ and } V_w < V_w^* \\
pV_w + (1 - p)V_l - (1 + \lambda)k & \text{if } p \geq m
\end{cases}
\]

- From Proposition 2.3

\( S(\pi) = \hat{s} \) and the project will always be proposed and adopted. Welfare is given by

\[
W(\pi_S^*) = pV_w - (1 - p)V_l - (1 + \lambda)k - \lambda\hat{s}.
\]

- From Proposition 2.4

\( S(\pi) = 0 \) and the project will be adopted only if \( p \geq m \). Hence the welfare level under constitution \( \Pi_{ST} \) is given by

\[
W(\pi_{ST})^* = \begin{cases} 
 0 & \text{if } p < m \\
pV_w + (1 - p)V_l - (1 + \lambda)k & \text{if } p \geq m
\end{cases}
\]

Proof of Proposition 2.6

First we note that \( p < 1 - m \) implies \( p < m \). Recall that under constitution \( \Pi_{ST} \) the project will never be implemented if \( p < m \) (see Proposition 2.4). Hence from an ex-ante perspective all citizens obtain a utility of zero under constitution \( \Pi_{ST} \).

Under constitution \( \Pi_T \) the project may be implemented if \( p < m \). More precisely, if \( p < m \), the project will be implemented if and only if \( V_w \geq V_w^* \) (see Proposition 2.2). Rewriting this conditions shows that the project will be implemented if and only if

\[
k \leq \frac{1 - (1 + \lambda)(m - p)}{1 + \lambda} V_w =: k^*.
\]

Hence a citizen’s expected utility in the constitutional stage is given by

\[
\mathbb{E}[W(\pi_T^*)] = \frac{1}{k} \int_0^{\min\{x,k^*\}} pV_w - (1 + \lambda)k - \lambda(m - p) \frac{(1 + \lambda)k}{1 - (1 + \lambda)(m - p)} dk, \quad (A.1)
\]
where we have used the facts that \( k \) is uniformly distributed on \([0; \overline{k}]\) and that welfare would be zero for realizations of \( k \) with \( k > k^* \). Equation A.1 can be transformed into

\[
\mathbb{E}[W(\pi_T^*)] = \frac{1}{\overline{k}} \left[ pV_w \min\{\overline{k}, k^*\} - \frac{1}{2} (1 + \lambda) \left( \frac{1 - (m - p)}{1 - (1 + \lambda)(m - p)} \right) \left( \min\{\overline{k}, k^*\} \right)^2 \right].
\]

Citizens weakly prefer constitution \( \Pi_T \) over constitution \( \Pi_{ST} \) if and only if \( \mathbb{E}[W(\pi_T^*)] \geq 0 \), which is equivalent to

\[
\min\{\overline{k}, k^*\} \leq 2p(1 - (1 + \lambda)(m - p))V_w =: \hat{k}. \quad (A.2)
\]

It is straightforward to show that \( k^* > \hat{k} \) for \( 1 - p > m \). As a consequence, utilitarian welfare is higher for \( \Pi_T \) if \( \overline{k} < \hat{k} \). It is higher for \( \Pi_{ST} \) if \( \overline{k} > \hat{k} \).

Proof of Proposition 2.7

In order to examine the agenda-setter’s incentives for improving the project, it will be useful to consider her utility for given project parameters and for each constitution. From Propositions 2.1 to 2.4 we obtain

\[
U_0(\pi_T^*) = \hat{s} + V_w \quad \text{(A.3)}
\]

\[
U_0(\pi_T^*) = \begin{cases} 
\hat{s} + V_w - V_w^* & \text{if } p < m \text{ and } V_w \geq V_w^* \\
\hat{s} & \text{if } p < m \text{ and } V_w < V_w^* \\
\hat{s} + V_w - (1 + \lambda)k & \text{if } p \geq m.
\end{cases} \quad (A.4)
\]

\[
U_0(\pi_S^*) = \hat{s} + V_w \quad \text{(A.5)}
\]

\[
U_0(\pi_{ST}^*) = \begin{cases} 
0 & \text{if } p < m \\
V_w - (1 + \lambda)k & \text{if } p \geq m.
\end{cases} \quad (A.6)
\]

Constitutions involving an arbitrary tax code (that is constitutions \( \Pi \) and \( \Pi_S \)) yield utility to the agenda-setter that is independent of the project parameters \( V_l, k, \) and \( p \). Hence exerting costly effort to enhance any project parameter other than \( V_w \) will never be profitable.

Under constitution \( \Pi_{ST} \) the agenda-setter may profit from exerting effort if \( p \) can be increased from \( \overline{p} < m \) to \( \overline{p} \geq m \). For sufficiently small costs \( c \), exerting effort in order to reduce \( k \) is optimal for \( p \geq m \).
Under constitution $\Pi_T$ the agenda-setter profits from increasing $p$ from $p < m$ to $p \geq m$ if $c$ is sufficiently small. Moreover, the agenda-setter has an incentive to increase $p$ even in the case $p < m$, as long as $V_w \geq V_w^*$. If $p < m$ and $V_w < V_w^*$, the agenda-setter has no incentive to enhance project efficiency. If $p \geq m$, the agenda-setter may have incentives to reduce project costs $k$ as under constitution $\pi_{ST}$. If $p < m$ and $V_w \geq V_w^*$, the agenda-setter has incentives to increase $V_l$ and to reduce $k$ (as $V_w^*$ is decreasing in $V_l$ and increasing in $k$).

Of course, the agenda-setter will enhance project efficiency if and only if the net gains from exerting effort exceed the costs involved in the effort.

$\square$
Appendix B

Proofs for Chapter 3

Proof of Proposition 3.1

Consider an arbitrary proposal $\pi^W$. Suppose that $\pi^*\mathcal{L}$ is a best response to $\pi^W$.

Then, proposal $\tilde{\pi}\mathcal{L}$, with

$$
\left\{ g(\tilde{\pi}\mathcal{L}) = g(\pi^*\mathcal{L}), (s_j(\tilde{\pi}\mathcal{L}) = s_j(\pi^*\mathcal{L}), t_j(\tilde{\pi}\mathcal{L}) = t_j(\pi^*\mathcal{L})) \right\}_{j \in [0,1]},
\left( s_{W}(\tilde{\pi}\mathcal{L}) = \hat{s}, t_{W}(\tilde{\pi}\mathcal{L}) = 0 \right),
$$

is also a best response to $\pi^W$.\(^1\) Moreover, proposal $\tilde{\tilde{\pi}}\mathcal{L}$, with

$$
\left\{ g(\tilde{\tilde{\pi}}\mathcal{L}) = g(\pi^*\mathcal{L}), (s_j(\tilde{\tilde{\pi}}\mathcal{L}) = s_j(\pi^*\mathcal{L}), t_j(\tilde{\tilde{\pi}}\mathcal{L}) = t_j(\pi^*\mathcal{L})) \right\}_{j \in [0,1]},
\left( s_{W}(\tilde{\tilde{\pi}}\mathcal{L}) = 0, t_{W}(\tilde{\tilde{\pi}}\mathcal{L}) = \hat{t} \right),
$$

is also a best response to $\pi^W$.

Now suppose that $\mathcal{L}$ applies the following selection device:

- If $\pi^W$ is chosen such that $\{ g(\pi^*\mathcal{L}) = 0, s_{\mathcal{L}}(\pi^*\mathcal{L}) = \hat{s}, t_{\mathcal{L}}(\pi^*\mathcal{L}) = 0 \}$, suggest $\tilde{\pi}\mathcal{L}$.
- Otherwise, suggest $\tilde{\tilde{\pi}}\mathcal{L}$.\(^2\)

For $\mathcal{W}$, this implies that if he chooses $\pi^W$ such that

$$
\left\{ g(\pi^*\mathcal{L}) = 0, s_{\mathcal{L}}(\pi^*\mathcal{L}) = \hat{s}, t_{\mathcal{L}}(\pi^*\mathcal{L}) = 0 \right\},
$$

he receives: $u_{W}(\tilde{\pi}\mathcal{L}) = \hat{s}$.

otherwise, he receives: $u_{W}(\tilde{\tilde{\pi}}\mathcal{L}) = g(\tilde{\tilde{\pi}}\mathcal{L})V_{w} - \hat{t} \leq V_{w} - \hat{t}$.

It is obvious that for sufficiently high $\hat{s}$ and $\hat{t}$, we find

$$
u_{W}(\tilde{\pi}\mathcal{L}) > V_{w} - \hat{t} \geq u_{W}(\tilde{\tilde{\pi}}\mathcal{L}).$$

\(^1\)The reason is that proposals $\pi^*\mathcal{L}$ and $\tilde{\pi}\mathcal{L}$ only differ with respect to subsidies and taxes to measure zero groups. Thus, the outcome for $\mathcal{L}$ is not affected by this change. More precisely, we find:

$$
u_{j}(\tilde{\pi}\mathcal{L}) = u_{j}(\pi^*\mathcal{L}), \forall j \in [0,1],$$

which implies that $I(\pi^W, \tilde{\pi}\mathcal{L}) = I(\pi^W, \pi^*\mathcal{L})$ and $u_{\mathcal{L}}(\tilde{\pi}\mathcal{L}) = u_{\mathcal{L}}(\pi^*\mathcal{L})$, and confirms that $\mathcal{L}$ is indifferent between $\tilde{\pi}\mathcal{L}$ and $\pi^*\mathcal{L}$. Thus, $\tilde{\pi}\mathcal{L}$ is also a best response to $\pi^W$.

\(^2\)In Section 3.5.1 we show that $\mathcal{W}$ is able to make proposals $\pi^W \in \Pi$ such that $\{ g(\pi^*\mathcal{L}) = 0, s_{\mathcal{L}}(\pi^*\mathcal{L}) = \hat{s}, t_{\mathcal{L}}(\pi^*\mathcal{L}) = 0 \}$ does not hold.
Appendix B. Proofs for Chapter 3

Thus, in equilibrium, the project is never implemented since $W$ prefers to renounce the project and receive high net-subsidies in return.

Obviously, the selection device is an ex-ante optimal for $L$, because in equilibrium he always obtains the highest possible utility, that is

$$u_L(\hat{\pi}^L) = \hat{s} \geq u_L(\pi^L), \ \forall \pi^L \in \Pi.$$

\[\square\]

Proof of Proposition 3.2

Consider an arbitrary proposal $\pi^W$. Suppose that $\pi^*L$ is a best response to $\pi^W$. Then, proposal $\tilde{\pi}^L$, with

$$\left\{ g(\tilde{\pi}^L) = g(\pi^*L), \left( s_j(\tilde{\pi}^L) = s_j(\pi^*L) \right)_{j \in [0,1]}, s_W(\tilde{\pi}^L) = \hat{s}, t(\tilde{\pi}^L) = t(\pi^*L) \right\},$$

is also a best response to $\pi^W$.\(^3\) Moreover, proposal $\tilde{\tilde{\pi}}^L$, with

$$\left\{ g(\tilde{\tilde{\pi}}^L) = g(\pi^*L), \left( s_j(\tilde{\tilde{\pi}}^L) = s_j(\pi^*L) \right)_{j \in [0,1]}, s_W(\tilde{\tilde{\pi}}^L) = 0, t(\tilde{\tilde{\pi}}^L) = t(\pi^*L) \right\},$$

is also a best response to $\pi^W$.

Now suppose that $L$ applies the following selection device:

- If $\pi^W$ is chosen such that $\left\{ g(\pi^*L) = 0, s_L(\pi^*L) = \hat{s}, t(\pi^*L) = 0 \right\}$, suggest $\tilde{\pi}^L$.
- Otherwise, suggest $\tilde{\tilde{\pi}}^L$.\(^4\)

For $W$, this implies that if he chooses $\pi^W$ such that

$$\left\{ g(\pi^*L) = 0, s_L(\pi^*L) = \hat{s}, t(\pi^*L) = 0 \right\},$$

he receives: $u_W(\tilde{\pi}^L) = \hat{s}$. Otherwise, he receives:

$$u_W(\tilde{\pi}^L) = g(\tilde{\pi}^L)V_w - t(\tilde{\pi}^L) \leq V_w.$$

It is obvious that for sufficiently high $\hat{s}$, we find

$$u_W(\tilde{\pi}^L) > V_w \geq u_W(\tilde{\pi}^L).$$

Thus, in equilibrium, the project is never implemented since $W$ prefers to renounce the project and receive high subsidies in return.

\(^3\)Again, the reason is that proposals $\pi^*L$ and $\tilde{\pi}^L$ only differ with respect to subsidies to measure zero groups. Thus, the outcome for $L$ is not affected by this change (see Footnote 1).

\(^4\)In Section 3.5.2 we show that $W$ is able to make proposals $\pi^W \in \Pi_T$ such that $\left\{ g(\pi^*L) = 0, s_L(\pi^*L) = \hat{s}, t(\pi^*L) = 0 \right\}$ does not hold.

146
Again, the selection device is ex-ante optimal for $L$, because it guarantees him the highest possible utility in equilibrium, that is

$$u_L(\tilde{\pi}^L) = \hat{s} \geq u_L(\pi^L), \forall \pi^L \in \Pi_T.$$ 

\[\square\]

**Proof of Proposition 3.3**

Consider an arbitrary proposal $\pi^W$. Suppose that $\pi^*L$ is a best response to $\pi^W$. Then, proposal $\tilde{\pi}^L$, with

$$\{g(\tilde{\pi}^L) = g(\pi^*L), (t_j(\tilde{\pi}^L) = t_j(\pi^*L))_{j \in \{0, 1\}}, t_W(\tilde{\pi}^L) = 0, s(\tilde{\pi}^L) = s(\pi^*L)\},$$

is also a best response to $\pi^W$.\(^5\) Moreover, proposal $\tilde{\tilde{\pi}}^L$, with

$$\{g(\tilde{\tilde{\pi}}^L) = g(\pi^*L), (t_j(\tilde{\tilde{\pi}}^L) = t_j(\pi^*L))_{j \in \{0, 1\}}, t_W(\tilde{\tilde{\pi}}^L) = \hat{t}, s(\tilde{\tilde{\pi}}^L) = s(\pi^*L)\},$$

is also a best response to $\pi^W$.

Now suppose that $L$ chooses the following selection device:

- If $\pi^W$ is chosen such that $\{g(\pi^*L) = 0, s(\pi^*L) = \hat{s}, t_L(\pi^*L) = 0\}$, suggest $\tilde{\pi}^L$.
- Otherwise, suggest $\tilde{\tilde{\pi}}^L$.\(^6\)

For $W$, this implies that if he chooses $\pi^W$ such that $\{g(\pi^*L) = 0, s(\pi^*L) = \hat{s}, t_L(\pi^*L) = 0\}$, he receives: $u_W(\tilde{\pi}^L) = \hat{s}$.

otherwise, he receives:

$$u_W(\tilde{\tilde{\pi}}^L) = s(\tilde{\tilde{\pi}}^L) + g(\tilde{\tilde{\pi}}^L)V_w - \hat{t} \leq \hat{s} + V_w - \hat{t}.$$

It is obvious that for sufficiently high $\hat{t}$, we find

$$u_W(\tilde{\pi}^L) > \hat{s} + V_w - \hat{t} \geq u_W(\tilde{\tilde{\pi}}^L).$$

Thus, in equilibrium, the project is never implemented since $W$ prefers to renounce the project and being exempted from taxation in return.

