Prices vs. Quantities with Fiscal Cushioning

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Abstract

Regulating international externalities, like climate change, raises various enforcement problems. It is often argued that international price-based regulations (e.g. emission taxes) are more difficult to enforce than quantity-based regulations (e.g. tradable pollution permits). In this paper, we analyze the relative performance of price-based and quantity-based instruments when costs and benefits are uncertain and enforcement of quantity regimes is stricter than that of price regimes. We show that under these conditions, instrument choice solely based on the relative slopes of the marginal curves can yield inefficient results. If policy enforcement differs, rational policy choice should also take into account the level of the marginal benefit curve, as well as institutional parameters. In contrast to earlier analyses on "Prices vs. Quantities", we find that the choice of instrument also depends on the variance of the marginal abatement costs. Numerical simulations of our stylized model suggest that, for climate policies, quantity-regulations might well be preferable to price-based approaches after all.

Keywords: market-based instruments, incomplete enforcement, uncertainty, environmental regulation

JEL classification: D8, L51, K42, Q58

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1 Introduction

In the last few decades, there has been an ongoing debate among environmental economists on the relative performance of price-based (e.g., a carbon tax) and quantity-based (e.g., tradable pollution permits) instruments. From the point of efficiency, both regulatory policies perform equally well if costs and benefits from pollution abatement are known. However, as Weitzman (1974) showed, the performance of price-based and quantity-based instruments can differ if costs and benefits are subject to uncertainty. Furthermore, the efficiency of environmental policies is altered significantly if they are considered under the more realistic assumption of incomplete enforcement (Montero (1999, 2002)).

Policy enforcement is of particular importance in the context of international regulations, such as the international climate policy framework. The Kyoto Protocol, for example, being based on a quantity approach has been severely criticized on the basis of the "relative slope criterion" derived in Weitzman (1974).\footnote{See for example Nordhaus (2007).} This criterion indicates that the choice of instrument is determined by the relative slopes of the marginal benefit and marginal cost curves. Given the estimated marginal curves in the context of regulating CO$_2$ emissions, a price regulation dominates a quantity-based approach in terms of efficiency.\footnote{See for example Hoel and Karp (2002), and Newell and Pizer (2003).} In contrast, it is often argued that enforcement of price-based instruments on the international level is significantly lower than that of quantity regulations.\footnote{See for example Wiener (1999).} Both arguments are strong and plausible. The net effect of uncertainty on the one hand, recommending the use of taxes to regulate CO$_2$ emissions, on the other hand differing enforceability, which would suggest quantity instruments, is unclear. Despite the fact that this has been frequently noted in the literature, a formalization of this problem has not yet been attempted. In this paper we address the question of instrument choice when: costs and benefits are uncertain and enforcement of quantity-based instruments is stricter than that of price-based instruments.

In environmental economics it is well known that the efficiency of price-based and quantity-based instruments can differ if benefits and costs are uncertain. The discussion on Prices vs. Quantities under uncertainty started with the seminal paper Weitzman (1974).\footnote{See Nordhaus (2007).} Weitzman uses linear approximations for the marginal curves and assumes uncorrelated and additive uncertainty, affecting the level but not the slope of the marginal curves. Weitzman demonstrated that the slopes of the marginal benefit and marginal cost curve determine the relative performance of the regulatory instruments. A price regime provides a higher expected social welfare than a quantity-based approach as long as the marginal cost curve is relatively steeper than the marginal benefit curve. Quantity-based instruments ought to be preferred in the opposite case. Yet, Weitzman pointed out that the relative slope criterion has to be altered if uncertainties are correlated\footnote{See Malcomson (1978) and Malcomson (1978) for a modification of the initial relative slope criterion if the slope of the marginal curves is uncertain.} and if the slopes of the marginal curves themselves are subject to uncertainty.\footnote{See Stavins (1996) for a detailed discussion on Prices vs. Quantities with correlated uncertainty.} Malcomson (1978) and Weitzman (1978) reconsider Weitzman’s initial approach if a linear approximation of the marginal curves is not appropriate. Later contributions to the Prices vs. Quantities debate stick to the assumptions of linearly approximable marginal curves with uncor-

There is a growing literature on the problem of incomplete enforcement of environmental regulations. The analyses take into account that polluters have incentives to misstate their actual emission budget. This strain of literature is considered as a direct extension of the economics of crime, which is based on Becker (1968), as well as on the subsequently established economics of tax evasion (Allingham and Sandmo (1972)). The first enforcement model on environmental policies is presented in Downing and Watson (1974). Harford (1978), Viscusi and Zeckhauser (1979), and Harford (1987) provide an analysis on firms' behavior under incompletely enforceable environmental tax schemes and pollution standards. Harrington (1988) models the interaction between enforcement agencies and firms in a repeated game under the assumption of limited possibility of sanctions. The effects of incomplete compliance on a market for tradable pollution permits are derived in Malik (1990). Livernois and McKenna (1999) consider a case of pollution standards with different fines for noncompliance. The efficiency issue of different policy instruments is analyzed in Sandmo (2002) for risk-neutral as well as for risk-averse actors.\(^7\)

A formal attempt to simultaneously analyze the choice of instrument under uncertainty and incomplete enforcement in the context of environmental regulations is presented in Montero (1999, 2002). In these analyses, the debate on Prices vs. Quantities takes into account that depending on the regime, polluters either evade taxes or hold an insufficient amount of pollution permits. The result is an altered version of the initial relative slope criterion, derived in Weitzman (1974). The relative slopes of the marginal curves remain the crucial variables for the choice of instrument. Yet, incomplete enforcement increases the set of situations in which quantity-based regulation ought to be preferred. Incomplete enforcement softens a quantity approach as the amount of emission reductions becomes uncertain in the presence of noncompliant actors.

In this paper we formally analyze the effect of an additional problem which arises in the context of instrument choice for international regulations. This is being applied to the regulation of \(CO_2\) emissions. It is often argued that on the international level the enforcement of quantity-based instruments is stricter than that of price-based instruments. Such preconditions alter significantly the relative performance of both policy instruments. Note, on the international level regulated actors are sovereign states and not firms or private entities. The sovereignty of countries implies that governments can define their fiscal policies independently of the decisions of a supra-national Regulatory Agency (RA hereafter) mandated to implement and enforce a global regulatory instrument. On the international level, in addition to

\(^6\)Yet, hybrid instruments perform better than either price-based or quantity-based instruments in their pure form (Roberts and Spence (1976)). Pizer (2002) shows the superiority of a combination of both instruments, e.g. a tradable permit system including a ‘safety valve’, in the case of regulating GHG emissions.

\(^7\)See Cohen (1999) for a detailed survey on monitoring problems and incomplete enforcement in environmental policies.
misrepresenting their actual emissions, polluters can also undermine the incentive effect of a price-based instrument: a situation known as 'fiscal cushioning' (Wiener (1999), Wiener (2001), Aldy et al. (2008)). For example, countries levy a global carbon tax and formally comply with the international obligations. At the same time, countries can use fiscal revenues to reduce other taxes which indirectly tax carbon (e.g. fuel duty) or increase subsidies for carbon intense production processes (e.g. coal subsidies). Yet, it can be reasonably assumed that a country's leeway to offset the incentive effect of an international levy is not without limits. Countries might still face international sanctions, like punitive tariffs, when their de facto noncompliance to the regulation is discovered. However, even if the RA observes a reduction of the emission levy's incentive effect, the respective country can argue that cuts in fuel taxes or a raise in subsidies on carbon intense production processes is motivated by domestic industrial policy objectives. Furthermore, national tax schemes are often quite inscrutable which further decreases the probability of detection (Aldy et al. (2008)). In contrast, the mechanism of quantity-based regulations, where a pre-determined amount of pollution permits is allocated to the individual countries, cannot be directly undermined by national governments. In such systems, once the emissions trading system is in place, the overall scarcity of pollution permits is fixed. This reduces the possibilities of a country's uncooperative behavior to the case where a country simply misstates its actual emissions. Therefore, it seems appropriate to represent the enforcement problem that is imminent on the international level by attributing a lower enforcement probability to the price-based instrument than to the quantity-based instrument.

