Experimental investigation of air injection in saturated unconsolidated porous media

Author(s):
Kong, Xiang-Zhao

Publication Date:
2010

Permanent Link:
https://doi.org/10.3929/ethz-a-006246547

Rights / License:
In Copyright - Non-Commercial Use Permitted
Experimental investigation of air injection in saturated unconsolidated porous media

A dissertation submitted to the
ETH ZURICH

for the degree of
Doctor of Sciences

presented by
XIANG-ZHAO KONG
M.Eng., University of Science & Technology of China
born in July 6, 1979
citizen of Guangdong, China

accepted on the recommendation of

Prof. Dr. Wolfgang Kinzelbach, examiner
Prof. Dr. Patrick Jenny, co-examiner
Prof. Dr. Fritz Stauffer, co-examiner

2010
# Contents

Abstract v  
Zusammenfassung vii  

1 Introduction 1  

2 Multiphase flow in porous media 3  
2.1 Microscopic physics 3  
2.1.1 Surface tension, wettability, and entry pressure 3  
2.1.2 Fluid forces and their combinations 5  
2.1.3 Imbibition and drainage 6  
2.1.4 Migration, fragmentation and coalescence 7  
2.2 State of research 8  

3 Air injection in two-dimensional experiments 17  
3.1 Experimental setup 17  
3.2 Morphodynamic hierarchy in water-saturated spherical granulates 18  
3.2.1 Experimental parameters 18  
3.2.2 Morphodynamic hierarchy and transition process 19  
3.2.3 Summary 31  
3.3 The channel migration phenomenon 32  
3.3.1 Experimental parameters 33  
3.3.2 Qualitative phenomenology 34
Abstract

The work described in this thesis is primarily concerned with the construction and study of laboratory scale models for the process of air injection into liquid-saturated grain packing. Experiments, both in two-dimensional (2D) and three-dimensional (3D) setups, were carried out using water-saturated packings of glass beads and/or packings of crashed fused silica glass grains saturated with a glycerin-water solution. High resolution digital images of the invasion patterns were recorded and analyzed.

During air injection into a vertically-placed 2D glass bead packing saturated with water, three stages were identified, a tree-like pattern, a fluidized pattern, and a migrating single-channel pattern. The expansion of the tree-like pattern behaves in a diffusion-like manner as the air branches advance upward randomly, and finally reach a more or less constant width. The starting position of the fluidized pattern was quantitatively estimated via balancing the pressure forces between the effective stress due to the weight of the grains and the pressure resistance on the displaced fluid combined with the capillary pressure. Four dynamic regimes were distinguished: regime (i) where the fluidization stops somewhere between the top of the packing and the injection orifice, a transition regime (ii), regime (iii) where the fluidization reaches the injection orifice, and regime (iv) where the deformation of the packing appears as soon as the air is injected. A critical injection rate $Q_f$ is defined to identify the transition regime. The value of $Q_f$ can be determined via $Q_a$, where $Q_a$ is calculated as averaged flux per channel. The regime (iv) is characterized by a characteristic injection rate $Q_c$, which is estimated by balancing the pressure gradient of the air flow and the overburden pressure gradient of the medium.

The phenomenon of the migrating channel is measured quantitatively in two parts, before and after breakthrough. Before breakthrough, the characteristic measurements concern maximum vertical advance, maximum horizontal advance, air volumetric fraction, ratio of total surface area to volume, specific surface area of the air phase, and box-counting dimension. After breakthrough, the characteristic measurements focus on mean horizontal position of air channel, horizontal shifting distance, lateral move-
ment distance, and lateral movement width. Before breakthrough, the maximum vertical height of the air structure approximately advances linearly with time. The maximum horizontal advance reaches a maximum value and then levels off for the rest of the time. Air volumetric fraction decreases monotonically with time, and finally levels off asymptotically to an approximate constant. In all cases, the air volumetric fraction for packings of small grains is larger than that for packings of large grains. The ratio of total surface area to volume varies in time similarly to the air volumetric fraction. However, the ratio of total surface area to volume can clearly be grouped according to the grain size, which is also true for the specific surface area of the air phase. Both can be scaled with the Bond number with a power of -0.5. After breakthrough, the migration process is studied by analyzing the mean horizontal position, horizontal shifting distance, lateral movement distance, and lateral movement width of the air channel. The results indicate that over 99% of the horizontal shifting distance is less than 10 mm. Furthermore, the probability density function indicates that the air channel oscillates more frequently in the packing of small grains than in the packing of large grains.

The interaction of the air flow with the grains and the liquid leads to a mobilization of the grains, in which air channels migrate and grain clusters undergo shearing. The channel migration comes to a stop after some time, leaving one thin and stable preferential channel for air flow. Assuming Hagen-Poiseuille's formula to be applicable, the size of the preferential channel should exceed a lower threshold $D_{ch}$ so that a mechanical equilibrium at the channel interface is maintained, but it should stay below an upper threshold $D_{max}$ so that a stable air channel is sustained. A rearrangement of the grains is observed which is caused by a pulsation effect. It induces a compaction process, in which the individual grains are disassembled from the region of non-zero shear rate and then reassembled into the compacted clusters of the region of zero shear rate. It also induces a size segregation process, in which smaller grains move into the spaces beneath larger grains.

By using high-speed image acquisition through laser scanning, the 3D dynamic air plume is recorded by sequential tomographic imaging. Due to the overlap between adjacent laser sheets and the light reflection, air bubbles are multiply exposed in the imaging along the scanning direction. A “curvature” method, based on a threshold on the curvature of grey-value in scanning direction, is proposed to remove the redundant pixels. The respective results are discussed by comparing the reconstructed air plume volume with the injected one and by evaluating the morphological consistency of the obtained air plume. The reconstructed air plume is further investigated with respect to its growth characteristics, such as breakthrough, air volume fraction, and air channel migration.
Zusammenfassung


Während der Luftinjektion in eine vertikale wassergesättigte 2D-Glaskugelpackung wurden drei typische Stadien identifiziert, eine baumähnliche Struktur, eine fluidisierte Struktur und eine Struktur mit einem wandernden Einzel-Luftkanal. Die Entwicklung der baumähnlichen Struktur verhält sich diffusionsähnlich durch zufälliges Wachstum der einzelnen Zweigkanäle und erreicht schließlich eine mehr oder weniger konstante Breite. Die Startposition der fluidisierten Struktur wurde quantitativ abgeschätzt. Dazu wurden Druckkräfte und die effektiven Spannungen infolge Gravitation und Strömungswiderstand mit dem vorherrschenden Kapillardruck kombiniert und abgeschätzt. Vier dynamische Regimes wurden unterschieden: Regime (i) in welchem die Fluidisierung zwischen oberer Packungsbegrenzung und Injektionsdüse endet, ohne die Düse zu erreichen, ein Übergangsregime (ii), Regime (iii) in welchem die Fluidisierung die Injektionsdüse erreicht und Regime (iv) wo eine Deformation der Packung sofort erfolgt, sobald Luft injiziert wird. Eine kritische Injektionsrate \( Q_f \) wurde definiert, die das Übergangsregime charakterisiert. Der Wert von \( Q_f \) kann mit Hilfe der mittleren Luftströmungsrate pro Luftkanal, \( Q_a \), bestimmt werden. Regime (iv) ist charakterisiert durch eine Injektionsrate \( Q_c \), welche mit Hilfe des Druckgradienten der Luftströmung sowie dem Gradienten des Überlagerungsdrucks der Packung abgeschätzt wird.

Das Phänomen des wandernden Luftkanals wurde vor und nach dem Durchbruch der Luftströmung durch die Packung quantitativ untersucht. Vor dem Durchbruch sind die charakteristischen Grössen die maximale vertikale Höhe und die maximale horizontale Höhe...


Mit Hilfe einer Hochgeschwindigkeits-Bilderfassung mit Laserabtastung wurde die 3D-Dynamik des Luftinjektionsschleiers durch sequentielle Bildtomographie erfasst. Infolge Überlagerung benachbarter Laser-Lichtebenen sowie durch Lichtreflexe werden
Chapter 1

Introduction

The study of gas movement following injection into liquid saturated porous media is an active area of exploration for theoretical and practical reasons [53, 58]. The remediation of volatile dissolved substances in groundwater by air injection (air-sparging [54, 81, 83]) is one of them, which has been proposed as a method to remove volatiles by partitioning to the air flow. Air injection is also used to emplace trapped gas into an aquifer to promote biological processes or to reduce hydraulic conductivity by clogging preferential flow paths [27, 33]. In stripping where the unwanted components are removed from a liquid stream by a vapor stream, air injection is applied to create the vapor streams which are favorable for the unwanted components. In urban water management, the aeration of filters for water treatment [98, 103] is traditionally used and gradually gaining popularity also in waste water treatment [15]. But also other fields such as gas-liquid flow in packed columns in chemical engineering, migration and escape of gas in permeable sediments in marine science, air pressure tests for the characterization of saturated porous and fractured media, and tertiary oil recovery in petroleum engineering rely on the same theoretical basis. Recently, carbon dioxide (CO$_2$) geological storage receives considerable attention since it is seen as a mitigation strategy against the rising CO$_2$ content of the atmosphere and the global warming caused by it. [13, 45]. Besides the engineering applications, great efforts in multiphase flow in porous media led to the applications of various theoretical concepts such as fractals [30], percolation [90], kinetic roughening [40], self-organized criticality [62], and quenched disorder [43]. For a more comprehensive review of recent relevant research in numerous experimental, numerical, and theoretical studies see Chapter 2.

The scientific challenge in all the areas mentioned above is the development of appropriate models for multiphase flow in porous media. However, further progress in the understanding and modeling of these phenomena can only be achieved through contin-
ued experimental work. The objective of this thesis is to contribute to the experimental investigations and physical understanding of dynamic mechanisms of air injection in unconsolidated porous media when changing the physical parameters of the fluids like viscosity contrast, interfacial tension and injection rate. Meanwhile, this work aims to develop and apply a planar laser optical scanning method for the investigation of air branches in high spatial and temporal resolution. The porous medium in my work is refraction-index matched with the liquid to make its interior accessible for visible light and thus optical scanning methods. Of all visualization methods currently used only the optical method allows high resolution of the air body as well as high temporal resolution of air branch dynamics. I investigated the parameters determining the mechanisms of interaction between air fingers, liquid, and the mobile porous medium.

The thesis is divided in the following chapters. First there is an independent chapter (Chapter 2) describing gas-liquid flow in porous media. A brief introduction of different physical aspects of multiphase flow in porous media is given, as well as a summary of the general understanding from the earlier research in air injection. This chapter is intended to bridge the gap between practicing engineers and applied physicists, providing summaries of the background materials. The subsequent chapters, Chapter 3 and Chapter 4, describe in detail the experimental studies, respectively in two dimensions and in three dimensions. Finally, there is a short closing Chapter 5, where I discuss the conclusions and the possible continuation of this work.
Chapter 2

Multiphase flow in porous media

In this chapter, I will first briefly introduce the readers to the different physical aspects of multiphase flow in porous media, mainly focusing on microscopic levels. Later on I will present a literature review of numerous experimental and theoretical topics relevant to my work.

2.1 Microscopic physics

The dynamics of the interface between the invading and defending fluids in a porous medium is dominated by the interplay among viscous, capillary, and gravity forces. At pore level, fluid properties like density, viscosity, wettability, and surface tension play essential roles in the dynamics of local structure of the interface.

2.1.1 Surface tension, wettability, and entry pressure

Surface tension is a property of the surface of a liquid. It is caused by cohesion between the molecules at the surface. Surface tension, denoted by the symbol \( \sigma \), can be defined in two ways. One definition is a force along a line of unit length on a given liquid surface, where the force is parallel to the surface but perpendicular to the line. Surface tension is therefore measured in forces per unit length. The other equivalent definition is work done per unit area, in which the work is stored as potential energy. Consequently, surface tension equals joules per unit area. When a tensioned surface is in equilibrium, it can be described by the Young-Laplace equation,

\[
\Delta p = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right),
\]  

(2.1)
where \( \Delta p \) denotes the pressure difference at the interface between immiscible liquids, \( R_1 \) and \( R_2 \) are the principal radiuses of curvature of the interface.

For multiphase flow in a porous medium, the surface tension at the curved interface between the two immiscible liquids shows as a capillary pressure, \( p_c \), at pore level. Given a porous medium with a pore size \( d \) (usually \( d \) is taken as the grain size \( D_{50} \) of the grain packing), the curvature of the interface is assumed to be \( d \). Thus, under the assumption of a static equilibrium of the interface the capillary pressure can be derived from the Young-Laplace equation with approximately \( R_1 = R_2 = d/2 \), yielding,

\[
p_c = \frac{4\sigma}{d}.
\]  

(2.2)

The above equation is value only when the wetting angle is \( 0^\circ \).

Wetting is the ability of a droplet of the liquid to maintain contact with a solid surface, resulting from intermolecular interactions when the two are brought together. The degree of wettability of a solid is determined by a force balance between adhesive (forces between a liquid and solid causing a liquid drop to spread across the surface) and cohesive forces (forces within the liquid cause the drop to become spherical and avoid contact with the surface). In general, the degree of wettability of a surface is measured by the contact angle \( \theta \) (shown in Fig. 2.1). The liquid is wetting when \( 0^\circ < \theta < 90^\circ \) and non-wetting when \( 90^\circ < \theta < 180^\circ \). \( \theta = 0^\circ \) indicates perfect wetting and makes a droplet spread to form a film on the surface. \( \theta = 180^\circ \) corresponds to perfect non-wetting, and the liquid forms a compact droplet.

![Figure 2.1: A sketch of a liquid droplet on a horizontal solid surface. The contact angle \( \theta \) between the horizontal surface and the liquid surface indicates the wettability of the liquid.](image)

For an equilibrium multiphase system in a porous medium the contact angle influences the strength of the capillary pressure \( p_c \). Usually, the capillary pressure \( p_c \) in a pore of size \( d \) is given by

\[
p_c = \frac{4\sigma}{d} \cos \theta.
\]  

(2.3)
By definition, the entry pressure $p_e$ is the minimum capillary pressure at which the non-wetting phase starts to displace the wetting phase. According to Equ. 2.3, the minimum capillary pressure locates at the largest pore throat within the local structure of the porous medium. Consequently, at any time the drainage favors the largest pore throat within the local structure. Once the entry pressure is overcome, the capillary breakthrough occurs.

### 2.1.2 Fluid forces and their combinations

The immiscible two-phase flow in a porous medium is governed mainly by three types of forces, viscous, capillary, and gravity forces. Consequently, the phenomena of the flow are characterized by a set of dimensionless numbers that describe the relative magnitude of these forces at pore level, commonly including the Bond number $B_o$, the capillary number $C_a$, and the viscosity ratio $M_v$.

The Bond number is a quantity representing the competition between gravity and capillary forces. Given a mean pore size of $d$ in a porous medium, the gravity difference due to the difference of fluid densities is proportional to $\Delta \rho g d$, where $\Delta \rho = \rho_1 - \rho_2$ the difference of fluid densities, $g$ the acceleration of gravity. Here the indices 1 and 2 denote the invading and defending fluid. The corresponding capillary pressure is proportional to $\sigma/d$. Consequently, the Bond number $B_o$ has the form

$$B_o = \frac{\Delta \rho g d}{\sigma/d} = \frac{\Delta \rho g d^2}{\sigma}.$$  \hspace{0.5cm} (2.4)

Most practical issues involve multiphase flow with fluids of different density, and therefore the Bond number needs to be taken into account. The effect of the gravity can lead to destabilization when the less dense fluid invades from the bottom, expressed by $B_o < 0$; or a stable configuration when the less dense fluid invades from the top, expressed by $B_o > 0$.

The capillary number describes the relation of viscous to capillary forces. When a flow appears with a velocity $u$ through pores of mean size $d$ inside a porous medium, the pressure drop due to the viscous force across an average pore can be expressed as $ud\mu/k$, where $k$ is the local permeability of the pore and $\mu$ is the viscosity of the fluid. The permeability of the pore is roughly given by $d^2$, which means $k \sim d^2$. Thus the capillary number is obtained as

$$C_a = \frac{ud\mu/k}{\sigma/d} = \frac{u\mu}{\sigma}.$$  \hspace{0.5cm} (2.5)
In general, the viscosity to be inserted into Eq. 2.6 would be the maximum one of the two fluids involved with the assumption that the more viscous fluid is the dominating one.

The viscosity ratio is defined as

\[
M_v = \frac{\mu_2}{\mu_1},
\]

where \(\mu_1\) and \(\mu_2\) are the viscosities of the defending and invading fluids respectively.

When viscous forces start to play a role in the flow dynamics, consideration of the viscosity ratio is necessary, since the viscous forces can either destabilize \((M_v < 1)\) or stabilize \((M_v > 1)\) the interface between the fluids.

### 2.1.3 Imbibition and drainage

Depending upon the wettability of the fluids the immiscible two-phase displacement in a porous medium essentially occurs in two forms, imbibition and drainage.

When a wetting fluid displaces a non-wetting fluid imbibition occurs. In imbibition, the phenomenon of displacement is affected by the pore geometry, the number of filled throats attached to a pore, and the film flow controlled by the roughness of the solid surface [68]. Typically, imbibition is characterized by pinch-off behavior of the defending fluid [104]. In imbibition, the displacing fluid will first invade the most narrow pore at low injection rate [55].

In the opposite case, drainage displacement, a non-wetting fluid displaces a wetting fluid. The mechanism of the drainage process is quite different from imbibition. In drainage, the non-wetting phase invades a pore only if the pressure difference between the non-wetting phase and the wetting phase exceeds the capillary pressure of that pore. Subsequently, the non-wetting phase first enters the largest of the adjacent pores. This invading behavior is called piston-type invasion [55]. Depending on different invading pattern structures, the drainage displacement can be divided into three major flow regimes [56]: viscous fingering, capillary fingering, and stable displacement. Viscous fingering typically consists of fingers propagating through the medium with a few trapped small clusters of defending fluid left behind. It is a drainage process in which a high viscosity fluid is displaced by a low viscosity fluid with high injection rate. The major resistance is due to viscous forces in the defending fluid while the pressure drop and capillary effects in the invading fluid can be neglected [2]. The opposite case, capillary fingering, occurs at very low injection rate, consisting of a rough and wide front. The size of the trapped clusters of defending fluid varies from the pore size to the length of
the system [2]. Its major resistance is due to capillary forces of the invading interface. In the stable displacement, a low viscosity fluid is displaced by a high viscosity fluid with high injection rate. It is characterized by a flat front of the invading interface. Compared to viscous fingering, its major resistance stems from the viscous forces in the invading fluid.

Figure 2.2: “Phase diagram” by Lenormand et al. [56] on a logarithmic plot with the capillary number $C_a$ along the $y$-axis and the viscosity ratio $M$ along the $x$-axis.

### 2.1.4 Migration, fragmentation and coalescence

Migration, fragmentation, and coalescence of non-wetting fluid clusters have been studied extensively both experimentally and numerically [7, 65, 105]. Migration generally describes a phenomenon that an isolated cluster of non-wetting fluid moves or elongates due to the buoyancy. The migration occurs through a sequence of steps consisting of drainage events at the upper front and imbibition events at the lower front of this isolated cluster [7]. Once the cluster elongates too much so that the capillary force somewhere along the cluster’s interface becomes competitive with the hydrostatic pressure drop, this cluster will break into a set of clusters. Thus fragmentation occurs. A rough estimate of the characteristic length scale $l_c$ of the newly formed clusters is provided by

$$\Delta \rho g l_c \sim \frac{2\sigma}{d},$$

(2.7)

which yields $l_c \sim |B_o|^{-1}d$. Essentially, the fragmentation process is governed by a competition between capillary and buoyancy forces. Subsequently, fragmentation reduces the buoyancy drive which may lead to blocking of new fragments. Coalescence occurs when a set of previously isolated invader fragments get into contact with each other.
and coalesces. Coalescence enhances migration since the buoyancy force acting on the new cluster is increased.

![Figure 2.3: A Sketch of migration process of gas clusters. During upward migration (a), the cluster might break up into fragments, leading to fragmentation (b). The isolated fragments might get into contact and coalesce with a cluster coming from below (c).]
effective than conventional vertical wells. An investigation by Reddy and Adams [82] using two-dimensional (2D) sand packs demonstrated that there was a strong impact of soil heterogeneity, i.e. soil layering, on airflow patterns and hydrocarbon removal efficiency. Furthermore, Semer et al. [88] concluded from their 2D experiments that soil type, groundwater flow conditions and systems controls were essential parameters for the formation of air flow patterns. Particularly, detailed knowledge of soil stratigraphy and heterogeneity is important to optimize air-sparging efficiency. Chen et al. [17] used X-ray computerized tomography scanning for three-dimensional (3D) imaging of air distribution patterns during air-sparging in high- and low-permeability sand packs. They successfully applied the concept of hydrodynamic stagnation saturation and fractional flow theory to explain their experiments. Van Dijke et al. [100] used numerical multiphase modeling to estimate the maximum radius of influence of air-sparging. Radial numerical multiphase simulations were performed by McCray and Falta [64] in order to define this radius of influence. The effect of sediment size on the area of influence was investigated by Peterson et al. [73] by laboratory studies in 2D visualization sand tanks. They confirmed the importance of grain size distribution on the type of air flow pattern. Burns and Zhang [14] investigated the effect of system parameters on the physical characteristics of bubbles produced through air-sparging by using image analysis in a rectangular (2D) flow chamber. 2D laboratory visualization experiments in fine-grained sand by Peterson et al. [74] revealed a previously unrecognized air-flow geometry, which they termed chamber flow. Compared to channel flow and pervasive bubbly flow, the chamber flow has the potential to penetrate and affect soils to a higher degree. Their results indicate that air-sparging in fine-grained sands could have an stronger impact on the soil than previously assumed. On the other hand it has been observed by Yang et al. [109] that pulsed air-sparging shows increased efficiency, either by bubble activation or induced instability. Tsai and Lin [96] confirmed the mobilization of sand grains and an increase in porosity directly proportional to the rate at which air is injected in their air-sparging experiments.

According to Anderson et al. [3], bubbles in liquid-saturated porous media can be classified into three types:(a) bubbles that wholly fit within a pore between grains; (b) bubbles that contain several adjacent pores; and (c) bubbles that push the surrounding grains aside as they grow. A bubble of type (a) will first form and grow into type (b) as gas is continually injected into the medium. Until a critical pressure inside the bubble is reached, a type (b) bubble is sufficient to overcome the shear strength of the surrounding medium. The medium then fractures, expanding a type (b) bubble into a type (c) bubble while lowering the pressure. In a fixed bed, the favorable types of bubble are types (a) and (b). However, type (c) of bubbles would be the favorable one in unconsol-
Chapter 2. Multiphase flow in porous media

idated porous media, such as biofilters.

A biofilter is a pollution control facility using living materials to capture and biologically degrade pollutants. Generally, the term biofilter is used twofold in the literatures: (i) for the treatment of contaminated wet air, which is streaming through a packed bed of biologically active material. When applied to air filtration and purification, biofilters use microorganisms to remove air pollution, which transfers into a thin biofilm on the surface of the packed material. Microorganisms, including bacteria and fungi are immobilized in the biofilm and degrade the pollutant. Therefore the gas phase occupies a continuum. (See e.g. a review of biofilter models by Devinny and Ramessh [25]) (ii) for the treatment of raw water, flowing through a packed bed, which grows biomass and may be aerated. Here the water forms the continuum and the air moves in the form of bubble trains through the bed. In my work the second option is the relevant case.

Biomass retained in aerated coarse media (1.5-4 mm) filters (Biofilters) has demonstrated to have excellent potential for biological treatment of water. Possible processes are degradation of organic waste, nitrification [72], denitrification, or even biological phosphorus removal, where all four processes may be integrated into one single reactor. Whereas model development was very active in the 90s, this activity almost came to a stop after 2000, possibly because the models never progressed beyond a very pragmatic, empirical state. A lack of understanding of hydraulic behavior of these filters may well be the reason. With the aid of hydraulic residence time distribution Tschui et al. [97] demonstrated in a pilot reactor that due to instable channeling of air, hydraulic flow distribution in biofilters is very inhomogeneous. Fluctuations of mean hydraulic residence time up to 40% were observed. This was later confirmed in a full scale reactor by Séguret and Racault [86]. Today ‘mechanistic’ models of the performance of such biofilters still rely on dispersed flow for the hydraulic regime [103]. Aeration is typically not included as an explicit process, but a sufficiently large, constant oxygen concentration is assumed. Since the wastewater load is highly variable and reaction kinetics is not typically zero order, neglecting the details of hydraulic flow patterns results in inaccurate prediction of reactor performance especially in view of ever more stringent effluent requirements. Energy consumption of aeration is a major cost factor in the operation of aerobic biofilters, therefore good understanding of oxygen transfer processes in biofilters is essential in process design. Deront et al. [24] found that $k_L a$ values (volumetric oxygen mass transfer coefficient) did not depend on the air flow but rather on hydraulic loading of the biofilter. Since air flow causes the cost of aeration this observation is remarkable and requires elucidation. Today’s development of biofilter technology tries to improve the filter media: Yang and Allen [108] designed a conical filter bed which allows for decreasing carrier particle size in the direction of flow and thereby improved physical
filtering performance. Di Ianconi et al. [26] make use of granular biomass (small, elastic and highly active beads of biomass which develop under specific operating conditions) in order to enhance volumetric activity. Both these arrangements (decreasing particle size and elastic carrier material) introduce additional parameters into the interaction of air, fluid and fixed bed. From a better understanding of air channeling through biofilters we expect improved designs of biofilters and more reliable models for filter performance. In addition aeration procedures might be optimized and thereby energy requirements and operating costs could be reduced. A possible extension of this study could be based on an idea of van den Heuvel et al. [99] who have used hydrodynamic pressure variation to improve mass transfer in a biological reactor with particulate biocatalysts. Similar procedures might be used to affect/control bubble movement and gas exchange in biofilters.