Again, the selection device is ex-ante optimal for $L$, because he can not do better:

$$u_L(\tilde{\pi}^L) = \hat{s} \geq u_L(\pi^L), \forall \pi^L \in \Pi_S.$$ 

\[\square\]

\(^5\)Again, the reason is that proposal $\pi^*L$ and $\tilde{\pi}^L$ only differ with respect to taxes to measure zero groups. Thus, the outcome is not affected by this change (see Footnote 1).

\(^6\)In Section 3.5.3 we show that $W$ is able to make proposals $\pi^W \in \Pi_S$ such that $\{g(\pi^*L) = 0, s(\pi^*L) = \hat{s}, t_L(\pi^*L) = 0\}$ does not hold.
Appendix B. Proofs for Chapter 3

Proof of Proposition 3.4

We proceed by backward induction: At the voting stage, all individuals vote for their preferred proposal. At the second stage, $L$ chooses his proposal given proposal $\pi^W$. His best-response function is a mapping from and into $\Pi_{ST}$, that is $R : \Pi_{ST} \rightarrow \Pi_{ST}$.

$$R(\pi^W) = \arg\max_{\pi^L \in \Pi_{ST}} \left\{ g(\pi^L)(V_l - (1 + \lambda)k) + s(\pi^L) - (1 + \lambda)s(\pi^L) \right\}, \text{ s.t. } I(\pi^W, \pi^L) = 1.$$  

(B.1)

$R(\pi^W)$ is characterized by:

(i) $s(R(\pi^W)) = 0$,

(ii) $g(R(\pi^W)) = \begin{cases} 1 & \text{if } u_j(\pi^W) > 0 \text{ for more than } 1/2 \text{ of the society} \\ 0 & \text{otherwise.} \end{cases}$

The first item immediately results from inspection of the maximization problem: Increasing $s(\pi^L)$ lowers the utility of all individuals, because the accruing taxes are higher than the subsidy. More precisely, one dollar more of subsidies raises additional per-capita taxes of $1 + \lambda$ dollars. Thus, by increasing $s(\pi^L)$, $L$ can neither increase his own utility, nor the one of any other individual. Consequently, subsidies cannot not be used as compensatory payments for other individuals. The second item requires some explanation: Knowing that $s(R(\pi^W)) = 0$, we obtain:

$$u_\ell(R(\pi^W)) = g(R(\pi^W))V_l.$$  

(B.2)

Because $V_l < 0$, it is obvious that $L$ prefers $g(R(\pi^W)) = 0$ whenever possible. However, we will show that he might be forced to choose $g(R(\pi^W)) = 1$ in order to fulfill the constraint $I(\pi^W, \pi^L) = 1$:

- Suppose $u_j(\pi^W) > 0$ for more than $1/2$ of the society:
  
  In this case, individuals receiving $u_j(\pi^W) > 0$ form a majority. Consequently, to fulfill the constraint $I(\pi^W, \pi^L) = 1$, $L$ needs the support of some of these individuals, that is $u_j(R(\pi^W)) \geq u_j(\pi^W) > 0$ has to hold for some $j$. However, since $s(R(\pi^W)) = 0$, the condition $u_j(R(\pi^W)) > 0$ can only hold if the project is suggested. Thus, $R(\pi^W)$ involves $g(R(\pi^W)) = 1$ in this case.

---

7In Corollary 3.1, we pointed out that it is always optimal for $L$ to chooses a counter-proposal that yields $I(\pi^W, \pi^L) = 1$. 

148
• Otherwise:

In that case, individuals receiving \( u_j(\pi^W) \leq 0 \) form a majority. Consequently, \( \mathcal{L} \) does not need the support of individuals receiving \( u_j(\pi^W) > 0 \) to fulfill the constraint \( I(\pi^W, \pi^L) = 1 \) and it is sufficient if \( u_j(R(\pi^W)) = 0 \geq u_j(\pi^W) \) holds for all \( j \). However, the condition \( u_j(R(\pi^W)) = 0 \) holds if \( g(R(\pi^W)) = 0 \).

Because \( \mathcal{L} \) prefers project rejection whenever possible, \( R(\pi^W) \) comprises \( g(R(\pi^W)) = 0 \) in that case.

Using \( R(\pi^W) \), we now turn to the first stage and explore optimal proposal-making of \( \mathcal{W} \):

\[
\pi^W = \arg\max_{\pi^W \in \Pi} u_{\mathcal{W}}(\pi^W, R(\pi^W)) = g(R(\pi^W))(V_w - (1 + \lambda)k). \tag{B.3}
\]

There exists a multitude of solutions to this problem. Thus \( \pi^W \) represents the set of maximizer. For all \( \pi^*W \in \pi^W \) we find:

1. \( g(\pi^W) \in \begin{cases} \{1\} & \text{if } p > \frac{1}{2} \\ \{0, 1\} & \text{otherwise.} \end{cases} \)

2. \( s(\pi^W) \in \begin{cases} \left[0, \frac{V_w - (1 + \lambda)k}{\lambda}\right] & \text{if } p > \frac{1}{2} \\ [0, \hat{s}] & \text{otherwise.} \end{cases} \)

Items (i) and (ii) immediately follow from the following considerations: Because \( V_w - (1 + \lambda)k > 0 \), it is obvious that \( \mathcal{W} \) tries to induce project adoption, that is \( g(R(\pi^W)) = 1 \), whenever possible. By the characterization of the best-response function, this requires \( u_j(\pi^W) > 0 \) for more than half of the society.

Because \( u_j(\pi^W) > 0 \) can only hold for project winners (remember that positive subsidies lower the utility of all individuals since the utility gain is smaller than the costs incurred by a one dollar increase in subsidies, that is \( 1 < 1 + \lambda \)), \( g(R(\pi^W)) = 1 \) is only attainable if project winners form a majority in the society, that is \( p > \frac{1}{2} \). In this case, a proposal as specified by items (i) and (ii) guarantees \( u_j(\pi^W) > 0 \) for all project winners and hence a majority of the society. Otherwise, if \( p \leq \frac{1}{2} \), \( u_j(\pi^W) > 0 \) cannot hold for more than half of the society because project winners are not sufficiently numerous. Thus, \( g(R(\pi^W)) = 1 \) is not attainable and \( \mathcal{W} \) is indifferent among all feasible proposals because a best response yields \( u_{\mathcal{W}}(R(\pi^W)) = 0 \) for all \( \pi^W \in \Pi \).
Summary: A first proposal is an element of the set $\pi^W$. An optimal counter-proposal is a best response to this first proposal and specified by $R(\pi^W)$. $\pi^*L = R(\pi^W)$ is implemented in equilibrium.

If $p > \frac{1}{2}$: $W$ is able to induce project adoption: He proposes the project and low (or zero) subsidies such that $u_j(\pi^W) > 0$ for all project winners. In his best response, $L$ has to suggest the proposal as well.

If $p \leq \frac{1}{2}$: It is not possible to find a proposal $\pi^W$ that induces project adoption. Consequently, $W$ is indifferent among all proposals in $\Pi_{ST}$. An optimal counter proposal involves $g(\pi^W) = 0$.

Finally, because $s(R(\pi^W)) = 0$ it follows that in both cases $S(\pi^*L) = 0$.

\[ \square \]

Proof of Proposition 3.5

We proceed by backward induction: At the voting stage, all individuals vote for their preferred proposal. At the second stage, $L$ chooses his proposal given proposal $\pi^W$. His best-response correspondence is a mapping from the set of feasible proposals $\Pi$ into its power set, denoted by $\mathcal{P}(\Pi)$, that is $\mathcal{R}: \Pi \to \mathcal{P}(\Pi)$.

\[ \mathcal{R}(\pi^W) = \arg\max_{\pi^L \in \Pi} \left\{ g(\pi^L)V_i + s_L(\pi^L) - t_L(\pi^L) \right\}, \text{ s.t. } I(\pi^W, \pi^L) = 1. \quad (B.4) \]

For all $R(\pi^W) \in \mathcal{R}(\pi^W)$ we find:

(i) $s_L(R(\pi^W)) = \hat{s}$ and $t_L(R(\pi^W)) = 0$,

(ii) $s_W(R(\pi^W)) = t_W(R(\pi^W)) = 0$,

(iii) $g(R(\pi^W)) = \begin{cases} 1 & \text{if } u_j(\pi^W) > \hat{s} \text{ for more than } 1/2 \text{ of the society} \\ 0 & \text{otherwise,} \end{cases}$

(iv) $\left(s_j(R(\pi^W)), t_j(R(\pi^W))\right)_{j \in [0,1]} \in [0, \hat{s}] \times [0, \hat{t}]$, such that $u_j(R(\pi^W)) \geq u_j(\pi^W)$ for at least half of the society and the budget is balanced.

\[ ^8 \text{In Corollary 3.1, we pointed out that it is always optimal for } L \text{ to choose a counter-proposal that yields } I(\pi^W, \pi^L) = 1. \]
Item (i) directly follows from the examination of the maximization problem\(^9\), and item (ii) is a consequence of Assumption 3.1. Items (iii) and (iv) require some explanation: Using \(s_L(R(\pi^W)) =  \hat{s}\) and \(t_L(R(\pi^W)) = 0\), we derive:

\[
u_L(R(\pi^W)) = \hat{s} + g(R(\pi^W))V_i. \tag{B.5}\]

Because \(V_i < 0\), it is obvious that \(L\) prefers a proposal involving \(g(R(\pi^L)) = 0\) whenever possible. However, he might be forced to choose \(g(R(\pi^L)) = 1\) in order to fulfill the constraint \(I(\pi^W, \pi^L) = 1\):

- Suppose that \(u_j(\pi^W) > \hat{s}\) for more than 1/2 of the society:
  In this case, individuals receiving \(u_j(\pi^W) > \hat{s}\) form a majority. Consequently, to fulfill the constraint \(I(\pi^W, \pi^L) = 1\), \(L\) needs the support of some of these individuals, that is \(u_j(R(\pi^W)) \geq u_j(\pi^W) > \hat{s}\) has to hold for some \(j\). However, the condition \(u_j(R(\pi^W)) > \hat{s}\) can only hold if the project is suggested (remember that \(\hat{s}\) represents the highest net-subsidy an individual can obtain). Thus, \(R(\pi^W)\) necessarily involves \(g(R(\pi^W)) = 1\) in this case.

- Otherwise:
  In this case, individuals receiving \(u_j(\pi^W) \leq \hat{s}\) form a majority. Consequently, \(L\) does not need the support of individuals receiving \(u_j(\pi^W) > \hat{s}\) to fulfill the constraint \(I(\pi^W, \pi^L) = 1\) and it is sufficient if \(\hat{s} \geq u_j(R(\pi^W)) \geq u_j(\pi^W)\) holds for half of the society. However, the condition \(u_j(R(\pi^W)) \leq \hat{s}\) can also hold if the project is not suggested (since \(\hat{s}\) represents the highest net-subsidy, \(L\) can fulfill the constraint by an adequate distribution of taxes and subsidies).
  Because \(L\) prefers project rejection whenever possible, \(R(\pi^W)\) involves \(g(R(\pi^W)) = 0\) in that case.

This constitutes the assertion in item (iii). Finally, Equation B.5 makes clear that \(L\) is not affected by an excessive redistribution of subsidies (his taxes are always zero, independent of the aggregate amount of subsidies). Thus, he is indifferent among all feasible distributions of subsidies and taxes that yield \(I(\pi^W, \pi^L) = 1\). This is the statement of item (iv).

\(^9\)Recall that adoption of proposal \(R(\pi^W)\) does not depend on the size of \(s_L(R(\pi^W))\) and \(t_L(R(\pi^W))\). Thus, \(L\) is free to choose his preferred tax and subsidy level without affecting the constraint \(I(\pi^W, \pi^L) = 1\).
Using $\mathcal{R}(\pi^W)$, we now turn to the first stage and explore optimal proposal-making of $\mathcal{W}$:

$$\pi^*/W = \arg\max_{\pi^W \in \Pi} u_W(\pi^W, R(\pi^W)) = g(R(\pi^W))V_w. \quad (B.6)$$

There exists a multitude of possible solutions to this problem. Thus $\pi^*/W$ represents the set of maximizer. For all $\pi^*/W \in \pi^*/W$ we have:

(i) $g(\pi^*/W) \in \begin{cases} \{1\} & \text{if } p > \frac{1}{2} \\ \{0, 1\} & \text{otherwise.} \end{cases}$

(ii) $(s_j(\pi^*/W), t_j(\pi^*/W))_{j \in [0,1]} \in [0, \hat{s}] \times [0, \hat{t}]$ such that the budget is balanced and additionally if $p > \frac{1}{2}$:

$$s_j(\pi^*/W) - t_j(\pi^*/W) > \hat{s} - V_w \text{ for more than } \frac{1}{2} \text{ project winners.}$$

Items (i) and (ii) result from the following considerations: Because $V_w > 0$, it is obvious that $\mathcal{W}$ tries to induce project adoption, that is $R(g(\pi^W)) = 1$, whenever possible. By the characterization of the best-response correspondence, $g(R(\pi^W)) = 1$ requires $u_j(\pi^W) > \hat{s}$ for more than half of the society.

Since $u_j(\pi^W) > \hat{s}$ can only hold for project winners, $g(R(\pi^W)) = 1$ is only attainable if project winners form a majority in the society, that is $p > \frac{1}{2}$. In this case, a proposal as specified by items (i) and (ii) guarantees $u_j(\pi^W) > \hat{s}$ for more than half of the society. Otherwise, if $p \leq \frac{1}{2}$, $u_j(\pi^W) > \hat{s}$ cannot hold for more than half of the society, because project winners are not sufficiently numerous. Thus, $g(R(\pi^W)) = 1$ is not attainable and $\mathcal{W}$ is indifferent among all feasible proposals because any best response yields $u_W(R(\pi^W)) = 0$ for all $\pi^W \in \Pi$.

**Summary:** In equilibrium, a first proposal $\pi^*/W$ is an element from the set $\pi^*/W$. An optimal counter-proposal is a best response to this first proposal and therefore an element of the set $\mathcal{R}(\pi^*/W)$. Let $\pi^*/L \in \mathcal{R}(\pi^*/W)$ be the chosen counter-proposal. $\pi^*/L$ is implemented in equilibrium.

If $p > \frac{1}{2}$, $\mathcal{W}$ is able to induce project adoption: He proposes the project and distributes subsidies and taxes such that $u_j(\pi^*/W) > \hat{s}$ for more than half of the society. In his best response, $\mathcal{L}$ has to suggest the project as well and distributes subsidies and taxes such that $u_j(\pi^*/L) \geq u_j(\pi^*/W) > \hat{s}$. Consequently, $g(\pi^*/L) = 1$ and $S(\pi^*/L) > 0$ (note that for any $\pi \in \Pi$, the aggregate amount of subsidies is bounded from above, that is $S(\pi) \leq \hat{s}$, $\forall \pi \in \Pi$). This constitutes the first part of the proposition.

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10 Note that $u_W(\pi^W, R(\pi^W)) = g(R(\pi^W))V_w$, $\forall R(\pi^W) \in \mathcal{R}(\pi^W)$. 