We therefore extend the framework established in Montero (1999) and derive the expected difference in social welfare of price-based over quantity-based regulations assuming stricter enforcement of quantity instruments. It is, a priori, not clear whether price-based or quantity-based instruments ought to be preferred in such a situation. On the one hand, stricter enforcement, i.e. an increase in the enforcement probability, reduces the advantage of quantity instruments identified by Montero (1999, 2002). On the other hand, stricter enforcement of a quantity approach increases net benefits from emission reductions in comparison to a price regime, as the second-best outcome under incomplete enforcement moves closer to the ex-ante first-best solution (complete enforcement). This latter effect is co-determined by level effects as well as institutionally determined variables. The main purpose of this paper is to analyze the relative strength of these two countervailing effects. It turns out that solely considering the relative slope criterion is insufficient for a rational instrument choice. Furthermore, we show that there exists a threshold level of the variance of the marginal abatement costs below which the relative-slope criterion is no longer relevant, as quantity instruments are always preferable.

The rest of the paper is organized as follows: In the next section we introduce the basic setup of the model. Section three determines the countries' compliance strategy under incomplete enforcement. The optimal policy design under incomplete enforcement is presented in section four. The central question of the relative...
difference in expected social welfare for both instruments is analyzed in section five. We present numerical simulations for plausible parameter values in section six and an extension of the model to a different enforcement policy in section seven. The last section concludes the paper.

2 Model

We develop an extension of the one-period model presented in Montero (1999) to analyze instrument choice under different enforcement probabilities for price- and quantity-based regulations.

We assume the existence of an international RA that implements either a price- or quantity-based instrument to induce a reduction in global emissions of a uniform pollutant. Several countries of mass 1 are affected by this policy measure. Let \( t \) be the corresponding tax under a price regime and \( l \) the amount of certificates distributed by the RA under a quantity regime.

For simplicity, following Montero (1999), each country discharges one unit of emissions. Countries can abate pollution at constant marginal costs \( c \), which differ across countries. The set of countries can be ranked continuously according to the level of abatement costs over the interval \([c_1, c_2] \). Marginal abatement costs are assumed to be uniformly distributed with the continuous distribution function \( g(c) \) and the cumulative distribution function \( G(c) \). Both functions are known to the welfare maximizing regulator and the countries, while specific marginal abatement costs are the countries’ private information.

The RA can calculate the expected aggregated abatement costs \( C \) and benefits \( B \). Costs and benefits depend on the expected amount of global emission reductions \( q \). Following Weitzman (1974), Baumol and Oates (1988) and Montero (1999), we use quadratic approximations for the abatement cost and benefit curve. Cost and benefit uncertainty enters additively into the linear marginal curves, affecting the level but not the slope of the marginal curves.\(^9\) The marginal cost and benefit curves under uncertainty are hence

\[
\frac{\partial C(q, \theta)}{\partial q} = (c + \theta) + C'' q \\
\frac{\partial B(q, \eta)}{\partial q} = (b + \eta) + B'' q,
\]

where \( C'' > 0, c \equiv C''(0), B'' < 0 \) and \( b \equiv B'(0) \) are fixed coefficients. Furthermore, it is assumed that \( B'(0) > C'(0) \) and \( C'(q) > B'(q) \) for a sufficiently large \( q \), which rules out corner solutions to the regulator’s optimization problem. \( \theta \) is a random shock to marginal costs and \( \eta \) respectively a random shock to marginal benefits. Let \( E[\theta] = 0, E[\theta^2] = \sigma_\theta^2 \) and \( E[\eta] = 0, E[\eta^2] = \sigma_\eta^2 \) be the expected value and the variance of the stochastic terms. Uncertainties are assumed to be uncorrelated, i.e. \( E[\theta \eta] = 0 \). Note that the linearity of the marginal cost curve is a result of the uniform distribution of \( c \).\(^10\)

The RA is also responsible for enforcing the regulation. Regardless of the implemented regulatory instrument, countries are required to submit a report, indicating

\(^9\)See Malcomson (1978) and Laffont (1977) for a discussion on Prices vs. Quantities if the slopes of the marginal benefit and cost curves are uncertain.

\(^10\)If \( g(c) \) is a uniform distribution function of \( c \), then \( g(c) = 1/C'' \) where \( C'' = (c_2 - c_1) \) for \( c \in [c_1, c_2] \).
whether they are compliant or not. However, the correctness of these reports can only be verified through costly inspections. The regulator randomly monitors those countries, which claim to be in compliance. Countries are penalized by the RA if inspections reveal a violation of the regulatory obligations. In this case, countries face a sanction with the monetary value $F$ and the requirement to return to compliance.\footnote{Penalty schemes forcing noncompliant participants to pay fines and to return to compliance are rather the norm than the exception. In the Kyoto-Protocol, countries which do not meet their targets have to over-fulfill their reduction target in the next commitment period by the respective amount, plus 30\% (see UNFCCC (2006)).} The latter implies that countries have to ultimately reduce one unit of pollution as their report indicates.\footnote{We discuss a modified enforcement policy in section 7.} All noncompliant countries face the same penalty $F$.\footnote{Livernois and McKenna (1999) imposes a different penalty scheme for noncompliance. Firms which truthfully report their noncompliance have to pay a low fine whereas firms claiming compliance but found to be in violation during costly inspections have to pay a higher fine.} Note that under these circumstances, it is the dominant strategy for all countries to send a report claiming compliance, whether this is true or not. As a consequence, all countries face the same constant probability $\alpha_i$, $i \in \{t, q\}$, of being inspected, where the subscript $"t"$ refers to taxes, $"q"$ to quantities. In order to model the disadvantages of price-based regulations laid out in the introduction, we assume that the enforcement probability for a quantity-based regime is higher than for a price-based regime, i.e. $\alpha_q > \alpha_t$.

In the setup presented here, we assume an exogenous enforcement policy under which $\alpha_q$, $\alpha_t$ and $F$ are given. The enforcement probabilities $\alpha_q$ and $\alpha_t$ are strictly smaller than one, e.g. because of the RA’s limited monitoring budget.\footnote{See among others Stigler (1970).} The sanction $F$ is insufficiently high to induce full compliance and is beyond the RA’s scope as the penalty is a result of political negotiations. International regulations and sanction mechanisms have to be accepted unanimously by all participants. Insufficiently large sanctions seem plausible, as some countries anticipate that they prefer non-compliance once their actual costs of abatement are revealed.\footnote{In reality, monetary sanctions for noncompliance are rather low as it is argued in Livernois and McKenna (1999) for national regulations. The imposition of severe sanctions on the international level seems even more challenging.} We therefore assume that the above mentioned limitations lead to a situation where the permit price, or the tax rate respectively, lie above the expected costs from noncompliance. We continue with determining the countries’ compliance strategy for both policies under incomplete enforcement before deriving the optimal policy design.

### 3 Compliance strategy under incomplete enforcement

Under the price-based regime, the RA levies a uniform emission tax $t$. Countries are monitored with probability $\alpha_t$. In the case of a quantity-based regulation, the RA distributes a total amount of $l$ pollution permits. It can be shown that there is no difference whether the RA allocates pollution allowances for free or auctions them off. Assume for the moment pollution permits are auctioned off. These allowances are tradable at a market clearing price $p$.\footnote{The market clearing condition will be established at the end of this section.} Countries are monitored with probability $\alpha_q$. 

\begin{enumerate}
\item[11] Penalty schemes forcing noncompliant participants to pay fines and to return to compliance are rather the norm than the exception. In the Kyoto-Protocol, countries which do not meet their targets have to over-fulfill their reduction target in the next commitment period by the respective amount, plus 30\% (see UNFCCC (2006)).
\item[12] We discuss a modified enforcement policy in section 7.
\item[13] Livernois and McKenna (1999) imposes a different penalty scheme for noncompliance. Firms which truthfully report their noncompliance have to pay a low fine whereas firms claiming compliance but found to be in violation during costly inspections have to pay a higher fine.
\item[15] In reality, monetary sanctions for noncompliance are rather low as it is argued in Livernois and McKenna (1999) for national regulations. The imposition of severe sanctions on the international level seems even more challenging.
\item[16] The market clearing condition will be established at the end of this section.
\end{enumerate}
Countries aim at minimizing costs but have to pay the sanction $F$ and have to reduce one unit of pollution if they are found to be in violation with the regulation. Under these circumstances, the single country’s compliance strategy derived below is the same as the one established in Montero (1999).

The set of countries can be divided in two subsets, depending on their marginal abatement costs relative to the ’price’ per unit of emission $x$. The emission price corresponds to the tax rate $t$ under a price-regime and to the permit price $p$ under a quantity regime.