Similar cases of gas migration in an unconsolidated porous medium happen in gas-liquid flow in packed columns in chemical engineering (See e.g. [52, 77, 93]), and in escape of gas in permeable sediments in marine science (See e.g. [8, 34, 47, 71]). It is worth mentioning that the pingo-like features (shown in Fig. 2.4, [71] and references therein) have been identified on the sea floor. In the reference [71], they explained that pressure generated by methane gas hydrate decomposition within layers in subsur-

Figure 2.4: A Sketch of pingo-like features (PLFs) and moat formation surrounding the PLF associated with gas hydrate decomposition. As the subsurface warms, the top of the gas hydrate stability zone will move downward, which is indicated by the black arrows. This picture is redrawn according to [71].

face permafrost extrude relatively cold, Pleistocene-aged ice-bound sediments upwards onto the seafloor and elevate them above the surrounding seafloor, forming PLFs.

Therefore, understanding the mechanics of the development and migration of bubbles, as well as the up-scaling dynamics of bubble clusters, in liquid-saturated porous media is a crucial step for a quantitative predication of numerical models. Boudreau et al. [8] assumed a spherical bubble shape, and a simple viscous interaction of the growing
bubble with the sediment matrix. However, back in 1994 in a study of bubbly sediments in Eckernförde Bay [1], bubbles were found to be often coin- or disk-shaped with their long axis oriented vertically. In gas injection studies with natural sediment samples, Johnson et al [47] observed two modes of bubble growth behavior: one is characterized by a saw-tooth record of pressure as the bubble grows, corresponding to bubbles that are coin- or disk-shaped; the other is described by the pressure increasing until bubble formation and then the pressure falling to a pressure intermediate between ambient starting pressure and the bubble formation pressure. The pressure then remains relatively constant during subsequent bubble growth. Furthermore, Boudreau et al. [9] offered another mode of bubble formation using X-ray computed tomography (CT) that bubbles in muddy cohesive sediments are highly eccentric oblate spheroids which grow either by fracturing the sediment or by reopening preexisting fractures, and bubbles in soft sandy sediment tend to be spherical. They explained that sand acts as a fluid or with a plastic behavior in response to bubble growth stresses.

During the migration process, besides fragmentation and coalescence [7,65,105] which are defined in section 2.1.4, bubbles may undergo “blocking” and “circumvention”, which were demonstrated by Stöhr and Khalili [95] (See Figure 2.5). They defined a relevant dimensionless number $K$ (the ratio of viscous and hydrostatic pressure gradients) to characterize different dynamic regimes: air migrates as isolated fragments for small $K$, and invades as an anisotropic finger for large $K$. With the assumption of Hagen-Poiseuille air flow inside a channel, $K$ is expressed as

$$K = \frac{\Delta p_v}{\Delta p_g} = \frac{128Q\mu_{air}}{\pi d^4 \Delta \rho g},$$

(2.8)

where $Q$ is the injection rate of air flow, $\Delta p_v$ the viscous pressure gradient expressed as $\Delta p_v = 128Q\mu_{air}/\pi d^4$, $\Delta p_g$ the hydrostatic pressure gradient, and $\mu_{air}$ the air viscosity.
2.2. State of research

Selker et al [87] defined two distinct geometries of gas plume, a “near source region”, where the gas pressure gradients exceed buoyant gradients and thus the gas shows largely radial flow, and a “far source region”, where the magnitude of the vertical gradient driving gas movement is identical to the hydrostatic gradient. They estimated the size of an unified channel of well-connected gas-filled pores to be

\[ D_{ch} = 2\sqrt{\frac{3\sigma}{\Delta \rho g}}, \]  

which is derived by the force balance over a simple geometry of a cylindrical channel with a hemispheric upper termination. The balance is between the buoyant force on the hemisphere and the surface tension on the hemisphere about its perimeter. Their result is consistent with the experimental values from Elder and Benson [29]. Moreover, a phase diagram (shown in Fig. 2.6) of gas flow patterns was proposed by Geistlinger et al. [36] summarizing the pre-experimental results including glass bead packings [10, 29, 51, 76, 84], homogeneous natural sand packings [73, 88], and macro-

![](image)

Figure 2.6: Classification of gas flow pattern dependent on flow rate and grain size. From [36].

heterogeneous sands [38, 75, 82]. They further experimentally demonstrated a transition from incoherent to coherent gas flow pattern both in grain-size (\(d\))- and flow-rate (\(Q\))-dependency, which can not be explained by the standard quasistatic criteria (in which flow rate approaches zero).

The extreme case of bubbling instability would be the fluidization process in two-phase systems, in which some intriguing aspects have been reported concerning bubble dynamics [4, 31, 57, 59, 60, 63, 101, 102, 107, 110], and macroscopic hydrodynamic properties in high pressure bubble columns [89]. Similar aspects have been studied in three-phase fluidization systems [20]. Other extensive studies addressed the characteristics
of the key macroscopic hydrodynamic properties, revealing the phenomena of moving packed bed [23], flow regime transition [61, 63], overall gas holdup [20]. Recently, air injection into a loosely packed granular medium [48, 50], a dense granular packing [49], or granular suspensions from sparse to dense [18] have been considered in a confined Hele-Shaw. Nevertheless, for such a system of coupled fluid-grain flows and interactions the situation is rather complicated, essentially due to the complexity of a granular material: it may act as a fluid or jam at high granular concentration and behave like a solid matrix.

As we can see from above, most experimental investigations of air injection have been performed in 2D porous media systems. Very few well-controlled 3D experiments have been performed since pore-scale air invasions are difficult to probe due to the fact that the medium is opaque. Visualization of two-phase flows in a non-destructive way can be realized in various ways. It is of course restricted to laboratory columns or boxes. For example, in problems of oil recovery X-ray CT has been applied frequently in laboratory setups. In air-sparging Chen et al. [17] used this technique. Nuclear Magnetic Resonance Imaging (NMRI) is another technique which allows spatial differentiation between the phases water and air, as air gives no resonance signal in contrast to the proton spins of the water molecule (e.g. [16]). 3D-NMRI is, however, slow and limited in resolution to voxels on the order of 1 mm side length. The iron content of natural soils often does not allow highly resolving NMRI as in the high magnetic fields artifacts are introducing too much disturbance into the image. An alternative in this case is the use of neutron scattering. In neutron scattering the protons in the water molecule are scattering the incident low energy neutrons from a reactor. Water is therefore more opaque in the image while air and the medium stay transparent. The temporal resolution of the method, though better than in NMRI, is still low. The spatial resolution is comparable to that of NMRI.

Besides the above non-destructive methods (X-ray CT, NMRI, neutron scattering), an alternative method exists, which follows the principle of light transmission in an optically homogeneous medium. It uses transparent granular materials saturated with a liquid of the same refractive index, and eventually combining it with a laser-induced fluorescence technique. With this method, the difficulty of accessing the information of displacement in the opaque porous media can be overcome to a certain degree. It has been used in very small models of multiphase flow in a spherical-bead matrix (1-2 cm-scale) to furnish data for constitutive relationships [21, 22, 39]. Walmann [106] used this method to observe a dispersion structure of miscible tracer molecules in porous media. Montemagno et al. [66] applied the photoluminescent volumetric imaging to visualize the multiphase flow and transport in a porous medium. Rashidi et al. [80] quantitatively
measured the pore geometry, fluid velocity, and solute concentration within a saturated 3D porous medium. Fontenot & Vigil [32] visualized the non-aqueous phase liquid dissolution and fragmentation events in porous media. A similar technique has been applied to visualize the imbibition of water into an oil-filled porous medium, the transport of a fluorescent dye [94] and the drainage displacement under combined capillary and gravity effects in index-matched porous media [68].
Chapter 3

Air injection in two-dimensional experiments

In this chapter, I am going to demonstrate air displacement inside an unconsolidated granular packing placed randomly in a 380 mm (width) × 600 mm (height) × 5.5 mm (depth) glass container. After the detail description of the experimental hardware configuration in section 3.1, the presentation and discussion of the 2D experiments is divided into two parts: section 3.2 deals with the air invasion in the glass bead packing which is saturated with water; sections 3.3 and 3.4 deal with the air invasion in a crushed grain packing saturated with a glycerine-water solution.

3.1 Experimental setup

All experiments were performed by injecting air into a 2D liquid-saturated porous medium, which was placed vertically in a 2D box model, which is indicated in Fig. 3.1. The 2D box is a transparent rectangular cell of width \( W = 380 \) mm, height \( H = 600 \) mm, and thickness \( a = 5.5 \) mm. Air is injected through a 0.3 mm-orifice placed on the central line, at a distance of \( H_{or} = 20 \) mm from the base of the box. The 0.3 mm-orifice was installed to diminish the pressure fluctuation produced by the fluctuating instability of bubbles forming and breaking at the orifice. This technique was used to constrain the feedback of the air flow in the system on the injection. A pressure buffer is installed to maintain a constant air pressure. The air flow is controlled by a valve between the pressure buffer and a GSM-A5TA-BN00 flow-meter which measures the flow rate. The air flow is visualized by illuminating the cell from behind with a light source, and pictures are registered by a fast camera (Photron Ultima APX) on the opposite side. The image
Chapter 3. Air injection in two-dimensional experiments

Figure 3.1: Hardware configuration for 2D experiments in the laboratory, and an inserted sketch of the 2D box model. The height of the packing is \( H_b \). The supernatant liquid has a constant surface elevation \( H_w \). The \( x-y \) co-ordinate is also shown, and its origin is set at the position of the orifice.

sequence with a frame rate of 2 Hz and a resolution of 1,024 × 1,024 pixels is continuously transferred to a computer over a FIREWIRE 1394 connection. At the same time, this computer records the sequence of the flow rate via a data acquisition device, which is triggered by the same trigger signal. Thus, the injection rate and the air trace are recorded simultaneously.

3.2 Morphodynamic hierarchy in water-saturated spherical granulates

3.2.1 Experimental parameters

The packing was packed by placing the spherical glass beads below the water table \( H_w = 400 \) mm. In order to reduce the layering effects, the pile of beads was first saturated with de-aerated water, then poured continuously below the water table. Afterwards a remixing process was applied by fluidizing the packings with de-aerated water, which was injected through a series of orifices at the base of the box. Before starting an experiment, the system was vibrated for a few minutes in order to get a dense packing under gravity. The density of the glass beads was measured to be \( \rho_g = 2.47 \times 10^3 \) kg/m³. The diameter of the glass beads is \( d = 0.4 - 0.6 \) mm. Therefore the initial packing with
3.2. Morphodynamic hierarchy in water-saturated spherical granulates

A constant mean height \( H_b = 225 \text{ mm} \) can be considered as practically homogeneous. Porosity was measured gravimetrically to \( \phi = 0.37 \). The hydraulic conductivity was measured as \( k_w = 4.05 \text{ mm/s} \) in a steady-state water flow experiment using a similar tank with a permeable bottom. Thus the permeability is \( k = 4.1 \times 10^{-10} \text{ m}^2 \).

The wetting fluid, water, has a dynamic viscosity of \( \mu_w = 0.001 \text{ Pa\cdot s} \) and a density of \( \rho_w = 998 \text{ kg/m}^3 \). Air serves as the invading non-wetting fluid. The corresponding parameters for the air are \( \mu_a = 1.83 \times 10^{-5} \text{ Pa\cdot s} \) and \( \rho_a = 1.2 \text{ kg/m}^3 \). The surface tension between these two fluids is \( \sigma = 7.2 \times 10^{-2} \text{ N/m} \). All values here refer to room temperature of 22°C.

A series of experiments has been performed where the injection rates of the air covered a range of \( 5 < Q < 350 \text{ ml/min} \). In the following, the term interface is used to describe the interface between air and water. The term front is used to describe the area between the upper fluidized pattern and the lower tree-like pattern. The term finger or branch is used to describe a single branch of the tree-like pattern, in which only pore-scale drainage exists. The length scale of a finger/branch is called the finger-scale, which covers several pores. The term channel is used to describe a pure air channel, which is formed by pushing the beads aside and is confined by the water-air interface.

3.2.2 Morphodynamic hierarchy and transition process

After air is injected into the bead packing, air displaces water inside the pores. We classify this stage as the first stage where the packing behaves like a consolidated medium. As we can see in Fig. 3.2, the first stage of the dynamic air flow pattern can be characterized as branching fingering with tree-like evolution behaviour, which is a well-known phenomenon in the field of air-sparging [10, 29, 36–38, 51, 87]. In this stage the dynamic process involves a drainage mechanism in which invading air overcomes the entry pressure of the pores between the glass beads, forming branching pore-scale air channels. The transverse width of the pattern at the top boundary of the packing increases with the air injection rate. Based on the bubble classification of Anderson et al. [3], the first stage is a process characterized by a bubble of type (a) growing into type (b) due to the continuous air injection. The dynamics of the fingering interface is an instability governed by the interplay between capillary, viscous, and gravity forces. The capillary forces are local and vary due to the randomness in distribution of the pore structure. They cause the local randomness in the tree-like pattern varying in pore-scale. The viscous forces act at all different scales. They are responsible for the macroscopic behavior of the tree-like pattern. A common way of describing the
relation of viscous to capillary forces is the capillary number $C_a$. Here I use the mean upward velocity of the upper tip of the tree-like pattern as the characteristic velocity to calculate $C_a$. This mean upward velocity was measured to be in a range of 2 cm/s to 10.4 cm/s. For small $C_a$, corresponding to very low injection rate, the viscous forces become negligible, and the invasion is dominated by the capillary forces. Consequently, the capillary fingering is present, where the invasion is characterized by a piston-like motion [55] following the path of least resistance. On the other hand, the viscous force destabilizes the interface due to the viscosity ratio $\mu_a/\mu_w = 0.018$, which leads to viscous fingering. The invasion is primarily in the direction of bulk flow. According to the experimental observation and the phase diagram of Clayton [19], the observed tree-like patterns in the experiments are within the realm of viscous fingering. This means that the capillary effect is assumed to be negligible, which is the basic assumption when analyzing the lateral width of the tree-like pattern in the following. Given the experimental configuration that the air is injected at the bottom, besides the viscous force another destabilizing force is the gravity force, which can be described by the Bond number $B_o < 0$, i.e., $(\rho_a - \rho_w)gd^2/\sigma < 0$, where $g$ is the acceleration of gravity. To characterize the tree-like pattern, we plotted the outer bound width, $W_t$, of the pattern, which is shown in Fig. 3.3. We can see that $W_t$ first increases with the vertical coordinate $Y$ near the injection orifice, and then becomes more or less constant. The position $Y$ of a given width $W_t$ above the orifice increases with $Q$. The data also tell us that $W_t$ has a non-zero value at the orifice, which is defined as the bottom width of the tree-like pattern. This non-zero bottom width is established in a very short time as soon as the air is injected, and then remains nearly constant. An example is shown in Fig. 3.2: the bottom width was established within less than 0.5 s. Together with the bottom width, the top width of the developed tree-like pattern is plotted against $Q$ in Fig. 3.4. Both the

Figure 3.2: Branching fingering with tree-like pattern of air injection rate $Q = 90$ ml/min evolving with time (physical size of the images: 219 mm$\times$228 mm): (a)$t = 0.5$ s, (b)$t = 1.5$ s, and (c)$t = 3.0$ s.
3.2. Morphodynamic hierarchy in water-saturated spherical granulates

Figure 3.3: The measured outer bound width, $W_t$, of the tree-like pattern changes with the vertical coordinate $Y$ and the air injection rate.

bottom width and the top width increase sharply while $Q$ is less than 90 ml/min, then remain constant. It is natural to assume that the near source region (air injection orifice) is fully saturated with air having a residual saturation of water $S_r=0.1$. The estimate for the residual water saturation is based on experience with similar packings [91]. The bottom width is indicated by $W_b$. As shown in the inset of Fig. 3.4, the tree-like pattern within the packing can be analyzed by considering local equilibrium of resistant forces within the invading phase, which is simplified as a one-dimensional air plume surrounded by hydrostatic water. This local equilibrium is acting on the entire interface which separates regions with and without air. In my model, a “flat” horizontal interface was assumed. At the bottom of the tree-like plume, this “flat” interface separates the region fully saturated with air from the region fully saturated with water. When the injection rate is small, the pressure distribution outside the air plume is assumed to be hydrostatic (due to well-connectedness of the liquid phase), which means the pressure distribution in the water phase is uniform at a given depth. Consequently, there is no horizontal water flow outside the tree-like plume. In order to drain the same volume of water vertically, the volume of air needs to overcome the viscous force $f_{ra}$ due to water displacement, and the viscous force $f_{rw}$ due to air flow, described as

$$
\begin{align*}
  f_{ra} & \sim Q \mu_a/(akW_t^b) \\
  f_{rw} & \sim Q \mu_w/(akW_t^b).
\end{align*}
$$
Chapter 3. Air injection in two-dimensional experiments

Figure 3.4: The bottom and top width of the tree-like pattern versus the air injection rate. (Inset: Schematic of transient air plume geometry with width of $W_t$ and thickness $a$, wherein $W_t = W^b_t$ for bottom width and $W_t = W^t_t$ for top width. The three principal forces are shown, the buoyant force density $f_g$, and resistant force densities $f_{rw}$ from water and $f_{ra}$ from air.)

Given $\mu_a/\mu_w << 1$, it is reasonable to ignore the viscous resistance due to the air flow when the injection rate $Q$ is small. As it is going to describe the macroscopic behavior of the tree-like pattern, the capillary forces can also be ignored. The reasons are: (a) the viscous forces dominate the macroscopic behavior of the tree-like pattern, while the capillary forces are responsible for the local randomness of the pattern; (b) the observed tree-like patterns in the experiments are within the realm of viscous fingering. Therefore, the magnitude of the vertical gradient driving the air movement is comparable to the hydrostatic gradient, $f_g$, written as,

$$f_g \sim (\rho_w - \rho_a)g.$$  \hspace{1cm} (3.2)

Now a macroscopic force balance (across the whole air-water interface) is assumed, that is,

$$f_{rw} = f_g,$$  \hspace{1cm} (3.3)

which provides a simple method to estimate the total cross sectional area ($a \times W^b_t$) of the plume for a given volume flux rate $Q$,

$$W^b_t = \frac{\mu_w Q}{ak(\rho_w - \rho_a)g}.$$  \hspace{1cm} (3.4)
This quantity shows a linear dependence of $W_b^t$ on $Q$, which gives a good estimate of the bottom width for the experiments when $Q$ is small (in our case less than 90 ml/min). Here the concept of relative permeability is ignored under the assumption that the near source region (air injection orifice) is fully saturated with air. A similar equation was derived by Selker et al. [87]. The difference is that here water viscosity is used instead of air viscosity. Their case is characterized as a static situation long after breakthrough in a rigid packing. As a result, their formula leads to considerable under-estimation in the case analyzed here.

As one can see in Fig. 3.2 the air plume developed from bulk flow without distinguishable air fingers to fingering flow. Assuming an inverse linear relationship between the grey value and the air saturation, the average grey value over the cross section of a single finger does not show a systematic decrease or increase along the flow path, although there are some variations. It means the air saturation does not show a systematic decrease or increase along the flow path either. Based on the volume conservation rule, the spreading of the plume is caused by a bifurcation process. The characteristic finger size will be presented below. From observation, one can roughly state that each finger has more or less the same size. Accordingly, there are no systematic changes in the local capillary pressure of a single finger along the flow path. Since we are far from the source, for each finger the only driving force is the hydrostatic pressure gradient which provides the buoyancy. We assume a “flat” interface at the top of the plume where all the fingers arrive at the same time, although it is not completely true in reality. This “flat” interface covers the whole plume at the top. This top width of the plume is indicated by $W_t^f$, which is the outer bound width of the plume at the top. The observed values of $W_b^k$ and $W_t^f$, show an approximately constant ratio (shown in Fig. 3.4). One can use this ratio to estimate the average saturation of air at the top of the tree-like pattern, $S_a$, assuming that the near source region (air injection orifice) is fully saturated with air having a residual saturation of water of $S_r=0.1$, i.e., $S_a = (1 - S_r) W_b^k / W_t^f$. The estimated result for air saturation at the top of the air plume is $S_a = 0.35 \pm 0.03$. Hence, the assumed “flat” interface separates the region fully saturated with water from the region with an air content of $S_a$. According to the Brooks-Corey model, the relative permeability of water is $k_{rw} = S_e^\varepsilon$, and the relative permeability of air is $k_{ra} = (1 - S_e)^2 (1 - S_e^{-2})$, where $S_e = (1 - S_a - S_r)/(1 - S_r)$ is the effective saturation, and $\varepsilon$ is an empirical exponent typically varying from 2 to 4. Here I take $\varepsilon=3$. In this case, the movement of air needed to overcome the viscous force density $f_{rw}$ due to water displacement, and the viscous force density $f_{ra}$ due to air flow, is implicitly contained in

$$
\begin{align*}
    f_{ra} &\sim Q \mu_a / (\alpha k_{rw} W_t^f) \\
    f_{rw} &\sim Q \mu_w / (\alpha k_{ra} W_t^f).
\end{align*}
$$

(3.5)
Considering \( \frac{\mu_a}{k_{ra}}/\left(\frac{\mu_w}{k_{rw}}\right) \ll 1 \), it is also reasonable to ignore the viscous resistance due to the air flow. When the injection rate \( Q \) is small, one can again assume the magnitude of the vertical gradient driving the air movement to be comparable to the hydrostatic gradient, \( f_g \), which provides an estimate of the total cross sectional area \((a \times W^t_i)\) for a given volume flux rate \( Q \),

\[
W^t_i = \frac{\mu_w Q}{a k_{rw} k (\rho_w - \rho_a) g}.
\] (3.6)

The estimated top width is plotted in Fig. 3.4. One can see that Equ. 3.6 can approximately estimate the top width for the experiments when \( Q \) is small (in the case presented here less than 90 ml/min). When \( Q \) is large, the gradient in gas pressure is considerably larger than the hydrostatic gradient, which means it is not suitable to use the hydrostatic gradient to approximate the gradient in gas pressure. That is the reason why both the bottom width and the top width will be much smaller than the predicted values in Equ. 3.4 and Equ. 3.6. When the injection rate is larger than 100 ml/min, both \( W^t_i \) and \( W^b_t \) reach nearly a constant value. Here one can see the difference between an air injection process in a mobile and in a rigid packing: in the rigid packing, a parabolic shape of the plume was observed \([36, 37, 51, 87, 92, 100]\). Their models gave an increasing width with the injection rate and the height of the packing. However, the experimental results show a constant width after a certain height (see Fig. 3.3) or above a certain injection rate (see Fig. 3.4). Moreover, this constant width of structures in the cases with \( Q > 90 \) ml/min is more or less half of the width of the cell. There might be two reasons: (1) a boundary effect, since the cell has wall boundary on both lateral sides; (2) a competition between channelized flow and pore flow, as the injection rate increases further the pure air channel occurs earlier, which constrains the ability for the tree-like pattern to grow further laterally.

After the tree breaks through at the top boundary of the packing, one can see a re-invasion process by water, which is visualized by the fading of the grey shade of the tree-like pattern in Fig. 3.5. This means that the air flow in these branches has not been draining any water since these branches have developed. The only resistant force is the viscous force in the air phase. Hence, the air-water system needs to move back to the equilibrium. For a steady-state case \( i.e., \) the grey shade of the tree-like pattern below the dashed line shown in Fig. 3.5 does not change any more), the magnitude of the vertical gradient driving the air movement is identical to the hydrostatic gradient, \( f_g \). In order to keep the outer shape of the tree-like pattern, the only way to increase the air flow resistance is to decrease the mean air saturation in the cross-sectional area \( A \) \( (A = a \times W^b_t \) for the near source region; \( A = a \times W^t_i \) for the top boundary region).
3.2. Morphodynamic hierarchy in water-saturated spherical granulates

Figure 3.5: Evolution of fluidized pattern in the experiment with the air injection rate \( Q = 90 \text{ml/min} \) versus time (physical size of the images: 219 mm × 228 mm): (a) \( t = 16.0 \) s, (b) \( t = 106.0 \) s, and (c) \( t = 356.0 \) s. The dashed line is the front between the fluidized and the tree-like patterns. The beads were fluidized and those staying at the top of the packing are indicated by an irregular zone in black.

Similar to the derivation of Equ. 3.4 and Equ. 3.6, one has

\[
Q = A \frac{k k_{ra} (S_a)}{\mu_a} (\rho_w - \rho_a) g. \tag{3.7}
\]

Compared to Equ. 3.4 and Equ. 3.6, one can approximately estimate the mean saturation of air to be \( S_a = 0.11 \) at the bottom of the tree-like pattern in the steady state. This result is roughly confirmed by calculating the average grey value over the cross section within the outer bound width, assuming an inverse linear relationship between the grey value and the air saturation. This average grey value was then compared with the average gray value over the cross section within the outer bound width at the bottom which was calculated before the breakthrough, assuming an air saturation of 0.9.