152
If $p \leq \frac{1}{2}$: It is not possible to find a proposal $\pi^W$ that induces project adoption. Consequently, $W$ is indifferent among all proposals in $\Pi$. An optimal counter-proposal involves $g(\pi^*L) = 0$ and $S(\pi^*L) \geq 0$. This yields the second part of the proposition.

**Proof of Lemma 3.6** An arbitrary proposal $\pi^W \in \Pi_T$ yields

\[
u_j(\pi^W) = g(\pi^W)(V_i - (1 + \lambda)k) - (1 + \lambda)S(\pi^W) \quad \forall j \in Q_{ns} \tag{B.7}
\]

\[
u_L(\pi^W) = s_L(\pi^W) + g(\pi^W)(V_i - (1 + \lambda)k) - (1 + \lambda)S(\pi^W). \tag{B.8}
\]

This allows us to write

\[
u_j(\pi^W) = \nu_L(\pi^W) - s_L(\pi^W) \quad \forall j \in Q_{ns}. \tag{B.9}
\]

Now, let us define the proposal $\tilde{\pi}^L$ with

\[
g(\tilde{\pi}^L) = g(\pi^W), \quad s_j(\tilde{\pi}^L) = \begin{cases} 
0 & \text{for } j = 0 \\
s_j(\pi^W) & \text{for } j \in [0, 1[ \quad t(\tilde{\pi}^L) = t(\pi^W). 
\end{cases}
\]

Except from $s_W(\tilde{\pi}^L)$ and $s_L(\tilde{\pi}^L)$, proposal $\tilde{\pi}^L$ is a “copy” of proposal $\pi^W$. Since $\pi^W$ is a feasible proposal with a balanced budget, it is obvious that $\tilde{\pi}^L$ is also a feasible proposal with a balanced budget. We obtain

\[
u_j(\tilde{\pi}^L) = \nu_j(\pi^W), \quad \forall j \in [0, 1[ \tag{B.10}
\]

\[
u_L(\tilde{\pi}^L) = \nu_L(\pi^W) + (\hat{s} - s_L(\pi^W)) \geq \nu_L(\pi^W). \tag{B.11}
\]

By Tie-breaking Rule 3.1, all $j \in [0, 1]$ vote for proposal $\tilde{\pi}^L$ which yields $I(\pi^W, \tilde{\pi}^L) = 1$. Since $\tilde{\pi}^L$ is feasible and would be adopted, $L$ would never suggest a proposal that yields lower utility for him. Thus, for $L$, the optimal counter-proposal $R(\pi^W)$ is never worse than $\tilde{\pi}^L$ and therefore satisfies

\[
u_L(R(\pi^W)) \geq \nu_L(\tilde{\pi}^L) = \nu_L(\pi^W) + (\hat{s} - s_L(\pi^W)).
\]

Because $s_L(R(\pi^W)) = \hat{s}$, we obtain similar to Equation B.9

\[
u_j(R(\pi^W)) = \nu_L(R(\pi^W)) - \hat{s} \quad \text{for all project losers not subsidized in } R(\pi^W). \tag{B.12}
\]
Hence, it is true that
\[ u_j(R(\pi^W)) + \hat{s} = u_L(R(\pi^W)) \geq u_L(\pi^W) + (\hat{s} - s_L(\pi^W)) = u_j(\pi^W) + \hat{s}, \quad (B.13) \]
which implies
\[ u_j(R(\pi^W)) \geq u_j(\pi^W), \quad (B.14) \]
for project losers not subsidized under $\pi^W$ and $R(\pi^W)$.

We conclude that project losers not subsidized under proposal $\pi^W$ do not have to be subsidized under $R(\pi^W)$ neither and nonetheless vote for $R(\pi^W)$. Hence in equilibrium, $L$ chooses a proposal that yields $u_j(R(\pi^W)) \geq u_j(\pi^W)$, $\forall j \in Q_{ns}$, because it is optimal for himself.

\[ \square \]

**Proof of Lemma 3.8**

Let $i \in Q_{g(R(\sigma^W))}$. Moreover, let $\sigma_i(g(R(\pi^W))) = \max_{j \in Q_{g(R(\pi^W))}} \{ \sigma_j(g(R(\pi^W))) \}$, that is individual $i$ is the individual with the highest $\sigma_j(g(R(\pi^W)))$ among all individuals in $Q_{g(R(\pi^W))}$. Hence, if $\sigma_i(g(R(\pi^W))) < \hat{s}$, it follows that $\sigma_j(g(R(\pi^W))) < \hat{s} \ \forall j \in Q_{g(R(\pi^W))}$.

We can distinguish two cases:

(i) $\sigma_i(g(R(\pi^W))) \leq 0$.

(ii) $\sigma_i(g(R(\pi^W))) > 0$.

In case (i), it is obvious that $\sigma_i(g(R(\pi^W))) < \hat{s}$. In the second case, we have
\[
\sigma_i(g(R(\pi^W))) = u_i(\pi^W) - g(R(\pi^W))(v_i - (1 + \lambda)k) + (1 + \lambda) \int_{j \in Q_{g(R(\pi^W))}} \max\{0, \sigma_j(g(R(\pi^W)))\} dj \\
= s_i(\pi^W) + (g(\pi^W) - g(R(\pi^W)))(v_i - (1 + \lambda)k) \\
+ (1 + \lambda) \int_{j \in Q_{g(R(\pi^W))}} \max\{0, \sigma_j(g(R(\pi^W)))\} dj - (1 + \lambda)S(\pi^W).
\]

Because $Q_{g(R(\pi^W))}$ consists of those individuals for which $\sigma_j(g(R(\pi^W)))$ is smallest, and $\sigma_i(g(R(\pi^W)))$ represents the highest $\sigma_j(g(R(\pi^W)))$ in $Q_{g(R(\pi^W))}$, there exists a fraction of $1/2$ individuals for which $\sigma_k(g(R(\pi^W))) \geq \sigma_i(g(R(\pi^W)))$, that is
\[
s_k(\pi^W) \geq s_i(\pi^W) - (g(\pi^W) - g(R(\pi^W))(v_k - v_i) =: \tilde{s}_k(\pi^W). \quad (B.15)
\]
Since \( s_k(\pi^W) \geq s_k(\pi^W) \) for 1/2 individuals, we obtain
\[
S(\pi^W) \geq \int_{j \in Q_{g(R(\pi^W))}} s_j(\pi^W)dj + \frac{1}{2}s_{k}(\pi^W) \tag{B.16}
\]
\[
= \int_{j \in Q_{g(R(\pi^W))}} s_j(\pi^W)dj + \frac{1}{2} \left[ s_i(\pi^W) - (g(\pi^W) - g(R(\pi^W)))(v_k - v_i) \right]. \tag{B.17}
\]

Thus, inserting the lower bound for \( S(\pi^W) \) (given by Equation B.17) into \( \sigma_i(g(R(\pi^W))) \) yields
\[
\sigma_i(g(R(\pi^W))) \leq s_i(\pi^W) + (g(\pi^W) - g(R(\pi^W)))(v_i - (1 + \lambda)k) + (1 + \lambda) \int_{j \in Q_{g(R(\pi^W))}} \max\{0, \sigma_j(g(R(\pi^W)))\}dj
\]
\[
- (1 + \lambda) \left[ \int_{j \in Q_{g(R(\pi^W))}} s_j(\pi^W)dj - \frac{1}{2} \left[ s_i(\pi^W) - (g(\pi^W) - g(R(\pi^W)))(v_k - v_i) \right] \right]
\]
\[
\leq s_i(\pi^W) + (g(\pi^W) - g(R(\pi^W)))(v_i - (1 + \lambda)k) + (1 + \lambda) \int_{j \in Q_{g(R(\pi^W))}} \max\{0, \sigma_j(g(R(\pi^W)))\}dj
\]
\[
- (1 + \lambda) \frac{1}{2} \left[ s_i(\pi^W) - (g(\pi^W) - g(R(\pi^W)))(v_k - v_i) \right]
\]
\[
\leq s_i(\pi^W) \left[ 1 - (1 + \lambda) \frac{1}{2} \right] + (g(\pi^W) - g(R(\pi^W)))(v_i - (1 + \lambda)k) + (1 + \lambda) \left( \frac{1}{2} - q_{ns} \right) \sigma_i(g(R(\pi^W))).
\]

The second inequality holds because \((1 + \lambda) \int_{j \in Q_{g(R(\pi^W))}} s_j(\pi^W)dj \geq 0\). The third one holds because \( \sigma_i(g(R(\pi^W))) \geq \sigma_j(g(R(\pi^W))), \forall j \in Q_{g(R(\pi^W))} \) and thus \((1 + \lambda)(\frac{1}{2} - q_{ns})\sigma_i(g(R(\pi^W))) \geq (1 + \lambda) \int_{j \in Q_{g(R(\pi^W))}} \max\{0, \sigma_j(g(R(\pi^W)))\}dj\).\(^{11}\)

Finally, we obtain
\[
\sigma_i(g(R(\pi^W))) \leq \frac{s_i(\pi^W) \left[ 1 - (1 + \lambda) \frac{1}{2} \right] + (g(\pi^W) - g(R(\pi^W))) \left( v_i - (1 + \lambda)k \right) + (1 + \lambda) \frac{1}{2} \left( v_k - v_i \right)}{1 - (1 + \lambda) \left( \frac{1}{2} - q_{ns} \right)}. \tag{B.18}
\]

Because
\[
\frac{s_i(\pi^W) \left[ 1 - (1 + \lambda) \frac{1}{2} \right] + (g(\pi^W) - g(R(\pi^W))) \left( v_i - (1 + \lambda)k \right) + (1 + \lambda) \frac{1}{2} \left( v_k - v_i \right)}{1 - (1 + \lambda) \left( \frac{1}{2} - q_{ns} \right)} \leq \frac{s \left[ 1 - (1 + \lambda) \frac{1}{2} \right] + (g(\pi^W) - g(R(\pi^W))) \left( v_i - (1 + \lambda)k \right) + (1 + \lambda) \frac{1}{2} \left( v_k - v_i \right)}{1 - (1 + \lambda) \left( \frac{1}{2} - q_{ns} \right)} < s
\]

\(^{11}\)Recall that \( |Q_{g(R(\pi^W))}| = \frac{1}{2} - q_{ns} \).
Appendix B. Proofs for Chapter 3

holds for sufficiently high \( \hat{s} \), we have shown that

\[
\sigma_j(g(R(\pi^W))) \leq \sigma_i(g(R(\pi^W))) < \hat{s}, \forall j \in Q_{g(R(\pi^W))}.
\]  

(B.19)

Proof of Lemma 3.9

Suppose \( g(\pi^W) = 0 \). We want to show that in this case

\[
u_j(\tilde{\pi}_L) \geq u_j(\pi^W), \forall j \in Q_{ns}.
\]  

(B.20)

Equation B.20 can be rewritten as

\[
(1 + \lambda) \int_{j \in Q_0} \max\{0, \sigma_j(0)\} dj \leq (1 + \lambda) S(\pi^W).
\]  

(B.21)

Equation B.21 holds because

\[
s_j(\tilde{\pi}_L) = 0 = s_j(\pi^W) \forall j \in Q_{ns}
\]  

(B.22)

\[
s_j(\tilde{\pi}_L) = 0 \leq s_j(\pi^W) \forall j \in Q^{others}_{g(R(\pi^W))}
\]  

(B.23)

Because the project is not suggested under both proposals and individuals in \( Q_{ns} \) and \( Q^{others}_{g(R(\pi^W))} \) receive less or equal subsidies under proposal \( \tilde{\pi}_L \) than under \( \pi^W \), it is obvious that the aggregate amount of subsidies (so far) is not higher under proposal \( \tilde{\pi}_L \). However, this implies that compensatory payments to individuals in \( Q_0 \) are not higher than the subsidies under proposal \( \pi^W \). Or put it differently: if \( s_j(\tilde{\pi}_L) = s_j(\pi^W) \) for \( j \in Q_0 \), then \( u_j(\tilde{\pi}_L) \geq u_j(\pi^W) \). This yields

\[
s_j(\tilde{\pi}_L) = \max\{0, \sigma_j(0)\} \leq s_j(\pi^W).
\]  

(B.24)

Thus, because subsidies to all individuals (except \( j = 1 \)) are not higher under \( \tilde{\pi}_L \) than under \( \pi^W \) (that is \( s_j(\tilde{\pi}_L) \leq s_j(\pi^W) \) holds for all \( j \in [0, 1[ \)), it is obvious that the aggregate amount of subsidies is lower under \( \tilde{\pi}_L \) than under \( \pi^W \). These considerations imply that Equations B.20 and B.21 hold. Finally, we obtain

\[
\max\{u_j(\tilde{\pi}_L), u_j(\tilde{\pi}_L)\} \geq u_j(\tilde{\pi}_L) \geq u_j(\pi^W), \forall j \in Q_{ns}.
\]

which confirms that \( \max\{u_j(\tilde{\pi}_L), u_j(\tilde{\pi}_L)\} \geq u_j(\pi^W) \) for all \( j \in Q_{ns} \) if \( g(\pi^W) = 0 \).
We can state a similar argument for \( \tilde{\pi}_L \): In case \( g(\pi^W) = 1 \), one can show that \( u_j(\tilde{\pi}_L) \geq u_j(\pi^W) \) for \( j \in Q_{ns} \), which again confirms that \( \max\{u_j(\tilde{\pi}_L), u_j(\tilde{\pi}_L)\} \geq u_j(\pi^W) \) for all \( j \in Q_{ns} \) if \( g(\pi^W) = 0 \).

\[ \square \]

**Proof of Lemma 3.11**

Consider proposal \( \tilde{\pi}_L \) for an arbitrary proposal \( \pi^W \). This yields for all \( i \in Q_1 \)

\[
u_i(\tilde{\pi}_L) = v_i - (1 + \lambda)k + \max\{0, \sigma_i(1)\} - (1 + \lambda) \int_{j \in Q_1} \max\{0, \sigma_j(1)\} dj \geq \nu_i(\pi^W).
\]

Recall that

\[
u_i(\tilde{\pi}_L) \begin{cases} > \nu_i(\pi^W) & \text{if } \sigma_j(1) < 0 \\ = \nu_i(\pi^W) & \text{if } \sigma_j(1) \geq 0. \end{cases}
\]

The aggregate amount of subsidies equals

\[
\int_{j \in Q_1} \max\{0, \sigma_j(1)\} dj.
\]

Starting from proposal \( \tilde{\pi}_L \), we construct an artificial proposal \( \tilde{\pi}'_L \) that comprises \( g(\tilde{\pi}'_L) = 0 \) and \( Q'_0 = Q_1 \). Since \( Q'_0 = Q_1 \) might not be the cheapest coalition in case \( g(\pi^C) = 0 \), that is \( Q'_0 \neq Q_0 \), it is obvious that proposal \( \tilde{\pi}'_L \) might not equal proposal \( \tilde{\pi}_L \). Nevertheless, proposal \( \tilde{\pi}'_L \) is an interesting boundary case.

First, we split the set \( Q_1 \) into the two disjoint subsets \( Q^W_1 \) and \( Q^C_1 \) where

\[
Q^W_1 := \{ j \in Q_1 \mid j \in ]0, p]\}
\[
Q^C_1 := \{ j \in Q_1 \mid j \in ]p, 1[\}
\]

Hence, the set \( Q^W_1 \) comprises all project winners and the set \( Q^C_1 \) all project losers from \( Q_1 \). It is obvious that \( Q_1 = Q^W_1 \cup Q^C_1 \).