First, countries whose marginal abatement costs are lower than the ’price’ per emission, $c < x$, never consider paying $x$ as part of their compliance strategy. In this case, the respective countries prefer reducing their emissions and submit a truthful compliance report as long as their marginal abatement costs are lower than their expected costs of noncompliance, that is if

$$c < \alpha_i(F + c),$$

with $i \in \{t, q\}$ and $i = t$ for $x = t$, respectively $i = q$ for $x = p$. The threshold level of marginal costs for a truthful compliance report is

$$\tilde{c}_i = \frac{\alpha_i}{1 - \alpha_i}F.$$  \hspace{1cm} (1)

Hence, countries with very low marginal abatement costs, i.e. $c \leq \tilde{c}_i$, reduce one unit of pollution and submit a truthful compliance report. In contrast, countries with higher marginal abatement costs, $\tilde{c}_i < c < x$, do not comply, submit a false report and claim to be in compliance.

Those countries whose marginal abatement costs are higher than the emission ’price’, i.e. $c \geq x$, never consider reducing their emissions. These countries pay the ’price’ for emissions and submit a truthful compliance report as long as $x$ is lower than the expected costs of noncompliance, that is if

$$x < \alpha_i(F + c).$$

Note, this implies that countries found in violation are forced to abate. The threshold level $\tilde{c}_i$ for which a truthful compliance report is filed by a high cost country is

$$\tilde{c}_i = \frac{x}{\alpha_i} - F.$$  \hspace{1cm} (2)

Those countries with very high marginal abatement costs, i.e. $\tilde{c}_i \leq c \leq \bar{c}$, pay the ’price’ per unit of emission and submit a truthful compliance report. Finally, countries with intermediate marginal abatement costs, i.e. $x \leq c < \tilde{c}_i$, prefer noncompliance. They submit a false report and claim to be in compliance.

In summary, only countries with either very low marginal abatement costs—i.e. $c \leq \tilde{c}_i$—or very high marginal abatement costs—i.e. $\tilde{c}_i \leq c \leq \bar{c}$—fulfill their legal obligations. They comply by reducing one unit of pollution or by paying the ’price’ $x$ for one unit of emission. All other countries, $\tilde{c}_i < c < \tilde{c}_i$, never comply. Those countries submit a false report and claim to be in compliance.

Note that these thresholds exist only if noncompliance is an attractive option for countries. That is, at least for some countries the expected costs for noncompliance have to be lower than the ’price’ per unit of emission $x$. For each regulatory regime, we can hence define a penalty $\bar{F}_i$ which solves

$$\alpha_i(c + \bar{F}_i) = x,$$
as the minimum sanction which is necessary to induce full compliance. All countries comply if the sanction $F$ is set prohibitively high, i.e. $F \geq \bar{F}$, as noncompliance would be too costly. In this case the threshold $\hat{c}$ becomes irrelevant, as $\hat{c} \geq \tilde{c}$.\(^{17}\) Yet, we already argued that the enforcement parameters, $\tilde{c}$ and $\hat{c}$, can be assumed to be insufficiently large to induce full compliance. As we are interested in the case where both instruments are subject to incomplete enforcement, we assume in the following that for both instruments there exist some countries for which the expected costs from violation are lower than the tax rate and the permit price respectively. More formally, we establish Assumption 1, which holds throughout this paper.

**Assumption 1.** The sanction $F$ is always lower than the minimum sanction, necessary to induce full compliance in both regimes,

$$F < \bar{F} = \min\{\bar{F}_q, \bar{F}_t\} \iff \frac{\alpha_i}{1-\alpha_i} F < x.$$ 

The fundamental difference between the policy instruments is the 'price' per unit of emission $x$. In a price-based regulation, the tax rate $t$ is exogenously set by the RA. In contrast, under a quantity approach, the permit price $p$ is endogenously determined by supply and demand of certificates. The equilibrium price of a permit under certainty about costs satisfies the market clearing condition (auctioning clearing price)

$$l = 1 \int_{\hat{c}}^{\bar{c}} g(c)dc.$$ 

The left hand side of (3) determines the supply side of certificates. The RA offers a total of $l$ certificates. The right hand side determines the demand side of the market. The demand for permits is driven by high cost countries that buy $1$ certificate to comply.\(^{18}\) Furthermore, the market clearing condition indicates that no noncompliant country returns to the market to buy certificates. Countries that are found to be in violation with the regulation are forced to reduce one unit of pollution as their report indicates.\(^{19}\)

Substituting (2) into (3) yields the equilibrium permit price, $p_c$, under certainty about marginal costs and benefits,

$$p_c = \alpha_q G^{-1}(1 - l) + \alpha_q F,$$ 

\(^{17}\)This is in line with the rationale presented in Becker (1968), according to which an increase in enforcement probability and/or higher fines will decrease the amount of violations.

\(^{18}\)Note that grandfathering is equivalent to auctioning. In case the RA allocates all pollution permits for free, each single country holds an initial amount of $l$ pollution permits. The threshold levels for a truthful compliance report, $\hat{c}_q$ and $\hat{c}_t$ remain unchanged. The market clearing condition changes to

$$l \int_{\hat{c}_q}^{\bar{c}_q} g(c)dc = (1 - l) \int_{\hat{c}_t}^{\bar{c}_t} g(c)dc$$

which is identical to (3). The left hand side determines the supply side of certificates, i.e. low cost compliant and all noncompliant countries. The right hand side determines the demand side of the market, consisting of high cost countries buying $(1 - l)$ certificates to comply. Hence, it is irrelevant for the compliance strategy of countries whether the RA allocates all pollution permits for free or auctions them off.

\(^{19}\)Of course, this is cost inefficient. Countries with marginal abatement costs higher than the 'price' per unit of emission are forced to reduce domestically rather than paying the tax or purchasing certificates.

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where $G^{-1}(1-l)$ reflects the marginal abatement costs just after $(1-l)$ emissions have been reduced. Hence, $G^{-1}(1-l)$ can be interpreted as the permit price $p_{full}$, which would be observed under full compliance, i.e. if $\alpha_q = 1$ and $F = 0$.

It is important to see that quantity instruments, in contrast to price instruments, are affected by abatement cost uncertainty as the permit price varies. Under uncertainty the market clearing condition (3) changes to

$$l = \int_{\theta}^{\tau+\theta} g(c-\theta) dc.$$

The permit price is therefore given by

$$p(l, \theta) = \alpha_q G^{-1}(1-l) + \alpha_q F + \alpha_q \theta = p_c + \alpha_q \theta.$$  \hspace{1cm} (5)

The permit price is now stochastic. Whether the price is higher or lower than the price under certainty, $p_c$, depends on the realization of $\theta$. Note further, incomplete enforcement reduces permit price fluctuations as noncompliance and the purchase of certificates become imperfect substitutes (Montero (1999, 2002)). We now determine the optimal policy design under incomplete enforcement and uncertainty about costs and benefits.

4 Optimal policy design with incomplete enforcement and uncertainty

In the following we assume that the benevolent RA is aware of the countries’ compliance strategy. Given the presence of incomplete enforcement, policy designs will be second-best optimal.\textsuperscript{20} Including uncertainty, both policy instruments are ex-ante second-best optimal. However, neither policy is likely to be ex-post optimal once uncertainty is resolved.

The RA’s objective is to maximize expected net social benefits from emission reductions. Under a price-based regulation the RA chooses the tax level $t$ that maximizes

$$E[W(t, \alpha_t, \theta, \eta)] = E[B(q_t(t, \alpha_t, \theta), \eta) - C(q_t(t, \alpha_t, \theta))].$$  \hspace{1cm} (6)

Under a quantity regime, the regulator chooses the amount of emission permits $l$ to maximize

$$E[W(l, \alpha_q, \theta, \eta)] = E[B(q_q(l, \alpha_q, \theta), \eta) - C(q_q(l, \alpha_q, \theta))].$$  \hspace{1cm} (7)

The expected abatement costs and benefits depend on the expected amount of global emission reductions, $q_i$, which are realized by low-cost countries and countries that are caught violating the regulation. The expected amount of global emission reductions is hence in the case of a price-based regulation

$$q_t(t, \alpha_t, \theta) = \int_{\tau+\theta}^{\tau} g(c-\theta) dc + \alpha_t \int_{\theta}^{\tau} g(c-\theta) dc.$$

\textsuperscript{20}Social welfare is reduced if policy makers implement a first best policy into a second-best world, see Montero (1999).
and in the case of a quantity-based regulation

$$q_q(l, \alpha_q, \theta) = \int_{c+\theta}^{\hat{c}_q} g(c - \theta)dc + \alpha_q \int_{c_q}^{\hat{c}_q} g(c - \theta)dc.$$  

As we assume $g(c)$ being a uniform distribution of $c$, the corresponding emission reductions can be reduced to

$$q_t(t, \alpha_t, \theta) = \frac{t - \xi - \theta}{\xi'^t}$$  

and to

$$q_q(l, \alpha_q, \theta) = \frac{p_c - \xi - (1 - \alpha_q)\theta}{\xi'^q}.$$  

Note, with incomplete enforcement expected emission reductions are uncertain under a quantity-based regulation which turns out to be a crucial advantage of quantity-based over price-based instruments (Montero (1999, 2002)).