Before the tree breaks through at the top boundary of the packing, an air channel is first formed by pushing the beads aside with a bubble of type (c) at the front tip of the tree-like pattern. It is observed that the vertical position of the onset of the air channel is approaching the orifice with increasing air injection rate. Now the second stage of bubble motion starts from the top boundary of the packing. This stage can be characterized as patch-scale air flow with fluidization in which grains are pushed aside, leading to mobilization of the medium. The patch-scale here refers to the characteristic width of a single air branch, which is around 10 to 20 pore sizes. This scale is roughly the same as the characteristic width of the bubbles of type (c) inside the fluidization range. The beads are uplifted and those staying at the top of the packing are indicated by an irregular zone shown in black in Fig. 3.5. Typically, the mobilization range is determined by the previous tree-like pattern. Following the evolution process from the start at the top of the packing, the front of the fluidized region successively moves
towards the air injection orifice. An example of the location of this front is shown as a
dashed line in Fig. 3.5. The dashed line can be detected by subtracting the successive
images pixel by pixel. Obviously, it separates the region of bubbles of type (c) and type
(b), as well as the fluidized and consolidated region. Figure 3.6 shows the evolution
process by tracing the position of the front, \( y \), which is defined as the lowest point of
the front along the coordinate \( Y \). Since the origin of the coordinate system is at the
orifice, \( y \) is the shortest distance from the orifice to the dashed line. Eventually, four
regimes can be identified: (i) \( Q \leq 20 \text{ ml/min} \), (ii) \( 20 \text{ ml/min} < Q < 60 \text{ ml/min} \), (iii) \( 60 \text{ ml/min} \leq Q < 300 \text{ ml/min} \), and (iv) \( Q \geq 300 \text{ ml/min} \). For (i), the front will finally stop
on its way towards the air injection orifice, staying somewhere below the top of the
packing. Correspondingly \( y \) will level off to a plateau, which can be seen in Fig. 3.6.
In this regime, the slow injection rate produces a shallow fluidization zone. However,
the final front positions are not simply determined by the injection rate. For example,
the final front position of 20 ml/min is slightly higher than that of 15 ml/min. It appears
that the final front position is determined by the average air flux in a single branch. The
larger the average flux, the further the front moves on. This will be discussed below and
shown in Fig. 3.9. Regime (ii) is an unstable regime for the front. It can halt somewhere
below the top of the packing (e.g. 30 ml/min and 40 ml/min), or reach the orifice (e.g.
25 ml/min and 33 ml/min). We name this regime transition regime which is shown in
Fig. 3.9. In regime (iii), the front can definitely reach the orifice. The arrival time is
simply sorted according to \( Q \). In the situation that the front can reach the orifice, \( y \)
decreases sharply before around 100 s. After that, the decrease of \( y \) slows down to
some extent. However, it speeds up again, shortly before the front reaches the orifice.
On the other hand, when \( Q \) is larger than 250 ml/min, we can see that the evolution
process quickly comes to an end, with the front reaching the air injection orifice. The
regime (iv) will be described below as \( Q \) increases further, in which the deformation of
the packing appears from the very beginning.

We defined \( y_o \) as the starting position of the second stage in the vertical coordinate. As
we described above, instead of penetrating between the grains at \( y_o \), air flow pushes
the grains aside to form a bubble of type (c). For pushing the grains aside at \( y_o \), air
needs to overcome a pressure force \( \delta p_1 \), which is equal to the effective stress due to
the weight of the grains,

\[
\delta p_1 = (1 - \phi)(\rho_g - \rho_w)(L - y_o),
\]

where \( L = H_b - H_{or} \) is the effective height of the packing. If grains are not pushed
aside, the airflow has to overcome the pressure resistance \( \delta p_2 \) on the displaced fluid
over the displaced distance \( L - y_o \) and the capillary pressure. A one-dimensional Darcy
3.2. Morphodynamic hierarchy in water-saturated spherical granulates

Figure 3.6: Here $y$ is defined as the lowest position of the front between the fluidized and the tree-like pattern. The zero of the vertical axis is located on the position of the orifice. The zero-point of time is set to be the start of the fluidization pattern.

Flow may be suggested, which is shown in Fig. 3.7. This yields

$$\delta p_2 = \left[ \frac{H_w}{k} U - (\rho_w - \rho_a) g (L - y_o) + \frac{2\sigma}{d} \right],$$

where the characteristic velocity $U$ can be described as

$$U = \frac{Q_{inj}}{a W_t(y_o)}.$$  \hspace{1cm} (3.10)

Now $y_o$ can be derived by balancing $\delta p_1$ and $\delta p_2$, yielding,

$$y_o = L - \frac{2\sigma}{d} \frac{1}{[(1 - \phi)(\rho_g - \rho_w) + (\rho_w - \rho_a)]g - \frac{\rho_w}{k} w_{inj} Q_{inj}}.$$  \hspace{1cm} (3.11)

Here the contact angle is $0^\circ$. Knowing the value of $W_t$ from Fig. 3.4, one can estimate $y_o$ via balancing $\delta p_1$ and $\delta p_2$, which is shown in Fig. 3.7. It shows that the predicted $y_o$ fits well with the experiments.

As $Q$ increases further (larger than 300 ml/min), the deformation of the packing appears from the very beginning, which is the regime (iv). One can use the well-known concept of critical velocity for a critical uplift velocity of a patch of grains (density $\rho_g$, porosity $\phi$) with hydraulic conductivity $k_w$ (density $\rho_w$). It is defined as

$$v_c = \frac{(\rho_g - \rho_w)(1 - \phi)}{\rho_w} k_w.$$  \hspace{1cm} (3.12)
This velocity $v_c$ is reached when the pressure gradient of flow, $\nabla P_f$, through the medium equals the overburden pressure gradient of the medium, $\nabla P_g$, at the critical point at which the medium is going to fluidize. According to Darcy’s law, $\nabla P_f = v_c \rho_w g / k_w$, where $\nabla P_f$ is the total pressure gradient, i.e., $\nabla P_f = \rho_w g \nabla Y + \nabla P_d$. In this expression, the first term indicates the hydrostatic pressure gradient, and the second term relates to the pressure drop associated to the viscous loss. As the vertical thickness of the packing is not too large, $\nabla P_g = (\rho_g - \rho_w)(1 - \phi) g$. In order to estimate the characteristic air injection rate, $Q_c / a$, a length scale is needed. Considering that the relevant pressure is related to the vertically load of the packing, the vertical thickness of the packing, $L = H_b - H_{or}$, is a proper length scale. This yields

$$Q_c = \frac{a(H_b - H_{or})(\rho_g - \rho_w)(1 - \phi)}{\rho_w} k_w. \tag{3.13}$$

Thus one can get $Q_c \approx 253$ ml/min. Although the theory here is restricted to single phase flow, the major resistance during a drainage process is exerted by the more viscous fluid, especially for a ration of $\mu_a / \mu_w << 1$ in our case. Compared with the experimental observation (around 300 ml/min), $Q_c \approx 253$ ml/min is a reasonable estimate.

As mentioned above, Fig. 3.6 tells us that there is a transition regime (ii) which separates the regime (i) for $Q$ smaller than a threshold injection rate $Q_f$, where the fluidization stops somewhere between the top of the packing and the orifice, and regime (iii) for $Q$ larger than $Q_f$, where the fluidization reaches the orifice. Based on our obser-

---

$^3$The properties of the hydraulic conductivity $k_w$ of fluid are related to the specific weight $\gamma = \rho_w g$ and the dynamic viscosity $\mu_w$ of the fluid. This can be expressed as $k_w = \frac{\mu_w}{\gamma}$. 

---

Figure 3.7: The vertical starting position $y_o$ of the fluidized region changes with the air injection rate. Inset: one-dimensional model for calculating $y_o$. 
Morphodynamic hierarchy in water-saturated spherical granulates

3.2. Morphodynamic hierarchy in water-saturated spherical granulates

vation, $Q_f$ is between 20 and 60 ml/min in our experiments. This means that when $Q$ is larger than $Q_f$, the evolution of the fluidization has the ability to reach the orifice, i.e., the fluidization has the ability to access the whole region created by the tree-like pattern. Specifically in our system, once there is a fracture (i.e., glass beads are fluidized), a bubble of type (c) is formed, which means the bubble interface attaches to the front and back plate of the 2D box. To get a rough estimate of the fracture width $w_f$ at the interface, we assume the simple geometry of a rectangular air-finger with an arch-shaped upper end having a curvature of $w_f/2$, which is shown in Fig. 3.8. Given a quasi-static assumption, the upward growth of the fracture above the interface is driven by the buoyancy, which is overcome by the surface tension. The buoyant force, $F_b$, on the arch with a volume of $\pi a w_f^2/8$ is given by

$$F_b = \frac{\pi}{8} a w_f^2 (\rho_w - \rho_a) g.$$

(3.14)

The surface tension restrains the growth of the fracture with a force, $F_s$,

$$F_s = 4a\sigma,$$

(3.15)

which is acting on the cross section $w_f \times a$. Balancing these two forces yields an estimated $w_f \approx 8.6$ mm (A similar value was also observed by [29] and predicted by [87]). This value is a proper approximation of the fracture aperture in the fluidized region. In most of our experiments, the width of the fracture is less than 10 mm. Based on this rough estimate, to open such a fracture of width $w_f$ the local fluidization flow rate $Q_a$ must be equal to $a w_f v_c$, yielding

$$Q_a = a w_f k_w \frac{(\rho_g - \rho_w)(1 - \phi)}{\rho_w},$$

(3.16)
or \( Q_a \approx 10 \text{ ml/min} \) in our case. This means that if the air flow rate in one single branch in the tree-like pattern is larger than \( Q_a \), this branch will become a channel of width \( w_f \). Figure 3.9 shows the number of channels in the fluidized region (above the front). One can see that the number of channels increases in two steps: in the first step it reaches around 3 and in the second step it reaches around 6. The number of channels is plotted against the averaged flow rate per channel in Fig. 3.9. The location of \( Q_a \) indicates a transition regime with injection rate \( Q_f \) between 23 and 55 ml/min. This result is consistent with the experimental data. Moreover, \( Q_f \) gives a clear separation of these two steps in the number of channels.

![Figure 3.9: The number of channels above the interface and the average flow rate per channel.](image)

Knowing that air flow cannot occur until water inside the pore has been displaced, we can classify the experimental observations into three scenarios using a dimensionless quantity, \( \chi = Q/(aLk_w) \). (1) When \( \chi \) is less than \( \chi = Q_f/(aLk_w) \), a narrow tree-like air plume appears. The fluidization inside the packing cannot reach the orifice, and the number of fractures inside the fluidized region is around 3. (2) When \( \chi \) is between \( \chi = Q_f/(aLk_w) \) and \( \chi = Q_c/(aLk_w) \), a wider tree-like air plume is developed. The fluidization can reach the orifice, and the number of the fractures inside the fluidized region is around 6. Finally a meandering single-channel is developed. (3) When \( \chi \) is beyond \( Q_c/(aLk_w) \), a deformation of the packing is occurring immediately after the air is injected.

After the fluidization process, the final evolutional stage is presented by a meandering single-channel flow pattern, which is shown in Fig.3.10. One can see that at turning points along the channel, a few dead-end air branches exist. From the image, one knows that most of the time this is an incoherent channel (or fracture) which is formed by
3.2. Morphodynamic hierarchy in water-saturated spherical granulates

Figure 3.10: A typical irregular oscillating channel. The air injection rate is $Q = 90$ ml/min. The image (physical size of 219 mm $\times$ 228 mm) is an integration of the differences over 2.5 s (from 1622.5 s to 1624.5 s). The deep grey areas are the areas where the channel changed during the 2.5 s. They indicate pre-existing air channels. Two convection cells were added manually in the image as dashed lines according to the observations in the experiments.

several discrete elongated bubbles. These bubbles are driven by their buoyant forces. Once there is a bubble reaching the top surface of the packing, a release event occurs. Some of these fractures disappear after each release event at the top of the packing while some new fractures appear before the next release event. Due to the release event, together with the capillary instability, the existing channel collapses occasionally so that it has the chance to migrate to another location in the vicinity. The characteristic width of the air channel can be estimated with Equ.3.14 and Equ.3.15 under the assumption of a balance between buoyancy and surface tension. The beads were carried by the rising bubbles along the channel. Due to the volume conservation, near the orifice region (the deepest position of the fluidization area) the beads will move from the sides towards the channel. As a result, two convection cells are developed (as shown in Fig.3.10). This migration phenomenon will be discussed in the following section.

3.2.3 Summary

In this section, I have demonstrated and characterized a pattern evolution when air was injected into a vertically-placed 2D glass beads packing saturated with water. Three stages can be identified, tree-like pattern, fluidized pattern, and migrating single-
Chapter 3. Air injection in two-dimensional experiments

channel pattern. The width of the tree-like pattern is affected linearly first by the injection rate near the injection region, and then becomes independent of the injection rate. The expansion of the air plume behaves in a diffusion-like manner as the air branches advance upward randomly, and finally reaches a more or less constant width. Assuming that the gradient in gas pressure is comparable to the hydrostatic gradient, quantitative expressions are presented to estimate the width of the air plume at its top and bottom while the injection rate is small. When \( Q \) is large, the gradient in gas pressure is considerably larger than the hydrostatic gradient. It is then not suitable to use the hydrostatic gradient to approximate the gradient in gas pressure. As a result, both the bottom width and the top width will be much smaller than the predicted values.

Based on pressure force balancing between the effective stress due to the weight of the grains and the pressure resistance on the displaced fluid combined with the capillary pressure, a quantitative prediction was proposed to estimate the starting position of the second stage, the fluidized pattern. It was found that during the evolution of the fluidized pattern the front between the fluidized pattern and tree-like pattern varies with time. Four dynamic regimes can be distinguished: regime (i) where the fluidization stops somewhere between the top of the packing and the injection orifice, a transition regime (ii), regime (iii) where the fluidization reaches the injection orifice, and regime (iv) where the deformation of the packing appears as soon as the air is injected. To identify these fluidized regimes, a critical injection rate \( Q_f \) is defined, at which an air channel of width \( w_f \) can be created with a flow rate of \( Q_a \) in a single finger. The predicted \( Q_f \) is determined via \( Q_a \) in Fig.3.9. The results are consistent with the experimental findings. A characteristic injection rate \( Q_c \) is defined to characterize at which injection rate the packing is going to be deformed from the very beginning.

3.3 The channel migration phenomenon

In this section, I will describe an instability phenomenon that occurs in the third stage of the evolution process shown in the previous section. As one can see from the previous section, the resolution of the invading structures is not high enough to capture a sharp interface at pore-scale level between the invading and defending fluids. This problem can be overcome by using a transparent porous matrix saturated with liquids having the same refractive index, which is the matching refractive index technique.

In this section, the phenomenon of channel migration will first be described qualitatively, from the onset of the air injection till the end of the migration. Then, this phe-
3.3. The channel migration phenomenon

The channel migration phenomenon will be measured in two parts, before and after breakthrough. In the former part, the characteristic measurements are studied via maximum vertical and horizontal advances, air volumetric fraction, ratio of total surface area to volume, specific surface area of the air phase, and box-counting dimension analysis. In the later part, the characteristic measurements comprise mean horizontal position of the air channel, horizontal shifting distance, lateral movement distance, and lateral movement width.

3.3.1 Experimental parameters

The defending wetting fluid used in all our experiments is a 90%-10% by volume glycerine-water solution. Air is used as the invading non-wetting fluid. The wetting fluid has a dynamic viscosity of $\mu_w=0.165$ Pa·s and a density of $\rho_w=1235$ kg/m$^3$ at room temperature of 22 °C. The surface tension between these two fluids is $\eta=6.4 \times 10^{-2}$ N/m. The porous medium is constructed by crushed fused silica glass with a density of $\rho_{gr}=2202$ kg/m$^3$, which is separated into two fractions with grain sizes of $d_1=0.6$-0.8 mm and $d_2=1.0$-1.6 mm. The crushed silica glass solution was chosen after we observed that glass beads of the required size do not have a homogeneous refractive index. Rather, the refractive index varies over the radius, due to the production method of glass beads. The solution makes a perfect index matching impossible. The wetting fluid and the fused glass have the same refractive index of $n=1.457$-1.458. Therefore the porous medium looks completely transparent. As a result, there is a high contrast between the invader (air with the refractive index of $n=1.0$) and the background (wetting fluid and porous medium).

The experiments were performed with different parameters: three different injection rates $Q$, 20 ml/min, 90 ml/min, and 150 ml/min; two different fractions of grains size, $d_1$ and $d_2$; and three different bed-heights $H_b$, 160 mm, 220 mm, and 285 mm. Thus, the effective bed-height is $H_b - H_{or}$, which yields 140 mm, 200 mm, and 265 mm, respectively. In each experiment, the grains of the crushed fused glass were first saturated with the glycerine-water solution and then filled into the box. After that, the box was vibrated for a few minutes, guaranteeing that the random porous medium was packed under gravity. The porosity of the packings was measured to be about $\phi=0.48$. For each group of parameters with the same grain size, fixed injection rate, and fixed bed height, the experiment was repeated three times, and each experiment was performed up to 15 minutes. In all experiments, the liquid table was set at a height $H_w=400$mm above the base.
3.3.2 Qualitative phenomenology

As addressed above, the 0.3 mm-orifice was used to keep the pressure fluctuations and therefore fluctuations in the flow rate small. The measurements of the injection rate indicate that the flux fluctuates with small amplitudes around a constant value. For the case of $Q=20\ \text{ml/min}$, the standard deviation of $Q$ is 1.5 ml/min; for the case of $Q=90\ \text{ml/min}$, it is 1.3 ml/min; and for the case of $Q=150\ \text{ml/min}$, it is 1.8 ml/min. The frequency of the fluctuations is typically larger than 1 Hz. In contrast, the migration of air flow fingers in the experiments is a rather slow process (the mean frequency is less than $1/60\ \text{Hz}$), which will be discussed in detail below.

![Figure 3.11: Images are taken from the experiment with $d_1$, $Q=150\ \text{ml/min}$, and $H_b=220\ \text{mm}$: Air channels or fractures are shown right after air is injected and air penetrates into pores around the branches (a) at $t=0.5\ \text{s}$; (b) at $t=3.0\ \text{s}$ (before air breakthrough); (c) at $t=3.5\ \text{s}$ (right after air breakthrough); and (d) at $t=5.5\ \text{s}$ (fragmentation).](image)

After air is injected into the porous medium, branching fingers of air flow are forming. The duration of this branching process depends on the height of the bed $H_b$, the injection rate $Q$, and the diameter of the grains $d$. The duration of the process increases with the height of the bed and grain size, but decreases with the air injection rate. During this period, air is transported through the medium network pores which have a size on the order of a grain diameter (See Fig. 3.11(a)). After the initial phase, due to the build-up of air pressure inside the air channel, grains are pushed aside by the liquid-air interface along the air branches, as shown in Fig. 3.11 (the boundaries of air branches are indicated in black color). It is observed that the meniscus of the interface is not necessarily a smooth curve, since the grains protrude from the interface. The surface tension of the interface then acts as the driving force pushing the grains aside. Before the front tip of the branches reaches the top of the medium, the width and length of
the branches grow continuously and stably, and the number of air branches is main-
tained. On the top of the packing, we observe a conical protrusion. This means a
deforation occurred inside the packing. As a result, the grains were sheared against
each other. In Fig. 3.11(b), the branch reaches its maximum width and length before
air breakthrough. After another 0.5 s (Fig. 3.11(c)), a breakthrough point is established
at the top boundary of the medium. Air contained in the various branches tends to
discharge via this breakthrough relieving the pressure built up in the fingers. Small
fingers at the peripheral boundary of the branch die out quickly. Therefore the periph-
eral boundary becomes smooth, partially because of surface tension. A few seconds
later (Fig. 3.11(d)), under the influence of gravity, a few big branches collapse allowing
sedimentation of those grains which were pushed aside along the channel. Due to the
complex pattern of the air channel, grains have different sedimentation time, resulting
in a reorganization of the medium. Some sections of air branches, which are unable
to release the air along their previous channel, become trapped air bubbles due to the
snap-off motion of surface tension. As one can see, the air channel shrinks after air
breakthrough, with only one main branch remaining. During the shrinking process, the
menisci of the interface form a smooth curve. Obviously, at this stage the surface ten-
sion smoothes the interface. The remaining branch is a coherent flow path from the
injection orifice to the breakthrough position at the top boundary of the grain packing.

Figure 3.12: Air slugs moving due to buoyancy and air supply through a tiny channel
(indicated by a black arrow). Images are taken from the experiment with $d_1$, $Q=150$
ml/min, and $H_b=220$ mm: (a) at $t=47.5$ s, (b) at $t=48.0$ s.

As the system evolves continuously, the remaining main air branch collapses into large
discrete bubbles, which are mostly creeping along the previous path created by the
original finger, as shown in Fig. 3.12. The flow pattern turns from channelized flow at
the beginning into slug flow. The slug flow is characterized by large bubbles creeping
upwards. Those bubbles can merge into still bigger bubbles, or can connect via tiny channels (indicated by a black arrow in Fig. 3.12) which open and close in a pulsating mode. That means air is transported at the time when the tiny channel is open. When it is closed, air bubbles are forced to move by buoyancy. During the slug flow, air bubbles mostly choose previous channels to move. The reason is that bubbles always move towards the biggest pore following the least capillary resistance. The biggest pore will probably be in a position where there was a channel before, since the grains take time to settle down again after they were pushed away by the previous bubbles. In the slug flow, air bubbles need to push aside the grains at the head front of the bubbles. It was again observed that at the top surface of the bubble the grains protruded from the interface so that the interface was again coarse. Nevertheless, at the rear end of the bubble the interface was rather smooth.

Figure 3.13: Air channel shifting inside the medium after air breakthrough is indicated by a superposition of air flow patterns at three time slices, 45 s (light blue), 66.5 s (purple), and 121.5 s (black) after the start of air injection. Image was taken from the experiment with $d_1$, $Q=150 \text{ ml/min}$, and $H_b=220 \text{ mm}$.

However, it is not always true that a bubble will follow the path of previous bubbles. Owing to the remixing process and reorganization of grains, the porosity of the medium near the previous channel changes stochastically at the grain patch level. This local change of porosity leads to an abrupt change in the distribution of pore sizes, causing air bubbles to migrate into a new channel. The process of channel migration is shown in Fig.3.13. From the images, one can see that the air channel shifts in a discrete and erratic manner from left to right. The interaction of the injected air flow and the medium...
structure leads to mobilization of the medium and an instability, which causes the air channel to migrate. The channel migration appears as a sequence of previous channels collapsing and new channels opening. However, the channel migration comes to a stop after some time, leaving one thin and stable channel at the end of each experiment.

### 3.3.3 Characteristic measurements before breakthrough

The characteristic measurements are extracted from a series of experiments, since quantitative analysis enables us to characterize the displacement patterns with measurable parameters in unique functional forms. These findings will be detailed in this section in two parts, before and after the first breakthrough. In the following text and figures in Sec. 3.3, $Q_1$, $Q_2$, and $Q_3$ refer to the different injection rates 20 ml/min, 90 ml/min and 150 ml/min, respectively, and $H_1$, $H_2$, and $H_3$ refer to the different effective bed-heights 140 mm, 200 mm and 265 mm, respectively.

**Maximum vertical advance**

Before the first breakthrough, the upward migration of air was measured according to the maximum vertical height of the invading structure, $h_{\text{max}}$, at different time intervals. Here the zero of $h_{\text{max}}$ is set to the position of the injection orifice. Figure 3.14(Left) shows the measurements of this maximum vertical height versus time for all the conducted experiments. Overall, $h_{\text{max}}$ increases linearly with time, although there can be considerable variations within one experiment over different time intervals, and among experiments under identical conditions (same $Q$, $H_b$, and $d$). One can see that the variations, especially in the early time intervals of the experiments, increase with the height of the packing $H_b$, but decrease with the injection rate $Q$. The data show only random differences for $d_1$ and $d_2$ with all other parameters being equal, which means grain size effects on $h_{\text{max}}$ are not apparent in the present experimental configurations.

**Maximum horizontal advance**

The transversal migration of air was measured according to the maximum horizontal width of the invading structure, $w_{\text{max}}$, at different time intervals. Figure 3.14(Right) shows the measurements of this maximum horizontal width versus time for all the conducted experiments. One can see that, for small injection rate (i.e., curves with square symbols in Fig. 3.14(Right)) $w_{\text{max}}$ reaches its maximum in a short time, and then levels
Figure 3.14: The maximum vertical advance of the invading air, $h_{\text{max}}$ (Left), and the maximum horizontal width, $w_{\text{max}}$ (Right), vary with time. Here $t_0$ is the breakthrough time.

off afterwards. Furthermore, for deeper beds (larger $H_0$) with small $Q$, $w_{\text{max}}$ even decreases after having reached its maximum. This is consistent with the observations of the phenomenon that, for small injection rate, the invading structure behaves in an incoherent manner forming discrete bubbles. It means that the vertical migration is driven by the buoyancy only. Once the bubble elongates in vertical direction, the transversal advance stops and even retreats. Therefore, $w_{\text{max}}$ decreases in such a situation. For larger injection rates, one can observe that the invading structure behaves in a relative coherent manner, since the air supply is large enough to maintain a relative coherent structure.