Let \( q^W_1 := |Q^W_1| \) and \( q^C_1 := |Q^C_1| \). Hence, \( q^W_1 + q^C_1 = |Q_1| = \frac{1}{2} - q_{ns} \).

\[ \text{12} \text{The proof of this claim follows the same lines as above. For that reason, the calculations are omitted.} \]
Let $\tilde{\pi}'L$ comprise the following subsidy scheme:

$$s_j(\tilde{\pi}'L) = \max\left\{0, \sigma_j(1)\right\} + \max\left\{0, \frac{(1 + \lambda)q_1V(W_w - (1 + \lambda)k) + (1 - (1 + \lambda)q_1^L(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q_1^V + q_1^L)}\right\}$$

for all $j \in Q_1^V$.

$$s_j(\tilde{\pi}'L) = \max\left\{0, \sigma_j(1)\right\} + \max\left\{0, \frac{(1 + \lambda)q_1^V(W_w - (1 + \lambda)k) + (1 - (1 + \lambda)q_1^L(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q_1^V + q_1^L)}\right\}$$

for all $j \in Q_1^L$.

Recall that $q_1^V + q_1^L = |Q_1| = \frac{1}{2} - q_{ns}$, which implies that $1 - (1 + \lambda)(q_1^V + q_1^L) > 0$ (for $\lambda < 1$). Thus, $s_j(\tilde{\pi}'L) > \max\{0, \sigma_j(1)\}$ for $j \in Q_1^V$ and $s_j(\tilde{\pi}'L) \geq \max\{0, \sigma_j(1)\}$ for $j \in Q_1^L$.

Finally, let $s_L(\tilde{\pi}'L) = \hat{s}$ and $s_j(\tilde{\pi}'L) = 0$ for all other individuals.

Given this subsidy scheme, two cases can be distinguished:

1. $(1 + \lambda)q_1^V < \hat{p}$
2. $(1 + \lambda)q_1^V \geq \hat{p}$

**Case 1: $(1 + \lambda)q_1^V < \hat{p}$**

Case 1 implies that

(i) $\frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)q_1^V} > \frac{(1 - (1 + \lambda)q_1^L(V_w - (1 + \lambda)k) + (1 + \lambda)q_1^L(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q_1^V + q_1^L)}$

(ii) $0 > \frac{(1 + \lambda)q_1^V(W_w - (1 + \lambda)k) + (1 - (1 + \lambda)q_1^L(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q_1^V + q_1^L)}$

As a consequence, the subsidies for $j \in Q_1^V$ become

$$s_j(\tilde{\pi}'L) = \max\{0, \sigma_j(1)\} + \frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)q_1^V} > 0. \quad (B.27)$$

The subsidies for $j \in Q_1^L$ become

$$s_j(\tilde{\pi}'L) = \max\{0, \sigma_j(1)\} \geq 0. \quad (B.28)$$
Hence, the aggregate amount of subsidies under proposal $\tilde{\pi}^c$ can be written as

$$\sum_{j \in Q_1} s_j(\tilde{\pi}^c) = \sum_{j \in Q_1} \max \{0, \sigma_j(1)\} dj + q_1^W V_w - (1 + \lambda)k \frac{1}{1 - (1 + \lambda)q_1^W}. \quad (B.29)$$

Next, we show that $u_j(\tilde{\pi}^c) \geq u_j(\pi^w)$ for all $j \in Q_1 = Q_1^w \cup Q_1^c$ and therefore all individuals in $Q_1$ vote for proposal $\tilde{\pi}^c$ when put to a vote against $\pi^w$.

First, consider $j \in Q_1^w$:

$$u_j(\tilde{\pi}^c) = s_j(\tilde{\pi}^c) - (1 + \lambda) \sum_{j \in Q_1} s_j(\tilde{\pi}^c) dj$$

$$= \max \{0, \sigma_j(1)\} + \frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)q_1^W} \sum_{j \in Q_1} \max \{0, \sigma_j(1)\} dj - (1 + \lambda) \sum_{j \in Q_1} \max \{0, \sigma_j(1)\} dj$$

$$= V_w - (1 + \lambda)k + \max \{0, \sigma_j(1)\} - (1 + \lambda) \sum_{j \in Q_1} \max \{0, \sigma_j(1)\} dj$$

$$= u_j(\tilde{\pi}^c) \geq u_j(\pi^w).$$

Now, consider $j \in Q_1^c$:

$$u_j(\tilde{\pi}^c) = s_j(\tilde{\pi}^c) - (1 + \lambda) \sum_{j \in Q_1} s_j(\tilde{\pi}^c) dj$$

$$= \max \{0, \sigma_j(1)\} - (1 + \lambda) \sum_{j \in Q_1} \max \{0, \sigma_j(1)\} dj - (1 + \lambda)q_1^W V_w - (1 + \lambda)k \frac{1}{1 - (1 + \lambda)q_1^W}$$

$$> V_i - (1 + \lambda)k + \max \{0, \sigma_j(1)\} - (1 + \lambda) \sum_{j \in Q_1} \max \{0, \sigma_j(1)\} dj$$

$$= u_j(\tilde{\pi}^c) \geq u_j(\pi^w).$$

It is obvious that the aggregate amount of subsidies under proposal $\tilde{\pi}^c$, given by Equation B.29, represents an upper bound for the aggregate amount of subsidies under proposal $\tilde{\pi}^c$, because $L$ would not choose higher subsidies than those in $\tilde{\pi}^c$. However, he might choose lower subsidies which decreases the aggregate amount of subsidies.

The aggregate amount of subsidies under $\tilde{\pi}^c$ might be lower than the one under $\tilde{\pi}^c$ because:

1. $s_j(\tilde{\pi}^c) < s_j(\tilde{\pi}^c)$ for (some) $j \in Q_1^c$:

Since $j \in Q_1^c$ receive $u_j(\tilde{\pi}^c) > u_j(\tilde{\pi}^c) \geq u_j(\pi^w)$, it might be possible to reduce

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13Recall that in proposal $\tilde{\pi}^c$ a share of $\frac{1}{2} - q_{ns}$ individuals vote for $\tilde{\pi}^c$ when put to a vote against $\pi^w$. Hence, it is not profitable for $L$ to choose a subsidy scheme that yields higher aggregate subsidies than those in proposal $\tilde{\pi}^c$. 

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159
Appendix B. Proofs for Chapter 3

subsidies for (some) \( j \in Q_1 \), such that \( u_j(\tilde{\pi}'^L) > u_j(\tilde{\pi}^L) \geq u_j(\pi^W) \).

However, because \( s_j(\tilde{\pi}'^L) = \max \{0, \sigma_j(1)\} \), the subsidy can only be reduced if \( \sigma_j(1) > 0 \). If \( \sigma_j(1) \leq 0 \), the subsidy \( s_j(\tilde{\pi}^L) \) remains at zero.

Note that if (some) subsidies could be reduced, subsidies of all other individuals would be affected because the aggregate amount of subsidies becomes smaller. Thus, subsidies of other individuals might be reduced as well.

2. \( s_j(\tilde{\pi}^L) < s_j(\tilde{\pi}'^L) \) for (some) \( j \in Q_1^W \):

Because \( u_j(\tilde{\pi}'^L) = u_j(\tilde{\pi}^L) \), the subsidy \( s_j(\tilde{\pi}'^L) \) can only be reduced if \( u_j(\tilde{\pi}^L) > u_j(\pi^W) \) (that is if \( \sigma_j(1) < 0 \) for (some) \( j \in Q_1^W \). Then, the subsidy \( s_j(\tilde{\pi}^L) < s_j(\tilde{\pi}'^L) \) could be chosen such that \( u_j(\tilde{\pi}^L) = u_j(\pi^W) < u_j(\tilde{\pi}'^L) \). Of course, individuals would still vote for \( \tilde{\pi}^L \).

Again, if (some) subsidies could be decreased, subsidies of other individuals might be reduced as well since the aggregate amount of subsidies becomes smaller.

3. It might not be optimal to choose \( Q_0 = Q_1 \), because there exists a cheaper coalition. This is the case if there are individuals \( i \in Q_1^{\text{others}} \) for which \( u_i(\pi^W) < u_j(\pi^W) \), \( j \in Q_1 \) (see Equation 3.15).

As shown above, these considerations may or may not apply (depending on proposal \( \pi^W \)). Consequently, we obtain

\[
\int_{j \in Q_0} \max\{0, \sigma_j(0)\} \, dj - \int_{j \in Q_1} \max\{0, \sigma(1)j\} \, dj \leq q_1^W V_w - (1 + \lambda)k \leq 1 - (1 + \lambda)q_1^W. \tag{B.30}
\]

Because

\[
(1 + \lambda)q_1^W V_w - (1 + \lambda)k < (1 + \lambda)k - V_t, \tag{B.31}
\]

for \( q_1^W < \frac{\hat{p}}{1 + \lambda} \), we immediately know that

\[
u_L(\tilde{\pi}'^L) > u_L(\tilde{\pi}^L). \tag{B.32}
\]

Since the aggregate amount of subsidies under proposal \( \tilde{\pi}^L \) is not higher than under \( \tilde{\pi}'^L \), we also know that \( u_L(\tilde{\pi}^L) \geq u_L(\tilde{\pi}'^L) \). Thus, if \( q_1^W < \frac{\hat{p}}{1 + \lambda} \), the project is not implemented in equilibrium.

Case 2: \( (1 + \lambda)q_1^W \geq \hat{p} \)

Case 2 implies that

\[
(i) \quad \frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)q_1^W} \leq \frac{1 - (1 + \lambda)q_1^W(V_w - (1 + \lambda)k)}{1 - (1 + \lambda)(q_1^W + q_1^L)}
\]

160
Appendix B. Proofs for Chapter 3

(ii) \(0 \leq \frac{(1 + \lambda)q_i^W(V_w - (1 + \lambda)k) + (1 - (1 + \lambda)q_i^W)(V_i - (1 + \lambda)k)}{1 - (1 + \lambda)(q_i^W + q_i^L)}\)

As a consequence, the subsidies for \(j \in Q_i^W\) become

\[s_j(\tilde{\pi}'^\xi) = \max\{0, \sigma_j(1)\} + \frac{(1 + \lambda)q_i^W(V_w - (1 + \lambda)k) + (1 - (1 + \lambda)q_i^W)(V_i - (1 + \lambda)k)}{1 - (1 + \lambda)(q_i^W + q_i^L)} > 0.\] (B.33)

The subsidies for \(j \in Q_i^L\) become

\[s_j(\tilde{\pi}'^\xi) = \max\{0, \sigma_j(1)\} + \frac{(1 + \lambda)q_i^L(V_w - (1 + \lambda)k) + (1 - (1 + \lambda)q_i^W)(V_i - (1 + \lambda)k)}{1 - (1 + \lambda)(q_i^W + q_i^L)} \geq 0.\] (B.34)

Hence, the aggregate amount of subsidies can be written as

\[
\int_{j \in Q_i} s_j(\tilde{\pi}'^\xi) dj = \int_{j \in Q_i} \max\{0, \sigma_j(1)\} dj + q_i^W(V_w - (1 + \lambda)k) + q_i^L(V_i - (1 + \lambda)k) + q_i^L(V_i - (1 + \lambda)k) + q_i^L(V_i - (1 + \lambda)k) + q_i^L(V_i - (1 + \lambda)k) + q_i^L(V_i - (1 + \lambda)k)
\]

\[\int_{j \in Q_i} \max\{0, \sigma_j(1)\} dj + \frac{q_i^W(V_w - (1 + \lambda)k) + q_i^L(V_i - (1 + \lambda)k)}{1 - (1 + \lambda)(q_i^W + q_i^L)}.\]

(B.35)

Again, we show that \(u_j(\tilde{\pi}'^\xi) \geq u_j(\pi^W)\) for all \(j \in Q_i = Q_i^W \cup Q_i^L\) and hence all individuals in \(Q_i\) vote for proposal \(\tilde{\pi}'^\xi\) when put to a vote against \(\pi^W\).

First, consider \(j \in Q_i^W\):

\[u_j(\tilde{\pi}'^\xi) = s_j(\tilde{\pi}'^\xi) - (1 + \lambda) \int_{j \in Q_i} s_j(\tilde{\pi}'^\xi) dj\]

\[= \max\{0, \sigma_j(1)\} + \frac{(1 - (1 + \lambda)q_i^L)(V_w - (1 + \lambda)k) + (1 + \lambda)q_i^L(V_i - (1 + \lambda)k)}{1 - (1 + \lambda)(q_i^W + q_i^L)} - (1 + \lambda) \int_{j \in Q_i} \max\{0, \sigma_j(1)\} dj - (1 + \lambda) \int_{j \in Q_i} \max\{0, \sigma_j(1)\} dj\]

\[= V_w - (1 + \lambda)k + \max\{0, \sigma_j(1)\} - (1 + \lambda) \int_{j \in Q_i} \max\{0, \sigma_j(1)\} dj = u_j(\tilde{\pi}'^\xi) \geq u_j(\pi^W).\]
Now, consider \( j \in Q^1_1 \):

\[
u_j(\tilde{\pi}'_1) = s_j(\tilde{\pi}'_1) - (1 + \lambda) \int_{j \in Q_1} s_j(\tilde{\pi}'_1) dj \\
= \max \{0, \sigma_j(1)\} + \frac{(1 + \lambda)q^W_1(V_w - (1 + \lambda)k) + (1 - (1 + \lambda)q^V_1)(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q^V_1 + q^L)} \\
- (1 + \lambda) \int_{j \in Q_1} \max \{0, \sigma_j(1)\} dj - \frac{q^W_1(V_w - (1 + \lambda)k) + q^L(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q^V_1 + q^L)} \\
= V_l - (1 + \lambda)k + \max \{0, \sigma_j(1)\} - (1 + \lambda) \int_{j \in Q_1} \max \{0, \sigma_j(1)\} dj \\
= u_j(\tilde{\pi}') \geq u_j(\pi').
\]

Again, it is obvious that the aggregate amount of subsidies under proposal \( \tilde{\pi}'_1 \), given by Equation B.35, represents an upper bound for the aggregate amount of subsidies under proposal \( \tilde{\pi}_1 \), because \( L \) would not choose higher subsidies than those in \( \tilde{\pi}'_1 \). The aggregate amount of subsidies under \( \tilde{\pi} \) might be lower than the one under \( \tilde{\pi}' \) because:

1. \( s_j(\tilde{\pi}_1) < s_j(\tilde{\pi}'_1) \) for (some) \( j \in Q_1 \):

Because \( u_j(\tilde{\pi}'_1) = u_j(\tilde{\pi}_1) \) for all \( j \in Q_1 \), the subsidy can only be reduced if \( u_j(\tilde{\pi}_1) > u_j(\pi') \) (that is if \( \sigma_j(1) < 0 \)) for (some) \( j \in Q_1 \). Then, the subsidy \( s_j(\tilde{\pi}_1) < s_j(\tilde{\pi}'_1) \) could be chosen such that \( u_j(\tilde{\pi}_1) = u_j(\pi') < u_j(\tilde{\pi}') \). Of course, individuals still vote for \( \tilde{\pi}_1 \).

If (some) subsidies could be decreased, the ones of other individuals might be reduced as well since the aggregate amount of subsidies becomes smaller.