The associated aggregate abatement costs will be

$$C(q_i, \alpha_i, \theta) = \int_{c+\theta}^{\hat{c}_i} c(g(c - \theta)dc + \alpha_i \int_{c_i}^{\hat{c}_i} c g(c - \theta)dc.$$  

Substituting (8) and (10) into (6), and taking the derivative with respect to $t$, yields the First Order Condition

$$E \left[(b + \eta) \frac{\partial q_t(t, \alpha_t, \theta)}{\partial t} + B'' q_t(t, \alpha_t, \theta) \frac{\partial q_t(t, \alpha_t, \theta)}{\partial t} - \left( \frac{t - \xi - F}{C''} \right) \frac{1}{C''} \right] = 0.$$  

Taking expectations and solving the FOC for $t$ yields the ex-ante second-best tax under uncertainty

$$t^* = \alpha_t \frac{bC'' - cB'' + FC''}{C'' - B''\alpha_t}.$$  

In the case of a quantity regime, we derive the optimal amount of certificates indirectly. Note that $p$ as determined by (5) is a function of $l$. We therefore maximize (7) over $p$ and then derive the optimal amount of emission allowances. Substituting (9) and (10) into (7), maximizing over $p$ yields the First Order Condition

$$E \left[(b + \eta) \frac{\partial q_q(l, \alpha_q, \theta)}{\partial p} + B'' q_q(l, \alpha_q, \theta) \frac{\partial q_q(l, \alpha_q, \theta)}{\partial p} - \left( \frac{p - \xi}{\alpha_q} - F \right) \frac{1}{C''} \right] = 0.$$  

Taking expectations and solving the FOC for $p$ yields the ex-ante second-best permit price under uncertainty

$$p^* = \frac{\alpha_q (bC'' - cB'' + FC'')}{{C'' - B''\alpha_q}}.$$  

Replacing $p$ by $p^*$ in (5), taking expectations, and solving for $l$, yields the ex-ante second-best optimal amount of pollution permits, $l^*$.

Given these insights, we can establish the following propositions.

**Proposition 1.** $t^* < p^*$
The optimal permit price $p^*$, which results from allocating $l^*$ pollution allowances, is higher than the optimal tax rate $t^*$ but lower than the ‘price’ per unit of emission under complete enforcement.

**Proof of Proposition 1**: The tax rate and the permit price are identical under complete enforcement (i.e. for $\alpha_i = 1$ and $F = 0$),

$$t_{full} = p_{full} = \frac{bC'' - cB''}{C'' - B''}$$

(13)

Substituting (2) into (5) and taking expectations yields $\hat{c}_q = G^{-1}(1 - l) = p_{full}$. Using $p < \hat{c}_q$ by construction, yields $p^* < p_{full}$. The fact that $p^* = t^*$ for $\alpha_q = \alpha_t$, $\partial p^*/\partial \alpha_q > 0$ and $0 < \alpha_t < \alpha_q < 1$ by construction, completes the proof.

**Proposition 2.** $q_t(t^*) < q_q(l^*)$

The expected amount of global emission reductions under incomplete enforcement is lower than under complete enforcement. Emission reductions under a price-based regulation, $q_t(t^*)$, are lower than under a quantity-based regulation, $q_q(l^*)$.

The proof of this proposition is, given the proof of Proposition 1, straightforward, and hence omitted here.

The fact that $t^* < p^*$ and that $q_t(t^*) < q_q(l^*)$ is an important feature of the approach presented here and a central difference to Montero (1999). As will be shown below, instrument choice is crucially affected by this feature. Most important for our analysis is the fact that Proposition 1 implies that Assumption 1 will only be fulfilled if $F < \frac{(1 - \alpha_q)(bC'' - cB'')}{\alpha_q(C'' - B'')}$. The latter inequality is hence the prerequisite for the plausible case of incomplete enforcement for both policy instruments. Having thusly specified the upper boundary for the range of possible sanctions considered here, we can now proceed with deriving the welfare difference for both policy instruments.

### 5 Instrument choice

We now turn to the central question of our paper and calculate the expected difference in social welfare of price-based over quantity-based instruments under incomplete enforcement, uncertain costs and benefits and different enforcement probabilities, i.e.

$$\Delta_{pq} = E[W(t^*, \alpha_t, \theta, \eta) - W(l^*, \alpha_q, \theta, \eta)].$$

(14)

Quantity-based regulation provides higher expected social welfare than price-based regulation and ought to be preferred as regulatory instrument if $\Delta_{pq} < 0$. Conversely, a price regime performs relatively better if $\Delta_{pq} > 0$.

Substituting (11), (12), $q_t(t^*)$, and $q_q(l^*)$ into (14), taking expectations, and rearranging terms yields the expected difference in social welfare of price-based over quantity-based regulations

$$\Delta_{pq} = \frac{\sigma^2 q^2 \sigma q}{2(C''')^2} \left[ C'' + B'' + (1 - \alpha_q)B'' \left( \frac{\alpha_q - \alpha_t}{2C''} \right) \right] +$$

$$\frac{(\alpha_q - \alpha_t)}{2C''} \left[ \frac{F^2}{(1 - \alpha_q)(1 - \alpha_t)} - \frac{(FC'' + bC'' - cB'')^2}{(C'' - \alpha_qB'')(C'' - \alpha_tB'')} \right]$$

(15)

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This equation is a main result of our paper and necessarily deserves to be discussed in more detail. First, note that under complete enforcement of both instruments (i.e. \( \alpha_q = \alpha_t = 1 \)), equation (15) is reduced to the welfare difference presented in Weitzman (1974). However, for incomplete enforcement with different enforcement probabilities, \( \Delta_{pq} \) features two additively separable effects. Interestingly, the cost uncertainty \( \sigma^2 \theta \) only enters into the first effect, while the enforcement probability for the price regime \( \alpha_t \) only occurs within the second term. For this reason we refer to the first term as uncertainty effect and to the second as differentiated enforceability effect.

Quite remarkably, the first term, i.e. \( \frac{\sigma^2 \alpha_q}{C''(1-\alpha_q)^2} [C'' + B'' + (1-\alpha_q)B''] \), is identical to the result derived in Montero (1999), yet specified for the enforcement probability of quantity-based instruments. The dependence of \( \alpha_q \) can be deduced from a specification of the effect’s rationale presented in Montero (1999, 2002). The amount of emission reductions under a quantity approach varies under incomplete enforcement and reacts to possible cost shocks. This affects the aggregated costs and benefits from pollution abatement under a quantity instrument. The reason for this is the multiplicative interaction of \( \alpha_q \) and \( \theta \), as specified by (9). Consequently, compared to the full enforcement scenario, the benefit advantage of a quantity approach reduces to \( (2-\alpha_q)\alpha_q \), while the cost advantage of a price approach reduces to \( \alpha_q \). The latter dominates the former effect, as \( (2-\alpha_q)\alpha_q > \alpha_q \). Hence, uncertain emission reductions under a quantity-based instrument, \( q_q(l, \alpha_q, \theta) \), increase the advantage of a quantity approach. The uncertainty effect is thus negative and in favor of a quantity regime as long as

\[
|B''| > \frac{C''}{(2-\alpha_q)}.
\]

The main novelty in equation (15) is in fact its lower term, which is neither dependent on cost nor on benefit uncertainty. This differentiated enforceability effect becomes zero and the overall welfare difference is identical to the one derived in Montero (1999), if no difference in enforcement exists, i.e. \( \alpha_q = \alpha_t \). However, as laid out in the introduction, it is plausible to assume that the enforcement of a quantity regime on the international level will be higher than that of a price scheme. Therefore, the differentiated enforceability effect reflects the welfare effects resulting from the difference in enforcement for quantities and taxes. The main driver of this welfare effect is the difference in the expected amount of emission reductions for the two policy regimes, as was summarized in Propositions 1 and 2. Hence, the sign of the differentiated enforceability effect is unambiguously negative, as the expected amount of pollution abatement under a quantity approach is higher than under a price approach. The differentiated enforceability effect hence measures the net gain in social welfare from implementing the stricter enforceable policy instrument. This result is summarized in the following proposition.