**Air volumetric fraction**

The air volumetric fraction, $f_a$, is determined by the ratio of air volume and a rectangular enclosing volume of the air plume, yielding

$$f_a = \frac{V_a}{V_e},$$

where $V_a$ is the injected volume of air, $V_e = w_{\text{max}} h_{\text{max}} a_{th}$ the rectangular enclosing volume of the invading structure, and $a_{th}$ the averaged thickness of the invading structure. At a given time, $a_{th}$ was determined as $a_{th} = V_a / A_{\text{str}}$, where $A_{\text{str}}$ was the total area of the invading structure on the 2D image. The choice of the enclosing volume enables consistency in analysis and comparison among the experiments. According to its definition, $f_a$ is in the range of $0 < f_a < 1$. It can be used to quantify the spatial distribution of the invading structure. As $f_a$ increases the invasion pattern can be described as
3.3. The channel migration phenomenon

Figure 3.15: Air volumetric fraction as a function of time. The solid line in the right graph (logarithmic scale both for the $x$- and $y$-axis) has a slope of -0.45.

“single”-channel flow in which a “fat” channel is formed; as it decreases the invasion pattern can be characterized as more multi-channel (or fingering-like) flow. Figure 3.15 shows the behavior of the air volumetric fraction for different $Q$, $H$, and $d$ as a function of time. Overall, $f_a$ decreases monotonically with time, and finally levels off asymptotically to an approximate constant. In all cases, $f_a$ for packings of small grains are larger than those for packings of large grains. It means that the invasion pattern for packings of large grains appears more like multi-channel flow. Correspondingly, one can see that the invading structure exhibits patterns of wide fractures (shown in Fig. 3.16(a)) in small grain packings, while the invading structure exhibits pore-filled patterns (shown in Fig. 3.16(b)) in large grain packings. It appears that the invading air favors to create fractures rather than penetrate into the pores in small grain packings, the opposite being the case in large grain packings.

Figure 3.16: (a) Structure of wide-fracture pattern, snapshot at $t = 1.0$ s from the experiment with $d_1$, $Q_3$, and $H_2$. (b) Structure of pore-filled pattern, snapshot at $t = 1.0$ s from the experiment with $d_2$, $Q_3$, and $H_2$. 
Figure 3.15 also shows $f_a$ in a double-log scale ($x$- and $y$-axis). One can see that when $t/t_o < 0.8$, $\log f_a$ decreases quasi-linearly with an average slope of -0.45, which is shown as a solid line in Fig. 3.15(Right). As shown in Fig. 3.16, there are pore-

![Graph](image-url)

Figure 3.17: Averaged thickness of the invading structure as a function of time. The thickness of the 2D model is marked by a thick dash-dotted line (at $a_{th} = 5.5$ mm).

filled air branches around the primary fractures. The fraction of pore-filled branches can be indicated by the average thickness $a_{th}$ to a certain degree. We know that, with a given air volume, the higher the fraction of pore-filled branches, the larger the total area $A_{str}$ of the invading structure on the 2D image. By the definition of $a_{th}$, while the fraction of pore-filled branches increases, $a_{th}$ decreases. Figure 3.17 shows $a_{th}$ as a function of time. Overall, $a_{th}$ increases linearly with time after a sharp decline at the very beginning of the experiment. It means that the fraction of pore-filled branches decreases during the experiment. It tells us that, as time passes, air branches favor to create fractures when they are approaching the top of the packing. From Fig. 3.17, one can see that in most of the cases, $a_{th}$ of small grain packings is larger than that of large grain packings. Again, it supports the conclusion that the invading air favors to create fractures rather than penetrate into the pores in small grain packings, the opposite being the case in large grain packings. Furthermore, most of the time, $a_{th}$ is less than half of the thickness $a$ of the 2D model. One can also roughly estimate the fraction of fracture area in the 2D image by the ratio of $a_{th}$ and $a$. Therefore, it can be concluded that the fraction of fracture area decreases with the grain size. It can be explained as a compromise between the stress inside the medium (here the mixture of grains and liquid) and the air entry pressure. For the air branch expanding from the pore-filled branch to the fracture-filled branch (at least a few grain sizes), the pressure exerted on the interface of air branches needs to satisfy two criteria, (i) the pressure has to overcome the stress inside the medium, and (ii) the stress must less than the air entry pressure along the whole interface of the branch. According to Bagnold [5],
3.3. The channel migration phenomenon

the stress from the grains is proportional to $d^2 \gamma^2$, where $\gamma$ is the shear rate. Meanwhile, the air entry pressure is proportional to $d^{-1}$ (according to Equ. 2.3). Therefore, one can see that the probability of satisfying both criteria decreases with grain size. This is consistent with Fig. 3.17.

**Ratio of total surface area to volume**

The total surface area, $A_t$, of the air phase is defined here as the sum of the areas between the air phase and the wetting fluid, and between the air phase and the solid grains [68, 69]. Values of $A_t$ can be calculated according to the boundary between air phase, and wetting fluid and solid grains from the 2D images. Thus in the 2D case, $A_t$ would be defined as the product of the perimeter of the invading structure and $a_{th}$. The ratio of $A_t$ to $V_a$, i.e. $f_{sv}$, describes the relationship between the total surface area and the total injected air volume. Increases in $f_{sv}$ values are related to the evolution of the invading structure, such as finger development, occurrence of fragmentation (random disconnection) and structure branching. Figure 3.18 shows $f_{sv}$ as a function of time with a given set of $Q$, $H_b$, and $d$. As one can see, $f_{sv}$ varies at early times, and then levels off finally. It is also noted that the values of $f_{sv}$ overall increase with $H_b$, while a variability of $f_{sv}$ due to the different injection rate is not evident. Clearly, the values of $f_{sv}$ fall into two groups, group (i) less than 0.5 mm$^{-1}$ and group (ii) larger than 0.5 mm$^{-1}$, depending on the grain size, $d_1$ and $d_2$, respectively. The two groups are roughly separated by the curve of the experiment with $d_2$, $H_3$, and $Q_1$. Note that $f_{sv}$ increases with $d$. This means that the evolution of the invading air, such as finger development,
occurrence of fragmentation or structure branching, occurs more easily in the packings of large grains. This is consistent with the observations of the growth activities of the invading air structure, e.g., the comparison of a transient status between the packings of small and large grains in Fig. 3.16. Again the fact that \( f_{sv} \) falls into different groups depending on grain size \( d \) is mainly related to a smaller air entry pressure in the packings of larger grains, which helps finger development and structure branching. Furthermore, according to Eq.2.7, the characteristic length scale of the new fragment is proportional to \( d^{-1} \). Given a certain amount of air, one can imagine that there will be more fragments based on the volume conservation of air in the packings of large grains, since there the fragments have smaller characteristic length scale. It means that increase of the grain size enhances the occurrence of fragmentation. By recalling the definition of the Bond number in Sec. 2.1.2, \( B_o \), the ratio of total surface area to volume, \( f_{sv} \), is scaled by \( B_o \) with a power of -0.5, which is shown in Fig. 3.18(Right). One can see that the data of \( f_{sv} \) overlap somewhat between the two groups of grains, \( d_1 \) and \( d_2 \).

### Specific surface area of the air phase

Interfacial area is defined as the area between two immiscible phases per unit bulk volume of porous medium. It can act as a surrogate for interface area between immiscible phases [68, 69]. This parameter, \( A_{sp} \), is calculated by dividing the total surface area of the invading phase (air), \( A_i \), by the enclosing volume of the structure, \( V_e \). Knowing \( A_{sp} \) is helpful in quantifying the processes of oxygen adsorption, dissolution, volatilization, and etc. Figure 3.19(Left) depicts the specific surface area of the air phase as a function of time with a given \( Q \), \( H_b \), and \( d \). As one can see, higher values of \( A_{sp} \) occurs at the beginning of the experiments, while they decrease monotonically to an approximately
3.3. The channel migration phenomenon

Figure 3.20: (Left) $A_{sp}$ versus time, and (Right) $A_{sp}$ versus $f_a$. Both are scaled with $B_o^{-0.5}$.

constant value as invasion continues. The variability of $A_{sp}$ with the different injection rates and packing heights is not evident. However, the curves of $A_{sp}$ clearly fall into two groups, group (i) of curves that are below the curve of the experiment with $d_2$, $H_3$, and $Q_1$, and group (ii) of curves that are above the curve of the experiment with $d_2$, $H_3$, and $Q_1$, depending on the grain size, $d_1$ and $d_2$, respectively. This grouping is similar to one of the the ratio of total surface area to volume, $f_{sv}$. One would also want to know whether there is a linear relationship [46,68,69] between $A_{sp}$ and the volumetric fraction of air, $f_a$. Figure 3.19(Right) shows the results. Overall, the specific surface area $A_{sp}$ approximately increases linearly with $f_a$ for all the experiments. The slopes of these linear relationship vary from 0.26 to 1.25. Similarly, the curves of $A_{sp}$ as function of $f_a$ are separated into two groups by the curve of the experiment with $d_2$, $H_3$, and $Q_1$, depending on the grain size, $d_1$ and $d_2$, respectively. In analogy to $f_{sv}$, one can also scale $A_{sp}$ by $B_o$ with a power of -0.5, which is shown in Fig. 3.20. The curves of $A_{sp}$ collapse independently with the grain size. Furthermore, the slopes of the approximately linear relationship between $A_{sp}B_o^{-0.5}$ and $f_a$ vary from 0.85 to 2.23, which is a narrower interval than seen in the Fig. 3.19(Right).

Box-counting dimension analysis

To quantify global features of patterns before the first breakthrough, the box-counting dimension was calculated by measuring the number of boxes, $N$, needed to cover the entire pattern, as a function of the box size, $r$. This method is so-called box-counting method, which is shown in Fig. 3.21. A representative sample of the box-counting data of the invading structure is shown in Fig. 3.22(Left). It depicts collapsing box-counting data except for the very beginning of the experiment, but even at that early
Chapter 3. Air injection in two-dimensional experiments

Figure 3.21: Illustration of the box-counting method acting on a representative snapshot of the air structure.

Figure 3.22: (Left) The box-counting data of the invading structure for different times in the experiment with \( d_1, Q_3, \) and \( H_2 \). (Right) The local slope of the box-counting results, corresponding to the local box-counting dimension, \( D_{local} \), as calculated by a linear least squares fit over the sliding interval \( \Delta \ln(r) = 0.69 \). Here the box size is measured in the unit of pixels, where the pixel size is about 0.3 mm.

Point in time they follow the same tendency. The local box-counting dimension, \( D_{local} \), calculated by the local slope of the box-counting results, is shown in Fig. 3.22(Right) for a set of parameters \( Q, H_b, \) and \( d \). It denotes that \( D_{local} \) of \( d_1 \) is slightly larger than that of \( d_2 \) as long as \( \ln(r) < 1.25 \). It means that when the covering box size is less than \( e^{1.25} \times 0.3 \) mm, i.e., 1.05 mm, \( D_{local} \) of \( d_1 \) is slightly larger than that of \( d_2 \). It is well known that increases of the roughness of the invading interface leads to increases of the box-counting dimension [67]. As shown in Fig. 3.16, one might expect that the small bumps (a structure element created by air penetration into the pores) along the invading interface have the pore scale. Thus, given \( d_1 < 1.05 \) mm < \( d_2 \) and a covering box size less than 1.05 mm, as expected, the results show that \( D_{local} \) of \( d_1 \) is slightly
larger than that of $d_2$, corresponding to a smaller bump size along the invading interface in the packings of $d_1$. This is consistent with what is observed in Fig. 3.16. Furthermore, for a covering box size between 1.05 mm and 3.65 mm (i.e., $e^{2.5 \times 0.3} = 3.65$ mm), the $D_{\text{local}}$ values are quite close to each other between $d_1$ and $d_2$. This means that in this range (1.05 mm to 3.65 mm), the roughness of the invading interface is approximately identical in the packings of these two grain sizes. This suggests that the roughness of the invading interface in the packings of these two grain sizes is generated by a structure element of a size between 1.05 mm and 3.65 mm, which is the width scale from a single finger to a fracture. Figure 3.16 confirms this result that in the range of 1.05 mm to 3.65 mm, the roughness of the invading interface is created by fingers or fractures.

### 3.3.4 Characteristic measurements after breakthrough

As described in Sec. 3.3.2, the process of channel migration proceeds in a discrete and erratic manner. The migration process is characterized by the previous channel collapsing and a new channel opening, and can simply be described in two stages. At the early stage, the new channel opens up along a previous branch. The intersection of the new channel and the previous one lies at a position close to the base of the bed. In general, the horizontal shifting distance resulting from this kind of branching is large. At the later stage, the shifting is the consequence of a new branch forming after the previous channel closes due to the reorganization of the grains. The intersection of the new channel and the previous one in this stage occurs in the upper part of the bed. One can observe that the horizontal shifting distance of the channel decreases with time. To quantify the migration process, the mean horizontal air channel position, $X$, is defined (Fig. 3.23). At a given time, a coherent channel was determined using the following procedure: images were first turned into black and white images by marking the air pixels with white color. A combined image was then built by superimposing the image with two previous and two later images. In the combined image, the isolated bubbles were removed, leaving only a continuous channel. The mean horizontal air channel position, $X$, is calculated by averaging the positions of white pixels along the horizontal direction at a given vertical $y$. One example of $X$ is shown as a red solid line in Fig. 3.23, and the values of $X$ as functions of time in three vertical positions are shown in Fig. 3.24.

A complementary measure, the total width of air channel, $W_{\text{tot}}$, was calculated in the following way: a counting procedure was applied to the combined image without the isolated bubbles, adding up all white pixels over the horizontal direction at a given $y$. The sampling data sets of $W_{\text{tot}}$ are shown in Fig. 3.24. It is clearly visible in Fig. 3.24
Chapter 3. Air injection in two-dimensional experiments

Figure 3.23: The mean horizontal air channel position (red solid line) at a given time.

Figure 3.24: (Left) Mean horizontal position ($X$) of air channel, as well as (Right) the corresponding total width ($W_{\text{tot}}$) of air channel, at different vertical position ($y$) is plotted versus time. The data set is taken from the experiment with $d_1$, $H_2$, and $Q_3$. Here the zero-point of time is set to the time of the first air breakthrough. The zero-point of $X$ is the horizontal position of the orifice.

that the magnitude of the horizontal shifting distance decreases in time. The number of “big jumps” in which the air channel shifts over a large distance is small. Most of time the air channel migrates in a peristaltic manner. The horizontal shifting distance is then quantified by the difference between the “jumps”, i.e. $\delta X = |X(y, t_{i+1}) - X(y, t_i)|$. The probability density of $\delta X$ is given in Fig. 3.25. Since the air channel will be fixed at the end of each experiment, $\delta X$ approaches zero in time. In order to avoid the time depen-
3.3. The channel migration phenomenon

Figure 3.25: Probability density function of $\delta X$. Here $\delta X = 0$ is not considered.

dency of the migration phenomenon, $\delta X = 0$ is not considered in the probability density function. One can see that over 99% of the horizontal shifting distance is less than 10 mm. Figure 3.25 also depicts that $\delta X$ increases both with $Q$ and $H_b$, but decreases with $d$. This means the probability to open new channels in the packing of fine grains is larger than that in the packing of coarse grains.

On the whole, the intersection of the new and the previous channel is transported vertically from the bottom to the top of the bed with time. At the end, the air channel develops into a narrow smooth curve with a series of connected bubbles, and does not shift anymore. From Fig. 3.24, one can see that the duration of the channel movement is different at different vertical positions ($y$). The lower the channel position, the earlier the motion ends. From the corresponding widths of the air channel, it can be seen that the air channel performs cycles between expansion and collapse. This succession of expansion and collapse provides the possibility to create a new channel. The amplitude of the width first increases to its maximum and then asymptotically decreases to the final value. In the end of this process, grains are settling, providing a fixed channel for the air to pass through the bed. From Fig. 3.24, the total width of a stable air channel is less than 5 mm.

In order to study the shifting movement of the air channel, the cumulated lateral movement distance, $L_{\delta X}$, and the lateral movement width, $W_{\delta X}$, of the area affected by air were calculated using a time series of $\delta X$ which is presented in Fig. 3.24. The cumulated lateral movement distance $L_{\delta X}$ which is presented in Fig. 3.26(a)(b)(c) is calcu-
Chapter 3. Air injection in two-dimensional experiments

Figure 3.26: The lateral movement distance, $L_{\delta X}$, and the lateral movement width, $W_{\delta X}$, are plotted versus $y$-coordinate with a vertical sampling distance of 5 mm. The data set is taken from the experiments with different $d$, $H_b$, and $Q$. Here the zero-point of time is set to the time of the first air breakthrough. The zero-point of $y$ is the vertical position of the orifice.
3.3. The channel migration phenomenon

lated as

\[ L_{\delta X} = \int_{t_0}^{t_N} \delta X dt, \]  

(3.18)

where \( t_0 \) is the time of the first breakthrough and \( t_N \) is the ending time of the experiment. As long as \( t_N \) is large enough, i.e., the experiment is running long enough to reach the final state (air channel is fixed), \( L_{\delta X} \) does not increase anymore, which means the leveling off of \( L_{\delta X} \) does not depend on the exact ending time of the phenomenon. The lateral width \( W_{\delta X} \) which is presented in Fig. 3.26(d)(e)(f) is calculated as the width of the area affected by air over horizontal stripes in the bed, i.e.,

\[ W_{\delta X} = \max_{t} (X(y, t)) - \min_{t} (X(y, t)). \]  

(3.19)

As shown in Fig. 3.26(a), (b) and (c), three regimes can be distinguished in \( L_{\delta X} \) for \( d_1 \), an increasing regime, a plateau regime, and a decreasing regime. The increasing regime starts from the orifice up to a certain height. This regime occupies up 60% of \( H \). Following the increasing regime, there is a regime where \( L_{\delta X} \) reaches its maximum. From Fig. 3.26, it can be seen that the range of this regime decreases sharply with the air injection rate, but increases monotonically with the height of the bed. In this regime, a bubble flow is established and the medium acts like an elastic gel. The smaller the injection rate, the more prominent is the bubble flow and the lower is the position at which the bubble flow starts. In the decreasing regime occurring close to the top of the bed, the range of the regime is almost the same in all the experiments for \( d_1 \). However, there is only an increasing regime left when the grain size increases to \( d_2 \). In all cases, the values of \( L_{\delta X} \) for \( d_2 \) are less than the value of \( L_{\delta X} \) for the cases of \( d_1 \) and \( Q_1 \). Overall, there is no obvious evidence to show that \( L_{\delta X} \) increases/decreases with air injection rate, \( Q \).

Figures 3.26(d), (e) and (f) relate \( W_{\delta X} \) to the vertical coordinate \( y \). As one can see, \( W_{\delta X} \) increases continuously with \( y \). Naturally, \( dW_{\delta X}/dy \) represents the angle of the plume area affected by the air channel shifting. Based on the experimental results, the plume angle is about constant once the injection rate is fixed. It neither changes with \( y \) nor with the height of the bed (the maximum height of the bed in the experiments is 285 mm). \( dW_{\delta X}/dy \) slightly increases with the injection rates. This indicates that an increase in the injection rate enhances the transversal transport of air.

3.3.5 Onset of the instability of the packing

As described in Sec. 3.2, the onset of the grain mobilization can be characterized by \( \chi = Q/ak_w(H_b - H_{or}) \), which is a dimensionless quantity consisting of the air injection
rate $Q$, the thickness of the 2D model $a$, the effective height of packing $H_b - H_{or}$, and the initial liquid conductivity of the packing $k_w$. Generally, the value of $k_w$ increases with $d$ and $\phi$, but decreases with $\mu_w$. The measured $k_w$ for the experiments using glycerin-water solution are $4.46 \times 10^{-5}$ m/s and $1.41 \times 10^{-4}$ m/s respectively for the packings with $d_1$ and $d_2$.

The dimensionless quantity $\chi$ can be interpreted by introducing the dipole velocity (see a simple sketch in the insert of Fig. 3.27)

$$v_d = \frac{Q}{\pi a (H_b - H_{or})}$$

as a characteristic flow velocity of the liquid-saturated porous packing, and the critical velocity [6]

$$v_c = \frac{(\rho_{gr} - \rho_w)(1 - \phi)}{\rho_w} k_w.$$  \hspace{1cm} (3.21)

Therefore $\chi$ is proportional to the ratio of $v_d$ and $v_c$. The critical velocity $v_c$ is related to the critical uplift velocity of a patch of grains (density $\rho_{gr}$, porosity $\phi$) with liquid conductivity $k_w$ (density $\rho_w$). At the very beginning, when an instability of the matrix occurs, a patch of grains must be uplifted by the air flow. This means that if $v_c$ is less than $v_d$, the grain patch is uplifted.

Based on an averaged initial porosity of $0.48 \pm 0.01$ measured in our experiments, the results of $v_d/v_c$ are shown in Table 3.1. The resulting ratio suggests that instability always happens in our experiments. This is consistent with the observation of a transfer from a transitory fixed-bed branching process to a grain-displacing branching process. Yet, the transfer time depends on the liquid conductivity of the local grain patch around
Table 3.1: The value of $v_d/v_c$

<table>
<thead>
<tr>
<th>$(H_b - H_{or})$ (mm)</th>
<th>$Q$ (ml/min)</th>
<th>$d_1$</th>
<th>$d_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>90</td>
</tr>
<tr>
<td>140</td>
<td>7.59</td>
<td>34.15</td>
<td>56.91</td>
</tr>
<tr>
<td>200</td>
<td>5.31</td>
<td>23.90</td>
<td>39.84</td>
</tr>
<tr>
<td>265</td>
<td>4.00</td>
<td>18.04</td>
<td>30.07</td>
</tr>
</tbody>
</table>

3.3.6 Summary

In this section, a set of two-dimensional laboratory visualization experiments reveals a gas-flow instability in a porous medium saturated with a glycerine-water solution. The interaction of the injected air flow and the medium structure leads to mobilization of the medium and an instability, which causes the air channel to migrate. This differs from the situation of air injection in a fixed bed. The channel migration appears as a sequence of previous channels collapsing and new channels opening. This novel phenomenon is significant for the aeration of filters. The channel migration comes to a stop after some time, leaving one thin and stable channel.

Before breakthrough, the maximum vertical height of the air structure approximately advances linearly with time. Yet, the maximum horizontal advance reaches its maximum value and then levels off for the rest of the time. For air volumetric fraction, it decreases monotonically with time, and finally levels off asymptotically to an approximate constant. In all cases, the air volumetric fractions for packings of small grains are larger than those for packings of large grains. The ratio of total surface area to volume varies with time in a similar fashion as the air volumetric fraction. However, the ratio of total surface area to volume can clearly be grouped by the grain size, which is also true for specific surface area of the air phase. Both can be scaled with the Bond number with a power of -0.5.

After breakthrough, the migration process is studied by analyzing the mean horizontal position, horizontal shifting distance, lateral movement distance, and lateral movement...
width of air channel. The results indicate that over 99% of the horizontal shifting distance is less than 10 mm. Furthermore, the probability density function of the horizontal shifting distance indicates that the air channel oscillates more frequently in the packing of small grains than in the packing of large grains. Three regimes, an increasing regime, a plateau regime, and a decreasing regime, can be found in the lateral movement distance for the packings with grain size \( d_1 \). While there is only an increasing regime in the lateral movement distance for the packings with grain size \( d_2 \). Both in the packings with grain size \( d_1 \) and \( d_2 \), the lateral movement width increases monotonically with the vertical position \( y \).

The mobilization of the grains is driven by the air flow. Inside the mobile zone, an instability appears related to the competition between the burden (the weight of the grains) above the air bubble and the surface tension of the bubble. The mobilization of the grains, as well as the final stable air channel, will be discussed in the next section.

### 3.4 Compaction and size segregation

As described in the above section, the interaction of the air flow injected at the bottom with the grains and the liquid leads to a mobilization of the grains, in which air channels migrate and grain clusters undergo shearing. The channel migration comes to a stop after some time, leaving one thin and stable preferential channel for air flow. One can go further in the investigation of the transportation of the medium during and after the mobilization process. In this section, two important phenomena, compaction and size segregation, will be described.

#### 3.4.1 Experimental parameters

The porous matrix is constructed by crushed fused silica glass grains with a density of \( \rho_{gr} = 2202 \text{ kg/m}^3 \). Two sets of contrasting experiments with layered and fully mixed packings, were conducted. Five types of saturated packings were used: (I) primary grains of size of \( d_{1p} = 0.8 - 1.0 \text{ mm} \) and tracer grains of size of \( d_{1t} = 0.8 - 1.0 \text{ mm} \); (II) primary grains of size of \( d_{1p} = 0.8 - 1.0 \text{ mm} \) and tracer grains of size of \( d_{2t} = 0.4 - 0.6 \text{ mm} \); (III) primary grains of size of \( d_{1p} = 0.8 - 1.0 \text{ mm} \) and tracer grains of size of \( d_{3t} = 2.0 - 2.5 \text{ mm} \); (IV) primary grains of size of \( d_{2p} = 0.4 - 0.6 \text{ mm} \) and tracer grains of size of \( d_{1t} = 0.8 - 1.0 \text{ mm} \); (V) primary grains of size of \( d_{3p} = 1.6 - 2.0 \text{ mm} \) and tracer grains of size of \( d_{3t} = 2.0 - 2.5 \text{ mm} \). In the visualization of the experiments in
the following figures, tracer grains are shown as solid black dots, while primary grains are transparent due to the matching of their refractive index with the one of the fluid. In order to match the refractive index of primary grains whose refractive index is about 1.457, the defending wetting fluid used in the experiments is a 90%-10% by volume glycerine-water solution. The wetting fluid has a dynamic viscosity of \( \mu_w = 0.165 \text{ Pa} \cdot \text{s} \) and a density of \( \rho_w = 1235 \text{ kg/m}^3 \). Air is used as the invading non-wetting fluid. The corresponding parameters for air are \( \mu_{nw} = 1.83 \times 10^{-5} \text{ Pa} \cdot \text{s} \) and \( \rho_{nw} = 1.2 \text{ kg/m}^3 \). The surface tension between these two fluids is \( \sigma = 6.4 \times 10^{-2} \text{ N/m} \). All values refer to room temperature of 22 °C.