2. It might not be optimal to choose \( Q_0 = Q_1 \), because there exists a cheaper coalition.

This is the case if there are individuals \( i \in Q^1_{\text{others}} \) for which \( u_i(\pi') < u_j(\pi') \), \( j \in Q_1 \) (see Equation 3.15).

Similar to Case 1, we obtain

\[
\int_{j \in Q_0} \max \{0, \sigma_j(0)\} dj - \int_{j \in Q_1} \max \{0, \sigma_j(1)\} dj \leq \frac{q^W_1(V_w - (1 + \lambda)k) + q^L_1(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q^V_1 + q^L)}.
\]

\[\text{(B.36)}\]

\[14\text{Recall that in proposal } \tilde{\pi}_1 \text{ a share of } \frac{1}{2} - q_{ns} \text{ individuals vote for } \tilde{\pi}_1 \text{ when put to a vote against } \pi'. \text{ Hence, it is not profitable for } L \text{ to choose a subsidy scheme that yields higher aggregate subsidies than those in proposal } Q_1.\]
Because
\[(1 + \lambda)q_1^W(V_w - (1 + \lambda)k) + q_1^L(V_l - (1 + \lambda)k) \geq (1 + \lambda)k - V_l,\]  
(B.37)
for \(q_1^W \geq \frac{\hat{p}}{1 + \lambda}\), we immediately know that
\[u_L(\tilde{\pi}'_L) \leq u_L(\tilde{\pi}_L).\]  
(B.38)
Because the aggregate amount of subsidies under proposal \(\tilde{\pi}_L\) is not higher than under \(\tilde{\pi}'_L\), it is unclear if \(u_L(\tilde{\pi}_L) \leq u_L(\tilde{\pi}'_L)\), either. Nevertheless, we can conclude that the project could be implemented in equilibrium if \(q_1^W \geq \frac{\hat{p}}{1 + \lambda}\).

\[\square\]

**Proof of Proposition 3.7**

We proceed by backward induction: At the voting stage, all individuals vote for their preferred proposal. At the second stage, \(L\) chooses his proposal given proposal \(\pi^W\). His best-response correspondence is a mapping from the set of feasible proposals \(\Pi_S\) into its power set, denoted by \(\mathcal{P}(\Pi_S)\), that is \(\mathcal{R}: \Pi_S \to \mathcal{P}(\Pi_S)\).

\[
\mathcal{R}(\pi^W) = \operatorname{argmax}_{\pi^L \in \Pi_S} \{g(\pi^L)V_l + s(\pi^L) - t_L(\pi^W, \pi^L)\}, \text{ s.t. } I(\pi^W, \pi^L) = 1.16
\]  
(B.39)
For all \(R(\pi^W) \in \mathcal{R}(\pi^W)\) we find:

(i) \(s(R(\pi^W)) = \hat{s}\) and \(t_L(\pi^L) = 0)\),

(ii) \(t_W(R(\pi^W)) = 0)\),

(iii) \(g(R(\pi^W)) = \begin{cases} 1 & \text{if } u_j(\pi^W) > \hat{s} \text{ for more than } 1/2 \text{ of the society} \\ 0 & \text{otherwise} \end{cases}\)

(iv) \(\{t_j(R(\pi^W))\}_{j \in [0,1] \in [0,1]}\) such that \(u_j(R(\pi^W)) \geq u_j(\pi^W)\) for at least half of the society and the budget is balanced.

\[\text{To see this: } (1 + \lambda)\frac{2q_1^W(V_w - (1 + \lambda)k) + q_1^L(V_l - (1 + \lambda)k)}{1 - (1 + \lambda)(q_1^W + q_1^L)} \geq (1 + \lambda)\frac{2q_1^W(V_w - (1 + \lambda)k)}{1 - (1 + \lambda)q_1^W} \geq (1 + \lambda)k - V_l \text{ for } q_1^W \geq \frac{\hat{p}}{1 + \lambda}.\]

\[\text{In Corollary 3.1, we pointed out that it is always optimal for } \mathcal{L} \text{ to chooses a counter-proposal that yields } I(\pi^W, \pi^L) = 1.\]
Item (i) follows directly from inspection of the maximization problem\textsuperscript{17}, and item (ii) is a consequence of Assumption 3.1. The explanations for items (iii) and (iv) are similar as in the proof of Proposition 3.5.

Using $\mathcal{R}(\pi^W)$, we now turn to the first stage and explore optimal proposal-making of $W$:

$$
\pi^*W = \arg\max_{\pi^W \in \Pi} u_W(\pi^W, R(\pi^W)) = g(R(\pi^W))V_w + \hat{s} \text{.} \textsuperscript{18}
$$

(B.40)

There exists a multitude of possible solutions to this problem. Thus $\pi^*W$ represents the set of maximizer. For all $\pi^*W \in \pi^*W$ we have:

(i) $g(\pi^*W) \in \left\{ \begin{array}{ll} 
\{1\} & \text{if } p > \frac{1}{2} \\
\{0, 1\} & \text{otherwise.} 
\end{array} \right.$

(ii) $(s(\pi^*W), t_j(\pi^*W))_{j \in [0, 1]} \in [0, \hat{s}] \times [0, \hat{t}]$ such that the budget is balanced and additionally if $p > \frac{1}{2}$:

$$
s(\pi^*W) - t_j(\pi^*W) > \hat{s} - V_w \text{ for more than } 1/2 \text{ project winners.}
$$

The explanation for these findings is completely similar as in the proof of Proposition 3.5.

**Summary**: In equilibrium, a first proposal is an element from the set $\pi^*W$. An optimal counter-proposal is a best response to this first proposal and therefore an element of the set $\mathcal{R}(\pi^W)$. Let $\pi^*L \in \mathcal{R}(\pi^W)$ be the chosen counter-proposal. $\pi^*L$ is implemented in equilibrium.

If $p > \frac{1}{2}$: $W$ is able to induce project adoption. He proposes the project and distributes subsidies and taxes such that $u_j(\pi^W) > \hat{s}$ for more than half of the society. In his best response, $L$ has to propose the project as well and distributes subsidies and taxes such that $u_j(\pi^*L) \geq u_j(\pi^*W) > \hat{s}$. Consequently, $g(\pi^*L) = 1$.

If $p < \frac{1}{2}$: It is not possible to find a proposal $\pi^W$ that induces project adoption. Thus, $W$ is indifferent among all proposals in $\Pi_s$. An optimal counter-proposal involves $g(\pi^*L) = 0$.

Finally, because $s(R(\pi^W)) = \hat{s}$, $\forall R(\pi^W) \in \mathcal{R}(\pi^W)$, it follows that $S(\pi^*L) = \hat{s}$.

\textsuperscript{17}Recall that adoption of proposal $R(\pi^W)$ does not depend on the size of $t_L(R(\pi^W))$. Thus, $L$ is free to choose his preferred tax level without affecting the constraint $I(\pi^W, \pi^C) = 1$. Moreover, because taxes can be distributed arbitrarily, $L$ can always fulfill the constraint $I(\pi^W, \pi^C) = 1$ if $s(R(\pi^W)) = \hat{s}$ (see item (iii)). Since $\hat{s}$ corresponds to the preferred subsidy level of $L$, he chooses $s(R(\pi^W)) = \hat{s}$.

\textsuperscript{18}Note that $u_W(\pi^W, R(\pi^W)) = g(R(\pi^W))V_w + \hat{s}$, $\forall R(\pi^W) \in \mathcal{R}(\pi^W)$.
Proof of Proposition 3.9

In order to examine the incentives of $\mathcal{W}$ for improving the project, it will be useful to consider his utility for given project parameters and for each constitution. From Propositions 3.5 to 3.8 we obtain

$$u_W(\pi^*) = u_W(\pi^*_S) = \begin{cases} V \text{ if } p > \frac{1}{2} \\ 0 \text{ otherwise.} \end{cases}$$

(B.41)

$$u_W(\pi^*_T) = \begin{cases} V - (1 + \lambda)k \text{ if } \hat{p} \leq (1 + \lambda)(p - \frac{1}{2}) \\ V - (1 + \lambda)k \text{ if } 1 - (1 + \lambda)\frac{1}{2} > \hat{p} > (1 + \lambda)(p - \frac{1}{2}) \text{ and } (1 + \lambda)p \geq \hat{p} \\ 0 \text{ if } (1 + \lambda)p < \hat{p}. \end{cases}$$

(B.42)

$$u_W(\pi^*_ST) = \begin{cases} V - (1 + \lambda)k \text{ if } p > \frac{1}{2} \\ 0 \text{ otherwise.} \end{cases}$$

(B.43)

Constitutions involving an arbitrary tax code (such as constitutions $\Pi$ and $\Pi_S$) yield utility to $\mathcal{W}$ that is independent of the parameters $V_l$ and $k$. Obviously, exerting costly effort to enhance these project parameters is not profitable for $\mathcal{W}$. On the contrary, these constitutions yield utility to $\mathcal{W}$ that depends on $p$. Hence, for sufficiently low effort costs $c$, exerting effort in order to increase $p$ from $p \leq \frac{1}{2}$ to $\hat{p} > \frac{1}{2}$ is profitable for $\mathcal{W}$.

A similar argument concerning $p$ can be applied to constitution $\Pi_{ST}$. Moreover, as long as $p > \frac{1}{2}$ and effort costs are sufficiently low, it is profitable for $\mathcal{W}$ to decrease $k$ as his utility also depends on $k$. However, exerting effort to increase $V_l$ is never profitable for $\mathcal{W}$ in constitution $\Pi_{ST}$.

Under constitution $\Pi_T$, $\mathcal{W}$ profits from decreasing $\hat{p}$ such that either $\hat{p} \leq (1 + \lambda)(p - \frac{1}{2})$, or $1 - (1 + \lambda)\frac{1}{2} > \hat{p} > (1 + \lambda)(p - \frac{1}{2})$ and $(1 + \lambda)p \geq \hat{p}$ hold. Moreover, as long as the project is implemented, $\mathcal{W}$ profits from decreasing $k$, too.

Note that there are different ways to lower $\hat{p}$. Most appropriately, $\mathcal{W}$ decreases $k$ from $\bar{k}$ to $\tilde{k}$ first.\(^{19}\) If the project is implemented with $\bar{k}$, he does not further enhance project efficiency. However, if the project is not implemented with $\bar{k}$, he might increase $p$ and/or $V_l$ such that the project is adopted with the new parameter-values.

\[\square\]

\(^{19}\)Decreasing $k$ has two positive effects for $\mathcal{W}$: First, it lowers $\hat{p}$ and second, it reduces project costs.

\(^{20}\)That is, if now either $\hat{p} \leq (1 + \lambda)(p - \frac{1}{2})$, or $1 - (1 + \lambda)\frac{1}{2} > \hat{p} > (1 + \lambda)(p - \frac{1}{2})$ and $(1 + \lambda)p \geq \hat{p}$.\]
Appendix C

Proofs for Chapter 4

Proof of Proposition 4.1

To prove Proposition 4.1 we proceed in ten steps (S1-S10):

S1: The fairness device implies that the problem of the agenda-setter is similar for all members of Party $W$. Let $s^W(\pi_F)$ denote the uniform subsidy distributed to members of Party $W$ under an arbitrary proposal $\pi_F$. For notational convenience we write $s^W$ and drop the argument $\pi_F$. Hence, the agenda-setter solves:

$$\max_{\pi_F \in \Pi_F} \left\{ \left( g(\pi_F)(V_w - (1 + \lambda)k) + s^W - (1 + \lambda)ps^W - (1 + \lambda) \int_p^1 s_j(\pi_F) dj \right) I(\pi_F) \right\}.$$  

We define

$$u_W(\pi_F) := g(\pi_F)(V_w - (1 + \lambda)k) + s^W - (1 + \lambda)ps^W - (1 + \lambda) \int_p^1 s_j(\pi_F) dj. \quad (C.1)$$

S2: We claim that any optimal proposal that yields $I(\pi_F) = 1$ is characterized by

$$s_j = \begin{cases} 
    s^L(\pi_F) & \text{for } j \in [p, m] \\
    0 & \text{for } j \in [m, 1]
\end{cases} \quad (C.2)$$

for some $s^L(\pi_F) \in [0, \hat{s}]$.\(^1\) We will specify $s^L$ in the next steps.

The claim follows from two observations:

First, the approval of a proposal requires the support of $m$ voters and thus $m - p$ project losers. Hence, it is sufficient to compensate project losers in $[p, m]$ for their incurred utility losses. Consequently, $s_j(\pi_F) = 0$ for $j \in [m, 1]$.

\(^1\)To make the notation more clear, we drop the argument $\pi_F$ and simply write $s^L$. 

166
Appendix C. Proofs for Chapter 4

Second, since higher subsidies for project losers reduce the utility of project winners, the agenda-setter has an incentive to pay the lowest amount of subsidies to project losers such that they vote for proposal \( \pi_F \). Thus, because project losers are identical, it is never profitable to offer different subsidy-rates for individuals in \([p, m]\).

S3: Let us now characterize the optimal \( s^L \) under \( I(\pi_F) = 1 \):

From the considerations in step S2 it follows that

\[
\int_p^1 s_j(\pi_F) \, dj = (m - p) s^L. \tag{C.3}
\]

In order to induce subsidized project losers to vote for proposal \( \pi_F \), it is required that \( u_j(\pi_F) \geq 0 \) for \( j \in [p, m] \), or equivalently

\[
g(\pi_F)(V_i - (1 + \lambda)k) + s^L - (1 + \lambda) p s^W - (1 + \lambda) \int_p^1 s_j(\pi_F) \, dj \geq 0. \tag{C.4}
\]

Inserting (C.3) into (C.4) and solving for \( s^L \) yields

\[
s^L \geq \frac{(1 + \lambda) p s^W}{1 - (1 + \lambda)(m - p)} + g(\pi_F) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)(m - p)}. \tag{C.5}
\]

Note that the right-hand side of (C.5) is non-negative for all feasible \( g(\pi_F) \) and \( s^W \), because of Assumption 4.1 and the fact that \( V_i - (1 + \lambda)k < 0 \). In order to maximize her utility under \( I(\pi_F) = 1 \), the agenda-setter chooses the smallest possible \( s^L \) such that \( I(\pi_F) = 1 \), that is:

\[
s^L = \frac{(1 + \lambda) p s^W}{1 - (1 + \lambda)(m - p)} + g(\pi_F) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)(m - p)}. \tag{C.6}
\]

Since \( s^L \) is a function of \( g(\pi_F) \) and \( s^W \), we will write \( s^L(g(\pi_F), s^W) \).