**Proposition 3.** For \( \alpha_q > \alpha_t \) and \( |B''| = \frac{C''}{(2-\alpha_q)} \), the expected welfare difference \( \Delta_{pq} \) is strictly negative.

*Proof of Proposition 3:* See Appendix A.1.

Hence, the welfare difference presented in (15) is, ceteris paribus, more likely to be negative compared to the simple incomplete enforcement case analyzed in Montero(1999, 2002). We can hence derive the following corollary.
Corollary 1. If enforcement of quantity-based instruments is stricter than that of price-based instruments the set of situations for which quantity-based instruments ought to be preferred increases.

This result provides some formal support to the often-expressed opinion that, as far as global externalities like climate change are concerned, instrument choice should not be solely based on the relative-slope criterion. In order to give a notion of the relative strength of the two potentially countervailing effects in the welfare difference, we provide numerical simulations in the following section. It is, however, useful to first proceed with a more thorough analysis of the analytical results.

Obviously, the choice of instrument is no longer solely determined by the relative slopes of the marginal curves but also by institutionally determined variables $\alpha_i$, as well as level effects. An increase in the intercepts of the marginal curves, i.e. $b_i$ and $c_i$, unambiguously strengthens the differentiated enforceability effect. An increase in $b_i$ shifts the marginal benefit curve upwards, which increases the benefits from emission reductions. Clearly, under incomplete enforcement, this becomes an important determinant of the expected welfare from the instruments, as each actor violating the regulation without being discovered decreases the welfare gains achieved. An increase in $c_i$ reduces ceteris paribus the optimal amount of pollution abatement. Yet, given $\alpha_q > \alpha_t$, the reduction in $q_t$ would be larger than in $q_q$. The differentiated enforceability effect becomes stronger with the level of the marginal curves, which leads to an overall increase of the relative performance of the quantity instrument. The effect of a change in the enforceability of quantity-based regulations on instrument choice is ambiguous. More countries initially comply and the difference in enforceability of the instruments increases with stricter enforcement of quantity-based instruments. Clearly, this leads to a strengthening of the differentiated enforceability effect as the second best outcome under incomplete enforcement moves closer to the first best solution (complete enforcement). The advantage of quantity-based instruments increases. On the other hand, an increase in $\alpha_q$ also increases the uncertainty effect. The advantage of quantity-based over price-based regulations, resulting from an uncertain amount of pollution abatement under a quantity approach, is reduced if the enforcement probability of quantity instruments increases. Which of these two countervailing effects dominates is in general ambiguous. In contrast, stricter enforcement of price-based instruments unambiguously increases the set of situations under which price-based regulations ought to be preferred. An increase in $\alpha_t$ reduces the difference in enforceability of the instruments. As a consequence, the disadvantage of a price approach reduces with stricter enforcement of price-based instruments as the optimal tax rate $t^*$ converges with the optimal permit price $p^*$. This unambiguously diminishes the differentiated enforceability effect, whereas the uncertainty effect is unaffected by a variation of $\alpha_t$. Hence, the intuitively plausible rationale that a lower enforcement probability for price-based regulations will render such instruments less attractive is confirmed.

Further insights can be gained if (15) is simplified and rearranged as follows. First, we assume $c = 0$. This implies that the country with the lowest marginal costs can abate one unit of pollution without cost.\(^{21}\) Second, we introduce the ratio $\beta$, measuring the relative slopes of the marginal benefit curve and the marginal cost

\(^{21}\) Note that this assumption is for many pollutants quite plausible. E.g., the intercept for the abatement cost curve for Greenhouse Gases presented in Enkvist et al. (2007), lies even below 0, at -150€/tCO$_2$e (Euros per ton of carbon dioxide equivalents). Note that departing from the assumption $c = 0$ would render quantity-based instruments even more favorable, as the differentiated enforceability effect strengthens with $c$.\(^{21}\)
curve, with $\beta = -B''/C''$, where $\beta \in \mathbb{R}^+$. Both curves have the same absolute slope if $\beta = 1$. The marginal abatement cost curve runs relatively steeper than the marginal benefit curve if $\beta < 1$ and is flatter in the opposite case. Furthermore, we express $\alpha_t$ as a share $k$ of the enforcement probability for quantity-based instruments, i.e. $\alpha_t = k\alpha_q$, with $k \in [0,1]$. Clearly, $k$ represents a measure for the relative enforceability of the two policy instruments. Taking all these modifications into account, equation (15) can be re-written as:

$$\Delta_{pq} = \frac{\sigma^2\alpha_q(1 - (2 - \alpha_q)\beta)}{2C''} + \frac{\alpha_q(1-k)}{2C''} \left[ \frac{F^2}{(1-\alpha_q)(1-k\alpha_q)} - \frac{(F + b)^2}{(1 + \alpha_q\beta)(1 + k\alpha_q\beta)} \right].$$ (16)

With this reformulation, the effects of a change in relative slopes, i.e. in $\beta$ become directly obvious. The first term, still representing the uncertainty effect, unambiguously increases with a decrease in $\beta$, which reduces the relative performance of quantity regulations. This comes to no surprise, as the uncertainty effect is in fact identical to the welfare difference calculated in Montero (1999), from which an altered version of Weitzman’s relative slope criterion was derived. More interesting is the effect of a change in $\beta$ on the second term in equation (16), which still represents the differentiated enforceability effect. It is now directly obvious that a decrease in $\beta$, i.e. a reduction of $|B''|$ relative to $C''$, strengthens the differentiated enforceability effect. This can be explained by the fact that a decrease in $\beta$ results in a larger increase in emission reductions for the quantity regime than for the price regime.\textsuperscript{22} As stated in Proposition 2, the emission reductions lie below the ex ante first-best level for both policy instruments. Hence, the expected second-best optimum under a quantity approach will be closer to the first best optimum than under a price approach. Given that cost uncertainty does not influence the differentiated enforcement effect, the influence of the relative slopes $\beta$ on this effect is hence quite intuitive.\textsuperscript{23}

The independence of the differentiated enforceability effect of cost uncertainty, also allows some further analytical considerations. Given that for $\beta < 1/(2 - \alpha_q)$ both effects in (16) have different signs, an increase in the variance also increases the uncertainty effect, which could outweigh the differentiated enforceability effect. However, if the variance is small enough the relative slope criterion becomes irrelevant and quantity-based instruments ought to be always preferred as regulatory instrument. Hence, it is possible to establish a level of the variance of costs $\sigma_\theta^2$, below which quantity-based regulation will be always preferable, independent of the relative slopes of the marginal cost and benefit curves. The result is summarized in the following proposition.

**Proposition 4.** For $\alpha_q > \alpha_t$, quantity-based regulation ought to be always preferred as regulatory instrument if

$$\sigma_\theta^2 < \sigma_\theta^2,$$

where

\textsuperscript{22}This can be easily seen by comparing $q_t(t^*)$ and $q_q(t^*)$ as determined by (8), (9),(11), and (12).

\textsuperscript{23}Note that if uncertainties are positively correlated, i.e. $E[\theta\eta] > 0$, an additional negative term enters into (15), which further increases the advantage of quantity-based instruments.
\[ \sigma_2^\theta = (1 - k) \left( \frac{(F + b)^2(2 - \alpha_q)^2}{2(2 - \alpha_q(1 - k))} - \frac{F^2}{(1 - \alpha_q)(1 - k\alpha_q)} \right) \] (17)

Proof: See Appendix A.2

An important implication of Proposition 4 is that the inclusion of different enforcement probabilities significantly alters the rationale commonly brought forward in the debate on Prices vs Quantities. First and most obviously, under the plausible assumption of different enforcement probabilities the relative slope criterion might become entirely irrelevant for rational instrument choice. Whether this is the case or not depends on the level of the variance of costs. Yet, this result is in itself in conflict with the general insights from the literature on Prices vs Quantities where the uncertainty is generally considered to have no influence on rational instrument choice.

Second, an increase in the level of the marginal benefit curve, \( b \), strengthens the differentiated enforceability effect and hence the advantage of quantity-based instruments. As a consequence, the level of \( \sigma_2^\theta \) increases with \( b \). Hence, it can be rightfully stated that if the level of marginal benefit curve is large enough, the importance of the relative slopes for instrument choice can become negligible. Again, this result is in contradiction with the facts established thus far within the debate on Prices vs Quantities, where the level of the marginal benefit curve is generally considered to have no influence on instrument choice.