In all experiments, the matrix is constructed with an initial height of \( H_b = 235 \pm 3 \text{ mm} \). The supernatant fluid has a constant surface elevation at 400 mm above the base of the box. The saturated medium was prepared by stirring the grains in the glycerin-water solution slowly until the medium looks transparent. Thus one can easily verify that there is no visible trapped air in the medium, since air has a refractive index of 1. Before starting an experiment, the system was vibrated for a few minutes, guaranteeing that a loose porous matrix was randomly packed under gravity with a measured porosity of 0.48±0.01. The injection rate \( Q_{inj} \) of air flow was fixed at 90 ml/min in the experiments.

### 3.4.2 Process description and discussion

After air injection into the saturated porous matrix starts, branching channels of pure air flow are forming by displacing the grains. Before breakthrough, some air branches are widened considerably due to the high air pressure and grains are displaced by the movement of the air-liquid interfaces along these air branches. The branch width is ranging from pore size to around 10 mm during the widening. According to the Young-Laplace equation, the air-liquid interface curvature is large under high capillary pressure, and thus it can penetrate into the pore space between grains. Correspondingly, the experimental observations depict a rough air-liquid interface, and grains protrude from the interface, some even being directly exposed to the air. After breakthrough, air pressure is released. Due to the pressure decrease, along the interface an imbibition of the liquid phase into the air channel and a collapse of the air channel are observed. Via a lowering of capillary pressure the pressure decrease leads to a small curvature of the interface. A smooth interface is always observed after breakthrough, which means that the interface then stays within the channel. Furthermore, some sections of air branches become isolated cavities (shown in Fig. 3.28) due to the snap-off displacement of the grains and the imbibition of the liquid.
Figure 3.28: Superposition of air flow patterns at three time slices, 97 s (light blue), 269 s (purple), and 351 s (black) after the start of air injection. Image was taken from the experiment with $d_{1p} = 0.8 - 1.0 \text{ mm}$, $Q = 90 \text{ ml/min}$. Three arrows (with corresponding colors) indicate the places where a “locking” phenomenon occurs.

Under the influence of gravity the grains creep downward to the bottom of a cavity created during the initial branching stage. However, due to the resistant viscous force of the liquid, the coherence force (caused by the interstitial liquid between the adjacent grains) and the friction between the grains, the front of the grains stays behind the liquid. This leaves a much larger space than the initial pore space. In this case, air flow continues to move through this pathway, since air needs less energy to drain the liquid from a larger space in which the capillary pressure is smaller. In some cases, the previous flow pathway undergoes a complete shutdown due to the creeping movement of the grains. Thus the air flow in the previous pathway is blocked. This creates a growing air bubble around the blocking position which after a while is filled by a bulk of grains. In this way a new pathway is established. The evolving process of air pathway migration is shown in a superposition of three consecutive images in Fig. 3.28.

As the air is injected continuously, the succession of pressure accumulation and pressure release leads to forward and backward movement of the grains, causing a shearing deformation and a rearrangement of the grain packing inside the liquid phase. This is confirmed by two sets of experiments, one with a layered arrangement of the tracer grains, in the following called layered packing, and one with a fully mixed arrangement of tracer grains called fully mixed in the following packing. We first focus on the experiments with layered packing.

First of all, from experiments using packings (I) to (V), which are shown in Fig. 3.29 to
Figure 3.29: Experiment with packing (I), primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm (image width: 180 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing. The small solid spots are tracers, while the large hollow spots are air-liquid interfaces which are visualized due to optical effects and not due to tracer accumulations. It is the same for the following figures.

Fig. 3.33, respectively, we see that the layers of tracer grains are drawn down a certain distance towards the injection orifice in all experiments, no matter whether the tracers are larger than, smaller than, or the same as the primary grains. Moreover, this distance increases with the distance to the injection orifice. This indicates that the grain packing has been compacted during the air injection process. We even see that the maximum drawdown distance always occurs at positions around a small air channel. This means that the compaction of the packing is not widespread but very much localized around the air flow. If we compare the experiments using packings (I) to (V), the drawdown distance decreases with the diameter of the primary grain. This is consistent with our experience that a finer grain packing is more compactable, given the same construction method to prepare the packing in our experiments. Here “finer grain” refers to grains with sizes in the range of 0.4 mm to 2.5 mm.
Secondly, the layers of tracer grains are breached. The breach of tracer grains increases with the distance to the injection orifice. However, from different experiments using packings (I) to (V), the patterns of breach can be divided into two types. For packing (I) where the tracer grains are the same size as the primary grains of the medium, packing (II) where the tracer grains are much smaller, and packing (V) where the tracer grains are coarser, the patterns of the breaches are more or less the same, forming a convex curve (shown in Fig. 3.34 as type I). This means that the tracer grains are mostly moving in a reverse direction relative to the air flow. In these experiments the layers of tracer grains stretch downwards around the breach. Moreover, the depth of the cone increases with the distance to the injection orifice. We conclude that the counter motion of the tracers is due to the compaction of the packing. In contrast, for packings (III) and (IV) where the tracer grains are coarser than the primary grains of the medium, the pattern of the breach is formed by first a convex curve and then a concave curve (shown in Fig. 3.34 as type II). Inside the breach, the tracer grains seem to adhere to
3.4. Compaction and size segregation

Figure 3.31: Experiment with packing (III), primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{3t} = 2.0 - 2.5$ mm (image width: 175 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing.

the air flow, so that some of them are moved to the top of the bed. We observe that the layers of tracer grains stretch upward around the breach, so that they form a contour of an upward facing cone. Furthermore, the height of the cone increases with the distance to the injection orifice. However, the height of the cone decreases sharply with the size of the primary grains. Finally, a preferential pathway of a thin air flow channel is established inside the breach.

We now discuss the experiments in a fully mixed packing. Figures 3.29(c) and (d) indicate the compaction of the packing after the air injection, since the distribution of the tracers is denser in Fig. 3.29(d). As a result, there is a drawdown distance at the top of the packing. As the tracers are relatively small grains as shown in Fig. 3.29, the quality of the image is less good than in the case of larger tracer grains because each tracer is roughly covered by one to two pixels only. Nevertheless, we clearly observe a crescent-shaped zone at the downstream end of the air flow where the tracers are less dense than in other areas. This means the majority of grains of this zone are
Chapter 3. Air injection in two-dimensional experiments

Figure 3.32: Experiment with packing (IV), primary grains with size $d_{2p} = 0.4 - 0.6$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm (image width: 182 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing.

the primary grains, i.e., the coarser grains. This tells us that a certain degree of size segregation was achieved inside this zone. This phenomenon was further proven by the experiments shown in Fig. 3.31 and Fig. 3.32, where tracer grains accumulate at the downstream end of the air flow, forming a crescent-shaped zone. Above this zone, there are quite a few tracer grains suspended around an air bubble. Below this zone, there is a concave belt with almost no tracer grains. This again indicates that the coarser grains (tracer grains), rather than the finer grains, tend to rise up to the top of the packing during the air injection. However, tracer grains of packing (V) do not show such a tendency, as can be seen in Fig. 3.33. We only observe that there is a concave belt at the top of the packing, containing almost no tracer grains. We can therefore conclude that in this experiment only the compaction occurred.

Accordingly, we note that: (a) in packings (III) and (IV) the relatively coarse grains accumulate at the top of the packing, following the direction of the air flow; (b) in packings (II) the relatively fine grains tend to move in the inverse direction of the air flow; (c) in
3.4. Compaction and size segregation

Figure 3.33: Experiment with packing (V), primary grains with size $d_{3p} = 1.6 - 2.0$ mm and tracer grains with size $d_{3t} = 2.0 - 2.5$ mm (image width: 184 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing.

Figure 3.34: Schematic shapes of a tracer grain layer observed in two types of patterns after the air injection. Type I: a convex curve; Type II: from a convex curve to a concave curve. Here the arrows indicate the direction of movement of the curve.
packings (I) and (V) no obvious motion tendency of tracers was observed. Now, we can understand the transition mechanism of the pattern change shown in Fig. 3.34. As we explained above, the convex curve is a result of the compaction. Without an upward motion of the tracers (e.g., in the packings (I), (II), and (V)), the pattern of type I forms. With an upward motion of the tracers (e.g., in the packings (III) and (IV)) against the downward motion in the compaction, an additional concave curve is formed besides the convex curve.

The segregation degree is quantified by a measurement of locally averaged grain size, which is shown in Fig. 3.35. Under the assumption of an ideal 2D experiment, the measurement is performed by calculating the averaged grain size of a small block (7 mm × 7 mm) over the whole observed field. The averaged grain size is $d_{av} = \beta d_{1t} + (1 - \beta)d_{2p}$, where $\beta$ is the fraction of tracer grains. The concave belt at the top of the packing holding a large fraction of tracer grains indicates that the relatively coarser grains (tracer grains) tend to rise up to the top of the packing after the air injection. Nevertheless, there is no direct evidence that elutriation due to the air flow plays an important role in the uplift of the grains. The experiment shown in Fig. 3.35 is used as an example. At the centre of the observed field, initially there is an area mostly containing relatively finer grains. If the elutriation due to the air flow played an important role, this area would have been uplifted by the end of the experiment. However, this area is pushed downward and compacted instead. The nature of the size segregation in our experiments is a “void-filling” mechanism [85] which will be explained in the following.

As we described above, the grains were pushed aside and a disturbed zone was created during the air injection process. With sufficiently high spatial resolution (about 0.3 mm/pixel) of the location of the tracers, this zone can be found using image analysis. By subtracting pixel-by-pixel, the image at a given time from an image taken before this

Figure 3.35: A rough estimate of locally averaged grain size of the experiment with a fully mixed packing with primary grain size $d_{2p} = 0.4 - 0.6$ mm and tracer grain size $d_{1t} = 0.8 - 1.0$ mm (image width: 275 mm). Left: initial status. Right: final status.
3.4. Compaction and size segregation

Figure 3.36: Experiment with packing (l), primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm in a fully mixed packing. Left: A snap shot at $t = 180.5$ s (image width: 320 mm). The black line is the lower boundary of the disturbed zone. Right: The lower boundary of the disturbed zone changes with time. Here the zero point of the coordinate is the position of the orifice.

Given time, a well-defined speckled region can be detected. An example of the lower boundary of this region is inserted into the image from the experiment in Fig. 3.36. The disturbed zone is more or less a reversed triangle with its lowest point around this “locking” position, a region (shown by arrows in Fig. 3.28) between a growing bubble inside this disturbed zone and a small air path below this disturbed zone. In the long run, this disturbed zone shrinks and stays finally on the top of the packing. However, in the short term, because of the air release events of air flow at the top of the packing, this zone can grow. Using a segmentation technique, we can separately calculate the area of air channel $S_c$ and the area of the disturbed zone $S_z$. The area of air channel $S_c$ is defined as the total area of isolated bubbles, air branches, and air channel in an instantaneous snapshot at a given time. Here the isolated bubbles are included because their rise also leads to the change of pore space geometry. The area of the disturbed zone $S_z$ is defined as the total area from the lower boundary of the moved zone to the top boundary of the packing. Both $S_c$ and $S_z$ are shown in Fig. 3.36. Obviously, $S_c$ will reach a local maximum before a release event, and a local minimum after a release event. This means a highly fluctuating profile will appear for $S_c$, which is shown in Fig. 3.37. Correspondingly, $S_z$ behaves similarly. We call this effect pulsation. This pulsation acts like a shaking applied to the disturbed zone. Thus the pulsation leads to a process of expansion/contraction of the packing size, which is associated with increment/decrement of $S_z$. Within this process, the pore space geometry changes due to the deformation of the packing, and shearing motion between the grains is occurring within this zone. Furthermore, the experimental observations show that except for some grains exposed at the air-liquid interface there are almost no grains inside the air channel, and the area
of the disturbed zone, $S_z$, is at least an order of magnitude larger than the area of the air channel, $S_c$, as shown in Fig. 3.37. Therefore, we can assume that the shearing motion of the grains and the size segregation only occur in the disturbed zone where the grains are saturated with the homogeneous liquid phase. The expansion creates voids for smaller grains to creep downward into the spaces beneath larger grains. This is the so-called “void-filling” mechanism for size segregation appearing in packings (III) and (IV). However, as the grain size increases, both the strength and the duration of the pulsation decrease sharply. As a result, the size of the crescent-shaped zone (in which the coarser grains accumulate) decreases with the grain size, and we finally did not observe size segregation in packing (V).

At the same time, slowly shearing motion due to the pulsation causes a change in the packing during the air injection process until grains find their place and are locked by an arch or micro-structure inside a force chain within the packing [79]. Consequently, a rearrangement of the packing is achieved after the process. As a result of the re-arrangement, the packing is locally compacted. As we noted above, after the air injection operation the top surface of the packing shows a V-shape depression in each experiment. If we compare the total volume of the packing before and after the experiment, a rough calculation shows that during the experiments the volume of the packing decreases by 4% to 8%, with the smaller injection rate of air flow generally causing a larger volume decrease. In most cases, a fine-grain packing exhibits a larger volume decrease. Given an averaged initial porosity of $0.48 \pm 0.01$ measured in our experiments, the resulting porosity slightly decreases to $0.435 \sim 0.446$. A way to quantify the local compaction of the packing is to roughly estimate the porosity change from the

![Figure 3.37: Left: The area of disturbed zone, $S_z$, is the area from the lower boundary of the disturbed zone up to the top. (Inset–the area of air channel, $S_c$, versus time.) Right: The normalized $S_z/S_z(0)$ as a function of dimensionless time $t/t^*$. The black continuous curve is a fit of the exponential law. (Inset–Both $S_z(0)$ and $t^*$ are linearly fitted as a function of the grain size.)](image-url)
3.4. Compaction and size segregation

Figure 3.38: Experiment of layered packing of primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm. Left: initial position of tracer lines (dashed line) and post-position of tracer lines (solid line). Right: estimate of the porosity change from the displacement between the tracer line before and after the experiment.

Empirical and theoretical experience with grain packing shows that there is a minimum value $\phi_m$ of porosity $\phi$:

$$0 < \phi_m \leq \phi \leq 1.$$  \hfill (3.22)

Experiments [28] on shearing motions also show that there is another value of $\phi$, the critical porosity $\phi_c$, below which shearing of the packing will cause expansion (Reynolds' dilatancy), and above which it will cause compaction [70]. Our experimental results indicate that $\phi_c$ will be less than 0.48 in our grain packing, since the packing has been compacted. As the grains come closer together, they become more closely interlocked, so that it is getting more difficult for them to move with respect to one another. This is called a “locking” phenomenon [70]. We can see that in the state where the minimum porosity has been approached, the grains are so closely interlocked that the packing is rigid. Passman et al. [70] also observe that grains tend to accumulate in the region of low shear rate. A theoretical description is given by the cluster model of Gavrilov et al. [35], which sees the dynamics of granular compaction as a competition between various clusters and a random environment inside the granular system. The cluster model explains the slow growth of the compacted region under vibration with the fragmentation of clusters and reassociation of the individual grains with surrounding clusters. Although the work of both authors only concerns grain packings under one-phase flow, it is to a
large degree applicable to our experiments as the grain-grain interaction in the dense packing itself plays the major role. Moreover, as we mentioned above, most of the grain-grain interactions can be assumed to occur within the homogeneous liquid phase. Only some grains are directly attached to the interface during the expansion, and according to the experimental observation the penetration of the interface between the grains is about one to two grains deep. Compared to grain-grain interactions occurring within the homogeneous liquid phase with an area of $S_z$ which is at least an order of magnitude larger than $S_{cv}$, it is reasonable to ignore cohesion effects or meniscus pinning due to surface tension.

In the experiments, the pulsation of air flow causes vibrations of the granular system. However, there are apparently two different regions in the system (shown in Fig. 3.36): the region of non-zero shear rate (the disturbed zone) and the region of zero shear rate (outside of the disturbed zone). Consequently, clusters inside the region of non-zero shear rate are fragmented and individual grains accumulate in the region of zero shear rate, reassociating with the surrounding immobile cluster. As the process progresses, this cluster grows from bottom to top, while the region of non-zero shear rate shrinks to zero. As it is shown in Fig. 3.37, $S_z(0)$ is defined as the mean $S_z$ at time zero, and $t^*$ is empirically defined as the characteristic time of the compaction process. Both $S_z(0)$ and $t^*$ are found to decrease linearly with the grain size. After $S_z$ and time $t$ were scaled by $S_z(0)$ and $t^*$ respectively, the experimental data collapsed to a line. We can see that the experimental data are well fitted by an exponential law,

$$\frac{S_z}{S_z(0)} = \exp\left(-\frac{t}{t^*}\right)^\alpha,$$

where $\alpha = 1.8$ is the stretching of the exponential.

During the process of compaction, the necessary local stability condition for the air flow developing into a preferential air channel below the “locking” position is,

$$\frac{128Q_{inj}\mu_{nw}}{\pi D_{ch}^4} = (\rho_w - \rho_{nw})g,$$  \hspace{1cm} (3.23)

assuming that the pressure gradient of the air flow along the air channel (left hand side of Eq. 3.23) equals the hydrostatic pressure gradient of the liquid (right hand side of Eq. 3.23). $D_{ch}$ is the critical diameter of the preferential air channel on the basis of Hagen-Poiseuille’s law. Only when the diameter of the preferential channel is larger than $D_{ch}$ will the mechanical equilibrium at the channel interface be maintained. This can be interpreted by the capillary instability at the channel interface. The capillary pressure at the channel throat increases with decreasing $D_{ch}$. Therefore, decreasing the channel diameter below $D_{ch}$ leads to collapse. It means $D_{ch}$ is the minimum necessary diameter for the preferential channel to form.

According to Eq. 3.23, the critical diameter of the preferential air channel in our experi-
3.4. Compaction and size segregation

ments is 0.6 mm for an air injection rate of 90 ml/min. However, the preferential pathway in our experiments is not an ideal pipe with uniform diameter. The measured values of $D_{ch}$ are in the range of 0.6 – 1.3 mm, 0.8 – 2.0 mm, and 1.2 – 2.4 mm, respectively for the packings with $d_{2p} = 0.4 – 0.6$ mm, $d_{2p} = 0.8 – 1.0$ mm, and $d_{3p} = 1.6 – 2.0$ mm. The results from the experiments show that the value of $D_{ch}$ increases slightly with the size of primary grains. Nevertheless, the results indicate a stable preferential pathway can develop, since the values of $D_{ch}$ in the experiments are larger than the critical values. Furthermore, the pore space of the rigid packing around the preferential pathways can be estimated to have a width equal to one typical grain diameter. It is obvious that air chooses $D_{ch}$ to reduce the energy dissipation caused by the entry pressure. Therefore, the preferential pathway for air flow is sustainable. One might ask whether there is a maximum size of the preferential channel under the conditions of our experiments. Based on the observed diameter of the channel, the maximum values are 1.3 mm, 2.0 mm, and 2.4 mm, respectively for the packings with $d_{2p} = 0.4 – 0.6$ mm, $d_{1p} = 0.8 – 1.0$ mm, and $d_{3p} = 1.6 – 2.0$ mm. If the channel has the chance to dilate, i.e. the grain-liquid mixture is pushed aside by the channel interface, the cross-sectional mean velocity of air flow inside the channel decreases sharply with the channel size. As a result, we can assume that a quasi-static equilibrium has been established. We designate the maximum size of air channel by $D_{max}$. Due to the difference in the densities of air and grain-liquid mixture, a buoyancy force is present and acts as a destabilizing force, which can be characterized by:

$$F_b = \frac{1}{6} \pi (\rho_{mix} - \rho_{nw}) g D_{max}^3,$$  \hspace{1cm} (3.24)

where $\rho_{mix} = (1 - \phi) \rho_{gr} + \phi \rho_{w}$ is the density of the grain-liquid mixture. Here $\phi$ is estimated as 0.48. In contrast, the surface tension force is always stabilizing and can be characterized by:

$$F_s = \pi \sigma D_{max}.$$  \hspace{1cm} (3.25)

The air channel enlarges continuously until the capillary instability occurs, tentatively expressed by:

$$F_s - F_b = 0,$$  \hspace{1cm} (3.26)

which yields a threshold diameter $D_{max} \approx 4.7$ mm. It means when the channel size is larger than $D_{max}$, the capillary instability causes a collapse or a split of the air channel. Compared to the experimental results, this estimate looks promising.
3.4.3 Summary

A rearrangement of the grains is observed which is caused by a pulsation effect. After applying the shearing motion of the packing induced by the pulsation effect, the packing is divided into two regions, a region of non-zero shear rate and a region of zero shear rate. The individual grains are disassembled from the region of non-zero shear rate and then reassembled into the compacted clusters of the region of zero shear rate. Consequently a compaction of the packing was observed. The compaction process was described via the decaying process of the area of the region of non-zero shear rate, which obeys an exponential law. Both the area of the region of non-zero shear rate at time zero and the characteristic time were found to be linearly dependent on the grain size. It needs larger energy to displace the compacted clusters of grains, since the so-called effective viscosity [70] of these clusters increases dramatically as the patch porosity approaches the minimum. As a result, a “locking” phenomenon occurs at the outer boundary of the region of non-zero shear rate, which is visualized by a stabilizing process of a preferential air channel.

Our experimental studies also demonstrate a size segregation process. Pulsation of the packing induces a process similar to shaking. This process helps smaller grains move into the spaces beneath larger grains. As it progresses, larger grains rise up to the top and segregation occurs.

Meanwhile, the local stabilizing process helps to develop a preferential air channel. Assuming Hagen-Poiseuille’s formula to be applicable, the size of the preferential channel should exceed a lower threshold $D_{ch}$ so that a mechanical equilibrium at the channel interface is maintained. However, in order to sustain a stable air channel, the channel size should not exceed an upper threshold $D_{max}$. 
Chapter 4

Air injection in three-dimensional experiments

As I described in the literature review in the last paragraph in section 2.2, very few well-controlled 3D experiments have been performed in drainage displacement. In this chapter, I am going to demonstrate air displacement inside an unconsolidated granular packing placed randomly in a 200 mm (width) × 200 mm (depth) × 300 mm (height) glass container, in which air is injected at the center of the 3D box’ bottom via an orifice. In the last Chapter (Chapter 3), it was shown that the refractive index matching technique has its advantage in permitting the visualization with visible light. By refining this technique and the measurement resolution in the current method, the contrast between air and the background (the 3D matrix) is so strong that direct imaging can be applied. The visualization principle of the air phase is light reflection at the irregular interface of the air bubble.

In order to quantitatively analyze the displacement patterns, a 3D data set must be extracted. This problem can be solved by subdividing the observation volume into a sequence of 2D slices, which then is converted into a 3D volume data set. This scanning concept is commonly used in tomography, and is also well-known in the whole-field velocity measurements [11,12,44]. The important parameters are slice thickness, maximum scan volume, overall achievable volume scan rate with a given camera, and slice location.
4.1 Experimental setup

4.1.1 Materials

The experimental principle here is based on the use of granular material and wetting liquid which have the same refractive index, making the system transparent. The porous matrix is constructed by crushed fused silica glass grains with a density of $\rho_s = 2202 \text{ kg/m}^3$. The material is separated into two fractions with grain sizes of $d_1 = 0.6\text{-}0.8 \text{ mm}$ and $d_2 = 1.0\text{-}1.6 \text{ mm}$ in the current study. The refractive index of the fused silica glass is $n = 1.457$ for a blue-light laser ($\lambda \simeq 490 \text{ nm}$). In order to match the refractive index of the grains, the defending wetting fluid used in the experiments is a 90%-10% by volume glycerine-water solution. The wetting fluid has a dynamic viscosity of $\mu_l = 0.165 \text{ Pa}\cdot\text{s}$ and a density of $\rho_l = 1235 \text{ kg/m}^3$ at room temperature. Air is used as the invading non-wetting fluid. The corresponding parameters for air are $\mu_a = 1.83 \times 10^{-5} \text{ Pa}\cdot\text{s}$ and $\rho_a = 1.2 \text{ kg/m}^3$. The surface tension between these two fluids is $\sigma = 6.4 \times 10^{-2} \text{ N/m}$.

The assemblies of liquid-saturated grain packings were randomly placed inside a 3D glass container of the size 200 mm (width) $\times$ 200 mm (depth) $\times$ 300 mm (height). The global porosity was measured to be on average of $\phi = 0.48$. A 0.3 mm orifice installed through the central point of the base serves as the injection point of the air flow $Q$. The orifice here was installed to diminish the pressure fluctuation produced by the fluctuating instability of bubbles forming and breaking at the orifice. This technique was used to constrain the feedback of the air flow in the system on the injection. Again the flow rate is measured using a GSM-A5TA-BN00 flow-meter.