S4: We next investigate under what condition the utility of the agenda-setter is increasing in \( s^W \) in the optimal proposal with \( I(\pi_F) = 1 \):

From considerations in steps S2 and S3 it follows that

\[
\int_p^1 s_j(\pi_F) \, dj = (m - p) \left[ \frac{(1 + \lambda) p s^W}{1 - (1 + \lambda)(m - p)} + g(\pi_F) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)(m - p)} \right]. \tag{C.7}
\]

Inserting (C.7) into \( u_W(\pi_F) \), we obtain the derivative of \( u_W(\pi_F) \) with respect to \( s^W \) as

\[
\frac{\partial u_W(\pi_F)}{\partial s^W} = 1 - (1 + \lambda)p - (1 + \lambda)(m - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)(m - p)}. \tag{C.8}
\]
The agenda-setter increases subsidies if and only if \( \frac{\partial u_W}{\partial s_W}(\pi_F) > 0 \) (according to Tie-breaking Rule 4.2, \( s^W = 0 \) is chosen in case \( \frac{\partial u_W}{\partial s_W}(\pi_F) = 0 \)). This implies that a positive \( s^W \) is chosen if

\[
1 - (1 + \lambda)p - (1 + \lambda)(m - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)(m - p)} > 0,
\]

which can also be written as

\[
1 - (1 + \lambda)m > 0. \tag{C.9}
\]

S5: We now examine whether for the agenda-setter \( g(\pi_F) = 1 \) is preferable over \( g(\pi_F) = 0 \) under the optimal proposal with \( I(\pi_F) = 1 \). We use the notation \( u_W(g(\pi_F) = 1, \cdot) \) to indicate the utility of members of Party \( W \) in case \( g(\pi_F) = 1 \), and we use \( u_W(g(\pi_F) = 0, \cdot) \) to indicate the utility of members of Party \( W \) in case \( g(\pi_F) = 0 \). Using (C.7), we obtain

\[
\begin{align*}
  u_W(g(\pi_F) = 1, \cdot) &= V_w - (1 + \lambda)k + s^W + (1 + \lambda)ps^W \\
  &\quad - (1 + \lambda)(m - p) \left[ \frac{(1 + \lambda)ps^W}{1 - (1 + \lambda)(m - p)} + \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)(m - p)} \right] \\
  u_W(g(\pi_F) = 0, \cdot) &= s^W - (1 + \lambda)ps^W - (1 + \lambda)(m - p) \frac{(1 + \lambda)ps^W}{1 - (1 + \lambda)(m - p)}
\end{align*}
\]

Comparing \( u_W(g(\pi_F) = 1, \cdot) \) and \( u_W(g(\pi_F) = 0, \cdot) \) makes clear that \( g(\pi_F) = 1 \) is preferable over \( g(\pi_F) = 0 \) iff

\[
V_w \geq V_w^* := (1 + \lambda) \frac{k - (m - p)V_i}{1 - (1 + \lambda)(m - p)} \tag{C.10}
\]

S6: Using (C.9) and (C.10), we can distinguish four different cases:

A: \( 1 - (1 + \lambda)m > 0 \) and \( V_w^* \geq V_w \) hold.
B: \( 1 - (1 + \lambda)m > 0 \) and \( V_w^* < V_w \) hold.
C: \( 1 - (1 + \lambda)m \leq 0 \) and \( V_w^* \geq V_w \) hold.
D: \( 1 - (1 + \lambda)m \leq 0 \) and \( V_w^* < V_w \) hold.

S7: Case A implies that the optimal proposal with \( I(\pi_F) = 1 \) comprises \( s^W > 0 \) and \( g(\pi_F) = 1 \). Because \( u_W(\pi_F) \) is linearly increasing in \( s^W \), the agenda-setter chooses \( s^W = \hat{s} \). It remains to check if a proposal comprising \( g(\pi_F) = 1 \) and \( s^W = \hat{s} \) is feasible:

\footnote{Recall that if indifferent, the agenda-setter chooses \( g(\pi_F) = 1 \) (see Tie-breaking Rule 4.1).}
Since \(g(\pi_F)\) and \(s^W\) are chosen within their respective domain and the budget is balanced, we only have to check whether \(s^c (1, \hat{s}) \in [0, \hat{s}]\):

From step S3 we know that \(s^c (g(\pi_F), s^W) \geq 0\) holds for all \(g(\pi_F) \in \{0, 1\}\) and \(s^W \in [0, \hat{s}]\).

\[ s^c (1, \hat{s}) \leq \hat{s} \text{ holds if } \]

\[
\frac{(1 + \lambda)p \hat{s}}{1 - (1 + \lambda)(m - p)} + \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)(m - p)} \leq \hat{s},
\]

which can be rewritten as

\[
\frac{1}{\hat{s}}((1 + \lambda)k - V_i) \leq 1 - (1 + \lambda)m.
\]

For sufficiently high \(\hat{s}\), Condition C.12 holds in Case A where \(1 - (1 + \lambda)m > 0\). Thus, the optimal proposal with \(I(\pi_F) = 1\) in case A is given by \(\pi_{F1}^{p < m}\) (see Proposition 4.1).

At last, we have to investigate whether it is optimal for the agenda-setter to choose \(\pi_{F1}^{p < m}\) instead of a proposal with \(I(\pi_F) = 0\).

Since the derivative of \(u_W(\pi_F)\) with respect to \(s^W\) is strictly positive, the agenda-setter experiences a utility gain from subsidization. Moreover, because \(u_W(\langle g(\pi_F) = 1, \cdot\rangle) \geq u_W(\langle g(\pi_F) = 0, \cdot\rangle)\), she is never worse off if the project is implemented. These considerations imply that \(u_W(\pi_{F1}^{p < m}) > 0\). Consequently, the agenda-setter chooses \(\pi_{F1}^{p < m}\).

**S8:** Case B implies that the optimal proposal with \(I(\pi_F) = 1\) comprises \(s^W = \hat{s}\) and \(g(\pi_F) = 0\).\(^3\) Again, it remains to check if such a proposal is feasible, that is, if \(s^c (0, \hat{s}) \leq \hat{s}\).\(^4\)

Since \(s^c (g(\pi_F), s^W)\) increases in \(g(\pi_F)\) (see Equation C.6), we find \(s^c (0, \hat{s}) < s^c (1, \hat{s})\). In step S7, we have shown that \(s^c (1, \hat{s}) \leq \hat{s}\) when \(1 - (1 + \lambda)m > 0\). This implies \(s^c (0, \hat{s}) < \hat{s}\) when \(1 - (1 + \lambda)m > 0\). Thus, the optimal proposal with \(I(\pi_F) = 1\) in case B is given by \(\pi_{F2}^{p < m}\) (see Proposition 4.1).

Again, we have to investigate whether it is optimal for the agenda-setter to choose \(\pi_{F2}^{p < m}\) instead of a proposal with \(I(\pi_F) = 0\).

Since the derivative of \(u_W(\pi_F)\) with respect to \(s^W\) is strictly positive, the agenda-setter experiences a utility gain from subsidization, which yields \(u_W(\pi_{F2}^{p < m}) > 0\). Consequently, the agenda-setter chooses \(\pi_{F2}^{p < m}\).

---

\(^3\)Recall that the agenda-setter chooses \(s^W = \hat{s}\) if \(1 - (1 + \lambda)m > 0\) because \(u_W(\pi_F)\) is linearly increasing in \(s^W\).

\(^4\)We already argued that \(s^c (g(\pi_F), s^W) \geq 0\) holds for all \(g(\pi_F) \in \{0, 1\}\) and \(s^W \in [0, \hat{s}]\) (see step S3).
Appendix C. Proofs for Chapter 4

S9: Case C implies that the optimal proposal with \( I(\pi_F) = 1 \) comprises \( s^W = 0 \) and \( g(\pi_F) = 1 \). To show that such a proposal is feasible, we have to check whether \( s^L(1,0) \leq \hat{s} \):

For \( s^W = 0 \), the subsidy \( s^L \) depends solely on project parameters (see Equation C.6). For sufficiently high \( \hat{s} \), the feasibility of \( s^L(1,0) \) is therefore guaranteed. Thus, the optimal proposal with \( I(\pi_F) = 1 \) in case C is given by \( \pi_{F_3}^{p<m} \) (see Proposition 4.1).

Again, we have to show that it is optimal for the agenda-setter to choose \( \pi_{F_3}^{p<m} \) over a proposal with \( I(\pi_F) = 0 \).

Since \( u_W(\langle g(\pi_F) = 1, \cdot \rangle) \geq u_W(\langle g(\pi_F) = 0, \cdot \rangle) \), the agenda-setter experiences a utility gain from project implementation, which yields \( u_W(\pi_{F_3}^{p<m}) \geq 0 \). Consequently, the agenda-setter chooses \( \pi_{F_3}^{p<m} \).

S10: Case D implies that the optimal proposal with \( I(\pi_F) = 1 \) comprises \( s^W = 0 \) and \( g(\pi_F) = 0 \). Such a proposal is equivalent to the status quo. Thus, the agenda-setter could either suggest the status quo or any other proposal that yields \( I(\pi_F) = 0 \) (especially, she could also suggest unconstitutional proposals). Hence, there is no unique equilibrium proposal in case D, but the status quo always prevails.

\[ \square \]

Proof of Proposition 4.3

To prove Proposition 4.3 we proceed in six steps (S1-S6):

S1: The agenda-setter’s problem is given by

\[
\max_{\pi_C \in \Pi_C} \left\{ \left( g(\pi_C)(V_w - (1 + \lambda)k) + s_0 - (1 + \lambda) \int_0^1 s_j(\pi_C) dj \right) I(\pi_C) \right\}.^5
\]

Recall that

\[ g(\pi_C)(V_w - (1 + \lambda)k) + s_0 - (1 + \lambda) \int_0^1 s_j(\pi_C) dj =: u_0(\pi_C). \quad (C.13) \]

S2: We claim that any optimal proposal that yields \( I(\pi_C) = 1 \) is characterized by

\[
s_j(\pi_C) = \begin{cases} 
0 & \text{for } j \in [0, p] \\
s^L(\pi_C) & \text{for } j \in [p, p + n(1 - p)] \\
0 & \text{for } j \in [p + n(1 - p), 1] 
\end{cases}
\quad (C.14)
\]

\[ ^5s_0 \text{ indicates the subsidy of the agenda-setter. For notational convenience we drop the argument } \pi_C. \]
for some $s^c(\pi_C) \in [0, \hat{s}]$.\footnote{To make the notation more clear, we drop the argument $\pi_C$ and simply write $s^c$.} We will specify $s^c$ in the next steps.

The claim follows from two observations:

First, the approval of a proposal requires the support of $m$ voters. Since party members are committed to vote uniformly and $p < m$, $1 - p \geq m$, the voting behavior of Party $\mathcal{L}$ is deciding for the adoption of a proposal. In order to make Party $\mathcal{L}$ vote for $\pi_C$, a fraction of $n(1 - p)$ project losers has to support $\pi_C$. Hence, it is sufficient to compensate project losers in $[p, p + n(1 - p)]$ for their incurred utility losses. Consequently, $s_j(\pi_{FC}) = 0$ for $j \in [0, p] \cup [p + n(1 - p), 1]$.

Second, since higher subsidies for project losers reduce the utility of the agenda-setter, she has an incentive to pay the lowest amount of subsidies to project losers such that they vote for proposal $\pi_C$. Thus, because project losers are identical, it is never profitable to offer different subsidy rates for individuals in $[p, p + n(1 - p)]$.

S3: Let us now characterize the optimal $s^c$ under $I(\pi_C) = 1$:

From the considerations in step S2 it follows that

$$\int_0^1 s_j(\pi_C) \, dj = n(1 - p)s^c. \tag{C.15}$$

In order to induce subsidized project losers to vote for proposal $\pi_C$, it is required that $u_j(\pi_C) \geq 0$ for $j \in [p, p + n(1 - p)]$, or equivalently

$$g(\pi_C)(V_i - (1 + \lambda)k) + s^c - (1 + \lambda) \int_0^1 s_j(\pi_C) \, dj \geq 0. \tag{C.16}$$

Inserting (C.15) into (C.16) and solving for $s^c$ yields

$$s^c \geq g(\pi_C) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)}. \tag{C.17}$$

Note that the right-hand side of (C.17) is non-negative for all feasible $g(\pi_C)$, because of Assumption 4.2 and the fact that $V_i - (1 + \lambda)k < 0$. In order to maximize her utility under $I(\pi_C) = 1$, the agenda-setter chooses the smallest possible $s^c$ such that $I(\pi_C) = 1$, that is:

$$s^c = g(\pi_C) \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)}. \tag{C.18}$$

Since $s^c$ depends solely on project parameters, it is feasible for $g(\pi_C) = 1$ and $g(\pi_C) = 0$ if $\hat{s}$ is sufficiently large (that is, $s^c \in [0, \hat{s}] \ \forall g(\pi_C) \in \{0, 1\}$).
Appendix C. Proofs for Chapter 4

S4: So far we characterized subsidies to voters other than the agenda-setter. For the agenda-setter it is obvious that \( s_0 = \hat{s} \) is the utility-maximizing subsidy in case \( I(\pi_C) = 1 \).

S5: We now examine whether for the agenda-setter \( g(\pi_C) = 1 \) is preferable over \( g(\pi_C) = 0 \) under the optimal proposal with \( I(\pi_C) = 1 \).

Given the subsidy scheme derived in steps S1-S4, it follows that 
\[
\int_0^1 s_j(\pi_C) dj = n(1-p) \left[ g(\pi_C) \frac{(1+\lambda)k - V_i}{1 - (1+\lambda)n(1-p)} \right]. \quad (C.19)
\]

We use the notation \( u_0((g(\pi_C) = 1, \cdot)) \) to indicate the agenda-setter’s utility when \( g(\pi_C) = 1 \), and analogously we use \( u_0((g(\pi_C) = 0, \cdot)) \) to indicate the agenda-setter’s utility when \( g(\pi_C) = 0 \). Inserting (C.19) into \( u_0((\pi_C) \) yields:
\[
u_0((g(\pi_C) = 1, \cdot)) = s_0 + V_w - (1+\lambda)k - (1+\lambda)n(1-p) \frac{(1+\lambda)k - V_i}{1 - (1+\lambda)n(1-p)}
\]
\[
u_0((g(\pi_C) = 0, \cdot)) = s_0.
\]

Comparing \( u_0((g(\pi_C) = 1, \cdot)) \) and \( u_0((g(\pi_C) = 0, \cdot)) \) makes clear that \( g(\pi_C) = 1 \) is preferable over \( g(\pi_C) = 0 \) iff
\[
V_w \geq V_w^{**} := (1+\lambda) \frac{k - n(1-p)V_i}{1 - (1+\lambda)n(1-p)}.
\quad (C.20)
\]

S6: Summary: The agenda-setter always chooses \( I(\pi_C) = 1 \), so she can ensure \( s_0 = \hat{s} \).

If \( V_w \geq V_w^{**} \), it is profitable for her to choose \( g(\pi_C) = 1 \) additionally. However, \( I(\pi_C) = 1 \) then requires \( s^C = \frac{(1+\lambda)k - V_i}{1 - (1+\lambda)n(1-p)} \) for \( n(1-p) \) project losers. This constitutes proposal \( \pi^p_{C1} \).

If \( V_w < V_w^{**} \), it is not profitable for her to suggest the project and she therefore chooses \( g(\pi_C) = 0 \). Since \( I(\pi_C) = 1 \) can be attained without paying further subsidies, \( s^C = 0 \). This constitutes proposal \( \pi^p_{C2} \).