Third, with the inclusion of different enforcement probabilities, optimal instrument choice is also dependent on institutional variables. For example, the threshold level \( \sigma_2^\theta \) unambiguously decreases in the difference in enforceability \( k \). Hence, the more difficult the enforcement of a price regulation relative to a quantity regime is, the larger is the set of situations for which the relative slope criterion is irrelevant for instrument choice.

In order to provide a notion on the relevance of the above-made analytical considerations, we present in the next section several numerical simulations of the expected difference in social welfare of price-based over quantity-based instruments by using plausible parameter ranges taken from the example of anthropogenic climate change.

6 Numerical Simulations

The above-presented analytical model is meant to conceptually identify in principle the relative effects of differentiated enforcement of environmental policies on the international level. The model is not meant to be directly explanatory for an actual real-world context. In order to get a notion of the relative strength of these effects in a specific situation numerical simulations with parameter combinations taken from a real-world example are quite useful. While the actual levels of the results presented below are to be interpreted with care, an observation of the general tendencies are surely particularly interesting. Hence, in the following, we present the results of numerical simulations based on parameter values derived from the context of international climate policy.

We calculate the expected difference in social welfare of price-based over quantity-based instruments \( \Delta_{pq} \) in dependence of the ratio of the slopes of the marginal curves \( \beta \). Table 1 gives an overview of the parameter values used for the numerical
The level of $C''$, $B''$, $\sigma^2_\theta$, and $b$ correspond to the data presented in Newell and Pizer (2003). Yet, the chosen level of the sanction $F$ deserves a further remark. We express the sanction $F$ as a share $\gamma$ of the sanction $\bar{F}$ defined as the smallest sanction which induces full compliance for at least one of both instruments, i.e. $F = \gamma \bar{F}$ with $\gamma \in [0,1]$. Following Assumption 1, considering (11), (12) and assuming $\xi = 0$, the smallest sanction which induces full compliance in at least one regime is specified as $F = \frac{(1-\alpha_q)b}{(1+\beta)\alpha_q}$.\footnote{See Appendix A.1 for a detailed derivation of $\bar{F}$.} Obviously the allowed sanction decreases with $\beta$. We therefore calculate the lowest sanction $\bar{F}$ for the relevant range of the ratio of the slopes of the marginal curves. The relevant range in which the assumption on different enforcement probabilities for the regulatory regimes may affect instrument choice is $\beta \in [0,1/(2-\alpha_q)]$. Given $b = 9$\$/t$ and assuming $\alpha_q = 0.8$, yields $\bar{F} \approx 1.23$\$/t$.

As already mentioned, in a real-world context the sanctioning mechanism established in international treaties is entirely determined through political negotiations. The scalar $\gamma$ can hence be interpreted as a parameter reflecting the level of the political determination of the member countries to actually commit to the agreed policies. Clearly, the sanction will be relatively close to the full enforcement level if polluters show a rather high level of commitment and expect to meet their regulatory obligations. Such a situation would correspond to a level of $\gamma$ close to 1. On the other hand, the sanction will be relatively low if a significant number of polluters expect to be in violation with the regulation. Such a political consensus would be reflected by lower $\gamma$-values. In our specification, we opted for choosing somewhat optimistically $\gamma = 0.8$, which yields $F \approx 0.98\$/t$.\footnote{This could lead to an interaction between the level of uncertainty and the penalty, which is not endogenously included in our model. However, such an endogenization would also have to take the differences in bargaining power of the different countries into account.}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope of marginal costs ($C''$)</td>
<td>$1.6 \times 10^{-13}$$/t^2$</td>
</tr>
<tr>
<td>Slope of marginal benefits ($B''$)</td>
<td>$-8.7 \times 10^{-13}$$/t^2$</td>
</tr>
<tr>
<td>Cost uncertainty ($\sigma^2_\theta$)</td>
<td>$13$$/t$</td>
</tr>
<tr>
<td>$b$</td>
<td>$98$$/t$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$98$$/t$</td>
</tr>
<tr>
<td>Enforcement probability of quantity-based instruments ($\alpha_q$)</td>
<td>0.8</td>
</tr>
<tr>
<td>Sanction (F)</td>
<td>$0.98$/t$</td>
</tr>
</tbody>
</table>

Table 1: Parameter values

Figure 1 illustrates the results of our numerical calculations based on (16). The expected difference in social welfare is positive, indicating the superiority of price-based regulation, if the curve is located in the first quadrant. Quantity-based regulations ought to be preferred if $\Delta p_q$ takes negative values. For comparison we depict also the results corresponding to the models presented in Weitzman (1974) (dotted...
curve) and Montero (1999) (dashed curve). According to the classical relative-slope criterion, price-based and quantity-based instruments perform equally if and only if the marginal benefit and marginal cost curves have the same slope in absolute values, i.e. \( \beta = 1 \), which is the case for the dotted curve. As already explained, incomplete enforcement increases the relative performance of quantity-based regulations (Montero (1999)), being illustrated in the dashed curve. The two curves below depict two different results derived from (16), assuming \( \alpha_q = 0.8 \). The curves differ in their assumption on the enforcement probability of price-based instruments in order to illustrate the importance of the difference in enforcement probabilities. We assume \( k = 0.75 \), corresponding to \( \alpha_t = 0.6 \), in the black curve and \( k = 0.375 \), corresponding to \( \alpha_t = 0.3 \), in the light gray curve. Obviously, the differentiated enforceability effect has a significant influence on instrument choice, by favoring quantity regulations, as the black and light gray curves are located strictly below the dashed curve. As discussed in the previous section, a lower enforcement probability of price-based instruments further increases the differentiated enforceability effect. Hence, the light gray curve lies even below the black curve. Furthermore, the differentiated enforceability effect reduces with \( \beta \). The light gray and black curves hence converge with the result from simple incomplete enforcement (dashed curve) for \( \beta \to \infty \).\(^{28}\)

Yet, in the special case of regulating CO\(_2\) emissions, the assumption of Newell and Pizer (2003) on the ratio of the slopes of the marginal curves corresponds to \( \beta \approx 5.4 \times 10^{-6} \). That is, price-based instruments still ought to be preferred to regulate CO\(_2\) emissions even though the enforcement of a quantity-based approach is stricter than that of a price-based approach.

Interestingly, the example of climate change is particularly useful to give a notion of an important feature of our result, which is the effect of the level of the marginal curves, in particular \( b \). Estimates for this parameter have varied over the years.

\[^{28}\lim_{\beta \to \infty} \frac{\alpha_q(1-k)F}{2(1-\alpha_q \beta)(1+\alpha_q \beta)}\] \( F(1-\alpha_q)(1-k\alpha_q) - \frac{(F+k)^2}{(1+\alpha_q \beta)(1+\alpha_q \beta)} \) = 0 if we substitute \( F = \gamma \cdot F \) with \( \gamma \in [0, 1] \) into the differentiated enforceability effect.
In the above-presented simulations we followed Newell and Pizer (2003) assuming \( b = 9\$/t \) for the initial marginal benefit from pollution abatement.\(^{29}\) More recent studies provide higher estimates for the level of marginal benefits. According to Downing et al. (2005) it is very likely that marginal benefits exceed 50\$/t. In Tol (2005), 103 estimates from 28 published studies were gathered to form a probability density function. Taking estimates only from peer-reviewed studies reduces the mean to 50\$/t. Further support for higher levels of \( b \) is given by the much discussed results of the Stern Review from 2006, which presents estimates of the marginal benefits for a 450–550 ppm \( CO_2 \) concentration target at about 25 to 30\$/t.\(^{30}\) \(^{31}\) \(^{32}\) In order to reflect the impact of changes in the level of the marginal benefit curve, suggested by these more recent estimates, we rerun our numerical simulations assuming \( b = 30\$/t \). All other parameters remain unchanged, in order to be able to compare the different scenarios. The results are illustrated in Figure 2. As before, we assume \( k = 0.75 \) in the black curve, respectively \( k = 0.375 \) in the light gray curve. The dotted and dashed lines again depict the (unchanged) results based on Weitzman (1974) and Montero (1999). Obviously, the results from (16) depicted in the light gray and black curve of Figure 2 are strictly negative. That is, quantity-based regulations ought to be always preferred, even though the benefit curve is close to being linear, i.e. \( \beta \to 0 \). Arguing in terms of Proposition 4, the threshold level \( \sigma^2_\theta \), which is solely determined by institutional variables, became relevant. In response to the increase in the intercept of the marginal benefit curve \( b \), renders \( \sigma^2_\theta < \sigma^2_\theta \).