In all experiments, the matrix is constructed with an initial height of $H_b = 119 \pm 2 \text{ mm}$. The supernatant fluid has a constant surface elevation at $H_w = 195 \text{ mm}$ above the base of the box. Two groups ($d_1$ and $d_2$) of experiments with the different injection rates of air flow, ranging from 15 ml/min to 300 ml/min, were performed in the current study. Before starting an experiment, the system was vibrated for a few minutes, guaranteeing that a random porous matrix was packed homogeneously under gravity.

4.1.2 Apparatus

In order to visualize the distribution of the air phase, a recording system with a high-speed camera and a laser scanning illumination system are employed. Figure 4.1 illustrates the current experimental setup for the 3D system. The light source for illumination is a continuous 20 W argon ion laser. The laser beam has a wavelength of $\lambda \simeq 490 \text{ nm}$
4.1. Experimental setup

Figure 4.1: Left: Hardware configuration for 3D experiments in the laboratory. Right: A sketch of the 3D experimental setup. The laser beam is expanded and thinned to a light sheet, then scanned through the observation volume using an eight-face prism. The coordinate system $x$-$y$-$z$ of the experiments is also shown.

(after passing a blue dichroic filter produced by Edmund Industrial Optics) and a diameter of around 2.0 mm. A series of lenses comprising a cylindrical plano-convex lens, a plano-concave, and then again a cylindrical plano-convex lens spreads the beam to a sheet of a “static” thickness of about 0.5 mm. Following this set of lenses, the vertical light sheet passes through an octagonal plexiglas cylinder (refractive index $n_c = 1.49$) composed of eight prisms. This cylinder can rotate at a set speed around its axis while the light sheet passes through the parallel facing planes of the cylinder. The octagonal Plexiglas cylinder produces a continuous scanning motion of the light sheet. It is flexible with respect to the thickness of the light sheet since the control of the thickness is accomplished through the camera timing. However, the maximum sweep of the light sheet is determined by the geometry of the octagonal cylinder (as shown in Fig. 4.2). The octagonal cylinder axis is mounted vertically and aligned carefully with the incom-

Figure 4.2: Sketch of the top view of the experimental setup. Here $\theta$ is the angle of incidence, and $z$ is the coordinate parallel to the scanning direction and in line with the symmetry axis of the camera arrangement. $D$ is the distance between two parallel faces of the octagon, i.e., the diameter of the inscribed circle of the octagonal cylinder).
ing light sheet. By its rotation, the angle of incidence, $\theta$, of the incoming light sheet varies with $\dot{\theta} = 2\pi f_v$ rad/s and is limited by $-\pi/8$ to $\pi/8$ due to the octagonal shape of the cylinder, where $f_v$ is the volume scan frequency. Each time the limiting angle is passed, the incident light sheet enters through the next face of the octagon, starting at a new angle shifted by $\pi/4$. For each eighth of a rotation of the octagonal cylinder, the $z$-coordinate of the exiting light sheet shifts from -24 mm to 24 mm, where $z$ is the coordinate along the scanning direction, coinciding with the symmetry axis of the camera arrangement (as shown in Fig. 4.2). The relation of $z$ to the other geometric parameters

![Diagram](image)

is shown in Fig. 4.3, which gives $z = (l^* - l) \cos \theta$, where $l^* = D \tan \theta$, $l = D \tan \theta_c$, and $n_c \sin \theta_c = n_a \sin \theta$ (Snell’s Law), yielding the parallel shift,

$$z = D \sin \theta \left[ 1 - \frac{\cos \theta}{\sqrt{(\frac{n_c}{n_a})^2 - \sin^2 \theta}} \right], \quad (4.1)$$

where $D=175$ mm is the inner circle diameter of the octagonal cylinder. The parallel shift $z$ is used in the determination of the slice position in later image processing (3D reconstruction). As $\theta$ varies from $-\pi/8$ to $\pi/8$, $z$ is approximately a linear function of $\theta$ (as shown in Fig. 4.4). Thus the slice position is assumed to vary linearly with $\theta$ in later image processing.

Through this scanning technique, the observation volume is subdivided into a series of slices. The slices were recorded by a high-speed digital camera (Photron Ultima APX)
4.1. Experimental setup

which is capable to achieve a maximum frame rate of 2 kHz at a full resolution of 1,024 × 1,024 pixels. With a built-in physical memory of 6 GB, the system is able to record a time series of 6,000 images. A Nikkor Micro 60 mm lens with an aperture of f/32 allows a good focus over the entire scanning depth of 50 mm. The recording frame rate \( f_r \) is related to the volume scanning rate \( f_v \) by the number of slices \( n_s \):

\[
f_r \geq n_s f_v. \tag{4.2}
\]

This means that, for a given \( f_r \), one can either choose small \( n_s \) and large \( f_v \) or large \( n_s \) and small \( f_v \), thereby increasing the spatial resolution by lowering the time resolution or vice versa. The exposure time is set to be the inverse of the frame rate. Therefore, the camera always integrates over the exposure time along the path of the moving light sheet. This means that the representative thickness of one single slice depends on the “static” thickness of the light sheet and the “dynamic” thickness, i.e., sweep distance during the exposure time. Figure 4.5 shows that the sweep distance \( \Delta z \) can be calculated as \( \Delta z = v/f_r \), where \( v \) is the scanning velocity. The scanning velocity in the experiments is around 50 cm/s with a volume scan rate of \( f_v = 10 \) Hz. From Fig. 4.5,
it is shown that between two adjacent slices there is an overlap distance of \( \delta z_p \) which is exactly the “static” thickness of the laser sheet.

### 4.1.3 Illumination principle

By refining the index matching technique and measurement resolution in our current method, the contrast between air (refractive index \( n = 1.0 \)) and the background (refractive index \( n = 1.457 \)) is so strong that the interface between air phase and the medium reflects enough light to visualize the invading structure. During most of the experiments, the structure invades the pores between the grains, thus the interface is rough. As a result, the reflection from the interface is diffuse, making the whole interface visible in the camera. Since we have an unconsolidated packing, at the later stage of the experiment, the grains are pushed aside by the air bubbles, and therefore there is a sharp transition to a behavior that is similar to a bubble in a liquid. In such a case, the interface is rather smooth, making the reflection specular. Therefore only parts of the interface are visible [42], and the performance of our method decreases, as will be discussed in the section below.

![Figure 4.6: An image from a single slice shows that a systematic attenuation of the measured grayvalue intensity in \( x \)-direction is not detected by the camera in the current configuration.](image)

In \( x \)-direction, the light intensity is approximately uniform when the laser sheet passes through the saturated medium. However, as the laser sheet crosses the injected air plume, light is reflected and refracted at the plume surface and one would therefore expect that the light intensity in the sheet should somewhat attenuate in \( x \)-direction. In
preliminary experiments we did not detect a systematic attenuation of the measured grayvalue intensity in \( x \)-direction even when the local number density of air bubbles was high, as it is shown in Fig. 4.6. This is also confirmed in the coming results that the reconstructed plume structures present a well balanced shape, i.e. without loss of information in \( x \)-direction.

In the present setup, with a fixed recording frame rate of \( f_r = 500 \) Hz, each volume scan consists of a maximum of 50 illuminated light sheets. Therefore, we have a total thickness of one single slice of \( \Delta z + \delta z_p = 1.5 \) mm and an overlap distance of \( \delta z_p = 0.5 \) mm between two adjacent slices. Considering that 50 slices are evenly placed over a distance of 50 mm (Fig. 4.4), the spacing between the slices is 1 mm. In the present configuration the observed volume has dimensions of about \( 85 \times 120 \times 50 \) mm³.

### 4.2 Data set extraction and reconstruction

Before coming to the dynamic morphologic description and measurement, a proper method for the data extraction and reconstruction must be developed. The process and results of the data extraction and reconstruction are demonstrated in detail through two transient experiments, which were carried out with air continuously injected into the medium at two different and approximately constant injection rates, 16.4 ml/min and 41.1 ml/min. Before coming to these results, a preliminary measurement was obtained from a static state, where the bubbles were stably trapped in the porous medium. The static state was obtained by stopping the injection after breakthrough and keeping the system at rest for one day. Thereafter the ‘frozen’ structure of the entrapped air was measured via the scanning method and visualized. The left-hand-side of Fig. 4.7 shows a photograph of the structure using a normal digital camera at room illumination. For comparison, a 3D reconstruction is shown on the right. For this reconstruction, the grayvalue images were first transformed into binary format, i.e. all pixels with a non-zero grayvalue are set to 1 and then visualized using the isosurface function in Matlab. From the figure we note that on the qualitative level all the important features of the structure can be satisfactorily reproduced without any further processing.

However, the structures appear unnaturally elongated in \( z \)-direction. As an illustrative example, in Fig. 4.8 we depict a sequence of grayvalue images (top) of a small air bubble with a diameter of less than 1 mm and associated contour slices (bottom). It can be seen that the maximum exposure is realized in slice nr. 4 while with smaller grayvalues the image of the bubble is still well visible in two neighboring slices on each side. Ideally,
Figure 4.7: Morphological consistency is verified in the case of discrete static bubbles. Left: 2D image of $y$-$x$ panel using a normal digital camera. Right: 3D reconstruction based on the raw images.

Figure 4.8: Sequence of images of a bubble with a diameter smaller than 1 mm. Top: raw gray images (12 pixels $\times$ 13 pixels with a resolution of about 0.1 mm/pixel) sequentially registering the bubble. Bottom: contours of the slices, where the distance between the center of two neighboring slices is 1 mm.

This bubble should only be visible in slice nr. 4. As mentioned in the previous section, the light sheets overlap with each other, and this leads to the redundant bubble images over several consecutive slices (Fig. 4.8). Due to the finite thickness of the light sheet, the overlap effect leads to redundant exposures in at most two neighboring slices on each side, which is the exact case shown in Fig. 4.8. It turned out that this is not the only effect causing the unnatural elongation, hereinafter referred to as multiple exposure. There is a second effect, namely, air bubbles which are located out of plane of the slice can be illuminated indirectly. This indirect illumination comes from light reflected by neighboring air bubbles which are themselves directly exposed to the light sheet. This
effect appears when the local number density of air bubbles is high. It can sometimes lead to structures that appear extremely elongated in the scanning direction. Typically, the grayvalue intensity of the bubble image due to indirect illumination is much lower than the consecutive exposures caused by the direct one, respectively. Precisely this feature will be used for the removal of the redundant exposures, as will be described in more detail below.

Due to multiply imaged bubbles, any quantitative measure like the volume or the shape of the structure in slice direction will be biased. Thus, an important pre-step in determining the invading structures is the choice of the methods used to identify and reconstruct their morphology properly. These methods are then verified by comparing the calculated reconstructed air volume with the injected one. In the following we present and test four methods on the data obtained from the two experiments: a level-based method where a threshold on the grayvalues intensity is used, an erosion method, a “two-step” method, and a “curvature” method based on the curvature of the grayvalue profile in the scanning direction.

From Fig. 4.8 we learn that a small threshold on the grayvalue intensity will help to remove the redundant pixels, while the main bubble image in slice nr. 4 will not change very much. This level-based method, i.e. based on a threshold on the grayvalue intensity, is well known in image processing. In order to proceed, a reasonable choice of the threshold value has to be made, keeping in mind that the experiments are transient. This choice might be based on a statistical measure such as the average grayvalue of the image. Figure 4.9 shows the variation of the average grayvalue, \( \langle \text{grayvalue} \rangle \), in time for the two dynamic experiments, where the angle brackets denote the average of all non-zero grayvalues in space, i.e. averaged both over the image plane and over 50 images belonging to a given volume scan. From Fig. 4.9 we note that \( \langle \text{grayvalue} \rangle \) varies in time and therefore we apply a time dependent threshold. We can also see that there are some variations between the experiments. A closer inspection revealed that this is due to a higher proportion of saturated pixels, i.e. pixels with a maximum grayvalue of 255. This is presumably due to the fact that the laser intensity also has an approximately Gaussian distribution in y-direction. As the plume structure grows in time more bubbles are exposed to higher light intensity.

The choice of the threshold is validated by comparing the injected volume with the one measured from the reconstructed plume structure after thresholding the images. The volume is calculated from the plume structure obtained as described above, with the addition that in a pre-step all pixels smaller than the threshold are set to zero. The result is shown in Fig. 4.10 for the two experiments. We observe that the volume
Figure 4.9: Averaged grayvalue (termed $\langle \text{grayvalue} \rangle$) in time for the two experiments. Inset: the recorded injection rates in time.

Figure 4.10: Comparison of the injected air volume (continuous line) with the volume calculated from the reconstructed plume structure over time using the level-based method, where the symbols refer to the selected threshold of $1.25 \langle \text{grayvalue} \rangle$ and the shaded columns represent a variation of the threshold by $\pm 0.25 \langle \text{grayvalue} \rangle$.

increases monotonically in time until a maximum is reached and thereafter it decreases to a plateau. One notes that the main trend of the initial increase is in agreement with the profile of the injected volume. The injected volume is obtained through integration of the recorded flow rates over time. Also the quantitative agreement between the two curves is generally satisfactory, with somewhat higher deviations notably in the early time of the experiment with the injection rate of 41.1 ml/min. The comparison is valid during the period when the structure remains within the limits of the field of view until breakthrough. For the two experiments, breakthrough occurs at $t = 7.5$ s and $t = 5.4$ s, respectively for $Q=16.4$ ml/min and 41.1 ml/min. The first relative maximum of the curve in Figure 4.10(bottom) roughly corresponds to the breakthrough time, whereas in the
4.2. Data set extraction and reconstruction

upper panel the second relative maximum of the curve corresponds to breakthrough. This means between the two relative maxima of Exp.1 volume is lost. This can be attributed to illumination in the sense that the interface of the plume in the proximity of the top of the packing became smooth and is therefore only partly visible. For Exp.2, there is a similar qualitative change of the interface. However, it occurs just at the breakthrough and is therefore not noticeable in the figure. Now we analyze how this method modifies the shape of the structure.

Figure 4.11 depicts the reconstructions of the experiments with $Q = 16.4$ and 41.1 ml/min for a given time and from two different views, namely the front and side views. The raw data are shown in Fig. 4.11(a,a') and the data after applying the level-based method on grayvalue intensity are shown in Fig. 4.11(b,b'). Apparently, the main features of the shape are conserved, even if many small clusters are erased. However, some elongated streaks are still clearly visible on the side view (Fig. 4.11(b')). It appears that the problem of multiple exposure is difficult to solve perfectly by using the level-based method on grayvalue intensity alone.

Another well-known image processing method is the so-called image erosion. This method takes the shape of objects into account only, irrespective of their grayvalue. Specifically, the procedure removes pixels on object boundaries of the structure. The number of pixels removed from the objects depends on the size and shape of a structuring element used to process the data. The structuring element is a matrix, which in general consists of 0's and 1's and that can have any arbitrary shape and size. We considered the simplest form of the matrix, which is a square when a 2D image is considered or a cubic matrix for our 3D structure and the matrix consists of 1's only. The side-length of the structuring element is the free parameter to be specified. We applied the erosion procedure to the raw structure (Fig. 4.11(a,a')). Figures 4.11(c,c') show the front and the side views of a snapshot of the eroded plume structure, where the side-length of the structuring element is 5. In order to test the sensitivity of the result on this parameter, elements of sizes of 3 and 7 were also tried, and the corresponding volume calculations are represented by the shading in Fig. 4.12. With the former choice, almost no effect on the plume was discernable and with the latter choice nearly the whole structure was erased. A choice of a side-length of 5 also makes sense if we recall the Fig. 4.8, where it is visible that the object is multiply imaged on two neighboring slices on either side of the object. From the side-view (Fig. 4.11(c')) we see that, compared to the level-based method (Fig. 4.11(b')), the elements of the structure are much less elongated and therefore the method seems to be more effective in correcting for the elongated streaks.
Figure 4.11: 3D reconstruction of the plume structure for the experiments with injection rate $Q = 16.4$ ml/min (upper panel) $Q=41.1$ ml/min (lower panel) at time $t = 4$ s, where the sub-upper and sub-lower panels show the front ($Y$-$X$) and side ($Y$-$Z$) view, respectively: raw data (a,a’), level-based method (b,b’), erosion method (c,c’), “two-step” method (d,d’), and “curvature” method (e,e’).
4.2. Data set extraction and reconstruction

Figure 4.12: Comparison of the injected air volume (continuous line) with the volume calculated from the eroded plume structure over time using the erosion method, where the symbols refer to the selected structuring element of $C_e\ (5 \times 5 \times 5$ pixels$^3$) and the shaded columns represent a variation of the side-length by $\pm 2$.

Figure 4.12 shows the comparison between the injected air volume and the one calculated from the eroded plume structure. Again, the main trend is correctly reproduced, however, quantitatively the agreement between the two curves is not as good as the one obtained via the level-based method. After the peak, we note again a decrease of the volume until it levels off. The volume in this last stage is unexpectedly low and can become close to zero. This very low volume is in disagreement with visual observation. It might be due to the fact that the plume structure becomes somewhat more fragmented in final stages of the experiment. A more fragmented structure exposes larger boundary surfaces and is therefore more strongly eroded. So the erosion corrects efficiently for the elongated streaks, but does not work satisfactorily when the structure becomes more fragmented.

A third alternative method comes to mind by looking at Fig. 4.8 again that subsequent exposures of the small bubble appear not only with smaller grayvalue but also smaller size. The multiple exposure results in elongated objects that appear as streaks on Fig. 4.11(e) and small, i.e. typically smaller than a pore size, patches in the front view Fig. 4.11(a). It seems therefore reasonable to think of a criterion that controls the patch size on the image of a slice. It is worthwhile to keep objects that have a size on the order of or larger than half an average pore size, which is about half a millimeter. On the image plane this corresponds to an area of the patch of roughly $A_p = 4 \times 4$ pixels$^2$.

This approach, in the following termed “two-step” method, is now applied as a step subsequent to the above level-based method. The procedure is that, first, a grayvalue threshold is applied using a lower level compared to the one proposed above, and sec-
Figure 4.13: Comparison of the injected air volume (continuous line) with the volume calculated from the reconstructed plume structure over time using the “two-step” method, where the symbols refer to the selected threshold of 1.25⟨grayvalue⟩ and the shaded area represents a variation of the threshold by ±0.25⟨grayvalue⟩.

ond, patches on the image with an area smaller than $A_p$ are erased. Figure 4.11(d,d’) depict the reconstructions of the plume structure after applying this approach, where the selected threshold is 1.25⟨grayvalue⟩. Comparing Fig. 4.11(b) with (d) we note that the two are almost identical, besides a few tiny objects. Comparing the side views Fig. 4.11(b’,d’) it appears that the streaks associated with multiple exposures were efficiently removed. Additionally, the shape of the structure in Fig. 4.11(d’) seems more realistic than the eroded structure, because the erosion method produces a more coherent structure due to the removal of smaller detached objects at the outer boundary of the plume (Fig. 4.11(c’)). The comparison between the volume calculated from the reconstructed plume structure and the injected volume is shown in Fig. 4.13 and it looks quite similar to Fig. 4.10. The agreement between the two curves is overall better than in Fig. 4.10 and Fig. 4.12. However, a drawback of this method is that the reconstructed plume appears too strongly fragmented. Some connections between the parts of the plume seem to be artificially removed. Since the “two-step” method is based on a fixed $A_p$, the redundant pixels are more efficiently erased for small structures than for the larger ones. This is why there is an underestimation at the very beginning and overestimation near the first peak of the volume profile (Fig. 4.13), respectively.

Up to now we used a threshold on the grayvalue intensity and did not consider its distribution in space. As mentioned above, the grayvalue intensity of a bubble image is high when directly exposed to the laser light (Fig. 4.8). Subsequent exposures due to overlapping slices or indirect illumination leave a particle image with comparatively lower grayvalue intensities. It is therefore possible to use a criterion based on the
grayvalue profile in scanning direction to distinguish between the two. We found that the
grayvalue curvature in scanning direction shows strongly negative values in its second
derivative, i.e. a marked concave curvature, when the bubble is directly exposed to the
laser light, whereas it attenuates to less negative or positive curvature values in the
case of subsequent exposures. For the example of the small bubble shown in Fig. 4.8,
only the image of slice nr. 4 corresponding to the local maximum of the grayvalue in
the scanning direction should remain and the neighboring images should be removed.
By knowing the second derivative of the grayvalue in $z$-direction one can easily remove
the multiple exposures. This method is termed “curvature” method.

Figure 4.14: Two examples of point-wise grayvalue distribution in $z$-direction and their
second derivative values according to three types of differentiator.

Let $S(x, y, z)$ be the raw data matrix which contains the grayvalues and $G_{zz}(x, y, z)$
its second derivative in $z$-direction. By applying a central difference approximation,
$G_{zz}(x, y, z)$ can be given by a 3-point differentiator

$$G_{zz}(x, y, z) = \frac{S(x, y, z + h) - 2S(x, y, z) + S(x, y, z - h)}{h^2}, \quad (4.3)$$

or a 5-point differentiator

$$G_{zz}(x, y, z) = \frac{-S(x, y, z + 2h) + 16S(x, y, z + h) - 30S(x, y, z) + 16S(x, y, z - h) - S(x, y, z - 2h)}{12h^2}, \quad (4.4)$$

where the spacing $h = 1$ mm. Figure 4.14 shows how the two differentiators act on the
point-wise grayvalue distribution in $z$-direction. Compared with the 3-point differentiator,
the 5-point differentiator gives larger values of $G_{zz}(x, y, z)$. However, as one can see,
the distribution of grayvalue in $z$-direction is not polynomially smooth. Furthermore,
the “noise” is not suppressed but amplified after applying the two above differentiators,
which shows as highly “wavy” values of $G_{zz}(x, y, z)$. These “waves” might then create
fake gaps between consecutive slices when a threshold value is set on $G_{zz}(x, y, z)$. In
order to avoid this situation, a noise-suppression differentiator must be applied. According to the work of Holoborodko [41], a 3-point noise-robust differentiator of second derivative can smooth out the “noise”. It is given by

\[ G_{zz}(x, y, z) = \frac{S(x, y, z + 2h) - 2S(x, y, z) + S(x, y, z - 2h)}{(2h)^2}. \]  

(4.5)

The result of this differentiator is shown in Figure 4.14. One can see that the “noise” is efficiently smoothed out. Therefore, later on the data are processed with the 3-point differentiator with noise-suppression. For the exceptional case, when over three consecutive slices the maximum grayvalue is reached, and \( G_{zz}(x, y, z) \) is zero identically (see Figure 4.14), these pixels are excluded from data reduction.

In order to proceed, a reasonable choice of the threshold value on \( G_{zz}(x, y, z) \) is needed. Considering that there might be isolated noise pixels in very low grayvalue intensity and that the experiments are transient, similar to the grayvalue threshold method described above, we use a statistical measure based on the mean of the negative part of \( G_{zz}(x, y, z) \). Figure 4.15 shows the variation of the average \( \langle G_{zz} < 0 \rangle \) in time for the two dynamic experiments, where \( \langle G_{zz} < 0 \rangle \) was obtained by averaging the negative part of \( G_{zz}(x, y, z) \) from 50 slices belonging to a given volume scan. One can see that \( \langle G_{zz} < 0 \rangle \) varies in time and between the experiments. Similarly to the level-based method, we apply a threshold which is proportional to \( \langle G_{zz} < 0 \rangle \).

For the thresholding, a binary matrix \( T(x, y, z) \) is defined as

\[ T(x, y, z) = \begin{cases} 
1 & \text{if } G_{zz} \leq c \langle G_{zz} < 0 \rangle \\
0 & \text{otherwise}
\end{cases} \]  

(4.6)

Figure 4.15: Mean of the negative part of \( G_{zz} \) (termed \( \langle G_{zz} < 0 \rangle \)) in time for the two experiments.
where \( c \) is a positive constant. This matrix \( T(x, y, z) \) is then multiplied element by element with \( S(x, y, z) \). Figure 4.16 shows the comparison between the injected air volume and the one calculated from the remaining structures after the “curvature” method was applied. The agreement between the two curves is quite good and overall better than the results from the other three methods. Figures 4.11(e,e') depict the reconstructions of the plume structure, where the selected threshold is \( 0.65 \langle G_{zz} < 0 \rangle \). Comparing Fig. 4.11(e) with (a) we note that the two are almost identical. All the visible objects are preserved. From the side views Fig. 4.11(e,e'), one can see that the streaks associated with multiple exposures were satisfactorily removed. Additionally, the structure in Fig. 4.11(e,e') keeps connections between coherent elements of the plume, which are very important during the growth of the plume.

Figure 4.16: Comparison of the injected volume (continuous line) with the volume calculated from the reconstructed plume structure in time using the gradient method, where the symbols refer to the selected threshold of \( 0.65 \langle G_{zz} < 0 \rangle \) and the shaded columns represent a variation of \( \pm 0.15 \langle G_{zz} < 0 \rangle \).

Since we have learnt that the “curvature” method performs quite well in terms of the morphology and volume of the reconstructed plume structure, we now use this method to show the evolution of the air plume by means of three representative time instances, which are shown in Fig. 4.17. The structure appears to be more coherent when the experiment is started and more elongated and fragmented at later times. Also from this example of a time evolution one can conclude that the reconstruction is morphologically consistent, in the sense that the dynamic growth of the main structures can be successfully recovered.
Figure 4.17: 3D reconstruction of the plume using the “curvature” method with the selected threshold is \( G_{zz} < 0 \) for three time instances, \( t = 1, 3 \) and \( 5 \) s. The injection rates in the two experiments are \( Q = 16.4 \) ml/min (upper panel) and \( Q = 41.1 \) ml/min (lower panel).