Proof of Proposition 4.4

To prove Proposition 4.4 we proceed in nine steps (S1-S9):

S1: Similar to Proposition 4.3, the agenda-setter solves
\[
\max_{\pi_C \in \Pi_C} \left\{ \left( g(\pi_C)(V_w - (1+\lambda)k) + s_0 - (1+\lambda) \int_0^1 s_j(\pi_C) dj \right) I(\pi_C) \right\}. \quad 7
\]

\[7\text{Recall that } s_0 \text{ indicates the subsidy of the agenda-setter, and that we drop the argument } \pi_C.\]
Appendix C. Proofs for Chapter 4

Again, we use

\[ g(\pi_C)(V_w - (1 + \lambda)k) + s_0 - (1 + \lambda) \int_0^1 s_j(\pi_C) dj =: u_0(\pi_C). \tag{C.21} \]

S2: We claim that any optimal proposal that yields \( I(\pi_C) = 1 \) is characterized by

\[
s_j(\pi_C) = \begin{cases} 
  s^W(\pi_C) & \text{for } j \in [0, np] \\
  0 & \text{for } j \in [np, p] \\
  s^L(\pi_C) & \text{for } j \in [p, p + n(1 - p)] \\
  0 & \text{for } j \in [p + n(1 - p), 1]
\end{cases}
\tag{C.22}
\]

for some \( s^W(\pi_C) \in [0, \hat{s}] \) and \( s^L(\pi_C) \in [0, \hat{s}] \).\(^8\) We will specify \( s^W \) and \( s^L \) in the next steps.

The claim follows from two observations:

First, the approval of a proposal requires the support of \( m \) voters. Since party members are committed to vote uniformly and \( p < m \), \( 1 - p < m \), the voting behavior of both parties is deciding for the adoption of a proposal. In order to bring Party \( W \) to vote for \( \pi_C \), a fraction of \( np \) project winners has to support \( \pi_C \). In order to bring Party \( L \) to vote for \( \pi_C \), a fraction of \( n(1 - p) \) project losers has to support \( \pi_C \). Hence, it is sufficient to compensate project winners in \([0, np]\) and project losers \([p, p + n(1 - p)]\) for their incurred utility losses. Consequently, \( s_j(\pi_C) = 0 \) for \( j \in [np, p] \cup [p + n(1 - p), 1] \).

Second, since higher subsidies for project winners or losers reduce the utility of the agenda-setter, she has an incentive to pay the lowest amount of subsidies to project winners and to project losers such that they vote for proposal \( \pi_C \). Thus, because project winners are identical, it is never profitable to offer different subsidy rates for individuals in \([0, np]\). Similarly, as project losers are identical, it is never optimal to offer different subsidy rates for individuals in \([p, p + n(1 - p)]\).

S3: Let us now characterize the optimal \( s^W \) under \( I(\pi_C) = 1 \):

From the considerations in step S2 it follows that

\[
\int_0^1 s_j(\pi_C) dj = np s^W + n(1 - p) s^L. \tag{C.23}
\]

\(^8\)To make the notation more clear, we drop the argument \( \pi_C \) and simply write \( s^W \) and \( s^L \).
In order to induce subsidized project winners to vote for \( \pi_C \), it is required that \( u_j(\pi_C) \geq 0 \) for \( j \in [0, np] \), or equivalently
\[
g(\pi_C)(V_w - (1 + \lambda)k) + s^W - (1 + \lambda) \int_0^1 s_j(\pi_C) \, dj \geq 0. 
\] (C.24)

Inserting (C.23) into (C.24) and solving for \( s^W \) yields
\[
s^W \geq \frac{(1 + \lambda)n(1 - p)s^C}{1 - (1 + \lambda)np} - g(\pi_C) \frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)np}. 
\] (C.25)

The right-hand side of (C.25) might be positive or negative depending on the values of \( g(\pi_C) \) and \( s^C \).

In order to maximize her utility under \( I(\pi_C) = 1 \), the agenda-setter chooses the smallest feasible \( s^W \) such that \( I(\pi_C) = 1 \), that is:
\[
s^W = \max \left\{ \frac{(1 + \lambda)n(1 - p)s^C}{1 - (1 + \lambda)np} - g(\pi_C) \frac{V_w - (1 + \lambda)k}{1 - (1 + \lambda)np}, 0 \right\}. 
\] (C.26)

S4: Let us now characterize the optimal \( s^C \) under \( I(\pi_C) = 1 \):

In order to induce subsidized project losers to vote for proposal \( \pi_C \), it is required that \( u_j(\pi_C) \geq 0 \) for \( j \in [p, p + n(1 - p)] \), or equivalently
\[
g(\pi_C)(V_l - (1 + \lambda)k) + s^C - (1 + \lambda) \int_0^1 s_j(\pi_C) \, dj \geq 0. 
\] (C.27)

Inserting (C.23) into (C.27) and solving for \( s^C \) yields
\[
s^C \geq \frac{(1 + \lambda)np s^W}{1 - (1 + \lambda)n(1 - p)} + g(\pi_C) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)}. 
\] (C.28)

Note that the right-hand side of (C.28) is non-negative for all feasible \( g(\pi_C) \) and \( s^W \), because of Assumption 4.2 and the fact that \( V_l - (1 + \lambda)k < 0 \). In order to maximize her utility under \( I(\pi_C) = 1 \), the agenda-setter chooses the smallest possible \( s^C \) such that \( I(\pi_C) = 1 \), that is:
\[
s^C = \frac{(1 + \lambda)np s^W}{1 - (1 + \lambda)n(1 - p)} + g(\pi_C) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)}. 
\] (C.29)

\footnote{For the lines of this proof we assume \( 1 > (1 + \lambda)n \) which implies \( 1 > (1 + \lambda)np \). It will turn out that this assumption becomes irrelevant as the agenda-setter always chooses a proposal with \( s^W = 0 \).}
Appendix C. Proofs for Chapter 4

S5: To obtain explicit solutions for \( s^W \) and \( s^L \), we solve the system of equations given by (C.26) and (C.29):

**Lemma C.1**

If \( V_w \geq V_w^{**} \) subsidies \( s^W \) and \( s^L \) are given by

\[
\begin{pmatrix}
  s^W \\
  s^L
\end{pmatrix} = g(\pi_C) \begin{pmatrix}
  0 \\
  \frac{1-1+(1+\lambda)n(1-p)}{(1+(1+\lambda)n(1-p))}
\end{pmatrix}.
\]

(C.30)

If \( V_w < V_w^{**} \) subsidies \( s^W \) and \( s^L \) are given by

\[
\begin{pmatrix}
  s^W \\
  s^L
\end{pmatrix} = -g(\pi_C) \begin{pmatrix}
  \frac{1}{1-(1+\lambda)n} \\
  \frac{1-(1+(1+\lambda)n(1-p))V_w+(1+(1+\lambda)n(1-p)V_l-(1+\lambda)k}{(1+(1+\lambda)n(1-p))V_w+(1+(1+\lambda)n(1-p)V_l-(1+\lambda)k}
\end{pmatrix}.
\]

(C.31)

The proof of Lemma C.1 is given at the end of this appendix.

Because subsidies \( s^W \) and \( s^L \) solely depend on project parameters, they are feasible for sufficiently large \( \hat{s} \), that is \( s^W \in [0, \hat{s}] \) and \( s^L \in [0, \hat{s}] \) \( \forall g(\pi_C) \in \{0, 1\} \).

S6: So far we characterized subsidies to voters other than the agenda-setter. For the agenda-setter, it is obvious that \( s_0 = \hat{s} \) is the utility-maximizing subsidy in case \( I(\pi_C) = 1 \).

S7: We now examine whether for the agenda-setter \( g(\pi_C) = 1 \) is preferable over \( g(\pi_C) = 0 \) under the optimal proposal with \( I(\pi_C) = 1 \), if \( V_w \geq V_w^{**} \).

Given the subsidy scheme derived in steps S1-S6, it follows that for \( V_w \geq V_w^{**} \)

\[
\int_0^1 s_j(\pi_C) dj = \frac{n(1-p)}{1-(1+\lambda)n(1-p)} \left[ \frac{(1+(1+\lambda)n(1-p))V_w+(1+(1+\lambda)n(1-p)V_l-(1+\lambda)k}{(1+(1+\lambda)n(1-p))V_w+(1+(1+\lambda)n(1-p)V_l-(1+\lambda)k} \right].
\]

(C.32)

We use the notation \( u_0(\langle g(\pi_C) = 1, \cdot \rangle) \) and \( u_0(\langle g(\pi_C) = 0, \cdot \rangle) \) analogously as in the proof of Proposition 4.3. Inserting (C.32) into \( u_0(\pi_C) \) yields:

\[
u_0(\langle g(\pi_C) = 1, \cdot \rangle) = s_0 + V_w - (1+\lambda)k - (1+\lambda)n(1-p) \frac{(1+(1+\lambda)n(1-p)\frac{1+(1+\lambda)n(1-p))V_w+(1+(1+\lambda)n(1-p)V_l-(1+\lambda)k}{(1+(1+\lambda)n(1-p))V_w+(1+(1+\lambda)n(1-p)V_l-(1+\lambda)k} \right].
\]

\[
u_0(\langle g(\pi_C) = 0, \cdot \rangle) = s_0.
\]

Comparing \( u_0(\langle g(\pi_C) = 1, \cdot \rangle) \) and \( u_0(\langle g(\pi_C) = 0, \cdot \rangle) \) makes clear that \( g(\pi_C) = 1 \) is preferable over \( g(\pi_C) = 0 \) because

\[
V_w \geq V_w^{**} := (1+\lambda)\frac{k-n(1-p)V_l}{1-(1+\lambda)n(1-p)}.
\]

(C.33)
At last, we examine whether for the agenda-setter $g(\pi_C) = 1$ is preferable over $g(\pi_C) = 0$ under the optimal proposal with $I(\pi_C) = 1$, if $V_w < V_w^{**}$.

Given the subsidy scheme derived in steps S1-S6, it follows that for $V_w < V_w^{**}$

\[
\int_0^1 s_j(\pi_C) \, dj = -n(1 - p) \left[ g(\pi_C) \frac{(1 + \lambda) npV_w + (1 - (1 + \lambda) np)V_l - (1 + \lambda)k}{1 - (1 + \lambda)n} \right] \]

\[
- \frac{np}{1 - (1 + \lambda)n} \left[ g(\pi_C) \frac{(1 - (1 + \lambda)n(1 - p))V_w + (1 + \lambda)n(1 - p)V_l - (1 + \lambda)k}{1 - (1 + \lambda)n} \right].
\]

Inserting (C.34) into $u_0(\pi_C)$ yields

\[
u_0 (\langle g(\pi_C) = 1, \cdot \rangle) = s_0 + V_w - (1 + \lambda)k \\
+ (1 + \lambda)n(1 - p) \left[ g(\pi_C) \frac{(1 + \lambda) npV_w + (1 - (1 + \lambda) np)V_l - (1 + \lambda)k}{1 - (1 + \lambda)n} \right] \\
+ (1 + \lambda) np \left[ g(\pi_C) \frac{(1 - (1 + \lambda)n(1 - p))V_w + (1 + \lambda)n(1 - p)V_l - (1 + \lambda)k}{1 - (1 + \lambda)n} \right].
\]

\[
u_0 (\langle g(\pi_C) = 0, \cdot \rangle) = s_0.
\]

Comparing $u_0 (\langle g(\pi_C) = 1, \cdot \rangle)$ and $u_0 (\langle g(\pi_C) = 0, \cdot \rangle)$ makes clear that $g(\pi_C) = 0$ is preferable over $g(\pi_C) = 1$ since

\[
V_w < V_w^{**} := (1 + \lambda) \frac{k - n(1 - p)V_l}{1 - (1 + \lambda)n(1 - p)}.
\]

Summary: The agenda-setter always chooses $I(\pi_C) = 1$, so she can ensure $s_0 = \hat{s}$.

If $V_w \geq V_w^{**}$, it is profitable for her to choose $g(\pi_C) = 1$ additionally. Since $V_w \geq V_w^{**}$, the subsidy $s^W$ can be set to zero and the subsidy $s^L = \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)}$. This constitutes proposal $\pi_{C1}^{p<m}$.

If $V_w < V_w^{**}$, it is not profitable for the agenda-setter to suggest the project and she chooses $g(\pi_C) = 0$. Since $I(\pi_C) = 1$ can be attained without paying further subsidies, $s^W = s^L = 0$. This constitutes proposal $\pi_{C2}^{p<m}$.

\[\square\]

**Proof of Proposition 4.6**

To prove Proposition 4.6 we proceed in ten steps (S1-S10):

S1: The fairness device implies that the problem of the agenda-setter is similar for all members of Party $W$. Let $s^W(\pi_{FC})$ denote the uniform subsidy distributed to members
of Party 𝐴̃ under an arbitrary proposal 𝑛𝐴̃. For national convenience we write 𝑠𝐴̃ and drop the argument 𝑛𝐴̃. Hence, the agenda-setter solves:

\[
\max_{𝑛𝐴̃ ∈ Π𝐴̃}\left\{ \left( g(𝑛𝐴̃)(𝑉 − (1 + 𝜆)𝑘) + 𝑠𝐴̃ − (1 + 𝜆)𝑝𝑠𝐴̃ − (1 + 𝜆) \int_0^1 𝑠_j(𝑛𝐴̃)dj \right) I(𝑛𝐴̃) \right\}.
\]

We define

\[
u𝐴̃(𝑛𝐴̃) := g(𝑛𝐴̃)(𝑉 − (1 + 𝜆)𝑘) + 𝑠𝐴̃ − (1 + 𝜆)𝑝𝑠𝐴̃ − (1 + 𝜆) \int_0^1 𝑠_j(𝑛𝐴̃)dj.
\]

(C.36)

S2: We claim that any optimal proposal that yields \(I(𝑛𝐴̃) = 1\) is characterized by

\[
s_j(𝑛𝐴̃) = \begin{cases} 
  s^L(𝑛𝐴̃) & \text{for } j ∈ [p, p + n(1 − p)] \\
  0 & \text{for } j ∈ [p + n(1 − p), 1]
\end{cases}
\]

(C.37)

for some \(s^L(𝑛𝐴̃) ∈ [0, s]\).\(^{10}\) We will specify \(s^L\) in the next steps.

The claim follows from two observations:

First, the approval of a proposal requires the support of \(m\) voters. Since party members are committed to vote uniformly and \(p < m\), \(1 − p ≥ m\), the voting behavior of Party \(L\) is deciding for the adoption of a proposal. In order to bring Party \(L\) to vote for \(𝑛𝐴̃\), a fraction of \(n(1 − p)\) project losers has to support \(𝑛𝐴̃\). Hence, it is sufficient to compensate project losers in \([p, p + n(1 − p)]\) for their incurred utility losses. Consequently, \(s_j(𝑛𝐴̃) = 0\) for \(j ∈ [p + n(1 − p), 1]\).

Second, since higher subsidies for project losers reduce the utility of project winners, the agenda-setter has an incentive to pay the lowest amount of subsidies to project losers such that they vote for proposal \(𝑛𝐴̃\). Thus, because project losers are identical, it is never profitable to offer different subsidy-rates for individuals in \([p, p + n(1 − p)]\).