Figure 3 gives further insights into the relevance of the threshold level of the variance

\(^{29}\)This estimation is comparable to estimates from Falk and Mendelsohn (1993) and Nordhaus (1994).

\(^{30}\)Following the “Business as usual” trajectory, estimates for the marginal benefits from emission reductions approach 85\$/t. (Stern (2006))

\(^{31}\)See Yohe et al. (2007) for an overview on recent estimates for the marginal benefits from reducing \( CO_2 \) emissions.

\(^{32}\)Estimates of the marginal benefits from pollution abatement of the above mentioned studies refer to near future marginal benefits. The initial marginal benefit from pollution abatement can hence be seen to be even higher.
of costs. We calculate $\sigma^2_\theta$ in dependence of the relative enforceability of the policy instruments $k$ and the level of the marginal benefit curve $b$, assuming $\alpha_q = 0.8$ and $\gamma = 0.8$. As already discussed in the previous section, the threshold level of the variance of costs increases with $b$ as the impact of the differentiated enforceability effect increases. The effect of a change in the enforcement probability of price-based instruments $\alpha_t$ is reflected in a variation of $k$. The threshold level of the variance of costs decreases with $k$ and converges to zero, if enforcement of both regimes is identical, corresponding to $k \to 1$. The threshold $\sigma^2_\theta$ thus increases with an increase in the level of the marginal benefit curve and a decrease in the enforcement probability of price-based instruments. In order to highlight the relevance of the threshold level of the variance of costs in the context of regulating $CO_2$ emissions, we include a plane representing the estimated variance presented in Newell and Pizer (2003), i.e. $\sigma^2_\theta = (13$/t$)^2$. All $\sigma^2_\theta$-values above this plane fulfill Proposition 4. That is, a quantity-based approach performs always better than a price-based approach irrespective of the ratio of the slopes of the marginal curves.

![Figure 3: $\sigma^2_\theta$ dependent on $k$ and $b$](image)

### 7 Extension: An Alternative Enforcement Policy

In the above-described sections, it is assumed that a country found in violation has to pay the sanction and is forced to comply with the regulation by reducing the respective emission domestically. Evidently, this is inefficient because countries with marginal abatement costs higher than the tax rate or the permit price are forced to abate pollution. In this section we extend the model by introducing a different enforcement policy. When found in violation, countries still have to pay a sanction but can now decide whether to comply through domestic abatement or by paying the tax or permit price instead. Hence, the model changes as follows:

The cut-off points for a truthful compliance report of low-cost countries ($c < p^n$...
respectively \( c < t^n \) remain

\[
\tilde{c}_i = \frac{\alpha_i}{1 - \alpha_i} F.
\]

In contrast to the previous enforcement policy, high-cost firms never consider reducing emissions but rather pay the tax or buy certificates to meet their obligations. The upper cut-off point for a truthful compliance report becomes

\[
\hat{c} = \tau,
\]

because Assumption 1 still has to be satisfied. Low cost countries, \( c \leq \tilde{c}_i \), comply whereas high costs countries, \( \tilde{c}_i < c \leq \tau \), never comply.

Due to the new enforcement policy, expected aggregated emission reductions amount to

\[
q(t, \alpha_t, \theta) = \int_{c+\theta}^{\tilde{c}_i} g(c-\theta)dc + \alpha_t \int_{\tilde{c}_i}^{c} g(c-\theta)
\]

under a price regime and to

\[
q^*(l, \alpha_q, \theta) = \int_{c+\theta}^{c_q} g(c-\theta)dc + \alpha_q \int_{c_q}^{l} g(c-\theta).
\]

under a quantity regime. The welfare maximizing tax under a price-based regulation is

\[
l^n = \frac{B'' F \alpha_q + C'' b - B'' \bar{c}}{C'' - B'' \alpha_q}.
\]

In case of a quantity-based regulation, the permit price again follows a market clearing condition which changes to

\[
l^n = \alpha_q \int_{p^n}^{p^n+\theta} g(c-\theta)dc
\]

and hence

\[
p^n(l^n, \theta) = G^{-1}(1 - \frac{l^n}{\alpha_q}) + \theta = p^n_c + \theta
\]

The demand for permits is driven by countries with high marginal abatement costs \( c \geq p \) which are found to be in violation and are forced back into compliance. These countries prefer to buy certificates rather than to abate domestically. If certificates are auctioned off, the RA supplies \( l^n \) allowances. In case of grandfathering, all types of countries sell their certificates, since low cost countries reduce emissions and high cost countries never comply. As before, we maximize the expected social welfare over \( p^n \), which yields

\[
p^{n*} = \frac{B'' F \alpha_q + C'' b - B'' \bar{c}}{C'' - B'' \alpha_q}.
\]

Replacing \( p^n \) by \( p^{n*} \) in (21), solving for \( l^n \), and taking expectations yields the second-best optimal amount of pollution permits, \( l^{n*} \).

There is a significant difference between this new and the previous enforcement policy. We therefore establish

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33 The superscript "n" refers to the new enforcement policy.

34 As in the previous enforcement policy, \( p^n_c \) denotes the equilibrium permit price under certainty.
Proposition 5. $p^{n^*} < t^{n^*}$

The distribution of $l^{\alpha_{q}}$ certificates results in a permit price $p^{n^*}$ which is lower than the optimal tax rate $t^{n^*}$, but higher than the ‘price’ per unit of emission under complete enforcement.

Proof of Proposition 5: Using the fact that $p^{\alpha_{full}} = t^{\alpha_{full}}$ under complete enforcement (e.g. if $F = 0$ and $\alpha_{t} = 1$), $p^{n^*} = t^{n^*}$ for $\alpha_{q} = \alpha_{t}$, considering Assumption 1, $\partial p^{n^*}/\partial\alpha_{q} < 0$ and $0 < \alpha_{t} < \alpha_{q} < 1$ by construction, completes the proof.

Note, despite the fact that Proposition 1 does no longer hold, Proposition 2 remains valid. The expected amount of pollution abatement is the same under both enforcement policies, i.e. $q^{n^*}(t^{n^*}) = q^{t^*}(t^*)$, respectively $q^{n^*}(l^{n^*}) = q^{l^*}(l^*)$.

We again calculate the expected difference in social welfare of price-based over quantity-based instruments under uncertainty and different enforcement probabilities, $\Delta_{pq}$. Substituting (20), (22), $q^{n^*}(t^{n^*})$, and $q^{n^*}(l^{n^*})$ into (14), taking expectations, assuming $E[\theta\eta] = 0$, and rearranging terms yields the expected difference in social welfare of price-based over quantity-based regulations

$$\Delta_{pq} = \frac{\sigma^{2}}{2C''} \left[ C'' + B'' + (1 - \alpha_{q})B'' \right]$$

$$\frac{(\alpha_{q} - \alpha_{t})}{2C''} \left[ \frac{F^2}{(1 - \alpha_{q})(1 - \alpha_{t})} - \frac{(FC'' + BC'' - CB'')^2}{(C'' - \alpha_{q}B'')(C'' - \alpha_{t}B'')} \right]$$

which is identical to (15). This result was to be expected as Proposition 2 still holds. We establish

Proposition 6. Incomplete enforcement with different enforcement probabilities affects instrument choice (see Proposition 3). Whether noncompliant countries found to be in violation can choose their compliance strategy or are forced to reduce domestically, does not affect instrument choice.

The proof is obvious and therefore omitted here.

Proposition 6 has important implications for policy recommendations. Compliance rules may enjoin the exclusion of polluters which failed to meet their regulatory obligations from certificate trading respectively tax payments. Polluters are then forced to reduce domestically regardless of their reduction costs. Other regulations let emitters decide how to meet their obligation. Yet, as Proposition 6 argues, enforcement rules do not alter instrument choice.

8 Conclusion

This paper compares the relative performance of price-based and quantity-based instruments to regulate an international externality, such as climate change. In this context it is often argued, that due to reasons of fiscal sovereignty, the enforceability of price-based regulations is lower than for tradable quantity restrictions. For our analysis, we extended the framework established in Montero (1999) to reflect different enforcement probabilities for international price and quantity regimes. As a contribution to the Prices vs. Quantities debate, we calculated the expected welfare difference of price-based over quantity-based instruments under uncertain...
abatement costs and benefits, incomplete enforcement, and different enforcement probabilities.