4.3 Qualitative description and characteristic measurements

In this section, the invasion patterns will first be qualitatively described. Following the qualitative morphologic description, the characteristic measurements concerning maximum vertical advance, maximum horizontal advance, air volumetric fraction, and box-counting dimension analysis are performed using the extracted data. The migration of the air channel after the breakthrough is also shown via the shifting distance of the centre of mass of the air plume structure, as will be described in more detail below.
4.3. Qualitative description and characteristic measurements

4.3.1 Invasion patterns

As an illustration, the invasion process is going to be described for a case of small injection rate. For a comparison, an invasion process in a rigid packing with the same injection rate is also shown. Yet, observations show a considerable difference between the low injection rate experiments and the high injection rate experiments. For low injection rate, the fragmentation of the invasion structure occurs repeatedly long before breakthrough. It means that during the experiment one can observe several isolated air bubbles (blobs) inside the medium before breakthrough (Fig. 4.18). It suggests that the invasion structure is more easily fragmented in lower injection rates. For high injection rate, the fragmentation usually occurs at the breakthrough. It means that during the experiment one can hardly observe any isolated bubbles inside the medium before breakthrough. In other words, one can generally observe a coherent invasion structure until it breaks through. One can conclude that before breakthrough the morphology of the invasion pattern shows a decrease of the fragmentation probability with high air injection rate.

As air is injected into the porous medium, a plume structure of air flow is developing (Fig. 4.18). However, the structures in a deformable/non-consolidated packing and a rigid packing are quite different from each other. Figure 4.18 depicts a typical invasion process of air injection in a deformable packing and in a rigid packing. In the rigid packing, air displaces liquid between the immobile grains. This displacement process is similar to the tree-like evolution behavior, which was described in Sec.3.2. The respective dynamic process involves a drainage mechanism in which invading air overcomes the entry pressure and the resistance of the displaced liquid. As shown in Fig. 4.18(a’)-(e’), the bottom part of the structure keeps growing steadily for the first 2 s. Then, however, the growth rate is becoming smaller and smaller, and finally approaches zero. As a result, after 4 s the bottom part of the structure remains unchanged until air breaks through. After breakthrough, air pressure is released. Accordingly, an imbibition of the liquid phase is observed, during which the liquid phase reoccupies the pores along the air-liquid interface. At the end of the imbibition process, a stable preferential channel is developed.

Compared with the development process of the air plume in a rigid packing, the displacement process in a deformable packing has a very different behavior, as shown in Fig. 4.18(a)-(e). Similar to the 2D experiments (Sec. 3.3), at t=0.1 s, one can observe a drainage process in which the air phase penetrates into the pores between the grains. However, by roughly comparing the plume structures of the deformable packing and the rigid packing, it can be noted that the plume structure in the deformable packing
Figure 4.18: The figure shows five consecutive images of the experiments in a deformable packing and a rigid packing after 0.1, 0.5, 1, 2, and 4 s, respectively (a)-(e) and (a')-(e'). The grain size is $d_1$. Both injection rates are about 20 ml/min. The 0.3-mm orifice is in the center of a 5-mm platform.
appears smaller. Based on the fact that the illustration examples have approximately the same injection rate, one can conclude that besides the drainage displacement of liquid in the pores, the grains were also displaced in the deformable packing to create enough volume. As the process continues, the plume structure is considerably widened due to the accumulation of air pressure, during which the grains are displaced by the movement of the air-liquid interface. According to the Young-Laplace equation, the air-liquid interface curvature is large under high capillary pressure, and thus can penetrate into the pore space between the grains. Correspondingly, one can observe a rough air-liquid interface where some grains are directly exposed to the air. At $t=2$ s, a characteristic difference between the development process of the air plume in the deformable packing and the rigid packing appears. This characteristic difference is shown in Fig. 4.18(d): the bottom part of the plume in the deformable packing shrinks even during the growth stage of the overall plume. At $t=4$ s, the plume takes a mushroom-like shape. Nonetheless, this mushroom-like plume still directly connects to the orifice with an air conical tube. Moreover, a smooth interface is always observed at the bottom part of the mushroom-like plume. After 4 s, a fragmentation of the plume occurs during which the conical tube is disconnected from the upper bulk of the plume (Fig. 4.19). Actually, this conical tube acts as a transport channel of air from the orifice to the upper bulk of the plume. It means that the migration of the upper bulk of the plume does not depend on the injection pressure. It is driven by its own buoyancy. Moreover, the interface of the conical tube appears much smoother than that of the upper bulk of the plume. This means that the interface of the conical tube is not directly in contact with the grains but stays within the fracture which was formed by pushing the grains aside during the growing stage at early time, i.e., before 4 s.

Figure 4.19(d) depicts the structure at the moment of breakthrough. After the breakthrough, the previous flow pathway undergoes a complete shutdown due to the creeping movement of the grains. Thus the air flow needs to find a new pathway. This is shown in Fig. 4.19(e) where a growing air bubble appears in the neighborhood of the previous flow pathway. This phenomenon was termed the migration of the air channel in Sec. 3.3 in the 2D experiments. Now it is confirmed in the 3D experiments.

As the air is injected continuously, the succession of the plume growing before its breakthrough and shrinking after its breakthrough leads to forward and backward movement of the grains. This causes a shearing deformation of the packing and a rearrangement of the grains. Due to the deformation and the rearrangement, the packing is compacted at the end of each experiment. This is confirmed by a conical drop of the top surface of the packing, which has also been found in the 2D experiments in Sec. 3.4. Due to the compaction, the packing is getting rigid, and the migration of the air channel comes
Figure 4.19: The figure shows another six consecutive images of the same experiments in the deformable packing after 4, 6, 8, 15.6, 22, 48.6 s, respectively (a)-(f). One can see the process of fragmentation ((a)-(c)), breakthrough (d), and air channel migration ((d)-(e)).

to a stop after some time, leaving one small and stable channel (Fig. 4.19(f)). One can roughly estimate that the cross-sectional diameter of this channel has the same length scale as the 5-mm platform.

As one can see from the above descriptions, all the phenomena seen in the 2D experiments have been confirmed to also occur in the 3D experiments. Similar to the characteristic measurements in the 2D experiments, the same quantitative analysis is going to be carried out for the 3D experiments in the following, including maximum vertical advance, maximum horizontal advance, air volumetric fraction, and box-counting dimension analysis. In order to describe the migration of the air channel after the breakthrough, an analogous shifting distance of the centre of the air plume structure is going to be calculated via mapping the 3D space into the 2D space, as will be described in more detail in Sec. 4.3.6. All the 3D data sets are extracted via the “curvature” method with a threshold of $0.65 \langle G_{zz} < 0 \rangle$.

### 4.3.2 Maximum vertical advance

As in the characteristic measurements for the 2D experiments in Sec. 3.3.3, before the first breakthrough, the upward migration of air was measured according to the max-
imum vertical height of the invading structure, $h_{\text{max}}$, at different time intervals. The zero level of $h_{\text{max}}$ is set to the position of the injection orifice. Figure 4.20 shows the measurements of this maximum vertical height versus time for all the evaluated experiments, which are confined by the field of view during the whole process. Overall, $h_{\text{max}}$ increases linearly with time with different slopes at different stages. These different stages are clearly shown in the scaled plot in which $h_{\text{max}}$ and $t$ are normalized by $H_b$ and $t_0$, respectively. When $t/t_0 < 0.4$, the profiles of $h_{\text{max}}$ show more or less the same slope, which means in this stage the air plume grows vertically with the same rate for all the evaluated experiments. When $t/t_0 > 0.5$, the profiles of $h_{\text{max}}$ are separated into two groups: group I in which $h_{\text{max}}/H_b$ first increases slowly and later speeds up quickly, and group II in which $h_{\text{max}}/H_b$ first increases quickly and later slows down.

![Figure 4.20](image)

**Figure 4.20:** (Left) The maximum vertical advance of the invading air, $h_{\text{max}}$, as a function of time. (Right) The scaled plot of $h_{\text{max}}/H_b$ v.s. $t/t_0$. Here $t_0$ is the breakthrough time.

closer inspection revealed that this is controlled by the aspect ratio between the vertical length $dh$ and the maximum horizontal width $dw$ of the upper isolated structure just before the breakthrough, as shown in Fig. 4.21. As it was described in Sec. 4.3.1, the invading structure is separated into several isolated structures during the growth before the breakthrough, wherein the upper isolated structure is responsible for the increase of $h_{\text{max}}$ in the later stage. Figure 4.21(Right) shows the classification of the profiles of $h_{\text{max}}/H_b$ v.s. $t/t_0$, which depends on the value of $dh/dw$. According to the group definition, one can see that the dashed line $dh/dw = 3$ is the group-separating line for group I and group II. Furthermore, the closer the values of $dh/dw$ is getting to 3, the closer the profile of $h_{\text{max}}/H_b$ is approaching the center of the ensemble of curves, as shown in Fig. 4.20(Right).

As one also can see in Fig. 4.20, the breakthrough time decreases both with $Q$ and $d$ in general. It means that increasing either the injection rate or the grain size makes
Figure 4.21: (Left) The definition of $dw$ and $dh$. This snapshot is the $xy$-view of the experiment with $d2$ and $Q= 20$ ml/min. (Right) The values of $dh/dw$ for all the evaluated experiments.

breakthrough occur earlier.

### 4.3.3 Maximum horizontal advance

Figure 4.22: The maximum horizontal width of the invading air, $x_{\text{max}}$ in $X$-direction, as a function of time. Here $t_0$ is the breakthrough time.

The horizontal migration of air was measured according to the maximum horizontal widths of the invading structure in $X$- and $Z$-directions, $x_{\text{max}}$ in Fig. 4.22 and $z_{\text{max}}$ in Fig. 4.23, at different time intervals for all the evaluated experiments. One can see that, both $x_{\text{max}}$ and $z_{\text{max}}$ show an asymptotic increase to their maximum values, and then level off afterwards. Although the invading structure behaves in an incoherent manner with discrete bubbles, in a later stage in which the upper isolated structure is driven by its buoyancy only, an increase of $x_{\text{max}}$ or $z_{\text{max}}$ is still possible, for example, in the
4.3. Qualitative description and characteristic measurements

Figure 4.23: The maximum horizontal width of the invading air, $z_{max}$, in Z-direction, as a function of time. Here $t_b$ is the breakthrough time.

experiment using the packing with grain size $d_1$ and the injection rate $Q = 64$ ml/min.

4.3.4 Air volumetric fraction

Figure 4.24: Air volumetric fraction as a function of time. Here $t_b$ is the breakthrough time.

The air volumetric fraction, $f_a$, is determined by the ratio of air volume and a rectangular enclosing volume of the air plume, yielding

$$ f_a = \frac{V_a}{V_e}, \tag{4.7} $$

where $V_a$ is the injected volume of air and, $V_e = x_{max}z_{max}h_{max}$ the rectangular enclosing volume of the invading structure. Here the enclosing rectangle is defined as $x_{max} \times z_{max} \times h_{max}$. As noted in Sec. 3.3.3, the choice of the enclosing volume allows...
consistency in analysis and comparison among the experiments. The air volumetric fraction \( f_a \) can be used to quantify the spatial distribution of the invading structure. The values of \( f_a \) in Fig. 4.24 show an increasing trend as \( 0 < t/t_0 < 0.15 \), which is quite different from the 2D experiments (Sec. 3.3.3), since the values of \( f_a \) for the 2D experiments decrease monotonically. The reason is that for each experiment we need a pre-injection with a tiny amount of air through the orifice to create a de-wetting process for the orifice. This de-wetting process precludes the capillary breakthrough at the orifice in the real injection. Hence, the pre-injection helps to avoid a rapid burst of the invasion at the very beginning of the injection. However, the pre-injection leads to the non-zero values for \( h_{\text{max}}, x_{\text{max}}, \) and \( z_{\text{max}} \) at \( t/t_0 = 0 \). As \( 0.15 < t/t_0 < 1 \), \( f_a \) levels off asymptotically from the local maximum to an approximate constant, which behaves the same as in the 2D experiments. In all the evaluated experiments, the values of \( f_a \) for packings of small grains are larger than those for packings of large grains. It means that the invasion pattern for packings of large grains appears more like multi-channel flow. As it was described in Sec. 4.3.1, the invading structure behaves like a mushroom-like pattern. Based on the aspect ratio between the vertical length \( dh \) and the maximum horizontal width \( dw \) of the upper isolated structure, i.e. \( dh/dw \), the invading structure is defined as a “fat” mushroom-like pattern as \( dh/dw < 3 \), while the invading structure is defined as a “slim” mushroom-like pattern as \( dh/dw > 3 \). For a “fat” mushroom-like pattern, there are almost no trapped air bubbles left behind. For a “slim” mushroom-like pattern, there is a certain amount of trapped air bubbles left behind, and these trapped air bubbles are also considered when \( x_{\text{max}} \) and \( z_{\text{max}} \) are measured. According to Fig. 4.21, one can easily observe a “fat” mushroom-like invading structure for packings of small grains, while the invading structure exhibits a “slim” mushroom-like pattern for packings of large grains. This is consistent with the conclusion that the values of \( f_a \) for packings of small grains are larger than those for packings of large grains, since a “fat” pattern should have a larger \( f_a \) than a “slim” pattern based on the definition of \( f_a \).

### 4.3.5 Box-counting dimension analysis

To quantify global features of patterns before the first breakthrough, the box-counting dimension was also calculated by measuring the number of boxes, \( N \), needed to cover the entire pattern, as a function of the box size, \( r \). As it was described in Sec. 4.2, there are limitations in the data extraction and reconstruction. The box-counting method would face a risk if it is calculated directly in 3D. An alternative way to solve this problem is calculating the box-counting dimension separately in two orthogonal views, \( xy \)-view and \( zy \)-view. The 3D box-counting dimension is then the result of the 2D box-counting
4.3. Qualitative description and characteristic measurements

dimension plus 1. This method can be found in the literature, for example, in reference [78].

Figure 4.25: (Left) The box-counting data of the invading structure for different times from the experiment with \(d_2\) and \(Q = 16\) ml/min. (Right) The local slope of the box-counting results, corresponding to the local box-counting dimension, \(D_{\text{local}}\), as calculated by a linear least squares fit over the sliding interval \(\Delta \ln(r) = 0.69\).

A representative sample of the box-counting data of the invading structure is shown in Fig. 4.25 for the experiments using grains of size \(d_2\) and injection rate \(Q = 16\) ml/min. It depicts the variations of the box-counting data during the invasion process before breakthrough. The local box-counting dimension, \(D_{\text{local}}\), calculated by the local slope of the box-counting results, is shown in Fig. 4.25. It shows that the profiles of \(N\) v.s. \(r\) expand into a band ordered according to time from bottom to top. That is because more air is injected over time, and therefore more boxes are needed to cover the structure. In this example, the local box-counting dimension is more scattered over time in the \(xy\)-view than in the \(zy\)-view. There are two choices to calculate the mean value of \(D_{\text{local}}\), averaging either over time or over box size. Both results are shown in Fig. 4.26.
we know that the size of the covering box should not be too large, the meaningful box size is less than 50 mm which is the size of the field of view in z-axis. Given the box size, the mean value of $D_{local}$, averaged over two views, falls into the same range, 1.4 to 1.6, both for the mean value over time and box size, as shown in Fig. 4.26. A closer inspection of the mean value of $D_{local}$ over box size reveals that there is a sudden drop at $t = 4.3$ s. This drop corresponds to a fragmentation of the air invading structure in which an isolated plume is formed and driven by its own buoyancy, similar to the behavior shown in Fig. 4.19(b).

Figure 4.27 depicts the mean value of $D_{local}$, averaged over two views, for all the evaluated experiments. It denotes that $D_{local}$ in the packing of grain size $d_1$ is slightly larger than that obtaining in the packing of grain size $d_2$ as $r < 2$ mm. Similar to the argument for the 2D experiments in Sec. 3.3.3, this is consistent with the conclusion that there is
4.3. Qualitative description and characteristic measurements

A smaller bump size along the invading interface in the packings of $d_1$. For a covering box size between 3 mm and 50 mm, the mean values of $D_{\text{local}}$ are quite close to each other, lying between $d_1$ and $d_2$. This means that in this range, the roughness of the invading interface is approximately identical in the packings of these two grain sizes. This suggests that the roughness of the invading interface in the packings of these two grain sizes is generated by a structure element of the same size, i.e. 2 mm to 50 mm, which is the width scale from a small trapped bubble to a big air branch.

For the mean value of $D_{\text{local}}$ over box size, there is not much difference between all the evaluated experiments. They all start from a relative low value, and then asymptotically increase to a stable value around 1.50. One can observe that there many drop events in the profiles of $D_{\text{local}}$, as shown in Fig. 4.27. I checked carefully that each drop event corresponds to a big fragmentation event as described above.

4.3.6 Migration of air channel

Similar to the 2D cases, the process of channel migration behaves in a discrete and erratic manner. The migration process is characterized by the previous channel collapsing and a new channel opening. In order to quantify the migration process, the position of the centre of mass of the air plume in each horizontal layer is defined as the representative position of the air channel, marked by a red dot in Fig. 4.28(Middle). At a given time, the centre of mass of the air plume was determined as follows: the field of view was first divided into layers along the vertical axis, and in each layer the air pixels (pixels containing air) were flagged as 1; second, the selected layer was collapsed along the vertical axis such that a 2D $xz$-view was obtained, as shown in Fig. 4.28(Middle). In this 2D $xz$-view, each pixel was weighted by summing up of the flagged pixels. Finally, the centre of mass of the air plume in this selected layer was calculated based on this weighted 2D $xz$-view.

Figure 4.28 shows an example of the air channel migration. The position of the air channel jumps over time in the selected layer (rendered in green) in the experiment using a packing with grain size $d_2$ and injection rate $Q = 20$ ml/min. It is clearly visible in this layer in Fig. 4.28(Right) that the number of “big jumps” in which the air channel shifts in a large distance is small. Most of the time the air channel migrates in a peristaltic manner. The horizontal shifting distance $\delta X$ is then quantified by the distance between two consecutive positions of the air channel in this layer, i.e. $\delta X = |CG(x, y, z, t_{i+1}) - CG(x, y, z, t_i)|$, where $CG(x, y, z, t_i)$ represents the position of the centre of mass $(x, z)$ in layer $y$ at a given time $t = t_i$. 
Figure 4.28: (Left) 3D reconstruction of the plume using the “curvature” method with the selected threshold $0.65 \langle G_{zz} < 0 \rangle$ before breakthrough. The selected experiment used the packing with grain size $d_2$ and the injection rate $Q = 20 \text{ ml/min}$. (Middle) The contour of the air volume in the selected layer rendered in green in the left figure. The position of the air channel is represented by the centre of mass (red dot) of the air plume in this layer. (Right) The trajectory of the center gravity in this layer versus time.

Figure 4.29: (Left) The maxima of migration distance, $\max(\delta X)$, along the $y$-coordinate. (Right) Probability density function of $\delta X$. Coding an experiment with grain size $d$ and injection rate $Q$ as $(d, Q)$, the labels, Exp1 to Exp5, refer to the experiments $(d_1, Q_1)$, $(d_2, Q_2)$, $(d_2, Q_3)$, $(d_2, Q_4)$, and $(d_2, Q_5)$, respectively. Here, $Q_i (i = 1, 2, 3, 4, 5)$ refers to 64, 16, 20, 40, and 170 ml/min, respectively.

Figure 4.29(Left) depicts the maxima of the migration distance, $\max(\delta X)$, over the vertical axis. Overall, $\max(\delta X)$ increases with $y$. The probability density of $\delta X$ is given in Fig. 4.29(Right). Since the air channel will be fixed at the end of each experiment, $\delta X = 0$ is not considered in the probability density function. One can see that over 99% of the horizontal shifting distance is less than 10 mm. This is the same as
what was found in the 2D experiments. However, there only one out of 15 conducted experiments using packings of grain size $d_1$ showed air channels migration after the breakthrough, against 4 out of 15 conducted experiments using packings of grain size $d_2$. The observations in the 3D case may lead to the opposite that the possibility of channel migration after breakthrough in the experiments using small grains is less than in the experiments using coarse grains.

4.4 Summary

In this chapter, a set of three-dimensional laboratory visualization experiments reveals a gas-flow instability in a porous medium saturated with a glycerine-water solution. The invading structure of the air plume was reconstructed via applying tomographic laser scanning and high speed imaging. A “curvature” method which is based on a threshold on the grayvalue curvature in the scanning direction was applied to remove the redundant pixels.

After the image processing, the characteristic measurements were conducted, including maximum vertical advance, maximum horizontal advance, air volumetric fraction, box-counting dimension analysis, and the migration distance. According to the growth rates, the vertical migration falls into two groups, the borderline between which seems to be controlled by the aspect ratio between the vertical length and the maximum horizontal width of the upper isolated structure. The criterion is that this ratio is equal to 3. The horizontal migration of the invading structure shows an asymptotic increase to a final value. The analysis of the air volume fraction shows a “fat” mushroom-like invading structure for packings of small grains, while the invading structure exhibits a “slim” mushroom-like pattern for packings of large grains. The box-counting method gave a box-counting dimension around 2.5 for all the evaluated experiments. Additionally, a few drop events in the value of the box-counting dimension were found to be related to fragmentation events.
Chapter 5

Conclusions and perspective

5.1 Concluding remarks

In this thesis, valuable new insights have been gained into the laboratory scale process of air injection into liquid-saturated unconsolidated grain packings, both for two- and three-dimensional setups. The investigation has been accomplished by the development of up-to-date experimental techniques, as well as new methods for data processing and characteristic measurements, which show the complex features of a system of coupled fluid-gas-grain flows.

As we can see in Chapter 3, given the non-wetting phase (air), the dimensionless quantity, \( \chi = Q/(aLk_w) \), plays an important role in the mobilization of the grains, the larger \( \chi \), the earlier (more easily) the mobilization of the grains. The following observations could be made:

- \( \chi \) increases with injection rate \( Q \): In any circumstance, the mobilization of the grains occurs earlier (more easily) with higher \( Q \);
- \( \chi \) increases with the viscosity of the wetting phase \( \mu_w \): Compared to the experiments with water (\( \mu_w = 0.001 \text{ Pa}\cdot\text{s} \)) serving as wetting phase, the experiments with glycerin-water solution (\( \mu_w = 0.165 \text{ Pa}\cdot\text{s} \)) serving as wetting phase show a much earlier (easier) start of the mobilization of the grains;
- \( \chi \) decreases with the effective height of the packing \( L = (H_b - H_{or}) \): In any circumstance, increasing the effective height of the packing delays the onset of the mobilization of the grains;
- \( \chi \) decreases with the grain size \( d \): The mobilization of the grains occurs more
easily in the packings of smaller grains. Furthermore, decreasing the grain size increases entry pressure, which in a way enhances the feasibility of the mobilization of grains.

Particularly, when a less viscous liquid serves as the wetting phase, the whole invasion process can be described by three successive dynamic processes: pore-scale tree-like drainage, finger-scale multi-channelized flow, and finger-scale single-channelized migration. Increasing either the injection rate or the viscosity of the wetting phase leads to the disappearance of the process of pore-scale tree-like drainage.

The measured width of the tree-like pattern shows a linear relationship as the injection rate is small. As the injection rate increases further, the width of the tree-like pattern reaches a final value. Assuming that the gradient in gas pressure is comparable to the hydrostatic gradient, quantitative expressions are presented to estimate this width, which shows a satisfactory result compared to the measured width.

The onset of grain mobilization has been discussed, including the mobilization occurring at the tip of a single finger and the mobilization occurring at the orifice once the air is injected. It was found that the occurrence of the mobilization is due to the fact that instead of penetrating between the pores the air chooses a more efficient way to pass through the medium. Besides the viscosity, surface tension, and gravity, the grain mobilization also induces instability to the air flow, which causes the air channel to migrate. Due to the release of the air at the top of the packing in a discrete way, the grain mobilization experiences a back and forward cycle. This leads to a macroscopic expansion and compaction of the whole packing in a pulsating way. However, the consequence of this pulsation effect to the packing was found to be determined by the viscosity of the defending phase. For a less viscous liquid, there is a conical swell at the top of the packing, causing an increase of the total volume of the packing. For a more viscous liquid, there is a conical indentation at the top of the packing, causing a decrease of the total volume of the packing. The compaction process was described via the decaying process of the area of the region of non-zero shear rate, which obeys an exponential law. Besides the compaction, a size segregation of the grains was also found, which was visualized by a series of experiments with tracers.

When using the glycerin-water solution as the defending phase in 2D experiments, the maximum vertical height of the air structure approximately advances linearly with time, while the maximum horizontal advance reaches its maximum value and then levels off for the rest of the time. The air volumetric fraction for packings of small grains are slightly larger than those for packings of large grains. Both the ratio of total surface area to volume and the specific surface area of the air phase can be scaled with the Bond
5.1. Concluding remarks

number with a power of -0.5. The migration process was quantified by the horizontal shifting distance. Its probability density function indicates that over 99% of the horizontal shifting distance is less than 10 mm. A stabilizing process was observed, which is related to a “locking” phenomenon, and a preferential air channel was finally developed. The size of the preferential channel was found to be confined by a lower and an upper threshold.