S3: Let us now characterize the optimal \(s^L\) under \(I(𝑛𝐴̃) = 1\):

From the considerations in step S2 it follows that

\[
\int_0^1 s_j(𝑛𝐴̃)dj = n(1 − p)s^L.
\]

(C.38)

In order to induce subsidized project losers to vote for proposal \(𝑛𝐴̃\), it is required that \(u_j(𝑛𝐴̃) ≥ 0\) for \(j ∈ [p, p + n(1 − p)]\), or equivalently

\[
g(𝑛𝐴̃)(𝑉_l − (1 + 𝜆)𝑘) + s^L − (1 + 𝜆)𝑝𝑠𝐴̃ − (1 + 𝜆) \int_0^1 𝑠_j(𝑛𝐴̃)dj ≥ 0.
\]

(C.39)

\(^{10}\)To make the notation more clear, we drop the argument \(𝑛𝐴̃\) and simply write \(s^L\).
Inserting (C.38) into (C.39) and solving for $s^L$ yields

$$s^L \geq \frac{(1 + \lambda)p s^W}{1 - (1 + \lambda)n(1 - p)} + g(\pi_{FC}) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)}. \tag{C.40}$$

Note that the right-hand side of (C.40) is non-negative for all feasible $g(\pi_{FC})$ and $s^W$, because of Assumption 4.2 and the fact that $V_l - (1 + \lambda)k < 0$. In order to maximize her utility under $I(\pi_{FC}) = 1$, the agenda-setter chooses the smallest possible $s^L$ such that

$$s^L = \frac{(1 + \lambda)p s^W}{1 - (1 + \lambda)n(1 - p)} + g(\pi_{FC}) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)}. \tag{C.41}$$

Since $s^L$ is a function of $g(\pi_{FC})$ and $s^W$, we will write $s^L(g(\pi_{FC}), s^W)$.

**S4:** We next investigate under what condition the utility of the agenda-setter is increasing in $s^W$ in the optimal proposal with $I(\pi_{FC}) = 1$:

From considerations in steps S2 and S3 it follows that

$$\int_p^1 s_j(\pi_{FC})dj = n(1 - p) \left[ \frac{(1 + \lambda)p s^W}{1 - (1 + \lambda)n(1 - p)} + g(\pi_{FC}) \frac{(1 + \lambda)k - V_l}{1 - (1 + \lambda)n(1 - p)} \right]. \tag{C.42}$$

Inserting (C.42) into $u_{W}(\pi_{FC})$, we obtain the derivative of $u_{W}(\pi_{FC})$ with respect to $s^W$ as

$$\frac{\partial u_{W}(\pi_{FC})}{\partial s^W} = 1 - (1 + \lambda)p - (1 + \lambda)n(1 - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)n(1 - p)}. \tag{C.43}$$

The agenda-setter increases subsidies if and only if $\frac{\partial u_{W}(\pi_{FC})}{\partial s^W} > 0$ (according to Tie-breaking Rule 4.2, $s^W = 0$ is chosen in case $\frac{\partial u_{W}(\pi_{FC})}{\partial s^W} = 0$). This implies that a positive $s^W$ is chosen if

$$1 - (1 + \lambda)p - (1 + \lambda)n(1 - p) \frac{(1 + \lambda)p}{1 - (1 + \lambda)n(1 - p)} > 0,$$

which can also be written as

$$1 - (1 + \lambda) [n + (1 - n)p] > 0. \tag{C.44}$$

**S5:** We now examine whether for the agenda-setter $g(\pi_{FC}) = 1$ is preferable over $g(\pi_{FC}) = 0$ under the optimal proposal with $I(\pi_{FC}) = 1$. We use $u_{W}(g(\pi_{FC}) = 1, \cdot)$ to indicate
implies that the optimal proposal with $g(\pi_{FC}) = 1$ and we use $u_W(\langle g(\pi_{FC}) = 0, \cdot \rangle)$ to indicate the utility of members of Party $W$ when $g(\pi_{FC}) = 0$. Using (C.42), we obtain

$$u_W(\langle g(\pi_{FC}) = 1, \cdot \rangle) = V_w - (1 + \lambda)k + s^W - (1 + \lambda)p s^W - (1 + \lambda)n(1 - p) \left[ \frac{(1 + \lambda)ps_W}{1 - (1 + \lambda)n(1 - p)} + \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \right].$$

$$u_W(\langle g(\pi_{FC}) = 0, \cdot \rangle) = s^W - (1 + \lambda)ps^W - (1 + \lambda)n(1 - p) \left[ \frac{(1 + \lambda)ps^W}{1 - (1 + \lambda)n(1 - p)} \right].$$

Comparing $u_W(\langle g(\pi_{FC}) = 1, \cdot \rangle)$ and $u_W(\langle g(\pi_{FC}) = 0, \cdot \rangle)$ makes clear that $g(\pi_{FC}) = 1$ is preferable over $g(\pi_{FC}) = 0$ iff

$$V_w \geq V_w^{**} := (1 + \lambda) \frac{k - n(1 - p)V_i}{1 - (1 - p)n(1 - p)}. \quad \text{(C.45)}$$

**S6:** Using (C.44) and (C.45), we can distinguish four different cases:

- **A:** $1 - (1 + \lambda)[n + (1 - n)p] > 0$ and $V_w \geq V_w^{**}$ hold.
- **B:** $1 - (1 + \lambda)[n + (1 - n)p] > 0$ and $V_w < V_w^{**}$ hold.
- **C:** $1 - (1 + \lambda)[n + (1 - n)p] \leq 0$ and $V_w \geq V_w^{**}$ hold.
- **D:** $1 - (1 + \lambda)[n + (1 - n)p] \leq 0$ and $V_w < V_w^{**}$ hold.

**S7:** Case A implies that the optimal proposal with $I(\pi_{FC}) = 1$ comprises $s^W > 0$ and $g(\pi_{FC}) = 1$. Because $u_W(\pi_{FC})$ is linearly increasing in $s^W$, the agenda-setter chooses $s^W = \hat{s}$. It remains to check if a proposal comprising $g(\pi_{FC}) = 1$ and $s^W = \hat{s}$ is feasible.

Since $g(\pi_{FC})$ and $s^W$ are chosen within their respective domain and the budget is balanced, we only have to check whether $s^c(1, \hat{s}) \in [0, \hat{s}]$:

From step S3 we know that $s^c(\langle g(\pi_{FC}), s^W \rangle) \geq 0$ holds for all $g(\pi_{FC}) \in \{0, 1\}$ and $s^W \in [0, \hat{s}]$.

$s^c(1, \hat{s}) \leq \hat{s}$ holds if

$$s^c(1, \hat{s}) = \frac{(1 + \lambda)p\hat{s}}{1 - (1 + \lambda)n(1 - p)} + \frac{(1 + \lambda)k - V_i}{1 - (1 + \lambda)n(1 - p)} \leq \hat{s}, \quad \text{(C.46)}$$

which can be rewritten as

$$\frac{1}{\hat{s}}((1 + \lambda)k - V_i) \leq 1 - (1 + \lambda)[n + (1 - n)p]. \quad \text{(C.47)}$$

**11**Recall that if indifferent, the agenda-setter chooses $g(\pi_{FC}) = 1$ (see Tie-breaking Rule 4.1).
Appendix C. Proofs for Chapter 4

For sufficiently high \( \hat{s} \), Condition C.47 holds in Case A where \( 1 - (1 + \lambda) [n + (1 - n)p] > 0 \). Thus, the optimal proposal with \( I(\pi_{FC}) = 1 \) in case A is given by \( \pi_{FC}^{p<m} \) (see Proposition 4.6).

At last, we have to investigate whether it is optimal for the agenda-setter to choose \( \pi_{FC}^{p<m} \) instead of a proposal with \( I(\pi_{FC}) = 0 \).

Since the derivative of \( u_W(\pi_{FC}) \) with respect to \( s^W \) is strictly positive, the agenda-setter experiences a utility gain from subsidization. Moreover, because \( u_W(\langle g(\pi_{FC}) = 1, \cdot \rangle) \geq u_W(\langle g(\pi_{FC}) = 0, \cdot \rangle) \), she is never worse off if the project is implemented. These considerations imply that \( u_W(h) > 0 \). Consequently, the agenda-setter chooses \( \pi_{FC}^{p<m} \).

S8: Case B implies that the optimal proposal with \( I(\pi_{FC}) = 1 \) comprises \( s^W = \hat{s} \) and \( g(\pi_{FC}) = 0 \).\(^{12}\) Again, it remains to check if such a proposal is feasible, that is, if \( s^L(0, \hat{s}) \leq \hat{s} \).\(^{13}\)

Since \( s^L(g(\pi_{FC}), s^W) \) increases in \( g(\pi_{FC}) \), we find \( s^L(1, \hat{s}) < s^L(0, \hat{s}) \). In step S7, we have shown that \( s^L(1, \hat{s}) \leq \hat{s} \) when \( 1 - (1 + \lambda) [n + (1 - n)p] \).

This implies \( s^L(0, \hat{s}) < \hat{s} \) when \( 1 - (1 + \lambda) [n + (1 - n)p] \). Thus, the optimal proposal with \( I(\pi_{FC}) = 1 \) in case B is given by \( \pi_{FC}^{p<m} \) (see Proposition 4.6).

Again, we have to investigate whether it is optimal for the agenda-setter to choose \( \pi_{FC}^{p<m} \) instead of a proposal with \( I(\pi_{FC}) = 0 \).

Since the derivative of \( u_W(\pi_{FC}) \) with respect to \( s^W \) is strictly positive, the agenda-setter experiences a utility gain from subsidization, which yields \( u_W(\pi_{FC}^{p<m}) > 0 \). Consequently, the agenda-setter chooses \( \pi_{FC}^{p<m} \).

S9: Case C implies that the optimal proposal with \( I(\pi_{FC}) = 1 \) comprises \( s^W = 0 \) and \( g(\pi_{FC}) = 1 \). To show that such a proposal is feasible, we have to check whether \( s^L(1, 0) \leq \hat{s} \).

For \( s^W = 0 \), the subsidy \( s^L \) depends solely on project parameters (see Equation C.41).

For sufficiently high \( \hat{s} \), the feasibility of \( s^L(1, 0) \) is guaranteed. Thus, the optimal proposal with \( I(\pi_{FC}) = 1 \) in case C is given by \( \pi_{FC}^{p<m} \) (see Proposition 4.6).

Again, we have to show that it is optimal for the agenda-setter to choose \( \pi_{FC}^{p<m} \) over a proposal with \( I(\pi_{FC}) = 0 \).

\(^{12}\)Recall that the agenda-setter chooses \( s^W = \hat{s} \) if \( 1 - (1 + \lambda) [n + (1 - n)p] > 0 \) because \( u_W(\pi_{FC}) \) is linearly increasing in \( s^W \).

\(^{13}\)We already argued that \( s^L(g(\pi_{FC}), s^W) \geq 0 \) holds for all \( g(\pi_{FC}) \in \{0, 1\} \) and \( s^W \in [0, \hat{s}] \) (see step S3).
Since \( u_W(\langle g(\pi_{FC}) = 1, \cdot \rangle) \geq u_W(\langle g(\pi_{FC}) = 0, \cdot \rangle) \), the agenda-setter experiences a utility gain from project implementation, which yields \( u_W(\pi_{FC}^{p\cdot m}) \geq 0 \). Consequently, the agenda-setter chooses \( \pi_{FC}^{p\cdot m} \).

S10: Case D implies that the optimal proposal with \( I(\pi_{FC}) = 1 \) comprises \( s^W = 0 \) and \( g(\pi_{FC}) = 0 \). Such a proposal is equivalent to the status quo. Thus, the agenda-setter could either suggest the status quo or any other proposal that yields \( I(\pi_{FC}) = 0 \) (especially, she could also suggest unconstitutional proposals). Hence, there is no unique equilibrium proposal in case D, but the status quo always prevails.

\[ \square \]

**Proof of Lemma C.1**

Suppose \( \frac{(1 + \lambda)n(1 - p) s^C - g(\pi_C)(V_w - (1 + \lambda)k)}{1 - (1 + \lambda) np} < 0 \): From Equation C.26 we know that this implies \( s^W = 0 \).

Using Equation C.29, we obtain

\[
s^C = g(\pi_C) \frac{V_l - (1 + \lambda)k}{1 - (1 + \lambda) n(1 - p)}. \tag{C.48}
\]

Inserting (C.48) into \( \frac{(1 + \lambda)n(1 - p) s^C - g(\pi_C)(V_w - (1 + \lambda)k)}{1 - (1 + \lambda) np} \) yields

\[
-g(\pi_C) \frac{(1 - (1 + \lambda)n(1 - p))V_w + (1 + \lambda)n(1 - p)V_l - (1 + \lambda)k}{(1 - (1 + \lambda) np)(1 - (1 + \lambda)n(1 - p))}. \tag{C.49}
\]

Thus, \( \frac{(1 + \lambda)n(1 - p) s^C - g(\pi_C)(V_w - (1 + \lambda)k)}{1 - (1 + \lambda) np} < 0 \) can hold if and only if the numerator of (C.49) is strictly positive,\(^\text{14}\) that is

\[
(1 - (1 + \lambda)n(1 - p))V_w + (1 + \lambda)n(1 - p)V_l - (1 + \lambda)k > 0. \tag{C.50}
\]

Rearranging terms of (C.50) yields

\[
V_w > \frac{(1 + \lambda)k - (1 + \lambda)n(1 - p)V_l}{1 - (1 + \lambda)n(1 - p)} =: V_w^{**}. \]

Suppose \( \frac{(1 + \lambda)n(1 - p) s^C - g(\pi_C)(V_w - (1 + \lambda)k)}{1 - (1 + \lambda) np} = 0 \): In this case, \( s^W = 0 \) and \( s^C \) is given by (C.48). Thus, \( \frac{(1 + \lambda)n(1 - p) s^C - g(\pi_C)(V_w - (1 + \lambda)k)}{1 - (1 + \lambda) np} = 0 \) holds for \( V_w = V_w^{**} \).

\(^{14}\)Recall that we assumed \( 1 - (1 + \lambda)n > 0 \) for the lines of the proof of Proposition 4.4. This implies \( 1 - (1 + \lambda)np > 0 \) and thus the denominator of (C.49) is strictly positive.
Suppose \( s^W = \frac{(1 + \lambda)n(1 - p)\pi C(V_w - (1 + \lambda)k)}{1 - (1 + \lambda)np} > 0 \): From (C.26) we know that
\[
\frac{(1 + \lambda)n(1 - p)\pi C}{1 - (1 + \lambda)np} > 0.
\]

Using Equation C.29, we obtain
\[
s^W = \frac{(1 + \lambda)n(1 - p)\pi C}{1 - (1 + \lambda)np}.
\]

Inserting (C.51) into \( s^W = \frac{(1 + \lambda)n(1 - p)\pi C}{1 - (1 + \lambda)np} \) yields
\[
s^W = -g(\pi C)\frac{(1 - (1 + \lambda)n)\pi C}{1 - (1 + \lambda)n}.
\]

Thus, \( \frac{(1 + \lambda)n(1 - p)\pi C}{1 - (1 + \lambda)np} > 0 \) can hold if and only if the numerator of (C) is strictly negative,\(^{15}\) that is
\[
(1 - (1 + \lambda)n(1 - p))V_w + (1 + \lambda)n(1 - p)V_i - (1 + \lambda)k < 0. \tag{C.52}
\]

Rearranging terms of (C.52) yields
\[
V_w < \frac{(1 + \lambda)k - (1 + \lambda)n(1 - p)V_i}{1 - (1 + \lambda)n(1 - p)} =: V_w^{**}.
\]

This completes the proof of Lemma C.1.

\(^{15}\)Recall that the denominator is strictly positive due to our assumption that \( 1 - (1 + \lambda)n > 0 \).
Bibliography


Bibliography


Curriculum Vitae

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