Interestingly, the effects of cost uncertainty under incomplete enforcement and of differences in the enforcement probabilities can be divided into two additively separable terms. The former 'uncertainty' effect is similar to the results presented in Montero (1999, 2002) for the enforcement probability of quantity-based instruments. This effect is crucial for the comparative advantage of a quantity approach resulting from uncertain emission reductions. The relative slopes of the marginal curves remain the main determinants for the direction of this effect. In contrast, the latter 'differentiated enforceability' effect, measuring the gain in net benefits from using the stricter enforceable policy instrument, always favors a quantity approach. This effect depends crucially on the level of the marginal curves and on institutionally determined parameters.

The relative slope criterion first derived in Weitzman (1974) is commonly brought forward in order to argue in favor of a price- (e.g. emission tax) or a quantity-based (e.g. tradable pollution permits) approach. We find that many facts established within the Prices vs. Quantities debate, have to be reconsidered as soon as differences in enforceability are taken into account. First of all, the level of cost uncertainty, which does not have an influence on optimal policy choice in Weitzman (1974) and Montero (1999), turns out to be a crucial factor. We show that there exists a threshold level of cost variance, below which a quantity instrument ought to be always preferred as regulatory instrument, irrespective of the relative slopes of the marginal benefit and marginal cost curve. This threshold level is determined by institutional variables and level effects. The level of the marginal benefit curve is also an important determinant of the relative performance of both instruments, which is, again in contradiction with the established facts of the Prices vs. Quantities debate.

In order to give a notion of the intensity of the aggregated effects, we present the results of numerical simulations based on data gathered in the climate change context. Our calculations show that the expected level of the marginal benefit curve, which is subject to dispute, significantly affects rational instrument choice. While with the estimates presented in Newell and Pizer (2003), a price-based regulation seems still preferable, the situations changes in favor of a quantity regime if the higher estimates of more recent meta-studies are taken into account. Moreover, the threshold level of the variance of costs becomes relevant for plausible parameter values, suggesting the use of quantity-based instruments to regulate CO₂ emissions for any ratio of the slopes of the marginal curves.
A Appendix

A.1 Differentiated enforceability effect

We prove that the differentiated enforceability effect under incomplete enforcement is always negative if $\alpha_q > \alpha_t$.

In order to prove this, we specify the sanction $F$ for which noncompliance is an attractive option for countries under both regimes. Following Assumption 1, using (11) and (12) requires

$$F < \frac{(1 - \alpha_q)(bC'' - cB'')}{\alpha_q(C'' - B'')} < \frac{(1 - \alpha_t)(bC'' - cB'')}{\alpha_t(C'' - B'')}$$

Replacing $B''$ by $-\beta C''$ and assuming $c = 0$ yields

$$F = \frac{(1 - \alpha_q)b}{(1 + \beta)\alpha_q}$$

as the sanction which induces full compliance in at least one of the regimes. We express $F$ as a function of $F$, i.e.

$$F = \gamma F$$

(23)

with $\gamma \in ]0, 1[$. Replacing $F$ by $\gamma F$ in (15) changes the differentiated enforceability effect to

$$\frac{(\alpha_q - \alpha_t)b^2}{2C''(1 + \beta)^2\alpha_q^2} \left\{ \frac{\gamma^2(1 - \alpha_q)}{(1 - \alpha_t)} - \frac{(\gamma(1 - \alpha_q) + (1 + \beta)\alpha_q)^2}{(1 + \alpha_q\beta)(1 + \alpha_t\beta)} \right\}.$$

As $(1 - \alpha_q)/(1 - \alpha_t) < 1$ for $\alpha_q > \alpha_t$, the differentiated enforceability effect is negative if

$$\frac{(\gamma(1 - \alpha_q) + (1 + \beta)\alpha_q)^2}{\gamma^2(1 + \alpha_q\beta)(1 + \alpha_t\beta)} \geq 1$$

which is always the case for $\alpha_q > \alpha_t, \gamma \in ]0, 1[ \text{ and } \beta \in \mathbb{R}^+.$

The differentiated enforceability effect is thus always negative. Q.e.d.
A.2 Cost variance

We show that \( \Delta_{pq} \) is always negative, if \( \sigma_\theta^2 < \sigma_\beta^2 \). For this, we define the uncertainty respectively the differentiated enforceability effect as a function of the relative slopes of the marginal curves \( \beta \), i.e.

\[
\Psi(\beta, \sigma_\theta^2) = \frac{\sigma_\theta^2 \alpha_q (1 - \beta (2 - \alpha_q))}{2C''} \tag{24}
\]

\[
\Omega(\beta) = \frac{(\alpha_q - \alpha_t)}{2C''} \left( \frac{F^2}{(1 - \alpha_q)(1 - \alpha_t)} - \frac{(b + F)^2}{(1 + \beta \alpha_q)(1 + \beta \alpha_t)} \right). \tag{25}
\]

For \( \Delta_{pq} \leq 0 \) it has to be shown, that

\[
\Psi(\beta, \sigma_\theta^2) + \Omega(\beta) \leq 0. \tag{26}
\]

We know from Appendix A.1 that

\[
\Omega(\beta) < 0, \quad \forall \beta \in \mathbb{R}^+. \tag{27}
\]

The sign of \( \Psi(\beta, \sigma_\theta^2) \) is ambiguous but strictly negative for \( \beta > \frac{1}{2 - \alpha_q} \). Hence, \( \forall \beta \in \mathbb{R}^+, \Delta_{pq} \leq 0 \), if

\[
\max_\beta \Psi(\beta, \sigma_\theta^2) \leq \max_\beta \Omega(\beta) \quad \forall \beta \in [0, \frac{1}{2 - \alpha_q}].
\]

Observe, that

\[
\arg\max_{\beta \in [0, \frac{1}{2 - \alpha_q}]} \Psi(\beta, \sigma_\theta^2) = 0 \tag{27}
\]

and

\[
\arg\max_{\beta \in [0, \frac{1}{2 - \alpha_q}]} \Omega(\beta) = \frac{1}{2 - \alpha_q}. \tag{28}
\]

The former is obvious from (24). In order to prove \( \arg\max_{\beta \in [0, \frac{1}{2 - \alpha_q}]} \Omega(\beta) = \frac{1}{2 - \alpha_q} \), consider the Kuhn-Tucker problem

\[
\max_\beta \{ \Omega(\beta); (\frac{1}{2 - \alpha_q} - \beta) \geq 0 \}.
\]

We set up the Lagrangian maximization problem

\[
\mathcal{L} = \Omega(\beta) + \lambda (\frac{1}{2 - \alpha_q} - \beta) \tag{29}
\]

where \( \lambda \) is a non-negative multiplier. The First-Order Conditions are

\[
\frac{\partial \mathcal{L}}{\partial \beta} = \frac{\partial \Omega(\beta)}{\partial \beta} - \lambda = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda} = \frac{1}{2 - \alpha_q} - \beta = 0,
\]

with

\[
\frac{\delta \Omega}{\delta \beta} = \frac{(\alpha_q - \alpha_t) (\alpha_q + \alpha_t + 2 \alpha_q \alpha_t \beta) (F + b)^2}{2C''(1 + \beta \alpha_q)^2(1 + \beta \alpha_t)^2}. \tag{30}
\]

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being strictly positive for $\alpha_q > \alpha_t$. Hence, $\lambda$ is positive and the constraint is binding. This implies $\arg\max_{\beta \in [0, \frac{1}{2-\alpha_q}]} \Omega(\beta) = \frac{1}{2-\alpha_q}$.

Substituting $\beta = 0$ in (24) and $\beta = \frac{1}{2-\alpha_q}$ in (25), replacing $\alpha_t = k\alpha_q$, assuming $c = 0$, and solving (26) for $\sigma_\theta^2$, yields a sufficient condition for the threshold level

$$\sigma_\theta^2 = (1-k) \left( \frac{(F+b)^2(1-\alpha_q)^2}{2(2-\alpha_q)(1-k)} - \frac{F^2}{(1-\alpha_q)(1-k)} \right)$$

below which $\Delta_{pq}$ is always negative. Q.e.d.
References


UNFCCC: 2006, Report of the conference of the parties serving as the meeting of the parties to the kyoto protocol on its first session, held at montreal from 28 november to 10 december 2005. addendum. part two: Action taken by the conference of the parties serving as the meeting of the parties to the kyoto protocol at its first session., *Document FCCC/KP/CMP/2005/8/Add.3*, UNFCCC.


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