The 3D invading structure of the air plume was reconstructed via applying tomographic laser scanning and high speed imaging. A “curvature” method which is based on a threshold for the grey value curvature in the scanning direction was applied to remove the redundant pixels. The following major achievements have been obtained in the present technique:

- The appropriate combination of solid and liquid for obtaining transparent and optically homogeneous porous media.
- The employment of high-speed imaging hardware which has the capability of measuring air volume elements in a matrix of \(1024 \times 512 \times 50\) volume elements at a given scanning rate, and illumination system.
- A detailed analysis by comparing the reconstructed air plume volume with the injected one and by evaluating the morphological consistency of the obtained air plume has led to an improved algorithm for image processing.

Similarly to the 2D experiments, the horizontal migration of the invading structure shows an asymptotic increase to a final value. However, unlike the 2D experiments, the vertical migration of the invading structure in the 3D experiments falls into two groups, which seems to be controlled by the aspect ratio between the vertical length and the maximum horizontal width of the upper isolated structure. The analysis of the air volume fraction shows a “fat” mushroom-like invading structure for packings of small grains, while the invading structure exhibits a “slim” mushroom-like pattern for packings of large grains. The box-counting method gave a box-counting dimension around 2.5 for all the evaluated experiments. The “jumps” in the value of the box-counting dimension averaged over box size are related to the fragmentation events.

The impact of thickness \(a\) of 2D cell has not been investigated by varying the value of \(a\) in a systematical way. Nonetheless, this impact can be discussed in an alternative way, which is shown by the averaged thickness \(a_{th}\) of the invading structure in Fig. 3.17. For the packings of small grains, the averaged number of grains between the two glass plates is around \(a/d_1 \approx 8\), which gives an invading structure of an averaged thickness
of around $a_{th}/d_1 \approx 3$ grains. For the packings of large grains, the averaged number of grains between the two glass plates is around $a/d_2 \approx 4$, which gives an invading structure of an averaged thickness of around $a_{th}/d_2 \approx 1$ grain. It might suggest that a moderate increase of $a/d$ provides a thicker invading structure.

By applying the composition index-matched fluid and medium, the opaqueness of the medium, like in the case of water in sand packings, is overcome, so that a pore-scale sharp air-liquid interface can be observed. The actually case of air injection in sand packings saturated with water and the index-matched medium have the following things in common:

- The phenomena of fragmentation, coalescence, and air channel migration. Therefore, the characteristic measurements of air channel migration in the index-matched medium are suitable for the case of water in sand packings.
- The onset of the instability of the medium, which can be described by $\chi$.
- The re-organization of the medium.

These findings can be used to assist in analyzing flow instability, pattern formation, and preferential flow in many practical configurations and applications, such as aeration of bio-filters, gas flow in packed beds, and migration and escape of gas in wetlands, soils and seabeds.

### 5.2 Perspective: current challenges and future work

As mentioned in the thesis, mobilization of the medium is caused by the interaction of the air flow and the medium. However, there are still challenges to overcome before a clear understanding. One of these is to analyze the stress distribution inside the medium before and after the mobilization. Although there is work about the mechanical response of sediments to bubble growth [8, 9, 47], it is necessary to link the mechanical stress distribution to the pressure-volume relationship of the bubble. In order to measure the stress in a non-destructive way, the medium could be constructed by force-sensitive polymers which respond to mechanical stress with color change.

In the thesis, I assumed that there was no horizontal flow in the water phase during the growth of the tree-like pattern. It would be better to perform a complementary experiment in which the flow of the water phase is also recorded at the same time. A fluorescent-particle seeded liquid is necessary to accomplish this measurement.
5.2. Perspective: current challenges and future work

In 3D experiments, the present technique is limited when the laser sheet encounters a smooth bubble surface on which a specular reflection occurs. This problem might be solved in an alternative way by illuminating the air body from both sides instead of one side only as in the present setup. As a complementary recording, a second camera can be placed on another side of the box.

A next important step is to set up a conceptual and numerical model of the process. This involves the following issues:

- A sharp air-liquid interface needs to be maintained, which the classical continuous model can not achieve.
- A multi-scale flow model for the fluids is needed, since the flow scale reaches from pore scale to channel scale.
- An efficient granular flow model is needed.
- A constitutive relation is needed to describe the coupled process of fluid-solid flow.
Acknowledgements

I would like to thank my supervisor Prof. Dr. Wolfgang Kinzelbach for his continuous support and constructive advice. Especially thanks for having given me the opportunity to accomplish my PhD in ETH, which greatly contribute to my life and my career.

I would like to thank Prof. Dr. Fritz Stauffer for his endless help and patience, for his effort in discussing problems with me, and for acting as co-examiner.

I appreciated very much the help and support of Prof. Dr. Patrick Jenny from the Institute of Fluid Dynamics, his great effort in providing inspiring ideas, and acting as co-examiner.

I am grateful to all of my colleagues at the Institute of Environmental Engineering for the pleasant work atmosphere. I would like to thank in particular Johannes Buehler who greatly helped me to improve my English, even if I had a hard time in discussing with him about all the Chinese issues. I appreciated the collaboration with Markus Holzner. I also wish to thank Toni Blunschi for the experimental setup and Klaus Hoyer for his brilliant suggestions. I would like to thank Beat Luethi for his great help in solving my tedious problems in the lab.

Finally, I wish to thank Wang Haijing and Li Haitao for sharing fun in daily life in Zurich, and my wife and my parents for their support and endless love and encouragement.

This work is supported by the ETH Research Grant TH-2606-1.
List of Tables

3.1 The value of $v_d/v_c$ .................................................. 51
List of Figures

2.1 A sketch of a liquid droplet on a horizontal solid surface. The contact angle $\theta$ between the horizontal surface and the liquid surface indicates the wettability of the liquid. .............................................. 4

2.2 "Phase diagram" by Lenormand et al. [56] on a logarithmic plot with the capillary number $C_a$ along the $y$-axis and the viscosity ratio $M$ along the $x$-axis. ................................................................. 7

2.3 A Sketch of migration process of gas clusters. During upward migration (a), the cluster might break up into fragments, leading to fragmentation (b). The isolated fragments might get into contact and coalesce with a cluster coming from below (c). ......................................................... 8

2.4 A Sketch of pingo-like features (PLFs) and moat formation surrounding the PLF associated with gas hydrate decomposition. As the subsurface warms, the top of the gas hydrate stability zone will move downward, which is indicated by the black arrows. This picture is redrawn according to [71]. ................................................................. 11

2.5 A Sketch of "blocking" and "circumvent" of bubbles. After the fragmentation Fig. 2.3(b), the isolated fragment might get trapped in a pore, rather than re-mobilized by a cluster coming from below, which circumvents that pore. ................................................................. 12

2.6 Classification of gas flow pattern dependent on flow rate and grain size. From [36]. ................................................................. 13

3.1 Hardware configuration for 2D experiments in the laboratory, and an inserted sketch of the 2D box model. The height of the packing is $H_b$. The supernatant liquid has a constant surface elevation $H_w$. The $x$-$y$ co-ordinate is also shown, and its origin is set at the position of the orifice. 18
3.2 Branching fingering with tree-like pattern of air injection rate $Q = 90 \text{ ml/min}$ evolving with time (physical size of the images: 219 mm$\times$228 mm): (a)$t = 0.5$ s, (b)$t = 1.5$ s, and (c)$t = 3.0$ s. 

3.3 The measured outer bound width, $W_t$, of the tree-like pattern changes with the vertical coordinate $Y$ and the air injection rate.

3.4 The bottom and top width of the tree-like pattern versus the air injection rate. (Inset: Schematic of transient air plume geometry with width of $W_t$ and thickness $a$, wherein $W_t = W_t^b$ for bottom width and $W_t = W_t^t$ for top width. The three principal forces are shown, the buoyant force density $f_g$, and resistant force densities $f_{rw}$ from water and $f_{ra}$ from air.)

3.5 Evolution of fluidized pattern in the experiment with the air injection rate $Q = 90 \text{ ml/min}$ versus time (physical size of the images: 219 mm$\times$228 mm): (a)$t = 16.0$ s, (b)$t = 106.0$ s, and (c)$t = 356.0$ s. The dashed line is the front between the fluidized and the tree-like patterns. The beads were fluidized and those staying at the top of the packing are indicated by an irregular zone in black.

3.6 Here $y$ is defined as the lowest position of the front between the fluidized and the tree-like pattern. The zero of the vertical axis is located on the position of the orifice. The zero-point of time is set to be the start of the fluidization pattern.

3.7 The vertical starting position $y_o$ of the fluidized region changes with the air injection rate. Inset: one-dimensional model for calculating $y_o$.

3.8 Schematic of buoyant channel geometry with an arch-shape upper end (curvature $w_f/2$) above the rectangular cross section ($w_f \times a$) imposing a buoyant driving force sufficient to elongate the air-water interface.

3.9 The number of channels above the interface and the average flow rate per channel.

3.10 A typical irregular oscillating channel. The air injection rate is $Q = 90 \text{ ml/min}$. The image (physical size of 219 mm$\times$228 mm) is an integration of the differences over 2.5 s (from 1622.5 s to 1624.5 s). The deep grey areas are the areas where the channel changed during the 2.5 s. They indicate pre-existing air channels. Two convection cells were added manually in the image as dashed lines according to the observations in the experiments.
3.11 Images are taken from the experiment with $d_1$, $Q=150$ ml/min, and $H_b=220$ mm: Air channels or fractures are shown right after air is injected and air penetrates into pores around the branches (a) at $t=0.5$ s; (b) at $t=3.0$ s (before air breakthrough); (c) at $t=3.5$ s (right after air breakthrough); and (d) at $t=5.5$ s (fragmentation). ................................. 34

3.12 Air slugs moving due to buoyancy and air supply through a tiny channel (indicated by a black arrow). Images are taken from the experiment with $d_1$, $Q=150$ ml/min, and $H_b=220$ mm: (a) at $t=47.5$ s, (b) at $t=48.0$ s. .... 35

3.13 Air channel shifting inside the medium after air breakthrough is indicated by a superposition of air flow patterns at three time slices, 45 s (light blue), 66.5 s (purple), and 121.5 s (black) after the start of air injection. Image was taken from the experiment with $d_1$, $Q=150$ ml/min, and $H_b=220$ mm. ................................................................. 36

3.14 The maximum vertical advance of the invading air, $h_{max}$ (Left), and the maximum horizontal width, $w_{max}$ (Right), vary with time. Here $t_0$ is the breakthrough time. ......................................................... 38

3.15 Air volumetric fraction as a function of time. The solid line in the right graph (logarithmic scale both for the $x$- and $y$-axis) has a slope of -0.45. 39

3.16 (a) Structure of wide-fracture pattern, snapshot at $t=1.0$ s from the experiment with $d_1$, $Q_3$, and $H_2$. (b) Structure of pore-filled pattern, snapshot at $t=1.0$ s from the experiment with $d_2$, $Q_3$, and $H_2$. ................. 39

3.17 Averaged thickness of the invading structure as a function of time. The thickness of the 2D model is marked by a thick dash-dotted line (at $a_{th}=5.5$ mm). ................................................................. 40

3.18 (Left) Ratio of total surface area to air volume, $f_{sv}$, and (Right) $f_{sv}$ is scaled with $B_o^{-0.5}$. ......................................................... 41

3.19 (Left) specific surface area of the air phase versus time, and (Right) specific surface area of the air phase versus the air volumetric fraction, $f_a$. 42

3.20 (Left) $A_{sp}$ versus time, and (Right) $A_{sp}$ versus $f_a$. Both are scaled with $B_o^{-0.5}$. ......................................................... 43

3.21 Illustration of the box-counting method acting on a representative snapshot of the air structure. ......................................................... 44
3.22 (Left) The box-counting data of the invading structure for different times in the experiment with $d_1$, $Q_3$, and $H_2$. (Right) The local slope of the box-counting results, corresponding to the local box-counting dimension, $D_{local}$, as calculated by a linear least squares fit over the sliding interval $\Delta \ln(r)=0.69$. Here the box size is measured in the unit of pixels, where the pixel size is about 0.3 mm.  

3.23 The mean horizontal air channel position (red solid line) at a given time.  

3.24 (Left) Mean horizontal position ($X$) of air channel, as well as (Right) the corresponding total width ($W_{tot}$) of air channel, at different vertical position ($y$) is plotted versus time. The data set is taken from the experiment with $d_1$, $H_2$, and $Q_3$. Here the zero-point of time is set to the time of the first air breakthrough. The zero-point of $X$ is the horizontal position of the orifice.  

3.25 Probability density function of $\delta X$. Here $\delta X = 0$ is not considered.  

3.26 The lateral movement distance, $L_{\delta X}$, and the lateral movement width, $W_{\delta X}$, are plotted versus $y$-coordinate with a vertical sampling distance of 5 mm. The data set is taken from the experiments with different $d$, $H_b$, and $Q$. Here the zero-point of time is set to the time of the first air breakthrough. The zero-point of $y$ is the vertical position of the orifice.  

3.27 A sketch of unstable packing and stable packing, separated by a grey dashed line. (Inset: a sketch definition of the dipole velocity, a superposition velocity at the midpoint between a source ($Q/a$) and a sink ($-Q/a$) with a distance of $2(H_b - H_0)$.)  

3.28 Superposition of air flow patterns at three time slices, 97 s (light blue), 269 s (purple), and 351 s (black) after the start of air injection. Image was taken from the experiment with $d_{1p} = 0.8 - 1.0$ mm, $Q = 90$ ml/min. Three arrows (with corresponding colors) indicate the places where a “locking” phenomenon occurs.  

3.29 Experiment with packing (I), primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm (image width: 180 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing. The small solid spots are tracers, while the large hollow spots are air-liquid interfaces which are visualized due to optical effects and not due to tracer accumulations. It is the same for the following figures.
3.30 Experiment with packing (II), primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{2t} = 0.4 - 0.6$ mm (image width: 182 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing. ........................................ 56

3.31 Experiment with packing (III), primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{3t} = 2.0 - 2.5$ mm (image width: 175 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing. ........................................ 57

3.32 Experiment with packing (IV), primary grains with size $d_{2p} = 0.4 - 0.6$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm (image width: 182 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing. ........................................ 58

3.33 Experiment with packing (V), primary grains with size $d_{3p} = 1.6 - 2.0$ mm and tracer grains with size $d_{3t} = 2.0 - 2.5$ mm (image width: 184 mm): initial status (a) and final status (b) of layered packing, initial status (c) and final status (d) of fully mixed packing. ........................................ 59

3.34 Schematic shapes of a tracer grain layer observed in two types of patterns after the air injection. Type I: a convex curve; Type II: from a convex curve to a concave curve. Here the arrows indicate the direction of movement of the curve. ........................................ 59

3.35 A rough estimate of locally averaged grain size of the experiment with a fully mixed packing with primary grain size $d_{2p} = 0.4 - 0.6$ mm and tracer grain size $d_{1t} = 0.8 - 1.0$ mm (image width: 275 mm). Left: initial status. Right: final status. ........................................ 60

3.36 Experiment with packing (I), primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm in a fully mixed packing. Left: A snapshot at $t = 180.5$ s (image width: 320 mm). The black line is the lower boundary of the disturbed zone. Right: The lower boundary of the disturbed zone changes with time. Here the zero point of the coordinate is the position of the orifice. ........................................ 61
3.37 Left: The area of disturbed zone, $S_z$, is the area from the lower boundary of the disturbed zone up to the top. (Inset–the area of air channel, $S_c$, versus time.) Right: The normalized $S_z/S_z(0)$ as a function of dimensionless time $t/t^*$. The black continuous curve is a fit of the exponential law. (Inset–Both $S_z(0)$ and $t^*$ are linearly fitted as a function of the grain size.)

3.38 Experiment of layered packing of primary grains with size $d_{1p} = 0.8 - 1.0$ mm and tracer grains with size $d_{1t} = 0.8 - 1.0$ mm. Left: initial position of tracer lines (dashed line) and post-position of tracer lines (solid line). Right: estimate of the porosity change from the displacement between the tracer line before and after the experiment.

4.1 Left: Hardware configuration for 3D experiments in the laboratory. Right: A sketch of the 3D experimental setup. The laser beam is expanded and thinned to a light sheet, then scanned through the observation volume using an eight-face prism. The coordinate system $x$-$y$-$z$ of the experiments is also shown.

4.2 Sketch of the top view of the experimental setup. Here $\theta$ is the angle of incidence, and $z$ is the coordinate parallel to the scanning direction and in line with the symmetry axis of the camera arrangement. $D$ is the distance between two parallel faces of the octagon, i.e., the diameter of the inscribed circle of the octagonal cylinder.

4.3 Principle of parallel shift when light is transmitted between two parallel faces of the octagon. $n_a$ is the refractive index of air. $\theta_c$ is the angle of refraction. $l$ is the horizontal displacement when the light reaches the opposite side of the octagon. $l^*$ is the virtual horizontal displacement when the light would reach the opposite side without refraction.

4.4 Shift distance $z$ as a function of the angle of incidence $\theta$ according to Equ. 4.1.

4.5 Schematic illustration of the light sheet intensity distribution through scanning with the velocity of $v$.

4.6 An image from a single slice shows that a systematic attenuation of the measured grayvalue intensity in $x$-direction is not detected by the camera in the current configuration.
4.7 Morphological consistency is verified in the case of discrete static bubbles. Left: 2D image of \( y-x \) panel using a normal digital camera. Right: 3D reconstruction based on the raw images. ........................................ 74

4.8 Sequence of images of a bubble with a diameter smaller than 1 mm. Top: raw gray images (12 pixels \( \times \) 13 pixels with a resolution of about 0.1 mm/pixel) sequentially registering the bubble. Bottom: contours of the slices, where the distance between the center of two neighboring slices is 1 mm. ................................................................. 74

4.9 Averaged grayvalue (termed \( \langle \text{grayvalue} \rangle \)) in time for the two experiments. Inset: the recorded injection rates in time. ................................................................. 76

4.10 Comparison of the injected air volume (continuous line) with the volume calculated from the reconstructed plume structure over time using the level-based method, where the symbols refer to the selected threshold of 1.25\( \langle \text{grayvalue} \rangle \) and the shaded columns represent a variation of the threshold by \( \pm 0.25 \langle \text{grayvalue} \rangle \). ................................................................. 76

4.11 3D reconstruction of the plume structure for the experiments with injection rate \( Q = 16.4 \) ml/min (upper panel) \( Q = 41.1 \) ml/min (lower panel) at time \( t = 4 \) s, where the sub-upper and sub-lower panels show the front \((Y-X)\) and side \((Y-Z)\) view, respectively: raw data (a,a'), level-based method (b,b'), erosion method (c,c'), “two-step” method (d,d'), and “curvature” method (e,e'). ................................................................. 78

4.12 Comparison of the injected air volume (continuous line) with the volume calculated from the eroded plume structure over time using the erosion method, where the symbols refer to the selected structuring element of \( C_e (5 \times 5 \times 5 \) pixels\(^3\)) and the shaded columns represent a variation of the side-length by \( \pm 2 \). ................................................................. 79

4.13 Comparison of the injected air volume (continuous line) with the volume calculated from the reconstructed plume structure over time using the “two-step” method, where the symbols refer to the selected threshold of 1.25\( \langle \text{grayvalue} \rangle \) and the shaded area represents a variation of the threshold by \( \pm 0.25 \langle \text{grayvalue} \rangle \). ................................................................. 80

4.14 Two examples of point-wise grayvalue distribution in \( z \)-direction and their second derivative values according to three types of differentiator. .................. 81

4.15 Mean of the negative part of \( G_{zz} \) (termed \( \langle G_{zz} < 0 \rangle \)) in time for the two experiments. ................................................................. 82
4.16 Comparison of the injected volume (continuous line) with the volume calculated from the reconstructed plume structure in time using the gradient method, where the symbols refer to the selected threshold of $0.65 \langle G_{zz} < 0 \rangle$ and the shaded columns represent a variation of $\pm 0.15 \langle G_{zz} < 0 \rangle$. "...

4.17 3D reconstruction of the plume using the "curvature" method with the selected threshold is $0.65 \langle G_{zz} < 0 \rangle$ for three time instances, $t = 1, 3$ and $5$ s. The injection rates in the two experiments are $Q = 16.4$ ml/min (upper panel) and $Q = 41.1$ ml/min (lower panel). "...

4.18 The figure shows five consecutive images of the experiments in a deformable packing and a rigid packing after $0.1, 0.5, 1, 2,$ and $4$ s, respectively (a)-(e) and (a')-(e'). The grain size is $d_1$. Both injection rates are about $20$ ml/min. The 0.3-mm orifice is in the center of a 5-mm platform. "...

4.19 The figure shows another six consecutive images of the same experiments in the deformable packing after $4, 6, 8, 15.6, 22, 48.6$ s, respectively (a)-(f). One can see the process of fragmentation ((a)-(c)), breakthrough (d), and air channel migration ((d)-(e)). "...

4.20 (Left) The maximum vertical advance of the invading air, $h_{\text{max}}$, as a function of time. (Right) The scaled plot of $h_{\text{max}}/H_b$ v.s. $t/t_0$. Here $t_0$ is the breakthrough time. "...

4.21 (Left) The definition of $dw$ and $dh$. This snapshot is the $xy$-view of the experiment with $d_2$ and $Q= 20$ ml/min. (Right) The values of $dh/dw$ for all the evaluated experiments. "...

4.22 The maximum horizontal width of the invading air, $x_{\text{max}}$ in $X$-direction, as a function of time. Here $t_0$ is the breakthrough time. "...

4.23 The maximum horizontal width of the invading air, $z_{\text{max}}$ in $Z$-direction, as a function of time. Here $t_0$ is the breakthrough time. "...

4.24 Air volumetric fraction as a function of time. Here $t_0$ is the breakthrough time. "...

4.25 (Left) The box-counting data of the invading structure for different times from the experiment with $d_2$ and $Q = 16$ ml/min. (Right) The local slope of the box-counting results, corresponding to the local box-counting dimension, $D_{\text{local}}$, as calculated by a linear least squares fit over the sliding interval $\Delta \ln(r)=0.69$. "...
4.26 The mean value of $D_{local}$ over box size (Left) and the mean value of $D_{local}$ over time (Right). The data set was extracted from the experiment with $d_2$ and $Q = 16$ ml/min.

4.27 The mean value of $D_{local}$ over time (Left) and over box size (Right), calculated by an average over two views.

4.28 (Left) 3D reconstruction of the plume using the “curvature” method with the selected threshold $0.65(G_{zz} < 0)$ before breakthrough. The selected experiment used the packing with grain size $d_2$ and the injection rate $Q = 20$ ml/min. (Middle) The contour of the air volume in the selected layer rendered in green in the left figure. The position of the air channel is represented by the centre of mass (red dot) of the air plume in this layer. (Right) The trajectory of the center gravity in this layer versus time.

4.29 (Left) The maxima of migration distance, $\max(\delta X)$, along the $y$-coordinate. (Right) Probability density function of $\delta X$. Coding an experiment with grain size $d$ and injection rate $Q$ as $(d, Q)$, the labels, $Exp1$ to $Exp5$, refer to the experiments $(d_1,Q_1)$, $(d_2,Q_2)$, $(d_2,Q_3)$, $(d_2,Q_4)$, and $(d_2,Q_5)$, respectively. Here, $Q_i (i=1,2,3,4,5)$ refers to 64, 16, 20, 40, and 170 ml/min, respectively.
Bibliography


dissolution kinetics of actanol in porous media. *J. Colloid Interface Sci.*, **210**, 261-
270.


during air injection into granular materials confined in a circular Hele-Shaw cell.

Flekkøy E. G., and Schmittbuhl J. (2008) Decompression and fluidization of a satu-
rated and confined granular medium by injection of a viscous liquid or gas. *Phys.
Rev. E*, **78**, 051302.


Experimental investigations of gas phase distribution. in: *Proceedings of the 2nd
European Bioremediation Conference Chania/Kreta, Greece*, March 30–April 7.

M., Peargin T., Bruce C. L., Amerson I. L., Coonfare C. T., Gillespie R. D. and Mc


Curriculum Vitae

Xiang-Zhao Kong

Personal

Born on July 6, 1979  
Citizen of Guangdong, China

Education

2006 - 2010  
PhD, Institute of Environmental Engineering, ETH Zurich, Switzerland.  
Under supervision of Prof. Wolfgang Kinzelbach.

2003 - 2006  
M.Eng., Department of Thermal Science & Energy Engineering,  
University of Science & Technology of China, Hefei, China. Under  
supervision of Prof. Qingsong Wu.

1999 - 2003  
B.Eng., Department of Thermal Science & Energy Engineering,  
University of Science & Technology of China, Hefei, China.  
B.Sci.(secondary), Department of Electronic Engineering & Information  
Science, University of Science & Technology of China, Hefei, China.

Journal Papers


Conference


X.-Z Kong, W. Kinzelbach, F. Stauffer, When air is injected into mobile liquid-saturated porous medium, EGU, Vienna, Austria, 2009. (Oral Presentation)

X.-Z Kong, W. Kinzelbach, F. Stauffer, When air is injected into mobile liquid-saturated porous medium, Preferential and Unstable Flow in Porous Media - From Water Infiltration to Gas Injection, Monte Verita, Switzerland, 2009. (Oral Presentation)


M.-B. Hu, Q.-S. Wu, X.-Z. Kong, Y.-H. Wu, Particle discharge process from a capillary pipe, Accepted by the conference on Traffic and Granular Flow ’05, Oct. 10-12, 2005, Berlin, Germany.

X.-Z Kong, M.-B. Hu, Q.-S. Wu, Effects of bottleneck on granular convection cells and segregation, Shanghai, China, 2004. (Poster)